Chapter – 3

3.1 Random Variable

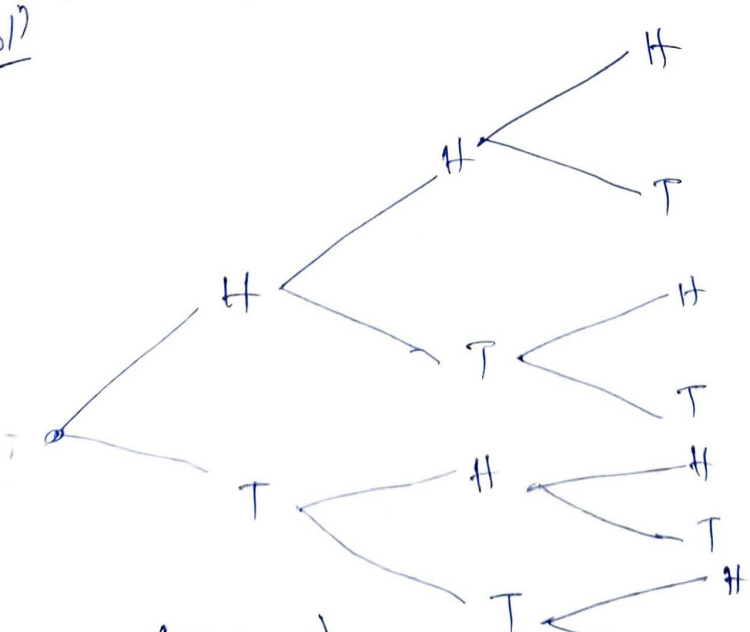
For a given sample space S of some experiment, a random variable (rv) is any rule that associates a number with each outcome in S.

In Mathematical language, a random variable is a function whose domain is the sample space S and whose range is the set of real numbers.

Ex-1

A fair coin tossed three times. find the sample points (S). If the random variable (X) counts no. of heads (H) in each sample point, then find the range or image set of r.v. X.

Soln



Sample points (S)

$HHH = S_1$

$HHT = S_2$

$HTH = S_3$

$HTT = S_4$

$THH = S_5$

$THT = S_6$

$TTH = S_7$

$TTT = S_8$

Sample point (S)	range of $X(S)$ = no. of heads in S	$P(S)$
$S_1 = HHH$	3	$1/8$
$S_2 = HHT$	2	$1/8$
$S_3 = HTH$	2	$1/8$
$S_4 = HTT$	1	$1/8$
$S_5 = THH$	2	$1/8$
$S_6 = THT$	1	$1/8$
$S_7 = TTH$	1	$1/8$
$S_8 = TTT$	0	$1/8$

The range set of X

$= X = X(S) = \{0, 1, 2, 3\}$

Let $A_x = \{S \in S \mid X = X(S) = x\}$

$A_0 = \{S_8\} = \{TTT\}$

$A_1 = \{S_4, S_6, S_7\} = \{HTT, THT, TTH\}$

$A_2 = \{S_2, S_3, S_5\} = \{HHT, HTH, THH\}$

$A_3 = \{S_1\} = \{HHH\}$

$A_0 \cup A_1 \cup A_2 \cup A_3 = S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$= \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$

i.e. $A_i, i = 0, 1, 2, 3$ are exhaustive

Since $A_i \cap A_j = \phi$ for $i \neq j$, they are mutually exclusive. where

$P(A_0) = \frac{1}{8}, P(A_1) = \frac{3}{8}, P(A_2) = \frac{3}{8}, P(A_3) = \frac{1}{8}$.

satisfied $\sum_{i=0}^{3} P(A_i) = P(A_0) + P(A_1) + P(A_2) + P(A_3)$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$

Examples

(1) A fair coin tossed ... If X counts no. of trials to get 1st head then find the range of r.v. X

Soln

Sample space $= S = \{H, TH, TTH, TTTH, \ldots\}$

sample pt (S)	H	TH	TTH	TTTH	...
$X = X(S)$ counts no. of trials	1	2	3	4	...

Range of $X = \{1, 2, 3, \ldots\}$ has countably infinite elements

(2) If a fair die is rolled once. success (S) is to get a six, then failure (F) is not getting a six. The r.v. X gives 1 if there is a success otherwise 0, then

sample pt (S)	S	F
$X = X(S)$	1	0

$\therefore X = \{1, 0\}$ is the range of X & has finite elements

(3) consider an experiment in which 9-volt batteries are tested until one with an acceptable voltage (S) is obtained. Find the range of r.v. X that counts the batteries tested before the experiment terminates.

Soln Sample space $= \{S, FS, FFS, \ldots\}$

$= \{S_1, S_2, S_3, \ldots\}$

S	S_1	S_2	S_3	...
$X(S)$	1	2	3	...

$X = X(S) = \{1, 2, 3, \ldots\}$

is countably infinite set.

Bernoulli trial A trial which gives only two possible outcomes is Bernoulli trial. The r.v. associated with it is Bernoulli r.v.

Ex Head (H) or Tail (T), Success (S) or failure (F), 0 or 1.

Note $P(S) + P(F) = 1$

Type of random variable

There are two types of r.v.s such as

1. Discrete r.v.
2. continuous r.v.

(1) **Discrete r.v.** If the range set of r.v. X is either finite or countably infinite, then it is discrete r.v.; i.e.,

$$X = \{x_1, x_2 \cdots x_n\}$$

or

$$\{x_1, x_2, \cdots\}$$

X is denumerable, $X \simeq \mathbb{N}$

(2) **continuous r.v.** If the range set of r.v. X is either a finite interval or real set $\mathbb{R}$, then it is continuous r.v.; i.e., $X = [a, b]$, $a < b \in \mathbb{R}$

or

$$X = (-\infty, \infty)$$