Let X and Y be jointly distributed DV's with ponfl p(a,y) or .pdf f(x,y) according to whether the variables are discrete or continious. Then to the expected value of a function h(x, Y), denoted by E[h(x,Y)], is given by $E[h(x,y)] = \begin{cases} \sum_{x} \left(\sum_{y} x \right) h(x,y) & \text{if } x \text{ and } y \text{ are discrete.} \end{cases}$ continious. continious. The covariance between two rus X and Y is Cov(X,Y) = E[(X - E(X))](Y - E(Y))]Eg An in(& eag dor (vM = V) (xM - x) & hot) quiz consisting duspules los ylandocor à rot stong out to tong to set of tooms string to radius, y 2 toom timious. Suppose that the joint pung of X and Y is given-(x) Cov(x,Y) = E(XY) - Mx·MY (4, r) (0 5 Correlation The correlation coefficient of X and Y, denoted by Corr (x, Y), fx, or just f, B defined by $\int_{X,Y} = \frac{Cov(X,Y)}{6x \cdot 6y}$

Proposition petrodiate de plans of y han x to sold in les la la contrata de la contrata del contrata de la contrata de la contrata del contrata de la contrata del contrata del contrata de la contrata de la contrata del contr sular hatagg Corr (axtbr cxtd) = Corr (x, Y) yd neville any two revision X and (Y) d nothinest of E[h(x,y)] = (x,x) sign) p(x,y) = (ternele 3. If X and Y (vare independent, then fx, y =0 but converse not true. 4. I Y XXX = 1 or -1 if and only if Y=ax+b

21. Y XXX X EVI GUT MOS VAX 6 - WIEL [(x, Y) = E[(x - E(x))](Y - E(Y))]E.g. An instructors v'has given a short quiz consisting of two parts. For a randomly selected student, let X= the number of points earined on the 1st part and in Your the number of points ceatined on the 2nd post. Suppose that the joint punt of X and Y is given in the table (x) Cov(x,Y) = E(xY) - Mx, My y (Y (X) x (X) X (Y (X) x (Y (X) x (X) X (Y (X) x (X) X (Y (X) x (X) $\int_{X_{x,Y}} = \frac{Cov(x,Y)}{o_{X} \cdot o_{Y}}$

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1) If the score recorded in the greader book is the
                       total no. of points earned on the two parts, what
                        Is the expected recorded, score E(X+Y)?
                                                        let h(x,y) = x + Y
                                     E[h(x,y)](x) = \sum_{x} \sum_{x} (x+y) \cdot p(x,y) \int_{-\infty}^{\infty} f(x-x) = \sum_{x}^{\infty} f(x+y) \cdot p(x,y)
                                                                                                        = (c-F-55) (c02+0.06+0.07 (0))+
   100+10 0+ 21 F. + (0+9); 0.02 + (0+5) 0.06 + (0+10); 0.02 + (0+15); 0.12)
                                                                                             + · · · + (10+10) · 0.14 + (10+15) · 0.01
    (b) If the maxim of 1 two scores is recorded, what is the
                                                                                    =14.1
                              expected recorded score? (M) (2-8) 7 2
                                                  E\left[\max\left(X,Y\right)\right] = \sum_{x} \left[\sum_{y} h(x,y) p(y,y) - \sum_{y} h(x,y) p(y,y)\right]
                                                                                            h (x, y)
                                                                                                                                               = max (0,0). 0.02 + max (0,5). 0.06 + · · · †
                                                                     1, max (10,15).0.14+max (10,15).0.06
                                                                                                                                                = 9.6.1
                                                              Es (a) Show that (ou (ax+b, cy+d) = ac (ov (x,y)).
    (b+(Y)=3) (6+(X)=1) \ \(\frac{1}{2}\) \(\frac{1}\) \(\frac{1}{2}\) \(\frac{1}{2}\) \(\frac{1}{2}\) \(\frac{1}{
                                                                                                            e^{-\alpha c} E(xY) + \frac{1}{c^{-\alpha}} E(x) + bc \cdot E(Y) + i_3 d
E(x) = \sum_{x} P_{x}(x) = O(0.02 + 0.06 + 0.02 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.2 + 0.1) + 5(0.04 + 0.15 + 0.15 + 0.1) + 5(0.04 + 0.15 + 0.15 + 0.15 + 0.1) + 5(0.04 + 0.15 + 0.15 + 0.15 + 0.15 + 0.1) + 5(0.04 + 0.15 + 0.15 + 0.15 + 0.15 + 0.1) + 5(0.04 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.1) + 5(0.04 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15 + 0.15
                                                                            = S.55. ((Y))(X)] - (YX)) 0 = to.01 + 0.14).
                                                                                                                                                                                                                       e ac (Cov(X,Y)
   E(Y) = > Y Py(y) = 8,55.
   E(\chi \chi) = \sum \chi y p(x,y) = 0.0.(0.02) + 0.5.(0.06) + ... + 10.10.0.14(d) + 10.15.
                         (corr (axtb) cyta) = (correct to cyta) (correct to correct to corr
                               Cov(X,Y) = 44.25 - 5.55 (8.55) = -3.2025. //
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(d) (H) Cornelations profor oxi and Y sprosen some odly the
                       Astal no. of points (X,X) = core (X,X) = Pents what all the form of the form o
                              -1.6^{2} = \sum (x-u^{2}) p_{x}(x) + (y+x) = E(x) = 5.55
                                                                                         = (0-5.55)2 (0.02 to.06 to.02 to.1) +
                      (5-5:53)20 (6.84)+0.15 +0.2+0.1)+0(10-5:55)2 (0.01+0:15+0.14+0.01
                                                                                   10:0: (31+01)+ Al.o · (01+01)
                                                                                                                                                                                                                                                                                                      1.01=
            90+ 6x = \sqrt{620} = \sqrt{12.4475} = 0
-6y = \sum (y-\mu)^2 P_y(y) = (0-8.55)^2 (0.02+0.04+0.01) + 0
-6y = \sum (y-\mu)^2 P_y(y) = (0-8.55)^2 (0.02+0.04+0.01) + 0
                                                                                                                               [((x,x))^{2}] = [((x,x))^{2}
6_{\gamma} = \sqrt{6_{\gamma}^{2}} = 19.1475.
(4, 0) \times 6_{\gamma} = 19.1475.
(4, 0) \times 6_{\gamma} = \frac{19.1475}{19.1475}.
(4, 0) \times 6_{\gamma} = \frac{19.1475}{19.1475} \approx -0.2074
            35. (a) Show that Cov (ax +b, cY+d) = ac Cov (x,Y).
                                 Cov(ax+b,cy+d) = E([ax+b]\cdot[cy+d]) - E(ax+b)\cdot E(cy+d)
                             ((x) y ( = ) E (acxy + adx + bc y + bd) - (a E(x) + b) (c E(y) + d)
                                                  = ac E(xY) + ad E(x) + bc E(Y) + bd

+ (10+50+21.0+10.00) = - (ac E(x)E(Y) + ad E(x) + bc E(Y) + bd)
                                 · ( Prof 10.0+ 21.0 + (E(XY) - E(X) E(Y))
                                                                                                           = ac Cov(x, Y).
                                                                                                                                                                                                                                                       E(Y) = [y R/(y) = 8.55.
                        (b) Show + that + (Sorr (ax+b) cx+d) = corr (x, x) = (x, y)
                                                            Corr (ax+b, cy+d) = Cor (ax+b, cy+d) = ac Cor (x,x)

(ax+b, cy+d) = ac Cor (x,x)
```

$$= \frac{ac}{|ac|} \frac{Cov(X,Y)}{6x \cdot 6y} = Corr(X,Y)$$

e) If a and e has opposite sign, then Corr(ax+b, cy+d) = -Corr(x,y).

3 Show that if
$$Y = ax + b$$
 $(a \neq 0)$, then $Corr(x,Y) = +1$ or -1 .

$$Corr(x,Y) = \frac{Cov(x,Y)}{6x 6y} = \frac{Cov(x,ax+b)}{6x 6ax+b}$$

$$Cov(x,Y) = E[xY] - E(x) \cdot E(Y)$$

$$= E[ax^{2}+bx] - E(x) \cdot E[ax+b]$$

$$= aE(x^{2}) + bE(x) - a(E(x))^{2} + bE(x)$$

$$= a[E(x^{2}) - (E(x))^{2}]$$

 $Cov(x,ax+b) = a 6x^2$

· Corr
$$(X,Y) = \frac{\alpha 6_{x}^{2}}{6_{x} \cdot |a| 6_{x}} = \frac{\alpha 6_{x}^{2}}{|a| 6_{x}^{2}} = \pm 1.$$