# Stack & actions its Applications

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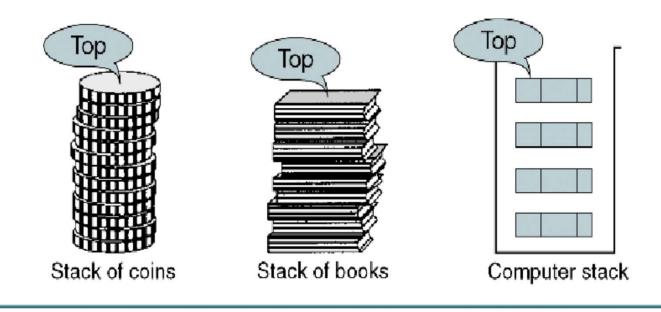
#### What is a stack?

- A stack is a linear data structure that stores a set of homogeneous elements in a particular order
- Stack principle: LAST IN FIRST OUT (LIFO)
- Means: the last element inserted is the first one to be removed
- Example:



Elements are removed in the reverse order in which they were inserted

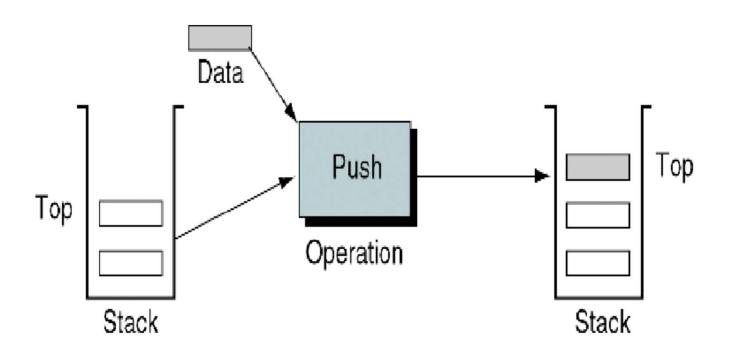
#### Examples of Stack



Stack

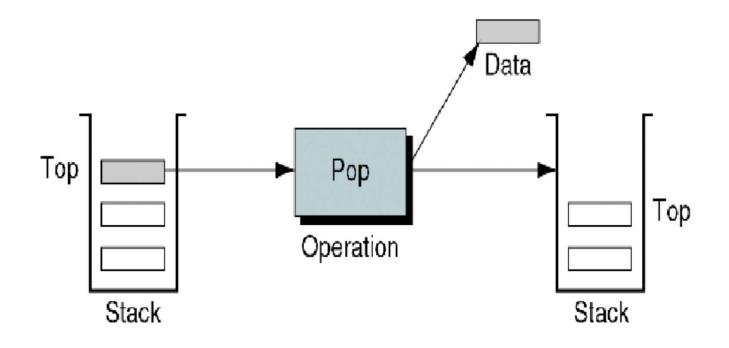
Elements are added to and removed from the top of the stack (the most recently added items are at the top of the stack).

#### **Operations on Stack**



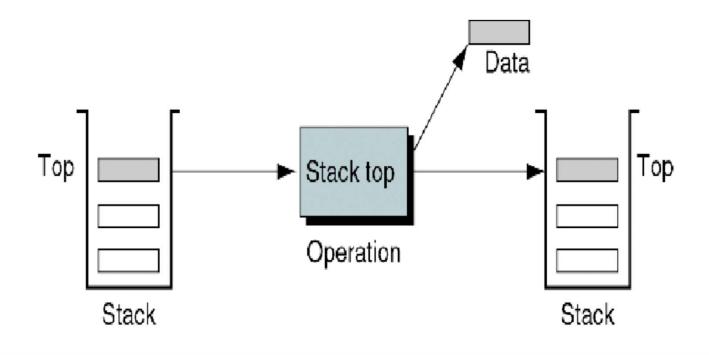
Push Stack Operation

#### Operations on Stack



Pop Stack Operation

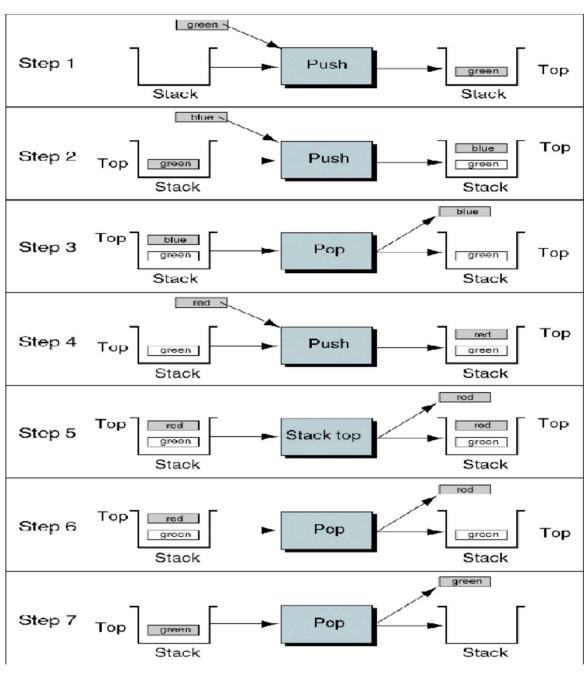
#### **Operations on Stack**



Stack Top Operation

#### **Examples of Stack**





#### Stack Applications

- Real life
  - Pile of books
  - Plate trays
- More applications related to computer science
  - Program execution stack
  - Evaluating expressions

#### Stack Implementation using Array

- Allocate an array of some size (pre-defined)
  - Maximum N elements in stack
- Bottom stack element stored at 0th position of array
- last element in the array is at the top
- Increment top when one element is pushed, decrement after pop

#### CreateS, isEmpty, isFull

#### Push

```
void push(element item) {
    /* add an item to the global stack */
    if (top == STACK_SIZE-1) {
        printf("Stack is Full");
        return;
    }
    stack[++top] = item;
}
```

### Pop

```
element pop() {
    /* return the top element from the stack */
    if (top == -1) {
        printf("Stack is Empty ...");
        return;
    }
    return stack[top--];
}
```

## Performance and Limitations (Implementation of stack ADT using Array)

- Performance
  - Let *n* be the number of elements in the stack
  - The space used is O(n)
  - Each operation runs in time O(1)
- Limitations
  - The maximum size of the stack must be defined *a priori*, and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

- Create a data structure that represents two stacks using only one array.
- Following functions must be supported by twoStacks:
  - push1(int x): pushes x to first stack
  - push2(int x): pushes x to second stack
  - pop1(): pops an element from first stack and return the popped element
  - pop2(): pops an element from second stack and return the popped element

Implementation of twoStack should be space efficient.

- Method 1 (Divide the space in two halves)
  - A simple way to implement two stacks is to divide the array in two halves and assign the half space to two each stack, i.e., use arr[0] to arr[n/2-1] for stack1, and arr[n/2] to arr[n-1] for stack2 where arr[] is the array to be used to implement two stacks and size of array be n.
- The problem with this method is <u>inefficient use of array space</u>.
- A stack push operation may result in stack overflow even if there is space available in arr[]

```
top1 = -1;

top2 = n/2 - 1;
```

```
// Method to push an element x to stack1
void push1(int x) {
  if(top1 == n/2-1) {
    printf("Stack Overflow...");
    return;
  }
  top1++;
  stack[top1] = x;
}
```

```
// Method to push an element x to stack2
void push2(int x) {
  if(top2 == n-1) {
    printf("Stack Overflow...");
    return;
  }
  top2++;
  stack[top2] = x;
}
```

```
// Method to pop an element from first stack
int pop1() {
  int x;
  if(top1 == -1) {
    printf("Stack Underflow...");
    return -9999;
  x = \text{stack}[\text{top1}];
  top1--;
  return(x);
```

```
// Method to pop an element from second stack
int pop2() {
  int x;
  if(top2 == n/2 - 1) {
   printf("Stack Underflow...");
   return -9999;
  x = stack[top2];
  top2--;
  return(x);
```

- Method 2: (A space efficient implementation)
- This method efficiently utilizes the available space.
- It doesn't cause an overflow if there is space available in arr[].
- The idea is to start two stacks from two extreme ends of arr[].
  - stack1 starts from starting of the array, the first element in stack1 is pushed at index 0.
  - The stack2 starts from end of the array, the first element in stack2 is pushed at index (n-1).
- Both stacks grow (or <u>shrink</u>) in opposite direction.
- To check for overflow, it needs to check for space between top elements of both stacks.

```
top1 = -1;top2 = n;
```

```
// Method to push an element x to stack1
void push1(int x) {
  if(top1 == top2-1) {
    printf("Stack Overflow...");
    return;
  }
  top1++;
  stack[top1] = x;
}
```

```
// Method to push an element x to stack2
void push2(int x) {
  if(top1 == top2-1) {
    printf("Stack Overflow...");
    return;
  }
  top2--;
  stack[top2] = x;
}
```

```
// Method to pop an element from first stack
int pop1() {
 int x;
 if(top1 == -1) {
   printf("Stack Underflow...");
   return -999;
  x = stack[top1];
  top1--;
  return(x);
```

```
// Method to pop an element from second stack
int pop2() {
 int x;
 if(top2 == n)  {
   printf("Stack Underflow...");
   return -999;
  x = stack[top2];
  top2++;
  return(x);
```

#### Reverse a String using Stack

```
#include<stdio.h>
#include<stdlib.h>
#define MAX 100
char str[MAX];
int top = -1;
int main() {
  char str[MAX];
  int i;
  printf("Input a string: ");
  scanf("\%[^\n]", str);
  for(i=0; i<strlen(str); i++) pushChar(str[i]);
  for(i=0; i<strlen(str); i++) str[i]=popChar();
  str[i] = '\0';
  printf("Reversed String is: %s\n", str);
  return 0;
```

```
void pushChar(char item) {
  if(top != MAX)  {
    top=top+1;
    strr[top]=item;
char popChar() {
  int item;
  if(top != -1) {
    item = str[top];
    top=top-1;
    return item;
```



## Infix, Prefix, & Postfix Expressions

#### **Infix Notation**

- Usually the algebraic expressions are written like this: a + b
- This is called infix notation, because the operator ("+") is in between operands in the expression
- A problem is that it needs parentheses or precedence rules to handle more complicated expressions:

#### For Example:

$$a + b * c = (a + b) * c ?$$
  
=  $a + (b * c) ?$ 

#### Infix, Postfix, & Prefix notation

- There is no reason to place the operator somewhere else.
- How ?
  - Infix notation : a + b
  - Prefix notation : + a b
  - Postfix notation: a b +

#### **Other Names**

Prefix notation was introduced by the Polish logician Lukasiewicz, and is sometimes called "Polish Notation".

Postfix notation is sometimes called "Reverse Polish Notation" or RPN.

## Why?

- Question: Why would anyone ever want to use anything so "unnatural," when infix seems to work just fine?
- Answer: With postfix and prefix notations, <u>parentheses are</u> no longer needed!
- Advantages of postfix:
  - Don't need rules of precedence
  - Don't need rules for right and left associativity
  - Don't need parentheses to override the above rules

#### Example

infix	postfix	prefix
(a + b) * c	a b + c *	* + a b c
a + (b * c)	a b c * +	+ a * b c

Infix form : <identifier> < operator> < identifier>

Postfix form : <identifier> <identifier> < operator>

Prefix form : <operator> <identifier> <identifier>

## Conclusion

Infix is the only notation that requires parentheses in order to change the order in which the operations are done.

## Infix to Postfix conversion (Intuitive Algorithm)

- An Infix to Postfix manual conversion algorithm is:
  - 1. Completely parenthesize the infix expression according to order of priority you want.
  - 2. Move each operator to its corresponding right parenthesis.
  - 3. Remove all parentheses.
- Examples:

#### **Evaluation of Expressions**

$$x = a / b - c + d * e - a * c$$

$$a = 4$$
,  $b = c = 2$ ,  $d = e = 3$ 

Interpretation 1: ((4/2)-2)+(3\*3)-(4\*2)=0+8+9=1

Interpretation 2: (4/(2-2+3))\*(3-4)\*2=(4/3)\*(-1)\*2=-2.66666...

How to generate the machine instructions corresponding to a given expression?

precedence rule + associative rule

### Precedence of Operators

Token	Operator	Precedence	Associativity
()	function call array element	17	left-to-right
-> .	struct or union member		
++	increment, decrement	16	left-to-right
! - + & * sizeof	logical not one's complement unary minus or plus address or indirection size (in bytes)	15	right-to-left
(type)	type cast	14	right-to-left
* / %	mutiplicative	13	Left-to-right

### Precedence of Operators

+ -	binary add or subtract	12	left-to-right
<<>>>	shift	11	left-to-right
>>= <<=	relational	10	left-to-right
==!=	equality	9	left-to-right
&	bitwise and	8	left-to-right
^	bitwise exclusive or	7	left-to-right
	bitwise or	6	left-to-right
&&	logical and	5	left-to-right
	logical or	4	left-to-right

### Precedence of Operators

?:	conditional	3	right-to-left
= += <u>-</u> = /= *- 0/ -	assignment	2	right-to-left
*= \%= <<= >>= &= ^=			
&= ^=			
,	comma	1	left-to-right

#### Postfix conversion

user

#### compiler

Infix	Postfix
2+3*4	234*+
a*b+5	ab*5+
(1+2)*7	12+7*
a*b/c	ab*c/
(a/(b-c+d))*(e-a)*c	abc-d+/ea-*c*
a/b-c+d*e-a*c	ab/c-de*+ac*-

Postfix: no parentheses, no precedence

#### Infix to postfix conversion

- 1. Read the tokens one by one from an infix expression using a loop.
- For each token x do the following:
  - When the token is an operand
    - Append to the end of the postfix expression
  - ii. When the *token* is a left parenthesis "("
    - Push the token into the stack.
  - iii. When the *token* is a right parenthesis ")"
    - Repeatedly pop a token from stack and append to the end of the postfix expression until "(" is encountered in the stack. Then pop "(" from stack.
    - If stack is empty before finding a "(", implies that expression is not a valid expression.

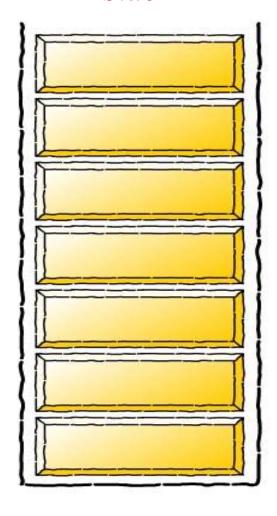
#### Infix to postfix conversion

- iv. When the *token* is an operator
  - Use a loop that checks the following conditions:
    - a) The stack is **not** empty
    - b) The token y currently at top of stack is an operator. In other words, it is not **not** a left parenthesis "(".
    - c) y is an operator of **higher or equal** precedence than that of x,
- As long as all these three conditions are true, repeatedly do the following:
  - Append the token \( \nslant \) into postfix expression
  - Call pop to remove another token, named y, from stack

- Note: The loop above will stops as soon as any of the three conditions is not true.
- Then, push the token x into stack.
- After all the tokens in infix expression are processed, then use another loop to repeatedly do the following as long as the stack is not empty:
  - Append the token from the top of the stack into postfixexpression.
  - Call pop to remove the top token y from the stack.

### Infix to postfix conversion: Example

#### stack



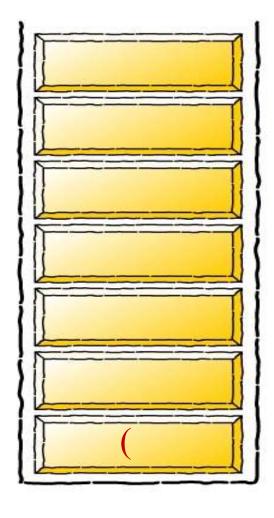
infix expression

$$(a+b-c)*d-(e+f)$$

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#### Infix to postfix conversion

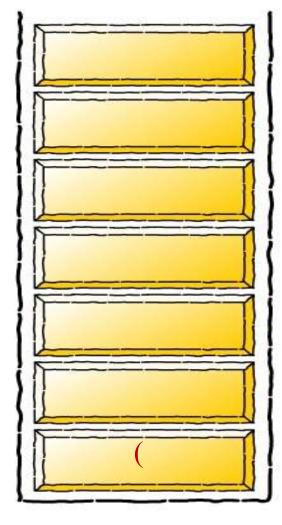
stack



infix expression

$$a + b - c$$
) \*  $d - (e + f)$ 

stack



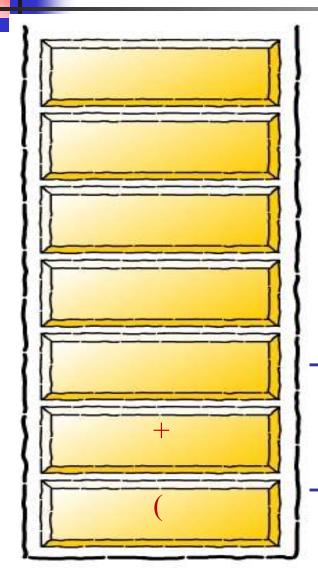
infix expression

$$+ b - c) * d - (e + f)$$

postfix expression

a

stack



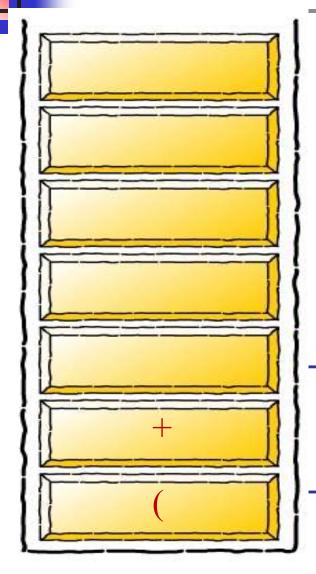
infix expression

$$b - c) * d - (e + f)$$

postfix expression

a

stack



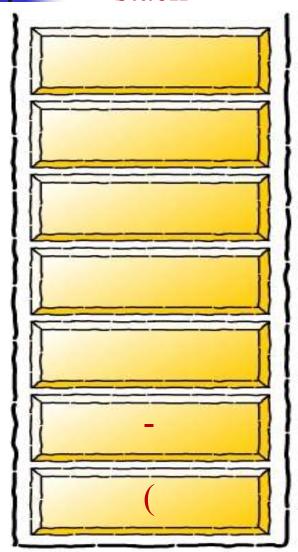
infix expression

$$-c)*d-(e+f)$$

postfix expression

a b

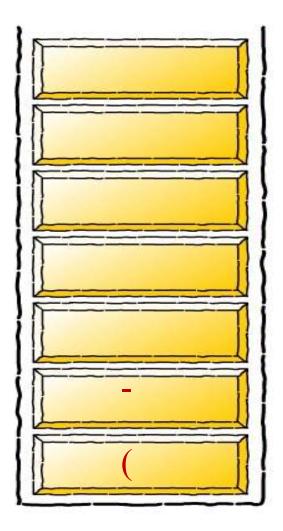
stack



infix expression

$$c)*d-(e+f)$$

stack

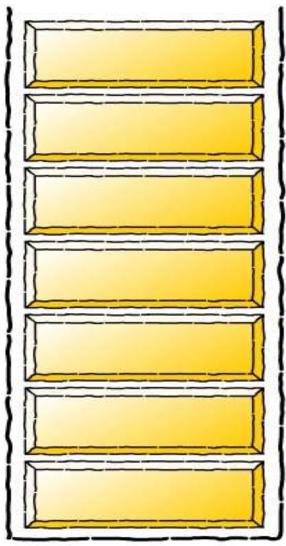


infix expression

$$) * d - (e + f)$$

$$ab+c$$

stack

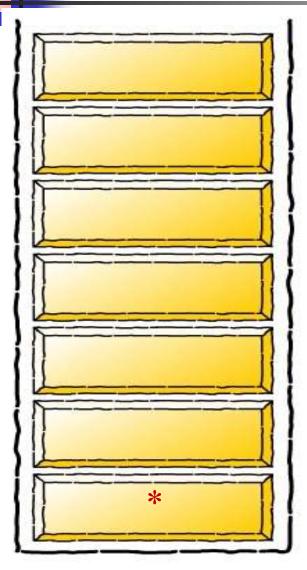


infix expression

$$*d - (e + f)$$

$$ab+c$$
-

stack

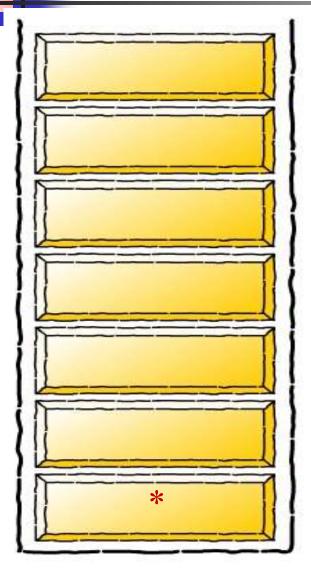


infix expression

$$d-(e+f)$$

$$ab+c$$

stack

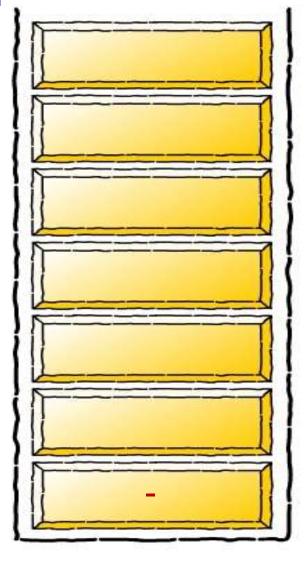


infix expression

$$-(e+f)$$

$$ab+c-d$$



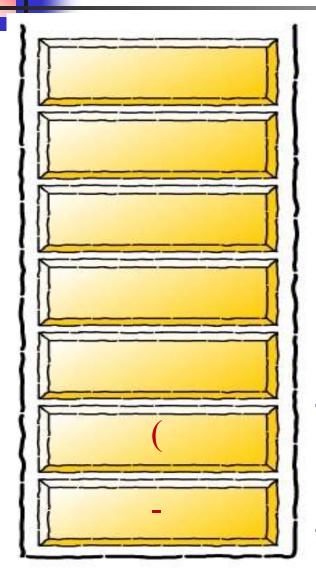


infix expression

$$(e+f)$$

$$ab+c-d*$$

stack

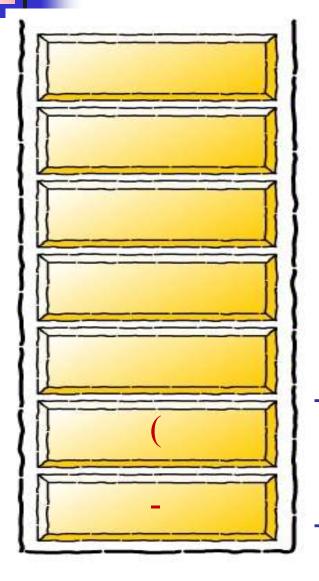


infix expression

$$e + f)$$

$$ab+c-d*$$

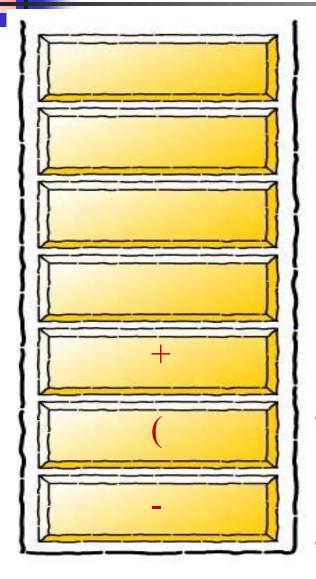
stack



infix expression

$$ab+c-d*e$$

stack

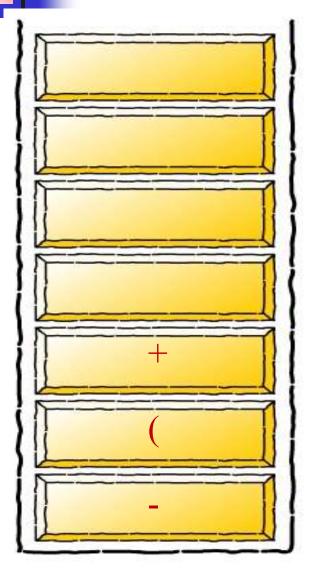


infix expression

f)

$$ab+c-d*e$$

stack

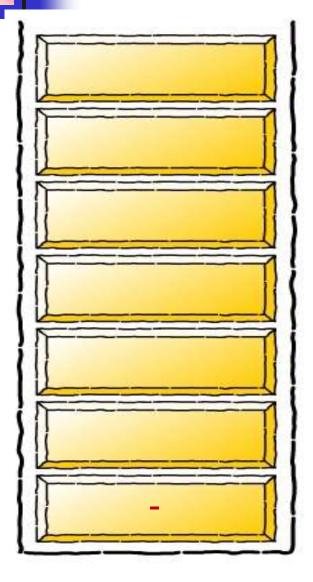


infix expression

postfix expression

ab+c-d\*ef

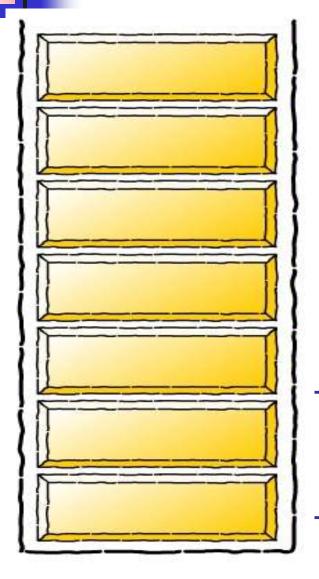
stack



infix expression

$$ab+c-d*ef+$$

stack



infix expression

$$a b + c - d * e f + -$$

#### Algorithm of Infix to Postfix

```
infixToPostfix(infixexpr):
    postfixList = []
    tokenList = infixexpr
    for token in tokenList:
        if token is an operand
            append(token) to postfixList
       elif token == '(':
            push(token)
        elif token == ')':
            topToken = pop()
            while topToken != '(':
                append(topToken) to postfixList
                topToken = pop()
        else
        while (!Empty()) and (prec[top()] >= prec[token])
            append(pop()) to postfixList
        push(token)
    while (!isEmpty())
            append(pop()) to postfixList
```

return (postfixList)

#### Convert 2\*3/(2-1)+5\*3 into Postfix form

Expression	Stack	Output
2	Empty	2
*	*	2
3	*	23
1	1	23*
(	/(	23*
2	/(	23*2
-	/(-	23*2
1	/(-	23*21
)	1	23*21-
+	+	23*21-/
5	+	23*21-/5
*	+*	23*21-/53
3	+*	23*21-/53
	Empty	23*21-/53*+

Postfix Expression is 23\*21-/53\*+

### **Examples of Prefix Expressions**

Infix

Prefix

A+B

A+B-C

(A+B)\*(C-D)

 $A^B*C-D+E/F/(G+H)$ 

 $((A+B)*C-(D-E))^{(F+G)}$ 

 $A-B/(C*D^E)$ 

+AB

-+ABC

\*+AB-CD

+-\*^ABCD//EF+GH

^-\*+ABC-DE+FG

 $-A/B*C^DE$ 

## Infix to Prefix conversion (Intuitive Algorithm)

- An Infix to Prefix manual conversion algorithm is:
  - 1. Completely parenthesize the infix expression according to order of priority you want.
  - 2. Move each operator to its corresponding left parenthesis.
  - 3. Remove all parentheses.
- Examples:

$$3 + 4 * 5 \longrightarrow (3 + (4 * 5)) \longrightarrow + 3 * 4 5$$

#### Algorithm of Infix to Prefix

```
Input an infix expression in a string 'infix'.
Reverse the string 'infix'
Create an empty stack and also create an empty list for prefix expression
while(!end of string(infix))
    ch= a character from 'infix' string
    if( ch == ')')
         push(ch)
    if( ch == \dot{r}(\dot{r})
       while(top() != ')')
         append(top()) to prefixList
         pop()
    if(ch is an operand)
         append(ch) to prefixList
    if(ch is an operator)
         while(!empty() && prec (top()) > prec(ch))
              append(top()) to prefixList
              pop()
         push(ch);
while(!empty())
    append(top()) to prefixList
    pop()
reverse the 'prefix' string
```

#### **Example Infix to Prefix**



Infix string: A+B\*C+D/E

<u>Ch</u>	<u>prefix</u>	stackop	
E	E		
/	E	/	
D	ED	/	
+	ED/	+	
C	ED/C	+	
*	ED/C	+, *	
В	ED/CB	+, *	
+	ED/CB*	+,+	
A	ED/CB*A	+,+	
	ED/CB*A+	+	
	ED/CB*A++		

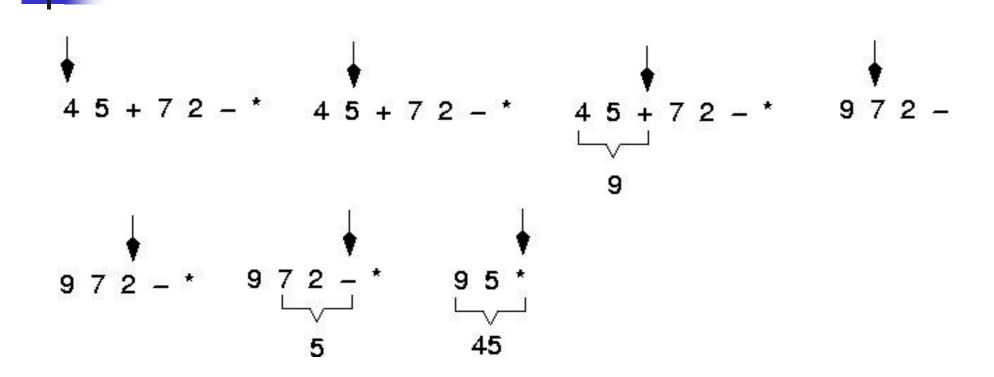
Reverse of is ++A\*BC/DE.

The prefix expression of A+B\*C+D/E is ++A\*BC/DE.

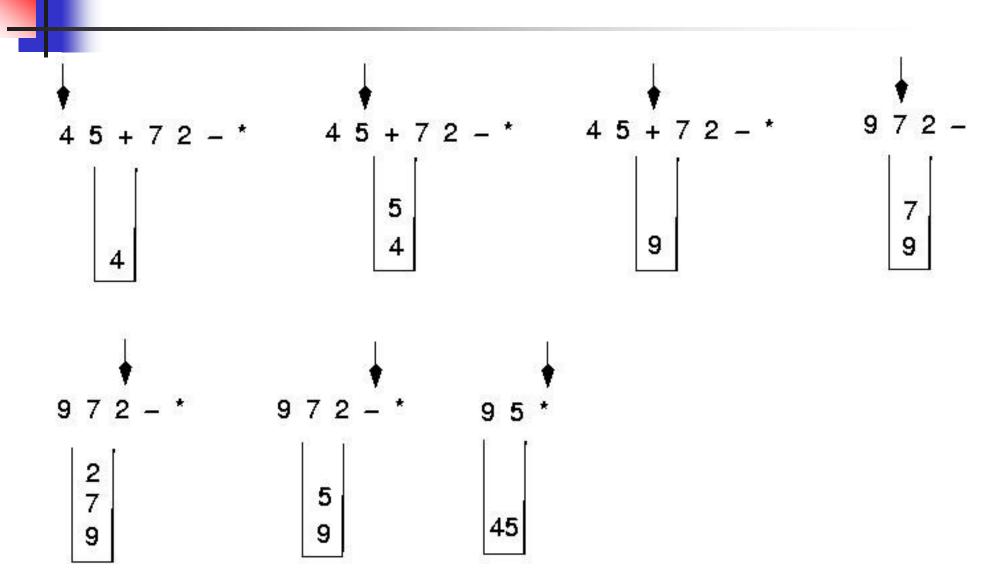
# Algorithm to Evaluate Postfix Expression

```
scan each character ch in the postfix expression if ch is an operator \odot, then
a = \text{pop first element from stack}
b = \text{pop second element from the stack}
\text{res} = b \odot a
\text{push res into the stack}
\text{else if } ch \text{ is an operand}
\text{push } ch \text{ into the stack}
\text{return top(stack)}
```

#### Example: postfix expressions



### Postfix expressions: Algorithm using stacks



# Algorithm for evaluating a postfix expression

### Question

Evaluate the following expression in postfix: 623+-382/+\*2^3+

- A. 49
- B. 51
- **C**. 52
- D. 7
- E. None of these

### Evaluate: 623+-382/+\*2^3+

<b>Symbol</b>	opnd1	opnd2	value	opndstk
6				6
2				6, 2
3				6, 2, 3
+	2	3	5	6, 5
-	6	5	1	1
3	6	5	1	1, 3
8	6	5	1	1, 3, 8
2	6	5	1	1, 3, 8, 2
/	8	2	4	1, 3, 4
+	3	4	7	1, 7
*	1	7	7	7
2	1	7	7	7, 2
^	7	2	49	49
3	7	2	49	49, 3
+	49	3	52	52

# Algorithm for evaluating a prefix expression

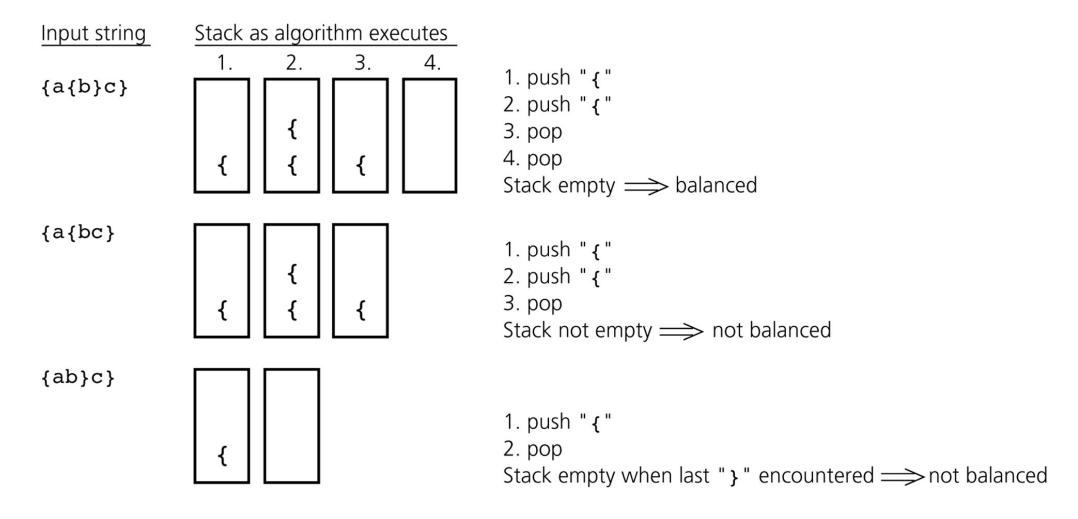
```
Reverse the prefix Expression
while more input items exist {
   If symb is an operand
        then push (symb)
   else { //symbol is an operator
        opnd1=pop()
        opnd2=pop()
        val = opnd1 symb opnd2
        push(val)
result = pop ()
```

#### Checking for Balanced Braces

- Requirements for balanced braces
  - Each time you encounter a "}", it matches an already encountered "{"
  - When you reach the end of the string, you have matched each "{"

# 4

#### Checking for Balanced Braces



#### Checking for Balanced Braces

```
createStack()
balancedSoFar = true
i = 0
while (balancedSoFar and i < lengthofaString) {
   ch = character at position i in aString
   ++i
   if (ch is '{') // push an open brace
         push( '{' )
   else if (ch is '}') // close brace
         if (!isEmpty())
                           // pop a matching open brace
                   pop()
                             // no matching open brace
         else
                   balancedSoFar = false
                             // ignore all characters other than braces
if (balancedSoFar and isEmpty())
   String has balanced braces
else
   String does not have balanced braces
```

#### Find Maximum depth of brackets

```
int maxDepth(char str[]) {
  int curr max = 0; // current count
  int max = 0; // overall maximum count
  int n = strlen(str);
  for (int i = 0; i < n; i++) {
     if (str[i] == '(') {
       curr max++;
       if (curr max> max)
          max = curr max;
     else if (str[i] == ')') {
       if (curr max>0)
          curr max--;
       else
          return -1;
```

```
// finally check for unbalanced string
if (current_max != 0)
   return -1;
return max;
}

int main() {
   char str[] = "( ((X)) (((Y))) )";
   printf("%d\n", maxDepth(str));
   return 0;
}
```

#### Recursion

- Process in which a function calls itself directly or indirectly is called recursion
- corresponding function is called as recursive function
- Using recursive algorithm, certain problems can be solved quite easily
- Example: Findout factorial of a number

```
int fact(int n) {
   if (n <= 1) return 1;
   else return n * fact(n - 1);
}</pre>
```

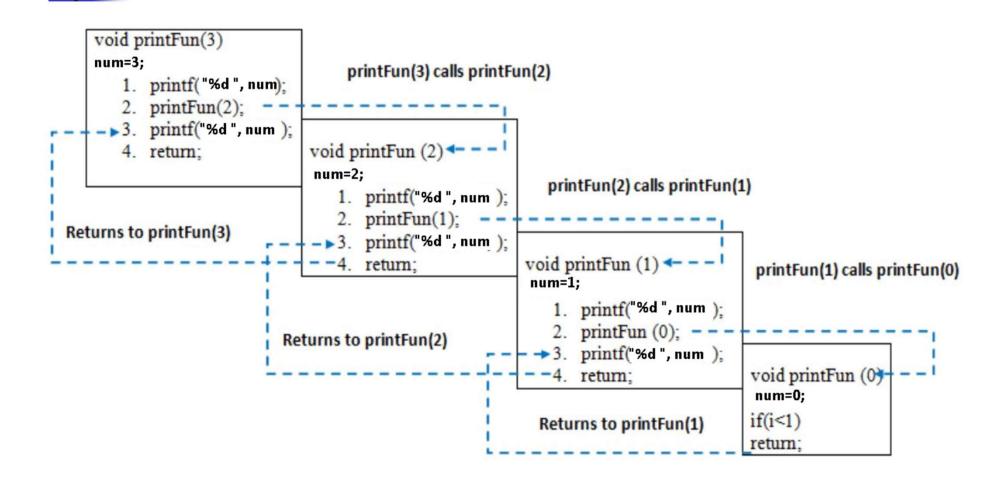
#### Demonstrate working of Recursion

```
void printFun(int num) {
  if (num < 1)
    return;
  else {
    printf("%d", num);
     printFun(num - 1); // statement 2
     printf("%d ", num);
     return;
```

```
int main() {
  int n = 3;
  printFun(n);
}
```

3 2 1 1 2 3

#### Demonstrate working of Recursion



#### Recursion

```
int fact(int n) {
   if (n <= 1) return 1;
   else return n * fact(n - 1);
 If call x = fact(3), stores n=3 on the stack
     fact calls itself, putting n=2 on the stack
       fact calls itself, putting n=1 on the stack
       fact returns 1
     fact has n=2, computes and returns 2*1 = 2
 fact has n=3, computes and returns 3*2 = 6
```

#### Classic: Factorial

```
argument n = 7.
main calls
fact(7)
value of n at this node: returned value
return (7*fact(6))
recursive call
n=6
return(6*fact(5))
n=5
return(5*fact(4))
n=4
return (4*fact(3))
n=3
return (3*fact(2))
n=2
return (2*fact(1))
return 2.
n=1 (return (1))
```

```
int fact (int n) {
                if (n<=1)
                     return (1) /* base case */
                else
                     return (n * fact (n-1));
                      /* recursive case
                     // end factorial()
return(7*720) = 5040 = answer!!
return(6*120) = 720
return(5*24) = 120
return(4*6) = 24
return (3 * 2) = 6
return(2 * fact(1)) = 2 * 1 = 2 Thus fact (2) = 2.
1 is substituted for the call (base case reached)
```



## Program: Linear implementation of stack Using Structure

```
#include<stdio.h>
#define MAX 50
typedef struct {
  int stk[MAX];
  int top;
}Stack;
void push(Stack *s, int item) {
  if(s\rightarrow top==MAX-1)
          printf("\nStack Overflow...\n");
          return;
  s->stk[++s->top]=item;
```

```
void pop(Stack *s, int *item) {
  if(s->top==-1)
          printf("\nStack Underflow...\n");
          return;
  *item=s->stk[s->top];
  s->top--;
```

### Program: Linear implementation of stack Using Structure

```
void display(Stack *s) {
  int i;
  if(s->top == -1) {
     printf("Stack is Empty...\n");
          return;
  printf("\nThe elements in the stack
           are...n");
  for(i=s->top; i>=1; i--)
     printf("%d->", s->stk[i]);
  printf("%d", s->stk[i]);
  printf("\n");
```

```
int main(){
  Stack s;
  int num;
  s.top=-1;
  int choice=0;
  do {
      printf("\nStack Options...\n");
      printf("\n1: Add item\n");
      printf("\n2: Remove item \n");
      printf("\n3: Display\n");
      printf("\n0: Exit\n");
      printf("\n\nEnter choice: ");
       scanf("%d",&choice);
```

## Program: Linear implementation of stack Using Structure

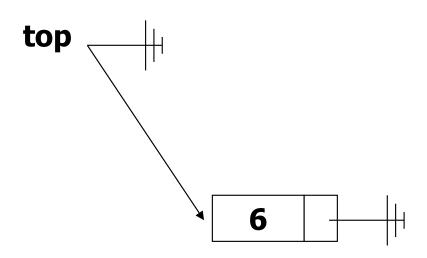
```
switch(choice) {
                                                  case 3:
                                                    display(&s);
  case 0:
                                                    break;
     break;
                                                  default:
  case 1:
                                                    printf("\nAn Invalid
    printf("\nEnter an item to be
                                                               Choice !!!\n");
             inserted: ");
     scanf("%d", &num);
                                             }while(choice!=0);
    push(&s, num);
                                             return 0;
     break;
  case 2:
    pop(&s, &num);
    printf("\nThe popped element
             is %d\n'', num);
     break;
```



#### Stack: Linked List Implementation

- Push and pop at the head of the list
  - New nodes should be inserted at the front of the list, so that they become the top of the stack
  - Nodes are removed from the front (top) of the list
- Straight-forward linked list implementation
  - push and pop can be implemented fairly easily, e.g. assuming that head is a reference to the node at the front of the list



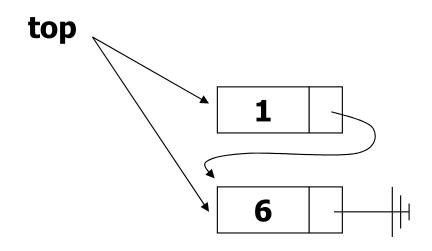


#### C Code

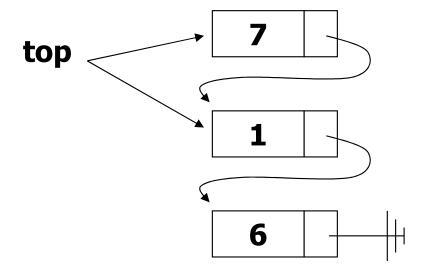
Stack s; s.push(6);



## C Code Stack s; s.push(6); s.push(1);

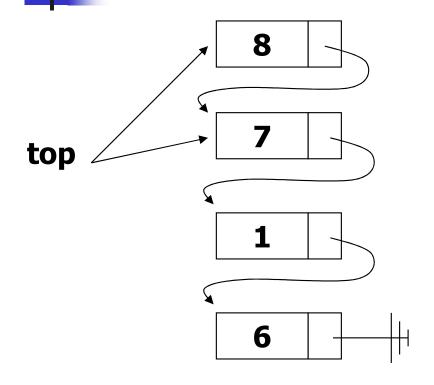






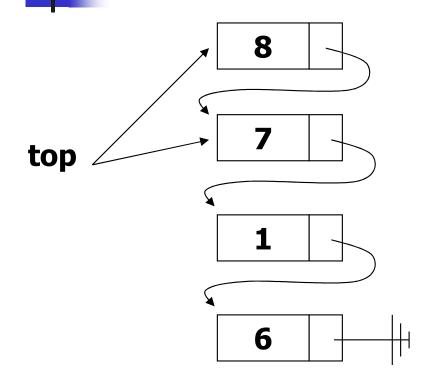
## C Code Stack s; s.push(6); s.push(1); s.push(7);





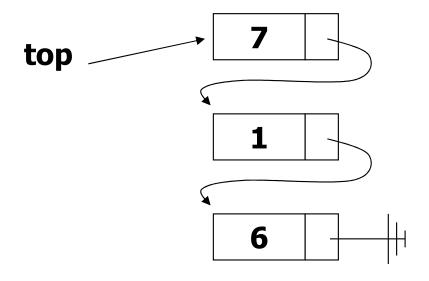
# C Code Stack s; s.push(6); s.push(1); s.push(7); s.push(8);





```
C Code
Stack s;
s.push(6);
s.push(1);
s.push(7);
s.push(8);
s.pop();
```





```
C Code

Stack s;
s.push(6);
s.push(1);
s.push(7);
s.push(8);
s.pop();
```

#### Stack Implementation

```
typedef struct stack {
  int data;
  struct stack *next;
} Stack;
```

## Stack Implementation: createStack, isEmpty

```
void createStack() { top = NULL; }
int isEmpty(Stack top) {
  if (top == NULL)
    return 1;
  else
    return 0;
}
```

#### Stack Implementation: push

```
void push(int value) {
    Stack *node;
    node=(struct stack *)malloc(sizeof(struct stack));
    node->data=value;
    if (top == NULL) {
        top=node;
        top->next=NULL;
    else {
        node->next=top;
        top=node;
```

#### Stack Implementation: pop

```
int Pop() {
    int item;
    stack *temp;
    if(top == NULL) {
        printf(" Empty Stack ...");
        return -1;
    else {
        temp=top;
        item=top->data;
        top=top->next;
        temp->next=NULL;
        free(temp);
        return(item);
```

#### Stack Implementation: display

```
void display(Stack *top) {
    Stack *temp;
    temp = top;
    if (top == NULL) {
    printf("Empty stack ...");
        return;
    else {
    while (temp != NULL) {
             printf("%d\n", temp->data);
             temp = temp->next;
```