Consider an experiment in which each of the three vehicles taking a particular free way exit turns left (L) or right (R) at the end of the exit ramp.

2^3

$S = \{LLL, RLL, LRL, LLR, RRR, RRL, RLR, LRR\}$

sample space

~~LLL~~ RRR $\quad T \to R$

~~LL~~ RRL $\quad F \to L$

⋮

$E_1 = \{LLL\}$ Let, E_1 be the event where all the vehicles will turn left.

$E_2 = \{RLL\}$,

$E_3 = \{LRL\}$,

...

$E_8 = \{LRR\}$

A: exactly one of the 3 vehicles turns right.

$A = \{RLL, LRL, LLR\}$

B: atmost one of the 3 vehicles turns right

$= \{RLL, LRL, LLR, LLL\}$

C: all three vehicles turn in the same direction.

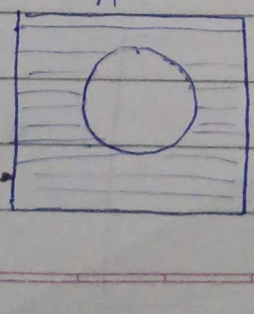
$= \{RRR, LLL\}$

Here, E_1 to E_8 are the simple events.

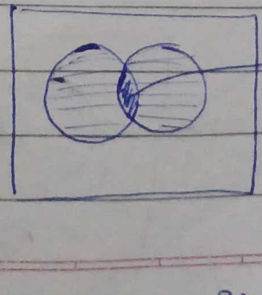
A, B and C are the compound events.

$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$ } De Morgan's law.

A' S



$A \cup B$



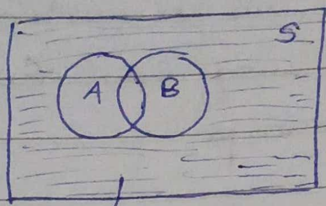
$\to A \cap B$

$P(B|A)$ 0.33 ✓

$P(B|A) = 0.55$

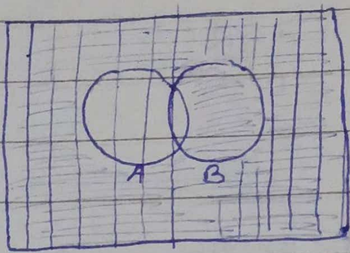
$P(B|A)$

9) $(A \cup B)'$



$(A \cup B)'$

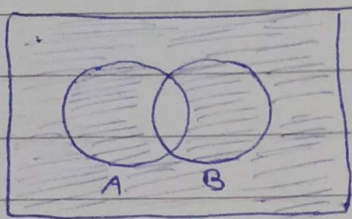
$A' \cap B'$



$A' \cap B'$

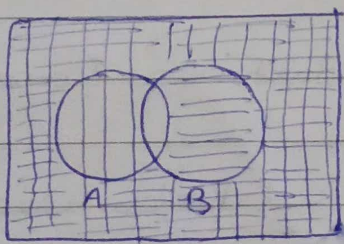
$\therefore (A \cup B)' = A' \cap B'$

$(A \cap B)'$

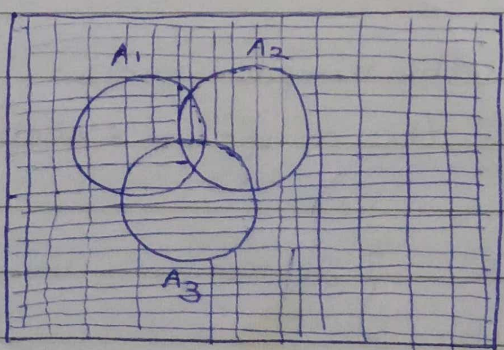


$\therefore (A \cap B)' = A' \cup B'$

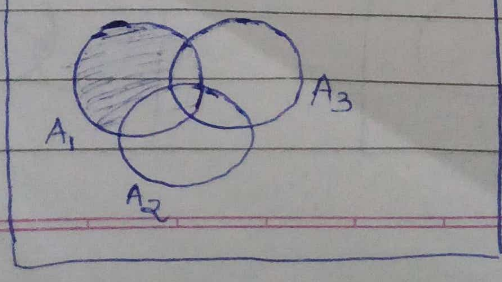
$A' \cup B'$



$A_1 \cap A_2' \cap A_3'$ $\rightarrow A_1 \cap (A_2 \cup A_3)'$

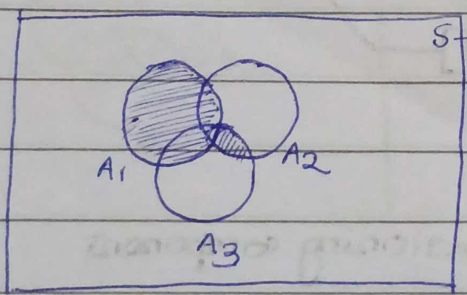


$\Downarrow$



$\rightarrow A_1 \cap A_2' \cap A_3'$

$A_1 \cup (A_2 \cap A_3)$

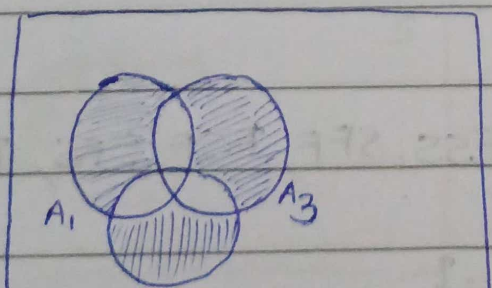


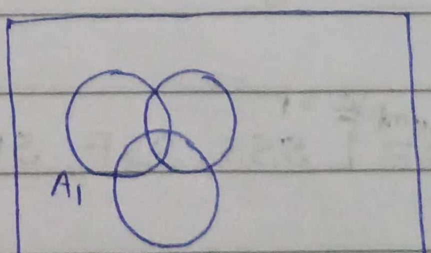
S → sample space

$\Rightarrow \cancel{(A \cup B)}'$

A

$(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)$





Probability

Definition

Let, 'E' be an event in a sample space 'S'

The probability of an event 'E' is denoted as $P(E)$ and is given by

$$P(E) = \frac{|E|}{|S|} = \frac{\text{number of favourable cases}}{\text{Total no. of possible outcomes}}$$

Axioms of probability

1. For any event 'A', $0 \le P(A) \le 1$
2. $P(\phi) = 0$, $P(S) = 1$
3. If A & B are any two events which are mutually exclusive in a sample space S, then:

$$P(A \cup B) = P(A) + P(B)$$

If $A_1, A_2, \ldots\ldots, A_n$ are mutually exclusive events in a sample space S

$$P\left(\bigcup_{i=1}^{n} A_i\right) = P(A_1) + P(A_2) + \ldots\ldots + P(A_n)$$

$$= \sum_{i=1}^{n} P(A_i)$$

It can also be generalised to infinite number of events.

$$\phi \subseteq A \subseteq S$$

Empty set is always a subset of every set

Any set is always a subset of sample space

$\phi \subseteq A \subseteq S$

$A \subseteq B \rightarrow |A| \leq |B|$

$\frac{|A|}{|S|} \leq \frac{|B|}{|S|}$

$\Rightarrow P(A) \leq P(B)$

$\phi \subseteq A \subseteq S$

$\Rightarrow P(\phi) \leq P(A) \leq P(S)$

$\Rightarrow 0 \leq P(A) \leq 1$

Q) Prove that $P(\phi) = 0$

$P(\phi) =$

Pf: Let, S & ϕ be any two sets.

Then $S \cup \phi = S$

$\Rightarrow P(S \cup \phi) = P(S)$

$\Rightarrow P(S) + P(\phi) = P(S)$ $\because$ S and ϕ are mutually exclusive.

$\Rightarrow P(\phi) = 0$

Q) Prove that, $P(A^c) = 1 - P(A)$ (Also called complimentation proof.)

A and A^c are mutually exclusive

Suppose A is any event in the sample space S.

$A^c = S - A$

we know, $A \cup A^c = S$

$P(A \cup A^c) = P(S)$

$\Rightarrow P(A) + P(A^c) = 1$

$\Rightarrow P(A^c) = 1 - P(A)$

Q) Prove that,

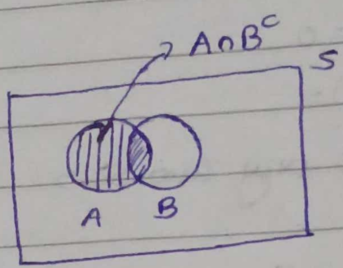
$$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$P(A \cap B^c) = P(A) - P(A) - P(B) + P(A \cup B)$$

$$\Rightarrow P(A \cap B^c) = P(A \cup B) - P(B)$$

$$\Rightarrow P(A) + P(B^c) - P(A \cup B^c)$$



P.F. we know, $A = (A \cap B) \cup (A \cap B^c)$

$(A \cap B)$ and $(A \cap B^c)$ are mutually exclusive (disjoint sets)

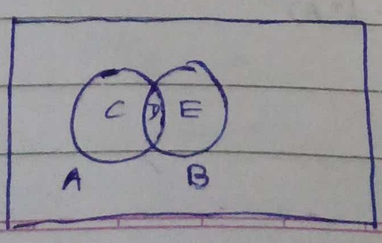
~~where, $(A \cap B)$ and $(A \cap B^c)$~~

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

$$\Rightarrow P(A \cap B^c) = P(A) - P(A \cap B)$$

Q) Prove that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ → Addition theorem for any two events A and B.

Because for mutually exclusive, $P(A \cap B)$ will be 0.



P.F. From the figure, $A = C \cup D$

$B = D \cup E$

C and D, D and E are mutually exclusive.

$A \cap B = D$

Now, $A \cup B = C \cup D \cup E$

$P(A \cup B) = P(C \cup D \cup E)$

$\Rightarrow P(A \cup B) = P(C) + P(D) + P(E)$ —(i)

Since, $A = C \cup D \Rightarrow P(A) = P(C) + P(D)$

$\Rightarrow P(C) = P(A) - P(D)$ —(ii)

$B = D \cup E \Rightarrow P(B) = P(D) + P(E)$

~~$\Rightarrow P(D) = P(B) - P(E)$~~

$\Rightarrow P(E) = P(B) - P(D)$ —(iii)

Putting (ii) and (iii) in eqn. (i), we get:-

$P(A \cup B) = P(A) - \cancel{P(D)} + \cancel{P(D)} + P(E)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(D)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $D = A \cap B$

H.W. Proof $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C)$
$+ P(A \cap B \cap C)$

To prove,

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$$

Let, $B\cup C = E$

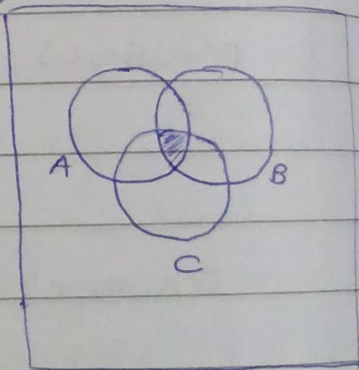
$\therefore P(B\cup C) = P(E)$

$$\therefore P(A\cup E) = P(A) + P(E) - P(A\cap E) \quad -(i)$$

$$P(A\cap E) = P(A\cap(B\cup C)) = P((A\cap B)\cup(A\cap C))$$

$$= P(A\cap B) + P(A\cap C) - P((A\cap B)\cap(A\cap C))$$

$$= P(A\cap B) + P(A\cap C) - P(A\cap B\cap C) \quad -(ii)$$



$$P(E) = P(B\cup C) = P(B) + P(C) - P(B\cap C) \quad -(iii)$$

Putting eqn. (ii) and (iii) in eqn. (i), we get:-

$\cancel{P(A\cup B\cup C) = P(A) + P(B)}$

$$P(A\cup E) = P(A) + P(B) + P(C) - P(B\cap C) - P(A\cap B) - P(A\cap C) + P(A\cap B\cap C)$$

$$\Rightarrow P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$$

Hence, proved.

Conditional probability

Sometimes we are asked to find the probabilities of an event when another event has already occured. Such type of probabilities are called the conditional probabilities.

Definition

Let, A and B be any two events in a sample space 'S'. Then the conditional probability of A given B is denoted as $P(A|B)$ and is defined as $P(A|B) = \frac{P(A\cap B)}{P(B)}$; $P(B) \neq 0$ ——— (i)

Similarly, $P(B|A) = \frac{P(A\cap B)}{P(A)}$, $P(A) \neq 0$ ——— (ii)

Note :- 1) $P(A|B) \neq P(A)$ 2) $P(B|A) \neq P(B)$

2) $P(A|B) = P(B|A)$ (Sometimes when $P(A\cap B) = 0$) $\rightarrow A\cap B = \phi$

or $P(A) = P(B) \neq 0$

3) $P(A\cap B) = P(A|B) \cdot P(B)$ (Multiplication rule) from eqn. (i)

$= P(B|A) \cdot P(A)$ and (ii)

4) When A and B are independent

$$P(A\cap B) = P(A) \cdot P(B)$$

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Example :- 1) In rolling a fair dice, if A be the event of getting an odd no. and B be the event of getting a no. less than or equal to 3, then find

$P(A|B)$ & $P(B|A)$

$P(A) = \{1,3,5\} = \frac{3}{6} = \frac{1}{2}$

$P(B) = \{1,2,3\} = \frac{3}{6} = \frac{1}{2}$

$P(A\cap B) = \{1,3\} = \frac{2}{6} = \frac{1}{3}$

$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{1/3}{1/2} = \frac{2}{3}$

$P(A|B) = \frac{P(A\cap B)}{P(B)}$

$P(B|A) = \frac{2}{3}$ because $(P(A) = P(B) \neq 0)$

whatsapp ASI a)

A: Blood group type A selected

$P(A) = 0.106 + 0.141 + 0.2 = 0.447$

B: Blood group type B selected.

C: The ethnic group 3 is selected.

$P(C) = 0.215 + 0.2 + 0.065 + 0.02 = 0.5$

$P(A\cap C) = 0.2$

b) $P(A|C) = \frac{P(A\cap C)}{P(C)} = \frac{0.2}{0.5} = 0.4$

$P(C|A) = \frac{P(A\cap C)}{P(A)} = \frac{0.2}{0.447} = 0.447$ (approx)

c)

$= 0.082 + 0.106 + 0.004 = 0.192$

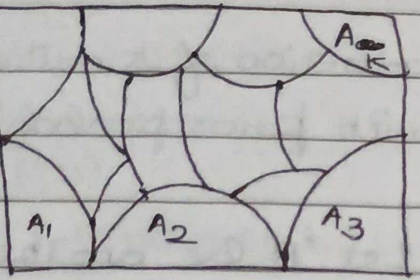
E: Individual is from ethinic group 1

$P(E\cap B') = P(E) - P(E\cap B)$

$= (0.082 + 0.106 + 0.008 + 0.004) - 0.008$

$= 0.192$

Partition of a sample space



Let, $A_1, A_2, \ldots\ldots, A_k$ be the k events in a sample space 'S', then they are called the partition of 'S' if

i) $A_i \cap A_j = \phi$ for $i \neq j$ & $i, j = 1, 2, \ldots, k$

ii) $\bigcup_{i=1}^{k} A_i = S$ (exhaustive events)

iii) $P(A_i) > 0 \; \forall \; i = 1, 2, \ldots\ldots, k$

All the events are Mutually exclusive events

$S = \{1, 2, \ldots, 6\}$

$A = \{1, 3, 5\}$

$B = \{2, 4, 6\}$

Here, the events are mutually exclusive and exhaustive events.

Law of total probability

Let, $A_1, A_2, \ldots\ldots, A_k$ be a collection of k mutually exclusive and exhaustive events with $p(A_i) > 0 \; \forall \; i = 1, 2, \ldots\ldots, k$.

Then for any event 'B' in the sample space 'S', the total probability of B is given by

$$P(B) = P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + \ldots\ldots + P(B|A_k) \cdot P(A_k)$$

Baye's theorem

Let, $A_1, A_2, \ldots\ldots, A_k$ be a collection of k mutually exclusive and exhaustive events with prior probability $P(A_i)$ $(\forall\ i = 1, 2, \ldots\ldots, k)$ in a sample space 'S'. Let, 'B' be another event in the sample space 'S'

Then, the posterior probability of A_j given that the event has already occured is given by

$$P(A_j \mid B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B \mid A_j) \cdot P(A_j)}{\sum_{i=1}^{k} P(B \mid A_i) \cdot P(A_i)}$$