Decidabilitate

$$\mathcal{L}_3 = \mathcal{L}(AF) = \mathcal{L}(\mathcal{G}_3)$$

$$\cap$$

$$\mathcal{L}_2 = \mathcal{L}(APD) = \mathcal{L}(\mathcal{G}_2)$$

$$\nparallel$$

$$\mathcal{L}(APD\,Det)$$

$$\cap$$

$$\mathcal{L}_1 = \mathcal{L}(ALM) = \mathcal{L}(\mathcal{G}_1)$$

$$\cap$$

$$\mathcal{L}_0 = \mathcal{L}(M.T.) = \mathcal{L}(\mathcal{G}_0)$$

$$\mathcal{L}_2 \not\ni \{a^p \mid p \text{ prim}\} \subset \{a^m \mid m \geq 0\} \in \mathcal{L}_3$$

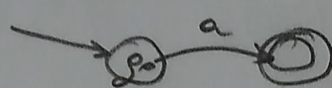
$$\mathcal{L}_2 \not\ni \{a^{n^2} \mid n \geq 0\} \subset$$

$\mathcal{L}_3$ este închisă la
- $\cup$, $\cdot$, *
- complementare $\overline{}$
- $\cap$ $\quad L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$

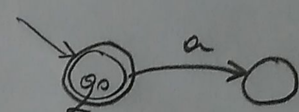
! – operația de complement – pentru $A = (Q, \Sigma, q_0, \delta, F)$ $\boxed{\text{complet definit}}$ $\rightsquigarrow \bar{A} = (Q, \Sigma, q_0, \delta, \underline{\underline{Q \setminus F}})$

ex: $L_2 = \{a, b\}^* \setminus \{a\} = \overline{\{a\}}$

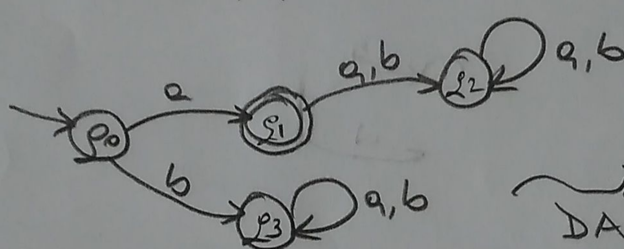
!NU!



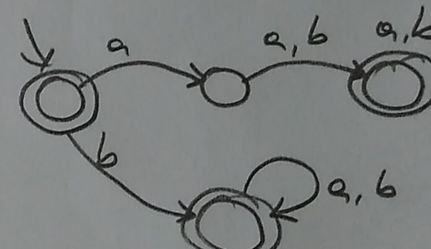
A_1



acceptă doar λ



$\xrightarrow{\text{DA}}$



$\mathcal{L}_2$ este închisă la
- $\cup$, $\cdot$, *
- $\cap$ cu limbaje regulate

$\left(\begin{array}{l} L_2 \in \mathcal{L}_2 \\ L_3 \in \mathcal{L}_3 \end{array} \right\} \Rightarrow L_2 \cap L_3 \in \mathcal{L}_2 \Big)$

Este decidabil pt $\mathcal{L}_3$

1. $\boxed{A \text{ AFD}, x \in \Sigma^* \quad x \in L(A)}$ - $\boxed{\text{DA}}$

2. $\boxed{A \text{ AFD} \quad L(A) = \emptyset?}$ $\boxed{\text{DA}}$

construiesc recursiv $M_0 = F, M_1 = M_0 \cup \{q \mid \delta(q, x) \in M_0\}$

$\dots M_{i+1} = M_i \cup \{q \mid \delta(q, x) \in M_i\}$

când $M_k = M_{k+1}$ - verific $q_0 \in M_k$ $\xrightarrow{\text{Da}} \neq \emptyset$, $\xrightarrow{\text{Nu}} = \emptyset$

[analog $N_0 = \{q_0\}$, $N_{i+1} = N_i \cup \{\delta(q, x) \; q \in N_i\}$
$N_k = N_{k+1}$ și verific $F \cap N_k \neq \emptyset$?]

3. $\boxed{A \text{ AFD} \quad L(A) = \Sigma^*} \Leftrightarrow \overline{L(A)} = \emptyset$ $\boxed{\text{DA}}$

4. $\boxed{L(A_1) \subseteq L(A_2)} \Leftrightarrow L(A_1) \cap \overline{L(A_2)} = \emptyset$ $\boxed{\text{DA}}$

5. $\boxed{L(A_1) = L(A_2)} \Leftrightarrow \begin{cases} L(A_1) \subseteq L(A_2) \\ L(A_2) \subseteq L(A_1) \end{cases}$ $\boxed{\text{DA}}$

$\mathcal{L}_2$ Decidabilitate

1. $L \in \mathcal{L}_2$ $\boxed{L = \emptyset}$ $\boxed{\text{DA}}$

$L = L(\underline{G})$ G în FNC — dacă S e terminabil (produce cuvânt doar din sb terminale) $\Rightarrow L \neq \emptyset$

2. $L \in \mathcal{L}_2$ L finit $\boxed{\text{DA}}$

$L = L(\underline{G})$ G în FNC — graf orientat pt producții

— dacă $\boxed{\text{nu are cicluri}}$ $\Rightarrow$ finit

($A \stackrel{*}{\Rightarrow} wA$)

3. $L \in \mathcal{L}_2$ $w \in L(G)$? $\boxed{\text{DA}}$

Alg Cocke – Younger – Kasami

Nedecidabile

- G CFG ambiguă ?
- G_1, G_2 CFG $L(G_1) \cap L(G_2) \neq \emptyset$?
- $G_3 \in \mathcal{L}_3$

$L(G_1) \neq L(G_2)$?

$L(G_1) \setminus L(G_2) \neq \emptyset$?

$L(G_1) \neq L(R)$

$L(G_1) \neq T^*$?

$L(R) \setminus L(G_1) \neq \emptyset$

1) $\left.\begin{array}{l} L_1 \subseteq L_2 \\ L_2 \in Reg \end{array}\right| \overset{?}{\Longrightarrow} L_1 \in Reg$

NU: $\underset{\substack{\notin \\ Reg}}{\{a^p \mid p \text{ prim}\}} \subseteq \{a^n \mid n \geq 0\} \in Reg$

2) $\left.\begin{array}{l} L_2 - L_1 = L_3 \\ L_2, L_3 \in Reg \end{array}\right| \overset{?}{\Longrightarrow} L_1 \in Reg$

NU: $\underset{\substack{\in \\ Reg}}{\{b^n \mid n \geq 0\}} \setminus \underset{\substack{\notin \\ Reg}}{\{a^p \mid p \text{ prim}\}} = \underset{\substack{\in \\ Reg}}{\emptyset}$

3) $\left.\begin{array}{l} \overline{L_1} \cap L_2 = L_3 \\ L_1, L_3 \in Reg \end{array}\right| \overset{?}{\Longrightarrow} L_2 \in Reg$

NU: $L_1 = \{a^n \mid n \geq 0\} \Rightarrow \overline{L_1} = \emptyset$

$\emptyset \cap \underset{\substack{\notin \\ Reg}}{\{a^p \mid p \text{ prim}\}} = \underset{\substack{\in \\ Reg}}{\emptyset}$

6

4. $L_1 \cup L_2 = L_3 \cap L_4$, $L_1, L_2, L_3 \in CFL$ $(\mathcal{L}_2)$ $\overset{?}{\Longrightarrow} L_4 \in CFL$

$L_1 = L_2 = L_3 = \emptyset$

$L_4 = \{a^p \mid p \text{ prim}\} \notin CFL$

$\emptyset = \emptyset \cap \{a^p \mid p \text{ prim}\}$

$\boxed{\text{NU}}$

5. $L_1 \cup \overline{L_2} = L_3 \cap L_4$, $L_2, L_3, L_4 \in Reg$ $\overset{?}{\Longrightarrow} L_1 \in Reg$

$Reg \not\ni L_1 = \{a^p \mid p \text{ prim}\}$

$L_2 = \emptyset \Rightarrow \overline{L_2} = \{a^n \mid n \geq 0\}$

$L_3 = L_4 = \{a^n \mid n \geq 0\}$

$\Rightarrow L_3 \cap L_4 = L_1 \cup \{a^n \mid n \geq 0\}$

$\{a^n \mid n \geq 0\}$

NU.

6. $L_2 - L_1 = L_4 - L_3$, $L_2, L_3, L_4 \in Reg$ $\overset{?}{\Rightarrow} L_1 \in Reg$

$L_2 = L_4 = L_3 = \emptyset$

$L_1 = \{a^p \mid p \text{ prim}\} \notin Reg$