

Proiectarea și Analiza Algoritmilor

S II - 2025

Curs 4

Arbori Binari Echilibrați. Arbori Binari Optimi

Analiza căutărilor în Arbori Binari Ordonăți

Programatorul nu are control asupra creșterii arborelui și nu poate estima cu precizie forma acestuia

Căutarea are complexitate între $O(\log_2 n)$ pentru un arbore perfect echilibrat (de înălțime minimă) și $O(n)$ pentru un arbore degenerat într-o listă

Înlocuind un arbore perfect echilibrat cu un arbore ordonat oarecare, statistic efortul de căutare crește cu 39%

Acest 39% impun o limită practică pentru efortul adițional de calcul pentru reorganizarea structurii după inserția cheilor

Un rol esențial în decizia de echilibrare îl joacă raportul dintre numărul de accese (căutări) și numărul de inserții / suprimări

Cu cât acest raport este mai mare cu atât reorganizarea structurii este mai justificată

Arbori AVL

Din analiza căutării în ABO este evident că o procedură de inserție / suprimare care restaurează structura de arbore astfel încât să fie tot timpul perfect echilibrată nu este viabilă, deoarece activitatea de restructurare este foarte complexă

Putem defini termenul “echilibrat” într-o manieră mai puțin strictă

Pot rezulta tehnici mai simple de reorganizare a structurilor arbori binari ordonați, al căror cost deteriorează într-o măsură mai redusă performanța de căutare

1962 - Adelson - Velskii, Landis

Un arbore binar ordonat este (parțial) echilibrat dacă și numai dacă pentru oricare not al arborelui, înălțimile celor doi subarbori diferă cu cel mult 1

Se numesc arbori AVL

Această definiție:

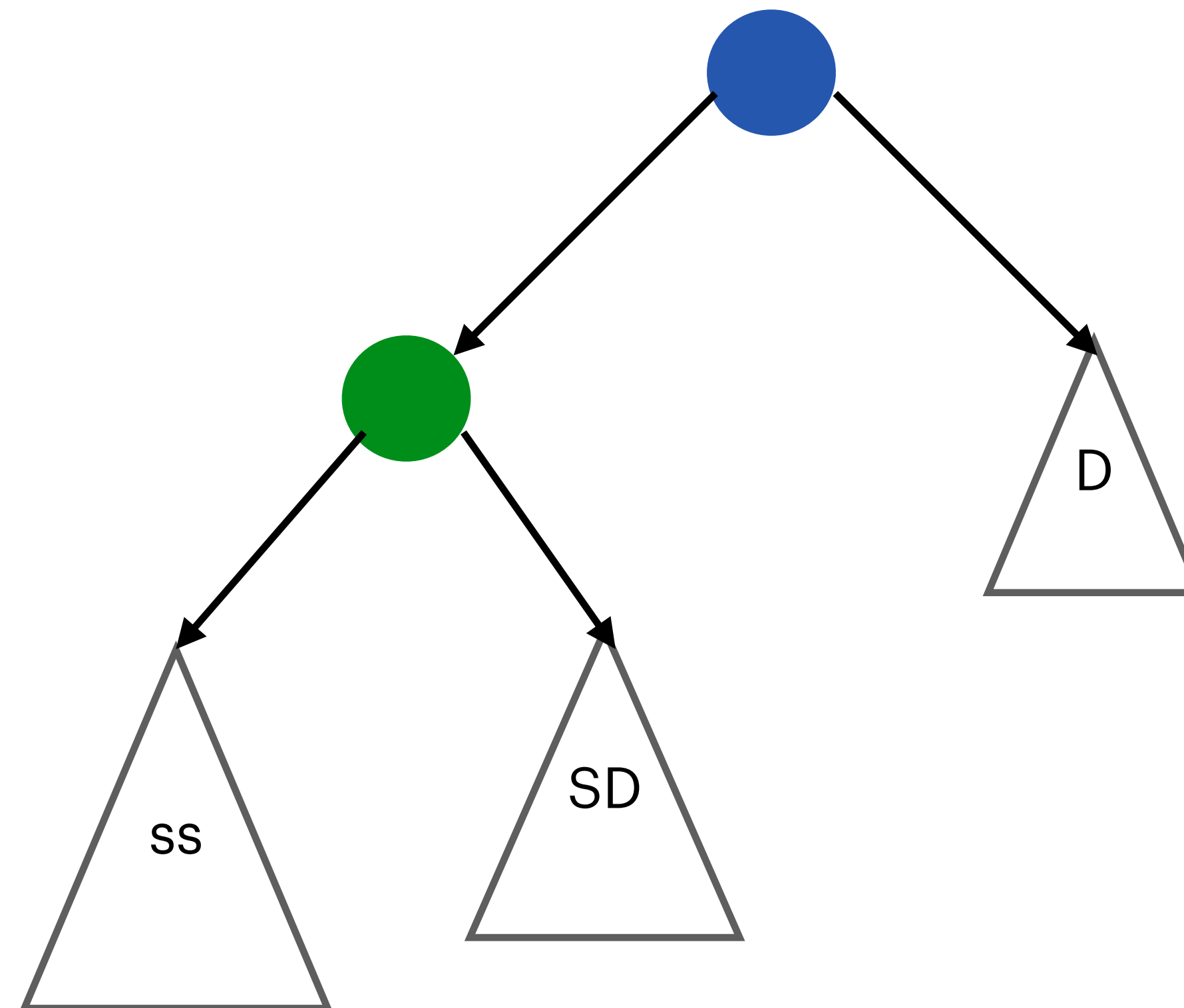
- Este simplă

- Conduce la o procedură viabilă de re-echilibrare

- Asigură o lungime medie a drumului de căutare practic identică cu cea a unui arbore perfect echilibrat

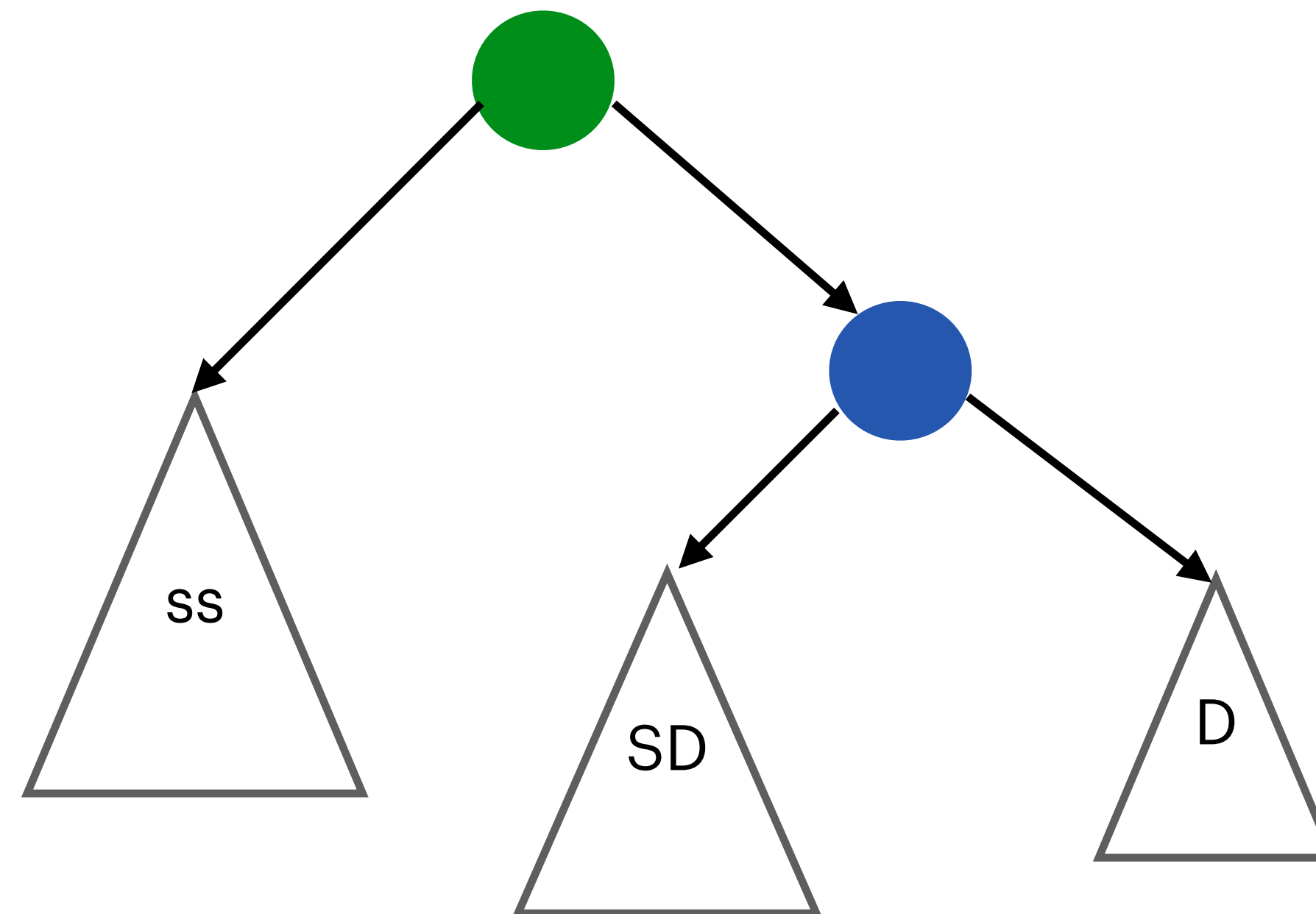
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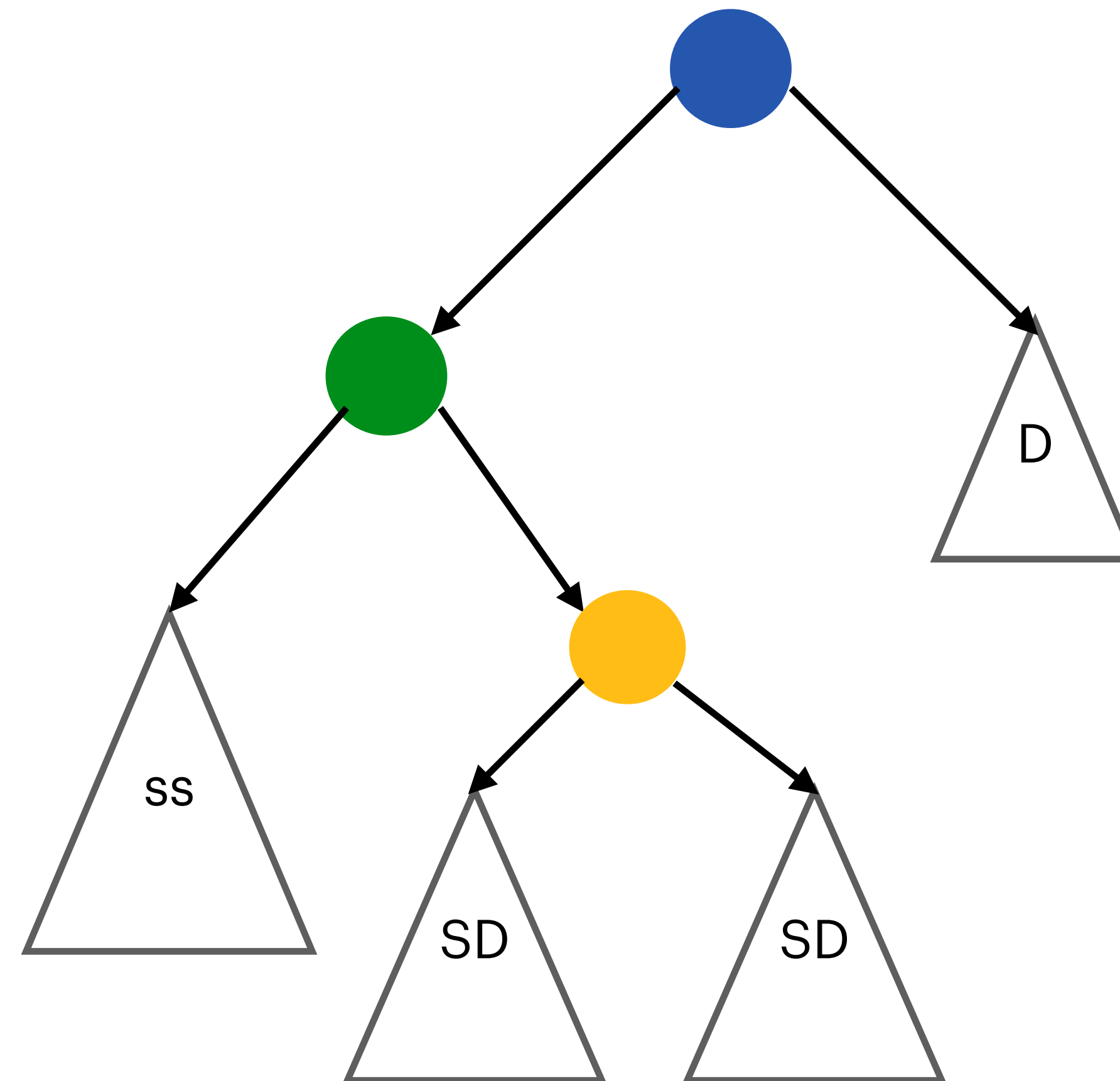
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Caz 2 - stânga



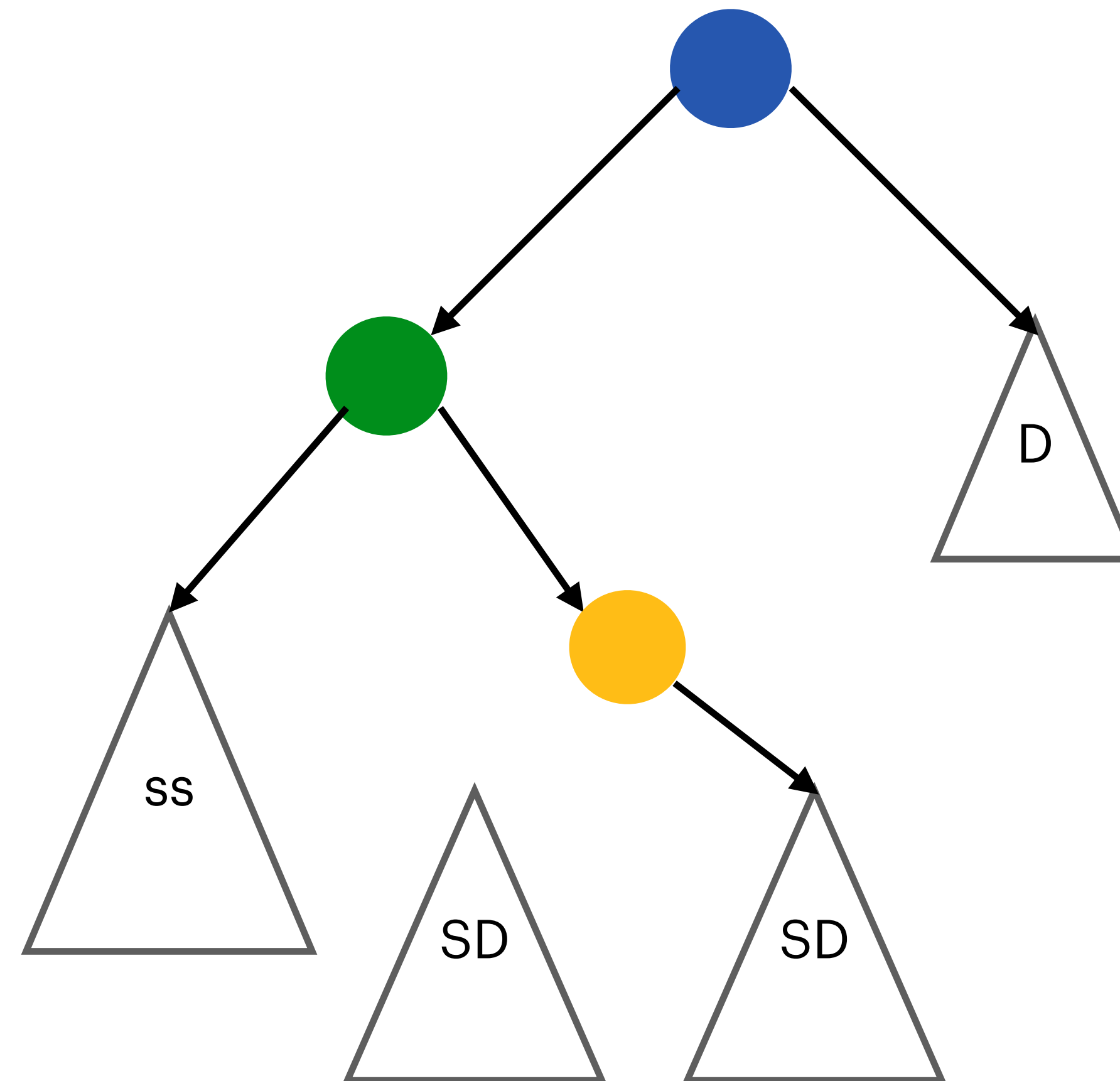
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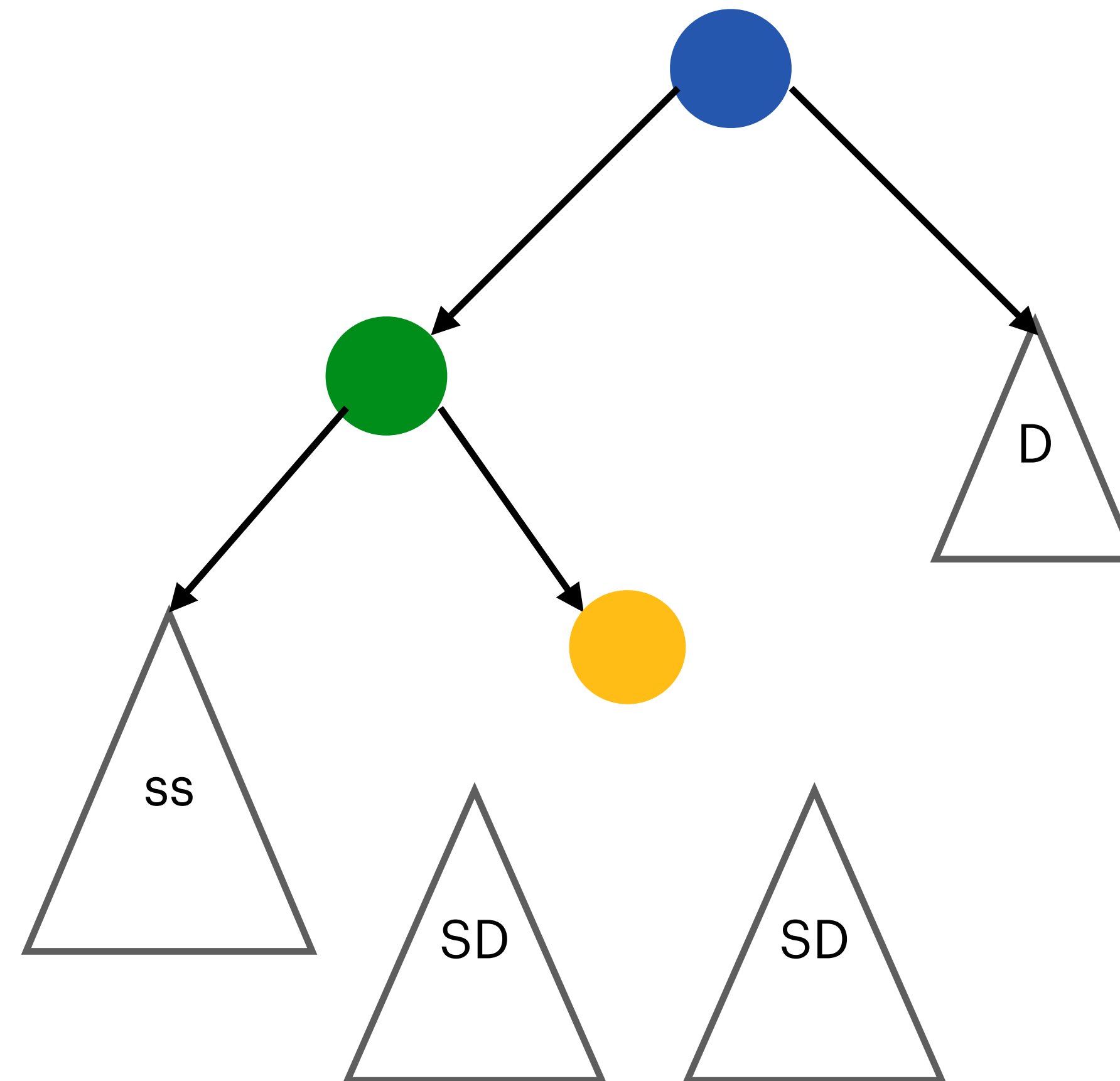
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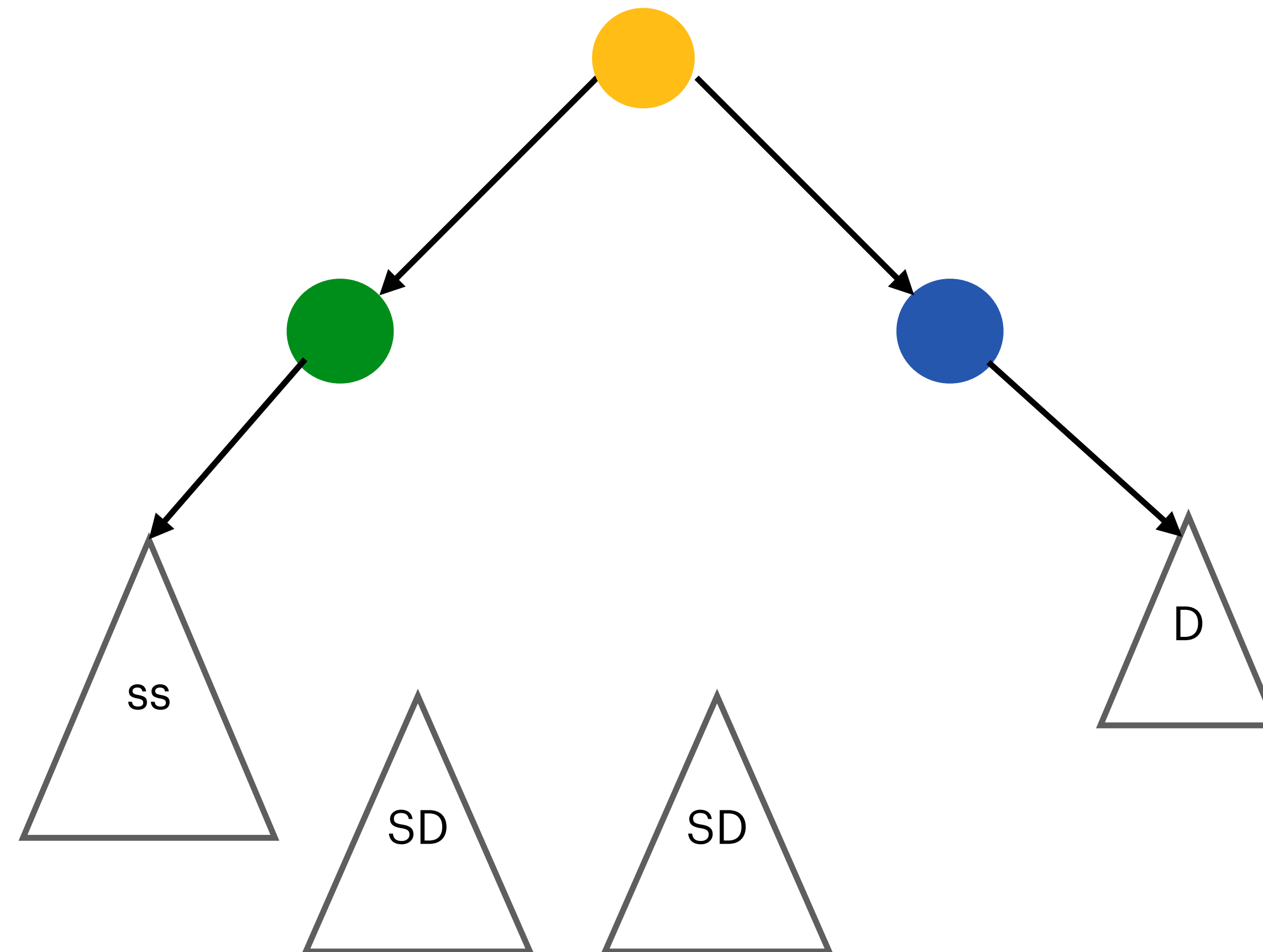
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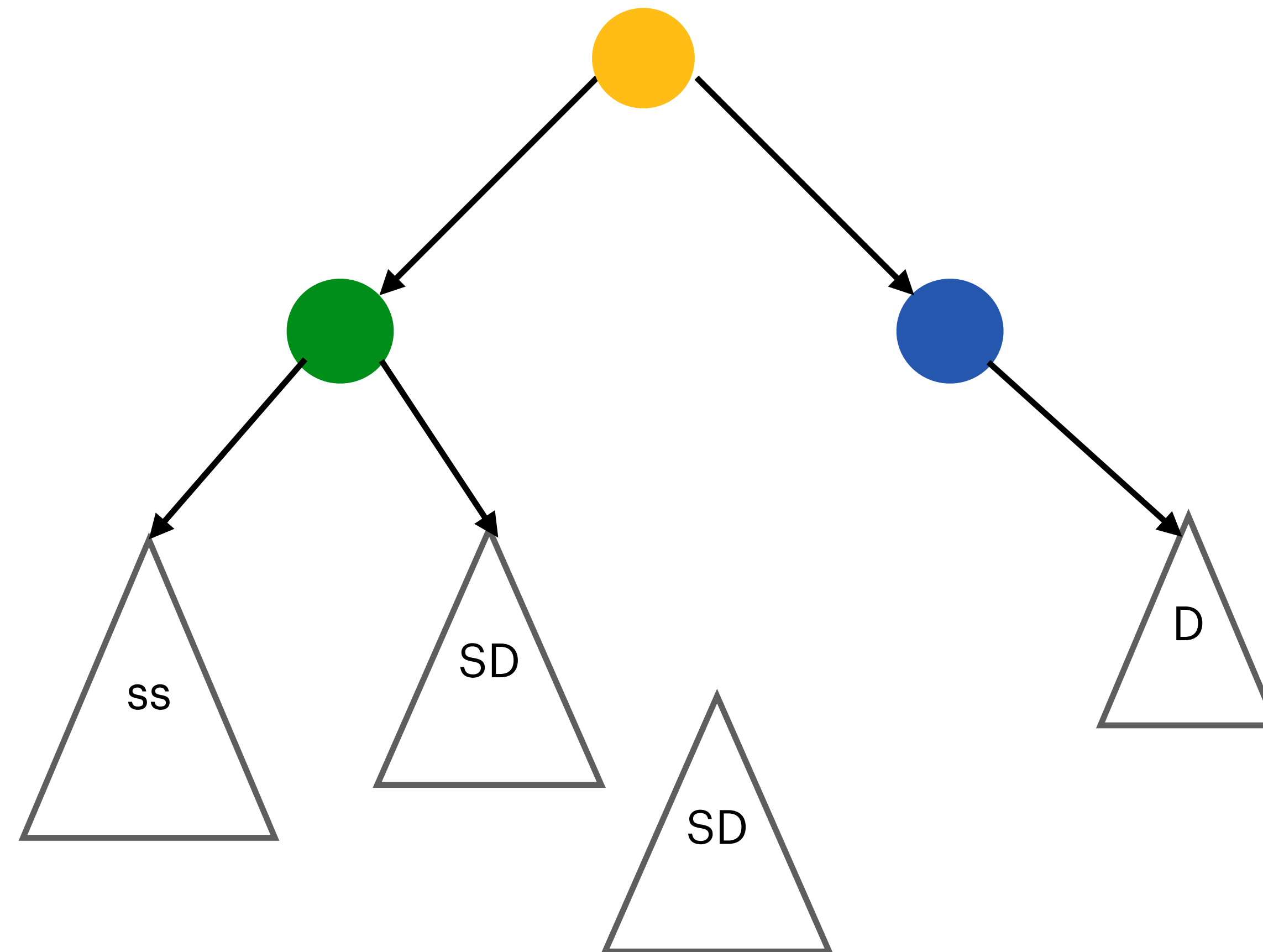
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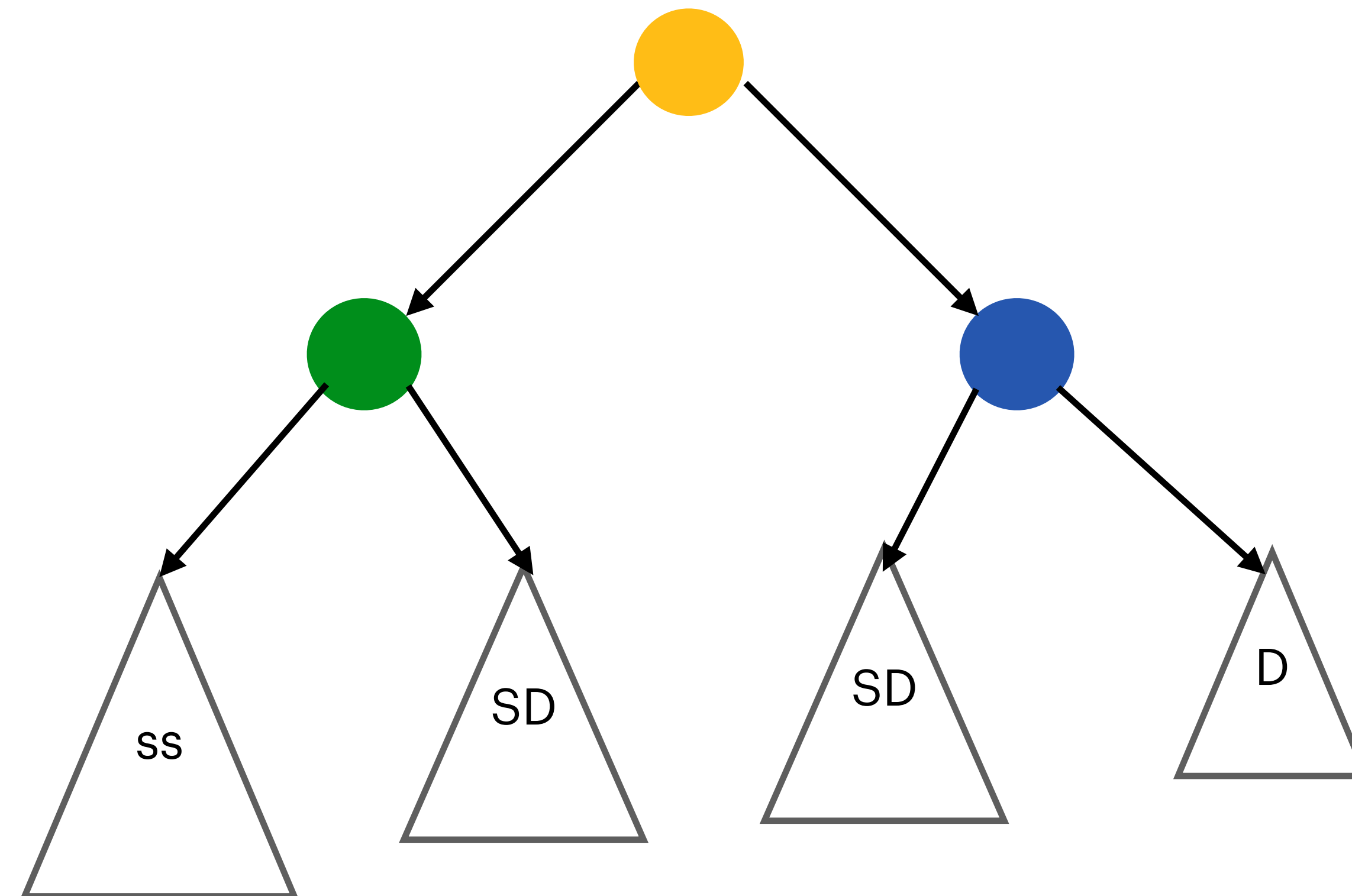
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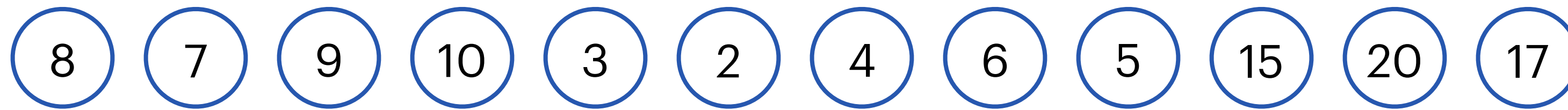


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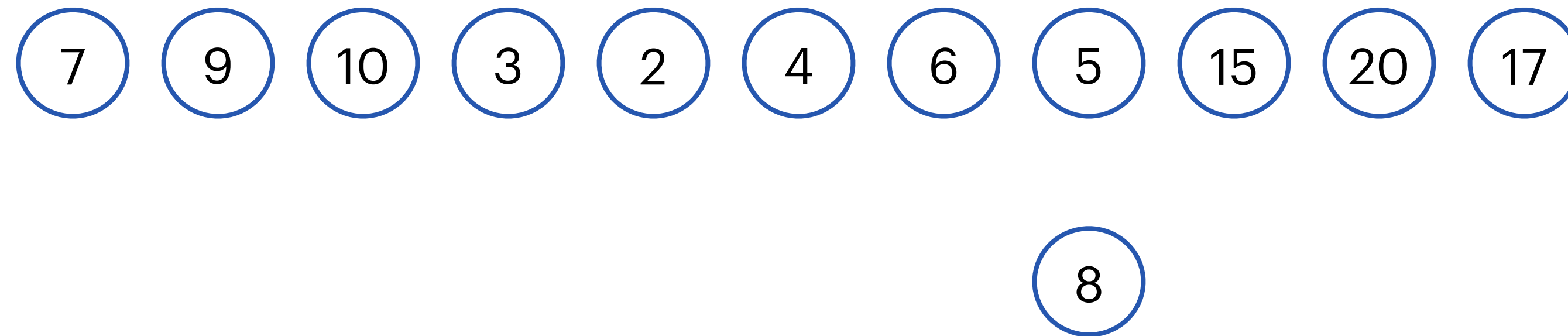
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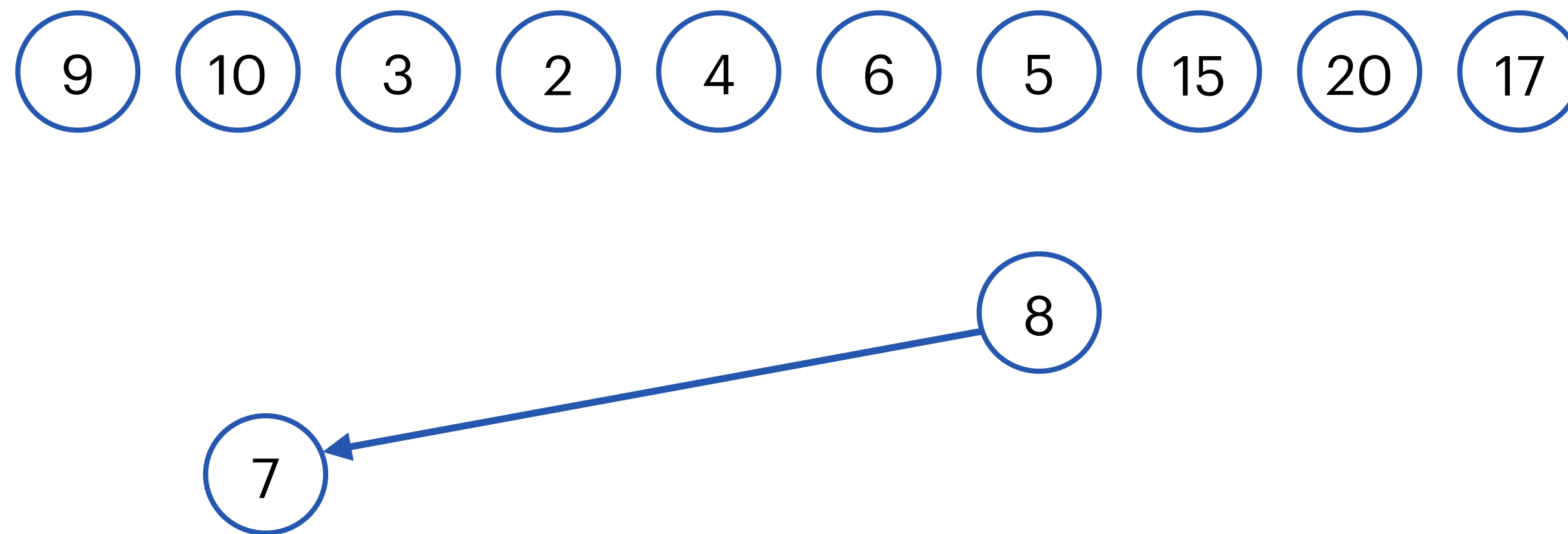
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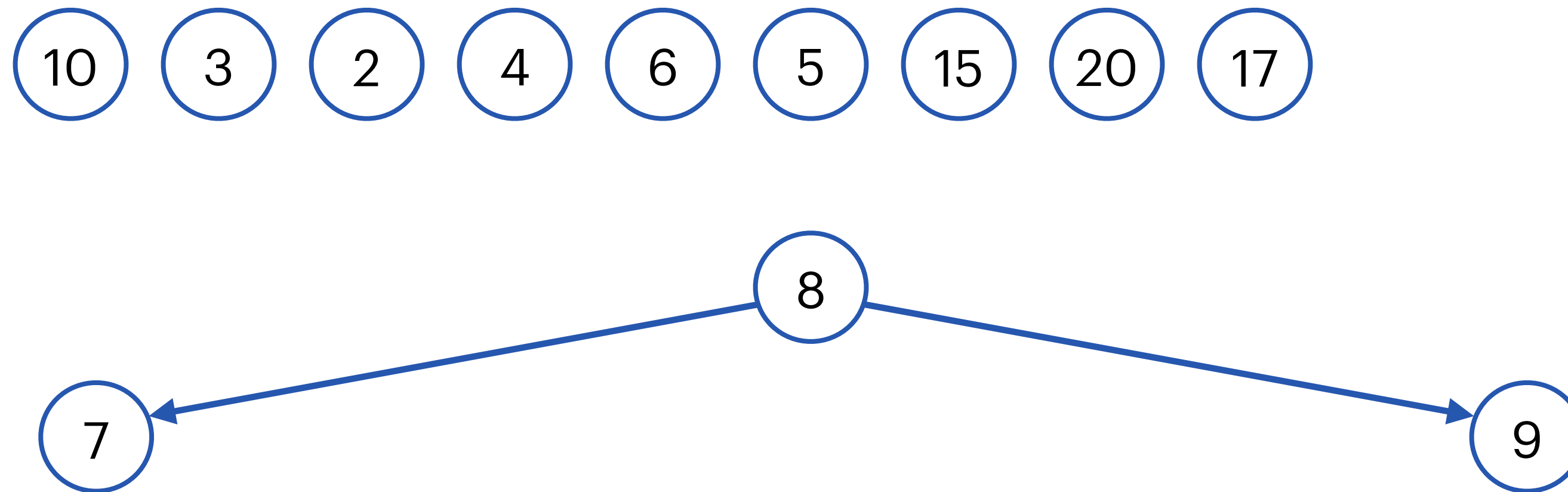
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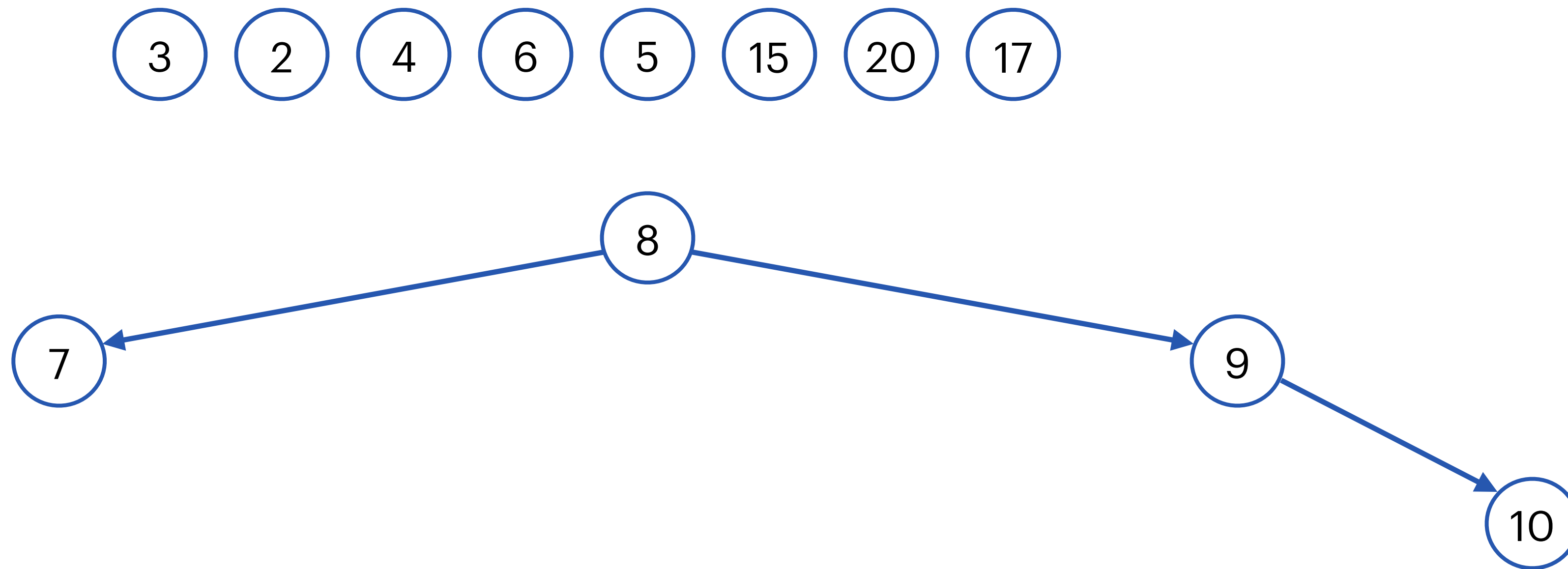
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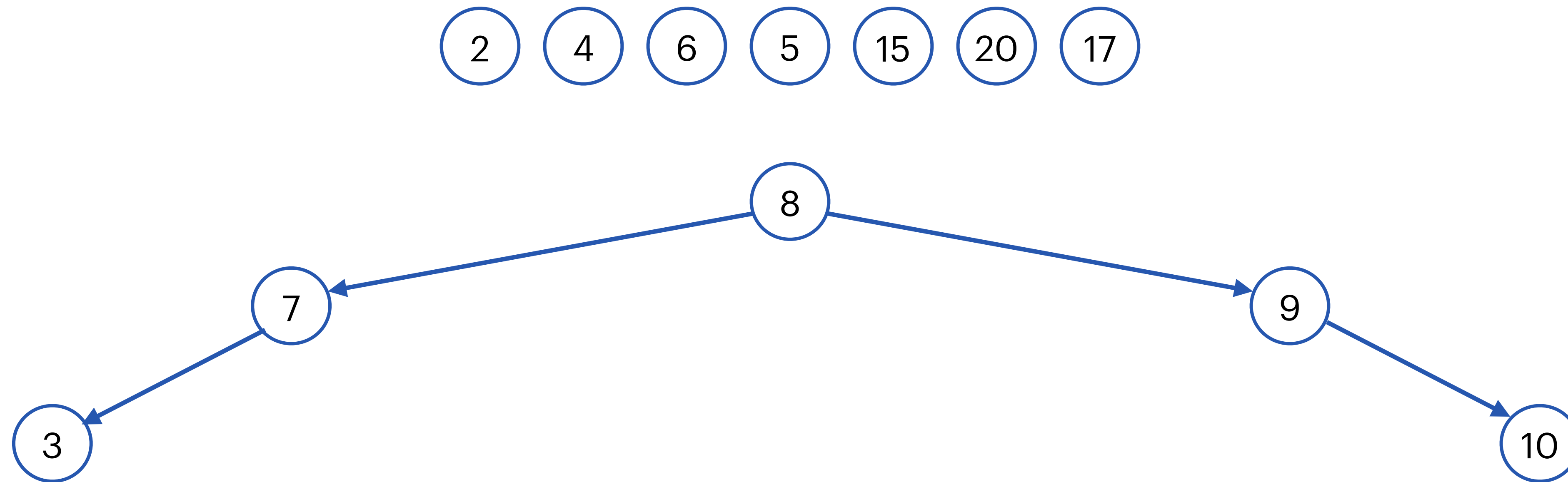
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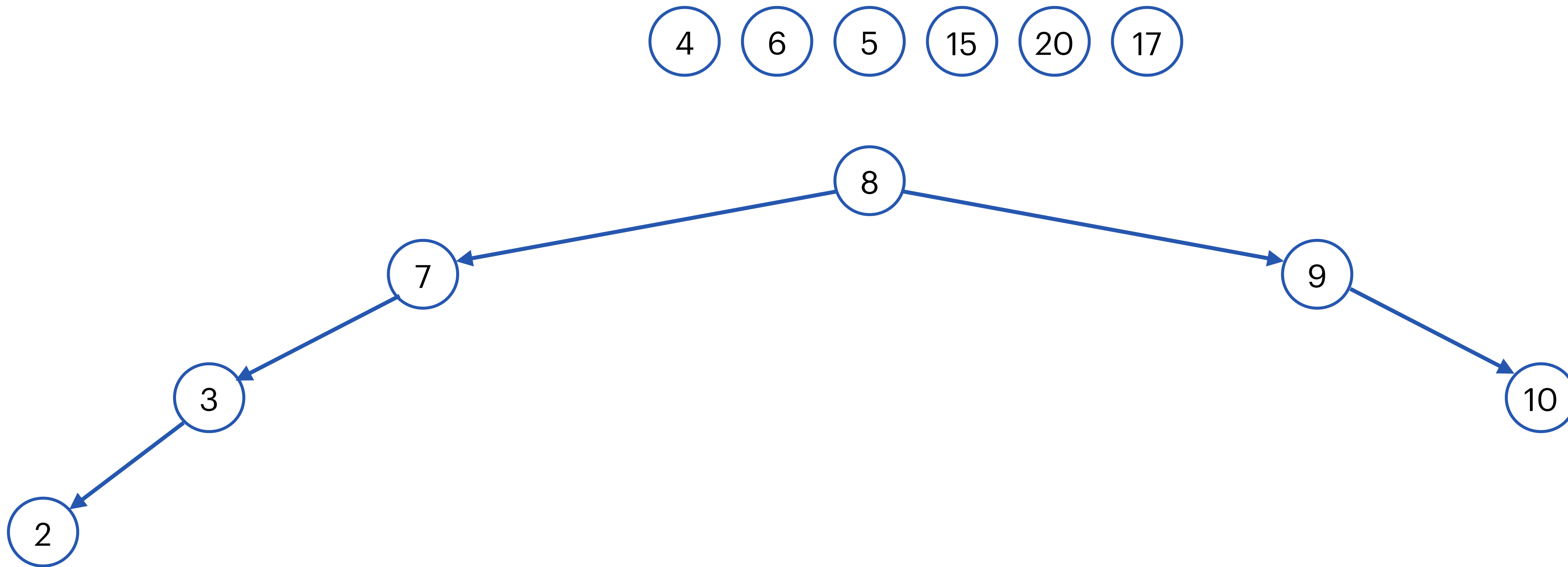
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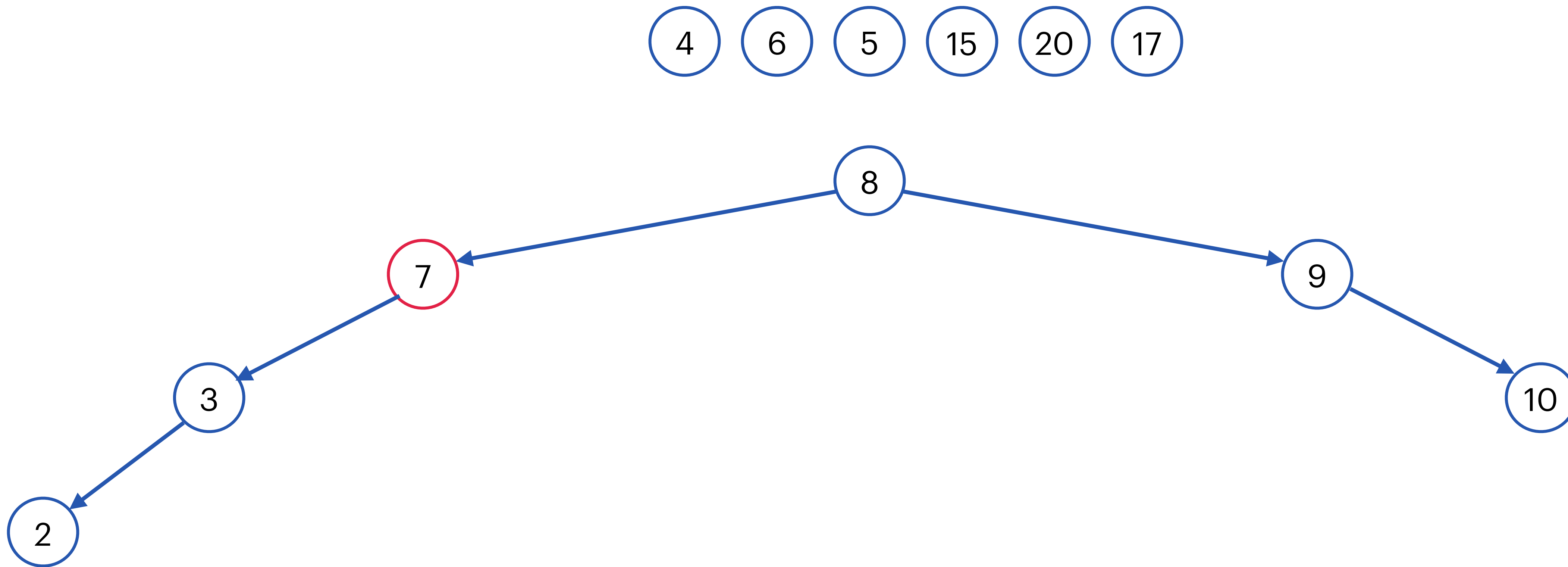
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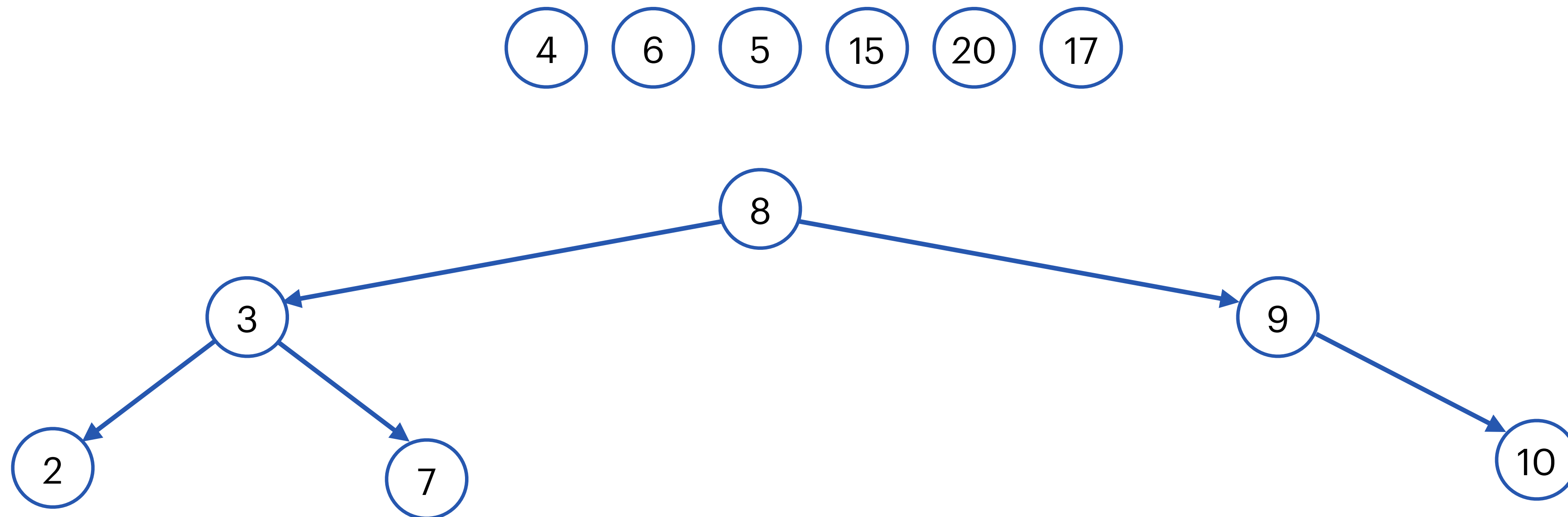
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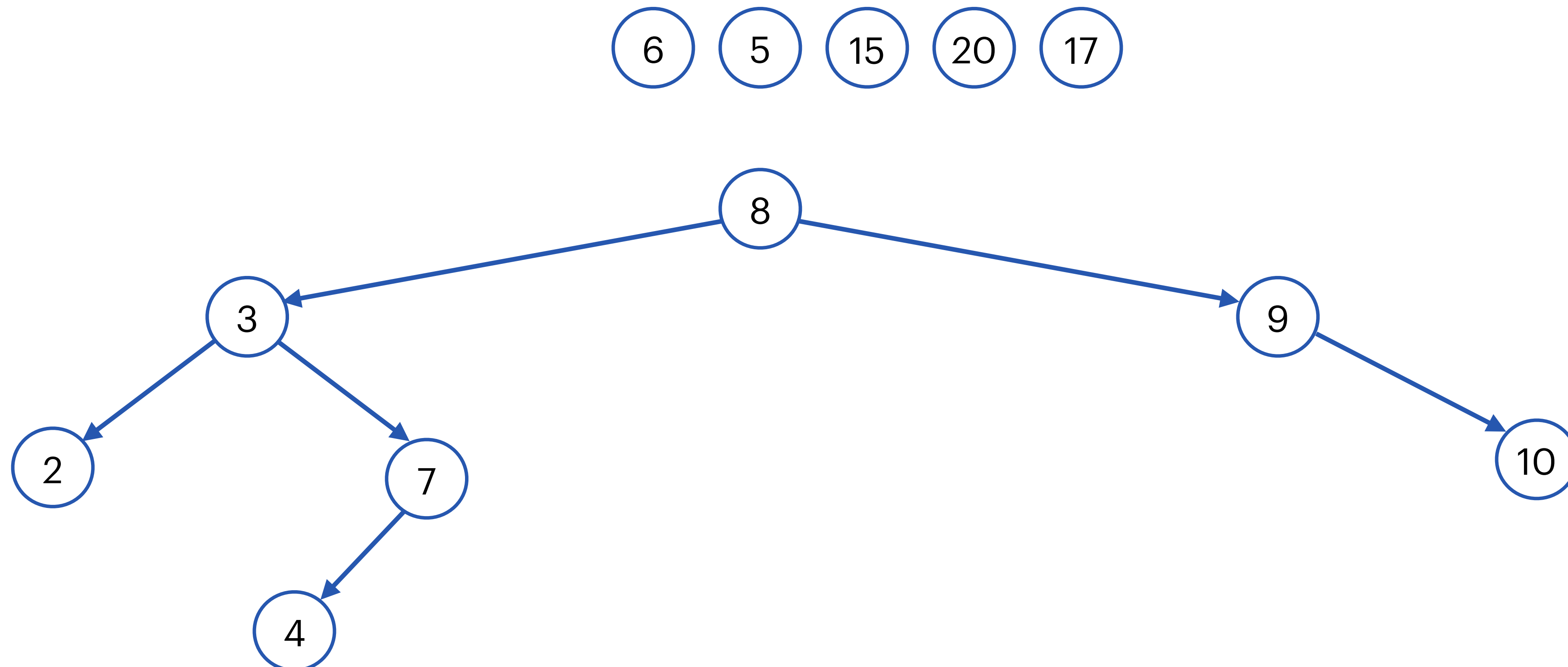
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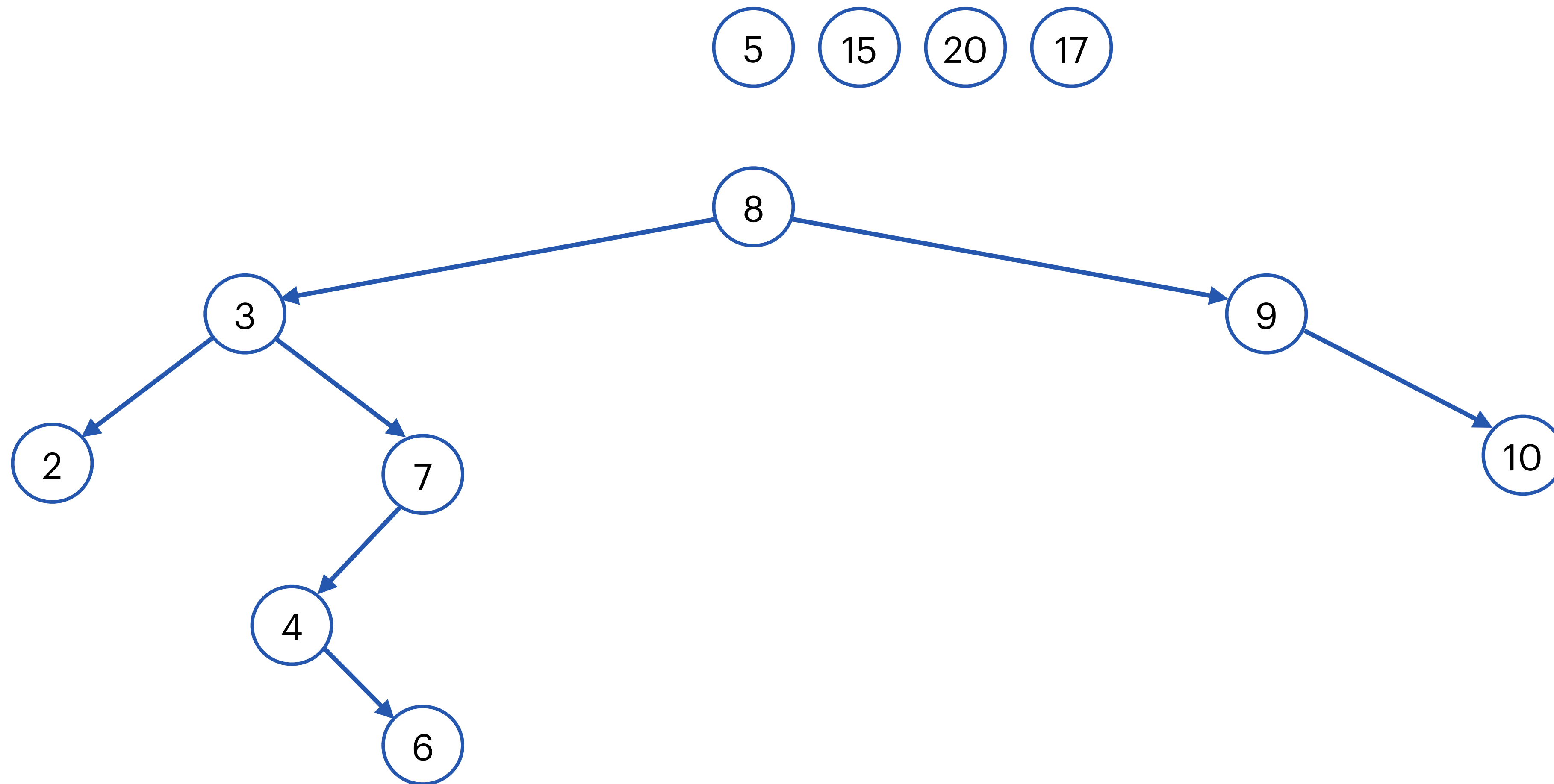
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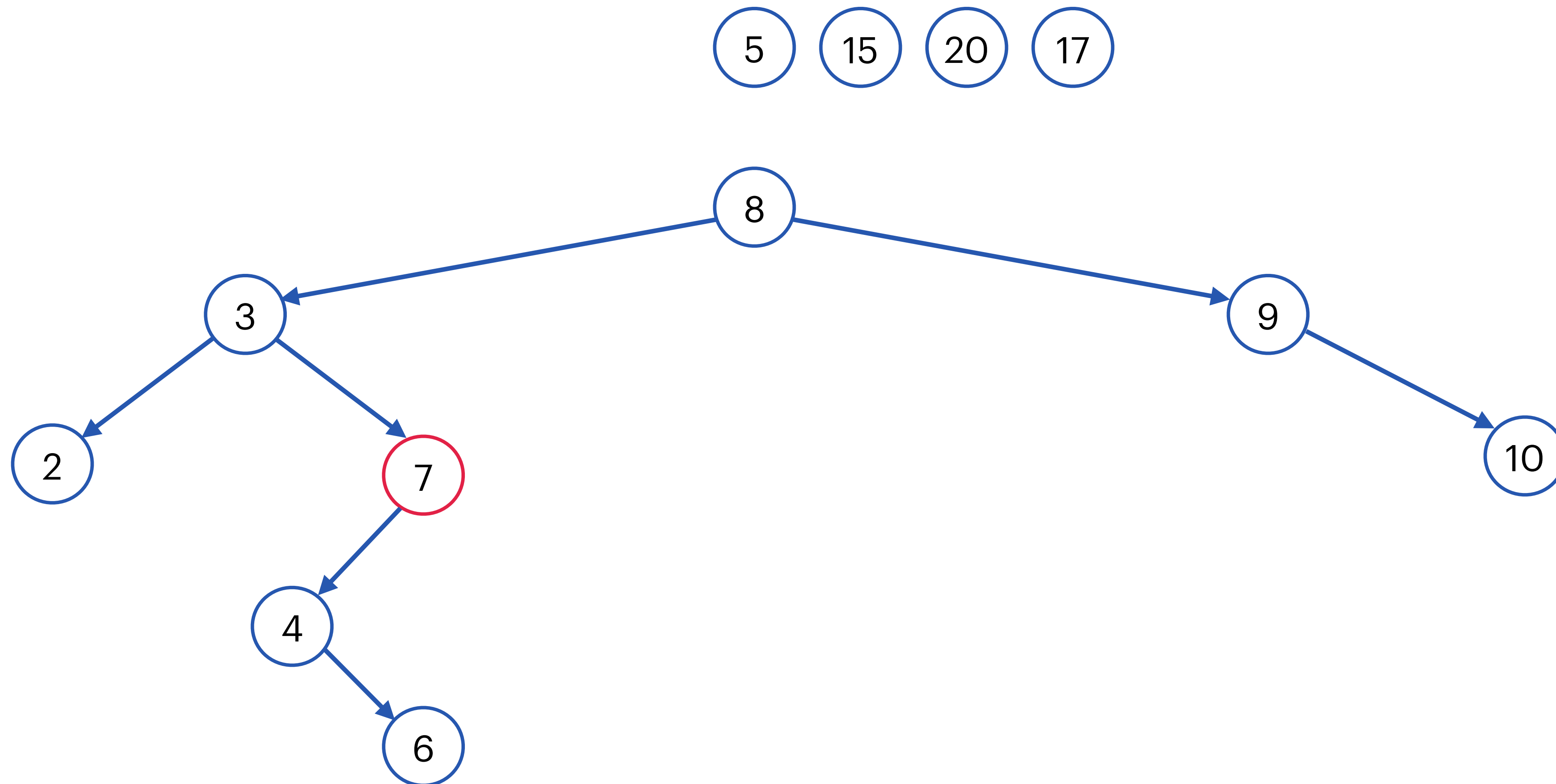
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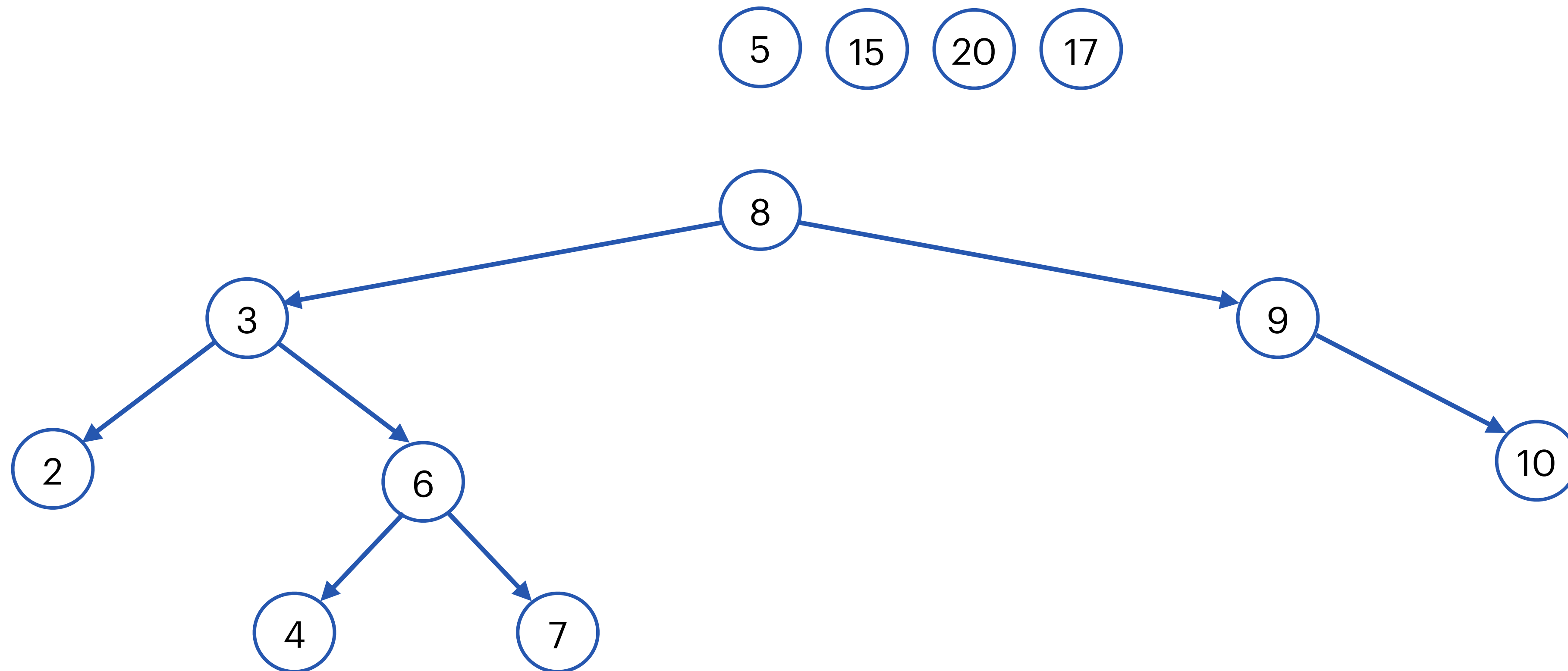
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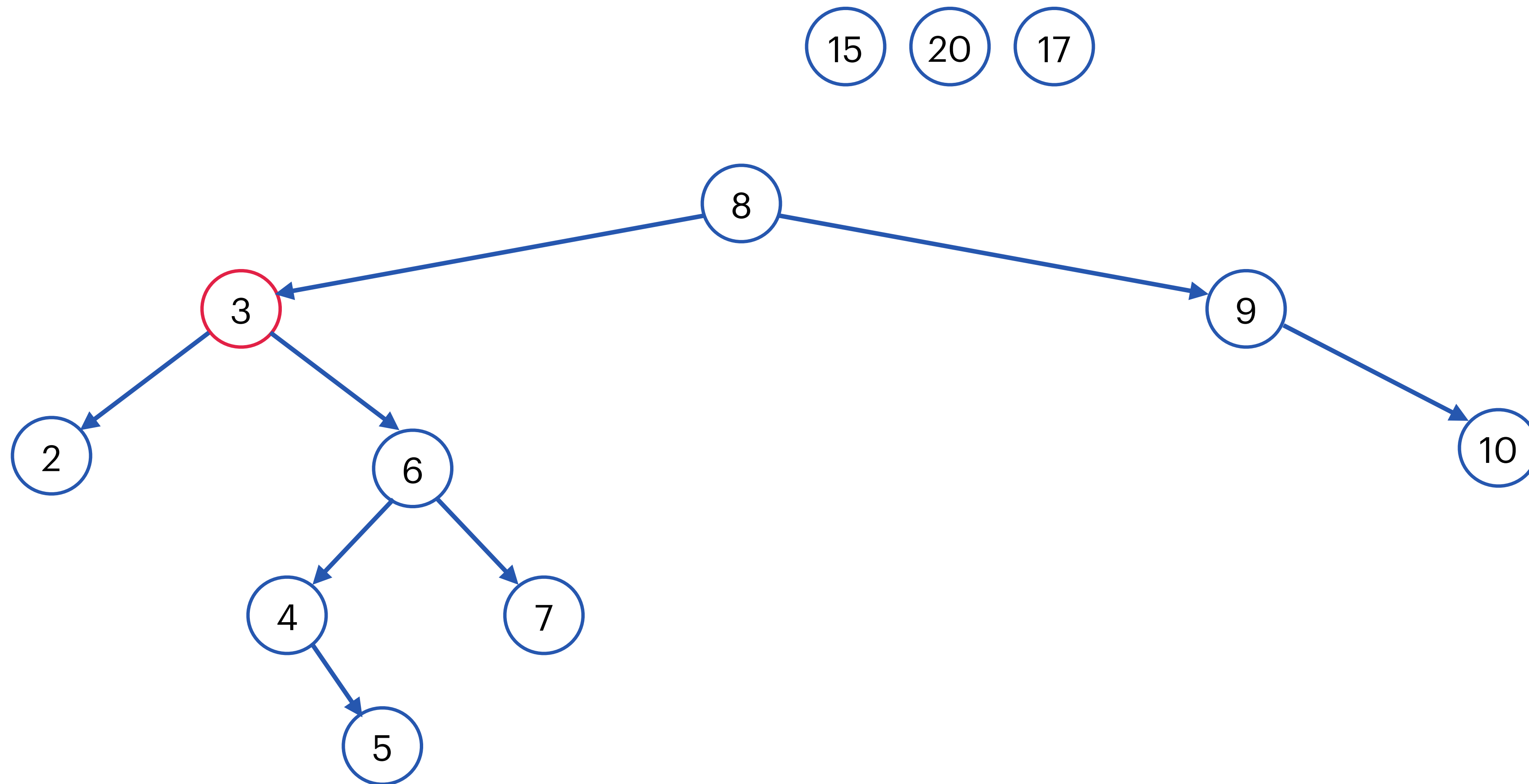
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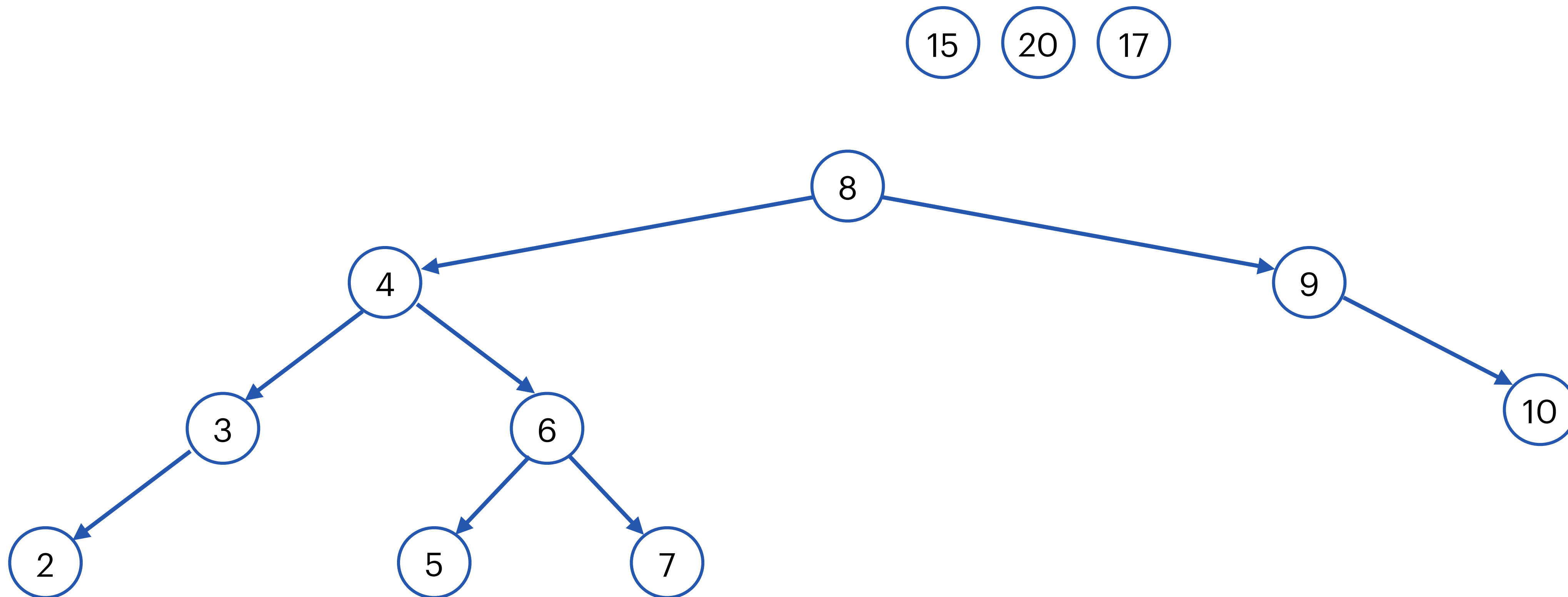
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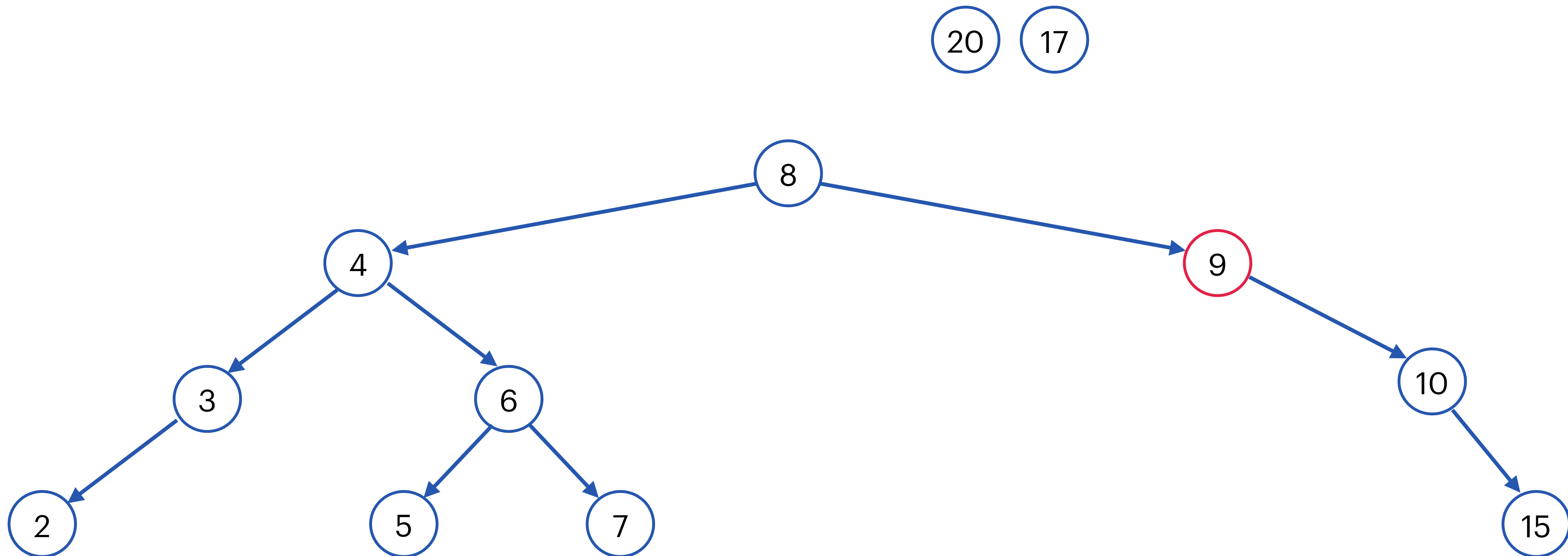
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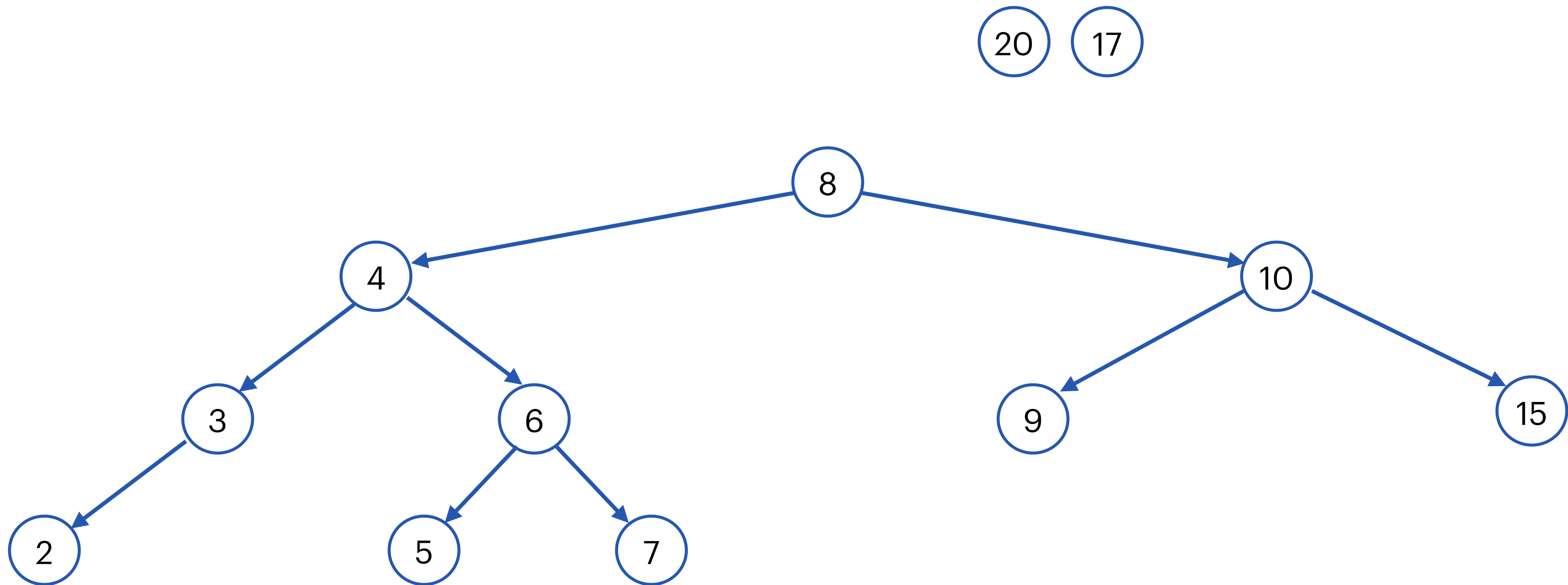
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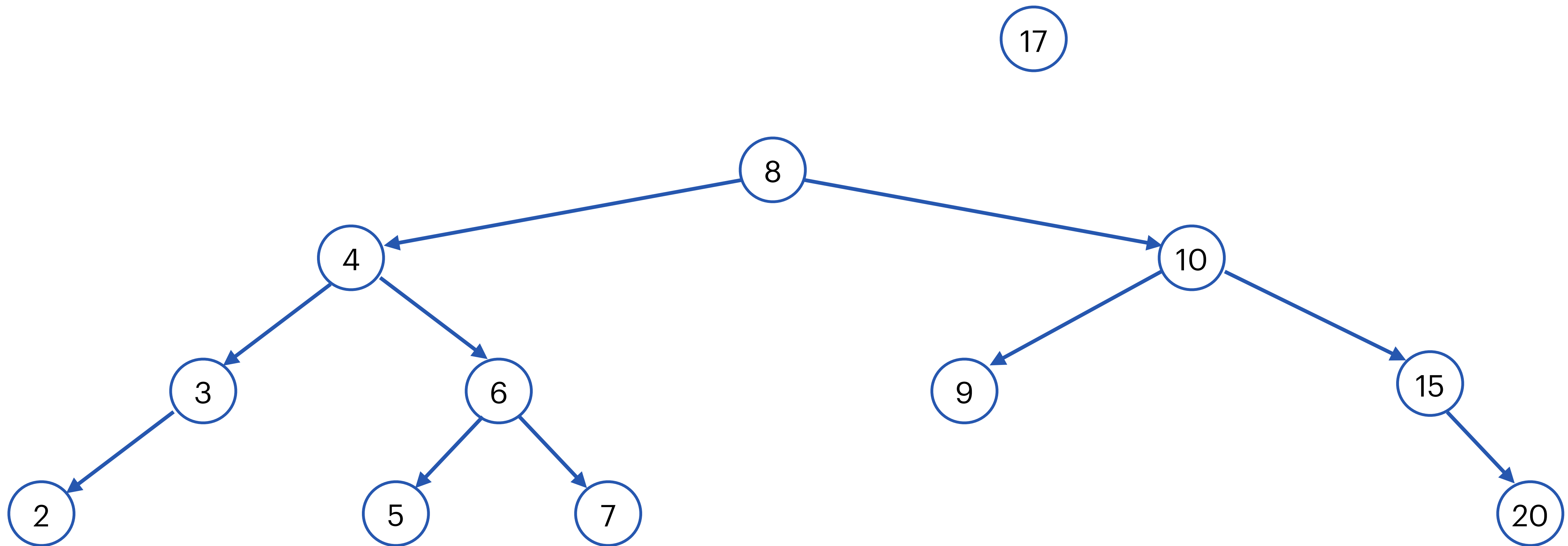
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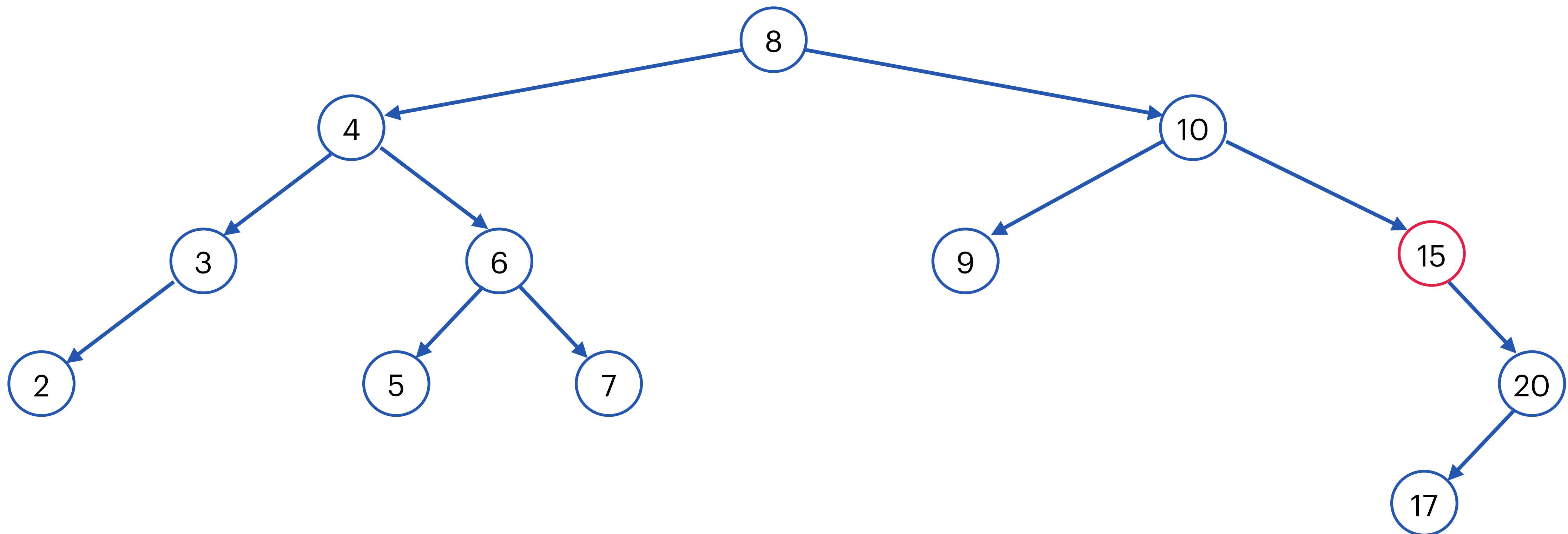
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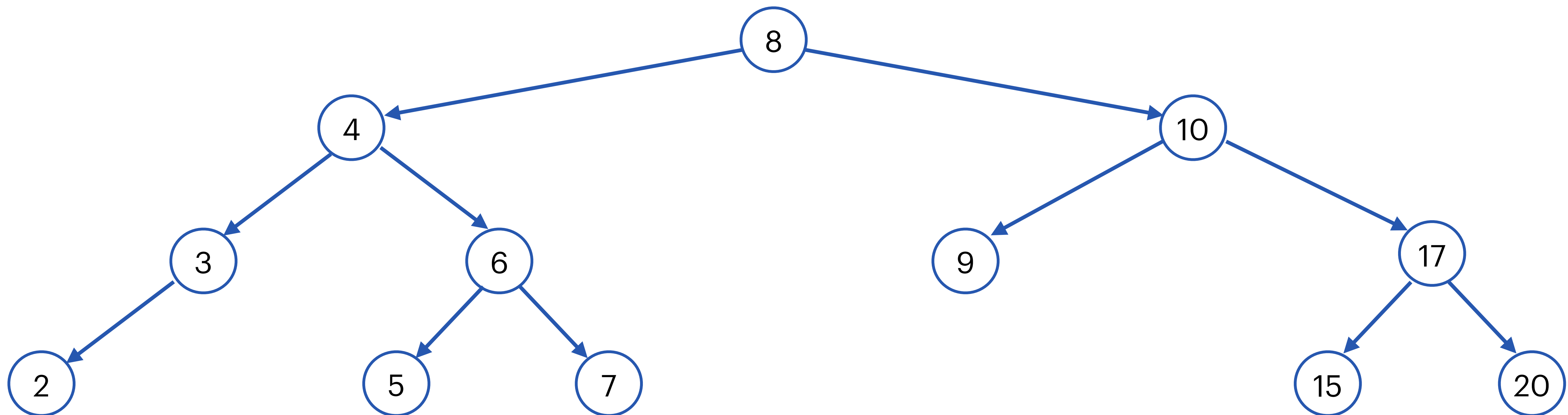
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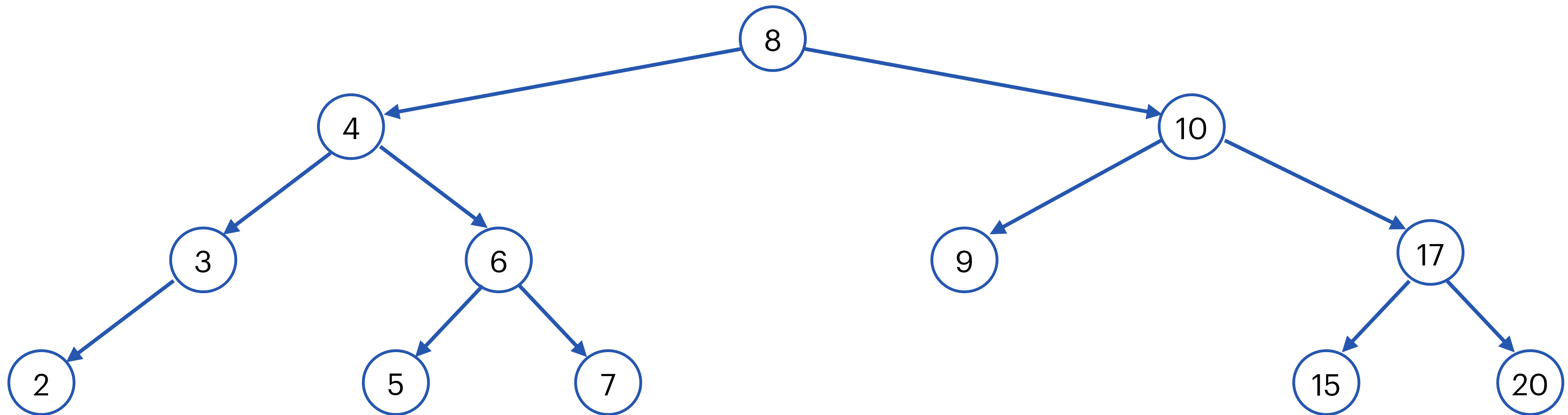
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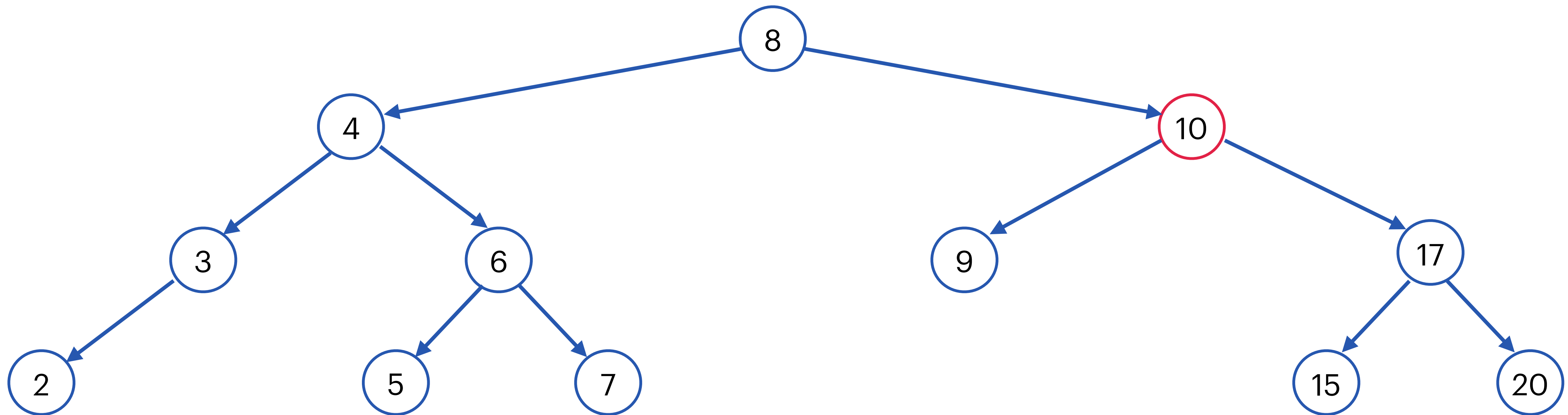
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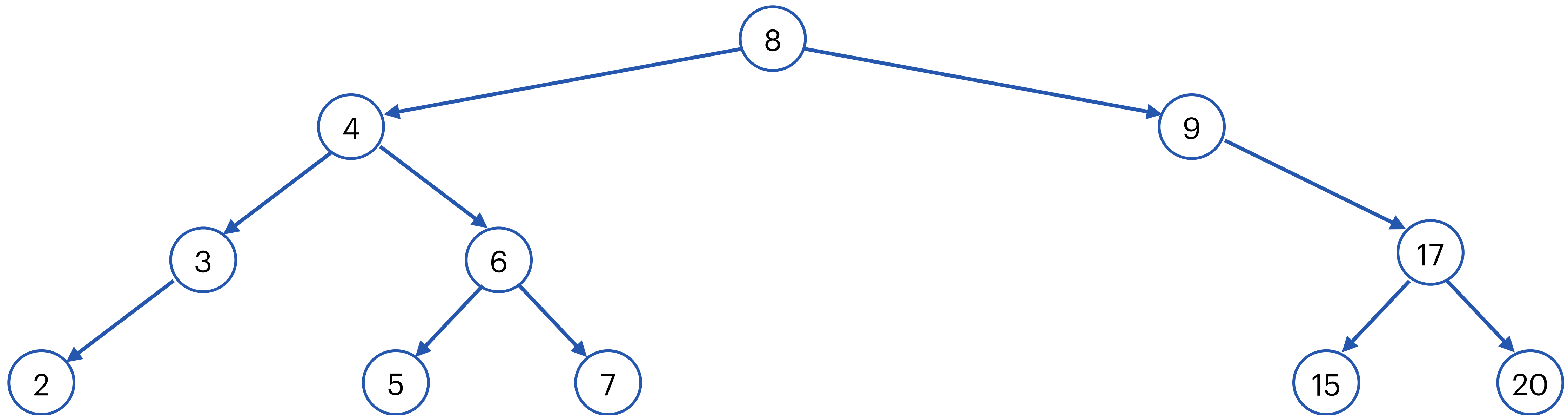
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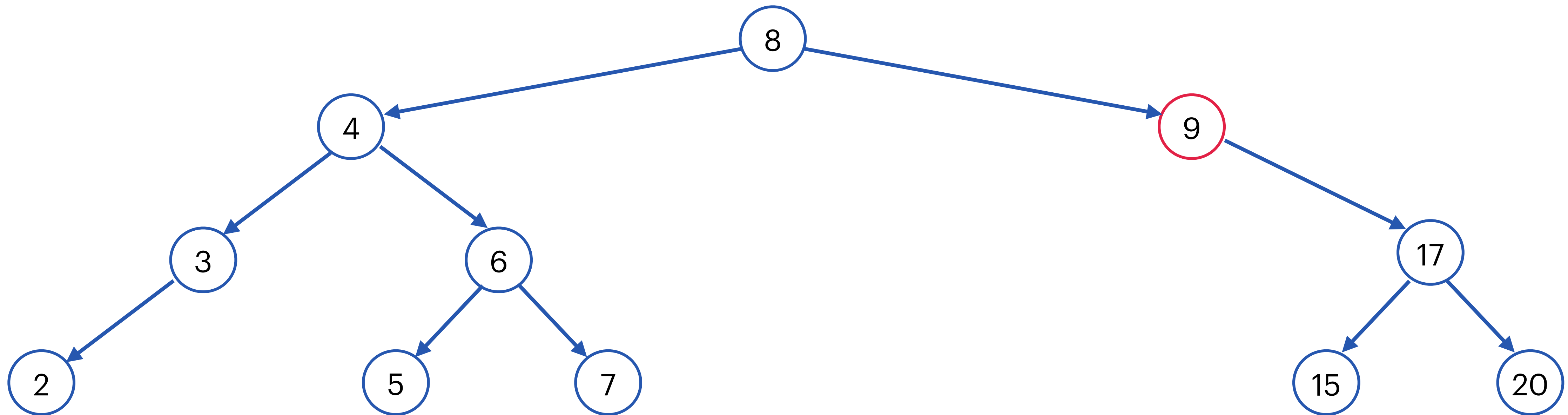
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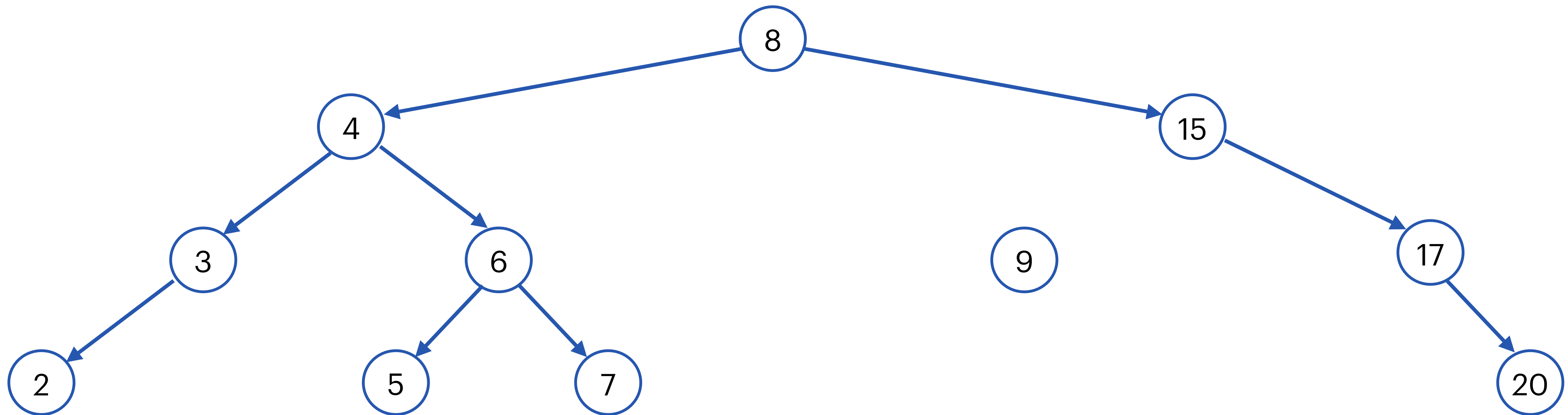
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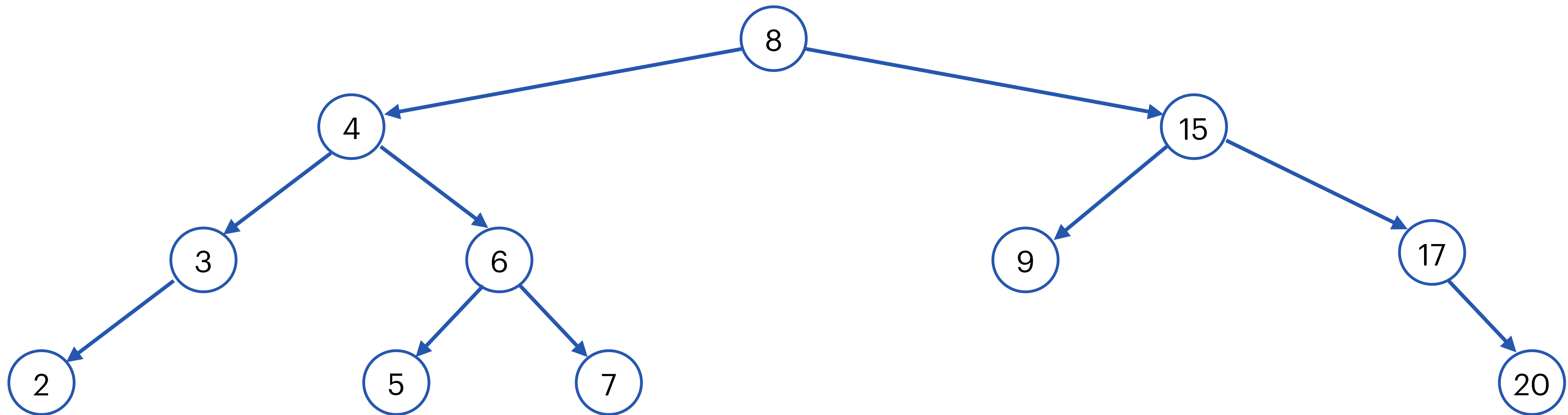
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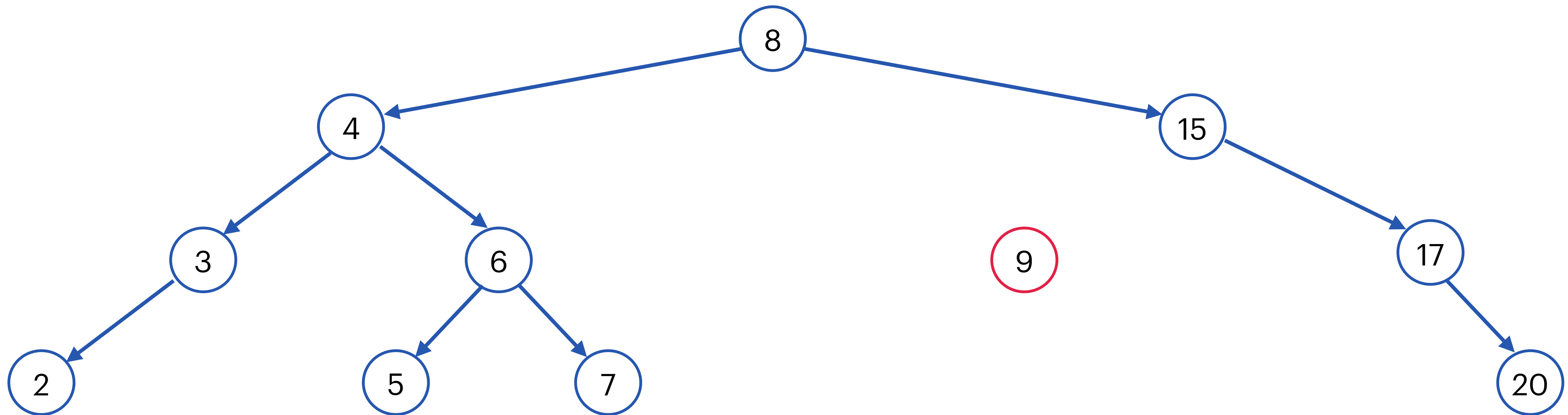
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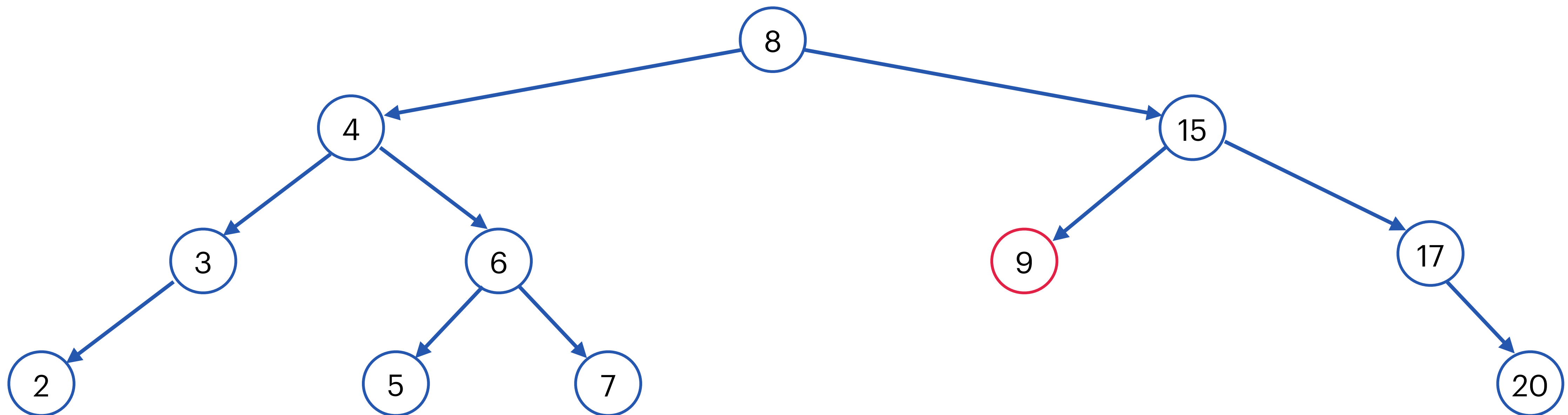
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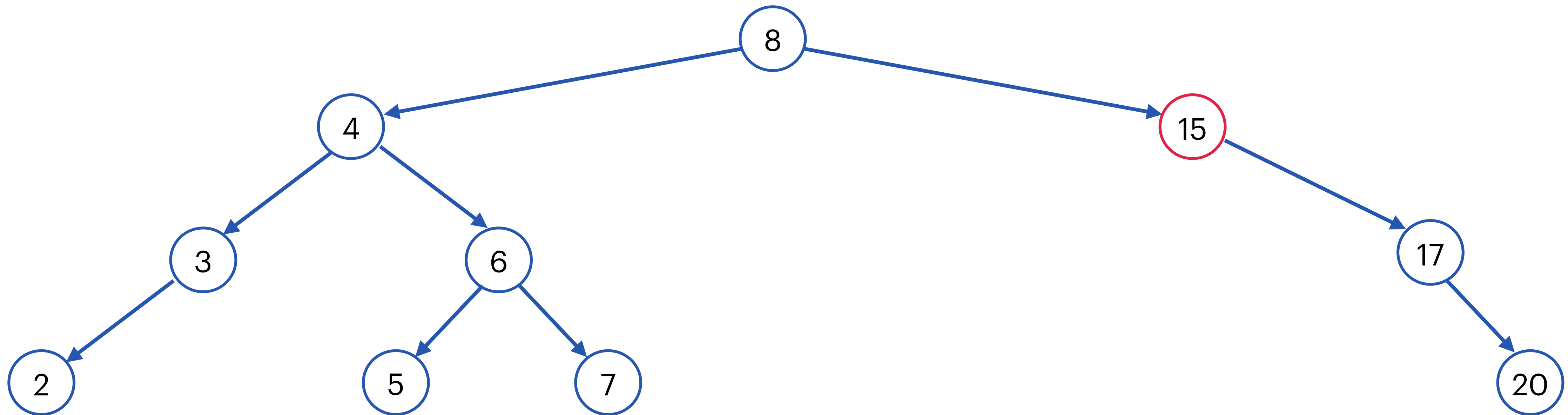
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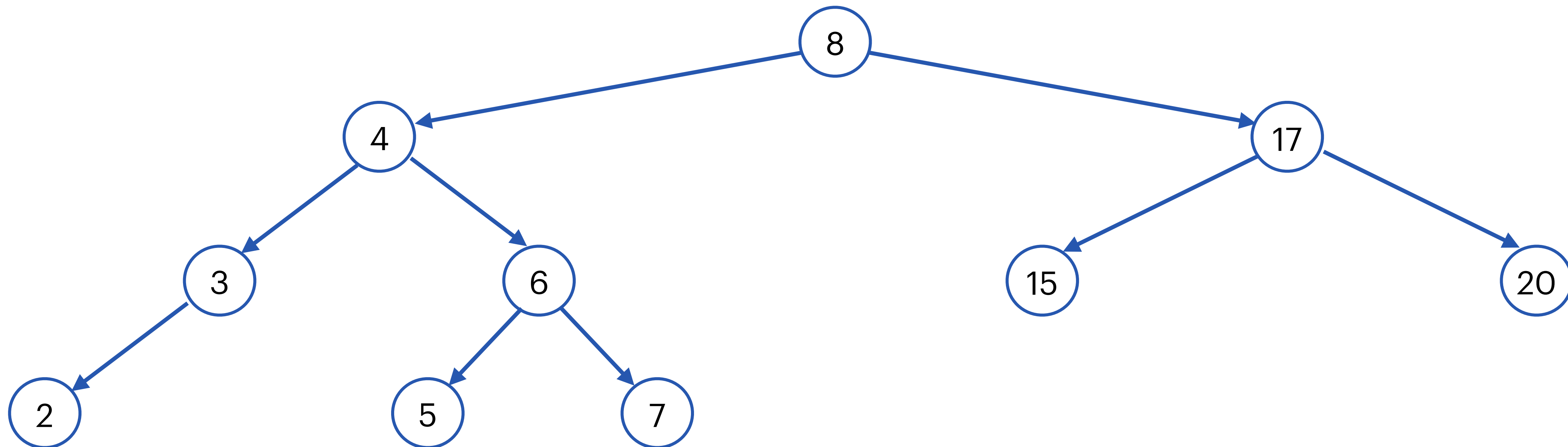
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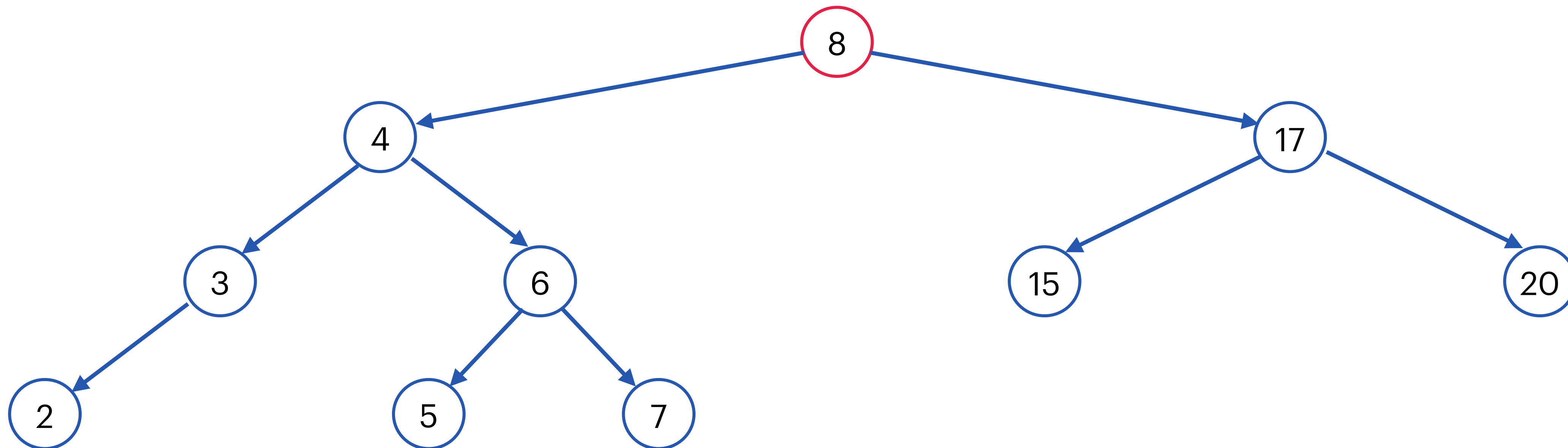
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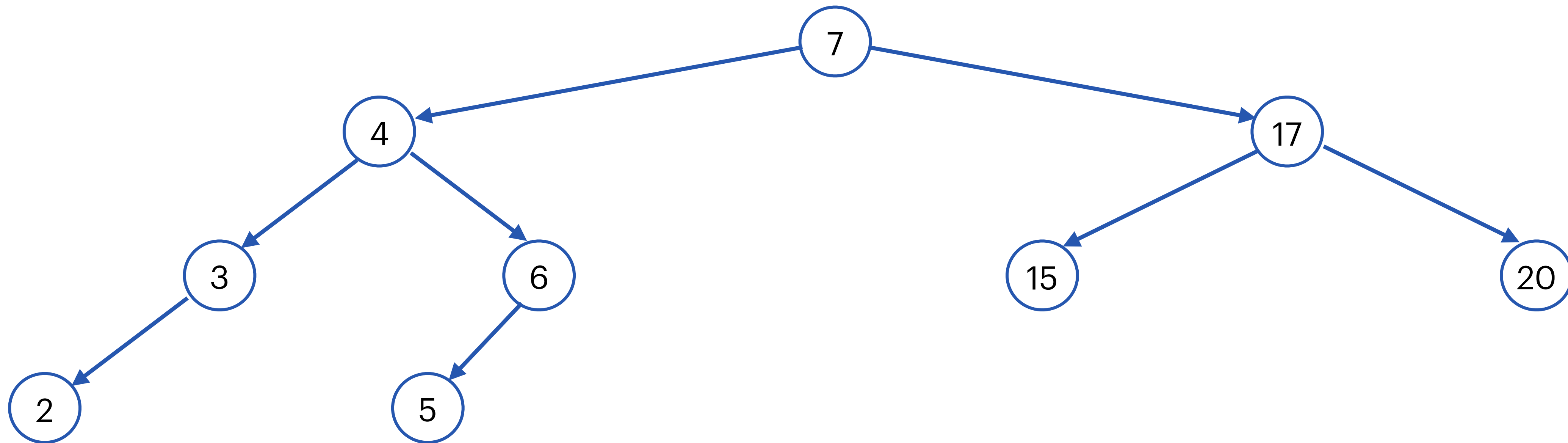
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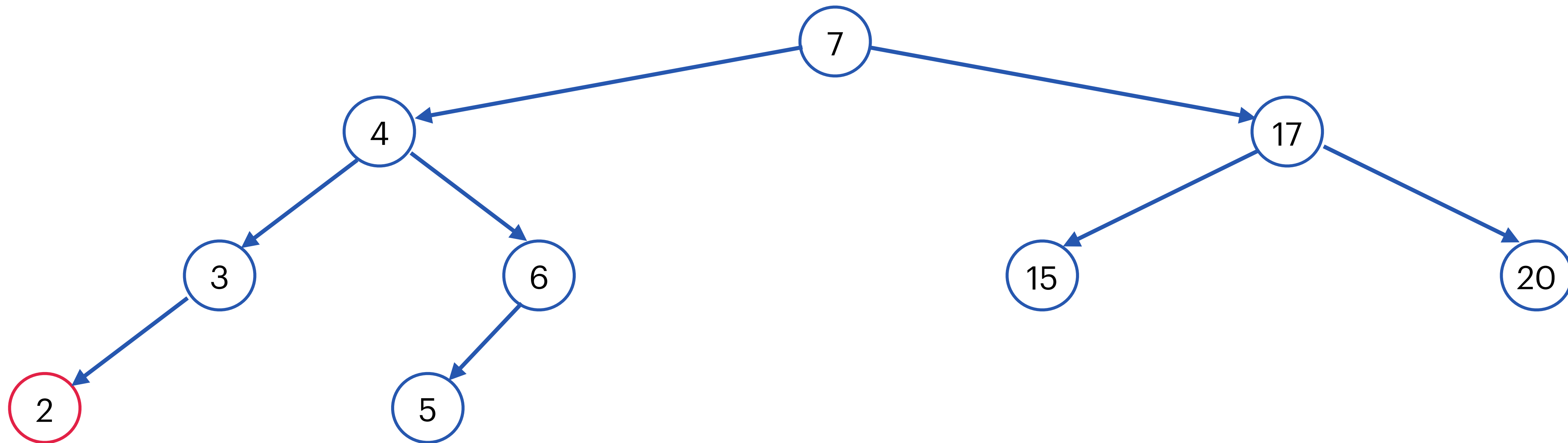
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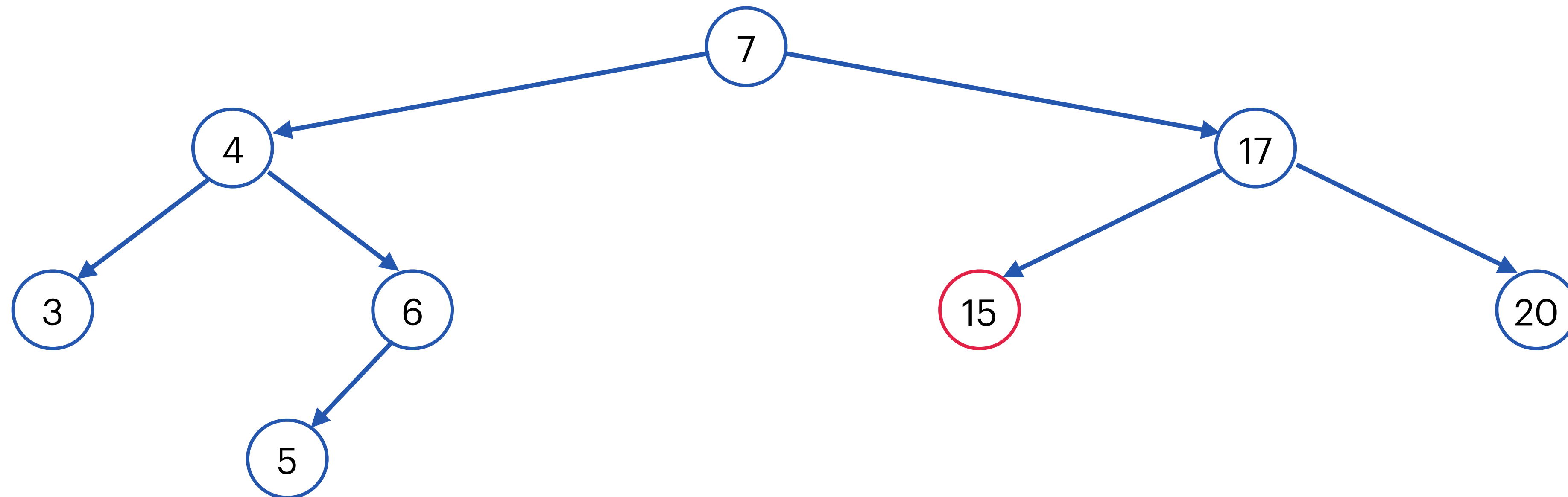
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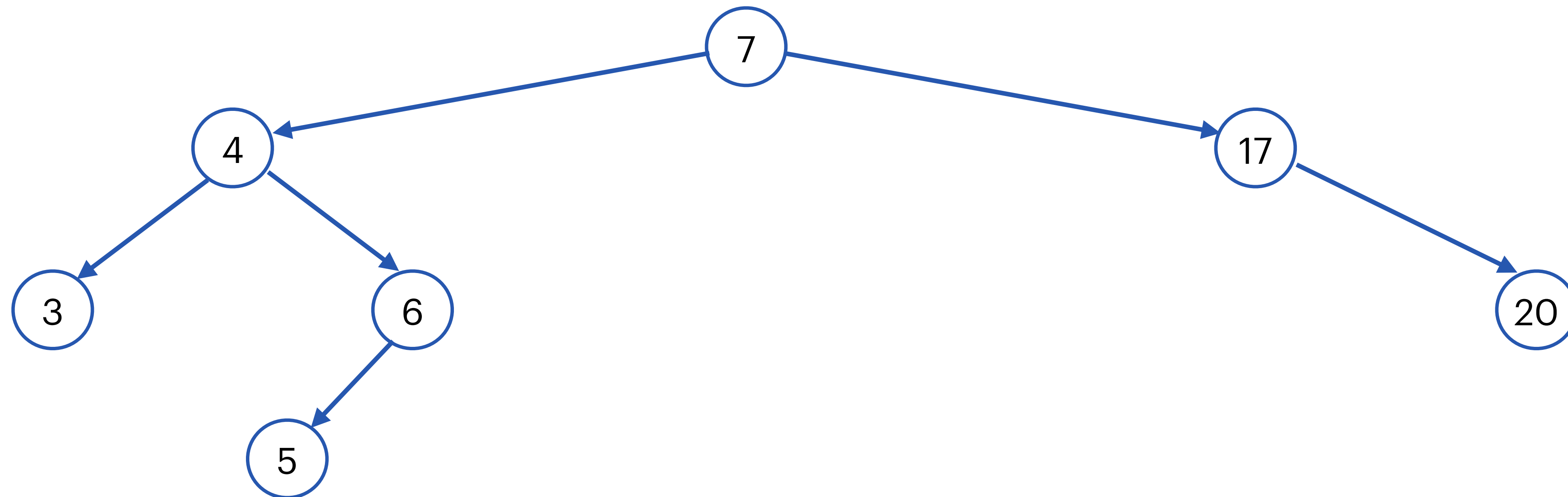
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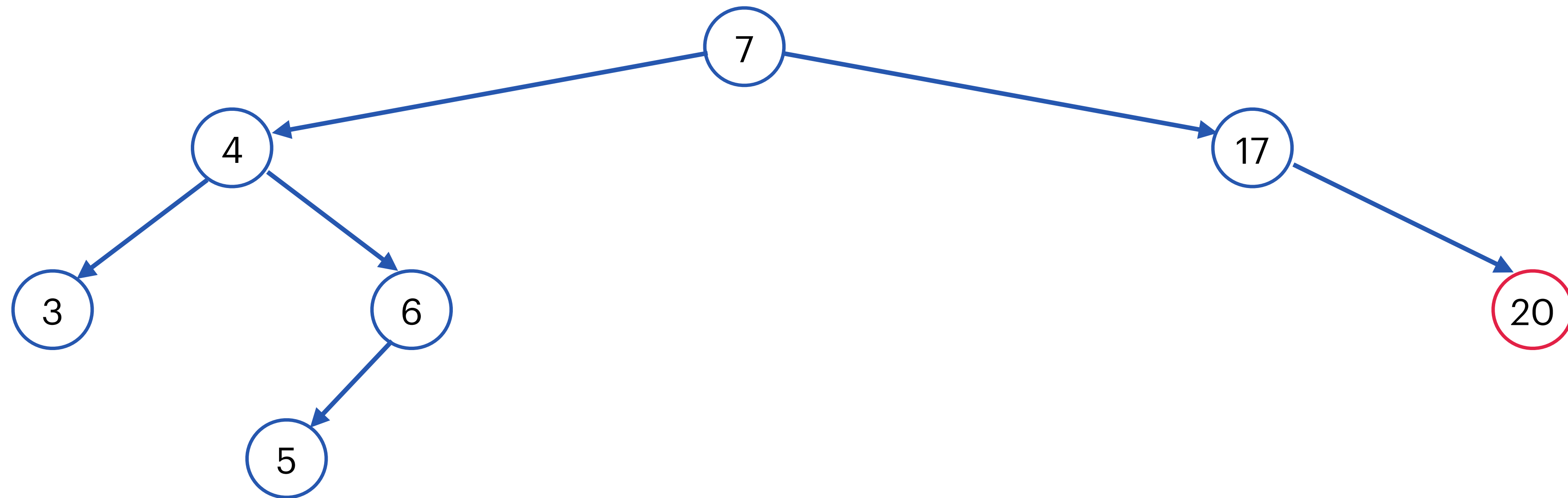
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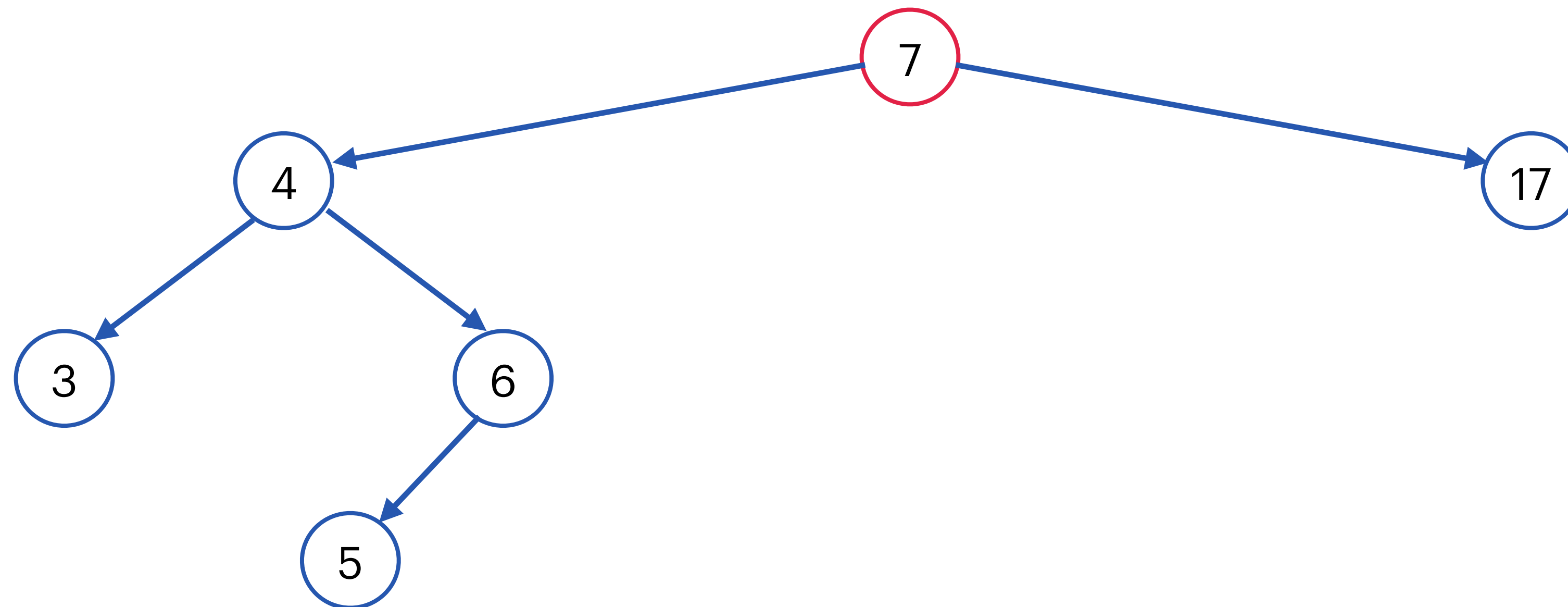
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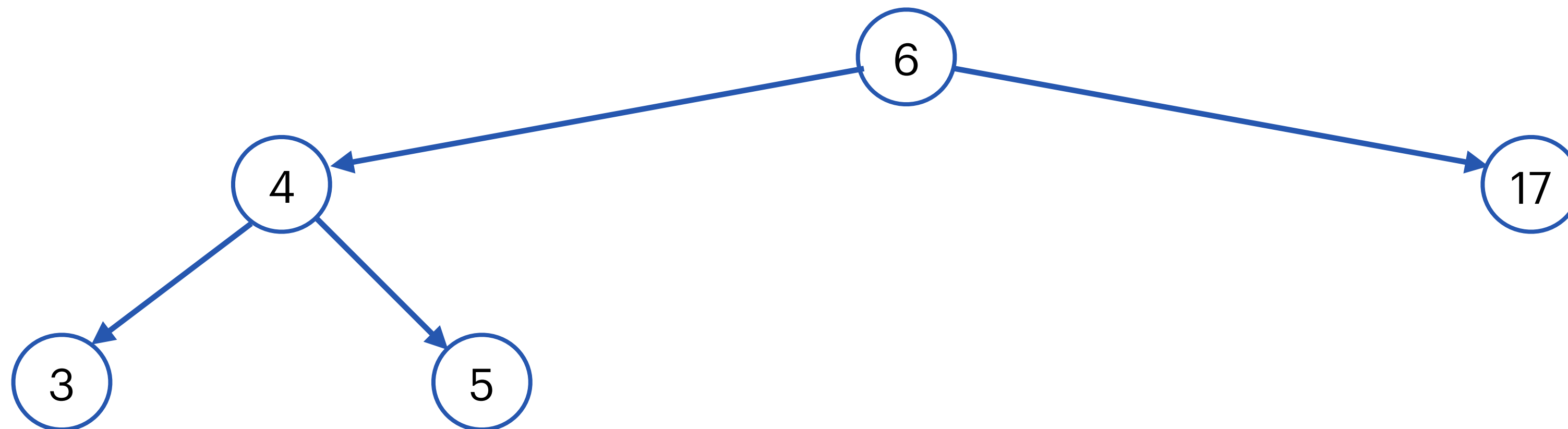
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A r b o r i B i n a r i O p t i m i

Situații în care se cunosc probabilitățile de acces ale diferitelor chei

Problema: organizarea arborelui binar ordonat astfel încât numărul total al pașilor de căutare pentru un număr suficient de încercări să fie minim

Lungimea drumului de căutare se modifică prin atribuirea unei ponderi (weight) fiecărui nod

Nodurile la care accesul se face mai frecvent devin noduri cu pondere mai mare, cele vizualizate mai rar, noduri cu pondere mai mică

Ponderea unui nod se asimilează cu probabilitatea de acces la acel nod

Se definește noțiunea de drum ponderat asociat unui arbore binar ordonat

Lungimea P_I a drumului ponderat asociat unui arbore binar este suma lungimii tuturor drumurilor de la rădăcină la fiecare nod al arborelui, fiecare drum fiind ponderat cu probabilitatea de acces la nodul respectiv

$$P_I = \sum_{i=1}^n p_i * h_i$$

Toți subarborii unui arbore binar optim sunt de asemenea optimi

Algoritm care construiește în mod sistematic arbori binari optimi din ce în ce mai mari, pornind de la nodurile individuale care sunt considerate drept cei mai mici subarbori.

Arborii binari optimi cresc de la frunze spre rădăcină, conform metodei “bottom-up” (“de jos în sus”)

A r b o r i H u f f m a n

Arbori / coduri Huffman: exemplu de utilizare ai arborilor binari cu ponderi

Se presupune că se prelucrează mesaje care constau din secvențe de caractere

În fiecare mesaj, caracterele sunt independente și pot apărea în orice poziție a mesajului

Se presupune că se cunoaște probabilitatea de apariție a fiecărui caracter, probabilitate care nu depinde de poziția caracterului în cadrul mesajului

Se dorește să se codifice fiecare caracter printr-o secvență de cifre binare 0 și 1, astfel încât codul unui caracter să nu fie prefix pentru codul nici unui alt caracter

Proprietatea de prefix: permite decodificarea unui mesaj binar prin ștergerea repetată a prefixelor șirului care sunt coduri de caractere

Una din tehnicile de aflare a codului de prefix optim este algoritmul lui Huffman

Coduri Huffman sunt folosite în diverse formate de fișiere, de ex: JPEG, MP3

Biblioteci open source cu defecte care implementau codificare Huffman folosite pentru formatele JPEG și WEBP în sistemul de operare iOS (Apple) au fost exploatare pentru a obține acces de la distanță pe dispozitive prin trimiterea unui fișier (poze) JPEG sau WEBP care aveau o structură creată pentru a exploata breșele implementare din aceste biblioteci.

Arbori Huffman

M A R E _ E _ M A R E A _ M A R M A R A

Arbori Huffman

M M M M 4

A R E _ E _

A R E A _

A R

A R A

Arbori Huffman

4
M

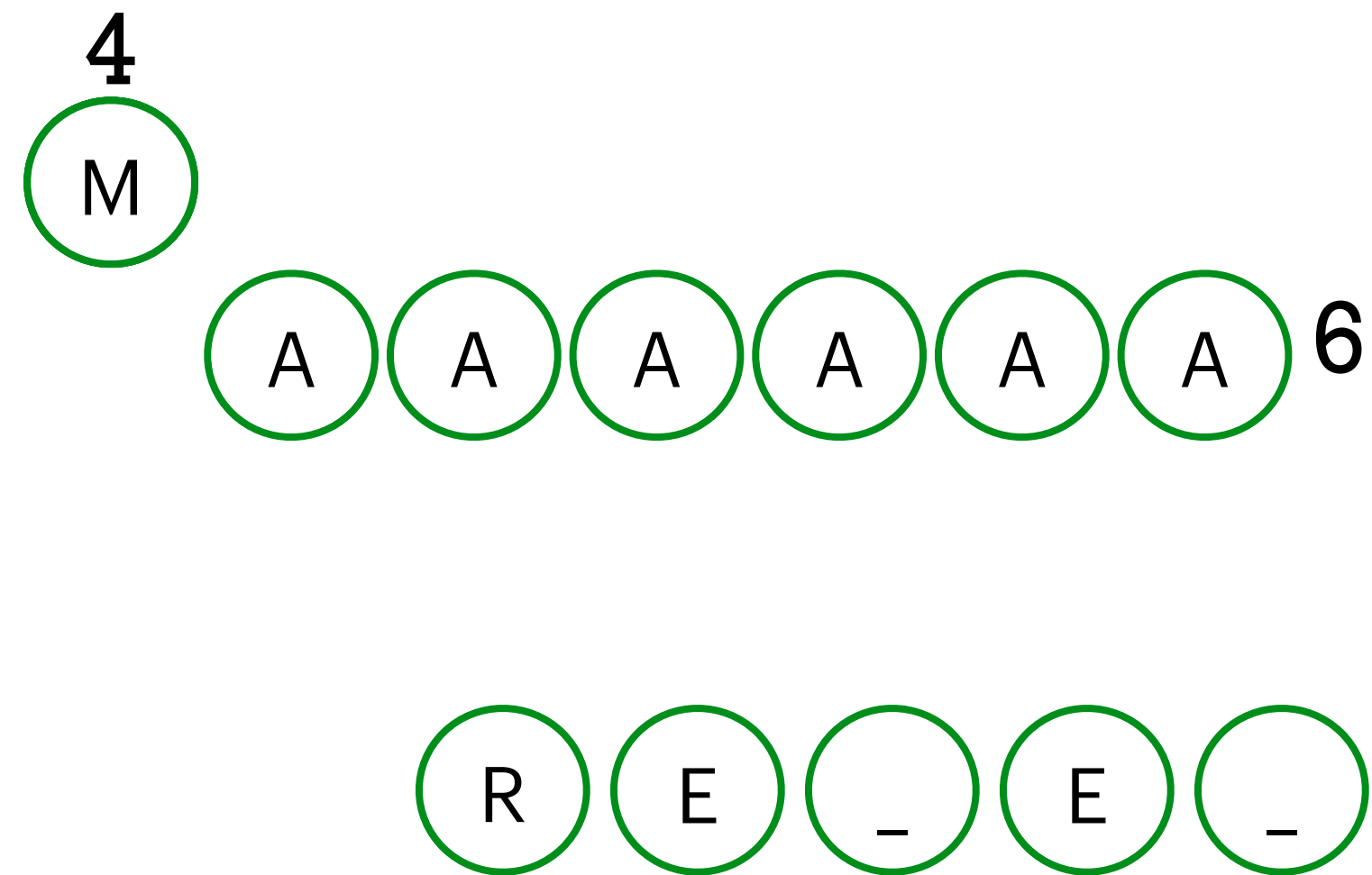
A R E _ E _

A R E A _

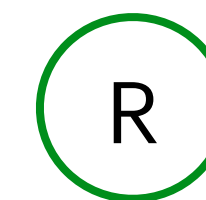
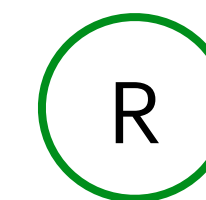
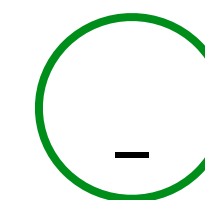
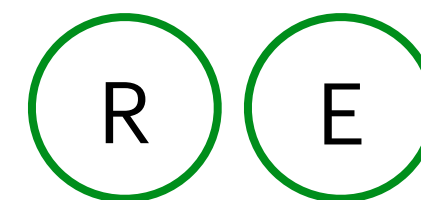
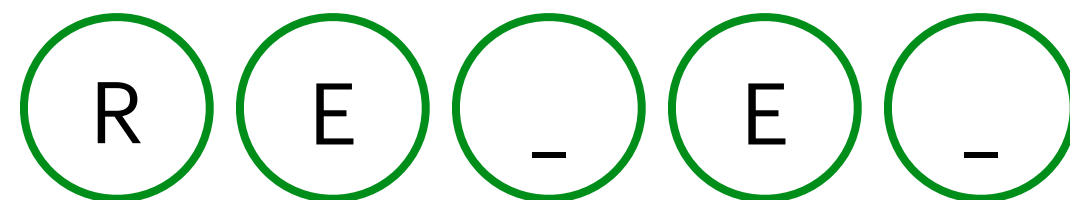
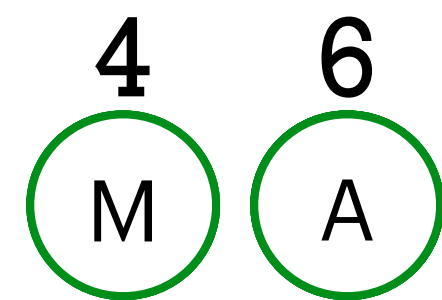
A R

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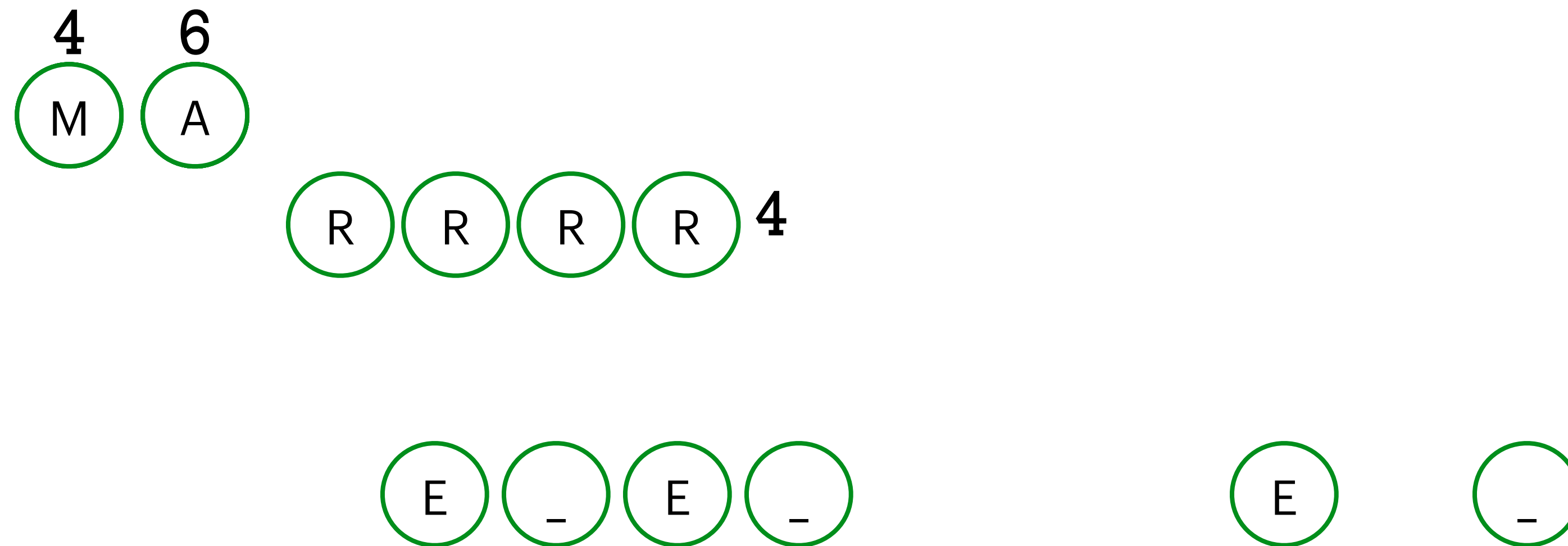
Arbori Huffman



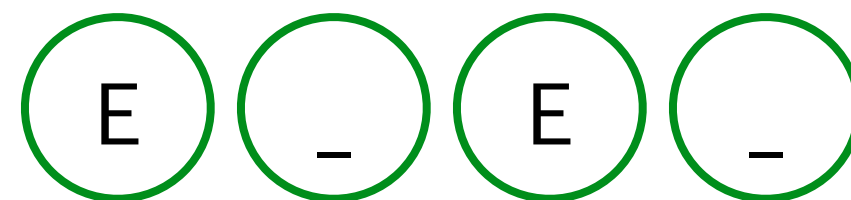
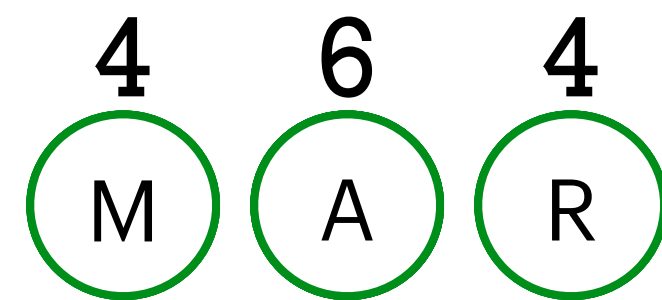
Arb ori Huffman



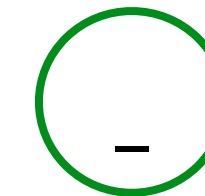
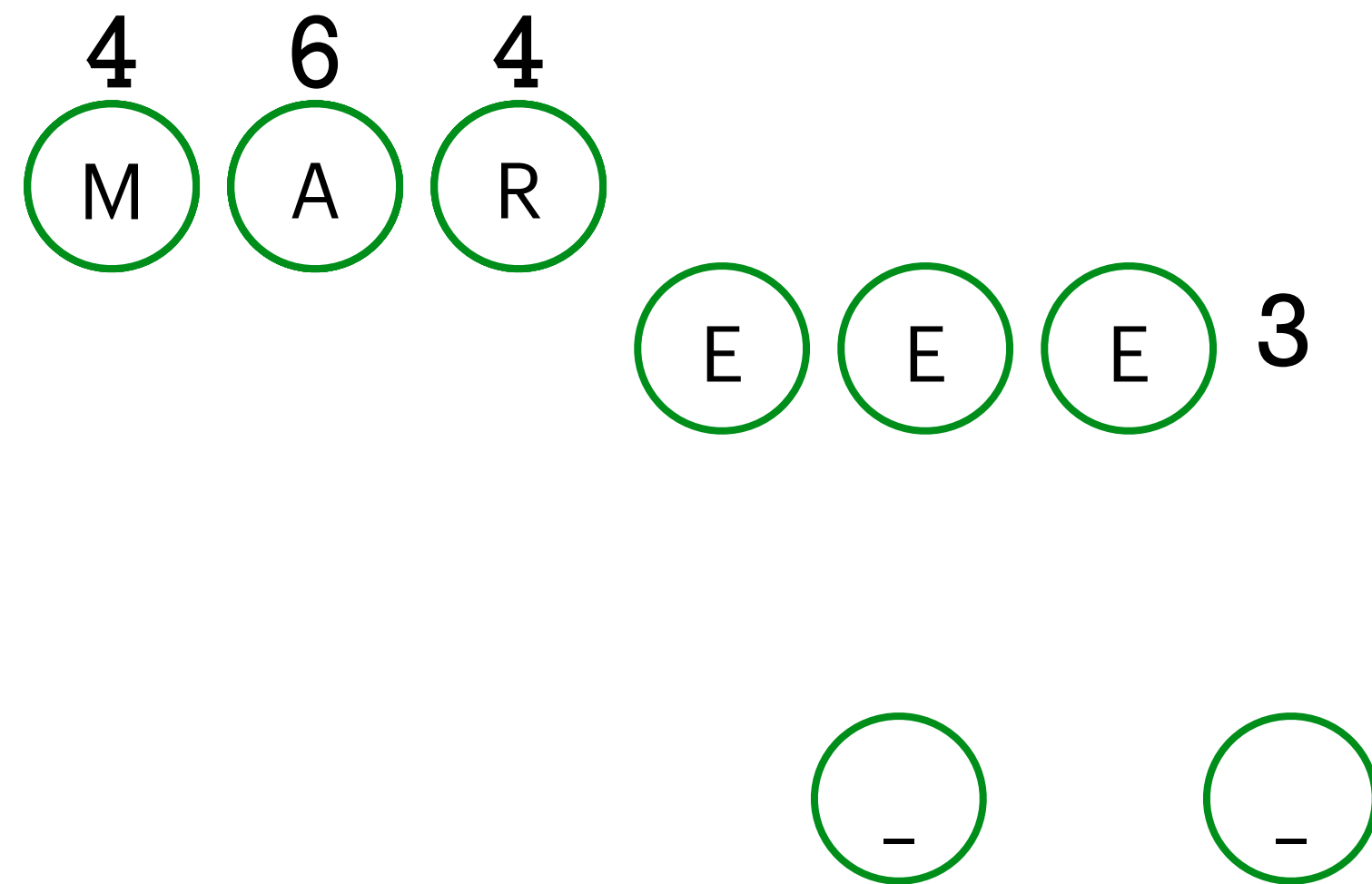
Arbori Huffman



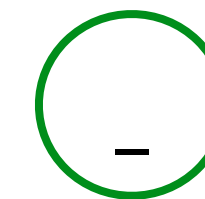
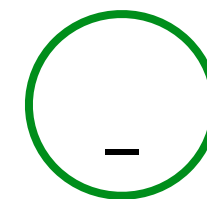
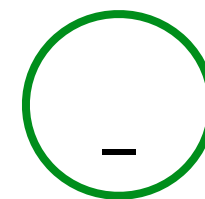
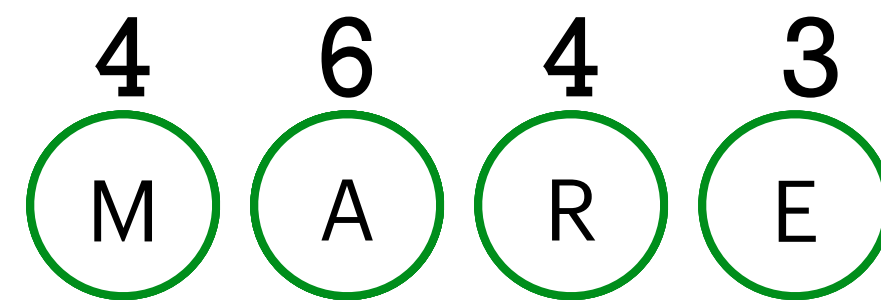
Arbori Huffman



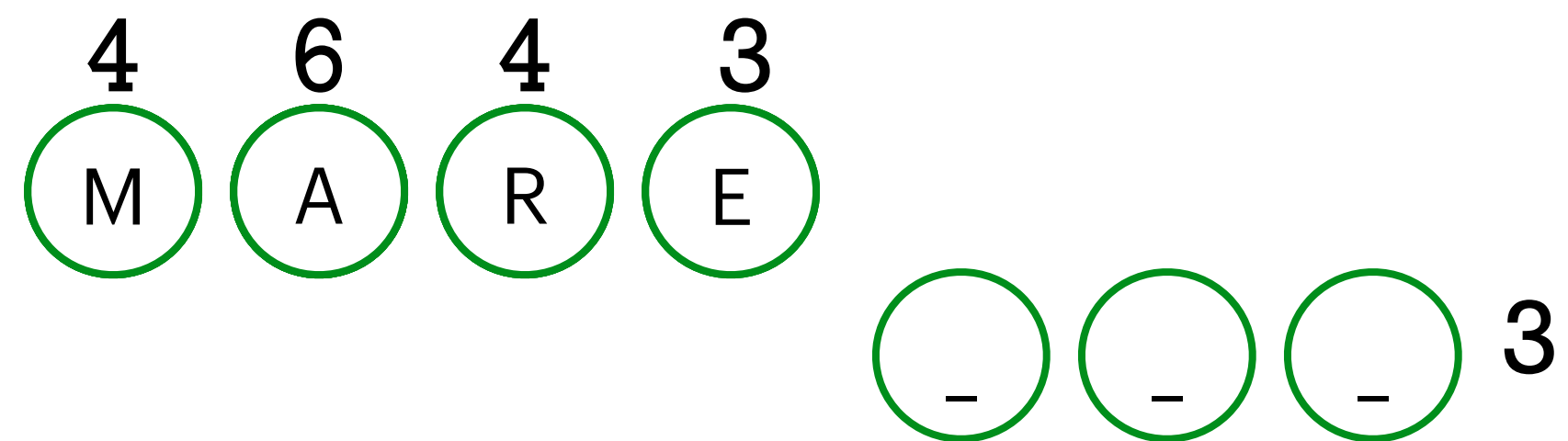
Arbori Huffman



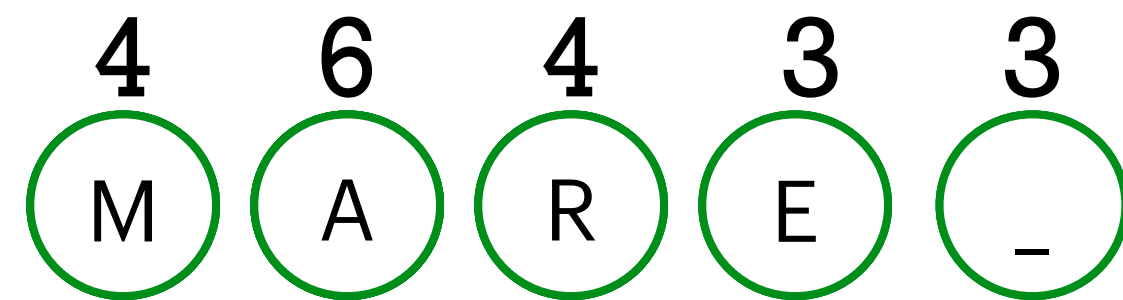
Arbori Huffman



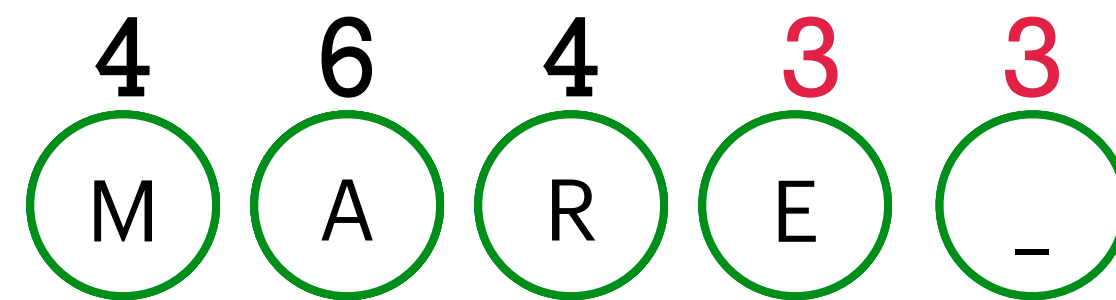
Arbori Huffman



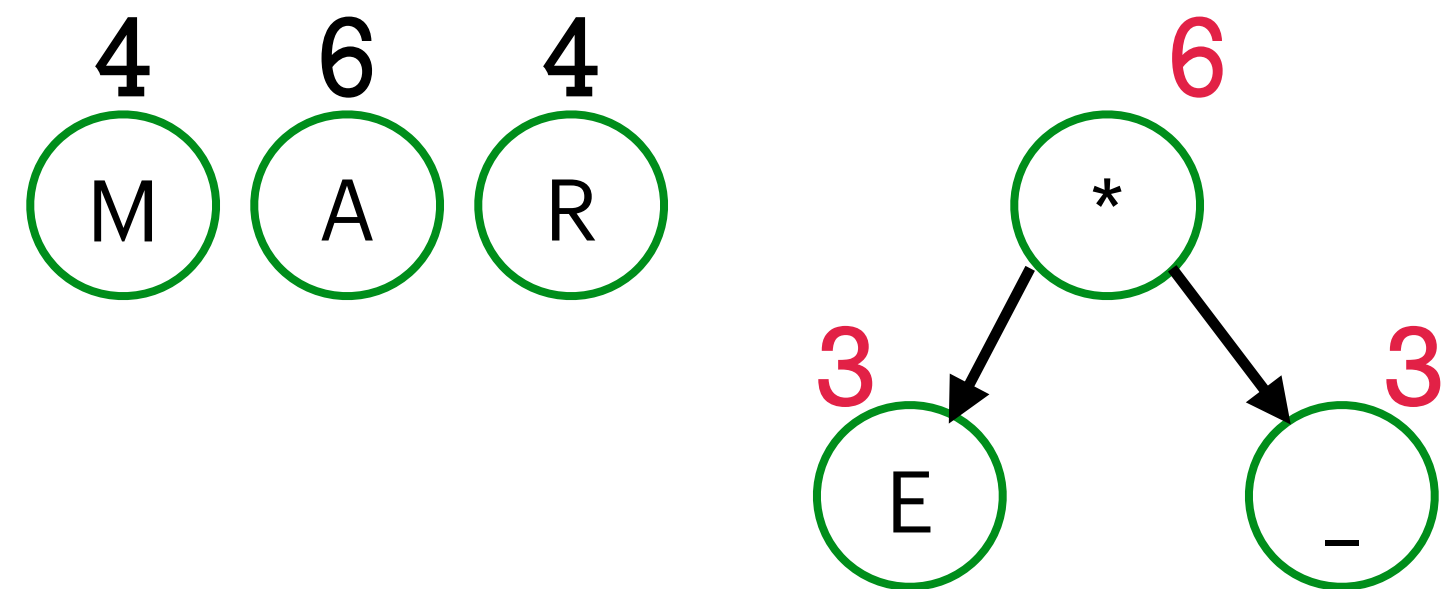
Arbori Huffman



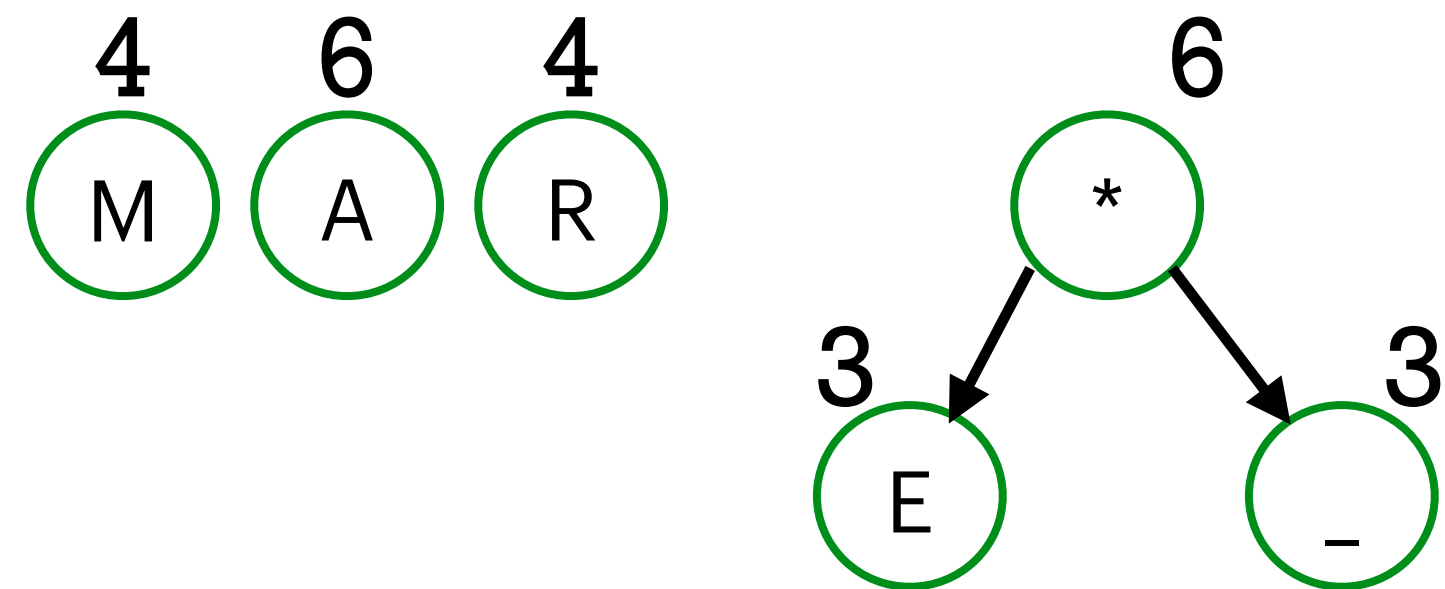
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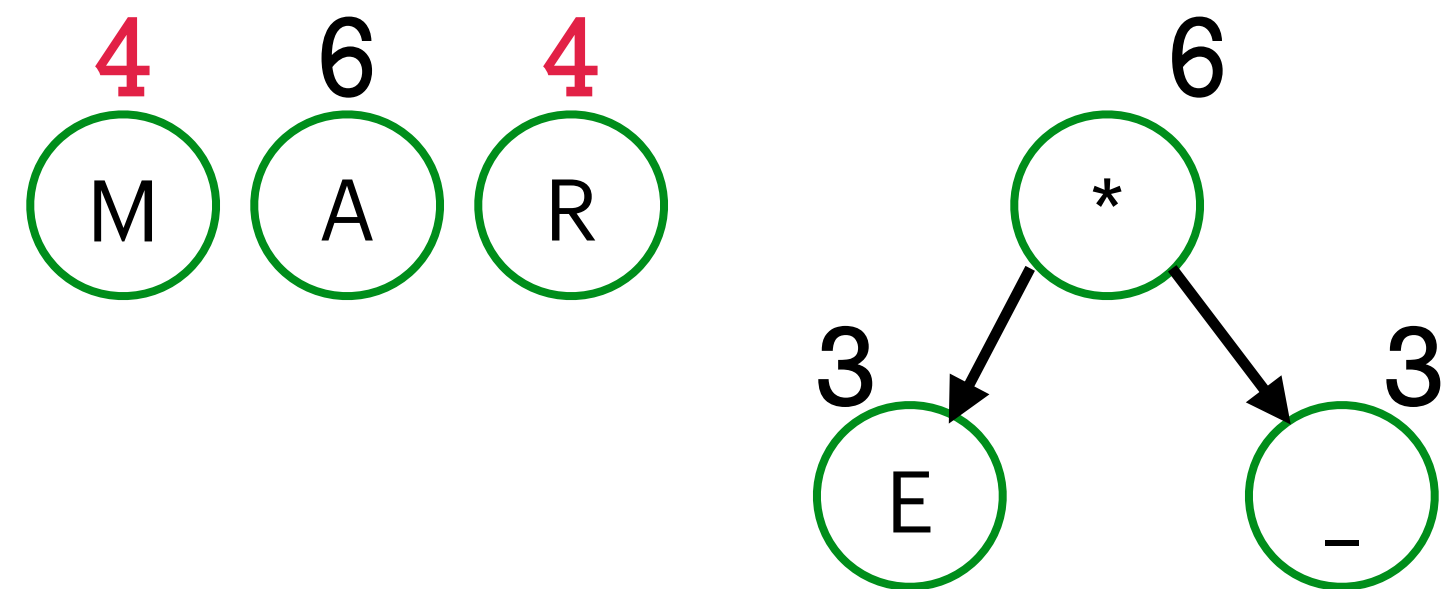
Arbori Huffman



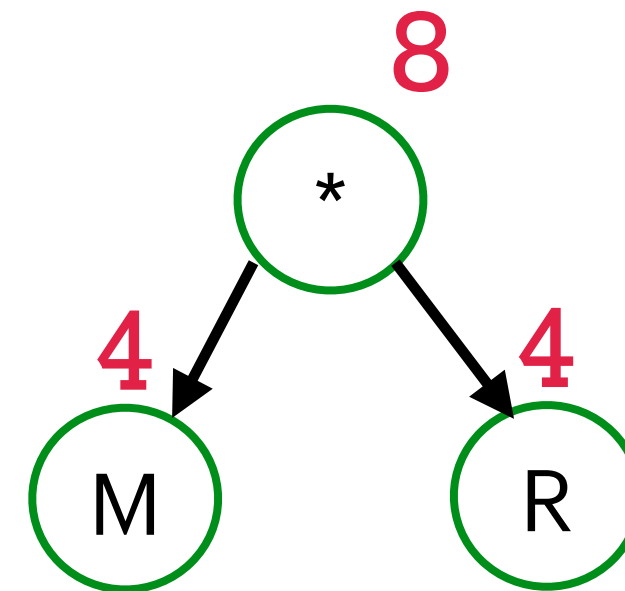
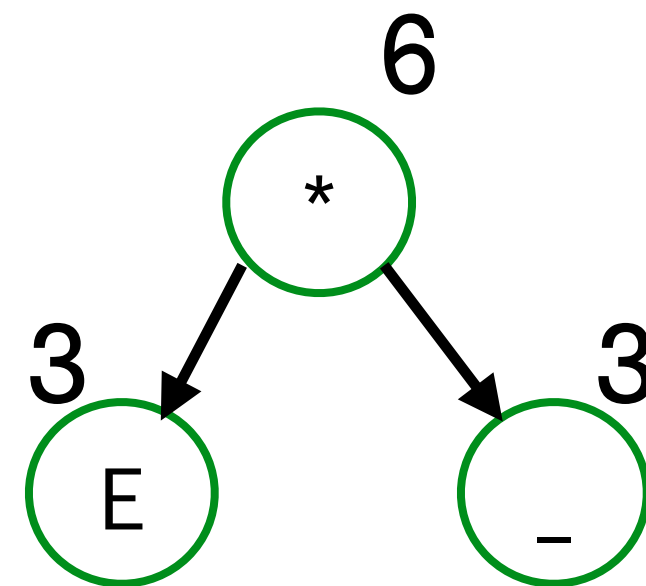
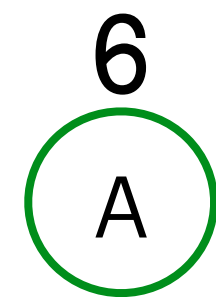
Arbori Huffman



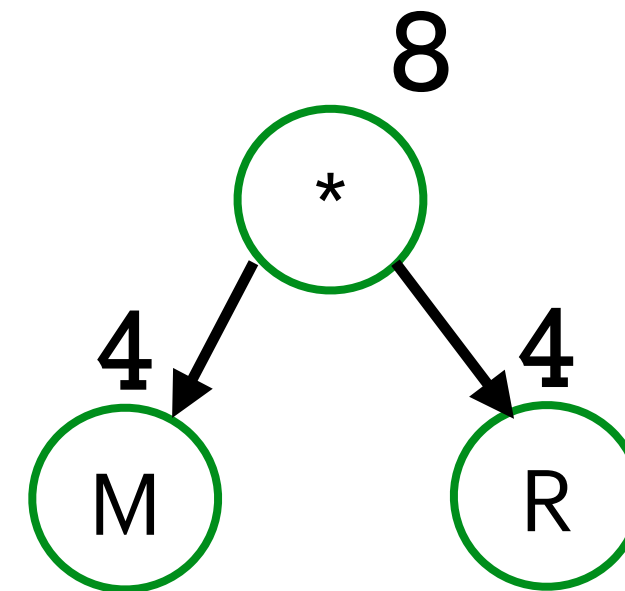
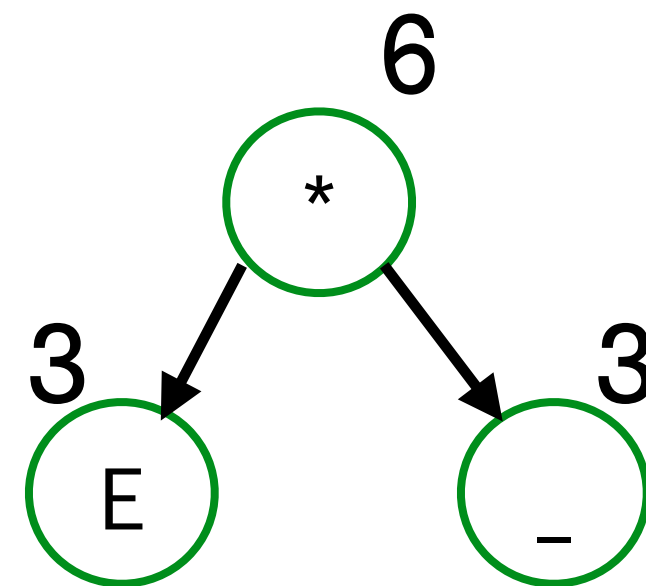
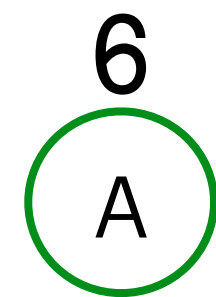
Arbori Huffman



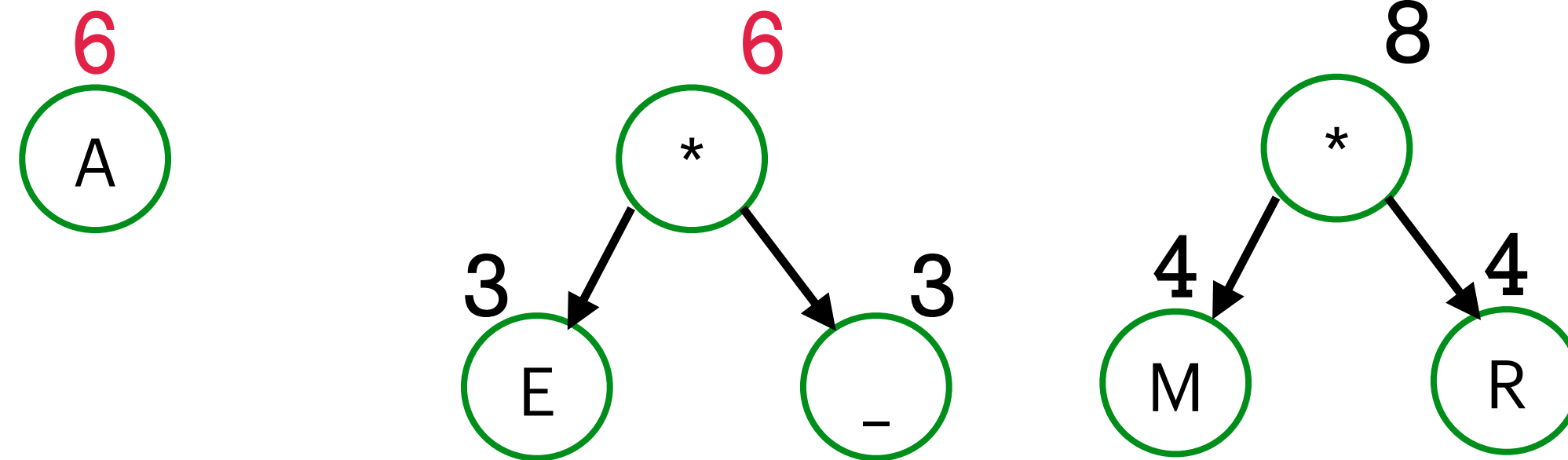
Arbori Huffman



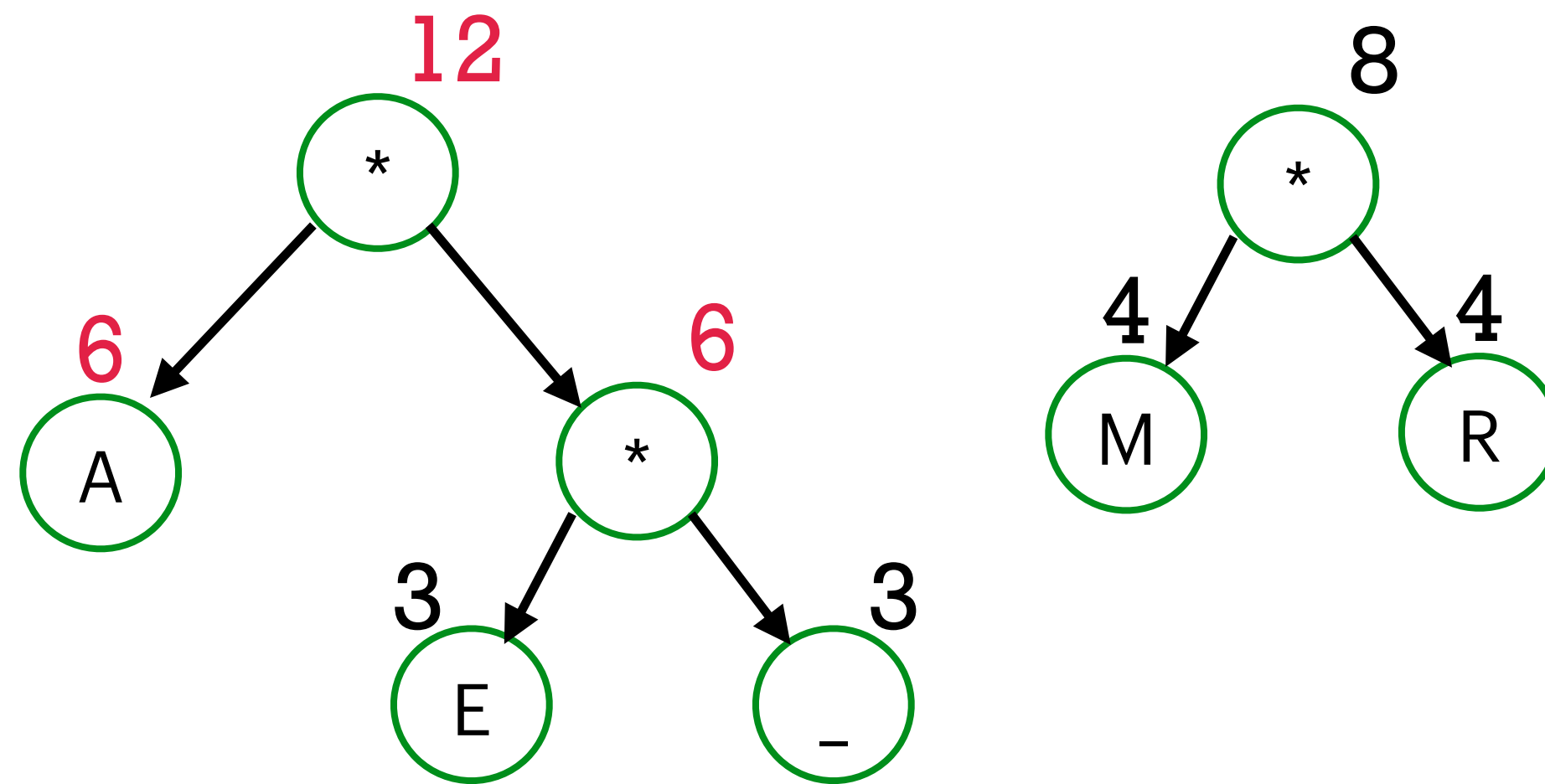
Arbori Huffman



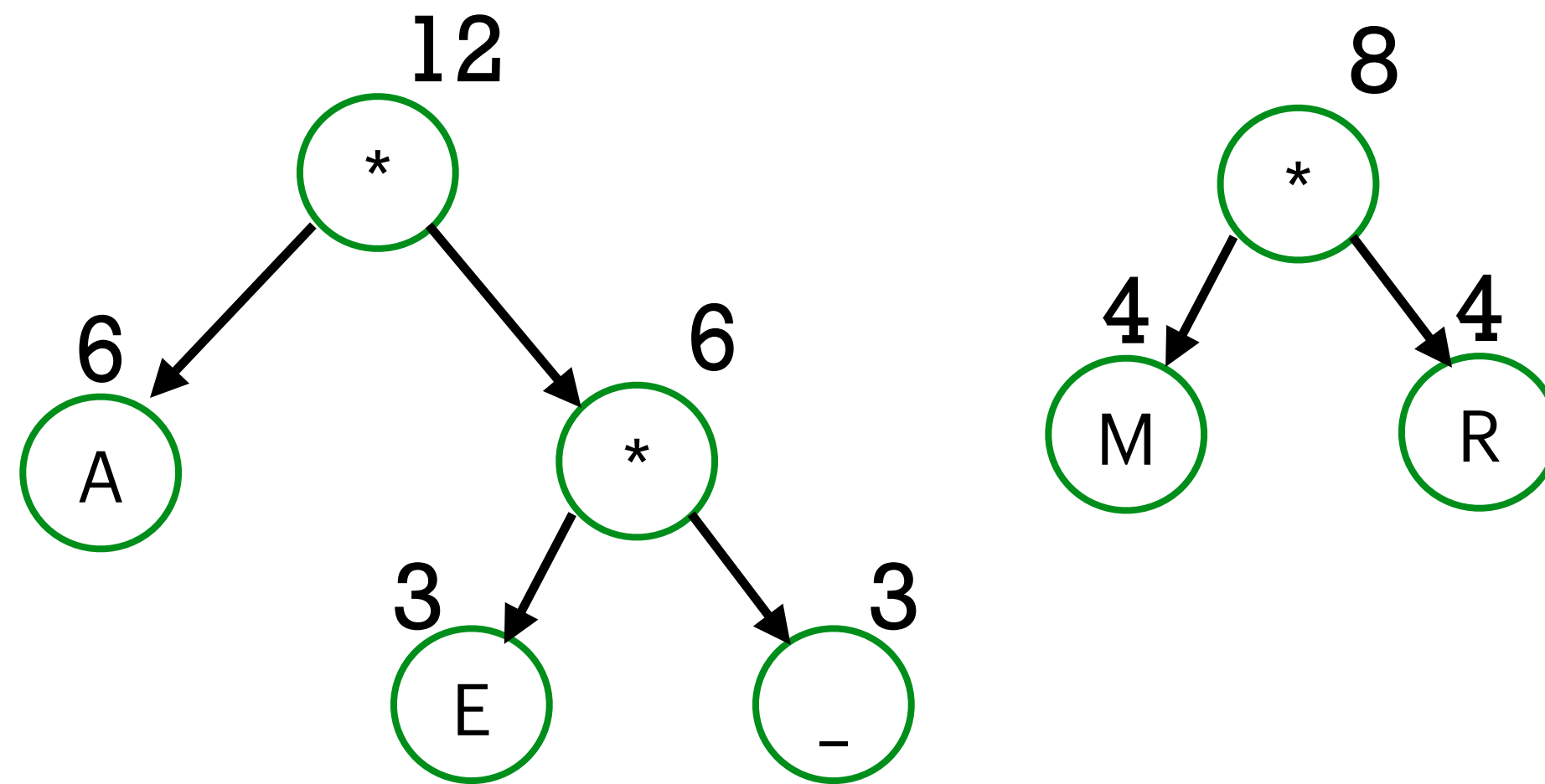
Arbori Huffman



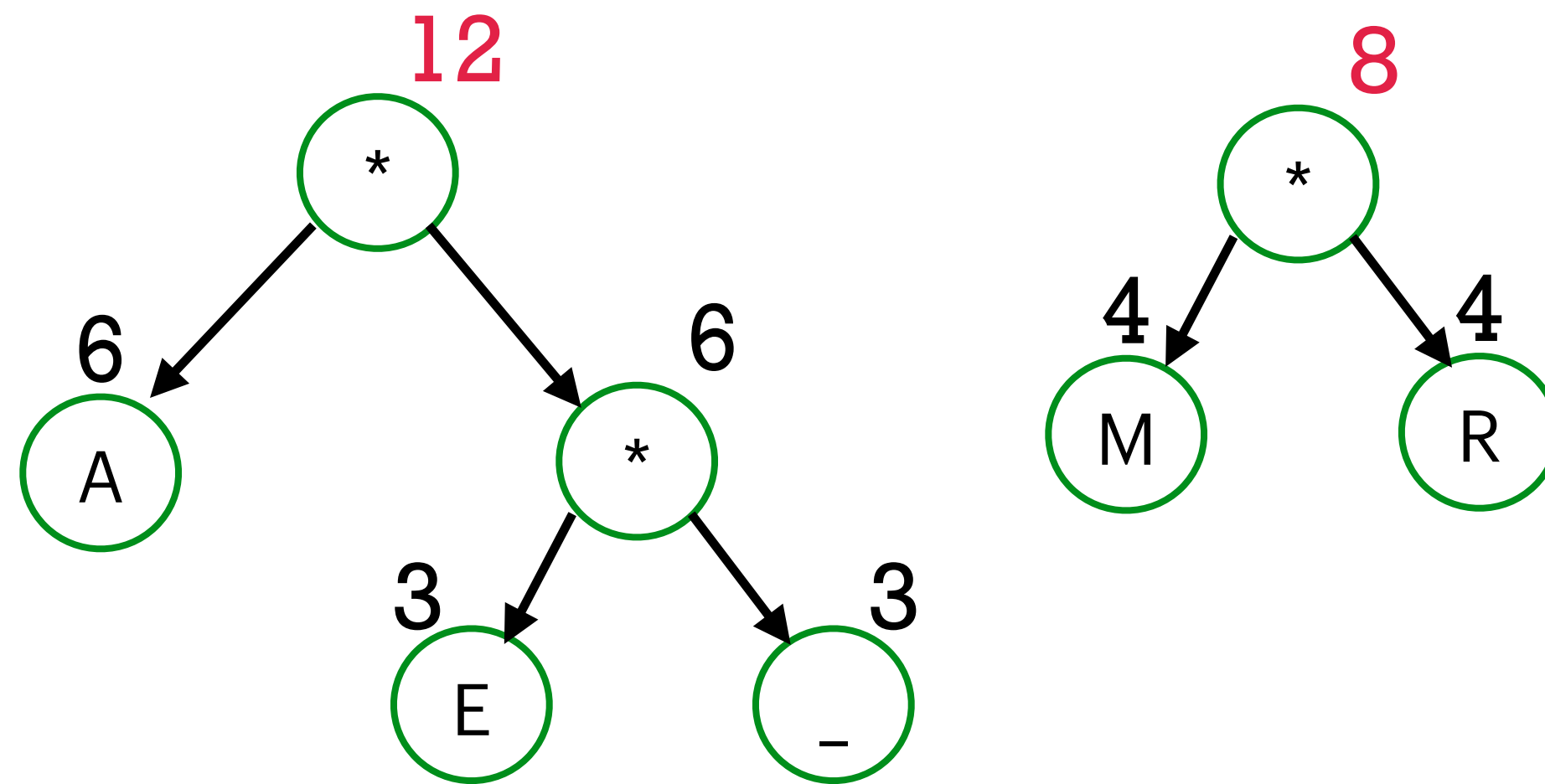
Arbori Huffman



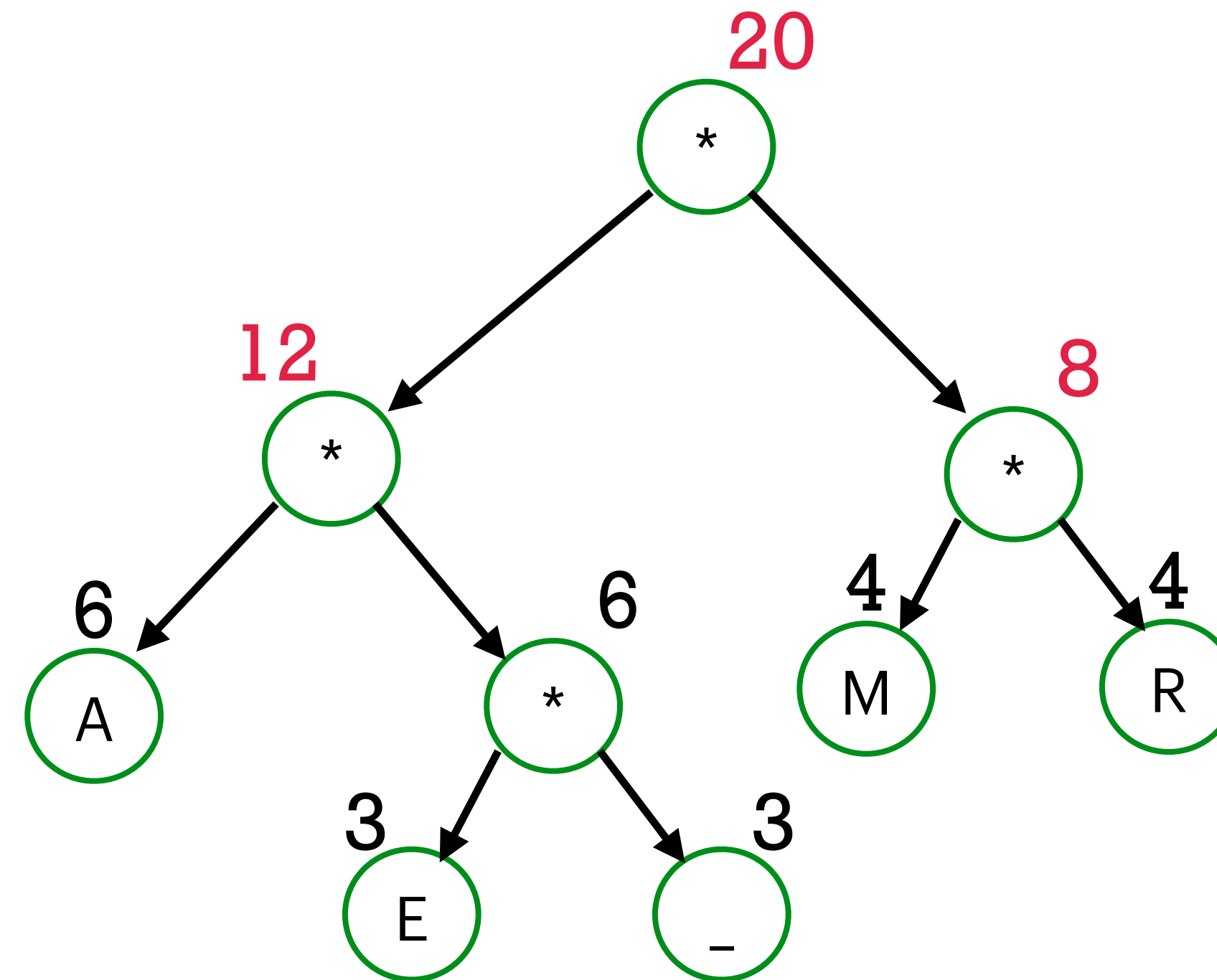
Arbori Huffman



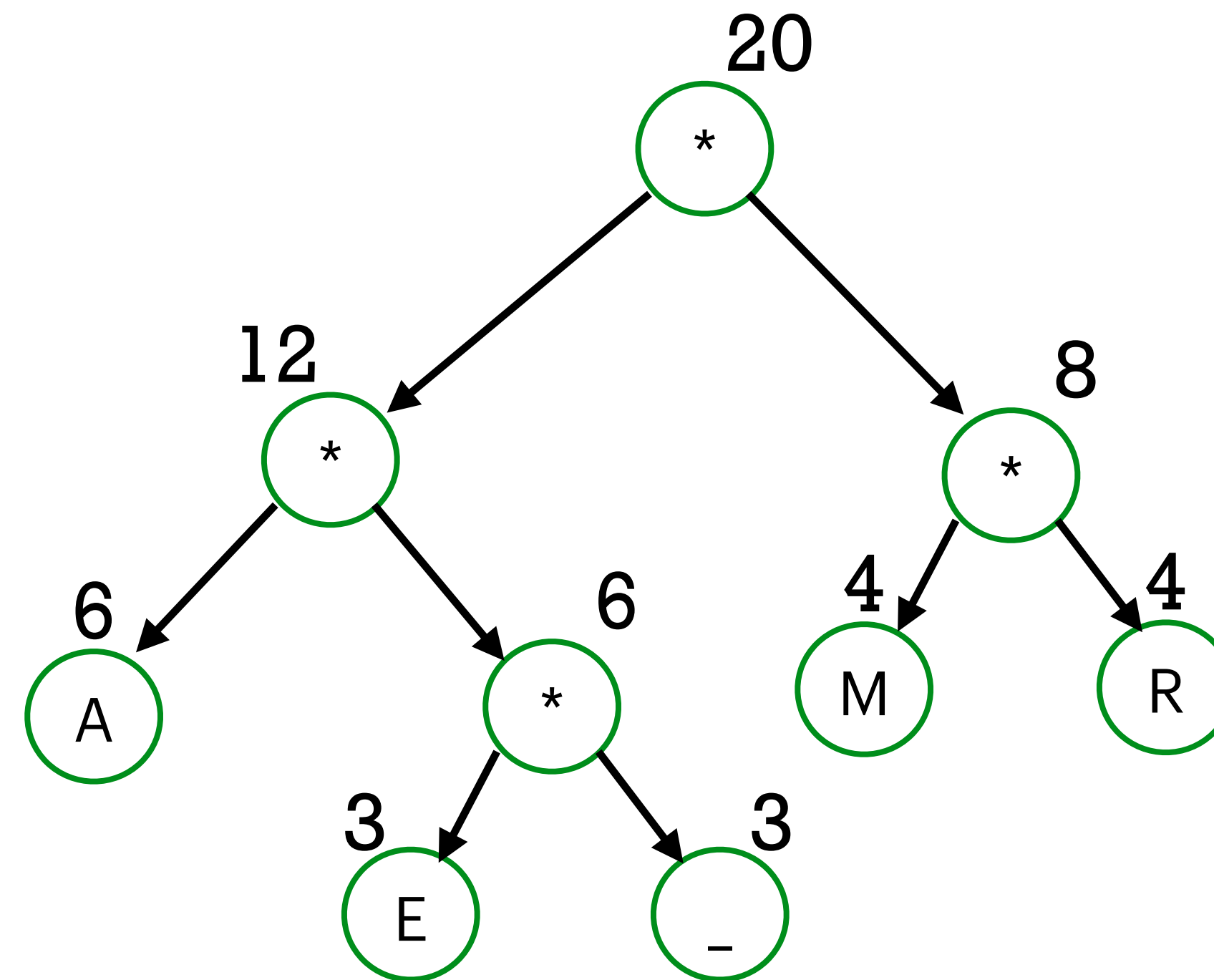
Arbori Huffman



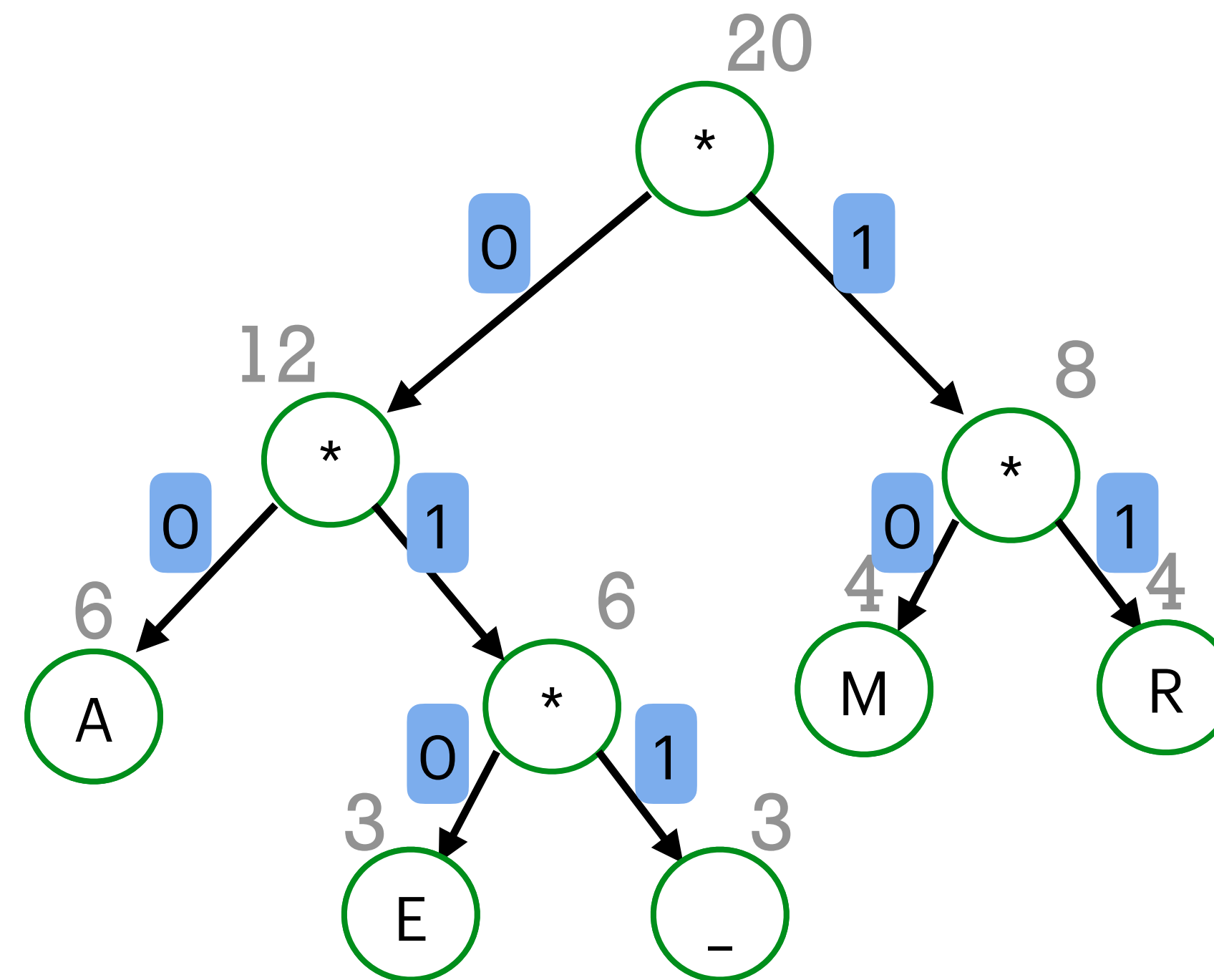
Arbori Huffman



Arbori Huffman

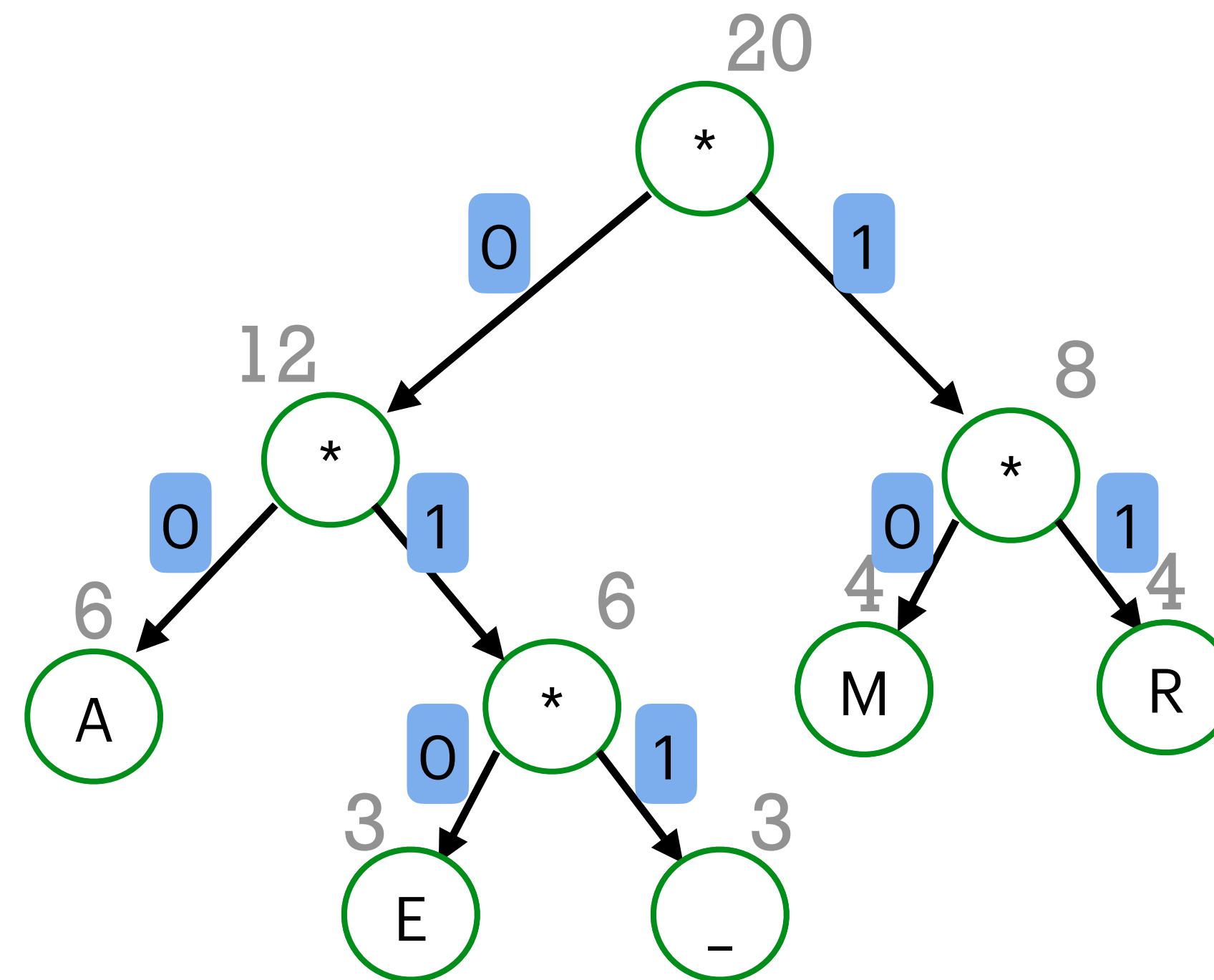


Arbori Huffman



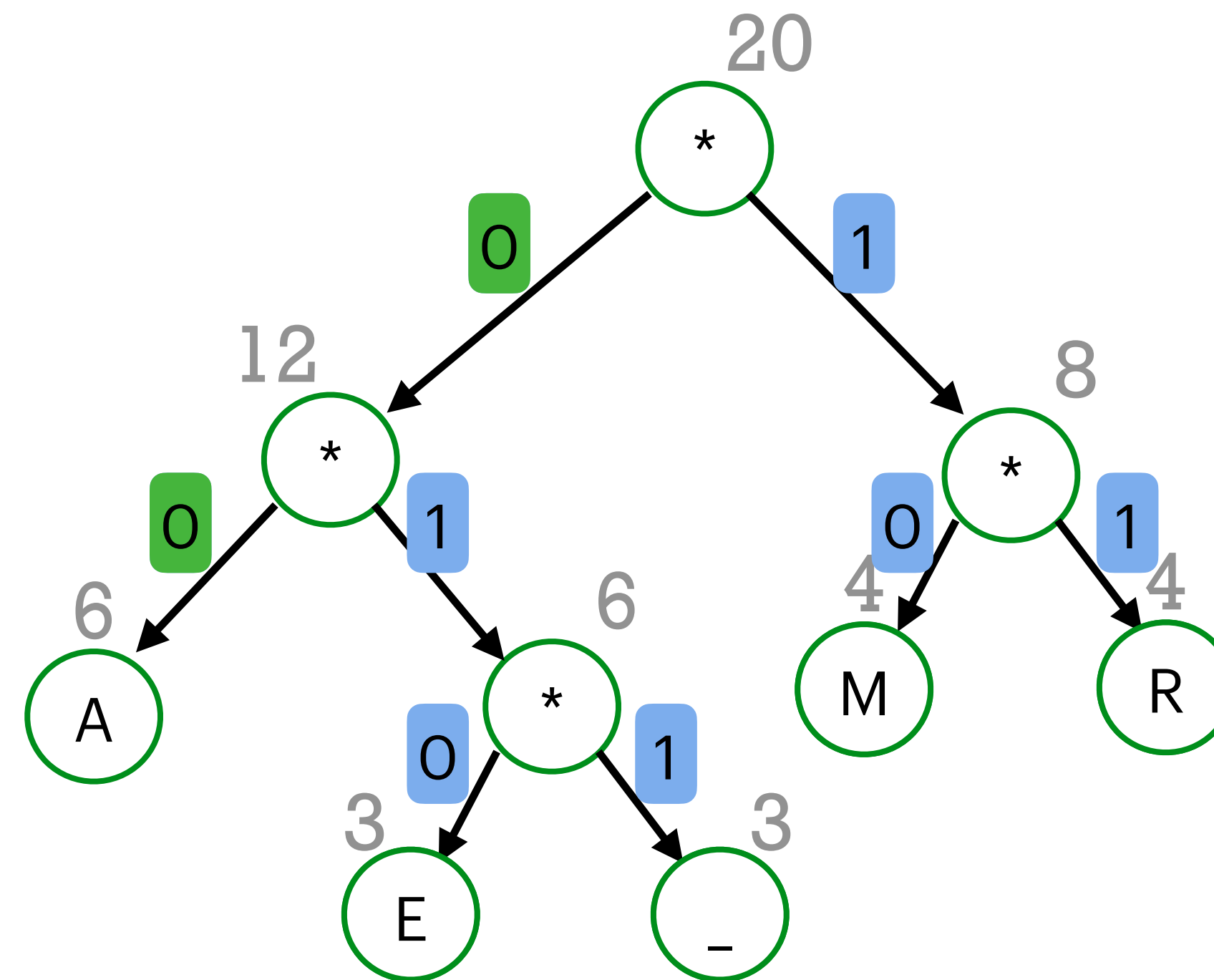
Arbori Huffman

A



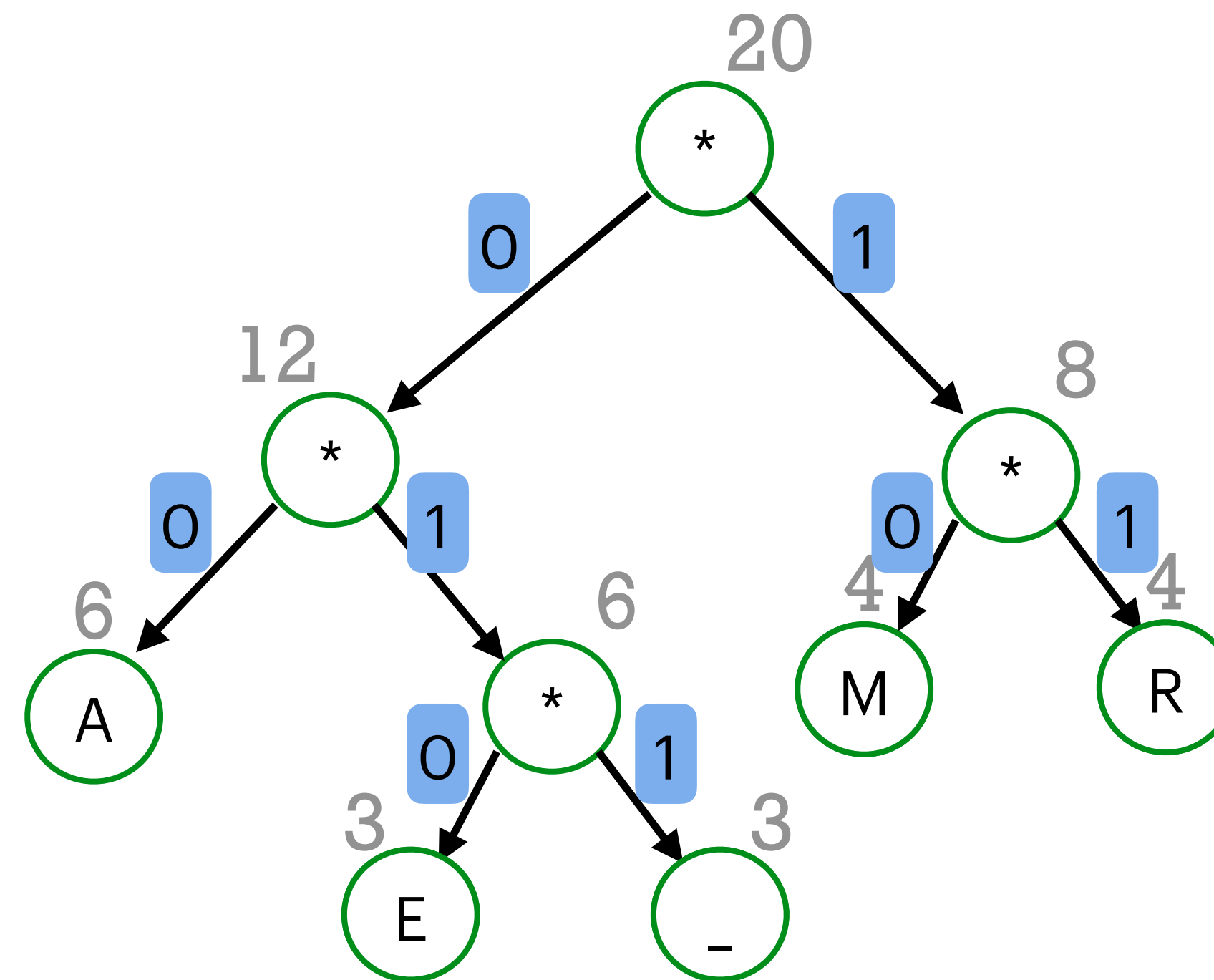
Arbori Huffman

A

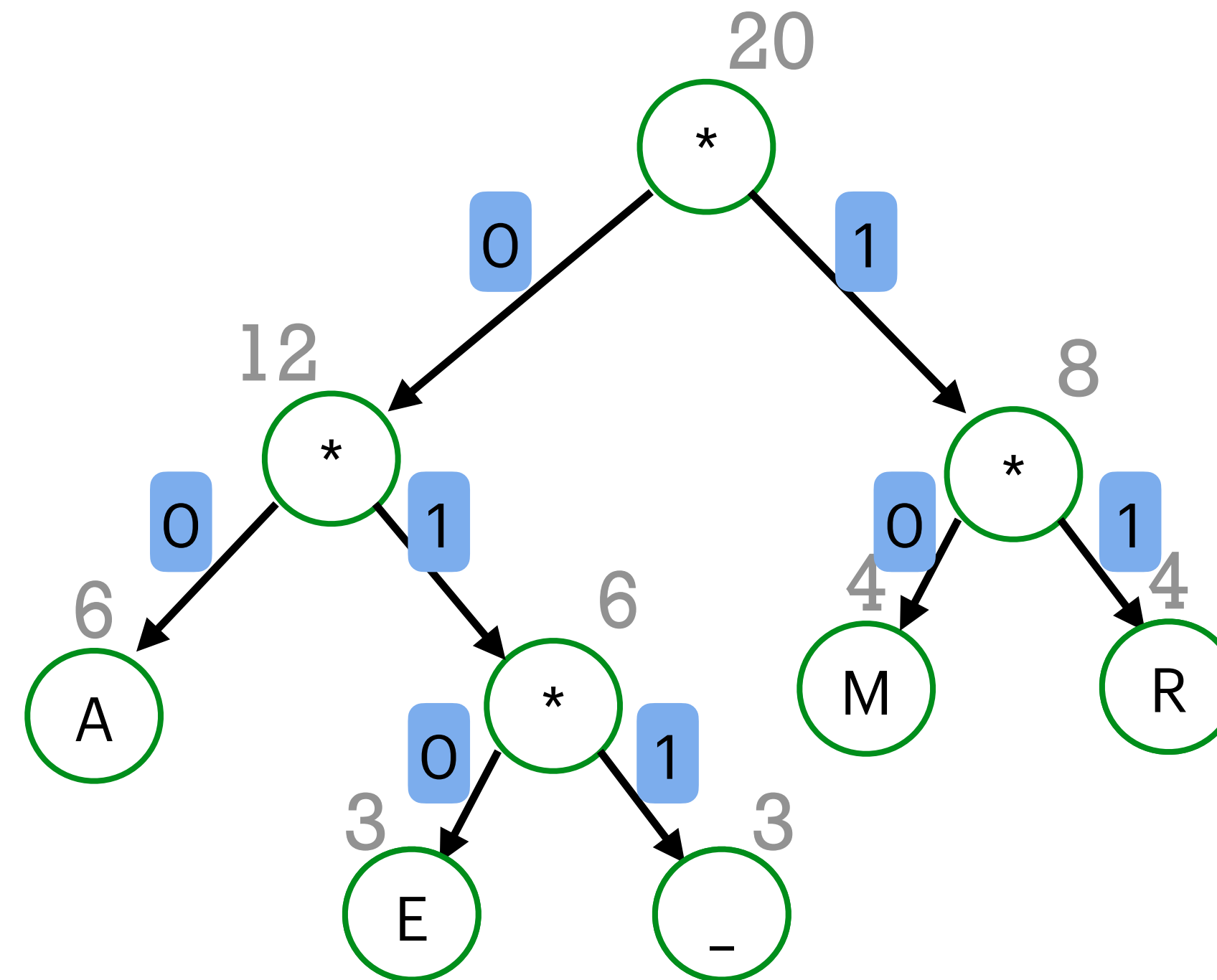
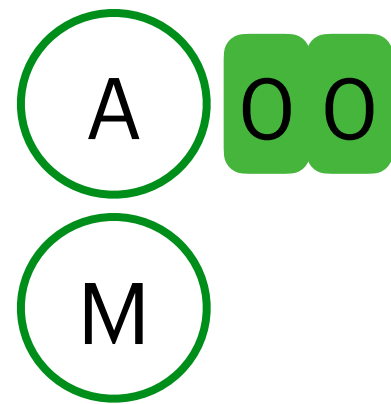


Arbori Huffman

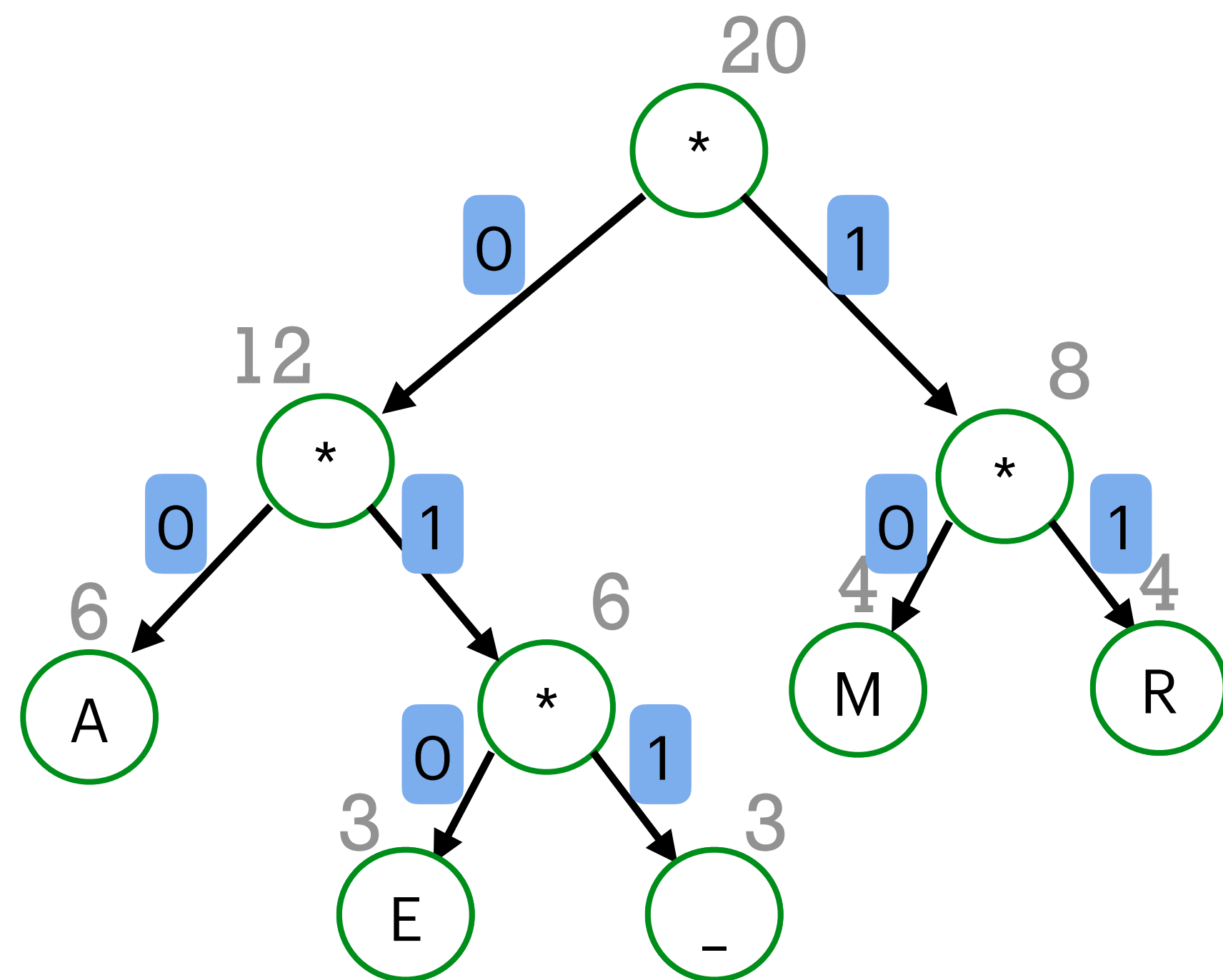
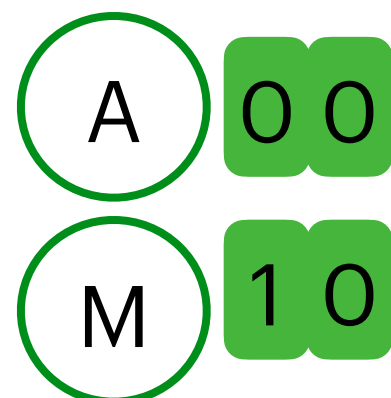
A 00



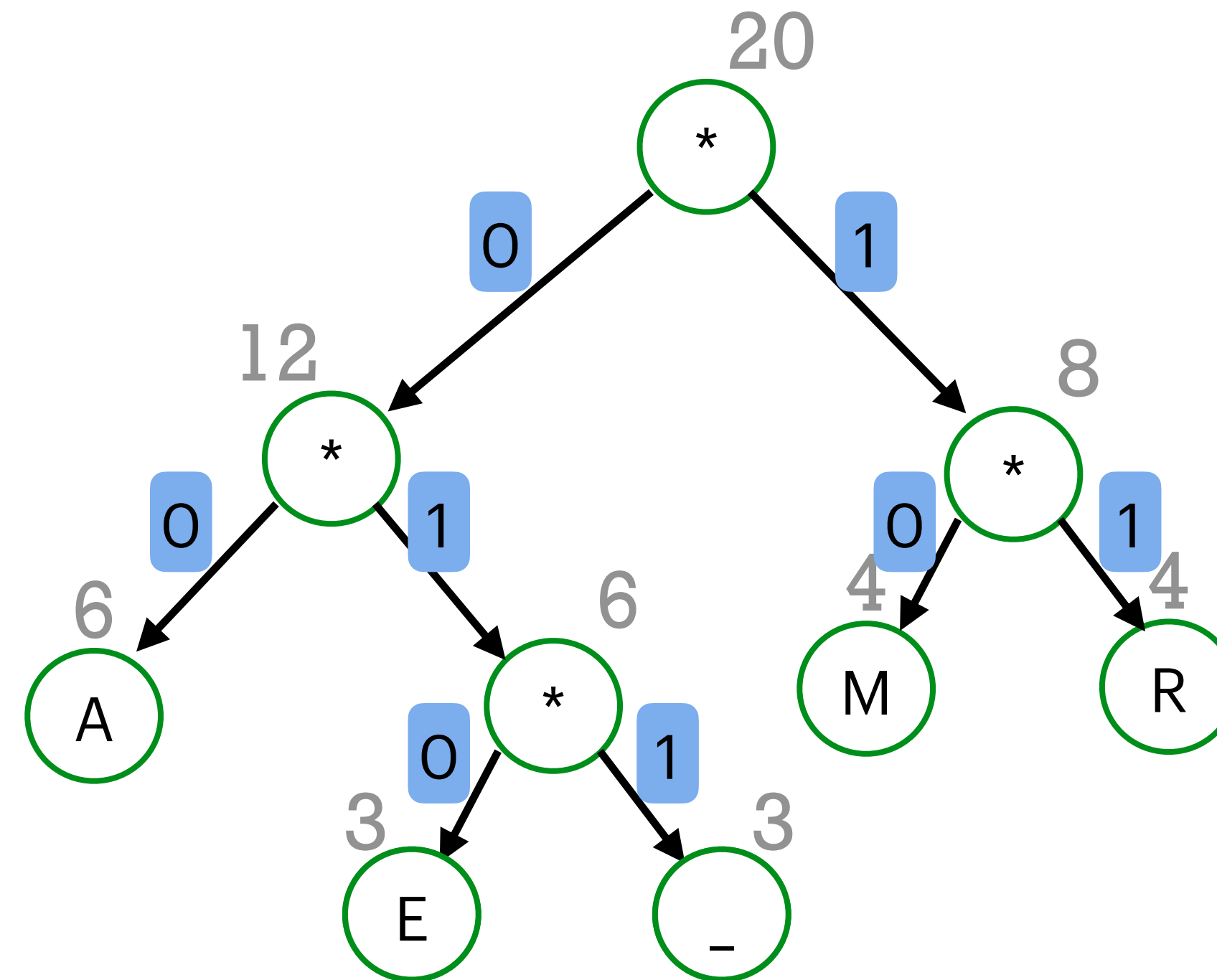
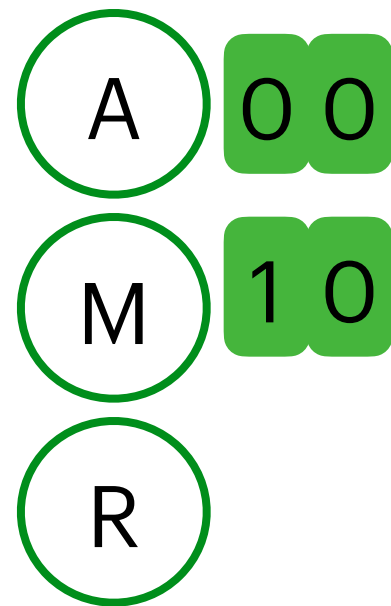
Arbori Huffman



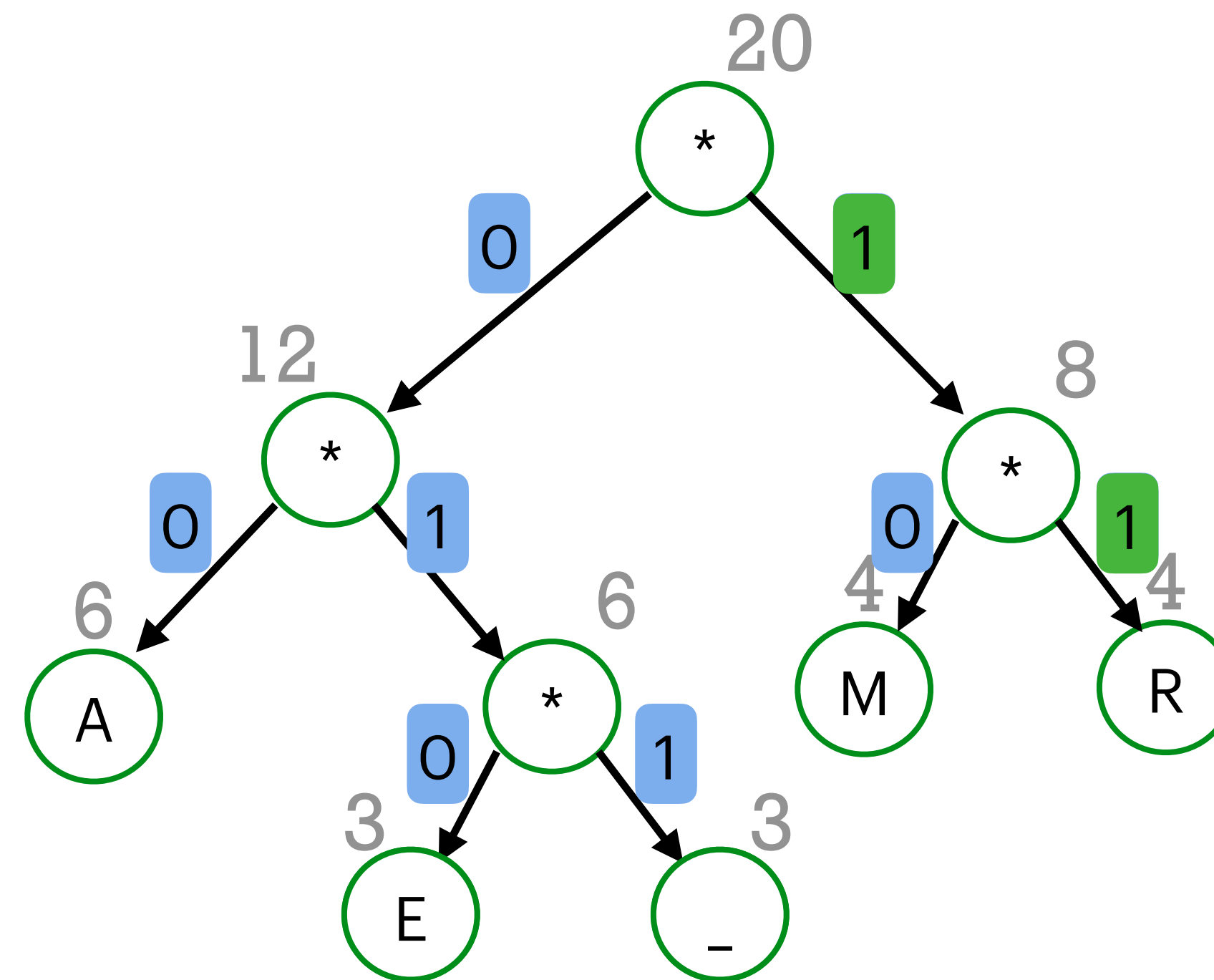
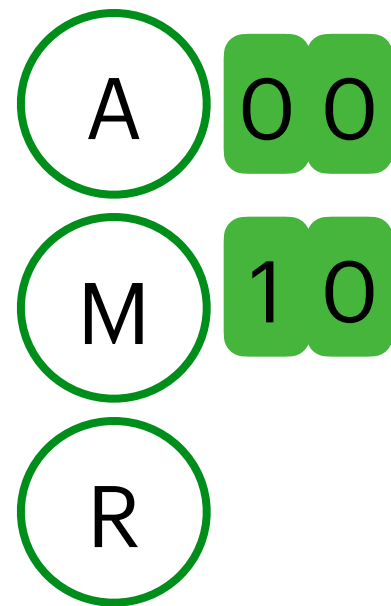
Arbori Huffman



Arbori Huffman

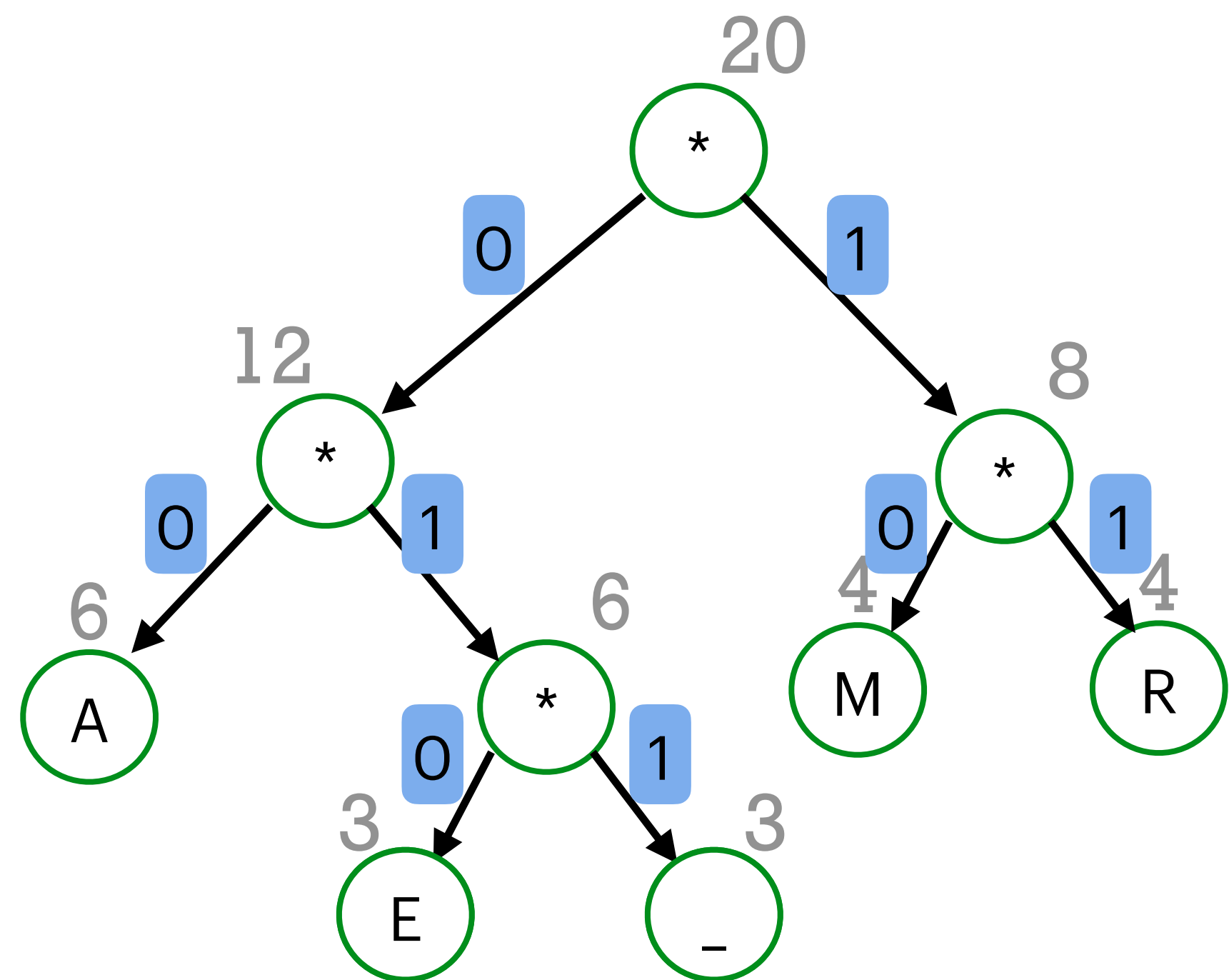


Arbori Huffman

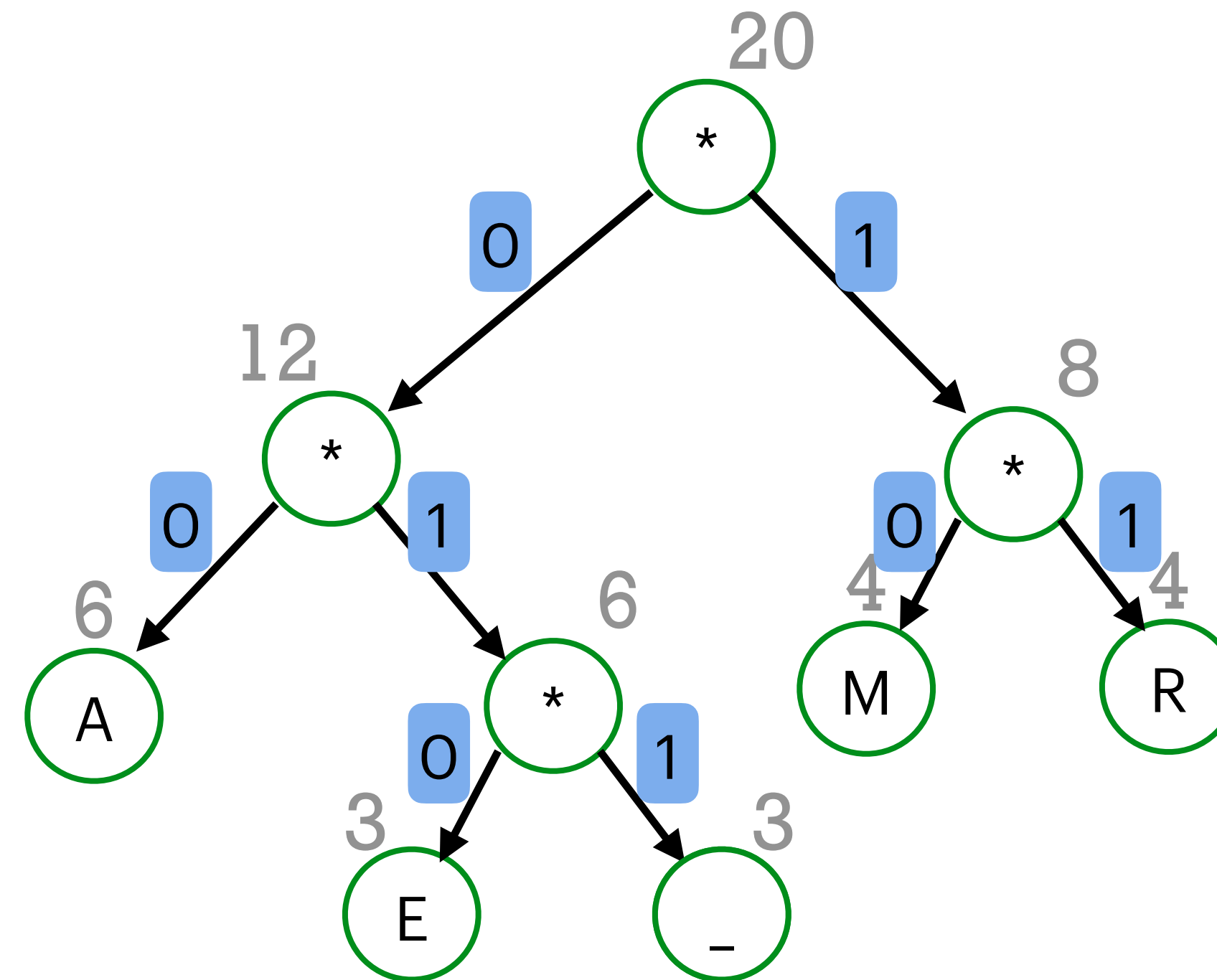
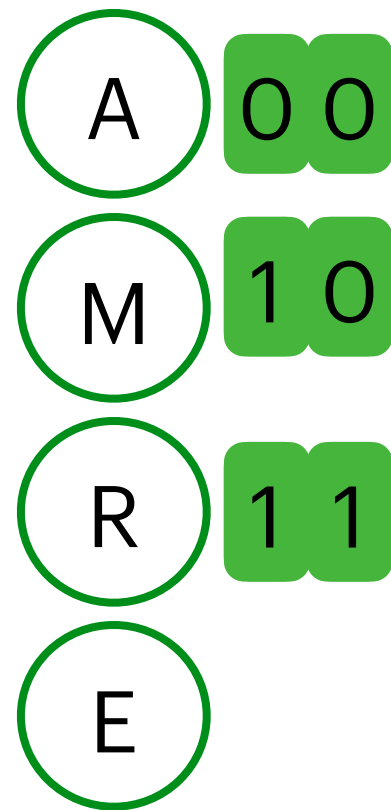


Arbori Huffman

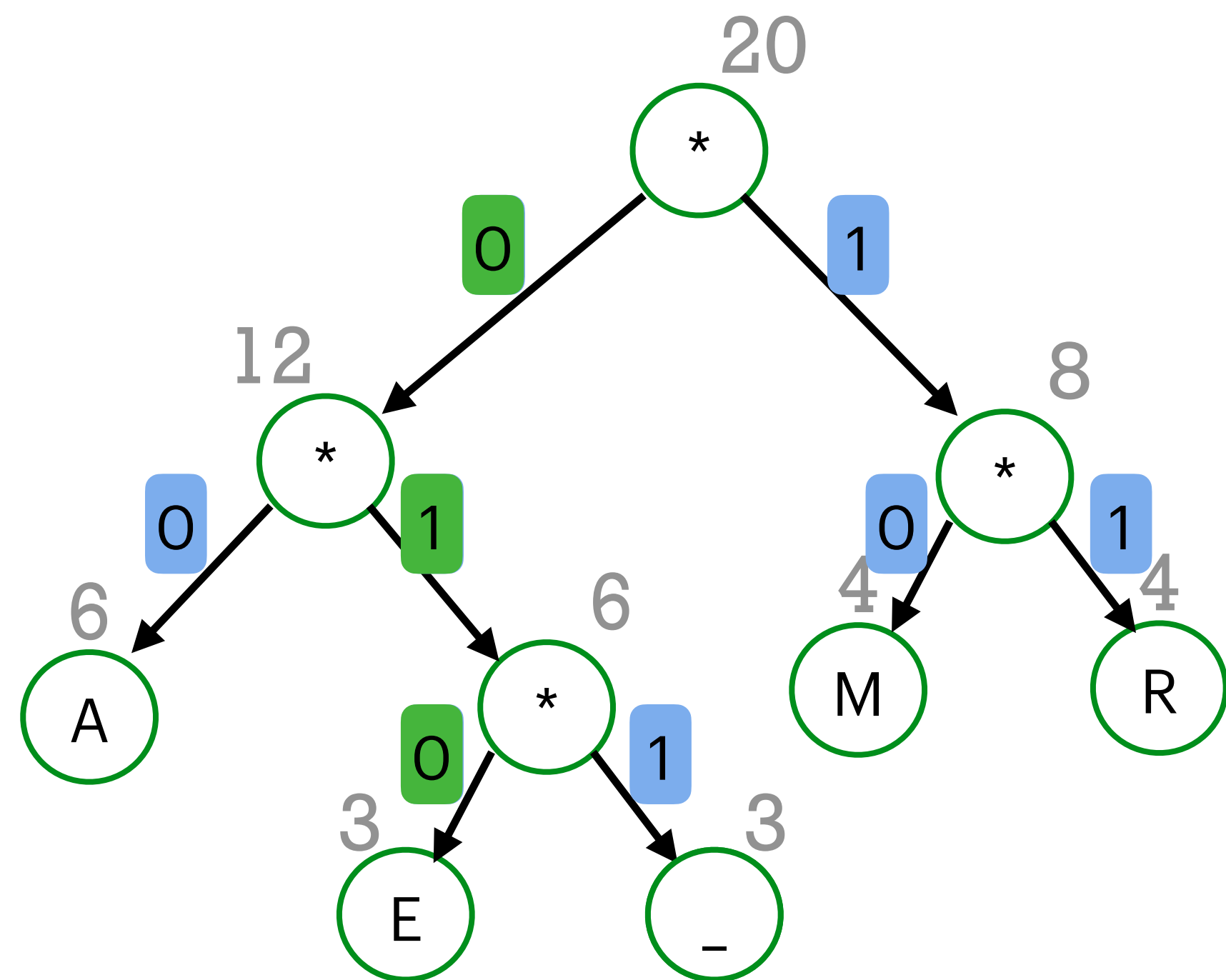
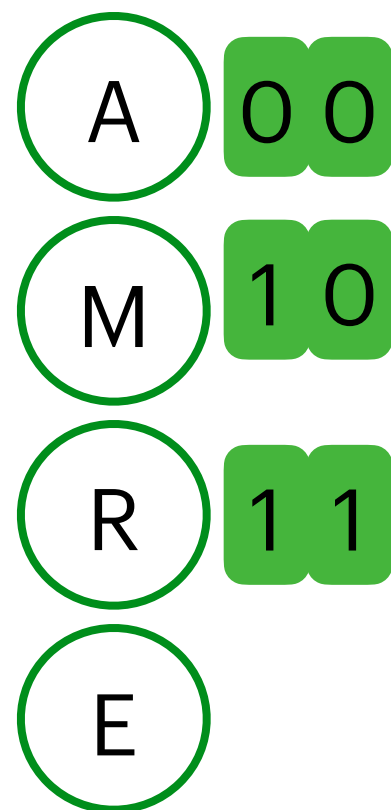
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M	1 0
R	1 1



Arbori Huffman

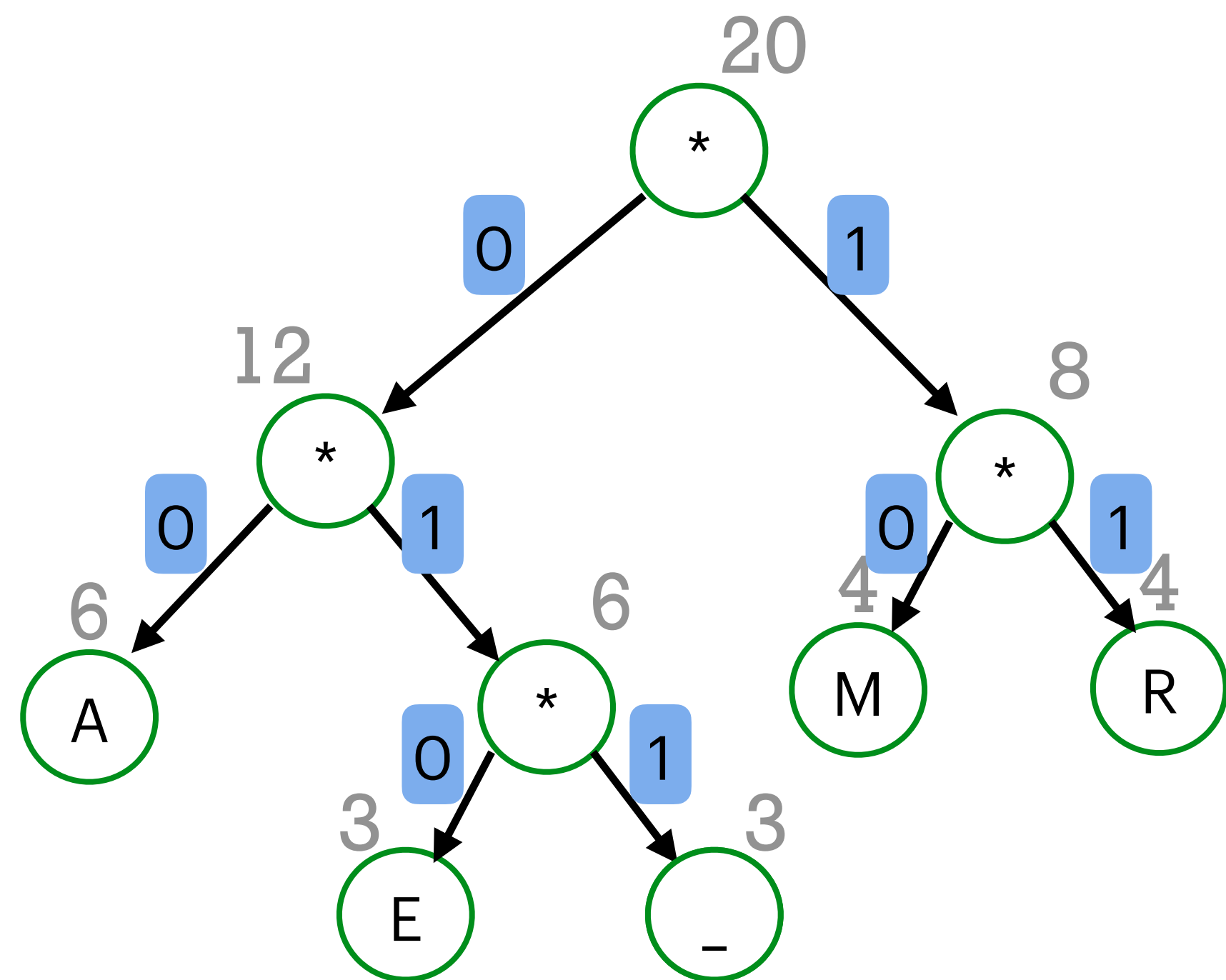


Arbori Huffman

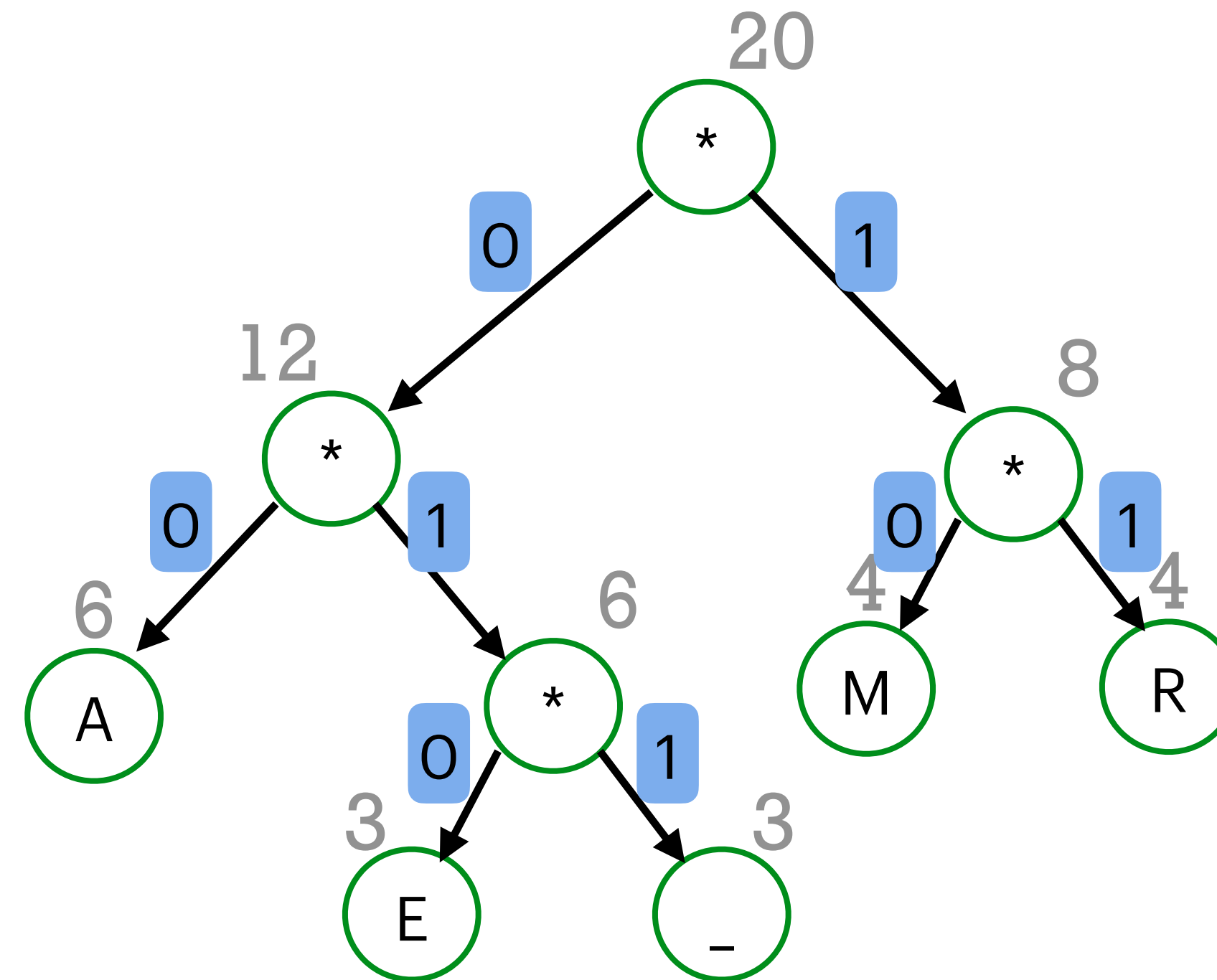
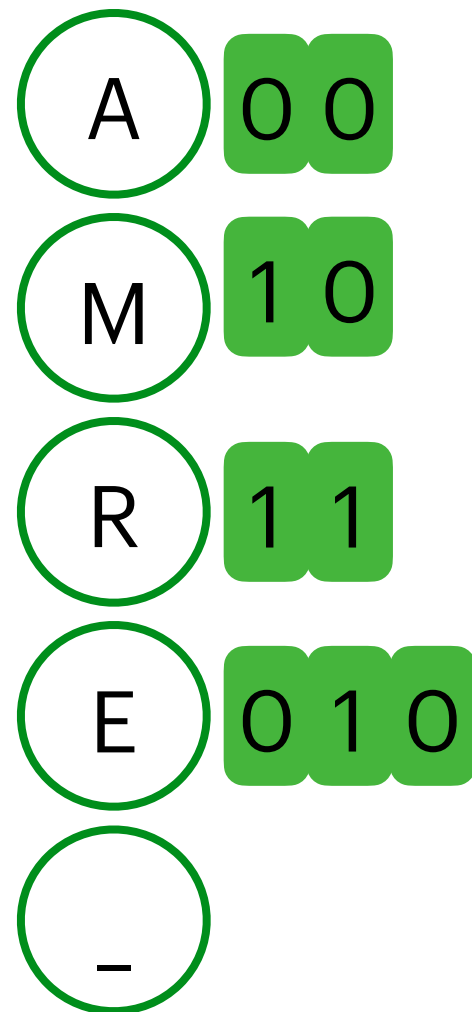


Arbori Huffman

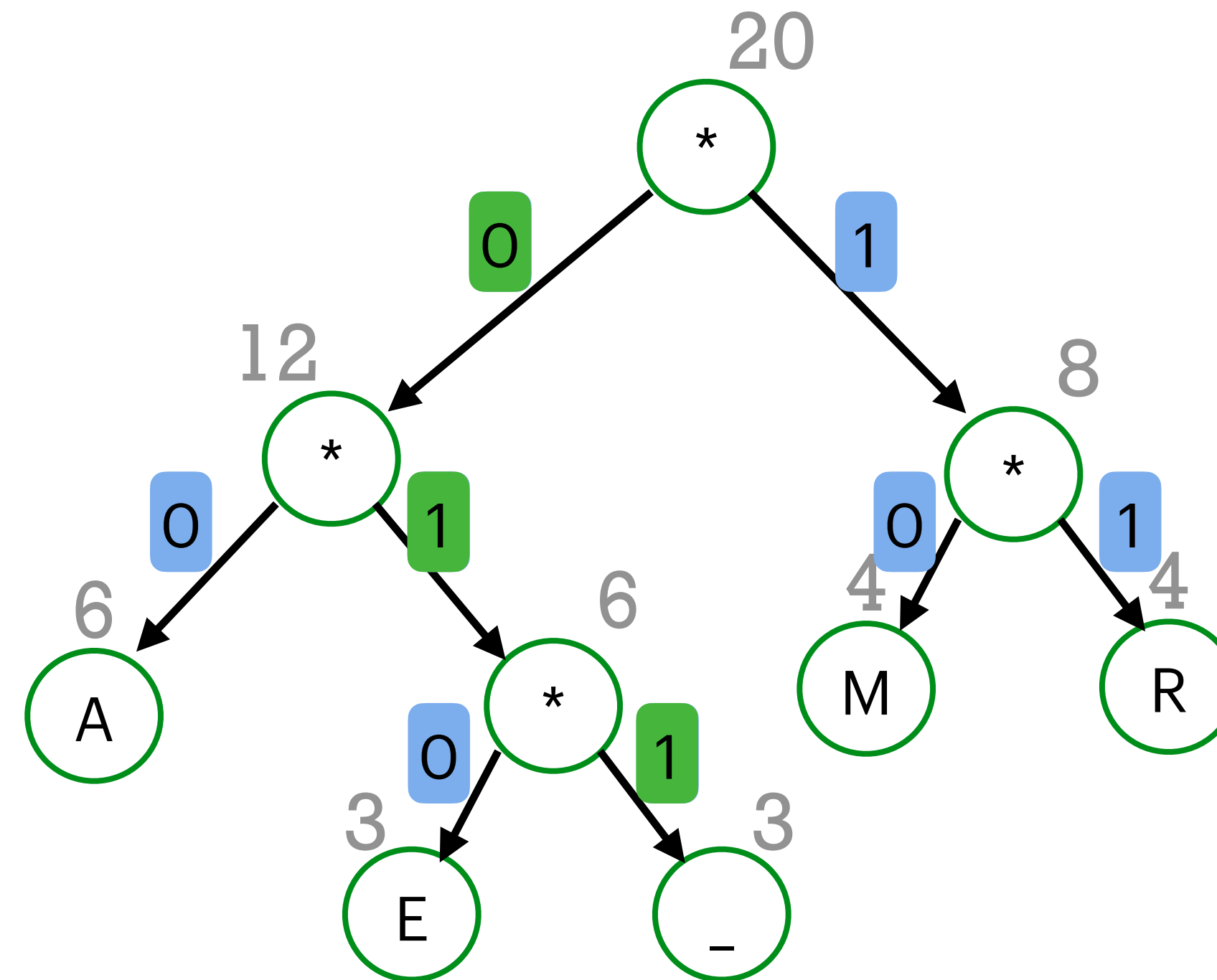
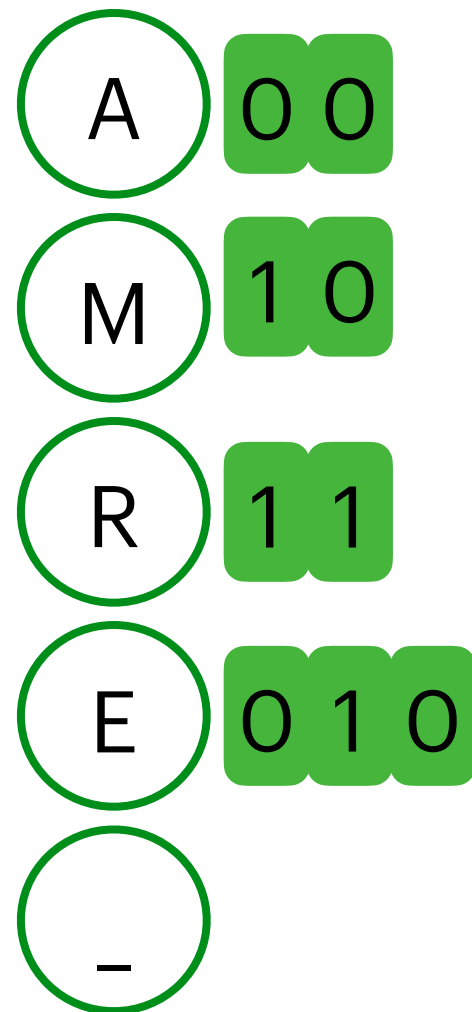
A	0 0
M	1 0
R	1 1
E	0 1 0



Arbori Huffman

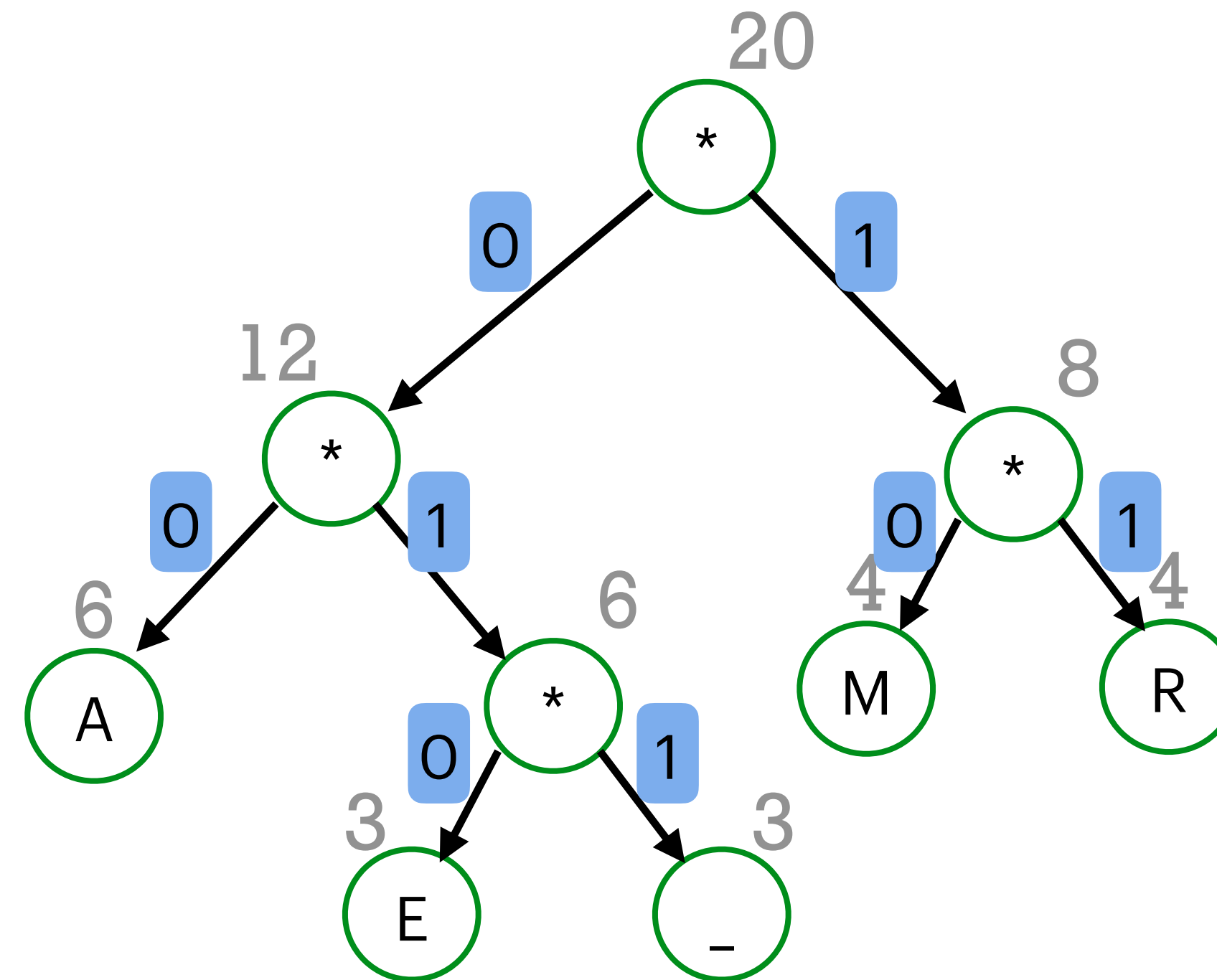


Arbori Huffman



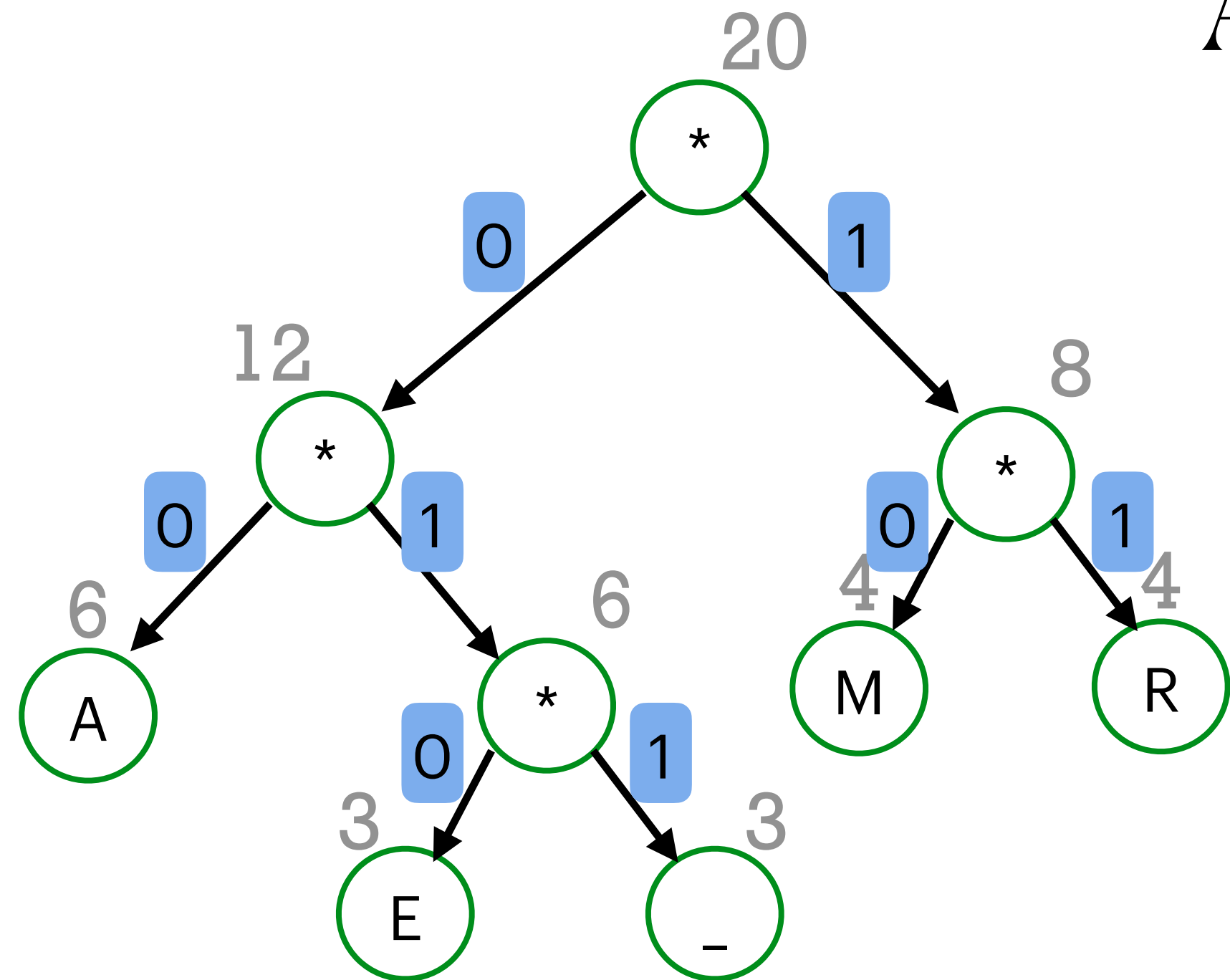
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



Arbori Huffman

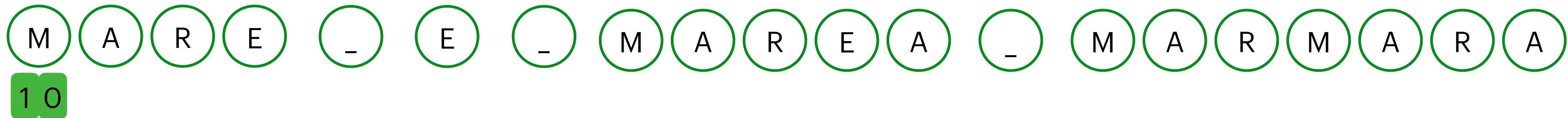
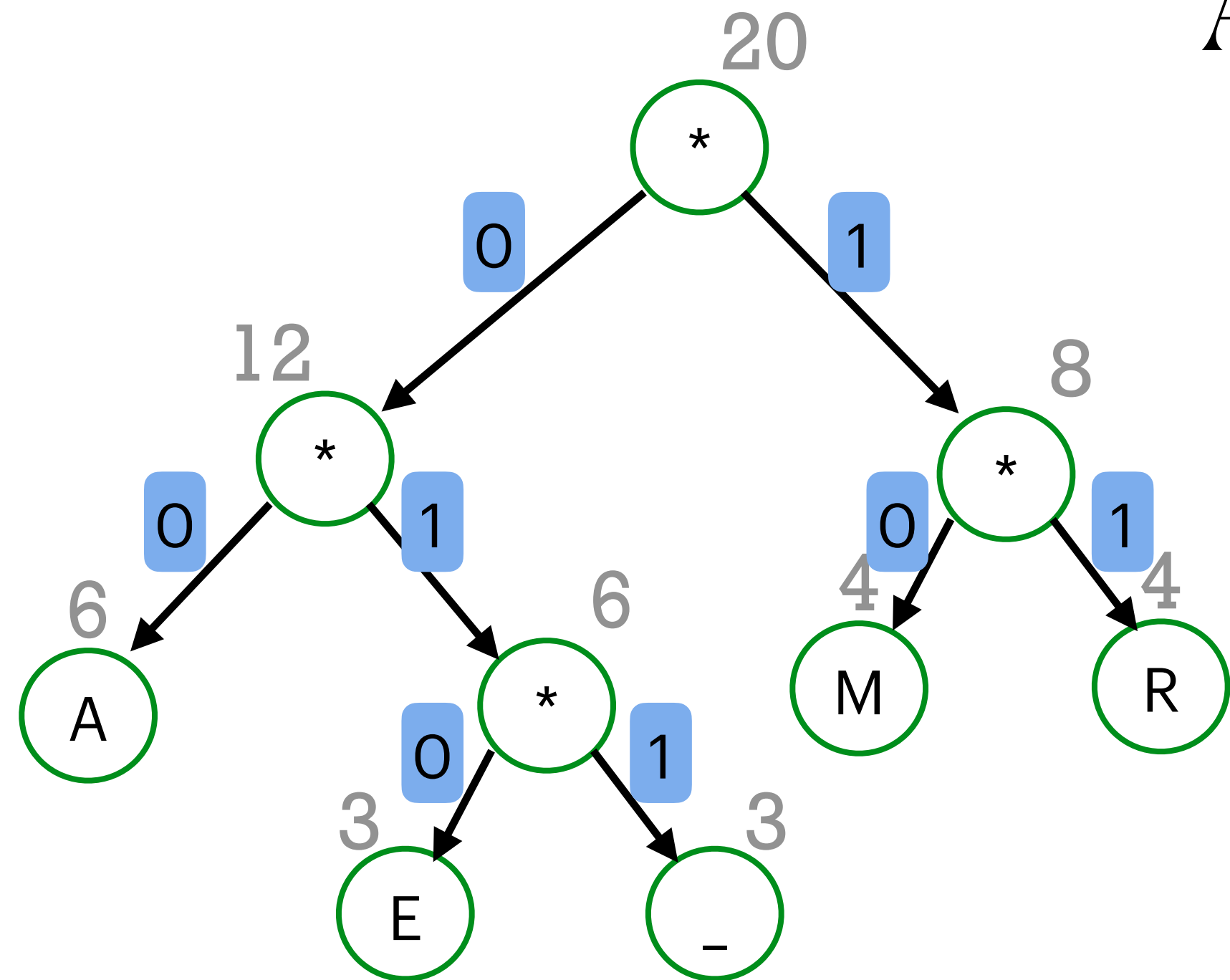
A	0 0
M	1 0
R	1 1
E	0 1 0
_	0 1 1



M A R E _ E _ M A R E A _ M A R M A R A

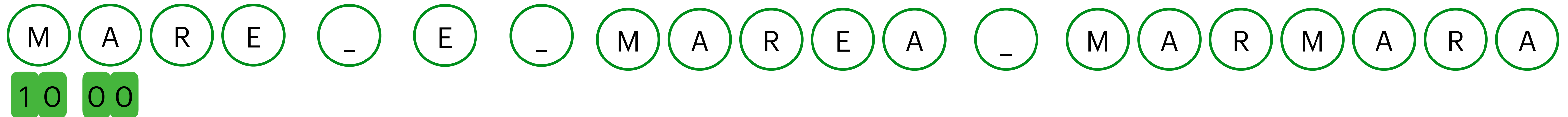
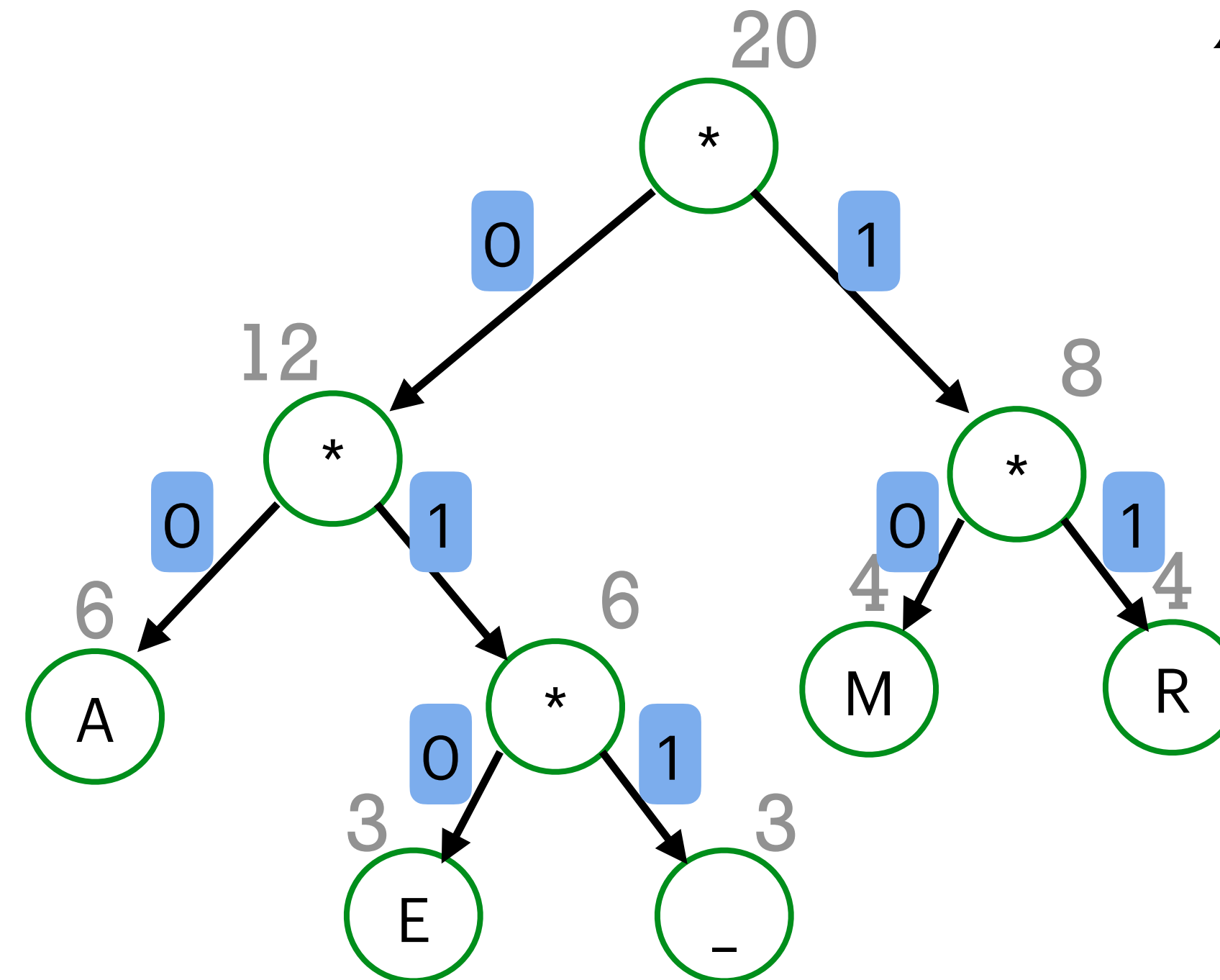
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
_	0 1 1



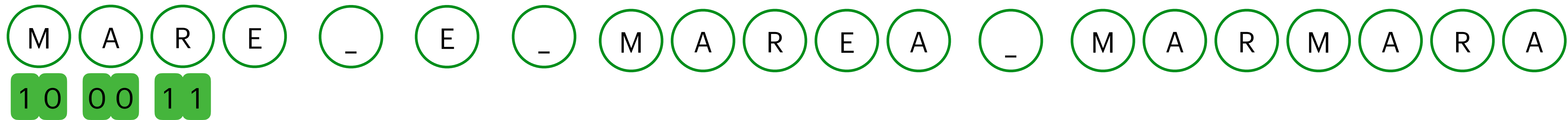
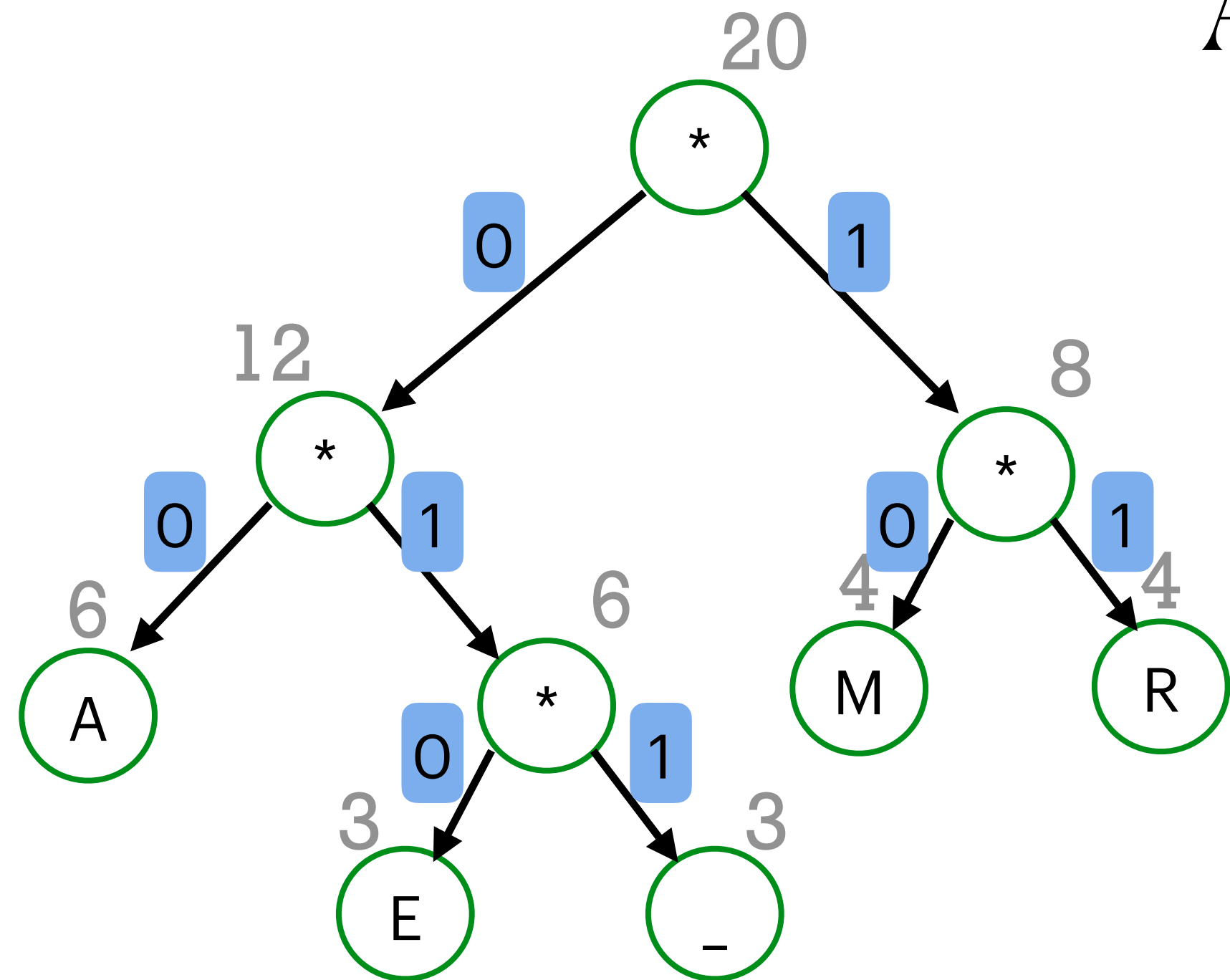
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
_	0 1 1



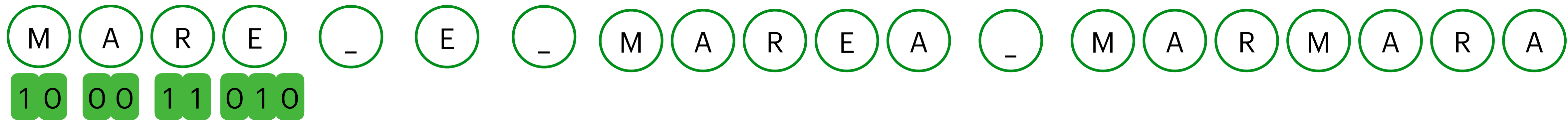
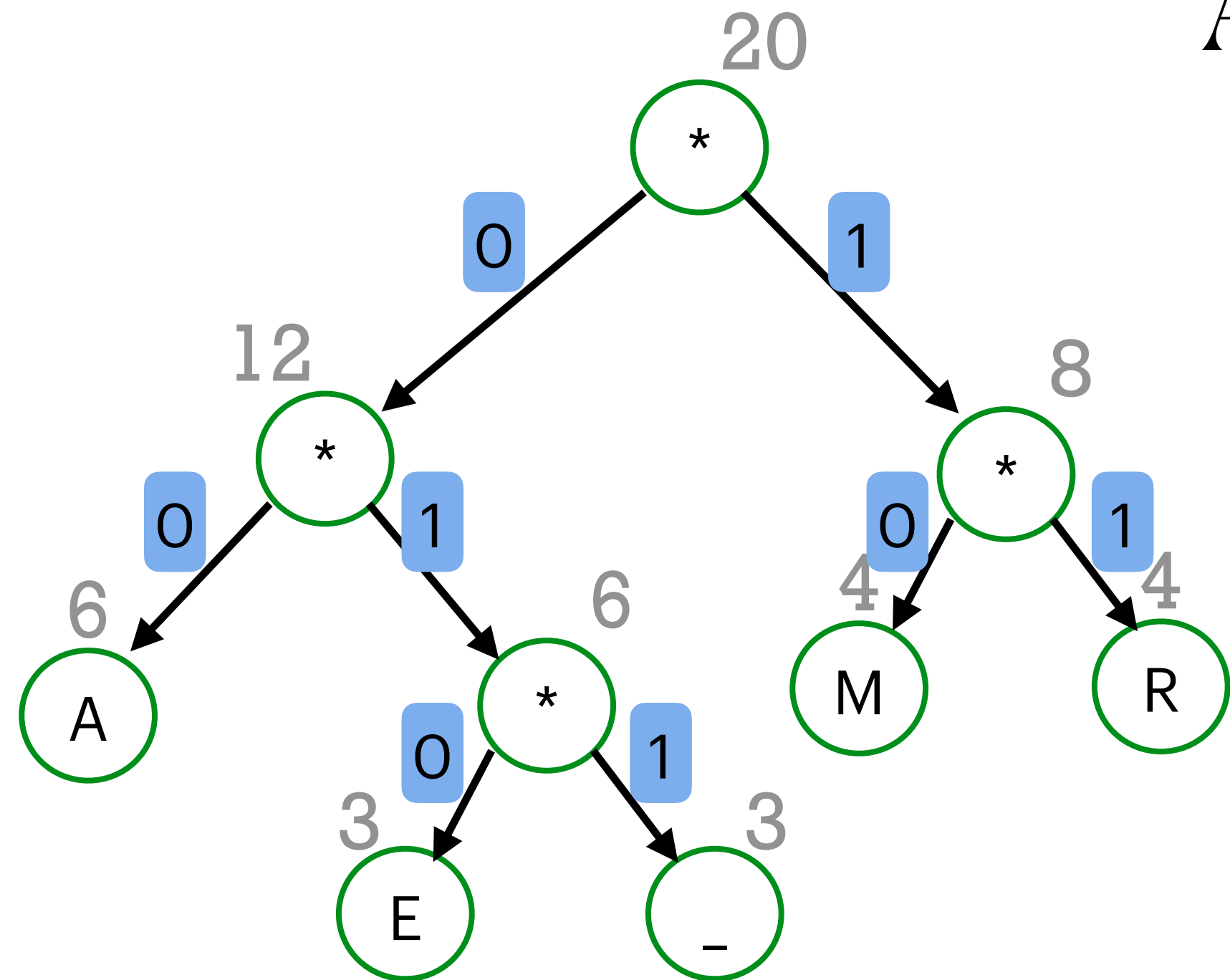
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
_	0 1 1



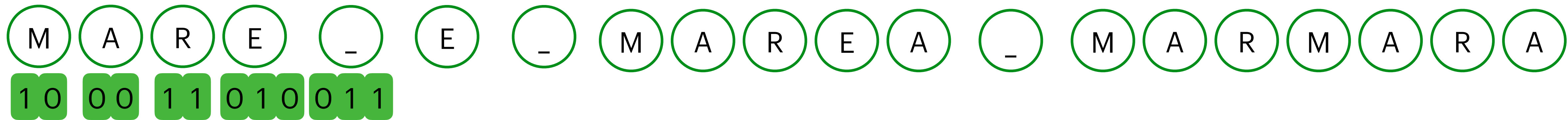
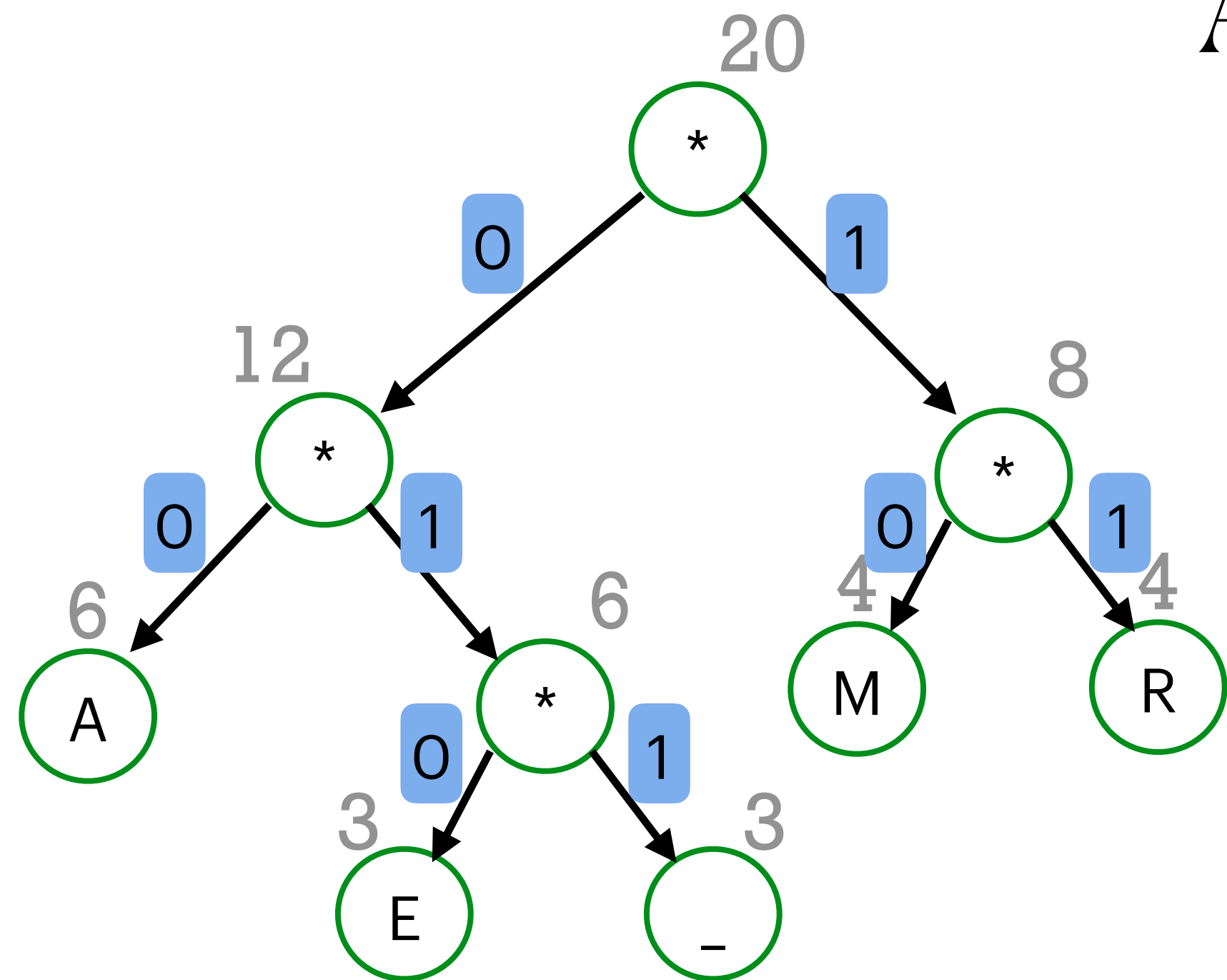
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
_	0 1 1



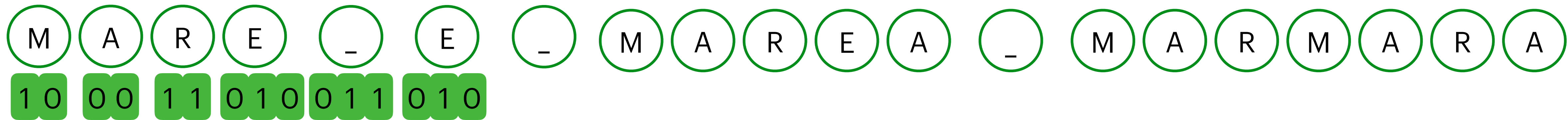
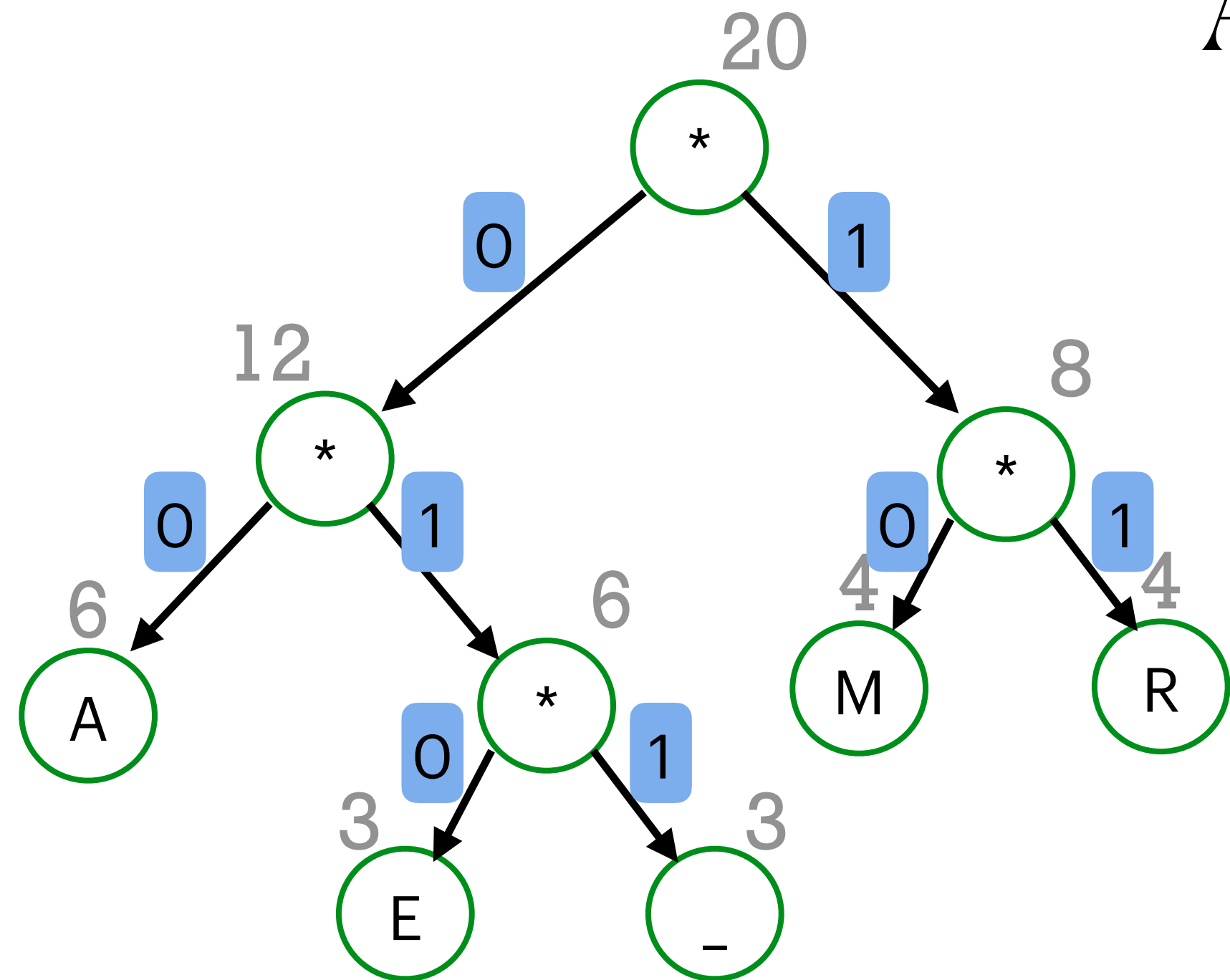
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
_	0 1 1



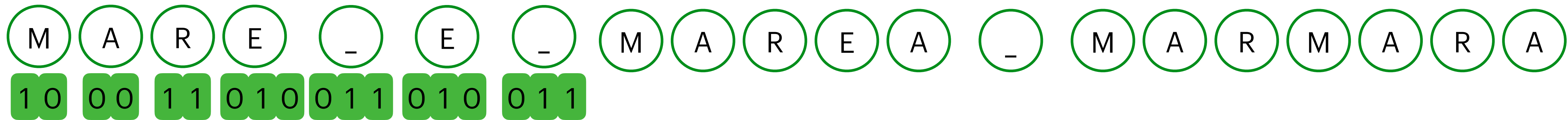
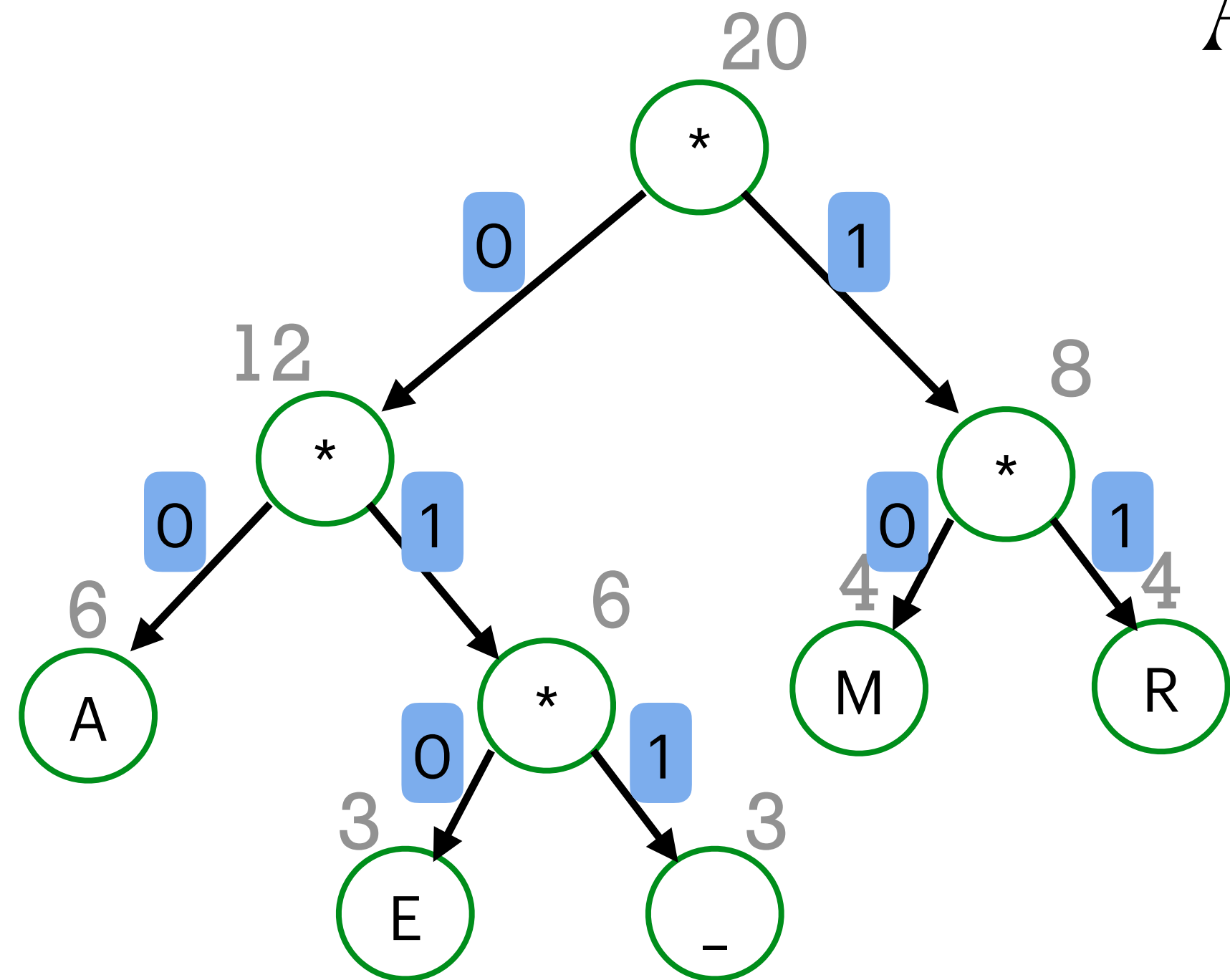
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
_	0 1 1



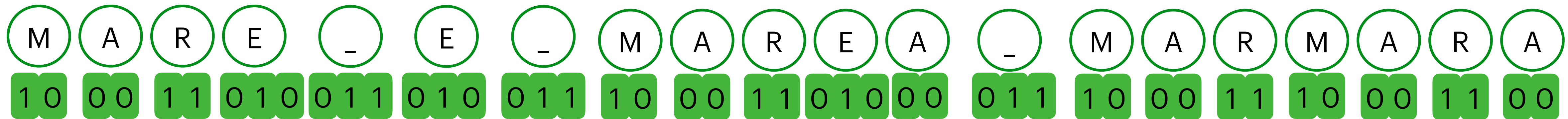
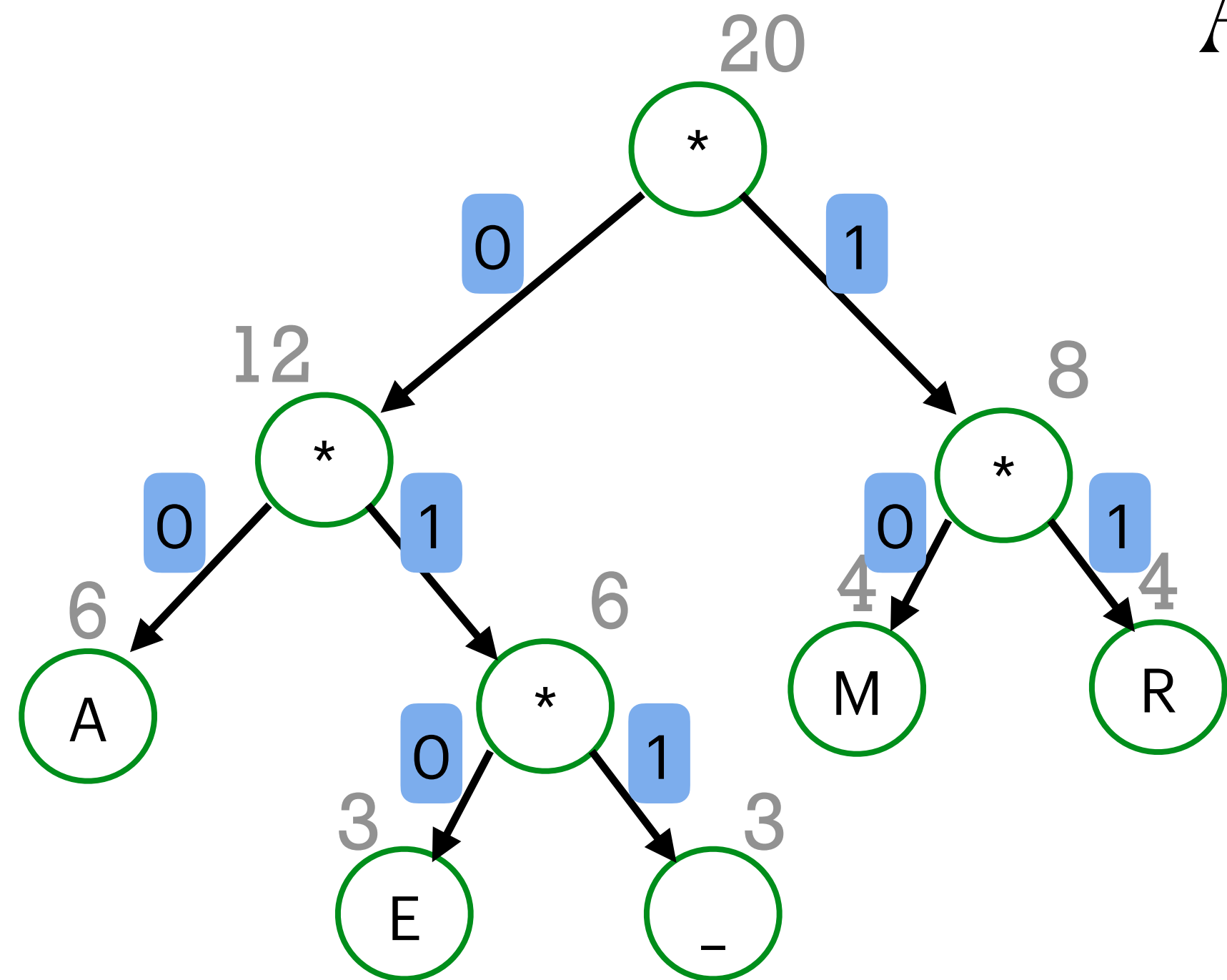
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
_	0 1 1



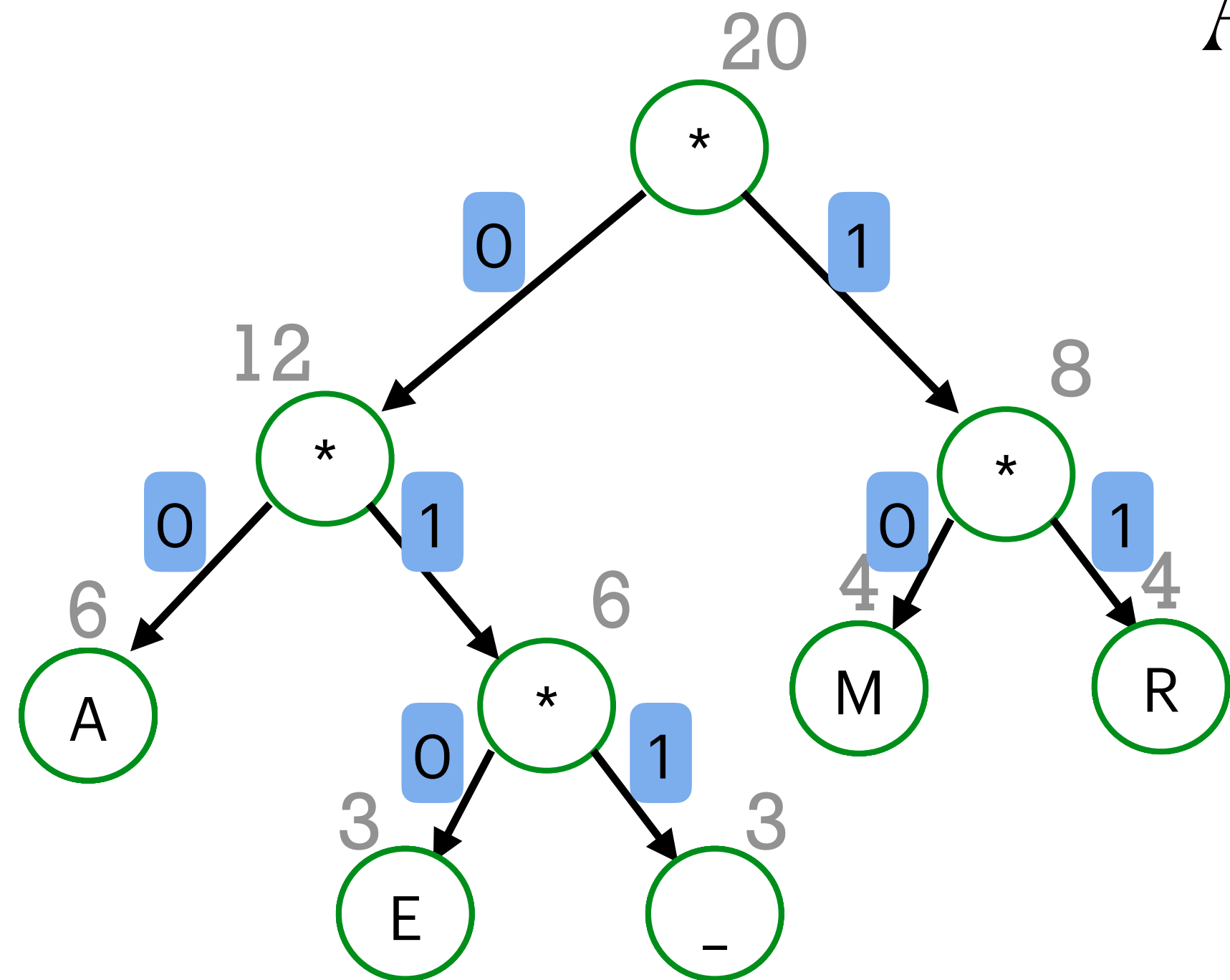
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
_	0 1 1



Arbori Huffman

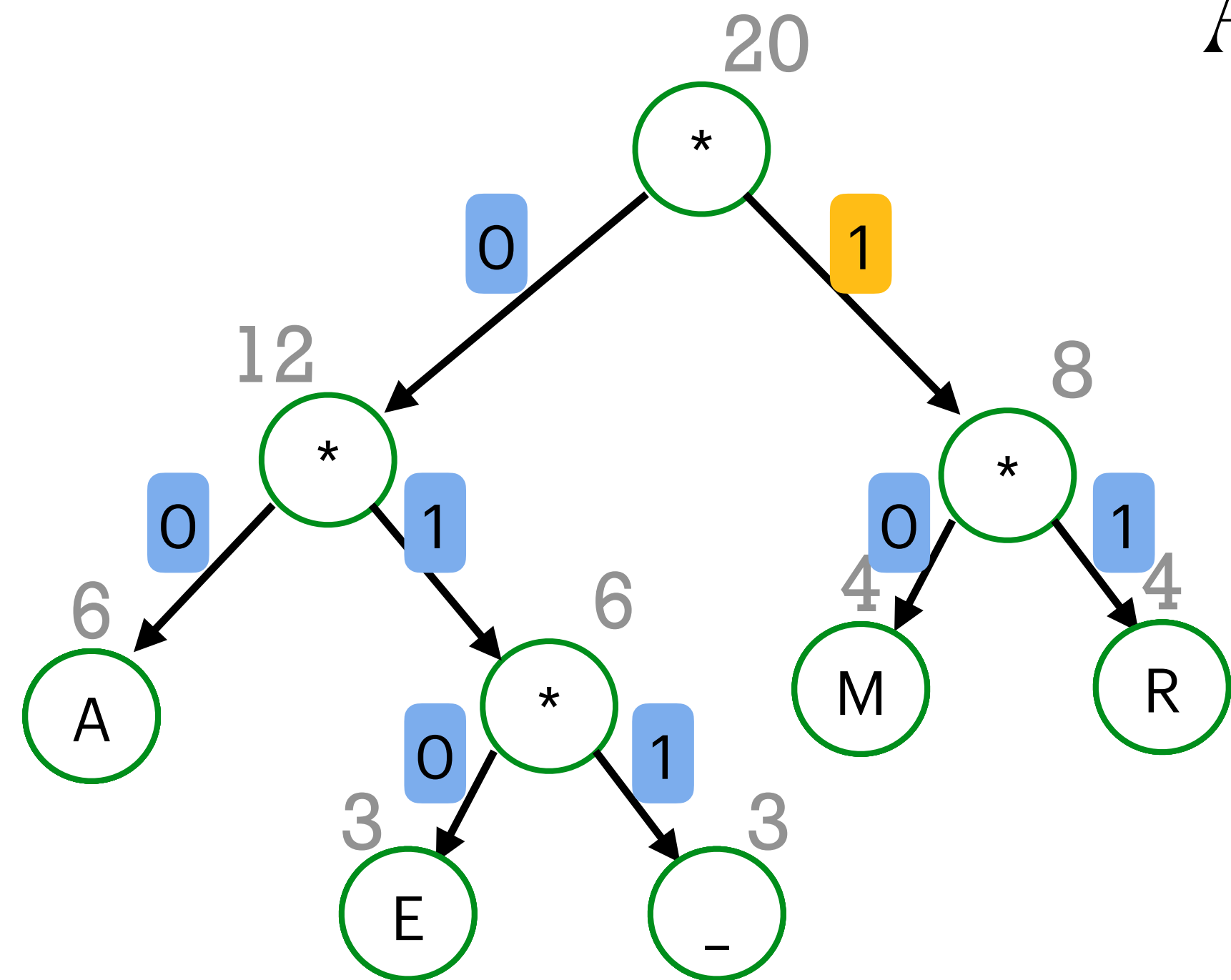
A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



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Arbori Huffman

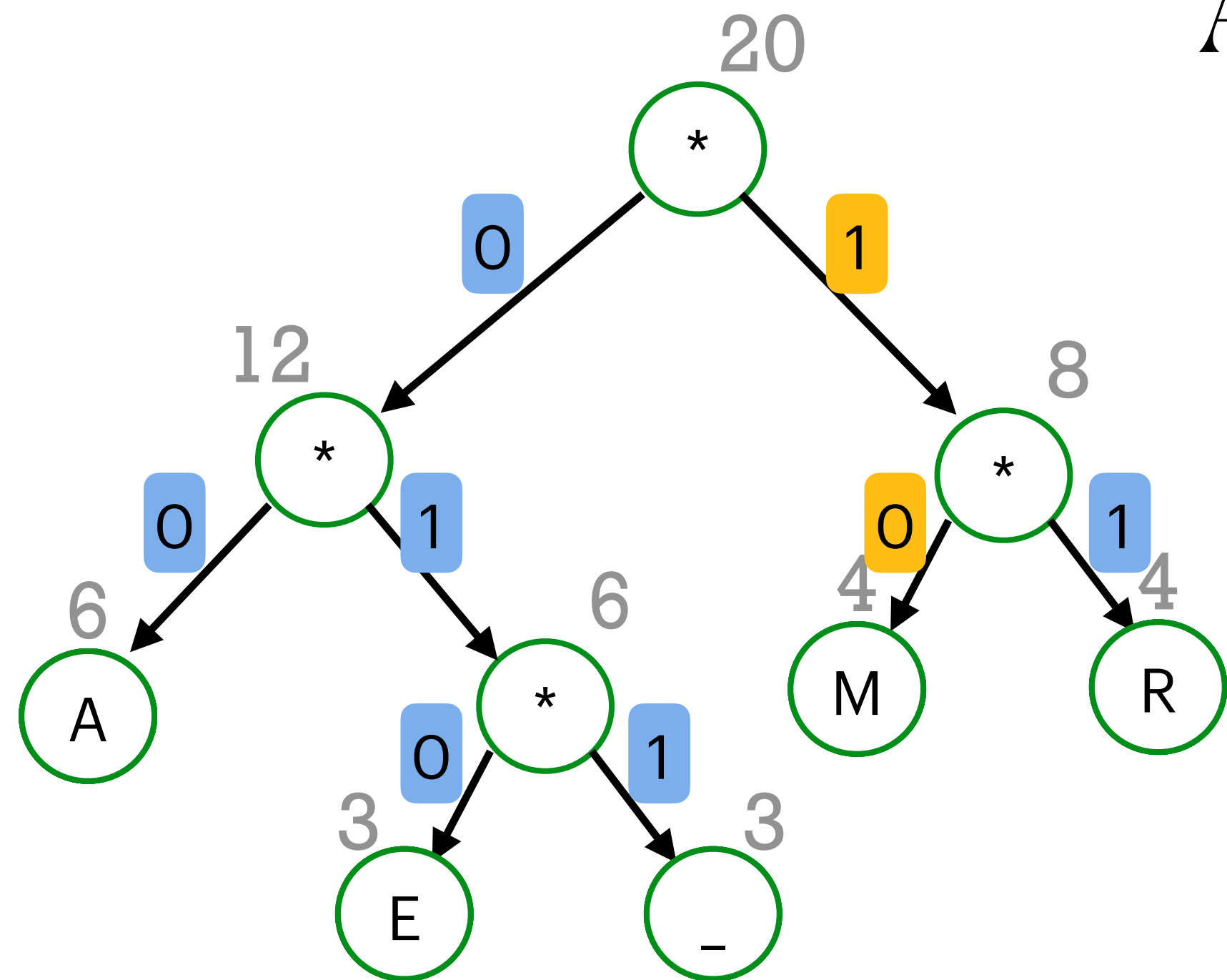
A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



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Arbori Huffman

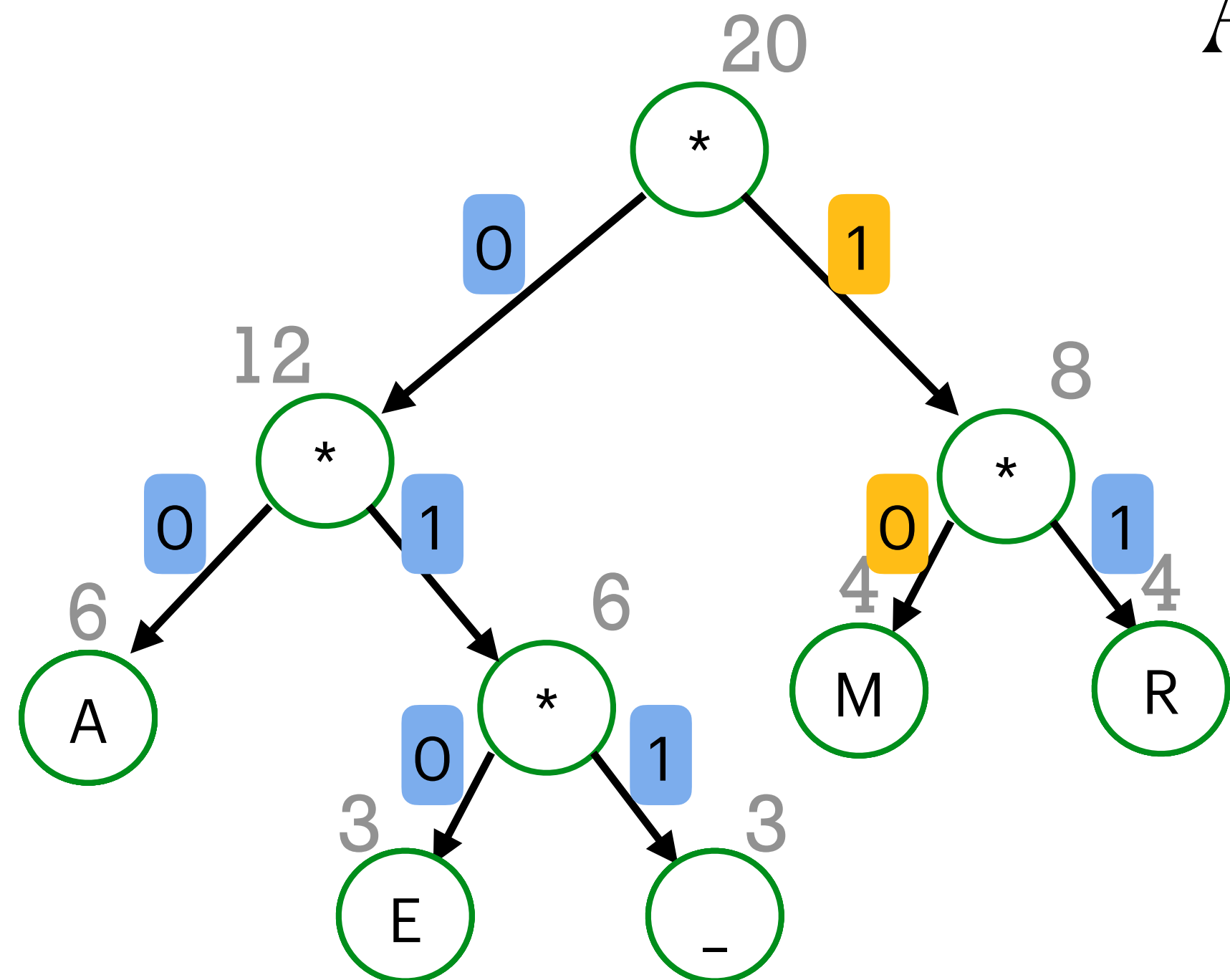
A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



1 0 0 0 1 1 0 1 0 0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

Arbori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1

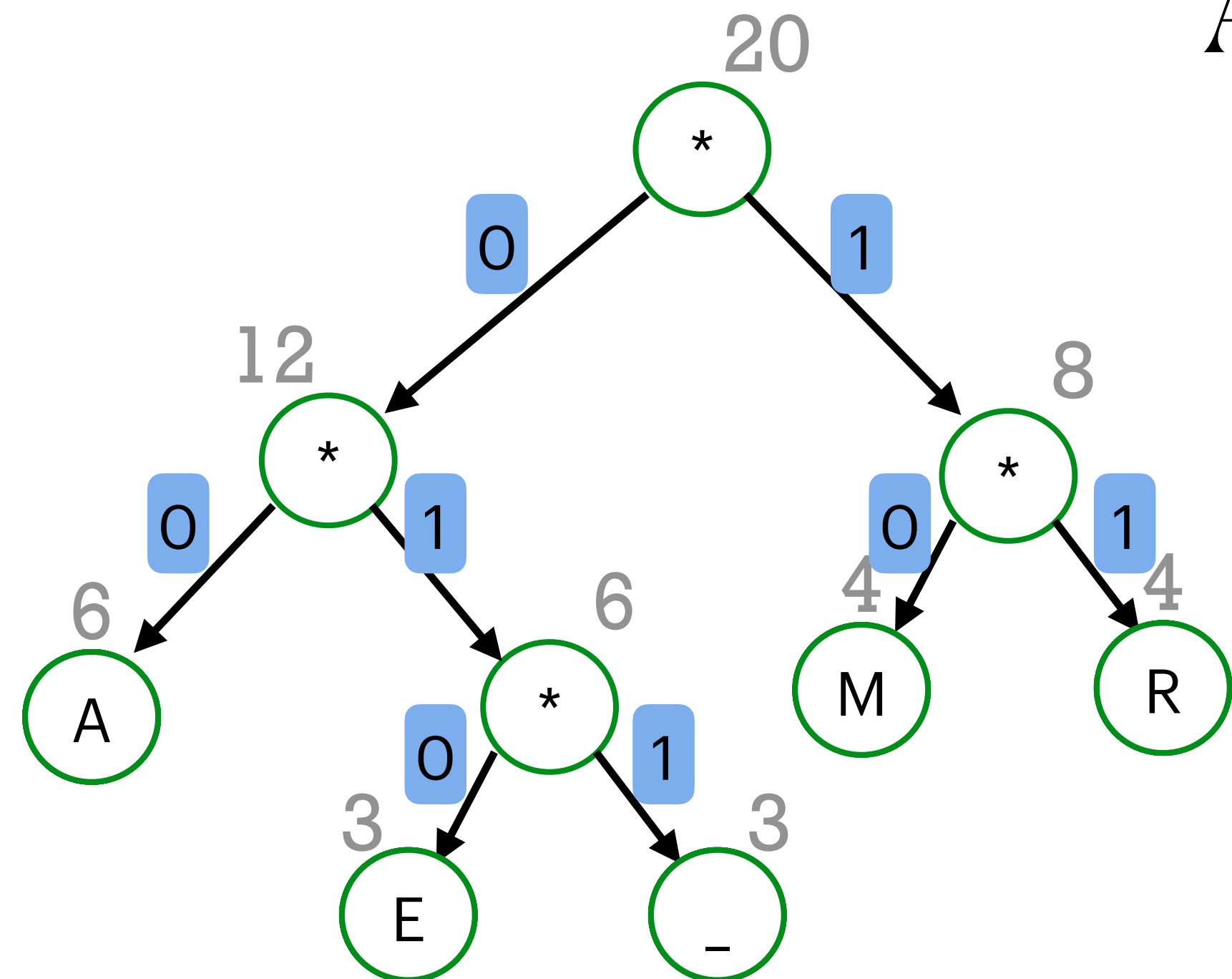


1 0 0 0 1 1 0 1 0 0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

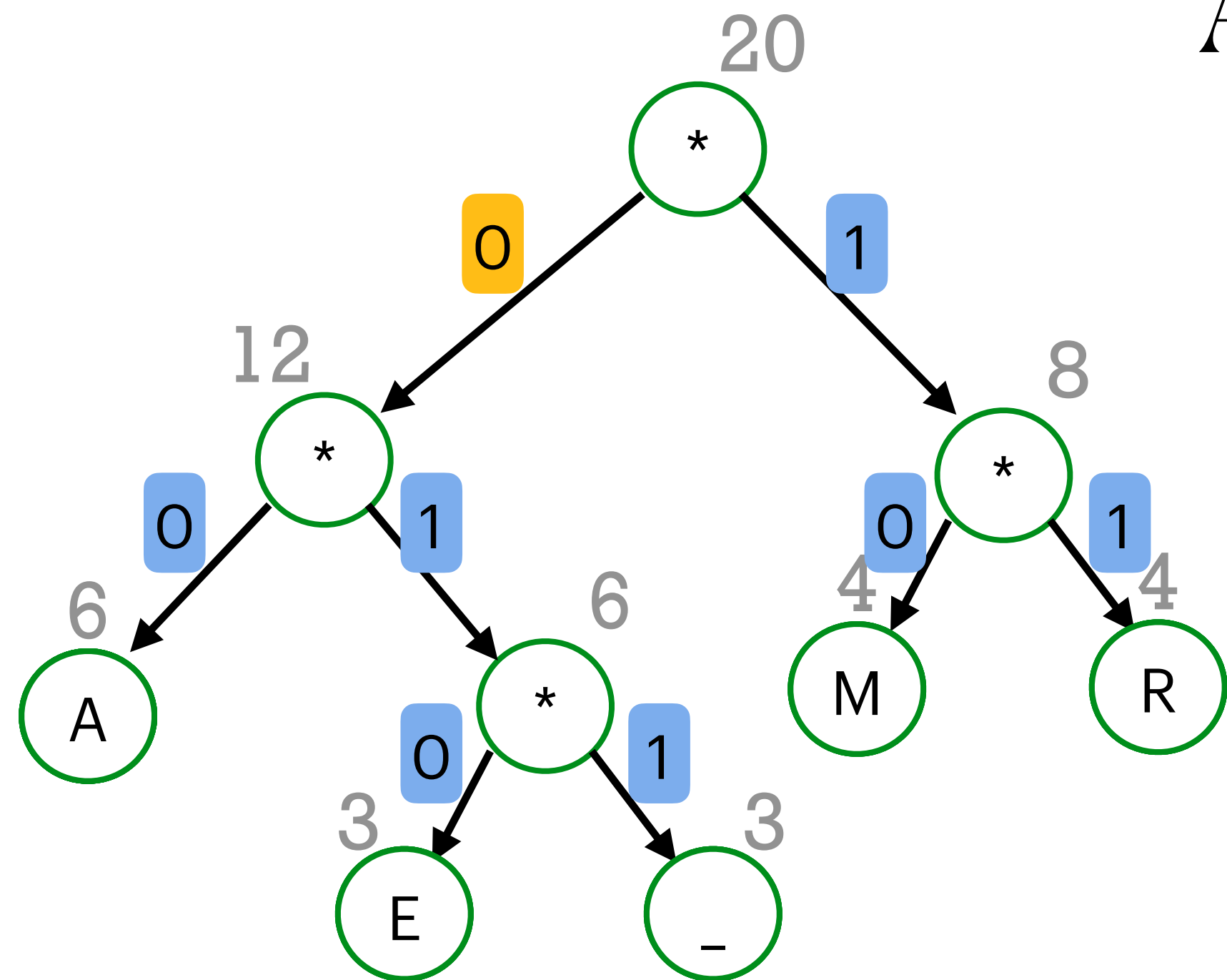


0 0 1 1 0 1 0 0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M

Arbori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1

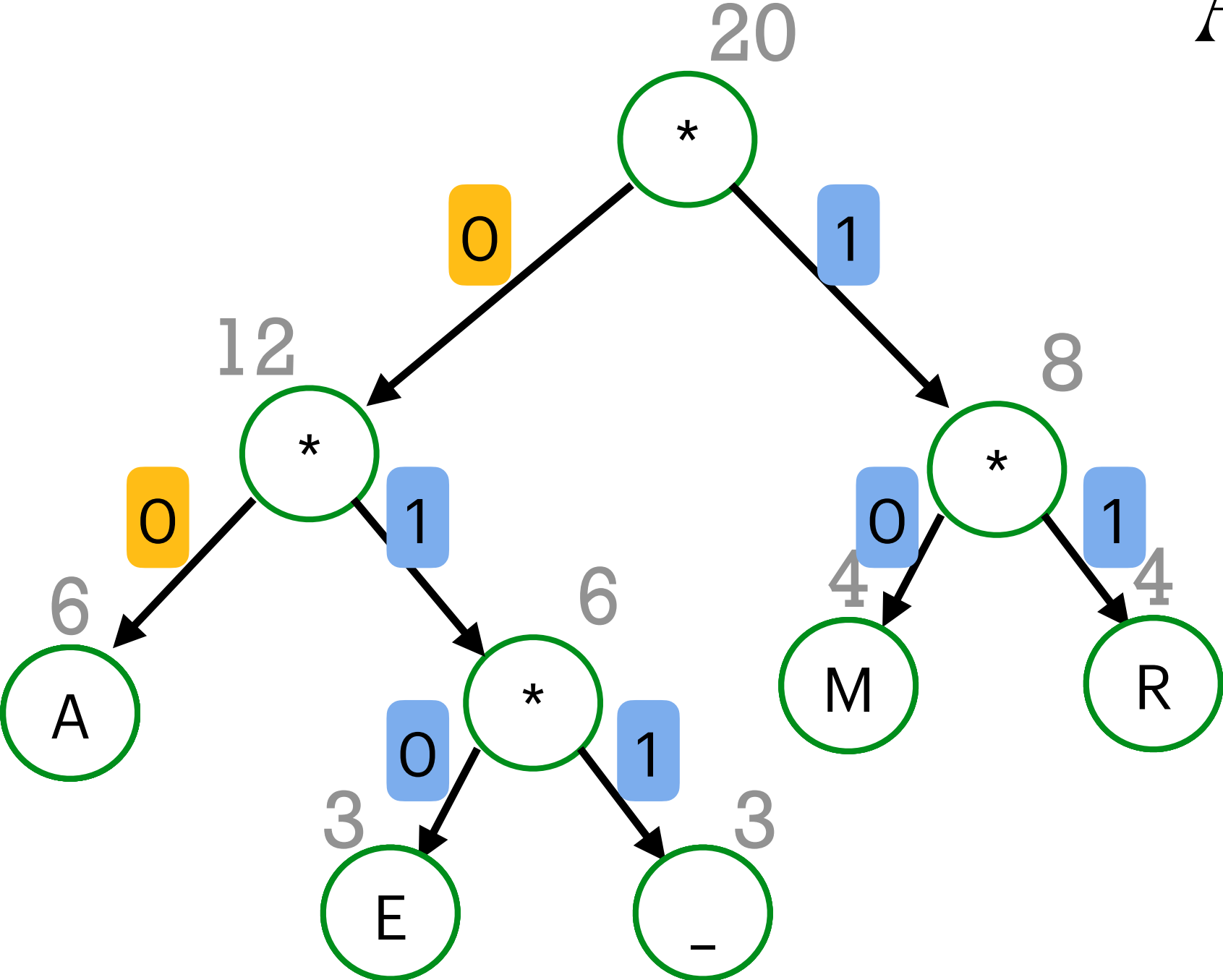


0 0 1 1 0 1 0 0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M

Arbori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1

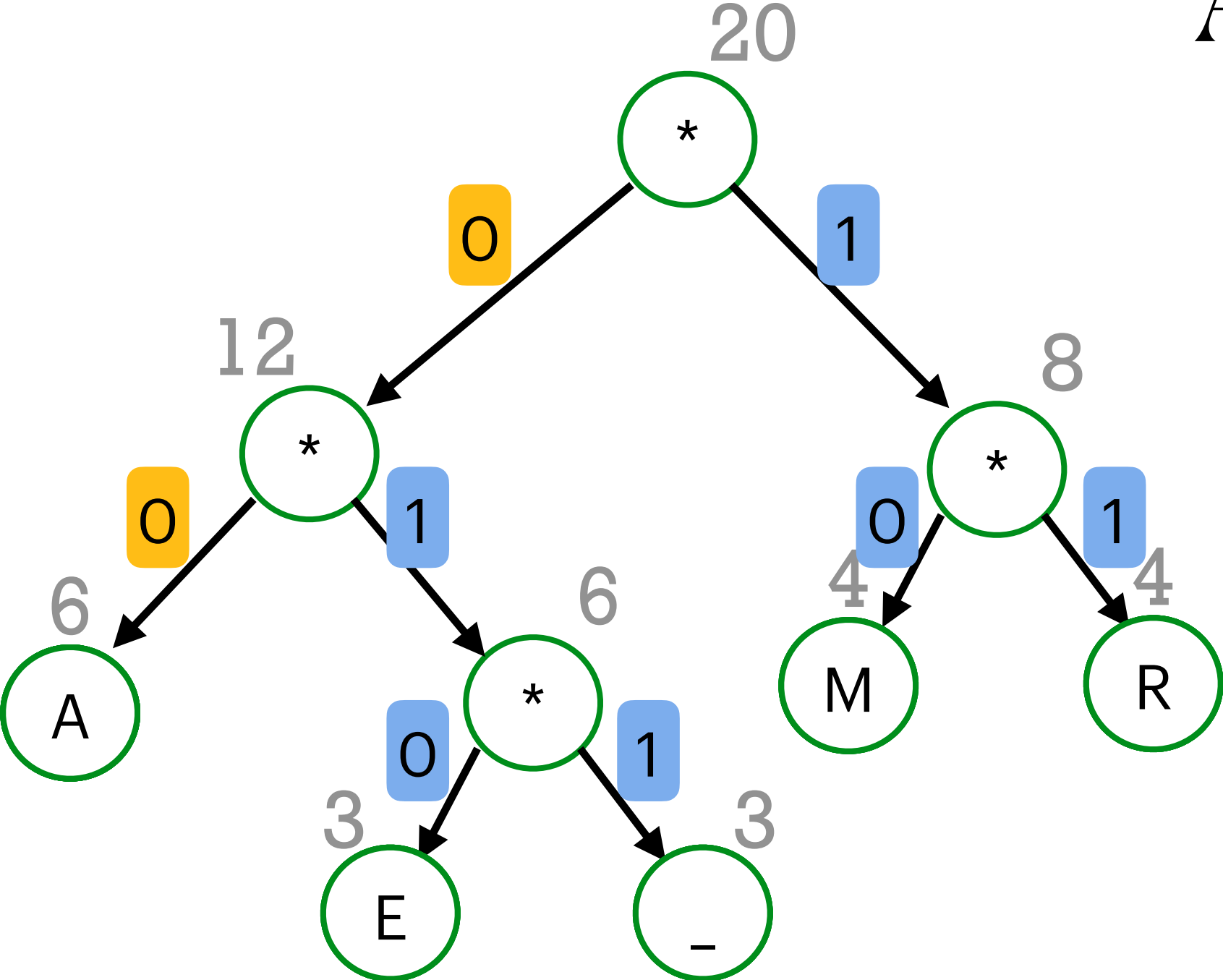


00110100110100111000110100001110001110001110001100

M

Arbori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1

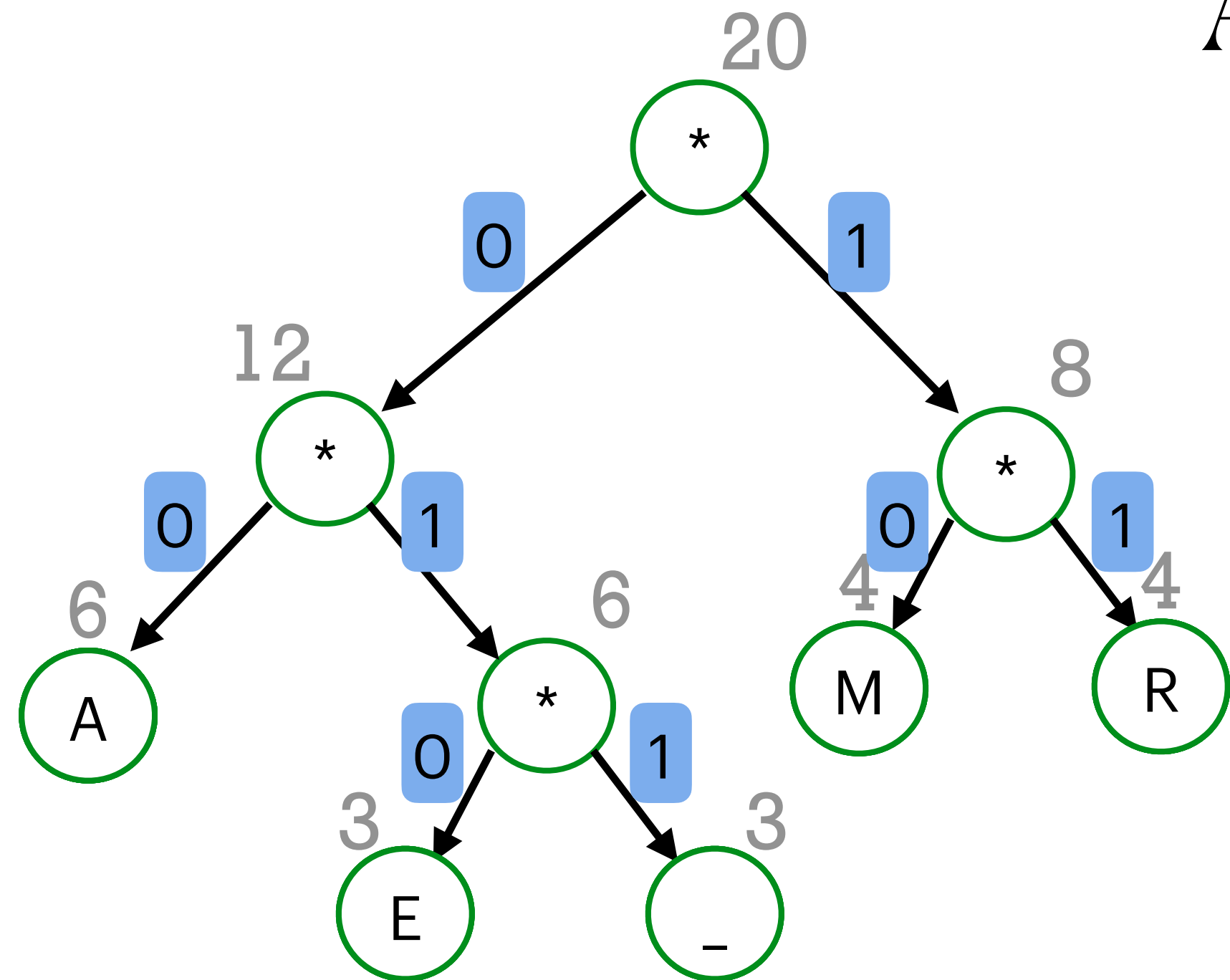


00110100110100111000110100001110001110001110001100

M

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

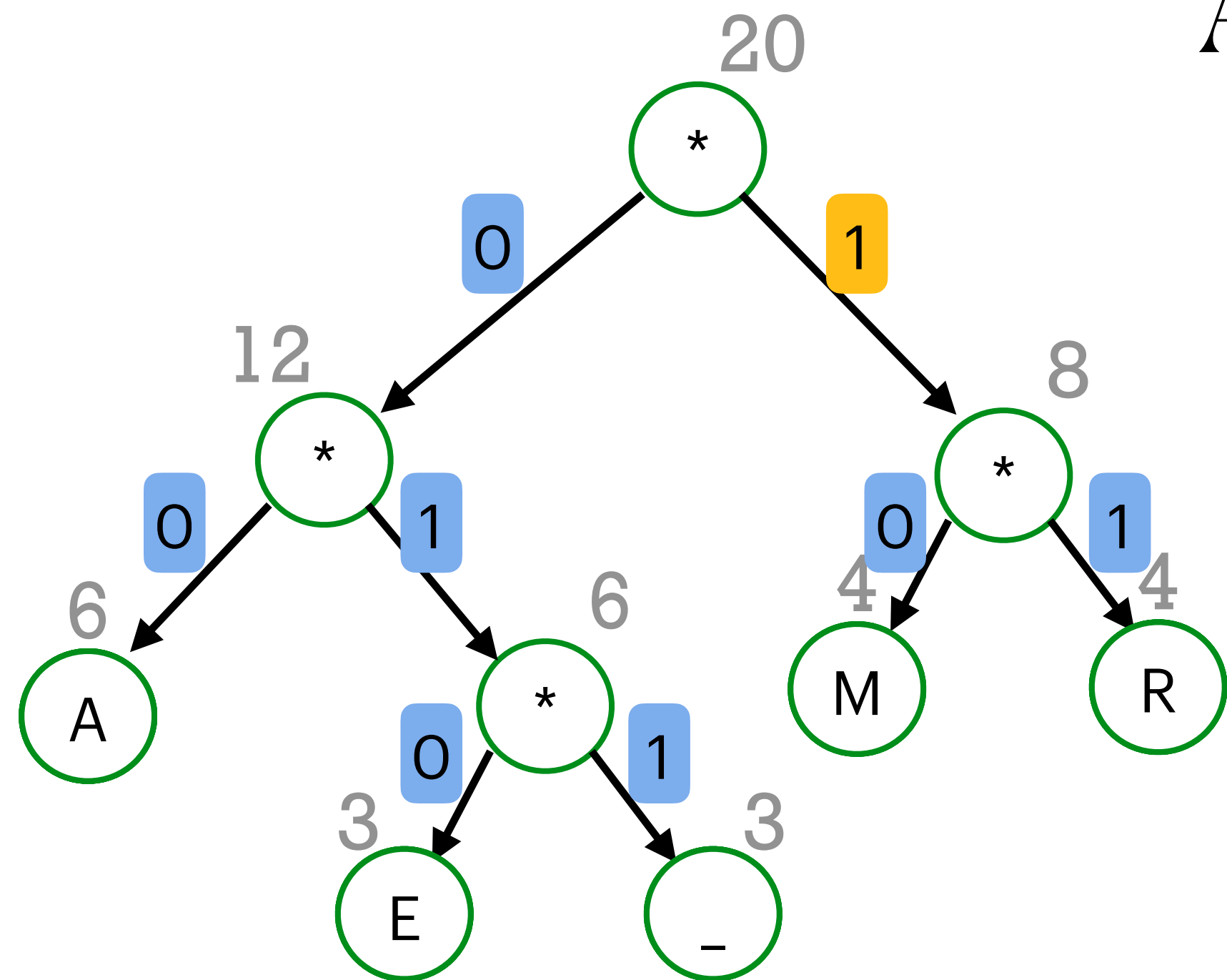


1 1 0 1 0 0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

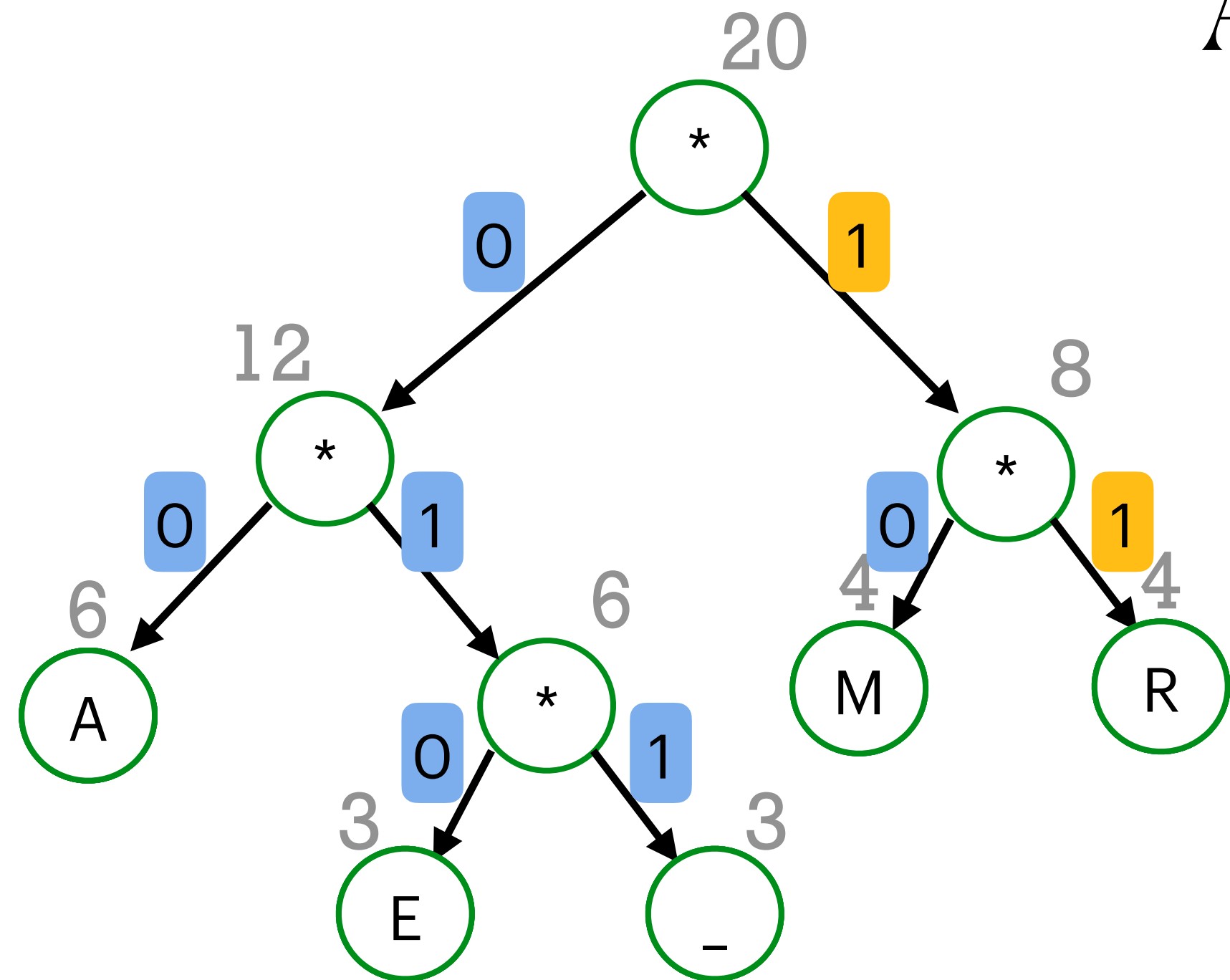


M A

1 1 0 1 0 0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

Arbori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1

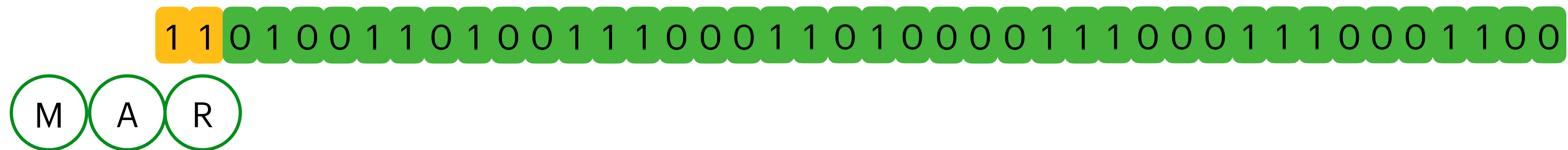
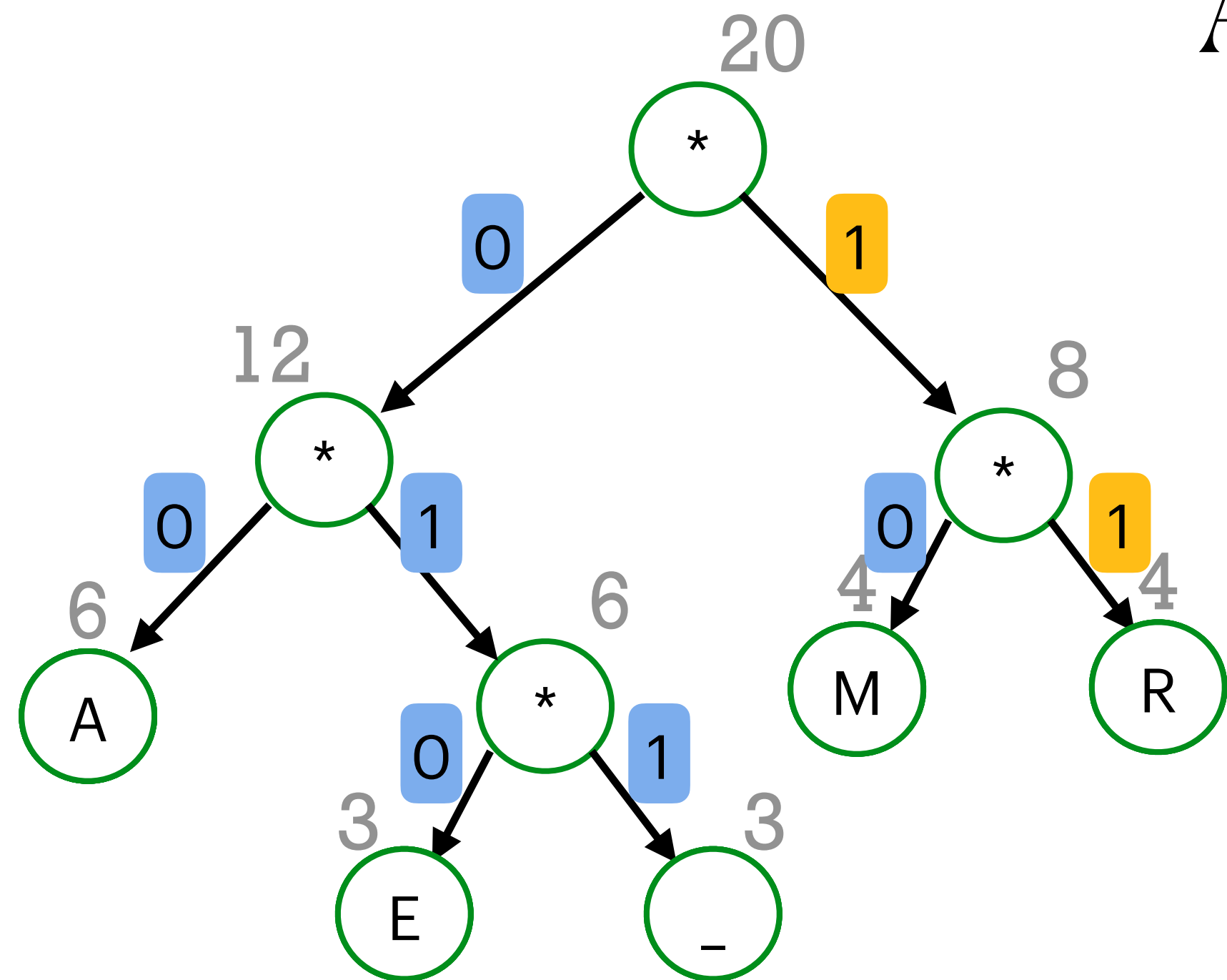


1 1 0 1 0 0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A

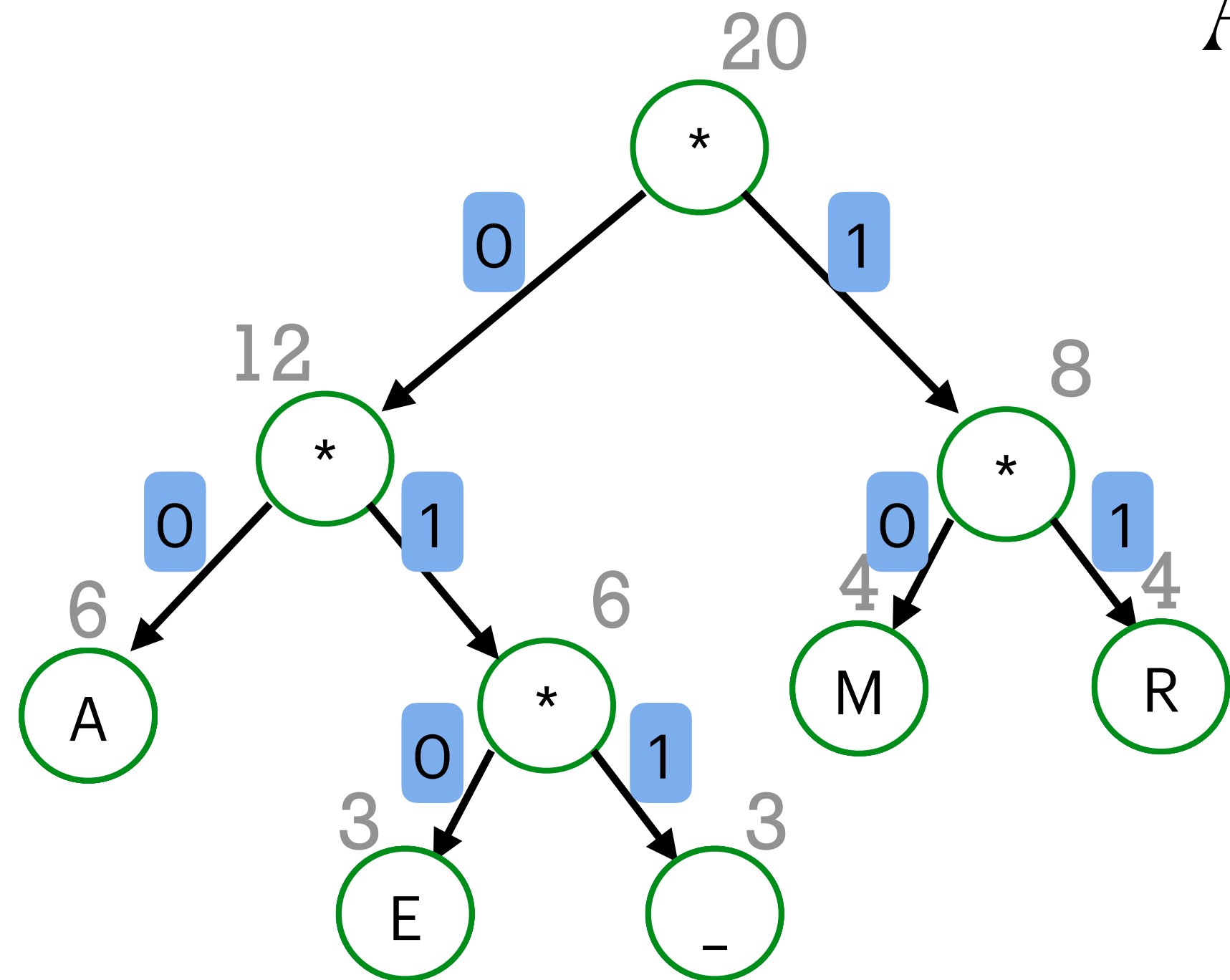
Arbori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1



Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

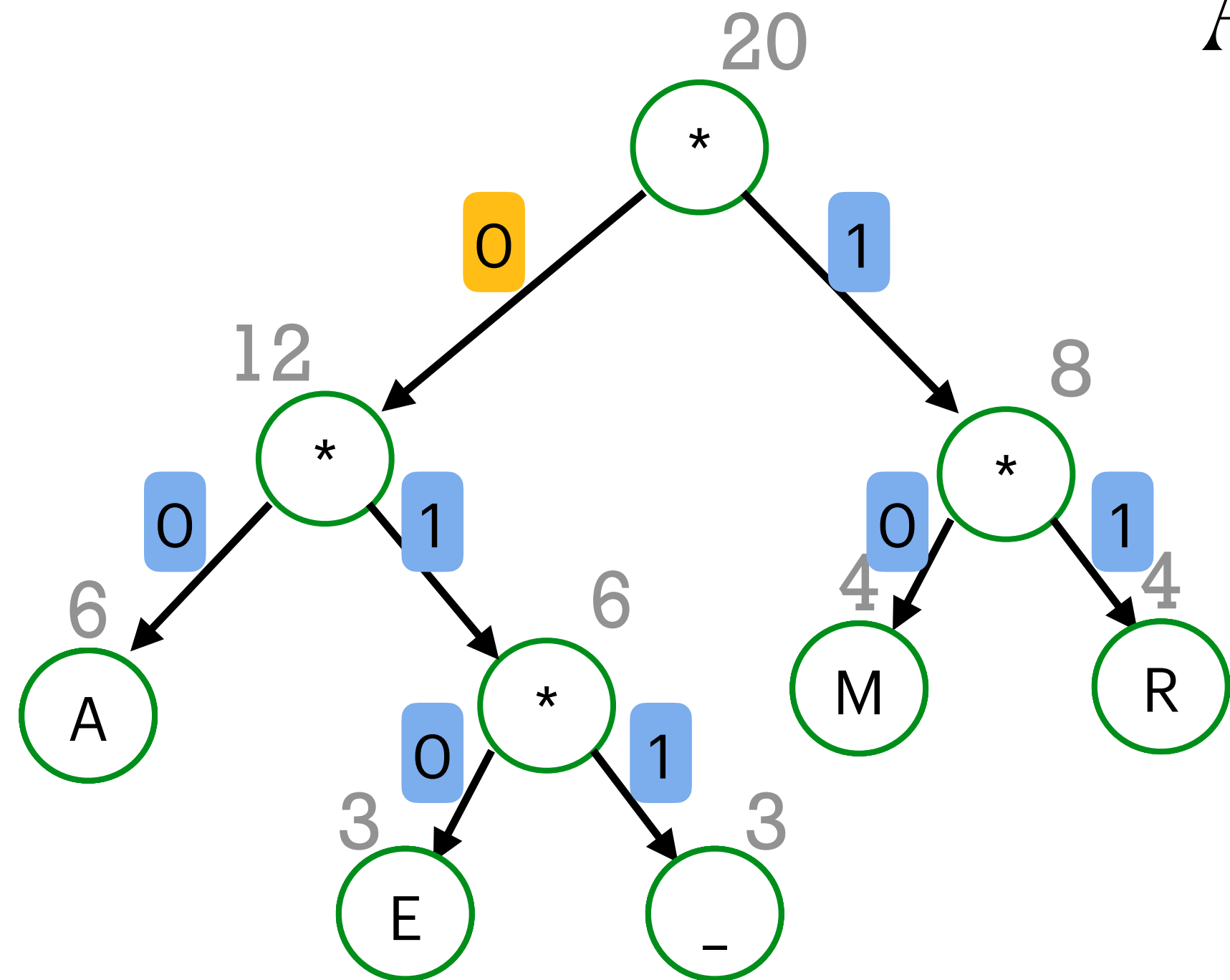


0 1 0 0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

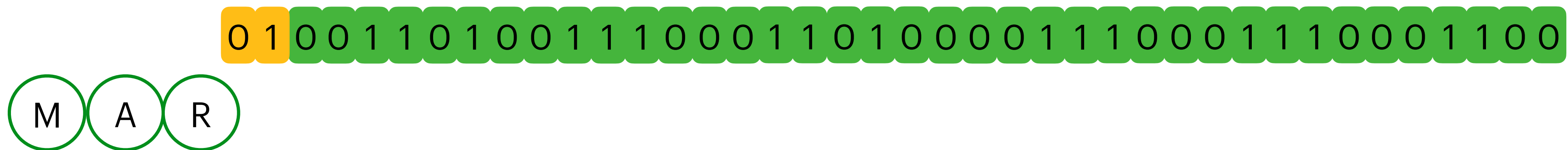
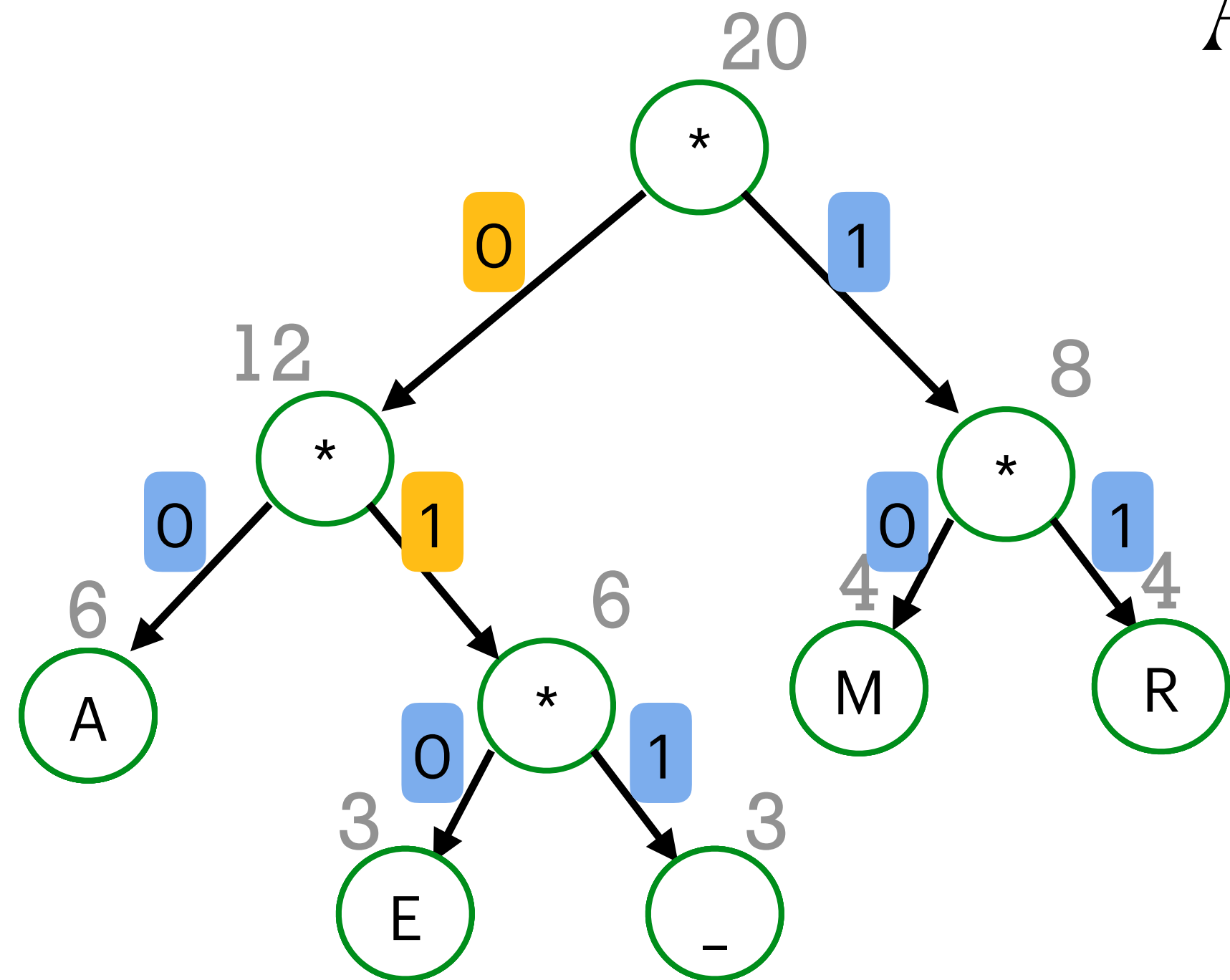


0 1 0 0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R

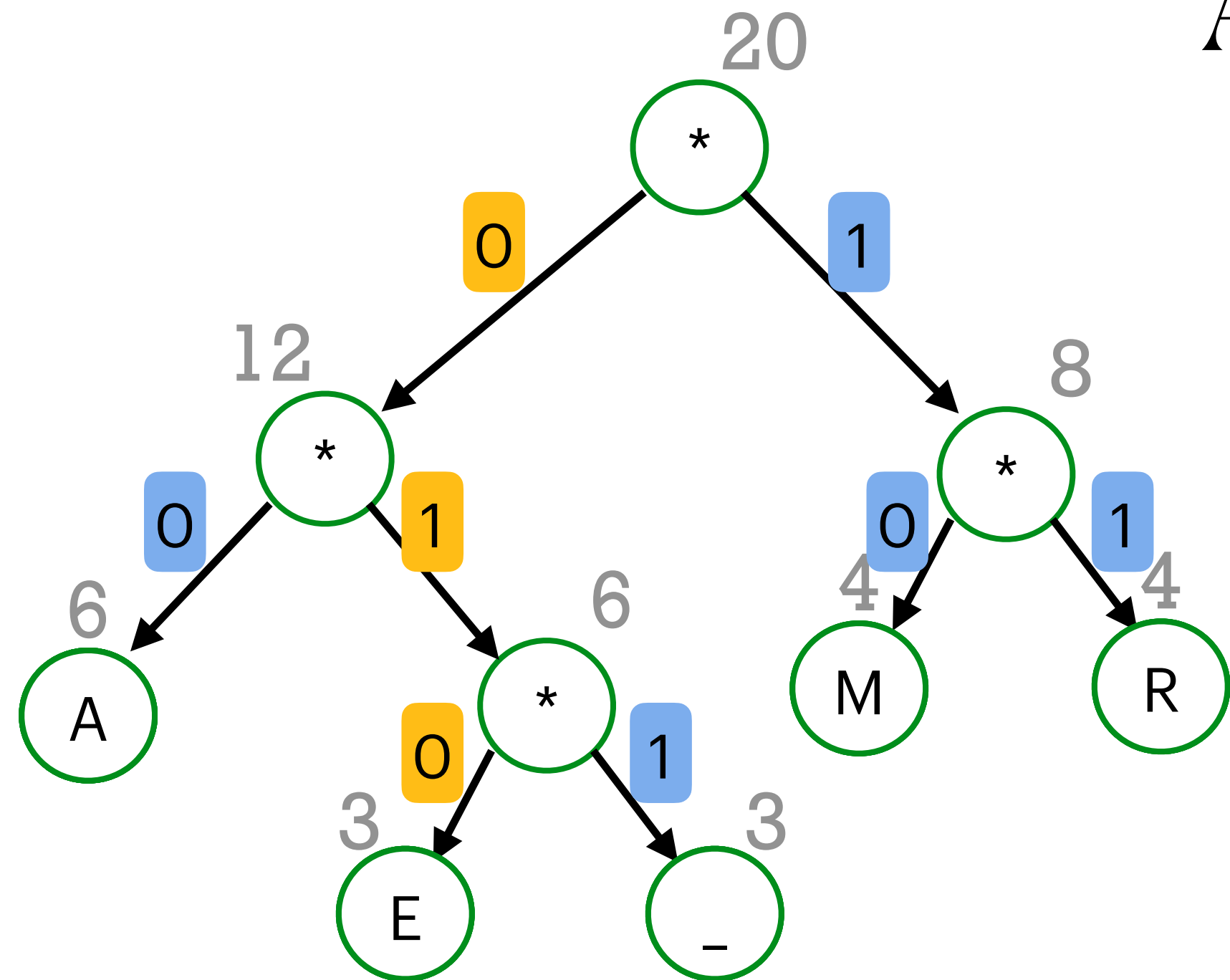
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



Arbori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1

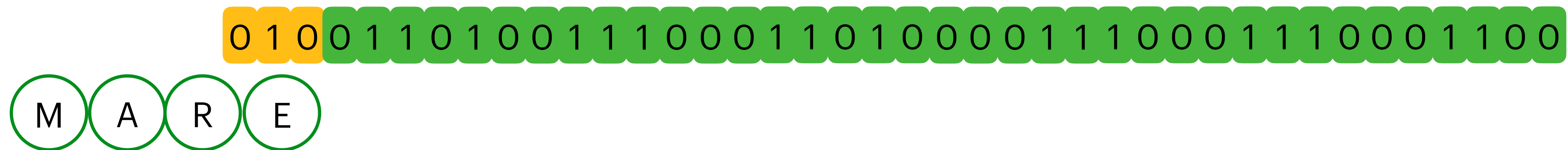
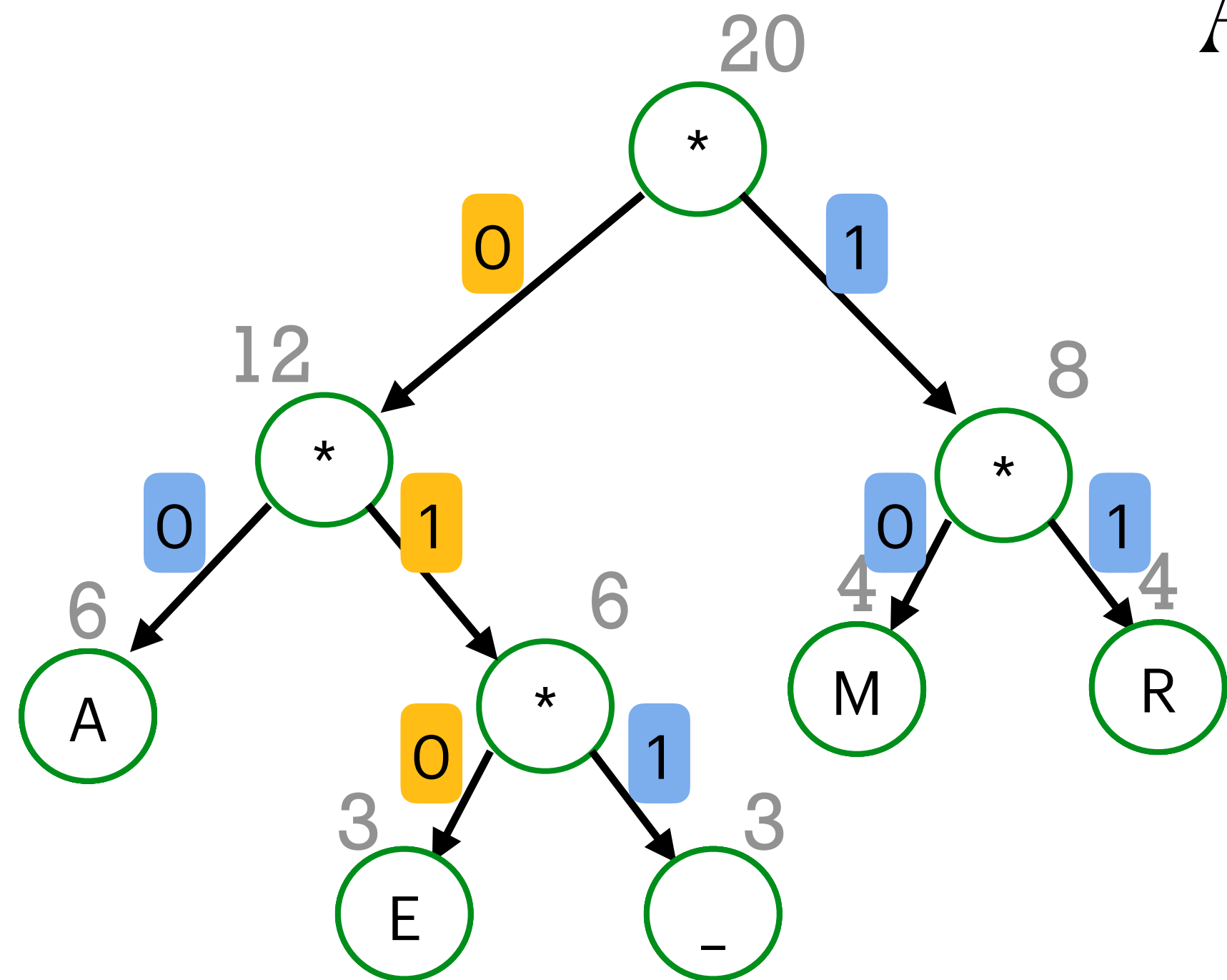


0 1 0 0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R

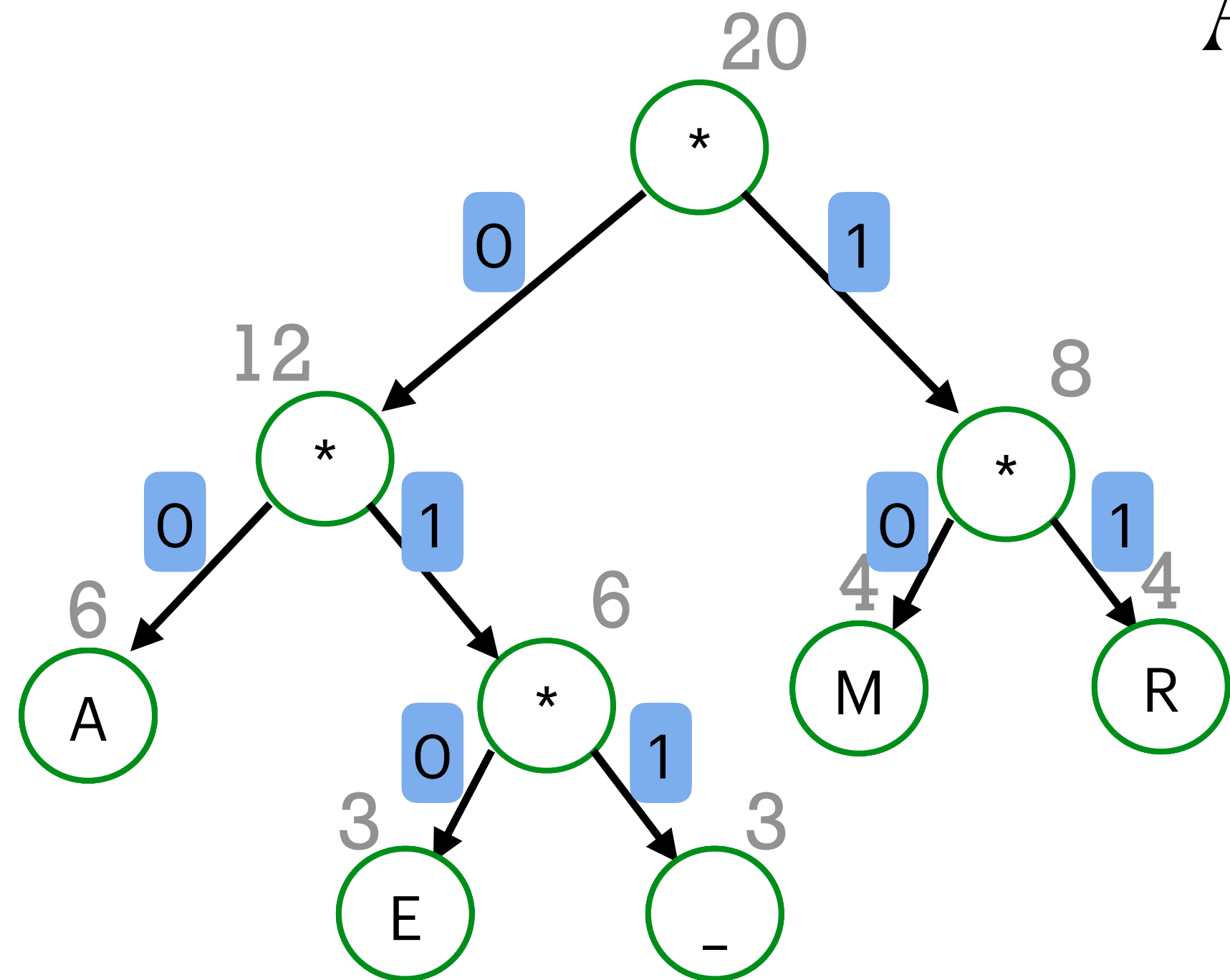
Arbori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1



Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

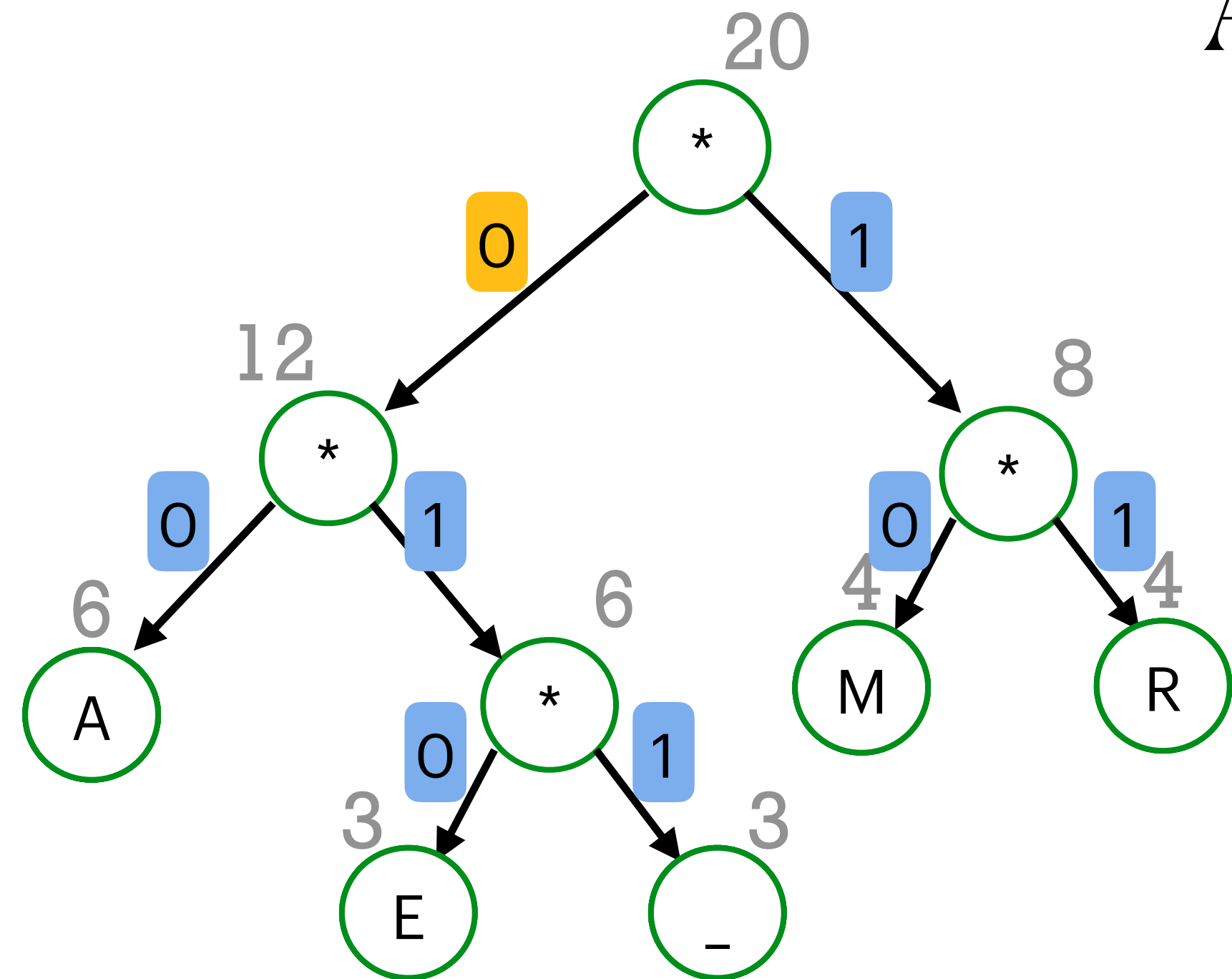


0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

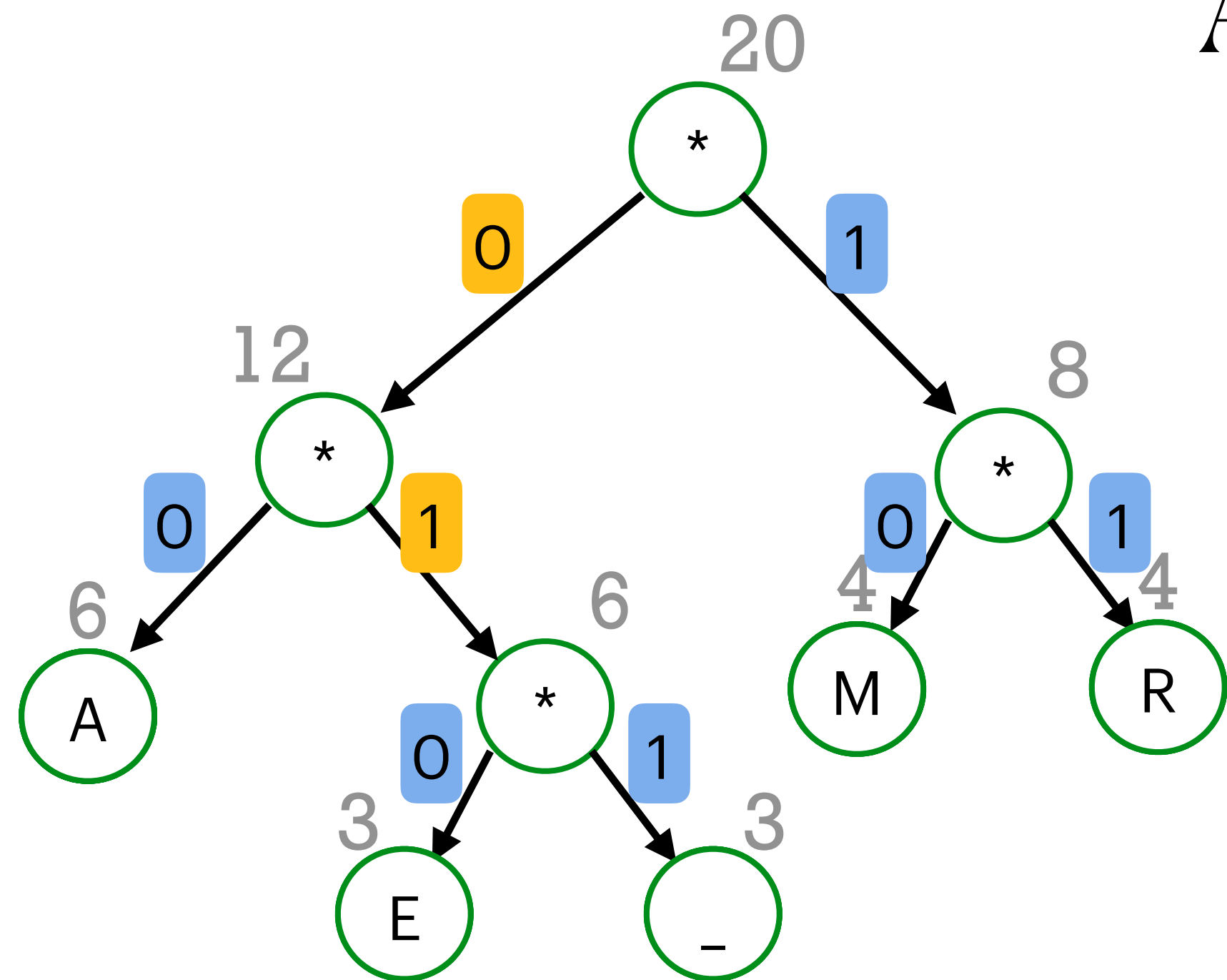


0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

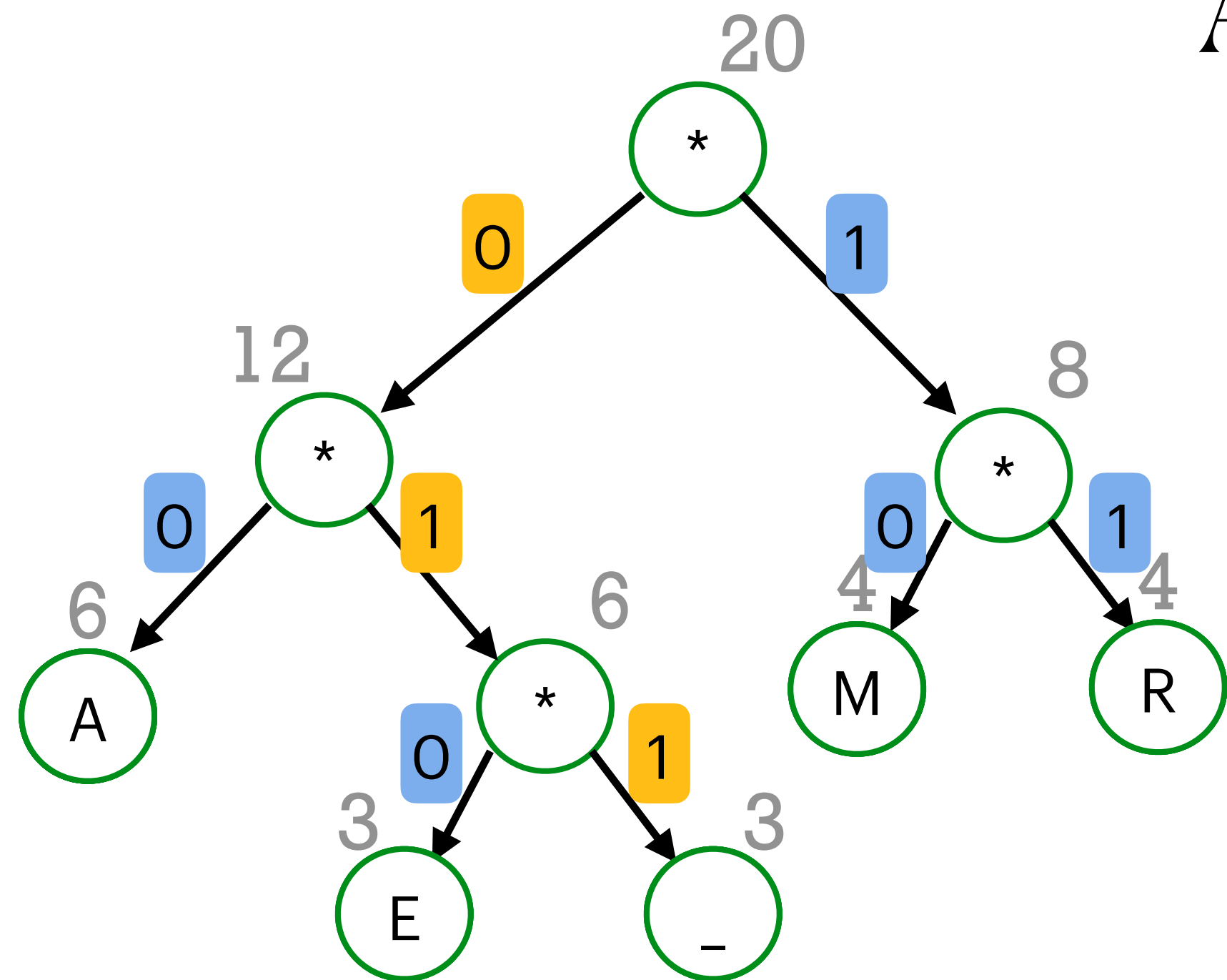


0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E

Arbori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1

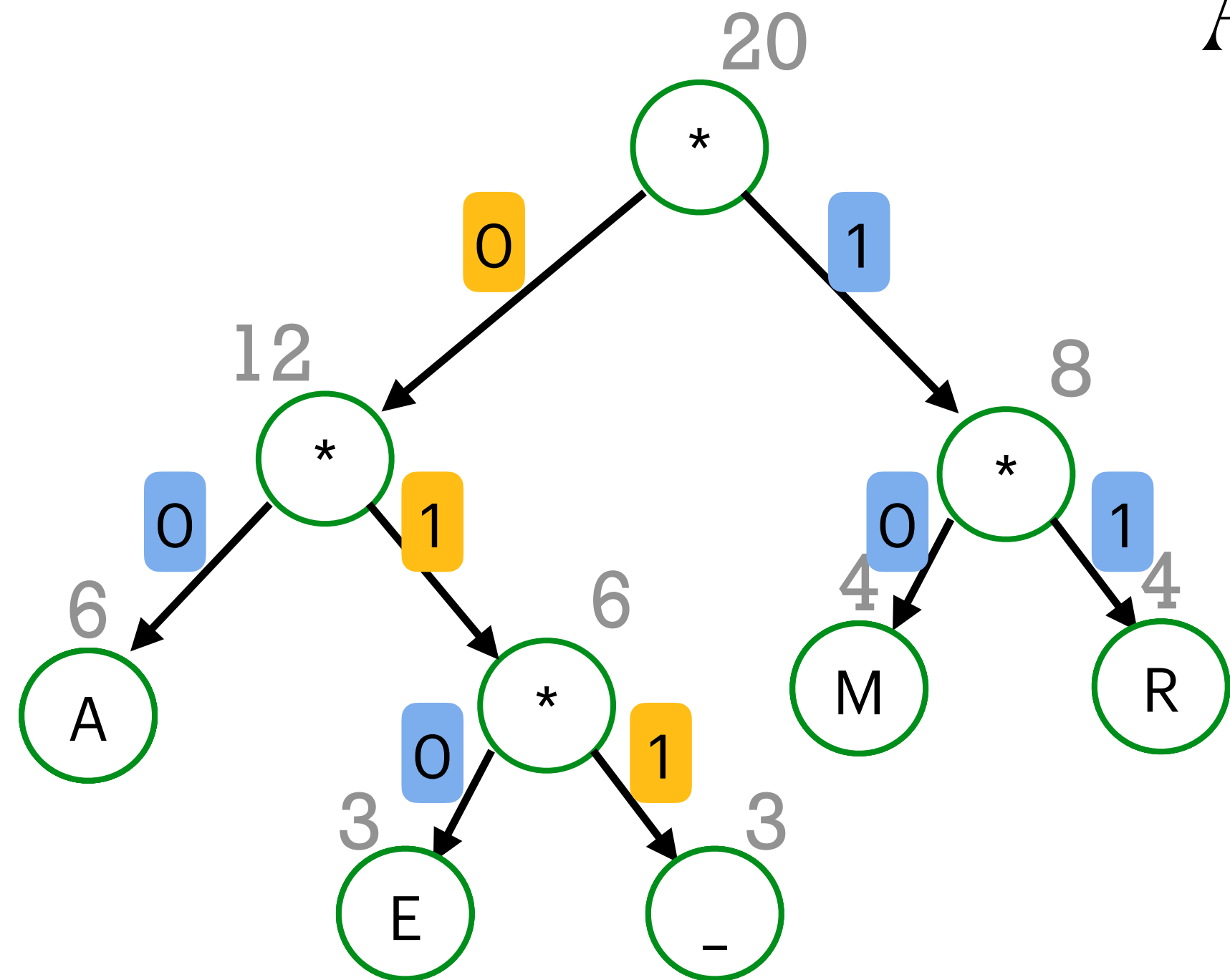


0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

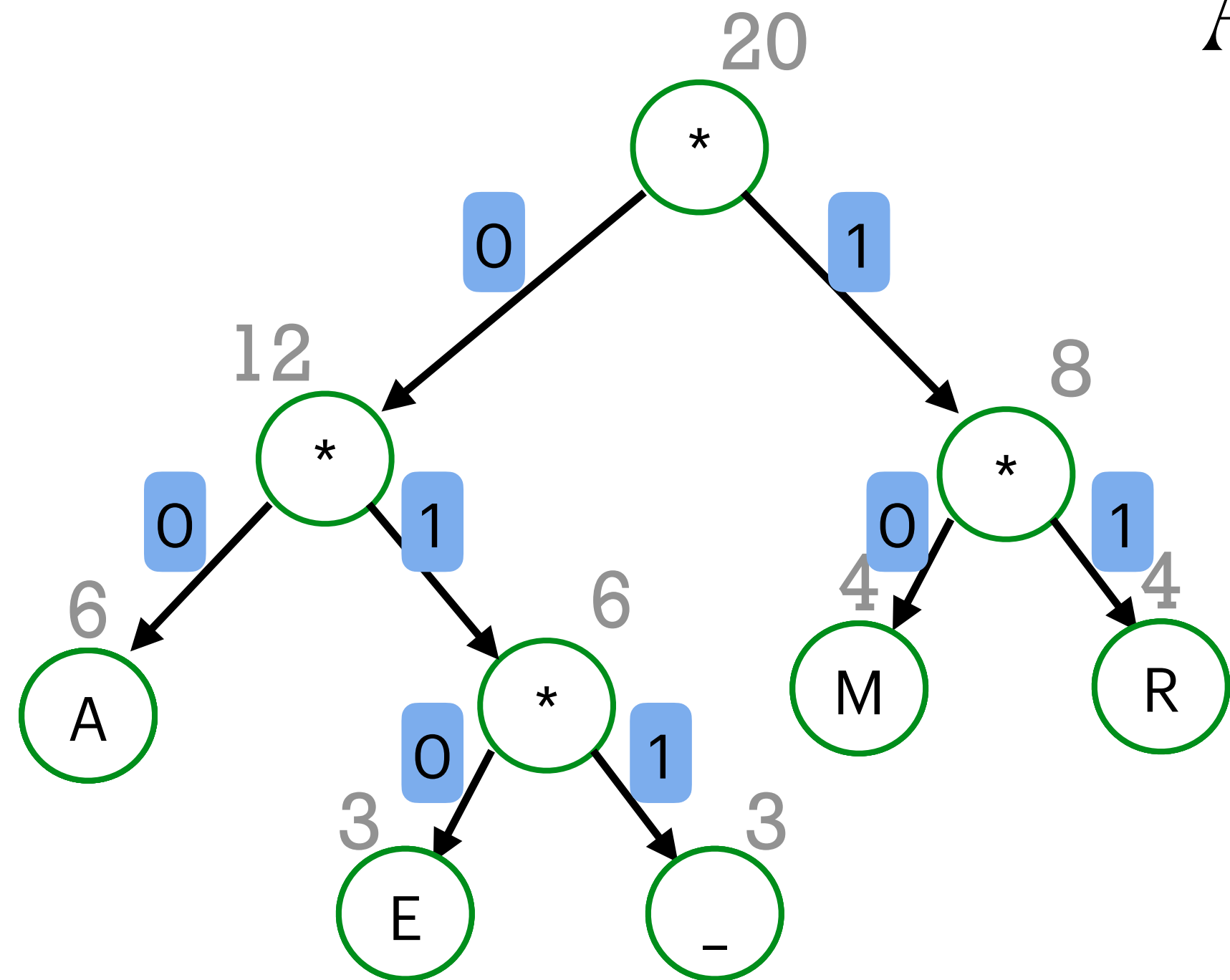


0 1 1 0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E -

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

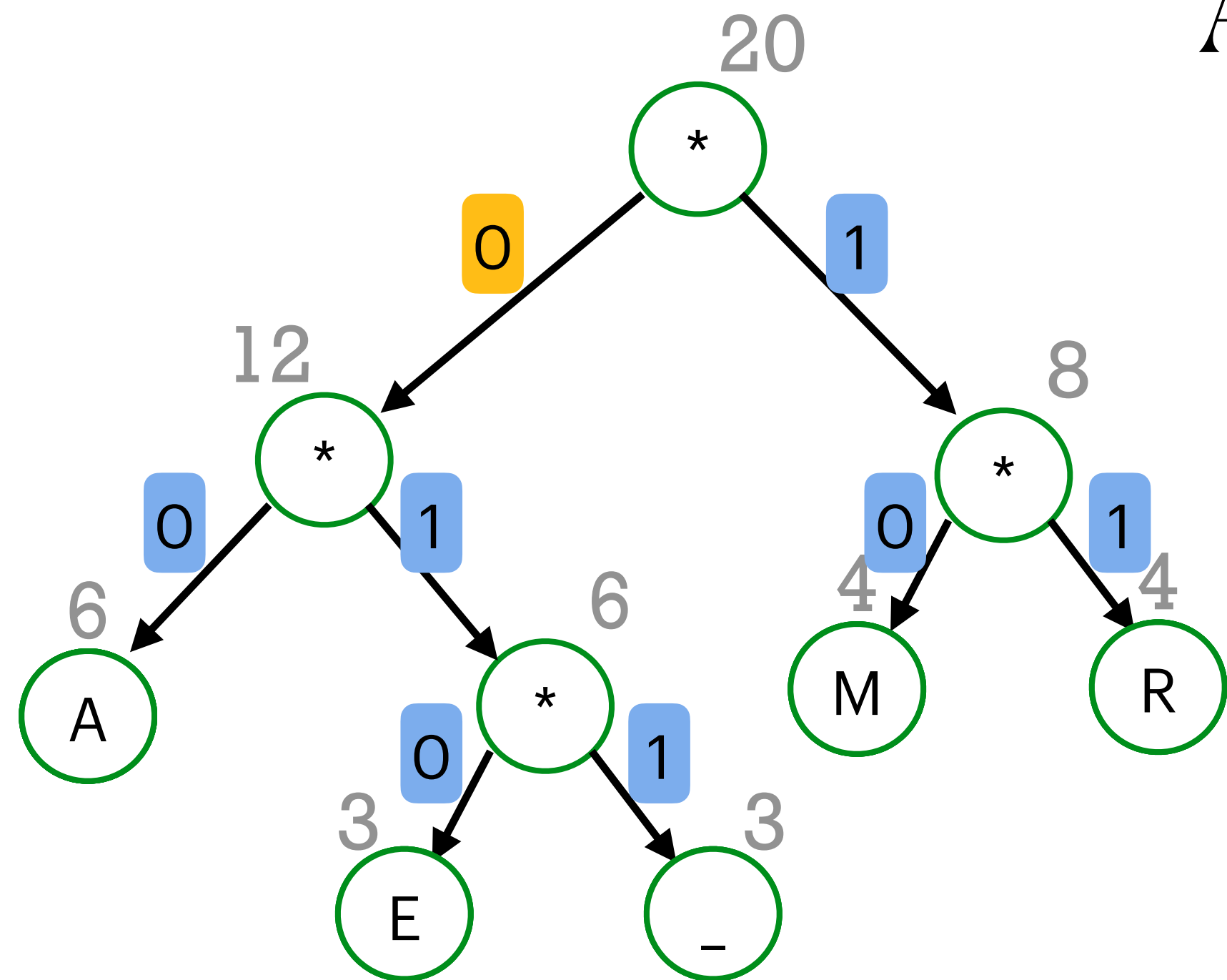


0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E -

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

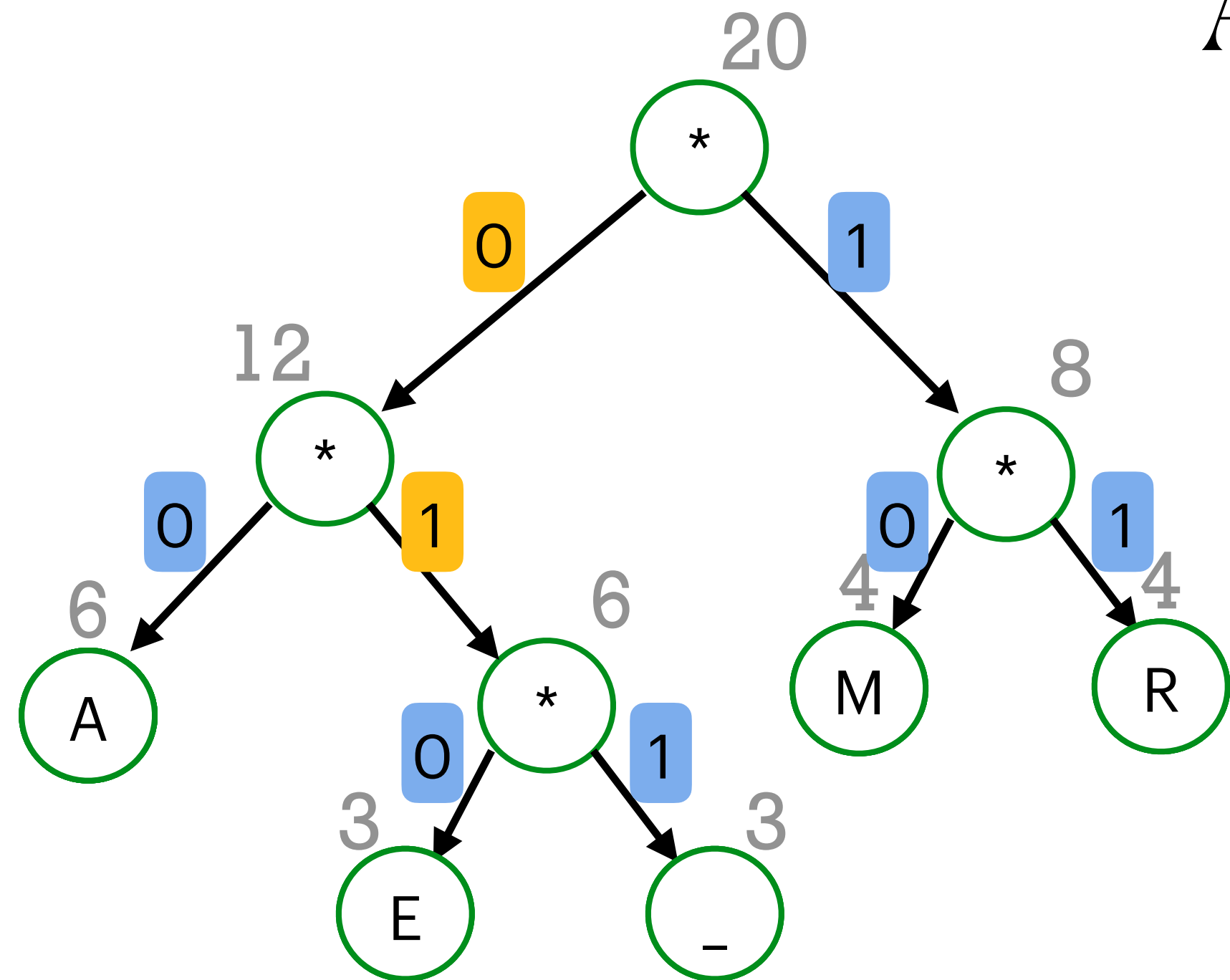


M A R E -

0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

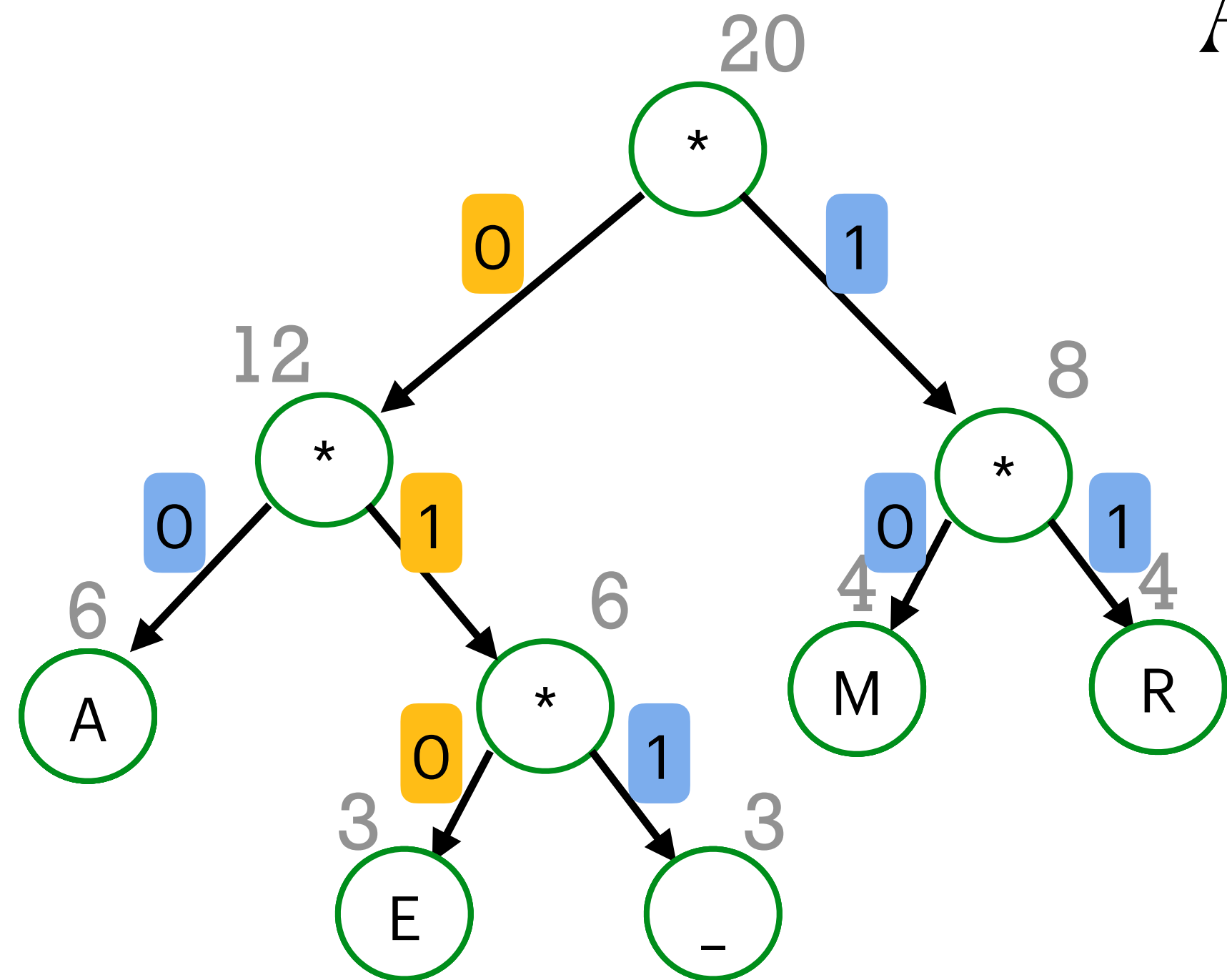


0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E -

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

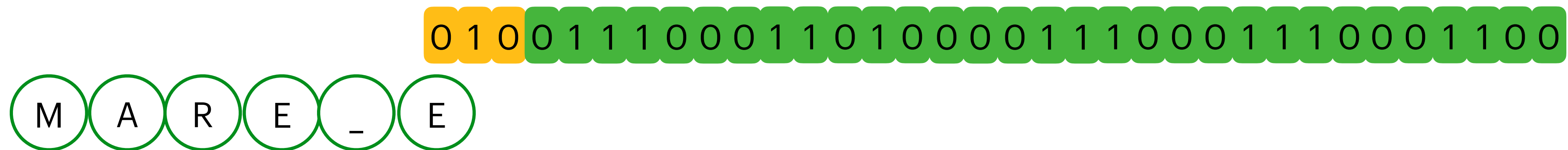
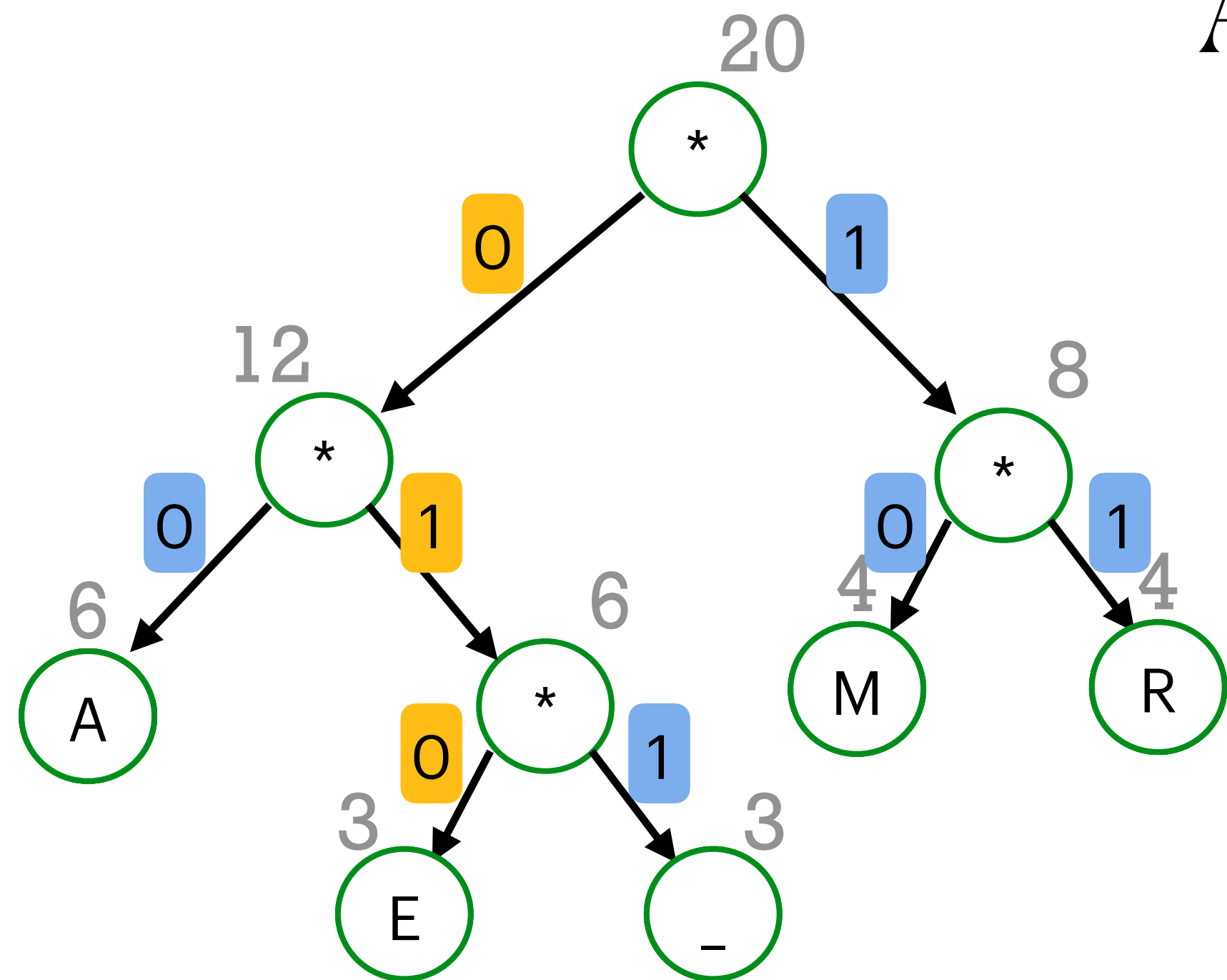


0 1 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E -

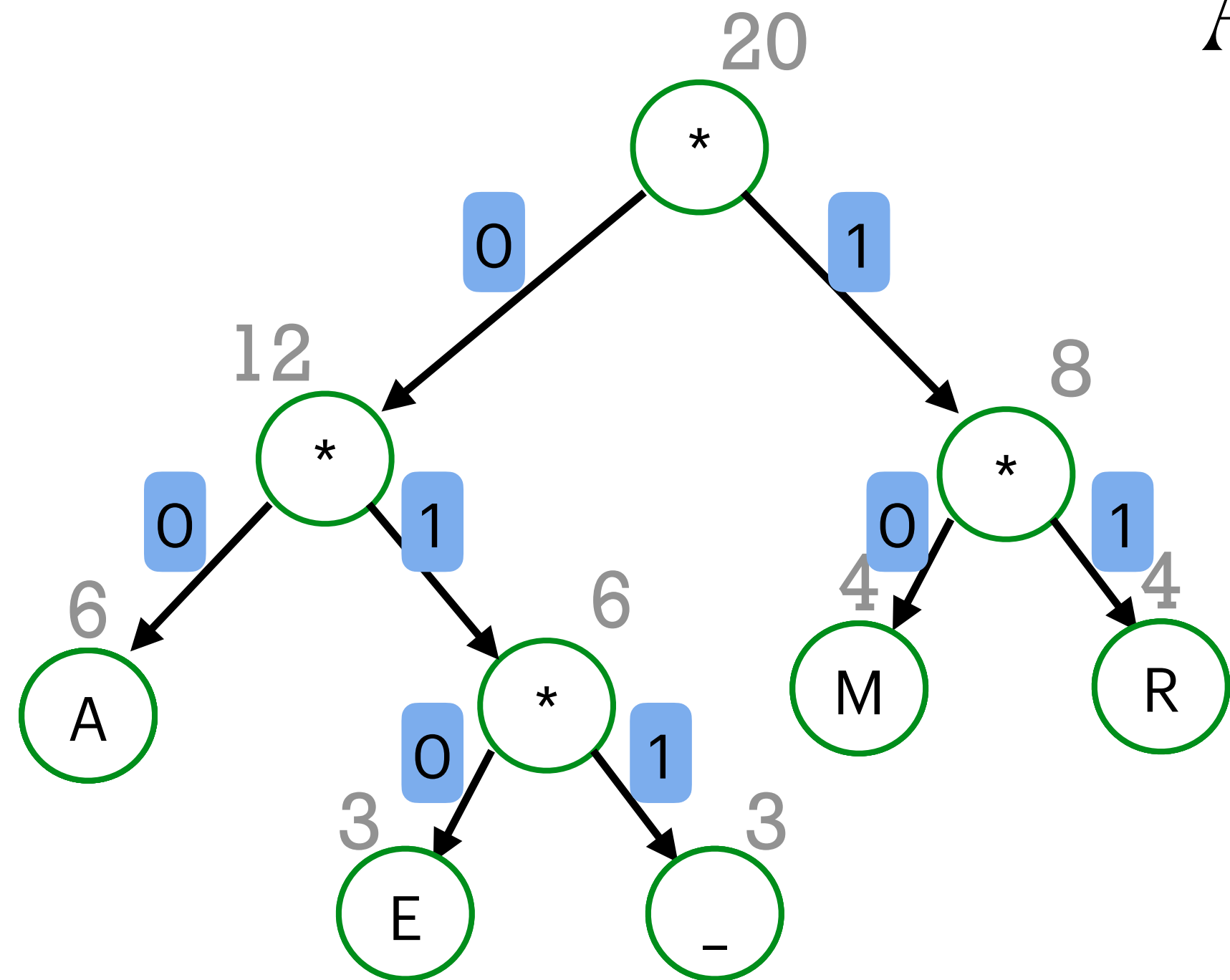
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

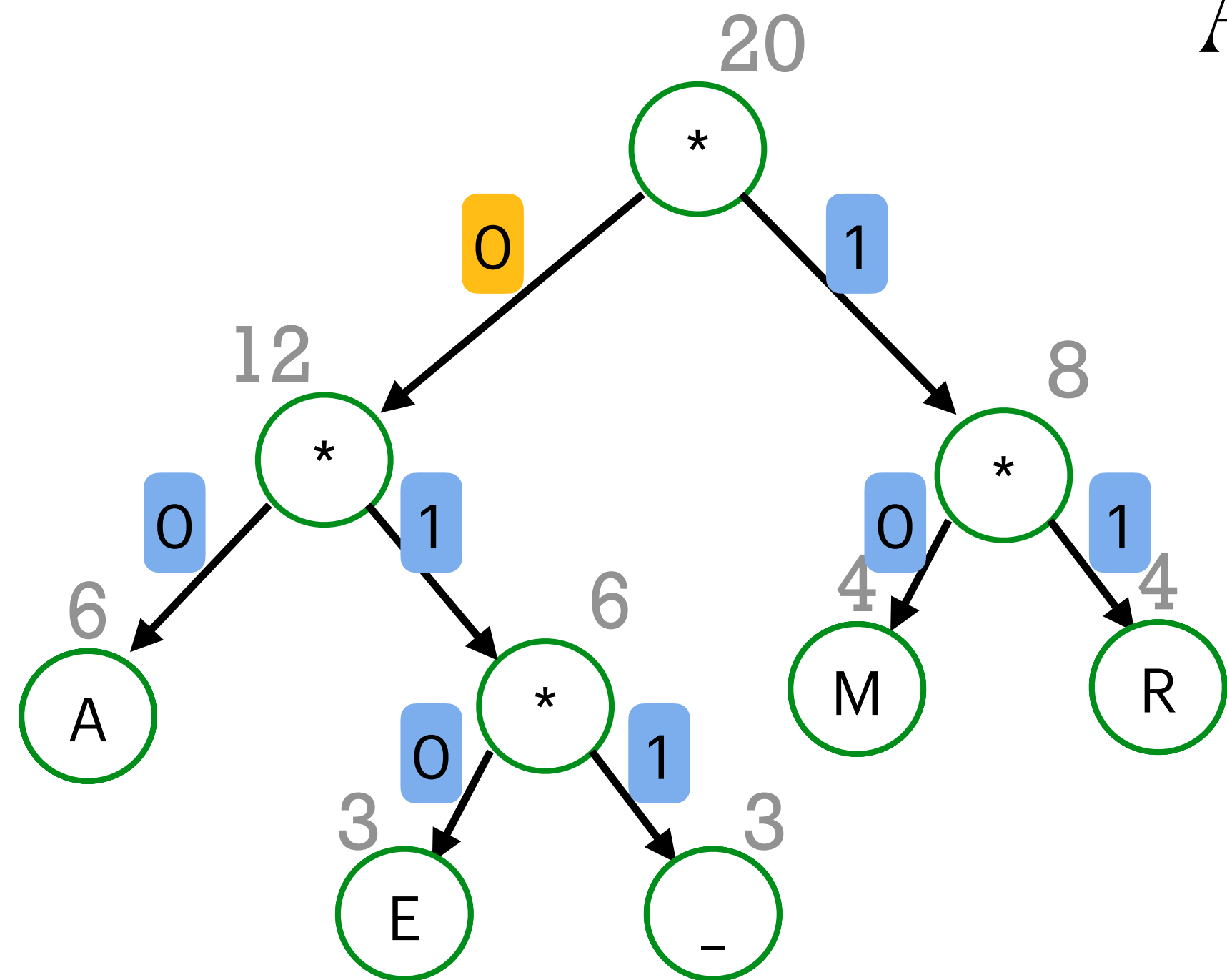


0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

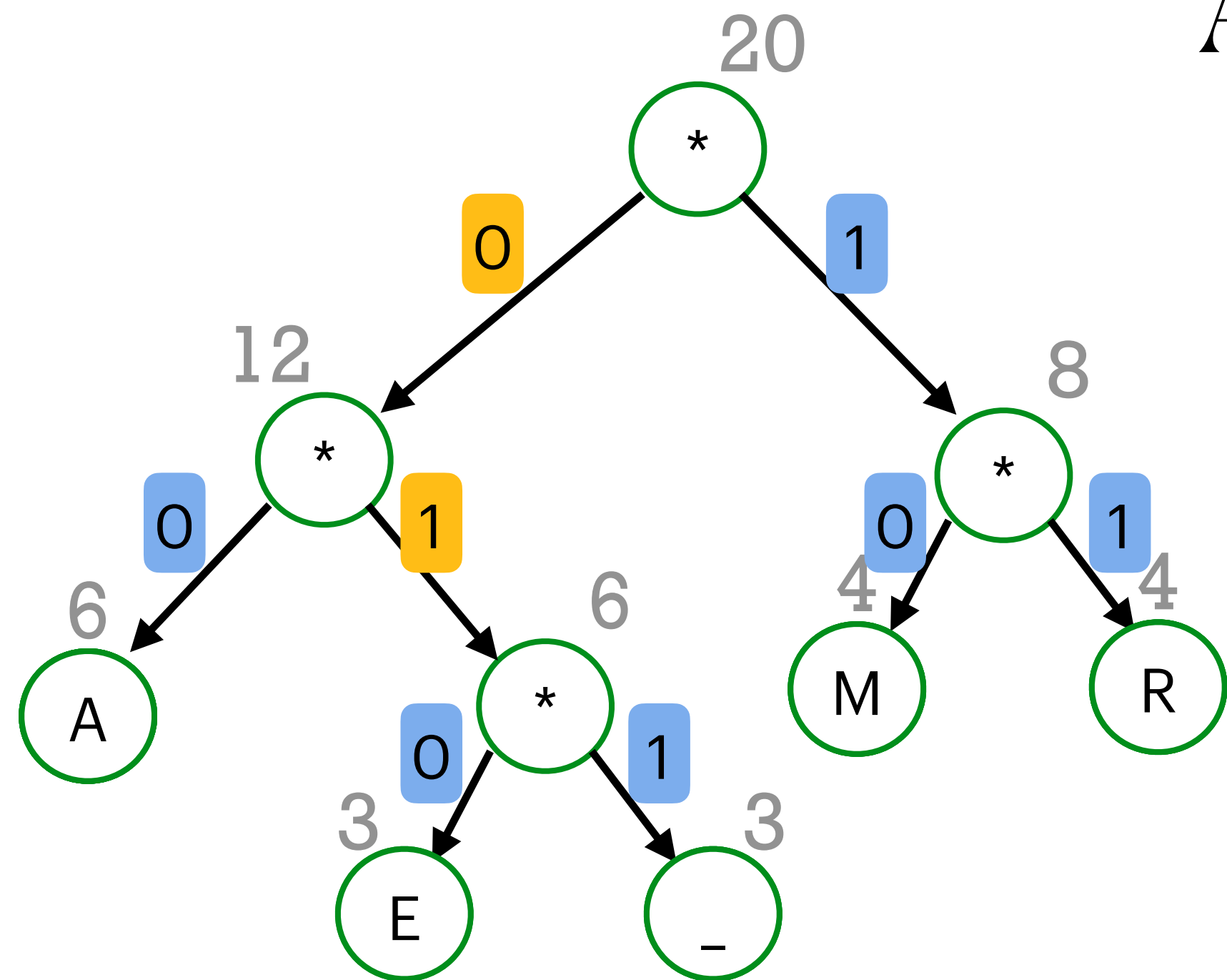


0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

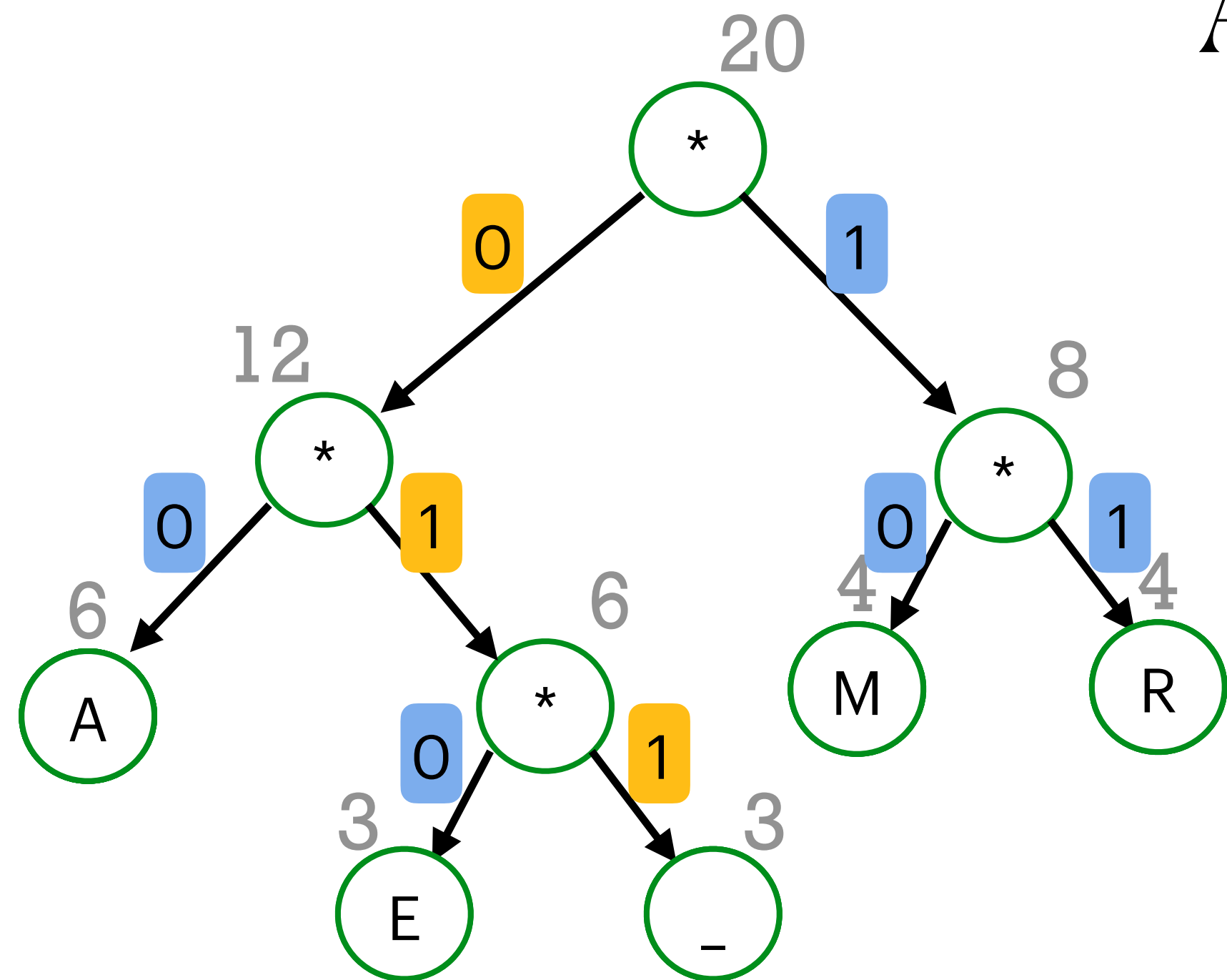


0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

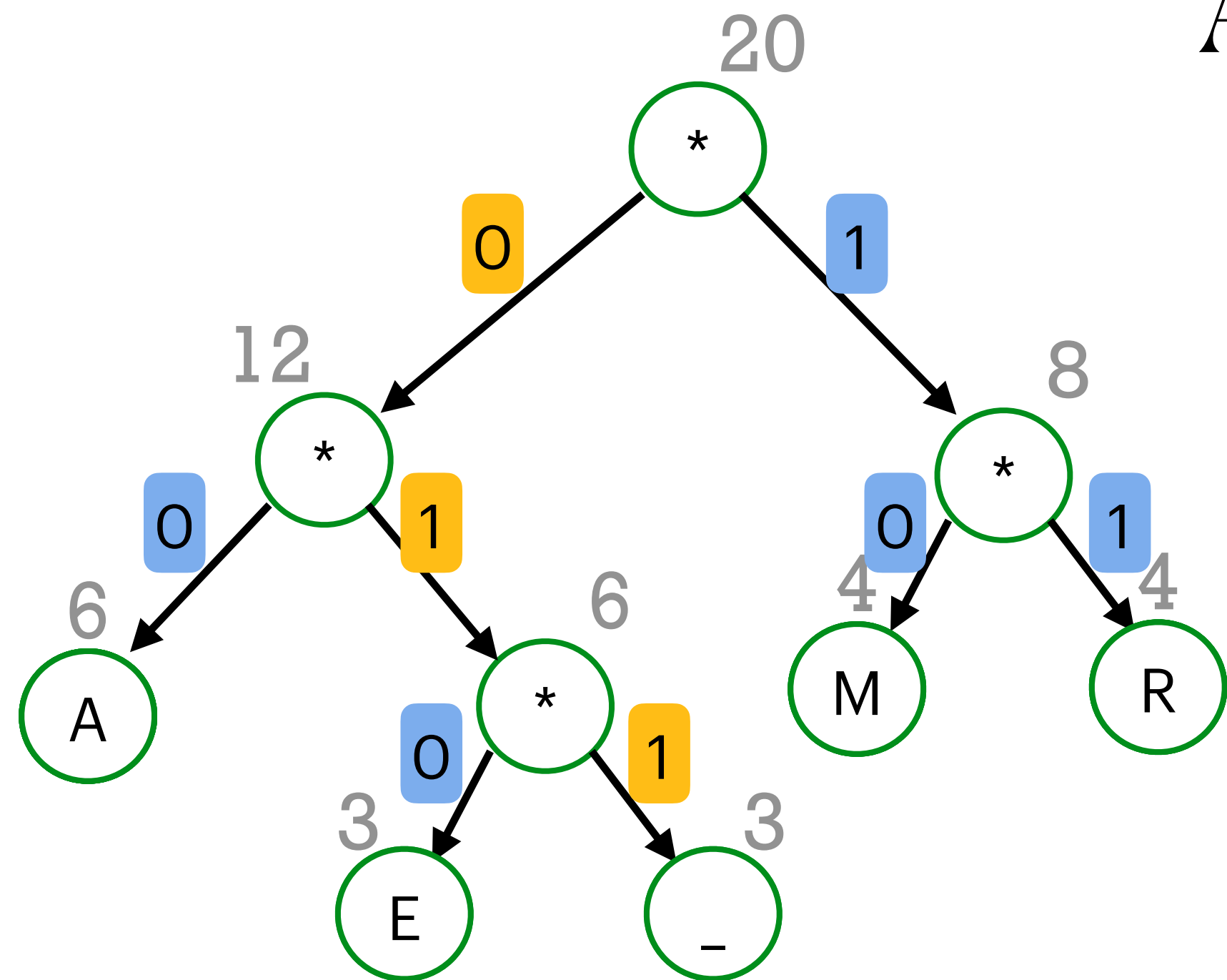


0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E

Arbori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1

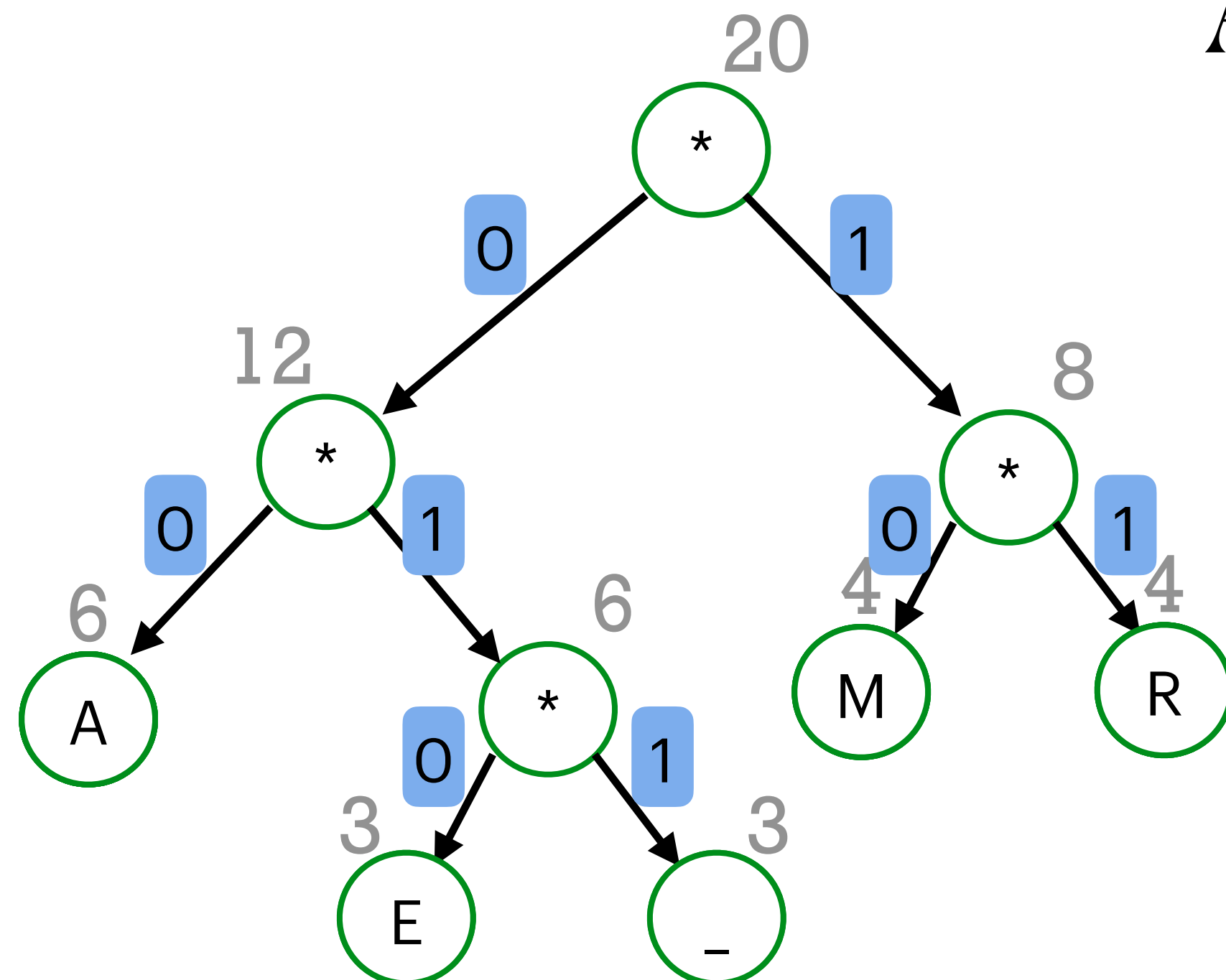


0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E _

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

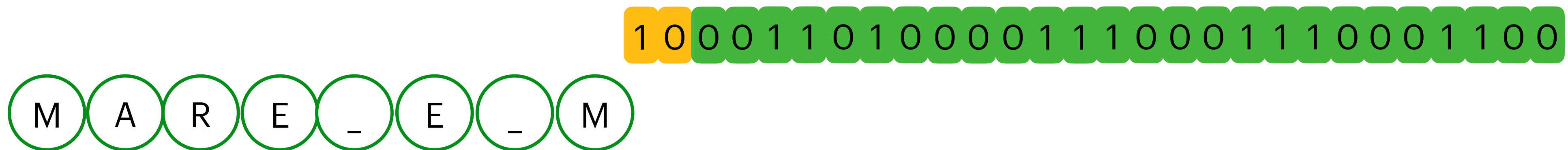
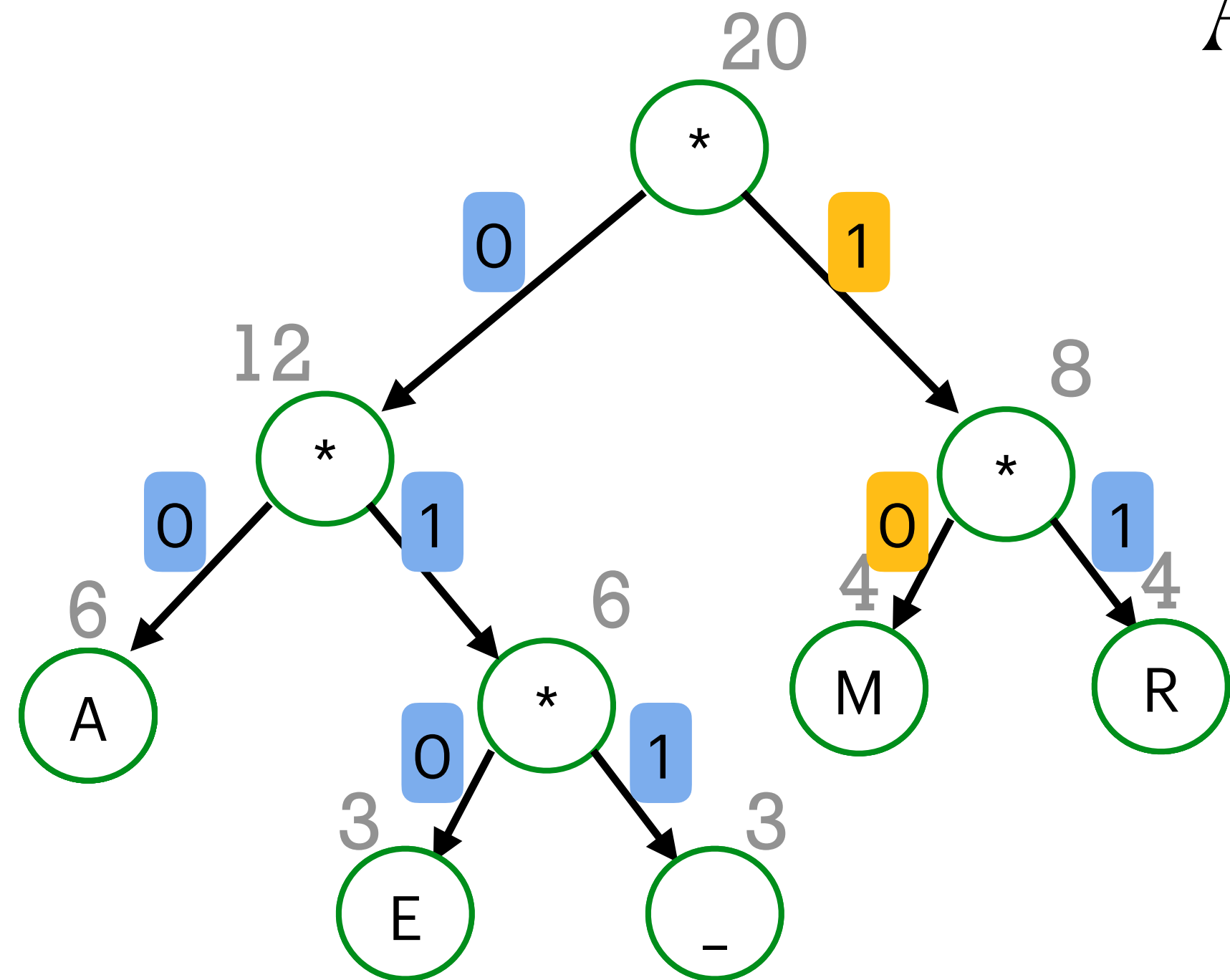


1 0 0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E _

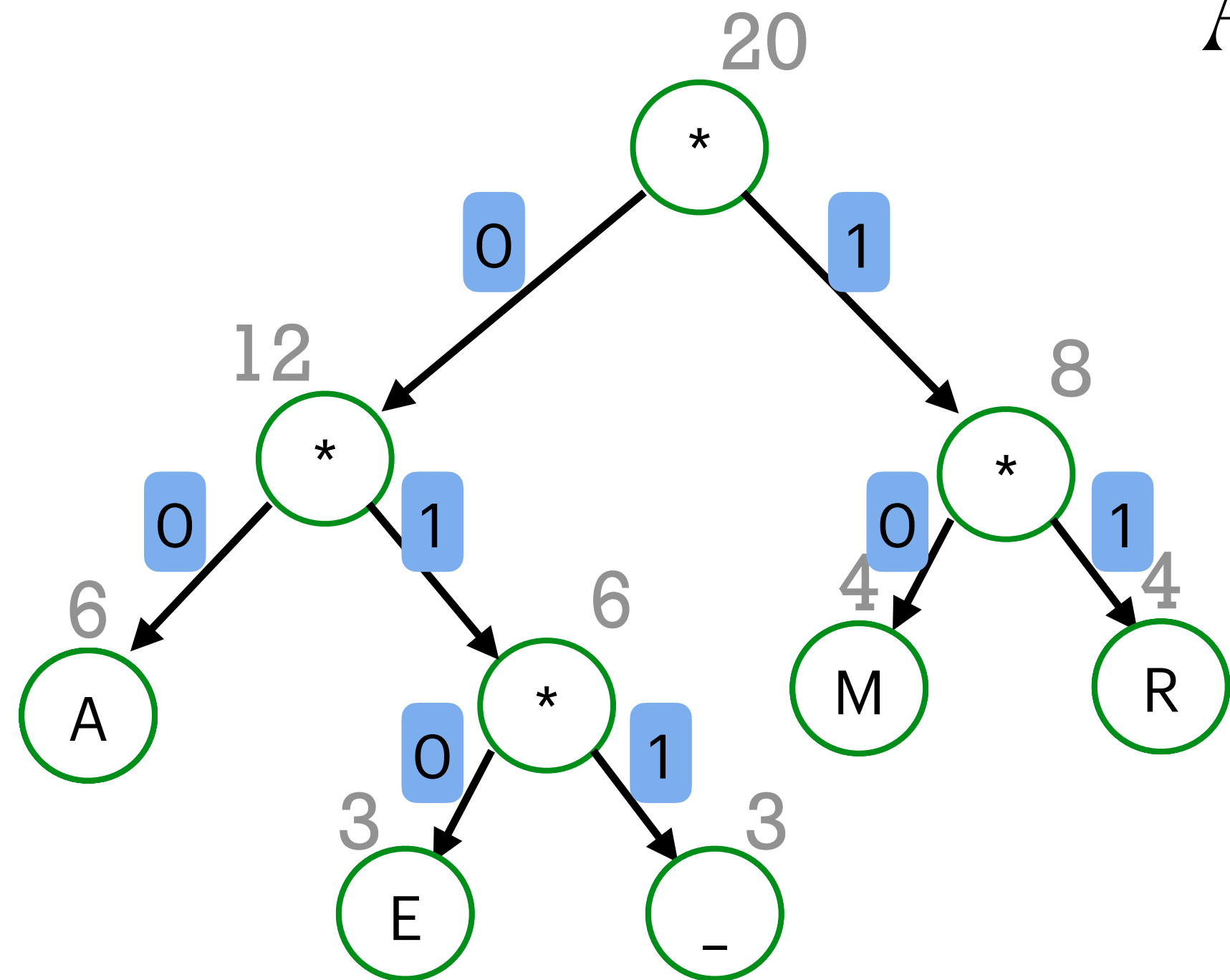
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
_	0 1 1

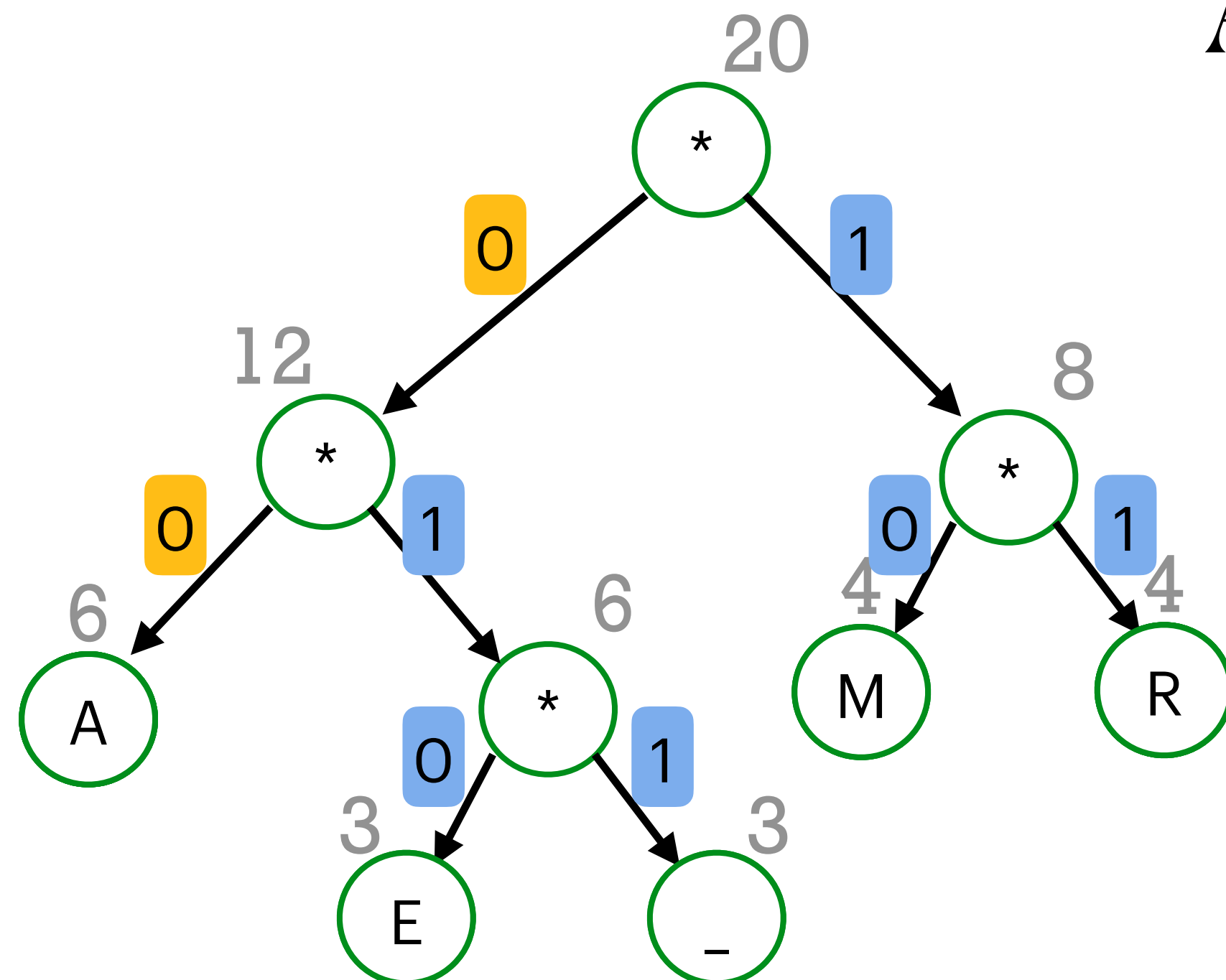


0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E _ M

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

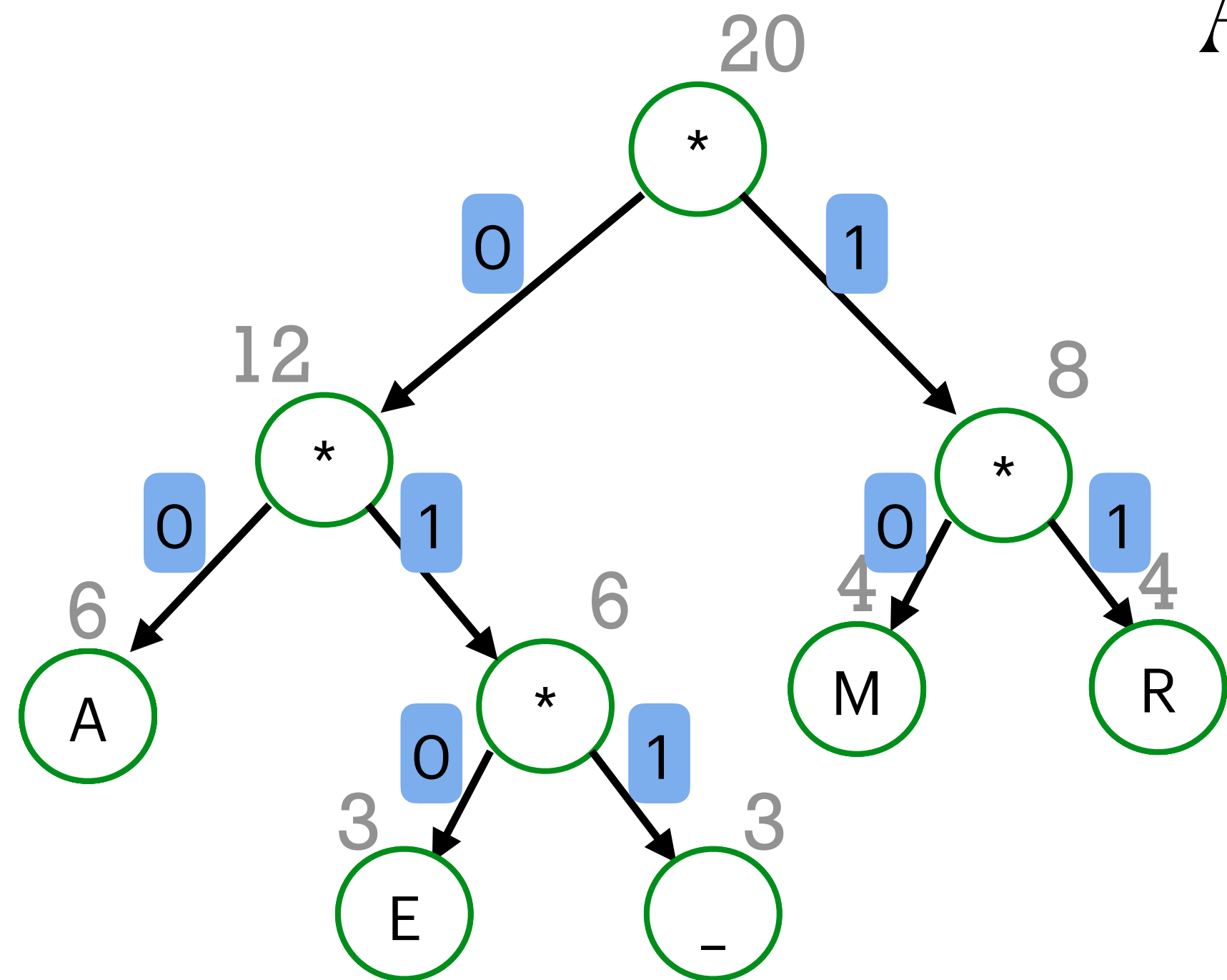


0 0 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E _ M A

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

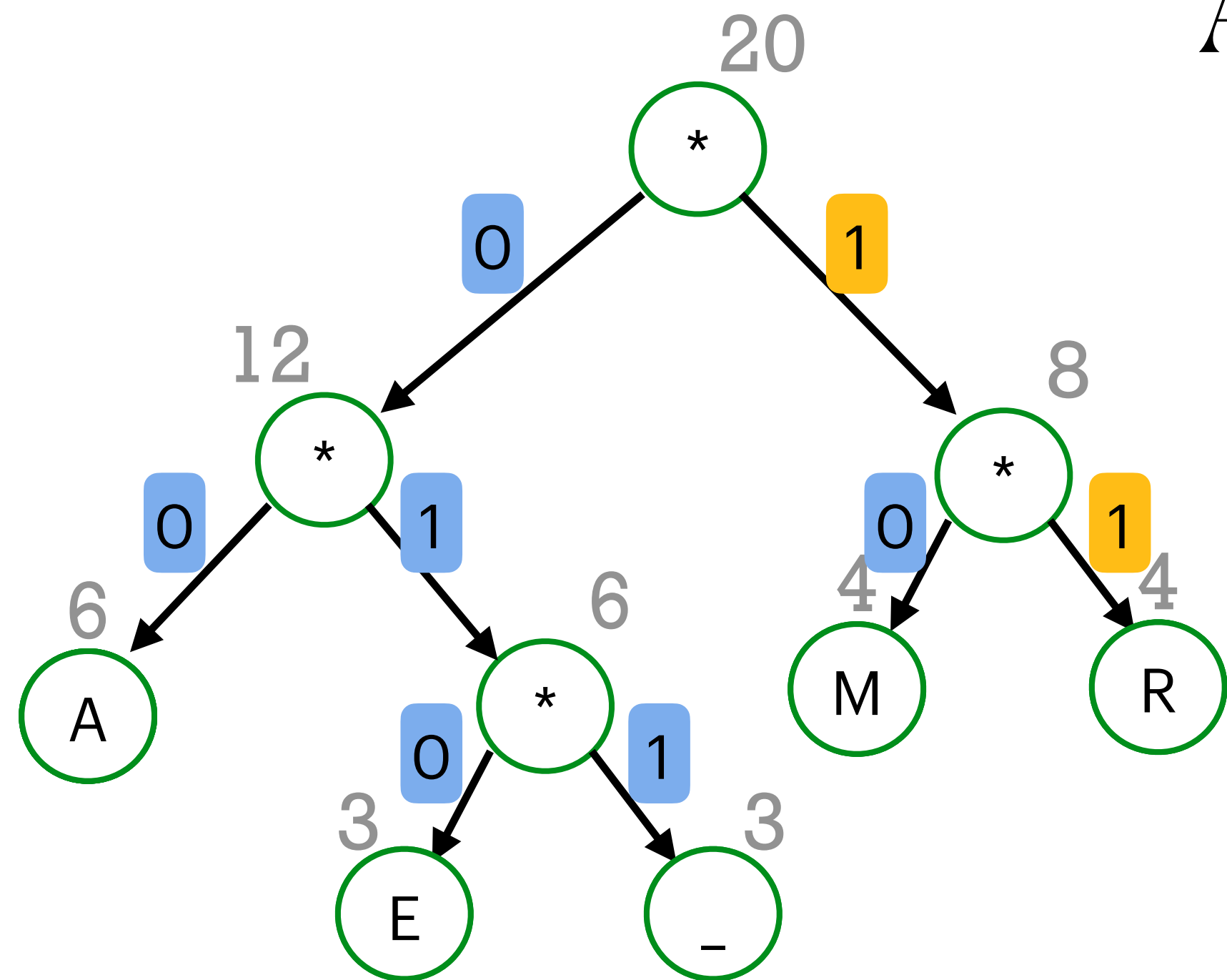


1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E _ M A

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

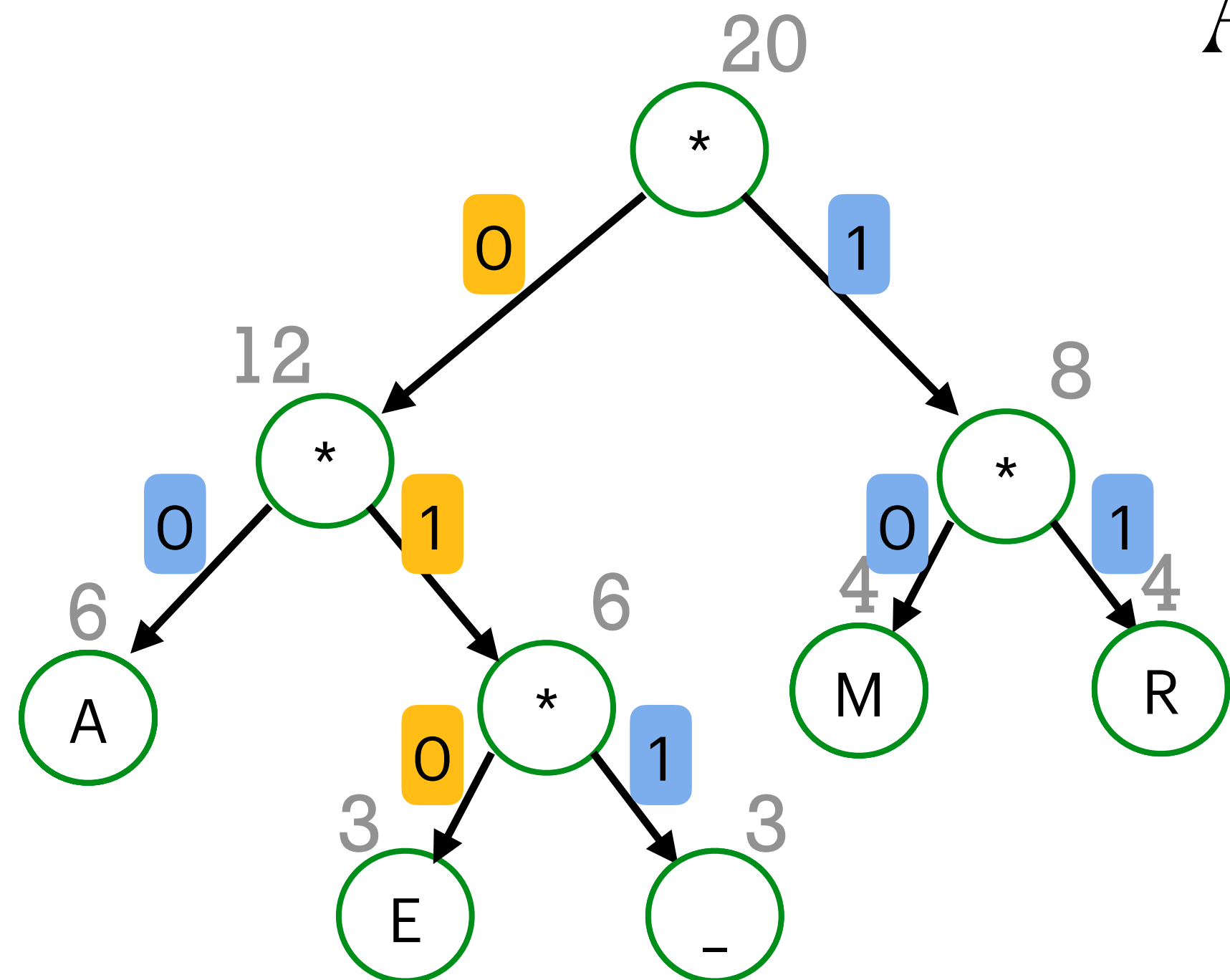


1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E _ M A R

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

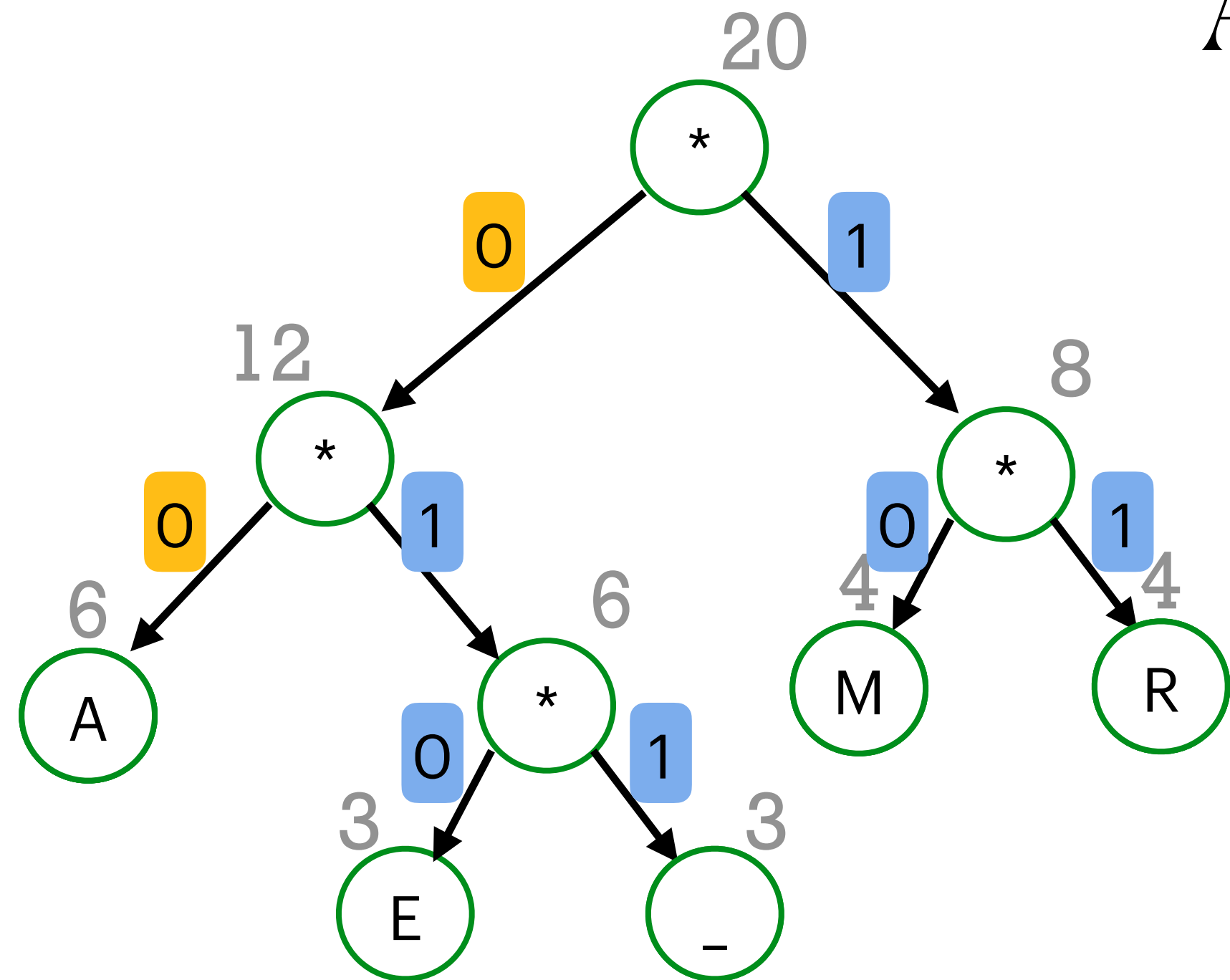


0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E _ M A R E

Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

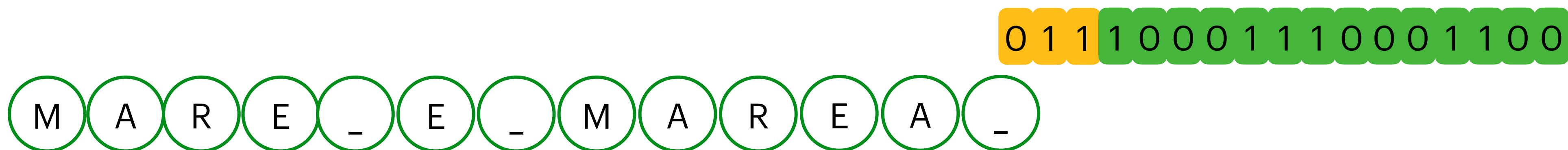
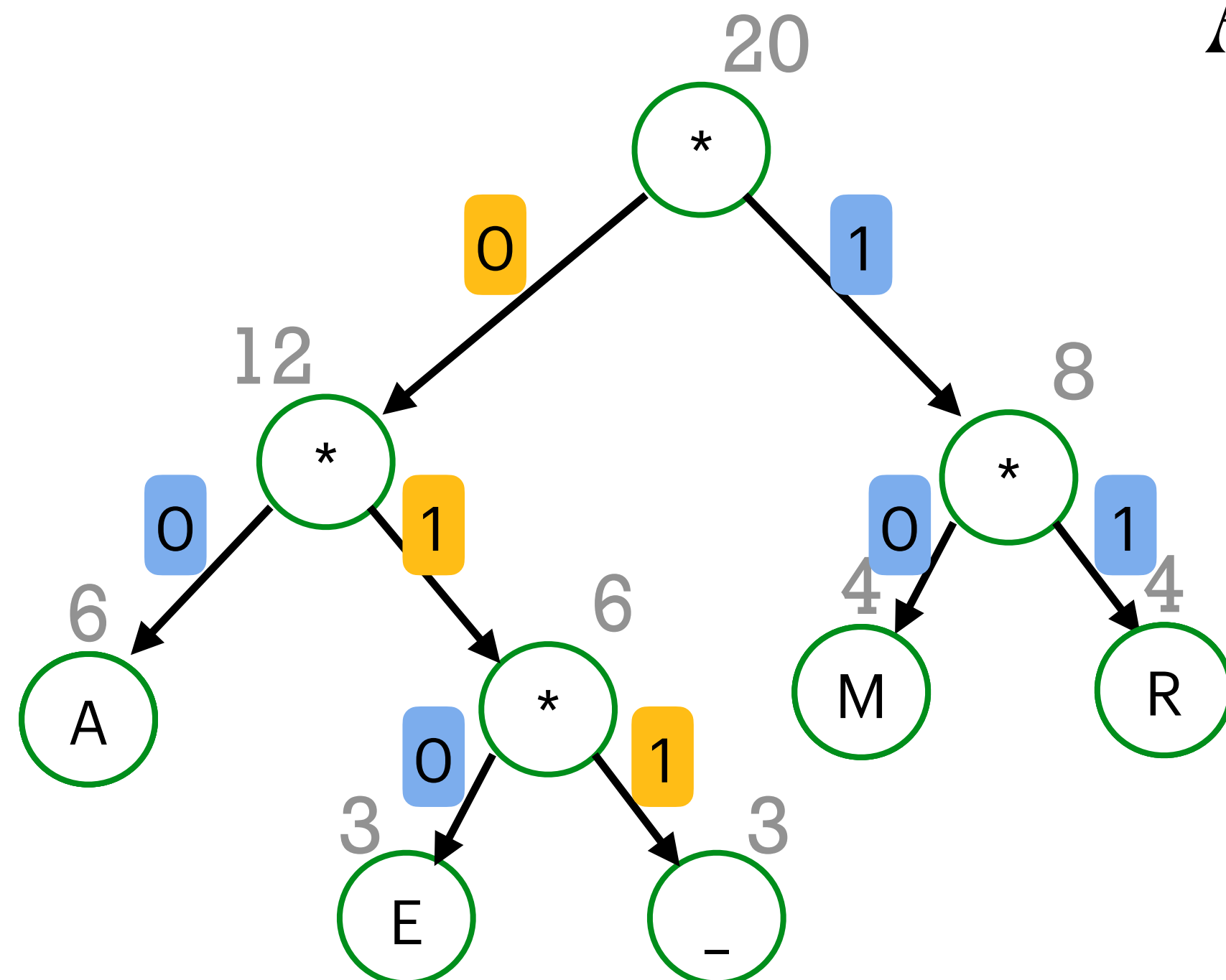


0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0

M A R E _ E _ M A R E A

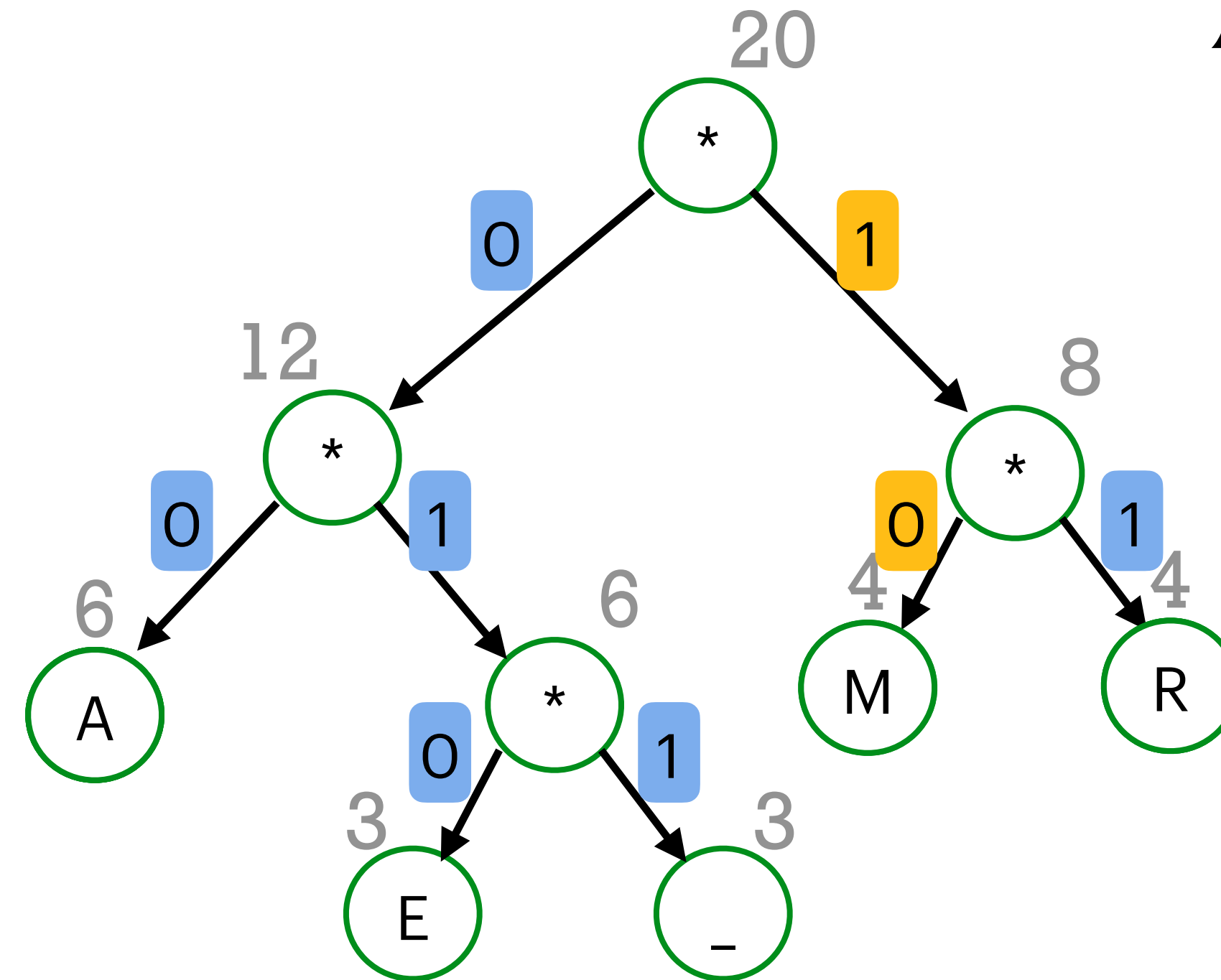
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



Arb ori Huffman

A	0	0	
M	1	0	
R	1	1	
E	0	1	0
-	0	1	1



1 0 0 0 1 1 1 0 0 0 1 1 0 0

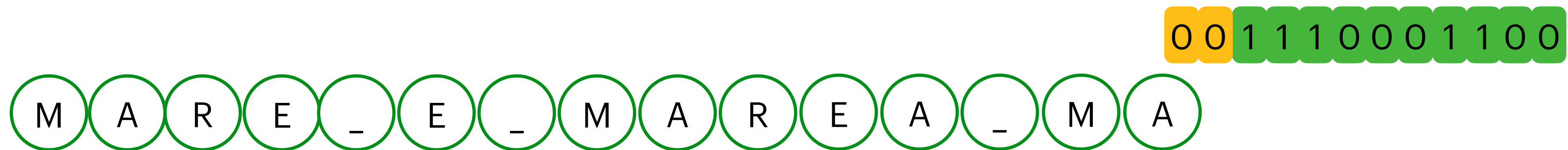
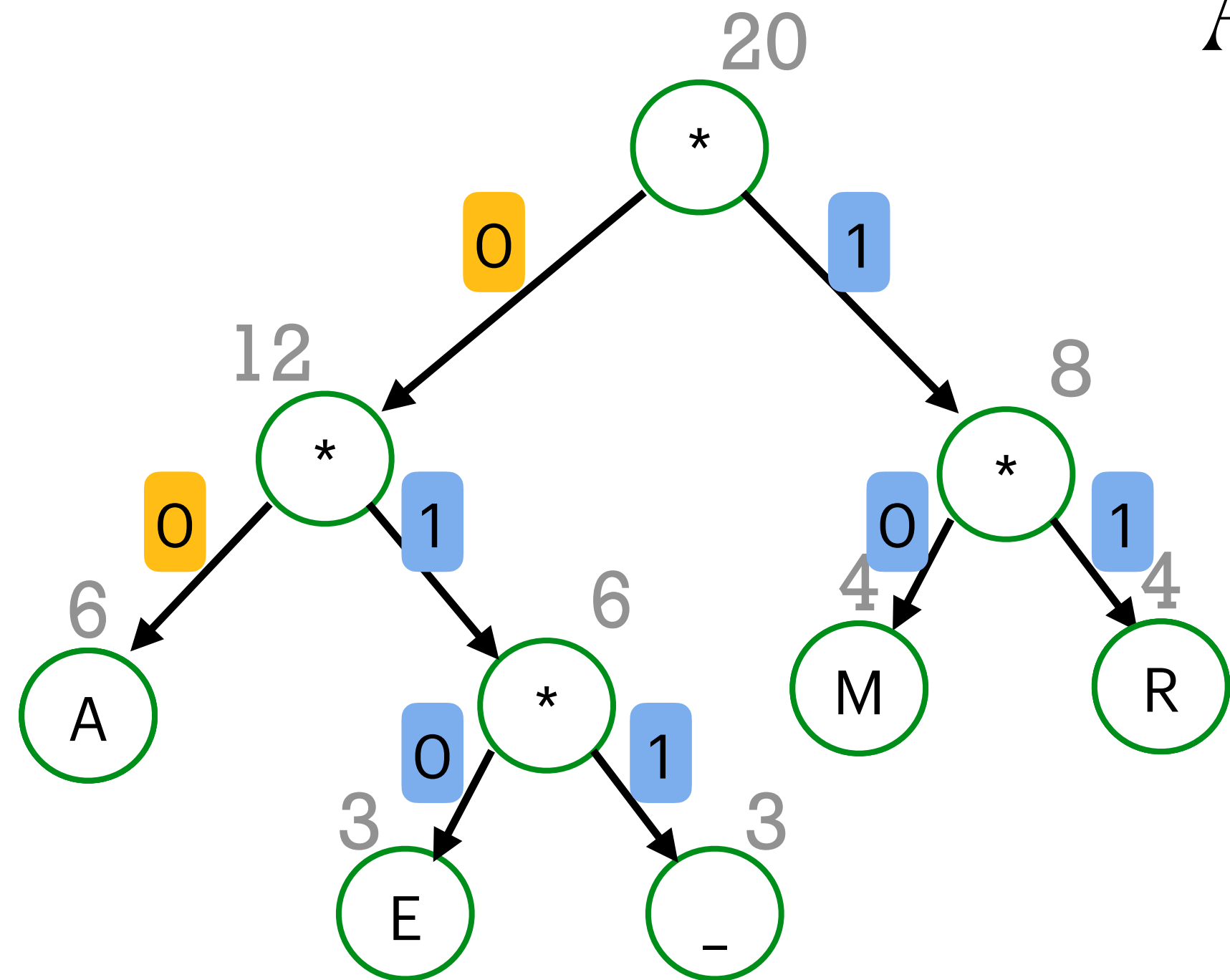
```

graph LR
    N1((M)) --> N2((A))
    N2 --> N3((R))
    N3 --> N4((E))
    N4 --> N5(( ))
    N5 --> N6((E))
    N6 --> N7(( ))
    N7 --> N8((M))
    N8 --> N9((A))
    N9 --> N10((R))
    N10 --> N11((E))
    N11 --> N12((A))
    N12 --> N13(( ))
    N13 --> N14((M))
    N14 --> End(( ))

```

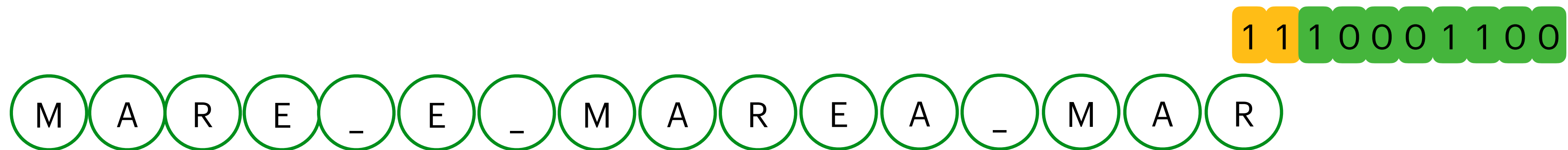
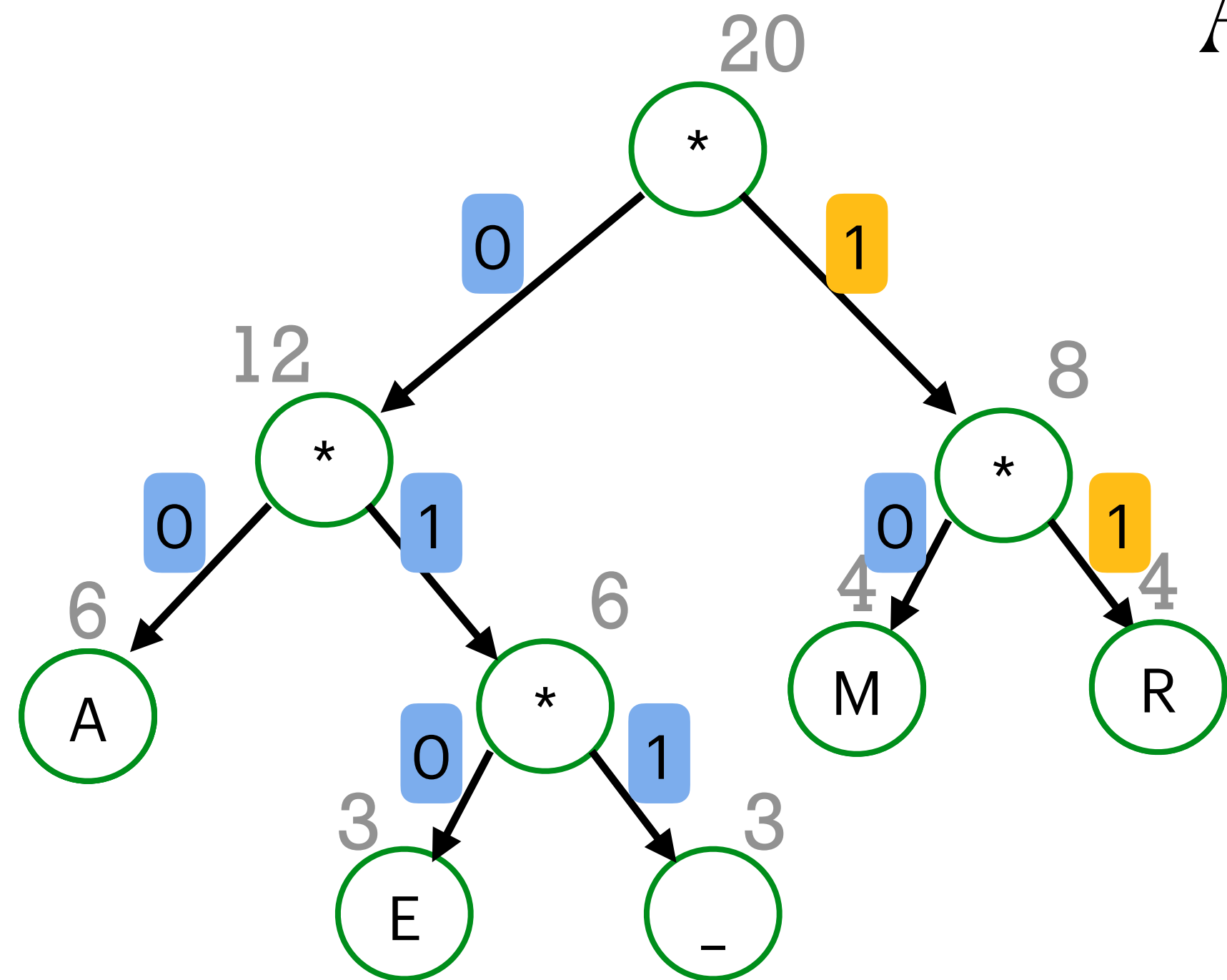
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



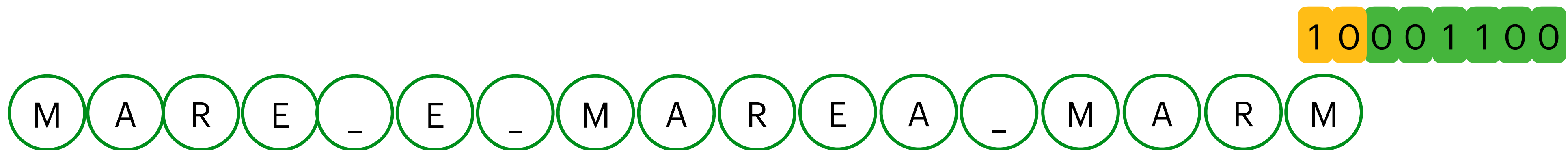
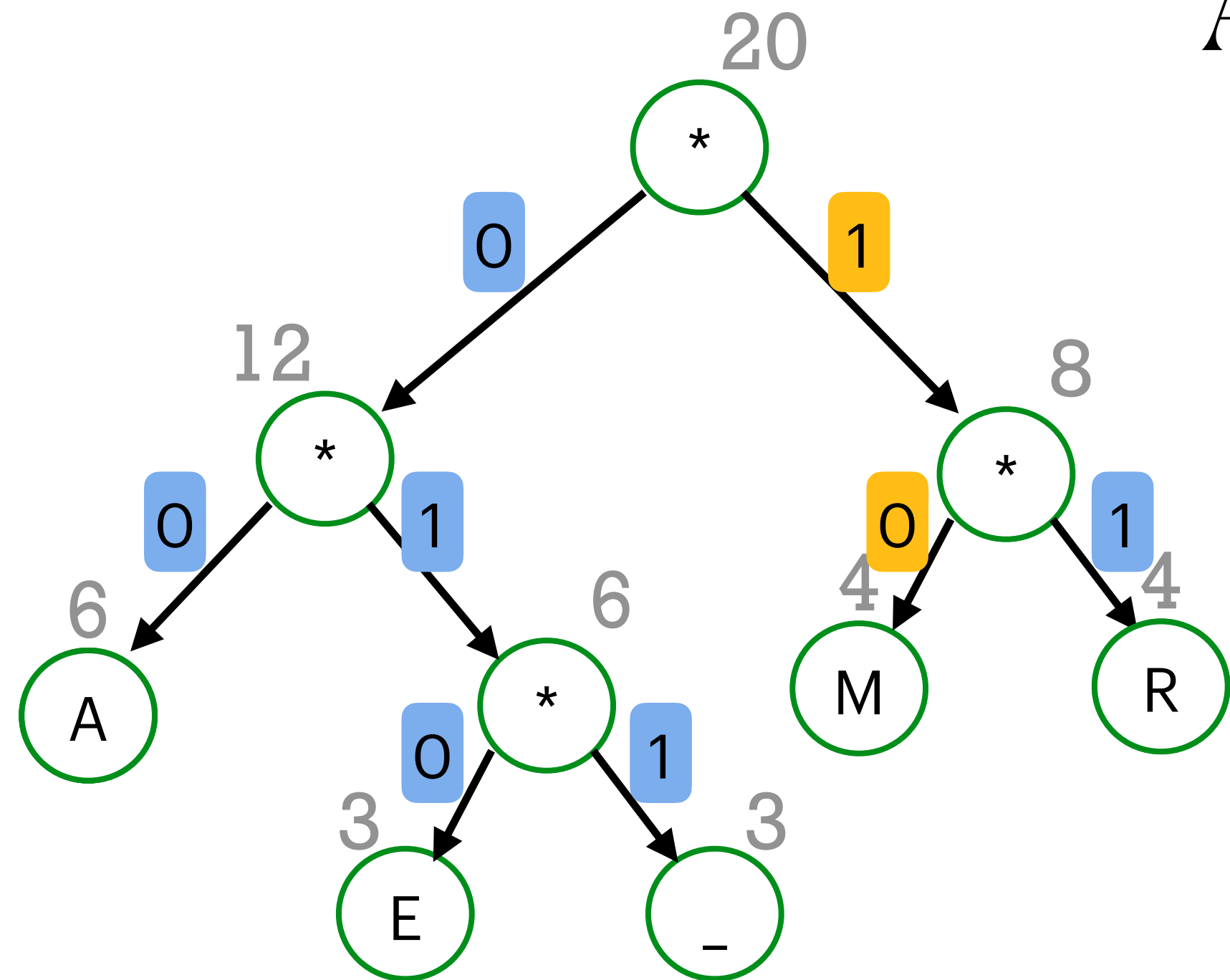
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



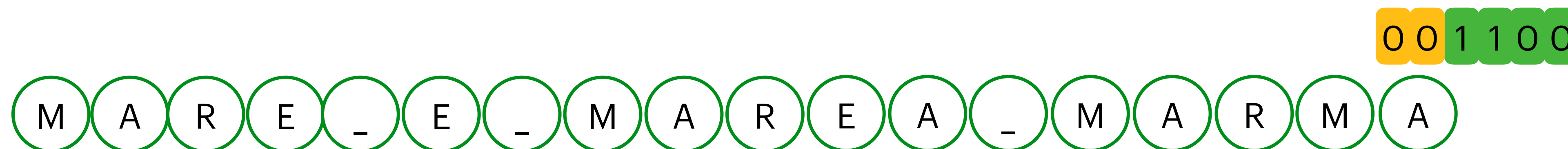
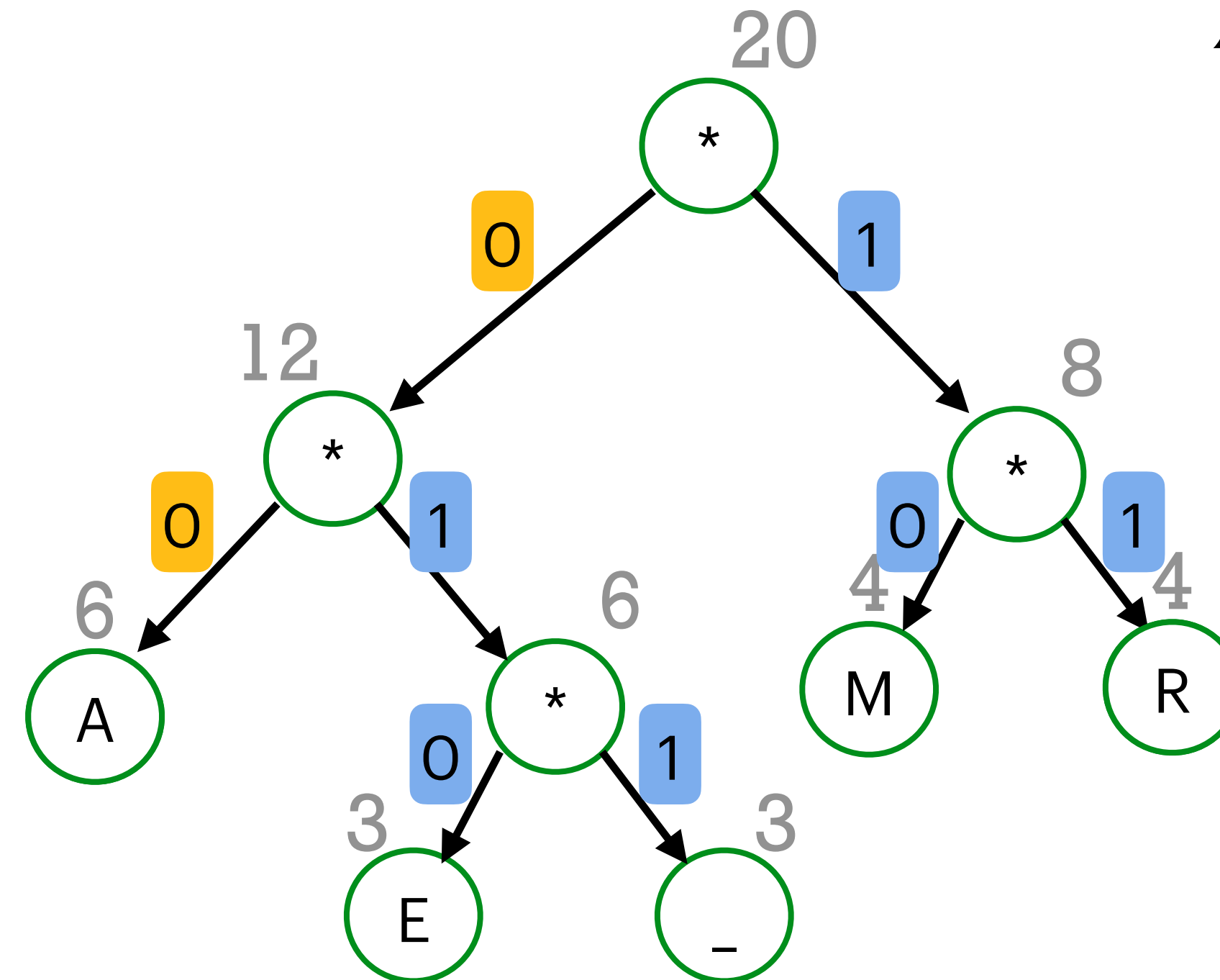
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



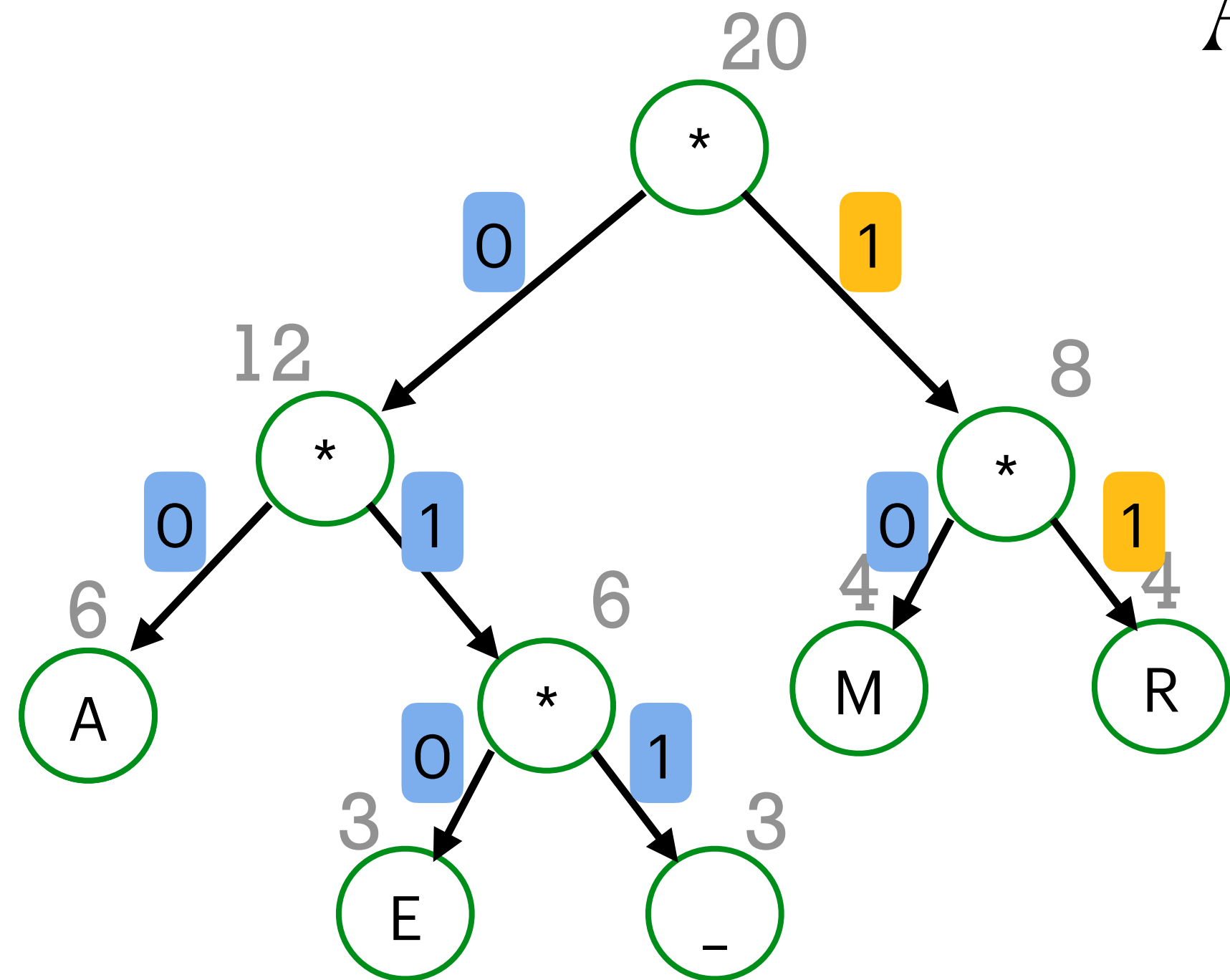
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1

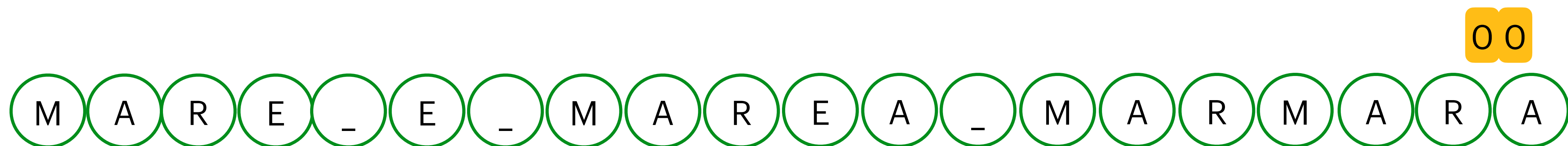
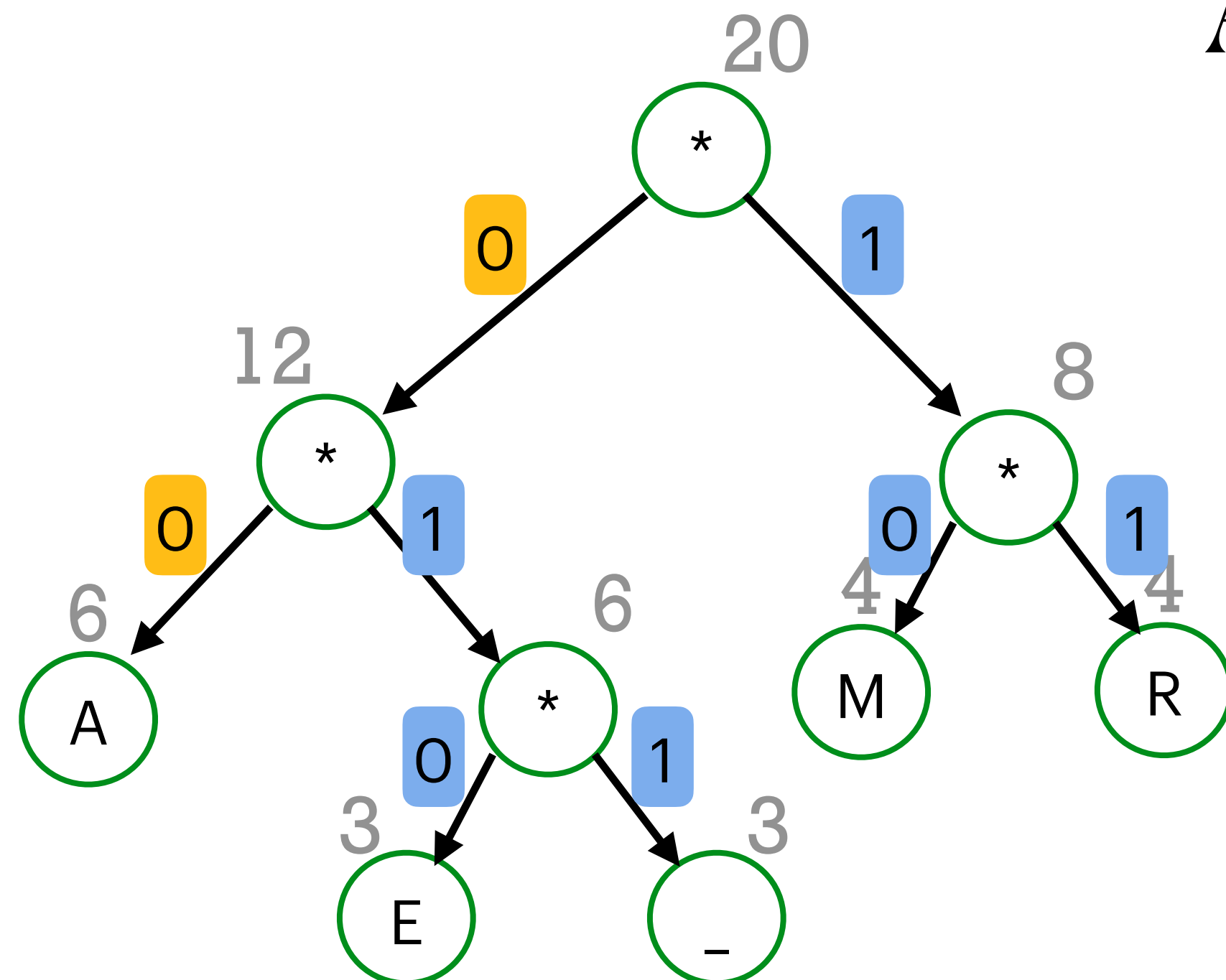


M A R E _ E _ M A R E A _ M A R M A R

1 1 0 0

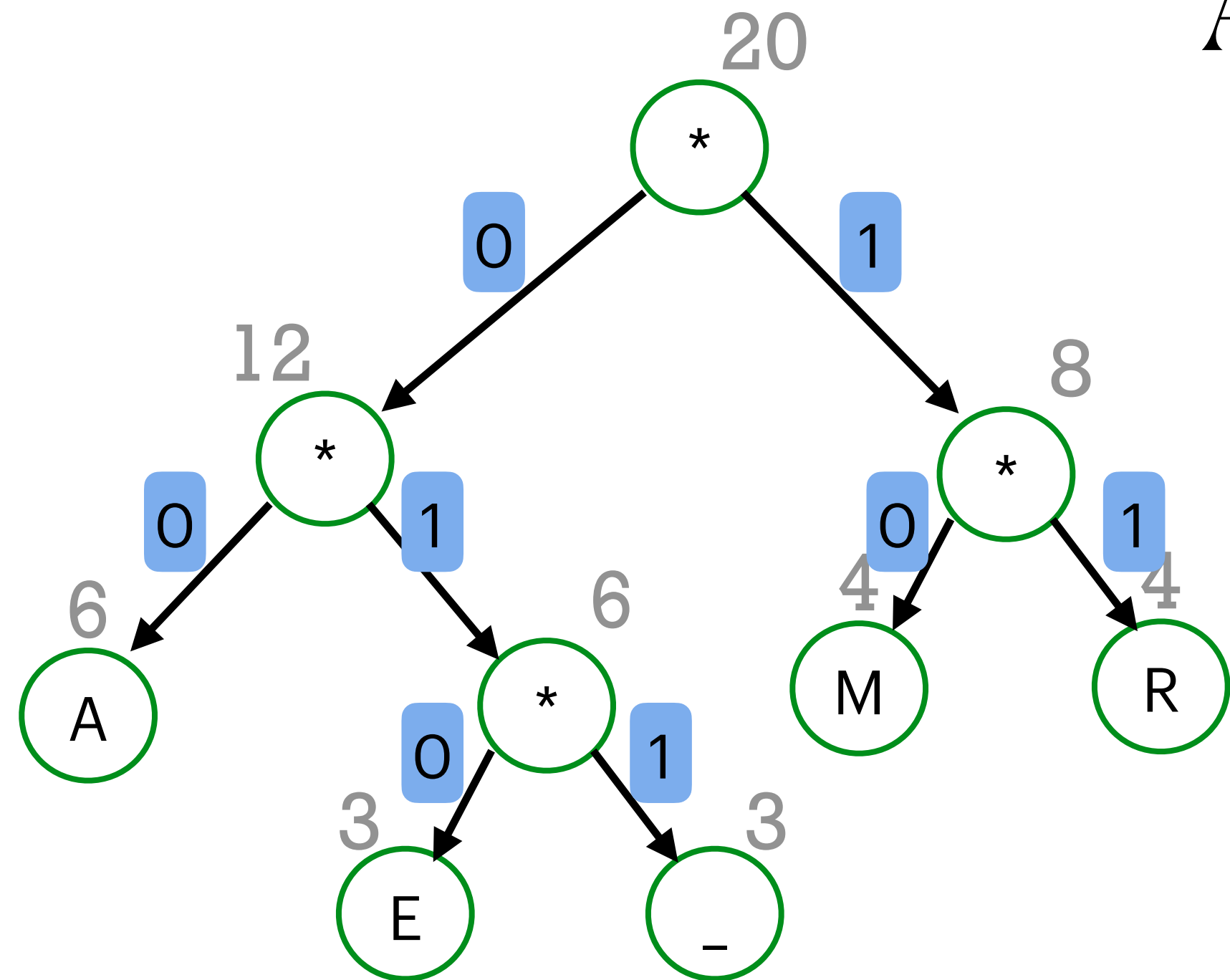
Arbori Huffman

A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



Arbori Huffman

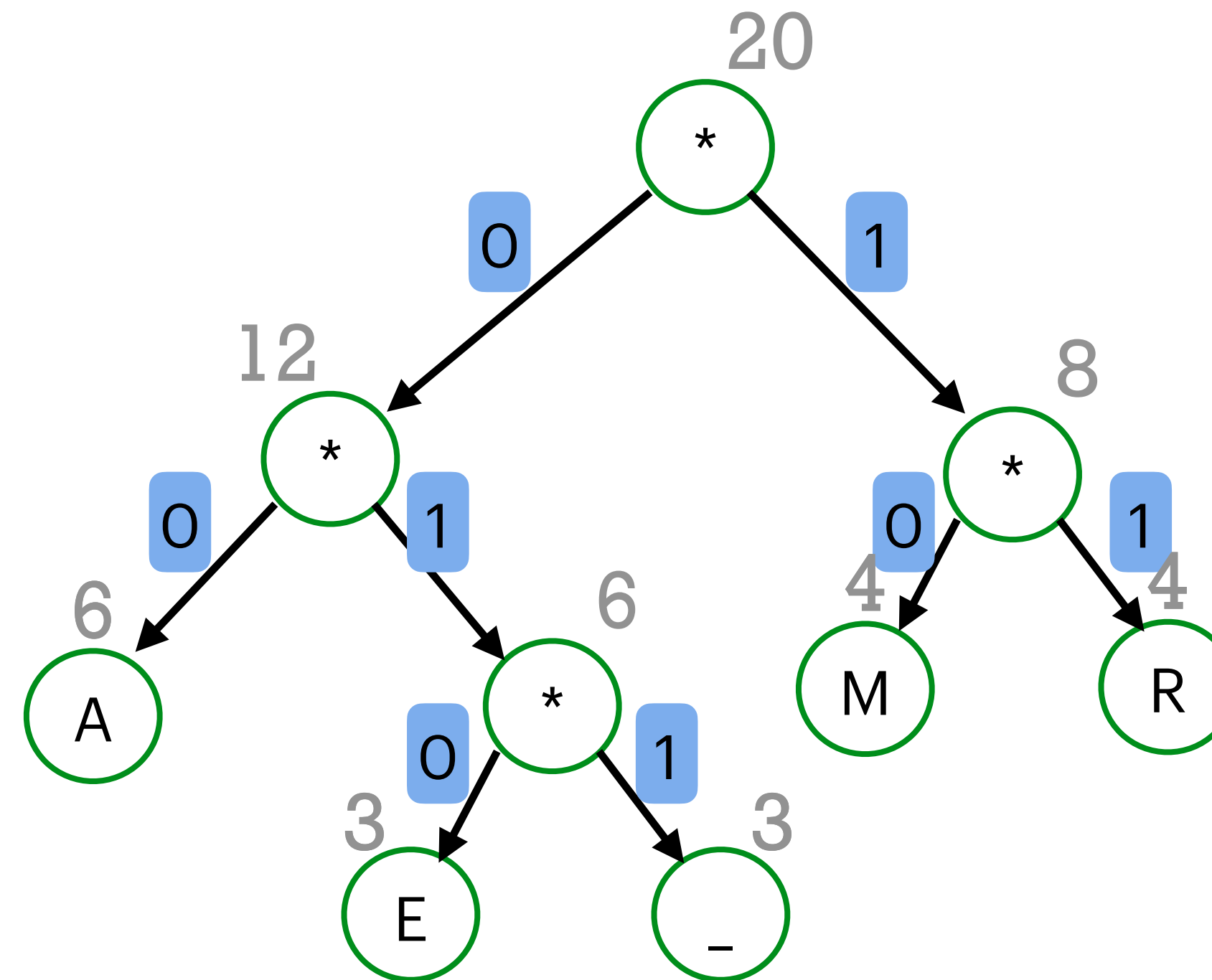
A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



M A R E _ E _ M A R E A _ M A R M A R A

Arbori Huffman

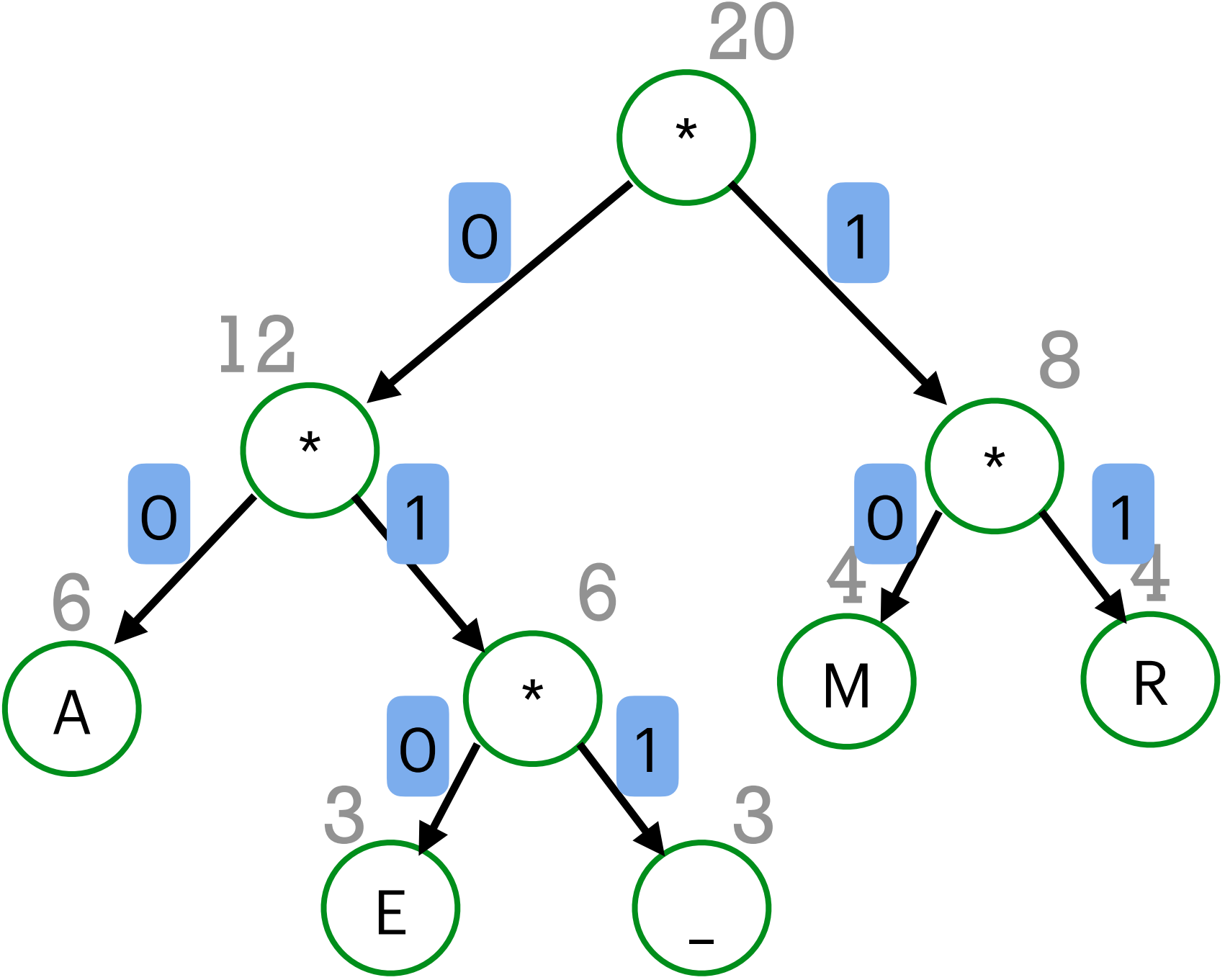
A	0 0
M	1 0
R	1 1
E	0 1 0
-	0 1 1



M A R E _ E _ M A R E A _ M A R M A R A

Arbori Huffman

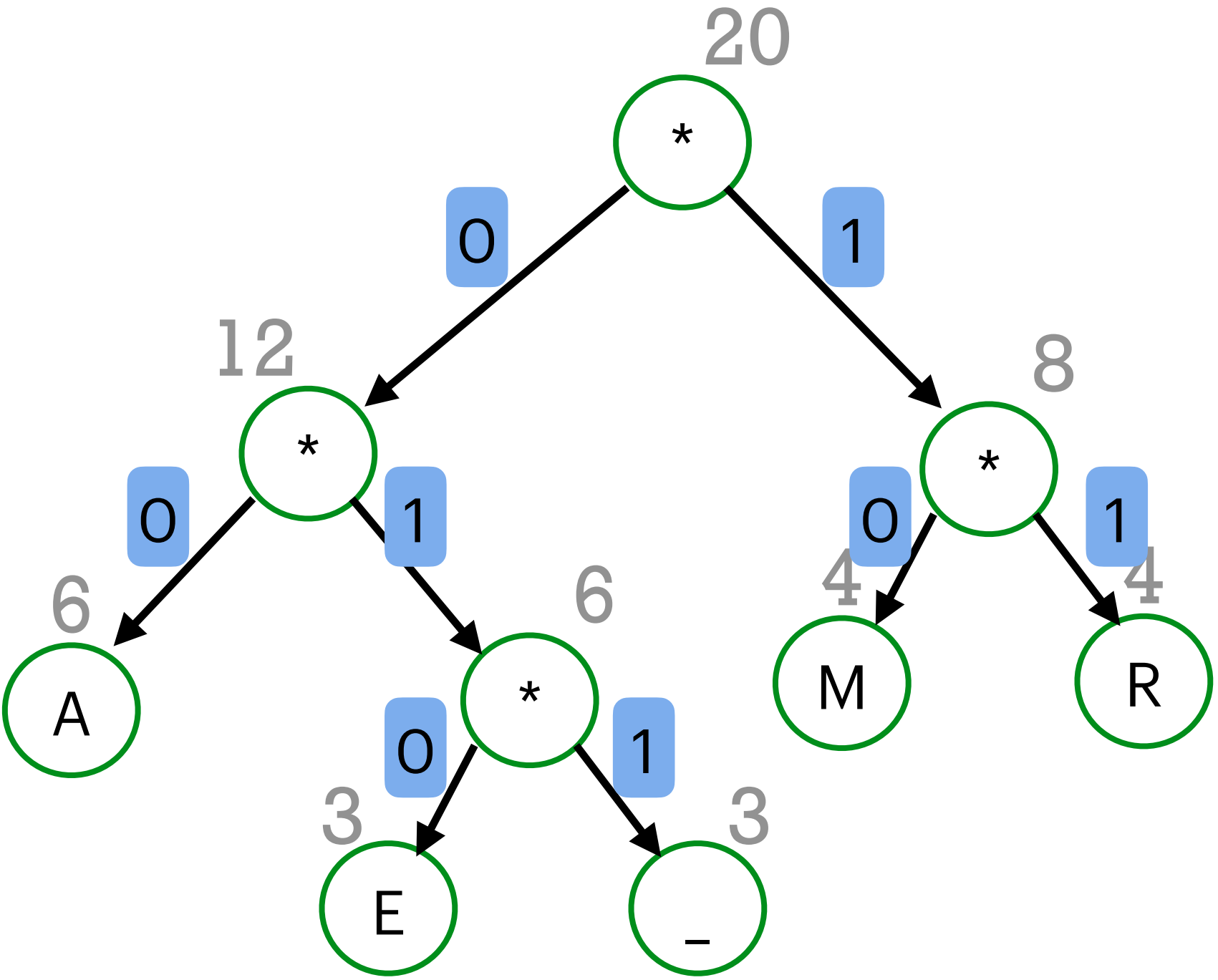
A	0 0	6
M	1 0	4
R	1 1	4
E	0 1 0	3
-	0 1 1	3



M A R E _ E _ M A R E A _ M A R M A R A

Arbori Huffman

A	0 0	$2 * 6 = 12$
M	1 0	$2 * 4 = 8$
R	1 1	$2 * 4 = 8$
E	0 1 0	$3 * 3 = 9$
-	0 1 1	$3 * 3 = 9$



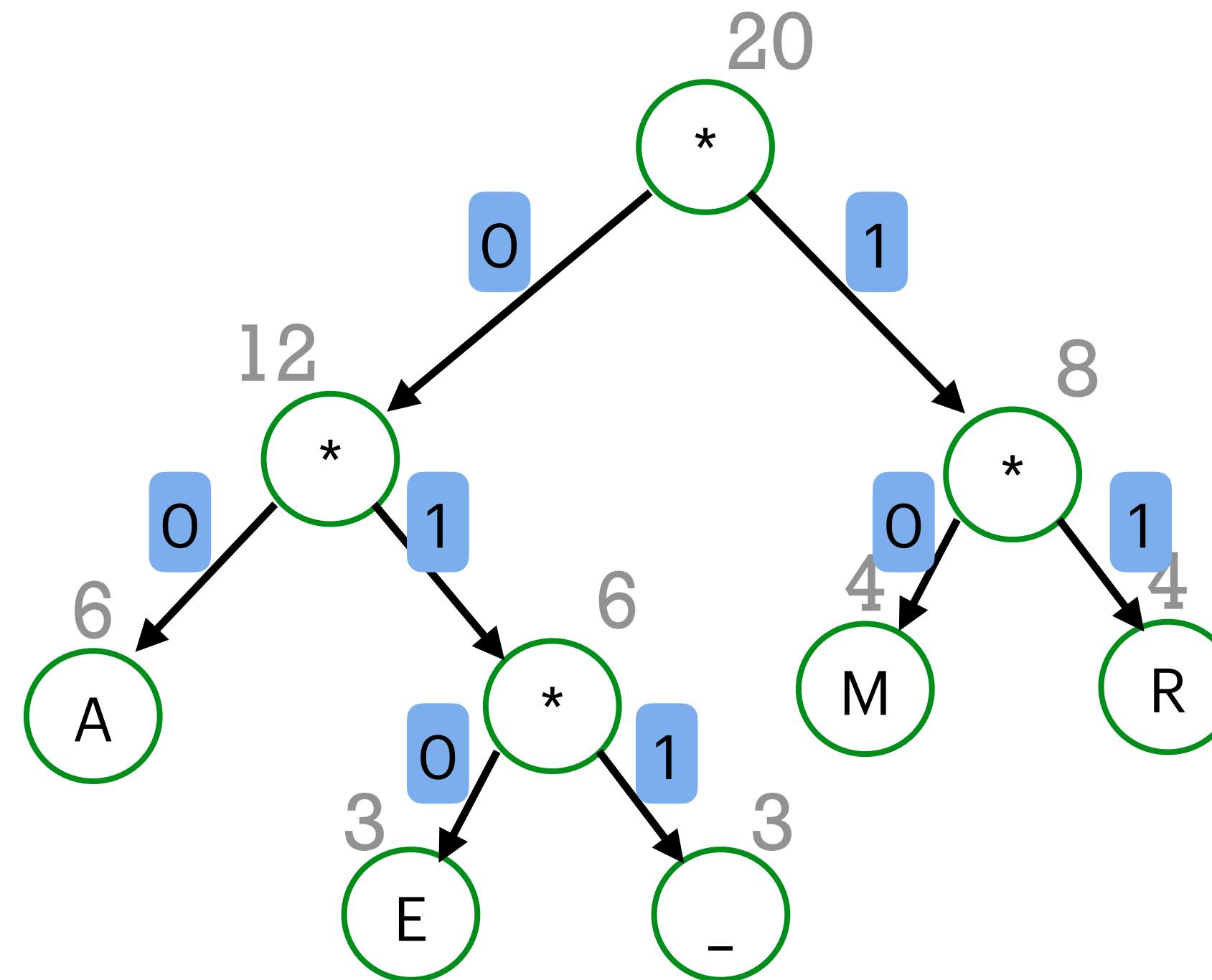
M A R E _ E _ M A R E A _ M A R M A R A

Arbori Huffman

A	00	$2 * 6 = 12$
M	10	$2 * 4 = 8$
R	11	$2 * 4 = 8$
E	010	$3 * 3 = 9$
-	011	$3 * 3 = 9$

$$12 + 8 + 8 + 9 + 9 = 46 \text{ biți}$$

ASCII: $20 * 8 = 80 \text{ biți}$



M A R E _ E _ M A R E A _ M A R M A R A