Department of Mathematics
KIIT Deemed to be University
Solution Manual
Assignment-I (MA 1003)

Q.1) Eliminate the arbitrary constants from the following equations and obtain the differential equation

$$e^{2y} + 2axe^{y} + a^2 = 0.$$

Solution: Given Equation is

$$e^{2y} + 2axe^{y} + a^2 = 0 \tag{1}$$

Differentiating Equation (1) w. r. to x

$$a = -\frac{e^{y}y'}{1+xy'}$$

Putting the value of a in Equation (1):

$$(1+xy')^2 - 2xy'(1+xy') + (y')^2 = 0$$

The required differential equation is:

$$(1-x^2)(y')^2 + 1 = 0.$$

Q.2) Solve the IVP by reducing to separable ODE.

$$yy' = x^3 + \frac{y^2}{x}, y(2) = 6.$$

Solution: Given ODE is:

$$yy' = x^3 + \frac{y^2}{x} \tag{2}$$

Let $y = ux$

Putting $y = ux$ and $y' = u + x\dfrac{du}{dx}$ in Eq. (2):

$$ux\left(u + x\frac{du}{dx}\right) = x^3 + \frac{u^2x^2}{x}$$

$$\Rightarrow \int u\,du = \int x\,dx$$

$\Rightarrow$ The general solution is: $y^2 = x^2(x^2 + c)$ (3)

Using initial condition in (3): $c = 5$

The required particular solution is:

$$y^2 = x^2(x^2 + 5)$$

Q.3) The tank contains 1000 gal of water in which 200lb of salt are dissolved. Fifty gal of brine, each containing $(1 + \cos t)$ lb of dissolved salt, run into the tank per minute. The mixture, kept uniform by stirring, runs out at the same rate. Find the amount of salt $y(t)$ in the tank at any time t.

Solution: Let $y(t)$ be the amount of salt in the tank at any time t.
The amount of brine runs into the tank per minute is 50 gallons.
Each gallon contains $(1 + \cos t)$ lb. of dissolved salt.
Hence 50 gallons contain $50(1 + \cos t)$ lb. of dissolved salt.
Thus the salt inflow rate per minute is $50(1 + \cos t)$.
Since 1000 gallons contain $y(t)$ amount of dissolved salt.
Hence 50 gallons contain $0.05y(t)$ of salt.
Thus salt outflow rate per minute is $0.05y(t)$.
We have
$\frac{dy}{dt} =$ Salt inflow rate-Salt outflow rate
The initial value problem is
$y' = 50(1 + \cos t) - 0.05y, \qquad y(0) = 200.$
The ODE can be written as
$y' + 0.05y = 50(1 + \cos t)$
The general solution is

$$y(t)e^{\int 0.05dt} = \int 50(1 + \cos t)e^{\int 0.05dt} dt + C$$

$$\Rightarrow y(t)e^{0.05t} = \frac{50}{0.05}e^{0.05t} + \frac{50e^{0.05t}}{1 + (0.05)^2}(0.05\cos t + \sin t) + C$$

$$\Rightarrow y(t) = 1000 + \frac{50}{1 + (0.05)^2}(0.05\cos t + \sin t) + Ce^{-0.05t}$$

Using the initial condition we get $C \cong -802.5$
The amount of salt in the tank at any time t is
$y(t) \cong 1000 + 2.494\cos t + 49.88\sin t - 802.5e^{-0.05t}$.

Q.4) Find an integrating factor and then find the particular solution.

$$y' = xy + 2x - x^3, y(0) = 0.$$

Solution: The given ODE can be written as

$(xy + 2x - x^3)dx - dy = 0$

$M(x, y) = xy + 2x - x^3$ and $N(x, y) = -1$

$\frac{\partial M}{\partial y} = x, \quad \frac{\partial N}{\partial x} = 0$

Since $M_y \neq N_x$, so ODE is not exact.

Integrating Factor

$\frac{M_y - N_x}{N(x, y)} = -x$

Thus I. F. is $e^{-x^2/2}$

Multiplying it in the ODE we get

$e^{-x^2/2}(xy + 2x - x^3)dx - e^{-x^2/2}dy = 0$

The general solution is

$\int e^{-x^2/2}(xy + 2x - x^3)dx = C$

$\Rightarrow x^2 - y = Ce^{x^2/2}$

Using the initial condition $y(0) = 0$ we get $C = 0$.

The required solution is $y(x) = x^2$.

Q.5) Find an integrating factor and then solve.

$$y\,dx - x\,dy + \ln x\,dx = 0.$$

Solution: Given ODE is

$$\left(y + \ln x\right)dx - x\,dy = 0 \tag{4}$$

$\Rightarrow M\left(x,\ y\right) = y + \ln x$ and $N\left(x,\ y\right) = -x$

$\Rightarrow M_y = 1$ and $N_x = -1$

Given ODE (4) is not exact.

Now, $\frac{M_y - N_x}{N(x,\ y)} = -\frac{2}{x}$

Hence, I. F. is $e^{-\int \frac{2}{x}\,dx} = \frac{1}{x^2}$

Multiplying Eq. (4) by the I. F.

$$\frac{1}{x^2}\left(y + \ln x\right)dx - \frac{1}{x}\,dy = 0$$

The general solution is

$$\int \left(\frac{y}{x^2} + \frac{\ln x}{x^2}\right) dx = C$$

The required solution is

$$y + \ln x + 1 = Cx$$

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