Association Rule Mining

Datamining and Data warehousing (CS 2004)

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Association Rule Mining

- What Is Pattern Discovery? Why Is It Important?
- Basic Concepts: Frequent Patterns and Association Rules
- Expressing Patterns in Compressed Forms: Closed Patterns vs. Max-Patterns
- Apriori
- Apriori : Improvements and Alternatives
- ECLAT
- FP Growth
- CLOSET+

What Is Pattern Discovery?

- What are patterns?
  - Patterns: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
  - Patterns represent intrinsic and important properties of datasets
- Pattern discovery: Uncovering patterns from massive data sets
- Motivation examples:
  - What products were often purchased together?
  - What are the subsequent purchases after buying an iPad?
  - What code segments likely contain copy-and-paste bugs?
  - What word sequences likely form phrases in this corpus?

Pattern Discovery: Why Is It Important?

- Finding inherent regularities in a data set
- Foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Mining sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: Discriminative pattern-based analysis
  - Cluster analysis: Pattern-based subspace clustering
- Broad applications
  - Market basket analysis, cross-marketing, catalog design, sale campaign analysis, Web log analysis, biological sequence analysis

Basic Concepts: Frequent Itemsets (Patterns)

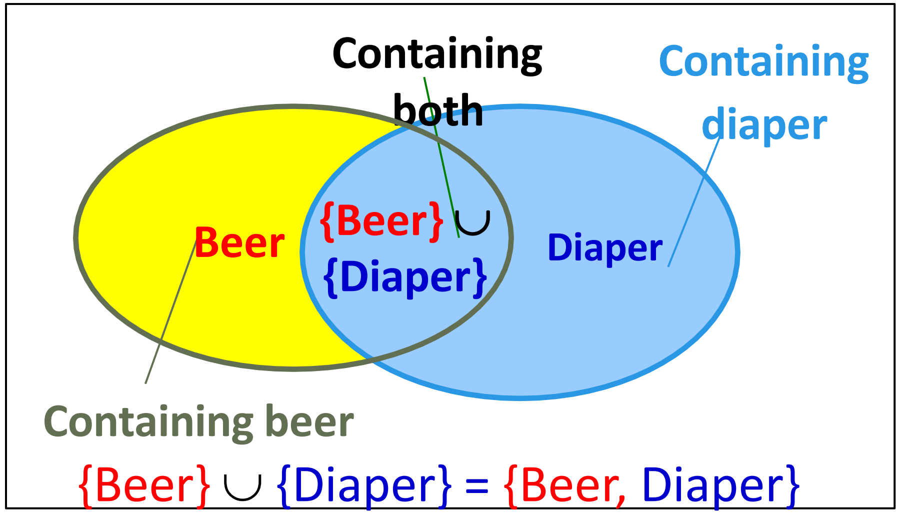
- Itemset: A set of one or more items
- k-itemset: $X = \{x_1, \ldots, x_k\}$
- *(absolute) support (count)* of X: Frequency or the number of occurrences of an itemset X
- *(relative) support*, *s:* The fraction of transactions that contains X (i.e., the probability that a transaction contains X)
- An itemset X is *frequent* if the support of X is no less than a *minsup* threshold (denoted as σ)

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

- Let *minsup = 50%*
- Freq. 1-itemsets:
  - Beer: 3 (60%); Nuts: 3 (60%)
  - Diaper: 4 (80%); Eggs: 3 (60%)
- Freq. 2-itemsets:
  - {Beer, Diaper}: 3 (60%)

From Frequent Itemsets to Association Rules

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



Note: Itemset: X ∪ Y, a subtle notation!

- Association rules: *X* → *Y* (s, c)
  - Support, *s*: The probability that a transaction contains X ∪ Y
  - Confidence, *c: The* conditional probability that a transaction containing X also contains *Y*
  - c = sup(X ∪ Y) / sup(X)
- **Association rule mining**: Find all of the rules, *X* → *Y*, with minimum support and confidence
- Frequent itemsets: Let *minsup = 50%*
  - Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
  - Freq. 2-itemsets: {Beer, Diaper}: 3
- Association rules: Let *minconf = 50%*
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

(Q: Are these all rules?)

Challenge: There Are Too Many Frequent Patterns!

- A long pattern contains a combinatorial number of sub-patterns
- How many frequent itemsets does the following TDB_1 contain?
  - $TDB_{1:}$ T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Assuming (absolute) *minsup* = 1
  - Let's have a try

    1-itemsets: (a_1): 2, (a_2): 2, …, (a_{50}): 2, (a_{51}): 1, …, (a_{100}): 1,

    2-itemsets: (a_1, a_2): 2, …, (a_1, a_{50}): 2, (a_1, a_{51}): 1 …, …, (a_{99}, a_{100}): 1,

    …, …, …, …

    99-itemsets: $\{a_1, a_2, \ldots, a_{99}\}$: 1, …, $\{a_2, a_3, \ldots, a_{100}\}$: 1

    100-itemset: $\{a_1, a_2, \ldots, a_{100}\}$: 1
  - In total: $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1$ sub-patterns!

A too huge set for any computer to compute or store!

Expressing Patterns in Compressed Form: Closed Patterns

- How to handle such a challenge?
- Solution 1: **Closed patterns**: A pattern (itemset) X is closed if X is *frequent,* and there exists *no super-pattern* Y ɔ X, *with the same support* as X
  - Let Transaction DB TDB_1: T_1: $\{a_1, …, a_{50}\}$; T_2: $\{a_1, …, a_{100}\}$
  - Suppose *minsup* = 1. How many closed patterns does TDB_1 contain?
    - Two: P_1: "(a_1, …, a_{50}): 2"; P_2: "(a_1, …, a_{100}): 1"
- Closed pattern is a lossless compression of frequent patterns
  - Reduces the # of patterns but does not lose the support information!
  - You will still be able to say: "(a_2, …, a_{40}): 2", "(a_5, a_{51}): 1"

Expressing Patterns in Compressed Form: Max-Patterns

- Solution 2: **Max-patterns**: A pattern X is a max-pattern if X is frequent and there exists no frequent super-pattern Y ↄ X
- Difference from close-patterns?
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB TDB_1: T_1: {a_1, …, a_{50}}; T_2: {a_1, …, a_{100}}
  - Suppose *minsup* = 1. How many max-patterns does TDB_1 contain?
    - One: P: "(a_1, …, a_{100}): 1"
- Max-pattern is a lossy compression!
  - We only know (a_1, …, a_{40}) is frequent
  - But we do not know the real support of (a_1, …, a_{40}), …, any more!
- Thus in many applications, mining close-patterns is more desirable than mining max-patterns

The Downward Closure Property of Frequent Patterns

- Observation: From $TDB_{1:}$ T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - We get a frequent itemset: $(a_1, \ldots, a_{50})$
  - Also, its subsets are all frequent: (a_1), (a_2), …, (a_{50}), (a_1, a_2), …, $(a_1, \ldots, a_{49})$, …
  - There must be some hidden relationships among frequent patterns!
- The downward closure (also called "Apriori") property of frequent patterns
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - Every transaction containing {beer, diaper, nuts} also contains {beer, diaper}
  - Apriori: Any subset of a frequent itemset must be frequent
- Efficient mining methodology
  - If any subset of an itemset S is infrequent, then there is no chance for S to be frequent—why do we even have to consider S!?

A sharp knife for pruning!

Apriori Pruning and Scalable Mining Methods

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not even be generated! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Scalable mining Methods: Three major approaches
  - Level-wise, join-based approach: **Apriori** (Agrawal & Srikant@VLDB'94)
  - Vertical data format approach: **Eclat** (Zaki, Parthasarathy, Ogihara, Li @KDD'97)
  - Frequent pattern projection and growth: **FPgrowth** (Han, Pei, Yin @SIGMOD'00)

Apriori: A Candidate Generation & Test Approach

- Outline of Apriori (level-wise, candidate generation and test)
  - Initially, scan DB once to get frequent 1-itemset
  - Repeat
    - Generate length-(k+1) candidate itemsets from length-k frequent itemsets
    - Test the candidates against DB to find frequent (k+1)-itemsets
    - Set k := k +1
  - Until no frequent or candidate set can be generated
  - Return all the frequent itemsets derived

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k

F_k : Frequent itemset of size k

K := 1;

F_k := {frequent items}; // frequent 1-itemset

While (F_k != $\emptyset$) **do {** // when F_k is non-empty

C_{k+1} := candidates generated from F_k; // candidate generation

Derive F_{k+1} by counting candidates in C_{k+1} with respect to *TDB* at minsup;

k := k + 1

}

return $\cup_k F_k$ // return F_k generated at each level

The Apriori Algorithm—An Example

minsup = 2

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

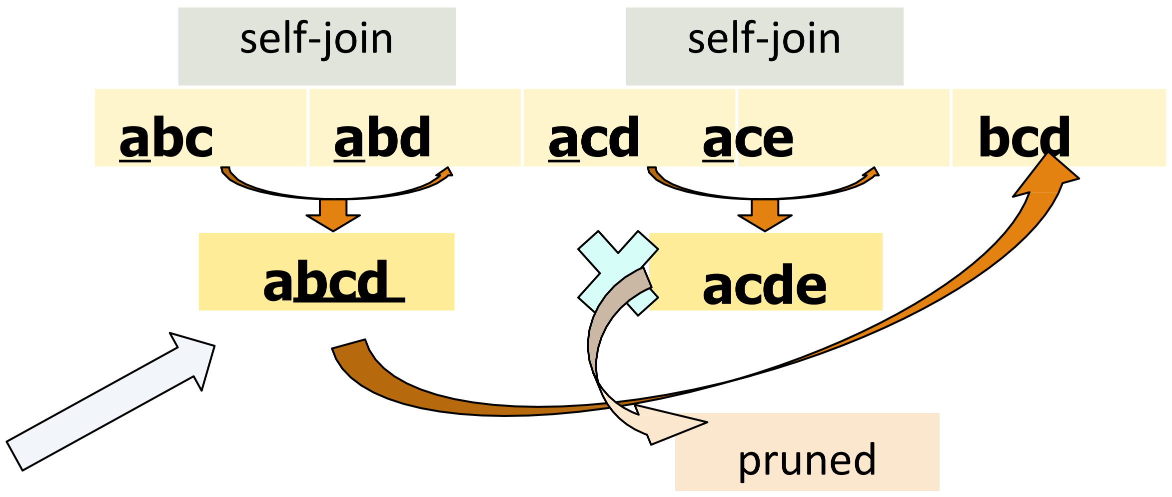
3rd scan

F_3

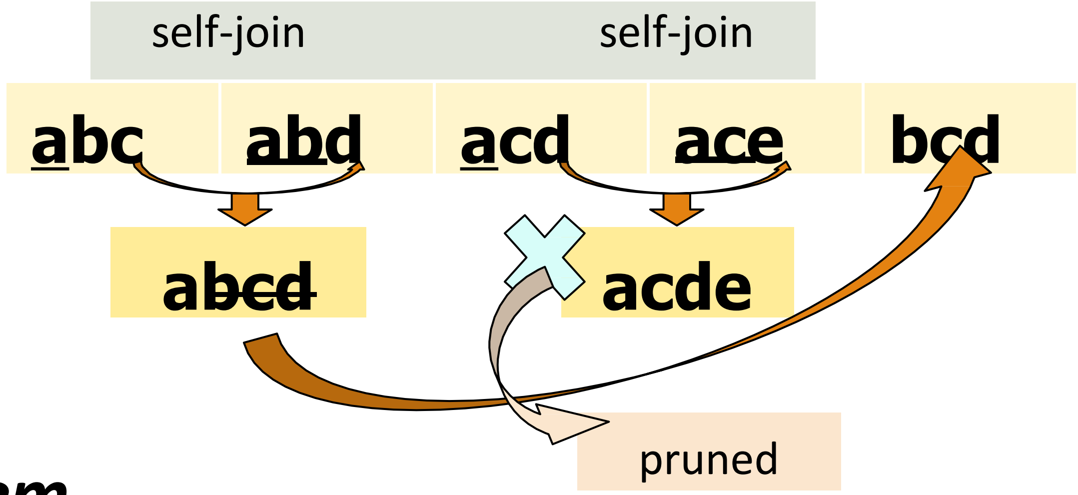
Itemset	sup
{B, C, E}	2

Apriori: Implementation Tricks

- How to generate candidates?
  - Step 1: self-joining F_k
  - Step 2: pruning
- Example of candidate-generation
  - $F_3 = \{abc, abd, acd, ace, bcd\}$
  - Self-joining: $F_3 * F_3$
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in F_3
  - $C_4 = \{abcd\}$



Candidate Generation: An SQL Implementation



- Suppose the items in F_{k-1} are listed in an order
- Step 1: self-joining F_{k-1}

  insert into $\boldsymbol{C_k}$
  select $\boldsymbol{p.item_1, p.item_2, \ldots, p.item_{k-1}, q.item_{k-1}}$
  from $\boldsymbol{F_{k-1}}$ ***as p***, $\boldsymbol{F_{k-1}}$ ***as q***
  where $\boldsymbol{p.item_1 = q.item_1, \ldots, p.item_{k-2} = q.item_{k-2}, p.item_{k-1} < q.item_{k-1}}$
- Step 2: pruning

  for all ***itemsets c in*** $\boldsymbol{C_k}$ do
  for all ***(k-1)-subsets s of c*** do
  if *(s is not in* F_{k-1}*)* **then delete** *c* **from** C_k

Apriori: Improvements and Alternatives

- Reduce passes of transaction database scans
  - Partitioning (e.g., Savasere, et al., 1995)
  - Dynamic itemset counting (Brin, et al., 1997)

To be discussed in subsequent slides

- Shrink the number of candidates
  - Hashing (e.g., DHP: Park, et al., 1995)
  - Pruning by support lower bounding (e.g., Bayardo 1998)
  - Sampling (e.g., Toivonen, 1996)

To be discussed in subsequent slides

- Exploring special data structures
  - Tree projection (Aggarwal, et al., 2001)
  - H-miner (Pei, et al., 2001)
  - Hypecube decomposition (e.g., LCM: Uno, et al., 2004)

Partitioning: Scan Database Only Twice

- Theorem: *Any itemset that is potentially frequent in TDB must be frequent in at least one of the partitions of TDB*

Here is the proof!

$$TDB_1 \quad + \quad TDB_2 \quad + \quad \cdots \quad + \quad TDB_k \quad = \quad TDB$$

$$sup_1(X) < \sigma|TDB_1| \qquad sup_2(X) < \sigma|TDB_2| \qquad \ldots \qquad sup_k(X) < \sigma|TDB_k| \qquad sup(X) < \sigma|TDB|$$

- Method: (A. Savasere, E. Omiecinski and S. Navathe, *VLDB'95*)
  - Scan 1: Partition database (how?) and find local frequent patterns
  - Scan 2: Consolidate global frequent patterns (how to?)
- Why does this method guarantee to scan TDB only twice?

Direct Hashing and Pruning (DHP)

- DHP (Direct Hashing and Pruning): Reduce the number of candidates (J. Park, M. Chen, and P. Yu, SIGMOD'95)
- Observation: A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
  - Candidates: a, b, c, d, e
  - Hash entries
    - {ab, ad, ae}
    - {bd, be, de}
    - …
  - Frequent 1-itemset: a, b, d, e
  - ab is not a candidate 2-itemset if the sum of count of {ab, ad, ae} is below support threshold

Itemsets	Count
{ab, ad, ae}	35
{bd, be, de}	298
……	…
{yz, qs, wt}	58

Hash Table

Exploring Vertical Data Format: ECLAT

- ECLAT (Equivalence Class Transformation): A depth-first search algorithm using set intersection [Zaki et al. @KDD'97]
- Tid-List: List of transaction-ids containing an itemset
- Vertical format: $t(e) = \{T_{10}, T_{20}, T_{30}\}$; $t(a) = \{T_{10}, T_{20}\}$; $t(ae) = \{T_{10}, T_{20}\}$
- Properties of Tid-Lists
  - $t(X) = t(Y)$: X and Y always happen together (e.g., $t(ac) = t(d)$)
  - $t(X) \subset t(Y)$: transaction having X always has Y (e.g., $t(ac) \subset t(ce)$)
- Deriving frequent patterns based on vertical intersections
- Using diffset to accelerate mining
  - Only keep track of differences of tids
  - $t(e) = \{T_{10}, T_{20}, T_{30}\}$, $t(ce) = \{T_{10}, T_{30}\}$ → Diffset $(ce, e) = \{T_{20}\}$

A transaction DB in Horizontal Data Format

Tid	Itemset
10	a, c, d, e
20	a, b, e
30	b, c, e

The transaction DB in Vertical Data Format

Item	TidList
a	10, 20
b	20, 30
c	10, 30
d	10
e	10, 20, 30

FPGrowth: Mining Frequent Patterns by Pattern Growth

- Idea: Frequent pattern growth (FPGrowth)
  - Find frequent single items and partition the database based on each such item
  - Recursively grow frequent patterns by doing the above for each partitioned database (also called *conditional database*)
  - To facilitate efficient processing, an efficient data structure, FP-tree, can be constructed
- Mining becomes
  - Recursively construct and mine (conditional) FP-trees
  - Until the resulting FP-tree is empty, or until it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

Example: Construct FP-tree from a Transational DB

TID	Items in the Transaction	Ordered, frequent items
100	*{f, a, c, d, g, i, m, p}*	*{f, c, a, m, p}*
200	*{a, b, c, f, l, m, o}*	*{f, c, a, b, m}*
300	*{b, f, h, j, o, w}*	*{f, b}*
400	*{b, c, k, s, p}*	*{c, b, p}*
500	*{a, f, c, e, l, p, m, n}*	*{f, c, a, m, p}*

1. Scan DB once, find single item frequent pattern: **Let min_support = 3**

   f:4, a:3, c:4, b:3, m:3, p:3

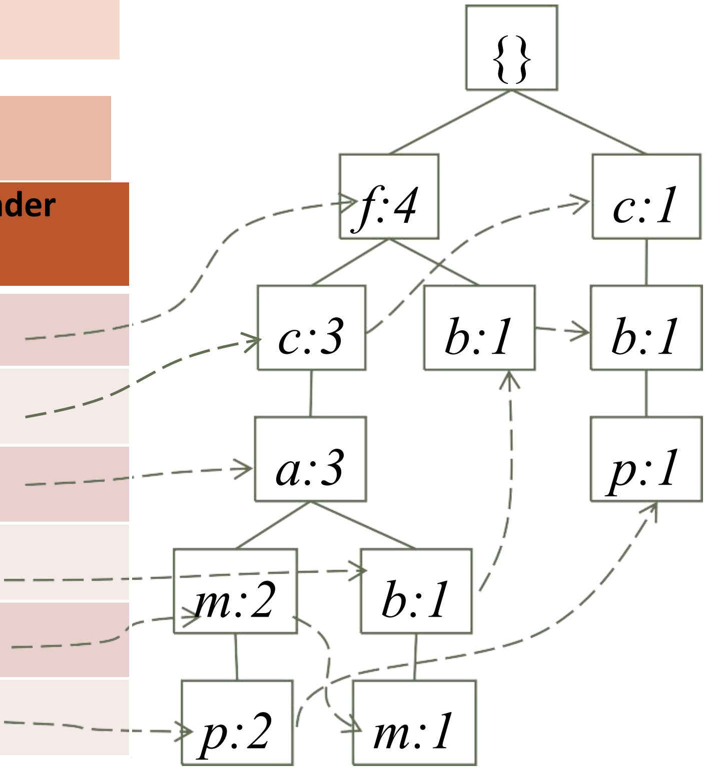
2. Sort frequent items in frequency descending order, f-list

   F-list = f-c-a-b-m-p

3. Scan DB again, construct FP-tree

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

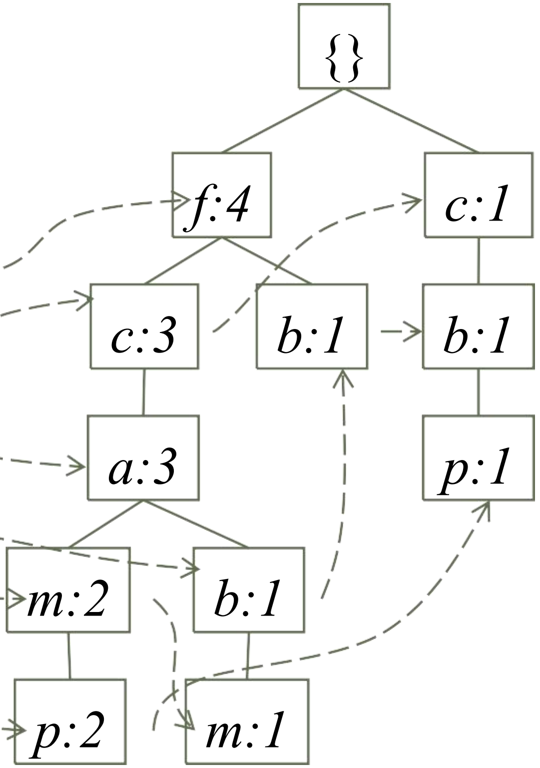


Divide and Conquer Based on Patterns and Data

- Pattern mining can be partitioned according to current patterns
  - Patterns containing p: p's conditional database: *fcam:2, cb:1*
  - Patterns having m but no p: m's conditional database: *fca:2, fcab:1*
  - …… ……
- *p*'s conditional pattern base: *transformed prefix paths* of item *p*

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Conditional pattern bases

Item	*Conditional pattern base*
c	f:3
a	fc:3
b	fca:1, f:1, c:1
m	fca:2, fcab:1
p	fcam:2, cb:1

Mine Each Conditional Pattern-Base Recursively

Conditional pattern bases

item	*cond. pattern base*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

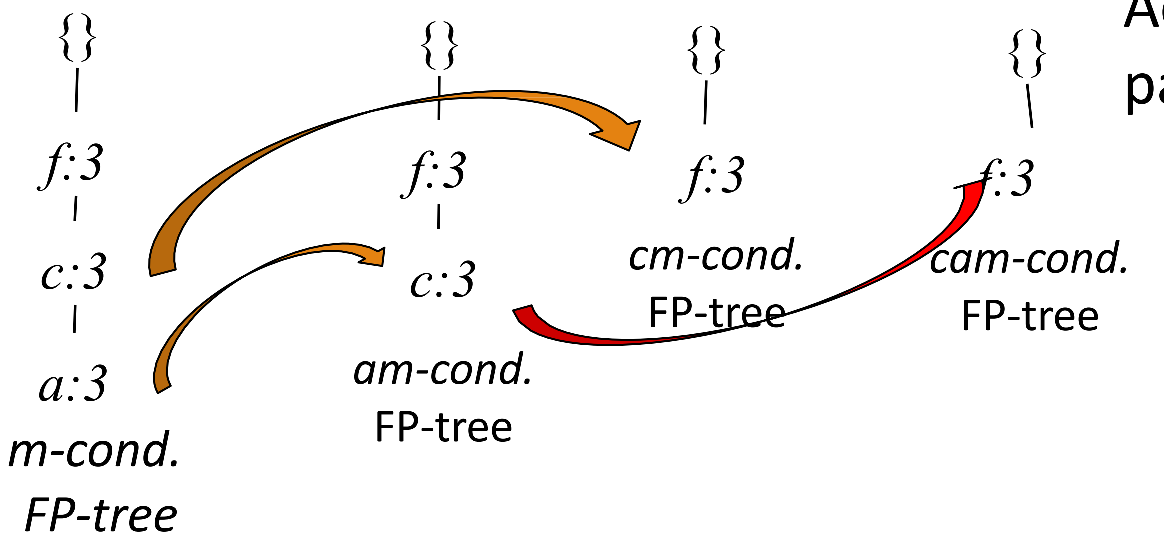
min_support = 3

- For each conditional pattern-base
  - Mine single-item patterns
  - Construct its FP-tree & mine it

p-conditional PB: ***fcam:2, cb:1 → c: 3***

m-conditional PB: ***fca:2, fcab:1 → fca: 3***

b-conditional PB: ***fca:1, f:1, c:1 → ϕ***



Actually, for single branch FP-tree, all frequent patterns can be generated in one shot

m: 3

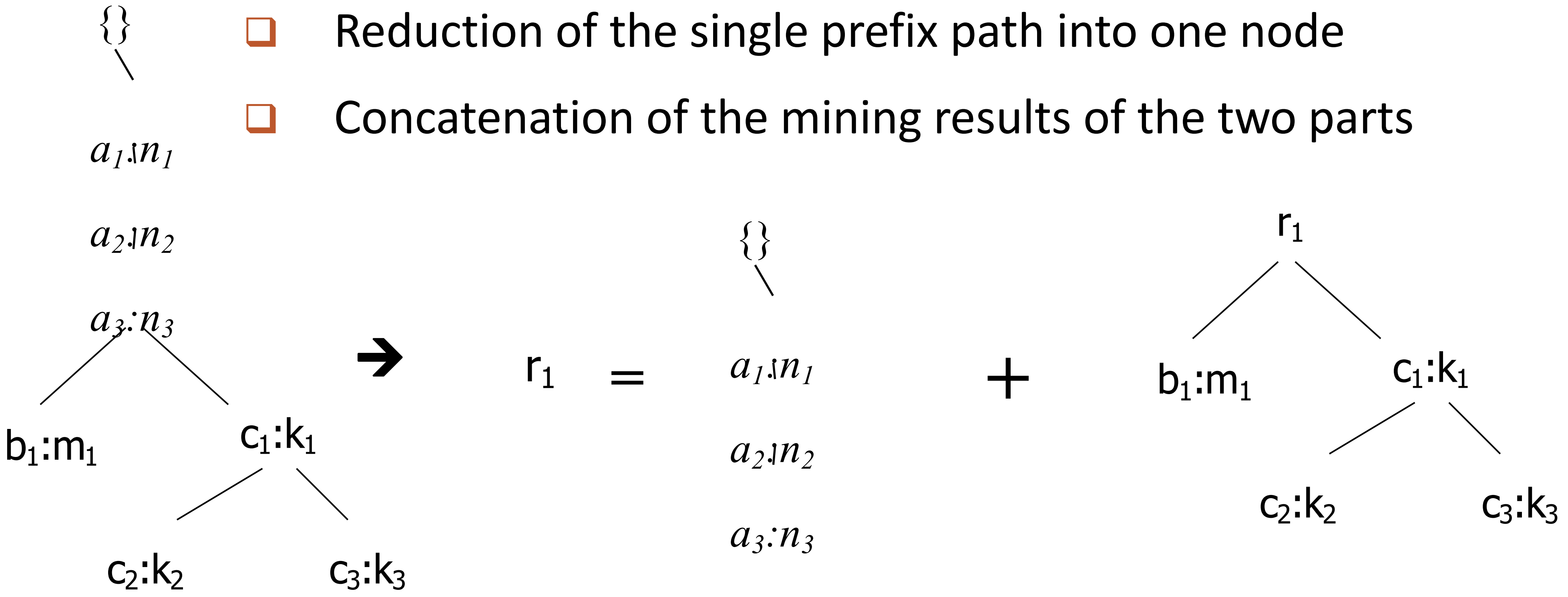
fm: 3, cm: 3, am: 3

fcm: 3, fam:3, cam: 3

fcam: 3

A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts



Scaling FP-growth by Database Projection

- What if FP-tree cannot fit in memory? — DB projection
  - Project the DB based on patterns
  - Construct & mine FP-tree for each projected DB
- Parallel projection vs. partition projection
  - Parallel projection: Project the DB on each frequent item
    - Space costly, all partitions can be processed in parallel
  - Partition projection: Partition the DB in order
    - Passing the unprocessed parts to subsequent partitions

Trans. DB

f_2 f_3 f_4 g h
f_3 f_4 i j
f_2 f_4 k
f_1 f_3 h
…

Assume only f's are freq. & the freq. item ordering is: f_1-f_2-f_3-f_4

Parallel projection

f_4-proj. DB

f_2 f_3
f_3
f_2
…

f_3-proj. DB

f_2
f_1
…

Partition projection

f_4-proj. DB

f_2 f_3
f_3
f_2
…

f_3-proj. DB

f_1
…

f_2 will be projected to f_3-proj. DB only when processing f_4-proj. DB

CLOSET+: Mining Closed Itemsets by Pattern-Growth

- Efficient, *direct* mining of closed itemsets
- Ex. Itemset merging: If Y appears in every occurrence of X, then Y is merged with X
  - d-proj. db: {acef, acf} → acfd-proj. db: {e}, thus we get: acfd:2
- Many other tricks (but not detailed here), such as
  - Hybrid tree projection
    - Bottom-up physical tree-projection
    - Top-down pseudo tree-projection
  - Sub-itemset pruning
  - Item skipping
  - Efficient subset checking
- For details, see J. Wang, et al., “CLOSET+: ……”, KDD'03

TID	Items
1	acdef
2	abe
3	cefg
4	acdf

Let minsupport = 2

a:3, c:3, d:2, e:3, f:3

F-List: a-c-e-f-d

Recommended Text and Reference Books

- **Text Book:**
  - J. Han and M. Kamber. Data Mining: Concepts and Techniques. Morgan Kaufmann, 3rd ed., 2011
- **Reference Books:**
  - H. Dunham. Data Mining: Introductory and Advanced Topics. Pearson Education. 2006.
  - I. H. Witten and E. Frank. Data Mining: Practical Machine Learning Tools and Techniques. Morgan Kaufmann. 2000.
  - D. Hand, H. Mannila and P. Smyth. Principles of Data Mining.Prentice-Hall. 2001.

THANK
YOU!