1.21 $\quad y' = \dfrac{x+y-3}{x-y-1}$

$$x = x_1 + h$$
$$y = y_1 + k$$

$$y' = \frac{(x_1+h)+(y_1+k)-3}{(x_1+h)-(y_1+k)-1} = \frac{x_1+y_1+h+k-3}{x_1-y_1+h-k-1}$$

$$\begin{cases} h+k-3=0 \\ h-k-1=0 \end{cases}$$

$$\left|\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & -1 & 1 \end{array}\right| \qquad \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2 \qquad \begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} = -4 \qquad \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = -2$$

$$h = -4/-2 = 2 \qquad k = -2/-2 = 1$$

$$\frac{dy_1}{dx_1} = \frac{x_1+y_1}{x_1-y_1} \qquad y_1 = u x_1$$

$$\frac{du}{dx_1} x_1 + u = \frac{x_1+y_1}{x_1-y_1} = \frac{x_1+ux_1}{x_1-ux_1} = \frac{x_1(1+u)}{x_1(1-u)}$$

$$\frac{du}{dx_1} x_1 = \frac{1+u}{1-u} - u = \frac{1+u-u(1-u)}{1-u} = \frac{1+u-u+u^2}{1-u} = \frac{1+u^2}{1-u}$$

$$\frac{du}{dx_1} x_1 = \frac{1+u^2}{1-u} \qquad \int \frac{1-u}{1+u^2}\, du = \int \frac{1}{x_1}\, dx_1$$

$$\operatorname{arctg} u - \frac{1}{2}\ln(1+u^2) = \ln(x_1) + \ln C$$

$$e^{\operatorname{arctg} u - \frac{1}{2}\ln(1+u^2)} = e^{Cx_1}$$

$$\frac{1}{\sqrt{1+u^2}}\, e^{\operatorname{arctg} u} = C x_1$$

$$e^{\operatorname{arctg}[(y-1)/(x-2)]} = C\sqrt{(x-2)^2+(y-1)^2}$$

1-22

$$\frac{dy}{dx} = \frac{2x+y-1}{4x+2y+5}$$

$$z = 2x+y \qquad \frac{dy}{dx} = \frac{z-1}{2z+5}$$

$$\frac{dz}{dx} - 2 = \frac{dy}{dx} \qquad \frac{dz}{dx} - 2 = \frac{z-1}{2z+5} \qquad \frac{dz}{dx} = \frac{z-1+2(2z+5)}{2z+5}$$

$$\frac{dz}{dx} = \frac{z-1+4z+10}{2z+5} = \frac{5z+9}{2z+5}$$

$$\frac{2z+5}{5z+9}\,dz = dx$$

$$\int \frac{2\left(z+\frac{5}{2}\right)}{5\left(z+\frac{9}{5}\right)}\,dz = \int \frac{2}{5}\left[\frac{z}{z+\frac{9}{5}} + \frac{5}{2}\left(\frac{1}{z+\frac{9}{5}}\right)\right] dz$$

$$= \frac{2}{5}\int \left[\frac{z+\frac{9}{5}-\frac{9}{5}}{z+\frac{9}{5}} + \frac{5}{2}\left(\frac{1}{z+\frac{9}{5}}\right)\right] dz$$

$$= \frac{2}{5}\left[z - \frac{9}{5}\ln\left(z+\frac{9}{5}\right) + \frac{5}{2}\ln\left(z+\frac{9}{5}\right)\right]$$

$$= x + c$$

L.M.

1.23

(7.d)

$(x+y+1)\,dx + (2x+2y-1)\,dy = 0$

Resolução

$a_1 b_2 - a_2 b_1 = 0$

$z = x+y$ $\qquad (z+1)\,dx + (2z-1)\,dy = 0$

$(z+1) + (2z-1)\frac{dy}{dx} = 0$

$dz = 1 + \frac{dy}{dx}$ $\qquad \frac{dy}{dx} = \frac{dz}{dx} - 1$

$(z+1) + (2z-1)\left(\frac{dz}{dx} - 1\right) = 0$

$\frac{-(z+1)}{(2z-1)} + 1 = \frac{dz}{dx}$

$\frac{dz}{dx} = \frac{(2z-1)-(z+1)}{2z-1} = \frac{z-2}{2z-1}\,dz$

$\frac{2z-1}{z-2}\,dz = dx$

$\int \frac{2z}{z-2}\,dz - \int \frac{1}{z-2}\,dz = \int dx$

$x = 2\left[\int \frac{z-2}{z-2}\,dz + \int \frac{2}{z-2}\,dz\right] - \int \frac{1}{z-2}\,dz$

$x = 2\left[z + 2\ln|z-2|\right] - \ln|z-2| +$

$x + 2y + 3\ln|x+y-2| = c$ [redacted] 2

(1.24)

$$\frac{dy}{dx} = \frac{3y-7x+7}{3x-7y-3}$$

$$\begin{cases} x = x_1 + h \\ y = y_1 + k \end{cases} \qquad \frac{dy}{dx} = \frac{-7x_1-7h+3y_1+3k+7}{3x_1+3h-7y_1-7k-3} =$$

$$= \frac{-7x_1+3y_1-7h+3k+7}{3x_1-7y_1+3h-7k-3}$$

$$\begin{cases} -7h+3k+7=0 \\ 3h-3k-3=0 \end{cases}$$

$$\begin{vmatrix} -7 & 3 \\ 3 & -7 \end{vmatrix} = 49-9=40$$

$$\begin{vmatrix} -7 & 3 \\ 3 & -7 \end{vmatrix} = 49-9=40 \qquad h=\frac{40}{40}=1$$

$$\begin{vmatrix} -7 & -7 \\ 3 & 3 \end{vmatrix} = -21+21=0 \qquad K=\frac{0}{40}=0$$

$$\begin{cases} x = x_1+1 \\ y = y_1+0 \end{cases}$$

$$\frac{dy}{dx} = \frac{3y_1-7x_1-7+7}{3x_1+3-7y_1-3} = \frac{3y_1-7x_1}{3x_1-7y_1}$$

$$\frac{dy_1}{dx_1} = \frac{3y_1-7x_1}{3x_1-7y_1}$$

$$y_1 = ux_1 \longrightarrow \frac{dy_1}{dx_1} = \frac{du}{dx_1}x_1+u = \frac{3(ux_1)-7x_1}{3x_1-7(ux_1)}$$

$$= \frac{u3x_1-7x_1}{3x_1-7ux_1}$$

$$\frac{du}{dx_1}x_1+u = \frac{\cancel{x_1}(3u-7)}{\cancel{x_1}(3-7u)}$$

$$\frac{du}{dx_1}x_1 = \frac{3u-7}{3-7u}-u = \frac{\cancel{3u}-7-\cancel{3u}+7u^2}{3-7u} = \frac{-7(1-u^2)}{3-7u}$$

$$\frac{3-7u}{-7(1-u^2)}du = \frac{1}{x_1}dx \qquad -\frac{1}{7}\int\frac{3-7u}{1-u^2}du = \frac{1}{x_1}dx$$

$$-\frac{1}{7}\left[\int \frac{3}{1-v^2} + \int \frac{7}{2}\,\frac{(-2v)}{1-v^2}\,dv\right] = -\frac{1}{7}\left[\left(3\int \frac{1/2}{1+v}\,dv + \frac{1}{2}\int \frac{1}{1-v}\,dv + \frac{7}{2}\ln(1-v^2)\right] = \int \frac{1}{x_1}\,dx\right.$$

$$-\frac{1}{7}\left[\underbrace{\frac{3}{2}\left[\ln(1+v)+\ln(1-v)\right] + \frac{7}{2}\ln(1-v^2)}\right]$$

$$-\frac{1}{7}\left[\frac{3}{2}\ln(1-v^2)+\underbrace{\frac{7}{2}\ln(1-v^2)}_{\frac{10}{2}\ln(1-v^2)}\right] = \ln cx_1$$

$$= -\frac{10}{14}\ln(1-v^2) = \ln cx_1$$

$$\frac{5}{7}\ln(1-v^2) = \ln cx_1$$

$$\ln(1-v^2)^{5/7} = -\ln cx_1$$

$$(1-v^2)^{5/7} = cx_1 \qquad \left(1-\frac{x_1^2}{y_1^2}\right)^5 = cx^7$$

$$\left(\frac{y_1^2-x_1^2}{x_1^2y_1^2}\right)^5 = c$$

L. MAD.

1.25 $(x+2y+1)\,dx-(2x+4y+3)\,dy=0$

Resolução

$a_1 b_2 - a_2 b_1 = 0$

$z = a_1 x + b_1 y \qquad z = x+2y$

$\frac{dz}{dx} = 1 + 2\frac{dy}{dx} \qquad \frac{dy}{dx} = \frac{1}{2}\left(\frac{dz}{dx}-1\right)$

$(z+1)\,dx-(2z+3)\,dy=0$

$(z+1) - (2z+3)\frac{dy}{dx} = 0$

$1 - \frac{(2z+3)}{(z+1)}\frac{dy}{dx} = 0$

$1 = \frac{2z+3}{z+1}\frac{dy}{dx} \qquad \frac{z+1}{2z+3} = \frac{dy}{dx} = \frac{1}{2}\left(\frac{dz}{dx}-1\right)$

$2\frac{z+1}{2z+3}+1 = \frac{dz}{dx}$

$\frac{2z+2+2z+3}{2z+3} = \frac{4z+5}{2z+3} = \frac{dz}{dx}$

$dx = \frac{2z+3}{4z+5}\,dz \qquad x = \int \frac{2z+3}{4z+5}\,dz$

$x = \frac{1}{2}\int \frac{4z+6-1+1}{4z+5}\,dz = \frac{1}{2}\int \frac{4z+5}{4z+5} + \frac{1}{4z+5}\,dz$

$x = \frac{1}{2}\left[z + \frac{1}{4}\ln|4z+5|\right] + c \qquad x = \frac{1}{2}\left[x+2y+\frac{1}{4}\ln|4x+8y+5|\right]+c$

$\boxed{1.26}$ $\underline{(x+2y+1)\,dx-(2x-3)\,dy=0}$

$$\frac{dy}{dx}=\frac{x+2y+1}{2x\quad-3}$$

$$\begin{cases} x=x_1+h \\ y=y_1+K \end{cases} \qquad \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} \Rightarrow D=0-4=-4$$

$$h=\frac{\begin{vmatrix} -1 & 2 \\ 3 & 0 \end{vmatrix}}{-4}=\frac{0-6}{-4}=\frac{3}{2}$$

$$K=\frac{\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}}{-4}=\frac{3-(-2)}{-4}=-\frac{5}{4}$$

$$\begin{cases} x=x_1+3/2 \\ y=y_1-5/4 \end{cases} \qquad \frac{dy}{dx}=\frac{(x_1+\frac{3}{2})+2(y_1-\frac{5}{4})+1}{2(x_1+\frac{3}{2})-3}=$$

$$\frac{dy}{dx}=\frac{dy_1}{dx_1}=\frac{x_1+2y_1}{2x_1}$$

$$y_1=ux_1 \to \frac{dy_1}{dx_1}=\frac{x_1+2ux_1}{2x_1}=\frac{1+2u}{2}$$

$$\frac{dy_1}{dx_1}=\frac{du}{dx_1}x_1+u=\frac{1+2u}{2} \qquad \frac{du}{dx_1}x_1=\frac{1+2u-2u}{2}=\frac{1}{2}$$

$$du=\frac{1}{2x_1}dx_1 \to u=\frac{1}{2}\ln x_1+c_1 \to \underline{y_1=\frac{1}{2}x_1\ln x_1+c_1x_1}$$

$$y_1=y+\frac{5}{4}=\frac{1}{2}(x-\tfrac{3}{2})\ln(x-\tfrac{3}{2})+c_1(x-\tfrac{3}{2})$$

$$4y+5=(2x-3)\ln(x-\tfrac{3}{2})+c_1(2x-3)$$

$$\boxed{\ln(x-\tfrac{3}{2})-\frac{4y+5}{2x-3}=c_1}$$

$\boxed{1.27}$ $\quad x+y-2+(1-x)y'=0$

$$\frac{dy}{dx}=\frac{2-x-y}{1-x}=\frac{-x-y+2}{-x\quad+1}\qquad \begin{cases} x=x_1+h \\ y=y_1+k \end{cases}$$

$$D=\begin{vmatrix}-1 & -1 \\ -1 & 0\end{vmatrix}=0-(1)=1$$

$$h=\frac{\begin{vmatrix}-2 & -1 \\ -1 & 0\end{vmatrix}}{-1}=\frac{0-(1)}{-1}=\frac{-1}{-1}=1 \qquad \begin{cases} x=x_1+1 \\ y=y_1+1 \end{cases}$$

$$K=\frac{\begin{vmatrix}-1 & -2 \\ -1 & -1\end{vmatrix}}{-1}=\frac{1-(2)}{-1}=\frac{-1}{-1}=1$$

$$\frac{dy_1}{dx_1}=\frac{-(x_1+1)-(y_1+1)+2}{-(x_1+1)+1}=\frac{-x_1-1-y_1-1+2}{-x_1-1+1}=\frac{x_1+y_1}{x_1}$$

$\boxed{y_1=ux_1}$ $\quad \frac{dy_1}{dx_1}=\frac{du}{dx_1}x_1+u=\frac{x_1+ux_1}{x_1}=\frac{1+u}{1}$

$$\frac{du}{dx_1}x_1=1+u-u=1 \qquad du=\frac{1}{x_1}dx_1$$

$$u=\ln x_1+c \rightarrow \frac{y_1}{x_1}=\ln x_1+c \quad \boxed{y_1=x_1\ln x_1+cx_1}$$

$$y-1=(x-1)\ln(x-1)+c(x-1)$$

$$\boxed{y=1+(x-1)\ln(x-1)+c(x-1)}$$

1.28

$$(x+y)\,dx + (x+y-1)\,dy = 0$$

Rj:

$$\frac{dy}{dx} = -\frac{x+y}{x+y-1} \qquad \text{sub } z = x+y$$

$$\frac{dz}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dz}{dx} = 1 - \frac{x+y}{x+y-1} = 1 - \frac{z}{z-1} = \frac{\cancel{z}-1-\cancel{z}}{z-1}$$

$$\frac{dz}{dx} = -\frac{1}{z-1}$$

$$(z-1)\,dz = -dx$$

$$\frac{z^2}{2} - z = -x + c$$

$$\frac{(x+y)^2}{2} - (x+y) = -x + c$$

$$(x+y)^2 - \cancel{2x} - 2y = -\cancel{2x} + c$$

$$\boxed{(x+y)^2 - 2y = c}$$

$\boxed{1.29}$ $2x+3y-5+(3x+2y-5)y'=0$

Res:

$2x+3y-5+(3x+2y-5)y'=0$

$2x+3y-5$ $\qquad \frac{dy}{dx}=-\frac{(2x+3y-5)}{3x+2y-5}$

$\frac{dy}{dx}=\frac{-2x-3y+5}{3x+2y-5}$ $\qquad x=x_1+h$, $y=y_1+k$

$\frac{dy_1}{dx_1}=\frac{-2(x_1+h)-3(y_1+k)+5}{3(x_1+h)+2(y_1+k)-5}=\frac{-2x_1-3y_1-2h-3k+5}{3x_1+3h+2y_1+2k-5}$

$\begin{cases}-2h-3k+5=0\\ 3h+2k-5=0\end{cases}$ $\qquad \begin{vmatrix}-2 & -3\\ 3 & 2\end{vmatrix}=-4+9=5$

$\frac{\begin{vmatrix}-5 & -3\\ +5 & 2\end{vmatrix}}{5}=h=\frac{(-10)+15}{5}=\frac{5}{5}=1$

$\frac{\begin{vmatrix}-2 & -5\\ 3 & 5\end{vmatrix}}{5}=k=\frac{-10+15}{5}=1$ $\qquad \boxed{\begin{matrix}h=1\\ k=1\end{matrix}}$

$\frac{dy_1}{dx_1}=\frac{-2x_1-3y_1}{3x_1+2y_1}$ $\qquad \boxed{y_1=ux_1}$ $\qquad \frac{dy_1}{dx_1}=\frac{du}{dx_1}x_1+u$

$\frac{du}{dx_1}x_1+u=\frac{-2x_1-3ux_1}{3x_1+2ux_1}$

$\frac{du}{dx_1}x_1=\frac{-2x_1-3ux_1}{3x_1+2ux_1}-\frac{u(3x_1+2ux_1)}{3x_1+2ux_1}$

$\frac{du}{dx_1}\cancel{x_1}=\frac{-2\cancel{x_1}-3u\cancel{x_1}-3u\cancel{x_1}-2u^2\cancel{x_1}}{3x_1+2ux_1}=\frac{-2u^2-6u-2}{x_1(3+2u)}$

$$\frac{(3+2v)}{-2v^2-6v-2}\,dv = \frac{dx_1}{x_1}$$

$$-\frac{1}{2}\,\frac{(3+2v)}{(v^2+3v+1)}\,dv = \frac{dx_1}{x_1}$$

$$-\frac{1}{2}\ln(v^2+3v+1) = \ln x_1 + \ln c$$

$$\ln(v^2+3v+1)^{-1/2} = \ln cx_1$$

$$(v^2+3v+1)^{-1/2} = cx_1 \qquad (v^2+3v+1)^{-1} = cx_1^2$$

$$\left(\left(\frac{y_1}{x_1}\right)^2+3\left(\frac{y_1}{x_1}\right)+1\right)^{-1} = cx_1^2$$

$$\frac{1}{\left(\frac{y_1}{x_1}\right)^2+3\left(\frac{y_1}{x_1}\right)+1} = cx_1^2$$

$$1 = cx_1^2\left(\left(\frac{y_1}{x_1}\right)^2+3\left(\frac{y_1}{x_1}\right)+1\right)$$

$$1 = c\left(y_1^2+3y_1x_1+x_1^2\right)$$

$$1 = c\left((y-1)^2+3(y-1)(x-1)+(x-1)^2\right)$$

$$1 = c\left(y^2-2y+1+3(yx-y-x+1)+x^2-2x+1\right)$$

$$1 = c\left(y^2-2y+1+3yx-3y-3x+3+x^2-2x+1\right)$$

$$1 = c\,(y^2+3yx+x^2-5x-5y$$

$$\boxed{y^2+3yx+x^2-5x-5y = c}$$