$\boxed{1.62}$ $xy' = x^3 - 2y$

Resolució:

$$xy' = x^3 - 2y$$

$$xy' + 2y = x^3$$

$$y' + \frac{2}{x}y = x^2$$

$$P(x) = 2/x \qquad Q(x) = x^2$$

$$y(x) = \frac{\int x^2 e^{\int 2/x \, dx} + c}{\int e^{\int 2/x \, dx}}$$

$$y(x) = \frac{\int x^2 e^{\ln x^2} + c}{x^2}$$

$$y(x) = \frac{x^5/5 + c}{x^2}$$

$$y(x) = \frac{x^3}{5} + c\,x^{-2}$$

$\boxed{1.63}$ $x(x+1)y' + y = x(x+1)^2 \operatorname{sen} x$

Resolução:

$$y' + \frac{1}{x(x+1)} y = (x+1) \operatorname{sen} x$$

$$y' + \frac{1}{x(x+1)} y = 0 \qquad \frac{dy}{y} = -\frac{dx}{x(x+1)}$$

$$\int \frac{dy}{y} = -\int \frac{1}{x(x+1)} dx = -\int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$$

$$\ln|y| = -\left[\ln|x| - \ln(x+1)\right] = -\ln\left|\frac{x}{x+1}\right| + c$$

$$y_h = c\left(\frac{x+1}{x}\right)$$

$$y = c(x)\left(\frac{x+1}{x}\right) \qquad y' = c'(x)\left(\frac{x+1}{x}\right) - c(x)\frac{1}{x^2}$$

$$c'(x)\left(\frac{x+1}{x}\right) - \cancel{c(x)\frac{1}{x^2}} + \cancel{\frac{1}{x(x+1)} c(x)\left(\frac{x+1}{x}\right)} = (x+1) \operatorname{sen} x$$

$$c'(x)\left(\frac{x+1}{x}\right) = (x+1) \operatorname{sen} x \Rightarrow c'(x) = x \operatorname{sen} x$$

$$c(x) = -x \cos x + \operatorname{sen} x + \bar{c}$$

$$y = c\,\frac{x+1}{x} - (x+1)\cos x + \frac{x+1}{x} \operatorname{sen} x$$ ✓

$\boxed{1.64}$ (a) $y' - 3y = e^{2x}$, $y=0$ para $x=0$ $x \in]-\infty, +\infty[$

Resol.

$$y(x) = \cancel{q}\, e^{-\int P(x)dx} \left[\int Q(x)\, e^{\int P(x)dx} dx \cdot \cancel{e^{\int P(x)dx}} + c \right]$$

$$y(x) = e^{-\int P(x)dx} \left[\int Q(x)\, e^{\int P(x)dx} + c \right]$$

$P(x) = -3 \qquad Q(x) = e^{2x}$

$$y(x) = e^{-\int -3dx} \left[\int e^{2x} e^{\int -3dx} + c \right]$$

$$y(x) = e^{3x} \left[e^{2x} e^{-3x} + c \right]$$

$$y(x) = e^{3x} \left[e^{-x} + c \right] = e^{2x} + ce^{3x}$$

$$y(0) = 0 = e^{0} + c\,e^{0} \Rightarrow c = -1$$

$$\boxed{y(x) = e^{2x} - e^{3x}}$$

$\boxed{1.64}$ (b) $\boxed{xy' - 2y = x^5}$ $y=1$ $x=1$ $I =]0, +\infty[$

Resol:

$$y' - \frac{2}{x} y = x^4$$

$$P(x) = -\frac{2}{x} \quad Q(x) = x^4$$

$$\int P(x)\,dx = \int -\frac{2}{x}\,dx = -2\ln x = \ln x^{-2}$$

$$y(x) = e^{-\int P(x)dx}\left[\int Q(x) e^{\int P(x)dx}\,dx + C\right]$$

$$y(x) = e^{-\ln x^{-2}}\left[\int x^4 e^{\ln x^{-2}}\,dx + C\right]$$

$$y(x) = x^2 \left[\int x^4 x^{-2}\,dx + c\right]$$

$$y(x) = x^2\left[\frac{x^3}{3} + c\right] = \frac{x^5}{3} + c x^2$$

$$y(1) = 1 \Rightarrow \frac{1}{3} + c\,1 = 1 \Rightarrow c = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\boxed{y(x) = \frac{x^5}{3} + \frac{2}{3} x^2}$$

$\boxed{1.64}$ (c) $\boxed{\dfrac{dx}{dt} + x = e^{2t}}$ $x=1$, $t=0$, $t \in]-\infty, +\infty[$

Resol.

$$P(t) = 1 \quad Q(t) = e^{2t}$$

$$\int P(t)\,dt = t$$

$$x(t) = e^{-\int P(t)\,dt}\left[\int Q(t)\, e^{\int P(t)\,dt}\,dt + C\right]$$

$$x(t) = e^{-t}\left[\int e^{2t} e^{t}\,dt + c\right]$$

$$x(t) = e^{-t}\left[\frac{e^{3t}}{3} + c\right] = \frac{e^{2t}}{3} + e^{-t} C$$

$t=0$ $x=1$

$$1 = e^{0}\left[\frac{e^{0}}{3} + c\right] = \frac{1}{\cancel{8}}\left[\frac{1}{3} + C\right] \Rightarrow c = 1 - \frac{1}{3}$$

$$\Rightarrow \boxed{C = \frac{2}{3}}$$

$$\boxed{x(t) = \frac{e^{2t}}{3} + \frac{2}{3} e^{-t}}$$

$$\boxed{x(t) = \frac{e^{2t}}{3} + \frac{2}{3} e^{-t}}$$

1.64 (d) $\boxed{y' + xy = x^3}$ $y=0$

Resol

$$P(x) = x \qquad Q(x) = x^3$$

$$\int P(x)\,dx = \int x\,dx = \frac{x^2}{2}$$

$$y(x) = e^{-\int P(x)\,dx}\left[\int Q(x)\, e^{\int P(x)\,dx} + c\right]$$

$$y(x) = e^{-\cancel{\int}\frac{x^2}{2}\cancel{dx}}\left[\int x^3 e^{\cancel{\int}\frac{x^2}{2}dx} + c\right]$$

$$y(x) = e^{-\frac{x^2}{2}}\left[\int x^3\, e^{\frac{x^2}{2}}\,dx + C\right]$$

$$y(x) = e^{-x^2/2}\left[\int x^2 e^{x^2/2}\, x\,dx + c\right]$$

$$u = \frac{x^2}{2} \quad du = \frac{1}{2}2x\,dx \quad \underline{du = x\,dx}$$

$$\left[2\int u\, e^{u}\,du + C\right]$$

$$\left[2\left(u\, e^{u} - \int e^{u}\,du\right) + C\right]$$

$$\left[2\left(u\, e^{u} - e^{u}\right) + C\right]$$

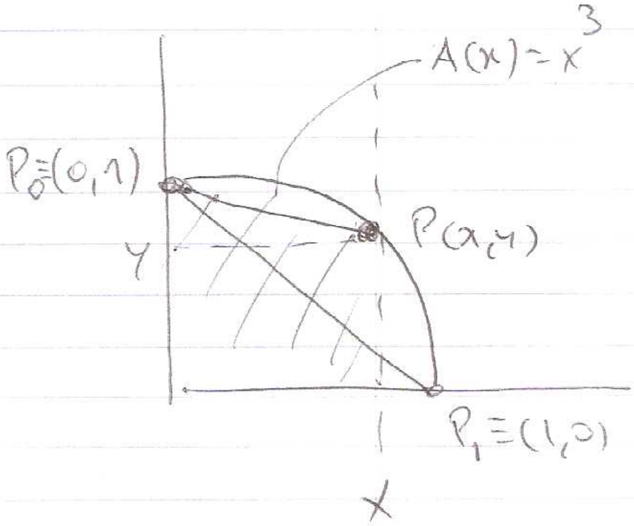
$$\left[2u\, e^{u} - 2e^{u} + C\right] = \left[\left(\frac{x^2}{2}\right) e^{x^2/2} - 2e^{x^2/2} + \right]$$

$$y(x) = e^{-x^2/2}\left[\quad\right] = 4\,\frac{x^2}{2}) - 2 + Ce^{-x^2/2}$$

$$\boxed{-2 + c = 0}$$

$$\boxed{y(x) = 2e^{-x^2/2} + x^2 - 2}$$

$\boxed{1.65}$



$$x^3 = \int_0^x f(x)\,dx - \left[xy + \frac{x(1-y)}{2}\right]$$

$$x^3 = F(x) - F(0) - \left(xy + \frac{x - xy}{2}\right)$$

$$\left(\frac{2xy + x - xy}{2}\right) = \frac{xy + x}{2}$$

$$x^3 = F(x) - F(0) - \frac{x}{2}(y+1)$$

$$3x^2 = F'(x) - 0 - \frac{1}{2}(y+1) - \frac{x}{2}y'$$

$$3x^2 = \underbrace{y - \frac{1}{2}y}_{} - \frac{1}{2} - \frac{x}{2}y'$$

$$3x^2 = \frac{y}{2} - \frac{1}{2} - \frac{x}{2}y'$$

$$-\frac{x}{2}y' + \frac{y}{2} - \frac{1}{2} - 3x^2 = 0$$

$$-xy' + y - 1 - 6x^2 = 0$$

$$y' - \frac{1}{x}y + \frac{1}{x} + 6x = 0$$

$$y' - \frac{1}{x}y = -\left(6x + \frac{1}{x}\right)$$

$$y = e^{-\int -\frac{1}{x}dx}\left[\int -\left(6x+\frac{1}{x}\right)e^{\int -\frac{1}{x}dx}\,dx + c\right]$$

$$y = e^{\ln x}\left[\int -\left(6x+\frac{1}{x}\right)e^{-\ln x}\,dx + c\right]$$

$$y = x\left[\int -\left(6x+\frac{1}{x}\right)x^{-1}\,dx + c\right]$$

$$y = x\left[-6x + \frac{1}{x} + c\right]$$

$$y = -6x^2 + 1 + cx \qquad \int -\frac{1}{x^2}$$

$$y(0) = -0 + 1 + 0$$

$$y(1) = -6 + 1 + c = 0 \qquad \boxed{c = 5}$$

$$\boxed{y = -6x^2 + 1 + 5x}$$

Prob. 166 $\boxed{y' \operatorname{sen} x + y \cos x = 1}$ C.M

Resol.:

$$y' + \underbrace{\frac{\cos x}{\operatorname{sen} x}}_{P(x)} y = \underset{Q(x)}{\frac{1}{\operatorname{sen} x}}$$

$$\mu(x) = \exp\left[\int P(x)\,dx\right] = \exp\left[\int \frac{\cos x}{\operatorname{sen} x}\,dx\right] =$$

$$= \operatorname{sen} x$$

$$\mu(x) = \exp\left[\ln|\operatorname{sen} x|\right]$$

$$\boxed{\mu(x) = \operatorname{sen} x}$$

$$y = \frac{\int \mu(x)\,Q(x)\,dx + C}{\mu(x)}$$

$$= \frac{\int \operatorname{sen} x \frac{1}{\operatorname{sen} x}\,dx + C}{\operatorname{sen} x}$$

$$y = \frac{\int dx + C}{\operatorname{sen} x} = \frac{x + C}{\operatorname{sen} x}$$

$$\boxed{y = \frac{x + C}{\operatorname{sen} x}}$$

1.66.

outra forma de resolver

$$\boxed{y' \operatorname{sen} x + y \cos x = 1}$$

Resol.:

$$\frac{dy}{dx} + \frac{\cos x}{\operatorname{sen} x} y = \frac{1}{\operatorname{sen} x}$$

$$\triangleright \quad \frac{dy}{dx} + \frac{\cos x}{\operatorname{sen} x} y = 0$$

$$\frac{dy}{y} + \frac{\cos x}{\operatorname{sen} x} dx = 0$$

$$\ln y + \ln|\operatorname{sen} x| = c$$

$$\ln y = c - \ln|\operatorname{sen} x| = \ln \frac{K}{\operatorname{sen} x}$$

$$\boxed{y_h = \frac{K}{\operatorname{sen} x}}$$

$$\triangleright \quad y_p = \frac{K(x)}{\operatorname{sen} x} \qquad y_p' = \frac{K'(x) \operatorname{sen} x - K(x) \cos x}{\operatorname{sen}^2 x}$$

$$\frac{K' \operatorname{sen} x - K \cos x}{\operatorname{sen}^2 x} + \frac{\cos x}{\operatorname{sen} x}\left(\frac{K}{\operatorname{sen} x}\right) = \frac{1}{\operatorname{sen} x}$$

$$K' \operatorname{sen} x - \cancel{K \cos x} + \cancel{K \cos x} = \operatorname{sen} x$$

$$K'(x) = 1 \longrightarrow \boxed{K = x} \longrightarrow y_p = \frac{x}{\operatorname{sen} x}$$

$$y = y_p + y_h = \frac{x}{\operatorname{sen} x} + \frac{K}{\operatorname{sen} x} = \boxed{\frac{K + x}{\operatorname{sen} x}}$$

$\boxed{1.67}$ $\quad xy' + 2(1-x^2)y = 1$

$$y' + \underbrace{\frac{2(1-x^2)}{x}}_{P(x)} y = \underbrace{\frac{1}{x}}_{Q(x)}$$

$$\exp\left[-\int P(x)\,dx\right] = \exp\left[-\int\left(\frac{2}{x} - 2x\right)dx\right] = \exp\left[-\left[\ln x^2 - x^2\right]\right]$$

$$= \exp\left[\ln x^{-2} + x^2\right] = x^{-2} \cdot e^{x^2}$$

$$\exp\left[\int P(x)\,dx\right] = \qquad = x^2 \cdot e^{-x^2}$$

$$y(x) = x^{-2} e^{x^2}\left[\int \frac{1}{x} x^2 e^{-x^2} dx + c\right]$$

$$= x^{-2} e^{x^2}\left[\int x e^{-x^2} dx + c\right] = x^{-2} e^{x^2}\left[-\frac{1}{2} e^{-x^2} + c\right]$$

$$\boxed{y(x) = -\frac{1}{2} x^{-2} + c\, e^{x^2} x^{-2}}$$

$$y = \frac{1}{x^2}\left(c\, e^{x^2} - \frac{1}{2}\right)$$

$\boxed{1.68}$ $y' + y \cot g\, x = \operatorname{sen} 2x$

Resolução:

$P(x) = \cot g\, x \qquad Q(x) = \operatorname{sen} 2x$

$$e^{\int P(x)\,dx} = e^{\int \cot g x \, dx} = e^{\int \frac{\cos x}{\operatorname{sen} x} dx} =$$

$$= e^{\ln|\operatorname{sen} x|} = \operatorname{sen} x$$

$$y = (\operatorname{sen} x)^{-1} \left[\int (\operatorname{sen} x)(\operatorname{sen} 2x)\,dx + c\right] =$$

$$y = (\operatorname{sen} x)^{-1} \left[\int \operatorname{sen} x \cdot 2 \cdot \operatorname{sen} x \cos x \,dx + c\right] =$$

$$y = (\operatorname{sen} x)^{-1} \left[\int 2\operatorname{sen}^2 x \cos x \,dx + c\right] =$$

$$y = \operatorname{sen}^{-1} x \left[\frac{2}{3}\operatorname{sen}^3 x + c\right] = \frac{2}{3}\operatorname{sen}^2 x + c\operatorname{sen}^{-1} x$$

$$\boxed{y = \frac{2}{3}\operatorname{sen}^2 x + c\operatorname{sen}^{-1} x}$$ ✓

$\boxed{1.69}$ $\boxed{(1-x^2)y'+xy=2x}$

$$y'+\underbrace{\frac{x}{1-x^2}}_{P}y=\underbrace{\frac{2x}{1-x^2}}_{Q}$$

$$\int P\,dx=\int\frac{x}{1-x^2}dx=-\frac{1}{2}\int\frac{-2x\,dx}{1-x^2}=-\frac{1}{2}\int\frac{du}{u}=$$

$u=1-x^2$

$du=-2x\,du$

$$=-\frac{1}{2}\ln u=$$

$$=-\frac{1}{2}\ln(1-x^2)$$

$$\exp\left[-\int P dx\right]=\exp\left[-\left(-\frac{1}{2}\ln(1-x^2)\right)\right]=\exp\left[\frac{1}{2}\ln(1-x^2)\right]$$

$$=(1-x^2)^{1/2}$$

$$\exp\left[\int P dx\right]=(1-x^2)^{-1/2}$$

$$y(x)=(1-x^2)^{1/2}\left[\int\frac{2x}{1-x^2}(1-x^2)^{-1/2}dx+c\right]$$

$$\left[\int 2x(1-x^2)^{-3/2}dx+c\right]$$

$$\left[-\frac{\cancel{(}1-x^2)^{-1/2}}{-1/2}+c\right]$$

$$y(x)=(1-x^2)^{1/2}\left[2(1-x^2)^{-1/2}+c\right]$$

$$\boxed{y(x)=2+c(1-x^2)^{1/2}}$$

$\boxed{1.10}$ $\quad \overline{\underline{xy' + 2y = 4e^{x^2}}}$

$$y' + \underbrace{\frac{2}{x}}_{P} y = \underbrace{\frac{4e^{x^2}}{x}}_{Q}$$

$$\int P\,dx = \int \frac{2}{x}\,dx = \ln x^2 \longrightarrow \exp\left[-\int P\,dx\right] =$$

$$= \exp \ln x^{-2}$$

$$= x^{-2}$$

$$\longrightarrow \exp\left[\int P\,dx\right] = x^2$$

$$\int Q\, e^{\int P(x)\,dx}\,dx = \int \frac{4e^{x^2}}{x} x^2\,dx = \int 4x e^{x^2}\,dx =$$

$$= 2\int 2x e^{x^2}\,dx = 2\int e^u\,du = 2e^u = 2e^{x^2}$$

$$\boxed{y(x) = x^{-2}\left[2e^{x^2} + c\right]}$$

$\boxed{1.71}$

$$xy' - 2y = x^3 e^x$$

$$y' \underbrace{- \frac{2}{x}}_{P} y = \frac{x^3}{x} e^x = \underbrace{x^2 e^x}_{Q}$$

$$\int P\,dx = \int -\frac{2}{x}\,dx = -2\ln x = \ln x^{-2}$$

$$\exp\left[-\int P\,dx\right] = \exp\left[-\ln x^{-2}\right] = x^2$$

$$\exp\left[\int P(dx)\right] = \qquad = x^{-2}$$

$$y(x) = x^2 \left[\int x^2 e^x \cdot x^{-2}\,dx + c\right]$$

$$y(x) = x^2 \left[\int e^x\,dx + c\right] = x^2\left[e^x + c\right]$$

$$\boxed{y(x) = x^2\left[e^x + c\right]}$$

$\boxed{1.72}$

$$\boxed{\frac{dy}{dt} = -K\,(y - M(t))}$$

$\boxed{1.73}$ $200\,°C \rightarrow 120\,°C$ $\quad$ ½ hora $\quad$ $M = 60\,°C$

a) $y(t) = ?$

$$\frac{dy}{dt} = -K\,(y - M) \qquad \frac{dy}{dt} = -K\,(y - 60)$$

$$y' + Ky = 60K$$

$$y(t) = e^{-Kt}\left[\int 60K\, e^{Kt}\,dt + c\right]$$

$$y(t) = e^{-Kt}\left[60\, e^{Kt} + c\right]$$

$$y(t) = 60 + c\,e^{-Kt}$$

$$y(0) = 200 = 60 + c \Rightarrow c = 140$$

$$\rightarrow \boxed{y(t) = 60 + 140\, e^{-Kt}}$$

$$T = 60 + 140\, e^{-Kt} \qquad \frac{T-60}{140} = e^{-Kt}$$

$$\therefore -Kt_T = \ln\left[\frac{T-60}{140}\right]$$

$$\boxed{t_T = -\frac{1}{K}\ln\left[\frac{T-60}{140}\right]}$$

Prob. 1.74 Ra desintegração

1600 anos $\rightarrow \frac{1}{2} Q_0$

Q: % desintegrada em 100 anos?

Resol.:

$$\frac{dQ}{dt} = -KQ$$

$$\frac{dQ}{Q} = -K\,dt \qquad \ln Q = -Kt + C$$

$$Q = C e^{-Kt} \qquad t_0 = 0 \Rightarrow Q = Q_0$$

$$Q_0 = C e^{-K0} \Rightarrow Q_0 = C$$

$$\boxed{Q = Q_0 e^{-Kt}}$$

$$\frac{1}{2} Q_0 = Q_0 e^{-K1600} \qquad \frac{1}{2} = e^{-K1600}$$

$$\ln \frac{1}{2} = -K1600 \qquad K = \frac{-\ln 1/2}{1600} = \frac{0,693}{1600}$$

$$\boxed{K = 4,332 \times 10^{-4}}$$

$$\begin{cases} r Q_0 = Q_0 e^{-K100} \\ \frac{1}{2} Q_0 = Q_0 e^{-K1600} \end{cases} \qquad r = \frac{1}{2} e^{K1500}$$

$$r = 95,76\%$$

logo em 100 anos desint. $100 - 95,76 = 4,24\%$