Qn. Set Code-1

Semester: 4th
Programme: B.Tech
Branch: CSCE, E&CSc

SPRING END SEMESTER EXAMINATION-2023

4th Semester B.Tech

DISCRETE MATHEMATICS

MA 2013

(For 2022 (L.E), 2021 & Previous Admitted Batches)

Time: 3 Hours Full Marks: 50

Answer any SIX questions.
Question paper consists of four SECTIONS i.e. A, B, C and D.
Section A is compulsory.
Attempt minimum one question each from Sections B, C, D.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A

1. Answer the following questions. $[1 \times 10]$

(a) What is the contrapositive of the conditional statement "It is necessary to have a valid password to log on to the server" in the form of "if… then….."

(b) Translate the statement "At least one of your friends is perfect" into logical expression using predicate and quantifiers.

(c) What is the reflexive and symmetric closures for the relation $R = \{(1,1),(1,3),(3,3)\}$ defined on the set $A = \{1,2,3\}$?

(d) A computer company must hire 20 programmers to handle system programming jobs and 30 programmers for applications programming out of those hired, 5 are expected to perform both the jobs. How many programmers must be hired?

(e) Define Poset. Give an example.

(f) Write generating function for the sequence
$< 1, -1, 1, -1, 1, \dots\dots >$.

(g) In the poset $(Z^+, |)$ (where Z^+ is the set of all positive integers and | is the divides relation) are the integers 9 and 351 comparable?

(h) Define cyclic group with an example.

(i) Mention self-inverse elements in the group $(\{-1, 1, i, -i\}, *)$ where* represents usual multiplication.

(j) If the ring $(\mathbb{Z}_n, \oplus_n, \otimes_n)$is always a field then n must be?

SECTION-B

2. (a) Verify using the truth table, whether the propositional statements $\sim p \to (q \to r)$ and $q \to (p \vee r)$ logically equivalent? [4]

(b) What is solution of recurrence relation [4]
$F_n = 6\,F_{n-1} - 9\,F_{n-2}$ where $F_0 = 1, F_1 = 6$.

3. (a) Suppose that there are 1807 freshmen at your school. Of these, 453 are taking a course in computer science, 567 are taking a course in mathematics, and 299 are taking courses in both computer science and mathematics. How many are not taking a course either in computer science or in mathematics? [4]

(b) Prove that $(\mathbb{Z}_n, \oplus_n)$ is an abelian group. [4]

SECTION-C

4. (a) Using Rules of Inference for predicates and quantifiers, validate the argument "All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic." [4]

(b) Use Warshall's algorithm to find the transitive closure of the relation [4]

$\{(a_{1,}a_1), (a_{1,}a_2), (a_{1,}a_4), (a_{2,}a_3), (a_{3,}a_3),$
$(a_{3,}a_5), (a_{4,}a_4), (a_{5,}a_2)\}$

on the set $A = \{a_{1\,,}a_{2\,,}a_{3\,,}a_{4\,,}a_{5,}\}$.

5\. (a) Using strong mathematical induction, show that for every integer n than 1, n can be written as product of primes. [4]

(b) Define units in a ring. Write all the units of the ring $(\mathbb{Z}_6, \oplus_6, \otimes_6)$. [4]

6\. (a) Using mathematical induction, show that $n^3 - n$ is divisible by 3. [4]

(b) Consider a poset $(\{3, 5, 9, 15, 24, 45\}, |)$ where $a|b$ means a divides b then answer the following questions: [4]

i. Find the maximal elements.
ii. Find the minimal elements.
iii. Is there a greatest element?
iv. Is there a least element?
v. Find all upper bounds of 3, 5.
vi. Find the least upper bound of 3, 5, if it exists.
vii. Find all lower bounds of 15, 45.
viii. Find the greatest lower bound of 15, 45, if it exists.

SECTION-D

7\. (a) Using Rules of Inference, show that the following argument is valid: If I eat spicy foods, then I have strange dreams. I have strange dreams if there is thunder while I sleep. I did not have strange dreams. Therefore, it did not thunder and I did not eat spicy food". [4]

(b) Let S be the set of all bit strings. Then sR_3t either when $s = t$ or both s and t are bit strings of length 3 or more that begin with the same three bits. Show that R_3 is an equivalence relation on S. Also, find the distinct equivalence classes. [4]

8. (a) What is solution of recurrence relation $F_n = F_{n-1} - 2\,F_{n-2}$ where $F_0 = 2, F_1 = 7$. [4]

(b) Let $G = \mathbb{R} - \{0\}$, and $a * b = \frac{ab}{2}$. Show that (G, *) is an abelian group. [4]

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