Continuing classwork

Exc: 3.5

68) $N = 18$, $M = 8$, ~~x=6~~ $n = 6$, $N-M = 10$

a) Hypergeometric distribution.

$$h(x; n, M, N) = \frac{{}^{M}C_x \times {}^{N-M}C_{n-x}}{{}^{N}C_n} = \frac{{}^{18}C_x \times {}^{10}C_{6-x}}{{}^{18}C_6}$$

$x = 0, 1, \dots, 6$

b) $$P(X=2) = h(2; 6, 8, 18) = \frac{{}^{18}C_2 \times {}^{10}C_4}{{}^{18}C_6}$$

$$= \frac{\frac{18\times17\times\cancel{16}}{2\times\cancel{16}} \times \frac{10\times9\times8\times7\times\cancel{16}}{2\times3\times4\times\cancel{16}}}{\frac{18\times17\times16\times15\times14\times13\times\cancel{12}}{\cancel{12}\times2\times3\times4\times5\times6}}$$

$$= \frac{\cancel{18}\times\cancel{17}}{\cancel{2}} \times \frac{10\times9\times8\times7}{\cancel{2}\times\cancel{3}\times\cancel{4}} \times \frac{\cancel{2}\times\cancel{3}\times\cancel{4}\times5\times\cancel{6}3}{\cancel{18}\times\cancel{17}\times16\times15\times14\times13}$$

$= 0.317$

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{{}^{18}C_0 \times {}^{10}C_6}{{}^{18}C_6} + \frac{{}^{18}C_1 \times {}^{10}C_5}{{}^{18}C_6} + \frac{{}^{18}C_2 \times {}^{10}C_4}{{}^{18}C_6}$$

$= 0.437$

$P(x \geq 2) = 1 - P(X<2)$

$= 1 - P(X \leq 1)$

$= 1 - \{P(X=0) + P(X=1)\}$

$= 1 - \left\{ \dfrac{18c_0 \times 10c_6}{18c_6} + \dfrac{18c_1 \times 10c_5}{18c_6} \right\}$

$= 0.88$

c) The mean value is $= np = 6 \times \dfrac{M}{N} = \cancel{6} \times \dfrac{8}{\cancel{18}_3} = 2.66$

Variance $= \left(\dfrac{N-n}{N-1}\right) n \dfrac{M}{N}\left(1 - \dfrac{M}{N}\right)$

$= \dfrac{\overset{2}{\cancel{6}}}{12} \times \cancel{6} \times \dfrac{8}{\cancel{18}_3} \times \left(1 - \dfrac{8}{18}\right)$

$= \dfrac{\overset{2}{\cancel{4}} \times 8}{12} \times \dfrac{10}{\cancel{18}_9}$

$= \dfrac{8 \times 10}{12 \times 9} = 0.522$

$= \dfrac{\cancel{12}^4}{12} \times \cancel{6} \times \dfrac{8}{\cancel{18}_3} \times \left(1 - \dfrac{8}{18}\right)$

$= \dfrac{4 \times 8}{12} \times \left(\dfrac{\overset{5}{\cancel{10}}}{\cancel{18}_9}\right)$

$= 1.045$

$\therefore$ Standard deviation $= \sqrt{1.045} = 1.022$

69) $n=6,\ N=12,\ M=7,\ N-M=5$

a) $P(X=4) = h(4; 6, 7, 12)$

$= \dfrac{7c_4 \times 5c_2}{12c_6}$

$= \dfrac{7 \times \overset{3}{\cancel{6}} \times 5 \times \cancel{\lfloor 4}}{\cancel{\lfloor 4} \times \cancel{2} \times \cancel{3}} \times \dfrac{5 \times \overset{2}{\cancel{4}} \times \cancel{\lfloor 3}}{\cancel{\lfloor 3} \times \cancel{2}}$

$\overline{\dfrac{\cancel{2}\ 12 \times 11 \times \overset{2}{\cancel{10}} \times \overset{3}{\cancel{9}} \times \overset{2}{\cancel{8}} \times 7 \times \cancel{\lfloor 6}}{\cancel{\lfloor 6} \times \cancel{2} \times \cancel{3} \times \cancel{4} \times \cancel{5} \times \cancel{6}}}$

$$= \frac{7 \times 5 \times 5 \times 2}{11 \times 2 \times 3 \times 2 \times 7} = 0.378$$

$$P(X \le 4) = 1 - P(X > 4) = 1 - (P(X=5) + P(X=6))$$

$$= 1 - \left[\frac{{}^7C_5 \times {}^5C_1}{{}^{12}C_6} + \frac{{}^7C_6 \times {}^5C_0}{{}^{12}C_6} \right]$$

$$= 1 - [0.11 + 0.0075]$$

$$= 0.8825$$

b) $P(X > E(x) + \sigma)$

$$E(x) = np = n \times \frac{M}{N} = \frac{6 \times 7}{12_2} = 3.5$$

$$V(x) = \left(\frac{N-n}{N-1}\right) n \frac{M}{N}\left(1 - \frac{M}{N}\right) = \frac{6}{11} \times 6 \times \frac{7}{12_2} \times \left(1 - \frac{7}{12}\right)$$

$$= \frac{6 \times 7}{11 \times 2} \times \frac{5}{12_2} = 0.7954$$

c) $N = 400$, $M = 40$, ~~PR~~ $N - M = 360$, $n = 15$

~~$P(X \le 5)$~~ $p = \frac{M}{N} = \frac{40}{400} = 0.1$ (If $\frac{n}{M} \le 0.05$, then binomial distribution else hypergeometric distribution)

~~B~~ $P(X \le 5) = B(5; 15, 0.1) = 0.998$

Ex: 3.5

2Q) $N=50, M=30, N-M=20, n=15$

a) $P(X=10) = \dfrac{\binom{30}{10}\binom{20}{5}}{\binom{50}{15}} = 0.2069$

d) Mean value $= \frac{nM}{N} = \frac{15 \times 30}{50} = 9$

Variance $= \frac{nM}{N}\left(1 - \frac{M}{N}\right)\left(\frac{N-n}{N-1}\right) = 9 \times \left(1 - \frac{30}{50}\right) \times \left(\frac{50-15}{50-1}\right)$

$= 2.571$

Standard deviation $= \sqrt{2.571} = 1.603$

e) Mean value $= \frac{15 \times 20}{50} = 6$

Variance $= 6 \times \left(1 - \frac{20}{50}\right) \times \left(\frac{50-15}{50-1}\right) = 2.571$

Standard deviation $= \sqrt{2.571} = 1.603$

Z1) $n=15,\ N=20,\ M=10,\ N-M=10$

a) $h(x;15,10,20) = \dfrac{{}^{10}C_x \times {}^{10}C_{15-x}}{{}^{20}C_{15}}$ $\quad x = 5, 6, \ldots, 10$

b) $P(X=10) = \dfrac{{}^{10}C_{10} \times {}^{10}C_5}{{}^{20}C_{15}} = \dfrac{\dfrac{10\times 9\times 8\times 7\times 6\times 15}{15\times 2\times 3\times 4\times 5}}{\dfrac{20\times 19\times 18\times 17\times 16\times 15}{15\times 2\times 3\times 4\times 5}}$

$\vec{P}(X=5)$

$= \dfrac{\cancel{8}^{3}\times \cancel{2}\times 7\times \cancel{2}}{19\times \cancel{3}\times 17\times \cancel{16}_{\cancel{8}\,4}}$

$= \dfrac{3\times 7}{19\times 17\times 4} = 0.0162 \times 2$

$= 0.0324$

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c) $\cancel{P(X \le E(x) + \sigma_x)}$

$$P(E(x) - \sigma_x < X < E(x) + \sigma_x) = P(|x - \mu| \le \sigma)$$

$$E(x) = n\frac{M}{N} = 15 \times \frac{10}{20} = 7.5$$

$$V(x) = \left(\frac{N-n}{N-1}\right) n\frac{M}{N}\left(1 - \frac{M}{N}\right)$$

$$= \frac{20-15}{20-1} \times 7.5 \times \left(1 - \frac{10}{20}\right)$$

$$= \frac{5}{19} \times 7.5 \times \frac{10}{20}$$

$$= 0.9868$$

$$\therefore \sigma_x = \sqrt{0.9868} = 0.9933$$

$$P(7.5 - 0.9933 < X < 7.5 + 0.9933)$$

$$= P(6.5067 < X < 8.4933)$$

$$= P(7 \le X \le 8)$$

$$= P(X=7) + P(X=8)$$

$$= 0.3483 + 0.3483$$

$$= 0.6966$$

Z2) $N=11, \; M=6, \; N-M=5, \; n=4$

a) $P(X=x) = \dfrac{\binom{6}{x}\binom{5}{4-x}}{\binom{11}{4}}$ $\quad x=0,1,2,3,4$

b) $E(x) = \dfrac{nM}{N} = \dfrac{4\times 6}{11} = 2.18$

Z3) $N=20, \; n=10, \; M=10, \; N-M=10$

a) $P(X=x) = \dfrac{\binom{10}{x}\binom{10}{10-x}}{\binom{20}{10}}$ $\quad x=0,1,2,\ldots,10$

23) b) $n = 10, \; N = 20, \; M = 5, \; N - M = 15$

Let X = The no. among the top 5 pairs who play east-west

$\therefore P$(All of the top 5 pairs end up playing in the same direction) $= P(X=0) + P(X=5)$

$$= h(0; 10, 5, 20) + h(5; 10, 5, 20)$$

$$= \frac{\binom{5}{0}\binom{15}{10}}{\binom{20}{10}} + \frac{\binom{5}{5}\binom{15}{5}}{\binom{20}{10}}$$

$$= \frac{\frac{15!}{10! \times 5!} + \frac{15!}{5! \times 10!}}{\binom{20}{10}}$$

$$= \frac{\frac{2 \times 15!}{\cancel{10!}\, 5!}}{\frac{20!}{\cancel{10!} \times 10!}}$$

$$= \frac{2 \times 15!}{5!} \times \frac{10!}{20!}$$

$$= 0.033$$

23) c) $N = 2n, \; M = n, \; N - M = n$

$$\therefore h(x; n, n, 2n) = \frac{\binom{n}{x}\binom{n}{n-x}}{\binom{2n}{n}}, \quad x = 0, 1, 2, \ldots, n$$

$$E(x) = n \times \frac{M}{N} = n \times \frac{n}{2n} = \frac{n}{2}$$

$$V(x) = \frac{nM}{N} \times \left(1 - \frac{M}{N}\right)\left(\frac{N-n}{N-1}\right) = \frac{n^2}{2n} \times \left(1 - \frac{n}{2n}\right)\left(\frac{2n-n}{2n-1}\right)$$

$$= \frac{n}{2} \times \left(\frac{\cancel{n}}{2\cancel{n}}\right)\left(\frac{n}{2n-1}\right)$$

$$= \frac{n^2}{8n-4}$$

24) $N = 50, \; n = 10, \; M = 15, \; N - M = 35$

a) X = The no. of firms visited by the inspector that are in violation of at least one regulation.

~~P(x) =~~

$$P(x) = \frac{\binom{15}{x}\binom{35}{10-x}}{\binom{50}{10}}$$

b) $N = 500, M = 150, n = 10$

$$p = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}} = \frac{150}{500} = 0.3$$

$$b(x; n, p) = b(x; 10, 0.3) = {}^{10}C_x \times (0.3)^x \times (0.7)^{10-x}$$

c) $E(x) = \frac{nM}{N} = \frac{10 \times 15}{50} = 3$

$$V(x) = \left(\frac{N-n}{N-1}\right) \frac{nM}{N} \times \left(1 - \frac{M}{N}\right) = \left(\frac{50-10}{50-1}\right) \times 3 \times \left(1 - \frac{15}{50}\right)$$

$$= 1.714$$

$$\frac{\quad}{p} \qquad \frac{\quad}{p}$$

$$\text{Variance} = V(x) = \frac{r(1-p)}{p^2} = \frac{1-p}{p^2} \quad [\because r = 1]$$

No. of successes

Q) 25 $p = 0.2, \; r = 2$

a) $P(X = x) = nb(x; 2, 0.2) = \binom{x+2-1}{2-1} (.2)^2 (.8)^x$

$$= \binom{x+1}{1} (.2)^2 (.8)^x$$

$$^nC_r = \frac{\lfloor n}{\lfloor r \times \lfloor n-r}$$

4 me be 2 boxes without prizes

b) $P(X=2) = nb(2; 2, 0.2) = \binom{3}{1} (.2)^2 (.8)^2$

$$= \frac{3 \times \cancel{\lfloor 2}}{\cancel{\lfloor 2}} \times (.2)^2 \times (.8)^2$$

$$= 0.0768$$

c) $P(X \leq 2) = nb(0; 2, 0.2) + nb(1; 2, 0.2) + nb(2; 2, 0.2)$

$$= \sum \binom{x+1}{1} (0.2)^2 \times (0.8)^x$$

$= 0.04 + 2(0.04)(0.8) + 3(0.4)(0.64)$

$= 0.1808$

The expected no of failures until we get r success

d) $E(x) = \frac{r(1-p)}{p} = \frac{2 \times (1-0.2)}{0.2} = 8$

The total no of boxes to be purchased $= E(x) + r = 8 + 2 = 10$

76) $p = 0.5,\ r = 3$

X = The no. of children in the family.

$nb(x; r, p) = nb(x, 3, 0.5) = \binom{x+2}{2} (0.5)^3 \times (0.5)^x$

77) $r = 6,\quad P(G) = 0.5 = P(B)$

$nb(x; r, p) = \binom{x+5}{5} (0.5)^6 (0.5)^x$

$E(x) = \frac{r(1-p)}{p} = \frac{6(1-0.5)}{0.5} = 6$

So, for each brother expected no of male children $= 2$

78) $p = 0.409$

i) $P(X=3) = (0.409)(1-0.409)^3 \approx 0.0844$

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= (0.409) + (0.409)(1-0.409) + (0.409)(1-0.409)^2 + 0.0844$

$= 0.8278$

Ex: 3.5

8) ii) $P(X > E(x) + \sigma_x) = ?$

$$E(x) = \frac{1-p}{p} = \frac{1-0.409}{0.409} = 1.44$$

$$V(x) = \frac{1-p}{p^2} = \frac{1-0.409}{(0.409)^2} = 3.533$$

$$\therefore \sigma_x = \sqrt{3.533} = 1.88$$

$$\therefore P(X > 1.44 + 1.88)$$

$$= P(X > 3.32)$$

$$= 1 - P(X \leq 3.32)$$

$$= 1 - P(X \leq 3)$$

$$= 1 - 0.878$$

$$= 0.122$$