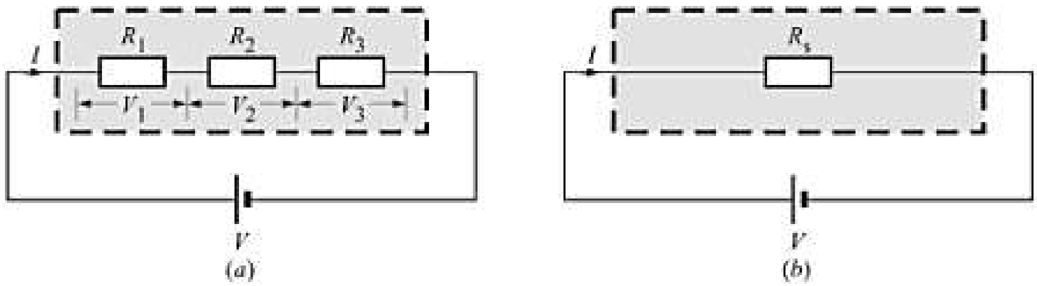

Fig. 2.5 *Resistances in series.*

The same current I flows through the three resistances. The applied voltage V must be equal to the sum of the three individual voltages, V_1, V_2, and V_3. That is,

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3) \tag{2.6}$$

From the equivalent circuit of Fig. 2.5*b*, we can write

$$V = IR_s \tag{2.7}$$

From Eqs. 2.6 and 2.7, it can be seen that

$$R_s = R_1 + R_2 + R_3$$

Thus, *the equivalent resistance of a number of resistances connected in series is equal to the sum of individual resistances*. In general, for n resistances in series, we can write

$$R_s = R_1 + R_2 + R_3 + \dots R_n = \sum_{j=1}^{n} R_j \tag{2.8}$$

Obviously, if n identical resistances, each of resistance R, are connected in series, the equivalent resistance will simply be nR. That is,

$$\boxed{R_{sn} = nR} \tag{2.9}$$

Parallel Combination of Resistances

Two or more resistances are said to be connected in ***parallel***, if *same* (*not* merely equal) voltage exists across them. Figure 2.6*a* shows three resistances in parallel, connected across a voltage source of emf V. Figure 2.6*b* shows its equivalent circuit, in which the three resistances are replaced by a single resistance R_p.

Same voltage V appears across all the three resistances. Hence, the three currents are given as

$$I_1 = \frac{V}{R_1}; \quad I_2 = \frac{V}{R_2} \quad \text{and} \quad I_3 = \frac{V}{R_3}$$

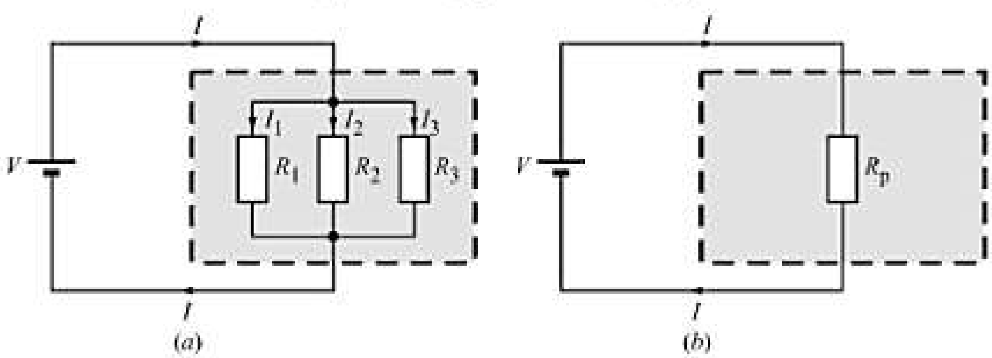

Fig. 2.6 *Resistances in parallel.*

Since the total current I entering the combination divides into I_1, I_2 and I_3, we have

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \tag{2.10}$$

From the equivalent circuit of Fig. 2.6*b*, we can write

$$I = \frac{V}{R_p} \tag{2.11}$$

Comparing Eqs. 2.10 and 2.11, we get

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Thus, *the reciprocal of equivalent resistance of a number of resistances connected in parallel is equal to the sum of the reciprocals of the individual resistances.* In general, for n resistances in parallel, we can write

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \frac{1}{R_n} = \sum_{j=1}^{n} \frac{1}{R_j} \tag{2.12}$$

Obviously, if n identical resistances, each of resistance R, are connected in parallel, the equivalent resistance will simply be R/n. That is,

$$\boxed{R_{pn} = \frac{R}{n}} \tag{2.13}$$

In case ***only two resistances*** are connected in parallel, the equivalent resistance is given as

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \tag{2.14}$$

or

$$\boxed{R_p = \frac{R_1 R_2}{R_1 + R_2}} \tag{2.15}$$

For two resistances in parallel, using Eq. 2.15 is much more convenient than using Eq. 2.14.

Voltage Divider

The concept of voltage divider is very useful in analysing electric circuits. Consider the circuit of Fig. 2.7, in which two resistances R_1 and R_2 are connected in series with a voltage source V. The current I is given as

$$I = \frac{V}{R_1 + R_2}$$

Therefore, the voltage V_1 across resistance R_1 is given as

$$V_1 = IR_1 = \frac{V}{R_1 + R_2} R_1$$

or

$$V_1 = V\frac{R_1}{R_1 + R_2} \tag{2.16}$$

Similarly, the voltage across R_2 is

$$V_2 = V\frac{R_2}{R_1 + R_2} \tag{2.17}$$

Thus, we find that *the voltage appearing across one of the series resistances is the total voltage times the ratio of its resistance to the total resistance.*

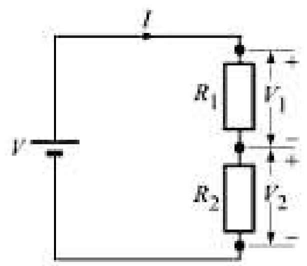

Fig. 2.7 *Illustration of voltage divider.*

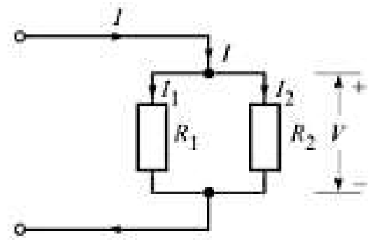

Fig. 2.8 *Illustration of current divider.*

Current Divider

Like voltage divider, the concept of current divider is also useful in analysing circuits. Consider the circuit in Fig. 2.8, in which two resistances R_1 and R_2 are connected in parallel. Same voltage V appears across both the resistances. The total current I entering the combination divides into I_1 and I_2, as shown. Therefore, we have

$$V = I_1 R_1 \quad \text{and} \quad V = I_2 R_2 \quad \text{or} \quad I_2 = \frac{I_1 R_1}{R_2}$$

$$I = I_1 + I_2 = I_1 + I_1 \frac{R_1}{R_2} = I_1\left(\frac{R_1 + R_2}{R_2}\right)$$

or

$$I_1 = I\frac{R_2}{R_1 + R_2} \tag{2.18}$$

Similarly, the current through R_2 is given as

$$I_2 = I\frac{R_1}{R_1 + R_2} \tag{2.19}$$

Thus, we find that *the current through one of the two parallel resistors is the total current times the ratio of the* ***other*** *resistance to the sum of resistances.*

EXAMPLE 2.3

Using voltage divider technique, determine the voltage across the four resistances in the circuit of Fig. 2.9*a*.

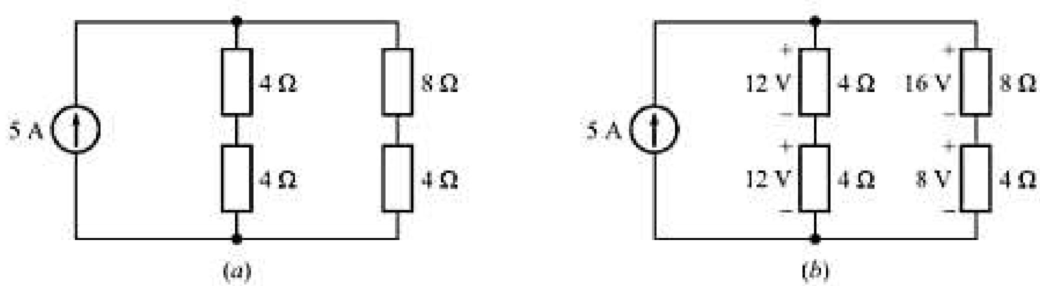

Fig. 2.9

Solution The voltage across the two parallel combinations divides independently in the two paths. The equivalent resistance seen by the current source is

$$R_p = (4 + 4) \,||\, (8 + 4) = \frac{8 \times 12}{8 + 12} = 4.8\ \Omega$$

Hence, the total voltage across the parallel combination is $5 \times 4.8 = 24$ V. This voltage equally divides between the two resistors in the first branch. In the second branch, the same voltage divides in the ratio of 8 : 4. The results are shown in Fig. 2.9*b*.

EXAMPLE 2.4

Using the voltage divider and current divider techniques, determine the unknown currents through and voltages across the resistances in the circuit of Fig. 2.10.

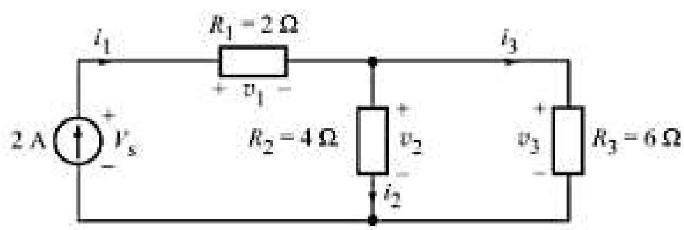

Fig. 2.10

Solution We first combine resistances to simplify the circuit. Thus, R_2 and R_3 can be replaced by their parallel equivalent $R_p = 4 \,||\, 6 = 2.4\ \Omega$ (Fig. 2.11*a*). The 2-Ω resistance is now combined with 2.4 Ω to give 4.4 Ω (Fig. 2.11*b*). Clearly, the voltage across the current source is $V_s = 2 \times 4.4 = 8.8$ V.

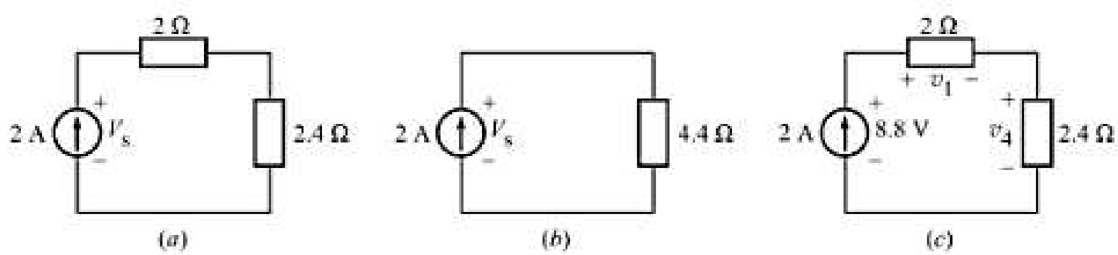

Fig. 2.11

We now restore the original circuit. Figure 2.11*c* is the same as Fig. 2.11*a*, but now we know that a voltage of 8.8 V is applied across the series combination of the 2 Ω and 2.4 Ω. The voltages across these resistances have been marked as v_1 and v_4. We can now use voltage divider technique to find these voltages. You may object to it and say, "It is a current source and not a voltage source which you are dividing." True, but the voltage created by the current source divides in a series circuit. It does not matter whether we have a 2-A current source producing 8.8 V or an 8.8-V voltage source producing 2 A—the circuit will respond in the same way. We are quite justified to divide the 8.8 V, and get

$$v_1 = 8.8 \times \frac{2}{2 + 2.4} = \mathbf{4\ V} \quad \text{and} \quad v_4 = 8.8 \times \frac{2.4}{2 + 2.4} = \mathbf{4.8\ V}$$

Voltage v_4 is the same as the voltages v_2 and v_3 in the original circuit (Fig. 2.10). Obviously, current $i_1 = \mathbf{2\ A}$, and according to Ohm's law,

$$i_2 = \frac{4.8\ \text{V}}{4\ \Omega} = \mathbf{1.2\ A} \quad \text{and} \quad i_3 = \frac{4.8\ \text{V}}{6\ \Omega} = \mathbf{0.8\ A}$$