

LECTURE NOTES OF WILLIAM CHEN

LINEAR ALGEBRA

This set of notes has been compiled over a period of more than 30 years. Some chapters were used in various forms and on many occasions between 1981 and 1990 by the author at Imperial College, University of London. The remaining chapters were written in Sydney.

The material has been organized in such a way to create a single volume suitable to take the reader to a reasonable level of linear algebra. Chapters 1 - 4 cover very basic material. The concept of vector spaces is then introduced in Chapters 5 - 7. More advanced topics, including the concept of linear transformations from one vector space to another and the concept of inner products, are covered in Chapters 8 - 12.

To read the notes, click the links below for connection to the appropriate PDF files.

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SECTION A --- ELEMENTARY TOPICS

Chapter 1 : LINEAR EQUATIONS >>

- Introduction
- Elementary Row Operations
- Row Echelon Form
- Reduced Row Echelon Form
- Solving a System of Linear Equations
- Homogeneous Systems
- Application to Network Flow
- Application to Electrical Networks
- Application to Economics
- Application to Chemistry
- Application to Mechanics

Chapter 2 : MATRICES >>

- Introduction
- Systems of Linear Equations
- Inversion of Matrices
- Application to Matrix Multiplication
- Finding Inverses by Elementary Row Operations
- Criteria for Invertibility
- Consequences of Invertibility
- Application to Economics
- Matrix Transformation on the Plane
- Application to Computer Graphics
- Complexity of a Non-Homogeneous System
- Matrix Factorization
- Application to Games of Strategy

Chapter 3 : DETERMINANTS >>

- Introduction
- Determinants for Squares Matrices of Higher Order

- Some Simple Observations
- Elementary Row Operations
- Further Properties of Determinants
- Application to Curves and Surfaces
- Some Useful Formulas
- Further Discussion

Chapter 4 : VECTORS >>

- Introduction
- Vectors in 2-Space
- Vectors in 3-Space
- Vector Products
- Scalar Triple Products
- Application to Geometry in 3-Space
- Application to Mechanics

SECTION B --- INTRODUCTION TO SOME ALGEBRAIC STRUCTURES

Chapter 5 : INTRODUCTION TO VECTOR SPACES >>

- Real Vector Spaces
- Subspaces
- Linear Combination
- Linear Independence
- Basis and Dimension

Chapter 6 : VECTOR SPACES ASSOCIATED WITH MATRICES >>

- Introduction
- Row Spaces
- Column Spaces
- Rank of a Matrix
- Nullspaces
- Solution of Non-Homogeneous Systems

Chapter 7 : EIGENVALUES AND EIGENVECTORS >>

- Introduction
- The Diagonalization Problem
- Some Remarks
- An Application to Genetics

SECTION C --- FURTHER TOPICS

Chapter 8 : LINEAR TRANSFORMATIONS >>

- Euclidean Linear Transformations
- Linear Operators on the Plane
- Elementary Properties of Euclidean Linear Transformations
- General Linear Transformations
- Change of Basis
- Kernel and Range
- Inverse Linear Transformations
- Matrices of General Linear Transformations
- Change of Basis
- Eigenvalues and Eigenvectors

Chapter 9 : REAL INNER PRODUCT SPACES >>

- Euclidean Inner Products

- Real Inner Products
- Angles and Orthogonality
- Orthogonal and Orthonormal Bases
- Orthogonal Projections

Chapter 10 : ORTHOGONAL MATRICES >>

- Introduction
- Eigenvalues and Eigenvectors
- Orthonormal Diagonalization

Chapter 11 : APPLICATIONS OF REAL INNER PRODUCT SPACES >>

- Least Squares Approximation
- Quadratic Forms
- Real Fourier Series

Chapter 12 : COMPLEX VECTOR SPACES >>

- Complex Inner Products
- Unitary Matrices
- Unitary Diagonalization