Analiza comportării SRA în RSC. Calculul VRSC

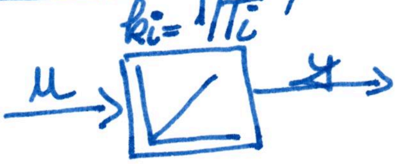
Regimul staționar constant (RSC) se poate stabili într-un sistem dacă:
- sistemul este stabil;
- intrările sistemului iau valoare constantă în timp ($w_\infty = ct$ și $v_\infty = ct$)

$\boxed{\text{Obs}}$: Cu indicele „∞" se marchează valorile de regim staționar constant din sistem.

VRSC aferente unui sistem se pot determina:
- pe cale analitică
- pe cale experimentală

Calculul VRSC pentru diferite elemente de transfer:

① Blocuri de tip integrator (ET-I, ET-PI, ...)

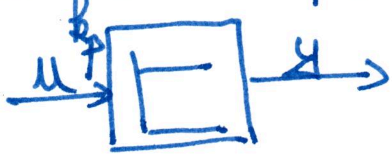


$\Rightarrow \begin{cases} \boxed{u_\infty = 0} \\ \boxed{y_\infty = ct} \end{cases}$

② Blocuri de tip derivativ (ET-D, ET-DT₁, ...)

$\Rightarrow \begin{cases} \boxed{u_\infty = ct} \\ \boxed{y_\infty = 0} \end{cases}$

③ Blocuri de tip proporțional (ET-P, ET-PT₁, ..., ET-PDT₁, ...)



$\boxed{y_\infty = k_p u_\infty}$

$\boxed{\text{TVF}}$: $y_\infty = \lim_{s \to 0} s y(s) = \lim_{s \to 0} s H(s) \cdot u(s)$

$H(s) = \dfrac{b_m s^m + \dots + b_1 s + b_0}{a_m s^n + \dots + a_1 s + a_0}$

$u(s) = \dfrac{1}{s} u_\infty$

$\Rightarrow y_\infty = \lim_{s \to 0} s H(0) \dfrac{1}{s} u_\infty \Rightarrow$

$y_\infty = H(0) u_\infty = \dfrac{b_0}{a_0} u_\infty = k_p u_\infty$

$\Rightarrow \boxed{y_\infty = k_p u_\infty}$

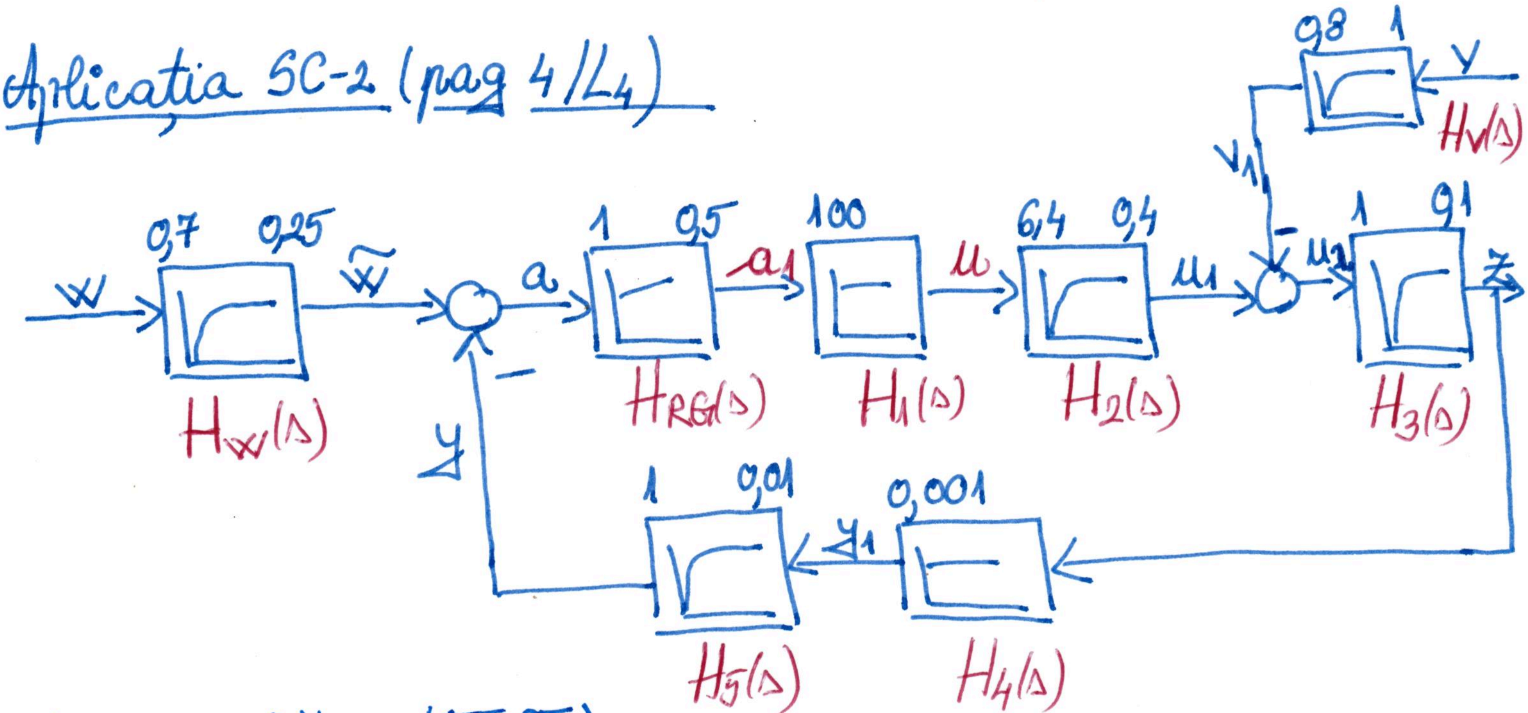
Statism natural:

$$\boxed{\delta_m = \frac{y_\infty}{v_\infty}\bigg|_{w_\infty=0} = \frac{k_v}{1+k_0}} \quad \text{sau} \quad \boxed{\delta_m = \frac{z_\infty}{v_\infty}\bigg|_{w_\infty=0} = \frac{k_v}{1+k_0}}$$

cu $\boxed{k_0 = k_R \cdot k_{PC}}$

- SRA cu RG de tip I, PI, PID : $e_\infty = 0$ și $\delta_m = 0$ (fără statism = astatic)
- SRA cu RG de tip P, PT1, PDT1, ...: $e_\infty \neq 0$ și $\delta_m \neq 0$ (cu statism)

Aplicația SC-2 (pag 4/L4)



$$H_w(s) = \frac{0{,}7}{1+0{,}25s} \quad (\text{ET-PT}_1)$$

$$H_{RG}(s) = \frac{k(1+sT)}{sT} = \frac{1(1+0{,}5s)}{0{,}5s} = \frac{1+0{,}5s}{0{,}5s} \quad (\text{ET-PI})$$

$$H_1(s) = 100 \quad (\text{ET-P})$$

$$H_v(s) = \frac{0{,}8}{1+s} \quad (\text{ET-PT}_1)$$

$$H_2(s) = \frac{6{,}4}{1+0{,}4s} \quad (\text{ET-PT}_1)$$

$$H_3(s) = \frac{1}{1+0{,}1s} \quad (\text{ET-PT}_1)$$

$$H_4(s) = 0{,}001 \quad (\text{ET-P})$$

$$H_5(s) = \frac{1}{1+0{,}01s} \quad (\text{ET-PT}_1)$$

a) Pentru $w_\infty = 10$ și $v_\infty = 1250$ să se determine VRSC ale celorlalte mărimi ale sistemului

$w_\infty = 10$ și $v_\infty = 1250$

$\tilde{w}_\infty = 0{,}7\, w_\infty = 7 \Rightarrow \boxed{\tilde{w}_\infty = 7}$

RG-PI : $\boxed{a_\infty = 0}$

$\left.\begin{array}{l} a_\infty = 0 \\ a_\infty = \tilde{w}_\infty - y_\infty \end{array}\right\} \Rightarrow \tilde{w}_\infty - y_\infty = 0 \Rightarrow y_\infty = \tilde{w}_\infty \Rightarrow \boxed{y_\infty = 7}$

$u_\infty = 100\, a_{1\infty} \Rightarrow \boxed{u_\infty = 1250}$

$u_{1\infty} = 6{,}4\, u_\infty = 640\, a_{1\infty} \Rightarrow \boxed{u_{1\infty} = 8000}$

$v_{1\infty} = 0{,}8\, v_\infty \Rightarrow \boxed{v_{1\infty} = 1000}$

$u_{2\infty} = u_{1\infty} - v_{1\infty} = 640\, a_{1\infty} - 1000 \Rightarrow \boxed{u_{2\infty} = 7000}$

$z_\infty = 1 \cdot u_{2\infty} = 640\, a_{1\infty} - 1000 \Rightarrow \boxed{z_\infty = 7000}$

$y_{1\infty} = 0{,}001 \cdot z_\infty = 0{,}64\, a_{1\infty} - 1 \Rightarrow \boxed{y_{1\infty} = 7}$

$\left.\begin{array}{l} y_\infty = 1 \cdot y_{1\infty} = 0{,}64\, a_{1\infty} - 1 \\ y_\infty = 7 \end{array}\right\} \Rightarrow 0{,}64\, a_{1\infty} - 1 = 7 \Rightarrow 0{,}64\, a_{1\infty} = 8 \Rightarrow$

$\boxed{a_{1\infty} = 12{,}5}$

b) Să se determine VRSC w_∞ care asigură la ieșire valoarea $z_\infty = 6000$ pentru $v_\infty = 1000$; de asemenea să se calculeze VRSC ale celorlalte mărimi ale sistemului.

$z_\infty = 6000$ și $v_\infty = 1000$

RG-PI : $\boxed{a_\infty = 0}$

$\left.\begin{array}{l} a_\infty = 0 \\ a_\infty = \tilde{w}_\infty - y_\infty \end{array}\right\} \Rightarrow y_\infty = \tilde{w}_\infty$

$\tilde{w}_\infty = 0{,}7\, w_\infty$

$\left.\begin{array}{l} y_\infty = 1 \cdot y_{1\infty} \\ y_{1\infty} = 0{,}001\, z_\infty = 6 \end{array}\right\} \Rightarrow \boxed{y_\infty = 6}$

$\Rightarrow \tilde{w}_\infty = 6$

$\tilde{w}_\infty = 0{,}7\, w_\infty$

$\Rightarrow w_\infty = \dfrac{6}{0{,}7} \Rightarrow \boxed{w_\infty = 8{,}5714}$

$u_\infty = 100\, a_{1\infty} \Rightarrow \boxed{u_\infty = 1062{,}5}$

$u_{1\infty} = 6{,}4\, u_\infty = 640\, a_{1\infty} \Rightarrow \boxed{u_{1\infty} = 6800}$

$u_{2\infty} = u_{1\infty} - v_{1\infty} \Rightarrow \boxed{u_{2\infty} = 6000}$

$V_{1\infty} = 0{,}8 \cdot V_{\infty} = 800 \Rightarrow \boxed{V_{1\infty} = 800}$

$u_{2\infty} = 640\, a_{1\infty} - 800$

$z_{\infty} = 1 \cdot u_{2\infty} = 640\, a_{1\infty} - 800 \Rightarrow 640\, a_{1\infty} - 800 = 6000 \Rightarrow a_{1\infty} = \frac{6800}{640} \Rightarrow$

$\boxed{a_{1\infty} = 10{,}625}$

c) Să se determine f.d.t. $H_{zw}(s)$ și $H_{zv}(s)$ și să se analizeze stabilitatea sistemului. Afectează creșterea de 10 ori a valorii lui k_R statismul sistemului?

$$H_{zw}(s) = \left.\frac{Z(s)}{W(s)}\right|_{v=0} = H_w(s) \cdot \frac{H_{RG}(s)H_1(s)H_2(s)H_3(s)}{1 + H_{RG}(s)H_1(s)H_2(s)H_3(s)H_4(s)H_5(s)}$$

$$H_{zw}(s) = \frac{0{,}7}{1+0{,}25s} \cdot \frac{\frac{1+0{,}5s}{0{,}5s} \cdot 100 \cdot \frac{6{,}4}{1+0{,}4s} \cdot \frac{1}{1+0{,}1s}}{1 + \frac{640(1+0{,}5s)}{0{,}5s(1+0{,}4s)(1+0{,}1s)} \cdot 0{,}001 \cdot \frac{1}{1+0{,}01s}} =$$

$$= \frac{0{,}7}{1+0{,}25s} \cdot \frac{\frac{640(1+0{,}5s)}{\cancel{0{,}5s(1+0{,}4s)(1+0{,}1s)}}}{\frac{0{,}5s(1+0{,}4s)(1+0{,}1s)(1+0{,}01s) + 0{,}64(1+0{,}5s)}{\cancel{0{,}5s(1+0{,}4s)(1+0{,}1s)}(1+0{,}01s)}} =$$

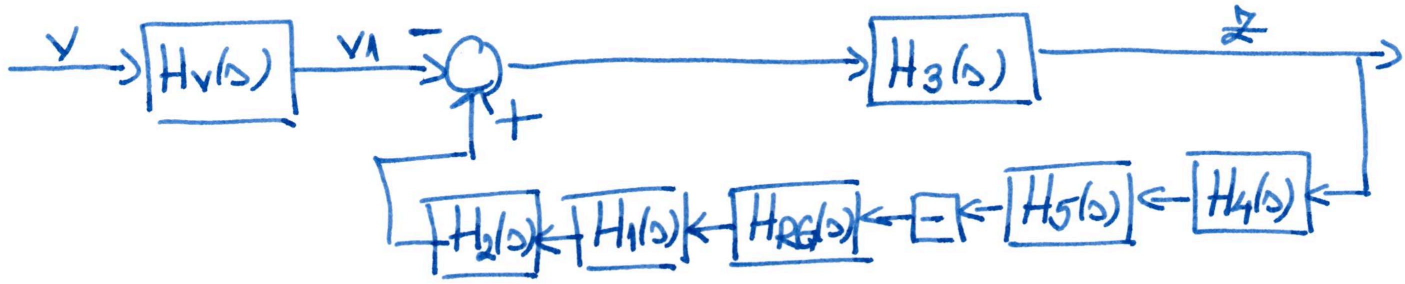
$$= \frac{0{,}7}{1+0{,}25s} \cdot \frac{640(1+0{,}5s)(1+0{,}01s)}{0{,}5\left[(0{,}4s^2+s)(0{,}001s^2+0{,}11s+1)+1{,}28(1+0{,}5s)\right]}$$

$$= \frac{0{,}7}{1+0{,}25s} \cdot \frac{1280(1+0{,}5s)(1+0{,}01s)}{0{,}0004s^4+0{,}044s^3+0{,}4s^2+0{,}001s^3+0{,}11s^2+s+1{,}28+0{,}64s}$$

$$= \frac{896(1+0{,}01s)(1+0{,}5s)}{(1+0{,}25s)(0{,}0004s^4+0{,}045s^3+0{,}51s^2+1{,}64s+1{,}28)} \Rightarrow$$

$$\boxed{H_{zw}(s) = \frac{896(1+0{,}01s)(1+0{,}5s)}{(1+0{,}25s)(0{,}0004s^4+0{,}045s^3+0{,}51s^2+1{,}64s+1{,}28)}}$$

$$H_{ZV}(s) = \frac{Z(s)}{V(s)}\Big|_{w=0} = -H_V(s)\cdot\frac{H_3(s)}{1-H_3(s)\left[(-)H_{RG}(s)H_1(s)H_2(s)H_4(s)H_5(s)\right]}$$



$$\Rightarrow H_{ZV}(s) = -H_V(s)\cdot\frac{H_3(s)}{1+H_{RG}(s)H_1(s)H_2(s)H_3(s)H_4(s)H_5(s)} =$$

$$= -\frac{0{,}8}{1+s}\cdot\frac{\cancel{\dfrac{1}{1+0{,}1s}}}{\dfrac{\cancel{0{,}5}(0{,}0004s^4+0{,}045s^3+0{,}51s^2+1{,}64s+1{,}28)}{\cancel{0{,}5}s(1+0{,}4s)\cancel{(1+0{,}1s)}(1+0{,}01s)}} =$$

$$= -\frac{0{,}8s(1+0{,}01s)(1+0{,}4s)}{(1+s)(0{,}0004s^4+0{,}045s^3+0{,}51s^2+1{,}64s+1{,}28)} \Rightarrow$$

$$\boxed{H_{ZV}(s) = -\frac{0{,}8s(1+0{,}01s)(1+0{,}4s)}{(1+s)(0{,}0004s^4+0{,}045s^3+0{,}51s^2+1{,}64s+1{,}28)}}$$

$$\Delta(s) = 1+H_{RG}(s)H_1(s)H_2(s)H_3(s)H_4(s)H_5(s) = 0{,}5(0{,}0004s^4+0{,}045s^3+0{,}51s^2+1{,}64s+1{,}28)$$

$\Rightarrow$ pentru simplitate se dă factor comun 0,0004 $\Rightarrow$

$$\Delta(s) = 0{,}0002(s^4+112{,}5s^3+1275s^2+4100s+3200)$$

$$\Delta_1(s) = s^4+112{,}5s^3+1275s^2+4100s+3200 = a_4s^4+a_3s^3+a_2s^2+a_1s+a_0$$

Sunt impuse condițiile necesare: $a_4 = 1 > 0$

$a_3 = 112{,}5 > 0$

$a_2 = 1275 > 0$

$a_1 = 4100 > 0$

$a_0 = 3200 > 0$

$$n=4 \Rightarrow H=\begin{bmatrix} a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_0 & 0 \\ 0 & a_3 & a_1 & 0 \\ 0 & a_4 & a_2 & a_0 \end{bmatrix} = \begin{bmatrix} 112{,}5 & 4100 & 0 & 0 \\ 1 & 1275 & 3200 & 0 \\ 0 & 112{,}5 & 4100 & 0 \\ 0 & 1 & 1275 & 3200 \end{bmatrix}$$

Sunt impuse <u>condițiile de stabilitate</u> (condițiile suficiente):

$$\left.\begin{array}{l} \det(H_1) = 112{,}5 > 0 \\ \det(H_2) = 112{,}5 \cdot 1275 - 4100 = 139337{,}5 > 0 \\ \det(H_3) = 112{,}5 \cdot 1275 \cdot 4100 - 112{,}5^2 \cdot 3200 \cdot - 4100^2 = 530783750 > 0 \\ \det(H_4) = \underbrace{a_0}_{>0} \underbrace{\det(H_3)}_{>0} \Rightarrow \det(H_4) > 0 \end{array}\right\} \Rightarrow$$

sistemul este <u>stabil</u>

$$Z(s) = H_{ZW}(s)\, W(s) + H_{ZV}(s) \cdot V(s) \xRightarrow{TVF} Z_\infty = \underset{H_{ZW}(0)}{\underset{\|}{k_W}}\, W_\infty + \underset{H_{ZV}(0)}{\underset{\|}{k_V}}\, V_\infty$$

$$\left.\begin{array}{l} k_W = H_{ZW}(0) = \dfrac{896}{1{,}28} = 700 \\ k_V = H_{ZV}(0) = 0 \end{array}\right\} \Rightarrow RG\text{-}PI: \boxed{Z_\infty = 700\, W_\infty + 0 \cdot V_\infty}$$

dacă $H_{RG}(s) = \dfrac{10(1+0{,}5s)}{0{,}5s}$

$$H_{ZW}(s) = \frac{0{,}7}{1+0{,}25s} \cdot \frac{\dfrac{10(1+0{,}5s)}{0{,}5s} \cdot 100 \cdot \dfrac{6{,}4}{1+0{,}4s} \cdot \dfrac{1}{1+0{,}1s}}{1 + \dfrac{6400(1+0{,}5s)}{0{,}5s(1+0{,}4s)(1+0{,}1s)} \cdot 0{,}001 \cdot \dfrac{1}{1+0{,}01s}} =$$

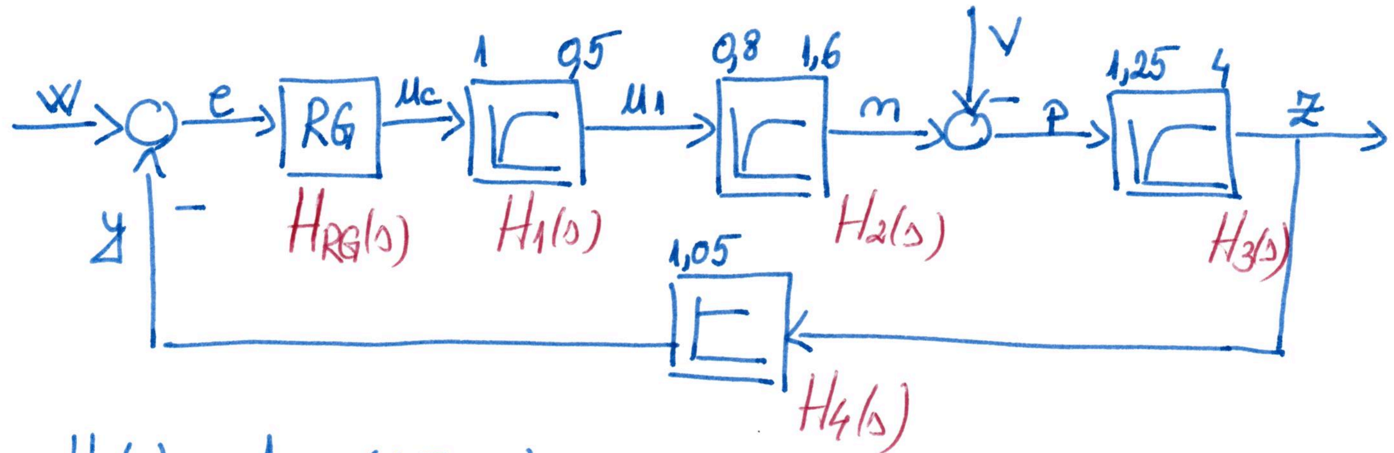
$$= \frac{0{,}7}{1+0{,}25s} \cdot \frac{\dfrac{6400(1+0{,}5s)}{\cancel{0{,}5s(1+0{,}4s)(1+0{,}1s)}}}{\dfrac{0{,}5s(1+0{,}4s)(1+0{,}1s)(1+0{,}01s) + 6{,}4(1+0{,}5s)}{\cancel{0{,}5s(1+0{,}4s)(1+0{,}1s)}(1+0{,}01s)}} =$$

$$= \frac{0{,}7}{1+0{,}25s} \cdot \frac{6400(1+0{,}5s)(1+0{,}01s)}{0{,}5s(1+0{,}4s)(1+0{,}1s)(1+0{,}01s) + 6{,}4(1+0{,}5s)} \Rightarrow$$

$$H_{ZW}(0) = 0{,}7 \cdot \frac{\cancel{6400}^{1000}}{\cancel{6{,}4}} = 700 \Rightarrow$$ creșterea lui k_R nu modifică statismul sistemului $\Rightarrow \boxed{\gamma_M = 0}$

=> oricât ar fi valoarea lui k_R datorită faptului că RG are componentă I statismul natural va fi zero.

Aplicația SC-4 (pag 5/L4)



$H_1(s) = \dfrac{1}{1+0{,}5s}$ (ET-PT_1)

$H_2(s) = \dfrac{0{,}8}{1+1{,}6s}$ (ET-PT_1)

$H_3(s) = \dfrac{1{,}25}{1+4s}$ (ET-PT_1)

$H_4(s) = 1{,}05$ (ET-P)

a) În cazul RG-PDT_1, pentru $w_\infty = 7$ și $v_\infty = 1{,}25$ să se determine toate VRSC din sistem.

$w_\infty = 7$ și $v_\infty = 1{,}25$

RG-PDT_1 : $\left.\begin{aligned} &e_\infty \neq 0 \\ &e_\infty = w_\infty - y_\infty \end{aligned}\right\} \Rightarrow e_\infty = 7 - y_\infty \Rightarrow \boxed{y_\infty = 7 - e_\infty} \Rightarrow \boxed{y_\infty = 4{,}2127}$

$u_{c\infty} = H_{RG}(0)\, e_\infty = 2e_\infty \Rightarrow \boxed{u_{c\infty} = 5{,}5746}$

$u_{1\infty} = 1 \cdot u_{c\infty} = 2e_\infty \Rightarrow \boxed{u_{1\infty} = 5{,}5746}$

$m_\infty = 0{,}8 \cdot u_{1\infty} = 1{,}6\, e_\infty \Rightarrow \boxed{m_\infty = 4{,}4597}$

$p_\infty = m_\infty - v_\infty = 1{,}6 e_\infty - 1{,}25 \Rightarrow \boxed{p_\infty = 3{,}2097}$

$z_\infty = 1{,}25 p_\infty = 1{,}25(1{,}6 e_\infty - 1{,}25) = 2e_\infty - 1{,}5625 \Rightarrow \boxed{z_\infty = 4{,}0121}$

$\left.\begin{aligned} &y_\infty = 1{,}05 z_\infty = 1{,}05(2e_\infty - 1{,}5625) = 2{,}1 e_\infty - 1{,}6406 \\ &y_\infty = 7 - e_\infty \end{aligned}\right\} \Rightarrow 2{,}1 e_\infty - 1{,}6406 = 7 - e_\infty$

$\Rightarrow 3{,}1 e_\infty = 8{,}6406 \Rightarrow$

$\boxed{e_\infty = 2{,}7873}$

b) Să se calculeze statismul natural (γ_n) în ieșirea de măsură y.

$$\gamma_n = \frac{y_\infty}{v_\infty}\Big|_{w_\infty=0} = \frac{k_N}{1+k_0}, \quad k_0 = k_R \cdot k_{pc}$$

<u>Varianta 1</u>: $\gamma_n = \frac{y_\infty}{v_\infty}\Big|_{w_\infty=0}$

$w_\infty = 0$ și $v_\infty = 1{,}25$

$$\text{RG-P}\Delta T_1:\ \left.\begin{array}{l} e_\infty \neq 0 \\ e_\infty = w_\infty - y_\infty \end{array}\right\} \Rightarrow e_\infty = 0 - y_\infty \Rightarrow e_\infty = -y_\infty$$

$u_{c\infty} = 2e_\infty$

$u_{1\infty} = 1 \cdot u_{c\infty} = 2e_\infty$

$m_\infty = 0{,}8 u_{1\infty} = 1{,}6 e_\infty$

$p_\infty = m_\infty - v_\infty = 1{,}6 e_\infty - 1{,}25$

$z_\infty = 1{,}25 p_\infty = 2e_\infty - 1{,}5625$

$$\left.\begin{array}{l} y_\infty = 1{,}05 z_\infty = 2{,}1 e_\infty - 1{,}6406 \\ y_\infty = -e_\infty \end{array}\right\} \Rightarrow 2{,}1 e_\infty - 1{,}6406 = -e_\infty \Rightarrow 3{,}1 e_\infty = 1{,}6406 \Rightarrow$$

$e_\infty = 0{,}5292 \Rightarrow y_\infty = -0{,}5292 \Rightarrow$

$$\gamma_n = \frac{y_\infty}{v_\infty} = -\frac{0{,}5292}{1{,}25} \Rightarrow \boxed{\gamma_n = -0{,}4234}$$

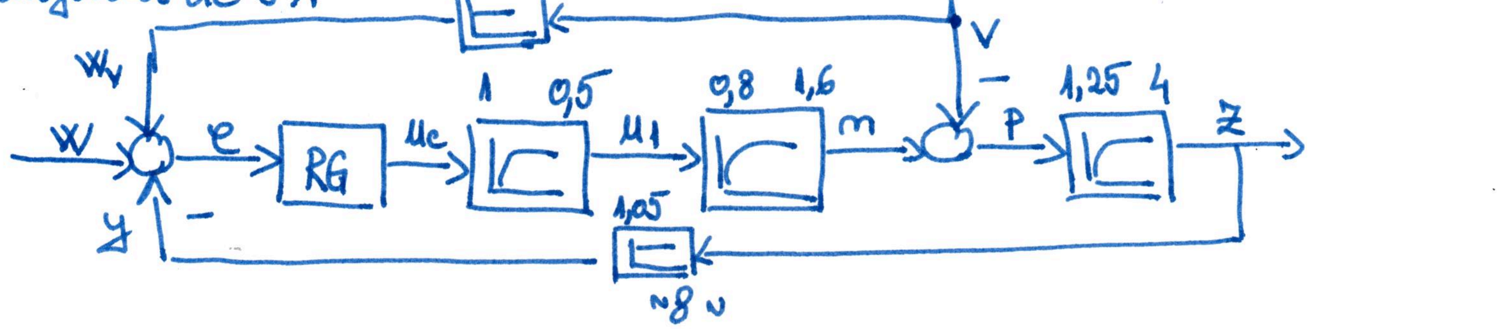
<u>Varianta 2</u>: $\gamma_n = \frac{k_N}{1+k_0} = \frac{k_N}{1+k_R k_{pc}} = -\frac{1{,}3125}{1+2\cdot 1{,}05} = -\frac{1{,}3125}{3{,}1} \Rightarrow$

$k_{pc} = 1 \cdot 0{,}8 \cdot 1{,}25 \cdot 1{,}05 = 1{,}05$

$$\boxed{\gamma_n \cong -0{,}4234}$$

$k_N = -1{,}25 \cdot 1{,}05 = -1{,}3125$

c) Să se calculeze expresia parametrului k_{bcv} în cadrul unei structuri de tip feedforward în raport cu perturbația v care asigură un statism artificial de 5%.



RG-PΔT1: $e_\infty \neq 0$

$$\left.\begin{array}{l} e_\infty = w_\infty + w_{v\infty} - y_\infty \\ w_{v\infty} = k_{bcv} v_\infty \end{array}\right\} \Rightarrow \left.\begin{array}{l} e_\infty = w_\infty + k_{bcv} v_\infty - y_\infty \\ y_\infty = 1{,}05 z_\infty \end{array}\right\} \Rightarrow e_\infty = w_\infty + k_{bcv} v_\infty - 1{,}05 z_\infty$$

$u_{c\infty} = 2 e_\infty \quad \left(H_{RG}(0) \cdot e_\infty \right)$

$u_{1\infty} = 1 \cdot u_{c\infty} = 2 e_\infty$

$m_\infty = 0{,}8\, u_{1\infty} = 1{,}6 e_\infty$

$p_\infty = m_\infty - v_\infty = 1{,}6 e_\infty - v_\infty$

$z_\infty = 1{,}25\, p_\infty = 2 e_\infty - 1{,}25 v_\infty = 2(w_\infty + k_{bcv} v_\infty - 1{,}05 z_\infty) - 1{,}25 v_\infty \Rightarrow$

$z_\infty = 2 w_\infty + 2 k_{bcv} v_\infty - 2{,}1 z_\infty - 1{,}25 v_\infty \Rightarrow 3{,}1 z_\infty = 2 w_\infty + (2 k_{bcv} - 1{,}25) v_\infty$

$\Rightarrow z_\infty = \frac{2}{3{,}1} w_\infty + \frac{(2 k_{bcv} - 1{,}25)}{3{,}1} v_\infty \Rightarrow z_\infty = 0{,}6452 w_\infty + \underbrace{(0{,}6452 k_{bcv} - 0{,}4032)}_{\gamma_a} v_\infty$

$$\left.\begin{array}{l} \gamma_a = 5\% \Rightarrow \gamma_a = 0{,}05 \\ \gamma_a = 0{,}6452 k_{bcv} - 0{,}4032 \end{array}\right\} \Rightarrow 0{,}6452 k_{bcx} = 0{,}4532 \Rightarrow$$

$\boxed{k_{bcv} = 0{,}7024}$