Physics (PH- 10001)

Waves and Interference

Topics to be covered

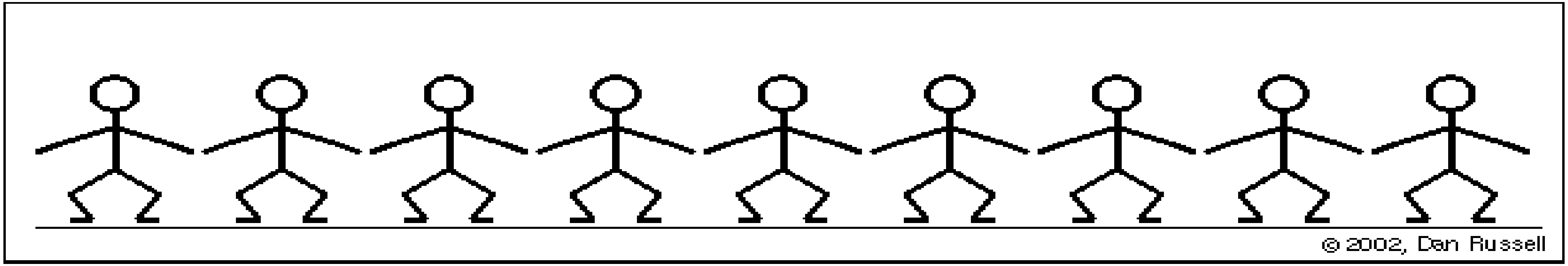
- Introduction to waves
  - Characteristics of wave motion
  - Types of waves
  - The wave equation
  - Differential equation of wave motion
- Concept of interference of light
  - Interacting waves and principle of superposition
  - Interference
  - Coherent sources of light
  - Types of interference based on the production of effective coherent sources
  - Conditions of interference
  - Analytical treatment of interference
    - ✓ Intensity distribution curve
    - ✓ Whether the 'law of conservation of energy' is satisfied in interference or not?

What is a wave ?

A disturbance or variation that transfers energy progressively from one place to another in a medium that may take the form of elastic deformation or variation of pressure/electric field/magnetic field intensity/electric potential/temperature etc.

Waves

Cooperative motion of the particles of the medium producing a wave.



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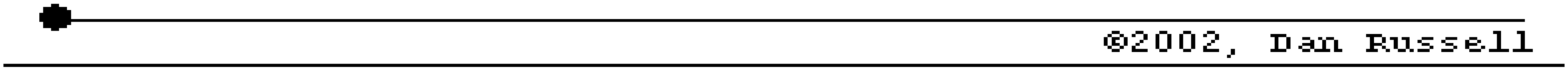
Waves in a string



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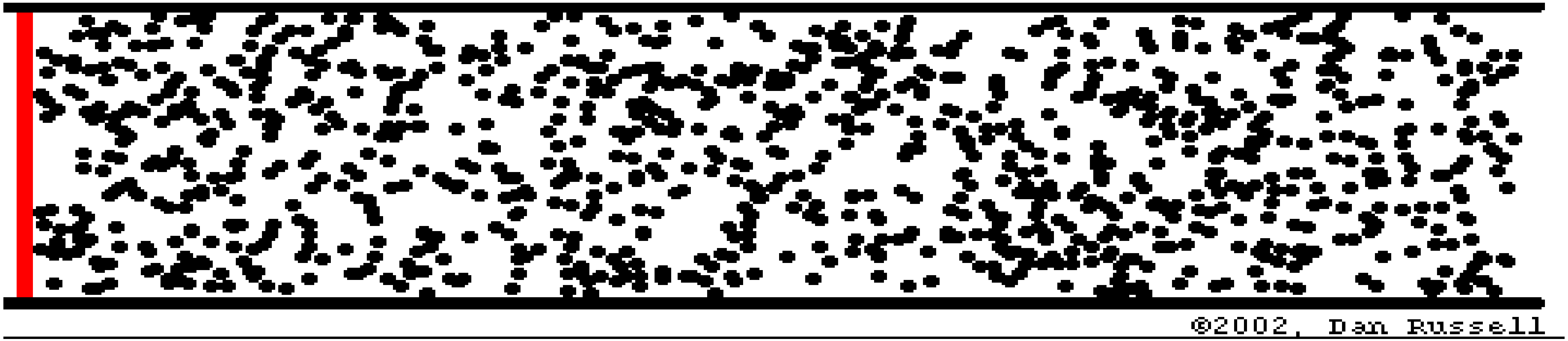
Pulsed Sound Wave



Summing up the characteristics of wave motion…………

- It is the disturbance produced in a medium due to repeated periodic motion of the particles of the medium.
- In the wave motion, wave travels in the forward direction while particles of the medium vibrate about their mean position.
- There is a regular phase difference between the particles of the medium.
- The velocity of the wave is different from the velocity of the particle. The velocity of the wave is uniform while the velocity of the particle is different at different positions.

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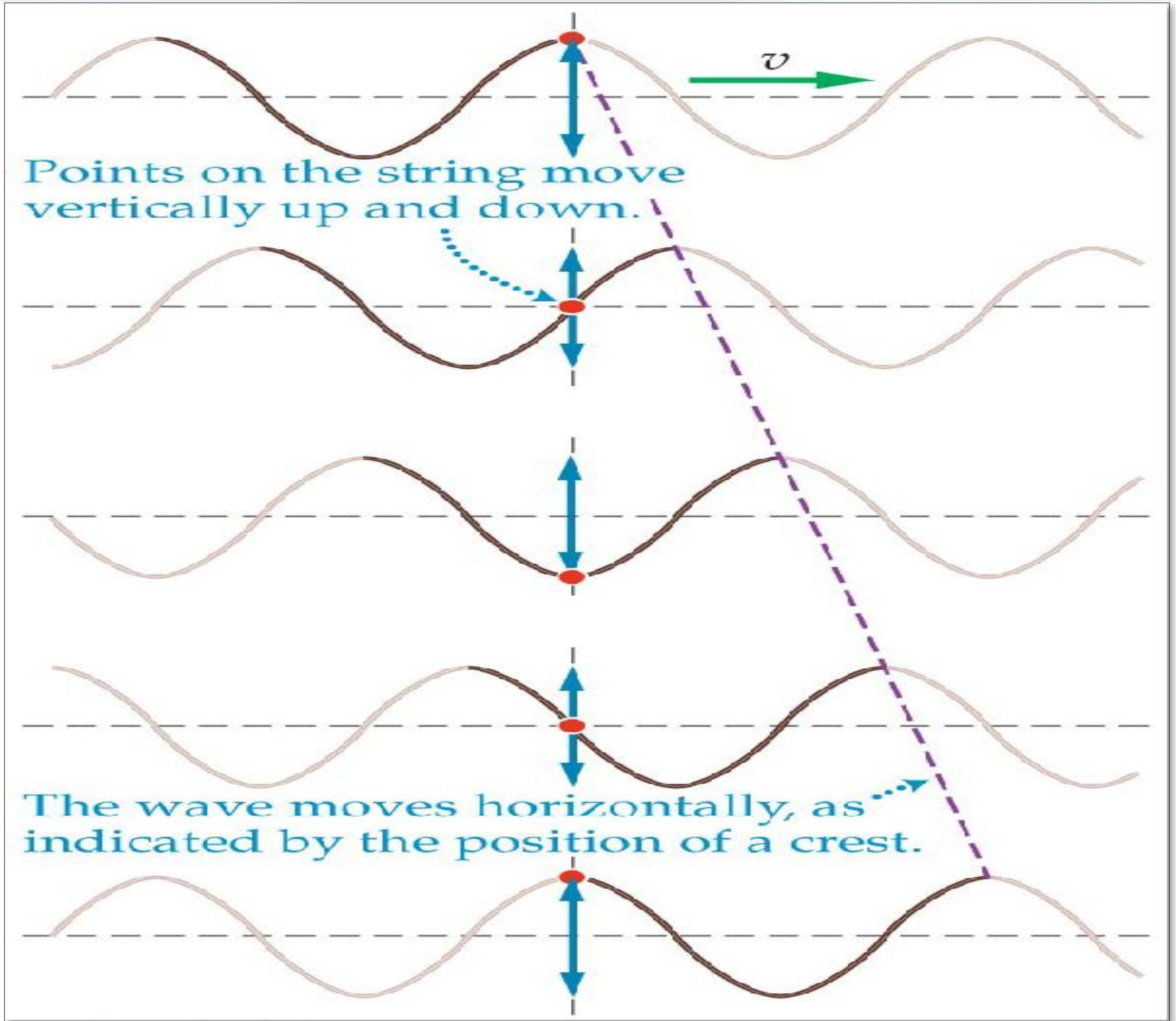

v
Points on the string move
vertically up and down.
The wave moves horizontally, as
indicated by the position of a crest.

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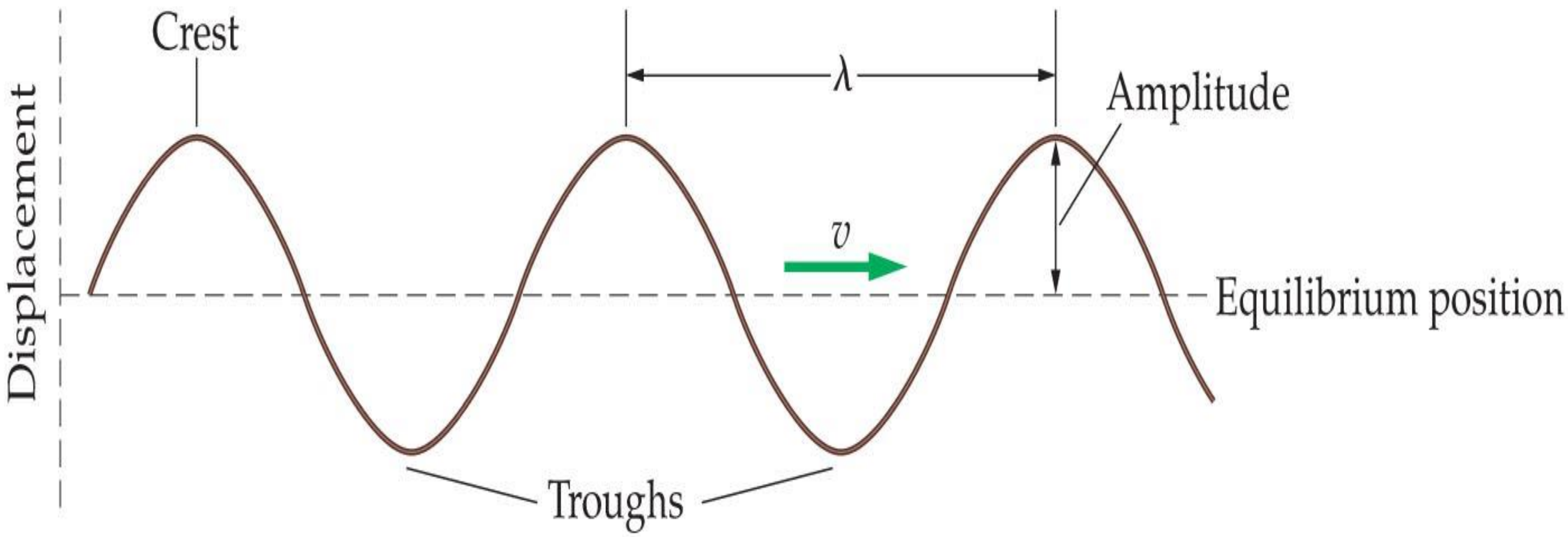
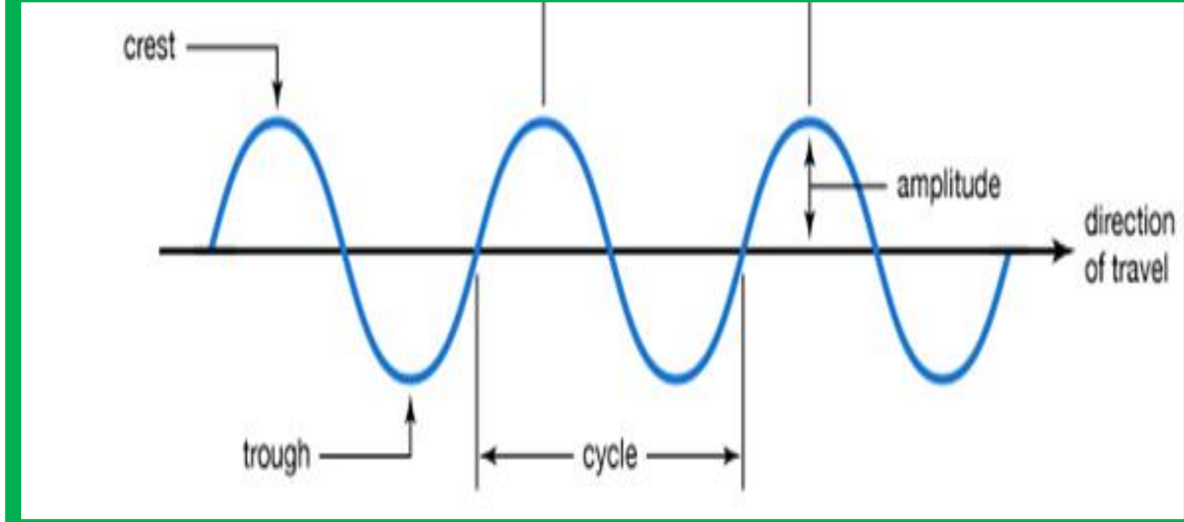


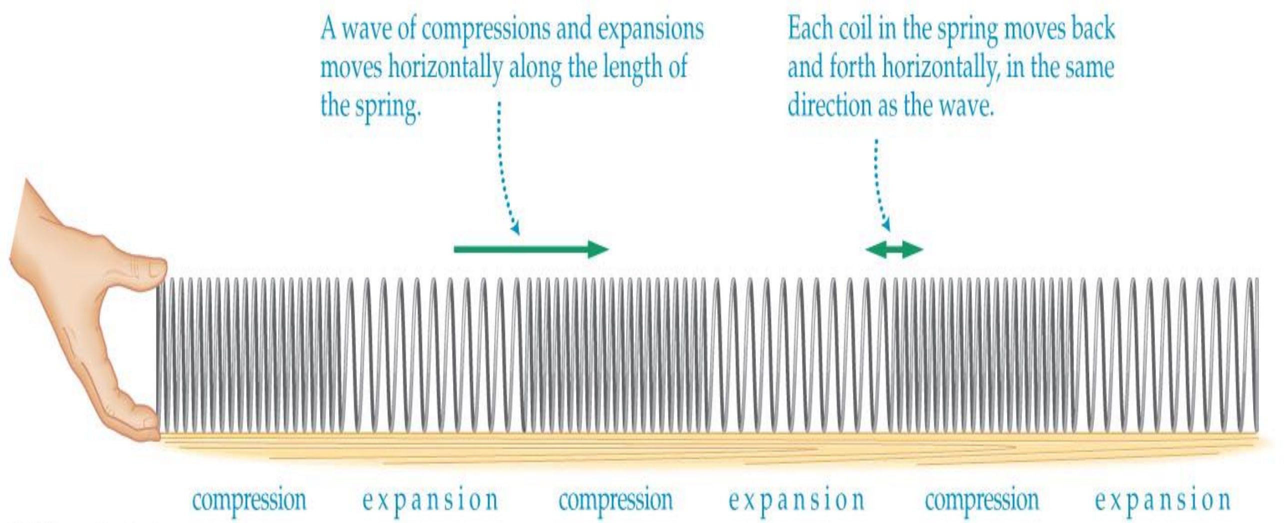
Crest
λ
Amplitude
Displacement
v
Equilibrium position
Troughs

Types of Waves

Transverse waves

Longitudinal waves





Follow the link:
https://tenor.com/view/longitudinal-wave-transverse-waves-waves-gif-13936583

Comparison between Transverse and longitudinal waves

Transverse waves	Longitudinal waves
➢ Particles of the medium vibrate about their mean position in a direction perpendicular to the direction of motion of the wave.	➢ Particles of the medium vibrate about their mean position in a direction parallel to the direction of the wave.
➢ Made up of crests and troughs	➢ Made up of compressions and rarefactions
➢ Electromagnetic waves and waves on the surface of water are transverse waves.	➢ Sound waves and seismic waves are longitudinal waves.

Follow the link:
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Wave expression

Consider the displacement of a particle at a point ‘P’ showing simple harmonic oscillation is given by,

$$y = a sin\omega t \qquad \text{...(1)}$$

Let there be another particle at ‘Q’ at a distance ‘x’ from ‘P’ and the wave is travelling with a velocity ‘v’ from P to Q, then the displacement of the particle at ‘Q’ may be given as,

$$y = a sin(\omega t - \varphi) \qquad \text{...(2)}$$

Where φ is the phase difference between the particles ‘P’ and ‘Q’.

$$\varphi = \frac{2\pi}{\lambda} \times path\ difference = \frac{2\pi x}{\lambda} \qquad \text{...(3)}$$

Contd….

By definition of angular frequency,

$$\omega = \frac{2\pi}{T} = 2\pi f = \frac{2\pi v}{\lambda} \quad \text{...(4)}$$

Substituting equations (3) and (4) in equation (2), we get

$$y = a\sin\left(\frac{2\pi v}{\lambda}t - \frac{2\pi}{\lambda}x\right) \quad \text{..(5)}$$

$$\boxed{y = a\sin\frac{2\pi}{\lambda}(vt - x)} \quad \text{..(6)}$$

Equation (6) represents the equation of displacement of a particle when the wave is travelling in + x direction. If the wave is travelling in –x direction such that the particle at Q is at a distance x in the negative direction, then the equation of displacement will be , $\boxed{y = a\sin\frac{2\pi}{\lambda}(vt + x)}$ ....(7)

Differential Equation of wave motion

Let the wave is travelling in +x direction, and can be represented as:

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) \qquad \text{...}(6)$$

Differentiating it w.r.t. time, we get

$$\frac{dy}{dt} = \frac{2\pi av}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x) \qquad \text{...}(8)$$

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 av^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt - x) \qquad \text{...}(9)$$

Again differentiating equation (6) wrt distance x,

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x) \qquad \text{...}(10)$$

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2 a}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt - x) \qquad \text{...}(11)$$

Comparing equations (9) and (11), we get, $\boxed{\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}}$ ....(12)

Equation (12) represents the general differential wave equation in one dimension

Summary

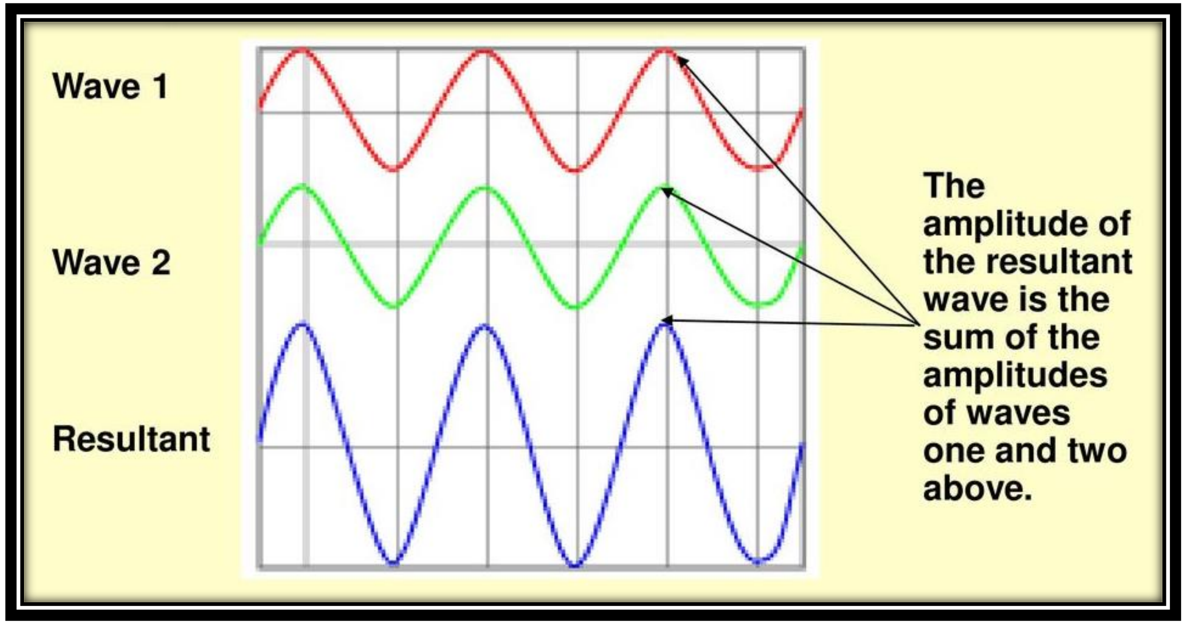
- We have discussed about waves, types of waves with examples.
- We have derived the differential equation of wave motion.
- We have also solved few sample numerical problems.

Next Class

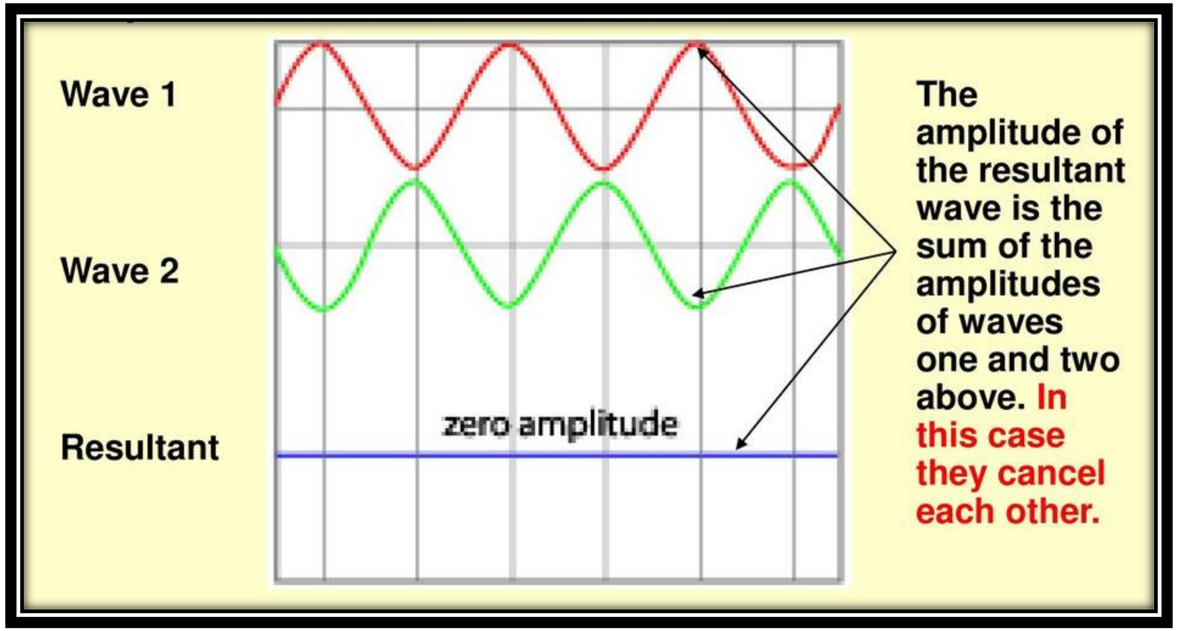
- We shall discuss about principle of superposition, concept of interference and its use in day to day life, conditions for interference and types of interferences with examples.
- We shall also discuss about coherent sources, production of coherent sources from a single source.

Principle of superposition

The resultant displacement of a particle of a medium when acted upon by two or more waves simultaneously is the algebraic sum of the displacements of the same particle due to individual waves in the absence of others.



Constructive Superposition

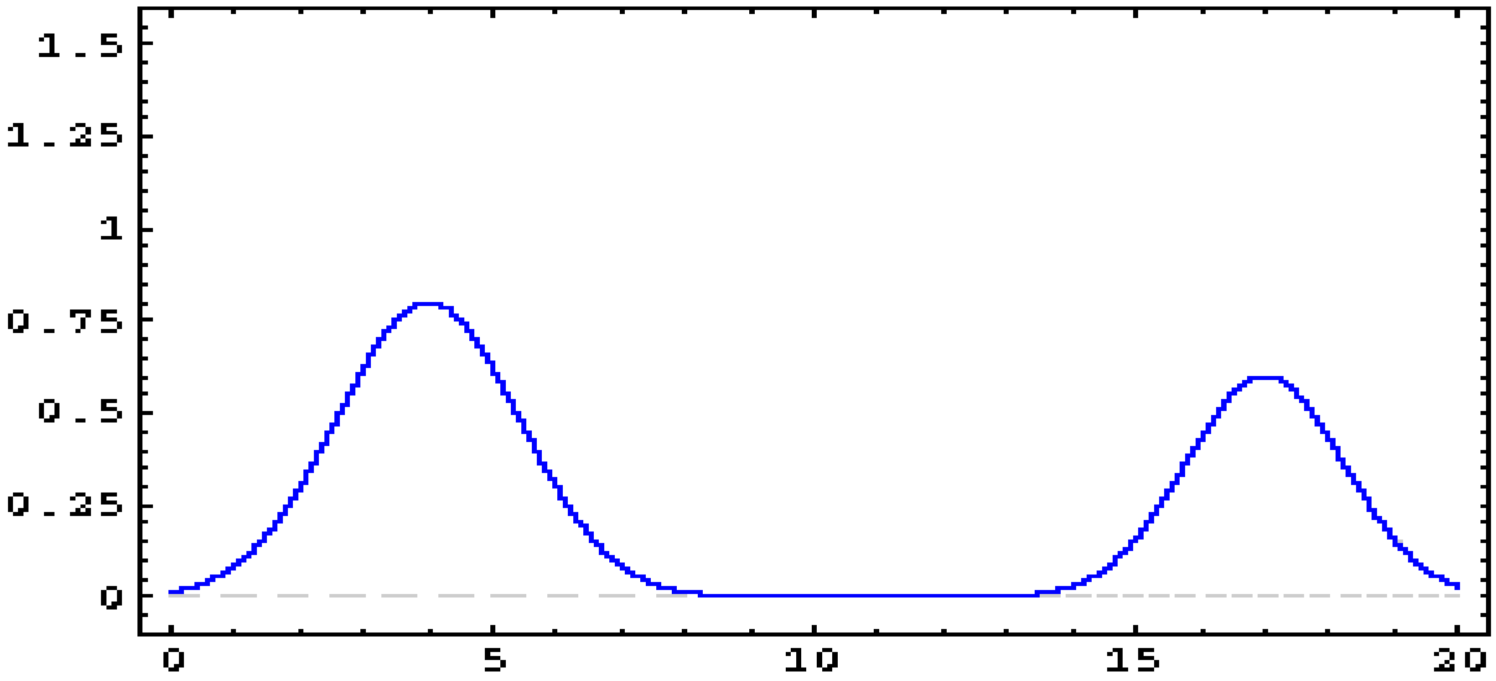


Destructive Superposition

Follow the link: https://www.youtube.com/watch?v=CAe3lkYNKt8

Superposition of waves

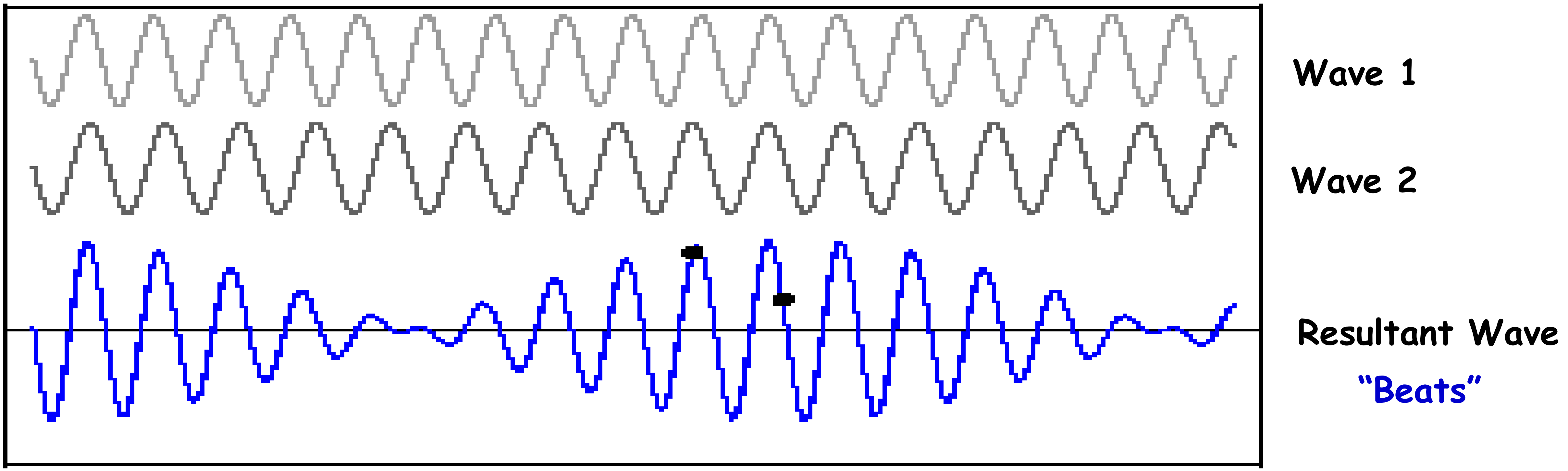
Pulsed Waves



Animations courtesy of Dr. Dan Russell, Kettering University

Superposition of waves

Two waves in same direction with slightly different frequencies



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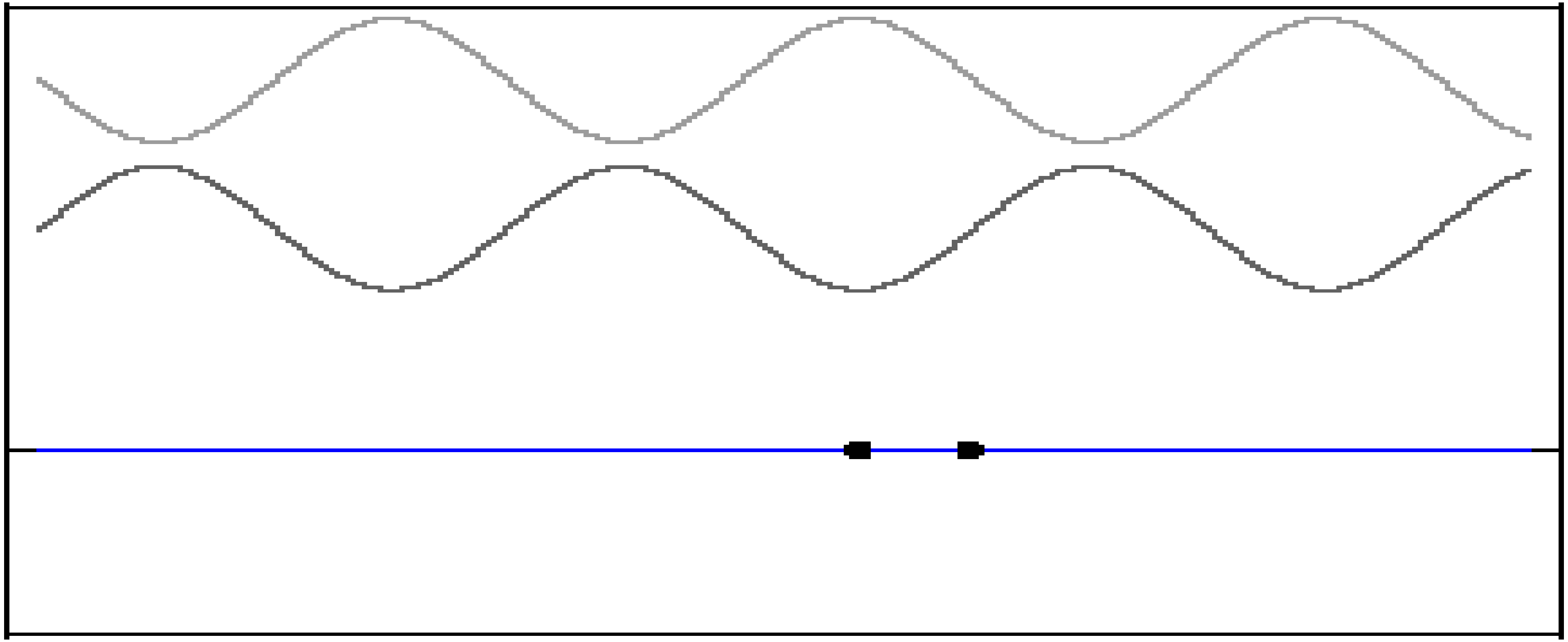
Superposition of waves

Harmonic waves in opposite directions



Incident Wave

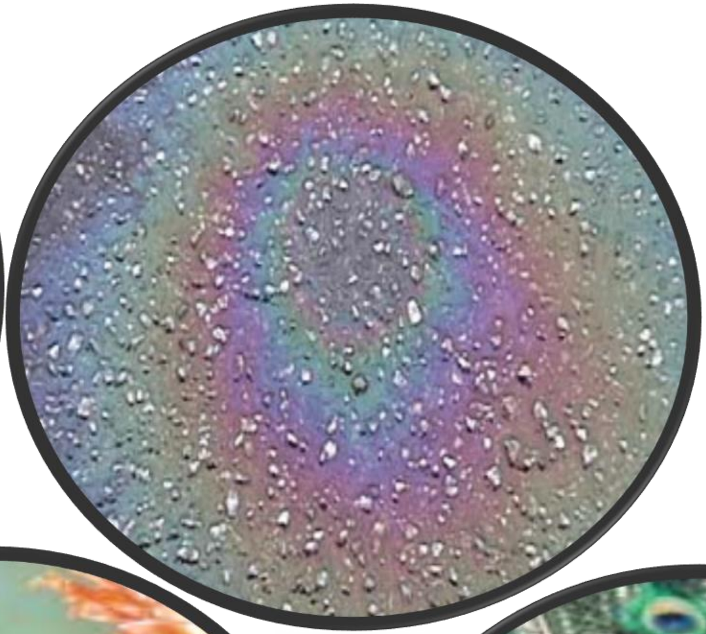
Reflected Wave

Resultant Wave

(Standing Wave)

Animations courtesy of Dr. Dan Russell, Kettering University

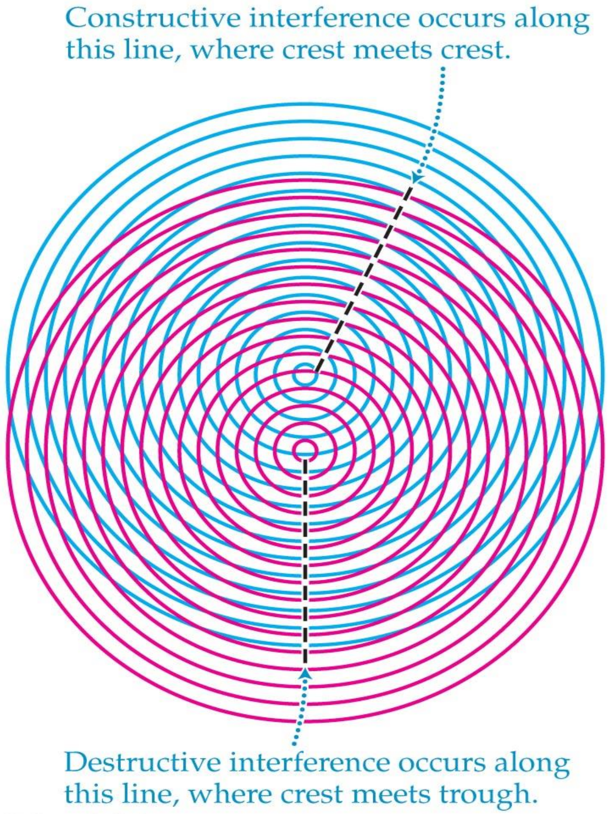
How these beautiful colored patterns are formed



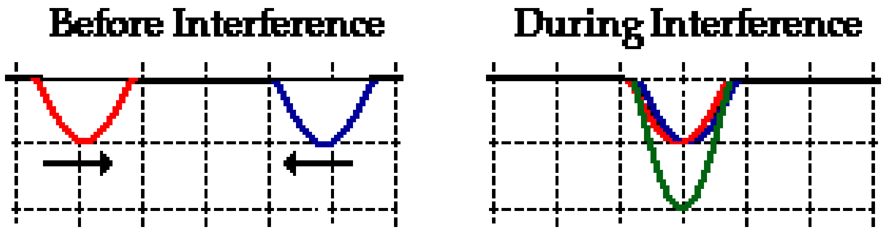
It is due to interference of light waves

Interference of light waves

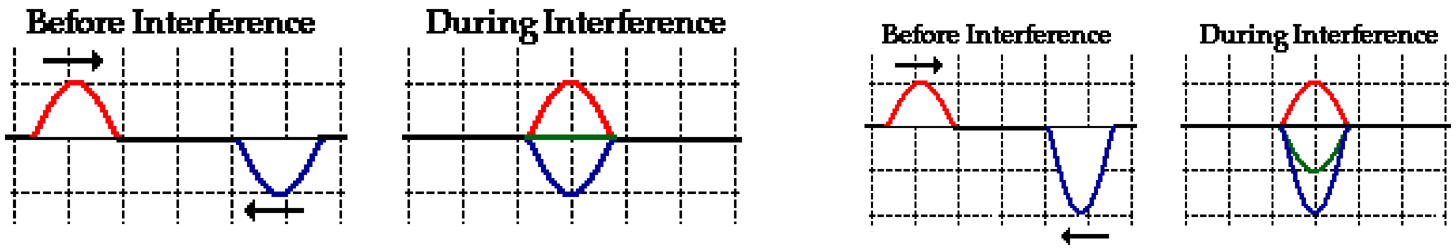
#Interference: When two or more waves of the same frequency travel in approximately the same direction and maintain a phase difference that remains constant with time, then the resultant intensity of light is not distributed uniformly in space. This non-uniform distribution of light intensity due to superposition of two or more waves is called **interference**.



#Constructive Interference:



#Destructive Interference:



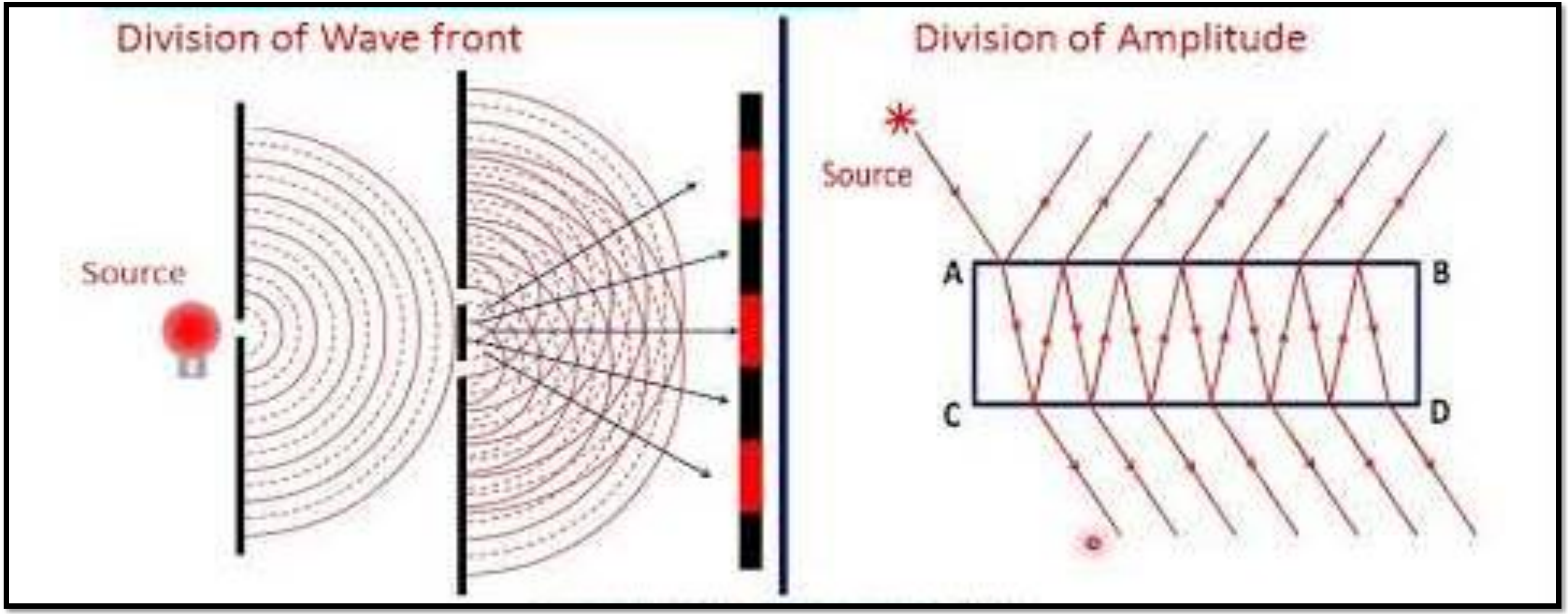
Follow the link: https://www.youtube.com/watch?v=PCYv0_qPk-4

Figure has taken from – https://www.physicsclassroom.com/class/waves/Lesson-3/Interference-of-Waves

Coherent sources of light

- Two or more sources of light are said to be coherent if there relative phases do not change with time.
- The phase difference between the sources remain constant :
  - the phase of each source remains constant in time
  - the phase of each source changes by the same amount
- Ordinary sources/completely independent sources of light (such as sun, star, electric bulb, candle, glowing solid etc.) are incoherent
- It is always possible to obtain two effectively coherent sources from a single incoherent source by reflection, refraction or other suitable process.
  - A narrow beam of light split into two components by one of the above mentioned process
  - These two beams are allowed to travel different optical path lengths
  - The two beams then superpose in a region to produce interference

Types of interferences based on production of coherent sources



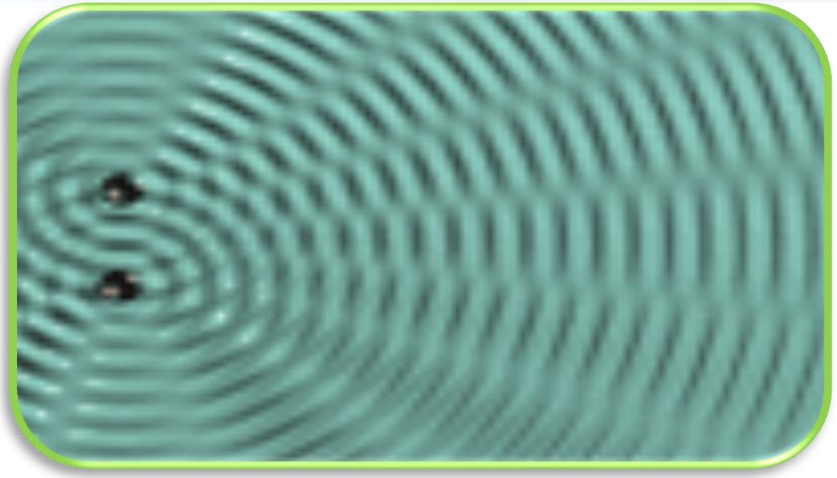
- ❖Young's double slit experiment
- ❖Fresnel's biprism
- ❖Lloyd's single and bimirror

- ❖Newton's ring
- ❖Michelson's Interferometer

Conditions of interference

#For sustained interference:

❖ Interfering waves must be coherent.
❖ Frequency of the waves must be same.
❖ Waves must be in the same state of polarization.



For better contrast of fringes:

❖ Amplitude of the waves must be equal or nearly equal (though this is not a necessary condition)
❖ Light source must be Monochromatic
❖ Narrow source of light is preferred

For clear observation of fringes:

❖ Distance between the sources and screen must be large
❖ Distance between the sources must be small.
❖ Background must be dark.

Analytical treatment of interference

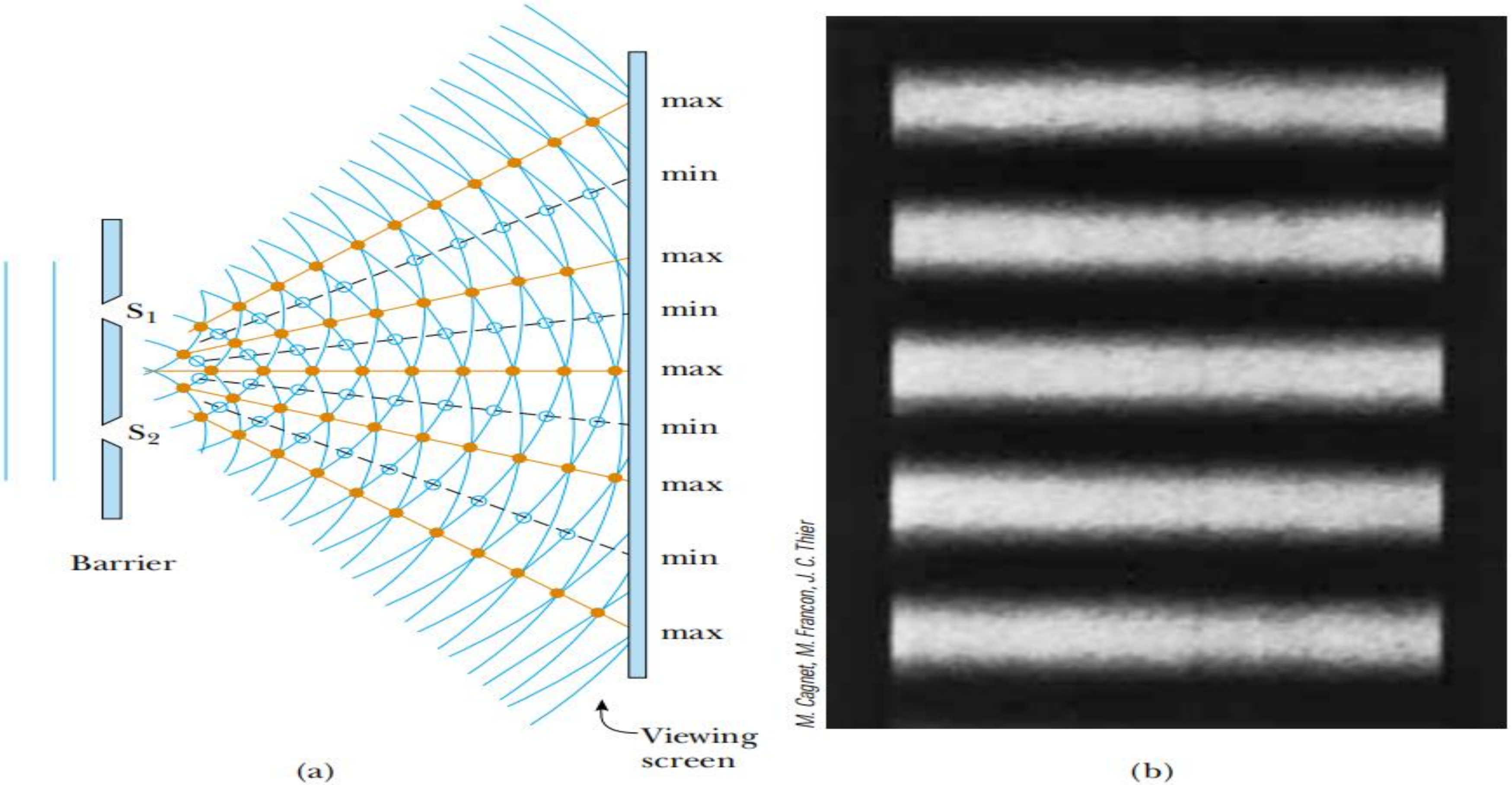


Fig. (a) Schematic diagram showing two slit interference. Slits S_1 and S_2 behave as coherent sources of light which produce an interference pattern on the screen; (b) enlarged view of the center of fringe pattern on the screen.

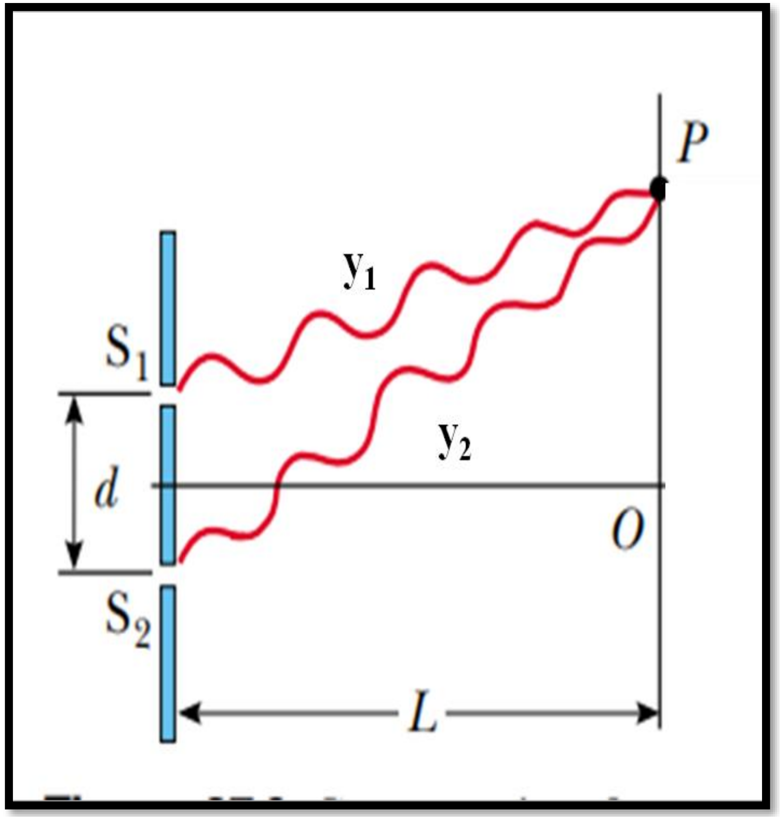
Contd……..

- Two coherent sources S_1 and S_2 , separated by a distance 'd' emit light waves of same angular frequency 'ω' and a constant phase difference 'δ'.
- y_1 and y_2 be the displacements produced by the individual waves at any point P on the screen, then

$$y_1 = a_1 sin\omega t \qquad \text{.......................}(1)$$

$$y_2 = a_2 \sin(\omega t + \delta) \qquad \text{.......................}(2)$$

where and are the amplitudes of the wave fronts from S_1 and S_2 respectively.



If 'y' be the resultant displacement at the point 'P' due to the waves from S1 and S2, then by the principle of superposition of waves, $y = y_1 + y_2$ ....(3)

Contd........

Using equation (1) and (2) in equation (3),

$$y = a_1 sin\omega t + a_2 \sin(\omega t + \delta) \quad ....(4)$$

$$= a_1 sin\omega t + a_2(sin\omega t cos\delta + cos\omega t sin\delta) = (a_1 + a_2 cos\delta)sin\omega t + (a_2 sin\delta)cos\omega t \quad ....(5)$$

Now let $a_2 sin\delta = Rsin\theta$ ....(6)

$$a_1 + a_2 cos\delta = Rcos\theta \quad ....(7)$$

Where R is the amplitude of the resultant wave due to the superposition of the two waves. Substituting equation (6) and (7) in equation (5) we get,

$$y = Rcos\theta sin\omega t + Rsin\theta cos\omega t \quad ....(8)$$

$$\boxed{y = Rsin(\omega t + \theta)} \quad ....(9)$$

Contd........

To find out the value of R, we have to square and add equations (6) and (7).

$$(a_1 + a_2 cos\delta)^2 + (a_2 sin\delta)^2 = R^2(cos^2\theta + sin^2\theta) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

$$\Rightarrow R^2 = a_1^2 + a_2^2 + 2a_1 a_2 cos\delta \qquad \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

$$\Rightarrow \boxed{R = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 cos\delta}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(12)$$

$$I \propto R^2$$

$I = R^2$ [keeping the constant of proportionality to be 1]

The resultant intensity 'I' at the point 'P' can be obtained as:

$$\boxed{I = a_1^2 + a_2^2 + 2a_1 a_2 cos\delta} \ldots\ldots\ldots\ldots\ldots\ldots(14) \text{ [utilizing equation (11)]}$$

Contd...

Expressing 'I' in terms of individual intensities I_1 and I_2,

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2} cos\delta} \quad \text{....................(15)}$$

Where $I_1 = a_1^2$ and $I_2 = a_2^2$ [taking the constant of proportionality to be 1]

Now let us find the conditions of maxima and minima and resultant intensity under those conditions.

Case I: (Constructive interference/Maxima/Bright fringes)

'I' will maximum (I_{max}) when $cos\delta = +1$

$$cos\delta = +1 \text{ for } \delta = 2n\pi, n = 0, \pm1, \pm2, \pm3, \pm4, \ldots..$$

Follow the link: https://i.gifer.com/4Qeo.gif

Contd........

The corresponding path difference between the interfering waves will be,

Path difference $= n\lambda, n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots..$

The maximum intensity, $\boxed{I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = (a_1 + a_2)^2}$ ....(16)

Case II (Destructive interference/Minima/Dark fringes) :

'I' will maximum (I_{min}) when $cos\delta = -1$

$cos\delta = -1$ for $\delta = (2n+1)\pi, n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots..$

For air medium

$phase\ difference\ \delta = \frac{2\pi}{\lambda} path\ difference$

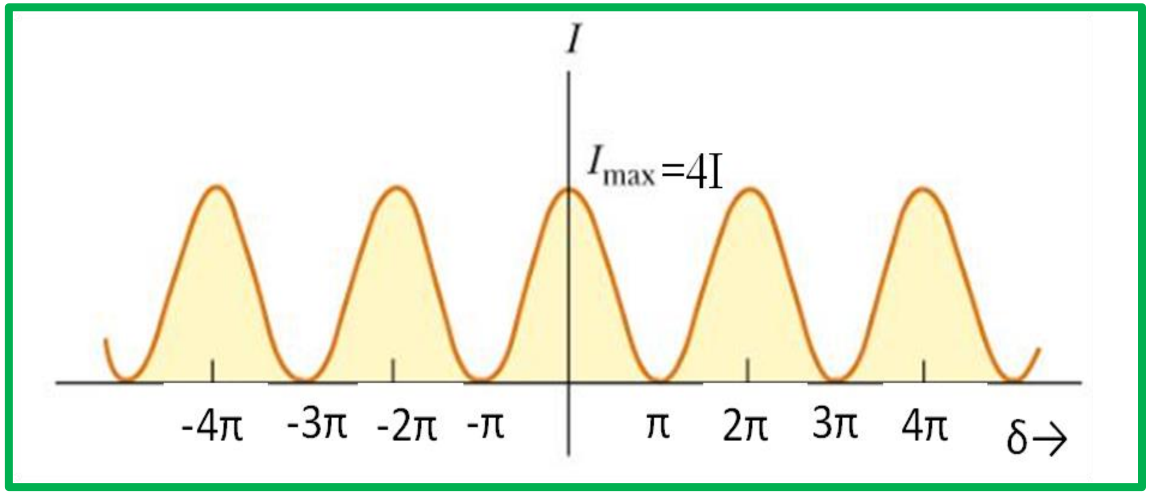
Path difference $= \frac{(2n+1)\lambda}{2}, n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots..$

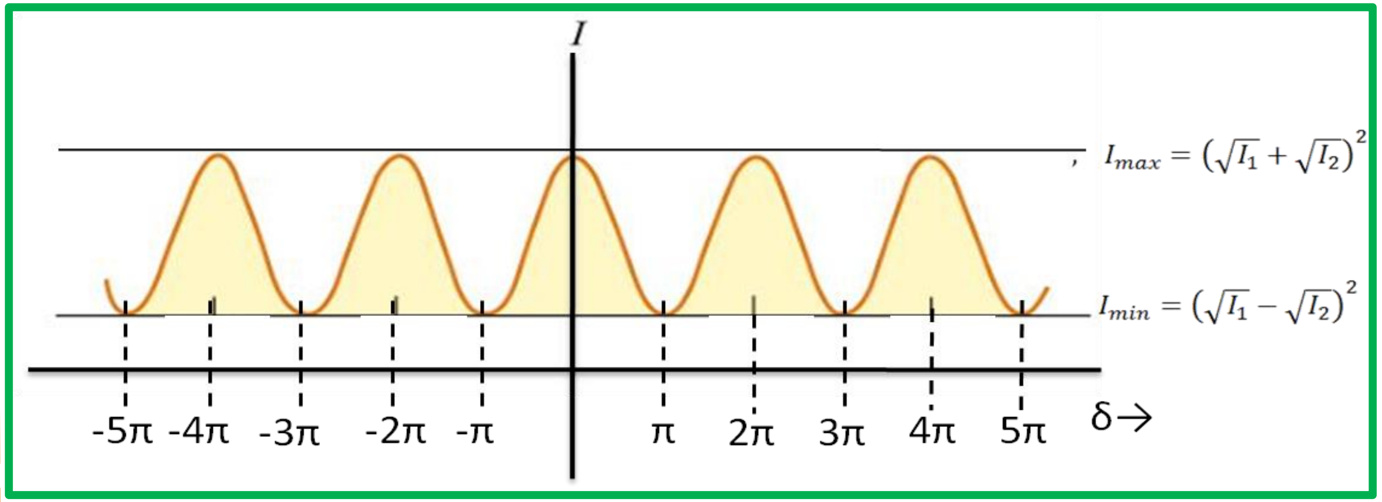
The minimum intensity, $\boxed{I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = (a_1 - a_2)^2}$ ....(17)

Intensity distribution curve

For $I_1 = I_2 = I,\quad I_{max} = 4I \text{ and } I_{min} = 0$

For $I_1 \neq I_2,\quad I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \text{ and } I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \neq 0$





Can energy be conserved in interference

So far it is known that after interference occurs, we observe a distribution of bright (maxima) and dark (minima) regions rather than uniform illumination. If we find out the average intensity post interference, we get,

$$I_{avg} = \frac{\int_0^{2\pi} I d\delta}{\int_0^{2\pi} d\delta} \qquad \text{..................(18)}$$

Substituting equation (15) in (18),

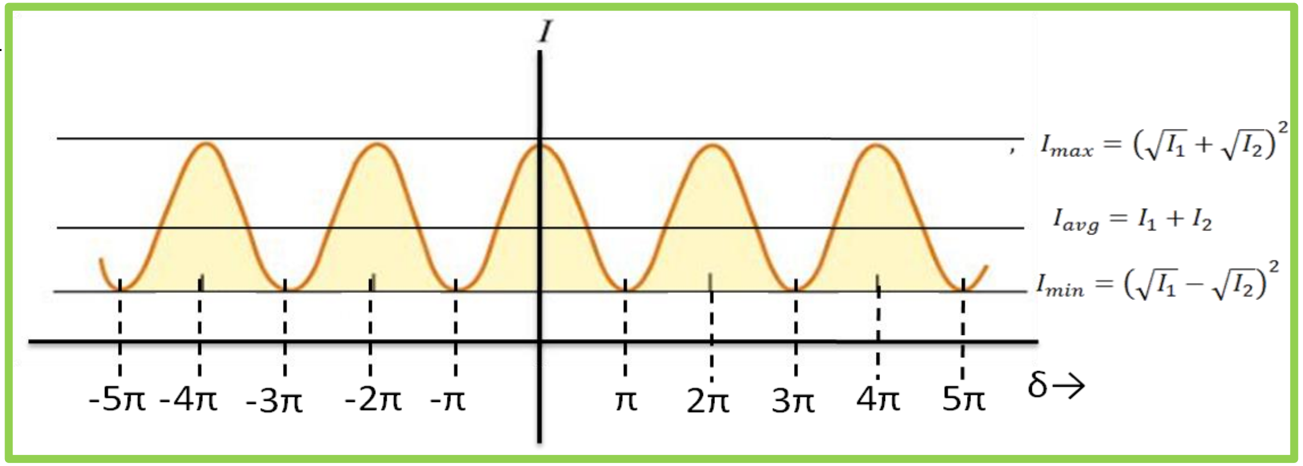
$$I_{avg} = \frac{\int_0^{2\pi} (I_1 + I_2 + 2\sqrt{I_1 I_2} cos\delta) d\delta}{2\pi} \qquad \text{..................(19)}$$

$$I_{avg} = \frac{\int_0^{2\pi} (I_1 d\delta + I_2 d\delta + 2\sqrt{I_1 I_2} cos\delta d\delta)}{2\pi}$$

Contd....

$$\Rightarrow I_{avg} = \frac{[I_1\delta]_0^{2\pi} + [I_2\delta]_0^{2\pi} - \left[2\sqrt{I_1 I_2}sin\delta\right]_0^{2\pi}}{2\pi}$$

$$\Rightarrow \boxed{I_{avg} = I_1 + I_2} \quad \text{.............(20)}$$



The total intensity of the wave fronts before interference occurs is also $I_1 + I_2$

$\Rightarrow$ '**Law of Conservation of Energy' is satisfied in the phenomena of interference**.

$\Rightarrow\Rightarrow$whatever energy apparently disappears at the minima, appears at the maxima.

Take Home

- We discussed about waves with some practical examples; their characteristics; transverse and longitudinal waves; and the differential wave equation.
- Next topic of interest was the phenomenon of interference and its real life applications; conditions of interference.
- We also had an idea upon the analytical treatment of interference by considering the two slit pattern.
- Further, the intensity distribution was elaborated along with the verification of law of conservation of energy in interference.