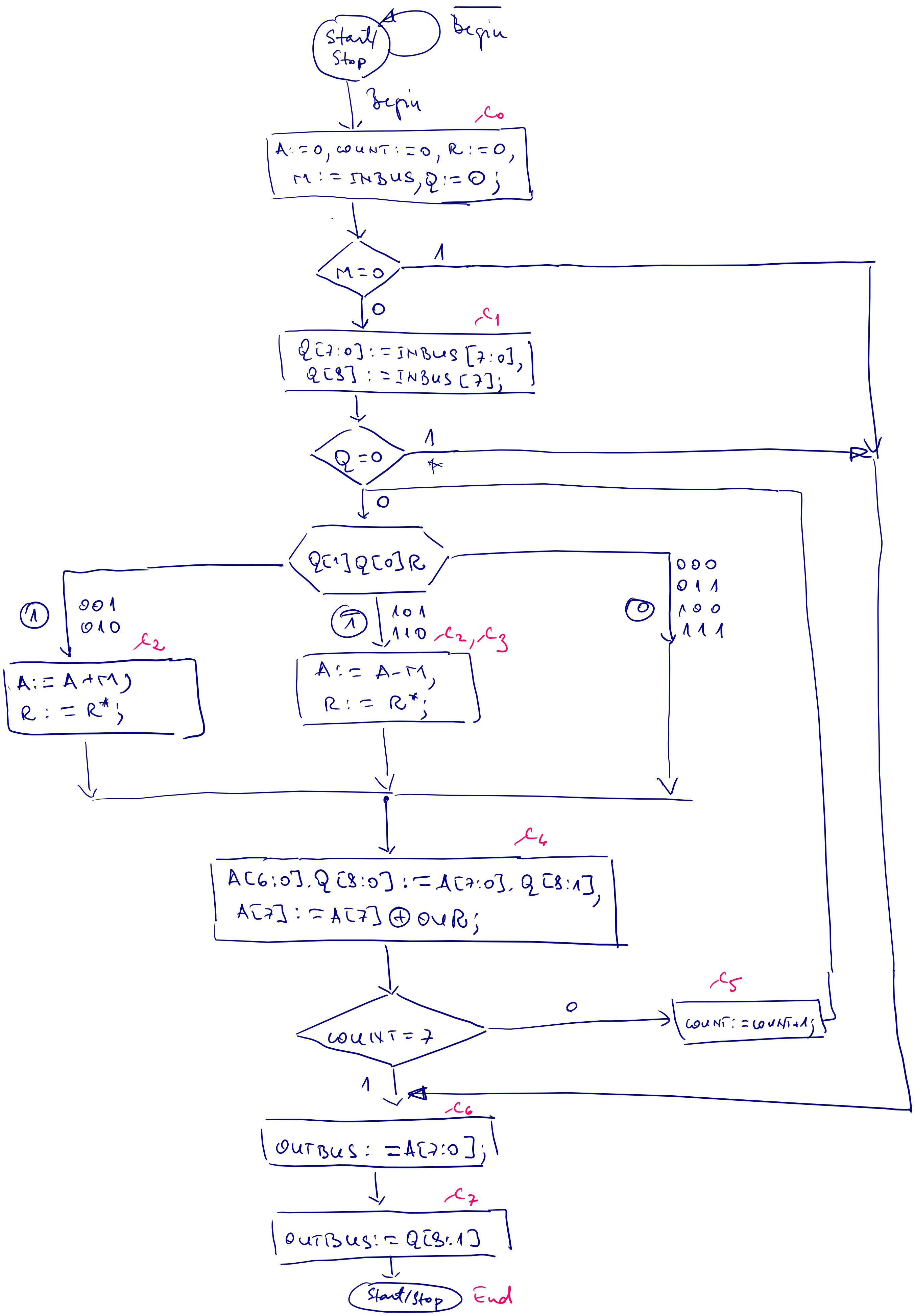
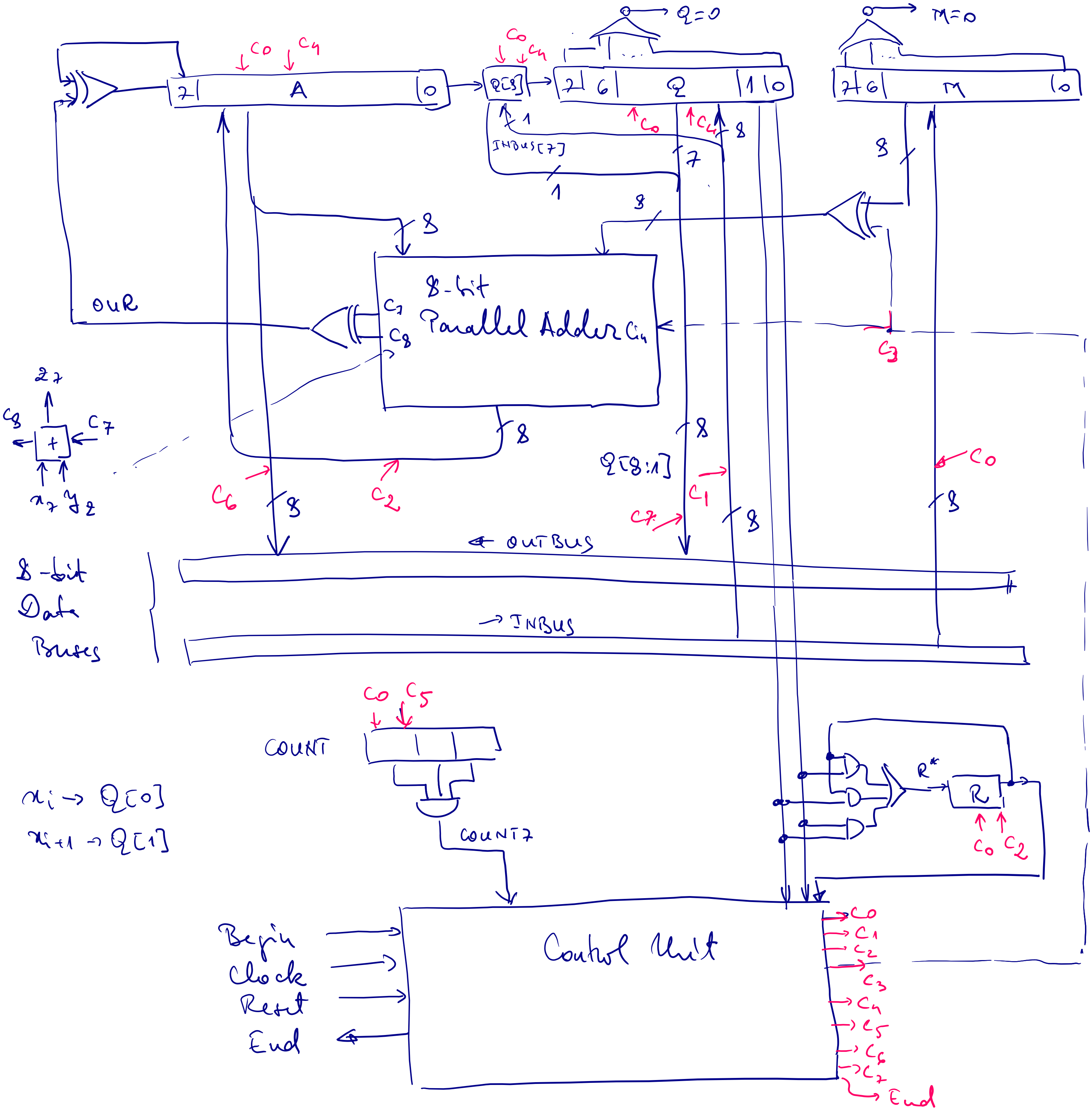
1.2.3 Modified Booth—HW implementation





1.3 Speeding up multiplication with the higher radix

1.3.1 Redundant sets of digits

<u>Radix-2</u> Conventional Redundant

$\{0, 1\}$ $\{\bar{1}, 0, 1\}$

<u>Example</u>

$$\begin{array}{ccccc} 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 0 & \bar{1} & 0 & 1 \end{array} = 1\times 2^0 - 1\times 2^2 + 1\times 2^4 = +13_{ten}$$

Binary representation

$1 \to \begin{smallmatrix}1\\0\end{smallmatrix}$; $\bar{1} \to \begin{smallmatrix}0\\1\end{smallmatrix}$; $0 \to \begin{smallmatrix}0\\0\end{smallmatrix}$

$$\begin{array}{r} 1\ 0\ 0\ 0\ 1\ - \\ 0\ 0\ 1\ 0\ 0 \\ \hline \underbrace{0\ 1\ 1\ 0\ 1}_{13_{ten}} \end{array}$$

Radix 4 Conventional Redundant

$\{0,1,2,3\}$ $\{\bar{2},\bar{1},0,1,2\}$

Example $\underbrace{\overset{4^4}{1}\ \overset{4^3}{0}\ \overset{4^2}{\bar{2}}\ \overset{4^1}{0}\ \overset{4^0}{\bar{1}}} = 1\times4^4 - 2\times4^2 - 1\times4^0 = 256 - 32 - 1 =$

$= 223_{ten}$

Binary representation

$0 \to \begin{smallmatrix}00\\00\end{smallmatrix}$; $1 \to \begin{smallmatrix}01\\00\end{smallmatrix}$; $2 \to \begin{smallmatrix}10\\00\end{smallmatrix}$;

$\bar{1} \to \begin{smallmatrix}00\\01\end{smallmatrix}$; $\bar{2} \to \begin{smallmatrix}00\\10\end{smallmatrix}$

$$\begin{array}{r} 31\ + \\ 64 \\ 128 \\ \hline 223 \end{array}$$

$$\begin{array}{l} 01\ 00\ 00\ 00\ 00\ - \\ 00\ 00\ 10\ 00\ 01 \\ \hline 00\ 11\ 01\ 11\ 11 = 223_{ten} \end{array}$$

1.3.2 Radix-4 Booth algorithm

x_i	x_{i-1}	OP
0	0	0
0	1	1
1	0	$\bar{1}$
1	1	0

x_{i+1}	x_i	x_{i-1}	OP	
0	0	0	0	
0	0	1	1	$+1\times2^i + 0\times2^{i+1}$
0	1	0	1	$-1\times2^i + 1\times2^{i+1} = +1\times2^i$
0	1	1	2	$0\times2^i + 1\times2^{i+1} = +2\times2^i$
1	0	0	$\bar{2}$	$0\times2^i - 1\times2^{i+1} = -2\times2^i$
1	0	1	$\bar{1}$	$1\times2^i - 1\times2^{i+1} = -1\times2^i$
1	1	0	$\bar{1}$	$-1\times2^i + 0\times2^{i+1} = -1\times2^i$
1	1	1	0	$0\times2^i + 0\times2^{i+1} = 0\times2^i$

$0 \to$ no. op

$1 \to +Y$ $\bar{2} \to -2Y$

$\bar{1} \to -Y$

$2 \to +2Y$

$$\begin{array}{r} 105\ - \\ 64 \\ \hline 41 \\ 32 \\ \hline =9 \end{array} \qquad \begin{array}{r} 79\ - \\ 64 \\ \hline 15 \end{array}$$

Example $X = -105$

$Y = -79$

$X = 1\,1\,1\,0\ 1\,0\,0\,1_{SM}$
$= 1\,0\,0\,1\ 0\,1\,1\,1_{C_2}$

$Y = 1\,1\,0\,0\ 1\,1\,1\,1_{SM}$
$= 1\,0\,1\,1\ 0\,0\,0\,1_{C_2}$

COUNT	A	Q	Q[-1]	M
00	0 0000 0000	1001 0111	0	1011 0001
+	0 0100 1111		−M	
	0 0100 1111			
	0 0001 0011	1110 0101	1	
01 +	1 0110 0010		+2M	
	1 0111 0101			
	1 1101 1101	0111 1001	0	
10	+ 1 1011 0001		+M	
	1 1000 1110			
	1 1110 0011	1001 1110	0	
11 +	0 1001 1110		−2M	
	0 1000 0001	1001 1110	0	
	0 0010 0000	0110 0111	1	

M = 11011 0001

−M = 00100 1111

2M = 10110 0010

−2M = 01001 1110

$$\begin{array}{r} -105 \times \\ -79 \\ \hline 945 \\ 735 \\ \hline +8295 \end{array}$$

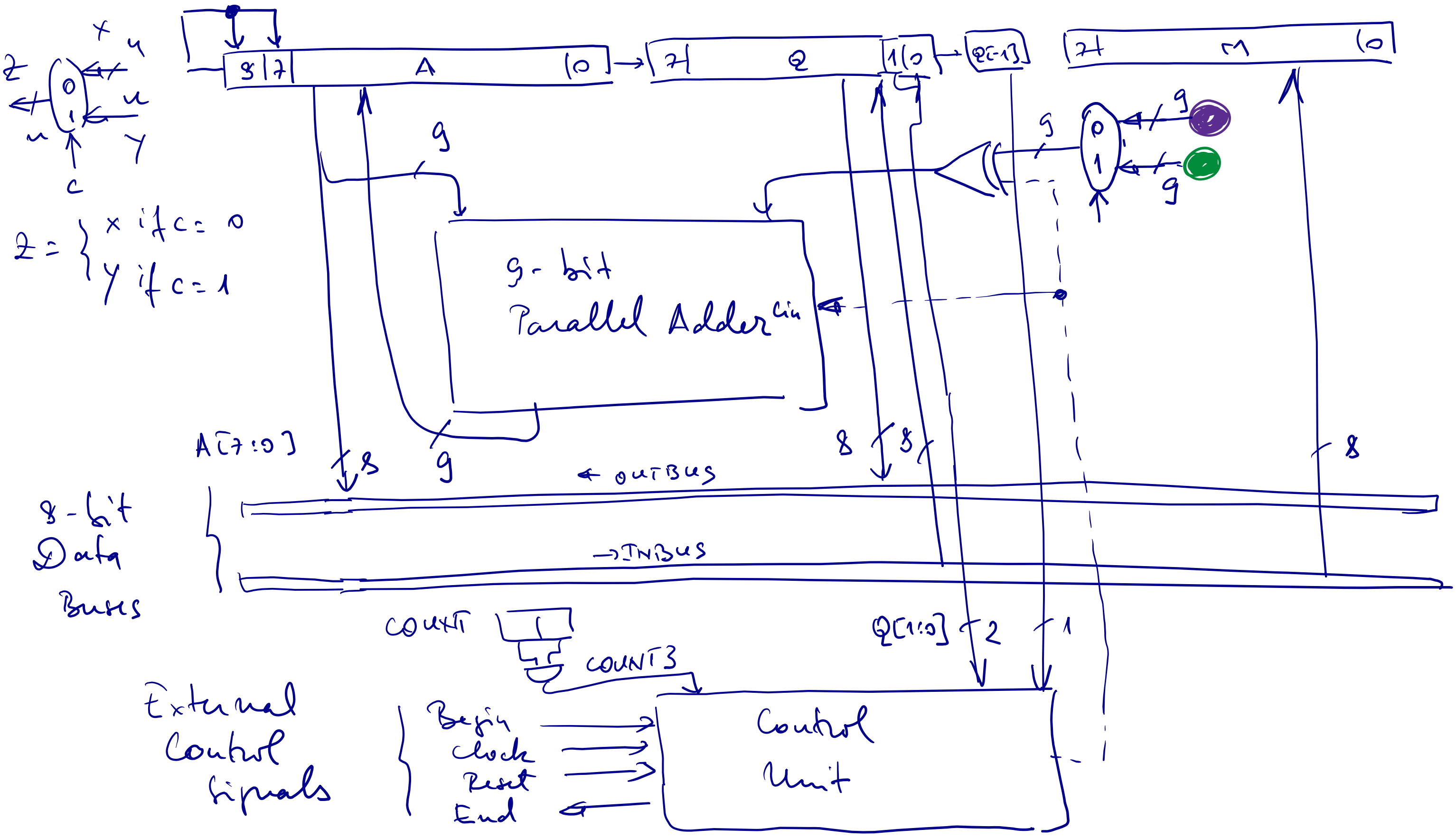
4096

81

$$\begin{array}{r} 39 + \\ 64 \\ 8192 \\ \hline +8295 \end{array}$$

✓

1.3.3 HW Implementation of the Radix-4 Booth algorithm



$z = \begin{cases} x & \text{if } c = 0 \\ y & \text{if } c = 1 \end{cases}$

8-bit Data Buses

External Control Signals