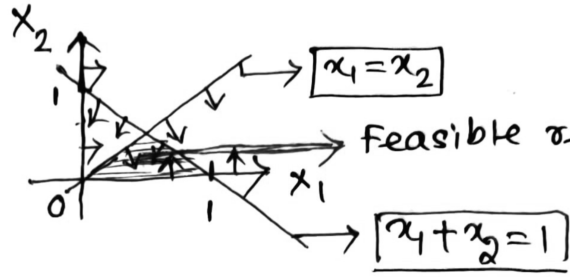
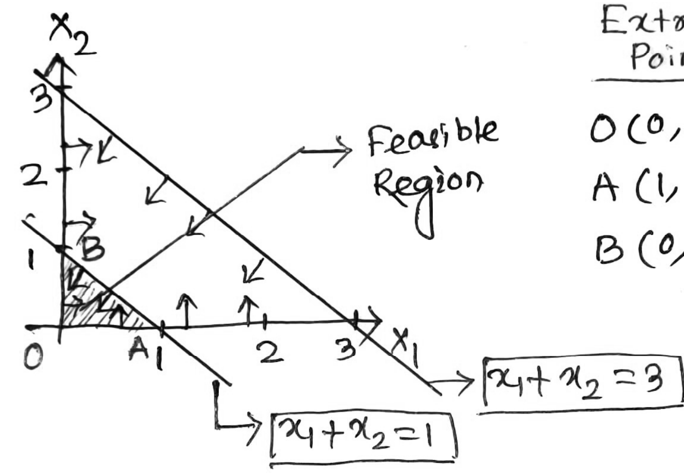
Autumn Mid Semester Examination - 2022

Optimization Technique (MA 10003) Solution

Q.1. a.



b.



Extreme Points	value of $Z = 2x_1 + 3x_2$
$O(0,0)$	$\longrightarrow$ 0 (minimum)
$A(1,0)$	$\longrightarrow$ 2
$B(0,1)$	$\longrightarrow$ 3

Thus, min. $Z = 0$ whenever $x_1 = 0,\ x_2 = 0$.

c. Max. (or Min.) $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

subject to

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \ (\leq = \geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \ (\leq = \geq)\ b_2$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \ (\leq = \geq)\ b_m$$

satisfying

$$x_1, x_2, \cdots, x_n \geq 0$$

where c_i, b_j & a_{ji} for $i = 1, 2, \cdots, n$; $j = 1, 2, \cdots, m$ are the constants.

d. Max. $Z = 2x_1 + 3x_2 + 0x_3 + 0x_4$

subject to

$$2x_1 + 3x_2 + x_3 = 1$$

$$-x_1 - 2x_2 + x_4 = 2$$

satisfying $x_1, x_2, x_3, x_4 \geq 0$

Here 'x_3' is the slack variable and 'x_4' is the surplus variable.

e. one of the initial basic feasible solution is

$$x_1 = 0,\ x_2 = 0,\ x_3 = 3,\ x_4 = 4.$$

Q.2. Let 'x_1' & 'x_2' be the number of 1st and 2nd type laptops ~~produced respectively~~ produced by the company.

As the profits per 1st and 2nd type Laptop are RS. 5000/- and RS. 8000/- respectively and the company wants to maximize its profit, the objective function is

$$\text{Max. } Z = 5000x_1 + 8000x_2$$

Let a laptop of 2nd type needs 't' units labour ~~manufacturing~~ time.

Since the laptop of 1st type requires twice as much labour time as the 2nd type and if all laptops are of 2nd type only, the company can produce 3000 laptops a day. Thus,

$$(2x_1 + x_2)t \leq 3000t$$

i.e; $2x_1 + x_2 \leq 3000$

Again since the market limits daily sales of the 1st and 2nd type to 1500 and 2500 laptops respectively,

$$x_1 \leq 1500$$

$$x_2 \leq 2500$$

Also since the no. of laptops of each type to be produced by the company are non negative

$$x_1 \geq 0,\ x_2 \geq 0.$$

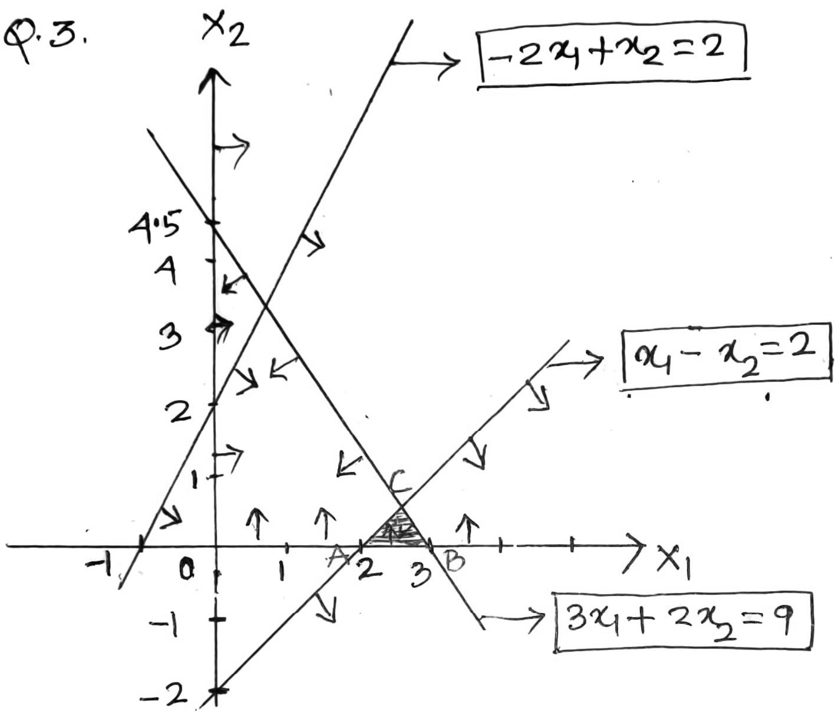
Thus the LPP is

$$\text{Max. } Z = 5000x_1 + 8000x_2$$

subject to

$$2x_1 + x_2 \leq 3000$$
$$x_1 \leq 1500$$
$$x_2 \leq 2500$$
$$x_1, x_2 \geqslant 0$$

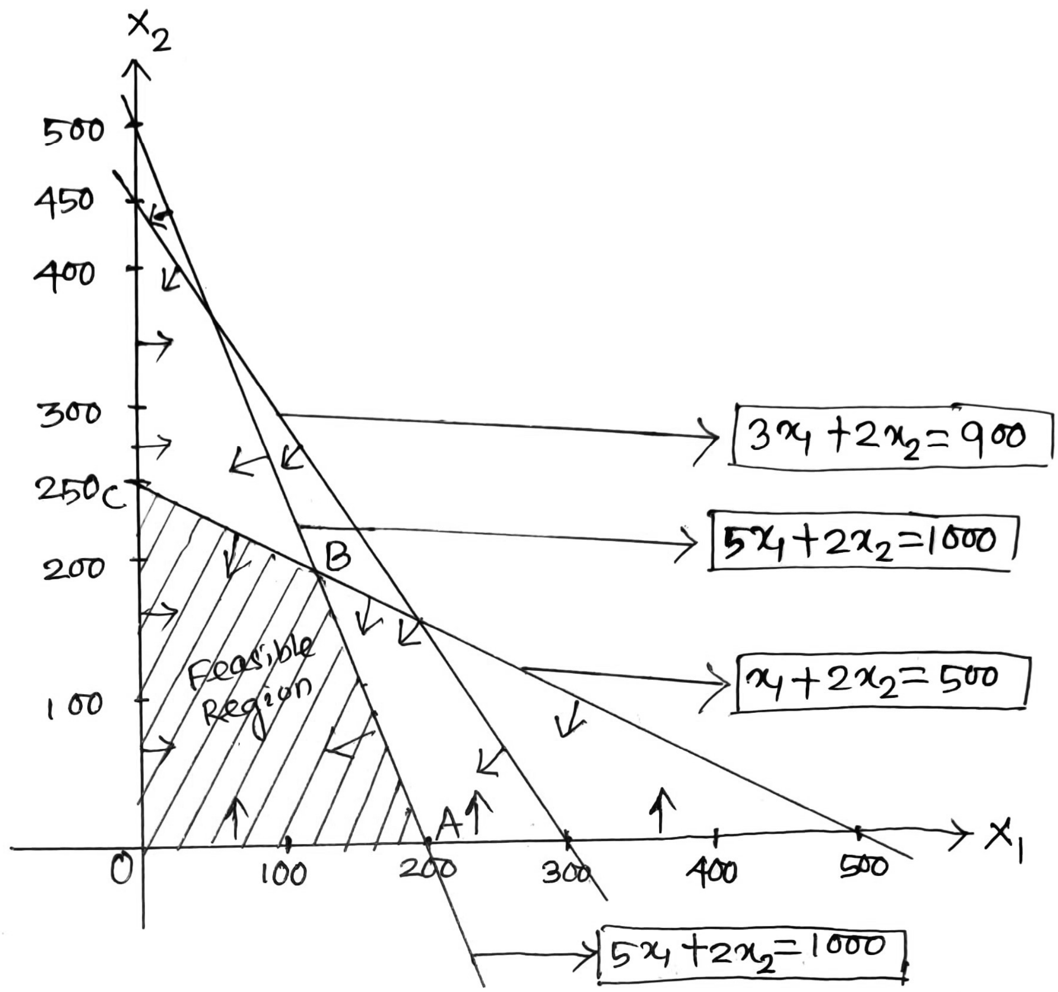
Q.3.



Extreme pts.	value of $Z (= 6x_1 + 4x_2)$
$A(2,0)$	12 (minimum)
$B(3,0)$	18
$C(\frac{13}{5}, \frac{3}{5})$	18

Thus, the minimum value of Z is 12 whenever $x_1 = 2, x_2 = 0$.

Q. 4.



Extreme Points	Value of $Z (= 100x_1 + 40x_2)$
O (0, 0)	0
A (200, 0)	20,000 (maximum)
B (125, 187.5)	20,000 (maximum)
C (0, 250)	10,000

It can be seen that the maximum value of Z occurs at the extreme points ~~of~~ 'A' and 'B' which are also the end points of the line segment AB. Since there are infinite number of points on the line segment AB ~~giving~~ which gives the same maximum value of Z, the given LPP has infinite number of optimal solutions.

Thus, max. $Z = 20,000$ with infinite no. of optimal solutions.

Q(5) Solve the LPP by simplex method

$$\text{Max } Z = 2x_1 + 3x_2$$

s.t

$$x_1 + 3x_2 \le 12$$

$$3x_1 + 2x_2 \le 12$$

$$x_1, x_2 \ge 0$$

Ans:

$$x_1 + 3x_2 + S_1 = 12$$

$$3x_1 + 2x_2 + S_2 = 12$$

$$x_1, x_2, S_1, S_2 \ge 0$$

$$\text{Max } Z = 2x_1 + 3x_2 + 0S_1 + 0S_2$$

		C_j	2	3	0	0	Min ratio
B.V.	C_B	X_B	x_1	x_2	S_1	S_2	Min $(X_B/$
← S_1	0	12	1	3	1	0	$\frac{12}{3} = 4$ ←
S_2	0	12	3	2	0	1	$\frac{12}{2} = 6$
		$Z_j - C_j$	-2	-3 ↑	0	0	
x_2	3	4	$1/3$	1	$1/3$	0	$\frac{4}{1/3} = 12$
← S_2	0	4	$7/3$	0	$-2/3$	1	$\frac{4}{7/3} = \frac{12}{7}$
		$Z_j - C_j$	-1 ↑	0	1	0	
x_2	3	$\frac{24}{7}$	0	1	$\frac{9}{21}$	$-1/7$	
x_1	2	$\frac{12}{7}$	1	0	$-2/7$	$\frac{3}{7}$	
		$Z_j - C_j$	0	0	$5/7$	$\frac{3}{7}$	

$R_2' = R_2 - 2R_1$

$3 - \frac{2}{3}$

$= \frac{9-2}{3} = 7/3$

$R_1' = R_1 - \frac{1}{3}R_2$

$4 - \frac{1}{3} \cdot \frac{12}{7} = 4 - \frac{4}{7}$

$= \frac{24}{7}$

$\frac{1}{3} - 1/3 = 0$

$\frac{1}{3} - \frac{1}{3} \times \frac{-2}{7}$

$= \frac{1}{3} + \frac{2}{21}$

$= \frac{7+2}{21}$

$= 9/21$

$0 - \frac{1}{3} \cdot \frac{3}{7} = -\frac{1}{7}$

$\frac{9}{7} - \frac{4}{7} = 5/7$

$-3/7 + \frac{6}{7} = \frac{3}{7}$

In the last table all $Z_j - C_j \ge 0$.
Hence the optimal solution is

$$x_1 = \frac{12}{7}, \quad x_2 = \frac{24}{7}$$

$$Z_{max} = 2 \times \frac{12}{7} + 3 \times \frac{24}{7} = \frac{24}{7} + \frac{72}{7}$$

$$= \frac{96}{7}$$

Scanned by CamScanner