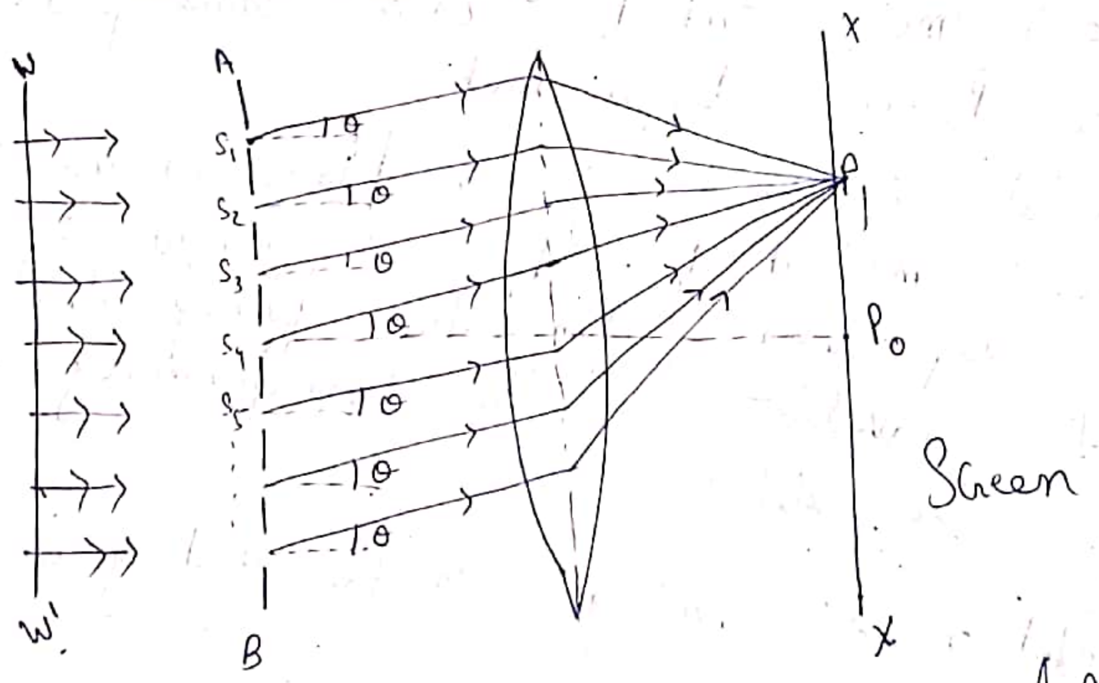
Plane Diffraction grating

A plane diffraction grating is an arrangement consisting of large number of parallel slits of equal width separated by equal opaque space over a plane surface.

→ Constructed by ruling equidistant lines by a fine diamond point on a plane transparent material such as glass.

→ Ruled portion becomes opaque & the space betn them serves as slits.



Let AB be the section of a plane diffraction grating, length of the slits being ⊥ to the plane of the paper.

e → width of each slit, d → width of each opaque space betn the slits.

$(e+d) \rightarrow$ grating element.

→ Points in two Consequetive Slits Separated by a distance $(e+d)$ are Called Corresponding point

→ A plane wavefront WW' illuminates the grating.

→ Parallel rays leaving the grating after diffraction are focussed on a Sceen in the focal plane of Convex lens L.

→ Wavelets leaving the grating plane normally $(\theta=0)$ meet at P_0 in the Same phase. Hence P_0 is a bright Point.

→ Wavelets leaving the grating plane at an angle θ meet at P_1. P_1 is bright or dark depending upon the phase diff betn the wavelets from the Corresponding points of the ~~Slits~~.

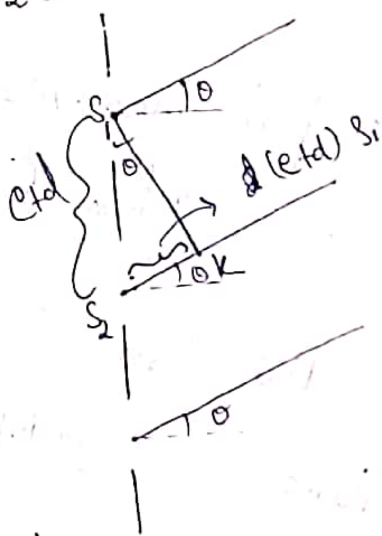
→ Total no. of rulings on the grating is N.

By the theory of Fraunhoffer diffraction at a Single Slit, the wavelets from all the points in a Slit, along the direction θ are equivalent to a Single wave of amplitude $A\frac{\sin\alpha}{\alpha}$, Starting from the middle point of the Slit, where $\alpha = \frac{\pi e \sin\theta}{\lambda}$.

Thus if N be the total no. of Slits in the grating the diffracted rays from all the Slits are equivalent to N parallel rays, one each from the middle points $S_1, S_2, S_3, \ldots$ of the Slits.

$\therefore$ The path diff betn wavelets from any two consecutive Slits,

$$(e+d)\sin\theta \quad (S_2K)$$



The corresponding phase diff,

$$= \frac{2\pi}{\lambda}(e+d)\sin\theta$$

$$= 2\beta \text{ (say)}$$

Hence, the resultant amplitude in the direction θ,

$$R = \frac{a \sin n\alpha/2}{\sin \alpha/2}$$

Here $a = \frac{A\sin\alpha}{\alpha}$, $n = N$, $\alpha = 2\beta$.

$$\therefore R = A\frac{\sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}$$

The resultant intensity,

$$I = R^2 = \left(A\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

The first term $\left(A\frac{\sin\alpha}{\alpha}\right)^2$ gives the distribution of intensity due a single slit while the factor $\left(\sin N\beta/\sin\beta\right)^2$ gives the distribution of intensity as a combined effect of all the slits.

Principal maxima.

When $\sin\beta = 0$, ie $\beta = \pm n\pi$

$n = 0, 1, 2, 3, \ldots$

But then $\sin N\beta = 0$

Thus $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ (indeterminate)

$\therefore$ By L'Hospital rule,

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$= \lim_{\beta \to \pm n\pi} \frac{N\cos N\beta}{\cos\beta} = \pm N$$

The intensity is then,

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 N^2$$

Sudhir Patel
9937220251

Which is a maximum. These maxima are most intense and are called principal maxima. They are obtained in the diffraction pattern as,

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$\Rightarrow \boxed{(e+d)\sin\theta = \pm n\lambda}$$

Where $n = 0, 1, 2, ---$

$n = 0 \rightarrow$ Zero order principal maxima

$n = 1, 2, 3, ---- \rightarrow$ first, second, third --- order principal maxima

$\pm \rightarrow$ denotes two principal max^m for each order lying on either side of the zero order maximum

Minima.

It can be shown that bet^n two consecutive principal maxima, a series of minima are observed.

When $\sin N\beta = 0$ but $\sin\beta \neq 0$, then

$$\frac{\sin N\beta}{\sin\beta} = 0.$$

Hence we get

$$I = A\left(\frac{\sin\alpha}{\alpha}\right) \times 0 = 0$$ which is a minimum

Scanned with CamScanner

These minima are obtained in the direction,

$$\sin N\beta = 0$$

$$\Rightarrow N\beta = \pm m\pi$$

$$\Rightarrow N\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

$$\Rightarrow \boxed{N(e+d)\sin\theta = \pm m\lambda}$$

Where m can take all integral values except 0, N, 2N, -----, mN, because these values of m make $\sin\beta = 0$ which gives principal maximum.

Thus, m = 1, 2, 3, ---- (N-1) give minima and then m = N gives again a principal maxima. Thus there are (N-1) minima bet^n two consequtive principal maxima.

Secondary Maxima.

Bet^n two successive principal maxima, there are (N-1) minima. Therefore bet^n two successive minima there should be one secondary maximum. As such, bet^n two principal maxima the no. of secondary maxima will be (N-2). Their positions are found by using the cond^n,

$$\frac{dI}{d\beta} = 0$$

$$\Rightarrow \left(\frac{A\sin\alpha}{\alpha}\right)^2 \cdot 2\left(\frac{\sin N\beta}{\sin\beta}\right) \cdot \left[\frac{N\cos N\beta \sin\beta - \sin N\beta \cos\beta}{\sin^2\beta}\right] = 0$$

$\Rightarrow$ Either $\sin N\beta = 0 \rightarrow$ Minima

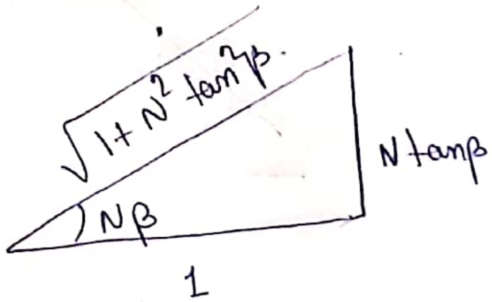
or $(N\cos N\beta \sin\beta - \sin N\beta \cos\beta) = 0$

$$N\cos N\beta \sin\beta - \sin N\beta \cos\beta = 0$$

$$\Rightarrow \boxed{N\tan\beta = \tan N\beta}$$

To find the value of $\frac{\sin^2 N\beta}{\sin^2\beta}$ under the above condition, we make use of triangle as shown in figure.

$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$



$$\therefore \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\tan^2\beta}{(1+N^2\tan^2\beta)\sin^2\beta}$$

$$= \frac{N^2}{(1+N^2\tan^2\beta)\cos^2\beta}$$

$$= \frac{N^2}{\cos^2\beta + N^2\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

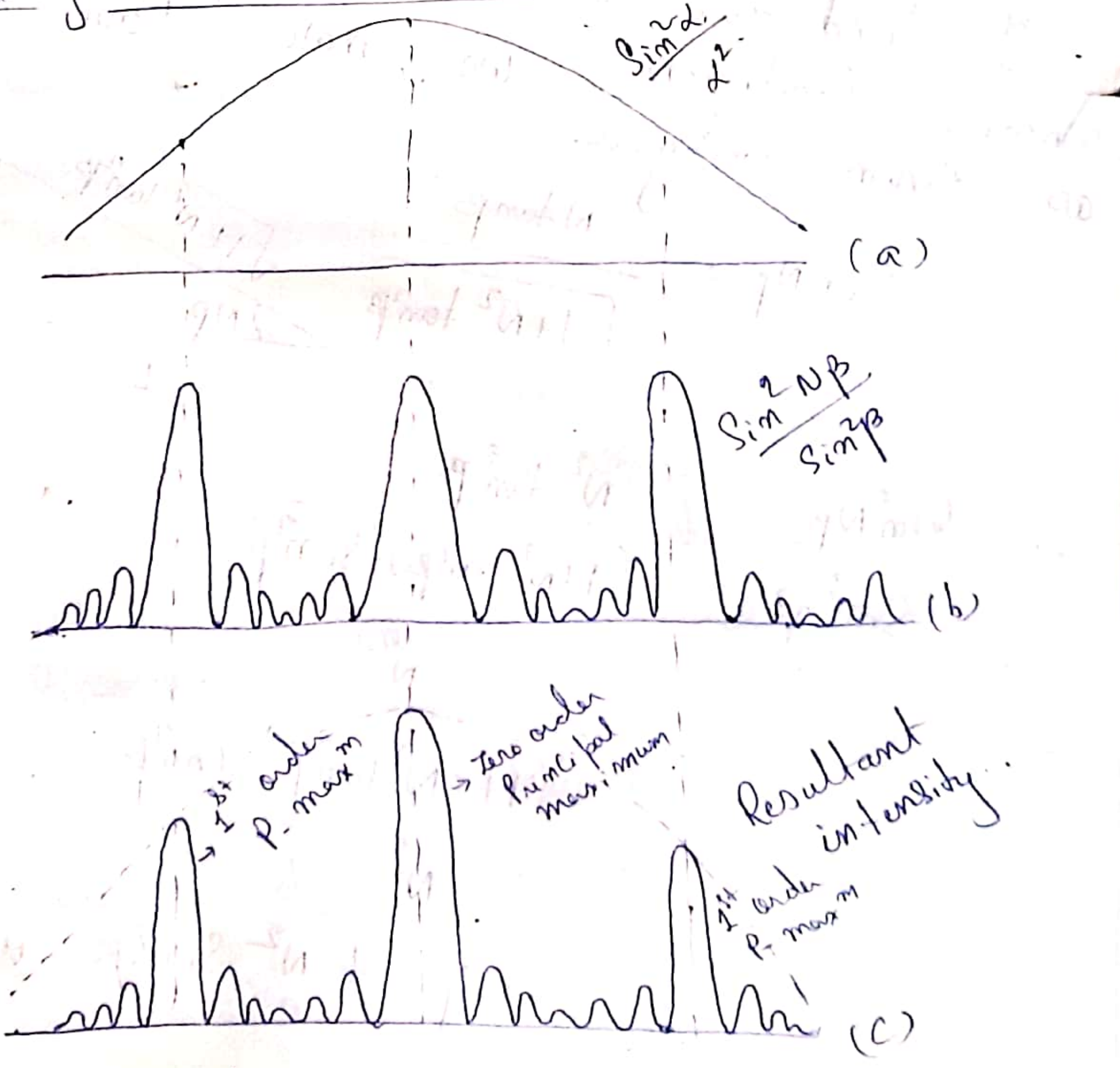
Intensity of Secondary maxima $\propto \dfrac{N^2}{1+(N^2-1)\sin^2\beta}$

" " Principal " $\propto N^2$

$$\therefore \frac{\text{Intensity of Secondary maxima}}{\text{Intensity of Principal maxima}} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

Hence, greater the value of N, weaker are secondary maxima. In actual grating, N is very large, hence the secondary maxima are not visible in the grating spectrum.

Intensity distribution Curve.



Absent Spectra in a diffraction grating

$$I = A^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}, \quad \alpha = \frac{\pi e \sin\theta}{\lambda}, \quad \beta = \frac{\pi (e+d)\sin\theta}{\lambda}$$

$\frac{\sin^2\alpha}{\alpha^2}$ → intensity due to diffractn at a single slit

$\frac{\sin^2 N\beta}{\sin^2\beta}$ → Combined effect of all the N slits.

If condn for maxima for both patterns are satisfied simultaneously, then spectra pattern remains unaffected.

However, when the condn for the principal maxm due to N slits & the condn for minimum for a single slit are satisfied simultaneously for a given value of θ, then the principal maxm of that order will be absent or missing. That spectra is called missing spectra or absent spectra.

$$(e+d)\sin\theta = n\lambda \quad \text{--- (1)}$$

(Condn for principal maxm; $n \to$ order of spectrum).

$$e\sin\theta = m\lambda \quad \text{--- (2)}$$

(Condn for minima in single slit pattern)

$m = 1, 2, 3, \ldots$

$$\frac{eq^n (1)}{eq^n (2)} \Rightarrow \boxed{\frac{e+d}{e} = \frac{n}{m}} \quad \text{--- (3)}$$

Condn for the spectrum of order n to be absent.

If $d = e$, eqn (3) becomes

$n = 2m \quad (m = 1, 2, 3, \ldots)$

$= 2, 4, 6, \ldots$

ie 2^{nd}, 4^{th}, 6^{th} order Spectra will be absent.

If $d = 2e$,

$n = 3m$

3^{rd}, 6^{th}, 9^{th} ---- order Spectra will be absent.

Dispersive power of a grating

If a light source contains more than one frequency (i.e a polychromatic source), then a number of bright lines corresponding to the spectra of the same order will be ~~visible~~ observed.

Thus, the lines for different colours will be dispersed. The degree of dispersion or the angular separation betn the spectra of the same order for two neighbouring frequencies gives the dispersive power of the grating.

Mathematically, dispersive power $= \frac{d\theta}{d\lambda}$

where $d\theta$ is the angular separation betn the spectral lines corresponding to a wavelength separation $d\lambda$.

Differentiating the relation,

$$(e+d)\sin\theta = n\lambda \text{, we get}$$

$$(e+d)\cos\theta \, d\theta = n \, d\lambda.$$

So, dispersive power, $\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$

Hence, the dispersive power $\propto n$ (order of spectra)

$\propto \frac{1}{e+d}$

$\propto \frac{1}{\cos\theta}$

Longer the value of θ, Smaller is $\cos\theta$, & higher is the dispersive power. Red light (more angle of diff), more dispersed than violet light.

Eg. Sodium D lines $\rightarrow$ more Separated in 2nd order than in 1st order.

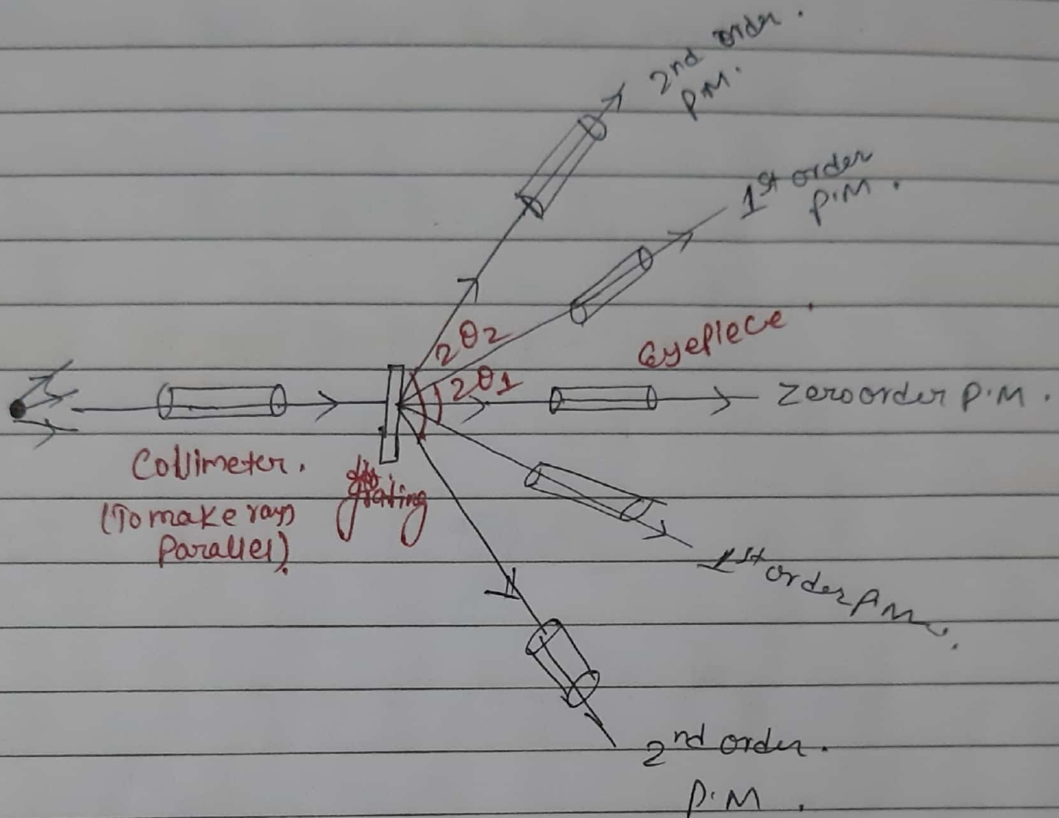
<u>Determination of wavelength of light</u>

30/4/

Date ___/___/___

* Application:-

(i) To determine wavelength of a monochromatic light using a Plane diffraction grating.



N lines/inch	N lines/cm.
$N(e+d) = 1$ inch $= 2.54$ cm,	$N(e+d) = 1$ cm.

Write yourself.