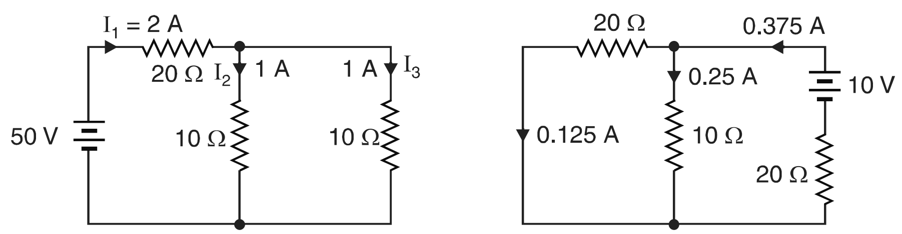
Or

If the resistance of any branch of a network is changed from R to (R + ΔR) where the current was originally I, then the change of current at any point in the network may be calculated by assuming than an e.m.f. – IΔR has been introduced into the modified branch while all other sources have their e.m.f.s. suppressed and are represented by their internal resistances only.

Illustration. Let us illustrate the compensation theorem with a numerical example. Consider the circuit shown in Fig. 3.235 (*i*). The various branch currents in this circuit are :

$$I_1 = \frac{50}{20+5} = 2\text{ A} \quad ; \quad I_2 = I_3 = 1\text{ A}$$



(*i*) (*ii*)

Fig. 3.235

Now suppose that the resistance of the right branch is increased to 20 Ω *i.e.* $\Delta R = 20 - 10 = 10\ \Omega$ and a voltage $V = -I_3\,\Delta R = -1 \times 10 = -10$ V is introduced in this branch and voltage source replaced by a short (∵ internal resistance is assumed zero). The circuit becomes as shown in Fig. 3.235 (*ii*). The compensating currents produced by this voltage are also indicated. When these compensating currents are algebraically added to the original currents in their respective branches, the new branch currents will be as shown in Fig. 3.236. The compensation theorem is useful in bridge and potentiometer circuits, where a slight change in one resistance results in a shift from a null condition.

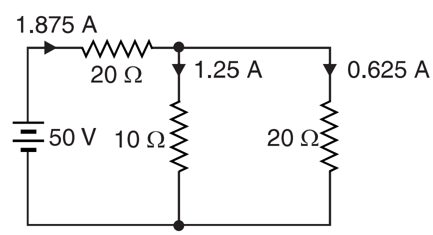


Fig. 3.236

3.23. Delta/Star and Star/Delta Transformation

There are some networks in which the resistances are neither in series nor in parallel. A familiar case is a three terminal network *e.g.* delta network or star network. In such situations, it is not possible to simplify the network by series and parallel circuit rules. However, converting delta network into star and *vice-versa* often simplifies the network and makes it possible to apply series-parallel circuit techniques.

3.24. Delta/Star Transformation

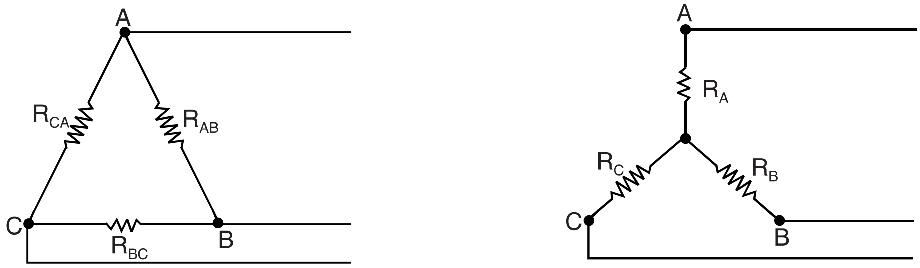
Consider three resistors R_{AB}, R_{BC} and R_{CA} connected in delta to three terminals *A, B* and *C* as shown in Fig. 3.237 (*i*). Let the equivalent star-connected network have resistances R_A, R_B and R_C. Since the two arrangements are electrically equivalent, the resistance between any two terminals of one network is equal to the resistance between the corresponding terminals of the other network.

Let us consider the terminals *A* and *B* of the two networks.

Resistance between *A* and *B* for star = Resistance between *A* and *B* for delta

or $$R_A + R_B = R_{AB} \parallel (R_{BC} + R_{CA})$$

or $$R_A + R_B = \frac{R_{AB}(R_{BC} + R_{CA})}{(R_{AB} + R_{BC} + R_{CA})} \qquad ...(i)$$



(*i*) (*ii*)

Fig. 3.237

Similarly, $$R_B + R_C = \frac{R_{BC}(R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \quad ...(ii)$$

and $$R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \quad ...(iii)$$

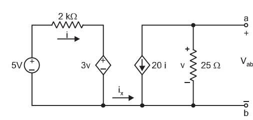
Subtracting eq. (*ii*) from eq. (*i*) and adding the result to eq. (*iii*), we have,

$$R_A = \frac{R_{AB}\, R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \quad ...(iv)$$

Similarly, $$R_B = \frac{R_{BC}\, R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \quad ...(v)$$

and $$R_C = \frac{R_{CA}\, R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \quad ...(vi)$$

How to remember ? There is an easy way to remember these relations. Referring to Fig. 3.238, star-connected resistances R_A, R_B and R_C are electrically equivalent to delta-connected resistances R_{AB}, R_{BC} and R_{CA}. We have seen above that :



$$R_A = \frac{R_{AB}\, R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

i.e. Any arm of star-connection $= \dfrac{\text{Product of two adjacent arms of } \Delta}{\text{Sum of arms of } \Delta}$

Fig. 3.238

Thus to find the star resistance that connects to terminal *A*, *divide the product of the two delta resistors connected to A by the sum of the delta resistors*. Same is true for terminals *B* and *C*.

3.25. Star/Delta Transformation

Now let us consider how to replace the star-connected network of Fig. 3.237 (*ii*) by the equivalent delta-connected network of Fig. 3.237 (*i*).

Dividing eq. (*iv*) by (*v*), we have,

$$R_A/R_B = R_{CA}/R_{BC}$$

$$\therefore \quad R_{CA} = \frac{R_A\, R_{BC}}{R_B}$$

Dividing eq. (*iv*) by (*vi*), we have,

$$R_A / R_C = R_{AB} / R_{BC}$$

$$\therefore \qquad R_{AB} = \frac{R_A R_{BC}}{R_C}$$

Substituting the values of R_{CA} and R_{AB} in eq. (*iv*), we have,

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

Similarly,

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

and

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

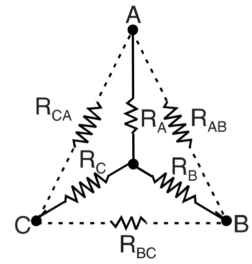


Fig. 3.239

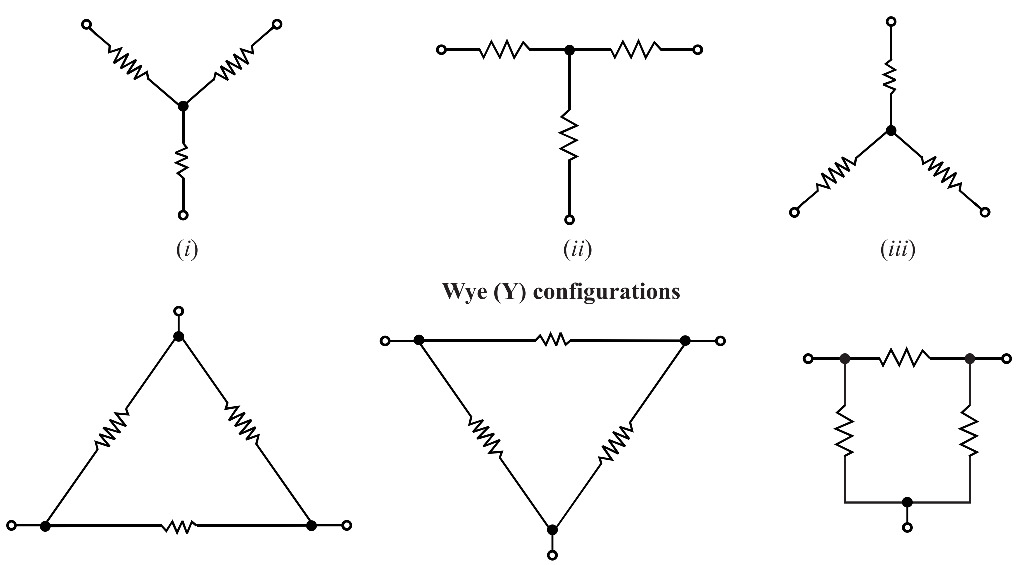
How to remember ? There is an easy way to remember these relations.

Referring to Fig. 3.239, star-connected resistances R_A, R_B and R_C are electrically equivalent to delta-connected resistances R_{AB}, R_{BC} and R_{CA}. We have seen above that :

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

i.e. Resistance between two terminals of delta = Sum of star resistances connected to those terminals *plus* product of same two resistances divided by the third star resistance

Note. Figs. 3.240 (*i*) to (*iii*) show three ways that a wye (Y) arrangement might appear in a circuit. Because the wye-connected components may appear in the equivalent form shown in Fig. 3.240 (*ii*), the arrangement is also called a *tee* (*T*) arrangement. Figs. 3.240 (*iv*) to (*vi*) show equivalent delta forms. Because the delta (Δ) arrangement may appear in the equivalent form shown in Fig. 3.240 (*vi*), it is also called a *pi* (π) arrangement. The figures show only a few of the ways the wye (*Y*) and delta (Δ) networks might be drawn in a schematic diagram. Many equivalent forms can be drawn by rotating these basic arrangements through various angles. Note that each network has three terminals.



(*i*) (*ii*) (*iii*)

Wye (Y) configurations

(*iv*) (*v*) (*vi*)

Delta (Δ) configurations

Fig. 3.240

Example 3.95. *Using delta/star transformation, find the galvanometer current in the Wheatstone bridge shown in Fig. 3.241 (i).*

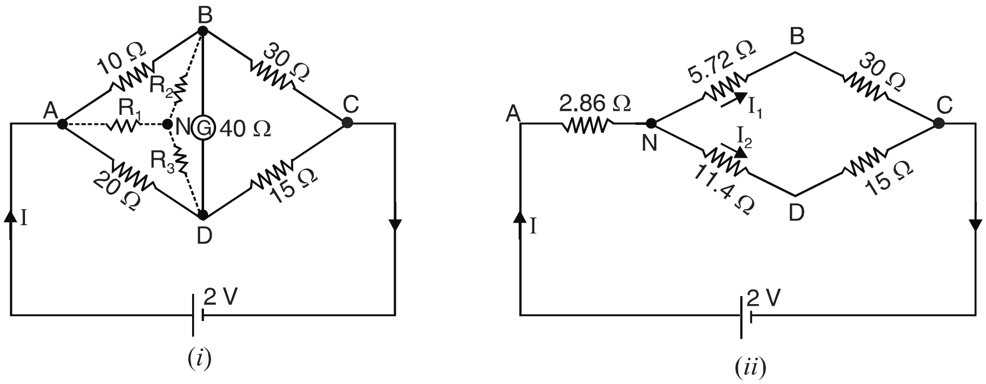


Fig. 3.241

Solution. The network *ABDA* in Fig. 3.241 (*i*) forms a delta. These delta-connected resistances can be replaced by equivalent star-connected resistances R_1, R_2 and R_3 as shown in Fig. 3.241 (*i*).

$$R_1 = \frac{R_{AB}\,R_{DA}}{R_{AB}+R_{BD}+R_{DA}} = \frac{10\times 20}{10+40+20} = 2.86\ \Omega$$

$$R_2 = \frac{R_{AB}\,R_{BD}}{R_{AB}+R_{BD}+R_{DA}} = \frac{10\times 40}{10+40+20} = 5.72\ \Omega$$

$$R_3 = \frac{R_{DA}\,R_{BD}}{R_{AB}+R_{BD}+R_{DA}} = \frac{20\times 40}{10+40+20} = 11.4\ \Omega$$

Thus the network shown in Fig. 3.241 (*i*) reduces to the network shown in Fig. 3.241 (*ii*).

$$R_{AC} = 2.86 + \frac{(30+5.72)\,(15+11.4)}{(30+5.72)+(15+11.4)} = 18.04\ \Omega$$

Battery current, $I = 2/18{\cdot}04 = 0{\cdot}11$ A

The battery current divides at N into two parallel paths.

$\therefore$ Current in branch *NBC*, $I_1 = 0.11 \times \dfrac{26.4}{26.4+35.72} = 0{\cdot}047$ A

Current in branch *NDC*, $I_2 = 0.11 \times \dfrac{35.72}{26.4+35.72} = 0{\cdot}063$ A

Potential of B *w.r.t.* C $= 30 \times 0{\cdot}047 = 1{\cdot}41$ V

Potential of D *w.r.t.* C $= 15 \times 0{\cdot}063 = 0{\cdot}945$ V

Clearly, point B is at higher potential than point D by

$$1{\cdot}41 - 0{\cdot}945 = 0{\cdot}465\ \text{V}$$

$\therefore$ Galvanometer current $= \dfrac{\text{P.D. between } B \text{ and } D}{\text{Galvanometer resistance}}$

$= 0{\cdot}465 / 40 = 11{\cdot}6 \times 10^{-3}$ A = **11·6 mA** *from B to D*

$$R'_1 = \frac{1 \times 8}{1+5+8} = \frac{4}{7}\,\Omega$$

$$R'_2 = \frac{5 \times 1}{1+5+8} = \frac{5}{14}\,\Omega$$

$$R'_3 = \frac{8 \times 5}{1+5+8} = \frac{20}{7}\,\Omega$$

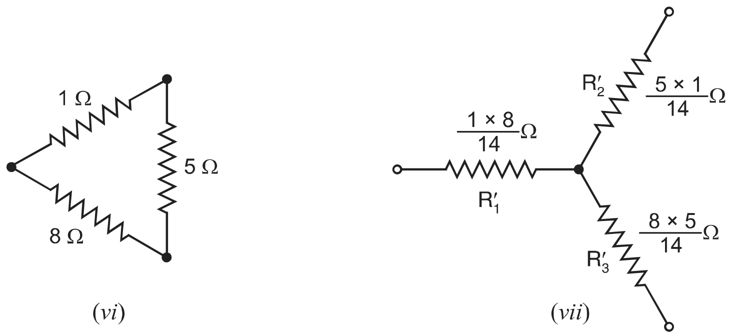


Fig. 3.242

After above delta-star conversion, the circuit reduces to the one shown in Fig. 3.242 (*viii*).

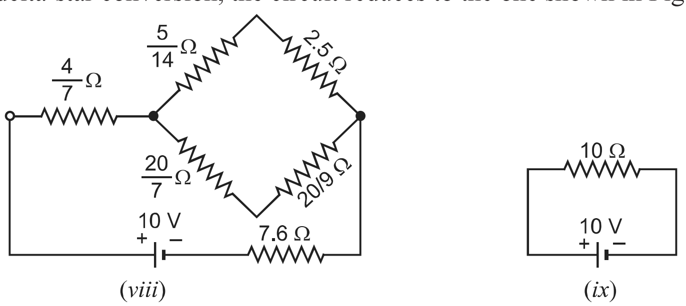


Fig. 3.242

Total resistance offered by the circuit to the battery is

$$R_T = \frac{4}{7} + \left[\left(\frac{5}{14} + 2.5\right) \| \left(\frac{20}{7} + \frac{20}{9}\right)\right] + 7.6$$

$$= \frac{4}{7} + \left(\frac{20}{7} \| \frac{320}{63}\right) + 7.6 = 10\ \Omega$$

$\therefore$ Current supplied by the battery [See Fig. 3.242 (*ix*)] is

$$I = \frac{V}{R_T} = \frac{10}{10} = \mathbf{1\ A}$$

Example 3.97. *A network of resistors is shown in Fig. 3.243 (i). Find the resistance (i) between terminals A and B (ii) B and C and (iii) C and A.*

Solution. The star-connected resistances 6 Ω, 3 Ω and 4 Ω in Fig. 3.243 (*i*) are shown separately in Fig. 3.243 (*ii*). These star-connected resistances can be converted into equivalent delta-connected resistances R_1, R_2 and R_3 as shown in Fig. 3.243 (*ii*).

$$R_1 = 4 + 6 + (4 \times 6/3) = 18\ \Omega$$

$$R_2 = 6 + 3 + (6 \times 3/4) = 13{\cdot}5\ \Omega$$

$$R_3 = 4 + 3 + (4 \times 3/6) = 9\ \Omega$$

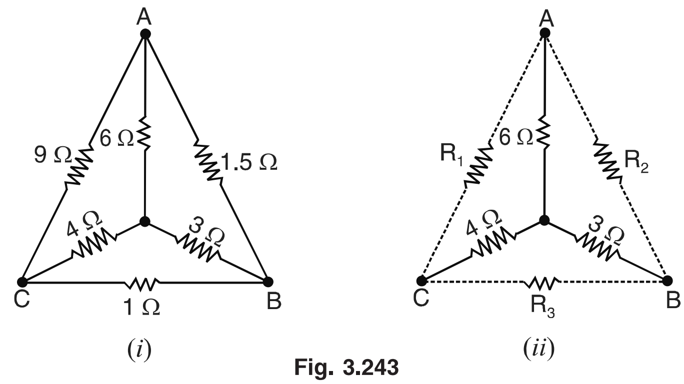


Fig. 3.243

These delta-connected resistances R_1, R_2 and R_3 come in parallel with the original delta-connected resistances. The circuit shown in Fig. 3.243 (*i*) reduces to the circuit shown in Fig. 3.244(*i*).

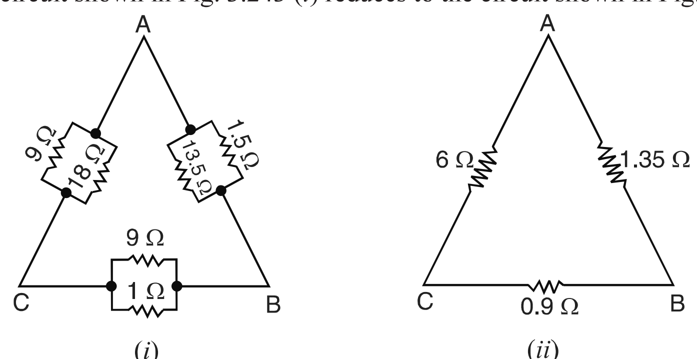


Fig. 3.244

The parallel resistances in each leg of delta in Fig. 3.244 (*i*) can be replaced by a single resistor as shown in Fig. 3.244 (*ii*) where

$$R_{AC} = 9 \times 18/27 = 6\ \Omega$$
$$R_{BC} = 9 \times 1/10 = 0{\cdot}9\ \Omega$$
$$R_{AB} = 1{\cdot}5 \times 13{\cdot}5/15 = 1{\cdot}35\ \Omega$$

(*i*) Resistance between A and B = $1{\cdot}35\ \Omega \parallel (6 + 0{\cdot}9)\ \Omega = 1{\cdot}35 \times 6{\cdot}9/8{\cdot}25 = \mathbf{1{\cdot}13\ \Omega}$

(*ii*) Resistance between B and C = $0{\cdot}9\ \Omega \parallel (6 + 1{\cdot}35)\ \Omega = 0{\cdot}9 \times 7{\cdot}35/8{\cdot}25 = \mathbf{0{\cdot}8\ \Omega}$

(*iii*) Resistance between A and C = $6\ \Omega \parallel (1{\cdot}35 + 0{\cdot}9)\ \Omega = 6 \times 2{\cdot}25/8{\cdot}25 = \mathbf{1{\cdot}636\ \Omega}$

Example 3.98. *Determine the load current in branch EF in the circuit shown in Fig. 3.245 (i).*

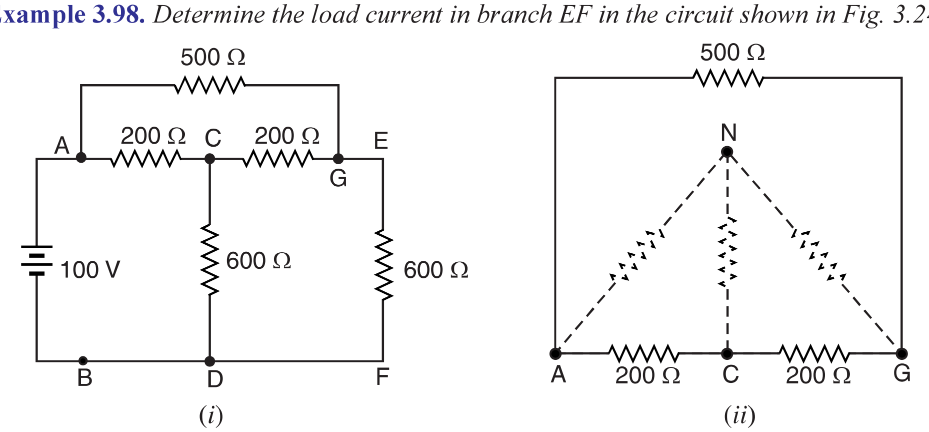


Fig. 3.245

Solution. The circuit *ACGA* forms delta and is shown separately in Fig. 3.245 (*ii*) for clarity. Changing this delta connection into equivalent star connection [See Fig. 3.245 (*ii*)], we have,

$$R_{AN} = \frac{500\times 200}{500+200+200} = 111.11\ \Omega \;;\quad R_{CN} = \frac{200\times 200}{500+200+200} = 44.44\ \Omega\;;$$

$$R_{GN} = \frac{500\times 200}{500+200+200} = 111.11\ \Omega$$

Thus the circuit shown in Fig. 3.245 (*i*) reduces to the circuit shown in Fig. 3.246 (*i*). The branch *NEF* (= 111·11 + 600 = 711·11 Ω) is in parallel with branch *NCD* (= 44·44 + 600 = 644·44 Ω) and the equivalent resistance of this parallel combination is

$$= \frac{711.11\times 644.44}{711.11+644.44} = 338\ \Omega$$

The circuit shown in Fig. 3.246 (*i*) reduces to the circuit shown in Fig. 3.246 (*ii*).

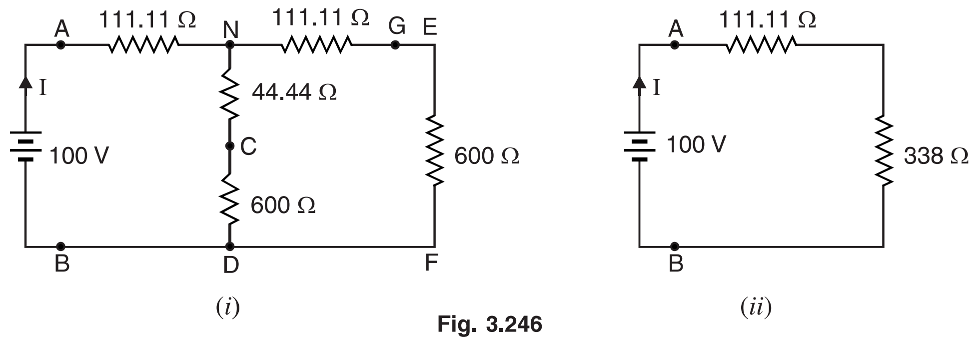


Fig. 3.246

$$\therefore \qquad \text{Battery current, } I = \frac{100}{338+111.11} = 0.222\text{ A}$$

This battery current divides into two parallel paths [See Fig. 3.246 (*i*)] *viz*. branch *NEF* and branch *NCD*.

∴ Current in branch *NEF i.e.* in branch *EF*

$$= 0.222\times\frac{644.44}{711.11+644.44} = \mathbf{0.1055\ A}$$

Example 3.99. *A square and its diagonals are made of a uniform covered wire. The resistance of each side is 1 Ω and that of each diagonal is 1·414 Ω. Determine the resistance between two opposite corners of the square.*

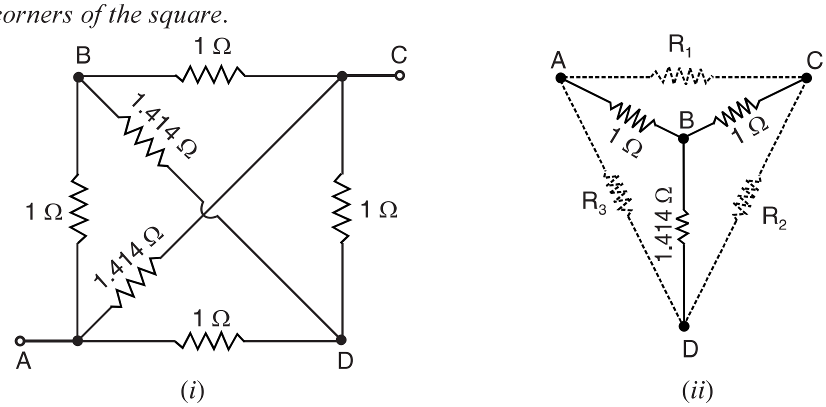


Fig. 3.247