NETWORK THEOREMS

4

OBJECTIVES

After completing this Chapter, you will be able to:

- *State and apply 'superposition theorem' to solve a circuit containing more than one source.*
- *State and apply 'Thevenin's theorem' to solve a circuit.*
- *State the benefits of 'Thevenin's theorem'.*
- *State and apply 'Norton's theorem' to solve a circuit.*
- *State, prove and apply 'maximum power transfer theorem'.*
- *Find a single equivalent voltage source for a number of voltage sources connected in parallel using 'Millman's theorem'.*
- *State and prove 'reciprocity theorem'.*
- *State and prove 'Tellegen's theorem'.*
- *State the importance of 'Tellegen's theorem'.*

4.1 INTRODUCTION

Simple circuits can be solved by using Ohm's law, Kirchhoff's laws, voltage divider, current divider, series and parallel combination of sources and resistors, etc. Special techniques, known as *network theorems* and *network reduction methods*, have been developed which drastically reduce the labour of solving a more complicated network. The network theorems provide simple conclusions and good insight into the problems. Some of these have universal applications, whereas some are limited to the networks containing linear* elements only.

4.2 SUPERPOSITION THEOREM

This theorem states that *the response in a linear circuit at any point due to multiple sources can be calculated by summing the effects of each source considered separately, all other sources being made inoperative* (or *turned* OFF). The theorem is applicable only to a linear network (containing independent and/or dependent sources).

How to Make a Source Inoperative

When a *voltage source* is made inoperative or turned OFF, no voltage drop exists across its terminals but *a current can still* flow through it. Hence, it acts like a *short-circuit* (Fig. 4.1*a*).

* A linear element obeys Ohm's law, e.g., resistance, inductance, etc. Semiconductor diodes and transistors are not linear elements.

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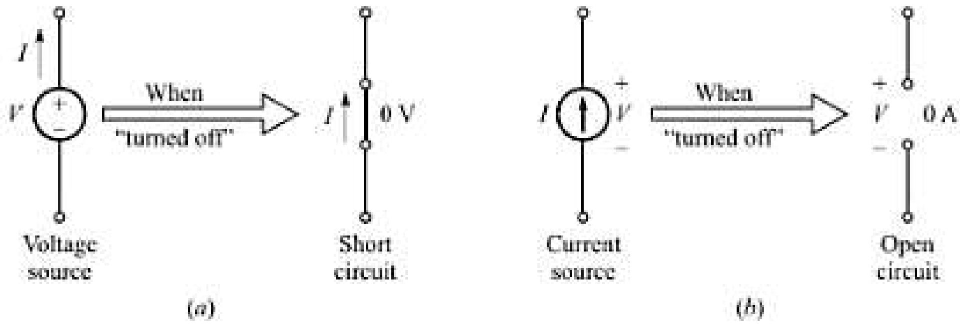

Fig. 4.1 *"Turning off" the sources.*

Similarly, when a *current source* is made inoperative or turned off, no current flows through it but a voltage can appear across its terminals. Hence, it acts like an *open-circuit* (Fig. 4.1*b*).

By making a source *inoperative* or turned OFF means that the voltage source is replaced by a short-circuit and the current source is replaced by an open-circuit.

EXAMPLE 4.1

Using superposition theorem, find the current I in the circuit shown in Fig. 4.2*a*.

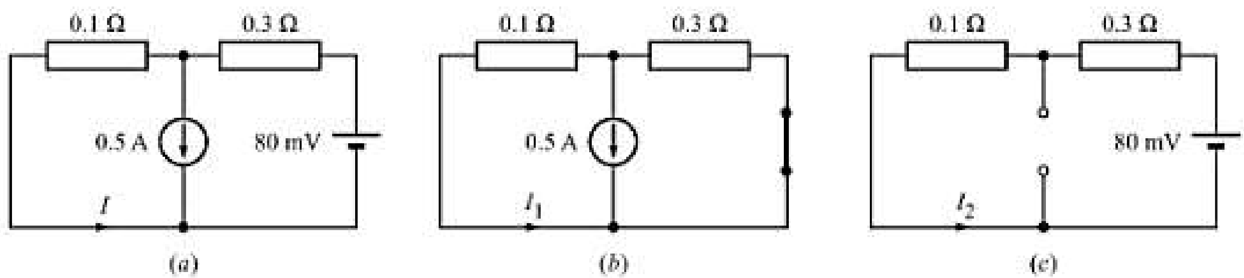

Fig. 4.2 *Principle of superposition illustrated.*

Solution Let us first consider the response I_1 due to the 0.5-A current source, and turn OFF the 80-mV voltage source by shorting it (Fig.4.2*b*). Applying current divider, we get

$$I_1 = -0.5 \times \frac{0.3}{0.1+0.3} = 0.375 \text{ A}$$

Next, consider the voltage source, and turn OFF the current source by opening it (Fig.4.2*c*). Ohm's law gives

$$I_2 = \frac{80 \times 10^{-3}}{0.1+0.3} + 0.2\text{A}$$

By the principle of superposition, the total current is given as

$$I = I_1 + I_2 = -0.375 + 0.2 = -\mathbf{0.175A}$$

Note that in redrawing the circuit for each source, we are always careful to mark the response current in the original direction and also assign a suitable subscript to indicate that we are not working with the original variables. This prevents the possibility of committing errors when we add the individual currents.

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EXAMPLE 4.2

Use superposition theorem to find current I_x in the network given in Fig. 4.3.

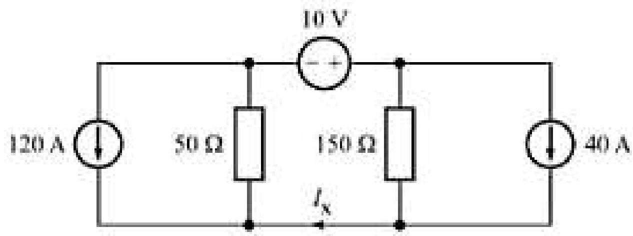

Fig. 4.3

Solution Consider first the voltage source alone. Both current sources are made inoperative by open-circuiting them (Fig. 4.4*a*). Current I_1 is

$$I_1 = \frac{10}{50 + 150} = 0.05 \text{ A}$$

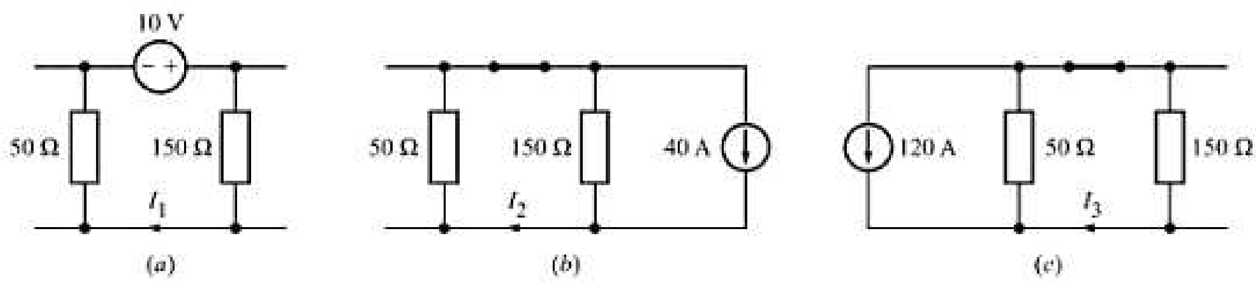

Fig. 4.4

Next consider the 40-A current source alone (Fig. 4.4*b*). Applying current divider, the current I_2 is given as

$$I_2 = 40 \times \frac{150}{50 + 150} = 30 \text{ A}$$

Lastly consider the 120-A current source alone (Fig. 4.4*c*). The current I_3 is

$$I_3 = -120 \times \frac{50}{50 + 150} = -30 \text{ A}$$

Applying the principle of superposition, we get

$$I_x = I_1 + I_2 + I_3 = 0.05 + 30 - 30 = \mathbf{0.05\ A}$$

EXAMPLE 4.3

Consider our benchmark example (Fig. 3.24*a*) discussed in Example 3.11, wherein we had calculated the voltage across 3-Ω resistor as 2.5 V, by using source transformation. The same circuit is shown in Fig. 4.5*a*. We again find voltage v across 3-Ω resistor by applying the principle of superposition.

Solution Consider first the **4-A current source**, while turning OFF the remaining two sources. The turned-OFF 5-A current source is replaced by an open-circuit and the turned-OFF 6-V voltage source is replaced by a short-circuit, as shown in Fig. 4.5*b*. We find that the current of 4 A divides into two parallel paths. Therefore, using current divider, we get

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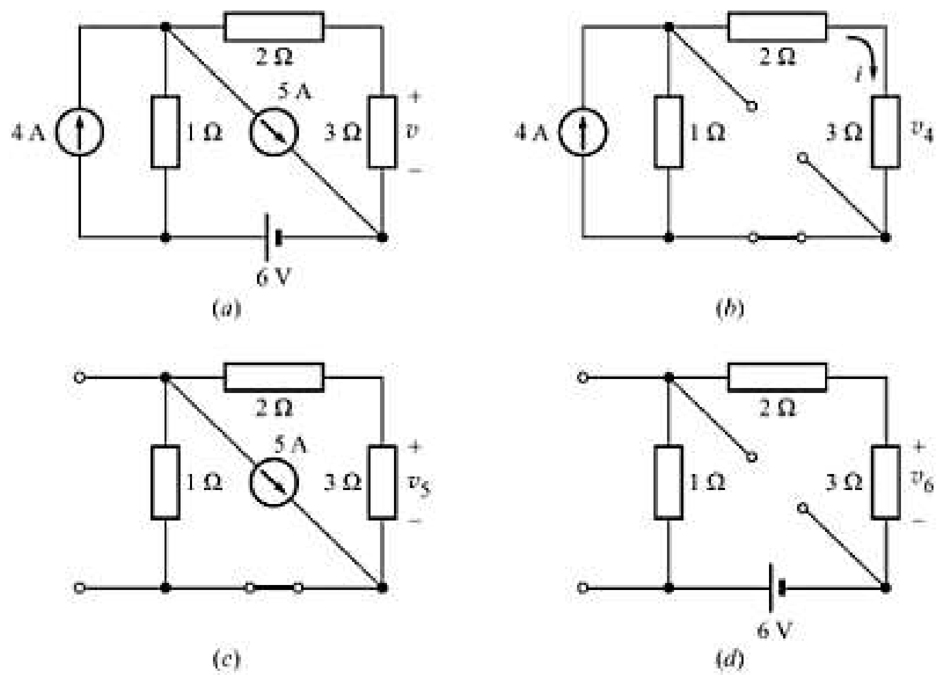

Fig. 4.5

$$i = 4 \times \frac{1}{1 + (2 + 3)} = \frac{2}{3}\text{A}$$

Thus, voltage v_4 across 3-Ω resistor due to 4-A current source is

$$V_4 = i \times R = (2/3\text{ A}) \times (3\Omega) = 2.0\text{V}$$

The polarity of this voltage is the same as the original polarity markings for v. Hence, this contribution to the final summation will appear with a + sign.

Next consider **5-A current source,** with 4-A and 6-V sources turned OFF (Fig. 4.5*c*). From the perspective of the 5-A source, the 2-Ω and 3-Ω resistances are in series and together in parallel with the 1-Ω resistor. Using current-divider, the voltage v_5 across 3-Ω resistor due to 5-A current source is given as

$$v_5 = \left[-5 \times \frac{1}{1 + (2 + 3)}\, A\right] \times (3\Omega) = -2.5\text{ V}$$

Note that the actual polarity of this voltage v_5 is opposite to the polarity marked on v. Hence, the minus sign.

We now consider **6-V voltage source,** with 4-A and 5-A sources turned OFF (Fig.4.5*d*). The three resistances are connected in series. The resulting voltage v_6 is calculated by voltage divider, as

$$v_6 = 6 \times \frac{3}{1 + 2 + 3} = 3.0\text{ V}$$

The polarity of this voltage is same as that of the original voltage v. Hence, the sign of v_6 will be positive in the final summation.

Using principle of superposition, we now obtain the total voltage v across 3-Ω resistor, as

$$v = +v_4 + v_5 + v_6 = +2.0 - 2.5 + 3.0 = +\mathbf{2.5\text{ V}}$$

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EXAMPLE 4.4

For the circuit shown in Fig. 4.6*a*, find the value of I_s to reduce the voltage across 4-Ω resistor to zero.

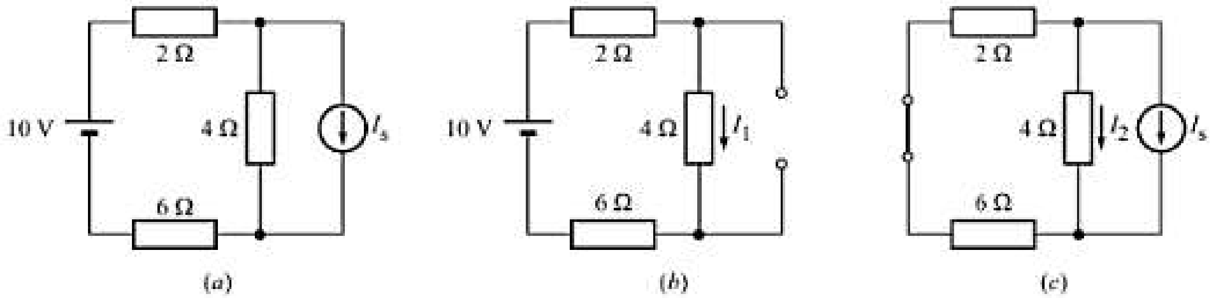

Fig. 4.6

Solution To solve this problem, we apply the principle of superposition. The current I_1 (from top to bottom) in 4-Ω resistor due to 10-V source in Fig. 4.6*b* is

$$I_1 = \frac{10}{2+4+6} = \frac{5}{6}\text{ A}$$

The current I_2 (from top to bottom) in 4-Ω resistor due to current source I_s in Fig. 4.6*c* is

$$I_2 = -I_s \times \frac{2+6}{2+6+4} = -\frac{2}{3}I_s$$

The voltage across 4-Ω resistor can be zero, only if the current through this resistor is zero. That is,

$$I_1 + I_2 = 0 \quad \text{or} \quad \frac{5}{6} + \left(-\frac{2}{3}I_s\right) = 0 \quad \Rightarrow \quad I_s = \frac{5\times 3}{6\times 2} = \mathbf{1.25\ A}$$

4.3 THEVENIN'S THEOREM

This theorem was first proposed by a French telegraph engineer M.L. Thevenin in 1883. Often, we need to find the response (*current*, *voltage* or *power*) in a single load resistance in a network. Thevenin's theorem enables us to do this without solving the entire network. It is specially very helpful and time-saving when we wish to find the response for different values of the load resistance.

Thevenin's Theorem states that *it is possible to simplify any linear circuit containing independent and dependent voltage and current sources, no matter how complex, to an equivalent circuit with just a single voltage source and a series resistance, between any two points of the circuit.*

Procedure

The procedure to apply Thevenin's theorem to a network will be explained in steps, by taking an example. Consider the circuit shown in Fig. 4.7*a*. Suppose that we are interested to find the current in resistor R_2. We proceed as follows.

1. Designate the resistor R_2 as "load" (Fig. 4.7*b*).
2. Pull out the load resistor and enclose the remaining network within a dotted box (Fig. 4.7*c*).
3. Temporarily remove the load resistor R_2, leaving the terminals A and B open (Fig. 4.7*d*).

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