Ex: 3.3

29)

x	1	2	4	8	16
$p(x)$	0.05	0.10	0.35	0.4	0.1

a) $E(x) = \sum_{x \in D} x\,p(x) = 1 \times 0.05 + 2 \times 0.10 + 4 \times 0.35 + 8 \times 0.4 + 16 \times 0.1$

$= 6.45$

b) $V(x) = \sum_{x \in D} (x-\mu)^2\, p(x) = (1-6.45)^2 \times 0.05 + (2-6.45)^2 \times 0.1 + (4-6.45)^2 \times 0.35$

$+ (8-6.45)^2 \times 0.4 + (16-6.45)^2 \times 0.1$

$= 15.6475$

c) $\sigma_x = \sqrt{\sigma_x^2} = \sqrt{15.6475} \approx 3.955$

d) $\cancel{V(x)} = E(x^2) = \sum_{x \in D} x^2 p(x) = 0.05 + 4 \times 0.1 + 16 \times 0.35 + 64 \times 0.4$

$+ 16^2 \times 0.1 = 57.25$

$\therefore V(x) = E(x^2) - \{E(x)\}^2$

$= 57.25 - 6.45^2$

$= 57.25 - 41.6025$

$= 15.6475$

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30)

Y	0	1	2	3
P(Y)	0·6	0·25	0·1	0·05

a) $E(Y) = \sum_{Y \in D} Y\,P(Y) = 1 \times 0{\cdot}25 + 2 \times 0{\cdot}1 + 3 \times 0{\cdot}05$
$= 0{\cdot}6$

b) $E(100Y^2) = 100\,E(Y^2) = 100 \times (0{\cdot}25 + 4 \times 0{\cdot}1 + 9 \times 0{\cdot}05)$
$= 110$

The expected amount of the surcharge is ₹110

31)

Y	45	46	47	48	49	50	51	52	53	54	55
P(y)	0·05	0·1	0·12	0·14	0·25	0·17	0·06	0·05	0·03	0·02	0·01

~~$V(Y) = \sum_{Y \in D}$~~

$E(Y) = \sum_{Y \in D} Y\,P(y) = 45 \times 0{\cdot}05 + 46 \times 0{\cdot}1 + 47 \times 0{\cdot}12 + 48 \times 0{\cdot}14 + 49 \times 0{\cdot}25$
$+ 50 \times 0{\cdot}17 + 51 \times 0{\cdot}06 + 52 \times 0{\cdot}05 + 53 \times 0{\cdot}03 + 54 \times 0{\cdot}02$
$+ 55 \times 0{\cdot}01$
$= 48{\cdot}84$

$E(Y^2) = \sum_{Y \in D} Y^2 p(y) = 45^2 \times 0{\cdot}05 + 46^2 \times 0{\cdot}1 + 47^2 \times 0{\cdot}12 + 48^2 \times 0{\cdot}14 + 49^2 \times 0{\cdot}25$
$+ 50^2 \times 0{\cdot}17 + 51^2 \times 0{\cdot}06 + 52^2 \times 0{\cdot}05 + 53^2 \times 0{\cdot}03 + 54^2 \times 0{\cdot}02$
$+ 55^2 \times 0{\cdot}01$
$= 2389{\cdot}84$

$\therefore V(Y) = E(Y^2) - \{E(Y)\}^2$
$= 2389{\cdot}84 - 48{\cdot}84^2$
$= 4{\cdot}4944$

$\therefore \sigma_Y = \sqrt{\sigma_Y^2} = \sqrt{4{\cdot}4944} = 2{\cdot}12$

The probability that Y is within 1 standard deviation of its mean value $= P(E(Y) - \sigma_Y < Y < E(Y) + \sigma_Y)$
$= P(48{\cdot}84 - 2{\cdot}12 < Y < 48{\cdot}84 + 2{\cdot}12)$
$= P(46{\cdot}72 < Y < 50{\cdot}96)$
$= P(X = 47) + P(X = 48) + P(X = 49) + P(X = 50)$
$= 0{\cdot}12 + 0{\cdot}14 + 0{\cdot}25 + 0{\cdot}17 = 0{\cdot}68$

Q) 32) $x \rightarrow$ The rated capacity of a freezer of this brand sold at a certain store.

x:	450	500	550
$p(x)$:	0.2	0.5	0.3

$$E(X) = \sum_{x \in D} x\, p(x) = (450 \times 0.2) + (500 \times 0.5) + (550 \times 0.3)$$
$$= 505$$

$$E(X^2) = \sum_{x \in D} x^2\, p(x) = 450^2 \times 0.2 + 500^2 \times 0.5 + 550^2 \times 0.3$$
$$= 256{,}250$$

$V(x) = E(x^2) - \{E(x)\}^2$

$= 256{,}250 - 505^2$

$= 1225$

$\mu = E(x)$

$$V(x) = \sum_{x \in D} (x-\mu)^2 p(x)$$

$$= (450-505)^2 \times 0.2 + (500-505)^2 \times 0.5 + (550-505)^2 \times 0.3$$

b) $E(2.5X - 650) = 2.5 \times E(x) - 650$

$= 2.5 \times 505 - 650$

$= 612.5$

c) ~~V(x) = E~~ $V(2.5X - 650) = (2.5)^2 V(x)$

$= (2.5)^2 \times 1225$

$= 7656.25$

d) $h(x) = X - 0.0003X^2$

$$E(X - 0.0003X^2) = \sum_{x \in D} (X - 0.0003X^2)\, p(x)$$

$$= \sum_{x \in D} x\,p(x) - 0.0003 \sum_{x \in D} x^2 p(x)$$

$= E(x) - 0.0003\, E(x^2)$

$= 505 - 0.0003 \times 256250$

$= 428.125$

{} ↓ don't denote by this ⇓ It represents GIF

33) $D = \{0, 1\}$

~~$x p(x) = p$~~

pmf

x:	0	1
$p(x)$:	$1-p$	p

a) $E(x^2) = \sum_{x \in D} x^2 p(x) = 0^2 \times (1-p) + 1^2 \times p$

$= p$

b) To prove that $V(x) = p(1-p)$

$V(x) = E(x^2) - \{E(x)\}^2 = p - p^2 = p(1-p)$

Note: $E(X^k) = E(x)$
(For a bernouli variable)

c) $E(X^{29}) = \sum_{x \in D} X^{29} \times p(x)$

$= p$ (For a bernouli random variable.)

34) $p(x) = \begin{cases} c/x^3, & x = 1,2,3,\ldots \\ 0, & \text{otherwise} \end{cases}$

$$E(x) = \sum_{x \in D} x\,p(x) = 1 \times \frac{c}{1^3} + 2 \times \frac{c}{2^3} + 3 \times \frac{c}{3^3} + \ldots$$

$$= c\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\right)$$

$$E(x) = \sum_{x=1}^{\infty} x\,p(x) = \sum_{x=1}^{\infty} x \times \frac{c}{x^3} = c\sum_{x=1}^{\infty} \frac{1}{x^2}$$

$$= c \times \frac{\pi^2}{6} \quad \left[\because \sum_{x=1}^{\infty} \frac{1}{x^2} = \frac{\pi^2}{6}\right]$$

So, $E(x)$ is finite.

35)

x	1	2	3	4	5	6
$p(x)$	1/15	2/15	3/15	4/15	3/15	2/15

Assuming, we order only 3 copies.

$$E(x) = \sum_{x=1}^{6} x\,p(x) = \frac{1}{15} + 2 \times \frac{2}{15} + 3 \times \frac{3}{15} + 3 \times \frac{4}{15} + 3 \times \frac{3}{15} + 3 \times \frac{2}{15}$$

$$\approx 2.733$$

The owner pays \$2.00 for each copy of the magazine

So, for 3 copies he has to pay $= 3 \times 2 = \$6.00$

The price to customers is \$4.00

$\therefore$ The net revenue $= 4X - 6$

$\therefore$ The expected revenue $= E(4X - 6)$

$$= 4E(X) - 6$$

$$= 4 \times 2.733 - 6$$

$$\approx 4.932$$

36)

X	0	1000	5000	10000
$p(x)$	0.8	0.1	0.08	0.02

The company offers a \$500 deductible policy

$G(X)$	0	500	4500	9500
$p(x)$	0.8	0.1	0.08	0.02

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$\therefore E(G(X)) = 500 \times 0{\cdot}1 + 4500 \times 0{\cdot}08 + 9500 \times 0{\cdot}02$

$= 600$

The company wishes its expected profit to be \$100

$\therefore$ The premium amount $= (100 + 600) = \$700$

37) $p(x) = \begin{cases} 1/n, & x = 1,2,3,\dots,n \\ 0, & \text{otherwise} \end{cases}$ → discrete uniform distribution.

$$E(x) = \sum_{x \in D} x \cdot p(x)$$

$$= \left(1 \times \frac{1}{n} + 2 \times \frac{1}{n} + 3 \times \frac{1}{n} \dots\dots n \times \frac{1}{n}\right)$$

$$= \frac{1}{n}\left(1+2+3+\dots\dots+n\right)$$

$$= \frac{1}{n} \times \frac{n(n+1)}{2}$$

$$= \frac{n+1}{2}$$

$$E(x^2) = \sum_{x \in D} x^2 p(x) \qquad \sum_{x=1}^{n}$$

$$= 1^2 \times \frac{1}{n} + 2^2 \times \frac{1}{n} + \dots\dots n^2 \times \frac{1}{n}$$

$$= \frac{1}{n}\left[1^2 + 2^2 + \dots\dots n^2\right]$$

$$= \frac{1}{n} \times \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{(n+1)(2n+1)}{6}$$

$$V(x) = E(x^2) - \{E(x)\}^2$$

$$= \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

$$\cancel{=(n+1)(2n} \left(\frac{n+1}{2}\right)\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

$$= \frac{(n+1)(n-1)}{12}$$

$$= \frac{n^2-1}{12}$$

38) x: 1 2 3 4

$p(x)$: 0.15 0.35 0.35 0.15

a) $E(x) = \sum_{x \in D} x p(x) = 1 \times 0.15 + 2 \times 0.35 + 3 \times 0.35 + 4 \times 0.15$

$= 2.5$

$E(5-X) = 5 - E(x)$

$= 5 - 2.5$

$= 2.5$

b) \$75 $\qquad \frac{150}{5-X}$

$E\left(\frac{150}{5-X}\right) = \sum_{x \in D} \left(\frac{150}{5-X}\right) \times p(x)$

$= \frac{150}{4} \times 0.15 + \frac{150}{3} \times 0.35 + \frac{150}{2} \times 0.35 + 150 \times 0.15$

$= 150\left(\frac{0.15}{4} + \frac{0.35}{3} + \frac{0.35}{2} + 0.15\right)$

$= \$\ 71.875$

$\therefore \frac{150}{5-X}$ will be better.

39)

x	1	2	3	4
$p(x)$	0.2	0.4	0.3	0.1

$E(x) = \sum_{x \in D} x p(x) = 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3 + 4 \times 0.1 = 2.3$

$E(x^2) = \sum_{x \in D} x^2 p(x) = 1 \times 0.2 + 4 \times 0.4 + 9 \times 0.3 + 16 \times 0.1 = 6.1$

$\therefore V(x) = E(x^2) - \{E(x)\}^2 = 6.1 - (2.3)^2 = 0.81$

$V(x) = E((x-\mu)^2) = \sum_{x \in D} (x - 2.5)^2 \times p(x) = 0.81$

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$E(40-2x) = 40 - 2E(x) = 40 - 2 \times 2.3 = 35.4$

$V(40-2x) = (-2)^2(0.81) = 3.24$

40) b) $V(aX+b) = \sigma^2_{ax+b} = a^2 \cdot \sigma_x^2$

$\therefore V(-X) = (-1)^2 \times V(x)$

$= V(x)$

$\therefore V(-X) = V(x)$

we know that, μ

41) $V(x) = \sum_{x \in D} (x-\mu)^2 p(x)$

μ_{ax+b}

$= a \cdot \mu_x + b$

$V(aX+b) = \sum_{x \in D} (ax+b - \mu_{ax+b})^2 p(x)$

$= \sum_{x \in D} (ax + \cancel{b} - a \cdot \mu_x - \cancel{b})^2 p(x)$

$= a^2 \sum_{x \in D} ((x-\mu_x)^2 p(x)$

$= a^2 \cdot V(x)$

$= a^2 \cdot \sigma_x^2$

42) a) $E(X) = 5$, $E(X(X-1)) = E(X^2 - X)$

$= \sum_{x \in D} (X^2 - X) p(x)$

$= \sum_{x \in D} X^2 p(x) - \sum_{x \in D} X p(x)$

$\Rightarrow E(X(X-1)) = E(X^2) - E(X)$

Given that, $E(X(X-1)) = 27.5$

$\Rightarrow E(X^2) = 27.5 + 5$

$\Rightarrow E(X^2) = 32.5$

b) $V(x) = E(x^2) - \{E(x)\}^2$

$= 32.5 - 25$

$= 7.5$

c) we know that, $E(x(x-1)) = E(x^2) - E(x)$

$\Rightarrow E(x) = E(x^2) - E(x(x-1))$

$\Rightarrow \{E(x)\}^2 = \{E(x^2) - E(x(x-1))\}^2$

$\therefore V(x) = E(x^2) - \{E(x^2) - E(x(x-1))\}^2$

$\Rightarrow E(x(x-1)) + E(x) = E(x^2)$

$\therefore V(x) = E(x^2) - \{E(x)\}^2$

$V(x) = E(x(x-1)) + E(x) - \{E(x)\}^2$

43) $E(x-c) = E(x) - c$, where c is a constant

When $c = \mu$,

$E(x-\mu) = \sum_{x \in D} (x-\mu)\, p(x)$

$= \sum_{x \in D} x p(x) - \mu \sum_{x \in D} p(x)$

$= E(x) - \mu \times 1 \quad [\because \sum_{x \in D} p(x) = 1]$

$= E(x) - E(x)$

$= 0$

45) $a \leq X \leq b$,

To prove $a \leq E(x) \leq b$

Given :- $a \leq X \leq b$

Multiplying by $p(x)$,

$a p(x) \leq x p(x) \leq b\, p(x)$

$\Rightarrow \sum_{x \in D} a p(x) \leq \sum_{x \in D} x p(x) \leq \sum_{x \in D} b\, p(x)$

$\Rightarrow a \sum_{x \in D} p(x) \leq \sum_{x \in D} x p(x) \leq b \sum_{x \in D} p(x)$

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$$\Rightarrow a \times 1 \leq \sum_{x \in D} x\,p(x) \leq b \times 1 \quad \left[\because \sum_{x \in D} p(x) = 1\right]$$

$$\Rightarrow a \leq E(x) \leq b$$

Hence, proved.

14) $P(|X-\mu| \geq k\sigma) \leq 1/k^2$

a) For, $k=3$

~~$P(|X-\mu| \geq 3\sigma)$~~

The value of upper bound $= \frac{1}{9}$

For $k=4$,

The value of upper bound $= \frac{1}{16}$

For $k=5$,

The value of upper bound $= \frac{1}{25}$

For $k=10$,

The value of upper bound $= \frac{1}{100}$

The upper bound for $k=2 = \frac{1}{k^2} = \frac{1}{4} = 0.25$

b) x : 0 1 2 3 4 5 6

$p(x)$: 0·10 0·15 0·20 0·25 0·20 0·06 0·04

Compute μ and σ. Then compute $P(|x-\mu| \geq k\sigma)$ for the values of k given in (a)

$\mu = 0{\cdot}15 + 0{\cdot}4 + 0{\cdot}75 + 0{\cdot}8 + 0{\cdot}3 + 0{\cdot}24$

$= 2.64$

$\sigma^2 = \sum (x - 2.64)^2 \times p(x)$

$= (2.64)^2 \times 0.1 + (-1.64)^2 \times 0.15 + (0.64)^2 \times 0.20 + (0.36)^2 \times 0.25$
$+ (1.36)^2 \times 0.2 + (2.36)^2 \times 0.06 + (3.36)^2 \times 0.04$

$= 2.3704$

$\sigma = \sqrt{2.3704} = 1.54$

Let us take $k = 2$

$P(|x - 2.64| \geq 2 \times 1.54)$

$= P(|x - 2.64| \geq 3.08)$

$= 1 - P(|x - 2.64| < 3.08)$

$= 1 - P\{-3.08 < x - 2.64 < 3.08\}$

$= 1 - P\{-3.08 + 2.64 < x < 3.08 + 2.64\}$

$= 1 - P\{-0.44 < x < 5.72\}$

see the probability distribution table.

$= P(x = 6) = 0.04$

$> 1 - (0.1 + 0.15 + 0.2 + 0.25 + 0.2 + 0.06)$

$= 0.64$

c)

$X: \quad -1 \quad 0 \quad 1$

$p(x): \quad 1/18 \quad 8/9 \quad 1/18$

$\mu = E(x) = \frac{-1}{18} + \frac{1}{18} = 0$

$E(x^2) = \frac{1}{18} + \frac{1}{18} = \frac{1}{9}$

$\therefore \sigma^2 = E(x^2) - \{E(x)\}^2 = \frac{1}{9} \Rightarrow \sigma = \frac{1}{3}$

$P|x-\mu| \geq 3\sigma$

$P(x-\mu) \geq 3\sigma$ and $P(x-\mu) \leq -3\sigma$

$\Rightarrow P(x) \geq 3 \times \frac{1}{3}$ and $P(x) \leq -3 \times \frac{1}{3}$

$\Rightarrow P(x) \geq 1$ and $P(x) \leq -1 \quad \therefore P(x) = \frac{1}{18} + \frac{1}{18} = \frac{1}{9}$

d) $P(|X-\mu| \geq 5\sigma) = 0.04$ —(i)

Let, X have the possible values $-1, 0, 1$

Let, us assume $P(x=-1)$ and $P(x=1)$ as half of the given probability as in part c

$\therefore P(x=-1) = \frac{0.04}{2} = 0.02$

$P(x=1) = \frac{0.04}{2} = 0.02$

$\therefore P(x=0) = 1 - P(x=-1) - P(x=1)$

$= 1 - 0.02 - 0.02$

$= 0.96$

Now, let us check whether it satisfies (i)

$\mu = \sum_{x \in D} x\,p(x) = -0.02 + 0.02 = 0$

$E(x^2) = \sum_{x \in D} x^2 p(x) = 0.02 + 0.02 = 0.04$

$\therefore \sigma^2 = E(x^2) - (E(x))^2 = 0.04$

$\sigma = \sqrt{0.04} = 0.2$

$\therefore P(|X-\mu| \geq 5\sigma) = P(|x| \geq 5 \times 0.2)$

$= P(x) \geq 1$ and $P(x) \leq -1$

$\therefore P(x) = 0.02 + 0.02 = 0.04$

$\therefore$ The distribution is:-

x	-1	0	1
$P(x)$	0.02	0.96	0.02