Eigen Basis (Basis of eigenvectors):
Let A be an $n \times n$ matrix. Then an eigen basis is a basis of $\mathbb{R}^n$ consisting of eigenvectors of A.
If A has n distinct eigenvalues, then A has a basis of eigenvectors $X_1, \;\; X_2, \ldots, X_n$ for $\mathbb{R}^n$.
Eigen space:
The eigenvectors corresponding to one and the same eigenvalue λ of A together with null vector 0, form a vector space, called the Eigen space of A corresponding to that λ. It is usually denoted by E_λ.
Algebraic Multiplicity and Geometric Multiplicity:
The order of an eigenvalue λ is called the algebraic multiplicity of λ and it is denoted by M_λ.
The number of linearly independent eigenvectors corresponding to an eigenvalue λ is called the geometric multiplicity of λ and it is denoted by m_λ. Thus, the geometric multiplicity is the dimension of the Eigen space corresponding to this λ.
An eigenvalue λ is regular if $M_\lambda = m_\lambda$.
Example: Find the algebraic multiplicity and geometric multiplicity of the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}$.
Solution: The characteristic equation of A is $\left|A - \lambda I\right| = 0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 2 \\ 0 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (\lambda - 3)^2 = 0$$

$$\Rightarrow \lambda = 3, 3$$

Hence the algebraic multiplicity of $\lambda = 3$ is $M_3 = 2$.
Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be an eigen vector corresponding to $\lambda = 3$. Then

$$AX = \lambda X$$

$$\Rightarrow \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 3 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow 2x_2 = 0, \;\; 3x_2 = 3x_2$$

$$\Rightarrow x_2 = 0$$

So, $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

The eigen vector corresponding to $\lambda = 3$ is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.
Hence the geometric multiplicity of $\lambda = 3$ is $m_3 = 1$.

Results:

1: If A is a $n \times n$ triangular matrix -upper triangular, lower triangular or diagonal, the eigenvalues of A are the diagonal entries of A.

2: $\lambda = 0$ is an eigenvalue of A if A is a singular (noninvertible) matrix.

3: A and A^T have the same eigenvalues.

4: $\det A$ is the product of the eigenvalues of A.

5: If λ be an eigenvalue of A, then

- **a)** $k\lambda$ is an eigenvalue of kA, $k =$ scalar.
- **b)** λ^2 is an eigenvalue of A^2**.**