SPRING END SEMESTER EXAMINATION-2019

2^{nd} Semester B.Tech & B.Tech Dual Degree

MATHEMATICS-II

MA-1004

(For 2018 Admitted Batch)

Time: 3 Hours | Full Marks: 50

Answer any SIX questions.
Question paper consists of four sections-A, B, C, D.
Section A is compulsory.
Attempt minimum one question each from Sections B, C, D.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A

1. $[1 \times 10]$

(a) Find the radius of convergence of the power series $\sum_{m=0}^{\infty} \left(\frac{2}{3}\right)^m x^{2m}$.

(b) Find $L\left(\cos t \;\; u\left(t - \frac{\pi}{2}\right)\right)$.

(c) Find the directional derivative of $f = \frac{1}{\sqrt{x^2+y^2+z^2}}$ at P:(3, 0, 4) in the direction of $\vec{a}$ =[1, 1, 3].

(d) Given $\vec{u} = [z, x, y], \vec{v} = [y + z,\; z + x,\; x + y]$, find $div\,(\vec{u} \times \vec{v})$.

(e) Obtain the Legendre polynomial $P_3(x)$ from Rodrigue's formula.

(f) Show that $J_1'(x) = J_0(x) - \frac{1}{x}J_1(x)$.

(g) Evaluate $\int_0^1 e^{-x^2} dx$ using Trapezoidal rule taking 5 sub intervals.

(h) Check whether $f(x) = sinhx - coshx$ is even, odd or neither even nor odd?

(i) Find $L^{-1}\left(\frac{e^{-\pi s}}{s^2+4}\right)$.

(j) If $x + y = u, x - y = v$, then what is the Jacobian $J = \frac{\partial(x,y)}{\partial(u,v)}$?

SECTION-B

2. (a) Find power series solution of $y'' + y' + x^2 y = 0$ in power of x. [4]

(b) Find Fourier series of given function defined as: [4]

$$f(x) = \begin{cases} x, & if -\pi < x < 0 \\ \pi - x, & if\ 0 < x < \pi \end{cases}$$

where $f(x + 2\pi) = f(x)$.

3. (a) Derive the Bonnet recursion formula. [4]

$(n+1)P_{n+1}(x)$=$(2n+1)xP_n(x) - nP_{n-1}(x), n =$ 1,2, ...

(b) Obtain a half range Fourier cosine series to represent the function $f(x) = 4 - x,\ 0 < x < 4$. [4]

SECTION-C

4. (a) Evaluate $\int \boldsymbol{F}(\boldsymbol{r}).\boldsymbol{dr}$ along the path C, where C is the quarter circle from $(2,0)$ to $(0,2)$ with center $(0,0)$ and $\boldsymbol{F}$=$[xy, x^2y^2]$. [4]

(b) Solve the integral equation by Laplace transformation [4]

$$y(t) - \int_0^t y(\tau)(t-\tau)d\tau = 2 - \frac{1}{2}t^2$$

5. (a) Find the value of $\int_0^1 \frac{x^2}{1+x^3}$ using Simpson's $\frac{1}{3}$rd rule with $h = 0.25$. [4]

(b) Evaluate the Surface integral $\iint \boldsymbol{F} \cdot \boldsymbol{n}\, dA$ over the Surface S, where, $S: \vec{r}=[u, v, 3u-2v], \vec{F} = [-x^2, y^2, 0]$, $0 \le u \le 1.5,\ 0 \le v \le 2$. [4]

6. (a) Evaluate $\int \boldsymbol{F}(\boldsymbol{r}).\boldsymbol{dr}$ counter clockwise around the boundary of the region $R: 1 \le y \le 2 - x^2$, where $\boldsymbol{F}=[x^2+y^2,\ x^2-y^2]$ using Green's theorem. [4]

(b) Use Lagrange interpolation to find $f(3)$ from the given data: [4]

x	0	1	2	5
$f(x)$	2.5	3.7	12.9	147.5

SECTION-D

7. (a) Using Laplace transformation, solve the following IVP [4]

$$y_1' - y_2 = 0$$

$$y_1 + y_2' = 2\cos t$$

$$y_1(0) = 1, y_2(0) = 0$$

(b) Find a solution to the given ODE by Frobenius method [4]

$$xy'' + y' - xy = 0$$

8. (a) Solve the following IVP using Laplace transformation [4]

$$y'' + 3y' + 2y = u(t-1) + \delta(t-2),$$
$$y(0) = 0, y'(0) = 1$$

(b) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} sinx$ [4]

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