Semester-3RD
(Regular & Back)(Back)
Sub & Code-DMS(MA-2003)
Branch(s)-CSE & IT

AUTUMN END SEMESTER EXAMINATION-2016
DISCRETE MATHEMATICAL STRUCTURES
[MA-2003]

Full Marks:60 **Time:03 Hours**

Answer any six questions including question No.1 which is compulsory.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. (2×10)

a. Find the negation of the statement:
$$2+3=5 \Longrightarrow 5-2=3.$$

b. What is the universal quantification of the sentence: x^2+x is even integer where x is an odd integer? Is the universal quantification is a true statement?

c. Find the equivalence relation corresponding to the partition set $P=$ { {a,b}, {c}, {d,e}} of the set $A=\{a,b,c,d,e\}$.

d. Let D_{20} denote the set of all positive divisor of 20. Draw the digraph of the divisibility relation and Hasse diagram of the POSET D_{20}.

e. Find generating functions corresponding to the numeric function
$$a_n=n; \quad n \geq 1.$$

f. Find the number of positive integers not exceeding 100 that are either odd or square of an integer.

g. The set $\mathbb{Z}_{20}=\{0,1,2,\cdots,19\}$ under addition and multiplication modulo 20 is a commutative ring. List all zero-divisors of $\mathbb{Z}_{20}$.

h. Let $U(10)$ be the set of all positive integer less than 10 and relatively prime to 10. Define the binary operation $*$ on $U(10)$ as $\forall\, a, b \in U(10)\; a*b = ab \; mod(10)$. Show that $*$ is a closed binary operation.

i. Define complete graph. If a complete graph has degree 8 for each vertex, then how many edges are there?

j. State the necessary and sufficient conditions for a graph to be an Eulerian graph.

2. (2×4)

a. Prove $2^n > n^3$ for $n \geq 10$ by method of induction.

b. Show that $\forall x\,(P(x) \wedge Q(x))$ and $\forall x\, P(x) \wedge \forall x\, Q(x)$ are equivalent.

3. (2×4)

a. Let H be a subgroup of a group G and define a relation R on G as $(a,b) \in R$ iff $ab^{-1} \in H$. Show that R is an equivalence relation.

b. Does D_{30} under divisibility relation is a complemented lattice? If yes, Draw its Hasse diagram and find complements of each of its elements; where D_{30} is the set of all positive divisors of 30.

Paper Setter: Dr. Manoranjan Sahoo

Moderator: Dr. P. K. Mohanty.

4. (2 × 4)

a. Solve the following recurrence relation

$$a_r - 4a_{r-1} = 8^r;\ \text{for } r \geq 1$$

with the initial condition $a_0 = 1$ by substitution method.

b. Solve the following recurrence relation using generating function.

$$a_n - a_{n-1} = n; \quad n \geq 1;\ a_0 = 0.$$

5. (2 × 4)

a. Let G be a group and $a, b, c \in G$. Then show that

(i) $ab = ac \Longrightarrow b = c,$

(ii) $(ab)^{-1} = b^{-1}a^{-1}.$

b. Prove that the mapping from the group $U(16)$ to itself given by $f(x) = x^3$ is an automorphism.

6. (2 × 4)

a. Find a homomorphism φ from $U(30)$ to itself with kernel $\{1,11\}$ and $\varphi(7) = 7$.

b. Let $\mathbb{Z}_5 = \{0,1,2,3,4\}$, $\oplus$ be addition modulo 5 and $\otimes$ be multiplication modulo 5 on $\mathbb{Z}_5$. Show that $(\mathbb{Z}_5; \oplus, \otimes)$ is a field.

7. (2 × 4)

a. For any Boolean algebra B, prove that $(a+b)(b+c)(c+a) = ab + bc + ca$.

b. Minimize the Boolean expression $xyz + xy'z + x'y'z + x'yz + x'yz' + xy'z'$.

8. (2 × 4)

a. Are these following graphs Isomorphic? Justify your answer.

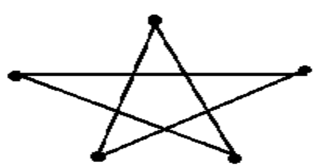


Fig.1

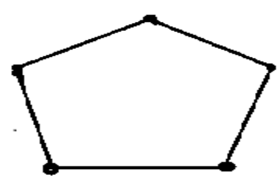
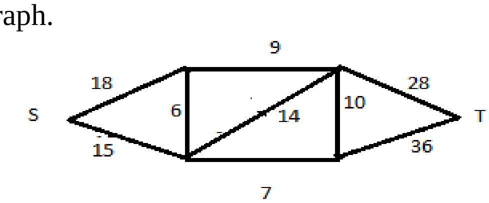


Fig.2

b. Using Dijkstra's algorithm find the shortest path from vertex S to T from the following weighted graph.



Paper Setter: Dr. Manoranjan Sahoo

Moderator: Dr. P. K. Mohanty.