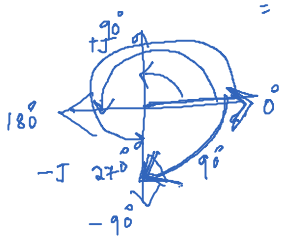
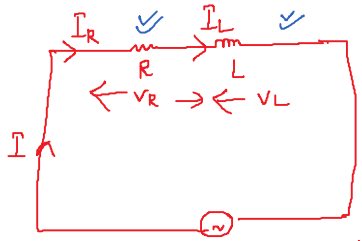
Monday, January 3, 2022
12:55 PM

$0+J = \tan^{-1}\left(\frac{1}{0}\right) = 90^\circ$

$0-J = \tan^{-1}\left(\frac{-1}{0}\right) = -90^\circ$



Series R-L Circuit

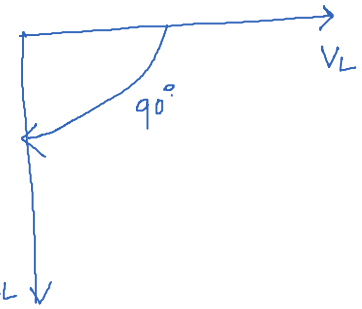


$v = V_m \sin\omega t$

$= |V| \angle 0^\circ$ ~ polar form.

R

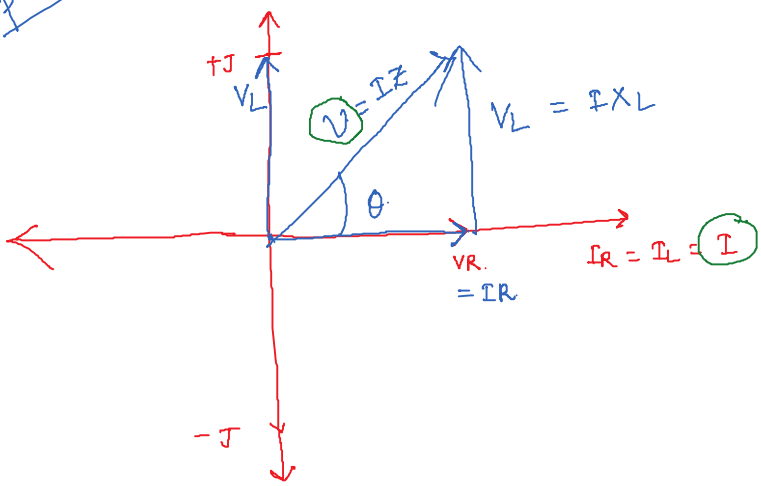
L

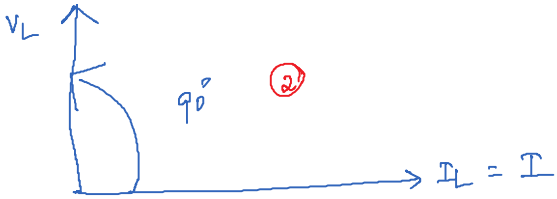


R-L

$V = \vec{V_R} + \vec{V_L}$

Phasor diagram





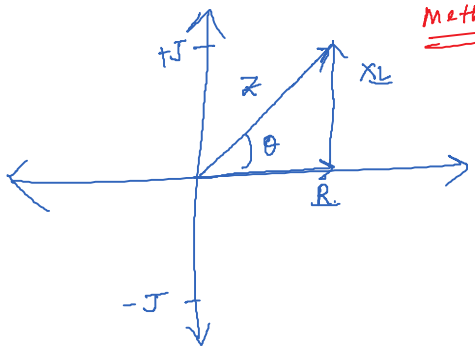
Impedance Triangle

Method I

$Z = R + jX_L$ (Cartesian)

$= |Z| \angle \theta$

$= \sqrt{R^2 + X_L^2} \angle \tan^{-1}\left(\frac{X_L}{R}\right)$



Method II

$V = \vec{V_R} + \vec{V_L}$

$= [IR + J0] + (0 + JIX_L)$

$$IZ = IR + jIX_L$$

$$Z = R + jX_L \quad \text{---} (2)$$

Current

$$v = |V| \angle 0^\circ$$

$$I = \frac{V}{Z} = \frac{|V| \angle 0^\circ}{|Z| \angle \theta}$$

$$I = \frac{|V|}{|Z|} \angle -\theta$$

$$I = \frac{V_m}{\sqrt{R^2 + X_L^2}} \angle -\tan^{-1}\left(\frac{X_L}{R}\right) \quad \text{---} (1)$$

$$v = V_m \sin \omega t$$

$$i = \frac{V_m}{\sqrt{R^2 + X_L^2}} \sin(\omega t - \tan^{-1}\left(\frac{X_L}{R}\right)) \quad \text{---} (2)$$

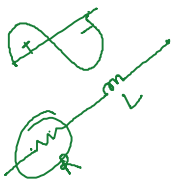
(i) $P_{inst} = vi = V_m \sin\omega t \; I_m \sin(\omega t - \theta) \quad \text{---} (3)$

$$= \frac{V_m I_m}{2} \left[2 \sin\omega t \sin(\omega t - \theta) \right]$$

(A = ωt, B = $\omega t - \theta$)

$$= \frac{V_m I_m}{2} \left[\cos(A - B) - \cos(A + B) \right]$$

$$= \frac{V_m I_m}{2} \left[\cos\theta - \cos(2\omega t - \theta) \right] \quad \text{---} (4)$$



(ii) $P_{avg} = \frac{V_m I_m \cos\theta}{2}$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos\theta = V_{rms} I_{rms} \cos\theta$$

$$= I_{rms} Z \; I_{rms} \frac{R}{Z}$$

$$= I_{rms}^2 R =$$

True power

(iii) $pf = \cos\theta = \frac{R}{Z}$

(iii) pf = $\cos\theta = \frac{R}{Z} = \frac{VI\cos\theta}{VI}$ = $\frac{\text{Active power}}{\text{Apparent power}}$ ← "True power"

Ex:1 A 50Hz sinusoidal voltage $v = 311\sin\omega t$ is applied to a RL Series ckt. of the $R = 5\Omega$ and $L = 0.02H$

Calculate (a) rms value of current and relative phase angle.

(b) expression for i

(c) avg power in the ckt.

(d) pf.

(e) Draw the phasor diagram?

(a) $I = \frac{V}{Z} = \frac{|V|\angle 0^\circ}{|Z|\angle\theta} = \frac{\frac{311}{\sqrt{2}}\angle 0^\circ}{5 + J\,2\pi\times 50 \times 0.02}$ ($2\pi\times 50\times 0.02 = 6.28$)

$= \frac{\frac{311}{\sqrt{2}}\angle 0^\circ}{\sqrt{5^2+6.28^2}\angle \tan^{-1}\left(\frac{6.28}{5}\right)}$

$= \frac{\frac{311}{\sqrt{2}}\angle 0^\circ}{8.03\angle 51.47^\circ}$

$I_{rms} = 27.39\,A$

$I_m = I_{rms}\sqrt{2}$

$= 27.39\angle -51.47^\circ$

(b) $i = I_m \sin(\omega t - \theta)$

$= 27.39\sqrt{2}\sin(\omega t - 51.47^\circ)$

(c) $P_{avg} = I_{rms}^2 R$

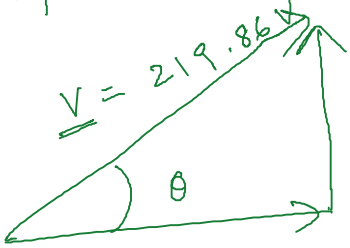
(c)

$= (27.39)^2 \times 5$

$= 3751.06\ W$

(d) $pf = \cos 51.47^\circ = 0.622$ Lagging

(e)



$V_L = IX_L$

$= 27.39 \times 6.28$

$= 172.0\ V$

$V_R = IR$

$= 27.39 \times 5$

$= 136.95\ V$

$V = \sqrt{172^2 + 136.95^2}$

$\approx \frac{311}{\sqrt{2}} = 219.86\ V$