Problems 2.2

2.2

Q.(12) consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a visa credit card and B be the analogous event for a master card. Suppose that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.25$.

a) Compute the probability that the selected individual has at least one of the two types of cards (i.e., the probability of the event $A \cup B$)

b) what is the probability that the selected individual has neither type of card?

c) Describe, in terms of A and B the event that the selected student has a visa card but not a MasterCard, and then calculate the probability of this event.

Ans:- Given that

A = "selected individual has a visa card"

B = " " " " Master Card"

$P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.25$

(a) $A \cup B$ = "selected individual has at least one of the two types of cards"

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.5 + 0.4 - 0.25 = \cancel{0.65}$$
$$= 0.65$$

(b) $A' \cap B'$ = "the event selected individual has neither type of card"

$$\therefore P(A' \cap B') = P\{(A \cup B)'\}, \quad \text{[using D' morgan's law]}$$
$$= 1 - P(A \cup B) = 1 - 0.65 = 0.35$$

(c) $A \cap B'$ = "selected student has a visa card but not a Master card"

$$\therefore P(A \cap B') = P(A) - P(A \cap B)$$
$$= 0.5 - 0.25 = 0.25$$

2.2 Q.(13)

A computer consulting firm presently has bids out on three project. Let A_i = {awarded project i}, for $i = 1,2,3$, and Suppose that $P(A_1) = 0.22$, $P(A_2) = 0.25$, $P(A_3) = 0.28$, $P(A_1 \cap A_2) = 0.11$, $P(A_1 \cap A_3) = 0.05$, $P(A_2 \cap A_3) = 0.07$, $P(A_1 \cap A_2 \cap A_3) = 0.01$. Express in words each of the following events, and compute the probability of each event:

a) $A_1 \cup A_2$ b) $A_1' \cap A_2'$ c) $A_1 \cup A_2 \cup A_3$ d) $A_1' \cap A_2' \cap A_3'$
e) $A_1' \cap A_2' \cap A_3$ f) $(A_1' \cap A_2') \cup A_3$

Ans: Given that A_i = {awarded project i}, $i = 1,2,3$.

(a) $A_1 \cup A_2$ = {awarded project 1 or project 2 or both projects},

$\therefore P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$
$= 0.22 + 0.25 - 0.11 = 0.36$

(b) $A_1' \cap A_2'$ = {awarded project neither 1 or 2},

$P(A_1' \cap A_2') = P\{(A_1 \cup A_2)'\}$, [using De morgan's law]
$= 1 - P(A_1 \cup A_2) = 1 - 0.36 = 0.64$
[$\because P(A') = 1 - P(A)$]

(c) $A_1 \cup A_2 \cup A_3$ = {awarded project 1 or project 2 or 3}

$\therefore P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$
$= 0.22 + 0.25 + 0.28 - 0.11 - 0.07 - 0.05 + 0.01$
$= 0.53$

$A_1 \cup A_2 \cup A_3$ = "awarded at least one of these 3 projects"

2.2

Q-(14) Suppose that 55 % of all adults regularly consume coeffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products.

a) What is the probability that a randomly selected adult regularly consumes both coeffee and soda?

b) What is the probability that a randomly selected adult does not regularly consume at least one of these two products?

Ans: Let $A = \{$adult regularly consumes coffee$\}$

$B = \{$adult " " carbonated soda$\}$

$C = \{$adult " " coeffee, soda or both$\}$

Then $P(A) = \frac{55}{100} = 0.55$, $P(B) = 0.45$, $P(C) = 0.70$

or $P(A \cup B) = 0.70$

(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.55 + 0.45 - 0.70 = 0.3$

(b) $P(C') = 1 - P(C) = 1 - 0.7 = 0.3$

or $P[(A \cup B)'] = 1 - P(A \cup B) = 1 - 0.7 = 0.3$

(d) $A_1' \cap A_2' \cap A_3' = \{$none of the three project was awarded$\}$

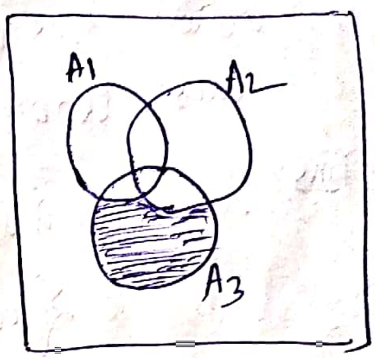
$P(A_1' \cap A_2' \cap A_3') = P\{(A_1 \cup A_2 \cup A_3)'\}$, [By De morgan's law]

$= 1 - P(A_1 \cup A_2 \cup A_3)$

$= 1 - 0.53 = 0.47$

(e) $A_1' \cap A_2' \cap A_3 = \{$awarded project 3 but neither 1 nor 2$\}$

From the venn diagram



$P(A_1' \cap A_2' \cap A_3) = P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

Here we add the last term i.e. $P(A_1 \cap A_2 \cap A_3)$ because we subtracted it twice from the probability of event A_3, once when we subtracted $A_1 \cap A_3$ and once when we subtracted $A_2 \cap A_3$. see the venn diagram.

$\therefore P(A_1' \cap A_2' \cap A_3) = 0.28 - 0.05 - 0.07 + 0.01 = 0.17$

(f) $(A_1' \cap A_2') \cup A_3 = \{$awarded neither project 1 and 2, or project 3$\}$

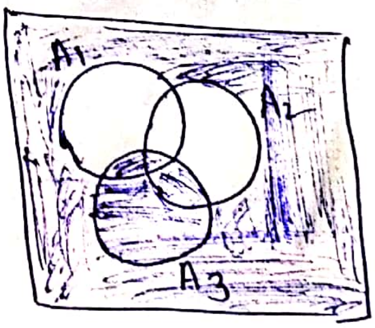
$\therefore P[(A_1' \cap A_2') \cup A_3]$

$= P(\text{none awarded}) + P(A_3)$

$= P(A_1' \cap A_2' \cap A_3') + P(A_3)$

$= 0.47 + 0.28$

$= 0.75$



Q. (25) (10th) The ~~the~~ three most popular options on a certain type of new car are a built-in GPS (A), a Sunroof (B), and an automatic transmission (C). If 40% of all purchasers request A, 55% request B, 70% request C, 63% request A or B, 77% request A or C, 80% request B or C and 85% request A or B or C, determine the probabilities of the following events.

a) The next purchaser will request at least one of the ~~following~~ three options.

b) The next purchaser will select none of the three options.

c) The next purchaser will request only an automatic transmission and not either of the other two options.

d) The next purchaser will select exactly one of three options.

Ans: $A \to$ GPS
$B \to$ Sunroof
$C \to$ Automatic transmission

Given $P(A) = 0.40$, $P(B) = 0.55$, $P(C) = 0.70$,
$P(A \cup B) = 0.63$, $P(A \cup C) = 0.77$, $P(B \cup C) = 0.80$,
$P(A \cup B \cup C) = 0.85$

Now $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.40 + 0.55 - 0.63 = 0.32$
$P(A \cap C) = P(A) + P(C) - P(A \cup C) = 0.40 + 0.70 - 0.77 = 0.33$
$P(B \cap C) = P(B) + P(C) - P(B \cup C) = 0.55 + 0.70 - 0.80 = 0.45$

(a) P(selected at least one of the three options A, B, C)

$= P(A \cup B \cup C) = 0.85$, (As given).

(b) P(none of the three options A, B, C selected)

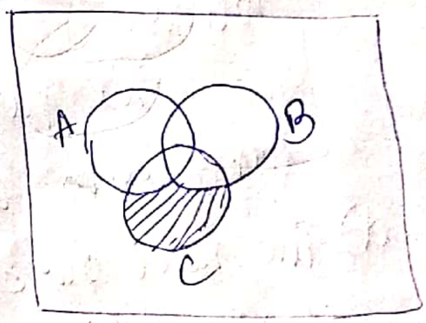
$= P\{(A \cup B \cup C)'\}$

$= 1 - P(A \cup B \cup C) = 1 - 0.85 = 0.15$

(c) P(only an automatic transmission and neither of GPS and sunroof)

$= P(\underline{C \cap A'} \cap B')$

$= P(C \cap A') - P(C \cap A' \cap B)$ $\quad \Big| \because P(A \cap B') = P(A) - P(A \cap B)$



$= P(C) - P(A \cap C) - P(\underline{B \cap C \cap A'})$

$= P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

~~Now P(A∩B∩C) = P(A~~

Now since $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$,

$\Rightarrow P(A \cap B \cap C) = P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(A \cap C)$

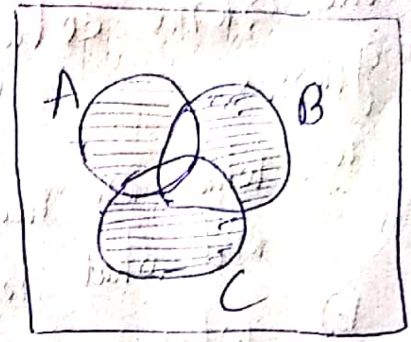
$= 0.85 - 0.4 - 0.55 - 0.7 + 0.32 + 0.33 + 0.45 = 0.30$

$\therefore P(C \cap A' \cap B') = 0.70 - 0.33 - 0.45 + 0.30 = 0.22$

(d) P (Select exactly one of these three options)

$= P(A \cap B' \cap C') + (A' \cap B \cap C') + P(A' \cap B' \cap C)$

Now, $P(A' \cap B' \cap C) = 0.22$ (from part (c))



$P(B \cap A' \cap C') = P(B) - P(B \cap A) - P(B \cap C) + P(B \cap A \cap C)$

$= 0.55 - 0.45 - 0.32 + 0.30$

$= 0.08$

$P(A \cap B' \cap C') = P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

$= 0.40 - 0.32 - 0.33 + 0.30$

$= 0.05$

$\therefore$ P (select exactly one of these three options)

$= 0.05 + 0.08 + 0.22$

$= 0.35$

Q. (16) An individual is presented with three different glasses of cola, labeled C, D and P. He is asked to taste all three and list them in order of preference. Suppose the same cola has actually been put into all three glasses.

a). What are the simple events in this ranking experiment and what probability would you assign to each one?

b). What is the probability that C is ranked first?

c). What is the probability that C is ranked first and D is ranked last?

Ans (a) The simple events are all corresponding outcomes of how do an individual list the three glasses of cola C, D and P. For example one simple event is CDP that means he likes cola C the most, then cola D and the least cola P.

∴ All simple events are

$S = \{CDP, CPD, DPC, DCP, PCD, PDC\}$.

P D C

We have 6 outcomes in S. Hence we could assign equal probabilities for each event which is $1/6$.

(b) $P(C \text{ is ranked } 1st) = \frac{2}{6} = 1/3$

(c) $P(C \text{ is ranked } 1st \text{ and } D \text{ is ranked last})$

$= P(\{CPD\}) = \frac{1}{6}$

2.2 Q.(26)

A Certain System can experience three different types of defects. Let A_i $(i=1,2,3)$ denote the event that the System has a defect of type i. suppose that
$P(A_1) = 0.12$, $P(A_2) = 0.07$, $P(A_3) = 0.05$, $P(A_1 \cup A_2) = 0.13$,
$P(A_1 \cup A_3) = 0.14$, $P(A_2 \cup A_3) = 0.10$, $P(A_1 \cap A_2 \cap A_3) = 0.01$

a) what is the probability that the system does not have a type 1 defect ?

b) what is the probability that the system has both type 1 and type 2 defects ?

c) what is the probability that the system has both type 1 and type 2 defects but not a type 3 defect ?

d) what is the probability that the system has at most two of these defects ?

Ans :-

(a) P(the system does not have a type 1 defect)
$= P(A_1') = 1 - P(A_1) = 1 - 0.12 = 0.88$

(b) P(the system has both type 1 and type 2 defects)
$= P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$
$= 0.12 + 0.07 - 0.13 = 0.06$

(c) P(the system has both type 1 and type 2 defects but not a type 3 defect)

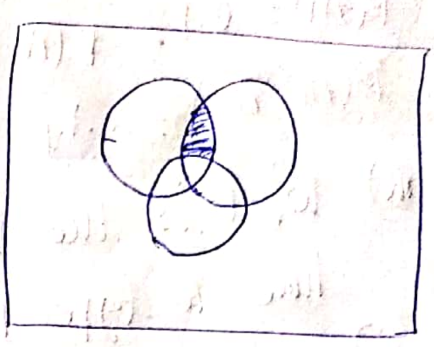
$= P(A_1 \cap A_2 \cap A_3')$

$= P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3)$

$= 0.06 - 0.01 = 0.05$

$\because P(A \cap B') = P(A) - P(A \cap B)$

or



(d) P(the system has at most two of these defects)

$= P\{(\text{no three defects})\}$

$= P\{(\text{three defects})'\} = 1 - P(A_1 \cap A_2 \cap A_3)$

$= 1 - 0.01 = 0.99$