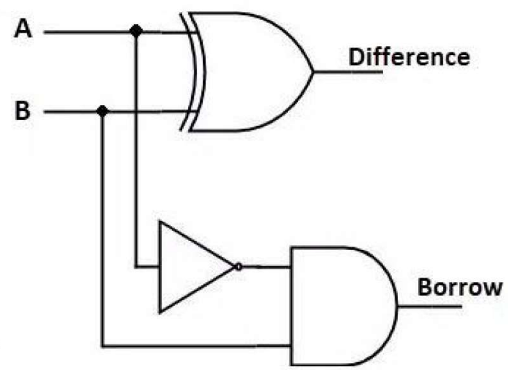
Subtractor- Half Subtractor

A	B	Difference (D)	Borrow (B_{out})
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0



K-Map Simplification :-

for Difference :-

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	$_0$	1 $_1$
A 1	1 $_2$	$_3$

$\therefore$ Diff. $= \bar{A}B + A\bar{B}$

for Borrow :-

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	$_0$	1 $_1$
A 1	$_2$	$_3$

$\therefore$ Borrow $= \bar{A}B$

Subtractor- Full Subtractor

A	B	B_{in}	Difference (D)	Borrow (B_{out})
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Full Subtractor

For Difference :-

A \ BB_{in}	$\bar{B}\bar{B}_{in}$ 00	$\bar{B}B_{in}$ 01	BB_{in} 11	$B\bar{B}_{in}$ 10
$\bar{A}$ 0		1		1
A 1	1		1	

$$\therefore \boxed{\text{Difference} = A \oplus B \oplus B_{in}}$$

$$\therefore \text{Difference} = \bar{A}\cdot\bar{B}B_{in} + \bar{A}B\bar{B}_{in} + A\bar{B}\bar{B}_{in} + ABB_{in}$$

$$= \bar{A}(\bar{B}B_{in} + B\bar{B}_{in}) + A(\bar{B}\cdot\bar{B}_{in} + BB_{in})$$

$$= \bar{A}(B \oplus B_{in}) + A(B \odot B_{in}) = \bar{A}(B \oplus B_{in}) + A(\overline{B \oplus B_{in}})$$

$$= A \oplus B \oplus B_{in} = A \oplus B \oplus B_{in}.$$

For B_{out} :-

A \ BB_{in}	$\bar{B}\bar{B}_{in}$ 00	$\bar{B}B_{in}$ 01	BB_{in} 11	$B\bar{B}_{in}$ 10
$\bar{A}$ 0		1	1	1
A 1			1	

$$\therefore \boxed{B_{out} = \bar{A}B + \bar{A}B_{in} + BB_{in}}$$

$$\therefore B_{out} = \bar{A}B + \bar{A}B_{in} + BB_{in}$$

Designing of Full Subtractor using Half-Subtractors

$$
\begin{aligned}
\text{Difference} &= A \oplus B \oplus B_{in} \\
\text{Borrow} &= (\overline{(A \oplus B)}).\ B_{in} + \bar{A}.\ B \\
&= (\bar{A}.\ \bar{B} + A.B).\ B_{in} + \bar{A}.\ B \\
&= \bar{A}.\ \bar{B}.\ B_{in} + A.B.\ B_{in} + \bar{A}.\ B \\
&= \bar{A}.\ \bar{B}.\ B_{in} + B\ (A.\ B_{in} + \bar{A}\ ) && [\text{Since, } A + BC = (A+B)\ (A+C)] \\
&= \bar{A}.\ \bar{B}.\ B_{in} + B.\ [(A + \bar{A})\ (B_{in} + \bar{A})] && [\text{Since, } A + \bar{A} = 1] \\
&= \bar{A}.\ \bar{B}.\ B_{in} + B.\ B_{in} + B.\ \bar{A} \\
&= B_{in}\ (\ \bar{A}.\ \bar{B} + B) + B.\ \bar{A} && [\text{Since, } A + BC = (A+B)\ (A+C)] \\
&= B_{in}\ [(\bar{A} + B)\ (B + \ \bar{B})] + B.\ \bar{A} && [\text{Since, } B + \bar{B} = 1] \\
&= \bar{A}.\ B_{in} + B.\ B_{in} + B.\ \bar{A}
\end{aligned}
$$