* Notes on the "Force of Mortality"
(Board notes on hazard function $h(t)$ (or $\lambda(t)$))

- The definition of the hazard function in survival analysis is

$$h(t) = \lim_{\Delta t \to 0} P(t < T \leq t + \Delta t \mid T > t)$$

"Instantanious rate of failure at time t, given subject has survived to time t."

- Using the rules of conditional probability we have

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$h(t) = \lim_{\Delta t \to 0} \frac{P(t < T \leq t + \Delta t)}{\Delta t \, P(T > t)}$$

$$= \lim_{\Delta t \to 0} \frac{P(t < T \leq t + \Delta t) / \Delta t}{P(T > t)}$$

(by rewriting, pulling Δt up)

$$= \lim_{\Delta t \to 0} \frac{\int_t^{t+\Delta t} f(x)\,dx \,/ \Delta t}{P(T > t)}$$

since $P(t < T \leq t + \Delta t)$ is by definition an integral

$$= \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t) / \Delta t}{P(T > t)}$$

by definition of cumulative distribution (CDF) function $\int_t^{t+\Delta t} f(x)\,dx = F(x)\Big]_t^{t+\Delta t}$,

where $F(x)$ is the antiderivative of $f(x)$.

$$= \frac{\partial F(t)}{\partial t} \Big/ P(T > t)$$

Since $\lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t}$ is just the definition of derivative.

$$= \frac{f(t)}{S(t)}$$

Since differentiating a CDF $F(t)$ gives you the density $f(t)$ at t, and since $P(T > t) = S(t)$ by definition.

* Useful to think about as we work more with hazard functions, and interpret hazard ratios → a hazard function is merely a derivative in t, over the survival function, which for interpretation purposes is simply an instantaneous rate of an event occurring at time t.