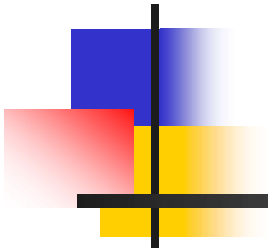


Introduction to Data Structures



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What is Algorithm

- An algorithm is “a finite set of precise instructions for performing a computation or for solving a problem”. It will take some value or set of values, as input, and produces some value or set of values, as output.
- Example:
 - Directions to somebody's house is an algorithm
 - A sorted, non-decreasing sequence of natural numbers of non-zero, finite length

Some algorithms are harder than others



- Some algorithms are easy
 - Finding the largest (or smallest) value in a list
 - Finding a specific value in a list
- Some algorithms are a bit harder
 - Sorting a list
- Some algorithms are very hard
 - Finding the shortest path between Miami and Seattle
- Some algorithms are essentially impossible
 - Factoring large composite numbers



Characteristics of an Algorithm

- Well-Defined Inputs: Inputs, it should be well-defined.
- Well-Defined Outputs: Must clearly define the output.
- Definiteness: the steps are defined precisely.
- Correctness: should produce the correct output.
- Finiteness: should end up with finite number of steps.
- Effectiveness: each step must be able to be performed in a finite amount of time.
- Generality: the algorithm should be applicable to all problems of a similar form.
- Language Independent: Just plain instructions that can be implemented in any language.



Algorithm: Example

Algorithm to add two numbers

Step 1: Start
Step 2: Declare variables num1, num2 and sum.
Step 3: Read values num1 and num2.
Step 4: Add num1 and num2 and assign the result to sum.
 $\text{sum} \leftarrow \text{num1} + \text{num2}$
Step 5: Display sum
Step 6: Stop

Algorithm to find all roots of a quadratic equation $ax^2+bx+c=0$.

Step 1: Start
Step 2: Declare variables a, b, c, D, x1, x2, rp and ip;
Step 3: Calculate discriminant
 $D \leftarrow b^2 - 4ac$
Step 4: If $D \geq 0$
 $r1 \leftarrow (-b + \sqrt{D}) / 2a$
 $r2 \leftarrow (-b - \sqrt{D}) / 2a$
 Display r1 and r2 as roots.
 Else
 Calculate real part and imaginary part
 $rp \leftarrow -b / 2a$
 $ip \leftarrow \sqrt{(-D)} / 2a$
 Display $rp + j(ip)$ and $rp - j(ip)$ as roots
Step 5: Stop



What is a Good Algorithm?

- Efficient:
 - Running time (**small**)
 - Space used (**less**)
- Efficiency as a function of input size:
 - The input size
 - Number of data elements



Space Complexity of Algorithms

For any algorithm memory may be used for the following:

- Variables (includes constant values, temporary values)
 - Program Instruction
 - Execution
-
- **Space complexity:** Amount of memory used by the algorithm (including the input values to the algorithm) to execute and produce the output.
 - While executing, algorithm uses memory space for three reasons:
 - **Instruction Space:** memory used to save the compiled version of instructions.
 - **Environmental Stack:** In function call, a system stack is maintained.
 - **Data Space:** Amount of space used by the variables and constants.
 - while calculating the Space Complexity of any algorithm,
 - **only Data Space is considered** (neglecting Instruction Space and Environmental Stack.)



Calculating the Space Complexity

Example:

```
{  
    int z = a + b + c;  
    return(z);  
}
```

In this expression,

- variables a, b, c and z are all integer types, take 4 bytes each.
- Total memory = $(4(4) + 4) = 20$ bytes,
- Additional 4 bytes is for return value.
- Because this space requirement is **fixed**, called **Constant Space Complexity**.

Example:

```
// n is the length of array a[]  
int sum(int a[], int n) {  
    int x = 0; // 4 bytes for x  
    for(int i = 0; i < n; i++) { // 4 bytes for i  
        x = x + a[i];  
    }  
    return(x);  
}
```

- In this code, $4*n$ bytes of space is required for the array a[] elements.
- 4 bytes each for x, n, i and the return value.
- Total memory requirement is $(4n + 16)$, **increasing linearly** with increase in the input value n. Called as **Linear Space Complexity**.



Data Structures and Algorithms

- **Algorithm:** Outline, the essence of a computational procedure, step-by-step instructions
- **Program:** an implementation of an algorithm in some programming language
- **Data structure:**
 - Organization of data needed to solve the problem
 - A way of organizing all data items that considers not only the elements stored but also their relationship to each other.



Time Complexity

- Time complexity of an algorithm signifies the **total time required** by the program to run till its completion.
- Time Complexity is most commonly estimated by **counting the number of elementary steps performed** by any algorithm to finish execution.
- Any problem can have number of solutions.
- **Two different algorithms** to find square of a number:

`m=0`

`for i=1 to n`

`do m = m + n`

`return m`

- loop will run n number of times
- time complexity will be n atleast
- As the value of n will increase the time taken will also increase

`return n*n`

- For this code, time complexity is constant
- Never be dependent on the value of n, so always give the result in 1 step.



Time Complexity

- Performance of algorithm may vary with different input data
- **worst-case Time complexity** is considered because that is the maximum time taken for any input size.
- The most common metric for calculating time complexity is **Big O notation**.
- This **removes all constant factors** so that the running time can be estimated in relation to n , as n approaches infinity.



Time Complexity

statement;

- A single statement.
- Its Time Complexity will be **Constant**.
- The running time of the statement will **not change in relation to n**.

```
for(i=0; i < n; i++) {  
    statement;  
}
```

- The time complexity for this algorithm will be **Linear**.
- The running time of the loop is **directly proportional to n**.
- When n doubles, so does the running time.



Time Complexity

```
for(i=0; i < n; i++) {  
    for(j=0; j < n; j++) {  
        statement;  
    }  
}
```

- Time complexity will be **Quadratic**.
- Running time of the two loops is **proportional to the square of n**.
- When n doubles, the running time increases by $n * n$.

```
while(low <= high) {  
    mid = (low + high) / 2;  
    if (target < list[mid])  
        high = mid - 1;  
    else if (target > list[mid])  
        low = mid + 1;  
    else break;  
}
```

- This is an algorithm to **break a set of numbers into halves**, to search a particular value.
- **Logarithmic Time Complexity**.
- The running time of the algorithm is proportional to the number of times n can be divided by 2.
- Because the algorithm **divides the working area in half with each iteration**.



Asymptotic Analysis

- Asymptotic analysis of an algorithm refers to defining the mathematical boundation of its run-time performance.
- Asymptotic analysis is input bound
 - no input to the algorithm, concluded to work in a **constant time**.
 - Other than the "*input*" all other factors are considered constant.
- Asymptotic analysis refers to computing the **running time of any operation in mathematical units of computation**.
 - Example: running time of one operation is computed as $f(n)$ and may be for another operation it is computed as $g(n^2)$
- Describes the algorithm efficiency and performance in a meaningful way.
- Describes the behaviour of time or space complexity for an algorithm

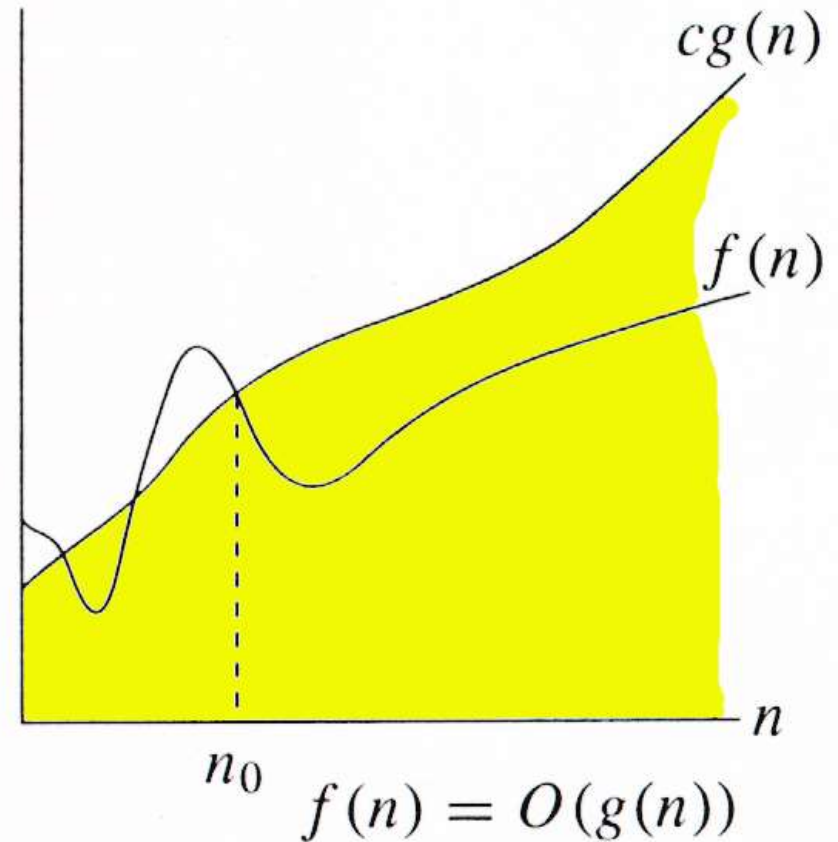


Asymptotic Notation

- Commonly used asymptotic notations to calculate the running time complexity of an algorithm.
 - O Notation
 - Ω Notation
 - θ Notation

Big Oh Notation, O

- The Big Oh notation is the formal way to express the **upper bound of an algorithm's running time**.
- It measures the worst case time complexity or the longest amount of time an algorithm can possibly take to complete.
- Definition: $O(g(n)) = \{ f(n) : \exists \text{ positive constants } c \text{ and } n_0, \text{ such that } \forall n \geq n_0, \text{ we have } 0 \leq f(n) \leq cg(n) \}$
- Intuitively: Set of all functions whose rate of growth is the same as or lower than that of $g(n)$.





Big Oh Notation: Example

Example:

Compute Big-Oh notation for

$$f(n) = 10n^2 + 4n + 2$$

$$\text{Given } f(n) = 10n^2 + 4n + 2$$

$$f(n) \leq c * g(n)$$

$$10n^2 + 4n + 2 \leq 10n^2 + 4n + n, \text{ for } n \geq 2$$

$$10n^2 + 4n + 2 \leq 10n^2 + 5n$$

$$10n^2 + 4n + 2 \leq 10n^2 + n^2, \text{ for } n \geq 5$$

$$10n^2 + 4n + 2 \leq 11n^2 \text{ where } c = 11, \\ g(n) = n^2 \text{ and } n_0 = 5$$

$$\text{Hence } f(n) = O(n^2)$$

Example:

Compute Big-Oh notation for

$$f(n) = 1000n^2 + 100n - 6$$

$$\text{Given } f(n) = 1000n^2 + 100n - 6$$

$$f(n) \leq c * g(n)$$

$$1000n^2 + 100n - 6 \leq 1000n^2 + 100n \text{ for all values of } n$$

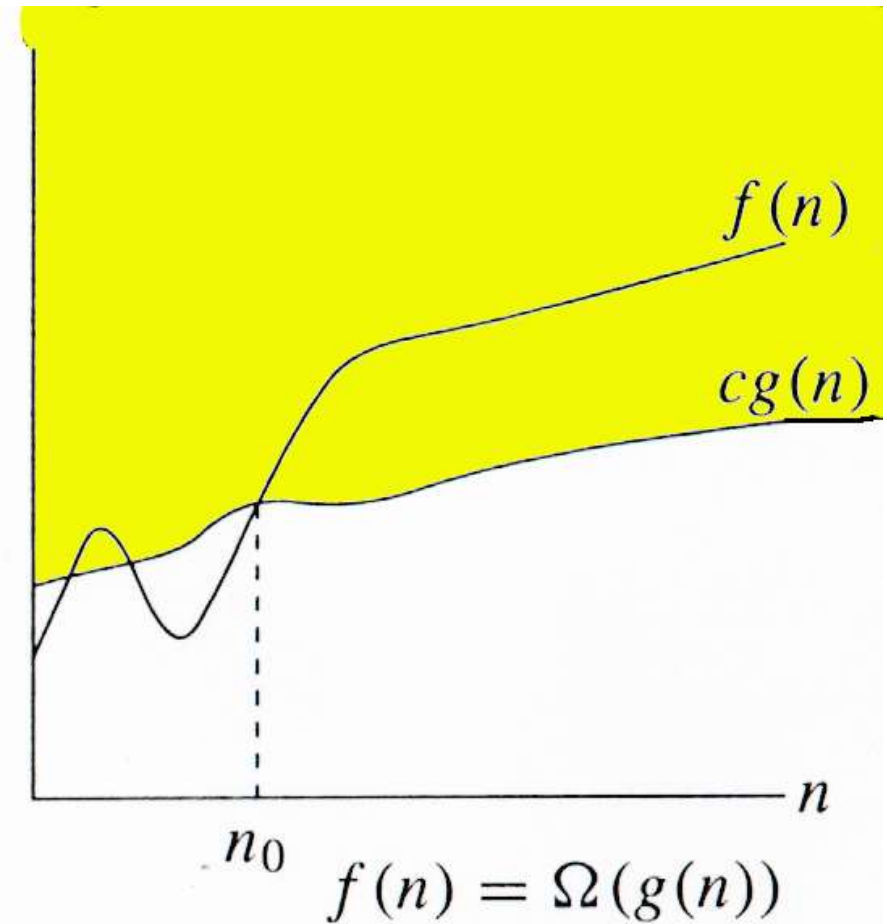
$$1000n^2 + 100n - 6 \leq 1000n^2 + n^2, \text{ for } n \geq 100$$

$$1000n^2 + 100n - 6 \leq 1001n^2, \text{ where } c = 1001, g(n) = \\ n^2 \text{ and } n_0 = 100$$

$$\text{Hence } f(n) = O(n^2)$$

Omega Notation, Ω

- The Omega notation is the formal way to express the lower bound of an algorithm's running time.
- It measures the best case time complexity or the best amount of time an algorithm can possibly take to complete.
- **Definition:** $\Omega(g(n)) = \{f(n) :$
 \exists positive constants c and n_0 , such that
 $\forall n \geq n_0,$
- we have $0 \leq cg(n) \leq f(n)\}$
- **Intuitively:** Set of all functions whose rate of growth is the same as or higher than that of $g(n)$.





Omega Notation, Ω

Example:

Compute omega notation for $f(n) = 10n^2 + 4n + 2$

Given $f(n) = 10n^2 + 4n + 2$

$$f(n) \geq c * g(n)$$

$$10n^2 + 4n + 2 \geq 10n^2 \text{ for all values of } n \text{ (} n \geq 0 \text{)}$$

where $c=10$, $g(n)=n^2$ and $n_0=0$

Hence $f(n) = \Omega(n^2)$

Example:

Compute omega notation for $f(n) = 4n^3 + 2n + 3$

Given $f(n) = 4n^3 + 2n + 3$

$$f(n) \geq c * g(n)$$

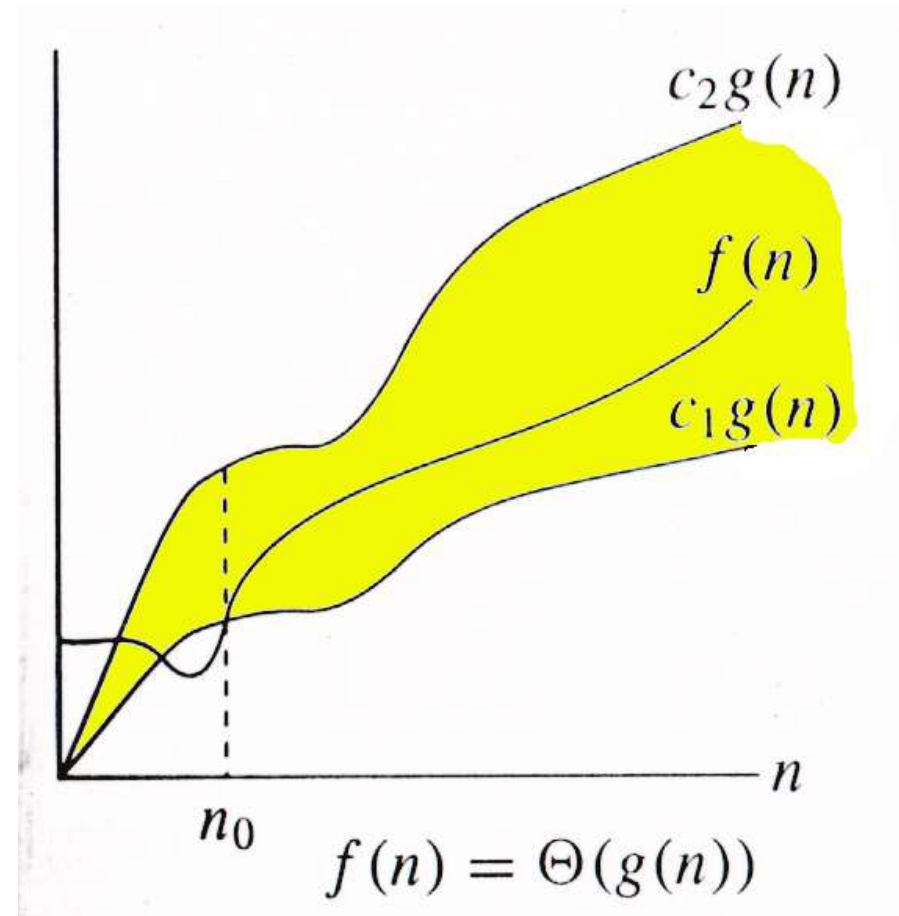
$$4n^3 + 2n + 3 \geq 4n^3 \text{ for all values of } n \text{ (} n \geq 0 \text{)}$$

where $c=4$, $g(n)=n^3$

and $n_0=0$

Theta Notation, Θ

- The Theta notation is the formal way to express **both the lower bound and the upper bound** of an algorithm's running time.
- **Definition:** $\Theta(g(n)) = \{f(n) :$
 \exists positive constants c_1, c_2 , and n_0 ,
such that $\forall n \geq n_0, 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)\}$
- **Intuitively:** Set of all functions that have the same *rate of growth* as $g(n)$.
- $g(n)$ is an asymptotically tight bound for $f(n)$.





Theta Notation, θ

Example:

Compute theta notation for $f(n)=3n+2$

Given $f(n)=3n+2$

$$c_1 * g(n) \leq f(n) \leq c_2 * g(n)$$

Compute $f(n) \leq c_2 * g(n)$

$$3n+2 \leq 3n+n, \text{ for } n \geq 2$$

$$3n+2 \leq 4n \text{ where } c_2=4 \text{ and } g(n)=n$$

Compute $c_1 * g(n) \leq f(n)$

$$3n \leq 3n+2 \text{ for all values of } n$$

$$\text{where } c_1=3, g(n)=n$$

Hence, $f(n)=\theta(n)$

Example:

Compute theta notation for $f(n)=10n^2+4n+2$

Given $f(n)=10n^2+4n+2$

$$c_1 * g(n) \leq f(n) \leq c_2 * g(n)$$

Compute $f(n) \leq c_2 * g(n)$

$$10n^2+4n+2 \leq 10n^2+4n+n, \text{ for } n \geq 2$$

$$10n^2+4n+2 \leq 10n^2+5n$$

$$10n^2+4n+2 \leq 10n^2+n^2, \text{ for } n \geq 5$$

$$10n^2+4n+2 \leq 11n^2, \text{ where } c_2=11 \text{ and } g(n)=n^2$$

Compute $c_1 * g(n) \leq f(n)$

$$10n^2 \leq 10n^2+4n+2 \text{ for all values of } n$$

$$\text{where } c_1=10, g(n)=n^2$$

Hence, $f(n)=\theta(n^2)$



Common Asymptotic Notations

Following is a list of some common asymptotic notations:

- constant — $O(1)$
- logarithmic — $O(\log n)$
- linear — $O(n)$
- $n \log n$ — $O(n \log n)$
- quadratic — $O(n^2)$
- cubic — $O(n^3)$
- polynomial — $n^{O(1)}$
- exponential — $2^{O(n)}$



Analysis of Algorithms

1. **O(1)**: Time complexity of a function (or set of statements) is considered as $O(1)$ if it doesn't contain **loop**, **recursion** and **call to any other non-constant time function**.

// set of non-recursive and non-loop statements

- Example: swap() function has $O(1)$ time complexity.
- A **loop** or **recursion** that runs a constant number of times is also considered as $O(1)$. For example the following loop is $O(1)$.

// Here c is a constant

```
for (int i = 1; i <= c; i++) {  
    // some O(1) expressions }  
}
```



Analysis of Algorithms

- 2) **$O(n)$** : Time Complexity of a loop is considered as $O(n)$ if the loop variables is **incremented** / **decremented** by a constant amount.

Example:

// Here c is a positive integer constant

```
for (int i = 1; i <= n; i += c) {  
    // some  $O(1)$  expressions
```

```
}
```

```
for (int i = n; i > 0; i -= c) {  
    // some  $O(1)$  expressions }
```




Analysis of Algorithms

3. $O(n^c)$: Time complexity of nested loops is equal to the number of times the innermost statement is executed.

Example:

```
for (int i = 1; i <= n; i += c) {  
    for (int j = 1; j <= n; j += c) {  
        // some O(1) expressions    }  
    }
```

```
for (int i = n; i > 0; i -= c) {  
    for (int j = i+1; j <= n; j += c) {  
        // some O(1) expressions  
    }
```

For example Selection sort and Insertion Sort have $O(n^2)$ time complexity.



Analysis of Algorithms

- 4) **$O(\text{Log } n)$** : Time Complexity of a loop is considered as $O(\log n)$ if the loop variables is divided / multiplied by a constant amount.

```
for (int i = 1; i <= n; i *= c) {  
    // some  $O(1)$  expressions    }  
for (int i = n; i > 0; i /= c) {  
    // some  $O(1)$  expressions    }
```

Example: Binary Search (refer iterative implementation) has $O(\log n)$ time complexity.



Analysis of Algorithms

5. **$O(\text{LogLog}n)$** Time Complexity of a loop is considered as $O(\log \log n)$ if the loop variables is reduced / increased exponentially by a constant amount.

```
// Here c is a constant greater than 1
for (int i = 2; i <= n; i = pow(i, c)) {
    // some O(1) expressions
}

// Here func() is sqrt or cuberooroot or any other constant root
for (int i = n; i > 0; i = func(i)) {
    // some O(1) expressions
}
```



Analysis of Algorithms

How to combine time complexities of consecutive loops?

When there are consecutive loops, we calculate time complexity as sum of time complexities of individual loops.

```
for (int i = 1; i <= m; i += c) {  
    // some O(1) expressions  
}  
for (int i = 1; i <= n; i += c) {  
    // some O(1) expressions  
}
```

- Time complexity of above code is $O(m) + O(n)$ which is $O(m+n)$
- If $m == n$, the time complexity becomes $O(2n)$ which is $O(n)$.



Question Discussion

What is time complexity of func()?

```
int func(int n) {  
    int count = 0;  
    for (int i = n; i > 0; i /= 2)  
        for (int j = 0; j < i; j++)  
            count += 1;  
    return count; }
```

- (A) $O(n^2)$
- (B) $O(n \log n)$
- (C) $O(n)$
- (D) $O(n \log n \log n)$



Question Discussion

What is time complexity of func()?

```
int func(int n) {  
    int count = 0;  
    for (int i = n; i > 0; i /= 2)  
        for (int j = 0; j < i; j++)  
            count += 1;  
    return count; }
```

- (A) $O(n^2)$
- (B) $O(n \log n)$
- (C) $O(n)$
- (D) $O(n \log n \log n)$

Answer: (C)

Explanation:

- For a input integer n , the innermost statement of func() is executed following times.
$$n + n/2 + n/4 + \dots 1$$
- So time complexity $T(n)$ can be written as: $T(n) = O(n + n/2 + n/4 + \dots 1) = O(n)$



Question Discussion

What is the time complexity of func()?

```
int func(int n) {  
    int count = 0;  
    for (int i = 0; i < n; i++)  
        for (int j = i; j > 0; j--)  
            count = count + 1;  
    return count;  
}
```

- (A) $O(n)$
- (B) $O(n^2)$
- (C) $O(n \cdot \log n)$
- (D) $O(n \log n \log n)$



Question Discussion

What is the time complexity of func()?

```
int func(int n) {  
    int count = 0;  
    for (int i = 0; i < n; i++)  
        for (int j = i; j > 0; j--)  
            count = count + 1;  
    return count;  
}
```

- (A) $O(n)$
- (B) $O(n^2)$
- (C) $O(n \cdot \log n)$
- (D) $O(n \log n \log n)$

Answer: (B)

Explanation:

- Time complexity can be calculated by counting number of times the expression “count = count + 1;” is executed.
- The expression is executed $0 + 1 + 2 + 3 + 4 + \dots + (n-1)$ times.
- Time complexity = $O(0 + 1 + 2 + 3 + \dots + n-1) = O(n \cdot (n-1)/2) = O(n^2)$



Question Discussion

Which of the given options provides the increasing order of asymptotic complexity of functions f_1 , f_2 , f_3 and f_4 ?

$$f_1(n) = 2^n$$

$$f_2(n) = n^{(3/2)}$$

$$f_3(n) = n \log n$$

$$f_4(n) = n^{(\log n)}$$

- (A) f_3, f_2, f_4, f_1
- (B) f_3, f_2, f_1, f_4
- (C) f_2, f_3, f_1, f_4
- (D) f_2, f_3, f_4, f_1



Question Discussion

Which of the given options provides the increasing order of asymptotic complexity of functions f_1 , f_2 , f_3 and f_4 ?

$$\begin{array}{ll} f_1(n) = 2^n & f_2(n) = n^{(3/2)} \\ f_3(n) = n \log n & f_4(n) = n^{(\log n)} \end{array}$$

- (A) f_3, f_2, f_4, f_1
- (B) f_3, f_2, f_1, f_4
- (C) f_2, f_3, f_1, f_4
- (D) f_2, f_3, f_4, f_1

Ans: (A)

Explanation:

$$\begin{array}{lll} f_1(n) = 2^n & f_2(n) = n^{(3/2)} & f_3(n) \\ = n \log n & f_4(n) = n^{(\log n)} & \end{array}$$

- Except f_3 , all other are polynomial/ exponential.
- f_3 is definitely first in output. Among remaining, $n^{(3/2)}$ is next.
- One way to compare f_1 and f_4 is to take Log of both functions.
 - Order of growth of $\log(f_1(n))$ is $O(n)$ and order of growth of $\log(f_4(n))$ is $O(\log n * \log n)$.
 - Since $O(n)$ has higher growth than $\Theta(\log n * \log n)$, $f_1(n)$ grows faster than $f_4(n)$.



Question Discussion

Which of the following is not $O(n^2)$?

(A) $(15^{10}) * n + 12099$

(B) $n^{1.98}$

(C) $n^3 / (\text{sqrt}(n))$

(D) $(2^{20}) * n$



Question Discussion

Which of the following is not $O(n^2)$?

(A) $(15^{10}) * n + 12099$

(B) $n^{1.98}$

(C) $n^3 / (\text{sqrt}(n))$

(D) $(2^{20}) * n$

Answer: (C)

Explanation: The order of growth of option c is $n^{2.5}$ which is higher than n^2 .



Question Discussion

What is the time, space complexity of following code:

```
int a = 0, b = 0;
for (i = 0; i < n; i++) {
    a = a + rand();
}
for (j = 0; j < m; j++) {
    b = b + rand();
}
```

- A. $O(n * m)$ time, $O(1)$ space
- B. $O(n + m)$ time, $O(n + m)$ space
- C. $O(n + m)$ time, $O(1)$ space
- D. $O(n * m)$ time, $O(n + m)$ space



Question Discussion

What is the time, space complexity of following code:

```
int a = 0, b = 0;
for (i = 0; i < n; i++) {
    a = a + rand();
}
for (j = 0; j < m; j++) {
    b = b + rand();
}
```

- A. $O(n * m)$ time, $O(1)$ space
- B. $O(n + m)$ time, $O(n + m)$ space
- C. $O(n + m)$ time, $O(1)$ space
- D. $O(n * m)$ time, $O(n + m)$ space

C. $O(n + m)$ time, $O(1)$ space

Explanation: The first loop is $O(n)$ and the second loop is $O(m)$. Since we don't know which is bigger, we say this is $O(n + m)$. This can also be written as $O(\max(n, m))$.

Since there is no additional space being utilized, the space complexity is constant / $O(1)$



Question Discussion

What is the time complexity of following code:

```
int a = 0;
for (i = 0; i < n; i++) {
    for (j = n; j > i; j--) {
        a = a + i + j;
    }
}
```

- A. $O(n)$
- B. $O(n \cdot \log(n))$
- C. $O(n * \text{sqrt}(n))$
- D. $O(n * n)$



Question Discussion

What is the time complexity of following code:

```
int a = 0;
for (i = 0; i < n; i++) {
    for (j = n; j > i; j--) {
        a = a + i + j;
    }
}
```

- A. $O(n)$
- B. $O(n \cdot \log(n))$
- C. $O(n \cdot \sqrt{n})$
- D. $O(n \cdot n)$

D. $O(n \cdot n)$

Explanation: The above code runs total no of times

$$= n + (n - 1) + (n - 2) + \dots 1 + 0$$

$$= n \cdot (n + 1) / 2$$

$$= 1/2 \cdot n^2 + 1/2 \cdot n$$

$O(n^2)$ times.



Question Discussion

What is the time complexity of following code:

```
int i, j, k = 0;
for (i = n / 2; i <= n; i++) {
    for (j = 2; j <= n; j = j * 2) {
        k = k + n / 2;
    }
}
```

- A. $O(n)$
- B. $O(n \log n)$
- C. $O(n^2)$
- D. $O(n^2 \log n)$



Question Discussion

What is the time complexity of following code:

```
int i, j, k = 0;
for (i = n / 2; i <= n; i++) {
    for (j = 2; j <= n; j = j * 2) {
        k = k + n / 2;
    }
}
```

- A. $O(n)$
- B. $O(n \log n)$
- C. $O(n^2)$
- D. $O(n^2 \log n)$

B. $O(n \log n)$

Explanation: If you notice, j keeps doubling till it is less than or equal to n . Number of times, we can double a number till it is less than n would be $\log(n)$.

Let's take the examples here.

for $n = 16$, $j = 2, 4, 8, 16$

for $n = 32$, $j = 2, 4, 8, 16, 32$

So, j would run for $O(\log n)$ steps.

i runs for $n/2$ steps.

So, total steps = $O(n/2 * \log(n)) = O(n * \log n)$



Question Discussion

What is the time complexity of following code:

```
int a = 0, i = n;  
while (i > 0) {  
    a += i;  
    i /= 2;  
}
```

- A. $O(n)$
- B. $O(\sqrt{n})$
- C. $O(n / 2)$
- D. $O(\log n)$



Question Discussion

What is the time complexity of following code:

```
int a = 0, i = n;  
while (i > 0) {  
    a += i;  
    i /= 2;  
}
```

D. $O(\log n)$

Explanation: We have to find the smallest x such that $n / 2^x \leq 1$
 $x = \log(n)$

- A. $O(n)$
- B. $O(\sqrt{n})$
- C. $O(n / 2)$
- D. $O(\log n)$



Question Discussion

```
for(i=1; i<=n; i=i*2)
    for(j=n; j>=1; j=j/2)
        statement;
```

Time Complexity $O((\log n)^2)$



Question Discussion

```
void function(int n){  
    int i,j,k;  
    for(i=n/2; i<=n; i++)  
        for(j=1; j + n/2<=n; j++)  
            for(k=1; k<=n; k= k * 2)  
                count++;  
}
```

Ans: $O(n^2 * \log n)$. In both these type of answers, we should give the marks because students do not know mathematical computation of time complexity.



Question Discussion

What is time complexity of the function func()?

```
void func(int arr[], int n) {  
    int i, j = 0;  
    for(i=0; i<n; i++)  
        while(j < n && arr[i] < arr[j])  
            j++;  
}
```

Ans: $O(n)$ (Note: Since the j is not initialized inside the outer while loop, the inner while loop will be executed nearly n number of times)



Question Discussion

What does it mean when we say that an algorithm X is asymptotically more efficient than Y?

- A. X will always be a better choice for small inputs
- B. X will always be a better choice for large inputs
- C. Y will always be a better choice for small inputs
- D. X will always be a better choice for all inputs

B. X will always be a better choice for large inputs

Explanation: In asymptotic analysis we consider growth of algorithm in terms of input size. An algorithm X is said to be asymptotically better than Y if X takes smaller time than Y for all input sizes n larger than a value n_0 where $n_0 > 0$.



Data Structures and Algorithms

- **Data structure** affects the design of both **structural** & **functional** aspects of a program.

Program=Algorithm + Data Structure

- Algorithm is a step by step procedure to solve a particular function.



Classification of Data Structure

- Data structure are normally divided into two broad categories:
 - Primitive Data Structure
 - Non-Primitive Data Structure



Classification of Data Structure

Data structure

Primitive DS

Non-Primitive DS

integer

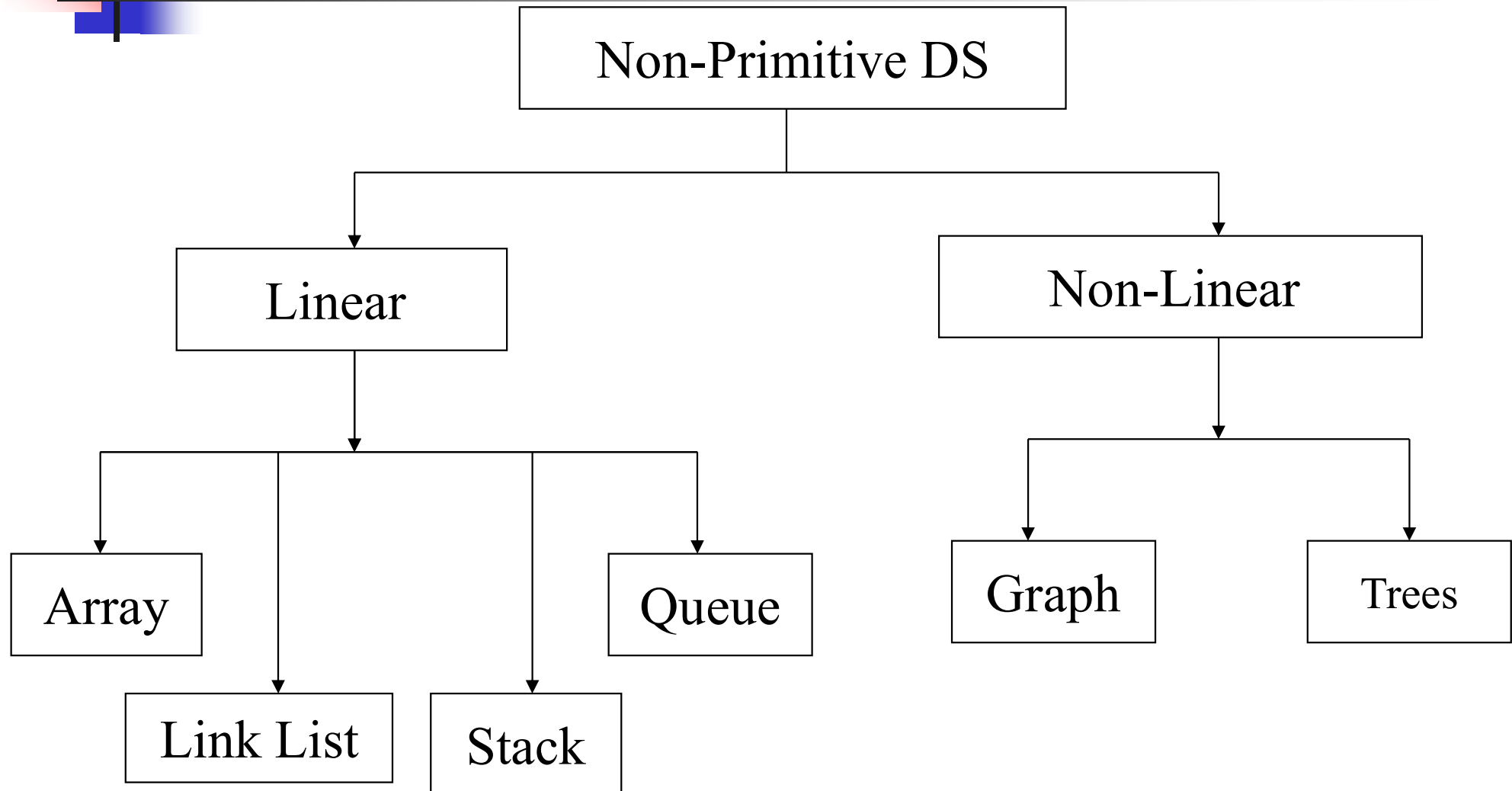
float

character

pointer



Classification of Data Structure





Primitive Data Structure

- There are basic structures and directly operated upon by the machine instructions.
- In general, there are different representation on different computers.
- Integer, Floating-point number, Character constants, string constants, pointers etc, fall in this category.



Non-Primitive Data Structure

- There are more sophisticated data structures.
- These are **derived** from the primitive data structures.
- The non-primitive data structures emphasize on structuring of a group of **homogeneous** (**same type**) or **heterogeneous** (**different type**) data items.



Non-Primitive Data Structure

- Lists, Stack, Queue, Tree, Graph are example of non-primitive data structures.



Non-Primitive Data Structure

- The most commonly used operation on data structure are broadly categorized into following types:
 - Create
 - Selection
 - Updating
 - Searching
 - Sorting
 - Merging
 - Destroy or Delete



Difference between them

- A primitive data structure is generally a basic structure that is usually built into the language, such as an integer, a float.
- A non-primitive data structure is built out of primitive data structures linked together in meaningful ways, such as
 - linked-list, binary search tree, AVL Tree, graph etc.



Description of various Data Structures: Arrays

- An array is defined as a set of finite number of homogeneous elements or same data items.
- It means an array can contain one type of data only, either all integer, all float-point number or all character.



Arrays

- Simply, declaration of array is as follows:

```
int arr[10]
```

- Where int specifies the data type or type of elements arrays stores.
- “arr” is the name of array & the number specified inside the square brackets is the number of elements an array can store, this is also called sized or length of array.



Arrays

- Following are some of the concepts to be remembered about arrays:
 - The individual element of an array can be accessed by specifying name of the array, following by index or subscript inside square brackets.
 - The first element of the array has index zero[0]. It means the first element and last element will be specified as:arr[0] & arr[9] respectively.



Arrays

- The elements of array will always be stored in the consecutive (continuous) memory location.
- The number of elements that can be stored in an array, that is the size of array or its length is given by the following equation:
$$(\text{Upperbound} - \text{lowerbound}) + 1$$



Arrays

- For this array it would be $(9-0)+1=10$, where 0 is the lower bound of array and 9 is the upper bound of array.
- Array can always be read or written through loop.
 - If we read a one-dimensional array it require one loop for reading and other for writing the array.



Arrays

- For example: Writing an array
for(i=0;i<=9;i++)
scanf("%d", &arr[i]);
- For example: Reading an array
for(i=0;i<=9;i++)
printf("%d", arr[i]);



Arrays

- If we are reading or writing two-dimensional array it would require two loops.
 - And similarly the array of a N dimension would required N loops.
- Some common operation performed on array are:
 - Creation of an array
 - Traversing an array



Arrays

- Insertion of new element
- Deletion of required element
- Modification of an element
- Merging of arrays



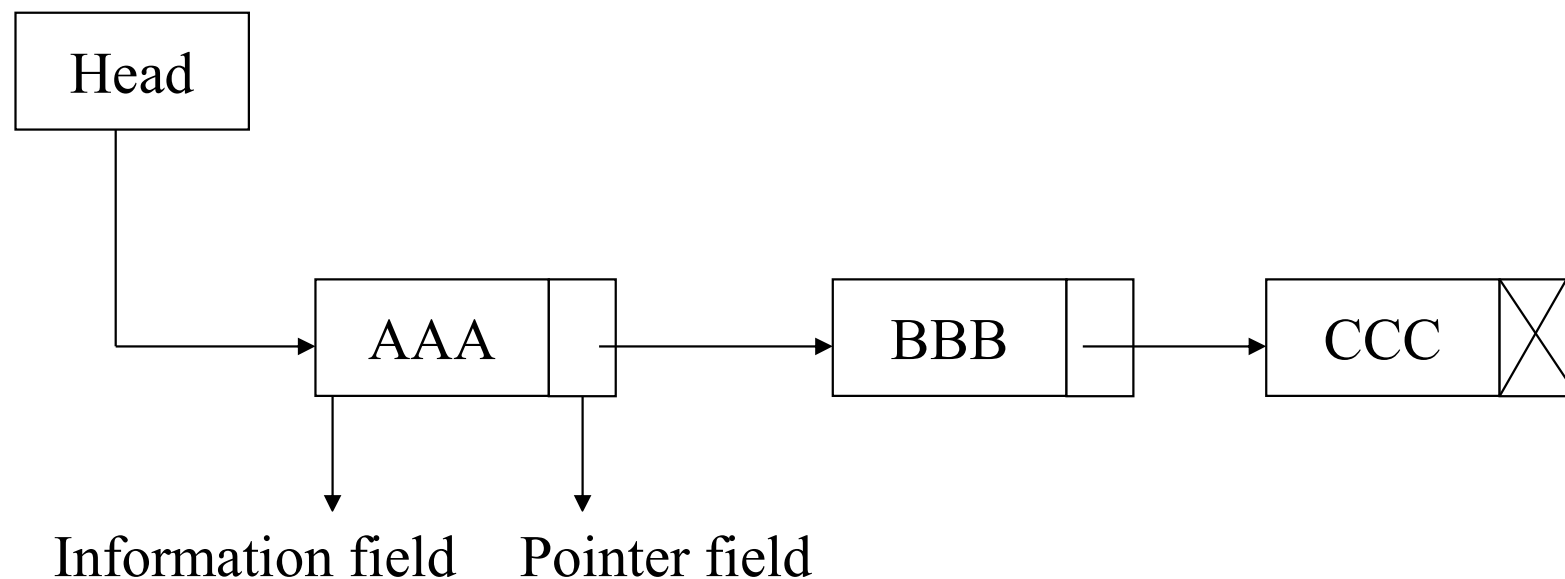
Lists

- A **lists** (**Linear linked list**) can be defined as a collection of variable number of data items.
- Lists are the most commonly used non-primitive data structures.
- An element of list must contain at least two fields:
 - one for storing data or information and other for storing address of next element.
- For storing address, a special data structure of list the address must be pointer type.

Lists

- Technically each such element is referred to as a node, therefore a list can be defined as a collection of nodes as show bellow:

[Linear Liked List]





Lists

- Types of linked lists:
 - Single linked list
 - Double linked list
 - Single circular linked list
 - Double circular linked list



Stack

- A stack is also an ordered collection of elements like arrays, but it has a special feature that deletion and insertion of elements can be done only from one end called the top of the stack (TOP)
- Due to this property it is also called as last in first out type of data structure (LIFO).



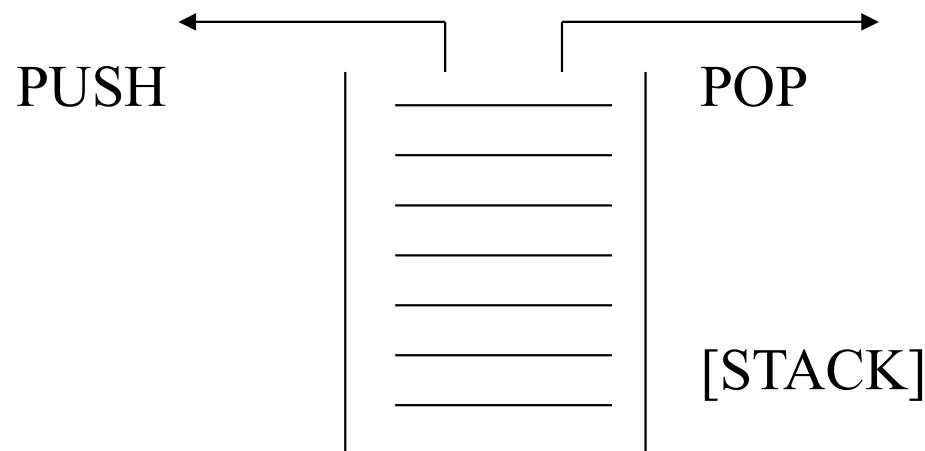
Stack

- It could be thought of just like a stack of plates placed on table in a party, a guest always takes off a fresh plate from the top and the new plates are placed on to the stack at the top.
- It is a non-primitive data structure.
- When an element is inserted into a stack or removed from the stack, its base remains fixed where the top of stack changes.



Stack

- Insertion of element into stack is called PUSH and deletion of element from stack is called POP.
- The bellow show figure how the operations take place on a stack:





Stack

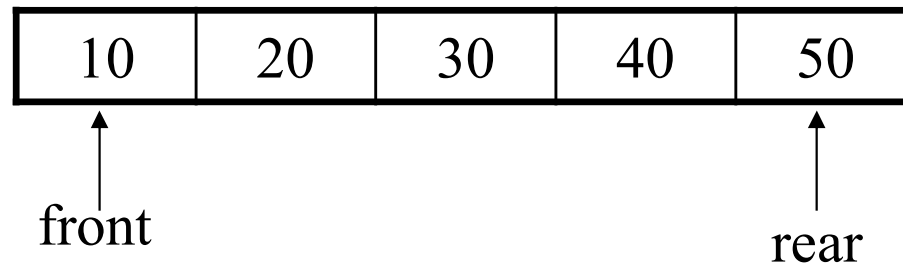
- The stack can be implemented into two ways:
 - Using arrays (Static implementation)
 - Using pointer (Dynamic implementation)



Queue

- Queue are first in first out type of data structure (i.e. FIFO)
- In a queue new elements are added to the queue from one end called REAR end and the element are always removed from other end called the FRONT end.
- The people standing in a railway reservation row are an example of queue.

-





Queue

- The queue can be implemented into two ways:
 - Using arrays (Static implementation)
 - Using pointer (Dynamic implementation)



Trees

- A tree can be defined as finite set of data items (nodes).
- Tree is non-linear type of data structure in which data items are arranged or stored in a sorted sequence.
- Tree represent the hierarchical relationship between various elements.



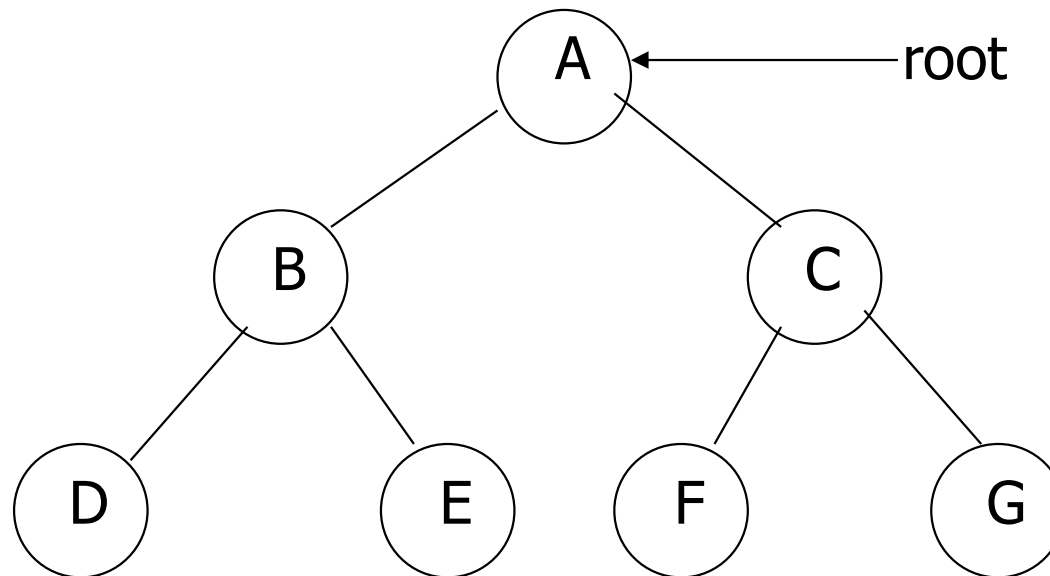
Trees

- In trees:
 - There is a special data item at the **top of hierarchy** called the *root* of the tree.
 - The remaining data items are partitioned into number of **mutually exclusive subset**, each of which is itself, a tree which is called the **sub tree**.
 - The tree always grows in length towards bottom in data structures, unlike natural trees which grows upwards.



Trees

- The tree structure organizes the data into branches, which related the information.





Graph

- Graph is a mathematical non-linear data structure capable of representing many kind of physical structures.
- It has found application in Geography, Chemistry and Engineering sciences.
- Definition: A graph $G(V,E)$ is a set of vertices V and a set of edges E .



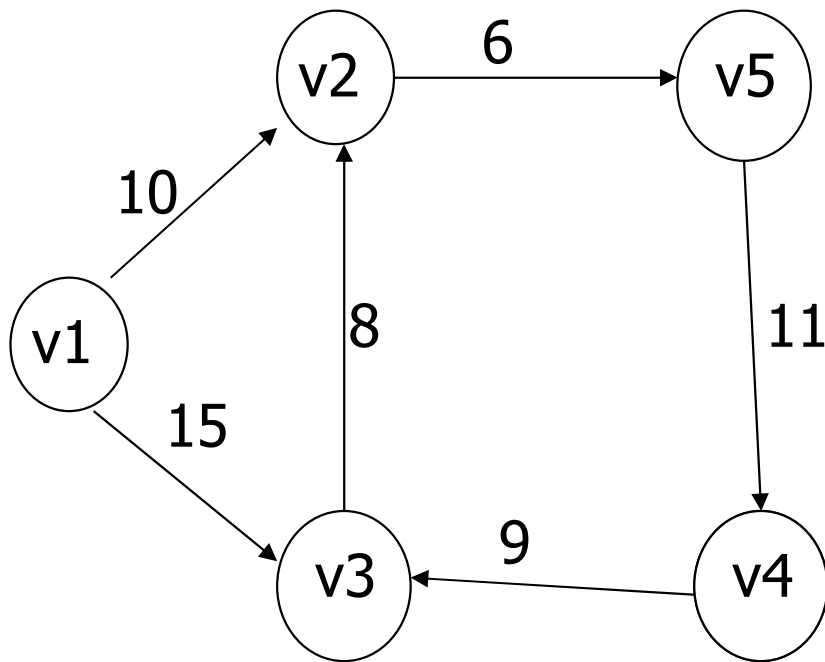
Graph

- An edge connects a pair of vertices and many have weight such as length, cost and another measuring instrument for according the graph.
- Vertices on the graph are shown as point or circles and edges are drawn as arcs or line segment.

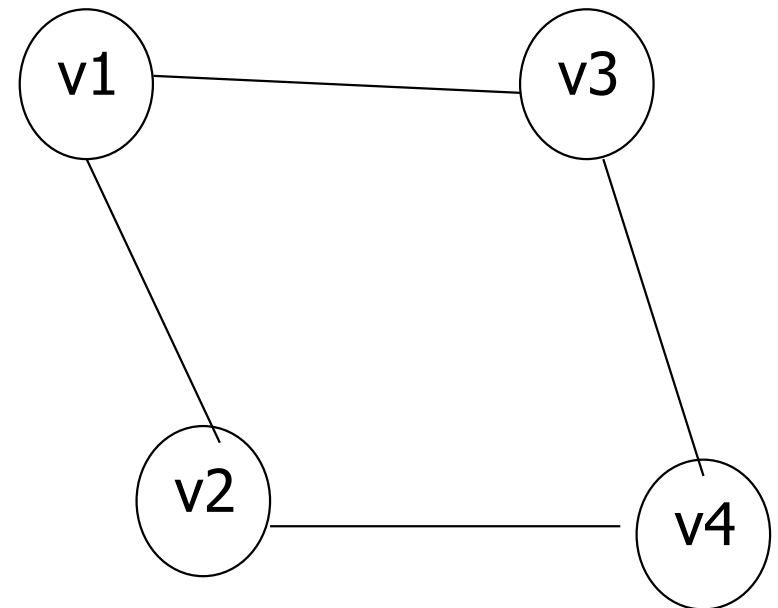


Graph

- Example of graph:



[a] Directed & Weighted Graph



[b] Undirected Graph



Graph

- Types of Graphs:
 - Directed graph
 - Undirected graph
 - Simple graph
 - Weighted graph
 - Connected graph
 - Non-connected graph