Semester: 4th ....
*Programme: ....**B.Tech**...................................*
*Branch/Specialization:**CSSE** ....*

SPRING END SEMESTER EXAMINATION-2023

4th Semester,B.Tech (Programme)

SUBJECT- PROBABILITY AND STATISTICS

CODE-MA2011

(For 2021 Admitted Batches)

Time: 3 Hours **Full Marks:50**

Answer any SIX questions.
Question paper consists of four SECTIONS i.e. A, B, C and D.
Section A is compulsory.
Attempt minimum one question each from Sections B,C, D.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A (Learning levels 1 and 2)				Learning levels as per Bloom's taxonomy	Course Outcomes (CO)
1.		Answer the following questions.	[1 × 10]		
	(a)	F-or two events A, B, if $P(A) = a_1$, $P(B) = a_2$, $P(A \cap B) = a_3$, then find $P(A \cup B\ )'$		understanding	CO1
	(b)	A continuous random variable X has a pdf $f(x) = 3x^2, 0 \leq x \leq 1$, then find the value of k such that $P(x > k) = 0.05$		evaluating	CO2
	(c)	If a random variable X has a Poisson distribution with mean 0.4, then find $P(X \leq 1)$.		evaluating	CO3
	(d)	A sample size $n = 14$ has the sample mean $\bar{x} = 5$ and variance 9. If the length of the confidence interval is $L = 4.1308$. Then find the confidence interval of the mean μ with confidence level γ.		applying	CO4
	(e)	If X_1 and X_2 are independent normal random variables with mean 23 and 4 and variance 3 and 1, respectively then find the mean and variance of the random variable $4X_1 + X_2$ x		applying	CO5
	(f)	If $X_1, X_2, \cdots X_{10}$ are 10 independent random variables with $E(X_i) = 3, V(X_i) = 2$ for $i = 1,2, \cdots, 10$, then find the mean and variance of the mean random variable $\bar{X}$.		understanding	CO6
	(g)	Find the third Moment of Exponential distribution with parameter lambda =3.		remembering	CO5
	(h)	An item is produced in large numbers. The machine is known to produce 5% defectives. A quality control inspector is examining the items by taking them at random. What is probability that at least four items are to be examined in order		applying	CO3

		to get 2 defectives ?			
	(i)	Show that the mean and variance corresponds to the standardized random variable $Z = \frac{X-\mu}{\sigma}$ are 0 and 1, respectively.		remembering	CO5
	(j)	(table below) Find $F(5,10)$.		evaluating	CO6

		Y			
X	$p(x_i, y_j)$	0	5	10	15
	0	0.02	0.06	0.02	0.10
	5	0.04	0.15	0.20	0.10
	10	0.01	0.15	0.14	0.10

Find $F(5,10)$.

		SECTION-B (Learning levels 1,2, and 3)		Learning levels as per Bloom's taxonomy	Course Outcomes (CO)
2.	(a)	Consider randomly selecting a single individual and having that person test drive 3 different vehicles. Define events A_1, A_2, and $A_3$by A_1=likes vehicle #1, A_2 =likes vehicle #2, A_3 =likes vehicle #3. Suppose that $P(A_1) = 0.65$, $P(A_2) = 0.55$, $P(A_3) = 0.70$, $P(A_1 \cup A_2) = 0.80$, $P(A_2 \cap A_3) = 0.40$, $P(A_1 \cup A_2 \cup A_3) = 0.88$. a. What is the probability that the individual likes both vehicle #1 and vehicle #2? Determine and interpret $P(A_1 / A_2)$. b. What is the probability that the individual likes either vehicle #2 or vehicle #3? Determine and interpret $P(A_2 / A_3)$. c. Are A_1 and A_2 independent events? Answer in two different ways.	[4]	Understanding	CO1
	(b)	a) Find mean and variance of Geometric distribution.	[4]	remembering	CO2
3.	(a)	In a commuting to work, a professor must first get on a bus near her house and then transfer to a second bus. If the waiting time (in minutes) at each stop has uniform distribution with $A = 0$ and $B = 5$, then it can be shown that the total waiting time Y has the pdf $f(y) = \begin{cases} \frac{1}{25}y & 0 \le y < 5 \\ \frac{2}{5} - \frac{1}{25}y & 5 \le y \le 10 \\ 0 & y < 0 \text{ or } y > 10. \end{cases}$ a. What is the probability that total waiting time is at most 3 min? b. What is the probability that total waiting time is at most 8 min? c. What is the probability that total waiting time is between 3 min and 8 min? d. What is the probability that total waiting time is either less than 2 min or more than 6 min?	[4]	evaluating	CO3

	(b)	Suppose n candidates for a job have been ranked $1, 2, 3, \ldots, n$. Let $X =$ rank of a randomly selected candidate, so that X haspmf $$f(x) = \begin{cases} \frac{1}{n}, & x = 1, 2 \cdots, n \\ 0, & \text{otherwise.} \end{cases}$$ Compute $E(X)$ and $V(X)$.	[4]	remembering	CO1
		SECTION-C (Learning Levels 3 and 4)		Learning levels as per Bloom's taxonomy	Course Outcomes (CO)
4.	(a)	Define covariance of two random variables X and Y and show that $Cov(X, Y) = E(X, Y) - E(X)E(Y)$.	[4]	remembering	CO3
	(b)	Find the maximum likelihood estimate of mean and variance of Normal distribution.	[4]	applying	CO4
5.	(a)	If (X,Y) is joint continuous random variables with joint pdf $f(x, y) = k(x + y),\ 0 < x < 1, 0 < y < 2$; and *0, otherwise* i. What is the value of k. *ii.* Determine F(0.5, 1.5). *iii.* Determine Mean *of X-Y.*	[4]	evaluating	CO4
	(b)	A service station has both self-service and full-service islands. One each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on self-service island at a particular time, and Y denote the number of hoses on the full-service island in use at that time. The joint pmf of X and Y appears in the accompanying tabulation. (table below) a. Compute $P(X \leq 1 \text{ and } Y \leq 1)$. b. Compute the marginal pmf of X and of Y. c. Are X and Y independent random variables? Explain.	[4]	applying	CO3
6.	(a)	Show that for any three events A , B, and C with $P(C) > 0$ $$P(A \cup B \mid C) + P(A \cap B \mid C) = P(A \mid C) + P(B \mid C).$$	[4]	remembering	CO3
	(b)	For any two random variables X, Y and arbitrary constants a and b, find $V(aX + b, cY + d)$	[4]	understanding	CO4

Table for 5(b):

$p(x, y)$		y		
		0	1	2
x	0	0.10	0.04	0.02
	1	0.08	0.20	0.06
	2	0.06	0.14	0.30

SECTION-D (Learning levels 4,5,6)				Learning levels as per Bloom's taxonomy	Course Outcomes (CO)
7.	(a)	Two components of a minicomputer have the following joint pdf for their useful lifetimes *X* and *Y*: $f(x,y)\begin{cases} xe^{-x(1+y)} & x\geq 0,\ y\geq 0 \\ 0 & \text{otherwise} \end{cases}$ a. Compute $F(2,3)$. b. Find the marginal of X and Y.	[4]	applying	CO6
	(b)	The actual tracking weight of a stereo cartridge that is set to track at 3 g on a particular changer can be regarded as continuous rv*X* with pdf $f(x) = \begin{cases} k[1-(x-3)^2] & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$ a. Find the value of k. b. Find the cumulative distribution function $F(x)$. c. What is the probability that the actual tracking weight is within 0.25 g of the prescribed weight?	[4]	Applying	CO5
8.	(a)	Define correlation coefficient of two random variables$X$ and Y and prove that $\text{Corr}(aX+b, cY+d) = \text{Corr}(X,Y)$ when a and c have the same sign. What happens if *a* and *c* have opposite sign?	[4]	remembering	CO4
	(b)	Find a 95% confidence interval for the mean μ of a normal population with standard deviation 4 from the sample 30, 42, 40, 34, 48, 50 then find the length of the interval [Given $P(Z > 1.96) = 0.025$)].	[4]	applying	CO5
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