Q. A binary communication channel series data as one of two types of signals denoted by '0' and '1'. Owing to noise, a transmitted '0' is sometimes received as '1' and a transmitted '1' is sometimes received as '0'. For a given channel, assume a prob. of 0.94 that a transmitted '0' is correctly received as a '0' and prob. of 0.91 that a transmitted '1' is transmitted as '1'. Further assume a prob. of 0.45 of transmitting a '0'. If a signal is sent determine

(i) Prob. that a '1' is received

(ii) prob. that a '0' is received

(iii) prob. that a '1' was transmitted, given that a '1' was received

(iv) prob. that a '0' was transmitted, given that a '0' was received

(v) prob. of an error.

Sol^n Define events, for $i = 0, 1$

T_i = i is transmitted

R_i = i is received

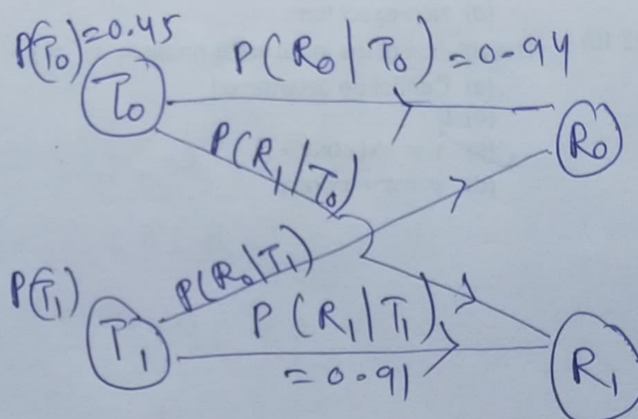
where $T_1 = 1$ was transmitted $= \bar{T_0}$ and
~~Given that~~ $R_1 = 1$ is received $= \bar{R_0}$

Given that 0 was transmitted, prob. that 0 is received $= P(R_0 | T_0) = 0.94$

Given that 1 was transmitted, prob. that 1 is received $= P(R_1 | T_1) = 0.91$

Prob. that transmitted 0 $= P(T_0) = 0.45$

The channel diagram is



Since $P(A|B) + P(A'|B) = 1$, we have
$$P(A'|B) = 1 - P(A|B).$$

$$P(R_1 | T_0) = P(\bar{R_0} | T_0) = 1 - P(R_0 | T_0)$$
$$= 1 - 0.94 = 0.06$$

$$P(R_0 | T_1) = P(\bar{R_1} | T_1) = 1 - P(R_1 | T_1)$$
$$= 1 - 0.91 = 0.09$$

$$P(T_1) = P(\bar{T_0}) = 1 - P(T_0) = 1 - 0.45 = 0.55$$

Again by total prob. Theorem, for any two events A & B,
$$P(B) = P(B|A)\,P(A) + P(B|A')\,P(A')$$
for $B = R_0$, $A = T_0$, $A' = T_1$
$$P(R_0) = P(R_0 | T_0)\,P(T_0) + P(R_0 | T_1)\,P(T_1)$$

$= 0.94 \times 0.45 + 0.09 \times 0.55$

$= 0.4725$

Thus $P(R_1) = P(\bar{R}_0) = 1 - P(R_0) = 1 - 0.4725$

$= 0.5275$

By multiplication rule

$$P(B|A)P(A) = P(A|B)P(B),$$

so $$P(T_1|R_1)P(R_1) = P(R_1|T_1)P(T_1)$$

$$\Rightarrow P(T_1|R_1) = \frac{P(R_1|T_1)P(T_1)}{P(R_1)}$$

$$= \frac{0.91 \times 0.55}{0.5275} = 0.9488$$

Again $$P(T_0|R_0)P(R_0) = P(R_0|T_0)P(T_0)$$

$$\Rightarrow P(T_0|R_0) = \frac{P(R_0|T_0)P(T_0)}{P(R_0)}$$

$$= \frac{0.94 \times 0.45}{0.4725} = 0.8952$$

(v) Prob. of an error

$= P(T_1 \cap R_0) + P(T_0 \cap R_1)$

$= P(T_1|R_0)P(R_0) + P(T_0|R_1)P(R_1)$

$= P(\bar{T}_0|R_0)P(R_0) + P(\bar{T}_1|R_1)P(R_1)$

$= [1 - P(T_0|R_0)]P(R_0) + [1 - P(T_1|R_1)]P(R_1)$

$= [1 - 0.8952](0.4725) + [1 - 0.9488](0.5275)$

$= 0.0765$