

		D. $P(A)P(B)$		
<u>Q.No:2</u>	<u>MCQ</u>	<p>If the probability of success for the random variable X of a Geometric distribution is 0.96, then its standard deviation is</p> <p>A. 0.2083 B. 0.1083 C. 0 D. 1</p>	CO 2	A
	<u>MCQ</u>	<p>If the mean of a Poisson distribution is 25, then its standard deviation is</p> <p>A. 25 B. 5 C. 0 D. 1</p>	CO 2	B
	<u>MCQ</u>	<p>If the mean of a Poisson distribution of rv X is one then $P(X = 1)$ is</p> <p>A. $1 - e$ B. $1 - e$ C. $1 - e^{-1}$ D. e^{-1}</p>	CO 2	D
	<u>MCQ</u>	<p>If the mean and variance of a random variable X are 0.5 and 3 respectively then the mean and variance of the random variable $Y = 3X + 2$ are</p> <p>A. 0 and 1 B. 3.5 and 0 C. 3.5 and 27 D. 0 and 27</p>	CO 2	C
<u>Q.No:3</u>	<u>MCQ</u>	<p>Sample mean and sample variance of the data 2, 7, 3, 4, 1 are</p> <p>A. $\bar{x} = 3.4, s^2 = 6.52$ B. $\bar{x} = 2.5, s^2 = 4.24$ C. $\bar{x} = 3.4, s^2 = 4.24$ D. $\bar{x} = 2.6, s^2 = 3.56$</p>	CO 5	C
	<u>MCQ</u>	<p>A sample is given with variance 9. To obtain 99% confidence interval of length $L = 0.4$, the sample size n is</p> <p>A. 1494 B. 1400 C. 1350 D. 1500</p>	CO 5	A

	<u>MCQ</u>	A sample size $n = 14$ has the sample mean $\bar{x} = 5$ and variance 9. If the length of the confidence interval is $L = 4.1308$. The confidence interval of the mean μ with confidence level γ is A. $\text{CONF}_{\gamma}(4.4124 \leq \mu \leq 5.5884)$ B. $\text{CONF}_{\gamma}(0.8692 \leq \mu \leq 9.1308)$ C. $\text{CONF}_{\gamma}(13.21 \leq \sigma^2 \leq 65.52)$ D. $\text{CONF}_{\gamma}(2.9346 \leq \mu \leq 7.0654)$	CO 5	D
	<u>MCQ</u>	A sample size $n = 14$ has the sample variance $s^2 = 25.3769$. If $c_1 = 5.01$ and $c_2 = 24.74$ are the percentiles of the normal distribution with degree of freedom $n - 1$, then confidence interval of the variance σ^2 with confidence level γ is A. $\text{CONF}_{\gamma}(12.25 \leq \sigma^2 \leq 60.24)$ B. $\text{CONF}_{\gamma}(13.21 \leq \sigma^2 \leq 65.25)$ C. $\text{CONF}_{\gamma}(13.34 \leq \sigma^2 \leq 64.28)$ D. $\text{CONF}_{\gamma}(13.21 \leq \sigma^2 \leq 65.52)$	CO 5	B
<u>Q.No:4</u>	<u>MCQ</u>	The range of correlation coefficient is A. $(-\infty, \infty)$ B. $(0, \infty)$ C. $(-1, 1)$ D. $(0, 1)$	CO 4	C
	<u>MCQ</u>	If X and Y are independent random variables then $\text{Cov}(X, Y) =$ A. 0 B. 1 C. -1 D. ± 1	CO 4	A
	<u>MCQ</u>	If X and Y are independent random variables then $\text{Corr}(X, Y) =$ A. 1 B. -1 C. 0 D. ± 1	CO 4	C
	<u>MCQ</u>	If $Y = aX + b$, $a \neq 0$ then $\text{Corr}(X, Y) =$ A. 1 B. -1 C. 0	CO 4	D

		D. ± 1		
Q.No:5	MCQ	<p>The value of the constant k, such that $f(x)$ defined by</p> $f(x) = \begin{cases} kx(1-x), & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$ <p>is a probability density function, is</p> <p>A. 4 B. 5 C. 6 D. 3</p>	CO 3	C
	MCQ	<p>The value of the constant k, such that $P(x)$ defined by</p> $P(x) = k \binom{3}{x}, (x = 0, 1, 2, 3)$ <p>is a probability mass function, is</p> <p>A. 3/8 B. 1/8 C. 1/3 D. 2/3</p>	CO 2	B
	MCQ	<p>The value of the constant k, such that $P(x)$ defined by</p> $P(x) = kx^2, (x = 0, 1, 2, 3)$ <p>is a probability mass function, is</p> <p>A. 3/8 B. 1/8 C. 1/14 D. 3/14</p>	CO 2	C
	MCQ	<p>The value of the constant k, such that $f(x)$ defined by</p> $f(x) = \begin{cases} kx^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$ <p>is a probability density function, is</p> <p>A. 4 B. 5 C. 6 D. 3</p>	CO 2	D
Q.No:6	MCQ	<p>If $\Phi(z)$ is the standardized normal distribution function then the value of $\Phi(-0.5)$ is</p> <p>A. 0.3085 B. 0.6915 C. 0.5 D. 0.1205</p>	CO 3	A

	MCQ	<p>If the density function for a uniform distribution is nonzero on the interval $[0,12]$, then the mean and variance of the distribution are</p> <p>A. 6,12 B. 12,6 C. 0,12 D. 6,0</p>	CO 3	A
		<p>If $\Phi(z)$ is the standardized normal distribution function then the value of $\Phi(-\infty)$ is</p> <p>A. 0.3085 B. 0.6915 C. 0 D. 0.1205</p>	CO 3	C
		<p>If the density function for a uniform distribution is nonzero on the interval $[0,16]$, then the mean and variance of the distribution are</p> <p>A. 6,12 B. 12,6 C. 8,4/3 D. 4/3,16</p>	CO 3	C
Q.No:7		<p>The current in a certain circuit as measured by an ammeter is a continuous random variable X with the following density function:</p> $f(x) = \begin{cases} 0.075x + 0.2 & 3 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$ <p>$P(X \geq 4)$ is</p> <p>A. 0.23 B. 0.50 C. 0.58 D. 0.78</p>	CO 3	B
		<p>Suppose the reaction temperature X (in $^{\circ}\text{C}$) in a certain chemical process has a uniform distribution with $A = -5$ and $B = 5$. For k satisfying $-5 < k < k + 4 < 5$, $P(k < X < k + 4)$ is</p> <p>A. 0.4 B. 0.5 C. 0.6</p>	CO 3	A

		D. 0.8		
		<p>The error involved in making a certain measurement is a continuous rv X with pdf</p> $f(x) = \begin{cases} 0.09375(4-x^2) & -2 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$ <p>Then $P(-1 < X < 1)$ is</p> <p>A. 0.5875 B. 0.6775 C. 0.6975 D. 0.6875</p>	CO 3	D
		<p>A college professor never finishes his lecture before the end of the hour and always finishes his lectures within 2 min after the hour. Let X = the time that elapses between the end of the hour and the end of the lecture and suppose the pdf of X is</p> $f(x) = \begin{cases} \frac{1}{4}x^3 & 0 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$ <p>The probability that the lecture ends within 80 seconds of the end of the hour is</p> <p>A. 60/83 B. 63/82 C. 64/81 D. 64/83</p>	CO 3	C

SECTION-B(Answer Any Three Questions. Each Question carries 12 Marks)

Time: 1 Hour and 30 Minutes

(3×12=36 Marks)

<u>Question No</u>	<u>Question</u>	<u>CO Mapping (Each</u>
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		<u>question should be from the same CO(s)</u>																																																								
<u>Q.No:8</u>	<p>Consider randomly selecting a single individual and having that person test drive 3 different vehicles. Define events A_1, A_2, and A_3 by A_1=likes vehicle #1, A_2 =likes vehicle #2, A_3 =likes vehicle #3. Suppose that $P(A_1) = 0.65$, $P(A_2) = 0.55$, $P(A_3) = 0.70$, $P(A_1 \cup A_2) = 0.80$, $P(A_2 \cap A_3) = 0.40$, $P(A_1 \cup A_2 \cup A_3) = 0.88$.</p> <p>a. What is the probability that the individual likes both vehicle #1 and vehicle #2? Determine and interpret $P(A_1 / A_2)$.</p> <p>b. What is the probability that the individual likes either vehicle #2 or vehicle #3? Determine and interpret $P(A_2 / A_3)$.</p> <p>c. Are A_1 and A_2 independent events? Answer in two different ways.</p> <p>d. If you learn that the individual did not like vehicle #2, what now is the probability that he/she liked at least one of the other two vehicles?</p> <p>A department store sells sport shirts in three sizes (small, medium, and large), three patterns (plaid, print, and stripe), and two sleeve lengths (long and short). The accompanying tables give the proportions of shirts sold in the various category combinations.</p> <table><tr><th colspan="4">Short-sleeved</th><th></th><th></th><th colspan="4">Long-sleeved</th></tr><tr><th colspan="4">Pattern</th><th></th><th></th><th colspan="4">Pattern</th></tr><tr><th>Size</th><th>Pl</th><th>Pr</th><th>St</th><th></th><th>Size</th><th>Pl</th><th>Pr</th><th>St</th></tr><tr><td>S</td><td>0.04</td><td>0.02</td><td>0.05</td><td></td><td>S</td><td>0.03</td><td>0.02</td><td>0.03</td></tr><tr><td>M</td><td>0.08</td><td>0.07</td><td>0.12</td><td></td><td>M</td><td>0.10</td><td>0.05</td><td>0.07</td></tr><tr><td>L</td><td>0.03</td><td>0.07</td><td>.08</td><td></td><td>L</td><td>0.04</td><td>0.02</td><td>.08</td></tr></table> <p>a. What is the probability that the next shirt sold is a medium, long-sleeved, print shirt?</p> <p>b. What is the probability that the next shirt sold is a short-sleeved shirt? A long-sleeved shirt?</p> <p>c. What is the probability that the size of the next shirt sold is medium? That the pattern of the next shirt sold is a print?</p> <p>d. Given that the shirt just sold was a short-sleeved plaid, what is the probability that its size was medium?</p>	Short-sleeved						Long-sleeved				Pattern						Pattern				Size	Pl	Pr	St		Size	Pl	Pr	St	S	0.04	0.02	0.05		S	0.03	0.02	0.03	M	0.08	0.07	0.12		M	0.10	0.05	0.07	L	0.03	0.07	.08		L	0.04	0.02	.08	CO1
Short-sleeved						Long-sleeved																																																				
Pattern						Pattern																																																				
Size	Pl	Pr	St		Size	Pl	Pr	St																																																		
S	0.04	0.02	0.05		S	0.03	0.02	0.03																																																		
M	0.08	0.07	0.12		M	0.10	0.05	0.07																																																		
L	0.03	0.07	.08		L	0.04	0.02	.08																																																		

	<p>Four customers purchasing a refrigerator at a certain appliance store, let A be the event that the refrigerator was manufactured in the U.S., B be the event that the refrigerator had an icemaker and C be the event that the customer purchased an extended warranty. Relevant probabilities are</p> <p>$P(A) = 0.75, P(B A) = 0.9, P(B A') = 0.8, P(C A \cap B) = 0.8,$ $P(C A \cap B') = 0.6, P(C A' \cap B) = 0.7, P(C A' \cap B') = 0.3$</p> <ol style="list-style-type: none"> Construct a tree diagram consisting of first, second, and third generation branches and place and event labeled and appropriate probabilities next to each branch. Compute $P(A \cap B \cap C)$. Compute $P(B \cap C)$. Compute $P(A B \cap C)$, the probability of a US purchase given that an individual icemaker and extended warranty are also purchased. 	
Q.No:9	<p>(i) Define Poisson distribution and show that mean and variance of Poisson distribution are same.</p> <p>(ii) Let X, the number of flaws on the surface of a randomly selected boiler of a certain type, have a Poisson distribution with parameter $\mu = 5$. Compute the following probabilities:</p> <ol style="list-style-type: none"> $P(X \leq 8)$ $P(9 \leq X)$ $P(5 \leq X \leq 8)$ $P(5 < X < 8)$ <p>(i) Define the pmf $b(x; n, p)$ of the Binomial distribution with the random variable X where n is the number of trials and p is the probability of getting success. Show that the mean of the Binomial distribution is np and variance is $np(1 - p)$.</p> <p>(ii) When circuit boards used in the manufacturer of compact disk players are tested, the long run percentage of defectives is 5%. Let X = the number of defective boards in a random sample of size $n = 25$, so $X \sim \text{Bin}(25, 0.05)$.</p> <ol style="list-style-type: none"> Determine $P(X \leq 2)$. Determine $P(X \geq 5)$. Determine $P(1 \leq X \leq 4)$. Calculate the expected value and standard deviation of X. <p>Define the pmf $nb(x; r, p)$ of Negative Binomial distribution where the random variable X is the number of trial out of which r number of trials are success with probability p. Write the formula for mean and variance of Negative Binomial distribution. Suppose that $p = p(\text{male birth}) = 0.5$. A couple wishes to have exactly two female children in their family. They will have children until</p>	CO2

	<p>this condition is fulfilled.</p> <ol style="list-style-type: none"> What is the probability that the family has x male children? What is the probability that the family has four children? What is the probability that the family has at most four children? How many male children would you expect this family to have? How many children would you expect this family to have? 	
<u>Q.No:10</u>	<p>The actual tracking weight of a stereo cartridge that is set to track at 3 g on a particular changer can be regarded as continuous rv X with pdf</p> $f(x; \theta) = \begin{cases} k[1 - (x - 3)^2] & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$ <ol style="list-style-type: none"> Find the value of k. Find the cumulative distribution function $F(x)$. What is the probability that the actual tracking weight is within 0.25 g of the prescribed weight? Find the median of X. <p>Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with $\mu = 8.8$ and $\sigma = 2.8$.</p> <ol style="list-style-type: none"> What is the probability that the diameter of a randomly selected tree will be at least 10 in.? What is the probability the diameter of a randomly selected tree is within one standard deviation of its mean value? What is the probability that the diameter of a randomly selected tree will be between 5 and 10 in.? What value c is such that the interval includes 98% of all diameter values? <p>Two components of a mini computer have the following joint pdf for their useful lifetimes X and Y</p> $f(x, y) = \begin{cases} xe^{-x(1+y)} & x \geq 0 \text{ and } y \geq 0 \\ 0 & \text{otherwise.} \end{cases}$ <ol style="list-style-type: none"> What is the probability that the lifetime X of the first component exceeds 3? What are the marginal pdf's of X and Y? 	CO3

	<p>c. Are the two lifetimes X and Y independent? Explain.</p> <p>d. What is the probability that the lifetime of at least one component exceeds 3?</p>	
Q.No:11	<p>a. Define covariance of two random variables X and Y and show that $\text{Cov}(aX + b, cY + d) = ac \text{Cov}(X, Y)$.</p> <p>b. Find the maximum likelihood estimate of θ if the density $f(x) = \begin{cases} \theta e^{-\theta x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$</p> <p>c. Find a 95% confidence interval for μ of a normal population with standard deviation 4 from the sample 30, 42, 40, 34, 48, 50.</p>	CO4,CO5, CO6
	<p>a. Define correlation coefficient of two random variables X and Y and prove that $\text{Corr}(aX + b, cY + d) = \text{Corr}(X, Y)$ when a and c have the same sign. What happens if a and c have opposite sign?</p> <p>b. Find the maximum likelihood estimate of mean and variance of Poisson distribution.</p> <p>c. Find a 99% confidence interval for μ of a normal population with standard deviation from the sample 66, 66, 65, 64, 66, 67, 64, 65, 63, 64.</p>	
	<p>a. Define covariance of two random variables X and Y and show that $\text{Cov}(X, Y) = E(X, Y) - E(X)E(Y)$.</p> <p>b. Find the maximum likelihood estimate of mean and variance of Normal distribution.</p> <p>c. Find a 90% confidence interval for μ of a normal population with standard variance 0.25, using the sample of 100 with the mean 212.3.</p>	