

Manual exec.: BFS, DFS, Ford's Alg., Dijkstra (+reverse), A*, Prim's Alg.,

DAG topo. sort by predecessor count, DAG topo. sort by DFS,
Floyd-Warshall, Ford-Fulkerson, Kruskal, Vertex cover

BFS - breadth-first search

- iterative w/ queue (FIFO)
- check if 7 paths
- same complexity
- add start node to queue →
- pull a node → process if not seen →
- add each unseen adjacent node to queue

DFS - depth-first search

- on a stack (LIFO) can use recursion
- complete search, finding all paths, ...
- Time: $O(V + E)$; Space: $O(V)$
- add start node
- ✓ Pull node from stack → if not seen:
- add to seen & process → add its children
- on the STACK (if unseen)

Ford's Algorithm

(Bellman-Ford) - shortest path from a source vertex to all the other vertices
- $V-1$ iterations (V is the number of vertices)

at start:

| S | V_1 | V_2 | ... |
|---|----------|----------|-----|
| 0 | ∞ | ∞ | |

Starting vertex

1st iteration

| | | | | | |
|---|----|----------|----------|----------|---|
| 0 | 10 | ∞ | ∞ | ∞ | 8 |
| S | A | B | C | D | E |

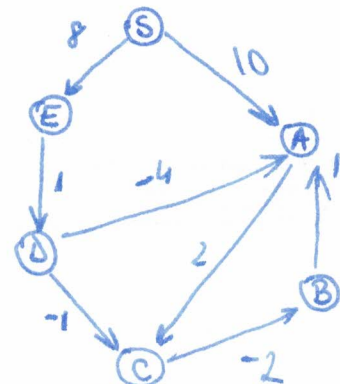
curr. node

| | | | | | |
|---|----|----------|----|----------|---|
| 0 | 10 | ∞ | 12 | ∞ | 8 |
| S | A | B | C | D | E |

| | | | | | |
|---|----|----|----|----------|---|
| 0 | 10 | 10 | 12 | ∞ | 8 |
| S | A | B | C | D | E |

| | | | | | |
|---|----|----|----|---|---|
| 0 | 10 | 10 | 12 | 9 | 8 |
| S | A | B | C | D | E |

- we then take node A and see we can reach to node C
- we take B, we don't know how to reach it so we skip
- we can get to B from C
- we skip D
- we get to D from E



(final: S A B C D E)
0 5 5 7 9 8

... iterate a total of $V-1$ times & update only when we get a shorter path

ex: Iteration 2, when reaching D, the paths to A & C change

| | | | | | |
|---|---|----|---|---|---|
| 0 | 5 | 10 | 8 | 9 | 8 |
| S | A | B | C | D | E |

!! if no values change during an iteration, the execution can be stopped

Dijkstra's Algorithm - shortest path between two vertices the starting vertex to all other vertices

(outbounds)

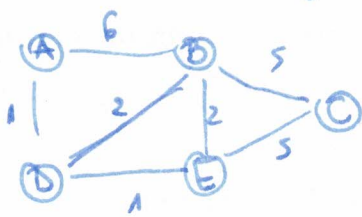
- take the neighbours of the starting vertex & compute the distances; update the values if they are smaller than the ones in the table
- mark the current node as 'visited'
- visit the vertex with the smallest known distance from the previous one
- re-calculate the distances from the starting vertex & update if necessary

starting vertex

| | |
|----------------|-----|
| S | → 0 |
| V ₁ | → ∞ |
| ... | |
| V _m | → ∞ |

initial table

ex: A - starting vertex



① A → D = 1
A → B = 6

| | |
|---|---|
| A | 0 |
| B | 6 |
| C | ∞ |
| D | 1 |
| E | ∞ |

A → visited

② we take vertex D

→ B: 1 + 2 = 3 < 6 - UPDATE

→ E: 1 + 1 = 2 < ∞ - UPDATE

| | |
|---|---|
| A | 0 |
| B | 3 |
| C | ∞ |
| D | 1 |
| E | 2 |

D → visited

we take E ③

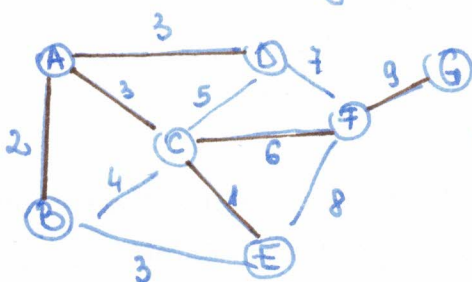
A* Algorithm - path finding & graph traversals

- an extended (faster) Dijkstra that also depends on a heuristic h (take the vertices based on the cost + h)

Prim's Algorithm - for finding a Minimum Spanning Tree (MST)

MST - min. weight connected graph with no cycles

- add starting vertex to 'visited'
- choose the smallest edge that connects to an unvisited node (mark that node as 'visited')
- take the smallest edge from the 'visited' nodes



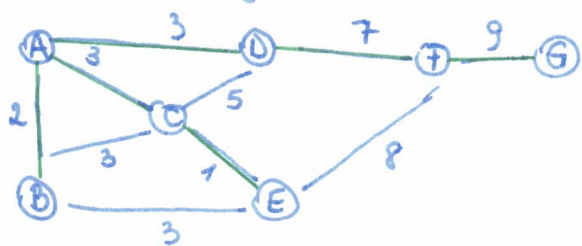
1) A - start, A - visited

2) Take [AB], B - visited

3) we can take [AD], [AC], [BE] (doesn't matter)

...

Kruskal's Algorithm \Rightarrow results in a MST



- \rightarrow pick the smallest edge: [EC]
- \rightarrow look for the smallest edge that doesn't create a cycle: [AB]
- \rightarrow choose any from [AD], [AC], [BE] (for ex: [AD])
- \rightarrow pick [AC] next
- \rightarrow picking them [BE] would result in a cycle
(nodes are alr. in the tree)
- \rightarrow next is [DF] & [FG]

DAG - Directed Acyclic Graph (dir. graph w/ no cycles)

Topological sorting using the predecessor counting algorithm

- \rightarrow we take a vertex w/ no predecessors, we put it on the sorted list & we eliminate it from the graph, and continue the same way
- \rightarrow ~~vertex not~~ if we cannot get a vertex w/ no predecessors \Rightarrow we have a cycle

DFS-Based topological sorting (?)

Floyd-Warshall Algorithm - find shortest path distance between every pair of vertices in a given directed graph

- \rightarrow create a $V \times V$ matrix initially having ∞ (a matrix \leftarrow dist)
(dist[i][i] = 0)

- \rightarrow we put the edges first (dist[u][v] = cost(u, v))

- \rightarrow

| | | |
|---|------------|--|
| for k from 1 to V for i from 1 to V for j from 1 to V | \nearrow | if dist[i][j] > dist[i][k] + dist[k][j] (replace the value) |
|---|------------|--|