2.22. Voltage and Current Sources

The term *voltage source* is used to describe a source of energy which establishes a potential difference across its terminals. Most of the sources encountered in everyday life are voltage sources *e.g.*, batteries, d.c. generators, alternators etc. The term *current source* is used to describe a source of energy that provides a current *e.g.*, collector circuits of transistors. Voltage and current sources are called active elements because they provide electrical energy to a circuit.

Unlike a voltage source, which we can imagine as two oppositely charged electrodes, it is difficult to visualise the structure of a current source. However, as we will learn in later sections, a real current source can always be converted into a real voltage source. In other words, we can regard a current source as a convenient fiction that aids in solving circuit problems, yet we feel secure in the knowledge that the current source can be replaced by the equivalent voltage source, if so desired.

2.23. Ideal Voltage Source or Constant-Voltage Source

An **ideal voltage source** (*also called constant-voltage source*) *is one that maintains a constant terminal voltage, no matter how much current is drawn from it.*

An ideal voltage source has zero internal resistance. Therefore, it would provide constant terminal voltage regardless of the value of load connected across its terminals. For example, an ideal 12V source would maintain 12V across its terminals when a 1 MΩ resistor is connected (so I = 12 V/1 MΩ = 12A) as well as when a 1 kΩ resistor is connected (I = 12 mA) or when a 1 Ω resistor is connected (I = 12A). This is illustrated in Fig. 2.86.

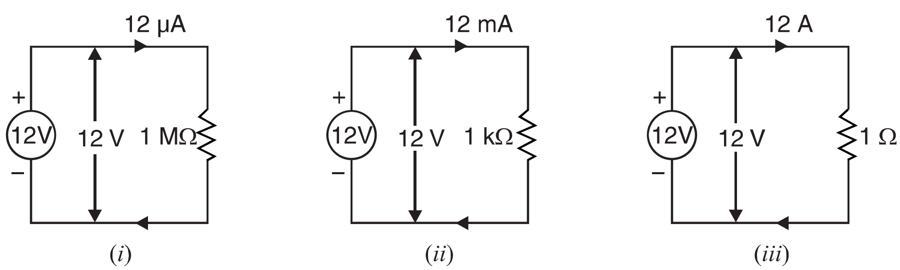

Fig. 2.86

It is not possible to construct an ideal voltage source because every voltage source has some internal resistance that causes the terminal voltage to fall due to the flow of current. *However, if the internal resistance of voltage source is very small, it can be considered as a constant voltage source.* This is illustrated in Fig. 2.87. It has a d.c. source of 6 V with an internal resistance R_i = 0.005 Ω. If the load current varies over a wide range of 1 to 10 A, for any of these values, the internal drop across R_i (= 0.005 Ω) is less than 0.05 volt. Therefore, the voltage output of the source is between 5.995 and 5.95 volts. This can be considered constant voltage compared with wide variations in load current. The practical example of a constant voltage source is the lead-acid cell. The internal resistance of lead-acid cell is very small (about 0.01 Ω) so that it can be regarded as a constant voltage source for all practical purposes. A constant voltage source is represented by the symbol shown in Fig. 2.88.

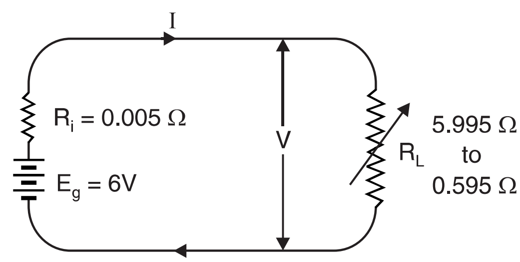

Fig. 2.87

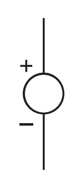

Fig. 2.88

2.24. Real Voltage Source

A real or non-ideal voltage source has low but finite internal resistance (R_{int}) that causes its terminal voltage to decrease when load current is increased and *vice-versa. A* **real voltage source** *can be represented as an ideal voltage source in series with a resistance equal to its internal resistance* (R_{int}) *as shown in Fig. 2.89.*

When load R_L is connected across the terminals of a real voltage source, a load current I_L flows through the circuit so that output voltage V_o is given by ;

$$V_o = E - I_L R_{int}$$

Here E is the voltage of the ideal voltage source *i.e.*, it is the potential difference between the terminals of the source when no current (*i.e.*, $I_L = 0$) is drawn. Fig. 2.90 shows the graph of output voltage V_o versus load current I_L of a real or non-ideal voltage source.

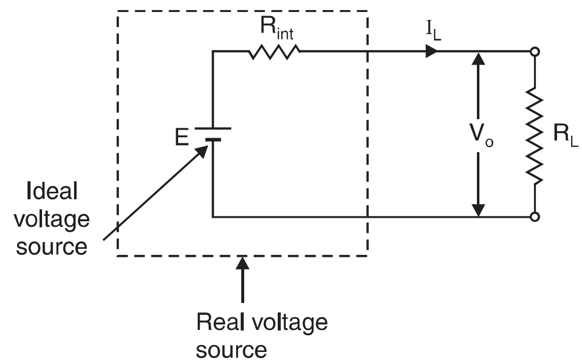

Fig. 2.89

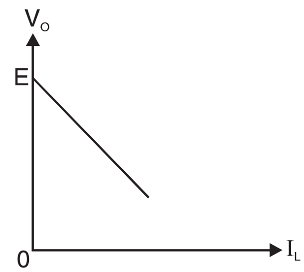

Fig. 2.90

As R_{int} becomes smaller, the real voltage source more closely approaches the ideal voltage source. Sometimes it is convenient when analysing electric circuits to assume that a real voltage source behaves like an ideal voltage source. This assumption is justified by the fact that in circuit analysis, we are not usually concerned with changing currents over a wide range of values.

2.25. Ideal Current Source

An **ideal current source** *or* **constant current source** *is one which will supply the same current to any resistance* (*load*) *connected across its terminals.*

An ideal current source has infinite internal resistance. Therefore, it supplies the same current to any resistance connected across its terminals. This is illustrated in Fig. 2.91. The symbol for ideal current source is shown in Fig. 2.92. The arrow shows the direction of current (conventional) produced by the current source.

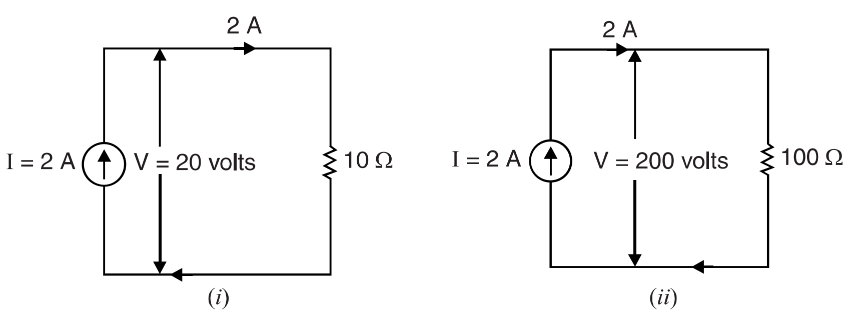

Fig. 2.91

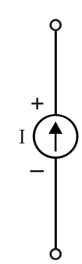

Fig. 2.92

Since an ideal current source supplies the same current regardless of the value of resistance connected across its terminals, it is clear that the terminal voltage V of the current source will depend on the value of load resistance. For example, if a 2 A current source has 10 Ω across its terminals, then terminal voltage of the source is $V = 2\text{ A} \times 10\ \Omega = 20$ volts. If load resistance is changed to 100 Ω, then terminal voltage of the current source becomes $V = 2\text{ A} \times 100\ \Omega = 200$ volts. This is illustrated in Fig. 2.91.

2.26. Real Current Source

A real or non-ideal current source has high but finite internal resistance (R_{int}). Therefore, the load current (I_L) will change as the value of load resistance (R_L) changes. *A* **real current source** *can be represented by an ideal current source* (I) *in parallel with its internal resistance* (R_{int}) *as shown in Fig. 2.93*. When load resistance R_L is connected across the terminals of the real current source, the load current I_L is equal to the current I from the ideal current source *minus* that part of the current that passes through the parallel internal resistance (R_{int}) *i.e.*,

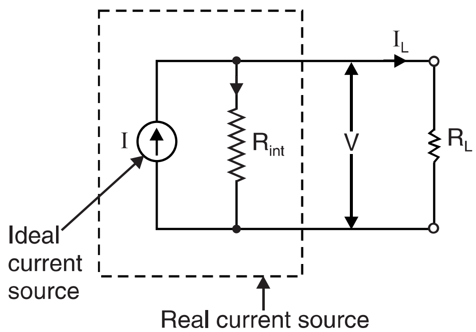

Fig. 2.93

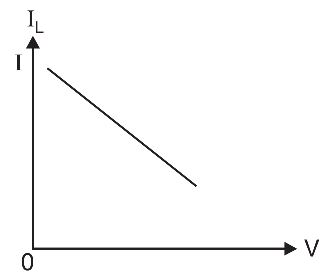

Fig. 2.94

$$I_L = I - \frac{V}{R_{int}}$$

where V = P.D. between output terminals

Fig. 2.94 shows the graph of load current I_L versus output voltage V of a real current source.

Note that load current I_L is less than it would be if the source were ideal. As the internal resistance of real current source becomes greater, the current source more closely approaches the ideal current source.

Note. Current sources in parallel add *algebraically*. If two current sources are supplying currents in the same direction, their equivalent current source supplies current equal to the sum of the individual currents. If two current sources are supplying currents in the opposite directions, their equivalent current source supplies a current equal to the difference of the currents of the two sources.

2.27. Source Conversion

A real voltage source can be converted to an *equivalent* real current source and *vice-versa*. When the conversion is made, the sources are equivalent in every sense of the word; it is impossible to make any measurement or perform any test at the external terminals that would reveal whether the source is a voltage source or its equivalent current source.

(*i*) Voltage to current source conversion. Let us see how a real voltage source can be converted to an equivalent current source. We know that a real voltage source can be represented by constant voltage E in series with its internal resistance R_{int} as shown in Fig. 2.95 (*i*).

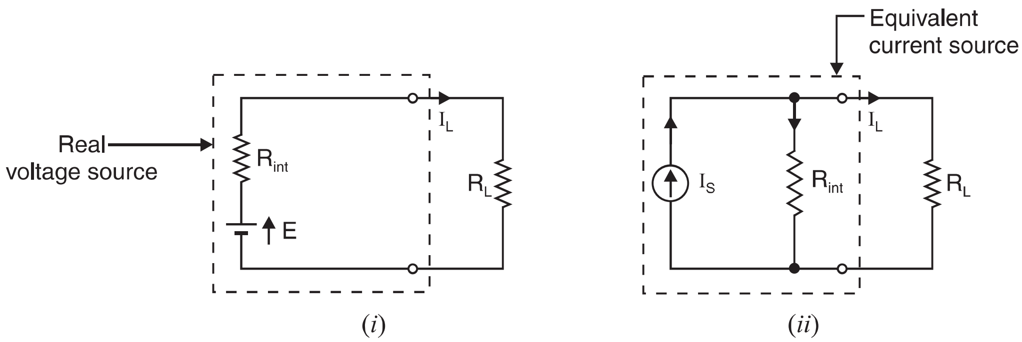

Fig. 2.95

It is clear from Fig. 2.95 (*i*) that load current I_L is given by ;

$$I_L = \frac{E}{R_{int} + R_L} = \frac{\dfrac{E}{R_{int}}}{\dfrac{R_{int} + R_L}{R_{int}}} = \frac{E}{R_{int}} \times \frac{R_{int}}{R_{int} + R_L}$$

$$\therefore \qquad I_L = I_S \times \frac{R_{int}}{R_{int} + R_L} \qquad \text{...(}i\text{)}$$

where $I_S = \dfrac{E^*}{R_{int}}$

= Current which would flow in a short circuit across the output terminals of voltage source in Fig. 2.95 (*i*)

From eq. (*i*), the voltage source appears as a current source of current I_S which is dividing between the internal resistance R_{int} and load resistance R_L connected in parallel as shown in Fig. 2.95 (*ii*). Thus the current source shown in Fig. 2.95 (*ii*) (dotted box) is equivalent to the real voltage source shown in Fig. 2.95 (*i*) (dotted box).

Thus a real voltage source of constant voltage E and internal resistance R_{int} is equivalent to a current source of current $I_S = E/R_{int}$ and R_{int} in parallel with current source.

Note that internal resistance of the equivalent current source has the same value as the internal resistance of the original voltage source but is in parallel with current source. The two circuits shown in Fig. 2.95 are equivalent and either can be used for circuit analysis.

(*ii*) Current to voltage source conversion. Fig. 2.96 (*i*) shows a real current source whereas Fig. 2.96 (*ii*) shows its equivalent voltage source. Note that series resistance R_{int} of the voltage source

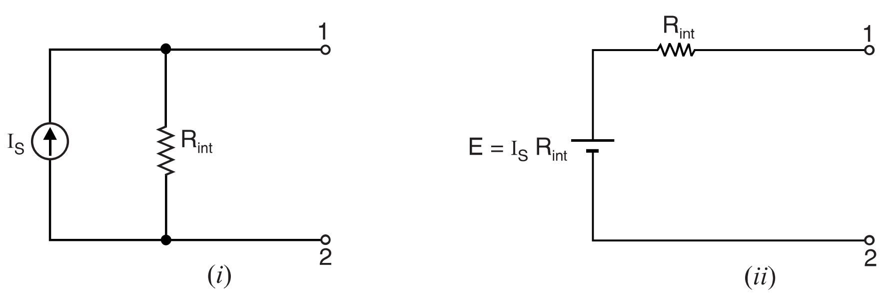

Fig. 2.96

* The source voltage is E and its internal resistance is R_{int}. Therefore, E/R_{int} is the current that would flow when source terminals in Fig. 2.95 (*i*) are shorted.

has the same value as the parallel resistance of the original current source. The value of voltage of the equivalent voltage source is $E = I_S R_{int}$ where I_S is the magnitude of current of the current source.

Note that the two circuits shown in Fig. 2.96 are equivalent and either can be used for circuit analysis.

Note. The source conversion (voltage source into equivalent current source and vice-versa) often simplifies the analysis of many circuits. Any resistance that is in series with a voltage source, whether it be internal or external resistance, can be included in its conversion to an equivalent current source. Similarly, any resistance in parallel with current source can be included when it is converted to an equivalent voltage source.

Example 2.49. *Show that the equivalent sources shown in Fig. 2.97 have exactly the same terminal voltage and produce exactly the same external current when the terminals (i) are shorted, (ii) are open and (iii) have a 500 Ω load connected.*

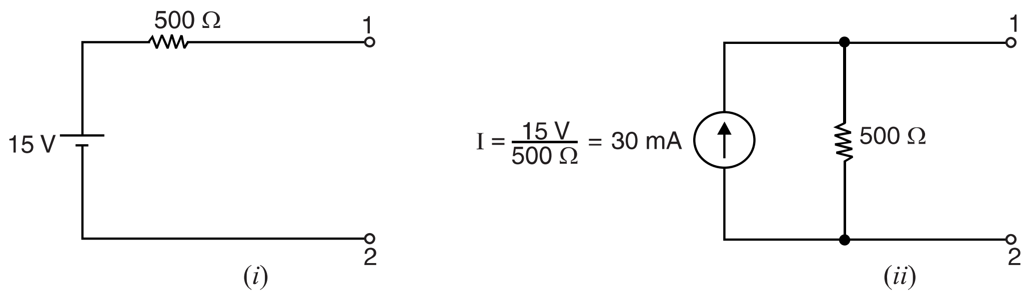

Fig. 2.97

Solution. Fig 2.97 (*i*) shows a voltage source whereas Fig. 2.97 (*ii*) shows its equivalent current source.

(*i*) When terminals are shorted. Referring to Fig. 2.98, the terminal voltage is 0 V in both circuits because the terminals are shorted.

$$I_L = \frac{15 \text{ V}}{500\ \Omega} = 30 \text{ mA} \text{ ... voltage source}$$

$$I_L = 30 \text{ mA} \text{ ...current source}$$

Note that in case of current source, 30 mA flows in the shorted terminals because the short diverts all of the source current around the 500 Ω resistor.

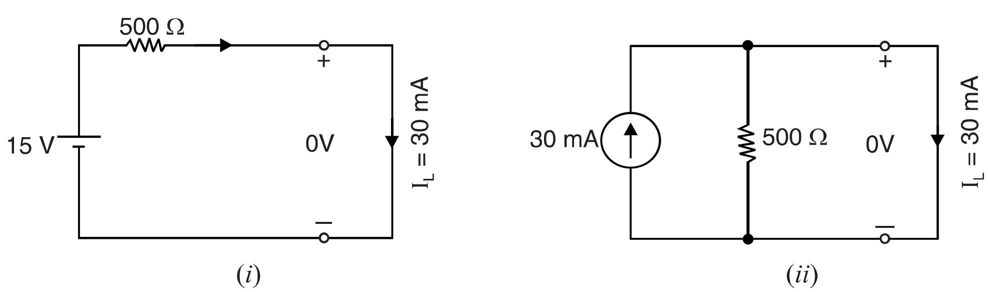

Fig. 2.98

(*ii*) When the terminals are open. Referring to Fig. 2.99 (*i*), the voltage across the open terminals of voltage source is 15 V because no current flows and there is no voltage drop across 500 Ω resistor. Referring to Fig. 2.99 (*ii*), the voltage across the open terminals of the current source is also 15 V ; $V = 30 \text{ mA} \times 500\ \Omega = 15$ V. The current flowing from one terminal into the other is zero in both cases because the terminals are open.

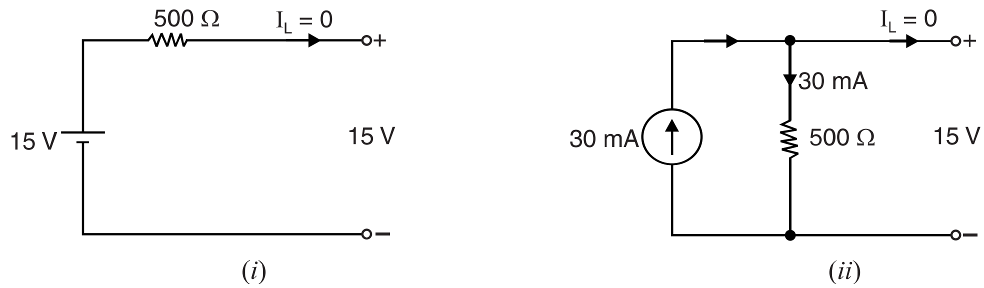

Fig. 2.99

(*iii*) Terminals have a 500 Ω load connected.

(*a*) Voltage source. Referring to Fig. 2.100 (*i*),

$$\text{Current in } R_L,\ I_L = \frac{15\text{ V}}{(500+500)\,\Omega} = 15\text{ mA}$$

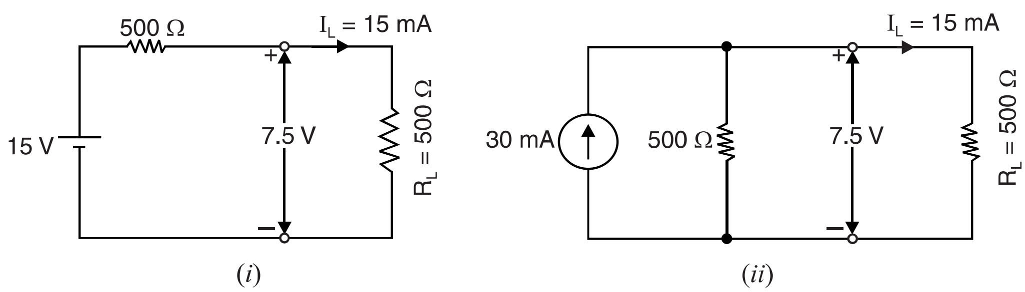

Fig. 2.100

Terminal voltage of source, $V = I_L R_L = 15\text{ mA} \times 500\ \Omega = 7.5\text{ V}$

(*b*) Current source. Referring to Fig. 2.100 (*ii*),

$$\text{Current in } R_L,\ I_L = 30 \times \frac{500}{500+500} = 15\text{ mA}$$

$$\text{Terminal voltage of source} = I_L R_L = 15\text{ mA} \times 500\ \Omega = 7.5\text{ V}$$

We conclude that equivalent sources produce exactly the same voltages and currents at their external terminals, no matter what the load and that they are therefore indistinguishable.

Example 2.50. *Find the current in 6 kΩ resistor in Fig. 2.101 (i) by converting the current source to a voltage source.*

Solution. Since we want to find the current in 6 kΩ resistor, we use 3 kΩ resistor to convert the current source to an equivalent voltage source. Referring to Fig. 2.101 (*ii*), the equivalent voltage is

$$E = 15\text{ mA} \times 3\text{ k}\Omega = 45\text{ V}$$

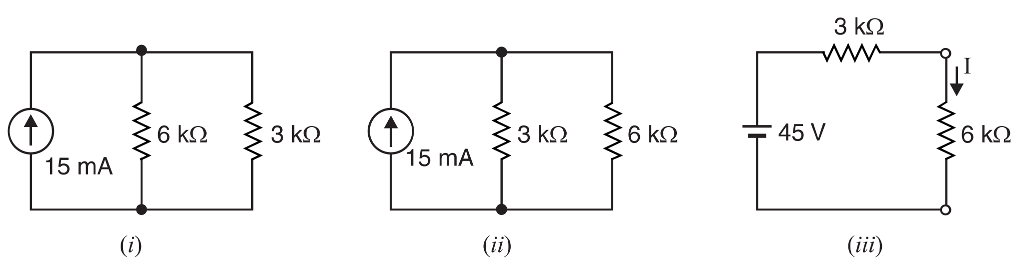

Fig. 2.101

The circuit then becomes as shown in Fig. 2.101 (*iii*). Note that polarity of the equivalent voltage source is such that it produces current in the same direction as the original current source.

Referring to Fig. 2.101 (*iii*), the current in 6 kΩ is

$$I = \frac{45\text{ V}}{(3+6)\text{ k}\Omega} = \mathbf{5\text{ mA}}$$

In the series circuit shown in Fig. 2.101 (*iii*), it would appear that current in 3 kΩ resistor is also 5 mA. However, 3 kΩ resistor was involved in source conversion, so we *cannot* conclude that there is 5 mA in the 3 kΩ resistor of the original circuit [See Fig. 2.101 (*i*)]. Verify that the current in the 3 kΩ resistor in that circuit is, in fact, 10 mA.

Example 2.51. *Find the current in the 3 kΩ resistor in Fig. 2.101* (*i*) *above by converting the current source to a voltage source.*

Solution. The circuit shown in Fig. 2.101 (*i*) is redrawn in Fig. 2.102 (*i*). Since we want to find the current in 3 kΩ resistor, we use 6 kΩ resistor to convert the current source to an equivalent voltage source. Referring to Fig. 2.102 (*i*), the equivalent voltage is

$$E = 15\text{ mA} \times 6\text{ k}\Omega = 90\text{ V}$$

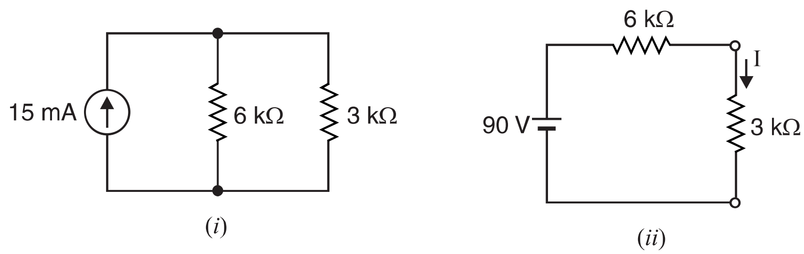

Fig. 2.102

The circuit then reduces to that shown in Fig. 2.102 (*ii*). The current in 3 kΩ resistor is

$$I = \frac{90\text{ V}}{(6+3)\text{k}\Omega} = \frac{90\text{ V}}{9\text{ k}\Omega} = \mathbf{10\text{ mA}}$$

Example 2.52. *Find the current in various resistors in the circuit shown in Fig. 2.103* (*i*) *by converting voltage sources into current sources.*

Solution. Referring to Fig. 2.103 (*i*), the 100 Ω resistor can be considered as the internal resistance of 15 V battery. The equivalent current is

$$I = \frac{15\text{ V}}{100\ \Omega} = 0.15\text{ A}$$

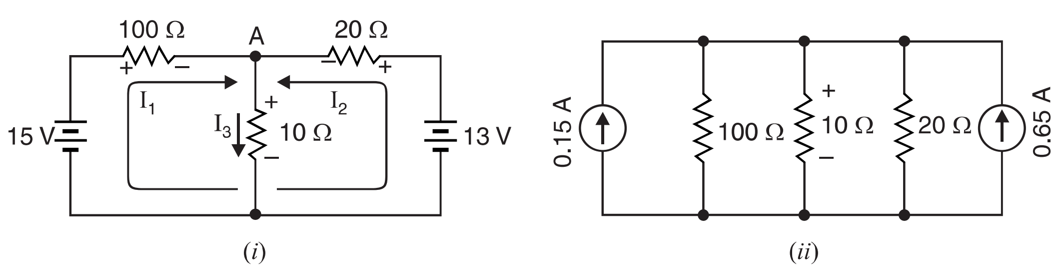

Fig. 2.103

Similarly, 20 Ω resistor can be considered as the internal resistance of 13 V battery. The equivalent current is

$$I = \frac{13\text{ V}}{20\,\Omega} = 0.65\text{ A}$$

Replacing the voltage sources with current sources, the circuit becomes as shown in Fig. 2.103 (*ii*). The current sources are parallel-aiding for a total flow = 0.15 + 0.65 = 0.8 A. The parallel resistors can be combined.

$$100\,\Omega \,||\, 10\,\Omega \,||\, 20\,\Omega = 6.25\,\Omega$$

The total current flowing through this resistance produces the drop :

$$0.8\text{ A} \times 6.25\,\Omega = 5\text{ V}$$

This 5 V drop can now be "transported" back to the original circuit. It appears across 10 Ω resistor [See Fig. 2.104]. Its polarity is negative at the bottom and positive at the top. Applying Kirchhoff's voltage law (KVL), the voltage drop across 100 Ω resistor = 15 – 5 = 10 V and drop across 20 Ω resistor = 13 – 5 = 8 V.

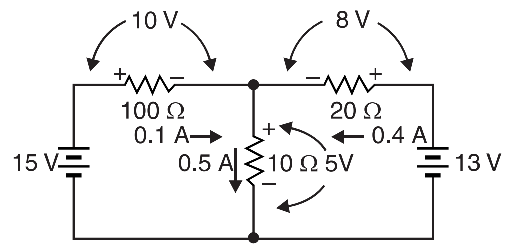

Fig. 2.104

$$\therefore \quad \text{Current in 100 Ω resistor} = \frac{10}{100} = \mathbf{0.1\ A}$$

$$\text{Current in 10 Ω resistor} = \frac{5}{10} = \mathbf{0.5\ A}$$

$$\text{Current in 20 Ω resistor} = \frac{8}{20} = \mathbf{0.4\ A}$$

Example 2.53. *Find the current in and voltage across 2 Ω resistor in Fig. 2.105.*

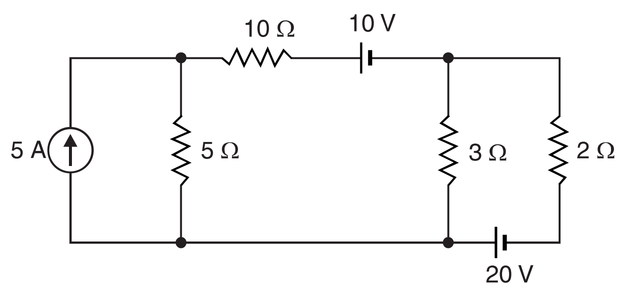

Fig. 2.105

Solution. We use 5 Ω resistor to convert the current source to an equivalent voltage source. The equivalent voltage is

$$E = 5\text{ A} \times 5\,\Omega = 25\text{ V}$$

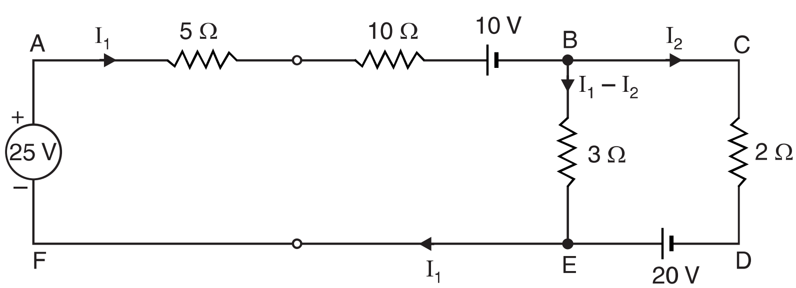

Fig. 2.106

The circuit shown in Fig. 2.105 then becomes as shown in Fig. 2.106.

Loop *ABEFA*. Applying Kirchhoff's voltage law to loop *ABEFA*, we have,

$$-5 I_1 - 10 I_1 - 10 - 3 (I_1 - I_2) + 25 = 0$$

or $$-18 I_1 + 3 I_2 = -15 \qquad ...(i)$$

Loop *BCDEB*. Applying Kirchhoff's voltage law to loop *BCDEB*, we have,

$$-2 I_2 + 20 + 3 (I_1 - I_2) = 0$$

or $$3 I_1 - 5 I_2 = -20 \qquad ...(ii)$$

Solving equations (*i*) and (*ii*), we get, $I_2 = 5$ A.

∴ Current through 2 Ω resistor = I_2 = **5 A**

Voltage across 2 Ω resistor = $I_2 \times 2 = 5 \times 2 =$ **10 V**

Example 2.54. *Find the current in 28 Ω resistor in the circuit shown in Fig. 2.107.*

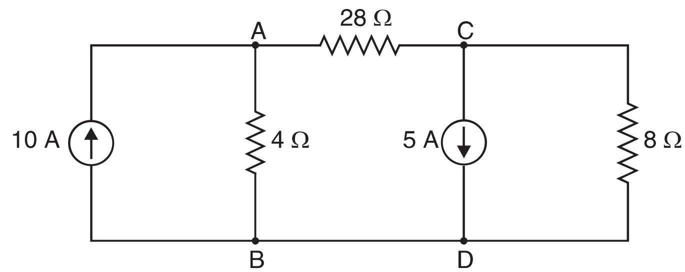

Fig. 2.107

Solution. The two current sources cannot be combined together because 28 Ω resistor is present between points *A* and *C*. However, this difficulty is overcome by converting current sources into equivalent voltage sources. Now 10 A current source in parallel with 4 Ω resistor can be converted into equivalent voltage source of voltage = 10 A × 4 Ω = 40 V in series with 4 Ω resistor as shown in Fig. 2.108 (*i*). Note that polarity of the equivalent voltage source is such that it provides current in the same direction as the original current source.

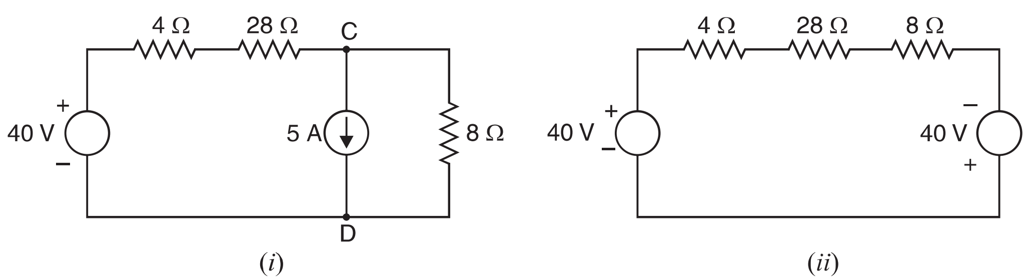

Fig. 2.108

Similarly, 5 A current source in parallel with 8 Ω resistor can be converted into equivalent voltage source of voltage = 5 A × 8 Ω = 40 V in series with 8 Ω resistor. The circuit then becomes as shown in Fig. 2.108 (*ii*). Note that polarity of the voltage source is such that it provides current in the same direction as the original current source. Referring to Fig. 2.108 (*ii*),

$$\text{Total circuit resistance} = 4 + 28 + 8 = 40\ \Omega$$

$$\text{Total voltage} = 40 + 40 = 80 \text{ V}$$

$$\therefore \quad \text{Current in 28 } \Omega \text{ resistor} = \frac{80}{40} = \mathbf{2\ A}$$

Tutorial Problems

1. By performing an appropriate source conversion, find the voltage across 120 Ω resistor in the circuit shown in Fig. 2.110. **[20 V]**

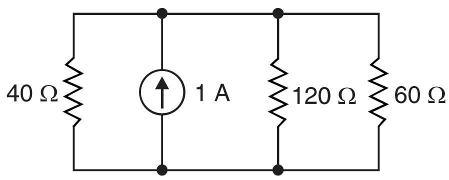

Fig. 2.110

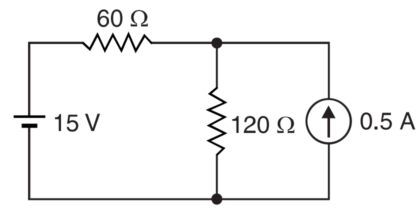

Fig. 2.111

2. By performing an appropriate source conversion, find the voltage across 120 Ω resistor in the circuit shown in Fig. 2.111. **[30 V]**

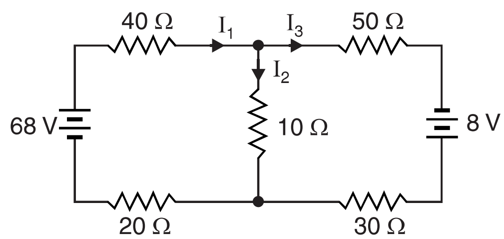

Fig. 2.112

3. By performing an appropriate source conversion, find the currents I_1, I_2 and I_3 in the circuit shown in Fig. 2.112. **[I_1 = 1 A; I_2 = 0.2 A; I_3 = 0.8 A]**

2.28. Independent Voltage and Current Sources

So far we have been dealing with independent voltage and current sources. We now give brief description about these two active elements.

(*i*) Independent voltage source. *An independent voltage source is a two-terminal element (e.g. a battery, a generator etc.) that maintains a specified voltage between its terminals.*

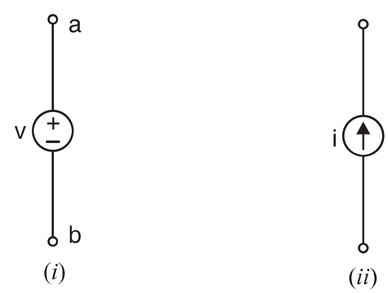

Fig. 2.113

An independent voltage source provides a voltage independent of any other voltage or current. The symbol for independent voltage source having v volts across its terminals is shown in Fig. 2.113. (*i*). As shown, the terminal a is v volts above terminal b. If v is greater than zero, then terminal a is at a higher potential than terminal b. In Fig. 2.113 (*i*), the voltage v may be time varying or it may be constant in which case we label it V.

(*ii*) Independent current source. *An independent current source is a two-terminal element through which a specified current flows.*

An independent current source provides a current that is completely independent of the voltage across the source. The symbol for an independent current source is shown in Fig. 2.113 (*ii*) where i is the specified current. The direction of the current is indicated by the arrow. In Fig. 2.113 (*ii*), the current i may be time varying or it may be constant in which case we label it I.