20)

$P(A) = 0 \cdot 6$ $\qquad$ $P(C) = 0 \cdot 2$

$P(B) = 0 \cdot 4$ $\qquad$ $P(B \cap C) = 0 \cdot 1$

$P(A \cap B) = 0 \cdot 3$ $\qquad$ $P(A \cap B \cap C) = 0 \cdot 08$

$P(A \cap C) = 0 \cdot 15$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0 \cdot 3}{0 \cdot 4} = 0 \cdot 75 \neq P(A)$$

$$P(A) \times P(B) = 0 \cdot 6 \times 0 \cdot 4 = 0 \cdot 24 \neq P(A \cap B)$$

$\therefore$ A and B are dependent events.

E1) a) $P(B'|A') = \frac{P(B' \cap A')}{P(A')} = \frac{P(B') \times P(A')}{P(A')} = \frac{0.3 \times 0.6}{0.6} = 0.3$

Ex: 2.5

21) $P(A) = 0.4$ A: Asian project is successful

$P(B) = 0.7$ B: European project is successful.

A and B are independent. (A' and B' are also independent)

~~P(A) = 0.4~~

$\therefore P(B'|A') = P(B')$

a) ~~$P(B'|A') = \frac{P(B' \cap A')}{P(A')} = P(B')$~~

~~$= 0.3 / (1 - 0.7)$~~

b) ~~$P(A \cup B) = P(A) + P(B) - P(A \cap B)$~~

~~$= 0.4 + 0.$~~

$P(A \cup B) = ?$ $P(A) + P(B) - P(A \cap B)$ → Since independent events.

$= 0.4 + 0.7 - P(A) \cdot P(B)$

$P(B'|A') = 0.3$ $= 1.1 - 0.4 \times 0.7$

$= 0.82$

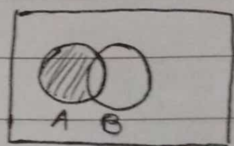
$\Rightarrow P(B' \cap A') = 0.3 \times P(A')$

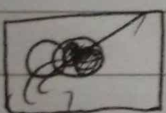
$\Rightarrow P((A \cup B)') = 0.3 \times 0.6$

$\Rightarrow 1 - P(A \cup B) = 0.18$

$\Rightarrow P(A \cup B) = 0.82$

c) $P(A|A \cup B) = \frac{P((A \cap B') \cap (A \cup B))}{P(A \cup B)}$ (written above: $P(A \cap B')$)





$= \frac{P(A \cap B')}{P(A \cup B)}$

$= \frac{P(A) \cdot P(B')}{0.82} =$

$= \frac{P(A) - P(A \cap B)}{0.82}$

$= \frac{P(A) - P(A) \times P(B)}{0.82}$

$= \frac{P(A)(1 - P(B))}{0.82}$

$= \frac{P(A) \cdot P(B')}{0.82} = \frac{0.4 \times (0.3)}{0.82} = 0.146$

Ex: 7.2

72) $P(A_1) = 0.22$, $P(A_2) = 0.25$, $P(A_3) = 0.28$, $P(A_1 \cap A_2) = 0.11$,
$P(A_1 \cap A_3) = 0.05$, $P(A_2 \cap A_3) = 0.07$

$P(A_1) \cdot P(A_2) = 0.22 \times 0.25 = 0.055 \neq P(A_1 \cap A_2)$
$P(A_2) \cdot P(A_3) = 0.25 \times 0.28 = 0.07 = P(A_2 \cap A_3)$
$P(A_1) \cdot P(A_3) = 0.22 \times 0.28 = 0.0616 \neq P(A_1 \cap A_3)$

$\therefore$ A_1 and A_2 are dependent events
A_1 and A_3 are dependent events
But, A_2 and A_3 are independent events as $P(A_2 \cap A_3)$
$= P(A_2) \times P(A_3)$

23) Given that, A and B are independent.

$\therefore P(A \cap B) = P(A) \cdot P(B)$

To prove $P(A'|B) = P(A')$

$$\Rightarrow P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$$

$$\cancel{= \frac{P(B)(1 - P(A \cap B))}{P(B)}}$$

$$= \frac{P(B)(1 - P(A) \cdot P(B))}{P(B)}$$

$$= \frac{P(B)(1 - P(A))}{P(B)}$$

$$= P(A')$$

$\therefore A'$ and B are independent.

74) $P(A) = 0{\cdot}4$, $P(B) = 0{\cdot}11$, $P(AB) = 0{\cdot}04$, $P(O) = 0{\cdot}45$

$P(\text{Both phenotypes are } O) = P(O \cap O) = P(O) \times P(O)$

$= 0{\cdot}45 \times 0{\cdot}45$

$= 0{\cdot}2025$

$P(\text{The phenotypes of two randomly selected individuals match})$

$= P(O \cap O) + P(AB \cap AB) + P(A \cap A) + P(B \cap B)$

$= P(O) \cdot P(O) + P(AB) \cdot P(AB) + P(A) \cdot P(A) + P(B) \cdot P(B)$

$= 0{\cdot}45 \times 0{\cdot}45 + 0{\cdot}04 \times 0{\cdot}04 + 0{\cdot}4 \times 0{\cdot}4 + 0{\cdot}11 \times 0{\cdot}11$

$= 0{\cdot}3762$

75) A: A particular point will signal a process with a the process

$P(A) = 0.05$

$P(A_1 \cup A_2 \cup \ldots A_{10}) = P(A_1' \cap A_2' \cap \ldots A_{10}')'$ [De Morgan's law]

$= 1 - P(A_1' \cap A_2' \cap \ldots \cap A_{10}')$

$= 1 - \{P(A_1') \times P(A_2)' \ldots P(A_{10})'\}$

$= 1 - \{(1-0.05) \times (1-0.05) \times \ldots\}$

$= 1 - 0.59873$

$= 0.40127$

$P(A_1 \cup A_2 \cup \ldots A_{25}) = 1 - \{0.95\}^{25} = 0.72261$

$$= \frac{\ldots}{1-\pi^2} \qquad \frac{\ldots}{1-\pi^2}$$

76) Let, A denote the event that the division is incorrect.

$P(A) = \frac{1}{9{,}000{,}000{,}000} = x$ (say)

1 billion $= 10^9$

~~Suppose~~ We have 1 billion divisions with the chip.

P(Atleast one error occurs in one billion divisions with this chip) $= P(A_1 \cup A_2 \cup \ldots\ldots A_{10^9})$

$= P(A_1' \cap A_2' \cap A_3' \ldots\ldots A_{10^9}')'$ (By De-morgans law)

$= 1 - P(A_1' \cap A_2' \cap A_3' \ldots A_{10^9}')$

$= 1 - \{P(A_1') \times P(A_2') \times \ldots\ldots \times A_{10^9}'\}$

$= 1 - \{(1-a) \times (1-a) \times \ldots$

$= 1 - \{0.99999998888889 \times \ldots\ldots$

$= 1 - 0.895$

$= 0.105$

77) a) P(atleast one rivet is defective) = 0.15

P(none of the rivets are defective) $= 1 - 0.15$

$= 0.85$

P(none of the rivets are defective)

= P(not defective) x P(not defective) ......

$= (P(\text{not defective}))^{25}$

$\Rightarrow (P(\text{not defective}))^{25} = 0.85$

$\Rightarrow P(\text{not defective}) = \sqrt[25]{0.85} = 0.9935$

P(a rivet is defective) = 1 − P(not defective)

$= 1 - 0.9935$

$= 0.0065$

b) P(atleast one rivet is defective) = 0.1

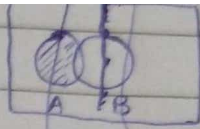
P(none of the rivets are defective) $= 1 - 0.1 = 0.9$

$(P(\text{not defective}))^{25} = 0.9$

$\Rightarrow P(\text{not defective}) = \sqrt[25]{0.9} = 0.9957$

$\therefore$ P(a rivet is defective) $= 1 - 0.9957 = 0.0043$

28) P(A valve opens) $= 0.96$ P(a valve doesn't open) $= 0.04$



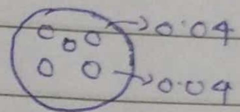
$A \cap B'$ | $A \cup B$

P(atleast one valve opens) $= ?$

$= 1 - P$(all 5 valves doesn't open)

$= 1 - (0.04)^5$

$= 0.99$



P(atleast one valve fails to open)

$= 1 - P$(all 5 value's open)

$= 1 - (0.96)^5$

$= 0.18$

29) Let, O be the event that the older pump will be failed

N be the event that the newer pump will fail

$P(O \cap N') = 0.1$

$P(O' \cap N) = 0.05$

$P(O \cap N) = x$

$P(A \cap B') = P(A) - P(A \cap B)$

$P(O \cap N') = P(O) - P(O \cap N)$

$\Rightarrow 0.1 = P(O) - x$

$\Rightarrow P(O) = 0.1 + x$

$0.05 = P(N) - P(O \cap N)$

$\Rightarrow 0.05 = P(N) - x$

$\Rightarrow P(N) = x + 0.05$

$x = P(O \cap N) = P(O) \cdot P(N)$ ($\because$ They are independent events)

$x = (0.1 + x)(x + 0.05)$

$\Rightarrow x = x^2 + 0.15x + 0.005$

$\Rightarrow x^2 - 0.85x + 0.005 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{0.85 \pm \sqrt{(0.85)^2 - 4 \times 0.005}}{2}$$

$x = 0.844, 0.005$

$P(O \cap N) \neq 0.844$ because $P(O' \cap N) = 0.05$
$P(N' \cap O) = 0.1$

$\Rightarrow$ P(system works) = P(1,2 subsystem works) $\cup$ P(3,4 subsystem works)

P(1,2 subsystem works) + P(3,4 system works) − P(1,2 subsystem works $\cap$ 3,4 subsystem works)

P(1,2 subsystem works) = P(1 works $\cup$ 2 works)
= P(1 works) + P(2 works)
− P(1 works $\cap$ 2 works)

$= 0.9 + 0.9 - 0.9 \times 0.9$

$= 0.99$

P(3,4 subsystem works) = P(3 works ∩ 4 works)

$= P(3 \text{ works}) \times P(4 \text{ works})$

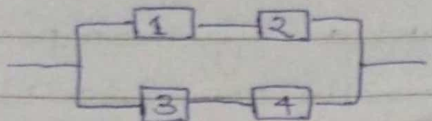
$= 0.8 \times 0.8$

$= 0.64$

∴ P(system works) $= 0.99 + 0.64 - 0.99 \times 0.64$

$= 0.9964$.

1) HW



P(System lifetime exceeds t_0)

$= P(A_1 \cap A_2) \cup P(A_3 \cap A_4)$

$= P(A_1 \cap A_2) + P(A_3 \cap A_4) - P((A_1 \cap A_2) \cap (A_3 \cap A_4))$

$= P(A_1) \cdot P(A_2) + P(A_3) \cdot P(A_4) - P(A_1) \cdot P(A_2) \cdot P(A_3) \cdot P(A_4)$

$= p^2 + p^2 - p^4$ $\quad [\because P(A_i) = p$ [Given]])

$= 2p^2 - p^4$

$2p^2 - p^4 = 0.99$

Let, $x = p^2$

$2x - x^2 = 0.99$

$\Rightarrow x^2 - 2x + 0.99 = 0$

$x = \dfrac{2 \pm \sqrt{4 - 4 \times 0.99}}{2}$

$= 1.1, 0.9$

$x = p^2$

$\therefore p^2 = 1.1$

$\Rightarrow p = \sqrt{1.1}$

$\Rightarrow p = 1.04$

which is not possible as $0 \leq P(E) \leq 1$

$p^2 = 0.9$

$\Rightarrow p = 0.948$

$\therefore p = 0.94$

$\therefore$ 0.9 would have to be changed to 0.94 in order to increase the system lifetime reliability from 0.9639 to 0.99

82) Let A: the red die shows 3 dots.

B: the green die shows 4 dots.

C: The total no. of dots showing on the dice is 7.

$A = \{31, 32, 33, 34, 35, 36\} = P(A) = \frac{6}{36} = \frac{1}{6}$

$B = \{ (1,4), (2,4), (3,4) \ldots, (6,4)\} = P(B) = \frac{6}{36} = \frac{1}{6}$

$C = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\} = \frac{1}{6} = P(C)$

$A \cap B = \{(3,4)\}$

$B \cap C = \{(3,4)\}$

$A \cap C = \{(3,4)\}$

$P(A\cap B) = \frac{1}{36}$, $P(B \cap C) = \frac{1}{36}$, $P(A \cap C) = \frac{1}{36}$

$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{36}$

$P(B \cap C) = P(B) \cdot P(C) = \frac{1}{36}$

$P(A \cap C) = P(A) \cdot P(C) = \frac{1}{36}$

$\Rightarrow$ A, B and C are pairwise independent.

$A \cap B \cap C = \{(3,4)\}$ $\qquad P(A \cap B \cap C) = \frac{1}{36}$

$\rightarrow P(A) \cdot P(B) \cdot P(C) = P(A \cap B \cap C)$ if 3 of them were independent.

∴ But, $P(A)\cdot P(B)\cdot P(C) \neq \frac{1}{36}$

Clearly A, B and C are not mutually disjoint

83) i) P(atleast one doesn't detect a defect) = 0.2

= 1 − P(1 detects the defect ∩ 2 detects the defect)

⇒ P(1 detects the defect ∩ 2 detects the defect)

= 1 − 0.2 = 0.8

P(1 detects the defect ∩ 2 doesn't detect the defect)

= P(1 detects the defect) − P(1 detects the defect ∩ 2 detects the defect)

= 0.9 − 0.8

= 0.1

P((1 detects the defect ∩ 2 doesn't detect the defect)

∪ (2 detects the defect ∩ 1 doesn't detect the defect))

P(1 doesn't ∩ 2 detects the defect)

= P(2 detects the defect) − P(1 detects the defect ∩ 2 detects the defect)

= 0.9 − 0.8 = 0.1

P(exactly one detects the defect) = P((1 detects the defect ∩ 2 doesn't)

∪ (1 doesn't ∩ 2 detects the defect))

= P(1 detects the defect ∩ 2 doesn't) + P(1 doesn't ∩ 2 detects the defect)

= 0.1 + 0.1 = 0.2 (Since both are mutually exclusive)

ii) P(neither of the inspectors detect a defect component)

$1 - \{P(1 \text{ detects} \cap 2) + P(\text{exactly one detects})\}$

$= 1 - \{0.8 + 0.2\} = 0$

P(all three components escape the detection)

$= (0) \cdot (0) \cdot (0) = 0$

84) A_1: Receiver functions properly.

A_2: The speakers function properly.

A_3: The CD player functions properly.

$P(A_1) = 0.95$, $P(A_2) = 0.98$, $P(A_3) = 0.8$

a) $P(A_1 \cap A_2 \cap A_3) = P(A_1) \times P(A_2) \times P(A_3)$ [$\because$ All the events are mutually independent]

$= 0.95 \times 0.98 \times 0.8$

$= 0.7448$

b) $P(A_1' \cup A_2' \cup A_3') =$ P(atleast one component needs service during the warranty period)

$= P((A_1 \cap A_2 \cap A_3)')$ [By De-Morgan's law]

$= 1 - P(A_1 \cap A_2 \cap A_3)$

$= 1 - 0.7448$

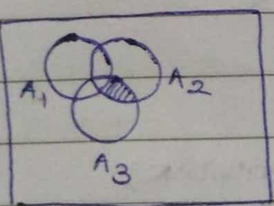
$= 0.2552$

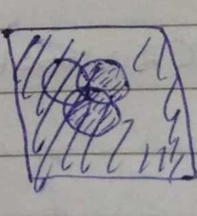
c) $P(A_1' \cap A_2' \cap A_3') = P(A_1') \times P(A_2') \times P(A_3')$

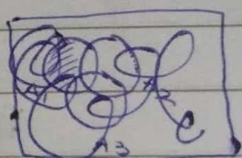
$= (1 - P(A_1)) \times (1 - P(A_2)) \times (1 - P(A_3))$

$= 0.05 \times 0.02 \times 0.2$

$= 0.0002$

d) 

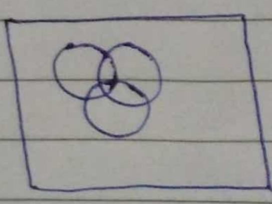




$P(A_1' \cap A_2 \cap A_3) = P(A_1') \times P(A_2) \times P(A_3)$

$= 0.05 \times 0.98 \times 0.8$

$= 0.0392$

e) 

$P(A_1' \cap A_2 \cap A_3) + P(A_2' \cap A_1 \cap A_3) + P(A_3' \cap A_1 \cap A_2)$

$= 0.05 \times 0.98 \times 0.8 + 0.02 \times 0.95 \times 0.8 + 0.2 \times 0.95 \times 0.98$

$= 0.2406$

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f) ~~PE~~ This probability cannot be determined because we do not know the warranty period.

Q7) a) $P(A_1) = 0.55,\ P(A_2) = 0.65,\ P(A_3) = 0.7$

i) $P(A_1 \cup A_2) = 0.8,\ P(A_2 \cap A_3) = 0.4$

A_1: likes vehicle 1

A_2: likes vehicle 2

A_3: likes vehicle 3

$\therefore P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$\Rightarrow 0.8 = 0.55 + 0.65 - P(A_1 \cap A_2)$

$\Rightarrow P(A_1 \cap A_2) = 0.55 + 0.65 - 0.8$

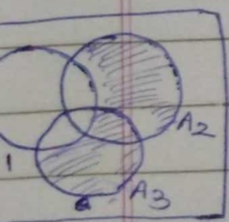
$\Rightarrow P(A_1 \cap A_2) = 0.4$

ii) $P(A_2 | A_3) = \dfrac{P(A_2 \cap A_3)}{P(A_3)} = \dfrac{0.4}{0.7} = 0.57 \neq P(A_2)$

$\therefore P(A_2 | A_3) \neq P(A_2)$

$\therefore P(A_2)$ and $P(A_3)$ are not ~~inke~~ dependent events.

iv) $\cancel{P(A_1' \cap A_2 \cap A_3}$ $P(A_2 \cup A_3 | A_1')$



$= \dfrac{P((A_2 \cup A_3) \cap A_1')}{P(A_1')} = \dfrac{P(A_2 \cup A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3)}{0.45}$

$P(A_2 \cup A_3) = P(A_2) + P(A_3) - P(A_2 \cap A_3)$

$= 0.65 + 0.7 - 0.4$

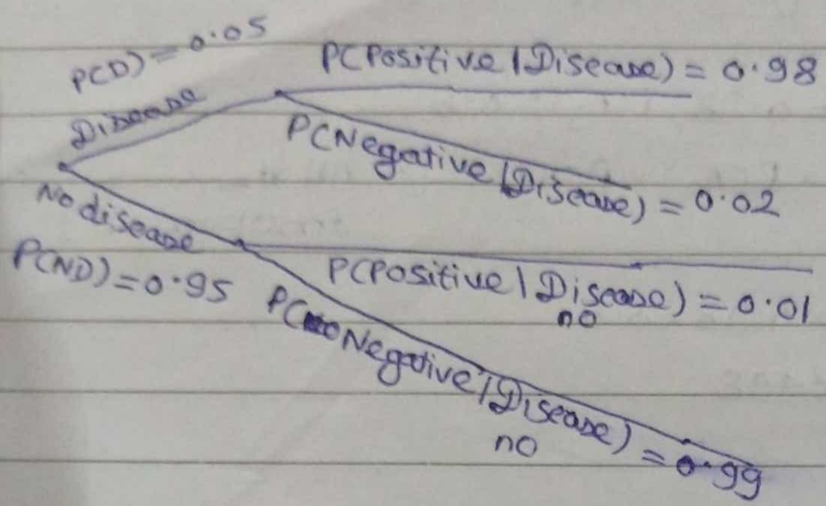
$= 0.95$

$P(A_1 \cap A_2)$

$P((A_2 \cap A_1') \cup (A_1' \cap A_3))$

$= \dfrac{P(A_1 \cup A_2 \cup A_3) - P(A_1)}{0.45} = \dfrac{0.88 - 0.55}{0.45} = 0.733$

85)



$P(\text{Positive} \cap \text{Disease}) = P(\text{Positive} \mid \text{Disease}) \times P(\text{Disease})$

$= 0.98 \times 0.05 = 0.049$

~~P(Positive~~

~~P(Negative ∩ Di~~

$P(\text{Positive} \cap \text{No disease}) = P(\text{Positive} \mid \text{No disease}) \times P(\text{Disease})$

$= 0.01 \times 0.985 = 0.00985$

$\therefore P(\text{Positive}) = 0.049 + 0.00985 = 0.05885$

$P(\text{Both are positive}) = 0.05885 \times 0.05885 = \cancel{0.00345744}$

$= 0.00342225$

$P(\text{Disease} \mid \text{Both are positive})$

$$= \frac{P(\text{Disease} \cap \text{Both are positive})}{P(\text{Both are positive})}$$

$$= \frac{0.049 \times 0.049}{0.00342225}$$

$= 0.7015$

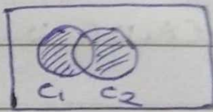
89)

C_1: {left ear tag is lost}

C_2: {right ear tag is lost}

A: {exactly one tag is lost}

B: {at most one tag lost}



$$A = (C_1' \cap C_2) \cup (C_2' \cap C_1)$$

$$B = (C_1' \cup C_2') = (C_1 \cap C_2)'$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P((C_1' \cap C_2) \cup (C_2' \cap C_1) \cap (C_1 \cap C_2)')}{P(C_1 \cap C_2)'}$$

$$= \frac{P((C_1' \cap C_2) \cup (C_2' \cap C_1))}{1 - P(C_1 \cap C_2)}$$

$$= \frac{P(C_1) + P(C_2) - P(C_1 \cap C_2)}{1 - P(C_1 \cap C_2)}$$

$$= \frac{\pi + \pi - P(C_1) \times P(C_2)}{1 - P(C_1) \times P(C_2)}$$

$$= \frac{2\pi - \pi^2}{1 - \pi^2} = \frac{2\pi(1-\pi)}{1-\pi^2}$$