1.52 $$\boxed{(x^4 \ln x - 2xy^3)\,dx + 3x^2y^2\,dy = 0}$$

Resoluçaõ:

$M = x^4 \ln x - 2xy^3 \qquad N = 3x^2y^2$

$\frac{\partial M}{\partial y} = -6xy^2 \qquad \frac{\partial N}{\partial x} = 6xy^2$

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -6xy^2 - 6xy^2 = -12xy^2$

Dividindo por N $\Rightarrow f(x) = -\frac{4}{x}$

$\mu(x) = e^{\int f(x)\,dx} = e^{\int -\frac{4}{x}\,dx}$

$\mu(x) = \exp[\ln x^{-4}]$

$\mu(x) = x^{-4}$

mult. ambos membros

$(\ln x - 2x^{-3}y^3)\,dx + 3x^{-2}y^2\,dy = 0$

integrando

$$\boxed{x\ln x - x + x^{-2}y^3 = c}$$

$\boxed{1.53}$ $\cos x \, dx + (y + \operatorname{sen} x)\, dy = 0$

Resolução:

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{\cos x}(-\cos x) = -1$$

$$\mu(y) = e^{\int dy} = e^{y}$$

multip. eq.

$$e^y \cos x \, dx + (e^y y + e^y \operatorname{sen} x)\, dy = 0$$

integrando

$$e^y \operatorname{sen} x + y e^y - e^y = c$$

$\boxed{1.54}$ $\quad y\,dx - (2x+y)\,dy = 0$

Resolucao:

$$\frac{\partial M}{\partial y} = 1 \quad \frac{\partial N}{\partial x} = -2 \quad \therefore \text{ ñ exacta!}$$

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{y}\left(1-(-2)\right) = \frac{3}{y}$$

$$\mu(y) = \exp\left[-\int \frac{3}{y}\,dy\right] = \exp\left[-3\ln y\right]$$

$$\mu(y) = \exp\left[\ln y^{-3}\right] = y^{-3}$$

$$y^{-3} \cdot y\,dx - y^{-3}(2x+y)\,dy = 0$$

$$y^{-2}\,dx - (2xy^{-3} + y^{-2})\,dy = 0$$

$$U(x,y) = \int_{x_0}^{x} M(t,y)\,dt + \int_{y_0}^{y} N(t_0,t)\,dt = c$$

$$U(x,y) = \int_{x_0}^{x} y^{-2}\,dt - \int_{y_0}^{y} (2x_0t^{-3} + t^{-2})\,dt = c$$

$$U(x,y) = y^{-2}x - y^{-2}x_0 - \left[\frac{2x_0t^{-3+1}}{-3+1}\right]_{y_0}^{y} - \left[\frac{t^{-2+1}}{-2+1}\right]_{y_0}^{y} = c$$

$x_0 = 0 \;;\; y_0 = 0$

$$U(x,y) = y^{-2}x - \frac{y^{-1}}{-1} = c \qquad U(x,y) = y^{-2}x + y^{-1} = c$$

$$\Rightarrow \boxed{x + y = cy^2}$$

$\boxed{1.56}$ $\boxed{y' - 3y = e^{2x}}$ $y = 0 \quad x = 0$

Resolução:

$$y = e^{\int -P\,dx}\left[\int Q\, e^{\int P\,dx}\,dx + c\right]$$

$$e^{\int -P\,dx} = e^{\int +3\,dx} = e^{+3x}$$

$$y = e^{+3x}\left[\int e^{2x}\, e^{-3x}\,dx + c\right] = e^{+3x}\left[\int e^{-x}\,dx + c\right] =$$

$$y = e^{+3x}\cdot\left[-\frac{1}{1}\, e^{-x} + c\right]$$

$$y = -e^{2x} + c\,e^{3x} \qquad x = 0 \quad y = 0 \Rightarrow \boxed{c = 1}$$

$$\boxed{y = e^{3x} - e^{2x}}$$ ✓

$\boxed{1.57}$ $$y' + xy = x^3$$

Resolução:

$$y' + xy = 0$$

$$\frac{dy}{dx} + xy = 0 \qquad dy + xy\,dx = 0 \qquad \frac{1}{y}dy + x\,dx = 0$$

$$\ln y = -\frac{x^2}{2} + c \Rightarrow y = c\,e^{-x^2/2}$$

$$y' = c'e^{-x^2/2} + c\,e^{-x^2/2}\left(-\frac{2x}{2}\right)$$

$$y' = e^{-x^2/2}(c' - xc)$$

$$e^{-x^2/2}(c' - \cancel{xc}) + \cancel{xc}\,e^{-x^2/2} = x^3$$

$$c' = x^3 e^{x^2/2}$$

$$c = \int x^3 e^{x^2/2} dx = \int 2u\,e^u du = 2\left[e^u(u-1)\right] + c^*$$

$$u = \frac{x^2}{2} \qquad x^2 = 2u \qquad 2x\,dx = 2\,du \qquad du = x\,dx$$

$$c = 2e^{x^2/2}\left(\frac{x^2}{2} - 1\right) + c^* \qquad y = \left(2e^{x^2/2}\left(\frac{x^2}{2}-1\right) + c^*\right)e^{-x^2/2}$$

$$y = 2\left(\frac{x^2}{2} - 1\right) + c^*\left(e^{-x^2/2}\right)$$

$$\boxed{y = c^* e^{-x^2/2} + x^2 - 1}$$

$\boxed{1.58}$ $(y/x)\,dx + (y^3 - \ln x)\,dy = 0$

Resolução:

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(\frac{y}{x}\right) = \frac{1}{x} \qquad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(y^3 - \ln x) = -\frac{1}{x}$$

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{x}{y}\left(\frac{1}{x} - \left(-\frac{1}{x}\right)\right) = \frac{2}{y}$$

$$\mu(y) = \exp\left[\int -\frac{2}{y}dy\right] = \exp\left[-2\ln y\right] = y^{-2}$$

$$y^{-2}\left(\frac{y}{x}\right)dx + y^{-2}(y^3 - \ln x)\,dy = 0$$

$$\frac{y^{-1}}{x}\,dx + (y - y^{-2}\ln x)\,dy = 0$$

$$\int_{x_0}^{x} \frac{y^{-1}}{x}\,dx + \int_{y_0}^{y}(y - y^{-2}\ln x_0)\,dy = c$$

$$\int_{1}^{x} \frac{y^{-1}}{x}\,dx + \int_{0}^{y}(y - \cancelto{0}{y^{-2}\ln 1})\,dy = c$$

$$y^{-1}\ln|x|\Big|_1^x + \frac{y^2}{2}\Big|_0^y = c \qquad y^{-1}\ln|x| + \frac{y^2}{2} = c$$

$$\boxed{\frac{1}{y}\ln|x| + \frac{y^2}{2} = c}$$

✓

Ln

$\boxed{1.59}$ $(e^{x+y} + 2\sqrt{x})\,dx + 2x\,e^{x+y}\,dy = 0$

R:

$M_y = e^{x+y} \qquad N_x = 2e^{x+y} + 2x\,e^{x+y}$

$(M_y - N_x)/N = \dfrac{e^{x+y} - 2e^{x+y} - 2x\,e^{x+y}}{2x\,e^{x+y}} = \dfrac{-\cancel{e^{x+y}}(1+2x)}{2x\,\cancel{e^{x+y}}}$

$\mu(x) = \exp\left[-\int\left(1+\frac{1}{2x}\right)dx\right] = \exp\left[-(x + \ln x^{1/2})\right]$

$\mu(x) = e^{-x}\,e^{-\ln(\sqrt{x})} = e^{-x}(\sqrt{x})^{-1} = \dfrac{1}{e^x\sqrt{x}}$

$\dfrac{1}{e^x\sqrt{x}}(e^{x+y} + 2\sqrt{x})\,dx + \dfrac{1}{e^x\sqrt{x}}(2x\,e^{x+y})\,dy = 0$

$\int_1^x \dfrac{1}{e^t\sqrt{t}}(e^{t+y} + 2\sqrt{t})\,dt + \int_1^y \dfrac{1}{e^x\sqrt{x}}(2xe^{(x+t)}\,dt = c$

$e^y\int_1^x t^{-1/2}\,dt + 2\int_1^x e^{-t}\,dt + \int_1^y \dfrac{1}{e^1\sqrt{1}}(2(1)\,e^{(1+t)}\,dt = c$

$e^y\left[2t^{1/2}\Big|_1^x + 2\left[-e^{-t}\Big|_1^x + 2\left[e^t\Big|_1^y = c\right.\right.\right.$

$e^y(2\sqrt{x} - \cancel{2(1)}) + 2(-e^{-x} + e^{-1}) + \cancel{2e^y} - 2e = c$

$e^y\,2\sqrt{x} + 2(-e^{-x}) = c_1 \qquad e^y\,2\sqrt{x} = c_1 + 2e^{-x}$

$e^y = \dfrac{c_1 + 2e^{-x}}{2\sqrt{x}} \qquad \boxed{y = \ln\left(\dfrac{c_1 + 2e^{-x}}{2\sqrt{x}}\right)}$ ✓