$\boxed{1.75}$ $$y' - 2y e^x = 2\sqrt{y}\, e^x$$

Resolução:

Eq. de Bernoulli c/ $p = 1/2$

$z = y^{1-p}$

$z = y^{1-1/2} = y^{+1/2}$ $\qquad z' = +\frac{1}{2} y^{-1/2} y'$

multip. p/ $\frac{1}{2} y^{-1/2}$

$$\frac{1}{2} y^{-1/2} \cdot y' - \frac{1}{2} y^{-1/2} \cdot 2 y e^x = \frac{1}{2} y^{-1/2} \cdot 2 y^{1/2} e^x$$

$z' - e^x z = e^x$ $\qquad$ eq. linear 1ª ordem em z

$z = c e^x - 1$

$y = (c e^x - 1)^2$; $y \equiv 0$ $\qquad$ solução ✓

$\boxed{1.76}$ $\quad y' - 4y = 2e^x y^{1/2}$

Ec. de Bernoulli $\quad y^p \quad p = \frac{1}{2}$

Sub. $\quad v = y^{1-\frac{1}{2}} = y^{1/2}$

$$v' = \underbrace{\frac{1}{2} y^{-1/2}} y'$$

$$\underbrace{\frac{1}{2} y^{-1/2}} y' - \underbrace{\frac{1}{2} y^{-1/2}} 4y = \underbrace{\frac{1}{2} y^{-1/2}} 2e^x y^{1/2}$$

$$v' - 2y = e^x$$

$$v' - 2v = e^x$$

$$v(x) = e^{-\int -2dx} \left[\int e^x e^{\int -2dx} dx + C \right] =$$

$$= e^{2x} \left[\int e^x e^{-2x} dx + c \right] =$$

$$= e^{2x} \left[-e^{-x} + c \right] = ce^{2x} - e^x$$

$$v(x) = ce^{2x} - e^x = y(x)^{1/2}$$

$$\boxed{y(x) = \left(ce^{2x} - e^x \right)^2}$$

$$\boxed{y = 0}$$

$\boxed{177}$ $\quad y' - y = -y^2(x^2+x+1)$

Rezol: Ec. Bernoulli $y^p = y^2$

Sub: $v = y^{1-p} = y^{1-2} = y^{-1}$

$v' = \underbrace{-y^{-2}}\, y'$

$\underbrace{(-y^{-2})\, y'} - \underbrace{(-y^{-2})\, y} = (-y^{-2})(-y^2)(x^2+x+1)$

$v' + v = +(x^2+x+1)$

$v(x) = e^{-\int dx}\left[\int (x^2+x+1)\, e^{\int dx} + C\right]$

$v(x) = e^{-x}\left[\int (x^2+x+1)\, e^{x} dx + C\right]$

$v(x) = e^{-x}\left[(\underbrace{e^x x^2 - 2xe^x + 2e^x} + \underbrace{xe^x - \cancel{e^x}} + \cancel{e^x}) + C\right]$

$v(x) = e^{-x}\left[e^x(x^2 - x + 2) + C\right] = (x^2 - x + 2) + Ce^{-x}$

$$\boxed{y(x) = (Ce^{-x} + x^2 - x + 2)^{-1} \;;\; y \equiv 0}$$

Prob. 1.49

$$xy' - 2y = 4x^3 y^{1/2}$$

Resol.:

$$xy' - 2y = 4x^3 y^{1/2}$$

$$y' - \frac{2}{x} y = 4x^2 y^{1/2}$$

Eq. Bernoulli c/ $p = 1/2$

$$z = y^{1-p} = y^{1-1/2} = y^{1/2}$$

$$z' = \underbrace{\frac{1}{2} y^{-1/2}} y'$$

(x) $\frac{1}{2} y^{-1/2}$ a eq.

$$\underbrace{\left(\frac{1}{2} y^{-1/2}\right)} y' - \left(\frac{1}{2} y^{-1/2}\right) \frac{2}{x} y = \left(\frac{1}{2} y^{-1/2}\right) 4x^2 y^{1/2}$$

$$z' - \frac{1}{x} z = 2x^2$$

$\triangleright$ $\mu(x) = \exp\left[\int -\frac{1}{x} dx\right] = \exp(-\ln x)$

$\mu(x) = +x^{-1}$

$\triangleright$ $\cancel{z} = \frac{\int \mu(x)(2x^2)\,dx + C}{-x} = \frac{\int +2x\,dx + C}{+x^{-1}}$

$\boxed{z = x^3 + cx}$ $\cancel{z} = \cancel{x^3 + cx}$ $\boxed{y = (cx + x^3)^2}$ ✓

1.79 $\quad y^2+2xy=x^2y'$

Resol:

$$x^2y'-2xy=y^2$$

$$y'-\frac{2}{x}y=\frac{1}{x^2}y^2$$

Bernoulli c/ $n=2$

Sub. $\quad v=y^{1-n}=y^{1-2}=y^{-1}$

$$v=y^{-1}$$

$$v'=(-1)\,y^{-2}y'=-y^{-2}y'$$

$$\underbrace{-y^{-2}y'}-\frac{2}{x}y(-y^{-2})=\frac{1}{x^2}y^2(-y^{-2})$$

$$v+\frac{2}{x}v=-\frac{1}{x^2}$$

$$\mu(t)=\exp\left[\int\frac{2}{x}\,dx\right]=\exp\left[2\ln x\right]=x^2$$

$$\mu(t)=x^2 \qquad v(t)=\frac{\int x^2\left(-\frac{1}{x^2}\right)dx+C}{x^2}$$

$$v(t)=\frac{-x+c}{x^2}=-\frac{1}{x}+\frac{c}{x^2}=\frac{c-x}{x^2}$$

$$y^{-1}=\frac{-x+c}{x^2} \qquad y=\frac{x^2}{-x+c}$$

$\boxed{1.80}$ $\boxed{y' + y = y^2}$

Rezol.

Ec. Bernolli $y^p = y^2$

$P(x) = 1$

$Q(x) = 1$

Sub. $v = y^{1-p} \rightarrow v' = \underbrace{-y^{-2}}\, y'$

$= y^{1-2} = y^{-1}$

$$\underbrace{-y^{-2}y'} + (-y^{-2})y = -y^{-2}y^2$$

$$v' - v = -1$$

$$v(x) = e^{-\int -1\,dx}\left[\int -1\, e^{\int -1\,dx}\,dx + c\right]$$

$$= e^{x}\left[\int -e^{-x} + c\right] = e^{x}\left[e^{-x} + c\right] =$$

$$v(x) = e^{0} + ce^{x} = 1 + ce^{x} = y^{-1}$$

$\therefore$ $\boxed{y(x) = \dfrac{1}{1+ce^{x}}}$; $y \equiv 0$

<u>1.81</u> $3y' + y = (1-2x)y^4$ <u>LM.</u>

<u>Resol.</u> Eq. Bernoulli

$$y' + \frac{1}{3}y = \frac{1-2x}{3}y^4$$

Sub. $v = y^{1-4} = y^{-3}$

$v' = -3y^{-4}y'$

$$\underbrace{-3y^{-4}y'} + \frac{1}{3}(-3\underbrace{y^{-4})y} = (-3y^{-4})\left(\frac{1-2x}{3}\right)y^4$$

$$v' + (-v) = 2x-1$$

$$\mu(t) = \exp\left[\int(-1)\,dt\right] = e^{-t}$$

$$v(t) = \frac{\int e^{-t}(2t-1)\,dt + C}{e^{-t}}$$

$$\int(e^{-t}2t-1)\,dt = 2\int e^{-t}t\,dt - t = 2\left[-e^{-t}t - \int -e^{-t}dt\right]$$

$$= 2\left[-e^{-t}t - e^{-t}\right] = -2e^{-t}(t+1)$$

$$v(t) = \frac{-2e^{-t}(t+1) + C}{e^{-t}} = -2(t+1) + Ce^{t}$$

$$y^{-3} = -2(t+1) + ce^{t}$$

$\boxed{1.82}$ $2y' \, tg\, x + y = y^{-1} \operatorname{sen} x$

Resolução:

$$2y' \, tg\, x + y = y^{-1} \operatorname{sen} x$$

$$y' + \frac{1}{2 \, tg\, x} y = \frac{1}{2 \, tg\, x} y^{-1} \operatorname{sen} x = \frac{1}{2} \cos x \, y^{-1}$$

Eq. Bernoulli, $p = -1$ $\quad z = y^{1-(-1)} = y^2$

$z' = 2 y y'$

multiplicar eq. p/ $2y$

$$2y \cdot y' + 2y \frac{1}{2 \, tg\, x} y = 2y \frac{1}{2} \cos x \, y^{-1}$$

$$z' + \frac{1}{tg\, x} z = \cos x \qquad \text{eq. linear 1ª ord. em } z$$

$$z = e^{-\int \frac{1}{tg x} dx} \left[\int e^{\int \frac{1}{tg x} dx} \cos x \, dx + c \right]$$

$$z = e^{-\int \frac{\cos x}{\operatorname{sen} x} dx} \left[\int e^{\int \frac{\cos x}{\operatorname{sen} x} dx} \cos x \, dx + c \right]$$

$$z = e^{-\ln|\operatorname{sen} x|} \left[\int e^{\ln|\operatorname{sen} x|} \cos x \, dx + c \right]$$

$$z = (\operatorname{sen} x)^{-1} \left[\int \operatorname{sen} x \cos x \, dx + c \right] = (\operatorname{sen} x)^{-1} \left(\frac{1}{2} \operatorname{sen}^2 x + c \right)$$

$$y^2 = \frac{1}{2} \operatorname{sen} x + \frac{c}{\operatorname{sen} x}$$

$\boxed{1.83}$ $\quad xy' - 2y = y^2x^2e^x$

Resolução:

$$xy' - 2y = y^2x^2e^x$$

$$y' - \frac{2}{x}y = y^2\frac{x^2}{x}e^x \qquad \text{Eq. Bernoulli c/ } p=2$$

$$z = y^{1-2} = y^{-1} \Rightarrow z' = (-1)y^{-2}y'$$

mult. eq. p/ $(-y^{-2})$

$$(-y^{-2})y' - (-y^{-2})\frac{2}{x}y = (-y^{-2})y^2xe^x$$

$$z' + \frac{2}{x}z = -xe^x \qquad \text{Eq. linear em } z \text{ Ford}$$

$$z = e^{-\int +\frac{2}{x}dx}\left[\int (-xe^x)\, e^{\int +\frac{2}{x}dx}dx + c\right]$$

$$z = e^{-2\ln x}\left[\int -xe^x e^{+2\ln x}dx + c\right]$$

$$z = x^{-2}\left[\int -xe^x x^{+2}dx + c\right]$$

$$z = x^{-2}\left[\int -x^{+3}e^x dx + c\right]$$

$$z = x^{-2}\left[-(x^3e^x - 3x^2e^x + 6xe^x - 6e^x) + c\right]$$

$$z = -xe^x + 3e^x - 6x^{-1}e^x + 6x^{-2}e^x + cx^{-2}$$

$$z = e^x(6x^{-2} - 6x^{-1} - x + 3) + cx^{-2}$$

$\boxed{1.84}$

$$xy' - 2y = y^2 \frac{e^x}{x}$$

Resolução:

$$y' - \frac{2}{x} y = y^2 \frac{e^x}{x^2} \qquad \text{Eq. de Bernoulli } p=2$$

$$z = y^{1-p} = y^{1-2} = y^{-1} \qquad z' = -y^{-2} y'$$

$$\underbrace{(-y^{-2}) y'}_{} - (-y^{-2}) \frac{2}{x} y = (-y^{-2}) y^2 \frac{e^x}{x^2}$$

$$z' + \frac{2}{x} z = -x^{-2} e^x \qquad \text{Eq. dif. 1.ª ord. em } z$$

$$z = e^{-\int 2/x \, dx} \left[\int (-x^{-2} e^x) e^{\int 2/x \, dx} dx + c \right]$$

$$= x^{-2} \left[\int (-x^{-2} e^x x^2 \, dx + c \right] = x^{-2} \left(\int -e^x dx + c \right)$$

$$z = -x^{-2} e^x + c x^{-2} = \frac{c - e^x}{x^2} = y^{-1}$$

$$\boxed{y = \frac{x^2}{c - e^x}} \; ; \; y \equiv 0$$

$\boxed{1.85}$

$$3xy' + y + x^2y^4 = 0$$

Resolução:

$$y' + \frac{1}{3x}y = \frac{-x^2y^4}{3x} = -\frac{x}{3}y^4 \quad \text{Eq. Bernoulli } p=4$$

$$z = y^{1-4} = y^{-3} \Rightarrow z' = -3y^{-4}y'$$

$$\underbrace{(-3y^{-4})y'} + (-\cancel{3}y^{-4})\frac{1}{\cancel{3}x}y = (-\cancel{3}y^{-4})\frac{x}{\cancel{3}}y^4$$

$$z' - \frac{1}{x}z = -x \qquad z' - \frac{1}{x}z = -x$$

$$z = e^{-(-1)\int\frac{1}{x}dx}\left[\int(-x)\,e^{-\int\frac{1}{x}dx} + c\right] = x\left[-\int x\,x^{-1}dx + c\right]$$

$$z = x(c-x) \qquad y^{-3} = x(c-x)$$

$$\boxed{y = \frac{1}{x(c-x)}\ ;\ y = 0}$$

$\boxed{1.86}$

$$y' + y\,\mathrm{tg}\,x + 2y^2\,\mathrm{sen}\,x = 0$$

Resolución:

$$y' + \mathrm{tg}\,x\,y = -2\,\mathrm{sen}\,x\,y^2 \qquad \text{Ec. Bernoulli } p=2$$

$$z = y^{1-2} = y^{-1} \Rightarrow z' = -y^{-2}y'$$

$$\underbrace{(-y^{-2})\,y'}_{z'} + (-y^{-2})\,\mathrm{tg}\,x\,y = (-\cancel{y^{-2}})(-2\,\mathrm{sen}\,x)\,\cancel{y^2}$$

$$z' - \mathrm{tg}\,x\,z = 2\,\mathrm{sen}\,x \qquad \text{Ec. Dif. 1º orden.}$$

$$z = e^{-\int(-\mathrm{tg}\,x)\,dx}\left[\int(2\,\mathrm{sen}\,x)\,e^{\int(-\mathrm{tg}\,x)\,dx} + C\right]$$

$$z = (\cos x)^{-1}\left[\int 2\,\mathrm{sen}\,x\,\cos x\,dx + c\right]$$

$$z = (\cos x)^{-1}(\mathrm{sen}^2 x + c) = (\cos x)^{-1}(\mathrm{sen}^2 x + c)$$

$$\boxed{y = \frac{\cos x}{c + \mathrm{sen}^2 x}\ ;\ y = 0}$$

✓

$\boxed{1.8t}$

$$yy' + xy^2 - x = 0 \quad ; \quad y(0) = -1$$

Resolução:

$$y' + xy - \frac{x}{y} = 0 \qquad y' + xy = \frac{x}{y} = xy^{-1}$$

$$z = y^{1-(-1)} = y^2 \Rightarrow z' = 2yy'$$

$$(2y)y' + (2y)xy = (2y)(y^{-1})x$$

$z' + 2xz = 2x$ Eq. Dif linear 1.ª ord. em z

$$z = e^{-\int 2x\,dx}\left[\int 2xe^{\int 2x\,dx}\,dx + c\right] = e^{-x^2}\left(\int 2xe^{x^2}dx + c\right)$$

$$z = e^{-x^2}\left(e^{x^2} + c\right) = 1 + e^{-x^2}C$$

$$y^2 = 1 + e^{-x^2}C \qquad 1 + c = (-1)^2 = 1$$

$$\Rightarrow \boxed{c = 0}$$

$$y^2 = 1 \Rightarrow y = \pm 1$$

$$y(0) = -1 \Rightarrow \boxed{y = -1}$$

$\boxed{1.88}$

$$2y' + 3\frac{y}{x\ln x} = 3y^{1/3} \quad ; \quad y(e)=0$$

<u>Resolução:</u>

$$y' + \frac{3}{2}\frac{1}{x\ln x}y = \frac{3}{2}y^{1/3} \quad \text{Eq. Bernoulli } q = 1/3$$

$$z = y^{1-1/3} = y^{2/3} \implies z' = \frac{2}{3}y^{-1/3}y'$$

$$\left(\frac{2}{3}y^{-1/3}\right)y' + \left(\frac{\cancel{2}}{\cancel{3}}y^{-1/3}\right)\frac{\cancel{3}}{\cancel{2}}\frac{1}{x\ln x}y = \left(\frac{\cancel{2}}{\cancel{3}}y^{-1/3}\right)\frac{\cancel{3}}{\cancel{2}}y^{1/3} = 1$$

$$z' \qquad + \frac{1}{x\ln x}z \qquad = 1$$

$$z' + \frac{1}{x\ln x}z = 1 \quad \text{Eq. Dif. 1ª Ord. em } z$$

$$z = e^{-\int\frac{1}{x\ln x}dx}\left[\int 1 \cdot e^{\int\frac{1}{x\ln x}dx} + c\right] =$$

$$\int\frac{1}{x\ln x}dx \underset{\substack{u=\ln x \\ du=\frac{1}{x}dx}}{=} \int\frac{1}{u}du = \ln u = \ln(\ln x)$$

$$z = (\ln x)^{-1}\left[\int 1 \cdot \ln x \, dx + c\right]$$

$$z = (\ln x)^{-1}\left[x\ln x - x + c\right] = y^{2/3}$$

$$y(e) = 0 \implies c = 0 \qquad \boxed{y^{2/3} = x\left(1 - \frac{1}{\ln x}\right)} \ \checkmark$$