f-plane

Na dinâmica dos fluidos geofísicos, a aproximação do f-plane é uma aproximação onde o parâmetro de Coriolis, denotado f, é ajustado para um valor constante.

Essa aproximação é freqüentemente usada para a análise de ciclones tropicais altamente idealizados. O uso de um parâmetro constante de Coriolis evita a formação de beta-giros que são amplamente responsáveis pela direção norte-oeste da maioria dos ciclones tropicais.

Plano beta f=fo +beta*y

Na dinâmica dos fluidos geofísicos, uma aproximação pela qual o parâmetro de Coriolis, f, é ajustado para variar linearmente no espaço é chamado de aproximação do plano beta.

Em uma esfera rotativa como a Terra, varia com o seno de latitude; na assim chamada aproximação f-plane, essa variação é ignorada, e um valor de f apropriado para uma determinada latitude é usado em todo o domínio.

3- Ondas de Gravidade Inercial e distribuição de Variáveis

Nesta seção nos discutiremos o efeito da diferença centrada no espaço sobre as ondas de gravidade. Assim, nos consideramos o sistema de equação linearizada.

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv \tag{3.1a}$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial x} - fu \tag{3.1b}$$

$$\frac{\partial h}{\partial t} = -H\nabla.\vec{V} \tag{3.1c}$$

Esta equação direre daquela da secção 2 no termo de coriolis f O termo de coriolis não contem derivadas . Entretanto, eles são difíceis de calcular sobre a grade C, que foi ideal para ondas de gravidade puras.

Assim , nos reconsideramos o problema da distribuição de variáveis.

Não é obivio como nos podemos analisar vários arranjos de variáveis. Nossa primeira opção é considerar (eq. 3.1) como parte de um sistema completo de equações primitivas. Nos estamos interessados no movimento de grande escala, por outro lado nos não devemos incluir o termo de Coriolis.

Sobre a grande escala, a equação primitiva admite dois tipos movimento distintos: baixa frequência e quase-geostrifico e escoamento quase-não divergente; e alta frequecia ondas de gravidade inercial. Ondas de gravidade inercial são continuamente excitada na atmosfera, entretanto, como elas são dispersiva, uma acumulação local de energia de ondas dispersa com o tempo. Estes processos é conhecid como ajustamento geostrofico; o movimento permanecente é um balanço aproximadamente geostrofico e muda somente lentamente com o tempo. Neste capitulo nos estaremos concentrado coma simulação correta destes processos, em que é essencialmente governado pela equação de endas de gravidade inercial(3.1).

Nos estamos interessadosem ambas ondas causadas pelo efeito físico, e que é causado por inadequados dados iniciais e procedimento numérico.

Entretanto o detalhes do processo de ajustamento não importa tanto quanto a correção do resutado do escoamento quase-geotrofico.

Nos devemos no entato investigar o efeito da distribuição do espaço de variáveis dependentes sobre a propriedade dispersiva da ondas de gravidade inercial. Este será feito usando a mais simples aproximação centrada para a derivada no espaço deixando a derivada no tempo em sua forma diferencial.

A discução é baseada sobre aquilo que Winninghoff e Arakawa como apresentado por Arakawa (Arakawa, 1972; Arakawa et al. 1974).

Nos consideramos 5 caminhos de distribuição de variáveis dependentes. No espaço.

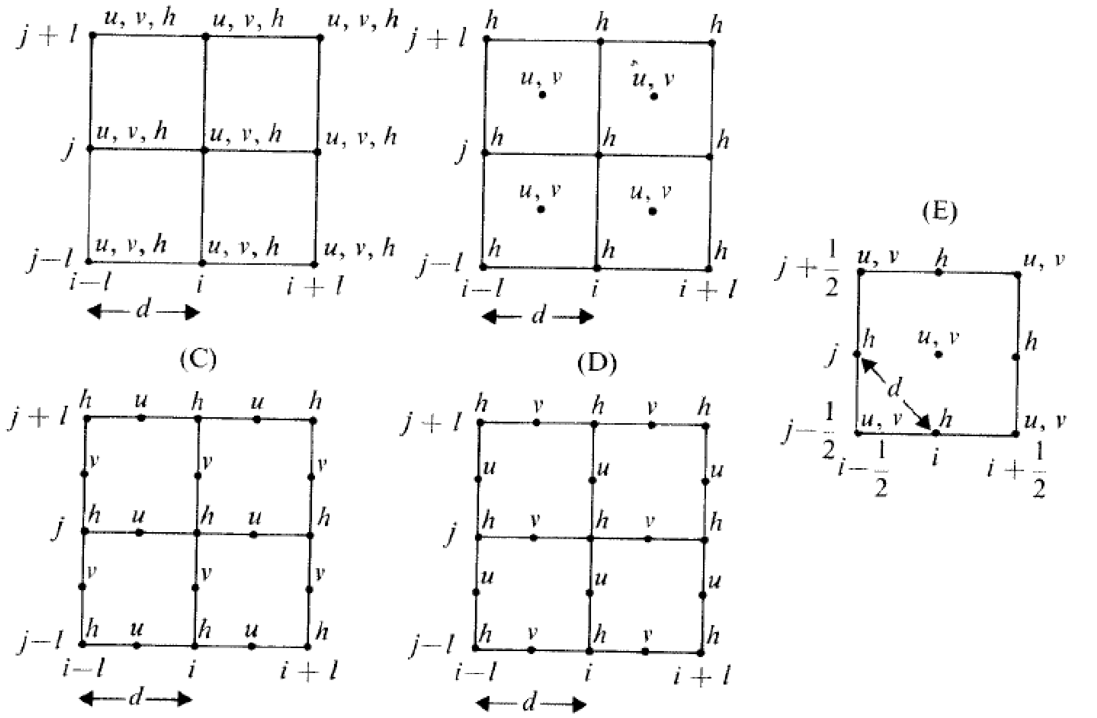

Fig 3.1

Nos Definimos d a distancia mais curta entre os pontos vizinhos carregando a mesma variável dependente. Na figura 3.1 d é o mesmo para cada uma das cinco grades. Assim, todas as grades tem o mesmo numero de variáveis dependentes por unidade de area. A tempo de computação necessário para um integração sobre cada uma das grade será sobre a mesma; propriedade da solução obtida embora , será diferente devido ao efeito do espaço de arrajamento das variáveis.

Usando o subscripts mostrado na figura 3.1, nos definimos um operador para a diferenciação no espaço centrado.

$$(\delta_x \alpha)_{i,j} \equiv \frac{1}{d'}\left(\alpha_{i+\frac{1}{2},j} - \alpha_{i-\frac{1}{2},j}\right)$$

Esta rotação é aplicável a todas as grades. Aqui d' é a distancia entre os pontos os quais a diferença finita é feita. Assim, para a grade A, embora D d' pe igual ao tamanho da grade d, e para a grade E é igual a $\sqrt{2}d$.

Nos também definimos uma media sobre o mesmo dois pontos por:

$$(\bar{\alpha}^x)_{i,j} \equiv \frac{1}{2}\left(\alpha_{i+\frac{1}{2},j} + \alpha_{i-\frac{1}{2},j}\right)$$

Assim, $\left(\delta_y \alpha\right)_{i,j}$ e $(\bar{\alpha}^y)_{i,j}$ são definido no mesmo caminho, mas com respeito ao eixo y. Finalmente,

$$(\bar{\alpha}^{xy})_{i,j} \equiv \left(\overline{\bar{\alpha}^x}^y\right)_{i,j}$$

Para cada uma das 5 grades nos usamos uma aproximação centrada simples para a derivada no espaço e temos de coriolis (3.1). Obtemo-nos os diferentes sistemas:

GRADE A

$$\begin{aligned}
\frac{\partial u}{\partial t} = -g\overline{\delta_x h}^x + fv &= -g\left(\frac{1}{d'}\left(\bar{h}^x_{i+\frac{1}{2},j} - \bar{h}^x_{i-\frac{1}{2},j}\right)\right) + fv \\
&= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2}+\frac{1}{2},j} + h_{i+\frac{1}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(h_{i-\frac{1}{2}+\frac{1}{2},j} + h_{i-\frac{1}{2}-\frac{1}{2},j}\right)\right)\right) + fv \\
&= -g\overline{\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+1,j} + h_{i,j}\right) - \frac{1}{2}\left(h_{i,j} + h_{i-1,j}\right)\right)\right)} + fv \\
&= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+1,j} + h_{i,j} - h_{i,j} - h_{i-1,j}\right)\right)\right) + fv \\
&= -g\left(\frac{1}{2d'}\left(h_{i+1,j} - h_{i-1,j}\right)\right) + fv
\end{aligned}$$

$$\frac{\partial v}{\partial t} = -g\overline{\delta_y h}^y - fu = -g\left(\frac{1}{d'}\left(\bar{h}^y_{i,j+\frac{1}{2}} - \bar{h}^y_{i,j-\frac{1}{2}}\right)\right) - fu$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i,j+\frac{1}{2}+\frac{1}{2}} + h_{i,j+\frac{1}{2}-\frac{1}{2}}\right) - \frac{1}{2}\left(h_{i,j-\frac{1}{2}+\frac{1}{2}} + h_{i,j-\frac{1}{2}-\frac{1}{2}}\right)\right)\right) - fu$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i,j+1} + h_{i,j}\right) - \frac{1}{2}\left(h_{i,j} + h_{i,j-1}\right)\right)\right) - fu$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i,j+1} + h_{i,j} - h_{i,j} - h_{i,j-1}\right)\right)\right) - fu$$

$$= -g\left(\frac{1}{2d'}\left(h_{i,j+1} - h_{i,j-1}\right)\right) - fu$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u}^x + \overline{\delta_y v}^y\right) = H\left(\left(\frac{1}{d'}\left(\bar{u}^x_{i+\frac{1}{2},j} - \bar{h}^x_{i-\frac{1}{2},j}\right)\right) + \left(\frac{1}{d'}\left(\bar{v}^y_{i,j+\frac{1}{2}} - \bar{v}^y_{i,j-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\frac{1}{2}\left(u_{i+\frac{1}{2}+\frac{1}{2},j} + u_{i+\frac{1}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(u_{i-\frac{1}{2}+\frac{1}{2},j} + u_{i-\frac{1}{2}-\frac{1}{2},j}\right)\right)\right)$$

$$+ H\left(\frac{1}{d'}\left(\frac{1}{2}\left(v_{i,j+\frac{1}{2}+\frac{1}{2}} + v_{i,j+\frac{1}{2}-\frac{1}{2}}\right) - \frac{1}{2}\left(v_{i,j-\frac{1}{2}+\frac{1}{2}} + v_{i,j-\frac{1}{2}-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\frac{1}{2}\left(u_{i+1,j} + u_{i,j}\right) - \frac{1}{2}\left(u_{i,j} + u_{i-1,j}\right)\right)\right)$$

$$+ H\left(\frac{1}{d'}\left(\frac{1}{2}\left(v_{i,j+1} + h_{i,j}\right) - \frac{1}{2}\left(v_{i,j} + v_{i,j-1}\right)\right)\right)$$

$$= H\left(\frac{1}{2d'}\left(u_{i+1,j} - u_{i-1,j}\right)\right) + H\left(\frac{1}{2d'}\left(v_{i,j+1} - v_{i,j-1}\right)\right)$$

GRADE B

$$\frac{\partial u}{\partial t} = -g\overline{\delta_x h}^y + fv = -g\left(\frac{1}{d'}\left(\bar{h}^y_{i+\frac{1}{2},j} - \bar{h}^y_{i-\frac{1}{2},j}\right)\right) + fv$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} + h_{i+\frac{1}{2},j-\frac{1}{2}}\right) - \frac{1}{2}\left(h_{i-\frac{1}{2},j+\frac{1}{2}} + h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) + fv$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} - h_{i-\frac{1}{2},j+\frac{1}{2}} + h_{i+\frac{1}{2},j-\frac{1}{2}} - h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) + fv$$

$$= -g\left(\frac{1}{2d'}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} - h_{i-\frac{1}{2},j+\frac{1}{2}} + h_{i+\frac{1}{2},j-\frac{1}{2}} - h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right) + fv$$

$$\frac{\partial v}{\partial t} = -g\overline{\delta_y h}^x - fu = -g\left(\frac{1}{d'}\left(\bar{h}^x_{i,j+\frac{1}{2}} - \bar{h}^x_{i,j-\frac{1}{2}}\right)\right) - fu$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} + h_{i-\frac{1}{2},j+\frac{1}{2}}\right) - \frac{1}{2}\left(h_{i+\frac{1}{2},j-\frac{1}{2}} + h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) - fu$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} - h_{i-\frac{1}{2},j+\frac{1}{2}} + h_{i+\frac{1}{2},j-\frac{1}{2}} - h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) - fu$$

$$= -g\left(\frac{1}{2d'}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} - h_{i-\frac{1}{2},j+\frac{1}{2}} + h_{i+\frac{1}{2},j-\frac{1}{2}} - h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right) - fu$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u}^y + \overline{\delta_y v}^x\right) = H\left(\left(\frac{1}{d'}\left(\bar{u}^y_{i+\frac{1}{2},j} - \bar{h}^y_{i-\frac{1}{2},j}\right)\right) + \left(\frac{1}{d'}\left(\bar{v}^x_{i,j+\frac{1}{2}} - \bar{v}^x_{i,j-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\frac{1}{2}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}}\right) - \frac{1}{2}\left(u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$+ H\left(\frac{1}{d'}\left(\frac{1}{2}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}}\right) - \frac{1}{2}\left(v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{2d'}\left(\left(u_{i+\frac{1}{2},j+\frac{1}{2}} - u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} - u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$+ H\left(\frac{1}{2d'}\left(\left(v_{i+\frac{1}{2},j+\frac{1}{2}} - v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} - v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

GRADE C

$$\frac{\partial u}{\partial t} = -g\delta_x h + f\bar{v}^{xy} = -g\left(\frac{1}{d'}\left(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j}\right)\right) + f\left(\frac{1}{2}\left(\bar{v}^y{}_{i+\frac{1}{2},j} + \bar{v}^y{}_{i-\frac{1}{2},j}\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j}\right)\right)$$

$$+ f\left(\frac{1}{2}\left(\frac{1}{2}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}}\right) + \frac{1}{2}\left(v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j}\right)\right)$$

$$+ f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial v}{\partial t} = -g\delta_y h - f\bar{u}^{xy} = -g\left(\frac{1}{d'}\left(h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}}\right)\right) - f\left(\frac{1}{2}\left(\bar{u}^y{}_{i+\frac{1}{2},j} + \bar{u}^y{}_{i-\frac{1}{2},j}\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}}\right)\right)$$

$$- f\left(\frac{1}{2}\left(\frac{1}{2}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}}\right) + \frac{1}{2}\left(u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}}\right)\right)$$

$$- f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\delta_x u + \delta_y v\right) = H\left(\left(\frac{1}{d'}\left(u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j}\right)\right) + \left(\frac{1}{d'}\left(v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\left(u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j} + v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial u}{\partial t} = -g\left(\frac{1}{d'}\left(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j}\right)\right) + f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial v}{\partial t} = -g\left(\frac{1}{d'}\left(h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}}\right)\right) - f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial h}{\partial t} = H\left(\frac{1}{d'}\left(\left(u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j} + v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}\right)\right)\right)$$

GRADE D

$$\frac{\partial u}{\partial t} = -g\overline{\delta_x h}^{xy} + f\bar{v}^{xy} = -g\left(\frac{1}{d'}\left(\bar{h}^{xy}_{i+\frac{1}{2},j} - \bar{h}^{xy}_{i-\frac{1}{2},j}\right)\right) + f\left(\frac{1}{2}\left(\bar{v}^{y}{}_{i+\frac{1}{2},j} + \bar{v}^{y}{}_{i-\frac{1}{2},j}\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{y}_{i+\frac{1}{2}+\frac{1}{2},j} + \bar{h}^{y}_{i+\frac{1}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(\bar{h}^{y}_{i-\frac{1}{2}+\frac{1}{2},j} + \bar{h}^{y}_{i-\frac{1}{2}-\frac{1}{2},j}\right)\right)\right)$$

$$+ f\left(\frac{1}{2}\left(\frac{1}{2}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}}\right) + \frac{1}{2}\left(v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{y}_{i+1,j} + \bar{h}^{y}_{i,j}\right) - \frac{1}{2}\left(\bar{h}^{y}_{i,j} + \bar{h}^{y}_{i-1,j}\right)\right)\right)$$

$$+ f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) =$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{y}_{i+1,j} - \bar{h}^{y}_{i-1,j}\right)\right)\right)$$

$$+ f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\frac{1}{2}\left(h_{i+1,j+\frac{1}{2}} + h_{i+1,j-\frac{1}{2}}\right) - \frac{1}{2}\left(h_{i-1,j+\frac{1}{2}} + h_{i-1,j-\frac{1}{2}}\right)\right)\right)\right)$$

$$+ f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{4d'}\left(\left(h_{i+1,j+\frac{1}{2}} - h_{i-1,j+\frac{1}{2}} + h_{i+1,j-\frac{1}{2}} - h_{i-1,j-\frac{1}{2}}\right)\right)\right)$$

$$+ f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial v}{\partial t} = -g\overline{\delta_y h}^{xy} - f\bar{u}^{xy} = -g\left(\frac{1}{d'}\left(\bar{h}^{xy}_{i,j+\frac{1}{2}} - \bar{h}^{xy}_{i,j-\frac{1}{2}}\right)\right) - f\left(\frac{1}{2}\left(\bar{u}^{y}{}_{i+\frac{1}{2},j} + \bar{u}^{y}{}_{i-\frac{1}{2},j}\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{x}_{i,j+\frac{1}{2}+\frac{1}{2}} + \bar{h}^{x}_{i,j+\frac{1}{2}-\frac{1}{2}}\right) - \frac{1}{2}\left(\bar{h}^{x}_{i,j-\frac{1}{2}+\frac{1}{2}} + \bar{h}^{x}_{i,j-\frac{1}{2}-\frac{1}{2}}\right)\right)\right)$$

$$- f\left(\frac{1}{2}\left(\frac{1}{2}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}}\right) + \frac{1}{2}\left(u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{x}_{i,j+1} + \bar{h}^{x}_{i,j}\right) - \frac{1}{2}\left(\bar{h}^{x}_{i,j} + \bar{h}^{x}_{i,j-1}\right)\right)\right)$$

$$- f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) =$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{y}_{i,j+1} - \bar{h}^{y}_{i,j-1}\right)\right)\right)$$

$$- f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\frac{1}{2}\left(h_{i+\frac{1}{2},j+1} + h_{i-\frac{1}{2},j+1}\right) - \frac{1}{2}\left(h_{i+\frac{1}{2},j-1} + h_{i-\frac{1}{2},j-1}\right)\right)\right)\right)$$

$$- f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{4d'}\left(\left(h_{i+\frac{1}{2},j+1} - h_{i+\frac{1}{2},j-1} + h_{i-\frac{1}{2},j+1} - h_{i-\frac{1}{2},j-1}\right)\right)\right)$$

$$- f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u}^{xy} + \overline{\delta_y v}^{xy}\right) = H\left(\left(\frac{1}{d'}\left(\bar{u}^{xy}_{i+\frac{1}{2},j} - \bar{u}^{xy}_{i-\frac{1}{2},j}\right)\right) + \left(\frac{1}{d'}\left(\bar{v}^{xy}_{i,j+\frac{1}{2}} - \bar{v}^{xy}_{i,j-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{u}^{y}_{i+\frac{1}{2}+\frac{1}{2},j} + \bar{u}^{y}_{i+\frac{1}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(\bar{u}^{y}_{i-\frac{1}{2}+\frac{1}{2},j} + \bar{u}^{y}_{i-\frac{1}{2}-\frac{1}{2},j}\right)\right)\right)$$

$$+ H\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{v}^{x}_{i,j+\frac{1}{2}+\frac{1}{2}} + \bar{v}^{x}_{i,j+\frac{1}{2}-\frac{1}{2}}\right) - \frac{1}{2}\left(\bar{v}^{x}_{i,j-\frac{1}{2}+\frac{1}{2}} + \bar{v}^{x}_{i,j-\frac{1}{2}-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{u}^{y}_{i+1,j} + \bar{u}^{y}_{i,j}\right) - \frac{1}{2}\left(\bar{u}^{y}_{i,j} + \bar{u}^{y}_{i-1,j}\right)\right)\right)$$

$$+ H\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{v}^{x}_{i,j+1} + \bar{v}^{x}_{i,j}\right) - \frac{1}{2}\left(\bar{v}^{x}_{i,j} + \bar{v}^{x}_{i,j-1}\right)\right)\right)$$

$$= H\left(\frac{1}{2d'}\left(\left(\bar{u}^{y}_{i+1,j} - \bar{u}^{y}_{i-1,j}\right)\right)\right) + H\left(\frac{1}{2d'}\left(\left(\bar{v}^{x}_{i,j+1} - \bar{v}^{x}_{i,j-1}\right)\right)\right)$$

$$= H\left(\frac{1}{2d'}\left(\frac{1}{2}\left(u_{i+1,j+\frac{1}{2}} + u_{i+1,j-\frac{1}{2}}\right) - \frac{1}{2}\left(u_{i-1,j+\frac{1}{2}} + u_{i-1,j-\frac{1}{2}}\right)\right)\right)$$

$$+ H\left(\frac{1}{2d'}\left(\frac{1}{2}\left(v_{i+\frac{1}{2},j+1} + v_{i-\frac{1}{2},j+1}\right) - \frac{1}{2}\left(v_{i+\frac{1}{2},j-1} + v_{i-\frac{1}{2},j-1}\right)\right)\right)$$

$$= H\left(\frac{1}{4d'}\left(\left(u_{i+1,j+\frac{1}{2}} + u_{i+1,j-\frac{1}{2}}\right) - \left(u_{i-1,j+\frac{1}{2}} + u_{i-1,j-\frac{1}{2}}\right)\right)\right)$$

$$+ H\left(\frac{1}{4d'}\left(\left(v_{i+\frac{1}{2},j+1} + v_{i-\frac{1}{2},j+1}\right) - \left(v_{i+\frac{1}{2},j-1} + v_{i-\frac{1}{2},j-1}\right)\right)\right)$$

GRADE E

$$(\delta_x\alpha)_{i,j} \equiv \frac{1}{d'}\left(\alpha_{i+\frac{1}{2},j} - \alpha_{i-\frac{1}{2},j}\right)$$

$$(\bar{\alpha}^x)_{i,j} \equiv \frac{1}{2}\left(\alpha_{i+\frac{1}{2},j} + \alpha_{i-\frac{1}{2},j}\right)$$

$$\frac{\partial u}{\partial t} = -g\delta_x h + fv = -g\left(\frac{1}{d'}\left(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j}\right)\right) + fv_{i,j}$$

$$\frac{\partial v}{\partial t} = -g\delta_y h - fu = -g\left(\frac{1}{d'}\left(h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}}\right)\right) - fu_{i,j}$$

$$\frac{\partial h}{\partial t} = -H\left(\delta_x u + \delta_y v\right) = H\left(\left(\frac{1}{d'}\left(u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j}\right)\right) + \left(\frac{1}{d'}\left(v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}\right)\right)\right)$$
$$= H\left(\frac{1}{d'}\left(\left(u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j} + v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}\right)\right)\right)$$

Nos podemos primeiro analisar um caso unidimensional, que na qual as variáveis u,v,h não variam com y, Assim nos temos

$$u, v, h = u(x,t), v(x,t), h(x,t)$$

A equação 3.1 se reduz a

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv$$ $$\frac{\partial v}{\partial t} = -fu$$ $$\frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$$	3.3

Substituindo a 1,2

$$u(x,t) = Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial u}{\partial t} = -iv\, Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = (-iv)(-iv)Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = -v^2 Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$h(x,t) = Re\left[\hat{h}e^{-i(kx-vt)}\right]$$

$\frac{\partial^2 u}{\partial^2 t} = -g\frac{\partial}{\partial x}\frac{\partial h}{\partial t} + f\frac{\partial v}{\partial t}$ $\frac{\partial v}{\partial t} = -fu$ $\frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$	3.3

$\frac{\partial^2 u}{\partial^2 t} = -g\frac{\partial}{\partial x}\left(-H\frac{\partial u}{\partial x}\right) + f(-fu)$	3.3

$\frac{\partial^2 u}{\partial^2 t} = gH\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) - f^2 u$	3.3

$$u(x,t) = Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial u}{\partial t} = iv\, Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = (iv)(iv)Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = -v^2 Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial u}{\partial x} = -ik\, Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 x} = (-ik)(-ik)Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 x} = -k^2 Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$-v^2 Re\left[\hat{u}e^{-i(kx-vt)}\right] = -gHk^2 Re\left[\hat{u}e^{-i(kx-vt)}\right] - f^2 Re\left[\hat{u}e^{-i(kx-vt)}\right]$	3.3

$-v^2 = -gHk^2 - f^2$	3.3

$v^2 = gHk^2 + f^2$	3.3
$\left(\frac{v}{f}\right)^2 = \frac{gH}{f^2}k^2 + 1$	3.3

$\left(\frac{v}{f}\right)^2 = \frac{\lambda^2}{f}k^2 + 1$	3.3

Assim, o raio de deformação:

$$\lambda = \frac{\sqrt{gH}}{f}$$

Nunca é igual a zero, a frequência da onda de gravidade inercial é uma função de k monotonicamente crescente. Portanto a velocidade de grupo $\frac{\partial v}{\partial k}$ nunca será igual a zero.

Isto é muito importante para os processos de ajustamento geotrópico. pois impede uma acumação local de energia das ondas.

No focaremos no efeito de diferenciação finita no espaço . As variaveis são assumidas não dependente de y, o sistema 3.2 reduz a:

$$\frac{\partial u}{\partial t} = -g\overline{\delta_x h}^x + fv$$

$$\frac{\partial v}{\partial t} = -fu$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u}^x\right)$$

$$\frac{\partial u}{\partial t} = -g\overline{\delta_x h}^x + fv$$

$$\frac{\partial u}{\partial t} = -g\frac{1}{d'}\left(\overline{h}^x{}_{i+\frac{1}{2},j} - \overline{h}^x{}_{i-\frac{1}{2},j}\right) + fv$$

$$\frac{\partial u}{\partial t} = -g\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2}+\frac{1}{2},j} + h_{i+\frac{1}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(h_{i-\frac{1}{2}+\frac{1}{2},j} + h_{i-\frac{1}{2}-\frac{1}{2},j}\right)\right) + fv$$

$$\frac{\partial u}{\partial t} = -g\frac{1}{2d'}\left(\left(h_{i+1,j} + h_{i,j}\right) - \left(h_{i,j} + h_{i-1,j}\right)\right) + fv$$

$$\frac{\partial u}{\partial t} = -g\frac{1}{2d'}\left(\left(h_{i+1,j} - h_{i-1,j}\right)\right) + fv$$

$$\frac{\partial^2 u}{\partial^2 t} = -g\frac{1}{2d'}\left(\left(\frac{\partial h_{i+1,j}}{\partial t} - \frac{\partial h_{i-1,j}}{\partial t}\right)\right) + f\frac{\partial v}{\partial t}$$

$$\frac{\partial^2 u}{\partial^2 t} = -g\frac{1}{2d'}\left(\left(\frac{\partial h_{i+1,j}}{\partial t} - \frac{\partial h_{i-1,j}}{\partial t}\right)\right) + f\frac{\partial v}{\partial t}$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u^x}\right)_{i+1,j} = -H\frac{1}{d'}\left(\bar{u}^x{}_{i+1+\frac{1}{2},j} - \bar{u}^x{}_{i+1-\frac{1}{2},j}\right) = -H\frac{1}{d'}\left(\bar{u}^x{}_{i+\frac{3}{2},j} - \bar{u}^x{}_{i+\frac{1}{2},j}\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u^x}\right)_{i+1,j} = -H\frac{1}{d'}\left(\frac{1}{2}\left(u^x{}_{i+\frac{3}{2}+\frac{1}{2},j} + u^x{}_{i+\frac{3}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(u^x{}_{i+\frac{1}{2}+\frac{1}{2},j} + u^x{}_{i+\frac{1}{2}-\frac{1}{2},j}\right)\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u^x}\right)_{i+1,j} = -H\frac{1}{2d'}\left(\left(u^x{}_{i+2,j} + u^x{}_{i+1,j}\right) - \left(u^x{}_{i+1,j} + u^x{}_{i,j}\right)\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u^x}\right)_{i+1,j} = -H\frac{1}{2d'}\left(\left(u^x{}_{i+2,j} - u^x{}_{i,j}\right)\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u^x}\right)_{i-1,j} = -H\frac{1}{d'}\left(\bar{u}^x{}_{i-1+\frac{1}{2},j} - \bar{u}^x{}_{i-1-\frac{1}{2},j}\right) = --H\frac{1}{d'}\left(\bar{u}^x{}_{i-\frac{1}{2},j} - \bar{u}^x{}_{i-\frac{3}{2},j}\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u^x}\right)_{i-1,j} = -H\frac{1}{d'}\left(\frac{1}{2}\left(u^x{}_{i-\frac{1}{2}+\frac{1}{2},j} + u^x{}_{i-\frac{1}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(u^x{}_{i-\frac{3}{2}+\frac{1}{2},j} + u^x{}_{i-\frac{3}{2}-\frac{1}{2},j}\right)\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u^x}\right)_{i-1,j} = -H\frac{1}{d'}\left(\frac{1}{2}\left(u^x{}_{i,j} + u^x{}_{i-1,j}\right) - \frac{1}{2}\left(u^x{}_{i-1,j} + u^x{}_{i-2,j}\right)\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u^x}\right)_{i+1,j} = -H\frac{1}{2d'}\left(\left(u^x{}_{i+2,j} - u^x{}_{i,j}\right)\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u^x}\right)_{i-1,j} = -H\frac{1}{2d'}\left(\left(u^x{}_{i,j} - u^x{}_{i-2,j}\right)\right)$$

$$\frac{\partial^2 u}{\partial^2 t} = -g\frac{1}{2d'}\left(\left(\frac{\partial h_{i+1,j}}{\partial t} - \frac{\partial h_{i-1,j}}{\partial t}\right)\right) + f\frac{\partial v}{\partial t}$$

$$\frac{\partial^2 u}{\partial^2 t} = -g\frac{1}{2d'}\left(\left(-H\frac{1}{2d'}\left(\left(u^x{}_{i+2,j} - u^x{}_{i,j}\right)\right) - \left(-H\frac{1}{2d'}\left(\left(u^x{}_{i,j} - u^x{}_{i-2,j}\right)\right)\right)\right)\right) + f\frac{\partial v}{\partial t}$$

$$\frac{\partial^2 u}{\partial^2 t} = -g\frac{1}{2d'}\left(\left(-H\frac{1}{2d'}\left(\left(u^x{}_{i+2,j} - u^x{}_{i,j}\right)\right) + H\frac{1}{2d'}\left(\left(u^x{}_{i,j} - u^x{}_{i-2,j}\right)\right)\right)\right) + f\frac{\partial v}{\partial t}$$

$$\frac{\partial^2 u}{\partial^2 t} = -\frac{gH}{4d'^2}\left(\left(-\left(\left(u^x{}_{i+2,j} - u^x{}_{i,j}\right)\right) + \left(\left(u^x{}_{i,j} - u^x{}_{i-2,j}\right)\right)\right)\right) + f\frac{\partial v}{\partial t}$$

$$\frac{\partial^2 u}{\partial^2 t} = -\frac{gH}{4d'^2}\left(\left(-\left(\left(u^x{}_{i+2,j} - u^x{}_{i,j}\right)\right) + \left(\left(u^x{}_{i,j} - u^x{}_{i-2,j}\right)\right)\right)\right) + f\frac{\partial v}{\partial t}$$

$$\frac{\partial^2 u}{\partial^2 t} = -\frac{gH}{4d'^2}\left(\left(-\left(\left(u^x{}_{i+2,j} - u^x{}_{i,j}\right)\right) + \left(\left(u^x{}_{i,j} - u^x{}_{i-2,j}\right)\right)\right)\right) + f\frac{\partial v}{\partial t}$$

$$\frac{\partial^2 u}{\partial^2 t} = \frac{gH}{4d'^2}\left(\left(\left(\left(u^x{}_{i+2,j} - u^x{}_{i,j}\right)\right) - \left(\left(u^x{}_{i,j} - u^x{}_{i-2,j}\right)\right)\right)\right) + f\frac{\partial v}{\partial t}$$

$$\frac{\partial^2 u}{\partial^2 t} = \frac{gH}{4d'^2}\left(\left(\left(u^x{}_{i+2,j} + u^x{}_{i-2,j}\right) - \left(2u^x{}_{i,j}\right)\right)\right) + f\frac{\partial v}{\partial t}$$

$$\frac{\partial^2 u}{\partial^2 t} = \frac{gH}{4d'^2}\left(\left(\left(u^x{}_{i+2,j} + u^x{}_{i-2,j}\right) - \left(2u^x{}_{i,j}\right)\right)\right) - f^2 u$$

$$(\delta_x \alpha)_{i,j} \equiv \frac{1}{d'}\left(\alpha_{i+\frac{1}{2},j} - \alpha_{i-\frac{1}{2},j}\right)$$

$$(\delta_x \delta_x \alpha)_{i,j} \equiv \frac{1}{d'^2}\left(\alpha_{i+1,j} + \alpha_{i-1,j} - 2\alpha_{i,j}\right)$$

$$(\bar{\alpha}^x)_{i,j} \equiv \frac{1}{2}\left(\alpha_{i+\frac{1}{2},j} + \alpha_{i-\frac{1}{2},j}\right)$$

$$u(x,t) = Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$h(x,t) = Re\left[\hat{h}e^{-i(kx-vt)}\right]$$

$$x = j\Delta x$$

$$t = m\Delta t$$

$$u(x,t) = Re\left[\hat{u}e^{-i\left(k(j\Delta x)-v(m\Delta t)\right)}\right]$$

$$h(x,t) = Re\left[\hat{h}e^{-i\left(k(j\Delta x)-v(m\Delta t)\right)}\right]$$

$$\frac{\partial u}{\partial t} = -i(-v)Re\left[\hat{u}e^{-i\left(k(j\Delta x)-v(m\Delta t)\right)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = -i(-mv)(-i(-mv))Re\left[\hat{u}e^{-i\left(k(j\Delta x)-v(m\Delta t)\right)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = (imv)(imv)Re\left[\hat{u}e^{-i\left(k(j\Delta x)-v(m\Delta t)\right)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = -(mv)^2 Re\left[\hat{u}e^{-i\left(k(j\Delta x)-v(m\Delta t)\right)}\right]$$

$$\frac{\partial^2 u(j=n, m=1)}{\partial^2 t} = -(v)^2 Re\left[\hat{u}e^{-i(k(n)\Delta x - v\Delta t)}\right]$$

$$\frac{\partial^2 u(j=n, m=1)}{\partial^2 t} = -(v)^2 Re\left[\hat{u}\left(e^{-i(nk\Delta x - v\Delta t)}\right)\right]$$

$$u(j=n, m=1) = Re\left[\hat{u}e^{-i\left(kn\Delta x - v(\Delta t)\right)}\right]$$

$$u(j=n+2, m=1) = Re\left[\hat{u}e^{-i\left(k(n+2)\Delta x - v(\Delta t)\right)}\right] = Re\left[\hat{u}e^{-i\left(nk\Delta x + 2k\Delta x - v(\Delta t)\right)}\right]$$
$$= Re\left[\hat{u}e^{-i(2k\Delta x)}e^{-i(nk\Delta x - v\Delta t)}\right]$$

$$u(j=n+2, m=1) = Re\left[\hat{u}\left(e^{-i(2k\Delta x)}e^{-i(nk\Delta x - v\Delta t)}\right)\right]$$

$$u(j=n-2, m=1) = Re\left[\hat{u}e^{-i\left(k(n-2)\Delta x - v(\Delta t)\right)}\right] = Re\left[\hat{u}e^{-i\left(nk\Delta x - 2k\Delta x - v(\Delta t)\right)}\right]$$
$$= Re\left[\hat{u}e^{i(2k\Delta x)}e^{-i(nk\Delta x - v\Delta t)}\right]$$

$$u(j=n+2, m=1) = Re\left[\hat{u}\left(e^{i(2k\Delta x)}e^{-i(nk\Delta x - v\Delta t)}\right)\right]$$

$$-(\upsilon)^2 Re\left[\hat{u}\left(e^{-i(nk\Delta x-\upsilon\Delta t)}\right)\right]$$
$$= \frac{gH}{4d'^2}\left(\left(\left(Re\left[\hat{u}\left(e^{-i(2k\Delta x)}e^{-i(nk\Delta x-\upsilon\Delta t)}\right)\right] + Re\left[\hat{u}\left(e^{i(2k\Delta x)}e^{-i(nk\Delta x-\upsilon\Delta t)}\right)\right]\right)\right.\right.$$
$$\left.\left. - \left(2Re\left[\hat{u}e^{-i\left(kn\Delta x-\upsilon(\Delta t)\right)}\right]\right)\right)\right) - f^2 Re\left[\hat{u}e^{-i\left(kn\Delta x-\upsilon(\Delta t)\right)}\right]$$

$$-(\upsilon)^2 = \frac{gH}{4d'^2}\left(\left(\left(\left[\left(e^{-i(2k\Delta x)}\right)\right] + \left[\left(e^{i(2k\Delta x)}\right)\right]\right) - (2)\right)\right) - f^2$$

$$(\upsilon)^2 = f^2 - \frac{gH}{4d'^2}\left(\left(\left(\left[\left(e^{-i(2k\Delta x)}\right)\right] + \left[\left(e^{i(2k\Delta x)}\right)\right]\right) - (2)\right)\right)$$

$$\left(\frac{\upsilon}{f}\right)^2 = 1 - \frac{gH}{4f^2d'^2}\left(\left(\left(\left[\left(e^{i(2k\Delta x)} + e^{-i(2k\Delta x)}\right)\right]\right) - (2)\right)\right)$$

$$\left(\frac{\upsilon}{f}\right)^2 = 1 - \frac{gH}{2f^2d'^2}\left(\left(\left(\frac{e^{i(2k\Delta x)} + e^{-i(2k\Delta x)}}{2}\right) - 1\right)\right)$$

$$\left(\frac{\upsilon}{f}\right)^2 = 1 - \frac{gH}{2f^2d'^2}\left(\left(\cos(2k\Delta x) - 1\right)\right)$$

$$\left(\frac{\upsilon}{f}\right)^2 = 1 + \frac{gH}{2f^2d'^2}\left(1 - \cos(2k\Delta x)\right)$$

$$\left(\frac{\upsilon}{f}\right)^2 = 1 + \frac{gH}{f^2d'^2}\frac{1}{2}\left(1 - \cos(2k\Delta x)\right)$$

$$\left(\frac{\upsilon}{f}\right)^2 = 1 + \frac{gH}{f^2d'^2}\sin^2(k\Delta x)$$

Assim, o raio de deformação :

$$\lambda = \frac{\sqrt{gH}}{f}$$

$$\left(\frac{\upsilon}{f}\right)^2 = 1 + \frac{\lambda^2}{d'^2}\sin^2(k\Delta x)$$

$$\left(\frac{v}{\lambda f}\right)^2 = \frac{1}{\lambda^2} + \frac{1}{d'^2}\sin^2(k\Delta x)$$

$$\left(\frac{v}{\lambda f}\right)^2 - \frac{1}{\lambda^2} = +\frac{1}{d'^2}\sin^2(k\Delta x)$$

$$(v)^2\frac{1}{f^2\lambda^2} - \frac{1}{\lambda^2} = +\frac{1}{d'^2}\sin^2(k\Delta x)$$

$$\frac{v^2 - f^2}{f^2\lambda^2} = +\frac{1}{d'^2}\sin^2(k\Delta x)$$

$\cos x = Re\left(e^{-ix}\right) = \frac{e^{ix}+e^{-ix}}{2}$	$\sin x = Im\left(e^{-ix}\right) = \frac{e^{ix}-e^{-ix}}{2i}$
$\sin^2 x + \cos^2 x = 1$	$\sin^2 x = \frac{1}{2}(1-\cos(2x))$

################3