$\boxed{4.1}$ Transf. de Laplace de $f(t)=1$

Resolução:

$$\mathcal{L}\{1\} = \int_0^{\infty} 1 \cdot e^{-st}\, dt =$$

$$= \lim_{M\to\infty} \int_0^M e^{-st}\, dt = \lim_{M\to\infty} \left(-\frac{1}{s} e^{-st}\right)_0^M =$$

$$= \lim_{M\to\infty} -\frac{1}{s} e^{-Mt} + \frac{1}{s} = 0 + \frac{1}{s}$$

$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}}$$

$\boxed{4.2}$ Transf. de Laplace de $f(t)= e^{-at}$

Resolução:

$$\mathcal{L}\{e^{-at}\} = \int_0^{\infty} e^{-at} e^{-st}\, dt =$$

$$= \int_0^{\infty} e^{-(a+s)t}\, dt = \lim_{M\to\infty} \left(\frac{-1}{(s+a)} e^{-(s+a)}\right)_0^M$$

$$= \lim_{M\to\infty} \left(-\frac{1}{s+a} e^{-(s+a)M} + \frac{1}{s+a}\right) =$$

$$= 0 + \frac{1}{s+a}$$

$$\boxed{\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}}$$

$\boxed{4.3}$ $f(t) = \cos at$

Resolución:

$$\mathcal{L}\{\cos at\} = \int_0^\infty \cos(at)\, e^{-st}\, dt =$$

$$\lim_{M\to\infty} \int_0^M \cos(at)\, e^{-st}\, dt = \lim$$

$$= \lim_{M\to\infty} \left[\left(\frac{1}{a}\,\text{sen}(at)\, e^{-st}\right)_0^M - \int_0^M -\frac{1}{a}\,\text{sen}(at)\, s\, e^{-st}\, dt \right] =$$

$$= \lim_{M\to\infty} \left[\left(\frac{1}{a}\,\text{sen}(aM)\, e^{-Mt} + \left(\left(-\frac{\cos at}{a^2}\right) s e^{-st}\right)_0^M - \int_0^M -\frac{\cos at}{a^2} \right.\right.$$

$$\left. (-s^2 e^{-st})\, dt \right] =$$

$$= \lim_{M\to\infty} \left(\frac{1}{a}\,\text{sen}(aM)\, e^{-sM} - \frac{s\cos(aM) e^{-st}}{a^2} + \frac{s}{a^2} - \int_0^M \frac{s^2}{a^2}\cos(at)\, e^{-st}\, dt \right) =$$

$$= \lim_{M\to\infty} \frac{e^{-sM}(a\,\text{sen}\, aM - s\cos aM)}{a^2} + \frac{s}{a^2} - \frac{s^2}{a^2}\mathcal{L}\{\cos at\} =$$

$$\left(1 + \frac{s^2}{a^2}\right)\mathcal{L}\{\cos at\} = \lim_{M\to\infty} \frac{e^{-sM}(a\,\text{sen}\, aM - s\cos aM)}{a^2} + \frac{s}{a^2}$$

$$\left(1 + \frac{s^2}{a^2}\right)\mathcal{L}\{\cos at\} = \frac{s}{a^2}$$

$$\boxed{\mathcal{L}\{\cos at\} = \frac{s}{s^2 + a^2}} \quad \text{se } s > 0$$

$\boxed{14.4}$ $f(t) = t^2$

Resolução:

$$\mathcal{L}\{t^2\} = \int_0^{\infty} t^2 e^{-st}\,dt = \lim_{M\to\infty} \int_0^M t^2 e^{-st}\,dt =$$

$$= \lim_{M\to\infty} \left[t^2 \frac{e^{-st}}{-s} \Big|_0^M - \int_0^M 2t\,\frac{e^{-st}}{-s}\,dt \right] =$$

$$(\ldots) \qquad + \frac{2}{s} \int_0^M t\,e^{-st}\,dt \Big] =$$

$$= \lim_{M\to\infty} \left[\left(t^2 \frac{e^{-st}}{-s} \right)_0^M + \frac{2}{s} \left(\left(t\frac{e^{-st}}{-s} \right)_0^M - \int_0^M 1 \cdot \frac{e^{-st}}{-s}\,dt \right) \right] =$$

$$\lim_{M\to\infty} \left[\left(\quad \right)_0^M + \frac{2}{s} \left(\left(\quad \right)_0^M - \frac{1}{-s} \left(\frac{e^{-st}}{-s} \right)_0^M \right) \right] =$$

$$\left[\left(\cancel{M^2 \frac{e^{-sM}}{-s}}^{\to 0} - 0 \right) + \frac{2}{s} \left(\cancel{M\frac{e^{-sM}}{-s}}^{\to 0} - \frac{1}{-s} \left(\cancel{\frac{e^{-sM}}{-s}}^{\to 0} - \frac{1}{-s} \right) \right) \right] =$$

$$= \qquad + \frac{2}{s} \left[-\frac{1}{-s} \left(-\frac{1}{-s} \right) \right] = \frac{2}{s^3}$$

$$\boxed{\mathcal{L}\{t^2\} = \frac{2}{s^3}}$$

4.5 $\mathcal{L}\{sen(at)\}$

Resolucao:

$$\mathcal{L}\{sen(at)\} = \lim_{M\to\infty} \int_0^M sen(at)\, e^{-st}\, dt =$$

$$= \lim_{M\to\infty} \left[sen(at) \frac{e^{-st}}{-s} \Big|_0^M - a \int_0^M cos(at) \frac{e^{-st}}{-s} dt \right] =$$

$$= \lim_{M\to\infty} \left[sen(aM) \frac{e^{-sM}}{-s} - sen(a0) \frac{e^{-sM}}{-s} - \frac{a}{-s} \left[cos(at) \frac{e^{-st}}{-s} \Big|_0^M - a \int_0^M - sen(at) \frac{e^{-st}}{-s} dt \right] \right]$$

$$= \frac{a}{s} \left[cos(aM) \frac{e^{-sM}}{-s} - cos(a.0).\frac{1}{-s} - \frac{a}{s} \mathcal{L}\{sen(at)\} \right] =$$

$$= \frac{a}{s} \left[\frac{1}{s} - \frac{a}{s} \mathcal{L}\{sen(at)\} \right] = \frac{a}{s^2} - \frac{a^2}{s^2} \mathcal{L}\{sen(at)\}$$

$$\left(1 + \frac{a^2}{s^2}\right) \mathcal{L}\{sen(at)\} = \frac{a}{s^2}$$

$$\mathcal{L}\{sen(at)\} = \left(\frac{s^2}{s^2+a^2}\right)\left(\frac{a}{s^2}\right) = \frac{a}{s^2+a^2} \checkmark$$

$$\boxed{\mathcal{L}\{sen(at)\} = \frac{a}{s^2+a^2}}$$

$\boxed{4.6}$ $\mathcal{L}\{\operatorname{senh}(at)\}$

Resolução:

$$\mathcal{L}\{\operatorname{senh}(at)\} = \frac{1}{2}\left[\mathcal{L}\{e^{at}\} - \mathcal{L}\{e^{-at}\}\right] =$$

$$= \frac{1}{2}\left[\lim_{M\to\infty}\int_0^M e^{at}e^{-st}\,dt - \lim_{M\to\infty}\int_0^M e^{-at}e^{-st}\,dt\right] =$$

$$= \frac{1}{2}\left[\lim_{M\to\infty}\left(\left(\frac{e^{-(s-a)t}}{-(s-a)}\right)_0^M - \left(\frac{e^{-(a+s)t}}{-(a+s)}\right)_0^M\right)\right] =$$

$$= \frac{1}{2}\left[\lim_{M\to\infty}\left(\underbrace{\frac{e^{-(s-a)M}}{-(s-a)}}_{\to 0} - \frac{1}{-(s-a)}\right) - \left(\underbrace{\frac{e^{-(a+s)M}}{-(a+s)}}_{\to 0} - \frac{1}{-(a+s)}\right)\right] =$$

$$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right] = \frac{1}{2}\left(\frac{(s+a)-(s-a)}{s^2-a^2}\right) =$$

$$= \frac{1}{2}\left(\frac{2a}{s^2-a^2}\right) = \frac{a}{s^2-a^2}$$

$$\boxed{\mathcal{L}\{\operatorname{senh}(at)\} = \frac{a}{s^2-a^2}}$$

✓

$\boxed{4.7}$

$$f(t) = \begin{cases} 5, & 0<t<2 \\ 0, & t>2 \end{cases}$$

Resolução:

$$\mathcal{L}\{f(t)\} = \lim_{M\to\infty}\left(\int_0^2 5e^{-st}dt + \int_2^M 0\cdot e^{-st}dt\right) =$$

$$= \lim_{M\to\infty}\left(\frac{5e^{-st}}{-s}\Big|_0^2 + 0\right) =$$

$$= \frac{5e^{-2s}}{-s} - \frac{5e^{-s\cdot 0}}{-s} = \frac{5}{s}(1-e^{-2s})$$

$\boxed{4.8}$

$$f(t) = \begin{cases} t, & 0<t<2 \\ 3, & t>2 \end{cases}$$

Resolução:

$$\mathcal{L}\{f(t)\} = \lim_{M\to\infty}\left(\int_0^2 te^{-st}dt + \int_2^M 3e^{-st}dt\right) =$$

$$= \lim_{M\to\infty}\left[t\frac{e^{-st}}{-s}\Big|_0^2 - \int_0^2 1\cdot\frac{e^{-st}}{-s}dt + 3\frac{e^{-st}}{-s}\Big|_2^M\right] =$$

$$= \lim_{M\to\infty}\left[2\frac{e^{-2s}}{-s} - 0 - \frac{1}{s^2}e^{-st}\Big|_0^2 + \cancel{3\frac{e^{-sM}}{-s}}^{\,0} - 3\frac{e^{-2s}}{-s}\right] =$$

$$= 2\frac{e^{-2s}}{-s} - \frac{1}{s^2}\left[e^{-2s} - 1\right] - 3\frac{e^{-2s}}{-s} = -\frac{e^{-2s}}{-s} - \frac{1}{s^2}e^{-2s} + \frac{1}{s^2}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^2} + \frac{e^{-2s}}{s} - \frac{e^{-2s}}{s^2}$$

$\boxed{4.9}$ $f(t) = t^{-1/2}$ sebendo $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$$\mathcal{L}\{f(t)\} = \int_0^\infty t^{-1/2} e^{-st} dt$$

$$\Gamma(\tfrac{1}{2}) = \int_0^\infty x^{-1/2} e^{-x} dx$$

$st = x \qquad s\,dt = dx$

$$\mathcal{L}\{f(t)\} = \frac{1}{s \times s^{-1/2}} \underbrace{\int (st)^{-1/2} e^{-st} (s\,dt)}_{= \Gamma(\frac{1}{2}) = \sqrt{\pi}}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^{1/2}} \sqrt{\pi} = \frac{\sqrt{\pi}}{\sqrt{s}} =$$

$$\mathcal{L}\{f(t)\} = \sqrt{\frac{\pi}{s}}$$ ✓

$\boxed{4\text{-}10}$ $f(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right), & t > \frac{2\pi}{3} \\ 0, & t < \frac{2\pi}{3} \end{cases}$

R:

$$\mathcal{L}\{f(t)\} = \int_0^{2\pi/3} 0 \cdot e^{-st}\,dt + \underbrace{\int_{2\pi/3}^{\infty} \cos\left(t-\frac{2\pi}{3}\right) e^{-st}\,dt}$$

$$\underbrace{\int_{2\pi/3}^{\infty} \cos\left(t-\frac{2\pi}{3}\right) e^{-st}\,dt}_{u \quad dv} = \underbrace{\cos\left(t-\frac{2\pi}{3}\right) \frac{e^{-st}}{s}}_{uv}\Bigg|_{2\pi/3}^{\infty} - \int_{2\pi/3}^{\infty} \overset{du \quad v}{-\sin\left(t-\frac{2\pi}{3}\right)\frac{e^{-st}}{-s}}\,dt$$

$$\lim_{M\to\infty}\left[\cancel{\cos\left(t-\frac{2\pi}{3}\right)}^{\,0} \frac{e^{-sM}}{s} - \underbrace{\cos\left(\frac{2\pi}{3}-\frac{2\pi}{3}\right)} \frac{e^{-s2\pi/3}}{s}\right] = \boxed{-\frac{e^{-s2\pi/3}}{s}}$$

$$\underbrace{\int_{2\pi/3}^{\infty} \cos\left(t-\frac{2\pi}{3}\right) e^{-st}\,dt} = \underbrace{-\frac{\overset{=1}{e^{-s2\pi/3}}}{s}} + \underbrace{\int_{2\pi/3}^{\infty} \sin\left(t-\frac{2\pi}{3}\right) \frac{e^{-st}}{-s}\,dt}$$

$$(*) = \frac{1}{-s}\left[\sin\left(t-\frac{2\pi}{3}\right)\frac{e^{-st}}{-s}\Bigg|_{2\pi/3}^{\infty} - \int_{2\pi/3}^{\infty} \cos\left(t-\frac{2\pi}{3}\right)\frac{e^{-st}}{-s}\,dt\right]^{(*)}$$

$$= -\frac{1}{s}\left[\left[\sin t - 0\right] + \frac{1}{s}\int_{2\pi/3}^{\infty} \cos\left(t-\frac{2\pi}{3}\right) e^{-st}\,dt\right]$$

$$= -\frac{1}{s^2}\int_{2\pi/3}^{\infty} \cos\left(t-\frac{2\pi}{3}\right) e^{-st}\,dt$$

$$\underbrace{\int_{2\pi/3}^{\infty} \cos\left(t-\frac{2\pi}{3}\right) e^{-st}\,dt} = -\frac{e^{s2\pi/3}}{s} - \frac{1}{s^2}\underbrace{\int_{2\pi/3}^{\infty} \cos\left(t-\frac{2\pi}{3}\right) e^{-st}\,dt}$$

$$\mathcal{L}\{f(t)\} = \int_{2\pi/3}^{\infty} \cos\left(t-\frac{2\pi}{3}\right) e^{-st}\, dt =$$

$$= -\frac{e^{s2\pi/3}}{s} - \frac{1}{s^2}\int_{2\pi/3}^{\infty} \cos\left(t-\frac{2\pi}{3}\right) e^{-st}\, dt =$$

$$= -\frac{e^{s2\pi/3}}{s}\,\frac{1}{\left(1+\frac{1}{s^2}\right)} = -\frac{e^{s2\pi/3}}{\cancel{s}}\,\frac{s^{\cancel{2}}}{s^2+1}$$

$$\left(1+\frac{1}{s^2}\right)\int = -\frac{e^{s2\pi/3}}{s}$$

$$\boxed{\mathcal{L}\{f(t)\} = e^{-\frac{2\pi}{3}s}\,\frac{s}{s^2+1}}$$

✓

$\boxed{4.13}$ $\mathcal{L}\{4t^3 - 3\cos 2t + 5e^{-t}\}$

R:

$$\mathcal{L}\{4t^3 - 3\cos 2t + 5e^{-t}\} =$$

$$= 4\mathcal{L}\{t^3\} - 3\mathcal{L}\{\cos 2t\} + 5\mathcal{L}\{e^{-t}\} =$$

$$= 4\,\frac{3!}{s^4} - 3\,\frac{s}{2^2+s^2} + 5\,\frac{1}{s+1} =$$

$$= \frac{24}{s^4} - \frac{3s}{2^2+s^2} + \frac{5}{s+1} =$$

$\boxed{4.14}$ $\mathcal{L}\{\cosh^2 2t\}$

R:

$$\mathcal{L}\{\cosh^2 2t\} = \mathcal{L}\left\{\left(\frac{e^{2t}+e^{-2t}}{2}\right)^2\right\} = \mathcal{L}\left\{\frac{e^{4t}+e^{-4t}+2}{4}\right\}$$

$$= \frac{1}{4}\left[\mathcal{L}\{e^{4t}\} + \mathcal{L}\{e^{-4t}\} + \mathcal{L}\{2\}\right] =$$

$$= \frac{1}{4}\left[\frac{1}{s-4} + \frac{1}{s+4} + \frac{2}{s}\right] = \frac{1}{4}\left[\frac{(s+4)s + (s-4)s + 2(s-4)(s+4)}{s(s-4)(s+4)}\right]$$

$$= \frac{1}{4}\left[\frac{4s^2-32}{s(s^2-16)}\right] = \frac{s^2-8}{s(s^2-16)}$$

$\boxed{4.15}$ $\mathcal{L}\{\cos^2 at\}$

Resol:

$$\cos(2x) = \cos^2 x - \operatorname{sen}^2 x$$
$$= \cos^2 x - [1 - \cos^2 x]$$
$$= 2\cos^2 x - 1$$
$$\cos^2 x = \frac{\cos(2x)+1}{2}$$

$$\mathcal{L}\{\cos^2 at\} = \frac{1}{2}\mathcal{L}\{\cos(2at)\} + \frac{1}{2}\mathcal{L}\{1\}$$

$$= \frac{1}{2}\left[\frac{s}{(2a)^2+s^2}\right] + \frac{1}{2}\left[\frac{1}{s}\right] = \frac{1}{2}\left[\frac{s}{(2a)^2+s^2} + \frac{1}{s}\right] = \frac{1}{2}\left[\frac{s^2+(4a^2+s^2)}{s(4a^2+s^2)}\right] =$$

$$= \frac{1}{2}\left[\frac{2s^2+4a^2}{s(4a^2+s^2)}\right] = \frac{s^2+2a^2}{s(4a^2+s^2)}$$ ✓

$\boxed{4.16}$ $\mathcal{L}\{\operatorname{sen}^2 at\}$

Resol

$$\operatorname{sen}^2 at = -\cos^2 at + 1$$

$$\mathcal{L}\{\operatorname{sen}^2 at\} = -\mathcal{L}\{\cos^2 at\} + \mathcal{L}\{1\}$$

$$= \frac{1}{s} - \left[\frac{s^2+2a^2}{s(4a^2+s^2)}\right] = \frac{4a^2+s^2-s^2-2a^2}{s(4a^2+s^2)}$$

$$\mathcal{L}\{\operatorname{sen}^2 at\} = \frac{2a^2}{s(4a^2+s^2)}$$ ✓

$\boxed{4.20}$ $\mathcal{L}\{t^3\}$

Resol:

$$\boxed{\mathcal{L}\{f'(t)\} = -f(0) + s\,\mathcal{L}\{f(t)\}}$$

$f(t)=t^3 \quad f'(t)=3t^2 \qquad 3\,\mathcal{L}\{t^2\} = 0 + s\,\mathcal{L}\{t^3\} \quad (*)$

$f(t)=t^2 \quad f'(t)=2t \qquad 2\,\mathcal{L}\{t\} = 0 + s\,\mathcal{L}\{t^2\} \quad (**)$

$f(t)=t \quad f'(t)=1 \qquad \mathcal{L}\{1\} = 0 + s\,\mathcal{L}\{t\} \quad (***)$

$f(t)=1 \quad f'(t)=0 \qquad \mathcal{L}\{0\} = -1 + s\,\mathcal{L}\{1\} \quad (IV)$

Por $(*)$ $\mathcal{L}\{t^3\} = \frac{3}{s}\mathcal{L}\{t^2\} = \frac{3}{s}\left[\frac{2}{s}\underbrace{\mathcal{L}\{t\}}_{(**)}\right] = \frac{3}{s}\frac{2}{s}\left[\frac{1}{s}\underbrace{\mathcal{L}\{1\}}_{(***)}\right] =$

$$= \frac{3}{s}\frac{2}{s}\frac{1}{s}\left[\frac{1}{s}\underbrace{\left(\mathcal{L}\{0\}+1\right)}_{(IV)}\right] =$$

$$\mathcal{L}\{t^3\} = \frac{6}{s^4}$$

4.21 $\mathcal{L}\{t\cos t\}$

R:

$$\mathcal{L}\{t\cos t\} = -\left[\mathcal{L}\{\cos t\}\right]' \qquad \boxed{\mathcal{L}\{t\,f(t)\} = -F'(s)}$$

$$= -\left(\frac{s}{1+s^2}\right)' = -\left(\frac{(1+s^2)-s(2s)}{(1+s^2)^2}\right) =$$

$$= -\frac{1+s^2-2s^2}{(1+s^2)^2} = -\frac{1-s^2}{(1+s^2)^2} = \frac{s^2-1}{(1+s^2)^2}$$

4.22 $\mathcal{L}\{t e^{at}\}$

R:

$$\mathcal{L}\{t\,f(t)\} = -F'(s)$$

$$\mathcal{L}\{t e^{at}\} = -\left(\frac{1}{s-a}\right)' = -\left(\frac{-1}{(s-a)^2}\right) =$$

$$= \frac{1}{(s-a)^2}$$

4.21 $\mathcal{L}\{t\cos t\}$ usando $\mathcal{L}\{f'(t)\} = -f(0) + s\,\mathcal{L}\{f(t)\}$

Resolução:

$f(t) = t\cos t \quad f'(t) = \cos t - t\,\mathrm{sen}\,t$

$\mathcal{L}\{\cos t - t\,\mathrm{sen}\,t\} = \mathcal{L}\{\cos t\} - \mathcal{L}\{t\,\mathrm{sen}\,t\} = 0 + s\,\mathcal{L}\{t\cos t\}$

$f(t) = t\,\mathrm{sen}\,t \quad f'(t) = \mathrm{sen}\,t + t\cos t$

$\mathcal{L}\{\mathrm{sen}\,t + t\cos t\} = \mathcal{L}\{\mathrm{sen}\,t\} + \mathcal{L}\{t\cos t\} = 0 + s\,\mathcal{L}\{t\,\mathrm{sen}\,t\}$

$$\mathcal{L}\{t\cos t\} = \frac{1}{s}\left[\mathcal{L}\{\cos t\} - \mathcal{L}\{t\,\mathrm{sen}\,t\}\right] =$$

$$= \frac{1}{s}\left[\mathcal{L}\{\cos t\} - \left[\frac{1}{s}\left(\mathcal{L}\{\mathrm{sen}\,t\} + \mathcal{L}\{t\cos t\}\right)\right]\right]$$

$$= \frac{1}{s}\left[\mathcal{L}\{\cos t\} - \frac{1}{s}\mathcal{L}\{\mathrm{sen}\,t\} - \frac{1}{s}\mathcal{L}\{t\cos t\}\right]$$

$$= \frac{1}{s}\mathcal{L}\{\cos t\} - \frac{1}{s^2}\mathcal{L}\{\mathrm{sen}\,t\} - \frac{1}{s^2}\mathcal{L}\{t\cos t\}$$

$$\mathcal{L}\{t\cos t\}\left(1+\frac{1}{s^2}\right) = \frac{1}{s}\left(\frac{s}{1+s^2}\right) - \frac{1}{s^2}\,\frac{1}{1+s^2} = \cancel{\frac{1}{s}\left(\frac{s}{s^2+1}\right) - \frac{1}{s^2}\,\frac{a}{a^2+s^2}}$$

$$= \frac{s^2-1}{s^2(1+s^2)}\,\frac{1}{\left(1+\frac{1}{s^2}\right)} = \frac{s^2-1}{\cancel{s^2}(1+s^2)\,\frac{(s^2+1)}{\cancel{s^2}}} =$$

$$= \cancel{\frac{s^4-1}{s^2+s^4+1+s^2}} = \cancel{\frac{s^2-1}{s^2+s^4+1+s^2}}$$

$$= \frac{s^2-1}{(s^2+1)^2}$$

$\boxed{4.28}$ $\mathcal{L}^{-1}\left\{\frac{9}{s^3+3s^2}\right\}$

R: $\mathcal{L}^{-1}\left\{\frac{9}{s^3+3s^2}\right\} =$

$$\frac{9}{s^3+3s^2} = \frac{9}{s^2(s+3)} = \frac{As+B}{s^2} + \frac{C}{s+3}$$

$$(As+B)(s+3) + s^2C = 9$$

$$As^2 + 3As + Bs + 3B + s^2C = 9$$

$$s^2(A+C) + s(3A+B) + 3B = 9$$

$$\left|\begin{array}{l} A+C=0 \\ 3A+B=0 \\ 3B=9 \end{array}\right| \begin{array}{l} C=1 \\ A=-1 \\ B=3 \end{array}\Bigg|$$

$$\mathcal{L}^{-1}\left\{\frac{-1s+3}{s^2} + \frac{1}{s+3}\right\} = \mathcal{L}^{-1}\left\{-\frac{1}{s} + \frac{3}{s^2} + \frac{1}{s+3}\right\}$$

$$= -1 + 3t + e^{-3t}$$

$$\mathcal{L}^{-1}\left\{\frac{9}{s^3+3s^2}\right\} = -1 + 3t + e^{-3t}$$ ✓

$\boxed{4.29}$ $\mathcal{L}^{-1}\left\{\frac{1}{s^2+25}\right\}$

R:

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+25}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{5}\frac{5}{s^2+5^2}\right\}$$

$$= \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{5}{s^2+5^2}\right\} = \boxed{\frac{1}{5}\operatorname{sen} 5t}$$

$\boxed{4.30}$ $\mathcal{L}^{-1}\left\{\frac{8s}{s^2+16}\right\}$

R:

$$\mathcal{L}^{-1}\left\{\frac{8s}{s^2+4^2}\right\} = 8\mathcal{L}^{-1}\left\{\frac{s}{s^2+4^2}\right\} =$$

$$= \boxed{8\cos 4t}$$

$\boxed{4.31}$ $\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\}$

R: $\mathcal{L}\{t^4\} = \frac{4!}{s^{4+1}}$ $\quad \mathcal{L}\{t^4\} = \frac{4!}{s^5}$

logo $\mathcal{L}^{-1}\left\{\frac{1}{4!}\frac{4!}{s^5}\right\} = \frac{1}{4!}\mathcal{L}^{-1}\left\{\frac{4!}{s^5}\right\} = \boxed{\frac{1}{4!}t^4}$

$\boxed{4.32}$ $\mathcal{L}^{-1}\left\{\frac{12}{4-3s}\right\}$

R:

$$= \mathcal{L}^{-1}\left\{\frac{12}{4-3s}\right\} = \mathcal{L}^{-1}\left\{\frac{-12\cdot\frac{1}{3}}{-\left(\frac{4}{3}-s\right)}\right\}$$

$$= -4\,\mathcal{L}^{-1}\left\{\frac{1}{\left(s-\frac{4}{3}\right)}\right\} = -4\,e^{\frac{4}{3}t}$$

$$\boxed{\mathcal{L}^{-1}\left\{\frac{12}{4-3s}\right\} = -4\,e^{4/3t}}$$

$\boxed{4.33}$ $\mathcal{L}^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$

R:

$$= \mathcal{L}^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\} = \mathcal{L}^{-1}\left\{\frac{4}{s-3}\right\} + \mathcal{L}^{-1}\left\{-\frac{1}{s+1}\right\} = (*)$$

$$\frac{3s+7}{s^2-2s-3} = \frac{A}{(s-3)} + \frac{B}{(s+1)}$$

$$A(s+1) + B(s-3) = 3s+7$$

$$As + A + Bs - 3B = 3s+7$$

$$(A+B)s + (A-3B) = 3s+7$$

$$\begin{cases} A+B = 3 \\ A-3B = 7 \end{cases}$$

$$\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} = -3-1 = -4$$

$$\begin{vmatrix} 3 & 1 \\ 7 & -3 \end{vmatrix} = -9-7 = -16$$

$$\begin{vmatrix} 1 & 3 \\ 1 & 7 \end{vmatrix} = 7-3 = 4$$

$$\begin{cases} A = -16/-4 = 4 \\ B = 4/-4 = -1 \end{cases}$$

$$(*) = \boxed{4e^{3t} - e^{-t}}$$

$\boxed{4.34}$ $\mathcal{L}^{-1}\left\{\dfrac{1}{s^3+4s^2+3s}\right\}$

R :

$$s^3+4s^2+3s = s(s^2+4s+3) = s(s+3)(s+1)$$

$$\frac{1}{s^3+4s^2+3s} = \frac{1}{s(s^2+4s+3)} = \frac{1}{s(s+3)(s+1)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+1}$$

$$A(s+3)(s+1) + B(s)(s+1) + C(s+3)(s+1) = 1$$

$$s^2(A+B+C) + s(4A+B+3C) + 3A = 1$$

$$\left|\begin{array}{l} A+B+C=0 \\ 4A+B+3C=0 \\ 3A=1 \end{array}\right. \qquad \left|\begin{array}{l} B+C=-1/3 \\ B+3C=-4/3 \\ A=1/3 \end{array}\right. \qquad \left|\begin{array}{l} A=1/3 \\ B=1/6 \\ C=-1/2 \end{array}\right.$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3+4s^2+3s}\right\} = \mathcal{L}^{-1}\left\{\frac{1/3}{s} + \frac{1/6}{s+3} + \frac{-1/2}{s+1}\right\}$$

$$= \boxed{\frac{1}{3} + \frac{e^{-3t}}{6} - \frac{e^{-t}}{2}}$$

$\boxed{4.35}$ $\quad \dfrac{d^2y}{dt^2} - 2\dfrac{dy}{dt} - 8y = 8 \qquad y(0) = 3 \quad y'(0) = 6$

R: $\quad \mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} - 8\mathcal{L}\{y\} = \mathcal{L}\{8\}$

$$\left[s^2\mathcal{L}\{y\} - s f(0) - f'(0)\right] - 2\left[s\mathcal{L}\{y\} - f(0)\right] - 8$$

$$-8\left[\mathcal{L}\{y\}\right] = \frac{8}{s}$$

$$\left[s^2 Y(s) - 3s - 6\right] - 2sY(s) + 6 - 8Y(s) = \frac{8}{s}$$

$$s^2Y(s) - 3s - \cancel{6} - 2sY(s) + \cancel{6} - 8Y(s) = \frac{8}{s}$$

$$Y(s)\left[s^2 - 2s - 8\right] = 3s + \frac{8}{s} = \frac{3s^2+8}{s}$$

$$Y(s) = \frac{3s^2+8}{s(s^2-2s-8)} = \frac{3s^2+8}{s(s+2)(s-4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s-4}$$

$$Y(s) = \frac{-1}{s} + \frac{5/3}{s+2} + \frac{7/3}{s-4}$$

$\dfrac{3\times4^2+8}{12} = \dfrac{56}{24} = \dfrac{7}{3}$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{-\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{5/3}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{7/3}{s-4}\right\}$$

$$y = -1 + \frac{5}{3}e^{-2t} + \frac{7}{3}e^{4t}$$

$\boxed{4.36}$ $\begin{cases} y' + x = \text{sen}\, t \\ x' + y = 0 \end{cases}$ $x(0) = 0$ $y(0) = 0$

R:

$$\begin{cases} \mathcal{L}\{y'\} + \mathcal{L}\{x\} = \mathcal{L}\{\text{sen}\, t\} \\ \mathcal{L}\{x'\} + \mathcal{L}\{y\} = 0 \end{cases}$$

$$\begin{cases} sY(s) - y(0) + X(s) = \dfrac{1}{1+s^2} \\ sX(s) - x(0) + Y(s) = 0 \end{cases}$$

$$\begin{cases} sY(s) - 0 + X(s) = \dfrac{1}{1+s^2} \\ sX(s) + Y(s) = 0 \end{cases}$$

$$\begin{cases} X(s) + sY(s) = \dfrac{1}{1+s^2} \\ sX(s) + Y(s) = 0 \end{cases}$$

$$X(s) = \begin{vmatrix} \frac{1}{1+s^2} & s \\ 0 & s \end{vmatrix} \Big/ \begin{vmatrix} 1 & s \\ s & 1 \end{vmatrix} = \frac{1}{(s^2+1)(1-s^2)}$$

$$Y(s) = \begin{vmatrix} 1 & \frac{1}{s^2+1} \\ s & 0 \end{vmatrix} \Big/ \begin{vmatrix} 1 & s \\ s & 1 \end{vmatrix} = \frac{-s}{(s^2+1)(1-s^2)}$$

$\boxed{4\text{-}36}$

$$\begin{cases} y' + x = \operatorname{sen} t & x(0)=0 \quad y(0)=0 \\ x' + y = 0 \end{cases}$$

R:

$$\begin{cases} \mathcal{L}\{y'\} + \mathcal{L}\{x\} = \mathcal{L}\{\operatorname{sen} t\} \\ \mathcal{L}\{x'\} + \mathcal{L}\{y\} = 0 \end{cases}$$

$$\begin{cases} sY(s) - y(0) + X(s) = \dfrac{1}{1+s^2} \\ sX(s) - x(0) + Y(s) = 0 \end{cases}$$

$$\begin{cases} sY(s) - 0 + X(s) = \dfrac{1}{1+s^2} \\ sX(s) + Y(s) = 0 \end{cases}$$

$$\begin{cases} X(s) + sY(s) = \dfrac{1}{1+s^2} \\ sX(s) + Y(s) = 0 \end{cases}$$

$$X(s) = \begin{vmatrix} \frac{1}{1+s^2} & s \\ 0 & s \end{vmatrix} \Big/ \begin{vmatrix} 1 & s \\ s & 1 \end{vmatrix} = \frac{1}{(s^2+1)(1-s^2)}$$

$$Y(s) = \begin{vmatrix} 1 & \frac{1}{s^2+1} \\ s & 0 \end{vmatrix} \Big/ \begin{vmatrix} 1 & s \\ s & 1 \end{vmatrix} = \frac{-s}{(s^2+1)(1-s^2)}$$

$\boxed{4.37}$ $\quad \dfrac{dy}{dt} - 2y = e^{5t} \quad , \; y(0) = 3$

$\underline{\underline{R}}$:

$$\mathcal{L}\left\{\frac{dy}{dt}\right\} = s\,Y(s) - y(0) = s\,Y(s) - 3$$

$$s\,Y(s) - 3 - 2\,Y(s) = \mathcal{L}\{e^{5t}\} = \frac{1}{s-5}$$

$$Y(s)\,(s-2) = \frac{1}{s-5} + 3 = \frac{1+3(s-5)}{s-5} = \frac{3s-14}{s-5}$$

$$Y(s) = \frac{3s-14}{(s-5)(s-2)}$$

$$\frac{3s-14}{(s-5)(s-2)} = \frac{A}{s-5} + \frac{B}{s-2} = \frac{A(s-2) + B(s-5)}{''}$$

$$= \frac{As - 2A + Bs - 5B}{''} = \frac{s(A+B) + (-2A-5B)}{''}$$

$$\left|\begin{array}{l} A+B = 3 \\ -2A - 5B = -14 \end{array}\right. \qquad \left|\begin{array}{l} A+B = 3 \\ 2A+5B = 14 \end{array}\right| \qquad \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} = 5-2 = 3$$

$$\begin{vmatrix} 3 & 1 \\ 14 & 5 \end{vmatrix} = 15-14 = 1$$

$$\left|\begin{array}{l} A = 1/3 \\ B = -12/3 = -4 \end{array}\right. \qquad \begin{vmatrix} 1 & 13 \\ 2 & 14 \end{vmatrix} = 14-26 = -12$$

$$Y(s) = \frac{1/3}{s-5} + \frac{-4}{s-2} \qquad y = \frac{1}{3}\,e^{5t} - 4\,e^{2t}$$

$\boxed{4.38}$ $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 6y = 0$, $y(0) = 1$

$y'(0) = 2$

R:

$s^2 Y(s) - s\,y(0) - y'(0) - 5\left[sY(s) - y(0)\right] + 6Y(s) = 0$

$Y(s)\left[s^2 - 5s + 6\right] - s - 2 + 5 = 0$

$Y(s) = \dfrac{s-3}{s^2-5s+6} = \dfrac{s-3}{(s-3)(s-2)} = \dfrac{1}{s-2}$

$$\boxed{y(x) = e^{-2t}}$$

$\boxed{4.39}$ $\quad \frac{d^3y}{dt^3}+2\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=10\cos t \quad \Bigg| \quad \begin{array}{l} y(0)=0 \\ y'(0)=0 \\ y''(0)=3 \end{array}$

R:

$$\mathcal{L}\{y'''\}+2\mathcal{L}\{y''\}-\mathcal{L}\{y'\}-2Y(s)=\mathcal{L}\{10\cos t\}$$

$$[s^3Y-s^2\cancel{f(0)}^{\,0}-s\cancel{f'(0)}^{\,0}-\cancel{f''(0)}^{\,3}]$$

$$2[s^2Y-s\cancel{f(0)}^{\,0}-\cancel{f'(0)}^{\,0}]-[sY-\cancel{f(0)}^{\,0}]-2Y=10\left(\frac{s}{s^2+1}\right)$$

$$[s^3Y-3]+2[s^2Y]-sY-2Y=10\left(\frac{s}{s^2+1}\right)$$

$$Y[s^3+2s^2-s-2]-3=10\left(\frac{s}{s^2+1}\right)$$

$$Y[s^3+2s^2-s-2]=10\frac{s}{s^2+1}+3=\frac{3s^2+10s+3}{s^2+1}$$

$$Y(s)=\frac{3s^2+10s+3}{(s^2+1)(s+2)(s+1)(s-1)}=\frac{As+B}{s^2+1}+\frac{C}{s+2}+\frac{D}{s+1}+\frac{E}{s-1}$$

$$(As+B)(s+2)(s+1)(s-1)=(As^2+2As+Bs+2B)(s^2-1)=(*)$$

$$C(s^2+1)(s+1)(s-1)=(Cs^2+C)(s^2-1)=Cs^4-C \checkmark \quad \checkmark$$

$$D(s^2+1)(s+2)(s-1)=(Ds^2+D)(s^2+s-2)=Ds^4+Ds^3-2Ds^2+Ds^2+Ds-2D$$

$$E(s^2+1)(s+2)(s+1)=(Es^2+E)(s^2+3s+2)=Es^4+3Es^3+2Es^2+Es^2+3sE+2E$$

$$(*)=As^4-As^2+2As^3-2As+Bs^3-Bs+2Bs^2-2B \checkmark$$

$$s^4[A+C+D+E]+s^3(2A+B+D+3E)+s^2(A+2B-D+3E)+s(-2A-B+D+3E)+$$

$$+(-2B-C-2D+2E)=$$

$$=3s^2+10s+3$$

$$\begin{cases} A+C+D+E=0 \\ 2A+B+D+3E=0 \\ -A+2B-D+3E=3 \\ -2A-B+D+3E=10 \\ -2B-C-2D+2E=3 \end{cases}$$

$A=-2$; $B=-1$; $C=-\frac{1}{3}$; $D=1$

$E=\frac{4}{3}$

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$$Y(s) = \frac{As+B}{s^2+1} + \frac{C}{s+2} + \frac{D}{s+1} + \frac{E}{s-1}$$

$$A = -2 \quad B = -1 \quad C = -\frac{1}{3} \quad D = 1 \quad E = \frac{4}{3}$$

$$Y(s) = -\frac{2s}{s^2+1} - \frac{1}{s^2+1} - \frac{1}{3}\frac{1}{(s+2)} + \frac{1}{s+1} + \frac{4}{3}\frac{1}{s-1}$$

$$Y(s) = -2\cos t - \sin t - \frac{1}{3}e^{-2t} + e^{-t} + \frac{4}{3}e^{t}$$

4.50 $f(t) = e^{-t}\cos 2t$

R: $\mathcal{L}\{e^{-t}\cos 2t\} = \dfrac{(s+1)}{(s+1)^2+2^2} = \dfrac{s+1}{s^2+2s+5}$

4.51 $f(t) = (t+2)^2 e^{t}$

R: $\mathcal{L}\{(t+2)^2 e^{t}\} = \mathcal{L}\{(t^2+4t+4)\,e^{t}\} =$

$$= \frac{2!}{(s-1)^3} + 4\,\frac{1}{(s-1)^2} + \frac{4}{(s-1)} = \frac{2+4(s-1)+4(s-1)^2}{(s-1)^3}$$

$$= \frac{2+4s-4+4s^2-8s+4}{(s-1)^3} \overset{!}{=} \boxed{\frac{4s^2-4s+2}{(s-1)^3}}$$

4.52 $f(t) = e^{-4t}\cosh 2t$

R: $\mathcal{L}\{e^{-4t}\cosh 2t\} = \dfrac{(s+4)}{(s+4)^2-4} \overset{!}{=} \boxed{\dfrac{s+4}{s^2+8s+12}}$

4.53 $\mathcal{L}^{-1}$ $\dfrac{s}{(s+3)^2+1}$

R: $e^{-3t}\cos t$ $\qquad \dfrac{s+3}{(s+3)^2+1} - \dfrac{3}{(s+3)^2+1}$

$$\underbrace{\frac{s+3}{(s+3)^2+1}}_{\cos} - 3\,\underbrace{\frac{1}{(s+3)^2+1}}_{\text{sen}}$$

$$\boxed{e^{-3t}(\cos t - 3\,\text{sen}\,t)}$$

4.54

$$\frac{2s+5}{s^2+4s+13}$$

R:

$$= \frac{2s+5}{(s+2)^2+9} = \frac{2(s+2)+1}{(s+2)^2+9} = 2\frac{(s+2)}{(s+2)^2+3^2} + \frac{1}{3}\frac{3}{(s+2)^2+3^2}$$

$$\mathcal{L}^{-1}\left\{2\underbrace{\frac{(s+2)}{(s+2)^2+3^2}}_{\cos t} + \frac{1}{3}\underbrace{\frac{3}{(s+2)^2+3^2}}_{\operatorname{sen} t}\right\}$$

$$= e^{-2t}\left(2\cos 3t + \frac{1}{3}\operatorname{sen} 3t\right)$$

4.55

$$\frac{\pi}{(s+\pi)^2}$$

R:

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+\pi)^2}\right\} = e^{-\pi t}\,t$$

$$\mathcal{L}^{-1}\left\{\frac{\pi}{(s+\pi)^2}\right\} = \boxed{\pi e^{-\pi t}\,t}$$

$\boxed{4.64}$ $\mathcal{L}\{(t-\tfrac{1}{2})\,u(t-\tfrac{1}{2})\}$

R:

recall $\mathcal{L}\{f(t-a)\,u(t-a)\} = e^{-as}F(s)$

$$e^{-\frac{1}{2}s}\mathcal{L}\{t\} = e^{-\frac{1}{2}s}\cdot\frac{1}{s^2} = \frac{e^{-\frac{s}{2}}}{s^2}$$ ✓

$\boxed{4.65}$ $\mathcal{L}\{(t-1)^2\,u(t-1)\}$

R: $$e^{-as}\mathcal{L}\{t^2\} = e^{-s}\frac{2!}{s^3}$$ ✓

$\boxed{4.66}$ $\mathcal{L}\{\cos(t-\pi)\,u(t-\pi)\}$

R: $$e^{-\pi s}\mathcal{L}\{\cos t\} = e^{-\pi s}\frac{s}{s^2+1}$$ ✓

$\boxed{4.67}$ $\mathcal{L}\{e^{t-2}\,u(t-2)\}$

R: $$e^{-2s}\mathcal{L}\{e^t\} = e^{-2s}\cdot\frac{1}{s-1}$$ ✓

$\boxed{4.68}$ $\mathcal{L}\{e^t\,u(t-\tfrac{1}{2})\}$

R: $$\mathcal{L}\{e^t\,u(t-\tfrac{1}{2})\} = \mathcal{L}\{e^{t-1/2}\cdot e^{1/2}\,u(t-\tfrac{1}{2})\} =$$

$$e^{1/2}e^{-\frac{1}{2}s}\frac{1}{s-1}$$ ✓

$\boxed{4.69}$ $\mathcal{L}\{\cos t\, u(t-\pi)\}$

R:

Lembrar $\cos(t-\pi) = \cos t \underbrace{\cos\pi}_{-1} + \operatorname{sen} t \underbrace{\operatorname{sen}\pi}_{0}$

$= -\cos t$

logo $\cos t = -\cos(t-\pi)$

$$\mathcal{L}\{\cos t\, u(t-\pi)\} = \mathcal{L}\{-\cos(t-\pi)\, u(t-\pi)\}$$

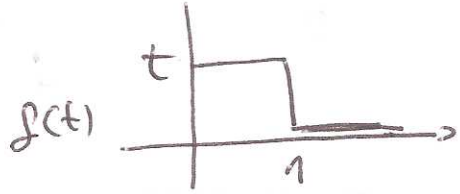
$$= -\mathcal{L}\{\cos(t-\pi)\, u(t-\pi)\} = -e^{-\pi s}\mathcal{L}\{\cos t\} =$$

$$= -e^{-\pi s}\frac{s}{s^2+1}$$

$\boxed{4.70}$ $\mathcal{L}\{g(t)\}$ $g(t)\begin{cases} t & , 0<t<1 \\ 0 & \text{outro}\end{cases}$

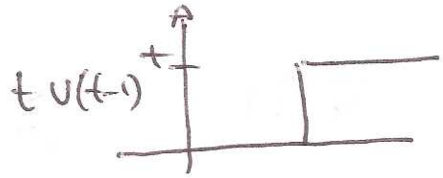
R:

graf.



$g(t) = t - t\,u(t-1)$

$\mathcal{L}\{g(t)\} = \mathcal{L}\{t - t\,u(t-1)\}$



$$= \mathcal{L}\{t - [t-1+1]\,u(t-1)\} =$$

$$= \mathcal{L}\{t - (t-1)\,u(t-1) - u(t-1)\} =$$

$$= \mathcal{L}\{t\} - \mathcal{L}\{(t-1)\,u(t-1)\} - \mathcal{L}\{u(t-1)\} =$$

$$= \frac{1}{s} - e^{-s}\cdot\frac{1}{s^2} - \frac{e^{-s}}{s} = \frac{1}{s} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$