Optimal Training Signals for MIMO OFDM Channel Estimation

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***Abstract*— This paper presents general classes of optimal training signals for the estimation of frequency-selective channels in MIMO OFDM systems. Basic properties of the discrete Fourier transform are used to derive the optimal training signals which minimize the channel estimation mean square error. Both single and multiple OFDM training symbols are considered. Several optimal pilot tone allocations across the transmit antennas are presented and classified as frequency-division multiplexing, time-division multiplexing, code-division multiplexing in the frequency-domain, code-division multiplexing in the time-domain, and combinations thereof. All existing optimal training signals in the literature are special cases of the presented optimal training signals and our designs can be applied to pilot-only schemes as well as pilot-data-multiplexed schemes.**

***Index Terms*— Training signal design, pilot tone allocation, channel estimation, MIMO, OFDM, DFT.**

I. Introduction

CHANNEL estimation is a critical component in many wireless communications systems. Training-signal-based channel estimation is widely used in packet-based communications. For single-carrier systems, optimal periodic or aperiodic sequences for channel estimation were studied in [1]-[6] and references therein. The optimal training sequences and pilot tones for orthogonal frequency division multiplexing (OFDM) channel estimation were investigated in [7][8]. Optimal placement and energy allocation of training symbols or pilot tones for both single-carrier and OFDM systems were considered in [9] for frequency-selective block-fading channel estimation. The training signal placement design is based on maximizing a lower bound on the training-based capacity with the assumption that all training symbols or pilot tones have the same energy. For OFDM systems, the optimal placement of pilot tones is equal spacing in the frequency domain. In [10], optimal design and placement of pilot symbols for frequency-selective block-fading channel estimation are addressed for single-input single-output (SISO) as well as multiple-input multiple-output (MIMO) single-carrier systems by minimizing the Cramer-Rao bound. The same problem was addressed in [11] by maximizing a lower bound on the average capacity.

In [12], optimal training signal design and power allocation for frequency-selective block-fading channel estimation in linearly-precoded OFDM systems (which include OFDM systems as well as single-carrier systems with cyclic prefix) were presented where it was shown that the L pilot tones (which is the minimum required to estimate an L-tap channel) are equi-powered and equi-spaced. For doubly-selective fading channels characterized by the basic expansion model, an optimal training structure was presented in [13] for SISO single-carrier systems by maximizing a lower bound on the average channel capacity (equivalently minimizing the minimum mean square error). In [14], MIMO training signal design for single-carrier systems was reduced into a SISO design with a longer training sequence using the space-time code structure. Furthermore, some sample training sequence constructions were presented.

In [15], optimal training signal design for frequency-selective block-fading channel estimation in MIMO OFDM systems was analyzed based on minimizing channel estimation mean square error (MSE). The optimal pilot tones for channel estimation based on one OFDM symbol were shown to be equi-powered and equi-spaced. Furthermore, pilot tones from different antennas must be phase-shift orthogonal. For channel estimation based on Q OFDM symbols, the conditions on pilot tones for the case of one OFDM symbol are just spread out over the Q symbols. Note that [16] also presented an optimal training signal design for MIMO OFDM systems where all sub-carriers are used as pilot tones with equal power and pilot tones from different antennas are phase-shift orthogonal. A similar design with BPSK pilot symbols (a phase-shift of $\pm\pi$ among pilot tones of different antennas) was used in [17] for two transmit antennas and later extended to more transmit antennas in [18].

In this paper, we revisit the problem of optimal training signal design for frequency-selective block-fading channel estimation in MIMO OFDM systems and present more general optimal training signals based on minimizing the channel estimation MSE. While we derive our results assuming all sub-carriers are used as pilot tones over the Q OFDM symbols, the corresponding results for pilot-data-multiplexed schemes can be obtained in a straight-forward manner (see Section V). A brief comparison with the existing approaches in the literature is now in order. Our approach is novel in that it is based on the discrete Fourier transform properties and could be useful for training signal designs with more constraints such as low peak-to-average energy ratio (PAR) and robustness to frequency offsets. In addition, all existing optimal OFDM training designs can be expressed as special cases of our design. Furthermore, our design introduces new optimal pilot structures and offers more insights as described next (see Section V for details). First, some of our new pilot structures can save training overhead (in terms of the numbers of pilot

Manuscript received May 11, 2004; revised December 7, 2004; accepted April 8, 2005. The associate editor coordinating the review of this paper and approving it for publication was C. Tellambura. This paper was presented in part at IEEE Globecom 2004. The work of H. Minn was supported in part by the Erik Jonsson School Research Excellence Initiative, the University of Texas at Dallas. The work of N. Al-Dhahir was supported in part by the Texas Advanced Technology Program (ATP) project no. 009741-0023-2003 and by NSF project no. 0430654.

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Digital Object Identifier 10.1109/TWC.2006.05024.

tones) for some system parameters and/or support a larger number of transmit antennas. Second, our design offers more flexibility in terms of applicability to different systems such ultra-wide band (UWB) systems. Third, some of our new pilot structures can give better channel estimation performance in the presence of PAR constraint and/or frequency offsets. Finally, our design shows that the condition "equi-powered, equi-spaced pilot tones", which has been considered in the literature as a necessary condition for the training signal optimality, can be relaxed.

The rest of this paper is organized as follows. In Section II, the signal model and the optimality condition for the training signals of multi-transmit-antenna systems are described. Section III presents optimal training signal designs over one OFDM symbol while Section IV generalizes them to the case of Q OFDM symbols. In Section V, the applicability of our designs to pilot-data-multiplexed scenarios and their relationships to the existing optimal training signal designs are discussed. Finally, the paper is concluded in Section VI.

II. Signal Model and Optimality Conditions

Consider a MIMO OFDM system where training signals from N_{Tx} transmit antennas are transmitted over Q OFDM symbols. Since the same channel estimation procedure is performed at each receive antenna, we only need to consider N_{Tx} transmit antennas and one receive antenna in designing optimal training signals. The channel impulse response (CIR) for each transmit-receive antenna pair (including all transmit/receive filtering effects) is assumed to have L taps and is quasi-static over Q OFDM symbols. Let $\boldsymbol{C}_{n,q}=[c_{n,q}[0], \ldots, c_{n,q}[K-1]]^T$ be the pilot tones vector of the n-th transmit-antenna at the q-th symbol interval where K is the number of OFDM sub-carriers and the superscript T denotes the transpose. Furthermore, let $\{s_{n,q}[k] : k = -N_g, \ldots, K-1\}$ be the corresponding time-domain complex baseband training samples, including N_g $(\geq L-1)$ cyclic prefix samples. Define $\boldsymbol{S}_n[q]$ as the training signal matrix of size $K \times L$ for the n-th transmit antenna at the q-th symbol interval whose elements are given by $[\boldsymbol{S}_n[q]]_{m,l} = s_{n,q}[m-l]$ for $m \in \{0, \ldots, K-1\}$ and $l \in \{0, \ldots, L-1\}$.

Let $\boldsymbol{s}_{n,q}$ represent the 0-th column of $\boldsymbol{S}_n[q]$. Then, the l-th column of $\boldsymbol{S}_n[q]$ is the l-sample cyclically-shifted version of $\boldsymbol{s}_{n,q}$ denoted by $\boldsymbol{s}_{n,q}^{((l))}$. Let $\boldsymbol{h}_n$ denote the length-L CIR vector corresponding to the n-th transmit antenna. After cyclic prefix removal at the receiver, denote the received vector of length K at the q-th symbol interval by $\boldsymbol{r}_q$. Then, the received vector over the Q OFDM symbol intervals is given by

$$\boldsymbol{r} = \boldsymbol{S}\boldsymbol{h} + \boldsymbol{n} \tag{1}$$

where

$$\boldsymbol{r} = [\boldsymbol{r}_0^T \; \boldsymbol{r}_1^T \; \ldots, \boldsymbol{r}_{Q-1}^T]^T \tag{2}$$

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_0[0] & \boldsymbol{S}_1[0] & \ldots & \boldsymbol{S}_{N_{\mathrm{Tx}}-1}[0] \\ \boldsymbol{S}_0[1] & \boldsymbol{S}_1[1] & \ldots & \boldsymbol{S}_{N_{\mathrm{Tx}}-1}[1] \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{S}_0[Q-1] & \boldsymbol{S}_1[Q-1] & \ldots & \boldsymbol{S}_{N_{\mathrm{Tx}}-1}[Q-1] \end{bmatrix} \tag{3}$$

$$\boldsymbol{h} = [\boldsymbol{h}_0^T \boldsymbol{h}_1^T \ldots \boldsymbol{h}_{N_{\mathrm{Tx}}-1}^T]^T, \tag{4}$$

and $\boldsymbol{n}$ is a length-KQ vector of zero-mean, circularly-symmetric, uncorrelated complex Gaussian noise samples with equal variance of σ_n^2.

The least-square channel estimate (also maximum likelihood in this case), assuming $\boldsymbol{S}^H\boldsymbol{S}$ has full rank, is given by [19]

$$\hat{\boldsymbol{h}} = (\boldsymbol{S}^H\boldsymbol{S})^{-1}\boldsymbol{S}^H\boldsymbol{r} \tag{5}$$

and the corresponding MSE is given by $\sigma_n^2 \, tr\{(\boldsymbol{S}^H\boldsymbol{S})^{-1}\}$. Let $\lambda_1, \ldots, \lambda_{LN_{\mathrm{Tx}}}$ be the eigen values (positive) of $\boldsymbol{S}^H\boldsymbol{S}$. Then, $tr\{(\boldsymbol{S}^H\boldsymbol{S})^{-1}\} = \lambda_1^{-1} + \ldots + \lambda_{LN_{\mathrm{Tx}}}^{-1}$. Since $tr\{(\boldsymbol{S}^H\boldsymbol{S})\} = \lambda_1 + \ldots + \lambda_{LN_{\mathrm{Tx}}} = L\sum_{m=0}^{N_{\mathrm{Tx}}-1} E_m$ is a constant, the minimum MSE is achieved if and only if $\lambda_1 = \ldots = \lambda_{LN_{\mathrm{Tx}}} = \frac{1}{N_{\mathrm{Tx}}}\sum_{m=0}^{N_{\mathrm{Tx}}-1} E_m = E_{\mathrm{av}}$. This is achieved when

$$\boldsymbol{S}^H\boldsymbol{S} = E_{\mathrm{av}}\boldsymbol{I} \tag{6}$$

$$\text{where} \quad E_{\mathrm{av}} = \frac{1}{N_{\mathrm{Tx}}}\sum_{m=0}^{N_{\mathrm{Tx}}-1} E_m \tag{7}$$

$$\text{and} \quad E_m = \sum_{q=0}^{Q-1}\sum_{k=0}^{K-1} |s_{m,q}[k]|^2. \tag{8}$$

The corresponding minimum MSE is $LN_{\mathrm{Tx}}\sigma_n^2/E_{\mathrm{av}}$. We will design training signals for N_{Tx} transmit antennas to achieve this minimum MSE, i.e. to satisfy Condition (6). In this paper, optimal training signals refer to those which achieve the minimum MSE. Condition (6) can be equivalently stated as

$$Condition-A: \; \sum_{q=0}^{Q-1} \boldsymbol{S}_m^H[q]\boldsymbol{S}_m[q] = E_{\mathrm{av}}\boldsymbol{I}, \; \forall m \tag{9}$$

$$Condition-B: \; \sum_{q=0}^{Q-1} \boldsymbol{S}_m^H[q]\boldsymbol{S}_n[q] = \boldsymbol{0}, \; \forall m \neq n. \tag{10}$$

III. Optimal Training Signal Design over One OFDM Symbol

This section investigates optimal training signal design when training signals from all transmit-antennas are transmitted over only one OFDM symbol. For notational simplicity, the symbol index q will be omitted in this section. For completeness, in the following we summarize the main DFT properties used in this paper. Let $X[n] = \sum_{k=0}^{K-1} x[k]e^{-j2\pi kn/K}$ and $x[k] = \frac{1}{K}\sum_{n=0}^{K-1} X[n]e^{j2\pi kn/K}$, i.e. $X[n] \stackrel{\mathcal{F}}{\longleftrightarrow} x[k]$.

Property-1: For any K, if $X[n] = a \; \forall n$, where $a \in \mathbb{C}$ and $\mathbb{C}$ is the field of complex numbers, then $x[k] = a\delta[k]$ where $\delta[k]$ is a discrete unit impulse function and vice versa.

Property-2: Assume that $K = ML_1$ for M=1, 2, ...

$$\text{If} \quad X[n] = \begin{cases} a, & n = iM; i = 0, \ldots, L_1 - 1; a \in \mathbb{C} \\ 0, & \text{elsewhere,} \end{cases} \tag{11}$$

$$\text{then} \quad x[k] = \begin{cases} aL_1/K, & k = iL_1; i = 0, \ldots, M-1 \\ 0, & \text{elsewhere,} \end{cases} \tag{12}$$

and vice versa.

Property-3: $X[(n-l)_K] \stackrel{\mathcal{F}}{\longleftrightarrow} e^{j2\pi lk/K}x[k]$ where $(\cdot)_K$ denotes the modulo-K operation, hence representing a cyclically-shifted version. Its dual form is given by $x[(k-m)_K] \stackrel{\mathcal{F}}{\longleftrightarrow} e^{-j2\pi mn/K}X[n]$.

In the following, our discussions based on Condition-A will be indexed as (A-i), (A-ii), etc., and those based on Condition-B will be denoted as (B-i), (B-ii), etc.

A. Optimality Conditions in Sub-Carrier Domain

Consider Condition-A in (9) with $Q = 1$. The following conditions can be derived.

(A-i) *Condition(A.1)*: The full rank condition in (9) implies that for every transmit antenna i, there must be at least L different nonzero tones.

(A-ii) The condition $(\boldsymbol{s}_m^{((l))})^H \boldsymbol{s}_m^{((l))} = E_{\rm av}$ from (9) implies that

$$Condition(A.2): \quad E_m = E_{\rm av} \ \ \forall m. \tag{13}$$

(A-iii) Consider the following condition from (9):

$$(\boldsymbol{s}_m^{((l))})^H \boldsymbol{s}_m^{((i))} = 0 \ \forall l \neq i; \text{ where } l, i \in \{0, \ldots, L-1\}. \tag{14}$$

Since $\boldsymbol{s}_m^{((l))} = F_K^{-1} W(l) \boldsymbol{C}_m$ where F_K is the K-point FFT matrix and $W(l)$=diag $\{1, e^{-j2\pi l/K}, \ldots, e^{-j2\pi (K-1)l/K}\}$ is a diagonal matrix, the following condition is obtained from (14):

$$Condition(A.3): \quad \sum_{k=0}^{K-1} |c_m[k]|^2 \, e^{j2\pi dk/K} = 0, \quad \text{for } d = \pm 1, \ldots, \pm(L-1). \tag{15}$$

(A-iii-a) By Property-1, Condition (15) is satisfied for any $K \geq L$ if

$$|c_m[k]|^2 = a_m, \quad \forall k, \ a_m > 0. \tag{16}$$

(A-iii-b) By Properties 2 and 3, Condition (15) is satisfied for $K = ML_1$, M = 1, 2, ... and $L_1 \geq L$, if

$$|c_m[k]|^2 = \begin{cases} a_m^{(l)}, & k = l + iM; l = 0, \ldots, M-1; \\ & i = 0, \ldots, L_1 - 1; a_m^{(l)} \geq 0 \\ 0, & \text{elsewhere.} \end{cases} \tag{17}$$

Consider Condition-B in (10). Using $\boldsymbol{s}_m^{((l))} = F_K^{-1} W(l) \boldsymbol{C}_m$, we obtain the following condition:

$$\sum_{k=0}^{K-1} c_m^*[k] \ c_n[k] \ e^{j2\pi dk/K} = 0 \quad \text{for } d = 0, \pm 1, \ldots, \pm(L-1), \forall m \neq n. \tag{18}$$

Let $G_{m,n}[k] = c_m^*[k] c_n[k]$ and $g_{m,n}[k] \stackrel{\mathcal{F}}{\longleftrightarrow} G_{m,n}[k]$. Then, Condition (18) can be expressed as

$$Condition(B.1): \sum_{k=0}^{K-1} G_{m,n}[k] \ e^{j2\pi dk/K} = 0, \quad \text{for } d = 0, \pm 1, \ldots, \pm(L-1), \forall i \neq j \tag{19}$$

$$\text{or} \quad g_{m,n}[k] = 0, \quad \text{for } k = 0, \ldots, L-1, K-L+1, \ldots, K-1. \tag{20}$$

The following definition will be useful in classifying the various training signal designs in the following (sub)sections.

Definition: Let L_0 be the smallest integer satisfying $L_0 = K/M$ (M is a positive integer) and $L_0 \geq L$.

B. Training Designs for $N_{\rm Tx} L_0 = K$

Consider the following cases:

(B-i) If $L \leq K < 2L$ (i.e., $M = 1$), then (20) implies that $g_{m,n}[k] = 0 \ \forall k$ and hence $G_{m,n}[k] = 0 \ \forall k$ which is possible only if pilots from different transmit antennas are frequency-division multiplexed (FDM). However, since each transmit antenna must have at least L tones according to Condition(A.1), we can only have one transmit antenna, i.e., $N_{\rm Tx}$=1. The optimal pilot tones are then given by (16) (constant amplitude pilot tones) with $N_{\rm Tx}$=1.

(B-ii) If $M = 2$, then $g_{m,n}[k]$ can have nonzero values at indices $\{L, \ldots, K-L\}$ while satisfying (20). Using Properties 2 and 3, we have the following two solutions which satisfy Condition (19) or (20):

(B-ii-a) All antennas use all pilot tones and have the following relationship:

$$G_{m,n}[k] = \begin{cases} a_{m,n}^{(l)}, & k = k_l + iM; i = 0, \ldots, L_0 - 1; \\ & l = 0, 1; k_0 = 0; k_1 = 1; \\ 0, & \text{elsewhere} \end{cases} \tag{21}$$

$$a_{m,n}^{(0)} = -a_{m,n}^{(1)}, \quad a_{m,n}^{(l)} \in \{\mathbb{C} \setminus 0\} \tag{22}$$

where $\setminus$ denotes the set difference operation. Note that $a_{m,n}^{(0)} = -a_{m,n}^{(1)}$ limits the number of transmit antennas to $N_{\rm Tx} = 2$. This allocation type will be called code-division multiplexing (CDM) pilot allocation.

(B-ii-b) Pilot tones of an antenna are disjoint from those of any other antenna resulting in

$$G_{m,n}[k] = 0 \ \forall k, \ m \neq n. \tag{23}$$

To satisfy Condition(A.3) from (15), each antenna's pilots must satisfy (17), i.e., they must be spread out with equal spacing over the indices $\{0, \ldots, K-1\}$. Since each antenna must have at least L tones and $K = 2L_0$, we can only have $N_{\rm Tx} = 2$ antennas, each having L_0 pilot tones. To satisfy Conditions (A.2) from (13) and (A.3) from (15), all pilot amplitudes must be the same. This type of allocation of equally-spaced disjoint pilot tones in the frequency-domain will be called FDM pilot allocation.

(B-iii) In general, consider $K = L_0 M$ for M = 1, 2, ... Let $L_1 = L_0 V$, $U = M/V$, and $V, U \in \{1, 2, \ldots, M\}$. We can have U FDM groups, each with L_1 pilot tones. For the u-th FDM group ($u \in \{0, 1, \ldots, U-1\}$), Condition (19) is satisfied if the following two conditions are met:

$$Condition(B.1.1): \quad G_{uV+m,uV+n}[k] = \begin{cases} a_{uV+m,uV+n}^{(l)}, & k = k_l^{(u)} + iM, \\ & i = 0, \ldots, L_0 - 1, \\ & l = 0, \ldots, V-1, \\ & \forall m \neq n, \\ 0, & \text{elsewhere} \end{cases} \tag{24}$$

$$\sum_{l=0}^{V-1} a_{uV+m,uV+n}^{(l)} = 0, \ \forall m \neq n \tag{25}$$

where $k_l^{(u)} \neq k_m^{(u)}$ if $l \neq m$, $k_n^{(u)} \in \{0, \ldots, M-1\}$, and $a_{uV+m,uV+n}^{(l)} \in \mathbb{C}$. For any $u \neq u'$, $k_l^{(u)}$ and $k_l^{(u')}$ are disjoint. The first condition (24) can be satisfied in infinitely many ways but the second condition (25) can be satisfied by only a few. Hence, (25) determines the number of antennas within an FDM group. If $a_{m,n}^{(l)}$ is restricted to have constant amplitude for any antenna pair (m, n), a simple solution to (25) is given by

$$\begin{aligned} a_{uV+m,uV+n}^{(l)} &= a_{uV+m,uV+n}^{(0)} e^{j2\pi l(n-m)/V}; \\ &\forall m \neq n;\ m, n \in \{0, \ldots, V-1\}. \end{aligned} \tag{26}$$

This solution indicates that the maximum number of antennas within each FDM group is V.

The corresponding optimal pilot tones within each FDM group satisfying Conditions (24) and (25), and hence Conditions (B.1) and (A.1), are then given by

$$\begin{aligned} c_{uV+m}[k] &= \begin{cases} b_{uV+m}^{(l)}[i]\ , & k = k_l^{(u)} + iM; \\ & i = 0, \ldots, L_0 - 1; \\ & l = 0, \ldots, V-1; \\ & u = 0,\ \ldots, U-1 \\ 0, & \text{elsewhere} \end{cases} \\ b_{uV+m}^{(l)}[i] &= p_m b_{uV}^{(l)}[i]\ e^{\frac{-j2\pi lm}{V}}, \\ & \quad p_m = 1;\ m = 0, \ldots, V-1 \\ |b_{uV}^{(l)}[i]| &= b_0 > 0. \end{aligned} \tag{27}$$

It can be readily checked that the solution (27) satisfies (17) and hence also satisfies Condition(A.3) from (15). Although Condition (25) is satisfied by any $p_m \in \{\mathbb{C} \setminus 0\}$, imposing the Condition(A.2) from (13) yields $p_m = 1$ and hence it is incorporated in (27). The pilot allocation within each FDM group may be considered as CDM type in the frequency domain and hence will be called CDM(F). The overall pilot allocation will be called U-FDM + V-CDM(F) pilot allocation. The maximum total number of antennas is $N_{\mathrm{Tx}} = UV = M$. If $U = 1$, we have a CDM(F) pilot allocation while if $V = 1$, we have an FDM pilot allocation. If $K = L_0$, i.e., $M = 1$, then $V = 1$ (one antenna within an FDM group), $U = 1$ (only one FDM group) and the result from (27) becomes identical to that in (B-i). For $K = 2L_0$, i.e., $M = 2$, if $V = 1$, then $U = 2$ and the result from (27) becomes identical to that in (B-ii-b). If $V = 2$, then $U = 1$ and the result from (27) becomes identical to that in (B-ii-a).

The condition (25) required for the CDM(F) allocation within each FDM group of the U-FDM + V-CDM structure is satisfied if $\{b_m^{(l)} :\ l = 0,\ 1,\ \ldots,\ V-1\}$ for all m are orthogonal sequences. Hence, if $\{c_m[k]\}$ are constrained to be binary (BPSK) symbols and V is a power of 2, optimal binary sequences $\{b_m^{(l)}\}$ for V antennas (m =0, 1, …, $V-1$) can be constructed from the Walsh-Hadamard sequences of length V as follows:

$$b_m^{(l)} = w_m[l]\ b_0^{(l)},\ m = 1,\ 2,\ \ldots, V-1 \tag{28}$$

where $w_m[l]$ is the l-th element of m-th Walsh-Hadamard sequences [1] of length V and $\{b_0^{(l)} :\ l = 0,\ 1,\ \ldots,\ V-1\}$ is any binary (± 1) sequence.

Other Training Structures

Note that for $V > 1$, all pilot tones of an antenna do not have to be equally spaced. The solution (27) suggests that within each FDM group, the pilot amplitude is the same. However, for $V = 2$, we can have the following alternative solution:

$$b_1^{(l)}[i] = \frac{1}{b_0^{(l)*}[i]}(-1)^l, \quad l = 0, 1;\ i = 0, \ldots, L_0 - 1 \tag{29}$$

$$|b_0^{(1)}[i]| = \frac{1}{|b_0^{(0)}[i]|} \tag{30}$$

$$|b_m^{(l)}[i]| = a_m^{(l)} > 0 \text{ for } i = 0, \ldots, L_0 - 1 \tag{31}$$

where $\{b_m^{(l)}[i]\}$ for different l do not necessarily have the same amplitude.

For $L_1 > L_0 \geq L$ and a positive integer U, if $K = L_0 L_1 U$ and there are disjoint sets of eqi-spaced, equi-energy L_0 and L_1 pilot tones, then (19) will give additional optimal pilot tones not covered by (27). These additional optimal pilot tones are composed of disjoint sets of L_0 and L_1 tones. Within each set of L_0 equi-energy pilot tones, the spacing is K/L_0 while within the set with L_1 equi-energy pilot tones, the spacing is K/L_1. Pilot amplitudes for different sets will be different but the total pilot energy for each antenna is the same for all antennas. An example is given in Table IV. This type of optimal pilot tones can be extended for $L \leq L_0 < L_1 < L_2 < \ldots < L_d$ as long as $K = U \prod_{i=0}^{d} L_i$ and there exist disjoint sets of equi-spaced L_i pilot tones with spacing K/L_i for all i.

C. Training Designs for $N_{\mathrm{Tx}} L_0 < K$

For U-FDM + V-CDM type pilot structures, which includes pure FDM or CDM structures, (27) gives optimal pilot vectors for M transmit antennas. If the actual number of transmit antennas N_{Tx} is smaller than M, we can use any N_{Tx} vectors from the available M optimal pilot vectors. In the following, we present more optimal pilot structures for $N_{\mathrm{Tx}} L_0 \leq K$.

For FDM-type structures, (19) also gives the following optimal pilot tones for the m-th transmit antenna as

$$\begin{aligned} c_m[k] &= \sum_{l=0}^{L_1 - 1} b_m^{(l)}\ \delta[k - \frac{lK}{L_1} - i_m];\ i_m \in [0, \frac{K}{L_1} - 1] \\ & \quad i_m \neq i_n \text{ if } m \neq n;\ m = 0, \ldots, N_{\mathrm{Tx}} - 1 \end{aligned} \tag{32}$$

where $\{b_m^{(l)}\}$ are constant-modulus symbols and L_1 is *any* integer such that K/L_1 is an integer while $L \leq L_1 \leq K/N_{\mathrm{Tx}}$. Similarly, we obtain from (19) optimal FDM pilot structures with unequal numbers of pilot tones (an integer multiple of L_0) as

$$\begin{aligned} c_m[k] &= \sum_{v=0}^{V_m - 1} \sum_{l=0}^{L_0 - 1} b_m^{(l,v)}\ \delta[k - \frac{lK}{L_0} - i_{m,v}]; \\ & m = 0, \ldots, N_{\mathrm{Tx}} - 1;\ i_{m,v} \in [0,\ \frac{K}{L_0} - 1]; \\ & i_{m_1,v_1} = i_{m_2,v_2} \text{ only if } (m_1 = m_2\ \&\ v_1 = v_2) \\ & \sum_{v=0}^{V_m - 1} \sum_{l=0}^{L_0 - 1} |b_m^{(l,v)}|^2 = K E_{\mathrm{av}}\ ;\ \sum_{m=0}^{N_{Tx} - 1} V_m L_0 \leq K \end{aligned} \tag{33}$$

where V_m is an integer greater than zero. In this case, due to the different numbers of pilot tones per antenna, the

[1]Walsh-Hadamard sequences are rows of the Walsh-Hadamard matrix and the 0-th Walsh-Hadamard sequence is an all-one sequence.

corresponding pilot amplitudes will be different so that the total pilot energy per antenna is the same (see (13)).

For CDM-type structures, similar to the development from (24) to (27), we can obtain optimal pilots defined by

$$c_m[k] = \sum_{v=0}^{V-1}\sum_{l=0}^{L_0-1} b_m^{(l,v)}\ \delta[k - \frac{lK}{L_0} - i_v]; \tag{34}$$
$$m = 0,\ldots,N_{\mathrm{Tx}}-1;\ i_v \in [0,\ \frac{K}{L_0}-1];$$
$$i_{v_1} = i_{v_2} \text{ only if } v_1 = v_2$$

$$\sum_{v=0}^{V-1} a_{m,n}^{(v)} = 0,\ \forall m \neq n;\ m,n \in \{0,\ldots,N_{\mathrm{Tx}}-1\} \tag{35}$$

where V is *any* integer satisfying $N_{\mathrm{Tx}} \leq V \leq K/L_0$ and $a_{m,n}^{(v)} = b_m^{(l,v)^*} b_n^{(l,v)}$. Note that each antenna has V groups of L_0 pilot tones per group. Within each group, L_0 equi-energy tones are separated by an equal spacing of K/L_0. But spacings between groups do not have to be the same. Equation (35) can be satisfied by

$$a_{m,n}^{(v)} = a_{m,n}^{(0)}\ e^{j2\pi v(i_n - i_m)/V},\quad \forall m \neq n; \tag{36}$$
$$m,n \in \{0,\ldots,N_{\mathrm{Tx}}-1\};$$
$$i_m, i_n \in \{0,\ldots,V-1\};\ i_m \neq i_n \text{ if } m \neq n.$$

There are other ways, not covered by (36), to satisfy (35). For example, for $V = 4$, and $N_{\mathrm{Tx}} = 3$, we can have the following design satisfying (35): $[b_0^{(0)}, b_0^{(1)}, b_0^{(2)}, b_0^{(3)}] = e^{j\phi_0}\ [1,1,1,1]$, $[b_1^{(0)}, b_1^{(1)}, b_1^{(2)}, b_1^{(3)}] = e^{j\phi_1}\ [e^{j\theta_1}, e^{j\theta_2}, -e^{j\theta_2}, -e^{j\theta_1}]$, $[b_2^{(0)}, b_2^{(1)}, b_2^{(2)}, b_2^{(3)}] = e^{j\phi_2}\ [-e^{j\theta_1}, e^{j\theta_2}, -e^{j\theta_2}, e^{j\theta_1}]$.

If all VL_0 pilot tones are equi-spaced (in this case, $K/(VL_0)$ is an integer), we have

$$G_{m,m}[k] = \sum_{l=0}^{VL_0-1} a\ \delta[k - \frac{lK}{VL_0} - n_0], \tag{37}$$
$$n_0 \in \{0,\ldots,\ \frac{K}{VL_0}-1\}; m = 0,\ldots,N_{\mathrm{Tx}}-1$$

and by the DFT Property-2, we obtain

$$g_{m,m}[k] = \frac{aVL_0}{K}\sum_{l=0}^{\frac{K}{VL_0}-1} \delta[k - lVL_0]\ e^{j2\pi k n_0/K}. \tag{38}$$

To satisfy the Condition B.1 from (20), $g_{m,n}[k]$ can be designed by cyclically shifting $g_{m,m}[k]$ such that

$$g_{m,n}[k] = g_{m,m}[(k - l_{m,n})_K];\ \forall m \neq n; \tag{39}$$
$$m,n \in \{0,\ldots,N_{\mathrm{Tx}}-1\};\ L \leq l_{m,n} \leq VL_0 - L.$$

A simple solution to (39) is to use an equal distance shifting as

$$g_{0,m}[k] = g_{0,0}[(k - mL_1)_K] \tag{40}$$

where L_1 is *any* integer satisfying $L \leq L_1 \leq \frac{VL_0 - L}{N_{\mathrm{Tx}}-1}$. The corresponding pilot tones for the m-th antenna are given by

$$c_m[k] = c_0[k]\ e^{-j2\pi k m L_1/K};\ m = 1,\ldots,N_{\mathrm{Tx}}-1 \tag{41}$$

where

$$c_0[k] = \sum_{l=0}^{VL_0-1} b_0^{(l)}\delta[k - \frac{lK}{VL_0} - n_0];\ n_0 \in \{0,\ldots,\frac{K}{VL_0}-1\} \tag{42}$$

and $\{b_0^{(l)}\}$ are constant-modulus symbols. An unequal distance shifting of $g_{m,m}[k]$ is also possible as long as the condition $L \leq l_{m,n} \leq VL_0 - L$, $\forall m \neq n$ is satisfied.

Note that for $VL_0 < K$, K must be an integer multiple of VL_0 in the above design for the CDM structure using VL_0 tones. However, if all subcarriers are used (i.e., $c_0[k] = b_0^{(k)},\ \forall k$), then K does not have to be an integer multiple of L_0 (but $K \geq L_0 N_{\mathrm{Tx}}$) and (41) can still be applied. In this case, the condition from (39) becomes $L \leq l_{m,n} \leq K - L$, $\forall m \neq n$. For the special case with equi-distance shifting, the condition becomes $L \leq L_1 \leq \frac{K-L}{N_{\mathrm{Tx}}-1}$. For $N_{\mathrm{Tx}}L_0 < K$, optimal pilot structures of U-FDM + V-CDM type can be similarly constructed by appropriately combining FDM and CDM optimal pilot structures described for $N_{\mathrm{Tx}}L_0 < K$.

D. Combined Training Structures

More complicated pilot allocations are also possible, for example, by not fixing the FDM boundaries and by combining FDM and CDM types. As an example, Table V presents a set of optimal pilot vectors for the estimation of N_{Tx} channels (each with $L = 2$ taps) over $Q = 1$ symbol interval in an OFDM system with $K = 16$ sub-carriers and $N_{\mathrm{Tx}} = 8$ transmit-antennas where α_k can be any constant-modulus symbol. Within the group of antennas 0, 1, 2, and 3, the pilot allocations are of FDM type. Within the group of antennas 4 and 5 or the group of antennas 6 and 7, the pilot allocation is of CDM(F) type while between the two groups it is of FDM type. Pilot allocations of the antenna pairs (0, 4), (1, 5), (2, 6), and (3, 7) are of CDM(F) type and those of antenna pairs (2, 4), (3, 5), (0, 6), and (1, 7) are of FDM type. Note that pilot amplitudes may be different for different antennas, e.g., between antenna 0 and antenna 4.

IV. Optimal Training Signal Design over Multiple OFDM Symbols

This section investigates training signal design for channel estimation based on observations over Q OFDM symbols.

A. Optimality Conditions in Sub-Carrier Domain

Based on Condition-A from (9), we observe the following:

(A-i) *Condition(A.1)*: The full rank condition in (9) implies that for every transmit antenna and within the Q OFDM symbols, there must be at least L different nonzero tones, each with at least one symbol duration.

(A-ii) The condition

$$\sum_{q=0}^{Q-1} \boldsymbol{s}_{m,q}^H \boldsymbol{s}_{m,q} = E_{\mathrm{av}}\boldsymbol{I}\ ,\ \forall m \tag{43}$$

means that

$$Condition(A.2):\ E_m = E_{\mathrm{av}}\ ,\ \forall m. \tag{44}$$

(A-iii) The condition

$$\sum_{q=0}^{Q-1} (\boldsymbol{s}_{m,q}^{((l))})^H \boldsymbol{s}_{m,q}^{((i))} = 0,\ \forall l \neq i;\ l,i \in \{0,1,\ldots,L-1\} \tag{45}$$

means that

$$\sum_{q=0}^{Q-1}\sum_{k=0}^{K-1}|c_{m,q}[k]|^2\, e^{\frac{j2\pi dk}{K}}=0 \text{ for } d=\pm1,\pm2,\ldots,\pm(L-1). \tag{46}$$

By defining

$$E_m[k]=\sum_{q=0}^{Q-1}|c_{m,q}[k]|^2,\ k=0,\ldots,K-1, \tag{47}$$

we can express (46) as

$$\begin{aligned}&\mathit{Condition(A.3)}:\\ &\sum_{k=0}^{K-1}E_m[k]\, e^{\frac{j2\pi dk}{K}} = 0 \text{ for } d=\pm1,\ldots,\pm(L-1).\end{aligned} \tag{48}$$

Note that $E_m=\sum_{k=0}^{K-1}E_m[k]$. Using Properties 2 and 3, we obtain the following condition satisfying (46) for $K=ML_0$:

$$E_m[k]=\begin{cases} a_m^{(l)}, & k=l+iM; l=0,\ldots,M-1;\\ & i=0,\ldots,L_0-1; a_m^{(l)}\geq 0\\ 0, & \text{elsewhere.}\end{cases} \tag{49}$$

At least one $a_m^{(l)}$ must be nonzero in order to get nonzero E_m. Note that pilot tone amplitudes of different antennas may not be necessarily the same but total pilot energies E_m must be the same for different antennas.

Now consider Condition-B from (10) which is given by

$$\begin{aligned}&\sum_{q=0}^{Q-1}\sum_{k=0}^{K-1}c^*_{m,q}[k]\ c_{n,q}[k]\ e^{j2\pi dk/K} = 0\\ &\text{for } d=0,\pm1,\ldots,\pm(L-1)\quad ;\quad \forall m\neq n.\end{aligned} \tag{50}$$

Let $G_{m,n}[q,k]=c^*_{m,q}[k]\ c_{n,q}[k]$ and $G_{m,n}[q,k]\overset{\mathcal{F}}{\longleftrightarrow} g_{m,n}[q,k]$. Then, (10) becomes

$$\begin{aligned}&\mathit{Condition(B.1)}:\\ &\sum_{k=0}^{K-1}\left(\sum_{q=0}^{Q-1}G_{m,n}[q,k]\right)e^{j2\pi dk/K}=0,\\ &\text{for } d=0,\pm1,\ldots,\pm(L-1);\forall m\neq n\end{aligned} \tag{51}$$

or

$$\begin{aligned}&\sum_{q=0}^{Q-1}g_{m,n}[q,k]=0,\\ &\text{for } k=0,\ldots,L-1,K-L+1,\ldots,K-1;\forall m\neq n.\end{aligned} \tag{52}$$

B. Training Designs over Q Symbols

(B-i-a) If we let $g_{m,n}[q,k]=0$, for $k=0,\ldots,L-1$, $K-L+1,\ldots,K-1$, $\forall m\neq n$ and $\forall q$, then the solution for the case of one OFDM symbol is applicable to each symbol of the Q-symbol case. To wit, for $K=ML_0$, we have Q sets of antennas where each set has M antennas whose pilot tone allocations are defined by the solution for the one-symbol case. Each set uses one out of Q symbols. Within each set, pilot tone allocation is of CDM(F) or FDM or FDM+CDM(F) type while different sets are of TDM type. The total number of antennas is $N_{\text{Tx}}=MQ$.

(B-i-b) Alternatively, using Properties 2 and 3, we have the following condition satisfying (10) for $K=ML_0$:

$$\sum_{q=0}^{Q-1}G_{m,n}[q,k] = \begin{cases} a_{m,n}^{(l)}, & k=l+iM,\forall m\neq n,\\ & i=0,\ldots,L_0-1,\\ & l=0,\ldots,M-1,\\ 0, & \text{elsewhere,}\end{cases} \tag{53}$$

$$\sum_{l=0}^{M-1}a_{m,n}^{(l)} = 0,\ \forall m\neq n;\ a_{m,n}^{(l)}\in\mathbb{C}. \tag{54}$$

Note that (53) satisfies $\sum_{q=0}^{Q-1}g_{m,n}[q,k]=0$ for $k=1,\ldots,L-1,\ K-L+1,\ldots,K-1,\ \forall m\neq n$ while (54) satisfies $\sum_{q=0}^{Q-1}g_{m,n}[q,k]=0$ for $k=0,\ \forall m\neq n$.

(B-i-b-1) If $G_{m,n}[q,k]=0$, $\forall q$ and $\forall k$ for some $m\neq n$ (a group of antennas), the corresponding pilot tones are disjoint. Within a group, the L_0 tones of each antenna are disjoint from those of any other antenna by means of multiplexing in time (TDM), in frequency (FDM) or in both time and frequency (TFDM). The pilot tones must also satisfy Conditions (A.2) and (A.3) through (44) and (49). Hence, the optimal L_0 pilot tones of one symbol duration for each antenna must be equally spaced (M tone spacing) with equal amplitude. They should be disjoint from pilot tones of any other antenna. All antennas have the same pilot amplitude.

(B-i-b-2) Consider the case where $G_{m,n}[q,k]\neq 0$, for some (or all) q, for some (or all) k, and for some $m\neq n$, (a group of antennas where $m,n\in\{i_0,i_1,\ldots\}$). Let N_f be the number of sets of equally-spaced (M tone spacing) L_0 tones over Q symbols assigned to an antenna from the above group (each set corresponds to tone indices $\{k_l+iM : i=0,\ldots,L_0-1\}$ where $k_l\neq k_m$ if $l\neq m$ and $k_n\in\{0,\ldots,M-1\}$). Let N_t be the number of repetitions in time (with symbol indices q_0, $q_1,\ldots,q_{N_t-1}$) of the above N_f sets of L_0 tones each. By using the same principle as in (26), within the q-th symbol interval, the N_f sets of pilot tones (each set has L_0 tones) can accommodate a set of N_f antennas if

$$\begin{aligned}&a_{i_m,i_n}^{(k_l)}[q_t]=a_{i_m,i_n}^{(k_0)}[q_t]\ e^{j2\pi l(n-m)/N_f},\\ &\text{for } l=0,\ldots,N_f-1; m\neq n; m,n\in\{0,\ldots,N_f-1\}.\end{aligned} \tag{55}$$

The corresponding pilot tones are given by

$$\begin{aligned} c_{i_m,q_t}[k] &= \begin{cases} b_m^{(l)}[q_t,d], & k=k_l+dM;\\ & d=0,\ldots,L_0-1;\\ & l=0,\ldots,N_f-1\\ 0, & \text{elsewhere}\end{cases}\\ b_m^{(l)}[q_t,d] &= b_0^{(l)}[q_t,d]\ e^{-j2\pi lm/N_f},\ m=0,\ldots,N_f-1\\ |b_0^{(l)}[q_t,d]| &= b_0>0,\end{aligned} \tag{56}$$

where the fulfillment of Conditions (A.1), (A.2), (A.3), and (B.1) is inherited from (27). This pilot allocation is of CDM(F) type.

Applying the CDM principle over the above N_t symbols can accommodate N_t sets of N_f antennas each. The optimal pilot tones for an antenna from the v-th set are related to those from the 0-th set by

$$\begin{aligned}&\{a_{i_{vN_f+m},i_{vN_f+n}}^{(k_l)}[q_t]:\forall l\}=\{a_{i_m,i_n}^{(k_l)}[q_0]:\forall l\}\ e^{j2\pi tv/N_t},\\ &\quad\text{for } v=0,\ldots,N_t-1; l=0,\ldots,N_f-1;\\ &\quad\forall m\neq n; m,n\in\{0,\ldots,N_f-1\}.\end{aligned} \tag{57}$$

TABLE I

OPTIMAL PILOT TONE VECTORS FOR A SISO OFDM SYSTEM WITH $K = 8, L = 2$

Sub-carrier Index							
0	1	2	3	4	5	6	7
$2\alpha_1$	0	0	0	$2\alpha_2$	0	0	0
$\sqrt{2}\alpha_1$	$\sqrt{2}\alpha_2$	0	0	$\sqrt{2}\alpha_3$	$\sqrt{2}\alpha_4$	0	0
$\sqrt{4/3}\alpha_1$	$\sqrt{4/3}\alpha_2$	0	$\sqrt{4/3}\alpha_3$	$\sqrt{4/3}\alpha_4$	$\sqrt{4/3}\alpha_5$	0	$\sqrt{4/3}\alpha_6$
α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8

TABLE II

OPTIMAL PILOT TONE VECTORS FOR A MIMO OFDM SYSTEM WITH $K > N_{\mathrm{Tx}}L$, $(K = 8, N_{\mathrm{Tx}} = 2, L = 2)$

Sub-carrier Index	Pilot Allocation							
	FDM		FDM		CDM(F)		CDM(F), $1 \leq m \leq 3$	
	Ant. 0	Ant. 1	Ant. 0	Ant. 1	Ant. 0	Ant. 1	Ant. 0	Ant. 1
0	$2\alpha_1$	0	$\sqrt{2}\alpha_1$	0	$\sqrt{2}\alpha_1$	$\sqrt{2}\alpha_1 e^{j\phi_1}$	α_1	$\alpha_1 e^{j\phi_1}$
1	0	0	0	$\sqrt{2}\alpha_5$	0	0	α_2	$\alpha_2 e^{-jm\pi/2} e^{j\phi_1}$
2	0	$2\alpha_3$	$\sqrt{2}\alpha_2$	0	0	0	α_3	$\alpha_3 e^{-j2m\pi/2} e^{j\phi_1}$
3	0	0	0	$\sqrt{2}\alpha_6$	$\sqrt{2}\alpha_2$	$-\sqrt{2}\alpha_2 e^{j\phi_1}$	α_4	$\alpha_4 e^{-j3m\pi/2} e^{j\phi_1}$
4	$2\alpha_2$	0	$\sqrt{2}\alpha_3$	0	$\sqrt{2}\alpha_3$	$\sqrt{2}\alpha_3 e^{j\phi_1}$	α_5	$\alpha_5 e^{-j4m\pi/2} e^{j\phi_1}$
5	0	0	0	$\sqrt{2}\alpha_7$	0	0	α_6	$\alpha_6 e^{-j5m\pi/2} e^{j\phi_1}$
6	0	$2\alpha_4$	$\sqrt{2}\alpha_4$	0	0	0	α_7	$\alpha_7 e^{-j6m\pi/2} e^{j\phi_1}$
7	0	0	0	$\sqrt{2}\alpha_8$	$\sqrt{2}\alpha_4$	$-\sqrt{2}\alpha_4 e^{j\phi_1}$	α_8	$\alpha_8 e^{-j7m\pi/2} e^{j\phi_1}$

The corresponding pilot tones are given by

$$c_{i_{vN_f+m},q_t}[k] = c_{i_m,q_0}[k]\ e^{-j2\pi tv/N_t},\ v = 0, \ldots, N_t - 1 \tag{58}$$

and $\{c_{i_m,q_0}[k] : m = 0, \ldots, N_f - 1\}$ are given by (56). The fulfillment of Conditions (A.1), (A.2), (A.3), and (B.1) is inherited from (56). This pilot allocation among the N_t sets is of CDM type in the time-domain and denoted by CDM(T). Hence, this overall pilot allocation over $N_f L_0$ tones and N_t symbols will be denoted by N_f-CDM(F) + N_t-CDM(T). For $N_{\mathrm{Tx}}L_0 < KQ$, the results from Sub-section III-C can be straight-forwardly applied.

C. Combined Training Structures

More complicated pilot allocation schemes are also possible by combining FDM, TDM, TFDM, CDM(F), and CDM(T) type allocations. An example of optimal pilot allocations of mixed types over $Q = 2$ OFDM symbols is shown in Table VI for the estimation of channels with $L = 4$ taps each in an OFDM system with $K = 16$ sub-carriers and $N_{\mathrm{Tx}} = 8$ transmit antennas. The group of antennas 0, 1, 2, 3 is disjoint from the others by FDM over 2 symbols. Antenna 4 is of TFDM type while antenna 5 is of purely FDM type over one symbol. The group of antennas 6 and 7 is disjoint from the others by TFDM. Within the group of antennas 0, 1, 2, and 3, pilot allocation is of 2-CDM(F) + 2-CDM(T) type. Within the group of antennas 6 and 7, pilot allocation is of CDM(F) type.

In Table VII, the pilot vector for each antenna is presented for the optimal training signal structure given in Table VI where α_k can be any constant-modulus symbol. Note that within one symbol, pilot amplitudes from different antennas may be different, e.g., compare pilot amplitudes of antennas 0, 4, and 6.

V. SIMULATION RESULTS AND DISCUSSIONS

A. Summary and Examples

In this section, we summarize our findings on the optimal training signals for MIMO OFDM channel estimation. For training signal design over one OFDM symbol, a simple optimal solution is the FDM+CDM(F) type pilot allocation given by (27) which includes FDM and CDM(F) types as special cases. The allocation in (27) can also be used for $N_{\mathrm{Tx}} < M$, $(M = K/L_0)$ by simply skipping any $(M - N_{\mathrm{Tx}})$ pilot vectors. More optimal pilot allocations for $N_{\mathrm{Tx}} < M$ are given by (32) and (33) for FDM structure and (41) for CDM(F) structure, and an appropriate combination of (32) or (33) and (41) for FDM+CDM(F) structure. In practice, K is a power of 2 for a simpler FFT implementation and hence L_0 can always be found. If all $K(\geq LN_{\mathrm{Tx}})$ sub-carriers are used in CDM(F) structure, then K need not be an integer multiple of L_0.

Some representative examples of optimal pilot tone vectors for SISO OFDM systems are given in Table I and those for MIMO OFDM systems are given in Tables II and III where $\{\alpha_i\}$ are constant-modulus symbols. An example of optimal pilot structure with unequal number of pilot tones per antenna is presented in Table IV. More complicated pilot allocations can be constructed by not fixing the FDM boundaries and by combining FDM and CDM(F). An example is given in Table V. For training signal design over Q OFDM symbols, two simple optimal solutions have been presented. The first one is composed of Q TDM groups where within each TDM group, the FDM+CDM(F) pilot allocation given in (27) is implemented. In the second solution, every antenna transmits on all pilot tones over all Q OFDM symbols and the optimal pilot tones are given by the CDM(F)+CDM(T) allocation defined in (56) and (58). Other more complicated solutions can be obtained by combining TDM (less than Q TDM groups), FDM, CDM(F), and CDM(T) allocations. Examples are given in Tables VI and VII.

TABLE III

OPTIMAL PILOT TONE VECTORS FOR A MIMO OFDM SYSTEM WITH $K = N_{\mathrm{Tx}}L$ ($K = 8, N_{\mathrm{Tx}} = 4, L = 2$)

Pilot Allocation	Sub-carrier Index	Ant. 0	Ant. 1	Ant. 2	Ant. 3
CDM(F)	0	α_1	$\alpha_1 e^{j\phi_1}$	$\alpha_1 e^{j\phi_2}$	$\alpha_1 e^{j\phi_3}$
	1	α_2	$\alpha_2 e^{-j\pi/2} e^{j\phi_1}$	$\alpha_2 e^{-j2\pi/2} e^{j\phi_2}$	$\alpha_2 e^{-j3\pi/2} e^{j\phi_3}$
	2	α_3	$\alpha_3 e^{-j2\pi/2} e^{j\phi_1}$	$\alpha_3 e^{-j4\pi/2} e^{j\phi_2}$	$\alpha_3 e^{-j6\pi/2} e^{j\phi_3}$
	3	α_4	$\alpha_4 e^{-j3\pi/2} e^{j\phi_1}$	$\alpha_4 e^{-j6\pi/2} e^{j\phi_2}$	$\alpha_4 e^{-j9\pi/2} e^{j\phi_3}$
	4	α_5	$\alpha_5 e^{-j4\pi/2} e^{j\phi_1}$	$\alpha_5 e^{-j8\pi/2} e^{j\phi_2}$	$\alpha_5 e^{-j12\pi/2} e^{j\phi_3}$
	5	α_6	$\alpha_6 e^{-j5\pi/2} e^{j\phi_1}$	$\alpha_6 e^{-j10\pi/2} e^{j\phi_2}$	$\alpha_6 e^{-j15\pi/2} e^{j\phi_3}$
	6	α_7	$\alpha_7 e^{-j6\pi/2} e^{j\phi_1}$	$\alpha_7 e^{-j12\pi/2} e^{j\phi_2}$	$\alpha_7 e^{-j18\pi/2} e^{j\phi_3}$
	7	α_8	$\alpha_8 e^{-j7\pi/2} e^{j\phi_1}$	$\alpha_8 e^{-j14\pi/2} e^{j\phi_2}$	$\alpha_8 e^{-j21\pi/2} e^{j\phi_3}$
FDM	0	$2\alpha_1$	0	0	0
	1	0	$2\alpha_3$	0	0
	2	0	0	$2\alpha_5$	0
	3	0	0	0	$2\alpha_7$
	4	$2\alpha_2$	0	0	0
	5	0	$2\alpha_4$	0	0
	6	0	0	$2\alpha_6$	0
	7	0	0	0	$2\alpha_8$
2-FDM + 2-CDM(F)	0	$\sqrt{2}\alpha_1$	$\sqrt{2}\alpha_1 e^{j\phi_1}$	0	0
	1	0	0	$\sqrt{2}\alpha_5$	$\sqrt{2}\alpha_5 e^{j\phi_3}$
	2	0	0	$\sqrt{2}\alpha_6$	$-\sqrt{2}\alpha_6 e^{j\phi_3}$
	3	$\sqrt{2}\alpha_2$	$-\sqrt{2}\alpha_2 e^{j\phi_1}$	0	0
	4	$\sqrt{2}\alpha_3$	$\sqrt{2}\alpha_3 e^{j\phi_1}$	0	0
	5	0	0	$\sqrt{2}\alpha_7$	$\sqrt{2}\alpha_7 e^{j\phi_3}$
	6	0	0	$\sqrt{2}\alpha_8$	$-\sqrt{2}\alpha_8 e^{j\phi_3}$
	7	$\sqrt{2}\alpha_4$	$-\sqrt{2}\alpha_4 e^{j\phi_1}$	0	0

TABLE IV

EXAMPLES OF FDM-TYPE OPTIMAL PILOT TONE VECTORS WITH UNEQUAL NUMBER OF PILOT TONES PER ANTENNA FOR AN OFDM SYSTEM WITH $K = 12$, $N_{\mathrm{Tx}} = 4$, $L = L_0 = 2$, $L_1 = 3$ AND $Q = 1$

Sub-carrier Index	Ant. 0	Ant. 1	Ant. 2	Ant. 3
0	$A_0\alpha_0$	0	0	0
1	0	0	$A_2\alpha_6$	0
2	0	$A_1\alpha_3$	0	0
3	0	0	0	0
4	$A_0\alpha_1$	0	0	0
5	0	0	0	$A_3\alpha_8$
6	0	$A_1\alpha_4$	0	0
7	0	0	$A_2\alpha_7$	0
8	$A_0\alpha_2$	0	0	0
9	0	0	0	0
10	0	$A_1\alpha_5$	0	0
11	0	0	0	$A_3\alpha_9$
$3\vert A_0\vert^2 = 3\vert A_1\vert^2 = 2\vert A_2\vert^2 = 2\vert A_3\vert^2,\ \vert\alpha_i\vert = 1$				

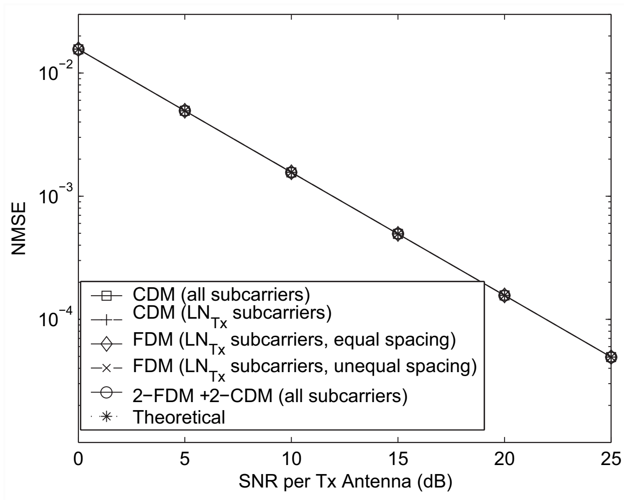

Fig. 1. The NMSEs of several optimal training structures for an MIMO OFDM system with $N_{\mathrm{Tx}} = 4$, $K = 64$ in an 8-tap multipath Rayleigh fading channel with an exponential power delay profile.

Although our discussion is based on pilot-only Q OFDM training symbols, the results can be easily adapted to pilot-data-multiplexed schemes. The optimal training signal design in pilot-data-multiplexed systems may be viewed as using data in place of pilot tones allocated to some transmit antennas with FDM pilot allocation in the original design for pilot-only training symbols (and removing those transmit antennas). The orthogonality between data and pilot tones is inherited from the original FDM pilot allocation.

B. Relationship to Existing Training Designs

Consider pilot-data-multiplexed schemes with $K = L_0M$ and $M = (M_d + M_p)$, where M_dL_0 sub-carriers are for data and M_pL_0 sub-carriers are for pilot tones. When $K = DM_pL_0$ where D is an integer (> 1) and pilot tones are equi-powered, equi-spaced and all transmit antennas use all pilot tones with CDM(F) pilot allocation (i.e., $U = 1$, $k_l = \tau + lD$ with $\tau \in \{0, \ldots, D-1\}$ and $l = 0, \ldots, M_p - 1$ in (27)), our results specialize to the optimal training signals for M_p transmit antennas over one OFDM symbol presented in [9] and [15]. Note that K need not be an integer multiple of M_pL_0 in our designs for pilot-data multiplexed schemes. This fact results in a better flexibility of our designs over the existing ones. For example, for $K = 64$, $L = 8$, and $N_{\mathrm{Tx}} = 3$, the

TABLE V

OPTIMAL PILOT TONE VECTORS IN AN OFDM SYSTEM WITH $N_{\mathrm{Tx}} = 8$, $K = 16$, $Q = 1$ FOR ESTIMATION OF CHANNELS WITH $L = 2$ TAPS EACH

Sub-carrier	Antenna Index							
Index	0	1	2	3	4	5	6	7
0	$\sqrt{2}\alpha_1$	0	0	0	α_1	α_1	0	0
1	0	$\sqrt{2}\alpha_3$	0	0	α_3	$-\alpha_3$	0	0
2	0	0	$\sqrt{2}\alpha_5$	0	0	0	α_5	α_5
3	0	0	0	$\sqrt{2}\alpha_7$	0	0	α_7	$-\alpha_7$
4	$\sqrt{2}\alpha_2$	0	0	0	$-\alpha_2$	$-\alpha_2$	0	0
5	0	$\sqrt{2}\alpha_4$	0	0	$-\alpha_4$	α_4	0	0
6	0	0	$\sqrt{2}\alpha_6$	0	0	0	$-\alpha_6$	$-\alpha_6$
7	0	0	0	$\sqrt{2}\alpha_8$	0	0	$-\alpha_8$	α_8
8	$\sqrt{2}\alpha_1$	0	0	0	α_1	α_1	0	0
9	0	$\sqrt{2}\alpha_3$	0	0	α_3	$-\alpha_3$	0	0
10	0	0	$\sqrt{2}\alpha_5$	0	0	0	α_5	α_5
11	0	0	0	$\sqrt{2}\alpha_7$	0	0	α_7	$-\alpha_7$
12	$\sqrt{2}\alpha_2$	0	0	0	$-\alpha_2$	$-\alpha_2$	0	0
13	0	$\sqrt{2}\alpha_4$	0	0	$-\alpha_4$	α_4	0	0
14	0	0	$\sqrt{2}\alpha_6$	0	0	0	$-\alpha_6$	$-\alpha_6$
15	0	0	0	$\sqrt{2}\alpha_8$	0	0	$-\alpha_8$	α_8

designs from [9] and [15] would not be applicable since they require that K be an integer multiple of LN_{Tx}. However, our design can still be applied; for example, by using unequi-spaced FDM or CDM(F), or FDM+CDM(F) structure. As another example, consider OFDM-based UWB systems where some tones may need to be turned off due to coexistence of other wireless devices (such as 802.11b and Bluetooth) within the UWB band. In this scenario, the existing equi-spaced pilot tones may not be feasible and our design has a clear advantage in terms of applicability/flexibility for different systems. Our optimal training signal designs over multiple OFDM symbols can be similarly linked to those of [15].

Now consider schemes where all sub-carriers are pilot tones. For $N_{\mathrm{Tx}}L \leq K$, our CDM design in (41) with $L_1 = \lfloor K/N_{\mathrm{Tx}} \rfloor$ gives the training design of [16]. If pilot symbols are constrained to be BPSK, then using a subset from the V pilot tone vectors of our CDM design with Walsh-Hadamard sequence (28) gives the training design from [18]. The corresponding subset is given by $w_m[l] = (-1)^{\lfloor l/2^{m-1} \rfloor}$ for m = 2, …, $N_{\mathrm{Tx}-1}$ in (28) with $VL_0 = K$ being an integer multiple of $2^{N_{\mathrm{Tx}}-1}L_0$. Note that for $K = 2^{m-1}L_0$ with BPSK pilot tones, the design from [18] can accommodate m transmit antennas while our design can accommodate upto 2^{m-1} transmit antennas.

C. Simulation Results

We have numerically evaluated the optimality condition in (6) for all the training signal designs discussed and confirmed their optimality with respect to minimizing MSE. Simulation results are also provided to corroborate the optimality of the proposed training signals. Simulation parameters assumed are $K = 64$, $N_{\mathrm{Tx}} = 4$, a multipath Rayleigh fading channel with $L = 8$ taps and an exponential power delay profile (3 dB per tap decaying factor). We evaluated five optimal training structures: a CDM-F structure using all subcarriers (same as [16]), an equi-spaced CDM-F structure using LN_{Tx} pilot tones (same as [15]), an equi-spaced FDM structure with a total of LN_{Tx} pilot tones, an unequi-spaced FDM structure with a total of LN_{Tx} pilot tones, and a 2-FDM + 2-CDM structure using all subcarriers. For BER evaluation, N_{Tx} independent streams of BPSK data symbols are transmitted simultaneously from N_{Tx} transmit antennas and we use maximum likelihod detection. Fig. 1 shows the NMSE simulation results as well as the theoretical ones and Fig. 2 presents the BER simulation results for the different optimal training signals. All optimal training signals evaluated have the same performance as expected.

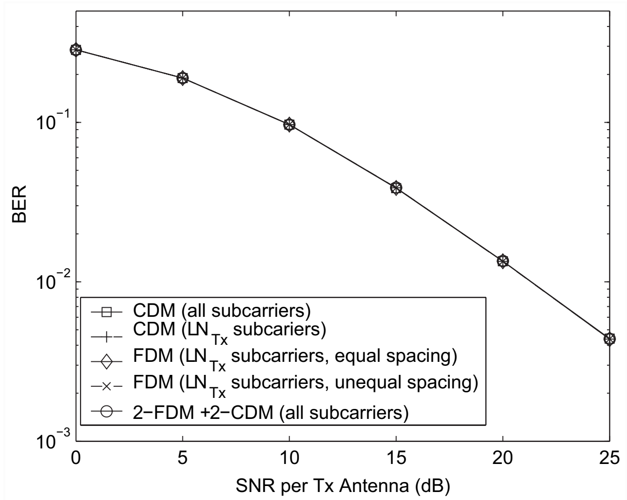

Fig. 2. The BERs of several optimal training structures for an MIMO OFDM system with $N_{\mathrm{Tx}} = 4$, $K = 64$ in an 8-tap multipath Rayleigh fading channel with an exponential power delay profile.

D. Other Properties of Optimal Training Signals

Training signals should be designed to have low PAR in order to avoid nonlinear distortion of the training signal at the transmit power amplifier. Depending on power amplifier design, allowable PAR of the training signal will vary. In the following, we provide a general discussion on the PAR of different optimal training structures. Optimal training signals

TABLE VII
OPTIMAL PILOT TONE VECTORS IN AN OFDM SYSTEM WITH $N_{\mathrm{Tx}} = 8$, $K = 16$, $Q = 2$ FOR ESTIMATION OF CHANNELS WITH $L = 4$ TAPS EACH

Symbol	Antenna Index							
index	0	1	2	3	4	5	6	7
	α_1	α_1	α_1	α_1	0	0	0	0
	α_2	$-\alpha_2$	α_2	$-\alpha_2$	0	0	0	0
	0	0	0	0	$2\ \alpha_9$	0	0	0
	0	0	0	0	0	$2\ \alpha_{13}$	0	0
	α_3	α_3	α_3	α_3	0	0	0	0
	α_4	$-\alpha_4$	α_4	$-\alpha_4$	0	0	0	0
	0	0	0	0	$2\ \alpha_{10}$	0	0	0
0	0	0	0	0	0	$2\ \alpha_{14}$	0	0
	α_5	α_5	α_5	α_5	0	0	0	0
	α_6	$-\alpha_6$	α_6	$-\alpha_6$	0	0	0	0
	0	0	0	0	0	0	$\sqrt{2}\alpha_{17}$	$\sqrt{2}\alpha_{17}$
	0	0	0	0	0	$2\ \alpha_{15}$	0	0
	α_7	α_7	α_7	α_7	0	0	0	0
	α_8	$-\alpha_8$	α_8	$-\alpha_8$	0	0	0	0
	0	0	0	0	0	0	$\sqrt{2}\alpha_{18}$	$\sqrt{2}\alpha_{18}$
	0	0	0	0	0	$2\ \alpha_{16}$	0	0
	α_1	α_1	$-\alpha_1$	$-\alpha_1$	0	0	0	0
	α_2	$-\alpha_2$	$-\alpha_2$	α_2	0	0	0	0
	0	0	0	0	0	0	$\sqrt{2}\alpha_{19}$	$\sqrt{2}\alpha_{19}$
	0	0	0	0	0	0	$\sqrt{2}\alpha_{19}$	$-\sqrt{2}\alpha_{19}$
	α_3	α_3	$-\alpha_3$	$-\alpha_3$	0	0	0	0
	α_4	$-\alpha_4$	$-\alpha_4$	α_4	0	0	0	0
	0	0	0	0	0	0	$\sqrt{2}\alpha_{20}$	$\sqrt{2}\alpha_{20}$
1	0	0	0	0	0	0	$\sqrt{2}\alpha_{20}$	$-\sqrt{2}\alpha_{20}$
	α_5	α_5	$-\alpha_5$	$-\alpha_5$	0	0	0	0
	α_6	$-\alpha_6$	$-\alpha_6$	α_6	0	0	0	0
	0	0	0	0	$2\ \alpha_{11}$	0	0	0
	0	0	0	0	0	0	$\sqrt{2}\alpha_{21}$	$-\sqrt{2}\alpha_{21}$
	α_7	α_7	$-\alpha_7$	$-\alpha_7$	0	0	0	0
	α_8	$-\alpha_8$	$-\alpha_8$	α_8	0	0	0	0
	0	0	0	0	$2\ \alpha_{12}$	0	0	0
	0	0	0	0	0	0	$\sqrt{2}\alpha_{22}$	$-\sqrt{2}\alpha_{22}$

TABLE VI
AN OPTIMAL PILOT ALLOCATION IN AN OFDM SYSTEM WITH $N_{\mathrm{Tx}} = 8$, $K = 16$, $Q = 2$ FOR ESTIMATION OF CHANNELS WITH $L = 4$ TAPS EACH (ANTENNA ASSIGNMENT ON THE GRID OF SUB-CARRIERS AND SYMBOLS)

Antennas assignments		
Sub-carrier	Symbol Index	
Index	0	1
0	0, 1, 2, 3	0, 1, 2, 3
1	0, 1, 2, 3	0, 1, 2, 3
2	4	6, 7
3	5	6, 7
4	0, 1, 2, 3	0, 1, 2, 3
5	0, 1, 2, 3	0, 1, 2, 3
6	4	6, 7
7	5	6, 7
8	0, 1, 2, 3	0, 1, 2, 3
9	0, 1, 2, 3	0, 1, 2, 3
10	6, 7	4
11	5	6, 7
12	0, 1, 2, 3	0, 1, 2, 3
13	0, 1, 2, 3	0, 1, 2, 3
14	6, 7	4
15	5	6, 7

of all transmit antennas with CDM(F) allocation have the same PAR since the time-domain signals are just cyclic-shifted versions of one another. For FDM allocation, optimal training signals of all antennas can be easily designed to have the same PAR by using the same pilot symbols on the assigned sub-carriers since a shift in the frequency-domain results in a phase rotation of the time-domain signal. Hence, as far as PAR is concerned, we just need to consider the training signal of the first antenna. For CDM(F) allocation, we can design all $\{c_0[n]:\ n = 0,\ 1,\ \ldots,\ K-1\}$. For FDM, we can design $\{c_0[iM]:\ i = 0,\ 1,\ \ldots,\ L_0 - 1; M = K/L_0\}$ which is equivalent to designing all sub-carrier symbols in an OFDM system with L_0 sub-carriers. Hence, generally there is no difference between CDM(F) and FDM if PAR of the training signals is the only concern. We can use zero-correlation or very low correlation sequences as pilot tones which will give very low PAR.

If pilot symbols are constrained to be from a finite alphabet signal constellation, FDM structure gives a much easier design since CDM design is associated with some phase-shifts.

Furthermore, training signals for channel estimation should be robust to frequency offsets. Different optimal training signals have different robustness to frequency offsets. For example, we simulated the channel estimation NMSE for the above five training signals in the presence of a frequency offset of 0.05 (normalized by the subcarrier spacing) at a SNR per antenna of 10 dB and the corresponding NMSE values are 5.06×10^{-3}, 4.79×10^{-3}, 4.75×10^{-3}, 5.14×10^{-3}, and 3.14×10^{-3}, respectively. Finding the best one(s) among the optimal training signals in the presence of a frequency offset and a PAR constraint is a challenging problem. Being a larger and more general set including the existing training structures, the proposed training structures will be useful in this quest.

VI. Conclusions

We presented general classes of optimal training signals for channel estimation in MIMO OFDM systems with single or multiple OFDM training symbols. The optimal pilot tone allocation among transmit antennas can be of frequency-division multiplexing, time-division multiplexing, code-division multiplexing in the time-domain, code-division multiplexing in the frequency-domain or combinations thereof. Depending on the pilot allocation, the pilot amplitudes of different antennas within an OFDM symbol can be different and all pilot tones of an antenna may not be equally spaced. The presented optimal training signal designs are applicable to pilot-only schemes as well as pilot-data-multiplexed schemes. Our proposed training designs include all existing training designs for OFDM as special cases and introduce new designs as well. Our approach based on the DFT properties facilitates new training designs. Our training designs provide a better flexibility in terms of system parameters and could be useful in training signal design in the presence of frequency offset and PAR constraints.

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