



MID-SEMESTER EXAMINATION-2016

DISCRETE MATHEMATICS

[MA-2003]

SEMESTER-3rd

BRANCH-CSE &IT

FULL MARKS:25

TIME: 02 Hours

Answer any five questions including question No.1 which is compulsory.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. Answers all. (1x5)
- a. Find converse of the conditional statement: "Sum of two odd integers is even".
- b. If the truth value of  $(p \rightarrow q)$  is False then find the truth value of  $p \wedge (p \rightarrow q)$ .
- c. Let  $R$  be a relation on the set of natural number such that

$$R = \{(a, b) \in N \times N : a \mid b\}.$$

Is it irreflexive? Antisymmetric?

- d. Find the equivalence relation on the set  $A = \{a, b, c, d\}$  corresponding to its partition:  $P = \{\{a, b\}, \{c, d\}\}$ .
- e. Give an example of relation which is symmetric and antisymmetric.

2. (2x2.5)

- a. Show that  $f_n > \alpha^{n-2}$ ; for  $n \geq 3$  and  $\alpha = \frac{1+\sqrt{5}}{2}$  where  $\langle f_n \rangle$  is sequence of Fibonacci numbers by strong induction.

- b. Construct the truth table for  $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ . Is it a tautology?

3. (2x2.5)

- a. Prove by method of induction.

$$1 + r + r^2 + \dots + r^n = \frac{1 - r^{n+1}}{1 - r}; r \neq 1.$$

- b. Show that  $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$  is a tautology without using the truth table.

4. (2x2.5)

- a. Test the validity of the following arguments:  
A student in this class has not read this book.  
Everybody in this class has passed the first examination.



Hence someone in this class has not read this book.

b. Prove or disprove the conclusion by rules of inference:

(i) Babies are illogical

(ii) Nobody is despised who can manage crocodile.

(iii) Illogical people are despised.

So, babies can't manage crocodile.

(2x4)

5.

a. Define; closer of properties of a relation? Find the symmetric closer of the relation

$$R = \{(a, b) \in \mathbb{N} \times \mathbb{N} : a > b\}.$$

b. Let  $S = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a = b \text{ or } a = -b\}$ . Show that  $S$  is an equivalence relation on the set of integers. Also find, how many number of elements will be in each of its equivalence classes.

(2x2.5)

6.

a. Find the transitive closure of the relation  $R = \{(1,2), (2,1), (2,3), (3,4), (4,1)\}$  on  $A = \{1, 2, 3, 4\}$ ; by Warshall's algorithm.

b. Let the set  $A = \{1, 2, 3, 4, 9, 16\}$  and  $R = \{(a, b) | a = b^2\}$  is a relation on the set  $A$ . Find  $M_R$ ,  $M_{R^{-1}}$ ,  $M_{R^c}$  and  $M_{R^2}$ .

(2x2.5)

7.

a. Let  $A \neq \emptyset$  be any set. Let  $R$  be a relation defined as  $XY \Leftrightarrow Y \subseteq X$  on  $P(A)$ ; the power set of  $A$ . Verify the properties of the relation  $R$ .

b. Does union of two equivalence relations an equivalence relation? Justify your answer.