BST 210 Homework 2

Wenjie Gu

Problem 1

```
#1(a)
dat = read.csv("SCCS2_v12.csv")
dat$bmi = dat$weight / (dat$height/100)^2
age2_gender.tc = lm(tc~age+I(age^2)+gender, data = dat)
age2_gender_bmi.tc = lm(tc~age+I(age^2)+gender+bmi, data =dat)
age2_bmi.tc = lm(tc~age+I(age^2)+bmi, data= dat)
summary(age2_gender.tc)
##
## Call:
## lm(formula = tc ~ age + I(age^2) + gender, data = dat)
##
## Residuals:
##
      Min
               1Q Median
                              3Q
                                     Max
## -2.6152 -0.6114 -0.0555 0.5250 4.1205
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.0390448 0.3913591 7.765 4.32e-14 ***
## age
              0.0909455 0.0186723 4.871 1.48e-06 ***
## I(age^2)
              -0.0007241 0.0002073 -3.494 0.000517 ***
## gender
               0.0944912 0.0851936
                                    1.109 0.267882
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9691 on 523 degrees of freedom
## Multiple R-squared: 0.1525, Adjusted R-squared: 0.1476
## F-statistic: 31.36 on 3 and 523 DF, p-value: < 2.2e-16
summary(age2_bmi.tc)
##
## Call:
## lm(formula = tc ~ age + I(age^2) + bmi, data = dat)
## Residuals:
      Min
               1Q Median
                              30
                                     Max
## -2.7195 -0.6100 -0.0762 0.5396 4.1447
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.6352317 0.4136506 6.371 4.13e-10 ***
              0.0798443 0.0189545
                                     4.212 2.98e-05 ***
## I(age^2)
              ## bmi
               0.0305255 0.0102095
                                    2.990 0.00292 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.9621 on 523 degrees of freedom
## Multiple R-squared: 0.1648, Adjusted R-squared:
## F-statistic: 34.39 on 3 and 523 DF, p-value: < 2.2e-16
summary(age2_gender_bmi.tc)
##
## Call:
## lm(formula = tc ~ age + I(age^2) + gender + bmi, data = dat)
##
## Residuals:
                1Q Median
                                3Q
##
       Min
                                       Max
  -2.6861 -0.5977 -0.0903 0.5483
                                   4.1049
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                2.6223982
                           0.4139550
                                       6.335 5.13e-10 ***
## age
                0.0792126
                           0.0189701
                                       4.176 3.48e-05 ***
## I(age^2)
               -0.0006227
                           0.0002087
                                      -2.984
                                             0.00298 **
## gender
                0.0774204
                           0.0847893
                                       0.913
                                              0.36162
## bmi
                0.0298810
                           0.0102355
                                       2.919
                                              0.00366 **
## ---
                 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
## Residual standard error: 0.9622 on 522 degrees of freedom
## Multiple R-squared: 0.1661, Adjusted R-squared: 0.1597
```

If bmi confounds the effect of gender, bmi needs to follow the following conditions:

- 1) bmi needs to be associated with gender in the study base (potentially yes)
- 2) bmi needs to be associated with total cholesterol in the absence of exposure (potentially yes)
- 3) bmi is not a downstream consequence of total cholesterol or gender (potentially yes)

Therefore, it is worth testing if bmi truly confounds the effect of gender.

F-statistic: 25.99 on 4 and 522 DF, p-value: < 2.2e-16

After adjusting for linear and quadratic age, adding bmi to the linear regression alters the coefficient of gender from 0.0945 to 0.0774, which decreases by 18%. The change in the coefficient of gender is larger than 10%, indicating that bmi confounds the effect of gender.

If gender confounds the effect of bmi, gender needs to qualify the following:

- 1) gender needs to be associated with bmi in the study base(potentially yes)
- 2) gender needs to be associated with total cholesterol in the absence of bmi (no, the gender predictor alone does not tell info about tc)
- 3) gender is not a downstream consequence of total cholesterol or bmi(yes)

According to the conventional definition of confounders, gender is not a confounder for the effect of bmi. Let's take a look at the data:

After adjusting for linear and quadratic age, adding gender to the linear regression alters the coefficient of bmi from 0.0305 to 0.0299, which decreases by 1.9% (< 10%), indicating that gender does not confound the effect of bmi.

#1(b)

```
age2_gender_bmi_emm.tc = lm(tc~age+I(age^2)+bmi+gender+gender*bmi, data = dat)
summary(age2_gender_bmi_emm.tc)
##
## Call:
## lm(formula = tc ~ age + I(age^2) + bmi + gender + gender * bmi,
##
       data = dat)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
## -2.7144 -0.6025 -0.0793 0.5501
                                    4.1128
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
                          0.4983415
                                       4.906 1.24e-06 ***
## (Intercept)
               2.4450100
                0.0790801
                           0.0189820
                                       4.166 3.63e-05 ***
## I(age^2)
               -0.0006209
                           0.0002088
                                      -2.973
                                              0.00308 **
## bmi
                0.0373623
                           0.0155395
                                       2.404
                                              0.01655 *
                                       0.791
                                              0.42942
## gender
                0.3810542
                           0.4818589
              -0.0125739
                           0.0196427
                                      -0.640
                                              0.52237
## bmi:gender
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9628 on 521 degrees of freedom
## Multiple R-squared: 0.1667, Adjusted R-squared: 0.1587
## F-statistic: 20.85 on 5 and 521 DF, p-value: < 2.2e-16
```

Since the p-value for the coefficient of bmi&gender interaction term is 0.5224, which is way larger than 0.05, bmi does not serve as an effect modifier of the effect of gender. Gender does not serve as an effect modifier either, because gender and bmi share the same interaction term and being an effect modifier is reciprocal.

Problem 2

```
\#2(a)
```

```
summary(age2_gender_bmi.tc)
```

```
##
## Call:
## lm(formula = tc ~ age + I(age^2) + gender + bmi, data = dat)
##
## Residuals:
##
       Min
                1Q Median
                                3Q
                                       Max
  -2.6861 -0.5977 -0.0903
                           0.5483
                                    4.1049
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
                                       6.335 5.13e-10 ***
## (Intercept)
                2.6223982
                           0.4139550
                                       4.176 3.48e-05 ***
## age
                0.0792126
                           0.0189701
## I(age^2)
               -0.0006227
                           0.0002087
                                      -2.984
                                              0.00298 **
## gender
                0.0774204
                           0.0847893
                                       0.913
                                              0.36162
## bmi
                0.0298810
                           0.0102355
                                       2.919
                                              0.00366 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
## ## Residual standard error: 0.9622 on 522 degrees of freedom ## Multiple R-squared: 0.1661, Adjusted R-squared: 0.1597 ## F-statistic: 25.99 on 4 and 522 DF, p-value: < 2.2e-16 tc = 2.6223 + 0.079*age - 0.0006227*age^2 + 0.0774*gender + 0.02988*bmi \\ tc (man,30-yo,bmi30) = 2.6223 + 0.079*30 - 0.0006227*(30^2) + 0.0774*0 + 0.02988*30 = 5.33 mmol/l \\ After unit conversion, predicted <math>tc = 5.33*38.67 = 206.11 \text{ mg/dl}.
```

#2(b)

In R, rstandard() gives the internally studentized residuals and rstudent() gives the externally studentized residuals.

Raw residuals(residuals() in R):

$$e_i = Y_i - \hat{Y}_i$$

Standardized residuals (no specific function in R, but can be calculated by dividing raw residuals by the estimated standard deviation):

$$z_i = \frac{e_i}{s_e} = \frac{e_i}{\sqrt{SSE/(N-p-1)}}$$

Internally studentized residuals(rstandard() in R):

$$r_i = \frac{e_i}{SE(e_i)} = \frac{e_i}{\hat{\sigma}\sqrt{1 - H_{ii}}}$$

Externally studentized residuals(rstudent() in R):

$$r_{(i)} = \frac{e_i}{\hat{\sigma_{(i)}}\sqrt{1 - H_{ii}}}$$

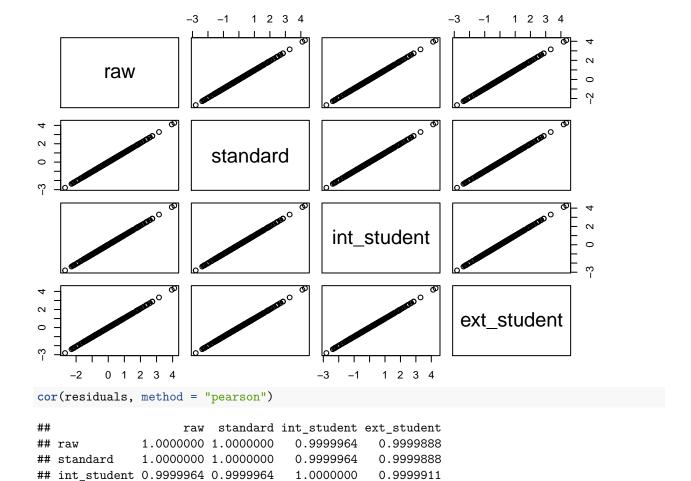
 $Reference: \ https://web.stanford.edu/class/stats191/notebooks/Diagnostics_for_multiple_regression.html \# Types-of-residuals and lab notes.$

#2(c)

```
standardized_res = function(model,n,p){
  raw = residuals(model)
  SSE = sum(raw^2)
  res = raw/(sqrt(SSE/(n-p-1)))
  return(res)
}
standard = standardized_res(age2_gender_bmi.tc, 527, 4)

raw = residuals(age2_gender_bmi.tc)
standard = standardized_res(age2_gender_bmi.tc, 527, 4)

int_student = rstandard(age2_gender_bmi.tc)
ext_student = rstudent(age2_gender_bmi.tc)
residuals = data.frame(raw,standard,int_student,ext_student)
plot(residuals)
```



Pearson correlation coefficient between raw and standardized residuals is 1 which makes sense because standardized residuals are directly derived from raw residuals. Pearson correlation coefficients between other pairs of residuals are just very close to 1, which indicates strong linear association between either pair of residuals, but not exactly 1 due to slight difference in the ways they're calculated.

1.0000000

0.9999911

For this SCCS2 example, we can use any residual but the scaled residuals (standardized and studentized) are preferred.

#2(d)

ext_student 0.9999888 0.9999888

```
summary(raw)
      Min. 1st Qu.
                   Median
                              Mean 3rd Qu.
                                              Max.
## -2.6861 -0.5977 -0.0903 0.0000 0.5483
                                           4.1049
summary(standard)
             1st Qu.
                                                    Max.
##
                       Median
                                        3rd Qu.
                                  Mean
## -2.79153 -0.62113 -0.09385 0.00000 0.56978 4.26604
summary(ext_student)
       Min.
               1st Qu.
                          Median
                                      Mean
                                             3rd Qu.
## -2.818656 -0.623985 -0.094891 0.000411
                                           0.571104
                                                      4.351831
```

summary(int_student)

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## -2.800091 -0.624351 -0.094982 -0.000089 0.571473 4.278930
```

The mean raw residuals is zero.

The mean standardized residuals is also zero.

The mean ext studentized residuals is 0.00041, which is close to zero.

The mean int studentized residuals is -0.000089, which is close to zero.

Assuming the regression line is the "best" fit, then the sum of residuals should be zero, thus the raw and standardized residuals means always equal zero in the "best" fit model.

While the internal and external studentized residuals should have non-zero means since each residual is scaled with respect to a specific term associated with each observation.

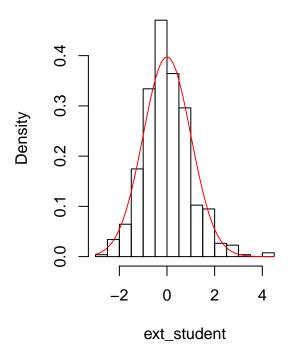
#2(e)

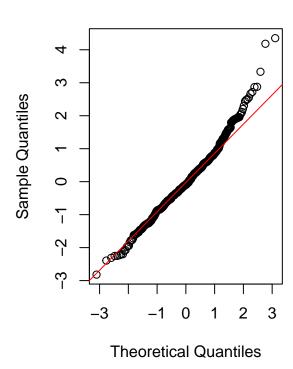
```
par(mfrow = c(1,2))
hist(ext_student, probability = T)
m = mean(ext_student)
std = sqrt(var(ext_student))
curve(dnorm(x, mean = m, sd = std), col = "red", add = T)

qqnorm(ext_student)
qqline(ext_student, col = "red")
```

Histogram of ext_student

Normal Q-Q Plot





According to the histogram, the median is slightly to the left of zero and the distribution is slightly right skewed.

The normal qq plot of the external studentized residuals has a longer right tail deviating up from the reference normal qq line.

Therefore, the external studentized residuals are not normally distributed.

#2(f)

```
dat$stdres = standard
resid2 = dat[dat$stdres > 2,c("age","gender","bmi","tc","stdres")]
resid3 = dat[dat$stdres > 3,c("age","gender","bmi","tc","stdres")]
resid2
##
            age gender
                                        stdres
                             bmi
                                   tc
## 43
       20.79398
                     0 21.56679 7.07 2.520523
## 66
       48.34497
                     0 20.34894 7.97 2.458422
## 92
       48.31485
                     0 21.87004 7.59 2.016860
## 96
       40.42163
                     0 22.99086 7.82 2.417567
                     0 29.83798 9.93 4.102769
## 103 54.57906
## 149 28.74196
                     0 23.27265 7.47 2.483763
## 191 39.44422
                     0 24.41728 8.10 2.694207
## 243 53.38809
                     0 28.77855 7.91 2.051174
                     0 23.05327 8.35 2.644643
## 256 57.27584
## 284 41.52772
                     0 25.11651 8.33 2.849190
## 387 63.32649
                     1 23.42319 8.02 2.183908
## 409 57.07050
                     1 24.02658 8.11 2.286241
## 451 46.19028
                     1 25.94075 9.91 4.266038
## 512 58.08898
                     1 29.40531 8.81 2.838760
## 514 67.12663
                     1 24.61937 9.10 3.277174
## 516 51.52088
                     1 31.25248 8.08 2.097499
resid3
##
            age gender
                                   tc
                     0 29.83798 9.93 4.102769
## 103 54.57906
## 451 46.19028
                     1 25.94075 9.91 4.266038
## 514 67.12663
                     1 24.61937 9.10 3.277174
predict(age2_gender_bmi.tc)[c(103,451,514)]
##
        103
                 451
                          514
```

There are three individuals having standardized residuals >3. For each of the cases, the predicted tc value is much lower than the true tc. For subject 103, the predicted tc is 5.98, while his real tc is 9.93. For subject 451, the predicted tc is 5.81, whereas her real tc is 9.91. And for subject 514, the predicted tc is 5.95 while her real tc is 9.10. The high residuals of these three observations are resulting from the large difference between the predicted and true values.

Problem 3

5.982244 5.805144 5.946646

```
#3(a)
hat_val = hat(model.matrix(age2_gender_bmi.tc))
sum(hat_val<0)
## [1] 0</pre>
```

```
mean(hat_val)
## [1] 0.009487666

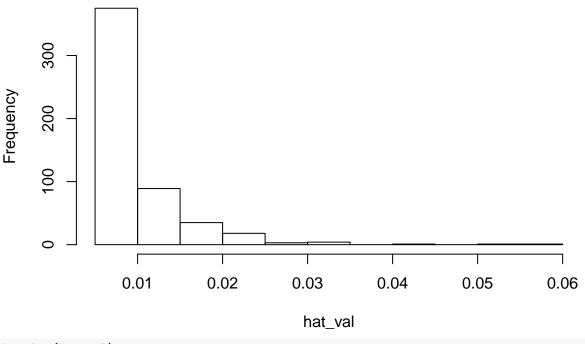
p = 4
n = nrow(dat)
(p+1)/n
```

[1] 0.009487666

After counting for negative elements, the function returns zero, which means no negative element in the hat values list. The mean value of hat_val is 0.00949, where (p+1)/n also equals 0.00949. Therefore, in this case, all hat values are positive and average out to equal (p+1)/n.

```
#3(b)
dat$hat = hat_val
hist(hat_val)
```

Histogram of hat_val



boxplot(hat_val)

```
0
                                           0
0.04
                                           0
                                           8
0.03
0.02
0.01
lev1 = dat[dat$hat > 2*(p+1)/n, c("age", "gender", "bmi", "tc", "hat")]
head(lev1)
             age gender
##
                             bmi
                                    tc
                      0 24.61002 4.40 0.02130464
## 22
       18.55989
       17.88912
                      0 18.99863 4.95 0.02088954
## 101 70.64203
                      0 21.43580 4.80 0.02089109
## 163 78.28063
                      0 24.52592 4.76 0.05294147
                      0 39.52477 6.18 0.02806296
## 171 54.45585
## 177 69.31691
                      0 31.04451 5.65 0.02068107
lev2 = dat[dat$hat > 4*(p+1)/n, c("age", "gender", "bmi", "tc", "hat")]
lev2
##
             age gender
                             bmi
                                    tc
## 163 78.28063
                      0 24.52592 4.76 0.05294147
## 242 78.74332
                      0 23.12740 6.42 0.05594461
                      1 40.49367 5.90 0.04034805
## 475 67.48528
attach(dat)
mean(age)
## [1] 43.48676
mean(bmi)
## [1] 24.16128
```

A total of 3 individuals have leverage >4(p+1)/n. Leverage measures how far observation i's covariates are from the overall covariate average. For the individuals with high leverage, they are of older age (>78 yo), much larger than average age of 43. The other individual with high leverage has extremely high bmi value (bmi = 40.49), much higher than average bmi of 24.16.

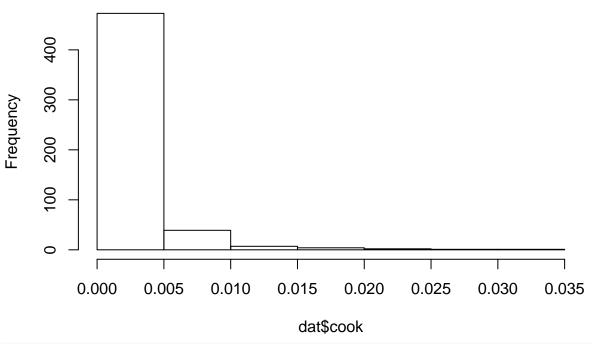
Problem 4

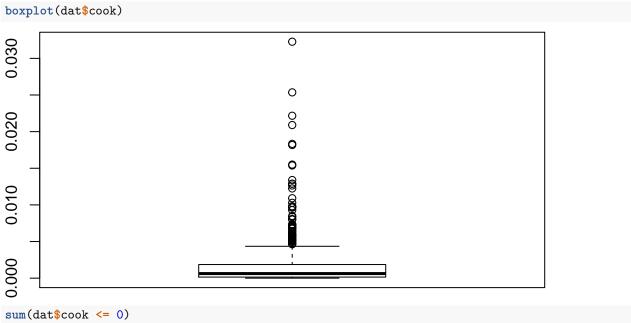
detach(dat)

#4(a)

```
dat$cook = cooks.distance(age2_gender_bmi.tc)
hist(dat$cook)
```

Histogram of dat\$cook





[1] 0

No non-positive cook's distance in this example.

#4(b)

```
cook1 = dat[dat$cook > 4/n, c("age", "gender", "bmi", "tc", "cook")]
cook2 = dat[dat$cook > 12/n, c("age", "gender", "bmi", "tc", "cook")]
head(cook1)
##
            age gender
                             bmi
                                   tc
                                              cook
## 25
                      0 16.40226 3.78 0.012648252
       63.53456
## 43
       20.79398
                      0 21.56679 7.07 0.018331201
## 66
       48.34497
                      0 20.34894 7.97 0.009814110
## 93
       24.73922
                      0 20.78217 2.54 0.009640282
## 99
       68.68720
                      0 25.80639 3.90 0.013392039
                      0 29.83798 9.93 0.025358620
## 103 54.57906
cook2
##
            age gender
                             bmi
                                   tc
                                             cook
## 103 54.57906
                      0 29.83798 9.93 0.02535862
## 514 67.12663
                      1 24.61937 9.10 0.03227442
```

There are two observations with high cook's distance (>4/n), this is probably due to their high total cholesterol level though their age and bmi are not too high, which could potentially have high influence on the regression model.

```
#4(c)
```

```
dat$dfit = dffits(age2_gender_bmi.tc)
high_dfit = dat[abs(dat$dfit)>(2*sqrt((p+1)/n)), c("age", "gender", "bmi", "tc", "dfit")]
high_dfit = high_dfit[order(-abs(high_dfit$dfit)),]
high_dfit
```

```
##
            age gender
                                           dfit
                            bmi
                                  tc
                                      0.4055829
## 514 67.12663
                     1 24.61937 9.10
## 103 54.57906
                     0 29.83798 9.93
                                      0.3616622
## 451 46.19028
                     1 25.94075 9.91
                                      0.3385871
## 468 32.07666
                     1 36.42918 3.72 -0.3242795
       20.79398
## 43
                     0 21.56679 7.07 0.3043415
## 455 70.35729
                     1 30.48458 7.90 0.3021814
## 459 38.62560
                     1 33.87477 3.70 -0.2795034
## 477 69.21835
                     1 37.40035 4.94 -0.2776338
                     0 25.80639 3.90 -0.2596078
## 99
       68.68720
## 512 58.08898
                     1 29.40531 8.81
                                     0.2556269
                     1 15.88121 7.48 0.2545604
## 498 56.22450
       63.53456
                     0 16.40226 3.78 -0.2521480
## 25
                     0 24.52592 4.76 -0.2474526
## 163 78.28063
## 273 73.29227
                     0 19.81879 4.43 -0.2337445
## 422 36.58864
                     1 37.53626 4.54 -0.2269843
## 66
       48.34497
                     0 20.34894 7.97
                                      0.2226094
## 387 63.32649
                     1 23.42319 8.02
                                      0.2219687
## 204 59.16496
                     0 27.67561 3.27 -0.2209856
## 93 24.73922
                     0 20.78217 2.54 -0.2205392
## 284 41.52772
                     0 25.11651 8.33 0.2169576
## 430 58.23135
                     1 18.95034 7.56
                                      0.2065473
## 516 51.52088
                     1 31.25248 8.08
                                      0.2053538
## 256 57.27584
                     0 23.05327 8.35
                                      0.2022190
## 191 39.44422
                     0 24.41728 8.10 0.2011222
```

Yes, there are a few individuals that have high influenc (

$$|DFFITS_i| > 2\sqrt{(p+1)/n}$$

). After sorting by dffits values, we found that the first two observations with the highest dffits values are the ones observed from #4(b) with high cook's distances.

Problem 5

##

Min

1Q

-2.65432 -0.57138 -0.07401 0.56057

Median

```
#5(a)
dat[(dat$stdres > 3) & (dat$cook > 12/n) & (dat$hat > 4*(p+1)/n), c("age", "gender", "bmi", "tc", "hat", "c
## [1] age
              gender bmi
                                    hat
                                            cook
                                                   stdres
## <0 rows> (or 0-length row.names)
dat[(dat$stdres > 3) & (dat$cook > 12/n), c("age", "gender", "bmi", "tc", "hat", "cook", "stdres")]
##
            age gender
                                   tc
                                               hat
                                                         cook
                      0 29.83798 9.93 0.007421154 0.02535862 4.102769
## 103 54.57906
## 514 67.12663
                      1 24.61937 9.10 0.014590264 0.03227442 3.277174
dat[(dat$stdres > 3) & (dat$hat > 4*(p+1)/n), c("age", "gender", "bmi", "tc", "hat", "cook", "stdres")]
## [1] age
              gender bmi
                                    hat
                                                   stdres
                                            cook
## <0 rows> (or 0-length row.names)
```

There is no individual meeting all of the three conditions (high standardized residual, high cook's distance and high leverage). There are two individuals having both high standardized residual and high cook's distance. There is no overlap between high standardized residual and high leverage.

```
#5(b)
dat[(dat$stdres > 3) | (dat$cook > 12/n) | (dat$hat > 4*(p+1)/n),c("age", "gender", "bmi", "tc", "hat", "co
##
            age gender
                             bmi
                                   tc
                                              hat
                                                          cook
                                                                   stdres
## 103 54.57906
                     0 29.83798 9.93 0.007421154 0.025358620
                                                               4.1027693
## 163 78.28063
                     0 24.52592 4.76 0.052941469 0.012244316 -1.0184298
## 242 78.74332
                     0 23.12740 6.42 0.055944608 0.007234465
                                                               0.7591132
                     1 25.94075 9.91 0.006016941 0.022166501
## 451 46.19028
                                                               4.2660375
## 475 67.48528
                     1 40.49367 5.90 0.040348053 0.002552495 -0.5397217
                     1 24.61937 9.10 0.014590264 0.032274418 3.2771738
## 514 67.12663
newdat = dat[!((dat\$stdres > 3) | (dat\$cook > 12/n) | (dat\$hat > 4*(p+1)/n)),c("age","gender","bmi","tc
nrow(newdat)
newmodel = lm(tc~ age +I(age^2) + gender + bmi, data = newdat)
summary(newmodel)
##
## lm(formula = tc ~ age + I(age^2) + gender + bmi, data = newdat)
##
## Residuals:
```

Max

3Q

```
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
               2.7164836 0.4078872
                                       6.660 7.03e-11 ***
## (Intercept)
## age
                0.0762966
                          0.0189559
                                       4.025 6.55e-05 ***
## I(age^2)
               -0.0006011
                           0.0002105
                                     -2.856
                                              0.00447 **
## gender
                0.0607624
                           0.0816723
                                       0.744
                                              0.45723
## bmi
                0.0288282
                           0.0099503
                                       2.897
                                             0.00392 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9221 on 516 degrees of freedom
## Multiple R-squared: 0.166, Adjusted R-squared: 0.1595
## F-statistic: 25.68 on 4 and 516 DF, p-value: < 2.2e-16
summary(age2_gender_bmi.tc)
##
## Call:
## lm(formula = tc ~ age + I(age^2) + gender + bmi, data = dat)
##
## Residuals:
##
      Min
                1Q Median
                                30
                                       Max
  -2.6861 -0.5977 -0.0903
                           0.5483
                                   4.1049
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
               2.6223982 0.4139550
                                       6.335 5.13e-10 ***
## age
                0.0792126
                           0.0189701
                                       4.176 3.48e-05 ***
               -0.0006227
                           0.0002087
                                      -2.984
                                              0.00298 **
## I(age^2)
                           0.0847893
## gender
                0.0774204
                                       0.913
                                              0.36162
## bmi
                0.0298810
                           0.0102355
                                       2.919
                                             0.00366 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9622 on 522 degrees of freedom
## Multiple R-squared: 0.1661, Adjusted R-squared: 0.1597
## F-statistic: 25.99 on 4 and 522 DF, p-value: < 2.2e-16
```

After eliminating 6 observations with standardized residual >3, cook's distance >12/n, or leverage >4(p+1)/n and fitting regression model to the new data, we found that the coefficient of age changes from 0.0792 to 0.0763(-3.7%), the coefficient for quadratic age from -0.0006227 to -0.0006011 (+3.4%), the coefficient for gender from 0.0774 to 0.0608(-21%), and the coefficient for bmi from 0.0299 to 0.0288(-3.7%). Only the change of coefficient for gender is significant. The overall findings do not change much.