Ex: 3.2

11) $p(0) = 0.2,\ p(1) = 0.25,\ p(2) = 0.3,\ p(3) = 0.15,\ p(4) = 0.10$

x :	0	1	2	3	4
$p(x)$:	0.2	0.25	0.3	0.15	0.10

1st check whether the sum of all probabilities is equal to 1.
Here, it is 1.

a) Draw the diagram.

b) $P(x \geq 2) = P(X=2) + P(X=3) + P(X=4)$

$= 0.3 + 0.15 + 0.10$

$= 0.55$

If it is not 1,
$1 - P(x<2)$
$1 - \{P(0) + P(1)\}$
$1 - 0.45$
$= 0.55$

$P(X > 2) = p(3) + p(4)$

$= 0.15 + 0.10$

$= 0.25$

c) $P(1 \leq X \leq 3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.25 + 0.3 + 0.15$

$= 0.70$

$$\begin{array}{r} 0.25 \\ 0.30 \\ 0.15 \\ \hline 0.70 \end{array}$$

d) The probability that the professor shows up ~~is 0.5~~
cannot be determined.

12)

y	45	46	47	48	49	50	51	52	53	54	55
$p(y)$	0.05	0.10	0.12	0.14	0.25	0.17	0.06	0.05	0.03	0.02	0.01

a) $P(Y \leq 50) = p(45)+p(46)+p(47)+p(48)+p(49)+p(50)$

$= 0.05+0.10+0.12+0.14+0.25+0.17$

$= 0.83$

b) $P(Y>50) = 1-P(Y \leq 50) = 1-0.83 = 0.17$

c) If I am the first person on the standby list,

$P(Y \leq 49) = p(45)+p(46)+p(47)+p(48)+p(49)$

$= 0.05+0.10+0.12+0.14+0.25$

$= 0.06$

... value of X is then

$\dfrac{\ldots + 3(1950) + \cdots + 7(300)}{15,000} = 4.57$ (3.7)

PAGE

DATE

If I am the third person on the standby list,

$P(Y \le 47) = p(45) + p(46) + p(47)$

$= 0.05 + 0.10 + 0.12$

$= 0.27$

13) a) $P(X \leq 3) = p(0) + p(1) + p(2) + p(3)$

$= 0.1 + 0.15 + 0.2 + 0.25$

$= 0.7$

Total no. of telephone lines = 6

$X \to$ No. of telephone lines in use.

X:	0	1	2	3	4	5	6
$P(x)$:	0.10	0.15	0.2	0.25	0.20	0.06	0.04

b) $P(X<3) = p(0)+p(1)+p(2)$
$= 0.1+0.15+0.2$
$= 0.45$

c) $P(X\geq 3) = p(3)+p(4)+p(5)+p(6)$ $\quad = 1-P(X<3)$
$= 0.25+0.20+0.06+0.04$ $\quad = 1-0.45$
$= 0.55$ $\quad = 0.55$

d) $P(2\leq X\leq 5) = p(2)+p(3)+p(4)+p(5)$
$= 0.71$

IMP e) $\cancel{1-P(2\leq x\leq 4) = 1-\{p(2)+p(3)+p(4)\}}$
$\cancel{= 1-0.65}$
$\cancel{= 0.35}$

$P(2\leq 6-x\leq 4) = P(-4\leq -x\leq -2) = P(2\leq x\leq 4)$
$= p(2)+p(3)+p(4)$
$= 0.65$

IMP f) $P(6-X\geq 4) = P(-X\geq -2) = P(X\leq 2)$
$= p(0)+p(1)+p(2)$
$= 0.45$

14) a) ~~Let, Y~~ $p(y) = ky$ for $y = 1, 2, \ldots 5$

$\sum_{y=1}^{5} p(y) = 1$

$= p(1)+p(2)+p(3)+p(4)+p(5) = 1$

$= k+2k+3k+4k+5k = 1$

$\Rightarrow 15k = 1 \quad \Rightarrow k = \frac{1}{15}$

b) $P(Y \leq 3) = p(1) + p(2) + p(3)$

$= k \times (1+2+3)$

$= \cancel{6}^{2} \times \frac{1}{\cancel{15}_{5}}$

$= \frac{2}{5} = 0.4$

c) $P(2 \leq Y \leq 4) = p(2) + p(3) + p(4)$

$= \frac{1}{15} \times (2+3+4)$

$= \frac{9}{15} = 0.6$

d) $p(y) = y^2/50 \quad , y = 1, \ldots, 5$

$p(1) + p(2) + p(3) + p(4) + p(5) = \frac{1}{50} + \frac{4}{50} + \frac{9}{50} + \frac{16}{50} + \frac{25}{50}$

$= \frac{55}{50} = 1.1 \neq 1$

So, this can't be the pmf.

$= 0.27$

15) a) $S = \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5),$ 0.36
$(3,4), (3,5), (4,5)\}$

b) X = { No. of defective boards in a batch }
$X = 0, 1, 2$
X=0 (when neither 1 or 2 are present in the batch)
X=1 (Either 1 or 2 are present in the batch)
X=2 (Both 1 and 2 are present in the batch)

$P(X=0) = \{(3,4), (3,5), (4,5)\} = \frac{3}{10} = 0.3$

$P(X=1) = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\} = \frac{6}{10} = 0.6$

$P(X=2) = \{(1,2)\} = \frac{1}{10} = 0.1$

c) $F(0) = P(X=0) = 0.3$
$F(1) = P(X=0) + P(X=1) = 0.3 + 0.6 = 0.9$
$F(2) = P(X=0) + P(X=1) + P(X=2) = 0.3 + 0.6 + 0.1 = 1$

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.3, & 0 \le x < 1 \\ 0.9, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$

16) $P(S) = 0.25$
$P(F) = 1 - P(S) = 0.75$

X = { No. of homeowners who have earthquake insurance }
$X = 0, 1, 2, 3, 4$

S = { SSSS, SFSS, SSSF, SSFS, FSSS, SSFF, FFSS, FFFF, SFSF, FFFS, FFSF, FSFF, SFFF, FSFS, FSSF, SFFS }

$P(X=0) = 0.75 \times 0.75 \times 0.75 \times 0.75 = 0.316 = \{FFFF\}$ $\frac{4!}{0!(4-0)!} \times 0.25^0$

$P(X=1) = \{FFFS, FFSF, FSFF, SFFF\}$ $(1-0.25)^4$

$= 0.75 \times 0.75 \times 0.75 \times 0.25 \times 4$

$= 4 \times 0.75^3 \times 0.25 = 0.421$

$P(X=2) = \{SSFF, FFSS, SFSF, FSFS, FSSF, SFFS\}$

$= 0.75^2 \times 0.25^2 \times 6$

$= 0.21$

$P(X=3) = \{SFSS, SSSF, SSFS, FSSS\}$

$= 0.25^3 \times 0.75 \times 4$

$= 0.046$

$P(X=4) = \{SSSS\} = 0.25^4 = 0.0039$

~~$F(x) = \begin{cases} 0, & x<0 \\ 0.316, & 0 \le x < 1 \end{cases}$~~

c) The most ~~value~~ likely value for X is 1 because it has the maximum probability among all the other values of x.

d) $P(X \geq 2) = P(2) + P(3) + P(4) = 0.21 + 0.046 + 0.0039 = 0.2599$

17) $P(A) = 0.9$

$P(U) = 1 - 0.9 = 0.1$

$Y \rightarrow$ The no. of batteries that must be tested.

a) $p(2) = P(Y=2) = (0.9 \times 0.9) = 0.81 = 1 \times (.1)^{2-2} \times (0.9)^2$

↓ because they are independent

UAA
AUA
AAU X → First do batteries R liye hogya tab third bat R liye nhi jayega.

b) $p(3) = (0.1 \times 0.9 \times 0.9) + 0.9 \times 0.1 \times 0.9$

$= 2 \times (0.1) \times (0.9)^2 = 0.162 = 2 \times (0.1)^{3-2} \times (0.9)^2$

c) $p(5) = (0.1 \times 0.1 \times 0.1 \times 0.9 \times 0.9)$

$+ (0.1 \times 0.9 \times 0.1 \times 0.1 \times 0.9)$

$+ (0.9 \times 0.1 \times 0.1 \times 0.1 \times 0.9)$

$+ (0.1 \times 0.1 \times 0.9 \times 0.1 \times 0.9)$

$= 4 \times (0.1)^3 \times (0.9)^2 = 4 \times (0.1)^{5-2} \times (0.9)^2$

$= 0.00324$

✓ UUUAA
UAUUA
AUUUA
✓ UUAUA

d) $p(y) = (y-1)(0.1)^{y-2} \times (0.9)^{2}$ Generalized probability mass function.
(pmf)

18) Total 36 outcomes are there.

a) $P(x=1) = \{1,1\} = \frac{1}{36}$

b) $P(x=2) = \{(1,1), (1,2), (2,1)\} = \frac{3}{36}$

c) $P(x=3) = \{(1,3), (2,3), (3,3), (3,1), (3,2)\} = \frac{5}{36}$

d) $P(x=4) = \{(1,4), (2,4), (3,4), (4,4), (4,1), (4,2), (4,3)\} = \frac{7}{36}$

e) $P(x=5) = \{(1,5), (2,5), (3,5), (4,5), (5,5), (5,1), (5,2), (5,3), (5,4)\}$
$= \frac{9}{36}$

f) $P(x=6) = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,6), (6,1), (6,2), (6,3), (6,4), (6,5)\}$
$= \frac{11}{36}$

b) $\cancel{F(0) = P(X \le 0) =}$

$F(1) = P(X \le 1) = \frac{1}{36}$

$F(2) = P(X \le 2) = \frac{1}{36} + \frac{3}{36} = \frac{4}{36}$

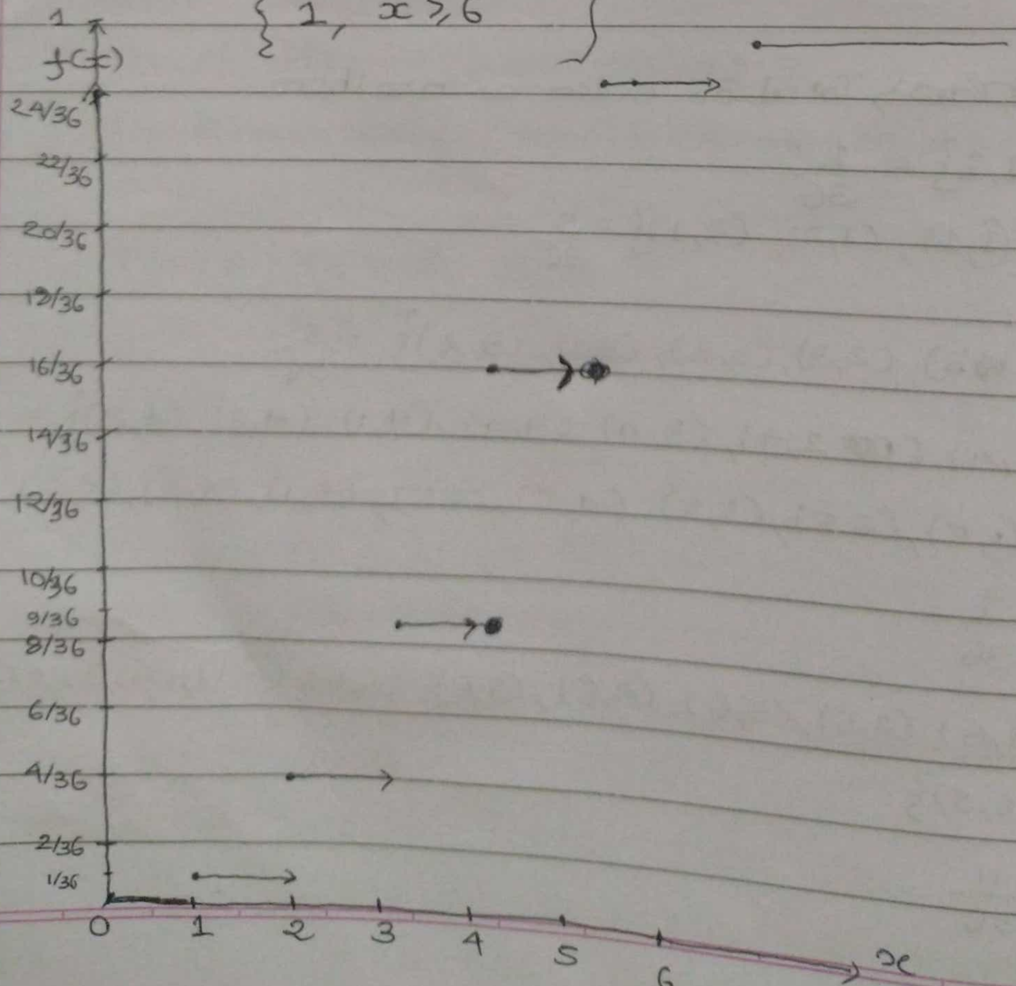
$F(3) = P(X \le 3) = \frac{1}{36} + \frac{3}{36} + \frac{5}{36} = \frac{9}{36}$

$F(4) = P(X \le 4) = \frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{7}{36} = \frac{16}{36}$

$F(5) = P(X \le 5) = \frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{7}{36} + \frac{9}{36} = \frac{25}{36}$

$F(6) = P(X \le 6) = \frac{25}{36} + \frac{11}{36} = 1$

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{36}, & 1 \le x < 2 \\ \frac{4}{36}, & 2 \le x < 3 \\ \frac{9}{36}, & 3 \le x < 4 \\ \frac{16}{36}, & 4 \le x < 5 \\ \frac{25}{36}, & 5 \le x < 6 \\ 1, & x \ge 6 \end{cases}$$



20) $C_1 = \{\text{couple one}\}$
$C_2 = \{\text{couple two}\}$
$C_3 = \{\text{couple three}\}$
$A = \{\text{First individual}\}$
$B = \{\text{Second individual}\}$

a) P(Individual arrives late) = 0.4

X = {No. of people who arrive late for seminar}
$= 0, 1, 2, 3, 4, 5.$

$P(X=0)$ = No individual arrives late
$= P\{C_1' C_2' C_3' A' B'\}$
$= (1-0.4)\times(1-0.4)\times(1-0.4)\times(1-0.4)\times(1-0.4)\times(1-0.4)$
$= 0.6^5$
$= 0.07776$

$P(X=1)$ = One individual arrives late.
$= P(C_1'C_2'C_3'A'B) + P(C_1'C_2'C_3'AB')$
$= 2\times0.6\times0.6\times0.6\times0.6\times0.4$
$= 2\times0.6^4\times0.4$
$= 0.10368$

$P(X=2)$ = Two individual arrives late
$= P(C_1'C_2'C_3'AB) + P(C_1C_2'C_3'A'B')$
$+ P(C_1'C_2C_3'A'B') + P(C_1'C_2'C_3A'B')$
$= 0.6\times0.6\times0.6\times0.4\times0.4$
$+ 3\times0.4\times0.6^4$
$= 0.6^3\times0.4^2 + 3\times0.4\times0.6^4$
$= 0.19008$

$P(X=3) = P(C_1C_2'C_3'A'B) + P(C_1C_2'C_3'AB')$
$+ P(C_1'C_2C_3'A'B) + P(C_1'C_2C_3'AB')$
$+ P(C_1'C_2'C_3A'B) + P(C_1'C_2'C_3AB')$
$= 6\times0.6\times0.6\times0.6\times0.4\times0.4$
$= 6\times0.6^3\times0.4^2$
$= 0.20736$

$P(X=4) = P(C_1 C_2' C_3' AB) + P(C_1' C_2 C_3' AB) + P(C_1' C_2' C_3 AB)$
$\quad + P(C_1 C_2 C_3' A'B') + P(C_1' C_2 C_3 A'B') + P(C_1 C_3 C_2' A'B')$
$\quad = 3\times 0.4^3 \times 0.6^2 + 3\times 0.4^2 \times 0.6^3$
$\quad = 0.1728$

$P(X=5) = P(C_1 C_2 C_3' AB') + P(C_1 C_2 C_3' A'B) + P(C_1' C_2 C_3 AB') + P(C_1' C_2 C_3 A'B)$
$\quad + P(C_1 C_2' C_3 AB') + P(C_1 C_2' C_3 A'B)$
$\quad = 6\times 0.4^3 \times 0.6^2$
$\quad = 0.13824$

$P(X=6) = P(C_1 C_2 C_3' AB) + P(C_1' C_2 C_3 AB) + P(C_2' C_1 C_3 AB)$
$\quad + P(C_1 C_2 C_3 A'B')$
$\quad = 3\times 0.4^4 \times 0.6 + 0.4^3 \times 0.6^2$
$\quad = 0.06912$

$P(X=7) = P(C_1 C_2 C_3 AB') + P(C_1 C_2 C_3 A'B)$
$\quad = 2\times 0.4^4 \times 0.6$
$\quad = 0.03072$

$P(X=8) = P(C_1 C_2 C_3 AB) = 0.4^5 = 0.01024$

$F(0) = p(0) = 0.07776$
$F(1) = p(1) + p(0) = \cancel{0.02592}\ 0.18144$
$F(2) = p(2) + p(1) + p(0) = \cancel{0.0864}\ 0.37152$
$F(3) = F(2) + p(3) = 0.57888$
$F(4) = F(3) + p(4) = 0.75168$
$F(5) = F(4) + p(5) = 0.88992$
$F(6) = F(5) + p(6) = 0.95904$
$F(7) = F(6) + p(7) = 0.98976$
$F(8) = F(7) + p(8) = 1$

$\therefore P(2 \le X \le 6) = F(6) - F(1) = 0.95904 - 0.18144 = 0.7776$

21) a) $p(x) = P(\text{1st digit is } x)$
$= \log_{10}\left(\frac{x+1}{x}\right), \; x = 1, 2, \ldots, 9$

Now, we'll find $\sum_{x=1}^{x=9} \log_{10}\left(\frac{x+1}{x}\right)$

$\left(\log_{10}\frac{2}{1} + \log_{10}\frac{3}{2} + \log_{10}\frac{4}{3} + \ldots \log_{10}\frac{10}{9}\right)$

$= \log_{10}\left(\cancel{2} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{4}{\cancel{3}} \times \ldots \frac{10}{\cancel{9}}\right)$

$= \log_{10} 10 = 1$

$\therefore$ It specifies a legitimate pmf.

b) $p(1) = \log_{10}\frac{2}{1} = 0.301$

$p(2) = \log_{10}\frac{3}{2} = 0.1761$

$p(3) = \log_{10}\frac{4}{3} = 0.1249$

$p(4) = \log_{10}\frac{5}{4} = 0.0969$

$p(5) = \log_{10}\frac{6}{5} = 0.0792$

$p(6) = \log_{10}\frac{7}{6} = 0.0669$

$p(7) = \log_{10}\frac{8}{7} = 0.0580$

$p(8) = \log_{10}\frac{9}{8} = 0.0512$

$p(9) = \log_{10}\frac{10}{9} = 0.0458$

c) $F(1) = p(1) = \log_{10} 2 = 0.301$

$F(2) = p(1) + p(2) = \log_{10} 2 + \log_{10}\frac{3}{2} = 0.4771$

$F(3) = 0.4771 + 0.1249 = 0.602$

$F(4) = 0.602 + 0.0969 = 0.6989$

$F(5) = 0.6989 + 0.0792 = 0.7781$

$F(6) = 0.7781 + 0.0669 = 0.845$

$F(7) = 0.845 + 0.058 = 0.903$

$F(8) = 0.903 + 0.0512 = 0.9542$

$F(9) = 0.9542 + 0.0458 = 1$

d) $P(X \le 3) = F(3) = 0.602$

$P(X \ge 5) = P(5 \le X \le 9) = F(9) - F(4) = 1 - 0.6989 = 0.3011$

22) $F(0) = p(0) = 0.1$

$F(1) = p(0) + p(1) = 0.1 + 0.15 = 0.25$

$F(2) = 0.25 + p(2) = 0.25 + 0.2 = 0.45$

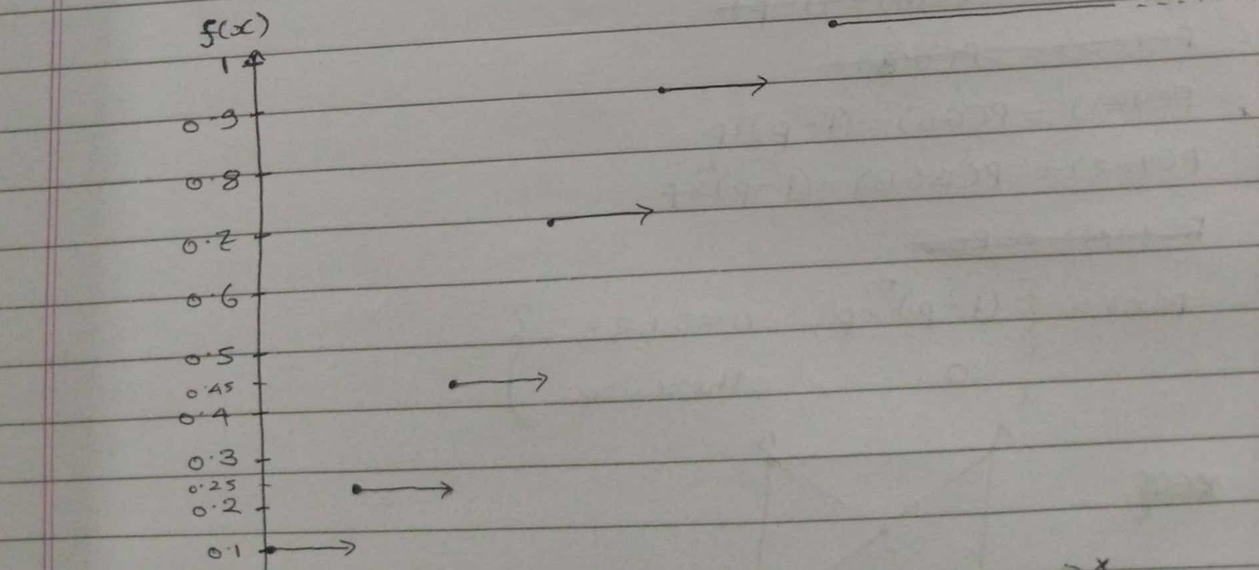
$F(3) = 0.45 + p(3) = 0.45 + 0.25 = 0.7$

$F(4) = 0.7 + p(4) = 0.7 + 0.2 = 0.9$

$F(5) = 0.9 + p(5) = 0.9 + 0.06 = 0.96$

$F(6) = 0.96 + p(6) = 0.96 + 0.04 = 1$

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.1, & 0 \le x < 1 \\ 0.25, & 1 \le x < 2 \\ 0.45, & 2 \le x < 3 \\ 0.7, & 3 \le x < 4 \\ 0.9, & 4 \le x < 5 \\ 0.96, & 5 \le x < 6 \\ 1, & x \ge 6 \end{cases}$$



a) $P(X \leq 3) = F(3) = 0.7$

b) $P(X < 3) = 1 - P(X \geq 3)$

$= 1 - P(3 \leq X \leq 6)$

$= 1 - (F(6) - F(2))$

$= 1 - 1 + 0.45$

$= 0.45$

c) $P(X \geq 3) = F(6) - F(2)$

$= 1 - 0.45 = 0.55$

d) $P(2 \leq X \leq 5) = F(5) - F(1)$

$= 0.96 - 0.25$

$= 0.71$

23) $F(x) = \begin{cases} 0, & x<0 \\ 0.06, & 0 \le x < 1 \\ 0.19, & 1 \le x < 2 \\ 0.39, & 2 \le x < 3 \\ 0.67, & 3 \le x < 4 \\ 0.92, & 4 \le x < 5 \\ 0.97, & 5 \le x < 6 \\ 1, & 6 \le x \end{cases}$

x:	0	1	2	3	4	5	6
$F(x)$:	0.06	0.19	0.39	0.67	0.92	0.97	1
$p(x)$:	0.06						

$P(x \le x)$

$= F(x)$

$P(x \le 0)$

$= F(0)$

$= P(X=0)$

$F(0) = 0.06$

$F(1) = 0.19$

$F(2) = 0.39$

$F(3) = 0.67$

$F(4) = 0.92$

$F(5) = 0.97$

$F(6) = 1$

$F(0) = p(0) = 0.06$

$P(x=1) = F(1) - F(0) = 0.19 - 0.06 = 0.13$

$P(X=2) = F(2) - F(1) = 0.39 - 0.19 = 0.2$

$P(X=3) = F(3) - F(2) = 0.67 - 0.39 = 0.28$

$P(X=4) = 0.25$

$P(X=5) = 0.05$

$P(X=6) = 0.03$

a) $P(X=2) = F(2) - F(1) = 0.2$

b) $P(X>3) = p(4) + p(5) + p(6) = 0.25 + 0.05 + 0.03 = 0.33$

$= 1 - P(X \le 3)$

$= 1 - F(3) = 1 - 0.67 = 0.33$

c) $P(2 \le x \le 5) = F(5) - F(1)$ $(p(2)+p(3)+p(5)+p(4))$

$= 0.78$

d) $P(2 < x < 5) = F(4) - F(2) = 0.92 - 0.39 = 0.53$

$p(3)+p(4) = 0.25+0.28 = 0.53$

24)

x :	~~0~~	1	3	4	6	12
$F(x)$:	~~0~~	0.3	0.40	0.45	0.6	1
$p(x)$		0.3	0.1	0.05	0.15	0.4

$$F(x) = \begin{cases} 0, & x < 1 \\ 0.30, & 1 \le x < 3 \\ 0.40, & 3 \le x < 4 \\ 0.45, & 4 \le x < 6 \\ 0.6, & 6 \le x < 12 \\ 1, & 12 \le x \end{cases}$$

$P(a \le x \le b)$
$= F(b) - F(a-1)$

$P(a \le x \le b)$
$= F(b) - F(a^-)$

strictly less than a but close to a

$P(3 \le X \le 6) = F(6) - F(1) = 0.6 - 0.3 = 0.3$

$= p(3) + p(4) + p(6)$

$= 0.1 + 0.05 + 0.15$

0.10
0.05
$+0.15$
0.30

$P(X \ge 4) = p(4) + p(6) + p(12)$

$= 0.6$

$P(4 \le X \le 12) = F(12) - F(3)$

$= 1 - 0.4 = 0.6$

25) $Y =$ The no. of girls observed

$p = P(B)$

$P(G) = 1 - P(B) = 1 - p$

$P(Y=0) = P(B) = p$

$\cancel{P(Y=1) = P(G) = (1-p)}$

$\cancel{P(Y=2) = P(GG) =}$

$P(Y=1) = P(GB) = (1-p) \times p$

$P(Y=2) = P(GGB) = (1-p)^2 \times p$

$\cancel{P(Y=N) = P(G}$

$$p(x) = \begin{cases} (1-p)^n \times p, & n = 0, 1, 2, 3, \ldots \\ 0, & \text{otherwise} \end{cases}$$

PAGE No.

DATE / /

27) a) $S = \{(1,2,3,4), (1,2,4,3), (1,3,2,4), (1,3,4,2), (1,4,2,3), (1,4,3,2), (2,1,3,4), (2,1,4,3), (2,3,1,4), (2,3,4,1), (2,4,1,3), (2,4,3,1), (3,1,2,4), (3,1,4,2), (3,2,1,4), (3,2,4,1), (3,4,1,2), (3,4,2,1), (4,1,2,3), (4,1,3,2), (4,2,1,3), (4,2,3,1), (4,3,1,2), (4,3,2,1)\}$

b) X: The no. of students who receive their own book.

$X = 0, 1, 2, 4$

$P(X=0) = \{(2,1,4,3), (2,3,4,1), (2,4,1,3), (3,1,4,2), (3,4,1,2), (3,4,2,1), (4,1,2,3), (4,3,1,2), (4,3,2,1)\}$

$= 9/24$

$P(X=1) = \{(1,3,4,2), (1,4,2,3), (3,2,4,1), (4,2,1,3), (2,4,3,1), (4,1,3,2), (2,3,1,4), (3,1,2,4)\}$

$= 8/24$

$P(X=2) = \{(1,2,4,3), (1,3,2,4), (1,4,3,2), (2,1,3,4), (3,2,1,4), (4,2,3,1)\}$

$= 6/24$

$P(X=4) = \{1,2,3,4\} = 1/24$

28