Lema de pompare CFL

① $L = \{ w w \mid w \in \{a,b\}^* \}$

Presupun că e CFL.

$\Rightarrow (\forall)\ n_0 \geq 1\ (\exists)\ |\alpha| \geq n_0,\ \alpha \in L$ a.î. $(\forall)$ descompunere

$\alpha = uvwxy$ are proprietățile:

1) $|vwx| \leq n_0$

2) $|vx| \geq 1$

3) $(\forall)\ i \geq 0\ \ uv^iwx^iy \in L$

aleg $\alpha = a^{n_0} b^{n_0} a^{n_0} b^{n_0}$

1) $\underbrace{aa\ldots a}_{vwx}\, b\ldots b\ a\ldots a\ b\ldots b$

$vwx = a^p,\ 1 \leq p \leq n_0$

$\Rightarrow vx = a^q,\ 1 \leq q \leq p \leq n_0$

$\Rightarrow \alpha' = \cancel{a^{n_0} b}\ uv^iwx^iy =$

$= a^{n_0-q}\, a^{i \cdot q}\, b^{n_0} a^{n_0} b^{n_0}$

aleg $i = 0 \Rightarrow \alpha' = a^{n_0-q} b^{n_0} a^{n_0} b^{n_0}$

~~dacă n_0 = par și q = impar $\Rightarrow |\alpha'|$ = impar~~

~~$\Rightarrow \alpha' \notin L$~~

~~dacă n_0 = impar și q = par $\Rightarrow |\alpha'|$ = impar~~

~~$\Rightarrow \alpha' \notin L$~~

~~dacă n_0 = par, q = par~~

~~n_0 = impar, q = impar~~ $\Rightarrow$

$|\alpha'| = n_0 - q + 3n_0 = 4n_0 - q$

dacă q = impar $\Rightarrow |\alpha'|$ = impar

$\Rightarrow \alpha' \notin L$

dacă q = par $\Rightarrow \dfrac{|\alpha'|}{2} = \dfrac{4n_0-q}{2} < 2n_0$ (pt. că $q \geq 2$)

$\Rightarrow$ prima jumătate: $\underbrace{a\ldots a}_{n_0-q}\, \underbrace{b\ldots b}_{2n_0 - q/2 - n_0 + q} = n_0 + q/2$

a doua jumătate: b...b a...a b...b

prima jumătate ≠ a doua jumătate

$\Rightarrow \alpha' \notin L$

2) $\underbrace{aa\ldots a\, b\ldots}_{uvx}$ $b\, a\ldots a\, b\ldots b$

$uvx = a^p b^q$, $1 \le p+q \le n_0$

$\Rightarrow vx = a^t b^r$, $1 \le t+r \le p+q \le n_0$

alegi $i=0 \Rightarrow \alpha' = a^{n_0-t} b^{n_0-r} a^{n_0} b^{n_0}$

$|\alpha'| = n_0 - t + n_0 - r + n_0 + n_0 = 4n_0 - r - t$

dacă $|\alpha'| =$ impar $\Rightarrow \alpha' \notin L$

dacă $|\alpha'| =$ par și $r+t \ge 1 \Rightarrow r+t \ge 2$

$\Rightarrow$ prima jumătate: $\underbrace{a\ldots a}_{n_0-t}\underbrace{bb\ldots b}_{n_0-r}\underbrace{a\ldots a}_{\frac{(r+t)}{2}}$

a doua jumătate: $\underbrace{a\ldots a}_{n_0-\frac{(r+t)}{2}}\underbrace{b\ldots b}_{n_0}$

$\Rightarrow \alpha' \in L$

3) $a\ldots a\, b\,\underbrace{\ldots}_{uvx} b\, a\ldots a\, b\ldots b$

$uvx = b^p$, $1 \le p \le n_0$

$\Rightarrow vx = b^q$, $1 \le q \le p \le n_0$

$\alpha' = uv^i w x^i y$

dacă $i=0 \Rightarrow \alpha' = a^{n_0} b^{n_0-q} a^{n_0} b^{n_0}$

$\Rightarrow a^{m-p} b^{m-q} a^{\cancel{2m} - \frac{p+q}{2} - \cancel{2m} + p + q} =$

$= a^{m-p} b^{m-q} a^{\frac{p+q}{2}}$ = prima jumătate

$a^{m - \frac{p+q}{2}} b^{m}$ = a doua jumătate

prima jumătate $\neq$ a doua jumătate

$\Rightarrow \alpha' \notin L$

3) $a \ldots a \underbrace{b \ldots b}_{uwx} a \ldots a b \ldots b$

$uwx \in b^{m}$

$\Rightarrow ux = b^{p}, 1 \leq p \leq m$

aleg $i = 2 \Rightarrow \alpha' = u v^{i} w x^{i} y =$

$= a^{m} b^{m+p} a^{m} b^{m}$

notez prima jumătate a lui α cu J_1 și a doua jumătate cu J_2

dacă p = par $\Rightarrow |\alpha'|$ = par

$J_1 = a^{m} b^{\frac{4m+p}{2} - m} =$

$= a^{m} b^{2m + \frac{p}{2} - m} = a^{m} b^{m + \frac{p}{2}}$

$J_2 = b^{\frac{p}{2}} a^{m} b^{m}$

$J_1 \neq J_2 \Rightarrow \alpha' \notin L$

dacă p = impar $\Rightarrow |\alpha'|$ = impar $\Rightarrow \alpha' \notin L$

4) $a \ldots a b \ldots \underbrace{b a \ldots}_{uwx} a b \ldots b$

$uwx \in b^{m} a^{m}$

$\Rightarrow ux = b^{p} a^{q}, 1 \leq p+q \leq m$

alog $i=0 \Rightarrow \alpha' = uv^i w x^i y = a^n b^{n-p} a^{n-q} b^n$

dacă $p+q$ = impar $\Rightarrow |\alpha'|$ = impar

$\Rightarrow \alpha' \notin L$

dacă $p+q$ = par $\Rightarrow \frac{|\alpha'|}{2} = \frac{4n-p-q}{2} = 2n - \frac{p+q}{2}$

$\cancel{n} + \cancel{n} - p \overset{?}{<} \cancel{2n} - \frac{p+q}{2}$

$-p \overset{?}{<} -\frac{p+q}{2} \quad | (-1)$

$p > \frac{p+q}{2} \Leftrightarrow 2p > p+q \Leftrightarrow p > q$

~~alog $i=2 \Rightarrow \alpha' = uv^i w x^i y = a^n b^{n+p} a^{n+q} b^n$~~

~~$= a^n b$~~

$\Rightarrow$ dacă $p > q$: $J_1 = a^n b^{n-p} a^{\cancel{2n} - \frac{p+q}{2} - \cancel{2n} + p} =$

$= a^n b^{n-p} a^{\frac{p-q}{2}}$

$J_2 = a^{n - \frac{p-q}{2}} b^n$

$J_1 \neq J_2$

dacă $p < q$: ~~$J_1 = a^n b^{\cancel{2n} - \frac{p+q}{2} - \cancel{n}}$~~ $= a^n b^{n - \frac{p+q}{2}}$

~~$J_2 = b^{\frac{p+q}{2}} a^n b^n$~~

~~dacă $p=q$~~ $\quad J_1 = a^n b^{2n - \frac{p+q}{2} - n} =$

$= a^n b^{n - \frac{p+q}{2}}$

$J_2 = b^{\cancel{n} - p - \cancel{n} + \frac{p+q}{2}} a^{n-q} b^n =$

$= b^{\frac{q-p}{2}} a^{n-q} b^n$

$J_1 \neq J_2 \Rightarrow \alpha' \notin L$

dacă $p = q \Rightarrow \frac{|\alpha'|}{2} = 2n - p$, $\alpha' = a^n b^{n-p} a^{n-p} b^n$

$\Rightarrow$ ~~prima jumătate:~~ $a^n b^{n-p} = J_1$

$a^{n-p} b^n = J_2$

$J_1 \neq J_2 \Rightarrow \alpha' \notin L$

5) $a\ldots a\,b\ldots b\,\underbrace{a\ldots}_{uvx}\,a\,b\ldots b$

$uvx \in a^n$

$\Rightarrow ux = a^p,\ 1 \le p \le n$

aleg $i=2 \Rightarrow \alpha' = uv^i w x^i y = a^n b^n a^{n+p} b^n$

notez J_1 = prima jum.

J_2 = a doua jum.

dacă p = par

$\Rightarrow J_1 = a^n b^n a^{\cancel{2n} + \frac{p}{2} - \cancel{2n}} = a^n b^n a^{p/2}$

$J_2 = a^{n+\frac{p}{2}} b^n$

$J_1 \neq J_2 \Rightarrow \alpha' \notin L$

dacă p = impar $\Rightarrow |\alpha'|$ = impar $\Rightarrow \alpha' \notin L$

6) $a\ldots a\,b\ldots b\,a\ldots\underbrace{a\,b}_{uvx}\ldots b$

$uvx \in a^n b^n$

$\Rightarrow ux = a^p b^q,\ 1 \le p+q \le n$

aleg $i=0 \Rightarrow \alpha' = uv^i w x^i y = a^n b^n a^{n-p} b^{n-q}$

$\Rightarrow \frac{|\alpha'|}{2} = 2n - \frac{p+q}{2}$

J_1 = prima jum

J_2 = a doua jum

dacă $p+q$ = par

$\Rightarrow J_1 = a^n b^{n-\frac{p+q}{2}}$

$J_2 = b^{\frac{p+q}{2}} a^{n-p} b^{n-q}$

$J_1 \neq J_2 \Rightarrow \alpha' \notin L$

dacă $p+q$ = impar $\Rightarrow |\alpha'|$ = impar $\Rightarrow$ ~~$\alpha' \notin L$~~ $\alpha' \in L$

7) $a\ldots a\,b\ldots b\,a\ldots a\,\underbrace{b\ldots}_{uvx}\,b$

$uvx \in b^n$

$\Rightarrow ux = b^p,\ 1 \le p \le n$

aleg $i=2 \Rightarrow \alpha' = a^n b^n a^n b^{n+p}$

⇒ dacă p e par:

$s_1 = a^n b^n a^{\frac{p}{2}}$

$s_2 = a^{n-\frac{p}{2}} b^{n+p}$ $\quad s_1 \neq s_2 \Rightarrow \alpha' \notin L$

dacă p = impar ⇒ $|\alpha'|$ = impar ⇒ $\alpha' \notin L$

⇒ $(\forall)$ descompunerea lui $\underset{\underset{a^n b^n a^n b^n}{\shortparallel}}{\alpha}$, $|\alpha| \geq n$, $(\exists)$ $i \geq 0$

a.î. $uv^i w x^i y \notin L$

⇒ L nu e CFL

(2) $L = \{ w c^i w \mid w \in \{a, b\}^*, i \geq 1 \}$ / CFL

aleg $\alpha = a^n c^n a^n$

1) $\underbrace{a \ldots}_{uvx} a \; c.ca \ldots a$

$uvx \in a^n$

⇒ $vx = a^p$, $1 \leq p \leq n$

⇒ aleg $i = 2$

notez s_1 = nr caractere dinaintea lui c

s_2 = nr caractere după c

⇒ $|s_1| = n+p$, $|s_2| = n$ ⇒ $s_1 \neq s_2$

⇒ $\alpha' \notin L$

2) $a \ldots \underbrace{a \; c}_{} \; a \ldots a$

$uvx \in a^n c$

⇒ $vx = a^p c^q$, $1 \leq p+q \leq n$

aleg $i = 0$ ⇒ $\alpha' = uv^i w x^i y = a^{n-p} c^{n-q} a^n$

$\frac{|\alpha'|}{2} = \frac{3n-p-q}{2}$ $\quad n-p-q > -2p$

~~$s_1 = a$~~

$w \in \{a,b\}^*$, $c \notin w$

⇒ $w = a^{n-p} = a^n$ (fals) ⇒ $\alpha' \in L$

3) ~~a ... a c a ... a~~ a

$\underbrace{\quad}_{uvx}$

$uvx \in a^m c a^m$

$\Rightarrow$ ~~$vx = a$~~ $\Rightarrow$ $\left| \begin{array}{l} u = a^p \\ v = a^q c a^r \\ x = a^t \end{array} \right.$ cazul 1 sau $\left| \begin{array}{l} u = a^p c a^q \\ v = a^r \\ x = a^t \end{array} \right.$ cazul 2 sau $\left| \begin{array}{l} u = a^p \\ v = a^q \\ x = a^r c a \end{array} \right.$ cazul 3

cazul 1: $\alpha' = a^{m-p} c a^{m-t}$

3) $a \ldots a \; \underbrace{c \ldots c}_{uvx} \; a \ldots a$

$uvx \in c^m$

$\Rightarrow ux = c^p, \; 1 \le p \le m$

(2) $L = \{ w c^i w \mid w \in \{a,b\}^*, i \ge 1 \}$

aleg $\alpha = a^m b^m c a^m b^m$

1) $ux = a^p$ (tratat la (1))

2) $ux = a^p b^q$ analog

3) $ux = b^q$ analog

4) $ux = b^p c$

aleg $i = 0 \Rightarrow c$ e del $\Rightarrow \alpha' \notin L$

5) $uvx \in b^m c a^m$

~~$\Rightarrow ux$~~

cazul 1:

$\left| \begin{array}{l} u = b^p \\ v = b^q c a^r \\ x = a^t \end{array} \right.$

aî $p + t \ge 1$

și $p + q + 1 + r + t \le m$

$\Rightarrow$ aleg $i = 0 \Rightarrow \alpha' = a^m b^{m-p} c a^{m-r} b^m$

w_1 = ce e înainte de c

w_2 = ce e după c

$w_1 = a^m b^{m-p}$

$w_2 = a^{m-r} b^m$ | $w_1 \ne w_2$ $\Rightarrow \alpha' \notin L$

cazul 2:

$u = b^p c\, a^t$ aleg $i = 0 \Rightarrow \alpha' = a^n b^{n-p} a^{n-r} b^n$

$w = a^q$ $c \notin \alpha' \Rightarrow \alpha' \in L$

$x = a^r$

$t+p+1+r \geq 1$

$t+p+1+q+r \leq n$

cazul 3:

$u = b^p$ $\Rightarrow$ aleg $i = 0 \Rightarrow a^n b^{n-p-r} a^{n-t} b^n$

$w = b^q$ $c \notin \alpha' \Rightarrow \alpha' \notin L$

$x = b^r c\, a^t$

$p+r+1+t \geq 1$

și $p+q+r+1+t \leq n$

6) restul cazurilor sunt tratate la exercițiul 1

(3) $L = \{ a^n b^m \mid n \geq 5$ și m pătrat perfect$\}$?

aleg $\alpha = a^5 b^{m^2}$

1) $ux = a^p$, $1 \leq p \leq 5$

alegi $i = 0 \Rightarrow \alpha' = a^{5-p} b^{m^2}$

$\Rightarrow i < 5 \Rightarrow \alpha' \notin L$

2) $ux = a^p b^q$ $1 \leq p$, $1 \leq q$, $1 \leq p+q \leq m$

$\Rightarrow$ alegi $i = 0 \Rightarrow$ $i < 5 \Rightarrow \alpha' \notin L$

3) $ux = b^p$, $1 \leq p \leq m$

alegi $i = 2 \Rightarrow$ $\alpha' = a^5 b^{m^2+p}$ $\cancel{m^2} - m \overset{?}{>} \cancel{m^2} - 2m + 1$

$m > 1$

$1 \leq p \leq m \mid (-1)$

$+1 \geq +p \geq +m \mid + m^2$

$m^2 + 1 \geq m^2 + p \geq m^2 + m$ ← pentru $m > 1$

$m^2 > m^2 + 1 \geq m^2 + p \geq m^2 + m > m^2 - 2m + 1$

$m^2 + 1 \leq m^2 + p \leq m^2 + m$

$m^2 < m^2 + 1 \leq m^2 + p \leq m^2 + m < m^2 + 2m + 1$

$\Rightarrow$ $m^2 + p$ nu e pp $\Rightarrow \alpha' \notin L$

(5) $L = \{a^n b^m c^k \mid n \geq m \geq k \geq 150\}$

aleg $\alpha = a^n b^n c^{150}$

1) $uv x = a^p$

$\Rightarrow$ $\alpha' = uv^i x^i y = a^{n-p+ip} b^n c^{150}$

alegi $i = 0 \Rightarrow \alpha' = a^{n-p} b^n c^{150}$

$p \geq 1$

$\Rightarrow n-p \leq n-1 < n$

$\Rightarrow$ ~~$\cancel{\ }$~~ $n-p < n \Rightarrow \alpha' \notin L$

2) $u\,vx \in a^n b^n$ $\Rightarrow$ ~~$u = a^p \quad x = b$~~

~~cazul 1:~~

cazul 1:

$u = a^p$

$x = b^q$

$\Rightarrow$ dacă $p < q$: aleg $i = 2$: $\alpha' = a^{n+p} b^{n+q} c^{150}$

$n+p < n+q$

$\Rightarrow \alpha' \notin L$

~~cazul 2:~~

~~$u = a^p b^q$~~

~~$x = b^r$~~

~~$\Rightarrow$ dacă~~

dacă $p > q \Rightarrow$ aleg $i = 0$: $\alpha' = a^{n-p} b^{n-q} c^{150}$

$\Rightarrow \alpha' \notin L$

cazul 2:

~~$u = a^p b$~~

$u = a^{t} b^r$

$x = b^z$

$\Rightarrow ux = a^t b^{r+z}$

notez ~~$t =$~~ $p = t$

$q = r+z$

cazul 3:

$u = a^t$

$x = a^r b^z$

notez $p = t$

$q = r+z$

dacă $p = q \Rightarrow$ aleg i a.î. ~~$a\ n-p <$~~

$n-p+ip < 150$

$\Rightarrow n+(i-1)p < 150$

$i-1 = \dfrac{149-n}{p}$

$\Rightarrow i = \dfrac{149-n}{p} - 1$

$\Rightarrow \alpha' = a^{n-(i-1)p} b^{n-(i-1)p}$

$n-(i-1)p = 149 < 150 \Rightarrow \alpha' \notin L$

~~cazul 2:~~

~~$u = a^p b^q$~~

~~$x = b^r$~~

~~analog cu cazul precedent:~~

~~$p \to p$~~

~~$q \to q+r$~~

~~Cazul 3:~~

~~$x = u z$~~

3) $uux \in b^n$

$\Rightarrow ux = b^p \Rightarrow$ ~~$\alpha' = a^n b^{n(i-1)p} c^{150}$~~

aleg $i = 2 \Rightarrow \alpha' = a^n b^{n+p} c^{150}$

$n+p > n$

$\Rightarrow \alpha' \notin L$

4) $uux \in b^n c^{150}$, $c \in x$ (altfel intram pe cazul 3) $\Leftrightarrow q \geq 1$

$\rightarrow$ cazul 1:

$u = b^p$

$x = c^q$, $q \leq 150$

$\rightarrow$ cazul 2:

$u = b^p c^t$

$x = c^r$, $t+r \leq 150$

notez $p = p$

$q = t+r$

$\rightarrow$ cazul 3:

$u = b^p$

$x = c^t c^r$, $t+r \leq 150$

not $p = p$

$q = t+r$

$\Rightarrow$ toate cele 3 cazuri pot fi tratate cu notatiile mentionate:

aleg $i = 0$

$\Rightarrow \alpha' = a^n b^{n-p} c^{150-q}$

$q \geq 1 \Rightarrow \alpha' \notin L$

5) $uux \in c^{150}$

$\Rightarrow ux = c^p \Rightarrow$ aleg $i = 0 \Rightarrow \alpha' \notin L$

(5) $L = \{a^{2k} b^{3k} c^{5k} \mid k \geq 2\}$

aleg $\alpha = a^{2n} b^{3n} c^{5n}$

1) $ux = a^p \Rightarrow$ aleg $i = 0 \Rightarrow \alpha' = a^{2n-p} b^{3n} c^{5n} \Rightarrow \alpha' \notin L$

2) $ux = a^p b^q \Rightarrow$ aleg $i = 0 \Rightarrow \alpha' = a^{2n-p} b^{3n-q} c^{5n} \Rightarrow \alpha' \notin L$

3) $ux = b^q$

$\vdots$

(5) DEMONSTRAȚIE CĂ $\{a^p \mid p \text{ prim}\}$ nu e CFL

Presupun că L este CFL

Lema Pompare $\Rightarrow$ $(\forall)\ n_0 \geq 0\ (\exists)\ \alpha \in L$ cu $|\alpha| \geq n_0$ a.î. $(\forall)$ descompunere $\alpha = uvwxy$ avem proprietățile:

1) $|vwx| \leq n_0$

2) $|vx| \geq 1$

3) $uv^iwx^iy \in L\ (\forall)\ i \geq 0$

~~aleg α un număr~~

aleg $\alpha = a^p \mid p$ un nr prim $\geq n_0$

~~cel mai mic nr prim $\geq 2 \Rightarrow |\alpha| \geq 2$~~

Avem ~~mai multe cazuri~~ un singur caz:

1. $u = a^p$
$v = a^q$
$w = a^r$ $\Rightarrow$ $uv^iwx^iy = a^{p+iq+r+si+t} =$
$x = a^s$
$y = a^t$

$= a^{p+r+t+i(q+s)}$

pentru $i = p+r+t$

$\Rightarrow uv^iwx^iy =$

$= a^{(p+r+t)(q+s+1)}$

din lema de pompare $\Rightarrow |vx| \geq 1$

$\Rightarrow q+s \geq 1$

$\Rightarrow (q+s+1) \geq 2$

$\Rightarrow$ $(p+r+t)(q+s+1)$ este compus

$\Rightarrow L$ nu este CFL

DEMONSTRAȚIE CĂ $L = \{a^p \mid p \text{ prim}\}$ nu este regulat

Pp că L este regulat.

$\Rightarrow (\forall)\ m_0 \geq 0\ (\exists)\ \alpha \in L$ cu $|\alpha| \geq m_0$ aî $(\forall)$ descompunere

$\alpha = \cancel{uvw}\ xyz$ $m_0 \in \mathbb{N}$ are propr:

1) $|xy| \leq m_0$

2) $|y| \geq 0$

3) $xy^iz \in L\ (\forall)\ i \geq 0,\ i \in \mathbb{N}$

Aleg $\alpha = a^p$ cu p un nr prim $\geq m_0$

$\Rightarrow \quad \alpha = a^p = xyz$

$\Rightarrow xy^iz = \cancel{a}\ \cancel{xy}\ |xz| + i|y|$

$\Rightarrow xy^iz = |xz| \pmod{|y|}$

~~$0\ i \pmod{|y|}$~~

~~$i|y| \pmod{|y|+1}$~~ $=$

~~$= i|y| - i \pmod{}$~~

$i|y| \pmod{|y|+1} = i|y| + i - i =$

$= \underbrace{(i(|y|+1) \pmod{|y|+1})}_{0} - i \pmod{|y|+1} =$

$= -i \pmod{|y|+1}$

$\Rightarrow xy^iz = |xz| - i \pmod{|y|+1}$

aleg $i = |xz|$

$\Rightarrow xy^iz = 0 \pmod{|y|+1}$

$\Leftrightarrow xy^iz$ divizibil cu $|y|+1$

din lema de pompare $\Rightarrow |y| \geq 1 \Rightarrow |y|+1 \geq 2$

$\Rightarrow$ ~~$xy^iz \Rightarrow$~~ $\Rightarrow xy^iz$ compus

$\Rightarrow xy^iz \notin L$

(6) ~~ex~~ Fie $A, B \in Reg$, $L = \{xy \mid x \in A, y \in B, |x| = |y|\}$

Arătați că L este CFL.

Presupunem că și A și B sunt pe același alfabet Σ.

$\Rightarrow$ Fie $(Q_A, q_{0A}, \delta_A, F_A)$ un DFA pt A^R și — inversul lui A

$(Q_B, q_{0B}, \delta_B, F_B)$ un DFA pt B.

Construim gramatica liniară cu neterminalele $\{S\} \cup Q_A \times Q_B$ după regulile:

- pentru fiecare $f_A \in Q_A$ și $f_B \in Q_B$, $S \to \langle f_A, f_B \rangle$
- pentru fiecare $q_A \in Q_A$ și $q_B \in Q_B$ și $\sigma_A, \sigma_B \in \Sigma$

  $\langle \delta_A(q_A, \sigma_A), \delta_B(q_B, \sigma_B) \rangle \to \sigma_A \langle q_A, q_B \rangle \sigma_B$
- $\langle q_{0A}, q_{0B} \rangle \to \lambda$

(7) $L = \{xyz \mid |x| = |y| = |z|, x \neq y, y \neq z, x \neq z\}$

Presupunem că e CFL

aleg $\alpha = a^n b^n c^n$

1) $\underbrace{a \dots a}_{uvx} b \dots b c \dots c$

$\Rightarrow ux = a^p$

aleg $i = 0 \Rightarrow \alpha' = u v^i w x^i y = a^{n-p} b^n c^n$

$\frac{|\alpha'|}{3} = \frac{3n-p}{3} = n - \frac{p}{3}$

dacă $p \not\vdots\, 3 \Rightarrow \alpha' \notin L$

dacă $p \,\vdots\, 3 \Rightarrow$ $J_1 = a^{n-\frac{p}{3}} b^{n - \frac{p}{3} - n + p} = a^{n-p} b^{\frac{2p}{3}}$

$J_2 = b^{n - \frac{2p}{3}} c^{n - \frac{p}{3} - n + \frac{2p}{3}} = b^{n - \frac{2p}{3}} c^{\frac{p}{3}}$

$J_3 = c^{n - \frac{p}{3}}$

J_1

⑦ $L = \{www \mid w \in \{a,b\}^*\}$

alig $\alpha = a^n b a^n a^n b a^n a^n b a^n$

1) $uvx \in a^n \Rightarrow i=0 \Rightarrow \alpha' = a^{n-p} b a^{n} b a^n b a^n a^n b a^n$

2) $uvx \in a^n b$ ~~iau i=0 ⇒ α' =~~

$u = a^p$	$u = a^p$	$u = a^p b$	~~$u = \emptyset$~~	$u = \emptyset$
$v = a^q$	$v = a^q b$	$v = \emptyset$	~~$v = a^p b$~~	$v = \emptyset$
$x = b$	$x = \emptyset$	$x = \emptyset$	~~$x = \emptyset$~~	$x = a^p b$

iau $i=0 \Rightarrow \alpha' = a^{n-p} a^n a^n b a^n a^n b a^n$

cum sunt doar 2 de b în cuvânt

$\Rightarrow \alpha' \notin L$

3) $uvx \in b$ analog

4) $uvx \in b a^n$ analog

5) $uvx \in a^n a^n$

6) $uvx \in a^n b$

7) $uvx \in b$

8) $uvx \in b a^n$

9) ~~uvx~~ se repetă cazurile

⑧ $L = \{\overline{a^n b^n c^n} \mid n \geq 1\}$

$\overline{a^* b^* c^*} \cup \{a^i b^j c^k \mid i \neq j\} \cup \{a^i b^j c^k \mid i \neq k\} \cup$

$\cup \{a^i b^j c^k \mid j \neq k\}$

toate cuvintele care nu sunt a ceva, b ceva, c ceva

toate cuvintele de forma asta care au $i \neq j$