FORMAL LANGUAGES AND AUTOMATA THEORY

CS 3003

AUTUMN END SEMESTER - 2018

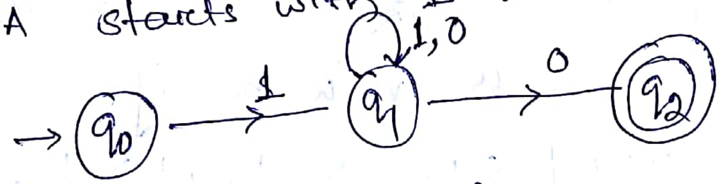
(1) (a) (i) DFA stands for Deterministic Finite Automata while NFA stands for Non-Deterministic finite Automata

(ii) In DFA the next possible state is distinctly set while in NFA each pair of state & i/p symbol can have many possible next states

(iii) DFA requires more space while NFA requires less space.

(iv) All DFA's are subset of NFA.

(b) NFA starts with 1 & ends with 0.



(c) CFG without λ - production

$S \to a \mid Ab \mid aBa \mid b \mid aa$

$A \to b$

$B \to b$

(d) Given CFG $S \to 0S1 \mid 0A \mid 0 \mid 1B \mid 1$

$\cancel{A \to 0A \mid 0 \to 0^+}$

$\cancel{B \to 1B \mid 1 \to 1^+}$

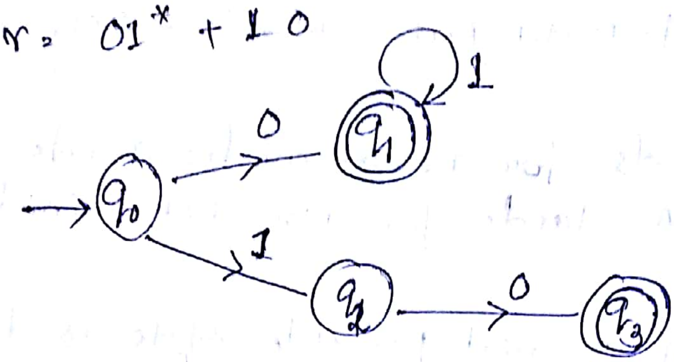
$\cancel{S \to 0S1 \mid 00^+ \mid 0 \mid 11^+ \mid 1}$

Language Accepted by the Grammar is

$\{0^n 1^m : m \neq n, \; m, n \geq 0\}$

e) NFA for the Regular Expression

$r = 01^* + 10$



(f) Pumping Lemma for context-free Language.

- Let L be an infinite context free Language.
- There exist some positive integer n, such that for any valid string $z \in L$ with $|z| \geq n$ can be decomposed as. $z = uvwxy$

z

u v w x y

with $|uvw| \leq n$ & $|vx| \geq 1$, such that

$uv^iwx^iy \in L$ for all $i \geq 0$.

if the given Language L is CFL, then uv^iwx^iy $\forall i \geq 0$ should be in L, otherwise L is not CFL.

(g) Recursive Language: A Language L over the alphabet Σ is called recurssive if there is a Turing machine that accepts every word in L and rejects every word in L'.

→ so the turing M/c will always halt in this case.

Recursive Enumerable: A Language L over the alphabet Σ is called recursively enumerable if there is a Turing Machine T that accepts every word in L and either rejects or loops forever for every word in L'.

h) → A Deterministic pushdown automata has atmost one legal transition for the same combination of input symbol, state, and top stack symbol. (ordinary)
→ if $\delta(q, \lambda, b)$ is not empty then $\delta(q, c, b)$ is empty for evry $c \in \Sigma$
NO, the family of DPDA are not equivalent to the family of NPDA [Ex: Even length pallindrome Pallindrome is accepted by NPDA not by DPDA]

(i) Let $L_1 = a^n b^n c^m$

$L_2 = a^m b^n c^n$

$L_1 \cap L_2 = a^n b^n c^m \cap a^m b^n c^n \Rightarrow a^n b^n c^n$

which is not CFL

(j) A PDA can only access the top of its stack, where as TM can access any possition on an infinite tape. The infinite tape cannot be simulated with a single stack, so a PDA is less computationally powerful — there are algorithms that can be programmed with a TM can not be programmed with a PDA.

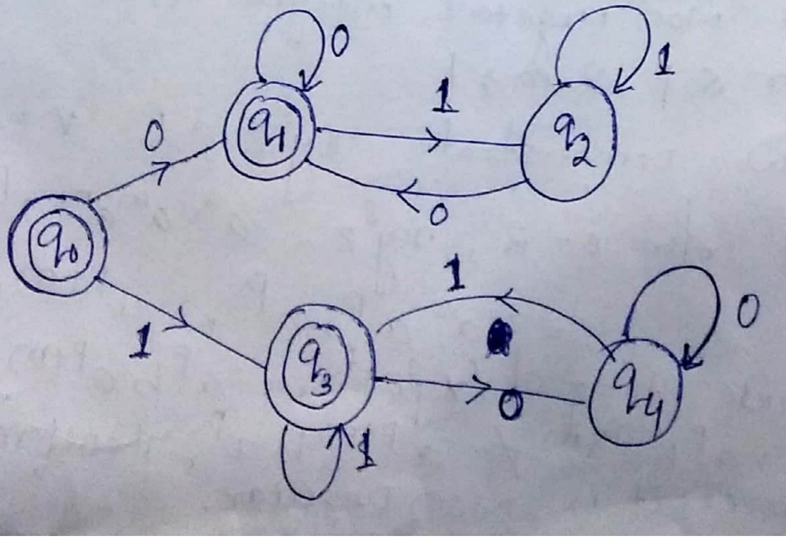
PDA : FA + 1 stack

TM : FA + 2 stack.

Q.2 (a) The NFA Recognizes all strings that contains two 0's separated by substring whose length is multiple of 3.

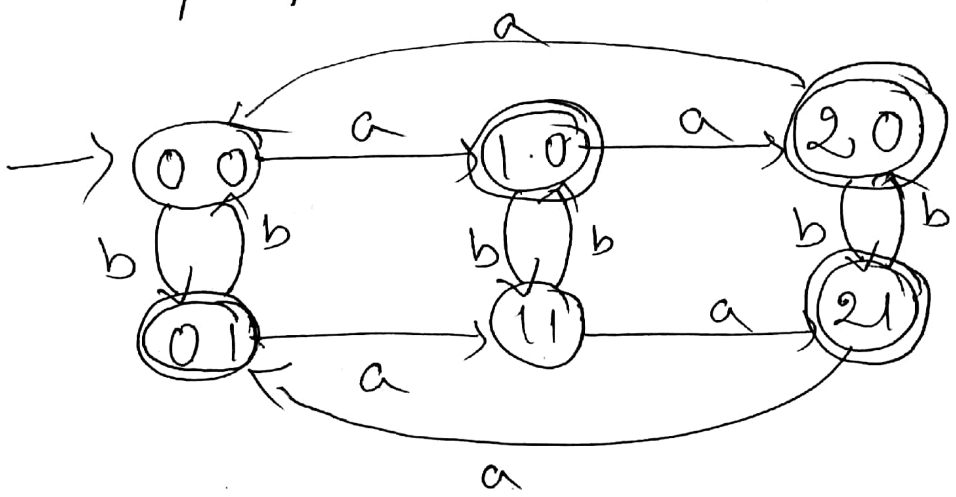
RE :- $(0+1)^* 0 ((0+1)(0+1)(0+1))^* 0 (0+1)^*$

(b)



(3)(a) $L = \{ v / v \in (a+b)^* \text{ and } n_a(v) \bmod 3 \neq n_b(v) \bmod 2 \}$ is regular

Ans If we can able to construct the DFA of a language, then we can say that that language is regular.



Hence, the above language is regular

Q. 3.(b) observe that $L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$

since Regular Language are closed under union and complementation, we have

- $\overline{L_1}$ & $\overline{L_2}$ are Regular
- $\overline{L_1} \cup \overline{L_2}$ is Regular
- Hence $L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$ is regular (proved)

Q. 4(a) Pumping Lemma for Regular Language:

↳ The pumping Lemma says that for any Regular Language L, there exist a constant p, such that any word w in L with length atleast p can be split into three substrings, $w = xyz$, where $|xy| \leq w$ & $|y| \geq 1$, such that

$xy^iz \in L \;\forall\, i > 0$

↳ if there exist atleast one value of i such that $xy^iz \notin L$, then L is not Regular.

$L = \{ S : S = S^R \}$ set of all pallindromes

→ Let $S = a^p b a^p$, where S is a pallindrome,

so $S \in L$, Also $|S| = 2p+1 \geq p$.

→ Divide S into three following parts, such that $x = a^k$, $y = a^m$, $z = a^{p-m-k} b a^p$, for some Non Negative integer k, m so that $k+m \leq p$ & $m \geq 1$

→ Now Demonstrate $xy^iz \in L \;\forall\, i > 0$.

but for $i = 2$, $xy^2z = a^k a^m a^m a^{p-m-k} b a^p$

$= a^m a^p b a^p = a^{p+m} b a^p$

→ Reverse string of $(xy^2z)^R$ is $a^p b a^{p+m}$

$a^p b a^{p+m} \neq a^{p+m} b a^p$, Therefore $xy^2z \notin L$

Hence it is Non Regular.

Q. 4.(b). Given CFG $S \to XaaX$
$X \to ax \mid bx$

step-I : Convert the CFG to GNF (2 mark)

$S \to axaaX \mid bxaaX$
$X \to ax \mid bx$

$S \to aB \mid bB$
$B \to xaaX$
$X \to ax \mid bx$

$\Rightarrow$

$S \to aB \mid bB$
$B \to xD$
$D \to aaX$
$X \to ax \mid bx$

$S \to aB \mid bB$
$B \to xD$
$D \to aE$
$E \to aX$
$X \to ax \mid bx$

$\Rightarrow$

$S \to aB \mid bB$
$B \to xD$
$D \to aE$
$E \to aX$
$X \to aF \mid bF$
$F \to x$.

PDA : (2 mark)

$\delta(q_0, \lambda, Z) = \{(q_1, SZ)\}$
$\delta(q_1, a, S) = \{(q_1, B)\}$
$\delta(q_1, b, S) = \{(q_1, B)\}$
$\delta(q_1, x, B) = \{(q_1, D)\}$
$\delta(q_1, a, D) = \{(q_1, E)\}$
$\delta(q_1, a, E) = \{(q_1, X)\}$
$\delta(q_1, a, X) = \{(q_1, F)\}$
$\delta(q_1, b, X) = \{(q_1, F)\}$
$\delta(q_1, x, F) = \{(q_1, \lambda)\}$
$\delta(q_1, \lambda, Z) = \{(q_2, \lambda)\}$

Q5.

(a)

$S \to 0S0 \mid 1S1 \mid 0A1 \mid 1A0$

$A \to 0A \mid 1A \mid \lambda$

(b)

(i)

$S \to AT \mid BV \mid AB$

$T \to SA$

$V \to SB$

$A \to a$

$B \to b$

(ii)

$S \to aBAD \mid bBAD \mid aD$

$B \to aBA \mid bBA \mid a$

$A \to a \mid b$

$D \to b$

Q 6)

(a) 1) λ-production Removal

$S \rightarrow aAA$

$B \rightarrow bBB \mid D \mid bB \mid b$

$B \rightarrow ab$

$C \rightarrow aB \mid a$

2) Unit production Removal

$S \rightarrow aAA$

$B \rightarrow bBB \mid bB \mid b$

$B \rightarrow ab$

$C \rightarrow aB \mid a$

3) Useless removal

No production is useful in the CFG

All productions are useless.

(b)(i) LMD:

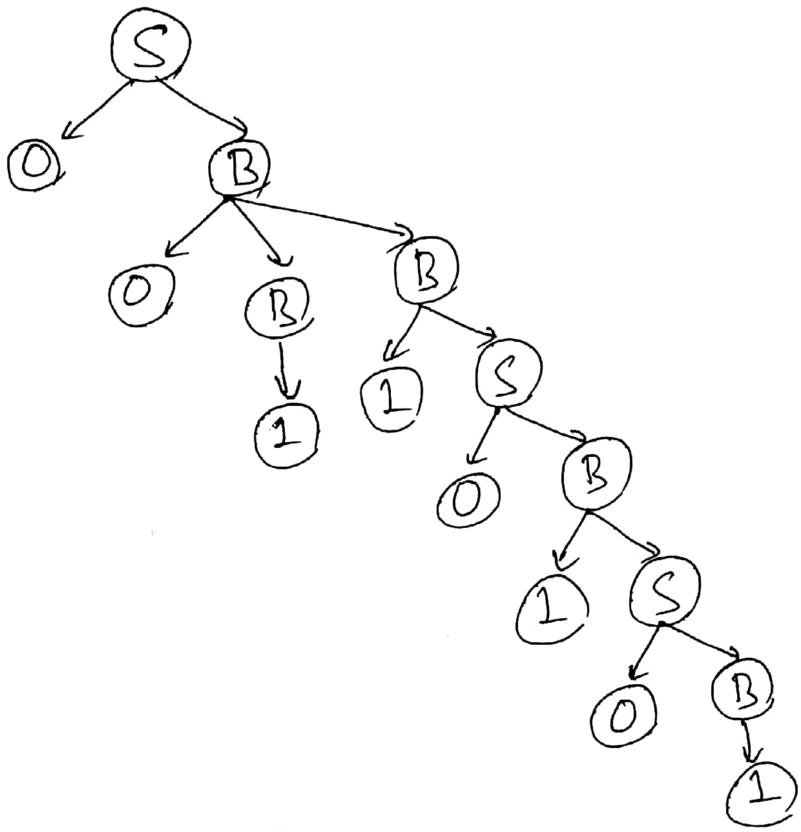
$S \Rightarrow 0B \Rightarrow 00BB \Rightarrow 001B \Rightarrow 0011S \Rightarrow 00110B \Rightarrow$
$001101S \Rightarrow 0011010B \Rightarrow 00110101$

(ii) RMD:

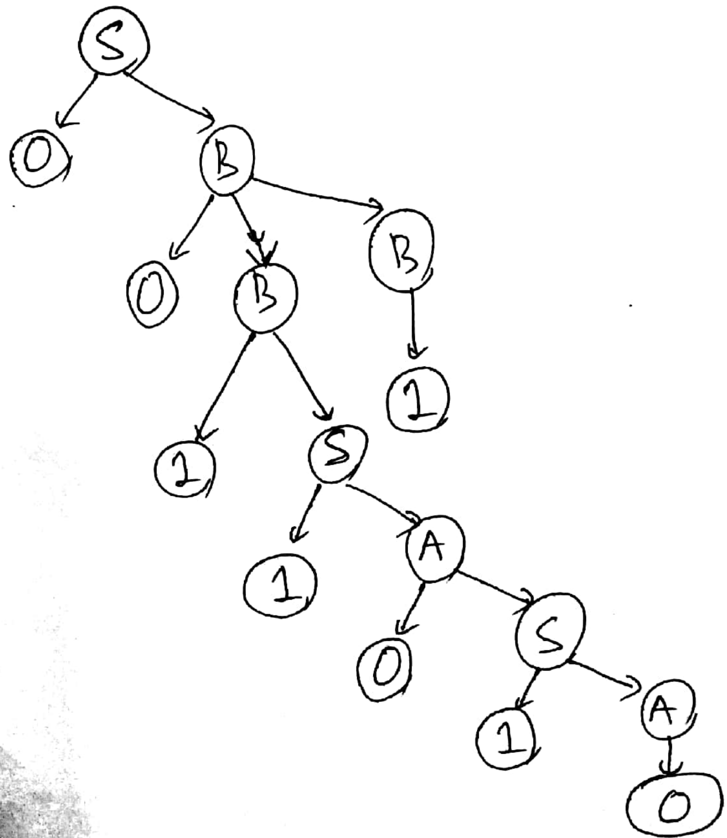
$S \Rightarrow 0B \Rightarrow 00BB \Rightarrow 00B1 \Rightarrow 001S1 \Rightarrow 0011A1$
$\Rightarrow 00110S1 \Rightarrow 001101A1 \Rightarrow 00110101$

Q6)

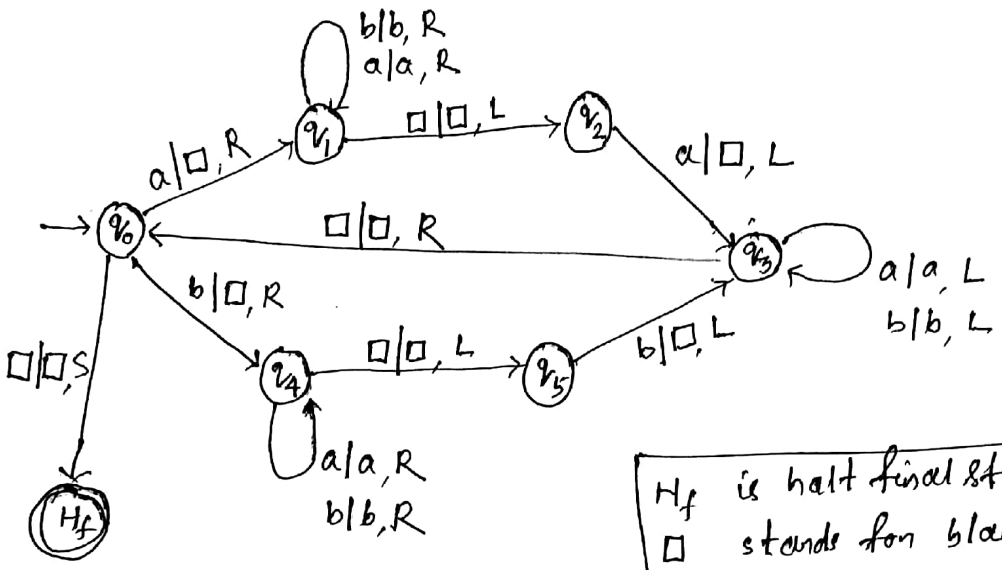
(b)(iii) Derivation Tree



(iv) The grammar is ambiguous because we can have 2 different derivation trees for the string 00110101, one given above and 2nd one,

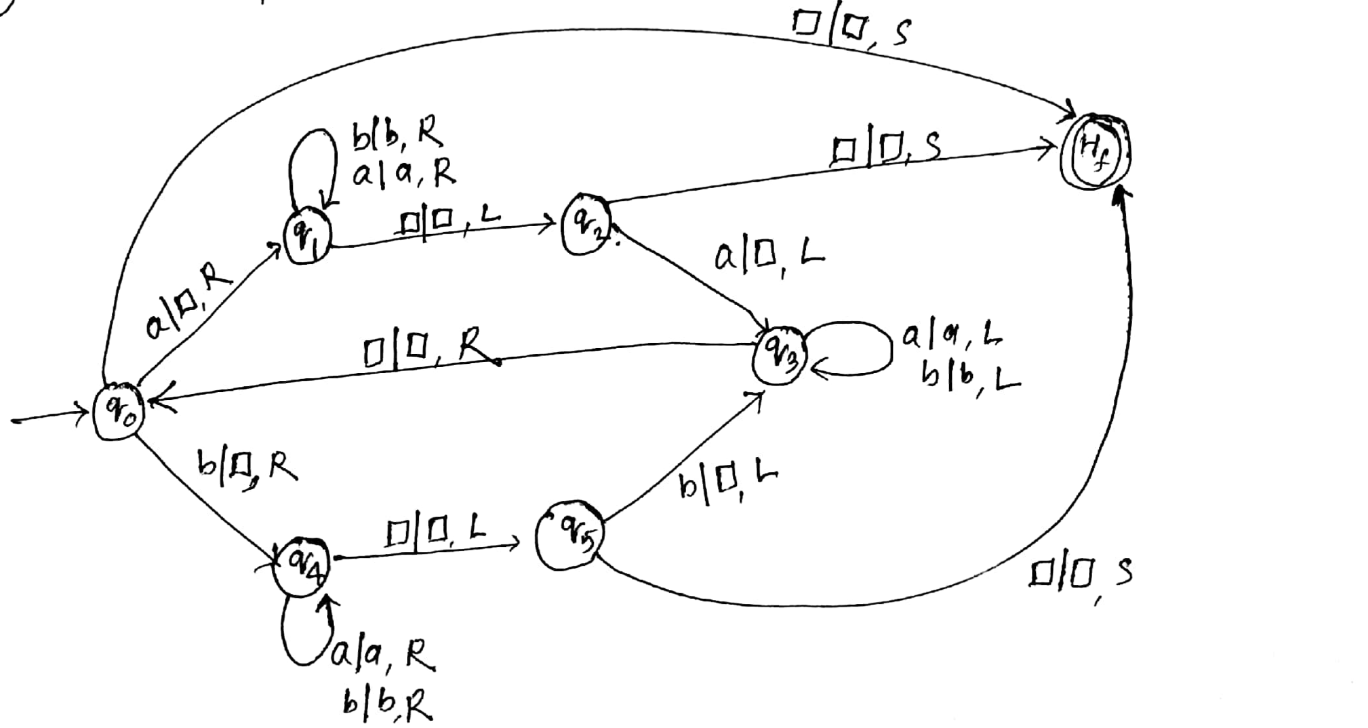


7 (a) $L = \{ww^R \mid w \in (0+1)^*\}$ ⇒ evenlength palindrome



Hf is halt final state
□ stands for blank
Replace a by 0 and b by 1

7 (b) $L = \{x \in \{a, b\}^* \mid x$ is a palindrome$\}$ ⇒ all palindromes (both even & odd)



Hf is Halt final state
□ stands for blank

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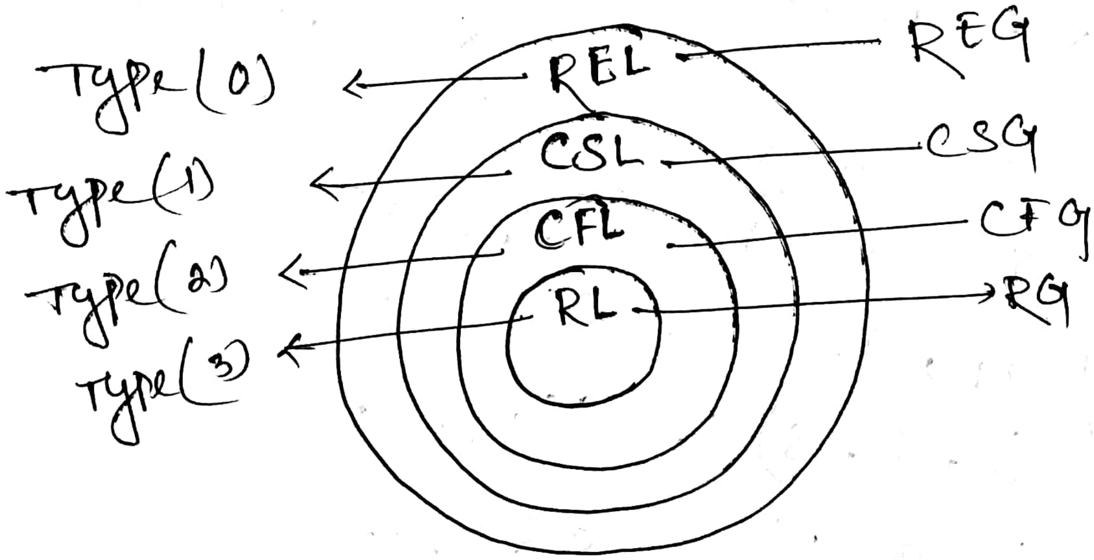
Q. 8 (a) Closure properties of CFL

→ The family of context-free languages is closed under union, concatenation, star-closure, & Reversal.

→ CFG's are not closed under intersection & Complementation.

Only properties – 1 (mark) proof :- (3 mark)
properties with proof : (4 mark)

(b) Chomsky hierarchy:



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