## BST210 HW3

#### Wenjie

#### Question 1

```
#1(a)
dat <- read.csv("/Users/BiankaUrsul/Documents/Class Info/Harvard HSPH/Fall2019/BST210/Homework#1/Data a
library(splines2)
library(gam)
## Loading required package: splines
## Loading required package: foreach
## Loaded gam 1.16.1
sorted_dat = dat[order(dat$age),]
head(sorted_dat)
##
         X caseID ethnic height weight waist
                                                 hip
                                                            tg hdl
                                                       tc
## 44
        44
              417
                       2 170.30
                                   55.1 68.00 88.75 4.95 0.68 1.14 3.50
## 343 343
             2818
                       1 147.05
                                   47.0 65.00 89.00 4.19 1.78 1.29 2.09
## 405 405
             3456
                       3 166.65
                                   58.0 64.75
                                               99.50 3.92 0.60 1.47 2.18
                                               94.00 4.23 0.89 1.81 2.02
## 342 342
             2817
                       1 164.45
                                   52.3 62.50
## 404 404
                                   59.5 69.00 101.00 4.24 0.73 1.25 2.66
             3452
                       3 161.05
## 22
        22
              227
                       1 170.45
                                   71.5 75.00 100.00 4.40 0.73 1.33 2.74
##
       diabetes hypertension educ drink smoke gender alcohol
                                                                    age ihd
## 44
                           1
                                 3
                                       0
                                             1
                                                    0
                                                            1 17.88912
## 343
              0
                                3
                                       0
                                             1
                                                    1
                                                            1 18.19028
                                                                          0
                           1
## 405
              0
                                4
                                       0
                                             1
                                                            1 18.22861
                           1
## 342
                                3
                                                            1 18.40931
              0
                                       0
                                             1
                                                    1
                                                                          0
                           1
## 404
                                2
                                                            2 18.47502
              0
                           1
                                       1
                                             1
                                                    1
## 22
              0
                           1
                                 3
                                       1
                                             1
                                                    Ω
                                                            2 18.55989
##
       Dummy2
## 44
## 343
            0
            2
## 405
## 342
            0
## 404
            0
## 22
mod1 = lm(tc~age, data = sorted_dat)
mod2 = lm(tc~age+I(age^2), data = sorted_dat)
mod3 = lm(tc~bSpline(age, knots = quantile(age, c(0.25,0.5,0.75)), degree =2), data = sorted_dat)
attach(sorted_dat)
plot(age, tc)
lines(age, fitted(mod1), col = "red")
lines(age, fitted(mod2), col = "blue")
lines(age, fitted(mod3), col = "green")
legend("topright", c("linear age", "linear & quadratic age", "Spline modeling of age"), col = c("red",
```

```
10
                                                        0
                                                                        linear age
                                                                        linear & quadratic age

    Spline modeling of age

                                                            0
     \infty
ဍ
                                                                                   0
                                            0
                                                             0
                     0
               20
                          30
                                      40
                                                                         70
                                                  50
                                                             60
                                                                                     80
                                               age
summary1 = summary(mod1)
summary2 = summary(mod2)
summary3 = summary(mod3)
summary1
##
## Call:
## lm(formula = tc ~ age, data = sorted_dat)
##
## Residuals:
##
       Min
                 1Q Median
## -2.6589 -0.6383 -0.0557 0.5009 4.3216
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4.376191
                           0.135848 32.214
                                               <2e-16 ***
## age
               0.026243
                           0.002966
                                       8.849
                                               <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9801 on 525 degrees of freedom
## Multiple R-squared: 0.1298, Adjusted R-squared: 0.1281
## F-statistic: 78.31 on 1 and 525 DF, p-value: < 2.2e-16
summary2
##
## Call:
## lm(formula = tc ~ age + I(age^2), data = sorted_dat)
##
## Residuals:
                                         Max
       Min
                 1Q Median
                                 3Q
```

## -2.6542 -0.6410 -0.0461 0.5151 4.1698

```
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.0658041 0.3907005
                                       7.847 2.41e-14 ***
## age
                0.0920305
                          0.0186508
                                       4.934 1.08e-06 ***
               -0.0007389 0.0002069 -3.572 0.000387 ***
## I(age^2)
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9693 on 524 degrees of freedom
## Multiple R-squared: 0.1505, Adjusted R-squared: 0.1472
## F-statistic: 46.41 on 2 and 524 DF, p-value: < 2.2e-16
summary3
##
## Call:
## lm(formula = tc ~ bSpline(age, knots = quantile(age, c(0.25,
       0.5, 0.75)), degree = 2), data = sorted_dat)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
## -2.6732 -0.6199 -0.0385 0.5465 4.0640
##
## Coefficients:
                                                                        Estimate
##
## (Intercept)
                                                                           4.4568
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                           0.7025
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                           0.7616
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                           1.6759
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                           1.3197
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                           1.2961
                                                                         Std. Error
## (Intercept)
                                                                             0.2402
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                             0.4027
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                             0.2427
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                             0.3012
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                             0.3183
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                             0.5712
##
                                                                         t value
## (Intercept)
                                                                          18.557
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                           1.745
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                           3.138
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                           5.563
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                           4.146
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                           2.269
##
                                                                        Pr(>|t|)
## (Intercept)
                                                                          < 2e-16
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                           0.0817
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                           0.0018
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3 4.24e-08
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4 3.94e-05
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                           0.0237
##
## (Intercept)
                                                                         ***
```

```
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1 .
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2 **
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3 ***
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4 ***
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9672 on 521 degrees of freedom
## Multiple R-squared: 0.1591, Adjusted R-squared: 0.1511
## F-statistic: 19.72 on 5 and 521 DF, p-value: < 2.2e-16

detach(sorted_dat)</pre>
```

For the spline modeling of age, I choose the knot points to be the points at 0.25, 0.5 and 0.75 percentile of age data and the order of polynomial set to be 2 (quadratic). Comparing the three sets of fitted values, the spline model curve has a similar pattern as the linear and quadratic age model with age larger than 60, and for age < 60, the spline model curve has a few more twists than the quadratic model. At age < 40, the trends of all three models look similar to each other. The  $R^2$  value is 0.1281 for the linear age model, 0.1472 for the linear and quadratic age model and 0.1511 for the spline model. Therefore, according to the  $R^2$  value, I would recommend the spline model as the "best".

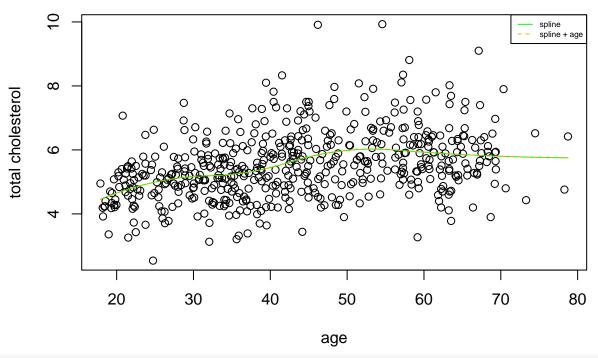
```
#1(b)
```

```
library(splines2)
mod4 = lm(tc~age + bSpline(age, knots = quantile(age, c(0.25,0.5,0.75)), degree = 2), data = sorted_dat
summary4 = summary(mod4)
summary4
##
## Call:
## lm(formula = tc ~ age + bSpline(age, knots = quantile(age, c(0.25,
       0.5, 0.75)), degree = 2), data = sorted_dat)
##
## Residuals:
##
       Min
                10 Median
                                3Q
                                       Max
## -2.6732 -0.6199 -0.0385
                            0.5465
                                    4.0640
##
## Coefficients: (1 not defined because of singularities)
##
                                                                         Estimate
## (Intercept)
                                                                         4.075773
## age
                                                                         0.021299
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1 0.551468
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2 0.349448
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3 1.006160
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4 0.263129
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
##
                                                                         Std. Error
## (Intercept)
                                                                           0.349471
                                                                           0.009386
## age
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                           0.379161
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                           0.253330
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                           0.294457
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                           0.578250
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                                 ΝA
```

```
##
                                                                         t value
## (Intercept)
                                                                          11.663
                                                                           2.269
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                           1.454
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                           1.379
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                           3.417
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                           0.455
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                              NA
##
                                                                         Pr(>|t|)
## (Intercept)
                                                                          < 2e-16
## age
                                                                         0.023660
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1 0.146426
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2 0.168357
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3 0.000683
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4 0.649267
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
##
## (Intercept)
                                                                         ***
## age
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3 ***
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9672 on 521 degrees of freedom
## Multiple R-squared: 0.1591, Adjusted R-squared: 0.1511
## F-statistic: 19.72 on 5 and 521 DF, p-value: < 2.2e-16
summary3
##
## Call:
## lm(formula = tc ~ bSpline(age, knots = quantile(age, c(0.25,
       0.5, 0.75)), degree = 2), data = sorted_dat)
##
##
## Residuals:
##
      Min
                1Q Median
                                       Max
## -2.6732 -0.6199 -0.0385 0.5465 4.0640
## Coefficients:
##
                                                                         Estimate
                                                                           4.4568
## (Intercept)
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                           0.7025
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                           0.7616
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                           1.6759
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                           1.3197
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                           1.2961
##
                                                                         Std. Error
## (Intercept)
                                                                             0.2402
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                             0.4027
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                             0.2427
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                             0.3012
```

```
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                            0.3183
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                            0.5712
##
                                                                        t value
## (Intercept)
                                                                         18.557
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                          1.745
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                          3.138
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                          5.563
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                          4.146
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                          2.269
##
                                                                        Pr(>|t|)
## (Intercept)
                                                                         < 2e-16
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                          0.0817
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                          0.0018
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3 4.24e-08
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4 3.94e-05
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
##
## (Intercept)
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1.
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2 **
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3 ***
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4 ***
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5 *
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9672 on 521 degrees of freedom
## Multiple R-squared: 0.1591, Adjusted R-squared: 0.1511
## F-statistic: 19.72 on 5 and 521 DF, p-value: < 2.2e-16
attach(sorted_dat)
plot(age, tc, main = "Total cholesterol vs. age", xlab = "age", ylab = "total cholesterol")
lines(age, fitted(mod3), col = "green", lty = "solid")
lines(age, fitted(mod4), col = "orange", lty = "dashed")
legend("topright", c("spline", "spline + age"), col = c("green", "orange"), lty = c("solid", "dashed"),
```

# Total cholesterol vs. age



difference = fitted(mod3)-fitted(mod4)
difference

| ## | 44            | 343           | 405          | 342           | 404          |
|----|---------------|---------------|--------------|---------------|--------------|
| ## | -8.881784e-15 | -3.375078e-14 | 4.263256e-14 | -8.881784e-16 | 8.881784e-16 |
| ## | 22            | 341           | 21           | 20            | 340          |
| ## | 8.881784e-16  | 8.881784e-16  | 8.881784e-16 | 8.881784e-16  | 8.881784e-16 |
| ## | 339           | 338           | 19           | 18            | 337          |
| ## | 8.881784e-16  | 1.776357e-15  | 8.881784e-16 | 8.881784e-16  | 8.881784e-16 |
| ## | 336           | 335           | 403          | 334           | 47           |
| ## | 8.881784e-16  | 8.881784e-16  | 8.881784e-16 | 0.000000e+00  | 8.881784e-16 |
| ## | 43            | 333           | 504          | 282           | 232          |
| ## | 0.000000e+00  | 8.881784e-16  | 0.000000e+00 | 8.881784e-16  | 8.881784e-16 |
| ## | 236           | 233           | 483          | 485           | 45           |
| ## | 1.776357e-15  | 8.881784e-16  | 8.881784e-16 | 8.881784e-16  | 8.881784e-16 |
| ## | 385           | 124           | 384          | 383           | 17           |
| ## | 8.881784e-16  | 8.881784e-16  | 8.881784e-16 | 0.000000e+00  | 8.881784e-16 |
| ## | 15            | 402           | 30           | 14            | 16           |
| ## | 8.881784e-16  | 0.000000e+00  | 0.000000e+00 | 0.000000e+00  | 8.881784e-16 |
| ## | 153           | 332           | 13           | 172           | 331          |
| ## | 8.881784e-16  | 8.881784e-16  | 0.000000e+00 | 0.000000e+00  | 8.881784e-16 |
| ## | 401           | 330           | 382          | 42            | 95           |
| ## | 8.881784e-16  | 0.000000e+00  | 0.000000e+00 | 8.881784e-16  | 0.000000e+00 |
| ## | 400           | 41            | 12           | 399           | 40           |
| ## | 8.881784e-16  | 8.881784e-16  | 8.881784e-16 | 0.000000e+00  | 8.881784e-16 |
| ## | 329           | 94            | 328          | 93            | 39           |
| ## |               | 0.000000e+00  |              |               | 8.881784e-16 |
| ## | 87            | 38            | 11           | 321           | 378          |
| ## |               | 8.881784e-16  |              |               | 0.000000e+00 |
| ## | 396           | 423           | 8            | 86            | 6            |

```
0.000000e+00 0.000000e+00
                                  8.881784e-16
                                                0.000000e+00
                                                                 8.881784e-16
##
                                              9
              318
                             320
                                                           190
                  -8.881784e-16
                                  0.000000e+00
                                                 0.000000e+00
##
    0.000000e+00
                                                                 8.881784e-16
              319
                                             29
                                                            85
                                                                           84
##
                              10
##
    0.000000e+00
                   0.000000e+00
                                  8.881784e-16
                                                 0.000000e+00
                                                                 0.000000e+00
                                            377
##
               37
                              83
                                                           316
##
    8.881784e-16
                   0.000000e+00
                                  0.000000e+00
                                                 0.000000e+00
                                                                 0.000000e+00
##
              463
                              36
                                              5
                                                           314
##
    8.881784e-16
                   0.000000e+00
                                  0.000000e+00
                                                 0.000000e+00
                                                                 0.000000e+00
##
               82
                              46
                                             79
                                                           149
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                 0.000000e+00
                                                                 0.000000e+00
##
               81
                             148
                                            313
                                                           147
                                                                          376
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                 0.000000e+00
                                                                 0.000000e+00
##
              509
                             356
                                            310
                                                           188
                                                                             3
    0.000000e+00
##
                   0.000000e+00
                                  0.000000e+00
                                                 0.000000e+00
                                                                 0.000000e+00
##
               78
                                            311
                                                            35
                                                                          375
    0.000000e+00
                   0.000000e+00
                                  0.00000e+00
                                                 0.000000e+00
##
                                                                -8.881784e-16
##
                             189
                                            312
                                                             4
              146
    0.000000e+00
                   8.881784e-16
                                  0.000000e+00
                                                 0.000000e+00
                                                                 0.000000e+00
##
##
               28
                             462
                                            186
                                                           307
                                  0.000000e+00
##
    0.000000e+00
                   0.00000e+00
                                                -8.881784e-16
                                                                 0.000000e+00
              309
##
                                            374
                                  0.000000e+00
##
    0.00000e+00
                   0.000000e+00
                                                 0.000000e+00
                                                               -8.881784e-16
                2
##
                                            381
                                                            73
##
   -8.881784e-16
                  -8.881784e-16
                                  0.000000e+00
                                                 0.000000e+00
                                                                 8.881784e-16
##
               76
                             305
                                            468
                                                            75
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00 -8.881784e-16
                                                                 0.000000e+00
##
              306
                              27
                                             26
                                                                          187
                                                                 0.000000e+00
    0.000000e+00
                   0.00000e+00
                                 -8.881784e-16 -8.881784e-16
##
##
                             486
                                             72
              184
                                                           145
##
   -8.881784e-16
                  -8.881784e-16
                                  0.000000e+00 -8.881784e-16
                                                                 0.000000e+00
##
              231
                             415
                                            503
                                                           496
                                                                          271
##
    0.00000e+00
                  -8.881784e-16
                                  0.000000e+00
                                                 0.000000e+00
                                                                 0.000000e+00
##
              501
                             345
                                            281
                                                           235
    -8.881784e-16
                   0.000000e+00
                                 -8.881784e-16
                                                 0.000000e+00
                                                                -8.881784e-16
##
##
              183
                             495
                                            494
                                                           304
##
   -8.881784e-16
                   0.000000e+00
                                 -8.881784e-16
                                                 0.000000e+00
                                                                 0.000000e+00
##
              371
                             251
                                            252
                                                           253
    -8.881784e-16
                   0.000000e+00
                                 -8.881784e-16
                                                 0.000000e+00
                                                               -8.881784e-16
##
                             241
##
              250
                                            135
                                                           136
                   0.000000e+00
                                  0.000000e+00
                                                 0.000000e+00
##
   -8.881784e-16
                                                               -8.881784e-16
##
              269
                             240
                                            270
                                                           482
                                                                          249
##
    0.000000e+00
                  -8.881784e-16
                                  0.00000e+00
                                                 0.000000e+00
                                                                -8.881784e-16
##
              279
                             182
                                            414
                                                           325
##
    0.000000e+00
                  -8.881784e-16
                                  0.00000e+00
                                                 -8.881784e-16
                                                                 0.000000e+00
                                            303
              230
                             395
                                                           492
##
                                                                          368
##
   -8.881784e-16
                   0.000000e+00
                                  0.000000e+00
                                                 0.000000e+00
                                                                 0.000000e+00
##
                1
                             491
                                            493
                                                           521
##
    0.000000e+00
                   0.000000e+00
                                 -8.881784e-16
                                                 0.000000e+00
                                                                -8.881784e-16
##
              370
                             154
                                            422
                                                           239
   -8.881784e-16 -8.881784e-16
                                  0.000000e+00
                                                 0.000000e+00
##
                                                                 0.000000e+00
##
              246
                             234
                                            128
                                                           247
##
   -8.881784e-16 -8.881784e-16 -8.881784e-16
                                                 0.000000e+00
                                                                 0.000000e+00
##
              524
                             511
                                             90
                                                           238
```

```
-8.881784e-16 -8.881784e-16 -8.881784e-16
                                                 0.000000e+00 -8.881784e-16
##
                             289
                                                                           364
              291
                                            461
                                                            366
                                                  0.000000e+00
                                                                -8.881784e-16
##
   -8.881784e-16 -8.881784e-16 -8.881784e-16
               53
                              89
                                            322
                                                            460
                                                                           459
##
##
    0.000000e+00
                   0.000000e+00
                                  -8.881784e-16
                                                  0.00000e+00
                                                                 0.000000e+00
                                            326
##
              120
                             297
                                                            420
                                                                           298
##
   -8.881784e-16
                   0.000000e+00
                                  -8.881784e-16
                                                  0.000000e+00
                                                                 0.000000e+00
##
               57
                             392
                                            179
                                                            150
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              142
                             440
                                             56
                                                            191
##
   -8.881784e-16
                  -8.881784e-16
                                  0.000000e+00
                                                  0.000000e+00
                                                                -8.881784e-16
##
              407
                             355
                                             32
                                                             52
                                                                           292
##
   -8.881784e-16
                   0.00000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              268
                             219
                                            294
                                                             64
   -8.881784e-16
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
##
               96
                             139
                                            441
                                                            413
    0.000000e+00
                   0.000000e+00
                                  -8.881784e-16
                                                 -8.881784e-16
                                                                 0.000000e+00
##
##
              299
                             449
                                             55
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
   -8.881784e-16
##
              140
                             464
                                             63
                                                            284
                                                                           302
                                  0.000000e+00
##
   -8.881784e-16
                  -8.881784e-16
                                                 -8.881784e-16
                                                                 0.000000e+00
##
              443
                             323
                                            185
##
    0.00000e+00
                   0.000000e+00
                                  0.000000e+00
                                                -8.881784e-16
                                                                -8.881784e-16
##
              466
                              62
                                             59
                  -8.881784e-16
##
    0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              237
                             365
                                             31
                                                            215
                                                                           344
   -8.881784e-16
                   0.00000e+00
                                  0.00000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
##
              419
                             193
                                            317
                                                            437
                                                                -8.881784e-16
    0.000000e+00
                   0.00000e+00
                                  0.00000e+00
                                                  0.00000e+00
##
##
               33
                              51
                                            180
                                                            424
                                                                           450
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              213
                             411
                                             97
                                                            467
                                                                           520
##
    0.00000e+00
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              359
                             152
                                                            214
                                                                            60
                                            143
##
    0.000000e+00
                   0.000000e+00
                                  8.881784e-16
                                                  0.000000e+00
                                                                 0.000000e+00
##
              442
                              61
                                            300
                                                            223
##
    0.000000e+00
                  -8.881784e-16
                                  0.000000e+00
                                                  0.000000e+00
                                                                 8.881784e-16
##
              391
                             397
                                             65
                                                                           465
                                                            144
    0.000000e+00
                   0.000000e+00
                                  -8.881784e-16
                                                  0.000000e+00
                                                                 0.000000e+00
##
                             386
                                                            220
##
              278
                                            141
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
##
                                                  8.881784e-16
                                                                -8.881784e-16
##
               67
                             380
                                            296
                                                             88
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.00000e+00
##
                              70
              451
                                            192
                                                            295
                                                                           301
                   0.000000e+00
##
   -8.881784e-16
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
                                            245
                                                             68
##
              489
                             444
                                                                           488
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              367
                              34
                                            379
                                                            324
                                                                           523
##
    0.000000e+00
                   0.00000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              277
                             390
                                             69
                                                             54
                                                                           133
                   8.881784e-16
                                                  0.000000e+00
                                                                 0.000000e+00
##
   -8.881784e-16
                                  8.881784e-16
##
              487
                             151
                                             92
                                                             66
                                                  0.000000e+00
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                                 0.000000e+00
##
              280
                             393
                                            425
                                                            484
                                                                           127
```

```
8.881784e-16
                   0.000000e+00
                                  0.000000e+00
                                                 0.000000e+00
                                                                 0.000000e+00
##
##
                                                            490
               71
                              98
                                            244
                                                                           134
                   0.000000e+00
                                   0.000000e+00
                                                  0.000000e+00
                                                                -8.881784e-16
##
    0.000000e+00
              445
                              49
                                            473
                                                            417
                                                                           225
##
##
    0.000000e+00
                   0.000000e+00
                                   0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
              137
##
                             258
                                            125
                                                            458
                                                                           206
##
    0.000000e+00
                   0.000000e+00
                                   0.00000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              516
                             260
                                            352
                                                            228
##
    8.881784e-16
                   0.000000e+00
                                   0.000000e+00
                                                  8.881784e-16
                                                                 0.000000e+00
              431
##
                             138
                                            515
                                                            109
                                                                           227
##
    8.881784e-16
                   8.881784e-16
                                   0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
                                            288
##
              508
                             507
                                                            203
                                                                           452
##
    0.000000e+00
                   0.000000e+00
                                   0.000000e+00
                                                  8.881784e-16
                                                                 0.000000e+00
##
              427
                             525
                                            160
                                                            360
                                                                           197
##
    0.000000e+00
                   8.881784e-16
                                   0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              255
                             243
                                                            388
                                                                           363
                                            112
                  -8.881784e-16
                                   0.00000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
    0.00000e+00
##
              500
                                            428
                                                            438
                             113
                                                                           171
    0.000000e+00
                   0.00000e+00
                                   0.000000e+00
                                                                 0.000000e+00
##
                                                  8.881784e-16
##
              103
                              48
                                            272
                                                            478
                                   8.881784e-16
##
    0.000000e+00
                   0.00000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              472
                                            259
                             129
                                   0.000000e+00
                                                 -8.881784e-16
##
    0.000000e+00
                   8.881784e-16
                                                                 0.000000e+00
##
              209
                                            456
##
    0.000000e+00
                   0.000000e+00
                                   0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              409
                             201
                                            261
                                                            226
                                                                           256
##
    0.00000e+00
                   0.000000e+00
                                   0.00000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              174
                             207
                                            499
                                                            354
                                                                           116
    0.00000e+00
                   0.00000e+00
                                   0.00000e+00
                                                  0.00000e+00
                                                                 0.000000e+00
##
##
              426
                             349
                                            476
                                                            156
##
    0.000000e+00
                   0.000000e+00
                                   0.000000e+00 -8.881784e-16
                                                                -8.881784e-16
##
              518
                             430
                                            114
                                                             24
                                                                           470
##
    0.00000e+00
                   0.00000e+00
                                   8.881784e-16
                                                 -8.881784e-16
                                                                 8.881784e-16
                             285
                                                            204
                                                                           205
##
              454
                                            102
##
    0.000000e+00
                   0.000000e+00
                                   0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
                                            389
##
              202
                             162
                                                            434
##
    0.000000e+00
                   0.00000e+00
                                   0.000000e+00
                                                  0.000000e+00
                                                                -8.881784e-16
##
              265
                                            347
                             166
                                                            111
    0.000000e+00
                   0.000000e+00
                                   8.881784e-16
                                                  8.881784e-16
                                                                 0.000000e+00
##
##
              132
                             479
                                            436
                                                            350
                                                                 0.000000e+00
    0.000000e+00
                   0.000000e+00
                                   0.000000e+00
                                                  0.000000e+00
##
##
              159
                             194
                                            155
                                                            198
                                                                           527
##
    8.881784e-16
                   0.00000e+00
                                   0.00000e+00
                                                  0.000000e+00
                                                                -8.881784e-16
##
                             100
              513
                                            286
                                                            358
    0.000000e+00
                   0.000000e+00
                                                                 0.000000e+00
##
                                   0.000000e+00
                                                  0.000000e+00
##
                             474
                                            263
                                                            211
              517
                                                                           110
##
    0.000000e+00
                   0.000000e+00
                                   0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              432
                             283
                                            199
                                                            453
                                                                           176
##
    0.000000e+00
                   0.000000e+00
                                  -8.881784e-16
                                                  0.000000e+00
                                                                 0.000000e+00
##
              471
                             221
                                            353
                                                            266
                                                                           519
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              481
                             433
                                            210
                                                            167
##
    0.000000e+00
                   0.000000e+00 -8.881784e-16
                                                  0.000000e+00
                                                                 0.000000e+00
##
              105
                             387
                                            497
                                                            107
                                                                           408
```

```
0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                 0.000000e+00
                                                                 0.000000e+00
##
##
                             212
                                                           208
               25
                                            170
                                                                           195
                  -8.881784e-16
                                                                 0.00000e+00
##
    0.000000e+00
                                  0.000000e+00
                                                  8.881784e-16
##
                             158
                                                                            23
              178
                                            131
                                                           115
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
                                            222
                                                           506
##
              410
                             275
                                                                           168
##
    0.000000e+00
                   0.000000e+00
                                  0.00000e+00
                                                  0.000000e+00
                                                                -8.881784e-16
                                                                           224
##
              522
                             104
                                            526
                                                           126
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.00000e+00
##
              161
                             264
                                            276
                                                           196
                                                                           287
##
    0.000000e+00
                   8.881784e-16
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              173
                             118
                                            514
                                                           108
                                                                           157
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
                                                                 0.000000e+00
##
              435
                             457
                                            475
                                                           267
                                                                           362
##
                   0.000000e+00
                                  0.00000e+00
    0.000000e+00
                                                  0.00000e+00
                                                                 0.000000e+00
##
              274
                                            346
                                                             99
    0.000000e+00
                                  0.00000e+00
                                                  0.000000e+00
##
                   0.00000e+00
                                                                 0.00000e+00
##
              477
                             429
                                            106
                                                                           348
                                                           177
                   0.000000e+00
                                  0.000000e+00
                                                  0.000000e+00
##
   -8.881784e-16
                                                                 0.000000e+00
##
              200
                             455
                                            101
                                                           273
                                                                           262
##
    0.000000e+00
                   0.000000e+00
                                  0.000000e+00 -8.881784e-16
                                                                 0.000000e+00
##
              163
                             242
## -8.881784e-16
                   0.000000e+00
```

#### detach(sorted\_dat)

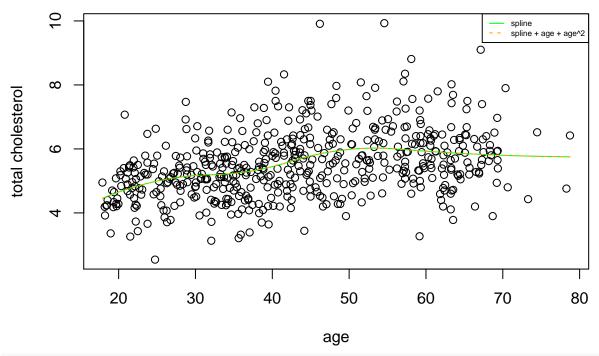
By plotting out the fitted line for spline(age) model and age+spline(age) model, we can see that the two curves overlap, and we found no difference in the predicted values in the two models, which means that the two models give the same prediction. Comparing the residual standard errors, multiple R-squared values and adjusted R-squared values, we found the two models having the exactly same statistics (RSE = 0.9672, multiple R^2 = 0.1591, adjusted R^2 = 0.1511). We can therefore conclude that the linear age model is nested within the spline model.

```
#1(c)
mod5 = lm(tc \sim age + I(age \sim 2) + bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2), data = s
summary5 = summary(mod5)
summary5
##
  Call:
##
  lm(formula = tc ~ age + I(age^2) + bSpline(age, knots = quantile(age,
##
       c(0.25, 0.5, 0.75)), degree = 2), data = sorted_dat)
##
##
  Residuals:
##
       Min
                1Q
                    Median
                                 3Q
                                         Max
   -2.6732 -0.6199 -0.0385
                            0.5465
                                     4.0640
##
##
##
   Coefficients: (2 not defined because of singularities)
                                                                             Estimate
##
##
   (Intercept)
                                                                            3.5341536
                                                                            0.0584534
## age
## I(age^2)
                                                                           -0.0003845
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                            0.3855986
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2 0.0303667
```

```
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3 0.6321756
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                        Std. Error
## (Intercept)
                                                                          1.1037063
                                                                          0.0739899
## age
## I(age^2)
                                                                          0.0008450
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                         0.4938852
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                         0.5458165
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                                 NA
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                                 NA
                                                                         t value
## (Intercept)
                                                                           3.202
                                                                           0.790
## age
## I(age^2)
                                                                          -0.455
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                           0.781
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                           0.056
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                           0.933
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                             NA
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                             NΑ
                                                                        Pr(>|t|)
                                                                          0.00145
## (Intercept)
                                                                          0.42988
## age
## I(age^2)
                                                                          0.64927
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                         0.43531
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                               ΝA
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                               NA
##
## (Intercept)
                                                                         **
## age
## I(age^2)
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9672 on 521 degrees of freedom
## Multiple R-squared: 0.1591, Adjusted R-squared: 0.1511
## F-statistic: 19.72 on 5 and 521 DF, p-value: < 2.2e-16
summary3
##
## Call:
## lm(formula = tc ~ bSpline(age, knots = quantile(age, c(0.25,
       0.5, 0.75)), degree = 2), data = sorted_dat)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
```

```
## -2.6732 -0.6199 -0.0385 0.5465 4.0640
##
## Coefficients:
                                                                        Estimate
##
## (Intercept)
                                                                          4.4568
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                          0.7025
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                          0.7616
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                          1.6759
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                          1.3197
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                          1.2961
                                                                        Std. Error
## (Intercept)
                                                                            0.2402
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                            0.4027
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                            0.2427
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                            0.3012
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                            0.3183
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                            0.5712
##
                                                                        t value
## (Intercept)
                                                                         18.557
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                          1.745
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
                                                                          3.138
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3
                                                                          5.563
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4
                                                                          4.146
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                          2.269
##
                                                                        Pr(>|t|)
## (Intercept)
                                                                         < 2e-16
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1
                                                                          0.0817
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3 4.24e-08
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4 3.94e-05
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5
                                                                          0.0237
##
## (Intercept)
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)1.
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)2 **
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)3 ***
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)4 ***
## bSpline(age, knots = quantile(age, c(0.25, 0.5, 0.75)), degree = 2)5 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9672 on 521 degrees of freedom
## Multiple R-squared: 0.1591, Adjusted R-squared: 0.1511
## F-statistic: 19.72 on 5 and 521 DF, p-value: < 2.2e-16
attach(sorted dat)
plot(age, tc, main = "Total cholesterol vs. age", xlab = "age", ylab = "total cholesterol")
lines(age, fitted(mod3), col = "green", lty = "solid")
lines(age, fitted(mod5), col = "orange", lty = "dashed")
legend("topright", c("spline", "spline + age + age^2"), col = c("green", "orange"), lty = c("solid", "d
```

### Total cholesterol vs. age



#### detach(sorted\_dat)

Similar as in 1(b), the fitted lines of the spline model and the linear + quadratic age model overlap and the residual standard error and  $R^2$  statistics are exactly the same for the two models (Residual standard error = 0.9672, multiple  $R^2 = 0.1591$ , adjusted  $R^2 = 0.1511$ ). Therefore, linear+quadratic age model is nested within the spline model.

```
\#1(d)
```

```
anova(mod2, mod5)
```

```
## Analysis of Variance Table
##
## Model 1: tc ~ age + I(age^2)
## Model 2: tc ~ age + I(age^2) + bSpline(age, knots = quantile(age, c(0.25,
## 0.5, 0.75)), degree = 2)
## Res.Df RSS Df Sum of Sq F Pr(>F)
## 1 524 492.35
## 2 521 487.34 3 5.0098 1.7853 0.1489
```

Because linear+quadratic age model is nested within the spline model, we can do F test to compare the two models. The p-value for the F test is 0.1489 (> 0.05), indicating that the spline model is not significantly better than the linear and quadratic age model. It is sufficient to use linear and quadratic age model to model the effects of age.

#### Problem 2

#2(a)

```
sorted_dat$bmi= sorted_dat$weight / (sorted_dat$height/100)^2
age.gender.bmi.1 = lm(tc~bmi, data = sorted_dat)
summary(age.gender.bmi.1)
##
## Call:
## lm(formula = tc ~ bmi, data = sorted_dat)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
## -2.7806 -0.6333 -0.1273 0.5735 4.2889
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
                          0.25007 16.435 < 2e-16 ***
## (Intercept) 4.11005
               0.05825
                          0.01019
                                    5.719 1.8e-08 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.019 on 525 degrees of freedom
## Multiple R-squared: 0.05864, Adjusted R-squared: 0.05685
## F-statistic: 32.7 on 1 and 525 DF, p-value: 1.803e-08
age.gender.bmi.2 = lm(tc~bmi + I(bmi^2) , data = sorted_dat)
summary(age.gender.bmi.2)
##
## Call:
## lm(formula = tc ~ bmi + I(bmi^2), data = sorted_dat)
## Residuals:
      Min
               1Q Median
                               30
                                      Max
## -2.7664 -0.6523 -0.1061 0.5651 4.2049
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.619210
                          0.996621
                                   1.625 0.10483
## bmi
               0.257627
                          0.077911
                                     3.307 0.00101 **
## I(bmi^2)
              -0.003859
                          0.001495 -2.581 0.01012 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.014 on 524 degrees of freedom
## Multiple R-squared: 0.07046,
                                   Adjusted R-squared: 0.06691
## F-statistic: 19.86 on 2 and 524 DF, p-value: 4.859e-09
age.gender.bmi.3 = lm(tc ~ bmi +I(bmi^2) + age, data = sorted_dat)
summary(age.gender.bmi.3)
##
## Call:
## lm(formula = tc ~ bmi + I(bmi^2) + age, data = sorted_dat)
## Residuals:
               1Q Median
                               3Q
##
      Min
                                      Max
```

```
## -2.7752 -0.6372 -0.0587 0.5478 4.2095
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.954701
                          0.951957
                                     2.053
                                             0.0405 *
                                     2.302
                                             0.0217 *
## bmi
               0.173209
                          0.075236
## I(bmi^2)
                          0.001436 -1.843
              -0.002647
                                             0.0659 .
                                    7.258 1.43e-12 ***
## age
               0.022385
                          0.003084
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9674 on 523 degrees of freedom
## Multiple R-squared: 0.1555, Adjusted R-squared: 0.1507
## F-statistic: 32.11 on 3 and 523 DF, p-value: < 2.2e-16
age.gender.bmi.4 = lm(tc ~ bmi +I(bmi^2) + gender, data = sorted_dat)
summary(age.gender.bmi.4)
##
## Call:
## lm(formula = tc ~ bmi + I(bmi^2) + gender, data = sorted_dat)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -2.7357 -0.6449 -0.0942 0.5621 4.1637
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.478818
                          1.013540 1.459 0.145149
## bmi
               0.266748
                          0.078836
                                    3.384 0.000769 ***
## I(bmi^2)
              -0.004044
                          0.001515 -2.669 0.007836 **
## gender
               0.069322
                          0.090022
                                   0.770 0.441613
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.014 on 523 degrees of freedom
## Multiple R-squared: 0.07151,
                                   Adjusted R-squared: 0.06618
## F-statistic: 13.43 on 3 and 523 DF, p-value: 1.875e-08
age.gender.bmi.5 = lm(tc~bmi + I(bmi^2) + age + I(age*bmi), data = sorted_dat)
summary(age.gender.bmi.5)
##
## Call:
## lm(formula = tc ~ bmi + I(bmi^2) + age + I(age * bmi), data = sorted_dat)
## Residuals:
               1Q Median
                               3Q
## -2.7573 -0.6371 -0.0585 0.5458 4.2104
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                1.7770981 1.0340123
                                       1.719
                                               0.0863 .
## bmi
                0.1745595 0.0753564
                                       2.316
                                               0.0209 *
## I(bmi^2)
               -0.0023859 0.0015544 -1.535
                                               0.1254
```

```
0.0302682 0.0181064
                                       1.672
                                               0.0952 .
## I(age * bmi) -0.0003324 0.0007522 -0.442
                                               0.6588
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9681 on 522 degrees of freedom
## Multiple R-squared: 0.1558, Adjusted R-squared: 0.1494
## F-statistic: 24.09 on 4 and 522 DF, p-value: < 2.2e-16
age.gender.bmi.6 = lm(tc~bmi + I(bmi^2) + age + I(gender*bmi), data = sorted_dat)
summary(age.gender.bmi.6)
##
## Call:
## lm(formula = tc ~ bmi + I(bmi^2) + age + I(gender * bmi), data = sorted_dat)
## Residuals:
##
                1Q Median
      Min
                               3Q
                                      Max
## -2.7220 -0.6087 -0.0610 0.5387
                                   4.1390
##
## Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                   1.747319
                              0.964539
                                         1.812
                                                 0.0706 .
## bmi
                                                 0.0136 *
                   0.188251
                              0.076067
                                         2.475
## I(bmi^2)
                   -0.003010
                              0.001462 -2.058
                                                 0.0400 *
## age
                   0.022650
                              0.003089
                                         7.333 8.64e-13 ***
## I(gender * bmi) 0.004604
                              0.003533
                                         1.303
                                                 0.1931
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9667 on 522 degrees of freedom
## Multiple R-squared: 0.1583, Adjusted R-squared: 0.1518
## F-statistic: 24.54 on 4 and 522 DF, p-value: < 2.2e-16
final.model = age.gender.bmi.3
summary(final.model)
##
## Call:
## lm(formula = tc ~ bmi + I(bmi^2) + age, data = sorted_dat)
## Residuals:
               1Q Median
                               3Q
## -2.7752 -0.6372 -0.0587 0.5478 4.2095
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
                          0.951957
                                     2.053
                                             0.0405 *
## (Intercept) 1.954701
## bmi
               0.173209
                          0.075236
                                     2.302
                                             0.0217 *
## I(bmi^2)
              -0.002647
                          0.001436
                                   -1.843
                                             0.0659 .
## age
               0.022385
                          0.003084
                                    7.258 1.43e-12 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9674 on 523 degrees of freedom
```

```
## Multiple R-squared: 0.1555, Adjusted R-squared: 0.1507 ## F-statistic: 32.11 on 3 and 523 DF, p-value: < 2.2e-16
```

We first compare the linear bmi model and the linear and quadratic bmi model, finding out that the coefficient for quadratic bmi term is significant. Therefore, we can make adjustment on top of the linear and quadratic bmi model. Now we're considering age as a confounder of bmi, which in conventional definition makes sense (age is a common cause of bmi and total cholesterol), and after including age term, the coefficients for linear and quadratic bmi terms change significantly, confirming that age is a confounder. But including gender does not effectively change the coefficients for linear or quadratic bmi. Therefore, we only consider age as a confounder.

Now we are considering whether age modifies the effect of bmi on total cholesterol. However the coefficient for the interaction term of age & bmi is not significant, indicating that age does not modify the effect of bmi on total cholesterol. Similarly, the interaction between gender and bmi is not significant, either.

Therefore, the final model I recommend is the model including linear bmi, quadratic bmi and age to predict total cholesterol. (Among the models I tried above, this model has the highest adjusted R<sup>2</sup> value as well.)

#### #2(b)

```
confint(final.model) *38.67
##
                     2.5 %
                                 97.5 %
## (Intercept)
                3.2703611 1.479062e+02
                 0.9824549 1.241351e+01
## bmi
                -0.2114889 6.753188e-03
## I(bmi^2)
                 0.6313305 1.099916e+00
## age
summary(final.model)$coefficients * 38.67
##
                  Estimate
                            Std. Error
                                          t value
                                                      Pr(>|t|)
## (Intercept) 75.5882942 36.81219501
                                         79.40302 1.567507e+00
                 6.6979836
                            2.90939118
                                         89.02585 8.397622e-01
## I(bmi^2)
                            0.05554619 -71.26619 2.548531e+00
               -0.1023679
                            0.11926258 280.67182 5.520816e-11
## age
                 0.8656231
Total Cholesterol = 1.9547 + 0.1732 * bmi - 0.002647 * bmi^2 + 0.02239 * age
```

My final model uses linear bmi(in  $kg/m^2$ ) (p = 0.0217), quadratic bmi(in  $(kg/m^2)^2$ ) (p = 0.0659) and age ( $p = 1.43 \times 10^{-12}$ ) to predict total cholesterol (in mg/dl) of subjects.

It is not sensible to interpret the intercept 75.59 mg/dl (95% confidence interval: [3.27, 147.91]), as bmi cannot be 0. After adjusting for age, every unit  $(kg/m^2)$  increase in the linear bmi term (bmi) only will contribute a 6.698 mg/dl (95% confidence interval: [0.983, 12.41]) increase in total cholesterol, and every unit( $(kg/m^2)^2$ ) increase in the quadratic bmi term (bmi^2) will contribute a -0.1024 mg/dl (95% confidence interval: [-0.211, -6.75 × 10<sup>-3</sup>]) decrease in total cholesteral.