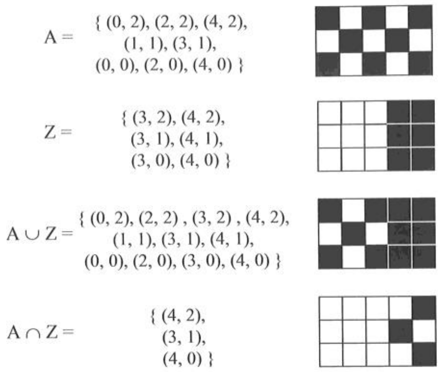
Morphological operations.

The morphological operations could be show in easy way in binary images in which pixels have only two values of 0 and 1. An object can be represented as a set of pixels which value is 1. In that case we can perform the set theory operations:



The most important concept used in mathematical morphology is the concept of a structural element B. This element is similar to the mask in the filtration. This is a small image of any size and shape in which pixels of value 1 (black) are used for calculations. It has its own coordinate system, which center is the highlighted, representative point.

Dilation

Dilation is an extension of the basic set.

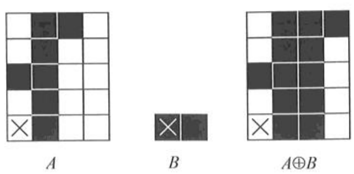
$$A \oplus B = \{p : p = a + b, a \in A, b \in B\}$$

Merge two sets by vector addition. It is a set of points of all possible vector addition of elements from A and B. It is used to fill holes and narrow “bays”.

Example:

A = {1,0; 1,1; 1,2; 0,2; 1,3; 1,4; 2,4}
B = {0,0; 1,0}

A⊕B = {1,0; 1,1; 1,2; 0,2; 1,3; 1,4; 2,4; 2,0; 2,1; 2,2; 1,2; 2,3; 3,4}



Erosion

The erosion operation is roughly the opposite operation to dilation.

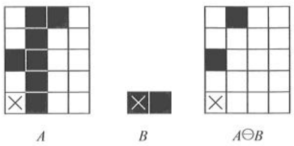
$$A \ominus B = \{a: (a+b) \in A, for\ all\ b \in B\}$$

The result is determined by those points for which the sum with the structural element is included in the examined set.

Example:

A = {1,0; 1,1; 1,2; 0,2; 1,3; 1,4; 2,4}
B = {0,0; 1,0}

A⊖B = {0,2; 1,4}



Opening and closing

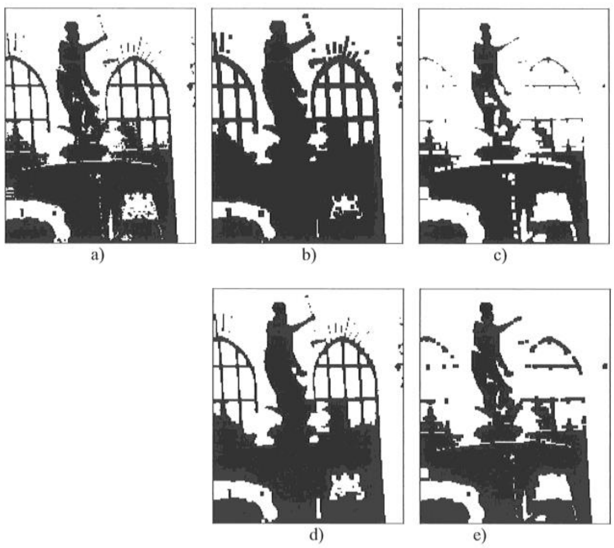
If the object is eroded and then dilated (or vice versa), then the result does not have to be an original object.

Performing an erosion operation on an object and then dilation creates an operation called an **opening**:

$$A \circ B = (A \ominus B) \oplus B$$

Performing an dilation operation on an object and then erosion creates an operation called an **closing**:

$$A \bullet B = (A \oplus B) \ominus B$$



Structural element 3x3 square.

a) original
b) dilation
c) erosion
d) closure
e) opening