$\boxed{1.1d}$ $$xy' = y(\ln y - \ln x) + y$$

Resol:

$$y' = \frac{y(\ln y - \ln x) + y}{x}$$

$$x \to \lambda x \qquad y \to \lambda y$$

$$y' = \frac{(\lambda y)(\ln(\lambda y) - \ln(\lambda x)) + (\lambda y)}{\lambda x}$$

$$= \frac{\cancel{\lambda} y(\cancel{\ln\lambda} + \ln y - \cancel{\ln\lambda} - \ln x) + \cancel{\lambda} y}{\cancel{\lambda} x}$$

$$y' = \frac{y(\ln y - \ln x) + y}{x} \quad \therefore \text{ Eq. é homogénea!}$$

Subst. $y = ux \qquad \frac{dy}{dx} = u + x\frac{du}{dx}$

$$u + x\frac{du}{dx} = \frac{ux\ln u + ux}{x}$$

$$u + x\frac{du}{dx} = u\ln u + u$$

$$\frac{du}{u\ln u} = \frac{dx}{x}$$

$\cancel{\frac{du}{u\ln u}}$ * $\qquad \ln|\ln u| = \ln|x| + C$

$$\ln|u| = Cx$$

$$u = e^{Cx}$$

$$\boxed{y = xe^{Cx}}$$ ✓

$\boxed{1.11}$ $\quad x\, e^{x^2/y^2}(x\,dy - y\,dx) = y^2\,dy$

Resol:

$x \to \lambda x \qquad y \to \lambda y$

$\lambda x\, e^{\lambda^2 x^2/\lambda^2 y^2}(\lambda x\,dy - \lambda y\,dx) = \lambda^2 y^2\,dy$

$\lambda^2 x\, e^{\lambda^2 x^2/\lambda^2 y^2}(x\,dy - y\,dx) = \lambda^2 y^2\,dy$ $\therefore$ Eq. homogénea!

$$\frac{dx}{dy} = \frac{x^2 e^{x^2/y^2} - y^2}{yx\, e^{x^2/y^2}} = \frac{\frac{x^2}{y^2}\, e^{x^2/y^2} - 1}{\frac{x}{y}\, e^{x^2/y^2}}$$

$x \to v y \qquad \frac{dx}{dy} = v + y\frac{dv}{dy}$

$$v + y\frac{dv}{dy} = \frac{v^2 e^{v^2} - 1}{v\, e^{v^2}}$$

$$y\frac{dv}{dy} = -\frac{1}{v e^{v^2}} \qquad v\, e^{v^2}\,dv = -\frac{1}{y}\,dy$$

$$\frac{1}{2} e^{v^2} = -\ln|y| + c$$

$$e^{v^2} + 2\ln|y| = c$$

$$e^{x^2/y^2} + 2\ln|y| = c$$

$\boxed{1.12}$

$$\boxed{y' = \frac{x^2+2y^2}{xy}}$$

Resolucar:

$$y' = \frac{(\lambda x)^2 + 2(\lambda y)^2}{(\lambda x)(\lambda y)} = \frac{\lambda^2 x^2 + 2\lambda^2 y^2}{\lambda^2 xy} = \frac{\lambda^2 (x^2+2y^2)}{\lambda^2 xy}$$

$$\frac{dy}{dx} = \frac{d}{dx}(y = ux) = \frac{du}{dx} x + u$$

$$\frac{du}{dx} x + u = \frac{x^2 + 2(ux)^2}{xux} = \frac{x^2}{x^2} \frac{(1+2u^2)}{u}$$

$$\frac{du}{dx} x = \frac{1+2u^2}{u} - u = \frac{1+2u^2-u^2}{u} = \frac{1+u^2}{u}$$

$$\frac{u}{1+u^2} du = \frac{dx}{x}$$

$$\frac{1}{2} \frac{2u}{1+u^2} du = \frac{dx}{x} \qquad \frac{1}{2} \ln|1+u^2| = \ln|x| + c$$

$$\ln|1+u^2| = \ln|cx^2|$$

$$1+u^2 = cx^2 \qquad 1 + \left(\frac{y}{x}\right)^2 = cx^2$$

$$x^2+y^2 = cx^4$$

$\boxed{1.13}$ $y^2 - 2xy + x^2y' = 0$

Resolução:

$$y^2 - 2xy + x^2y' = 0 \qquad \frac{dy}{dx} = \frac{2xy - y^2}{x^2}$$

$$= \frac{2(\lambda x)(\lambda y) - (\lambda y)^2}{(\lambda x)^2}$$

eq. homogénea

$$= \frac{2\cancel{\lambda^2}xy - \cancel{\lambda^2}y^2}{\cancel{\lambda^2}x^2}$$

$$y = ux \qquad \frac{dy}{dx} = \frac{du}{dx}x + u$$

$$\frac{du}{dx}x + u = \frac{2\cancel{x}(u\cancel{x}) - (u\cancel{x})^2}{\cancel{x^2}} = 2u - u^2$$

$$\frac{du}{dx}x = 2u - u^2 - u = u - u^2 = u(1-u)$$

$$\frac{du}{u(1-u)} = \frac{dx}{x} \qquad \int \frac{du}{u(1-u)} = \int \frac{dx}{x}$$

$$\int \frac{1-u+u}{u(1-u)}du = \ln|x| + C = \ln|cx|$$

$$\int \frac{1}{u}du + \int \frac{1}{(1-u)}du = \ln|cx|$$

$$\ln u - \ln(1-u) = \ln|cx|$$

$$\ln\left|\frac{u}{1-u}\right| = \ln|cx| \qquad \frac{u}{1-u} = cx$$

$$\frac{y/x}{1-y/x} = cx \longrightarrow \boxed{y = \frac{cx^2}{cx+1}}$$

$\boxed{1.14}$ $xy' = y - \sqrt{x^2+y^2}$

Resolução:

$$\frac{dy}{dx} = \frac{y - \sqrt{x^2+y^2}}{x}$$

homogénea? $\frac{dy}{dx} = \frac{(\lambda y) - \sqrt{\lambda^2 x^2 + \lambda^2 y^2}}{\lambda x} = \frac{y - \sqrt{x^2+y^2}}{x}$

$\therefore$ é homogénea!

$y = ux$ $\qquad \frac{dy}{dx} = \frac{du}{dx} x + u$

$$\frac{du}{dx} x + u = \frac{ux - \sqrt{x^2 + x^2 u^2}}{x} = \frac{u\cancel{x} - \cancel{x}\sqrt{1+u^2}}{\cancel{x}}$$

$\frac{du}{dx} x + \cancel{u} = \cancel{u} - \sqrt{1+u^2}$ $\qquad \frac{du}{dx} = -(\sqrt{1+u^2}) \frac{1}{x} dx$

$$\frac{1}{\sqrt{1+u^2}} du = -\frac{1}{x} dx$$

$\rightarrow$ $\int \frac{du}{\sqrt{a^2+u^2}} = \ln(u + \sqrt{a^2+u^2}) + c$

$$\ln(u + \sqrt{1+u^2}) = -\ln x + c = \ln\left(\frac{1}{xc}\right)$$

$$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = \frac{1}{xc}$$

$y + \sqrt{x^2+y^2} = c$ $\qquad \boxed{x^2 + 2cy = c^2}$

11.1

$$x(y+4x)y'+y(x+4y)=0$$

$$y'=-\frac{y(x+4y)}{x(y+4x)} \qquad \frac{\lambda y(\lambda x+4\lambda y)}{\lambda x(\lambda y+4\lambda x)}$$

a eq. é homogénea.

$$y=ux \qquad \frac{dy}{dx}=\frac{du}{dx}x+u$$

$$\frac{du}{dx}x+u=-\frac{u\cancel{x}(\cancel{x}+4u\cancel{x})}{\cancel{x}(u\cancel{x}+4\cancel{x})}=-\frac{u(1+4u)}{u+4}$$

$$\frac{du}{dx}x=-\frac{u(1+4u)}{u+4}-u=-\frac{u(1+4u)-u(u+4)}{u+4}$$

$$\frac{du}{dx}x=\frac{-u-4u^2-u^2-4u}{u+4}=\frac{-5u^2-5u}{u+4}$$

$$-\frac{u+4}{5u(u+1)}du=\frac{1}{x}dx$$

$$-\frac{1}{5}\left[\frac{(u+1)+3}{u(u+1)}\right]du=\frac{1}{x}dx$$

$$-\frac{1}{5}\left[\frac{1}{u}+\frac{3}{u(u+1)}\right]du=\frac{1}{x}dx$$

$$-\frac{1}{5}\left[\int\frac{1}{u}du+\underbrace{\int\frac{3}{u(u+1)}du}\right]=\int\frac{1}{x}dx$$

$$\int\frac{3}{u(u+1)}du=\int\left(\frac{A}{u}+\frac{B}{u+1}\right)du=\int\frac{3}{u}-\frac{3}{u+1}du=3\ln u-3\ln(u+1)$$

$$A(u+1)+Bu=3 \qquad Au+A+Bu=3$$

$$u(A+B)+A=3 \qquad \begin{cases}A+B=0\\A=3\end{cases} \qquad \begin{cases}B=-3\\A=3\end{cases}$$

$$-\frac{1}{5}\left[\ln v + 3\ln v - 3\ln(v+1)\right] = \ln x$$

$$-\frac{1}{5}\left[4\ln v - 3\ln(v+1)\right] = \ln x$$

$$-\left[4\ln v - 3\ln(v+1)\right] = 5\ln x$$

$$3\ln(v+1) - 4\ln v = \ln\frac{(v+1)^3}{v^4} = \ln x^5 + \ln c$$

$$\frac{(v+1)^3}{v^4} = cx^5 \qquad v = \frac{y}{x} \qquad \frac{\left(\frac{y}{x}+1\right)^3}{\left(\frac{y}{x}\right)^4} = cx^5$$

$$\frac{\frac{(y+x)^3}{\cancel{x^3}}}{\frac{y^4}{x^4}} = cx^5 \qquad (y+x)^3 = cx^5y^4\frac{1}{x}$$

$$(y+x)^3 = cx^4y^4 \qquad \blacksquare$$

1.16

$$(y^2 - x^2 e^{x/y})\,dy + (y^2 + xy)e^{x/y}\,dx = 0$$

$$\frac{dx}{dy} = \frac{-(y^2 - x^2 e^{x/y})}{(y^2 + xy)e^{x/y}}$$

$x = vy$

$$\frac{dx}{dy} = \frac{-(y^2 - v^2 y^2 e^v)}{(y^2 + vy^2)e^v} = \frac{-(1 - v^2 e^v)}{(1+v)e^v}$$

$$\frac{dx}{dy} = \frac{dv}{dy}y + v = \frac{-(1 - v^2 e^v)}{(1+v)e^v}$$

$$\frac{dv}{dy}y = \frac{-(1 - v^2 e^v)}{(1+v)e^v} - v = \frac{-(1 - v^2 e^v) - v(1+v)e^v}{(1+v)e^v}$$

$$\frac{dv}{dy}y = \frac{-1 + \cancel{v^2 e^v} - ve^v - \cancel{v^2 e^v}}{(1+v)e^v} = \frac{-1 - ve^v}{(1+v)e^v}$$

$$\frac{(1+v)e^v}{-1 - ve^v}\,dv = \frac{dy}{y}$$

$$-\frac{f'(B)}{f(B)}\,dv = \frac{dy}{y}$$

$$-\ln(1 + ve^v) = \ln y + \ln c = \cancel{\ln cy}$$

$$-(1 + ve^v) = cy$$

$$-\left(1 + \frac{x}{y}e^{x/y}\right) = cy$$

1.17

$x^{1/2}\,dx + (xy)^{1/4}\,dy = 0$, $y(0)=1$

$(xy)^{1/4}\,dy = -x^{1/2}\,dx$

$y^{1/4}\,dy = -x^{1/4}\,dx$

$\frac{y^{1/4+1}}{\cancel{1/4+1}} = -\frac{x^{1/4+1}}{\cancel{1/4+1}} + c$ $\quad y^{\frac{5}{4}} = -x^{\frac{5}{4}} + c$ $\quad \left.\begin{matrix} x=0 \\ y=1 \end{matrix}\right| \to c=1$

$\boxed{y = \left(1 - x^{5/4}\right)^{\frac{4}{5}}}$

??

$\boxed{1.19}$ $xy' = y - \sqrt{x^2+y^2}$

Resolução:

$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases}$ $x^2+y^2=r^2$

$xy' = y - \sqrt{x^2+y^2}$

$(r\cos\theta)\dfrac{dy}{dx} = r\,\text{sen}\,\theta - \sqrt{r^2} = r(\text{sen}\,\theta - 1)$

$\dfrac{dy}{dx} = \dfrac{dy}{d\theta}\dfrac{d\theta}{dx} = r\cos\theta\,\dfrac{d\theta}{dx}$

$(r\cos\theta)(r\cos\theta)\dfrac{d\theta}{dx} = r(\text{sen}\,\theta - 1)$

$\dfrac{r\cos^2\theta}{(\text{sen}\,\theta-1)}\,d\theta = dx$ $\dfrac{\cos\theta}{(\text{sen}\,\theta-1)}\,d\theta = \dfrac{dx}{x}$

$\displaystyle\int \frac{\cos\theta}{(\text{sen}\,\theta-1)}\,d\theta = \int \frac{dx}{x}$

$\ln|\text{sen}\,\theta - 1| = \ln|x| + c = \ln|cx|$

$\text{sen}\,\theta - 1 = cx$

$= cr\cos\theta$

$cr = \dfrac{\text{sen}\,\theta - 1}{\cos\theta}$

no livro $cr = \dfrac{1}{1+\text{sen}\,\theta}$

??

$\boxed{1.18}$ $$x y' = y - \sqrt{x^2+y^2}$$

Resolução:

$$\left| \begin{array}{l} x = r\cos\theta \\ y = r \operatorname{sen}\theta \end{array} \right. \qquad x^2+y^2=r^2$$

$$x y' = y - \sqrt{x^2+y^2}$$

$$(r\cos\theta)\frac{dy}{dx} = r\operatorname{sen}\theta - \sqrt{r^2} = r(\operatorname{sen}\theta - 1)$$

$$\frac{dy}{dx} = \frac{dy}{d\theta}\frac{d\theta}{dx} = r\cos\theta\frac{d\theta}{dx}$$

$$(r\cos\theta)(r\cos\theta)\frac{d\theta}{dx} = r(\operatorname{sen}\theta - 1)$$

$$\frac{r\cos^2\theta}{(\operatorname{sen}\theta-1)}d\theta = dx \qquad \frac{\cos\theta}{(\operatorname{sen}\theta-1)}d\theta = \frac{dx}{x}$$

$$\int\frac{\cos\theta}{(\operatorname{sen}\theta-1)}d\theta = \int\frac{dx}{x}$$

$$\ln|\operatorname{sen}\theta - 1| = \ln|x| + c = \ln|cx|$$

$$\operatorname{sen}\theta - 1 = cx$$

$$= c\,r\cos\theta$$

$$cr = \frac{\operatorname{sen}\theta - 1}{\cos\theta}$$

no livro $cr = \dfrac{1}{1+\operatorname{sen}\theta}$

$\boxed{1.20}$

$$\boxed{\frac{dy}{dx} = \frac{y(y^2+x^2)+2x^2y}{2x^3}} \qquad y = ux \qquad \frac{dy}{dx} = \frac{du}{dx}x + u$$

$$= \frac{ux(u^2x^2+x^2)+2x^2ux}{2x^3} = \frac{du}{dx}x+u$$

$$= \frac{u^3\cancel{x^3}+u\cancel{x^3}+2\cancel{x^3}u}{2\cancel{x^3}} = \frac{u^3+u+2u}{2} = \frac{u^3+3u}{2}$$

$$= \frac{u^3+3u-2u}{2} = \frac{du}{dx}x \qquad \frac{u^3+u}{2} = \frac{du}{dx}x$$

$$\frac{2}{u^3+u}du = \frac{dx}{x} \qquad \frac{2}{u(u^2+1)}du = \frac{dx}{x}$$

$$\frac{A}{u} + \frac{Bu+c}{u^2+1} = \frac{A(u^2+1)+(Bu+c)u}{\dots} = \frac{Au^2+A+Bu^2+cu}{\dots}$$

$$= \frac{u^2(A+B)+cu+A}{\dots} = \left|\begin{array}{r} A+B=0 \\ c=0 \\ A=2 \end{array}\right| \begin{array}{l} A=2 \\ C=0 \\ B=-2 \end{array}$$

$$\int\left(\frac{2}{u}+\frac{-2u}{u^2+1}\right)du = \int\frac{dx}{x}$$

$$2\ln u + (-\ln(u^2+1)) = \ln x + c$$

$$\ln u^2 - \ln(u^2+1) = \ln cx$$

$$\ln\frac{u^2}{u^2+1} = \ln cx \qquad cx = \frac{u^2}{u^2+1} = \frac{\frac{y^2}{x^2}}{\frac{y^2}{x^2}+1}$$

$$cx = \frac{1}{1+\frac{y^2}{x^2}} = \frac{x^2}{x^2+y^2}$$

$$c = \frac{x}{x^2+y^2} \qquad c(x^2+y^2) = x \qquad x^2+y^2 = \frac{\cancel{x}}{c}$$

$$y^2 = \frac{x}{c} - x^2 = \cancel{x}\left(\frac{1}{c}-x\right) \qquad \boxed{y^2 = \frac{x}{c} - x^2}$$