RLC Series Circuit

Example 3.1.1. Find the amplitude, u_m, rms value, U, angular frequency, ω, frequency, f, period, T and phase angle, ϕ of the sinusoidal voltage: $u(t) = 15\ cos(100t - 35^0)\ V$.

Solution:

Analyzing the sinusoidal voltage function, we have: the amplitude u_m=15V; *the rms value* $U = \frac{u_m}{\sqrt{2}} = 10.60V$; *the angular frequency is* ω=100 rad/s; *the frequency* $f = \frac{\omega}{2\pi} = 15.92\ Hz$; *the period* $T = \frac{1}{f} = \frac{2\pi}{\omega} = 0.0628\ s = 62.8\ ms$; *the phase angle* $\varphi = -35^0$.

Example 3.1.2. Calculate the phase angle between $u_1(t) = -10\ cos(\omega t + 40^0)$ and $u_2(t) = 12\ sin(\omega t - 20^0)$.

Solution:

In order to compare u_1 *and* u_2*, we have to express both functions in the same form, i.e. in terms of sin functions. Using the identity* $-\cos\alpha = sin(\alpha - 90^0)$*, we can write* $u_1(t) = 10\ sin(\omega t + 40^0 - 90^0) = 10\ sin(\omega t - 50^0)$. *Comparing now both functions, the phase angle between the two functions is* $\varphi_{12} = \varphi_1 - \varphi_2 = -50^0 - (-20^0) = -30^0$. *We say that the* $u_1(t)$ *lags the* $u_2(t)$ *by the angle of* 30^0*, or conversely, the* $u_2(t)$ *leads the* $u_1(t)$ *by the angle of* 30^0.

Example 3.1.3. A sinusoidal current at the moment t=0 has the value i(0)=5A and at t_1=2.5 ms reaches peak value. Knowing the period T=20 ms, calculate: a) the phase angle, ϕ; the instantaneous form of the sinusoidal current, i(t).

Solution:

We will consider a sinusoidal current in form $i(t) = i_m \sin(\omega t + \varphi)$. *For* t_1=2.5 ms *the function reaches the peak, means that* $\sin(\omega t_1 + \varphi) = 1$*, and as a consequence* $\omega t_1 + \varphi = \frac{\pi}{2}$*, and* $\varphi = \frac{\pi}{2} - \omega t_1 = \frac{\pi}{2} - \frac{2\pi}{T} t_1 = 0.787\ (rad) \cong 45^0$. *For t=0 we have i(0)=5A, which means* $5 = i_m \sin 0.787$*, and* $i_m = 7.071 \cong 5\sqrt{2}\ (A)$. *Finally, the instantaneous current is:* $i(t) = 7.071 \sin(314t + 45^0) = 5\sqrt{2} \sin(314t + 45^0)\ (A)$.

Example 3.2.1. The sinusoidal current through a 5Ω's resistance is: $i(t) = 2\sqrt{2}\ sin(100t + 60^0)\ A$.Calculate the rms value and instantaneous value of the voltage on the resistor. Find the value of the current and voltage for t=0.01s.

Solution:

According to the relation between the current and voltage on a resistance, the rms value of the voltage is $U_R = I \cdot R = 2 \cdot 5 = 10\ (V)$*, and the instantaneous voltage* $u_R(t) = R \cdot i(t) = 10\sqrt{2}\sin(100t + 60^0)\ \ (V)$*.The value of the current for t=0.01s is* $i(0.01) = 2\sqrt{2}\sin(1 + \frac{\pi}{3}) = 2.51\ (A)$*, while the voltage value is* $u(0.01) = 10\sqrt{2}\sin(1 + \frac{\pi}{3}) = 12.57\ (V)$.

Example 3.2.2. The sinusoidal current through a 250 mH's inductance is $i(t) = 4\sqrt{2}\ sin(100t - 30^0)\ A$. Calculate the rms value and the instantaneous value of the voltage on the inductance.

Solution:

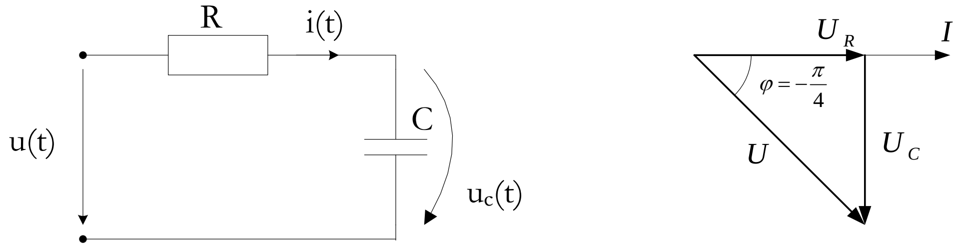
The inductive reactance is $X_L = \omega L = 25\ \Omega$*. As a remark, the inductive reactance depends on the inductance but also on the angular frequency of the supplying current. The rms value of the voltage is* $U_L = X_L \cdot I = 100\ (V)$*, and the instantaneous voltage is* $u_L(t) = L\ \frac{di}{dt} = 100\sqrt{2}\cos(100t - 30^0) = 100\sqrt{2}\sin(100t + 60^0)\ (V)$.

Example 3.2.3. The sinusoidal voltage across a 10 µF's capacitance is $u_C(t) = 150\sqrt{2}\ sin(20t + 15^0)\ V$. Calculate the rms value and the instantaneous value of the current through the capacitance.

Solution:

The capacitive reactance is $X_C = \frac{1}{\omega C} = 50\ \Omega$*. As a remark, the same as for the inductance, the capacitive reactance depends on the capacitance but also on the angular frequency of the supplying current. The rms value of the current is* $I = \frac{U_C}{X_C} = 3\ (A)$*, and the instantaneous voltage is* $i(t) = C\ \frac{du_C}{dt} = 3\sqrt{2}\cos(20t + 15^0) = 3\sqrt{2}\sin(20t + 105^0)\ (A)$.

Example 3.3.1. The circuit parameters for the RC series circuit below are R=100Ω, C=31.8 μF. We know the current amplitude i_m=1.41 A and the phase angle ϕ_i=0. Determine: a) the instantaneous voltage on the capacitance, $u_C(t)$; b) the instantaneous voltage on the resistance, $u_R(t)$.

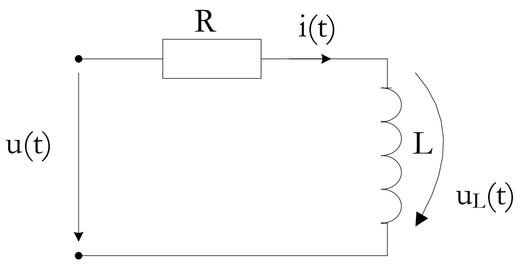


Example 3.3.1.

Solution:

The form of the sinusoidal current is: $i(t) = i_m \sin \omega t$. *We can calculate the circuit impedance,* $Z = \sqrt{R^2 + \left(-\frac{1}{\omega C}\right)^2} = 141.4\ (\Omega)$, *and the supplying voltage amplitude,* $u_m = Z \cdot i_m = 200\ (V)$. *From* $\tan \varphi = \frac{-X_C}{R} = -\frac{1}{\omega RC} \cong -1$, *the voltage phase angle is* $\varphi = -45^0$. *The instantaneous value of the supplying voltage is* $u(t) = 200 \sin\left(100\pi t - \frac{\pi}{4}\right)\ (V)$. *The peak of the capacitance-voltage is* $u_{Cm} = i_m \frac{1}{\omega C} = 141.13\ (V)$ *and the instantaneous voltage on capacitance is* $u_C(t) = 141.13 \sin\left(100\pi t - \frac{\pi}{2}\right)\ (V)$. *The pick of the resistance voltage is* $u_{Rm} = i_m R = 141\ (V)$ *and the instantaneous voltage on the resistance is* $u_R(t) = 141 \sin(100\pi t)\ (V)$. *We take into account that the voltage on capacitance lags the current by an angle of* 90^0, *while the voltage on the resistance is in phase with the current. The phasor diagram was drawn in the figure above.*

Example 3.3.2. The circuit parameters for the RL series circuit below are R=10Ω, L=100 mH, and the instantaneous voltage on the inductance $u_L(t) = 100\, sin(314t + 30^0)V$. Determine: a) the instantaneous value of the current in the circuit, i(t); b) the instantaneous value of the supplying voltage, u(t).

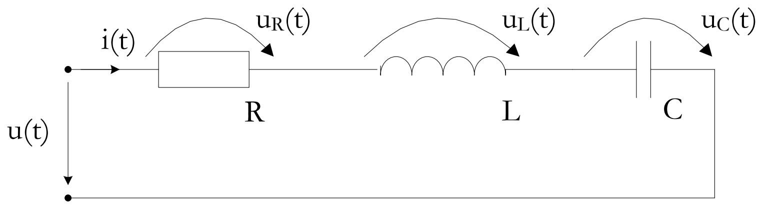


Example 3.3.2.

Solution:

At first, from $u_{Lm} = i_m \cdot X_L$ *we calculate* $i_m = \frac{u_{Lm}}{X_L} = 3.183\ A$. *The current lags the voltage on the inductor by an angle of 90*^{0}, $i(t) = 3.183\sin(314t + 30^0 - 90^0) = 3.183\sin(314t - 60^0)\ (A)$. *The circuit impedance is* $Z = \sqrt{R^2 + (\omega L)^2} = 33\ (\Omega)$, *and the peak of the supplying voltage* $u_m = Z \cdot i_m = 105\ (V)$. *From* $\tan\varphi = \frac{X_L}{R} \cong 3.14$, *the phase angle between the supplying voltage and current is* $\varphi = \arctan 3.14 \cong 72^0$. *The voltage leads the current by the angle of 72*^{0} *and, as a consequence, the instantaneous supplying voltage is* $u(t) = 105\sin(314t + 12^0)\ V$.

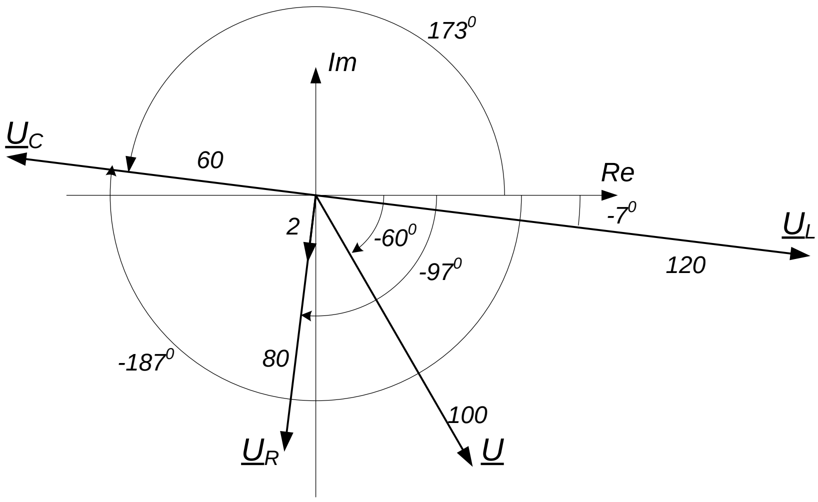
Example 3.3.3. The circuit parameters for the RLC series circuit below are R=40Ω, $L = \frac{6}{10\pi}H$, $C = \frac{1}{3\pi}\mu F$, and the supplying voltage: $u(t) = 100\sqrt{2}\ sin(100\pi t - 60^0)\, V$. Calculate: a) inductive and capacitive reactance and the impedance of the circuit, X_L, X_C, Z; b) rms and instantaneous value of the current, I, i(t); c) rms and instantaneous value of the voltage on the resistance, U_R, $u_R(t)$; d) rms and instantaneous value of the voltage on inductance, U_L, $u_L(t)$; rms and instantaneous value of the voltage on the capacitance, U_C, $u_C(t)$.



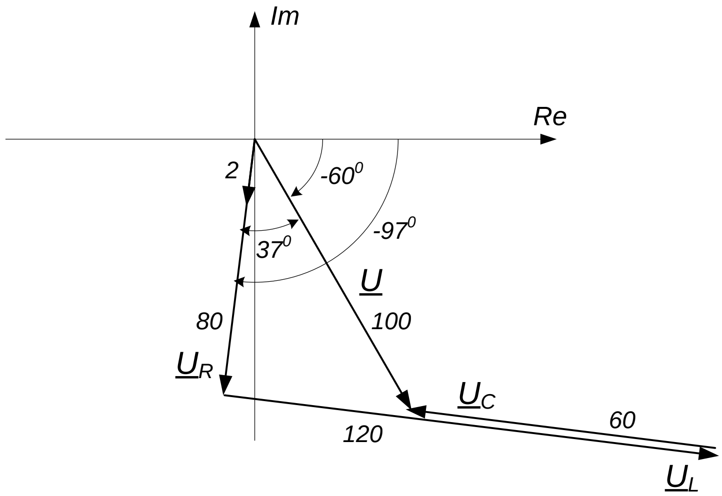
Example 3.3.3.

Solution:

The inductive reactance is $X_L = \omega L = 60\ \Omega$, *the capacitive reactance* $X_C = \frac{1}{\omega C} = 30\ \Omega$, *and the circuit impedance* $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{R^2 + (X_L - X_C)^2} = 50\ \Omega$. *The rms value of the current is* $I = \frac{U}{Z} = 2\ A$, *and from* $\tan\varphi = \frac{X_L - X_C}{R} = \frac{3}{4}$, *the phase angle between supplying voltage and current is* $\varphi = \mathrm{atan}\frac{3}{4} \cong 37^0$. *Because* $X_L > X_C$, *the circuit behaves inductively, i.e. the current lags the voltage by the angle* $\varphi = 37^0$. *The instantaneous value of the current is* $i(t) = 2\sqrt{2}\sin(100\pi t - 60^0 - 37^0) = 2\sqrt{2}\sin(100\pi t - 97^0)\ (A)$. *The rms value of the resistance voltage is* $U_R = I \cdot R = 80\ V$, *and the instantaneous value* $u_R(t) = 80\sqrt{2}\sin(100\pi t - 97^0)\ V$. *The rms value of the inductance voltage is* $U_L = X_L \cdot I = 120\ V$, *and the instantaneous value* $u_L(t) = 120\sqrt{2}\sin(100\pi t - 7^0)\, V$ *(the voltage on the inductance lags the current by an angle of 90*^{0}*). The rms value of the capacitance-voltage is* $U_C = X_C \cdot I = 60\, V$, *and the instantaneous value* $u_C(t) = 60\sqrt{2}\sin(100\pi t - 187^0)\, V$ *(the voltage on the capacitance leads the current by an angle of 90*^{0}*).*



Phasor Diagram



Phasor Diagram