$\boxed{1.43}$ $$\left(x + e^{x/y}\right) dx + e^{x/y}\left(1 - \frac{x}{y}\right) dy = 0$$

Resolução:

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(x + e^{x/y}\right) = e^{x/y}\left(-\frac{1}{y^2}\right)$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left[e^{x/y}\left(1 - \frac{x}{y}\right)\right] = e^{x/y}\left(1 - \frac{x}{y}\right) + e^{x/y}\left(-\frac{1}{y}\right)$$

$$= e^{x/y}\left(1 - \frac{x}{y} - \frac{1}{y}\right) \qquad \frac{\partial M(x,y)}{\partial y} \neq \frac{\partial N}{\partial x}(x,y)$$

$$U(x,y) = \int_{x_0}^{x} M(t,y)\,dt + \int_{y_0}^{y} N(x_0,t)\,dt \qquad \left|\begin{array}{l} x_0 = 0 \\ y_0 = 2 \end{array}\right.$$

$$U(x,y) = \int_0^x \left(t + e^{t/y}\right) dt + \int_2^y \left(1 - \frac{0}{t}\right) dt =$$

$$= \frac{t^2}{2} + \frac{e^{t/y}}{1/y}\Bigg|_0^x + t\Bigg|_2^y =$$

$$= \frac{x^2}{2} + y e^{x/y} - \cancel{\frac{0^2}{2}} - \cancel{y e^{0/y}} + \cancel{y} - 2 =$$

$$U(x,y) = \frac{x^2}{2} + y e^{x/y} - 2 = C$$

$$\boxed{\frac{x^2}{2} + y e^{x/y} = -2}$$

$\boxed{1.44}$ $(3x^2+6xy^2)dx+(6x^2y+4y^3)dy=0$

Resolução:

$\frac{\partial M}{\partial y} = 12xy$ $\frac{\partial N}{\partial x} = 12xy$

$\therefore$ Eq. homogénea

$\frac{\partial U}{\partial x} = 3x^2+6xy^2$

$U(x,y) = \int 3x^2+6xy^2\,dx = x^3+2xy^3+c(y)$

$\frac{\partial U(x,y)}{\partial y} = 6xy^2+c'(y) = N = 6x^2y+4y^3$

$c'(y) = 4y^3$ $c(y) = y^4$

$U(x,y) = x^3+2xy^3+y^4 = c$

$$\boxed{x^3+2xy^3+y^4=c}$$

$\boxed{1.45}$ $(x+y)dx + (x+2y)dy = 0$

exacte!

Sub. $y = ux \quad dy = du\,x + u\,dx$

$(x+ux)dx + (x+2ux)(du\,x + u\,dx) = 0$

$x(1+u)dx + x(1+2u)(\qquad) = 0$

$(1+u+u+2u^2)dx + (x+2ux)du = 0$

$\frac{1}{x}dx + \frac{1+2u}{1+2u+2u^2}du = 0$

$\ln x + \frac{1}{2}\ln(2u(u+1)+1) = C$

$\ln\left[x^2(2u(u+1)+1)\right] = c$

$x^2\left[2\frac{y}{x}\left(\frac{y}{x}+1\right)+1\right] = c$

$x^2\left[2\frac{y^2}{x^2} + 2\frac{y}{x} + 1\right] = c$

$$\boxed{2y^2 + 2xy + x^2 = c}$$

Prob. 46

$$(x^3 - 3xy^2 + 2)\,dx + (-3x^2y + y^2)\,dy = 0$$

Resol.:

▷ $\frac{\partial M}{\partial y} = -6xy \quad \frac{\partial N}{\partial x} = -6xy \Rightarrow$ exacte!

▷ $\int x^3 dx - \int 3xy^2 dx + \int 2 dx - \int 3x^2y\,dy + \int y^2 dy = 0$

$$\frac{x^4}{4} - \frac{3x^2}{2}y^2 + 2x - \frac{3}{2}x^2y^2 + \frac{y^3}{3} = C$$

L.M

Prob. 1.47

$$x\,dx + y\,dy = \frac{(x\,dy - y\,dx)}{(x^2+y^2)}$$

Resol.:

$$\underbrace{\left(x + \frac{y}{(x^2+y^2)}\right)}_{M}dx + \underbrace{\left(y - \frac{x}{(x^2+y^2)}\right)}_{N}dy = 0$$

▷ exack?

$$\frac{\partial M}{\partial y}\left(x + \frac{y}{x^2+y^2}\right) = \frac{y2y - 1x(x^2+y^2)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2} \checkmark$$

$$\frac{\partial N}{\partial x}\left(y - \frac{x}{x^2+y^2}\right) = \frac{-x2x - (\cancel{*}1)(x^2+y^2)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2-y^2)^2} \checkmark$$

• exack!

$$U(x,y) = \int_{x_0}^{x} M(t,y)\,dt + \int_{y_0}^{y} N(x_0,t)\,dt = C$$

$$= \int_0^x \left(t + \frac{y}{t^2+y^2}\right)dt + \int_0^y t - \frac{0}{0^2+y^2}\Big)\,dt = C$$

$$= \left[\frac{t^2}{2} + \operatorname{arctg}\left(\frac{t}{y}\right)\right]_0^x + \left[\frac{t^2}{2}\right]_0^y = C$$

$$= \frac{x^2}{2} + \operatorname{arctg}\left(\frac{x}{y}\right) + \frac{y^2}{2} = C$$

$$\boxed{x^2 + 2\operatorname{arctg}\left(\frac{x}{y}\right) + y^2 = C} \checkmark$$

$\boxed{1.48}$ $(2x/y^3)\,dx + (1/y - 3x^2/y^4)\,dy = 0$

R:

exacta?

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2xy^{-3}) = -6xy^{-4}$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(1/y - 3x^2/y^4) = -6xy^{-4}$$

logo é exacta!

$$U(x,y) = \int_{x_0}^{x} M(t,y)\,dt + \int_{y_0}^{y} N(x_0,t)\,dt = c$$

$$U(x,y) = \int_0^x 2t/y^3\,dt + \int_0^y \frac{1}{t}\,dt = c$$

$$= \int_0^x \frac{2}{y^3}t\,dt + \int_0^y \frac{1}{t}\,dt = c$$

$$= \frac{2t^2}{2y^3}\Big|_0^x + \ln t\Big|_0^y = c$$

$$\boxed{\frac{x^2}{y^3} + \ln y = c}$$

✓

$\boxed{1.48}$ $(x^2-y)\,dx-(x+\operatorname{sen}^2 y)\,dy=0$

R:

exacta ?

$M_y=-1$

$N_x=-1$ $\Big\rangle$ logo é exacta !

$$U(x,y)=\int_{x_0}^{x} M(t,y)\,dt+\int_{y_0}^{y} N(x_0,t)\,dt$$

$$U(x,y)=\int_0^x (t^2-y)\,dt+\int_0^y \operatorname{sen}^2 t\,dt$$

$$U(x,y)=\left.\frac{t^3}{3}-yt\right|_0^x+\frac{1}{2}(y-\sin t\cos t)$$

$$\frac{x^3}{3}-y\cancel{x}+\frac{1}{2}(y-\sin y\cos y)=c$$

$\boxed{1.53}$ $(4 \operatorname{sen} x \operatorname{sen} 3y \cos x)dx = (3 \cos 3y \cos 2x)\, dy$

<u>Resolução:</u>

$$\frac{\partial M}{\partial y} = 4 \operatorname{sen} x \cos x \cos 3y \, . \, 3$$

$$\frac{\partial N}{\partial x} = -3 \cos 3y \, (-\operatorname{sen} 2x) \, . \, 2$$

$=$ $\therefore$ Eq. é <u>exacta</u>!

$$u(x,y) = \int_{x_0}^{x} M(t,y)\,dt + \int_{y_0}^{y} N(x_0,t)\,dt$$

$$u(x,y) = \int_0^x \operatorname{sen} 3y \, . \, 2 \operatorname{sen} 2t \, dt - \int_0^y 3 \cos 3t \, . \, \cancel{\cos(2.0)}^{=1} \, dt$$

$x_0 = 0, y_0 = 0$ $\qquad$ $u = 3t \quad du = 3dt$

$$u(x,y) = \int_0^x \operatorname{sen} 3y \, . \operatorname{sen} u \, . du - \int_0^y \cos u \, du$$

$$u(x,y) = \operatorname{sen} 3y \left[-\cos u\right]_0^x - \left[\operatorname{sen} u\right]_0^y$$

$$= \operatorname{sen} 3y \left[-\cos 2x - (-\underset{1}{\cancel{\cos 0}})\right] - \left[\operatorname{sen} 3y - \overset{0}{\cancel{\operatorname{sen} 0}}\right]$$

$$= \operatorname{sen} 3y \left[-\cos 2x \cancel{+1}\right] - \cancel{\operatorname{sen} 3y}$$

$$u(x,y) = -\operatorname{sen} 3y \cos 2x = c$$

$$\boxed{u(x,y) = \operatorname{sen} 3y \cos 2x = C}$$

$\boxed{1.51}$ $$(3x^2y + 8xy^2)\,dx + (x^3 + 8x^2y + 12ye^y)\,dy = 0$$

Resolução:

$$\frac{\partial M}{\partial y} = 3x^2 + 16x \qquad \frac{\partial N}{\partial x} = 3x^2 + 16x$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ∴ Eq. é dif. exacta

$$U(x,y) = \int_{x_0}^{x} (3t^2y + 8ty^2)\,dt + \int_{y_0}^{y} (x_0^3 + 8x_0^2t + 12ye^y)\,dt$$

$x_0 = 0$; $y_0 = 0$

$$U(x,y) = \frac{3t^3}{3}y + \frac{8t^2}{2}y^2 \Big|_0^x + \int_0^y 12te^t\,dt$$

$$\int \underset{g}{t}\, \underset{f'}{e^t}\,dt = e^t t - \int 1 \cdot e^t\,dt = e^t t - e^t = e^t(t-1)$$

$$U(x,y) = x^3y + 4x^2y^2 + 12e^y(y-1) - 12e^0(0-1)$$

$$U(x,y) = x^3y + 4x^2y^2 + 12ye^y - 12e^y + 12 = C$$

$$\boxed{x^3y + 4x^2y^2 + 12ye^y - 12e^y = C}$$ ✓