$\boxed{2.32}$ $$y''' - 2y'' + y' = xe^x$$

Resolução:

$$r^3 - 2r^2 + r = 0 \qquad r(r^2 - 2r + 1) = 0$$

$r = 0$; 1 (dupla)

$$y_h = c_1 + c_2 e^x + c_3 x e^x$$

$$y = c_1(x) + c_2(x) e^x + c_3(x) x e^x$$

$$\begin{cases} c_1'(x) + c_2'(x) e^x + c_3'(x) x e^x = 0 \\ 0 + c_2'(x) e^x + c_3'(x)(e^x + xe^x) = 0 \\ 0 + c_2'(x) e^x + c_3'(x)(2e^x + xe^x) = 0 \end{cases}$$

$$W(x) \begin{vmatrix} 1 & e^x & xe^x \\ 0 & e^x & (x+1)e^x \\ 0 & e^x & (x+2)e^x \end{vmatrix} = e^{2x}$$

$$c_1'(x) = \begin{vmatrix} 0 & e^x & xe^x \\ 0 & e^x & (x+1)e^x \\ xe^x & e^x & (x+2)e^x \end{vmatrix} / e^{2x}$$

$$c_2'(x) = \begin{vmatrix} 1 & 0 & xe^x \\ 0 & 0 & (x+1)e^x \\ 0 & xe^x & (x+2)e^x \end{vmatrix} / e^{2x}$$

$$c_3'(x) = \begin{vmatrix} 1 & e^x & 0 \\ 0 & e^x & 0 \\ 0 & e^x & xe^x \end{vmatrix} / e^{2x}$$

$$c_1'(x) = x e^x$$

$$c_1(x) = \int x e^x \, dx = x e^x - e^x + \bar{c}_1$$

$$c_2'(x) = -(1+x)x$$

$$c_2(x) = \int -(1+x) x \, dx = -\frac{x^2}{2} - \frac{x^3}{3} + \bar{c}_2$$

$$c_3'(x) = x$$

$$c_3(x) = \int x \, dx = \frac{x^2}{2} + \bar{c}_3$$

$$y = x e^x - e^x + \bar{c}_1 + e^x\left(-\frac{x^2}{2} - \frac{x^3}{3} + \bar{c}_2\right) + x e^x\left(\frac{x^2}{2} + \bar{c}_3\right)$$

$$y = c_1 + c_2 e^x + c_3 x e^x - \frac{1}{2} x^2 e^x + \frac{1}{6} x^3 e^x \quad \checkmark$$

$$y = -\frac{1}{2} x^2 e^x + \frac{1}{6} x^3 e^x \quad \text{sol. particular}$$

$\boxed{2.35}$ $$y'' + 2y' + y = \frac{e^{-x}}{x}$$

Resolução:

$$r^2 + 2r + 1 = 0 \qquad r_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4(1)(1)}}{2(1)} = -1$$ (duple)

$$y_h = c_1 e^{-x} + c_2 x e^{-x}$$

$$W(x) = \begin{vmatrix} e^{-x} & x e^{-x} \\ -e^{-x} & (1-x) e^{-x} \end{vmatrix} = e^{-2x}(1-x) + e^{-2x} x = e^{-2x}$$

$$c_1'(x) = \begin{vmatrix} 0 & x e^{-x} \\ e^{-x} x^{-1} & (1-x) e^{-x} \end{vmatrix} / e^{-2x} = 0 - e^{-2x} x^0 / e^{-2x} = -1$$

$$c_2'(x) = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & e^{-x} x^{-1} \end{vmatrix} / e^{-2x} = e^{-2x} x^{-1} - 0 / e^{-2x} = x^{-1}$$

$$c_1(x) = \int -1 \, dx = -x$$

$$c_2(x) = \int x^{-1} dx = \ln x$$

$$y_p = -x e^{-x} + x e^{-x} \ln x$$

$$y = c_1 e^{-x} + c_2 x e^{-x} - x e^{-x} + x e^{-x} \ln x$$

$$y = c_1 e^{-x} + (c_2 - 1) x e^{-x} + x e^{-x} \ln x$$

$$y = c_1 e^{-x} + c_2 x e^{-x} + x e^{-x} \ln x$$

$\boxed{2.33}$ $y'' + y = tg\,x$

Resolución:

$r^2 + 1 = 0 \qquad r^2 = -1 \qquad r = \pm\sqrt{-1} = \pm i$

$y_h = c_1 \cos x + c_2 \operatorname{sen} x$

$y(x) = c_1(x) \cos x + c_2(x) \operatorname{sen} x$

$y'(x) = c_1'(x) \ldots$

$W(x) = \begin{vmatrix} \cos x & \operatorname{sen} x \\ -\operatorname{sen} x & \cos x \end{vmatrix} = \cos^2 x + \operatorname{sen}^2 x = 1$

$c_1'(x) = \begin{vmatrix} 0 & \operatorname{sen} x \\ tg\,x & \cos x \end{vmatrix} / 1 = -tg\,x \operatorname{sen} x = -\dfrac{\operatorname{sen}^2 x}{\cos x}$

$c_2'(x) = \begin{vmatrix} \cos x & 0 \\ -\operatorname{sen} x & tg\,x \end{vmatrix} / 1 = tg\,x \cos x = \operatorname{sen} x$

$c_1(x) = \displaystyle\int -\frac{\operatorname{sen}^2 x}{\cos x}\,dx = -\int \left(\frac{1-\cos^2 x}{\cos x}\right) dx =$

$= -\left[\displaystyle\int \frac{1}{\cos x}\,dx - \int \cos x\,dx\right] = -\left[\int \sec x\,dx - \operatorname{sen} x\right] =$

$= -\left[\ln(\sec x + \tan x) - \operatorname{sen} x\right]$

$c_2(x) = \displaystyle\int \operatorname{sen} x\,dx = -\cos x$

$y_h = (\operatorname{sen} x - \ln(\sec x + \tan x)) \cos x - \cos x \operatorname{sen} x$

$y(x) = y_h + y_p$

$y(x) = c_1 \cos x + c_2 \operatorname{sen} x - \ln|\sec x + \tan x|$ ✓

$\boxed{2.34}$ $y''' + y' = \sec x$

<u>Resolução:</u>

$r^3 + r = 0 \qquad r(r^2+1) = 0 \qquad r = 0 ; \pm i$

$y_h = c_1 e^{0x} + c_2 e^{0x} \cos x + c_3 e^{0x} \operatorname{sen} x$

$y_h = c_1 + c_2 \cos x + c_3 \operatorname{sen} x$

$$W(x) = \begin{vmatrix} 1 & \cos x & \operatorname{sen} x \\ 0 & -\operatorname{sen} x & \cos x \\ 0 & -\cos x & -\operatorname{sen} x \end{vmatrix} = (-\operatorname{sen} x)^2 + \cos^2 x = 1$$

$$c_1'(x) = \begin{vmatrix} 0 & \cos x & \operatorname{sen} x \\ 0 & -\operatorname{sen} x & \cos x \\ \sec x & -\cos x & -\operatorname{sen} x \end{vmatrix} / 1 = -\cos x(-\sec x \cos x) + \operatorname{sen} x(-\sec x(-\operatorname{sen} x))$$

$$= \sec x(\cos^2 x + \operatorname{sen}^2 x) = \sec x$$

$$c_2'(x) = \begin{vmatrix} 1 & 0 & \operatorname{sen} x \\ 0 & 0 & \cos x \\ 0 & \sec x & -\operatorname{sen} x \end{vmatrix} / 1 = 1[0 - \sec x \cos x]$$

$$= -\sec x \cos x$$

$$c_3'(x) = \begin{vmatrix} 1 & \cos x & 0 \\ 0 & -\operatorname{sen} x & 0 \\ 0 & -\cos x & \sec x \end{vmatrix} / 1 = (-\operatorname{sen} x \sec x)$$

$c_1(x) = \int \sec x \, dx = \ln|\sec x + \operatorname{tg} x|$

$c_2(x) = -\int \sec x \cos x \, dx = -x$

$c_3(x) = -\int \frac{\operatorname{sen} x}{\cos x} dx = -\int \operatorname{tg} x \, dx = \ln|\sec x|$

$y(x) = c_1 + c_2 \cos x + c_3 \operatorname{sen} x + \ln|\sec x + \operatorname{tg} x| -$

$- x \cos x + \operatorname{sen} x \ln|\cos x|$

$\boxed{2.37}$ $y'' + 9y = \sec 3x$

R:

$$y'' + 9y = 0$$

$$r^2 + 9 = 0 \qquad r_{1,2} = \pm 3i$$

$$y_h = c_1 e^0 \cos 3x + c_2 e^0 \sin 3x$$

$$y_h = c_1 \cos 3x + c_2 \sin 3x$$

$$y_p = c_1(x) \cos 3x + c_2(x) \sin 3x$$

$$W = \begin{vmatrix} \cos(3x) & \sin(3x) \\ -3\sin(3x) & 3\cos(3x) \end{vmatrix} = 3\cos^2(3x) + 3\sin^2(3x) = 3$$

$$c_1'(x) = \begin{vmatrix} 0 & \sin(3x) \\ \sec(3x) & 3\cos(3x) \end{vmatrix} / 3 = \frac{-\sin(3x)\sec(3x)}{3} =$$

$$c_2'(x) = \begin{vmatrix} \cos(3x) & 0 \\ -3\sin(3x) & \sec(3x) \end{vmatrix} / 3 = \frac{\cos(3x)\sec(3x)}{3} = \frac{1}{3}$$

$$c_1(x) = \int \frac{1}{3} \frac{(-\sin(3x))}{\cos(3x)} dx = \frac{1}{9} \ln|\cos(3x)|$$

$$c_2(x) = \int \frac{1}{3} dx = \frac{1}{3} x$$

$$y = y_h + y_p =$$

$$y = c_1 \cos(3x) + c_2 \sin(3x) + \frac{1}{9} \ln|\cos(3x)| \cos 3x + \frac{1}{3} x \sin(3x)$$

$$y = c_1 \cos(3x) + c_2 \sin(3x) + \frac{1}{9} \ln|\cos(3x)| \cos(3x) + \frac{1}{3} x \sin(3x)$$

função	anulador
1	D
x	D^2
Kx^n	D^{n+1}
$Ke^{\alpha x}$	$D-\alpha$
$Kx^n e^{\alpha x}$	$(D-\alpha)^{n+1}$
$\cos\beta x$	$D^2+\beta^2$
$\operatorname{sen}\beta x$	$D^2+\beta^2$
$Ke^{\alpha x}\cos\beta x$	$D^2-2\alpha D+\alpha^2+\beta^2$
$Ke^{\alpha x}\sin\beta x$	$D^2-2\alpha D+\alpha^2+\beta^2$
$Kx^n e^{\alpha x}\cos\beta x$	$(D^2-2\alpha D+\alpha^2+\beta^2)^{n+1}$
$Kx^n e^{\alpha x}\sin\beta x$	$(D^2-2\alpha . D+\alpha^2+\beta^2)^{n+1}$

$\boxed{2.43}$ Det. anuladores

(a) $y = e^x$ R: $\boxed{(D-1)} e^x = e^x - e^x = 0$

(b) $y = 3e^x$ R: $\boxed{(D-1)} 3e^x = 3e^x - 3e^x = 0$

(c) $y = 2x^2$ R: $(D^{n+1})y = \boxed{D^3}(2x^2) = D^2(4x) = D(4) = 0$

(d) $y = \cos 2x$ R: $(D^2+\beta^2)y = D^2 \cos 2x + \beta^2 \cos 2x$

$\boxed{(D^2+4)}$ $= D(-\sin 2x \cdot 2) + \beta^2 \cos 2x$

$= -4\cos(2x) + \beta^2 \cos 2x$

$= -4\cos(2x) + 4\cos(2x)$

$= 0$

(e) $y = x\cos 2x$ R: $(D^2+\beta^2)^2 y = \boxed{(D^2+4)^2}$

$(D^2+2^2)(D^2+2^2)\, x\cos 2x =$

$(D^2+2^2)\, D^2 x\cos 2x + 4x\cos 2x =$

$(\quad)\, D(\cos 2x - x\, 2\sin 2x) + 4x\cos 2x =$

$(\quad)(4\cos 2x - 2\sin 2x - x\cos 2x) + 4x\cos 2x =$

$(-16\cos 2x + 8\sin 2x + 2\sin 2x - 4x\cancel{\cos 2x} + 4x\cancel{\cos 2x} = 0$

(f) $y = 6 + e^{2x}$ R: $\boxed{D(D-2)}(6 + e^{2x}) =$

$D \cdot [\cancel{2e^{2x}} - 12 - \cancel{2e^{2x}}] = 0$

(g) $y = (\sin 3x + \cos 3x)e^x$ R: $(D^2 - 2D + 1 + 3^2)$

$\boxed{2.44}$ $y''' - y' = e^x$

R:

$y''' - y' = 0$

$r^3 - r = 0 \qquad r(r^2-1)=0 \quad r_{1,2,3} = 0, \pm 1$

$y_h = c_1 + c_2 e^x + c_3 e^{-x}$

Aniquilador p/ 2º membro $(D-1)$

$(D-1)e^x = e^x - e^x = 0$

$(D-1)(D(D^2-1))y = (D-1)e^x = 0$

$(D-1)(D^3-D) = D^4 - D^2 - D^3 + D =$

$r^4 - r^3 - r^2 - r = 0 \qquad r=0 \quad r=1$ (dupla)

$r=-1$

$y_* = c_1 + c_2 e^{-x} + c_3 e^x + c_4 x e^x$

$y_p = y_* - y_h = \cancel{c_1} + \cancel{c_2 e^{-x}} + \cancel{c_3 e^x} + c_4 x e^x -$

$- (\cancel{c_1} + \cancel{c_2 e^x} + \cancel{c_3 e^{-x}}) =$

$y_p = d x e^x$

$(dxe^x)''' - (dxe^x)' = e^x$

$(de^x + dxe^x)'' - (de^x + dxe^x) = e^x$

$(de^x + de^x + dxe^x)' - (de^x + dxe^x) = e^x$

$de^x + de^x + \cancel{de^x} + \cancel{dxe^x} - \cancel{de^x} - \cancel{dxe^x} = e^x$

$2de^x = e^x \qquad 2d = 1 \quad \boxed{d = 1/2} \qquad \boxed{y_p = \frac{1}{2} x e^x}$

$y = c_1 + c_2 e^x + c_3 e^{-x} + \frac{1}{2} x e^x$