sim

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1 Atividade 4

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1.0.1 Exercício 1

Neste exercício, obterá-se a solução númerica e visualizações do sistema de Lorenz para diversos parametros visando ilustrar a transicão de um sistema com um único ponto de equilibrio para até um sistema com atratores caóticos e sem ponto de equilibrio.

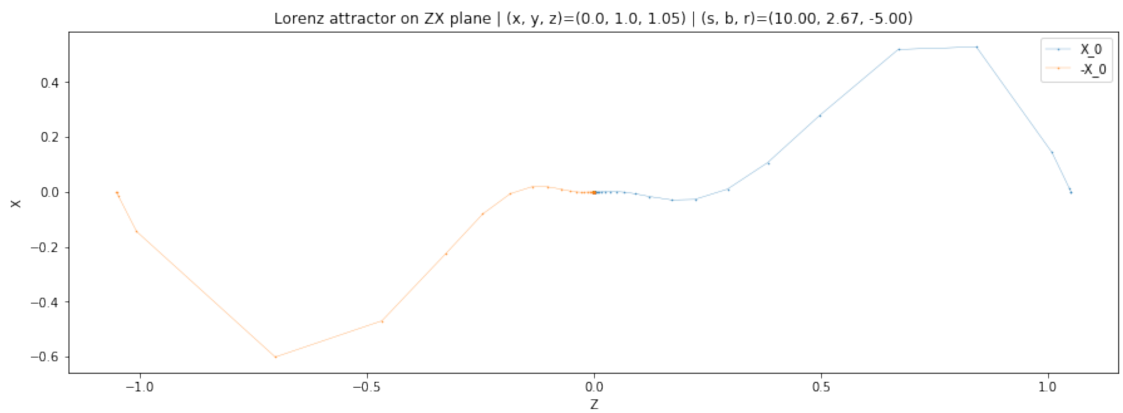
```
In [1]: from scipy.integrate import solve_ivp
        import pandas as pd
        import seaborn as sns
        import matplotlib.pyplot as plt
        import numpy as np

        def lorenz(t, X, sigma, beta, rho):
            x, y, z = X
            dx = sigma * (y - x)
            dy = rho * x - x * z - y
            dz = x * y - beta * z
            return dx, dy, dz
```

```
In [2]: sigma = 10
        b = 8 / 3
        T = (0, 1000)
```

1.0.2 Ponto de equilibrio na origem

```
In [3]: T = (0, 30)
        r = -5
        X_0 = (0.0, 1.0, 1.05)

        plt.figure(figsize=(15, 5))

        f = solve_ivp(lorenz, T, X_0, args=(sigma, b, r), dense_output=True)
        (x, y, z) = f.y
        t = f.t
```

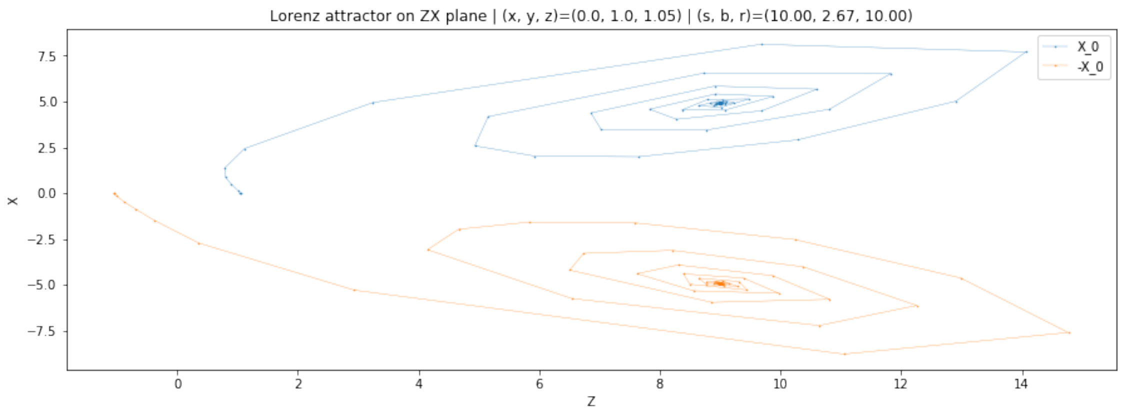
```
plt.title(f"Lorenz attractor on ZX plane"
          f" | (x, y, z)={X_0}"
          f" | (s, b, r)=({sigma:.2f}, {b:.2f}, {r:.2f})")
plt.plot(z, x, '.-', linewidth=0.3, alpha=0.9, markersize=1, label='X_0')

f = solve_ivp(lorenz, T, [-el for el in X_0], args=(sigma, b, r), dense_output=True)
(x, y, z) = f.y
t = f.t
plt.plot(z, x, '.-', linewidth=0.3, alpha=0.9, markersize=1, label='-X_0')

plt.legend()
plt.xlabel("Z")
plt.ylabel("X")
plt.show()
```



1.0.3 Dois pontos de equilíbrio

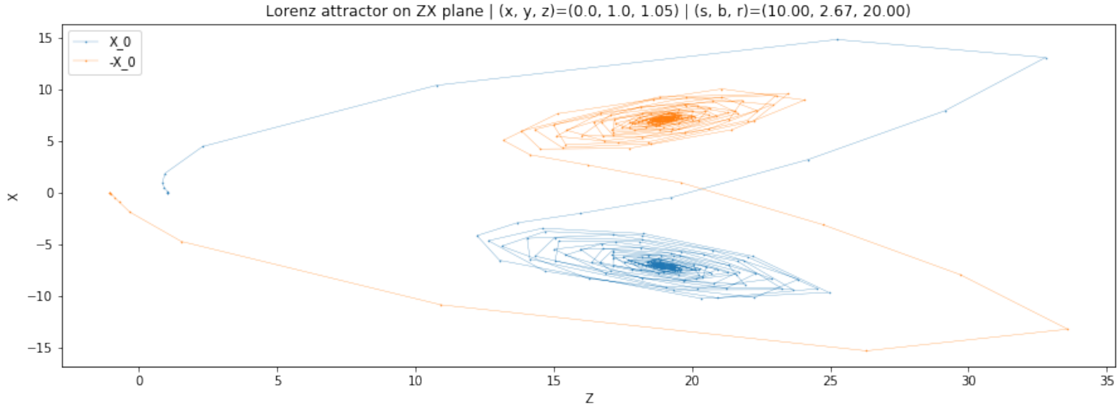
```
In [4]: X_0 = (0.0, 1.0, 1.05)
        T = (0, 50)
        r = 10

        f = solve_ivp(lorenz, T, X_0, args=(sigma, b, r), dense_output=True)
        (x, y, z) = f.y
        t = f.t
        plt.figure(figsize=(15, 5))
        plt.title(f"Lorenz attractor on ZX plane"
                  f" | (x, y, z)={X_0}"
                  f" | (s, b, r)=({sigma:.2f}, {b:.2f}, {r:.2f})")
        plt.plot(z, x, '.-', linewidth=0.3, alpha=0.9, markersize=1, label='X_0')

        f = solve_ivp(lorenz, T, [-el for el in X_0], args=(sigma, b, r), dense_output=True)
        (x, y, z) = f.y
        t = f.t
```

```
plt.plot(z, x, '.-', linewidth=0.3, alpha=0.9, markersize=1, label='-X_0')

plt.legend()
plt.xlabel("Z")
plt.ylabel("X")
plt.show()
```



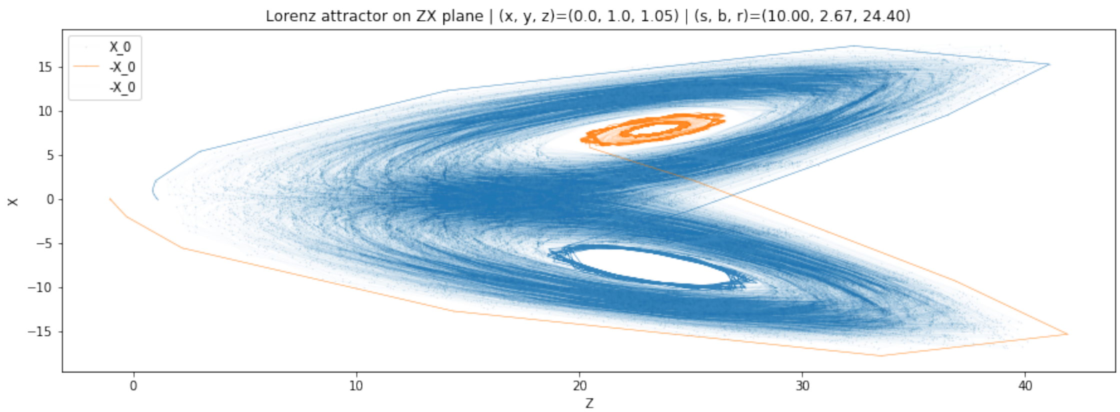
1.0.4 Órbitas caóticas

```
In [5]: X_0 = (0.0, 1.0, 1.05)
        T = (0, 50)
        r = 20
        f = solve_ivp(lorenz, T, X_0, args=(sigma, b, r), dense_output=True)
        (x, y, z) = f.y
        t = f.t
        plt.figure(figsize=(15, 5))
        plt.title(f"Lorenz attractor on ZX plane"
                  f" | (x, y, z)={X_0}"
                  f" | (s, b, r)=({sigma:.2f}, {b:.2f}, {r:.2f})")
        plt.plot(z, x, '.-', linewidth=0.3, alpha=0.9, markersize=1.0, label='X_0')

        f = solve_ivp(lorenz, T, [-el for el in X_0], args=(sigma, b, r), dense_output=True)
        (x, y, z) = f.y
        t = f.t
        plt.plot(z, x, '.-', linewidth=0.3, alpha=0.9, markersize=1, label='-X_0')

        plt.legend()
        plt.xlabel("Z")
        plt.ylabel("X")
        plt.show()
```



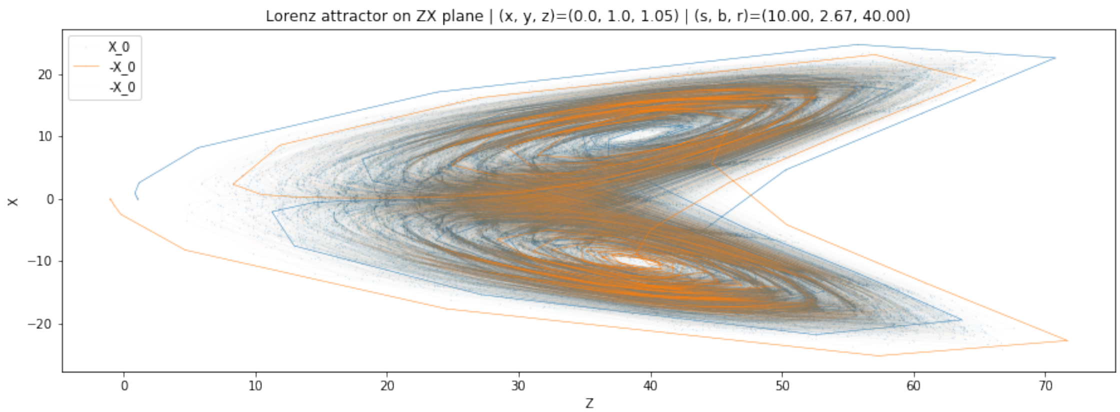
1.0.5 Atratores caóticos

```
In [6]: X_0 = (0.0, 1.0, 1.05)
        T = (0, 5000)
        r = 24.40
        f = solve_ivp(lorenz, T, X_0, args=(sigma, b, r), dense_output=True)
        (x, y, z) = f.y
        t = f.t
        plt.figure(figsize=(15, 5))
        plt.title(f"Lorenz attractor on ZX plane"
                  f" | (x, y, z)={X_0}"
                  f" | (s, b, r)=({sigma:.2f}, {b:.2f}, {r:.2f})")
        plt.plot(z, x, '.-', linewidth=0.01, alpha=0.9, markersize=0.1, label='X_0')
        plt.plot(z[:100], x[:100], '.-', linewidth=0.5, alpha=0.9, markersize=0.1, color='C0')

        f = solve_ivp(lorenz, T, [-el for el in X_0], args=(sigma, b, r), dense_output=True)
        (x, y, z) = f.y
        plt.plot(z[:100], x[:100], '.-', linewidth=0.5, alpha=0.9, markersize=0.1, label='-X_0
        plt.plot(z, x, '.-', linewidth=0.005, alpha=0.9, markersize=0.01, label='-X_0', color=

        plt.legend()
        plt.xlabel("Z")
        plt.ylabel("X")
        plt.show()
```



1.0.6 Atratores caóticos sem atratores pontuais

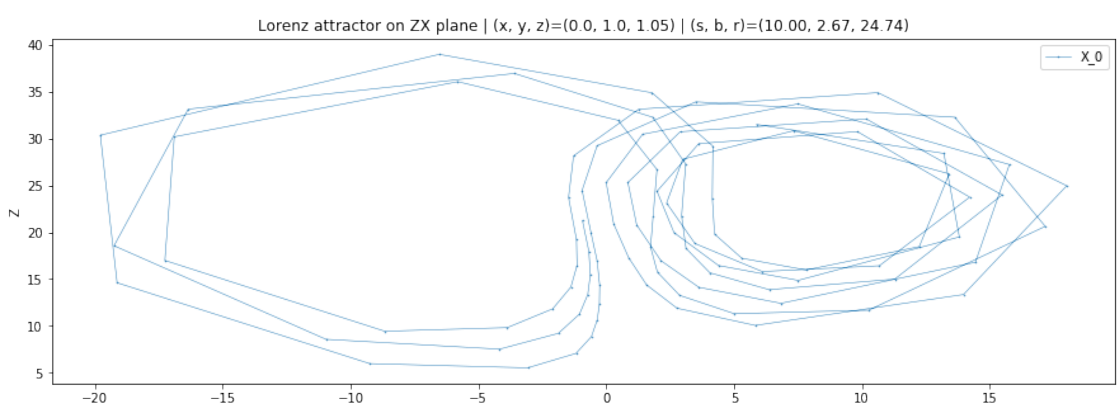
```
In [7]: X_0 = (0.0, 1.0, 1.05)
        T = (0, 5000)
        r = 40
        f = solve_ivp(lorenz, T, X_0, args=(sigma, b, r), dense_output=True)
        (x, y, z) = f.y
        t = f.t
        plt.figure(figsize=(15, 5))
        plt.title(f"Lorenz attractor on ZX plane"
                  f" | (x, y, z)={X_0}"
                  f" | (s, b, r)=({sigma:.2f}, {b:.2f}, {r:.2f})")
        plt.plot(z, x, '.-', linewidth=0.01, alpha=0.9, markersize=0.1, label='X_0')
        plt.plot(z[:100], x[:100], '.-', linewidth=0.5, alpha=0.9, markersize=0.1, color='C0')

        f = solve_ivp(lorenz, T, [-el for el in X_0], args=(sigma, b, r), dense_output=True)
        (x, y, z) = f.y
        plt.plot(z[:100], x[:100], '.-', linewidth=0.5, alpha=0.9, markersize=0.1, label='-X_0
        plt.plot(z, x, '.-', linewidth=0.005, alpha=0.9, markersize=0.01, label='-X_0', color=

        plt.legend()
        plt.xlabel("Z")
        plt.ylabel("X")
        plt.show()
```



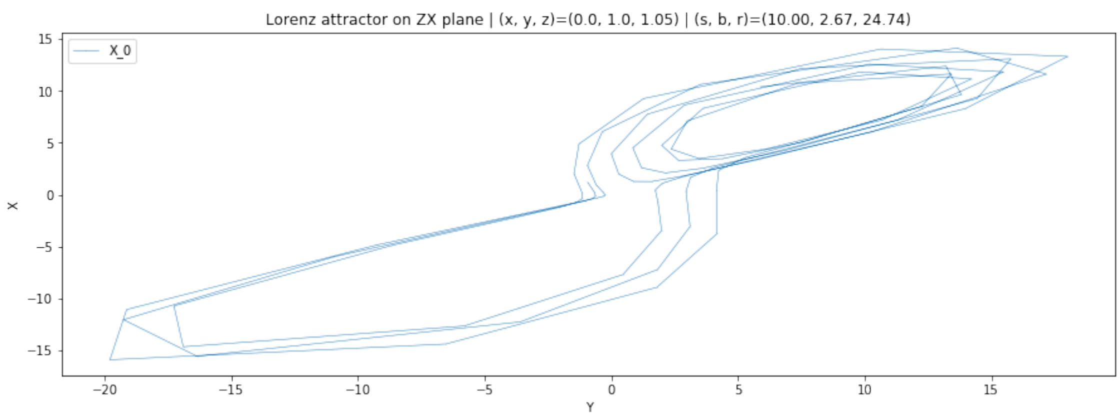
1.1 Exercício 2

1.1.1 Figure 2

```
In [8]: X_0 = (0.0, 1.0, 1.05)
        T = (0, 2000)
        r = 24.74
        f = solve_ivp(lorenz, T, X_0, args=(sigma, b, r), dense_output=True)
        (x, y, z) = f.y.T[1400:1500].T

        t = f.t

        plt.figure(figsize=(15, 5))
        plt.title(f"Lorenz attractor on ZX plane"
                  f" | (x, y, z)={X_0}"
                  f" | (s, b, r)=({sigma:.2f}, {b:.2f}, {r:.2f})")
        plt.plot(y, z, '.-', linewidth=0.5, alpha=0.9, markersize=1, label='X_0')
        plt.legend()
        plt.xlabel("Y")
        plt.ylabel("Z")
        plt.show()
```



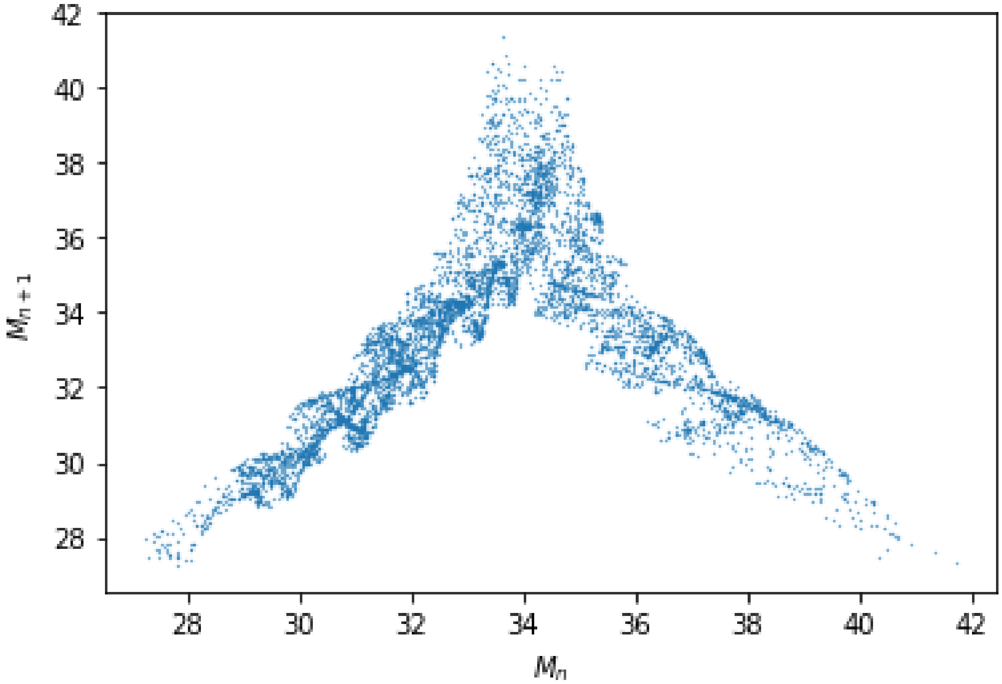
In [9]:
```
X_0 = (0.0, 1.0, 1.05)
T = (0, 2000)
r = 24.74
f = solve_ivp(lorenz, T, X_0, args=(sigma, b, r), dense_output=True)
(x, y, z) = f.y.T[1400:1500].T

t = f.t

plt.figure(figsize=(15, 5))
plt.title(f"Lorenz attractor on ZX plane"
          f" | (x, y, z)={X_0}"
          f" | (s, b, r)=({sigma:.2f}, {b:.2f}, {r:.2f})")
plt.plot(y, x, '.-', linewidth=0.5, alpha=0.9, markersize=0.1, label='X_0')
plt.legend()
plt.xlabel("Y")
plt.ylabel("X")
plt.show()
```



1.1.2 Figure 4

In [10]:
```
X_0 = (0.0, 1.0, 1.05)
T = (0, 5000)
r = 24.74
f = solve_ivp(lorenz, T, X_0, args=(sigma, b, r), dense_output=True)
(x, y, z) = f.y
t = f.t

z = pd.Series(z)
larger_than_next = (z.diff() > 0)
larger_than_previous = (z[::-1].diff() > 0)
```

```
relative_max_indices = (larger_than_next & larger_than_previous)
relative_max = z.loc[relative_max_indices]
plt.plot(relative_max[:-1], relative_max[1:], '.', markersize=0.5)

plt.xlabel("$M_n$")
plt.ylabel("$M_{n+1}$")
plt.show()
```



1.1.3 Figura 5

```
In [11]: X_0 = (0.0, 1.0, 1.05)
         T = (0, 5000)
         r = 24.74
         f = solve_ivp(lorenz, T, X_0, args=(sigma, b, r), dense_output=True)
         (x, y, z) = f.y
         t = f.t

         z = pd.Series(z)
         larger_than_next = (z.diff() > 0)
         larger_than_previous = (z[::-1].diff() > 0)
         relative_max_indices = (larger_than_next & larger_than_previous)
         relative_max = z.loc[relative_max_indices]
         relative_max -= relative_max.min()
         relative_max /= relative_max.max()
```

```python
f = lambda M: 2 * M if M < 0.5 else 2 * (1 - M)

plt.plot(relative_max, relative_max.map(f), '.', markersize=0.5)

plt.xlabel("$M_n$")
plt.ylabel("$M_{n+1}$")
plt.show()
```