Exc: 25.4 ~~To aver~~

1) Given sample: 1, −1, 1, 3, −8, 6, 0 ; $\alpha = 5\%$

$H_0: \mu = 0 \ (= \mu_0)$ (say)

$H_1: \mu > 0 \ (= \mu_1)$

$P(T > c) = \alpha = 0.05$, 6 degrees of freedom

$= 1 - P(T \le c) = 0.05$

$F(c) = 0.95$

$\Rightarrow c = 1.94 = t_{\alpha, n-1}$

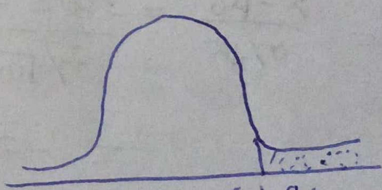
$0.95 \rightarrow$ 6 degrees of freedom 1.94

$0.05 = -1.94$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{7}(1-1+1+3-8+6+0) = \frac{2}{7}$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 = \frac{1}{6}\left(\left(1-\frac{2}{7}\right)^2 + \left(-1-\frac{2}{7}\right)^2 + \left(1-\frac{2}{7}\right)^2 + \left(3-\frac{2}{7}\right)^2 + \left(-8-\frac{2}{7}\right)^2 + \left(6-\frac{2}{7}\right)^2 + \left(0-\frac{2}{7}\right)^2\right)$$

$$= 18.57$$

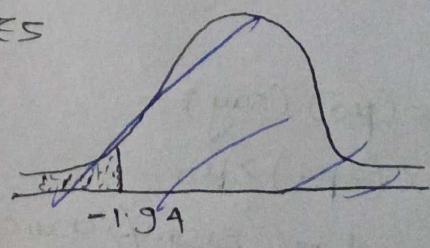


1.94

Right one tailed tree

$\Rightarrow S = 4.31$

$$\therefore T = \frac{\bar{x} - \mu_0}{(S/\sqrt{n})} = \frac{\frac{2}{7} - 0}{\frac{4.31}{\sqrt{7}}} = 0.175$$



−1.94

As $T < c$, accept the null hypothesis.

2) $n = 4040$, ~~$\alpha = 5\%$~~ $\alpha = 5\%$

we'll refer binomial distribution

$p = \frac{1}{2}$, $n = 4040$

$\mu = np = \frac{4040}{2} = 2020$, $\sigma = \sqrt{npq} = \sqrt{2020 \times \frac{1}{2}} = 31.78$

$H_0: \mu = 2020 \ (= \mu_0)$ right tailed test

$H_1: \mu = 2048 \ (= \mu_1) > \mu_0$

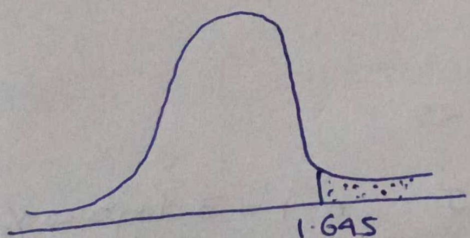
$P(Z > c) = \alpha = 0.05$

$\Rightarrow 1 - P(Z \leq c) = 0.05$

$\Rightarrow P(Z \leq c) = 0.95$

$\Rightarrow \phi(c) = 0.95$

$\Rightarrow c = 1.645$



$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

When $n > 80$, sample variance = population variation $\sigma_{\bar{x}}^2 = \sigma^2$

when $n < 80$

$$Z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_0}{\sigma}$$

$$= \frac{2048 - 2020}{31.78}$$

$$= 0.881$$

As $Z < c$, accept the null hypothesis.

Q) 4) Given, $\sigma^2 = 4$, $H_0 : \mu = 30$, $n = 10$, $\bar{X} = 28.5$, $\alpha = 5\%$

a) $H_1 : \mu = 28.5$ b) $\mu = 30.7$

a) $H_0 : \mu = 30 = (\mu_0)$ (Say)

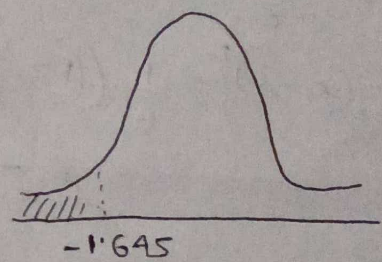
$H_1 : \mu = 28.5 (= \mu_1) < \mu_0$

For 5% Level of significance

$P(Z < c) = 0.05 = \alpha$

$\Rightarrow \phi(c) = 0.05$

$\Rightarrow c = -1.645 = -Z_\alpha$



Again, $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{28.5 - 30}{2/\sqrt{10}}$

$= -2.372$

As, $Z < c$, reject the null hypothesis.

b) $H_0 : \mu = 30 = (\mu_0)$ (say)

$H_1 : \mu = 30.7 (= \mu_1) > \mu_0$

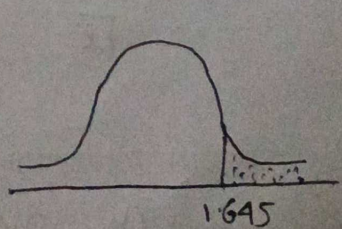
For 5% Level of significance

$P(Z > c) = 0.05 = \alpha$

$\Rightarrow P(Z \leq c) = 0.95$

$\Rightarrow \phi(c) = 0.95$

$\Rightarrow c = 1.645 = Z_\alpha$



As $z < c$, accept the null hypothesis.

Ex: 25.4

8) Sample: 0.8, 0.81, 0.81, 0.82, 0.81, 0.82, 0.8, 0.82, 0.81, 0.81

$n = 10$, $\alpha = 0.05$,

$$\bar{x} = \frac{1}{10}(0.8+0.81+0.81+0.82+0.81+0.82+0.8+0.82+0.81+0.81)$$

$$= 0.811$$

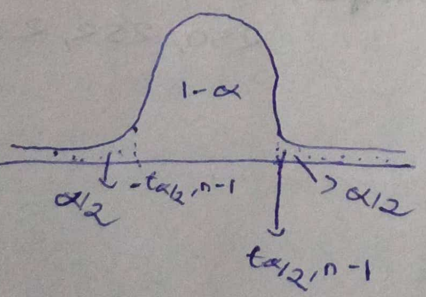
$$S^2 = \frac{1}{n-1}\sum(x_i-\bar{x})^2$$

$$= \frac{1}{9}\times\{(0.8-0.811)^2+(0.81-0.811)^2+(0.81-0.811)^2+(0.82-0.811)^2$$
$$+(0.81-0.811)^2+(0.82-0.811)^2+(0.8-0.811)^2$$
$$+(0.82-0.811)^2+(0.81-0.811)^2+(0.81-0.811)^2\}$$

$$S = 0.007$$

$H_0: \mu = 0.8 (=\mu_0)$

$H_1: \mu_1 \neq \mu_0$



$$P(-c<T<c) = 1-\alpha$$

$$\Rightarrow F(c) - F(-c) = 1-\alpha$$

$$\Rightarrow 2F(c) - 1 = 0.95$$

$\Rightarrow F(c) = 0.975$, 9 degrees of freedom

$$c = 2.26$$

$$T = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{0.811 - 0.8}{\frac{0.007}{\sqrt{10}}} = 4.97$$

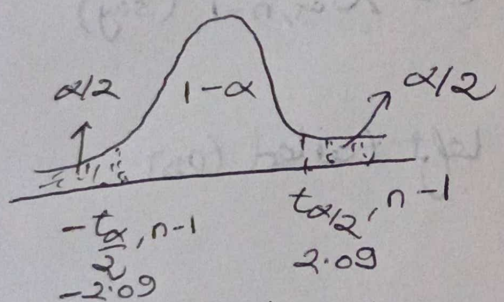
As $T \notin (-2.26, 2.26)$, reject the null hypothesis.

9) $\alpha = 0.05$, $n = 20$, $\bar{X} = 996$, $S = 5g$

$S \downarrow$ Sample standard deviation.

$H_0 : \mu = 1000 \, (= \mu_0)$

$H_1 : \mu \neq \mu_0$ → Differs significantly.



→ Doro jagah nhi nikalo R ek hi ka nikal

$P(-c < T < c) = 1 - \alpha$

$\Rightarrow F(c) - F(-c) = 0.95$

$\Rightarrow 2(F(c)) - 1 = 0.95$

$\Rightarrow F(c) = 0.975$ with 19 degrees of freedom.

$\Rightarrow c = 2.09$

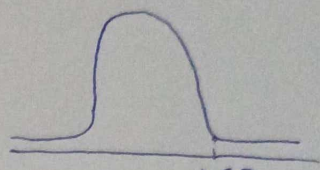
$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{996 - 1000}{5/\sqrt{20}} = -3.58$$

As $T \notin (-2.09, 2.09)$, reject the null hypothesis.

10) $n = 50,\ \alpha = 0.05,\ \bar{x} = 32000,\ S = 4000$

$H0: \mu = 30000 (= \mu_0)$

$H1: \mu > \mu_0$



$P(T > c) = \alpha$

$\Rightarrow 1 - P(T \leq c) = \alpha$

$\Rightarrow P(T \leq c) = 1 - \alpha$

$\Rightarrow F(c) = 0.95$, 49 degrees of freedom

$\Rightarrow c = 1.68$

$$T = \frac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{32000 - 30000}{\frac{4000}{\sqrt{50}}} = 3.53$$

As $T > c$, reject the null hypothesis.

12) $\alpha = 0.05,\ n = 200$

$\mu = np$, variance $= \sigma^2$

$p = 0.7$

$\bar{x} = np$

$= 200 \times 0.7$

$= 140$

$\sigma^2 = npq$

$= 200 \times 0.7 \times 0.3$

$= 140 \times 0.3$

$= 42$

Here, $n > 80$

$\therefore \sigma_{\bar{x}}^2 = \sigma^2$

$\boxed{\begin{array}{l} H0: \mu = 140 (= \mu_0) \\ H1: \mu > \mu_0 \\ P(X > c) = \alpha \\ \Rightarrow 1 - P(X \leq c) = 1 - \alpha \end{array}}$

$1 - P(X \leq c) = 140$

$\Rightarrow 1 - \alpha = \phi\left(\frac{c - \bar{x}}{\sigma}\right)$

$\Rightarrow 1 - 0.05 = \phi\left(\frac{c - 140}{\sqrt{42}}\right)$

$\Rightarrow 0.95 = \phi\left(\frac{c - 140}{\sqrt{42}}\right)$

$\Rightarrow \frac{c - 140}{\sqrt{42}} = 1.645$

$\Rightarrow c = 150.66$

As $148 < c = 150.66$, we do not reject the hypothesis.

14) $n = 28,\ s = 3.5,\ \alpha = 0.05$

$\sigma^2 = \sigma_0^2 \rightarrow$ Null hypothesis or $\sigma = \sigma_0$ → Isme kabhi v $<, >, \neq$ nhi lagana hai, only =

Test for this $\sigma < \sigma_0$, here $\sigma_0 = 5$

$H_0 : \sigma = 5 \,(= \sigma_0)$ (say)

$H1 : \sigma < 5 \,(= \sigma_0)$

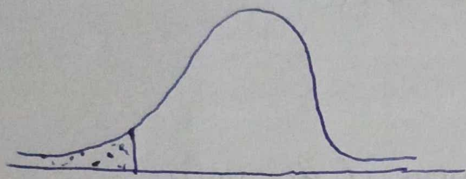
$c = \chi^2_{\alpha, n-1}$ (say)

$P(Y < c) = 0.05$

$\Rightarrow F(c) = 0.05$, 27 degrees of freedom.

Left tailed test

$\Rightarrow c = 16.2$



16.2

Now, $Y = \dfrac{(n-1)S^2}{\sigma_0^2} = \dfrac{27 \times 3.5^2}{5^2} = 13.23$

As $Y < c$, reject the null hypothesis.

13) $n = 10,\ S = 0.5,\ \alpha = 0.05$

$H0 : \sigma = 0.4 = (\sigma_0)$

$H1 : \sigma > 0.4 \Rightarrow \sigma > \sigma_0$ → Right tailed test

$c = \chi^2_{1-\alpha, n-1}$

~~$Y(c) > 0.0$~~

$P(Y > c) = 0.05$

~~$\Rightarrow F(c) = 0.05$, 9 degrees of~~

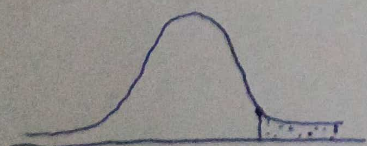
$\Rightarrow 1 - P(Y \leq c) = \alpha$

$\Rightarrow P(Y \leq c) = 1 - \alpha$

$\Rightarrow P(Y \leq c) = 0.95$

$\Rightarrow F(c) = 0.95$, 9 degrees of freedom

$\Rightarrow c = 16.92$



16.92

Now, $Y = \dfrac{(n-1)S^2}{\sigma_0^2} = \dfrac{9 \times 0.5^2}{0.4^2} = 14.06$

As $Y < C$, accept the null hypothesis.