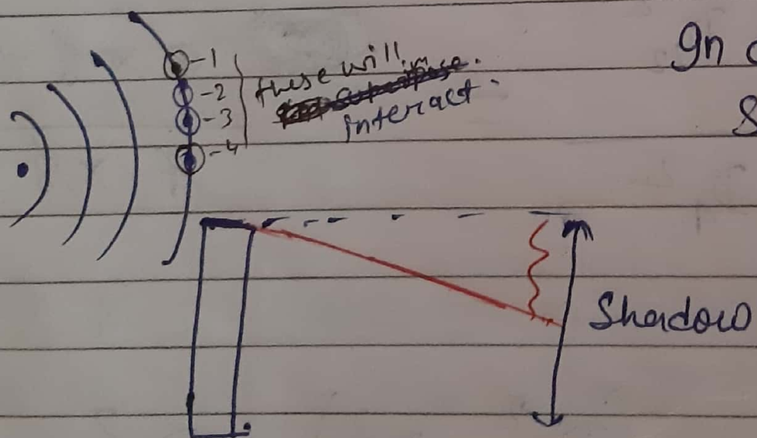
* Diffraction :-

When light falls on obstacles or small apertures whose dimensions are comparable to the wavelength of light, there is a departure from the straight line propagation and light bends around the corners of the obstacles or apertures and enters into the region of geometrical shadow. This bending of light is called diffraction.

Ex:- Closely spaced CD tracks act as a diffraction grating.

Hologram on a Credit Card, books.

Silver lining on the edges of Clouds.



In diffraction wave fronts of same primary wave ~~super~~ ~~impose~~ interact

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→ Source and Screen are present at finite distance → Fresnel's diffraction
Wavefront → Circular or spherical.

→ Source and Screen are at ∞ distance → Fraunhoffer's
Wave front type → Plane

Interference	Diffraction
① It is the result of interaction of light coming from different wave fronts originating from the same source.	It is the result of interaction of light coming from different parts of the same wavefront.
② Interference fringes may or may not be of the same width	Diffraction fringes are not of the same width.
③ All the bright bands are of the same intensity	The different maximas are of varying intensities with the maximum intensity for the central maximum.

→ **Diffraction:-**

- Fresnel's diffraction. (not in Syllabus)
- Fraunhoffer's diffraction.

Fresnel's	Fraunhoffer's
① The source and the screen are placed at finite distances from the slit or obstacle having sharp edges	The source and the screen are placed at ∞ distances from the slit or the obstacle having sharp edges.

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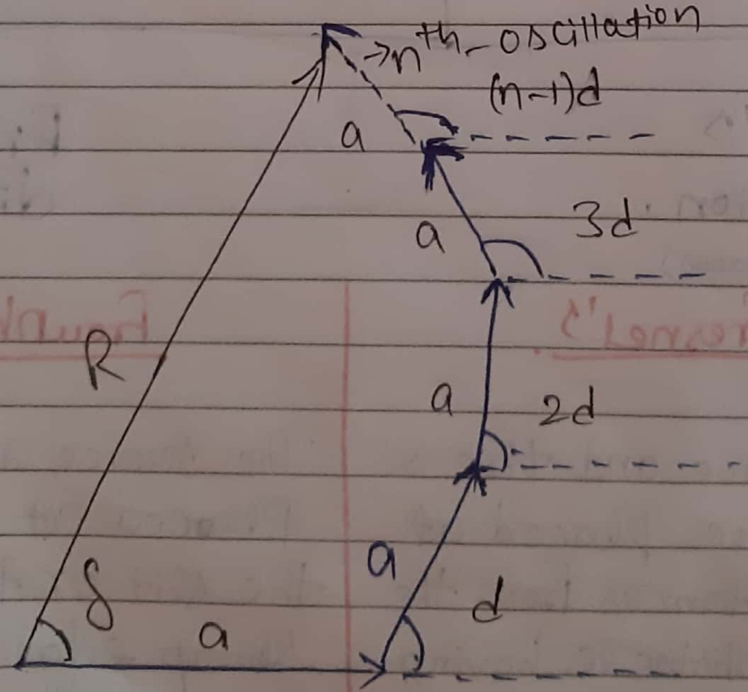
2	No lenses are required for making the rays Parallel or Converted	Minimum two convex lenses are required, One to make the light from the source Parallel and the other to focus the light after diffraction on to the Screen.
3	Incident wavefront is spherical or cylindrical	Incident wavefront is Plane

* Resultant of 'n' Simple harmonic oscillations :-



No. of simple harmonic oscillation = n
amplitude of each oscillation = a
Phase diff. b/w the successive oscillations = d.

Let there be 'n' oscillation of same time period, and same amplitude 'a' and the same phase diff. 'd' b/w the successive oscillations which act on a particle simultaneously.

To find out the Resultant ampiltude 'R', we construct a vector polygon.

Resolving the amplitudes into their parallel and perpendicular ~~direction~~ components :-

$$R\cos\delta = \left[a + a\cos d + a\cos 2d + \cdots + a\cos(n-1)d\right]$$

$$= a\left[1 + \cos d + \cos 2d + \cdots + \cos(n-1)d\right]$$

$$R\sin\delta = a\left[0 + \sin d + \sin 2d + \cdots + \sin(n-1)d\right]$$

Solving it:-

$$\boxed{R = a\,\frac{\sin\frac{nd}{2}}{\sin\frac{d}{2}}}$$

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23/9/19 * Fraunhoffer's diffraction at a single slit.

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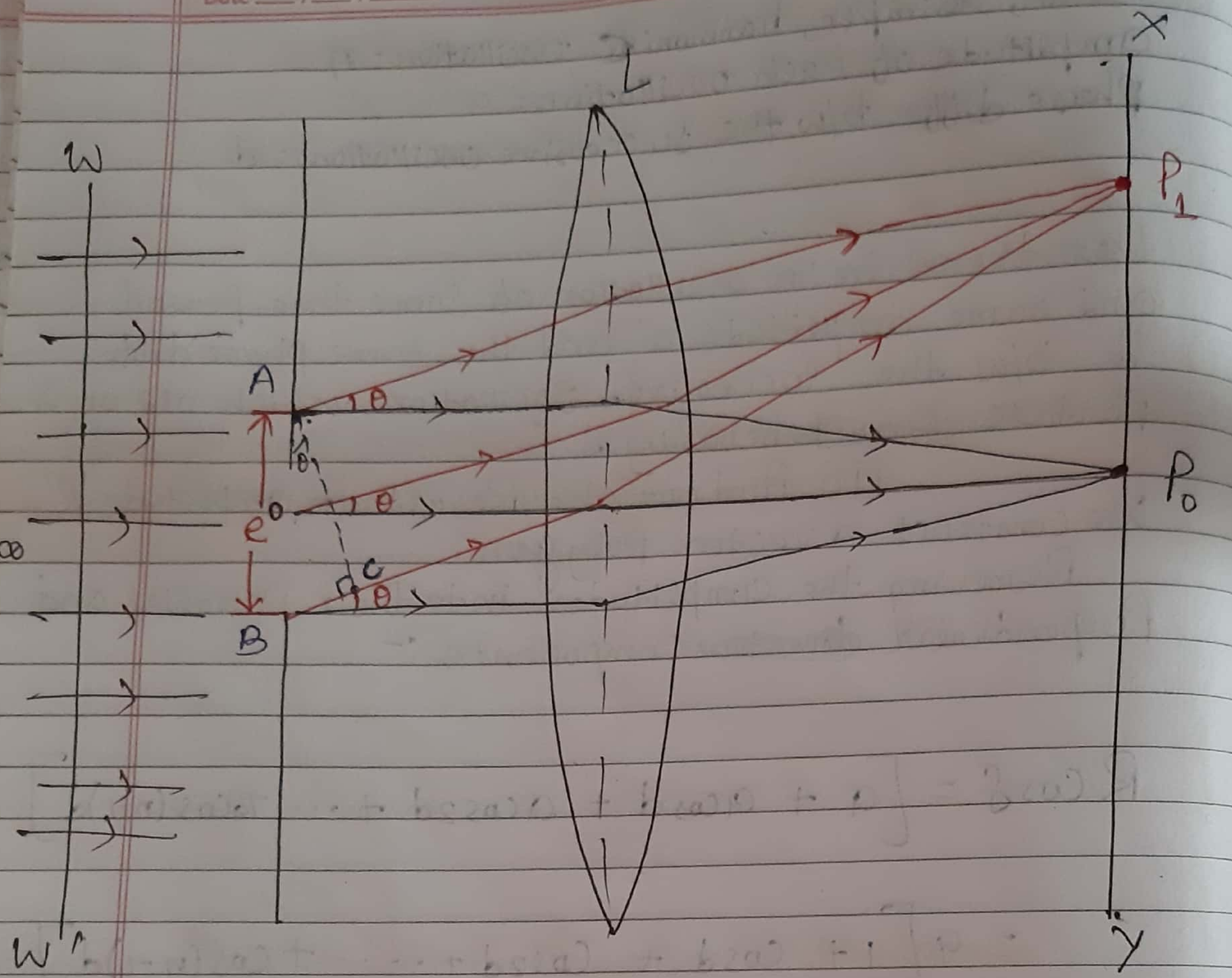

* The figure represents a section AB of narrow slit of width 'e' and a plane wavefront WW' of monochromatic light of wavelength 'λ' is incident on it

* The diffracted light after passing through the slit is focused onto a screen placed in the focal plane of convex lens 'L'.

* The secondary wavelets after diffraction which are travelling normally to the slit along OP' are -

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brought to focus at a point P_0 on the screen.

* P_0 is the central bright maxima.

* The secondary wavelets travelling at an angle θ with the normal are brought to focus at P_1 on the screen.

* P_1 is a point of min^m or max^m intensity depending upon the path diff. b/w the secondary wavelets, originating from the corresponding points of the wavefront.

→ The path diff. b/w the secondary wavelets from 'A' and 'B' in the direction θ = BC

$$BC = AB \sin\theta = e \sin\theta$$

→ The corresponding phase diff. :-

$$= \frac{2\pi}{\lambda} e \sin\theta$$

Let us consider the width of the slit is divided into 'n' equal parts and the amplitude of the wave from each part is 'a'. The phase diff. b/w any two consecutive waves from these parts would be,

$$\frac{1}{n}\left(\frac{2\pi}{\lambda} e \sin\theta\right) = d \ (\text{let say})$$

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To find out the Resultant amplitude we apply the method of vector addition of amplitudes and hence, the Resultant 'R' :-

$$R = a \frac{\sin \frac{nd}{2}}{\sin \frac{d}{2}}$$

$$= a \frac{\sin\left(\frac{\pi e \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e \sin\theta}{n\lambda}\right)}$$

(Substituting $d = \frac{1}{n}\frac{2\pi}{\lambda} e \sin\theta$)

Let $\frac{\pi e \sin\theta}{\lambda} = \alpha$

($e \sin\theta$ = Path diff.)

c) $$R = a \frac{\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

$\because n \to \infty, \frac{\alpha}{n} \to 0$ (as 'n' is very large)

So, $\sin\frac{\alpha}{n} \simeq \frac{\alpha}{n}$

Hence,

$$R = na \frac{\sin\alpha}{\alpha}.$$

$$\boxed{R = A \frac{\sin\alpha}{\alpha}} \quad \text{—— (1)}$$

($A = na$)

($n \to \infty$
$a \to 0$
$na \Rightarrow A$ finite)

The Resultant Intensity;

$$\boxed{I = |A|^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2} \quad \text{--- (2)}$$

where; $I_0 = |A|^2$

* Principle Maxima:- (Max^m intensity).

From equation (1) & (2) 'I' will be max^m when 'R' is max^m.

and 'R' will be max^m when $\frac{\sin\alpha}{\alpha} = 1$ under the limit $\alpha \to 0$

$$\Rightarrow \lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$$

Then, $\boxed{R = A}$ and $\boxed{I_{P.M} = I_0}$ (Intensity at Principle maxima i.e. max^m intensity).

Under the limiting value $\alpha \simeq 0$

$$\Rightarrow \frac{\pi e \sin\theta}{\lambda} = 0 \qquad \left\{\because \frac{\pi e \sin\theta}{\lambda} = \alpha\right\}$$

$$\Rightarrow \sin\theta = 0$$

$$\Rightarrow \boxed{\theta = 0^\circ}$$

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The Cond^n $[\theta = 0°]$ Corresponds to the Principle max^m and this max^m is formed by those Secondary wavelets which travel normally (⊥) to the Slit.

→ To find out the other secondary maxima's and minimas we differentiate the intensity expression w.r.t α and equate it to 0.

So,

$$\frac{dI}{d\alpha} = 0$$

$$\Rightarrow \frac{d}{d\alpha}\left(I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\right) = 0$$

$$\Rightarrow I_0 \frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right)^2 = 0$$

$$\Rightarrow I_0 \, 2\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right) = 0$$

Either; $\sin\alpha = 0$ or $(\alpha\cos\alpha - \sin\alpha) = 0$

$\Rightarrow$

→ **Minima:-**

From eq^n (2) intensity will be minimum when
$\sin\alpha = 0$

$\Rightarrow \alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

or, $\boxed{\alpha = \pm m\pi}$, $m = 1, 2, 3, \ldots$

$\Rightarrow \dfrac{\pi e \sin\theta}{\lambda} = \pm m\pi$

$\Rightarrow \boxed{e\sin\theta = \pm m\lambda}$, $m = 1, 2, 3, \ldots$

Path. diff.

$m=0$ is not accepatable because this corresponds to $\alpha = 0$ where, we get the principle maxima

The above equation expresses the condition that the rays diffracted at an angle θ, superpose destructively if the path difference b/w the extreme diffracted rays is an integral multiple of λ then,

$\boxed{I_{min} = 0}$

$\Rightarrow$

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→ Secondary maxima:-

The condⁿ $\alpha\cos\alpha - \sin\alpha = 0$ Corresponds to the Secondary maximum.

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\Rightarrow \alpha = \frac{\sin\alpha}{\cos\alpha}$$ (α is not Constant)

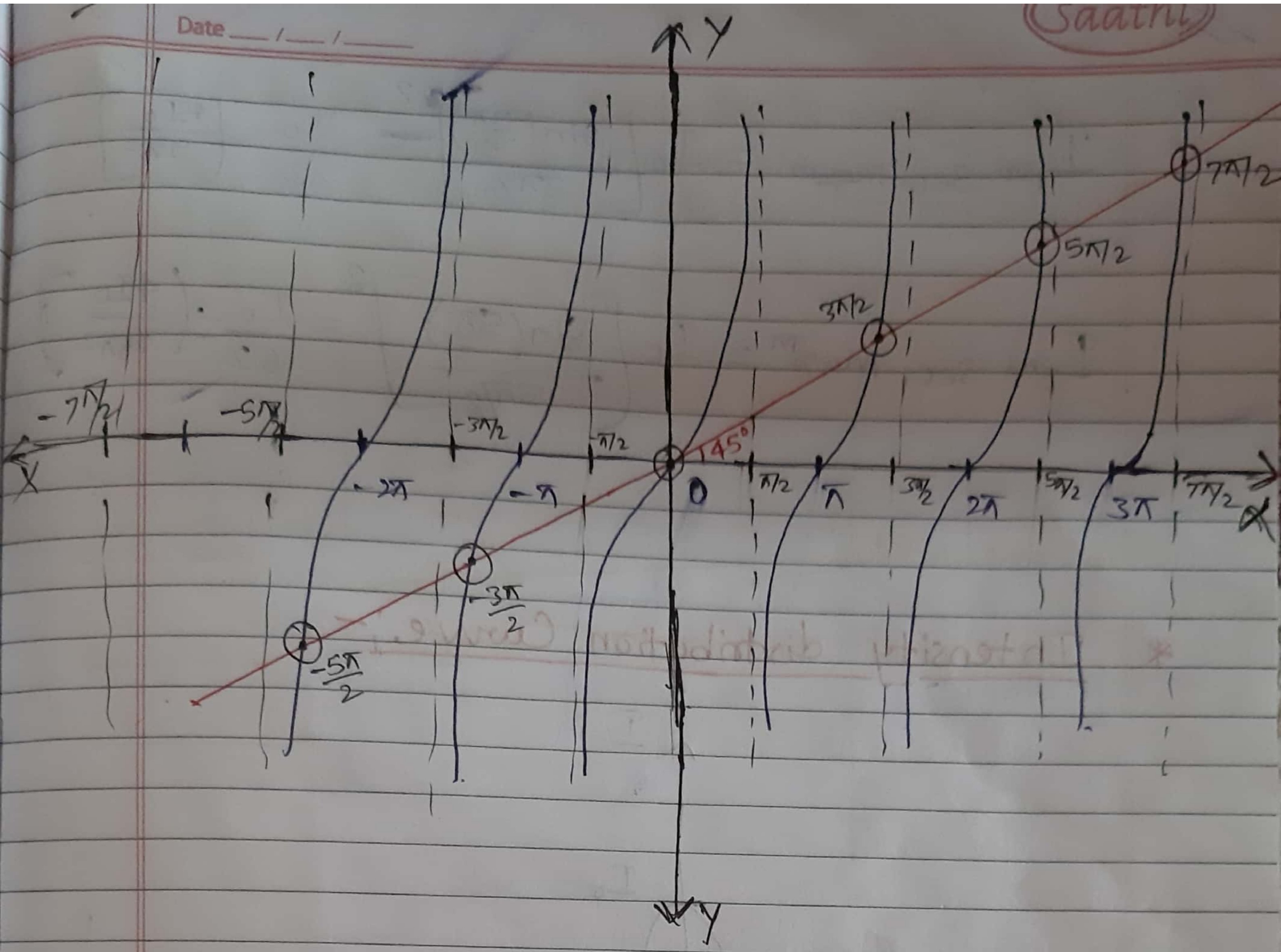
$$\Rightarrow \boxed{\alpha = \tan\alpha} \rightarrow y = \alpha, \quad y = \tan\alpha$$

This eqn can be solved graphically by plotting the curves $y = \alpha$ and $y = \tan\alpha$ on the same graph.

→ The relation $y = \alpha$ represents a straight line.- Passing through the origin and making an angle 45° with both the axis.

→ $y = \tan\alpha$ represents discontinous curves having a number of brenches with asymptotes at intervals of 'π'. The points of intersections of both the curves will give the value of α that will satisfy the relation $\alpha = \tan\alpha$.

⇒



→ The secondary max^m occur at

$$\alpha = \pm\frac{3\pi}{2},\ \pm\frac{5\pi}{2},\ \pm\frac{7\pi}{2},\ \ldots$$

$$\boxed{\alpha = \pm(2n+1)\frac{\pi}{2}}\ ; \quad n = 1, 2, 3, \ldots$$

$\alpha = 0$ is neglected since we get a principal max^m there.

$\Rightarrow$

$$I_{1st\ sec.\ max^m} = I_0 \left[\frac{\sin\left(\frac{3\pi}{2}\right)}{3\pi/2}\right]^2 = \frac{I_0}{22} = \left(\frac{4I_0}{9\pi^2}\right)$$

$$I_{2nd\ sec.\ max^m} = I_0 \left[\frac{\sin\left(\frac{5\pi}{2}\right)}{5\pi/2}\right]^2 = \frac{I_0}{62} = \left(\frac{4I_0}{25\pi^2}\right)$$

* Intensity distribution Curve :-