Ex: 25.3

1) Given,

$\gamma = 95\%$, $\sigma = 4$

Given sample is 30, 42, 40, 34, 48, 50

we'll calculate z(D)

For $\gamma = 95\%$, $c = 1.960$

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$= \frac{1}{6} \times (30+42+40+34+48+50)$$

$$= 40.67$$

$$k = \frac{c\sigma}{\sqrt{n}} = \frac{1.96 \times 4}{\sqrt{6}} = 3.2$$

So, the confidence interval for 'μ' is

$$\text{CONF}_\gamma \{ 40.67 - 3.2 \le \mu \le 40.67 + 3.2 \}$$

i.e; $\text{CONF}_\gamma \{ 37.47 \le \mu \le 43.87 \}$

Ex: 25.3

4) $\gamma = 90\%$, $\sigma^2 = 0.25$, $\sigma = \sqrt{0.25} = 0.5$

$\phi(c) = 0.9$

$\Rightarrow c = 1.645$

$n = 100$

$\bar{x} = 212.3$

$k = \frac{c\sigma}{\sqrt{n}} = \frac{1.645 \times 0.5}{\sqrt{100}} = 0.082$

$\therefore \text{CONF}_\gamma (\bar{x} - k \le \mu \le \bar{x} + k)$

$= \text{CONF}_\gamma (212.3 - 0.082 \le \mu \le 212.3 + 0.082)$

$= \text{CONF}_\gamma (212.218 \le \mu \le 212.382)$

8) $\gamma = 99\%$

$n = 4$

Sample: 425, 420, 425, 435

$\bar{x} = \frac{1}{4}(425 + 420 + 425 + 435) = 426.25$

$F(c) = \frac{1}{2}(1+\gamma) = \frac{1}{2}(1 + 0.99) = 0.995$, $\therefore n-1 = 3$ degrees of freedom.

$c = 5.84$

$S^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$

$= \frac{1}{3}\{(425 - 426.25)^2 + (420 - 426.25)^2 + (425 - 426.25)^2 + (435 - 426.25)^2\}$

$= \frac{1}{3}(118.75)$

$\Rightarrow S^2 = 39.58$

$\Rightarrow S = \sqrt{39.58} = 6.29$

$k = \frac{cS}{\sqrt{n}} = \frac{5.84 \times 6.29}{\sqrt{4}} = 18.367$

$\text{CONF}_\gamma (\bar{x} - k \le \mu \le \bar{x} + k) = \text{CONF}_\gamma (426.25 - 18.367 \le \mu \le 426.25 + 18.367)$

$= \text{CONF}_\gamma (407.883 \le \mu \le 444.617)$

11) $\gamma = 99\%$

$F(c) = \frac{1}{2}(1+\gamma) = \frac{1}{2}(1+0.99) = 0.995$

Sample: 66, 66, 65, 64, 66, 67, 64, 65, 63, 64

$n = 10$

$\bar{x} = \frac{1}{10}(66+66+65+64+66+67+64+65+63+64)$

$= 65$

$s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$

$= \frac{1}{9}\{(66-65)^2 + (66-65)^2 + (65-65)^2 + (64-65)^2 + (66-65)^2 + (67-65)^2$
$+ (64-65)^2 + (65-65)^2 + (63-65)^2 + (64-65)^2\}$

$= \frac{1}{9}\{1+1+1+1+4+1+4+1\}$

$= \frac{14}{9} = 1.56$

$s = \sqrt{1.56} = 1.25$

$F(c) = 0.995$, $n-1 = 9$ degrees of freedom

$\Rightarrow c = 3.25$

$k = \frac{cs}{\sqrt{n}} = \frac{3.25 \times 1.25}{\sqrt{10}} = 1.28$

$\text{CONF}_\gamma\{\bar{x} - k \le \mu \le \bar{x} + k\}$

$= \text{CONF}_\gamma\{65 - 1.\cancel{35}\,28 \le \mu \le 65 + 1.\cancel{35}\,28\}$

~~$= \text{CONF}_\gamma\{63.65 \le \mu \le 66.35\}$~~ $= \text{CONF}_\gamma\{63.72 \le \mu \le 66.28\}$

~~18) Find~~

18) $\gamma = 95\%$, Sample: 17.3, 17.8, 18.0, 17.7, 18.2, 17.4, 17.6, 18.1

$F(c_1) = \frac{1}{2}(1-0.95)$ | $n=8$ $\qquad$ $F(c_2) = \frac{1}{2}(1+0.95)$

$F(c_1) = 0.025 = 2.5\%$ $\qquad$ $F(c_2) = 0.975 = 97.5\%$ } Degree of freedom $= n-1 = 7$

$\Rightarrow c_1 = 1.69$ $\qquad$ $c_2 = 16.01$

$$\bar{x} = \frac{1}{8} \times (17.3+17.8+18.0+17.7+18.2+17.4+17.6+18.1)$$

$$= 17.76$$

$$s^2 = \frac{1}{7} \times \Big((17.3-17.76)^2 + (17.8-17.76)^2 + (18-17.76)^2 + (17.7-17.76)^2 + (18.2-17.76)^2 + (17.4-17.76)^2 + (17.6-17.76)^2 + (18.1-17.76)^2\Big)$$

$$= 0.1055$$

$$k_1 = \frac{(n-1)s^2}{c_1} = \frac{7 \times 0.1055}{1.69} = 0.437$$

$$k_2 = \frac{(n-1)s^2}{c_2} = \frac{7 \times 0.1055}{16.01} = 0.046$$

If not given, take either $\gamma = 99$ or $\gamma = 99.5$

Confidence interval for σ^2 is

$\therefore \mathrm{CONF}_{0.95}\{0.046 \le \sigma^2 \le 0.437\}$.

20) $\gamma = 95\%$

Sample: 251, 255, 258, 253, 253, 252, 250, 252, 255, 256

$F(c_1) = \frac{1}{2}(1-0.95) = 0.025$

$F(c_2) = \frac{1}{2}(1+0.95) = 0.975$

$n = 10$

$n-1 = 9$ degrees of freedom.

$\therefore c_1 = 2.7, \; c_2 = 19.02$

$\bar{x} = \frac{1}{10}(251+255+258+253+253+252+250+252+255+256)$

$= \frac{1}{10}(2535) = 253.5$

$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$

$= \frac{1}{9}\{(251-253.5)^2 + (255-253.5)^2 + (258-253.5)^2 + (253-253.5)^2 + (253-253.5)^2 + (252-253.5)^2 + (250-253.5)^2 + (252-253.5)^2$

$$+(255-253.5)^2+(256-253.5)^2\}$$

$$= 6.05$$

$$k_1 = \frac{(n-1)s^2}{c_1} = \frac{9 \times 6.05}{2.7} = 20.17$$

$$k_2 = \frac{(n-1)s^2}{c_2} = \frac{9 \times 6.05}{19.02} = 2.86$$

$$\mathrm{CONF}_\gamma\{k_2 \leq \sigma^2 \leq k_1\}$$

$$\Rightarrow \mathrm{CONF}_\gamma\{2.86 \leq \sigma^2 \leq 20.17\}$$