Data Structures and Algorithms (CS 2001)

KALINGA INSTITUTE OF INDUSTRIAL TECHNOLOGY

School of Computer Engineering

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4 Credit

Lecture Note

Motivating Quotes

- “Every program depends on algorithms and data structures, but few programs depend on the invention of brand new ones.” -- Kernighan & Pike
- “I will, in fact, claim that the difference between a bad programmer and a good one is whether he considers his code or his data structures more important. Bad programmers worry about the code. Good programmers worry about data structures and their relationships” -- Linus Torvalds
- “Smart data structures and dumb code works a lot better than the other way around.” - Eric S. Raymond
- “It's easy to make mistakes that only come out much later, after you've already implemented a lot of code. You'll realize Oh I should have used a different type of data structure. Start over from scratch” -- Guido van Rossum

Importance of the Course

- Data Structure and algorithms are the foundation of computer programming.

Program = Data Structure + Algorithm

- Almost every computer program, even a simple one, uses data structure and algorithms. Example -
  - Program – To print student list
  - Data Structure – Array
  - Algorithm - Loop
- Algorithm thinking, problem solving and data structures are the vital for the software engineers. How ?
  - Solve the problem more efficiently
  - Use right tool to solve the problem
  - Run program more efficiently
- **The interviewer test and evaluate the candidates performance using Data structure and algorithm.**

Course Description

- Provide **solid foundations** in **basic concepts of problem solving** – both data structures and algorithms
- **Select and design data structure & algorithms** that are **appropriate** for the given problems
- **Demonstrate** the **correctness** of the algorithm and **analyzing** their **computational complexities**

How?

Blend of Theory and Practical

Prerequisites

- Programming in C (CS 1001)
- Mathematics for Computer Science

Course Contents

Sr #	Major and Detailed Coverage Area	Hrs
1	**Introduction**	4
	Structures and Unions, Pointers, Dynamic Memory Allocation, Algorithm Specification, Space and Time Complexity	
2	**Arrays**	5
	Arrays, Abstract Data Type, Dynamically Allocated Arrays, Polynomials, Two-dimensional Array, Address Calculation, Matrix Addition and Multiplication, Sparse Matrix, Upper & Lower Triangular Matrix, Tridiagonal Matrix	
3	**Linked List**	8
	Singly Linked Lists and Chains, Representing Chains in C, Polynomials, Sparse Matrix, Doubly Linked Lists, Circular & Header Linked lists	
4.1	**Stacks**	4
	Stacks, Stacks using Dynamic Arrays and Linked List, Evaluation of Expressions	
	Mid Semester	
4.2	**Queues**	4
	Queues, Queue using Dynamic Arrays and Linked List, Circular Queues using Dynamic Arrays and Linked List, Evaluation of Expressions, Priority Queue, Dequeue	
5	**Trees**	12
	Introduction, Binary Trees, Binary Tree Traversals, Threaded Binary Trees, Binary Search Trees, AVL Trees, m-way Search Trees, B-Trees, Introduction to B+-Trees, Tree Operation, Forests	

Course Contents continue…

Sr #	Major and Detailed Coverage Area	Hrs
6	**Graphs**	4
	Graph ADT, Graph Operation – DFS, BFS	
7	**Sorting**	4
	Insertion Sort, Quick Sort, Merge Sort, Heap Sort, Bubble Sort, Selection Sort, Radix Sort	
8	**Searching**	3
	Linear Search, Binary Search, Hashing – Hash Function, Collision Resolution Techniques	
	End Semester	

Textbook

- Data Structures using C by Aaron M. Tenenbaum, Yedidyah Langsam, Moshe J. Augenstein

Reference Books

- Data Structures, Schaum's OutLines, Seymour Lipschutz, TATA McGRAW HILL
- Fundamentals of Data Structures in C, 2nd edition, Horowitz, Sahani, Anderson-Freed, Universities Press.
- Data Structures A Pseudocode Approach with C, 2nd Edition, Richard F. Gilberg, Behrouz A. Forouzan, CENGAGE Learning, India Edition
- Data Structures and Algorithm Analysis in C, Mark Allen Weiss, Pearson Education, 2nd Edition.
- Data Structures and Algorithms (Addison-Wesley Series in Computer Science and Information Pr) by Alfred V. Aho, Jeffrey D. Ullman, John E. Hopcroft.

Outcome based Learning Objectives

By the end of this course, students will be able to

- Understand the concepts of data structure, data type and abstract data type (ADT)
- Analyse algorithms and determine their time complexity
- Implement linked data structure to solve various problems
- Understand and apply various data structures such as arrays, stacks, queues, trees and graphs to solve various computing problems
- Implement and apply standard algorithms for searching and sorting
- Effectively choose the data structure that efficiently models the information in a problem.

Evaluation

Grading:

- Internal assessment – 30 marks
  - 2 quizzes & each worth 2.5 marks = 5 marks
  - 5 group/individual assignments and each worth 3 marks = 15 marks
  - Mini-Project = 10 marks
- Midterm exam - 20 marks
- Endterm exam - 50 marks

Definition - ADT

Abstract Data Type, abbreviated **ADT**, is a logical description of how we view the data and the operations that are allowed without regard to how they will be implemented or how it does the job. This means it is like a black box where users can only see the syntax and semantics of operation and hides the inner structure and design of the data type.

The definition of ADT only mentions what operations are to be performed but not how these operations will be implemented. It does not specify how data will be organized in memory and what algorithms will be used for implementing the operations. It is called “abstract” because it gives an implementation independent view.

An ADT consists of 2 parts: (1) declaration of data, (2) declaration of operations

Commonly used ADTs: Arrays, List, Stack, Queues, Trees, Graphs etc along with their operations.

Definition – ADT cont'd

Stack ADT: A Stack contains elements of same type arranged in sequential order. All operations takes place at a single end that is top of the stack and following operations can be performed:

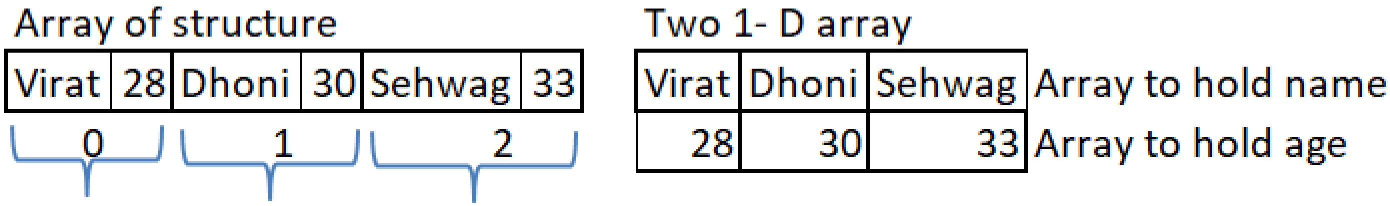
- ❑ push() – Insert an element at one end of the stack called top.
- ❑ pop() – Remove and return the element at the top of the stack, if it is not empty.
- ❑ peek() – Return the element at the top of the stack without removing it, if the stack is not empty.
- ❑ size() – Return the number of elements in the stack.
- ❑ isEmpty() – Return true if the stack is empty, otherwise return false.
- ❑ isFull() – Return true if the stack is full, otherwise return false.

Definition – Data Structure

It is basically a group of data elements that are put together under one name and defines a particular way of storing and organizing data in a computer so that it can be used efficiently.

The implementation of ADT, often referred to as a data structure.

For example, we have cricket player's name "Virat" & age 28. "Virat" is of string data type and 28 is of integer data type. This data can be organized as a record like Player record. Now players record can be collected and store in a file or database as a data structure. For example: "Dhoni" 30, "Gambhir" 31, "Sehwag" 33.



Array of structure

Virat	28	Dhoni	30	Sehwag	33

Two 1- D array

Virat	Dhoni	Sehwag	Array to hold name
28	30	33	Array to hold age

Data Structure Classification

Characteristics	Description
Linear	The data items are assessed in a linear sequence, but it is not compulsory to store all elements sequentially. Example: Array
Non-Linear	The data items are stored/accessed in a non-linear order. Example: Tree, Graph
Homogeneous	All the elements are of same type. Example: Array
Heterogeneous	The elements are variety or dissimilar type of data Example: Structures
Static	Are those whose sizes and structures associated memory locations are fixed, at compile time. Example: Array
Dynamic	Are those which expands or shrinks depending upon the program need and its execution. Also, their associated memory locations changes. Example: Linked List created using pointers

Need of Data Structure

As applications are getting complexed and amount of data is increasing day by day, there may arise the following problems:

- **Processor speed:** To handle very large amount of data, high speed processing is required, but as the data is growing day by day to the billions of files per entity, processor may fail to deal with that much amount of data.
- **Data Search:** Consider an inventory size of 1L items in a store, If our application needs to search for a particular item, it needs to traverse 1L items every time, results in slowing down the search process.
- **Multiple requests:** If thousands of users are searching the data simultaneously on a web server, then there are the chances that a very large server can be failed during that process

In order to solve the above problems, data structures are used. Data is organized to form a data structure in such a way that all items are not required to be searched and required data can be searched instantly.

Algorithm

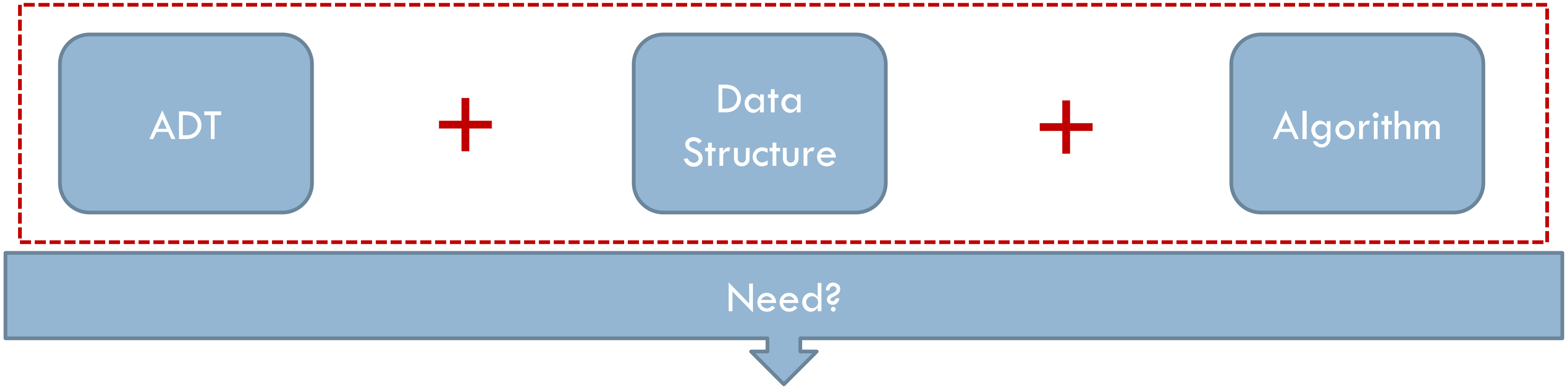
Let us consider the problem of preparing an omelette. To prepare an omelette, we follow the steps given below:

1) Get the frying pan.
2) Get the oil.
    a. Do we have oil?
        i. If yes, put it in the pan.
        ii. If no, do we want to buy oil?
            1. If yes, then go out and buy.
            2. If no, we can terminate.
3) Turn on the stove, etc...
4) …

What we are doing is, for a given problem (preparing an omelette), we are providing a step-by step procedure for solving it.

Definition - Algorithm

Algorithm: It is a self-contained step-by-step set of operations to be performed for calculation, data processing and/or automated reasoning tasks etc. Algorithm is not the complete code or program, it is just the core logic(solution) of a problem, which can be expressed either as an informal high level description as pseudo code or using a flowchart.



To become a computer scientist and stopping yourself from monkey coder.

Structure

Arrays allow to define type of variables that can hold several data items of the same kind. Similarly **structure** is another user defined data type available in C that allows to combine data items of **different kinds**.

Structures are used to **represent** a **record**. Suppose you want to keep track of your books in a library. You might want to track the following attributes about each book –

- ❑ Title
- ❑ Author
- ❑ Subject
- ❑ Book ID

Defining the Structure:

```
struct [structure tag]
{

  member definition;
  member definition;
  ...
  member definition;
} [one or more structure variables];
```

Book Structure

```
struct Books
{
  char  title[50];
  char  author[50];
  char  subject[100];
  int   book_id;
} book;
```

Access Structure Elements

```
/* Declare Book1 of type Book */
struct Books Book1;

/* Declare Book2 of type Book */
struct Books Book2;
Book1.title =“DSA”;
Book2.book_id = 6495700;
```

Union

Unions are quite similar to the structures in C. Union is also a **derived type** as structure. Union can be defined in same manner as structures just the keyword used in defining union in **union** where keyword used in defining structure was **struct**.

You can define a union with many members, but only one member can contain a value at any given time. Unions provide an efficient way of using the same memory location for multiple-purpose.

Defining the Union:

```
union [union tag]
{

  member definition;
  member definition;
  ...
  member definition;
} [one or more union variables];
```

Data Union

```
union Data
{
  int i;
  float f;
  char str[20];
} data;
```

Access Union Elements

```
/* Declare data of type Data */
union Data data;

data.i =10;
data.f = 34.72;
data.str =“C Programming”
```

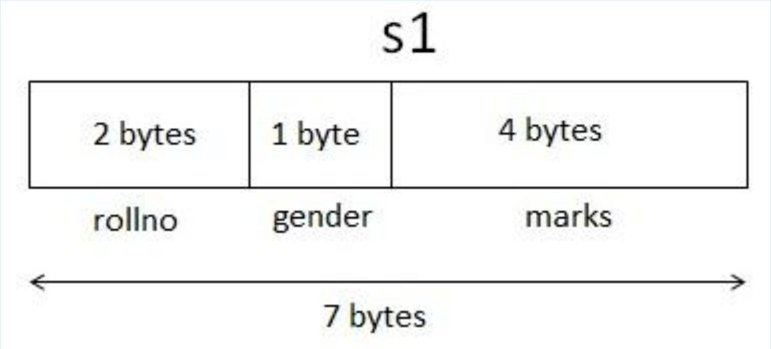
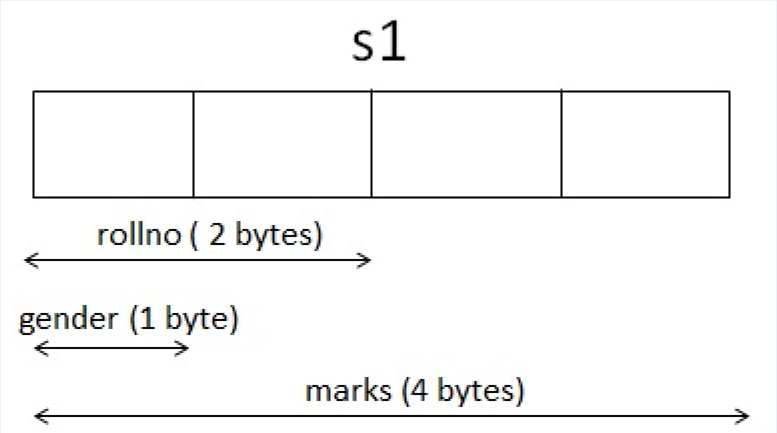
Difference b/w Structure & Union

Structure	Union

In structure each member get separate space in memory. Take below example.
struct student
{
int rollno;
char gender;
float marks;
} s1;
The total memory required to store a structure variable is equal to the sum of size of all the members. In above case 7 bytes (2+1+4) will be required to store structure variable s1.
	In union, the total memory space allocated is equal to the member with largest size. All other members share the same memory space. This is the biggest difference between structure and union.
	union student
	{
	int rollno;
	char gender;
	float marks;
	}s1;
	In above example variable marks is of float type and have largest size (4 bytes). So the total memory required to store union variable s1 is 4 bytes.

Difference b/w Structure & Union continue…

Pictorial Representation

Structure	Union
 s1 2 bytes 1 byte 4 bytes rollno gender marks 7 bytes	 s1 rollno (2 bytes) gender (1 byte) marks (4 bytes)

Difference b/w Structure & Union cont…

Structure	Union

We can access any member in any sequence.
 	
s1.rollno = 20;
s1.marks = 90.0;
printf("%d",s1.rollno);
	We can access only that variable whose value is recently stored.
	
	s1.rollno = 20;
	s1.marks = 90.0;
	printf("%d",s1.rollno);
	
	The above code will show erroneous output. The value of rollno is lost as most recently we have stored value in marks. This is because all the members share same memory space.

All the members can be initialized while declaring the variable of structure.	Only first member can be initialized while declaring the variable of union. In above example, we can initialize only variable rollno at the time of declaration of variable.

Dynamic Memory Allocation

The exact size of array is **unknown** until the **compile time** i.e. time when a compiler compiles code written in a programming language into a executable form. The size of array you have declared initially can be sometimes **insufficient** and sometimes **more than required**.

What?

The process of allocating memory during program execution is called dynamic memory allocation. It also allows a program to obtain more memory space, while running or to release space when no space is required.

Difference between static and dynamic memory allocation

Sr #	Static Memory Allocation	Dynamic Memory Allocation
1	User requested memory will be allocated at compile time that sometimes insufficient and sometimes more than required.	Memory is allocated while executing the program.
2	Memory size can't be modified while execution.	Memory size can be modified while execution.

Dynamic Memory Allocation cont…

Functions available in C for memory management

Sr #	Function	Description
1	void *calloc(int num, int size)	Allocates an array of num elements each of which size in bytes will be size.
2	void free(void *address)	Releases a block of memory block specified by address.
3	void *malloc(int num)	Allocates an array of num bytes and leave them uninitialized.
4	void *realloc(void *address, int newsize)	Re-allocates memory extending it up to new size.

Dynamic Memory Allocation Example

```
#include <stdio.h>
#include <stdlib.h>
int main()
{
  int *pi;
  float *pj;
  pi = (int *) malloc(sizeof(int));
  pj = (float *) malloc(sizeof(float));
  *pi = 10;
   *pj = 3.56;
   printf(“integer = %d, float = %f”, *pi, *pj);
   free(pi);
   free(pj);
   return 0;
}
```

IMH

```
#include <stdio.h>
#include <stdlib.h>
int main()
{
  int *pi;
  float  *pj;
  pi = (int *) malloc(sizeof(int));
  pj = (float *) malloc(sizeof(float));
  if (!pi || !pj)
  {
     printf(“Insufficient Memory”);
     return;
  }
  *pi = 10;
   *pj = 3.56;
    printf(“integer = %d, float = %f”, *pi, *pj);
    free(pi);
    free(pj);
   return 0;
}
```

IMH : Insufficient Memory Handling

DMA realloc Example

C Program illustrating the usage of realloc

```
#include <stdio.h>
#include <stdlib.h>
int main()
{
    char *mem_allocation;
   /* memory is allocated dynamically */
   mem_allocation = malloc( 20 * sizeof(char) );
   if( mem_allocation == NULL )
   {
      printf("Couldn't able to allocate requested memory\n"); return;
   }
   else
   {
     strcpy( mem_allocation,“dynamic memory allocation for realloc function");
   }
   printf("Dynamically allocated memory content  : %s\n", mem_allocation );
  mem_allocation=realloc(mem_allocation,100*sizeof(char));
   if( mem_allocation == NULL )
   {
      printf("Couldn't able to allocate requested memory\n");
   }
   else
   {
      strcpy( mem_allocation,"space is extended upto 100 characters");
   }
   printf("Resized memory : %s\n", mem_allocation );
   free(mem_allocation);
   return 0;
}
```

Difference between calloc and malloc

Sr #	malloc	calloc
1	It allocates only single block of requested memory	It allocates multiple blocks of requested memory
2	doesn't initializes the allocated memory. It contains garbage values.	initializes the allocated memory to zero

3	int *ptr;
	ptr = malloc(20 * sizeof(int));
	For the above, 20*2 bytes of memory only allocated in one block.
	Total = 40 bytes
		int *ptr;
		Ptr = calloc(20, 20 * sizeof(int));
		For the above, 20 blocks of memory will be created and each contains 20*2 bytes of memory.
		Total = 800 bytes

Algorithm Specification

An algorithm is a **finite** set of instructions that, if followed, **accomplishes a particular task**. In addition, all algorithms must satisfy the following criteria:

1. **Input**. There are zero or more quantities that are externally supplied.
2. **Output**. At least one quantity is produced.
3. **Definiteness**. Each instruction is clear and unambiguous.
4. **Finiteness**. If we trace out the instructions of an algorithm, then for all cases, the algorithm terminates after a finite number of steps.
5. **Effectiveness**. Every instruction must be basic enough to be carried out, in principle, by a person using only pencil and paper. It is not enough that each operation be definite as in (3); it also must be feasible.

Difference between an algorithm & program – program does not have to satisfy the 4th condition.

Describing Algorithm

1. **Natural Language –** e.g. English, Chinese - Instructions must be definite and effectiveness.
2. **Graphics representation – e.g. Flowchart -** work well only if the algorithm is small and simple.
3. **Pseudo Language -**
   - Readable
   - Instructions must be definite and effectiveness
4. **Combining English and C**

Algorithm Specification cont…

Describing Algorithm – Natural Language

Problem - Design an algorithm to add two numbers and display result.

```
Step 1 – START
Step 2 – declare three integers a, b & c
Step 3 – define values of a & b
Step 4 – add values of a & b
Step 5 – store output of step 4 to c
Step 6 – print c
Step 7 – STOP
```

Note *- Writing step numbers, is optional.*

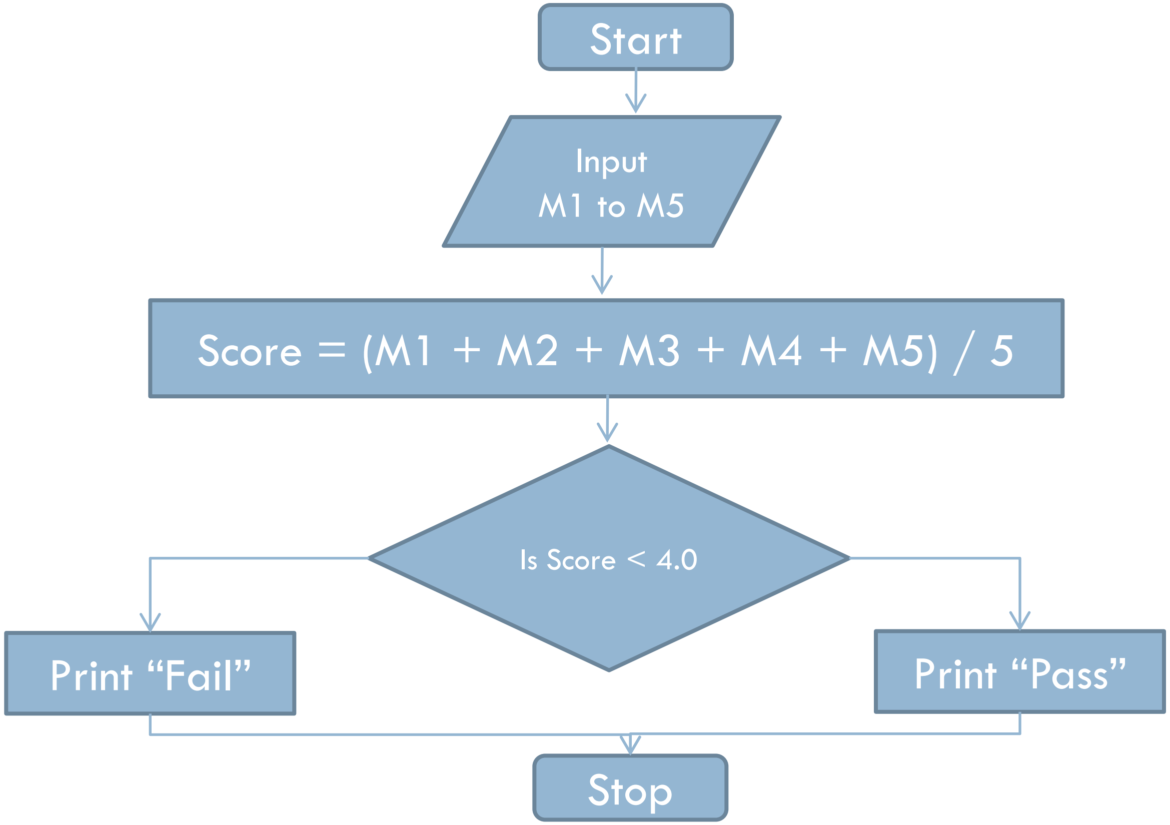
Problem - Design an algorithm to find the largest data value of a set of given positive data values.

```
Step 1 – START
Step 2 – input NUM
Step 3 – LARGE = NUM
Step 4 – While (NUM >=0)
            if (NUM > LARGE) then
                LARGE = NUM
            input NUM
Step 5 – display "Largest Value is:", LARGE
Step 6 – STOP
```

Algorithm Specification cont…

Describing Algorithm – Flowchart

Problem – Write an algorithm to determine a student's final grade and indicate whether it is passing of failing. The final grade is calculated as the average of five marks.



Pseudo Language

The Pseudo language is neither an algorithm nor a program. It is an abstract form of a program. It consists of English like statements which perform the specific operations. It employs **programming-like** statements to depict the algorithm and no standard format (language independent) is followed. The statements are carried out in a order & followings are the commonly used statements.

<table>
<tr><th>Statement</th><th>Purpose</th><th colspan="2">General Format</th></tr>
<tr><td>Input</td><td>Get Information</td><td colspan="2"><b>INPUT:</b> Name of variable
e.g. INPUT: user_name</td></tr>
<tr><td>Process</td><td>Perform an atomic activity</td><td colspan="2">Variable ← arithmetic expression
e.g. x ← 2 or x ← x + 1 or a ← b * c</td></tr>
<tr><td rowspan="2">Decision</td><td rowspan="2">Choose between different alternatives</td><td>IF (condition is met) then
statement(s)
ENDIF</td><td>IF (condition is met) THEN
statement(s)
ELSE
statements(s)
ENDIF</td></tr>
<tr><td>e.g.
IF (amount < 100) THEN
interestRate ← .06
ENDIF</td><td>e.g.
IF (amount < 100) THEN
interestRate ← .06
ELSE
interest Rate ← .10
ENDIF</td></tr>
</table>

Pseudo Language cont…

Statement	Purpose	General Format	

Repetition	Perform a step multiple times	**REPEAT**
		statement(s)
		UNTIL (condition is met)
			WHILE (condition is met)
			statement(s)
			ENDWHILE

		e.g. count ← 0 REPEAT ADD 1 to count OUTPUT: count UNTIL (count < 10) OUTPUT: “The End”	**e.g.** count ← 0 WHILE (count < 10) ADD 1 to count OUTPUT: count ENDWHILE OUTPUT: “The End”
Output	Display information	**OUTPUT:** Name of variable e.g. OUTPUT: user_name	**OUTPUT**: message OUTPUT: ‘Credit Limit’ limit

Pseudo Language Guidelines

Guidelines	Explanation
Write only one statement per line	Each statement in pseudo code should express just one action for the computer. If the task list is properly drawn, then in most cases each task will correspond to one line of pseudo code. **Task List** — **Pseudo code** Read name, hours worked, rate of pay — INPUT: name, hoursWorked, payRate gross = hours worked * rate of pay — gross ← hoursWorked * payRate Write name, hours worked, gross — OUTPUT: name, hoursWorked, gross
Capitalize initial keyword	In the example above note the words: **INPUT** and **OUTPUT**. These are just a few of the keywords to use, others include: **IF, ELSE, REPEAT, WHILE, UNTIL, ENDIF**
Indent to show hierarchy	Each design structure uses a particular indentation pattern. ❑ **Sequence** - Keep statements in sequence all starting in the same column ❑ **Selection** - Indent statements that fall inside selection structure, but not the keywords that form the selection ❑ **Loop** - Indent statements that fall inside the loop but not keywords that form the loop INPUT: name, grossPay, taxes IF (taxes > 0) net ← grossPay – taxes ELSE net ← grossPay ENDIF OUTPUT: name, net

Pseudo code Guidelines cont…

Guidelines	Explanation

End multiline structures	INPUT: name, grossPay, taxes
	IF (taxes > 0)
	        net ← grossPay – taxes
	ELSE
	        net ← grossPay
	ENDIF
	OUTPUT: name, net
	
	Watch the IF/ELSE/ENDIF as constructed above, the ENDIF is in line with the IF. The same applies for WHILE/ENDWHILE etc…

Keep statements language independent	Resist the urge to write in whatever language you are most comfortable with, in the long run you will save time. Remember you are describing a logic plan to develop a program, you are not programming!

Pseudo Language Example

1. Problem - Design the pseudo code to add two numbers and display the average.

```
INPUT: x, y
sum ← x + y
average ← sum / 2
OUTPUT: 'Average is:' average
```

2. Problem - Design the pseudo code to calculate & display the area of a circle

```
INPUT: radius
area← 3.14 * radius * radius
OUTPUT: 'Area of the circle is ' area
```

2. Problem - Design the pseudo code to calculate & display the largest among 2 numbers

```
INPUT: num1, num2
max ← num1
IF (num2 > num 1) THEN
  max ← num2
ENDIF
OUTPUT: 'Largest among 2 numbers is' max
```

Algorithm Specification cont…

Example - Translating a Problem into an Algorithm – Combining English and C

- **Problem -** Devise a program that sorts a set of n>= 1 integers
- Step I – **Concept** – looks good but not an algorithm
  - ✓ From those integers that are currently unsorted, find the smallest and place it next in the sorted list – selection sort
- Step II – An **algorithm**, written in C and English

```
for (i= 0; i< n; i++)
{
    Examine list[i] to list[n-1] and suppose that the smallest integer is list[min];
    Interchange list[i] and list[min];
}
```

- Optional Step III – **Coding** – translating the algorithm to C program

```
void sort(int *a, int n) {
   for (i= 0; i< n; i++) {
     int j= i;
     for (int k= i+1; k< n; k++) {
       if (a[k]< a[j]) {
         j = k; int temp=a[i];  a[i]=a[j];  a[j]=temp;
       }
     }
   }
}
```

Algorithm Specification cont…

Translating a Problem into an Algorithm cont… Correctness Proof

- Theorem
  - ✓ Function sort(a, n) correctly sorts a set of n>= 1 integers. The result remains in a[0], ..., a[n-1] such that a[0]<= a[1]<=...<=a[n-1].
- Proof

For i= q, following the execution of lines, we have a[q]<= a[r], q< r< =n-1.

For i> q, observing, a[0], ..., a[q] are unchanged.

Hence, increasing i, for i= n-2, we have a[0]<= a[1]<= ...<=a[n-1]

Recursive Algorithm

A recursive algorithm is an algorithm which calls itself. In general, recursive computer programs require more memory and computation compared with iterative algorithms, but they are simpler and for many cases a natural way of thinking about the problem. There are 2 types of recursive functions.

<table>
<tr><th>Direct recursion Function</th><th>Indirect recursion Function</th></tr>
<tr><td>Functions call themselves e.g. function α calls α</td><td>Functions call other functions that invoke the calling function again e.g. a function α calls a function β that in turn calls the original function α.</td></tr>
<tr><td><b>Example –</b>
int fibo (int n)
{
   if (n==1 || n==2)
      return 1;
   else
      return (fibo(n-1)+fibo(n-2));
}</td><td><b>Example –</b>
int func1(int n)
{
   if (n<=1)
      return 1;
   else
      return func2(n);
}

int func2(int n)
{
   return func1(n);
}</td></tr>
</table>

Recursive Algorithm cont…

- When is **recursion an appropriate mechanism**?
  - ✓ The problem itself is defined recursively
  - ✓ Statements: if-else and while can be written recursively
  - ✓ Art of programming
- **Why recursive algorithms** ?
  - ✓ Powerful, express an complex process very clearly
- **Properties** - A recursive function can go infinite like a loop. To avoid infinite running of recursive function, there are two properties that a recursive function must have –
  - ✓ **Base criteria** – There must be at least one base criteria or condition, such that, when this condition is met the function stops calling itself recursively.
  - ✓ **Progressive criteria**– The recursive calls should progress in such a way that each time a recursive call is made it comes closer to the base criteria.
- **Implementation** - Many programming languages implement recursion by means of **stack.**

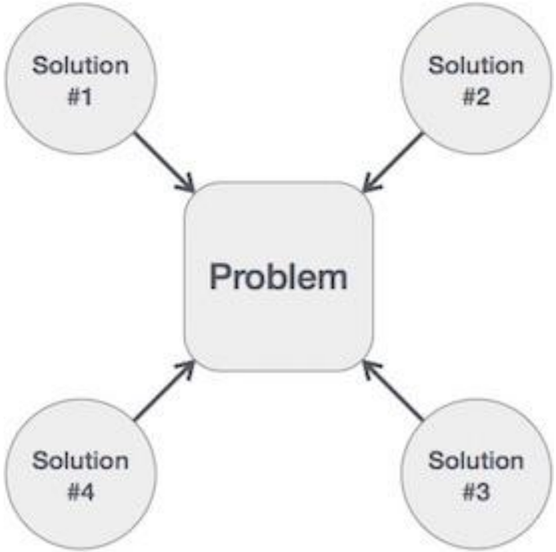
Recursive Implementation of Fibonacci

```
#include<stdio.h>
void Fibonacci(int);
int main()
{
   int k,n;
   long int i=0,j=1,f;

   printf("Enter the range of the Fibonacci series: ");
   scanf("%d",&n);

   printf("Fibonacci Series: ");
   printf("%d %d ",0,1);
   Fibonacci(n);
   return 0;
}
```

```
//continuation of program
void Fibonacci(int n)
{
   static long int first=0,second=1,sum;

   if(n>0)  Base Criteria
   {
       sum = first + second;
       first = second;
       second = sum;
       printf("%ld ",sum);
       Fibonacci(n-1);  Progressive Criteria
   }
}
```

Algorithm Analysis

We design an algorithm to get solution of a given problem. A problem can be solved in more than one ways.



Hence, many solution's algorithms can be derived for a given problem. Next step is to analyze those proposed solution algorithms and implement the best suitable.

Why the Analysis of Algorithms?

To go from city “A” to city “B”, there can be many ways of accomplishing this: by flight, by bus, by train and also by bicycle. Depending on the availability and convenience, we choose the one that suits us. Similarly, in computer science, multiple algorithms are available for solving the same problem (for example, a sorting problem has many algorithms, like insertion sort, selection sort, quick sort and many more). Algorithm analysis helps us to determine which algorithm is most efficient in terms of time and space consumed.

Goal of the Analysis of Algorithms

The goal of the analysis of algorithms is to compare algorithms (or solutions) mainly in terms of running time but also in terms of other factors (e.g., memory, developer effort, etc.)

What is Running Time Analysis?

It is the process of determining how processing time increases as the size of the problem (input size) increases. Input size is the number of elements in the input, and depending on the problem type, the input may be of different types. The following are the common types of inputs.

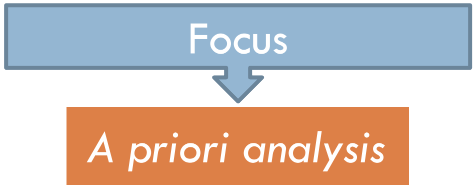
- Size of an array
- Number of elements in a matrix
- Number of bits in the binary representation of the input

Algorithm Comparison

Efficiency of an algorithm can be analyzed at two different stages, before implementation and after implementation, as mentioned below –

- **A priori analysis** – This is **theoretical analysis** of an algorithm. Efficiency of algorithm is measured by assuming that all other factors e.g. processor speed, are constant and have no effect on implementation.

- **A posterior analysis** – This is **empirical analysis** (by means of direct and indirect observation or experience) of an algorithm. The selected algorithm is implemented using programming language. This is then executed on target computer machine. In this analysis, actual statistics like running time and space required and are collected.


Focus

A priori analysis

Algorithm Complexity

Suppose X is an algorithm and n is the size of input data, the time and space used by the Algorithm X are the two main factors which decide the efficiency of X.

- **Time Factor –** The time is measured by counting the number of key operations such as comparisons in sorting algorithm
- **Space Factor –** The space is measured by counting the maximum memory space required by the algorithm.

The complexity of an algorithm f(n) gives the running time and / or storage space required by the algorithm in terms of n as the size of input data.

Space Complexity

Space complexity of an algorithm represents the amount of memory space required by the algorithm in its life cycle. Space required by an algorithm is equal to the sum of the following two components –

- A fixed part that is a space required to store certain data and variables, that are independent of the size of the problem. For example simple variables & constant used, program size etc.
- A variable part is a space required by variables, whose size depends on the size of the problem. For example dynamic memory allocation, recursion stack space etc.

Space complexity SC(P) of any algorithm **P** is SC(P) = FP + VP (I) where FP is the fixed part and VP(I) is the variable part of the algorithm which depends on instance characteristic I. Following is a simple example that tries to explain the concept –

```
Algorithm: SUM(A, B)
Step 1 -  START
Step 2 -  $C \leftarrow A + B + 10$
Step 3 -  Stop
```

3 variables and data type of each variable is int, So VP(3) = 3*2 = 6 bytes (No. of characteristic i.e. I = 3)
1 constant (i.e. 10) and it is int, so FP = 1*2 = 2 bytes
So - SC(SUM) = FP + VP (3) = 2 + 6 = 8 bytes

Space Complexity cont…

Example 1

```
int square(int a)
{
    return a*a;
}
```

In above piece of code, it requires 2 bytes of memory to store variable 'a' and another 2 bytes of memory is used for return value.

That means, totally it requires 4 bytes of memory to complete its execution. And this 4 bytes of memory is fixed for any input value of 'a'. This space complexity is said to be **Constant Space Complexity**.

If any algorithm requires a fixed amount of space for all input values then that space complexity is said to be Constant Space Complexity

Example 2

```
int sum(int a[], int n)
{
  int sum = 0, i;
  for(i = 0; i < n; i++) { sum = sum + a[i];}
  return sum;
}
```

In above piece of code it requires -

- 2*n bytes of memory to store array variable ‘a[]’
- 2 bytes of memory for integer parameter 'n‘
- 4 bytes of memory for local integer variables 'sum' and 'i' (2 bytes each)
- 2 bytes of memory for return value.

That means, totally it requires '2n+8' bytes of memory to complete its execution. Here, the amount of memory depends on the input value of 'n'. This space complexity is said to be **Linear Space Complexity.**

If the amount of space required by an algorithm is increased with the increase of input value, then that space complexity is said to be Linear Space Complexity

Time Complexity

Time Complexity of an algorithm represents the amount of time required by the algorithm to run to completion. Time requirements can be defined as a numerical function T(n), where T(n) can be measured as **number of steps** * **time taken by each steps**

For example, addition of two n-bit integers takes n steps. Consequently, the total computational time is **T(n) = c*n**, where c is the time taken for addition of two bits. Here, we observe that T(n) grows linearly as input size increases.

Asymptotic analysis

Asymptotic analysis of an algorithm, refers to defining the mathematical foundation/framing of its run-time performance.

Asymptotic analysis are input bound i.e., if there's no input to the algorithm it is concluded to work in a constant time. Other than the "input" all other factors are considered constant.

Time Complexity cont… Asymptotic analysis

Asymptotic analysis refers to computing the running time of any operation in mathematical units of computation.

For example,

- Running time of one operation is computed as **f(n)**
- May be for another operation it is computed as **g(n^2)**

Usually, time required by an algorithm falls under three types –

❑ **Worst Case (Usually done)**– Upper bound on running time of an algorithm.

❑ **Average Case (Sometimes done)**– Average time required for running an algorithm.

❑ **Best Case (Bogus)** – Lower bound on running time of an algorithm.

Time Complexity cont… Best, Average and Worst Case examples

Example 1: Travel from Kanyakumari to Srinagar

❑ **Worst Case** – You go to Ahemdabad and then you take a east go to Guhwati and then again take a west and go to Srinagar.

❑ **Average Case** – You go normal route highways only to reach there.

❑ **Best Case** – Take every short cut possible leave the national highway if you need to take state highway, district road if you need to but you have to get to Srinagar with least possible distance

Example 2: Sequential Search for k in an array of n integers

❑ **Best Case** – The 1st item of the array equals to k

❑ **Average Case** – Match at n/2

❑ **Worst Case** – The last position of the array equals to k or unsuccessful search

Time Complexity cont… Asymptotic Notations

Following are commonly used asymptotic notations used in calculating running time complexity of an algorithm.

- ❑ **O Notation – “Big Oh”** - The O(n) is the formal way to **express the upper bound** of an algorithm's running time. It **measures the worst case time complexity or longest amount of time** an algorithm can possibly take to complete.

- ❑ **Ω Notation – “Omega”** - The Ω(n) is the formal way to **express the lower bound** of an algorithm's running time. It **measures the best case time complexity or best amount of time** an algorithm can possibly take to complete.

- ❑ **θ Notation – “Theta” -** The θ(n) is the formal way to **express both the lower bound and upper bound** of an algorithm's running time.

Time Complexity cont… Asymptotic Notations cont…

Sr #	Algorithm	Time Complexity		
		Best	Average	Worst
1	Bubble Sort	$\Omega(n)$	$\Theta(n^2)$	$O(n^2)$
2	Insertion Sort	$\Omega(n^2)$	$\Theta(n^2)$	$O(n^2)$
3	Quick Sort	$\Omega(n \log n)$	$\Theta(n \log n)$	$O(n^2)$
4	Merge Sort	$\Omega(n \log n)$	$\Theta(n \log n)$	$O(n \log n)$
5	Heap Sort	$\Omega(n \log n)$	$\Theta(n \log n)$	$O(n^2)$

Time-Space Trade Off

The best algorithm to solve a given problem is one that requires less space in memory and takes less time to complete its execution. But in practice it is not always possible to achieve both these objectives. As we know there may be more than one approach to solve a particular problem. One approach may take more space but takes less time to complete its execution while the other approach may take less space but takes more time to complete its execution. We may have to sacrifice one at the cost of the other. If space is our constraint, then we have to choose a program that requires less space at the cost of more execution time. On the other hand if time is our constraint then we have to choose a program that takes less time to complete its execution at the cost of more space.

Algorithm Type

Sr #	Notation	Name
1	$O(1)$	Constant
2	$O(\log(n))$	Logarithmic
3	$O(\log(n)^c)$	Poly-logarithmic
4	$O(n)$	Linear
5	$O(n^2)$	Quadratic
6	$O(n^c)$	Polynomial
7	$O(c^n)$	Exponential

Note: c is constant

Time Complexity Rules

- Time complexity of a function (or set of statements) is considered as O(1) if it doesn't contain **loop**, **recursion** and **call to any other non-constant time function**. **Example:**

```
void swap(int *x, int *y)
{
   int temp;
   temp=*x;
   *x=*y;
   *y=temp;
}
```

Note -

Constant Time - algorithm runs in a fixed amount of time, it just means that it isn't proportional to the length/size/magnitude of the input. i.e., for any input, it can be computed in the same amount of time (even if that amount of time is really long).

Time complexity of swap function= Total number of simple statements = 4 = constant = **O(1)**

Time Complexity Rules for Loops

- A loop or recursion that runs a constant number of times is also considered as **O(1). Example -**

```
int i;
for (i = 1; i <= c; i++) // c is a constant
{
   // some O(1) expressions
}
```

- Time complexity of a loop is considered as **O(n)** if the loop variables i is incremented / decremented by a constant amount c. **Example -**

```
for (i = 1; i <= n; i = i+c)
{
  // some O(1) expressions
}
```

Time Complexity Rules for Loops cont…

- Time Complexity of a loop is considered as **O(log n)** if the loop variables i is multiplied or divided by a constant amount c. **Example -**

```
for (i = 1; i <= n; i=i*c)
{
  // some O(1) expressions
}
```

Let us assume that the loop is executing some k times. At k^{th} step $c^k = n$, and at $(k + 1)^{th}$ step we come out of the loop. Taking logarithm on both sides, gives

$\log(c^k) = \log n \rightarrow k \log c = \log n \rightarrow k = \log n$

- Time Complexity of a loop is considered as **O(log log n)** if the loop variables i if the loop variables is reduced / increased exponentially by a constant amount c. **Example -**

Example 1	Example 2

Here c is a constant greater than 1
for (i = 2; i <= n; i=pow(i ++,c))
{
  // some O(1) expressions
}
	//Here fun is sqrt or cuberoot or any other constant root
	for (i = n; i > 0; i = fun(i))
	{
	  // some O(1) expressions
	}

Time Complexity Rules for Loops cont…

- Time Complexity of a loop is considered as $\mathbf{O(\sqrt{n})}$ if the loop variables i is multiplied by itself. **Example** -

```
for (i = 1; i*i <= n; i++)
{
  // some O(1) expressions
}
```

When i^2 is greater than n, the loop will terminate. So, this means i = $O(\sqrt{n})$

- Time Complexity of a loop is considered as **O(1)** if the loop variables i is encountered with break statement. **Example** –

```
for (i = 1; i*i <= n; i++)
{
  // some O(1) expressions
  break;
}
```

Time Complexity Rules for Nested Loops

- Time complexity of nested loops is equal to the number of times the innermost statement is executed that is nothing but the multiplication of outer loop complexity into inner loop complexity.

Sr #	Program Segment	Time Complexity	Explanation

1	int i, j, k;
	for (i = 1; i <=m; i ++)
	{
	  for (j = 1; j <=n; j ++)
	  {
	    k=k+1;
	  }
	}
		O(m*n)
		If m = n, then $O(n^2)$
			Outer for m times and inner for n times, so total m*n times

2	int i, j, k;
	for (i = n; i >1; i =i-2)
	{
	  for (j = 1; j <=n; j=j+3)
	  {
	    k=k+1;
	  }
	}
		$O(n^2)$	Outer loop approx n times and inner approx n times, so total n^2 times.

Time Complexity Rules for Loops cont…

Sr #	Program Segment	Time Complexity	Explanation

3	int i, j, k;
	for (i = n; i >1; i =i/2) {
	  for (j = 1; j <=n; j++) {
	    k=k+1;
	  }
	}
		O(n log n)	Outer loop approx log n times, inner loop n times, so total n log n times.

4	int i, j, k;
	for (i = 1; i<=n; i=i*2) {
	  for (j =n; j>=1; j=j/2) {
	     k = k +1;
	  }
	}
		O((log n)^{2})	Outer loop approx log n times and inner loop approx log n times, so total ((log n)^{2} times.

5	int i, j;
	for (i = 1; i<=n; i=i++) {
	  for (j =1; j <=n; j=j+i) {
	     // some O(1) expressions
	  }
	}
		O(n log n)	The inner loop executes n/i times for each value of i. Its running time is n * ∑i=1 to n of n/i = O(n * log n)

Time Complexity Rules for Loops cont…

Sr #	Program Segment	Time Complexity	Explanation

6	for (i = 1; i<=n; i++)
	{
	for (j =1; j<=100; j++)
	{
	Simple-Statements;
	}
	}
		O(n)	The outer loop executes approx. n times, and the innermost 100 times. Here n=O(n) and 100=O(1) constant time. So total = O(n)*O(1) = O(n)

- When there are consecutive loops, the time complexity is calculated as sum of time complexities of individual loops and final time complexity is the higher order term in terms of n (the one which is larger than others for larger value of n)

Sr #	Program Segment	Time Complexity	Explanation

1	for (int i = 1; i <=m; i += c)
	{
	// some O(1) expressions
	}
	for (int i = 1; i <=n; i += c)
	{
	// some O(1) expressions
	}
		O(m+n)
		If m=n, then
		O(2n)=O(n)
			First for outer i loop O(m), second for inner i loop O(n) times, so total O(m+n) times

Time Complexity Rules for Loops cont…

Sr #	Program Segment	Time Complexity	Explanation

2	for (int i = 1; i <=n; i *= 2)
	{
	  // some O(1) expressions
	}
	for (int i = 1; i <=n; i ++)
	{
	  // some O(1) expressions
	}
		O(n)	First for outer i log n times, second inner i it is n times. Now total=log n + n = O(n) as n is asymptotically larger than log n.

3	int i, j, k,l;
	for (i = 1; i <=n; i ++)
	{
	   for (j = 1; j <=n; j=j*2) {p=i+j;}
	}
	
	for (k = n; k >=1; k=k/3)
	{ q=k+p; }
	
	for (l = n; l >=1; l=l-2)
	{ q=k+p; }
		O(n log n)	Nested for-ij loop is executed n log n times. k loop log n times, l loop n times. So total= n log n + log n + n = O(n log n) as n log n > n > log n

Time Complexity Rules for Recursion

Sr #	Program Segment	Time Complexity	Explanation

1	**Factorial of a number**
	int Factorial (int n)
	{
	if n >= 1 then
	return n * Factorial(n – 1);
	else
	return 1;
	}
		O(n)	The algorithm is linear, running in O(n) time. This is the case because it executes once every time it decrements the value n, and it decrements the value n until it reaches 0, meaning the function is called recursively n times Both decrementation and multiplication are constant operations..

2	**Fibonacci Sequence**
	int Fib(int n)
	{
	if n == 1 then
	return 1;
	else
	return Fib(n-1) + Fib (n – 2)
	}
		??	??

3	void recursiveFun(int n, int m, int o)
	{
	if (n <= 0)
	{
	printf("%d, %d\n",m, o);
	}
	else
	{
	recursiveFun(n-1, m+1, o);
	recursiveFun(n-1, m, o+1);
	}
	}
		$O(2^n)$	Each function call calls itself twice unless it has been recursed n times and hence exponential in nature.

Time Complexity Class Work

Sr #	Program Segment	Time Complexity	Explanation

1	int j=0,i;
	for (i = 0; i<n ; i++)
	{
	  while (j<n && arr[i] <arr[j])
	  {
	    j++;
	  }
	}
		??	??

2	int i,j;
	for (i=1;i<=m;i=i*2)
	{
	 for (j=1;j<=i;j++)
	 {
	   printf("%d %d ",i,j);
	 }
	}
		??	??

3	int count = 0,i,j;
	for (i = n ; i>0; i /= 2)
	{
	 for (j = 0; j<i ; j++)
	  count += 1;
	}
		??	??

Space Complexity Rules

Sr #	Program Segment	Space Complexity	Explanation

1	int sum(int x, int y, int z)
	{
	  int r = x + y + z;
	  return r;
	}
		O(1)	requires 3 units of space for the parameters and 1 for the local variable, and this never changes, so this is O(1).

2	int sum(int a[], int n) {
	  int r = 0;
	  for (int i = 0; i < n; ++i) {
	    r += a[i];
	  }
	  return r;
	}
		O(n)	requires n units for a, plus space for n, r and i, so it's O(n).

3	void matrixAdd(int a[], int b[], int c[], int n)
	{
	  for (int i = 0; i < n; ++i)
	 {
	    c[i] = a[i] + b[j]
	  }
	}
		O(n)	requires n units for a, b and c plus space for n, and i, so it's O(n).

Summary

Detailed Lessons

Functions, Structures and Unions, Pointers, Dynamic Memory Allocation, Algorithm Specification, Space and Time Complexity

How was the journey?

THANK YOU!

Assignments

1. Find the time complexity of the following code segment

✓
```
for (i = 1; i <= n; i = i*2)
 {
   for (j = n; j >= 1; j = j/2)  { some statement }
 }
```

✓
```
for (j = c; j > 0; j--)   // consider c as const
{
  some statement
}
```

✓
```
for (i = n; i <= 1; i = i/c) // consider c as const
 {
    // some statement
 }
```

2. Write an recursive algorithm to print from 1 to n (where n > 1)
3. Write an recursive algorithm to find the kth smallest element of a set S
4. Write an recursive algorithm to sum the list of numbers

Assignments cont…

5. Find the space complexity of the following code segment

```
int Factorial (int n)
{
  if n == 0 then
    return 0;
  else
    return n * Factorial(n – 1);
}
```

6. Find the time and space complexity of the following code segment

```
int Fib(int n)
{
  if n <= 1 then
    return 1;
  else
    return Fib(n-1) + Fib (n – 2);
}
```

Assignments cont…

7. Find the space complexity of the following code segment

```
 void fun()
{
   int i=1, j=1;
   for (; i<=n; i++)
      for (; j<=log(i); j++)
         printf("KIIT");
}
```

8. What is the time, space complexity of following code:

```
int a = 0, b = 0;
for (i = 0; i < N; i++) {
    a = a + rand();
}
for (j = 0; j < M; j++) {
    b = b + rand();
}
```

Assignments cont…

9. What are the operations can be performed on various data structure?
10. What are the advantages of using ADT?
11. Explain Top-Down and Bottom-Up approach in designing the algorithm
12. Explain little o and little omega (w) notation.
13. In how many ways can you categorize data structures? Explain each of them.
14. What do you understand by time–space trade-off?
15. Explain the criteria that you will keep in mind while choosing an appropriate algorithm to solve a particular problem.
16. Write best case, worst case and average case time complexity of following categories of algorithm –
    - Constant time
    - Linear time
    - Logarithmic time
    - Polynomial time
    - Exponential time

Assignments cont…

17. Discuss the significance and limitations of the Big O notation.
18. Design an algorithm whose worst case time complexity is $O(m + n \log n)$
19. Design an algorithm whose worst case time complexity is $O(m + n^2 \log m)$
20. Design an algorithm whose worst case time complexity is $O(m^2 + n (\log m)^2)$

Home Work (HW)

- ❑ Find the time and space complexity of the following code segment
  - ✓ int GCD(int x, int y)

```
{
  if y == 0 then
    return x
  else if  x >= y AND y > 0
    return GCD (y, x % y)
   else
     return 0;
}
```

Home Work (HW)

- Find the time and space complexity of the following code segment

```
void matrixAddition(int a[][], int b[][], int c[][], int n)
{
  int i, j;
  for (i = 0; i < n; ++i)
  {
    for (j = 0; j < n; ++j)
    {
      c[i][j] = a[i][j] + b[i][j];
    }
  }
}
```

Supplementary Reading

- ❑ Watch the following video
  - ✓ https://www.youtube.com/watch?v=8syQKTdgdzc
  - ✓ https://www.youtube.com/watch?v=AL7yO-I5kFU
  - ✓ http://nptel.ac.in/courses/106102064/1
- ❑ Read the following
  - ✓ https://www.tutorialspoint.com/data_structures_algorithms/

FAQ

- **What is Information** - If we arrange some data in an appropriate sequence, then it forms a Structure and gives us a meaning. This meaning is called Information . The basic unit of Information in Computer Science is a bit, Binary Digit. So, we found two things in Information: One is Data and the other is Structure.
- **What is Data Structure?**
  1. A data structure is a systematic way of organizing and accessing data.
  2. A data structure tries to structure data!
     - Usually more than one piece of data
     - Should define legal operations on the data
     - The data might be grouped together (e.g. in an linked list)
  3. When we define a data structure we are in fact creating a new data type of our own.
     - i.e. using predefined types or previously user defined types.
     - Such new types are then used to reference variables type within a program

FAQ cont…

- ❑ **Why Data Structures?**

1. Data structures study how data are stored in a computer so that operations can be implemented efficiently
2. Data structures are especially important when you have a large amount of information
3. Conceptual and concrete ways to organize data for efficient storage and manipulation.

- ❑ **ADT**

1. Abstract Data Types (ADT's) are a model used to understand the design of a data structure
2. 'Abstract' implies that we give an implementation-independent view of the data structure
3. ADTs specify the type of data stored and the operations that support the data
4. Viewing a data structure as an ADT allows a programmer to focus on an idealized model of the data and its operations

FAQ cont…

- **Time Complexity of algorithm -** Time complexity of an algorithm signifies the total time required by the program to run till its completion. The time complexity of algorithms is most commonly expressed using the **big O** notation. Time Complexity is most commonly estimated by counting the number of elementary functions performed by the algorithm. And since the algorithm's performance may vary with different types of input data, hence for an algorithm we usually use the worst-case Time complexity of an algorithm because that is the maximum time taken for any input size.
- **Types of Notations for Time Complexity**
  1. Big Oh denotes "fewer than or the same as" <expression> iterations.
  2. Big Omega denotes "more than or the same as" <expression> iterations.
  3. Big Theta denotes "the same as" <expression> iterations.
  4. Little Oh denotes "fewer than" <expression> iterations.
  5. Little Omega denotes "more than" <expression> iterations. idealized model of the data and its operations