MID SEMESTER EXAMINATION-2018
DISCRETE MATHEMATICAL STRUCTURES
[MA-2003]

Full Marks: 20 | Time: 1.5 Hours

Answer any five questions including question No. 1 which is compulsory.
The figures in the margin indicate full marks.

1. Answer all the following (1x4)

a. Let p: You drive over 65 miles per hour and q: You get a speeding ticket.
Write the following propositions using logical expressions:
(i) You drive over 65 miles per hour, but you do not get a speeding ticket.
(ii) You will get a speeding ticket if you drive over 65 miles per hour.

b. Find the negations of the statement "Some old dogs can learn new tricks."

c. List the ordered pairs in the equivalence relation R produced by the partition $A_1 = \{1,2,3\}$, $A_2 = \{4,5\}$, $A_3 = \{6\}$ of $S = \{1,2,3,4,5,6\}$.

d. What are the sets in the partition of the integers arising from the congruence modulo 3 relation?

2. (2x2)

a. Using method of induction prove that

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{n\times(n+1)}=\frac{n}{n+1}.$$

b. Let $f_0 = 0, f_1 = 1$ and $f_n = f_{n-1} + f_{n-2};\ n \geq 2$. Then using Strong Induction show that $f_n \geq \alpha^{n-2}; n \geq 3$ where $\alpha = \frac{1+\sqrt{5}}{2}$.

3. (2x2)

a. Check the validity of the following Argument.
Everyone who eats granola everyday is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

b. Construct the truth table for $[(p \to q)\land(q \to r)] \longrightarrow (p \to r)$. Is it a tautology?

4. (2x2)

a. If R and S are equivalence relation, show that $R \cap S$ is also an equivalence relation.

b. Let R be a binary relation on the set of all positive integers such that $R = \{(a, b): a - b$ is an odd positive integer $\}$. Check whether R is reflexive? symmetric? Anti symmetric? Transitive?

5. (2x2)

a. How many elements are in the union of four sets if each of the sets have 50, 60, 70 and 80 elements respectively, each pair has 5 elements in common, each triple of the sets has 1 common element and no element in all four sets?

b. Let $R = \{(1,2),(2,1),(2,2),(2,3),(3,1)\}$ and $S = \{(1,2),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ be relations defined on the set $A = \{1,2,3\}$. Then find the relations $R \cup S, R \cap S$, and $S \circ R$.

6. For a given set S, show that $(P(S), \subseteq)$ is a poset, where $P(S)$ is the power set of S. Hence draw the digraph of the $(P(S), \subseteq)$ where $S = \{a, b, c\}$. (4)

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