Exe: 5.5

58) a) X_i ($i = 1, 2, 3$) denote the number of type i containers shipped during a given week.

X_1, X_2 and X_3 are independent variables.

$\mu_1 = 200 \qquad \mu_2 = 250 \qquad \mu_3 = 100$

$\sigma_1 = 10 \qquad \sigma_2 = 12 \qquad \sigma_3 = 8$

V_0 = Volume $= 27X_1 + 125X_2 + 512X_3$

$$E(V_0) = 27\,E(X_1) + 125\,E(X_2) + 512\,E(X_3)$$

$$= 27 \times 200 + 125 \times 250 + 512 \times 100$$

$$= 87850$$

$$V(V_0) = 27^2\,V(X_1) + 125^2 \times V(X_2) + 512^2 \times V(X_3)$$

$$= 27^2 \times 10^2 + 125^2 \times 12^2 + 512^2 \times 8^2$$

$$= 1,91,00,116$$

b) If x_i's were not independent, the expected value we calculated will remain the same but the variance we calculated will change (because covariance will be included)

59) X_1, X_2 and X_3 are independent and normal r.v's

a. $\mu_1 = \mu_2 = \mu_3 = 60$

b. $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 15$

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$P(T_0 \leq 200)$ $\quad$ $P(150 \leq T_0 \leq 200)$

$T_0 = x_1 + x_2 + x_3 \rightarrow$ linear combination

$a_1 = 1, a_2 = 1, a_3 = 1$

$E(T_0) = \mu_{T_0} = \mu_1 + \mu_2 + \mu_3 = 180$

$V(T_0) = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 15 \times 3 = 45$

$\sigma_{T_0} = \sqrt{45} = 6.71$

$$P(T_0 \leq 200) = P\left(Z \leq \frac{200-180}{6.71}\right)$$

$$= P(Z \leq 2.98) = \phi(2.98)$$

$$= 0.9986$$

$P(150 \leq T_0 \leq 200)$

$$= P\left(\frac{150-180}{6.71} \leq T_0 \leq 2.98\right)$$

$$= P(-4.47 \leq T_0 \leq 2.98) \rightarrow -(1-\phi(4.47))$$

$$= \phi(2.98) - \phi(-4.47)$$

$$= 0.9986 - 1 + \phi(4.47)$$

$$= 0.9986$$

b) $P(55 \leq \bar{x})$ and $P(58 \leq \bar{x} \leq 62)$

$P(\bar{x} \geq 55)$

$$P\left(Z \geq \frac{55-\mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$= P\left(Z \geq \frac{55-60}{2.24}\right)$$

$$= 1 - P(Z < 2.23)$$

$$= 1 - \phi(2.23)$$

$$= 0.9875$$

$\mu_{\bar{x}} = \mu_{x_1} = 60$

$\sigma_{\bar{x}}^2 = \frac{\sigma_{x_1}^2}{n} = \frac{15}{3} = 5$

$\sigma_{\bar{x}} = \sqrt{5} = 2.24 \left(\frac{\sigma}{\sqrt{n}}\right)$

d) $\mu_1 = 40,\ \mu_2 = 50,\ \mu_3 = 60,\ \sigma_1^2 = 10,\ \sigma_2^2 = 12,\ \sigma_3^2 = 14,$

$P(X_1 + X_2 + X_3 \leq 160)$

Let, $Y = X_1 + X_2 + X_3 \quad a_1 = 1,\ a_2 = 1,\ a_3 = 1$

$\mu_Y = \mu_1 + \mu_2 + \mu_3 = 150$

$\sigma_Y = \sqrt{10 + 12 + 14} = 6$

$P(Y \leq 160)$

$= P\left(Z \leq \frac{160 - 150}{6}\right)$

$= P(Z \leq 1.67)$

$= \phi(1.67)$

$= 0.9525$

$P(X_1 + X_2 \geq 2X_3)$

$P(X_1 + X_2 - 2X_3 \geq 0)$

Let, $\tilde{Y} = X_1 + X_2 - 2X_3$

$\mu_{\tilde{Y}} = \mu_1 + \mu_2 - 2\mu_3$

$= 40 + 50 - 2 \times 60$

$= -30$

$\sigma_{\tilde{Y}} = \cancel{\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}}$ $\quad a_3 = -2$ $\quad a_3^2 = 4$

$= \sqrt{10 + 12 + 56} = \sqrt{78} = 8.83$

$P(\tilde{Y} \geq 0) = P\left(Z \geq \frac{30}{8.83}\right)$

$= 1 - P(Z < 3.40)$

$= 1 - \phi(3.40)$

$= 1 - 0.9997$

$= 0.0003$

60) $Y = \frac{1}{2}x_1 + \frac{1}{2}x_2 - \frac{1}{3}x_3 - \frac{1}{3}x_4 - \frac{1}{3}x_5$

~~$P(Y \geq 0) = P$~~

$E(Y) = \frac{1}{2}E(x_1) + \frac{1}{2}E(x_2) - \frac{1}{3}E(x_3) - \frac{1}{3}E(x_4) - \frac{1}{3}E(x_5)$

$\mu_1 = 22,\ \mu_2 = 22,\ \mu_3 = 26,\ \mu_4 = 26,\ \mu_5 = 26$

$\sigma_1 = 1.2,\ \sigma_2 = 1.2,\ \sigma_3 = 1.5,\ \sigma_4 = 1.5,\ \sigma_5 = 1.5$

$E(Y) = \frac{1}{2} \times 22 + \frac{1}{2} \times 22 - \frac{1}{3} \times 26 - \frac{1}{3} \times 26 - \frac{1}{3} \times 26$

$= -4$

$\sigma_y^2 = \frac{1}{4} \times \sigma_1 + \frac{1}{4} \times \sigma_2 - \frac{1}{3} \times \sigma_3 - \frac{1}{3} \times \sigma_4 - \frac{1}{3} \times \sigma_5$

$= \frac{1}{4} \times 1.2^2 + \frac{1}{4} \times 1.2^2 + \frac{1}{9} \times 1.5^2 + \frac{1}{9} \times 1.5^2 + \frac{1}{9} \times 1.5^2$

$= 1.47$

$\sigma_y = \sqrt{\sigma_y^2} = \sqrt{1.47} = 1.212$

$P(Y \geq 0) = P\left(\frac{y - \mu_y}{\sigma_y} \geq \frac{0 - \mu_y}{\sigma_y}\right)$

$= P\left(Z \geq \frac{0+4}{1.212}\right)$

$= 1 - P(Z < 3.3)$

$= 1 - \phi(3.3)$

$= 1 - 0.9995$

$= 0.0005$

$P(Y > -2) = P\left(\frac{y - \mu_y}{\sigma_y} \geq \frac{-2 - \mu_y}{\sigma_y}\right)$

$= P\left(Z \geq \frac{-2+4}{1.212}\right)$

$= 1 - P(Z < 1.65)$

$= 1 - \phi(1.65)$

$= 1 - 0.9505$

$= 0.0495$

~~63) a) cov(x1,x2)~~

63) a)

		x_2			
		0	1	2	3
	0	0.08	0.07	0.04	0.00
	1	0.06	0.15	0.05	0.04
x_1	2	0.05	0.04	0.1	0.06
	3	0	0.03	0.04	0.07
	4	0	0.01	0.05	0.06

$p_{x_1}(0) = p(0,0) + p(0,1) + p(0,2) + p(0,3) = 0.08 + 0.07 + 0.04 + 0 = 0.19$

$p_{x_1}(1) = p(1,0) + p(1,1) + p(1,2) + p(1,3) = 0.06 + 0.15 + 0.05 + 0.04 = 0.3$

$p_{x_1}(2) = 0.05 + 0.04 + 0.1 + 0.06 = 0.25$

$p_{x_1}(3) = 0.03 + 0.04 + 0.07 = 0.14$

$p_{x_1}(4) = 0.01 + 0.05 + 0.06 = 0.12$

$p_{x_2}(0) = 0.08 + 0.06 + 0.05 = 0.19$

$p_{x_2}(1) = 0.07 + 0.15 + 0.04 + 0.03 + 0.01 = 0.3$

$p_{x_2}(2) = 0.04 + 0.05 + 0.1 + 0.04 + 0.05 = 0.28$

$p_{x_2}(3) = 0.04 + 0.06 + 0.07 + 0.06 = 0.23$

$$E(x_1) = \sum x_1 p_{x_1} = 0.19\times 0 + 0.3\times 1 + 0.25\times 2 + 0.14\times 3 + 0.12\times 4$$
$$= 1.7$$

$$E(x_2) = \sum x_2 p_{x_2} = 0.19\times 0 + 0.3\times 1 + 0.28\times 2 + 0.23\times 3$$
$$= 1.55$$

~~cov(x1,x2) = Σ(x1 − μx1)(x2 − μx2)~~

$$E(x_1 x_2) = \sum x_1 x_2\, p(x_1,x_2) = 0.15 + 2\times 0.05 + 3\times 0.04$$
$$+ 2\times 0.04 + 2\times 0.1 \times 2 + 2\times 0.06\times 3$$
$$+ 3\times 0.03 + 3\times 2\times 0.04 + 3\times 3\times 0.07$$
$$+ 4\times 0.01 + 4\times 2\times 0.05 + 4\times 3\times 0.06$$
$$= 3.33$$

$$\mathrm{cov}(x_1,x_2) = E(x_1 x_2) - E(x_1)\cdot E(x_2)$$
$$= 3.33 - 1.7\times 1.55$$
$$= 0.695$$

b) $V(x_1+x_2) = V(x_1) + V(x_2) + 2\,\mathrm{cov}(x_1,x_2)$

$E(x_1^2) = \sum x_1^2 p_{x_1}$

$= 0.3 + 4\times0.25 + 9\times0.14 + 16\times0.12$

$= 4.48$

$V(x_1) = E(x_1^2) - \{E(x_1)\}^2$

$= 4.48 - (1.7)^2$

$= 1.59$

$E(x_2^2) = \sum x_2^2 p_{x_2}$

$= 0.3 + 0.28\times4 + 0.23\times9$

$= 3.49$

$V(x_2) = E(x_2^2) - \{E(x_2)\}^2$

$= 3.49 - (1.55)^2$

$= 1.0875$

$\therefore V(x_1+x_2) = 1.59 + 1.0875 + 2\times 0.695$

$= 4.0675$

$V(x_1) + V(x_2) = 1.59 + 1.0875 = 2.6775$

$\therefore V(x_1+x_2)$ is much larger than $V(x_1)+V(x_2)$