AUTUMN END SEMESTER EXAMINATION-2013

3rd Semester B.Tech

DISCRETE MATHEMATICAL STRUCTURES MA-302

(Back-2008 Admitted Batch & Previous)

Full Marks: 70 Time: 3 Hours

Answer any SIX questions including Question No.1 which is compulsory.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. Answer all the followings: [2 × 10

a) Find the generating function of the sequence $a_n = n+1$.

b) Let $f : Z \to Z$ be such that $f(x) = x+1$. Is f invertible? and if it is, what is its inverse?

c) By using Havel-Hakimi Theorem check whether the following degree sequence is graphical or not.

1, 1, 1, 2, 3, 3, 4, 7

d) What are the equivalence classes of 0 and 1 for congruence modulo 4 over set of Integers?

e) Let p and q be the propositions

p : It is below freezing.

q : It is snowing.

Write the following propositions using p and q and logical connectives.

(i) It is either below freezing or it is snowing, but it is not snowing if it is below freezing.

(ii) That it is below freezing is necessary and sufficient for it to be snowing.

f) Determine the truth values of each of these statements if the domain for all variables consists of all integers.

(i) $\forall n(n^2 \geq 0)$ (ii) $\exists n(n^2 = 2)$

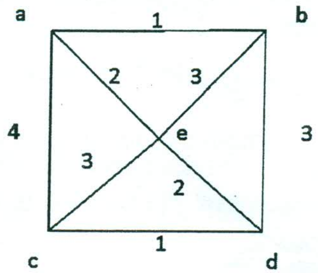
g) Test whether the given permutation is even or odd?

$$p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 2 & 4 & 1 & 3 \end{pmatrix}$$

h) Give an example of a subgroup of a non-abelian group which is normal subgroup.

i) Is the group G = {1,-1,I,-i} cyclic? If yes then find its generator.

j) Find out the minimum spanning tree of the following graph.



2. a) For each of these sets of premises, what relevant conclusions can be drawn? Explain the rules inference used to obtain conclusion from the premises. [5

"All rodents gnaw their food."

"Mice are rodents."

"Rabbits do not gnaw their food".

"Bats are not rodents."

b) Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer. [5

3\. a) Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = (n+1)3^n$, $n \geq 2$. [5

b) Solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 2, n \geq 2$ by the generating function method with the initial conditions $a_0 = 1$ and $a_1 = 1$. [5

4\. a) Let $R = \{(1,2), (2,3), (3,1)\}$ and $A = \{1,2,3\}$. Find reflexive, symmetric and transitive closure of R. [5

b) Show that the groups $(G,\times)$ and $(\overline{G},\circ)$ are Isomorphic. [5
Where G=$\{1,-1\}$ be the group w.r.t. ordinary multiplication $\overline{G}$ be the group of permutation defined over A=$\{1, 2\}$ w.r.t. composition.

5\. a) Show that the set $R = \{0,1,2,3,4,5\}$ is a commutative ring with respect to addition modulo 6 and multiplication modulo 6. [5

b) For any group G, prove that G is abelian if and only if $(ab)^2 = a^2b^2$ *for all* $a,b \in G$. [5

6\. a) Suppose that $g: A \to B$ *and* $f: B \to C$. [5

(i) Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.

(ii) Show that if both f and g are onto functions, then $f \circ g$ is also onto.

b) Find the number of primes less than 100 using principle of inclusion and exclusion. [5

6. a) Let $f: B^3 \rightarrow B^6$ be an encoding function as given below. [5

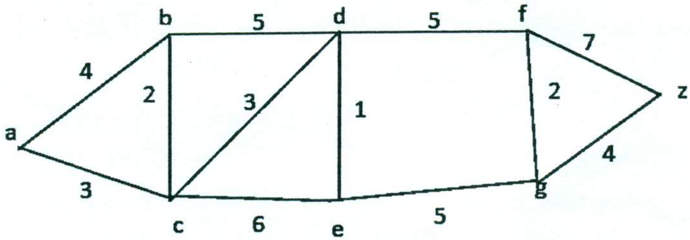
$$f(000)=000000$$
$$f(001)=001111$$
$$f(010)=010011$$
$$f(011)=011100$$
$$f(100)=100110$$
$$f(101)=101001$$
$$f(110)=110101$$
$$f(111)=111010$$

Check whether the encoding function 'e' is a group code or not. If yes how many errors 'e' can detect?

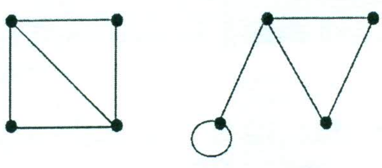
b) State and prove Lagrange's theorem on Group. [5

8. a) Prove that in an undirected graph there are even number of vertices of odd degree. [2

b) Find the length of the shortest path from vertex 'a' to vertex 'z' by *Dijkstra's* algorithm. [5



c) Check whether the following two graphs are Isomorphic or not? [3



X X X X X