$\boxed{1.89}$ $xy' - y + y^2 = x^2$

Resolução:

$$\underbrace{y' - \frac{1}{x}}_{P(x)} y + \underbrace{\frac{1}{x}}_{Q(x)} y^2 = \underbrace{x}_{R(x)}$$

Eq. de Ricatti

sol. partic. $y = x$

$$y = x + \frac{1}{z} \qquad y' = 1 + \left(-\frac{1}{z^2}\right) z'$$

$$\left(1 - \frac{1}{z^2} z'\right) - \frac{1}{x}\left(x + \frac{1}{z}\right) + \frac{1}{x}\left(x + \frac{1}{z}\right)^2 = x$$

$$\cancel{1} - \frac{1}{z^2} z' - \cancel{1} - \frac{1}{xz} + \frac{1}{x}\left(x^2 + 2x\frac{1}{z} + \frac{1}{z^2}\right) = x$$

$$-\frac{1}{z^2} z' - \frac{1}{xz} + \cancel{x} + \frac{2}{z} + \frac{1}{xz^2} = \cancel{x}$$

$$-\frac{x}{z^2} z' - \frac{1}{z} + \frac{2x}{z} + \frac{1}{z^2} = 0$$

$$z' - \frac{2x-1}{x} z = \frac{1}{x}$$

$$z' - \frac{2x-1}{x} z = 0 \qquad \frac{z'}{z} = \frac{x}{2x-1} \qquad z_h = c\,\frac{e^{2x}}{x}$$

método de variação de constante

$$z = c(x)\,\frac{e^{2x}}{x} \qquad z' = c'\,\frac{e^{2x}}{x} + c\left(-\frac{e^{2x}}{x^2} + \frac{2e^{2x}}{x}\right)$$

$$c'\,\frac{e^{2x}}{x} + c\left(-\frac{e^{2x}}{x^2} + \frac{2e^{2x}}{x}\right) - \frac{2x-1}{x}\,c\,\frac{e^{2x}}{x} = 0$$

$$d(x) = e^{-2x}$$

$$c(x) = -\frac{1}{2} e^{-2x} + \frac{\bar{c}}{2}$$

$$z(x) = -\frac{1}{2x} + \frac{\bar{c} e^{2x}}{2x}$$

$$y = x + \frac{2x}{\bar{c} e^{2x} - 1} \; ; \quad y = x$$

$\boxed{1.90}$ $\qquad y' + y + y^2 = 2$

R:

de forma

$y' + P(x)y + Q(x)y^2 = R(x)$ $\qquad$ eq. de Riccati

Substituição

$y = v_p(x) + \frac{1}{z}$ $\qquad$ neste caso c/ $\boxed{v_p(x) = c}$

$y = c + \frac{1}{z}$ $\qquad y' = 0 - \frac{1}{z^2} z'$ $\qquad c = 1 \vee -2$ c é solução

$-\frac{1}{z^2} z' + c + \frac{1}{z} + \left(c + \frac{1}{z}\right)^2 = 2$ $\qquad \frac{-1 \pm \sqrt{1 - 4(-2)}}{2} = \frac{-1 \pm 3}{2}$

$-\frac{1}{z^2} z' + c + \frac{1}{z} + c^2 + 2\frac{c}{z} + \frac{1}{z^2} = 2$

$-\frac{1}{z^2} z' + \frac{1}{z}(1 + 2c) + \frac{1}{z^2} = (2 - c - c^2)$

$\boxed{c = 1}$ $\quad -\frac{1}{z^2} z' + \frac{1}{z}(3) + \frac{1}{z^2} = 0$ $\qquad -z' + 3z + 1 = 0$

$z' - 3z - 1 = 0$

$z = e^{-\int -3\,dx} \left[\int e^{\int -3\,dx} \cdot (+1)\,dx + c\right]$

$z = e^{3x}\left[+\int e^{-3x} dx + c\right] = e^{3x}\left[+\frac{e^{-3x}}{-3} + c\right] = -\frac{1}{3} + c e^{3x}$

$y = 1 + \frac{3}{-1 + 3ce^{3x}} = \frac{ce^{3x} + 2}{ce^{3x} - 1}$; $y = 1$; $y = -2$

$\boxed{1.91}$ $\quad y' + xy + xy^2 = 6x$

R:

A ec. e de forma $y' + P(x)y + Q(x)y^2 = R(x)$

ec. de Riccatti

$u_p(x) = \text{const.} \qquad u' = 0 \qquad 0 + xu + xu^2 = 6x$

$$u + u^2 = 6$$

$u^2 + u - 6 = 0 \qquad u_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4(6)}}{2} = \frac{-1 \pm \sqrt{25}}{2} = \frac{-6}{2}; \frac{4}{2}$

$u_{1,2} = -3; 2$

Subst. $y = -3 + \frac{1}{z} \qquad y' = -\frac{1}{z^2} z'$

$$\left(-\frac{1}{z^2} z'\right) + x\left(-3 + \frac{1}{z}\right) + x\left(-3 + \frac{1}{z}\right)^2 = 6x$$

$$-\frac{1}{z^2} z' \cancel{- 3x} + x\frac{1}{z} + \cancel{9x} - 6x \cdot \frac{1}{z} + x\frac{1}{z^2} = \cancel{6x} \to 0$$

$$-z' + zx - 6xz + x = 0$$

$$-z' + z(\overset{-5x}{x - 6x}) + x = 0 \qquad z' - z(x - 6x) - x = 0$$

$\boxed{z' + (5x)z = x} \qquad z = \exp\left(-\int 5x\,dx\right)\left[\int e^{\int 5x\,dx} \cdot x + c\right]$

$$z = e^{-5x^2/2}\left[\frac{1}{5} e^{5x^2/2} + c\right] = \frac{1}{5} + c e^{-5x^2/2}$$

$$y = -3 + \frac{5}{1 + ce^{-5x^2/2}} = \frac{-3 - 3ce^{-5x^2/2} + 5}{1 + ce^{-5x^2/2}}$$

$$y = \frac{2 - 3c\exp(-5x^2/2)}{1 + c\exp(-5x^2/2)}$$

1.94 $\quad y' = e^{-x}y^2 + y - e^x \; ; \; \boxed{u = e^x}$

$$y' - y - e^{-x}y^2 = -e^x$$ Ecuac. de Riccatti

$P(x) = -1 \quad Q(x) = -e^{-x} \quad R(x) = -e^x$

$$y(x) = u(x) + \frac{1}{z} \qquad y'(x) = u'(x) - \frac{1}{z^2}z'$$

$$y'(x) = e^x - \frac{1}{z^2}z'$$

$$\left(e^x - \frac{1}{z^2}z'\right) - \left(e^x + \frac{1}{z}\right) - e^{-x}\left(e^x + \frac{1}{z}\right)^2 = -e^x$$

$$e^x - \frac{1}{z^2}z' - e^x - \frac{1}{z} - e^{-x}\left(e^{2x} + 2\frac{e^x}{z} + \frac{1}{z^2}\right) = -e^x$$

$$\cancel{e^x} - \frac{1}{z^2}z' - \cancel{e^x} - \frac{1}{z} - \cancel{e^x} - \frac{2}{z} - \frac{e^{-x}}{z^2} = \cancel{-e^x}\ 0$$

$$-\frac{1}{z^2}z' - \frac{1}{z} - \frac{2}{z} - \frac{e^{-x}}{z^2} = \cancel{2e^x}\ 0$$

$$-\frac{1}{z^2}z' - \frac{3}{z} - \frac{e^{-x}}{z^2} = \cancel{2e^x}\ 0$$

$$z' + 3z + e^{-x} = \cancel{2e^x z^2}\ 0$$

$$z' + 3z = -e^{-x}$$

$$z = e^{-\int 3\,dx}\left[\int -e^{-x} e^{\int 3\,dx}\,dx + c\right]$$

$$z = e^{-3x}\left[-\int e^{-x}e^{3x}\,dx + c\right] =$$

$$z = e^{-3x}\left[-\frac{e^{2x}}{2} + c\right] = -\frac{e^{-x}}{2} + c\,e^{-3x}$$

$y = e^x + 1/z \longrightarrow \boxed{y = e^x + \frac{2}{ce^{-3x} - e^x}}$

$\boxed{1\cdot 98}$ $2y'^2 - 2xy' - 2y + x^2 = 0$

$$y = y'^2 - xy' + \frac{x^2}{2}$$

$$y' = p$$

$$y = p^2 - xp + \frac{x^2}{2}$$

$$y' = p = 2p\frac{dp}{dx} - p - x\frac{dp}{dx} + x$$

$$(2p - x)\frac{dp}{dx} = 2p - x$$

1º caso

$$2p - x \neq 0 \Rightarrow \frac{dp}{dx} = 1 \Rightarrow p = x + c$$

$$\Rightarrow y = (x+c)^2 - x(x+c) + \frac{x^2}{2}$$

ou

$$y = cx + \frac{x^2}{2} + c^2$$

2º caso

$$2p - x = 0 \Rightarrow p = \frac{x}{2} \Rightarrow y \cancel{= \left(\frac{x}{2}\right)^2}$$

$$\Rightarrow y = \left(\frac{x}{2}\right)^2 - x\frac{x}{2} + \frac{x^2}{2}$$

$$y = \frac{x^2}{4}$$