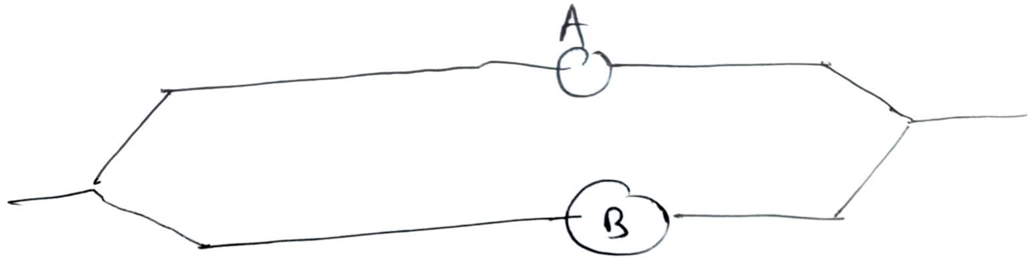
Ex A system consists of two components A & B connected parallely in the following way:



The event E_1 denotes that the component A works with probability 0.9 and the event E_2 denotes that the component B works with probability 0.8. Find the prob. that the system works

Sol^n Given $P(E_1) = P(\{A \text{ works}\}) = 0.9$

$P(E_2) = P(\{B \text{ works}\}) = 0.8$

Since E_1 & E_2 are independent, we have

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = 0.9 \times 0.8 = 0.72$$

The prob. that system works is

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = 0.9 + 0.8 - 0.72 = 0.98$$

Alternative

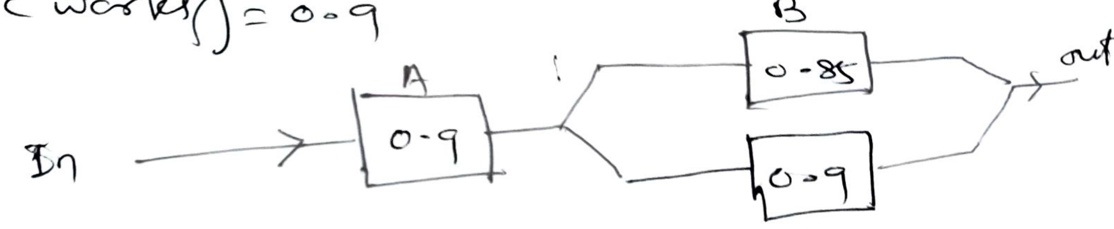
$P(\text{System does not work}) = P(E_1' \cap E_2')$

$= P(E_1')\, P(E_2') = 0.1 \times 0.2 = 0.02$

$\Rightarrow P(\text{system works}) = 1 - 0.02 = 0.98$

4

Ex (4) In the following system A, B & C are the components works with probabilities $P(\{A \text{ works}\}) = 0.9$, $P(\{B \text{ works}\}) = 0.85$
$P(\{C \text{ works}\}) = 0.9$



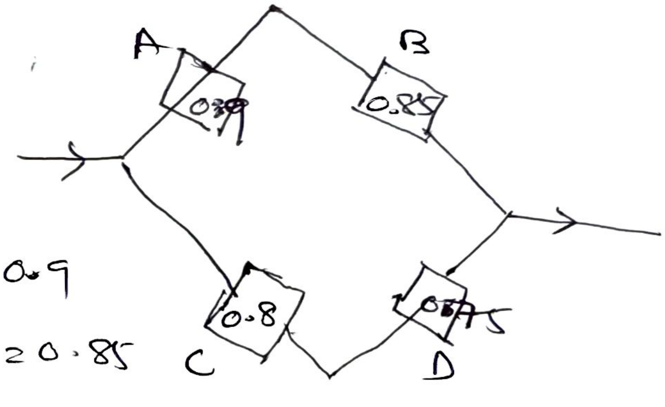
Find the prob. that the system works

Soln

Probability of system works is

$$= P(A \cap (B \cup C)) = P[(A \cap B) \cup (A \cap C)]$$
$$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$
$$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$$
$$= 0.9 \times 0.85 + 0.9 \times 0.9 - 0.9 \times 0.85 \times 0.9$$
$$= 0.8895$$

Ex



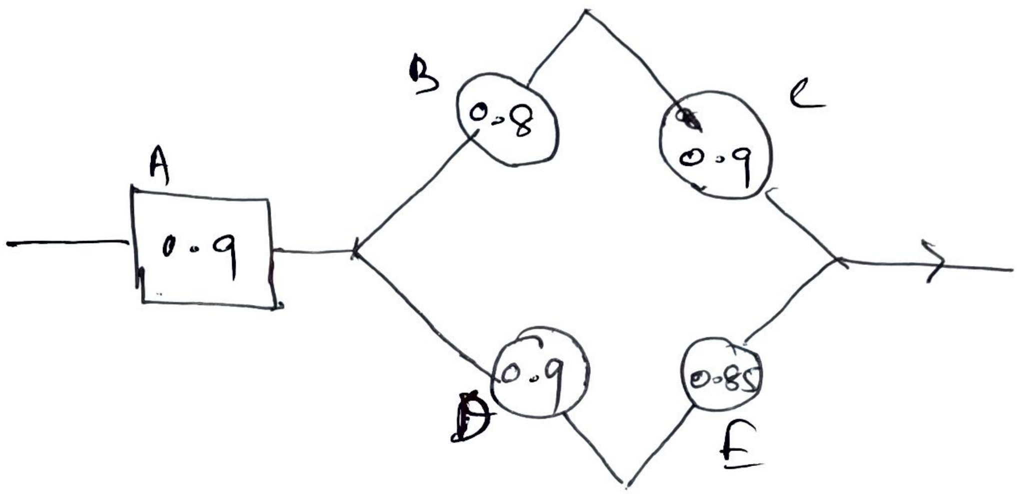
In the system

$P_A = P(\{A \text{ works}\}) = 0.9$
$P_B = P(\{B \text{ works}\}) = 0.85$
$P_C = P(\{C \text{ works}\}) = 0.8$
$P_D = P(\{D \text{ works}\}) = 0.75$. Find the prob. that system works

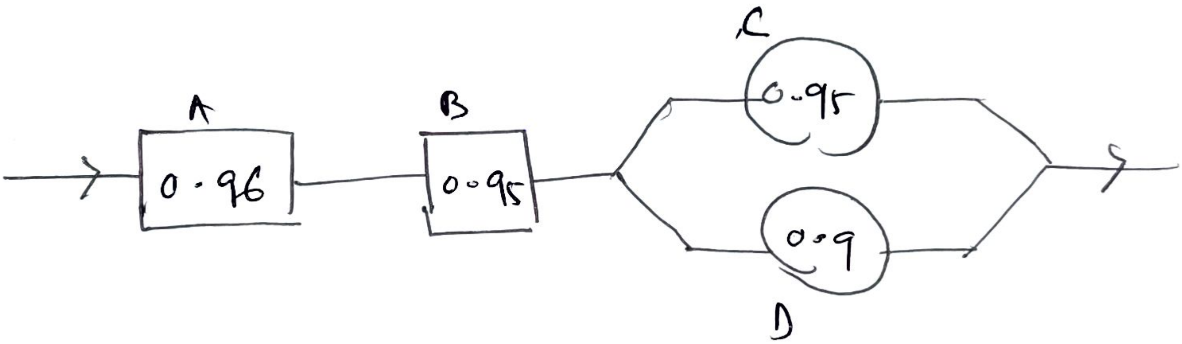
Soln $P(\{\text{system works}\}) = P((A \cap B) \cup (C \cap D))$
$$= P(A \cap B) + P(C \cap D) - P(A \cap B \cap C \cap D) = 0.9 \times 0.85 + 0.8 \times 0.75 - 0.9 \times 0.85 \times 0.8 \times 0.75$$
$$= 0.906$$

Task

① From that system, given in the figure, find the prob. that the system does not work



②



Find the prob. that the system does not work

③ For any mutually exclusive events A & B and for any event C,

$$P(A \cup B | C) = A(A|C) + P(B|C)$$

prove it.