(*ii*) *The above theorems and techniques are applicable only to networks that have linear, bilateral circuit elements.*

(*iii*) The network theorem or technique to be used will depend upon the network arrangement. The general rule is this. *Use that theorem or technique which requires a smaller number of independent equations to obtain the solution or which can yield easy solution.*

(*iv*) *Analysis of a circuit usually means to determine all the currents and voltages in the circuit.*

3.4. Maxwell's Mesh Current Method

In this method, Kirchhoff's voltage law is applied to a network to write mesh equations in terms of **mesh currents** *instead of branch currents*. Each mesh is assigned a separate mesh current. This mesh current is assumed to flow *clockwise* around the perimeter of the mesh without splitting at a junction into branch currents. Kirchhoff's voltage law is then applied to write equations in terms of unknown mesh currents. The branch currents are then found by taking the algebraic sum of the mesh currents which are common to that branch.

Explanation. Maxwell's mesh current method consists of following steps :

(*i*) Each mesh is assigned a separate mesh current. For convenience, all mesh currents are assumed to flow in *clockwise direction. For example, in Fig. 3.2, meshes *ABDA* and *BCDB* have been assigned mesh currents I_1 and I_2 respectively. The mesh currents take on the appearance of a mesh fence and hence the name mesh currents.

(*ii*) If two mesh currents are flowing through a circuit element, the actual current in the circuit element is the algebraic sum of the two. Thus in Fig. 3.2, there are two mesh currents I_1 and I_2 flowing in R_2. If we go from B to D, current is $I_1 - I_2$ and if we go in the other direction (*i.e.* from D to B), current is $I_2 - I_1$.

(*iii*) **Kirchhoff's voltage law is applied to write equation for each mesh in terms of mesh currents. Remember, while writing mesh equations, rise in potential is assigned positive sign and fall in potential negative sign.

(*iv*) If the value of any mesh current comes out to be negative in the solution, it means that true direction of that mesh current is anticlockwise *i.e.* opposite to the assumed clockwise direction.

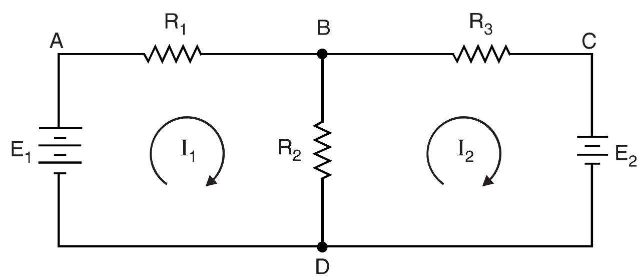


Fig. 3.2

Applying Kirchhoff's voltage law to Fig. 3.2, we have,

Mesh ABDA.

$$-I_1R_1 - (I_1 - I_2)R_2 + E_1 = 0$$

or $$I_1(R_1 + R_2) - I_2R_2 = E_1 \qquad ...(i)$$

* It is convenient to consider all mesh currents in one direction (clockwise or anticlockwise). The same result will be obtained if mesh currents are given arbitrary directions.

** Since the circuit unknowns are currents, the describing equations are obtained by applying *KVL* to the meshes.

Mesh BCDB.

$$-I_2R_3 - E_2 - (I_2 - I_1)R_2 = 0$$

or\` $$-I_1R_2 + (R_2 + R_3)I_2 = -E_2 \quad ...(ii)$$

Solving eq. (*i*) and eq. (*ii*) simultaneously, mesh currents I_1 and I_2 can be found out. Once the mesh currents are known, the branch currents can be readily obtained. *The advantage of this method is that it usually reduces the number of equations to solve a network problem.*

Note. Branch currents are the real currents because they actually flow in the branches and can be measured. However, mesh currents are fictitious quantities and cannot be measured except in those instances where they happen to be identical with branch currents. Thus in branch *DAB*, branch current is the same as mesh current and both can be measured. But in branch *BD*, mesh currents (I_1 and I_2) cannot be measured. Hence mesh current is a concept rather than a reality. However, it is a useful concept to solve network problems as it leads to the reduction of number of mesh equations.

3.5. Shortcut Procedure for Network Analysis by Mesh Currents

We have seen above that Maxwell mesh current method involves lengthy mesh equations. Here is a shortcut method to write mesh equations simply by inspection of the circuit. Consider the circuit shown in Fig. 3.3. The circuit contains resistances and independent voltage sources and has three meshes. Let the three mesh currents be I_1, I_2 and I_3 flowing in the clockwise direction.

Loop 1. Applying *KVL* to this loop, we have,

$$100 - 20 = I_1(60 + 30 + 50) - I_2 \times 50 - I_3 \times 30$$

or $$80 = 140I_1 - 50I_2 - 30I_3 \quad ...(i)$$

We can write eq. (*i*) in a shortcut form as :

$$E_1 = I_1R_{11} - I_2R_{12} - I_3R_{13}$$

Here E_1 = Algebraic sum of e.m.f.s in Loop (1) in the direction of I_1

$= 100 - 20 = 80$ V

R_{11} = Sum of resistances in Loop (1)

= Self*-resistance of Loop (1)

$= 60 + 30 + 50 = 140\ \Omega$

R_{12} = Total resistance common to Loops (1) and (2)

= Common resistance between Loops (1) and (2) = 50 Ω

R_{13} = Total resistance common to Loops (1) and (3) = 30 Ω

It may be seen that the sign of the term involving self-resistances is positive while the sign of common resistances is negative. It is because the positive directions for mesh currents were all chosen clockwise. Although mesh currents are abstract currents, yet mesh current analysis offers the advantage that resistor polarities do not have to be considered when writing mesh equations.

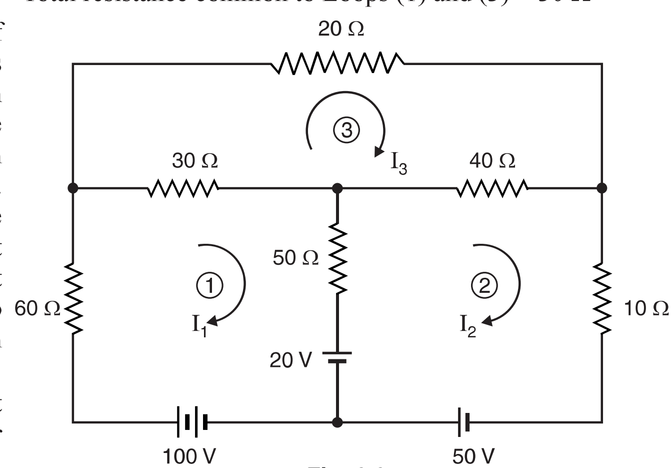


Fig. 3.3

Loop 2. We can use shortcut method to find the mesh equation for Loop (2) as under :

* The sum of all resistances in a loop is called self-resistance of that loop. Thus in Fig. 3.3, self-resistance of Loop (1) = 60 + 30 + 50 = 140 Ω.

$$E_2 = -I_1R_{21} + I_2R_{22} - I_3R_{23}$$

or $$50 + 20 = -50I_1 + 100I_2 - 40I_3 \qquad ...(ii)$$

Here, E_2 = Algebraic sum of e.m.f.s in Loop (2) in the direction of I_2

$= 50 + 20 = 70$ V

R_{21} = Total resistance common to Loops (2) and (1) = 50 Ω

R_{22} = Sum of resistances in Loop (2) = 50 + 40 + 10 = 100 Ω

R_{23} = Total resistance common to Loops (2) and (3) = 40 Ω

Again the sign of self-resistance of Loop (2) (R_{22}) is positive while the sign of the terms of common resistances (R_{21}, R_{23}) is negative.

Loop 3. We can again use shortcut method to find the mesh equation for Loop (3) as under :

$$E_3 = -I_1R_{31} - I_2R_{32} + I_3R_{33}$$

or $$0 = -30I_1 - 40I_2 + 90I_3 \qquad ...(iii)$$

Again the sign of self-resistance of Loop (3) (R_{33}) is positive while the sign of the terms of common resistances (R_{31}, R_{32}) is negative.

Mesh analysis using matrix form. The three mesh equations are rewritten below :

$$E_1 = I_1R_{11} - I_2R_{12} - I_3R_{13}$$

$$E_2 = -I_1R_{21} + I_2R_{22} - I_3R_{23}$$

$$E_3 = -I_1R_{31} - I_2R_{32} + I_3R_{33}$$

The matrix equivalent of above given equations is :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

It is reminded again that (*i*) all self-resistances are positive (*ii*) all common resistances are negative and (*iii*) by their definition, $R_{12} = R_{21}$; $R_{23} = R_{32}$ and $R_{13} = R_{31}$.

Example 3.1. *In the network shown in Fig. 3.4 (i), find the magnitude and direction of each branch current by mesh current method.*

Solution. Assign mesh currents I_1 and I_2 to meshes *ABDA* and *BCDB* respectively as shown in Fig. 3.4 (*i*).

Mesh ABDA. Applying *KVL*, we have,

$$-40I_1 - 20(I_1 - I_2) + 120 = 0$$

or $$60I_1 - 20I_2 = 120 \qquad ...(i)$$

Mesh BCDB. Applying *KVL*, we have,

$$-60I_2 - 65 - 20(I_2 - I_1) = 0$$

or $$-20I_1 + 80I_2 = -65 \qquad ...(ii)$$

Multiplying eq. (*ii*) by 3 and adding it to eq. (*i*), we get,

$$220I_2 = -75 \qquad \therefore \quad I_2 = -75/220 = -0{\cdot}341 \text{ A}$$

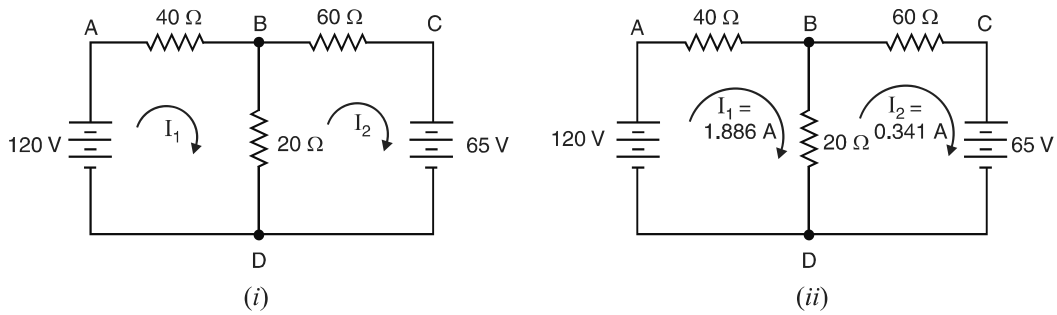


Fig. 3.4

The minus sign shows that true direction of I_2 is anticlockwise. Substituting $I_2 = -0{\cdot}341$A in eq. (*i*), we get, $I_1 = 1{\cdot}886$ A. The actual direction of flow of currents is shown in Fig. 3.4 (*ii*).

By determinant method

$$60I_1 - 20I_2 = 120$$
$$-20I_1 + 80I_2 = -65$$

$$\therefore \qquad I_1 = \frac{\begin{vmatrix} 120 & -20 \\ -65 & 80 \end{vmatrix}}{\begin{vmatrix} 60 & -20 \\ -20 & 80 \end{vmatrix}} = \frac{(120\times 80)-(-65\times -20)}{(60\times 80)-(-20\times -20)} = \frac{8300}{4400} = 1{\cdot}886 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 60 & 120 \\ -20 & -65 \end{vmatrix}}{\text{Denominator}} = \frac{(60\times -65)-(-20\times 120)}{4400} = \frac{-1500}{4400} = -0{\cdot}341 \text{ A}$$

Referring to Fig. 3.4 (*ii*), we have,

Current in branch $DAB = I_1 =$ **1·886 A** ; Current in branch $DCB = I_2 =$ **0·341 A**

Current in branch $BD = I_1 + I_2 = 1{\cdot}886 + 0{\cdot}341 =$ **2·227 A**

Example 3.2. *Calculate the current in each branch of the circuit shown in Fig. 3.5.*

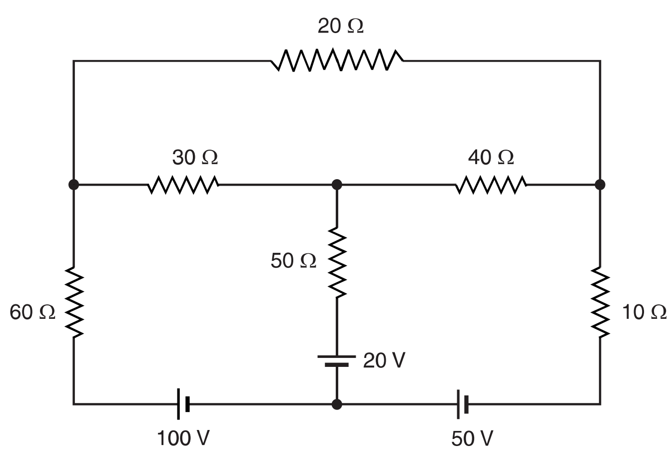


Fig. 3.5

Solution. Assign mesh currents I_1, I_2 and I_3 to meshes *ABHGA*, *HEFGH* and *BCDEHB* respectively as shown in Fig. 3.6.

Mesh ABHGA. Applying *KVL*, we have,

$$-60I_1 - 30(I_1 - I_3) - 50(I_1 - I_2) - 20 + 100 = 0$$

or $$140I_1 - 50I_2 - 30I_3 = 80$$

or $$14I_1 - 5I_2 - 3I_3 = 8 \quad ...(i)$$

Mesh GHEFG. Applying *KVL*, we have,

$$20 - 50(I_2 - I_1) - 40(I_2 - I_3) - 10I_2 + 50 = 0$$

or $$-50I_1 + 100I_2 - 40I_3 = 70$$

or $$-5I_1 + 10I_2 - 4I_3 = 7 \quad ...(ii)$$

Mesh BCDEHB. Applying *KVL*, we have,

$$-20I_3 - 40(I_3 - I_2) - 30(I_3 - I_1) = 0$$

or $$30I_1 + 40I_2 - 90I_3 = 0$$

or $$3I_1 + 4I_2 - 9I_3 = 0 \quad ...(iii)$$

Solving for equations (*i*), (*ii*) and (*iii*), we get, $I_1 = 1{\cdot}65$ A ; $I_2 = 2{\cdot}12$ A ; $I_3 = 1{\cdot}5$ A

By determinant method

$$14I_1 - 5I_2 - 3I_3 = 8$$
$$-5I_1 + 10I_2 - 4I_3 = 7$$
$$3I_1 + 4I_2 - 9I_3 = 0$$

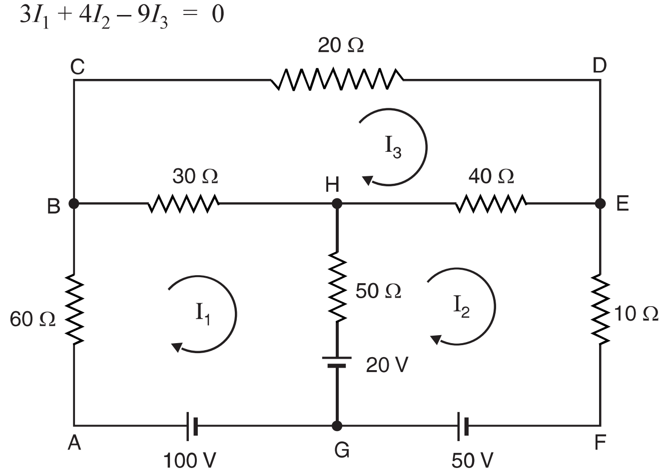


Fig. 3.6

$$\therefore \quad I_1 = \frac{\begin{vmatrix} 8 & -5 & -3 \\ 7 & 10 & -4 \\ 0 & 4 & -9 \end{vmatrix}}{\begin{vmatrix} 14 & -5 & -3 \\ -5 & 10 & -4 \\ 3 & 4 & -9 \end{vmatrix}} = \frac{8\begin{vmatrix} 10 & -4 \\ 4 & -9 \end{vmatrix} + 5\begin{vmatrix} 7 & -4 \\ 0 & -9 \end{vmatrix} - 3\begin{vmatrix} 7 & 10 \\ 0 & 4 \end{vmatrix}}{14\begin{vmatrix} 10 & -4 \\ 4 & -9 \end{vmatrix} + 5\begin{vmatrix} -5 & -4 \\ 3 & -9 \end{vmatrix} - 3\begin{vmatrix} -5 & 10 \\ 3 & 4 \end{vmatrix}}$$

$$= \frac{8[(10\times -9) - (4\times -4)] + 5[(7\times -9) - (0\times -4)] - 3[(7\times 4) - (0\times 10)]}{14[(10\times -9) - (4\times -4)] + 5[(-5\times -9) - (3\times -4)] - 3[(-5\times 4) - (3\times 10)]}$$

$$= \frac{-592 - 315 - 84}{-1036 + 285 + 150} = \frac{-991}{-601} = 1{\cdot}65 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 14 & 8 & -3 \\ -5 & 7 & -4 \\ 3 & 0 & -9 \end{vmatrix}}{\text{Denominator}} = \frac{14[(-63)-(0)]-8[(45)-(-12)]-3[(0)-(21)]}{-601}$$

$$= \frac{-882-456+63}{-601} = \frac{-1275}{-601} = 2{\cdot}12 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 14 & -5 & 8 \\ -5 & 10 & 7 \\ 3 & 4 & 0 \end{vmatrix}}{\text{Denominator}} = \frac{14[(0)-(28)]+5[(0)-(21)]+8[(-20)-(30)]}{-601}$$

$$= \frac{-392-105-400}{-601} = \frac{-897}{-601} = 1{\cdot}5 \text{ A}$$

∴ Current in 60 Ω = I_1 = **1·65 A** from *A* to *B*

Current in 30 Ω = $I_1 - I_3$ = 1·65 – 1·5 = **0·15 A** from *B* to *H*

Current in 50 Ω = $I_2 - I_1$ = 2·12 – 1·65 = **0·47 A** from *G* to *H*

Current in 40 Ω = $I_2 - I_3$ = 2·12 – 1·5 = **0·62 A** from *H* to *E*

Current in 10 Ω = I_2 = **2·12 A** from *E* to *F*

Current in 20 Ω = I_3 = **1·5 A** from *C* to *D*

Example 3.3. *By using mesh resistance matrix, determine the current supplied by each battery in the circuit shown in Fig. 3.7.*

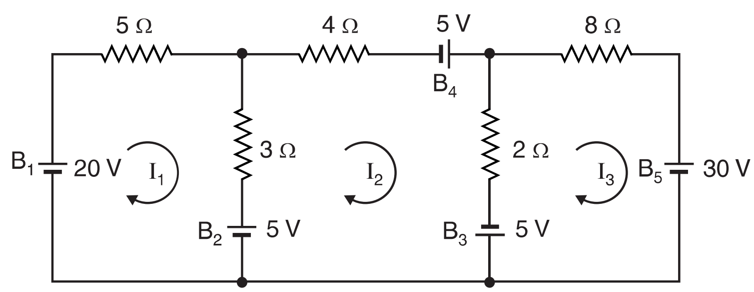


Fig. 3.7

Solution. Since there are three meshes, let the three mesh currents be I_1, I_2 and I_3, all assumed to be flowing in the clockwise direction. The different quantities of the mesh-resistance matrix are :

$$R_{11} = 5 + 3 = 8\ \Omega \quad ; \quad R_{22} = 4 + 2 + 3 = 9\ \Omega \quad ; \quad R_{33} = 8 + 2 = 10\ \Omega$$
$$R_{12} = R_{21} = -3\ \Omega \quad ; \quad R_{13} = R_{31} = 0 \quad ; \quad R_{23} = R_{32} = -2\ \Omega$$
$$E_1 = 20 - 5 = 15\text{ V} \quad ; \quad E_2 = 5 + 5 + 5 = 15\text{ V} \quad ; \quad E_3 = -30 - 5 = -35\text{ V}$$

Therefore, the mesh equations in the matrix form are :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

or

$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

By determinant method, we have,

$$I_1 = \frac{\begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix}}{\begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix}} = \frac{1530}{598} = 2{\cdot}56 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix}}{\text{Denominator}} = \frac{1090}{598} = 1{\cdot}82 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix}}{\text{Denominator}} = \frac{-1875}{598} = -3{\cdot}13 \text{ A}$$

The negative sign with I_3 indicates that actual direction of I_3 is opposite to that assumed in Fig. 3.7. Note that batteries B_1, B_3, B_4 and B_5 are discharging while battery B_2 is charging.

$\therefore$ Current supplied by battery $B_1 = I_1 =$ **2·56 A**

Current supplied to battery $B_2 = I_1 - I_2 = 2{\cdot}56 - 1{\cdot}82 =$ **0·74 A**

Current supplied by battery $B_3 = I_2 + I_3 = 1{\cdot}82 + 3{\cdot}13 =$ **4·95 A**

Current supplied by battery $B_4 = I_2 =$ **1·82 A**

Current supplied by battery $B_5 = I_3 =$ **3·13 A**

Example 3.4. *By using mesh resistance matrix, calculate the current in each branch of the circuit shown in Fig. 3.8.*

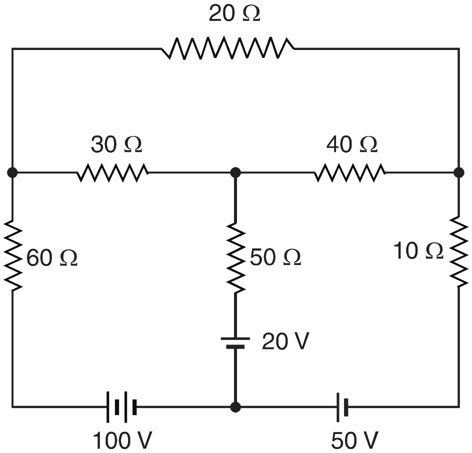


Fig. 3.8

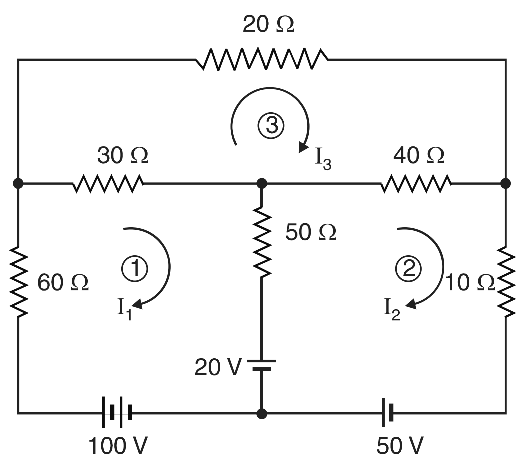


Fig. 3.9

Solution. Since there are three meshes, let the three mesh currents be I_1, I_2 and I_3, all assumed to be flowing in the clockwise direction as shown in Fig. 3.9. The different quantities of the mesh resistance-matrix are :

$$R_{11} = 60 + 30 + 50 = 140\ \Omega\ ;\ R_{22} = 50 + 40 + 10 = 100\ \Omega\ ;\ R_{33} = 30 + 20 + 40 = 90\ \Omega$$

$$R_{12} = R_{21} = -50\ \Omega\ ;\ R_{13} = R_{31} = -30\ \Omega\ ;\ R_{23} = R_{32} = -40\ \Omega$$

$$E_1 = 100 - 20 = 80\text{ V}\ ;\ E_2 = 50 + 20 = 70\text{ V}\ ;\ E_3 = 0\text{ V}$$

Therefore, the mesh equations in the matrix form are :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

or

$$\begin{bmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 70 \\ 0 \end{bmatrix}$$

By determinant method, we have,

$$I_1 = \frac{\begin{vmatrix} 80 & -50 & -30 \\ 70 & 100 & -40 \\ 0 & -40 & 90 \end{vmatrix}}{\begin{vmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{vmatrix}} = \frac{991000}{601000} = 1{\cdot}65\text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 140 & 80 & -30 \\ -50 & 70 & -40 \\ -30 & 0 & 90 \end{vmatrix}}{\text{Denominator}} = \frac{1275000}{601000} = 2{\cdot}12\text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 140 & -50 & 80 \\ -50 & 100 & 70 \\ -30 & -40 & 0 \end{vmatrix}}{\text{Denominator}} = \frac{897000}{601000} = 1{\cdot}5\text{ A}$$

$\therefore$ Current in 60 Ω = I_1 = **1·65 A in the direction of I_1**

Current in 30 Ω = $I_1 - I_3$ = **0·15 A in the direction of I_1**

Current in 50 Ω = $I_2 - I_1$ = **0·47 A in the direction of I_2**

Current in 40 Ω = $I_2 - I_3$ = **0·62 A in the direction of I_2**

Current in 10 Ω = I_2 = **2·12 A in the direction of I_2**

Current in 20 Ω = I_3 = **1·5 A in the direction of I_3**

Example 3.5. *Find mesh currents i_1 and i_2 in the electric circuit shown in Fig. 3.10.*

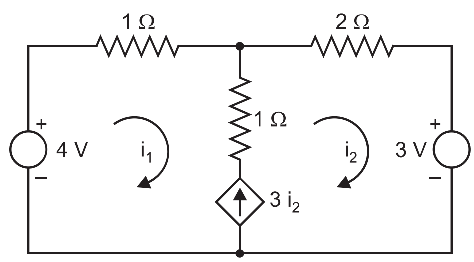


Fig. 3.10

Solution. We shall use mesh current method for the solution. Mesh analysis requires that all the sources in a circuit be voltage sources. If a circuit contains any current source, convert it into equivalent voltage source.

Outer mesh. Applying *KVL* to this mesh, we have,

$$-i_1 \times 1 - 2i_2 - 3 + 4 = 0 \quad \text{or} \quad i_1 + 2i_2 = 1 \qquad ...(i)$$

First mesh. Applying *KVL* to this mesh, we have,

$$-i_1 \times 1 - (i_1 - i_2) \times 1 - 3i_2 + 4 = 0 \quad \text{or} \quad i_1 + i_2 = 2 \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we have $i_1 = \mathbf{3A}$; $i_2 = \mathbf{-1\ A}$

Example 3.6. *Using mesh current method, determine current I_x in the circuit shown in Fig. 3.11.*

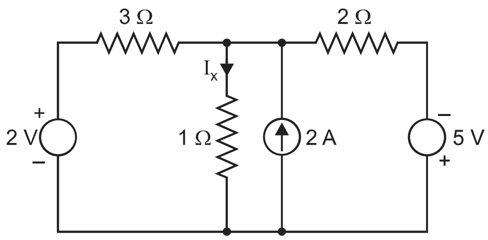


Fig. 3.11

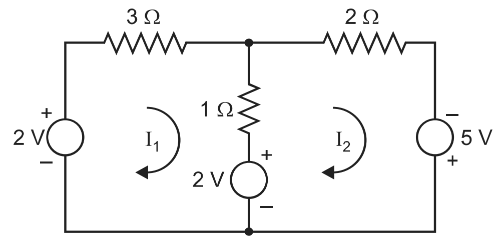


Fig. 3.12

Solution. First convert 2A current source in parallel with 1Ω resistance into equivalent voltage source of voltage 2A × 1Ω = 2V in series with 1Ω resistance. The circuit then reduces to that shown in Fig. 3.12. Assign mesh currents I_1 and I_2 to meshes 1 and 2 in Fig. 3.12.

Mesh 1. Applying *KVL* to this mesh, we have,

$$-3I_1 - 1 \times (I_1 - I_2) - 2 + 2 = 0 \quad \text{or} \quad I_2 = 4I_1$$

Mesh 2. Applying *KVL* to this mesh, we have,

$$-2I_2 + 5 + 2 - (I_2 - I_1) \times 1 = 0$$

or $$-2(4I_1) + 7 - (4I_1 - I_1) = 0 \quad (\because I_2 = 4I_1)$$

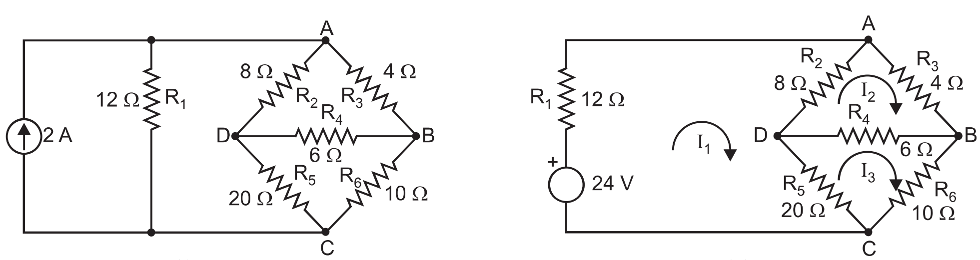
$$\therefore \quad I_1 = \frac{7}{11}\text{A} \text{ and } I_2 = 4I_1 = 4 \times \frac{7}{11} = \frac{28}{11}\text{A}$$

$$\therefore \quad \text{Current in } 3\Omega \text{ resistance, } I_1 = \frac{7}{11}\text{A} \ ; \ \text{Current in } 2\Omega \text{ resistance, } I_2 = \frac{28}{11}\text{A}$$

Referring to the original Fig. 3.11, we have,

$$I_x = I_1 + (2 - I_2) = \frac{7}{11} + \left(2 - \frac{28}{11}\right) = \mathbf{\frac{1}{11}\ A}$$

Example 3.7. *Using mesh current method, find the currents in resistances R_3, R_4, R_5 and R_6 of the circuit shown in Fig. 3.13 (i).*



(*i*) (*ii*)

Fig. 3.13

Solution. First convert 2 A current source in parallel with 12Ω resistance into equivalent voltage source of voltage = 2A × 12Ω = 24V in series with 12Ω resistance. The circuit then reduces to the one shown in Fig. 3.13 (*ii*). Assign the mesh currents I_1, I_2 and I_3 to three meshes 1, 2 and 3 shown in Fig. 3.13 (*ii*).

Mesh 1. Applying *KVL* to this mesh, we have,

$$-12I_1 - 8 \times (I_1 - I_2) - 20 \times (I_1 - I_3) + 24 = 0$$

or $$10I_1 - 2I_2 - 5I_3 = 6 \quad ...(i)$$

Mesh 2. Applying *KVL* to this mesh, we have,

$$-4I_2 - 6 \times (I_2 - I_3) - 8(I_2 - I_1) = 0$$

or $$-4I_1 + 9I_2 - 3I_3 = 0 \quad ...(ii)$$

Mesh 3. Applying *KVL* to this mesh, we have,

$$-10I_3 - 20 \times (I_3 - I_1) - 6 \times (I_3 - I_2) = 0$$

or $$-10I_1 - 3I_2 + 18I_3 = 0 \quad ...(iii)$$

From eqs. (*i*), (*ii*) and (*iii*), $I_1 = 1.125$ A ; $I_2 = 0.75$ A ; $I_3 = 0.75$ A

∴ Current in R_3 (= 4Ω) = I_2 = **0.75 A *from* A to B**

Current in R_4 (= 6Ω) = $I_2 - I_3 = 0.75 - 0.75$ = **0A**

Current in R_5 (= 20Ω) = $I_1 - I_3 = 1.125 - 0.75$ = **0.375A *from* D to C**

Current in R_6 (= 10Ω) = I_3 = **0.75A *from* B to C**

Example 3.8. *Use mesh current method to determine currents through each of the components in the circuit shown in Fig. 3.14 (i).*

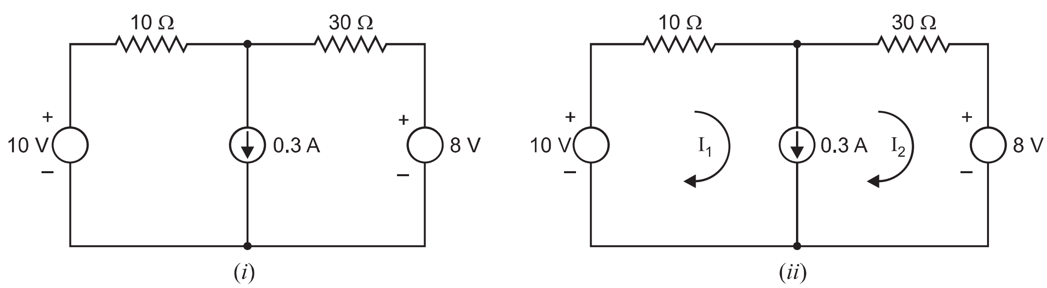


Fig. 3.14

Solution. Suppose voltage across current source is v. Assign mesh currents I_1 and I_2 in the meshes 1 and 2 respectively as shown in Fig. 3.14 (*ii*).

Mesh 1. Applying *KVL* to this mesh, we have,

$$10 - 10I_1 + v = 0 \quad ...(i)$$

Mesh 2. Applying *KVL* to this mesh, we have,

$$-30I_2 - 8 - v = 0 \quad ...(ii)$$

Adding eqs. (*i*) and (*ii*), $2 - 10I_1 - 30I_2 = 0$ ...(*iii*)

Also current in the branch containing current source is

$$I_1 - I_2 = 0.3 \quad ...(iv)$$

From eqs. (*iii*) and (*iv*), $I_1 = 0.275$ A ; $I_2 = -0.025$A

∴ Current in 10Ω = I_1 = **0.275A**

Current in 30Ω = I_2 = **– 0.025 A**

Current in current source = $I_1 - I_2 = 0.275 - (-0.025)$ = **0.3A**

Note that negative sign means current is in the opposite direction to that assumed in the circuit.

Tutorial Problems

1. Use mesh analysis to find the current in each resistor in Fig. 3.15.

[in 100 Ω = 0·1 A from *L* to *R* ; in 20 Ω = 0·4 A from *R* to *L* ; in 10 Ω = 0·5 A downward]

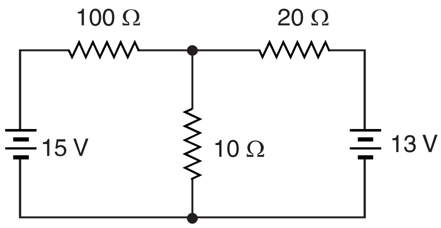


Fig. 3.15

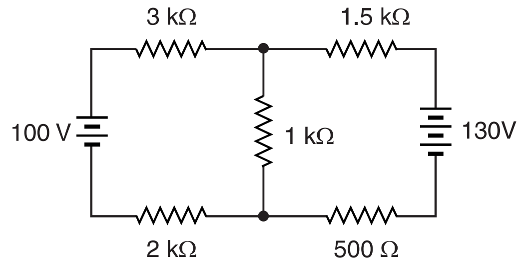


Fig. 3.16

2. Using mesh analysis, find the voltage drop across the 1 kΩ resistor in Fig. 3.16. **[50 V]**
3. Using mesh analysis, find the currents in 50 Ω, 250 Ω and 100 Ω resistors in the circuit shown in Fig. 3.17. **[*I*(50 Ω) = 0·171 A → ; *I*(250 Ω) = 0·237 A ← ; *I*(100 Ω) = 0·408 A ↓]**

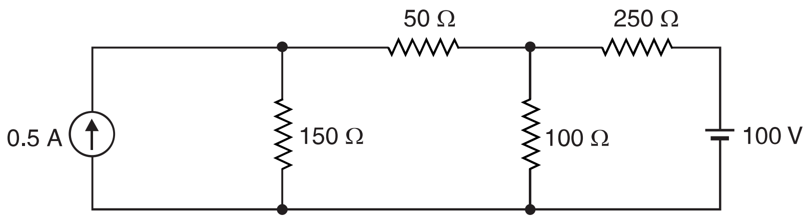


Fig. 3.17

4. For the network shown in Fig. 3.18, find the mesh currents I_1, I_2 and I_3. **[5 A, 1 A, 0·5 A]**

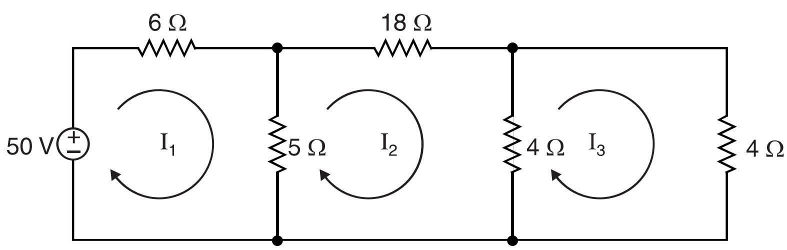


Fig. 3.18

5. In the network shown in Fig. 3.19, find the magnitude and direction of current in the various branches by mesh current method. **[*FAB* = 4 A ; *BF* = 3 A ; *BC* = 1 A ; *EC* = 2 A ; *CDE* = 3 A]**

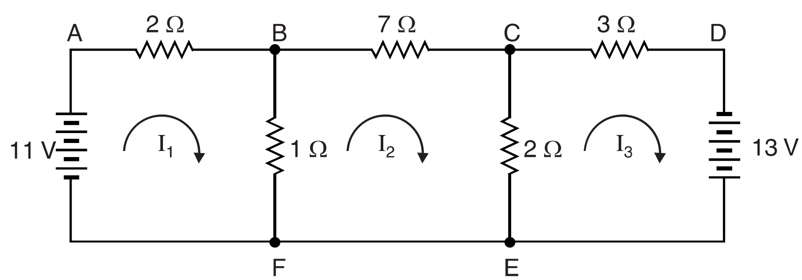


Fig. 3.19

3.6. Nodal Analysis

Consider the circuit shown in Fig. 3.20. The branch currents in the circuit can be found by Kirchhoff's laws or Maxwell's mesh current method. There is another method, called *nodal analysis* for determining branch currents in a circuit. In this method, one of the nodes (Remember a node is a point in a network where two or more circuit elements meet) is taken as the *reference node*. The