LOGISTICS PROJECT

Health Care Provider of Florida Problem

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Health Care Provider of Florida (HCPF) is planning to build a number of new emergency-care clinics in central Florida. HCPF management has divided a map of the area into seven regions. They want to locate the emergency centers so that all seven regions will be conveniently served by at least one facility. Five candidate sites are available for constructing the new facilities. The regions that can be served conveniently by each site are indicated by X below:

	Candidate Building Sites				
Regions	Sanford	Altamonte	Apopka	Casselberry	Maitland
1	X		X		
2	X	X		X	X
3		X		X	
4			X		X
5	X	X			
6			X		X
7				X	X
Cost ($1,000s)	$450	$650	$550	$500	$525

1. Formulate an ILP model to determine which sites should be selected so as to provide convenient service to all locations in the least costly manner.
2. Implement the model via the modelling language AMPL and solve it by means of the optimization solver CPLEX.
3. Assume that a budget of $1000 is available to HCPF for constructing the new facilities. Modify the model, and solve it by means of AMPL and CPLEX, in order to determine where to construct the facilities so as to maximize the number of the regions which will be served conveniently.
4. Modify the model in 3 with the following additional constraint: if a facility is constructed in Sanford, then a facility must necessarily be constructed also in Casselberry. Solve the proposed model by means of AMPL and CPLEX, and compare the optimal solution found with the ones determined at points 2 and 3.

1. **Formulate an ILP model to determine which sites should be selected so as to provide convenient service to all locations in the least costly manner.**

At this first point, in order to provide the service to all the regions in the least cost manner, we implement a **Set Covering Location Model**. To do so, firstly we need to set the input data of the problem, which are:

- I = *set of* ***Regions*** (or demand nodes)
- J = *set of* ***candidate sites***
- C_j = *Cost for opening the emergency center at the site j*, ∀j ∈ J
- N_i = *set of candidate building sites that can cover the Region i*, ∀i ∈ I

  - N_1 = {Sanford, Apopka}
  - N_2 = {Sanford, Altamonte, Casselberry, Maitland}
  - N_3 = {Altamonte, Casselberry}
  - N_4 = {Apopka, Maitland}
  - N_5 = {Sanford, Altamonte}
  - N_6 = {Apopka, Maitland}
  - N_7 = {Casselberry, Maitland}

Decision variables:
In this case we have to decide where to locate the emergency centers in order to cover all the clients at the minimum cost, so it's indeed to introduce a **binary variable** x_j that indicates in which building sites j we locate the emergency center.

$$x_j = \begin{cases} 1 \; if \; we \; decide \; to \; locate \; the \; emergency \; center \; at \; the \; site \; j \\ 0 \; otherwise \end{cases}$$

Objective function:
The **objective function** is the minimization of the cost of locating the emergency centers, which is given by the sum over *j* belonging to J of the **cost of opening** the facility at the site j times the binary variable which indicates if a facility is opened.

$$min \sum_{j \in J} c_j \, x_j$$

Covering constraints:
It's indeed to set a **covering constraint**. Since the problem requires that all the regions must be covered, it means that at least one facility that can serve a Region must be opened.
That constraint is given by the sum of the decision binary variables x_j which must be greater than or equal to 1.

$$\sum_{j \in N_j} x_j \geq 1 \, , \; \forall i \epsilon I$$

$$X_j \in \{0,1\}, \; \forall j \epsilon J$$

ILP model for the HCPF problem:

$$\min\ 450x_{Sanford} + 650x_{Altamonte} + 550x_{Apopka} + 500x_{Casselberry} + 525x_{Maitland}$$

$x_{Sanford} + x_{Apopka} \geq 1$ (covering Region 1)
$x_{Sanford} + x_{Altamonte} + x_{Casselberry} + x_{Maitland} \geq 1$ (covering Region 2)
$x_{Altamonte} + x_{Casselberry} \geq 1$ (covering Region 3)
$x_{Apopka} + x_{Maitland} \geq 1$ (covering Region 4)
$x_{Sanford} + x_{Altamonte} \geq 1$ (covering Region 5)
$x_{Apopka} + x_{Maitland} \geq 1$ (covering Region 6)
$x_{Casselberry} + x_{Maitland} \geq 1$ (covering Region 7)

$x_{Sanford}, x_{Altamonte}, x_{Apopka}, x_{Casselberry}, x_{Maitland} \in \{0,1\}$

Since the covering constraints of Region 4 and 6 are the same, the one of Region 6 is redundant. This is because, if an emergency center is located in one of the two sites that can cover Region 4, also Region 6 will also be conveniently served as both Apopka and Maitland can cover this Region.

2. **Implement the model via the modelling language AMPL and solve it by means of the optimization solver CPLEX.**

By implementing the model described above using the optimization solver CPLEX, we get the following results:

```
Opening [*] :=
  Altamonte  0
     Apopka  0
Casselberry  1
   Maitland  1
    Sanford  1
;

Total_cost = 1475
```

2. Optimal Solution

Given the cost of opening the emergency centers at a certain site, in order to minimize the total cost and cover all the Regions the optimal solution is to open the facilities at Casselberry, Maitland and Sanford.
As previously seen:

- Casselberry can serve Region n. 2, 3 and 7;
- Maitland can serve Region n. 2, 4, 6 and 7;
- Sanford can serve Region n. 1, 2 and 5.

The total cost is $1475, which is the minimum cost for covering all the regions.

3. **Assume that a budget of $1000 is available to HCPF for constructing the new facilities. Modify the model, and solve it by means of AMPL and CPLEX, in order to determine where to construct the facilities so as to maximize the number of the regions which will be served conveniently.**

At this third point, the problem presents a budget constraint: the HCPF cannot spend more than $1000 in order to open the new facilities.

Given that information, we implement a **Maximal Covering Location Model (MCLM)**, where the goal is to maximise the number of covered Regions.
Looking at the cost of opening an emergency center at each site, the budget constraint gives an upper limit on the number of facilities to be opened, therefore not all regions can be covered.
However, our case is slightly different from the standard MCLM. The two key differences are:

- The number of facilities to be opened (p) is not known a priori.
- In order to respect the budget constraint, we have to consider the cost of opening a facility at each site. We can use the c_j variable presented in the previous point.

The budget constraint can be expressed as follows:

$$\sum_{j \in J} c_j x_j \leq 1000$$

Since not all regions will be covered, we have to introduce a new binary decision variable z_i that indicates which regions are covered by the emergency centers:

$$z_i = \begin{cases} 1 \text{ if node } i \text{ is covered} \\ 0 \text{ otherwise} \end{cases} , \forall i \in I$$

$$Z_i \in \{0,1\}, \forall i \in I$$

The fact that not all regions will be covered implies that also the covering constraints need to be modified:

$$\sum_{j \in N_i} x_j \geq z_i , \qquad \forall i \in I$$

We also have to express a new objective function: if in the SCLM the goal was to minimize the cost, the goal of the MCLM is to maximize the covered regions by staying within the budget. The objective function can be expressed as follows:

$$\max \sum_{i \in I} z_i$$

ILP model:

max $z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7$

$450x_{Sanford} + 650x_{Altamonte} + 550x_{Apopka} + 500x_{Casselberry} + 525x_{Maitland} \leq 1000$

$x_{Sanford} + x_{Apopka} \geq z_1$ (covering Region 1)
$x_{Sanford} + x_{Altamonte} + x_{Casselberry} + x_{Maitland} \geq z_2$ (covering Region 2)
$x_{Altamonte} + x_{Casselberry} \geq z_3$ (covering Region 3)
$x_{Apopka} + x_{Maitland} \geq z_4$ (covering Region 4)
$x_{Sanford} + x_{Altamonte} \geq z_5$ (covering Region 5)
$x_{Apopka} + x_{Maitland} \geq z_6$ (covering Region 6)
$x_{Casselberry} + x_{Maitland} \geq z_7$ (covering Region 7)

$x_{Sanford}, x_{Altamonte}, x_{Apopka}, x_{Casselberry}, x_{Maitland} \in \{0,1\}$
$z_1, z_2, z_3, z_4, z_5, z_6, z_7 \in \{0,1\}$

As for the previous case, the sixth covering constraint is redundant and it can be removed.

The results show that the optimal solution is to open the emergency centers at Sanford and Maitland, since it is the only solution within the budget that allows to cover 6 regions.

```
Opening [*] :=
  Altamonte  0
     Apopka  0
Casselberry  0
   Maitland  1
    Sanford  1
;

CoveredRegion [*] :=
1  1
2  1
3  0
4  1
5  1
6  1
7  1
;
```

3. Optimal Solution

The total cost of opening at Sanford and Maitland is \$975, which is within the budget.

4. **Modify the model in 3 with the following additional constraint: if a facility is constructed in Sanford, then a facility must necessarily be constructed also in Casselberry. Solve the proposed model by means of AMPL and CPLEX, and compare the optimal solution found with the ones determined at points 2 and 3.**

We can implement the new model by adding a new constraint to the previous model. Since we know that if a facility is built in Sanford we also have to locate a facility in Casselberry, we can express the constraint as follows:

$$x_{Sanford} \leq x_{Casselberry}$$

The results show that the optimal solution is to open the emergency centers at Sanford and Casselberry.

```
Opening [*] :=
  Altamonte  0
     Apopka  0
Casselberry  1
   Maitland  0
    Sanford  1
;

CoveredRegion [*] :=
1  1
2  1
3  1
4  0
5  1
6  0
7  1
;
```

4. Optimal solution

The results of point 3 and point 4 are coherent with the result of point 2. Without a budget, the optimal solution would be to open at Sanford, Casselberry and Maitland.
By imposing the budget constraint of $1000 we imply that HCPF should be opening 2 facilities, since 3 would exceed the budget and 1 would not cover enough regions to be the optimal solution.
Also, if HCPF would not open at Sanford it would be impossible to open more than one facility, since the costs would inevitably exceed the budget. This explains why for point 4 the optimal solution is to open at Sanford and consequently at Casselberry. Since for point 3 we do not have the constraint of opening at Casselberry if we open at Sanford, HCPF should open at Maitland in place of Casselberry.
Overall, the optimal solution is the one presented in point 3, since opening at Sanford and Maitland would cost $975 and they would cover 6 regions, while in the solution of point 4 (opening at Sanford and Casselberry) only 5 regions would be covered with an expense of $950.