$\boxed{1.32}$ $\quad y^2+2ax=0 \qquad a>0$

$2yy'+2a=0 \qquad 2yy'+\left(-\frac{y^2}{x}\right)=0$

$y'=\frac{y^{\cancel{2}}}{x}\cdot\frac{1}{2\cancel{y}}=\frac{y}{2x}$

$y'_{ort}=-\frac{1}{y'}=-\frac{2x}{y} \qquad y\,dy=-2x\,dx$

$\frac{1}{2}y^2=-\frac{2x^2}{2}+c \qquad \boxed{y^2+2x^2=c}$

$\boxed{1.33}$ $\quad \boxed{y=ax^n}$

$y'=nax^{n-1} \longrightarrow y'=n\left(\frac{y}{x^n}\right)x^{n-1}=nyx^{-1}$

$y'_{ort}=\frac{-1}{y'}=-\frac{1}{nyx^{-1}}=-\frac{x}{ny}$

$\frac{dy}{dx}=-\frac{x}{ny} \qquad y\,dy=-\frac{x}{n}dx$

$\frac{1}{2}y^2=-\frac{x^2}{2n}+c \qquad \boxed{y^2+\frac{1}{n}x^2=c}$

1.34 $x^K + y^K = a^K$ trajectórias ortogonais

R: $K x^{K-1} + K y^{K-1} y' = 0$

$$y' = -\frac{K x^{K-1}}{K y^{K-1}} \qquad y' = -\frac{x^{K-1}}{y^{K-1}}$$

$$y'_{\text{ortogonal}} = \frac{y^{K-1}}{x^{K-1}}$$

$$\frac{dy}{dx} = \frac{y^{K-1}}{x^{K-1}} \qquad \int \frac{dy}{y^{K-1}} = \int \frac{dx}{x^{K-1}}$$

$$\frac{y^{2-K}}{2-K} = \frac{x^{2-K}}{2-K} + c \qquad K \neq 2$$

$K = 2$?

$\boxed{1.35}$ Det. eq. traj. ortogonais

$$y^2+x^2=ax^4$$

R:

$$y^2+x^2=ax^4 \qquad 2yy'+2x=4ax^3$$

$$2yy'+2x=4\left(\frac{y^2+x^2}{x}\right)$$

$$2yy'=\frac{4(y^2+x^2)-2x^2}{x}$$

$$\boxed{y'=\frac{2y^2+x^2}{xy}} \Leftrightarrow \quad yy'=\frac{2y^2+2x^2-x^2}{x}$$

$$y'_{ortog.}=-\frac{1}{y'}=-\frac{xy}{2y^2+x^2} \qquad y'_{ortog}=\frac{-xy}{2y^2+x^2}$$

$$(2y^2+x^2)\frac{dy}{dx}+(xy)=0$$

$$(xy)\,dx+(2y^2+x^2)\,dy=0 \qquad M_y=x \quad N_x=2x$$

$$(M_y-M_x)/M=(x-2x)/xy=-x/xy=-1/y$$

$$\mu(y)=\exp\left[-\int -1/y\,dy\right]=y$$

$$xy^2\,dx+(2y^3+x^2y)\,dy=0$$

$$\int_0^x ty^2\,dt+\int_0^y 2t^3+x_0^2t\,dt=c$$

$$y^2\frac{t}{2}\Big|_0^x+\frac{2t^4}{4}\Big|_0^y=c \qquad y^2\frac{x^2}{2}+\frac{y^4}{2}=c$$

$$\boxed{y^2x^2+y^4=c}$$

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$x^2y^2 - 4ax^2 = 0$ Det. ef. trajectórias ortogonais

R:

$$y^2 + 2xyy' - 8ax = 0$$

$$2xyy' = 8ax - y^2$$

$$y' = \frac{8ax - y^2}{2xy} = \frac{8\frac{y^2}{4x}x - y^2}{2xy}$$

$$4ax^2 = xy^2$$

$$a = \frac{xy^2}{4x^2} = \frac{y^2}{4x}$$

$$y' = \frac{2y^2 - y^2}{2xy} = \frac{y^2}{2xy} = \frac{y}{2x}$$

$$y'_{ortog.} = -\frac{1}{y'} = -\frac{2x}{y}$$

$$y\,y'_{ortog.} + 2x = 0$$

$$2x\,dx + y\,dy = 0$$

$$\int_0^x 2t\,dt + \int_0^y t\,dt = c$$

$$2\frac{t^2}{2}\Big|_{t=0}^{x} + \frac{t^2}{2}\Big|_0^y = c$$

$$2\frac{x^2}{2} + \frac{y^2}{2} = c$$

$$\boxed{x^2 + \frac{y^2}{2} = c}$$ ✓

$\boxed{1.39}$ $\boxed{y = axe^x}$

solução:

$$y = axe^x$$

$$y' = ae^x + axe^x = a(e^x + xe^x)$$

$$y' = a(e^x + xe^x)$$

$$a = \frac{y'}{(e^x + xe^x)}$$

$$y = \frac{y'}{(e^x + xe^x)} x e^x$$

$$y' = \frac{y(\cancel{e^x}(1+x))}{x\cancel{e^x}} = \frac{y(1+x)}{x}$$

$$y'_{ort.} = -\frac{x}{y(1+x)} \qquad y\,y'_{ort} = -\frac{x}{1+x}$$

$$y\,dy = -\left(\frac{1+x-1}{1+x}\right)dx = -\left(1 - \frac{1}{x+1}\right)dx$$

$$\frac{y^2}{2} = -x + \ln(x+1) + c$$

$$\boxed{y^2 = -2x + \ln(x+1)^2 + c}$$