TUTORIAL PROBLEMS

1. Three similar coils are star connected to a 3-phase, 400 V, 50 Hz supply. If the ind... and resistance of each coil are 38.2 mH and 16 Ω respectively, determine (*i*) line cu... (*ii*) power factor, and (*iii*) power consumed. [(*i*) 11.55 A (*ii*) 0.8 *lag* (*iii*) 6.4...
2. The voltage measured between the terminals of a 3-phase, 3-wire alternator is recorded... 208V. A 3-phase load consisting of three 10-ohm resistors in star is connected to... terminals of the alternator. If one of the resistors should become open-circuited, what w... be the current, the voltage and the power of the remaining resistors ?

   [10.4 A; 104 V; 1081...
3. A 6600 volts three-phase,star-connected alternator supplies 4000 kW at a p.f. of 0.7... ging. Calculate the line current. If the load p.f. is raised to 0.95, the total current rema... the same, find the new output.

   [437.5 A; 47...
4. Three 20 μF capacitors are star-connected across 420 V, 50 Hz, three-phase, three-wire...
   (*i*) Calculate the current in each line.
   (*ii*) If one of the capacitors is short-circuited, calculate the line currents.
   (*iii*) If one of the capacitors is open-circuited, calculate the line currents and p.d. ac... each of the other two capacitors.

   [(*i*) 1.525 A (*ii*) 2.64 A, 2.64 A, 4.57 A (*iii*) 1.32 A, 132 A, 0 A; 21...
5. Three similar coils, connected in star, take a total power of 3 kW at a p.f. of 0.8 lagging fr... a 3-phase, 400 V, 50 Hz supply. Calculate the resistance and reactance of each coil.

   [33.92 Ω ; 25.4...

19.8. DELTA (Δ) OR MESH CONNECTION

In this method of interconnection, the *dissimilar ends* of the three phase windings are join... together, *i.e.* finishing end of one phase is connected to the starting end of the other phase and so... to obtain mesh or delta as shown in Fig. 19.18. The three line conductors are taken from the thr... junctions of the mesh or delta and are designated as *R*, *Y* and *B*. This is called 3- *phase*, 3-wire *delta-connected system.* Since no neutral exists in a Δ-connection, only 3-phase, 3-wire system ca... be formed.

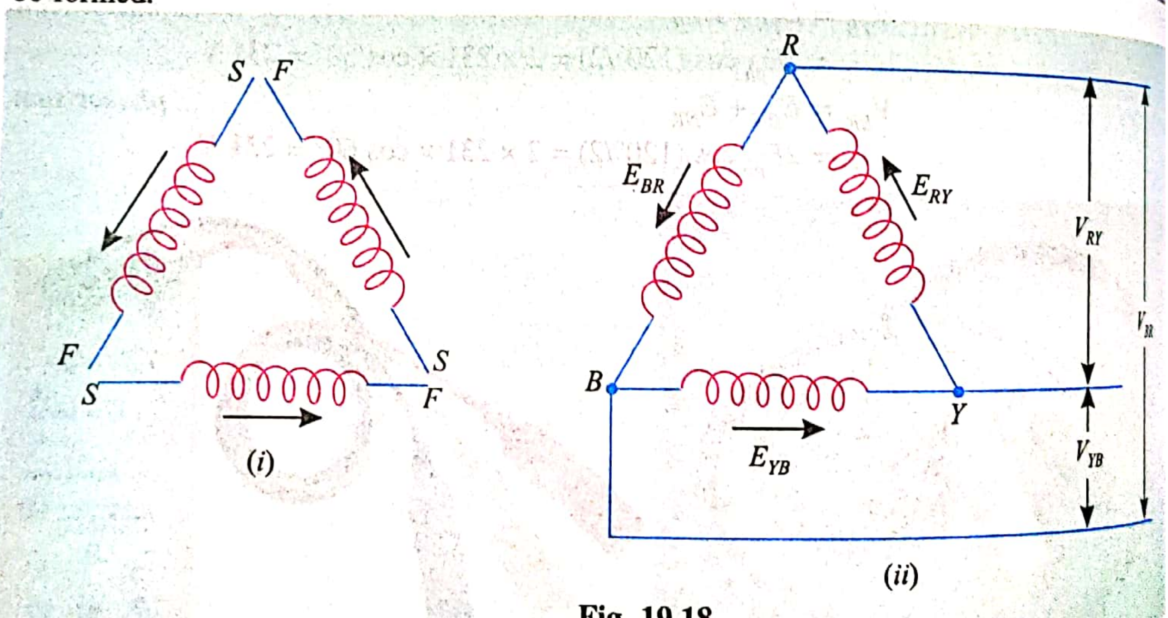

Fig. 19.18

In Fig. 19.18 (*ii*), it may appear that the three phases are short-circuited on themselves. But th... is not the case. The finishing end of one phase is connected to the starting end of the other phase...

that the resultant voltage round the mesh is the phasor sum of the three phase voltages. Since the three phase voltages are equal in magnitude and displaced 120° from one another, their phasor sum is zero (See Art. 19.3). Consequently, no current can flow round the mesh when the terminals are open.

Note. At no instant will all the three line currents flow in the same direction either outwards or inwards. This is expected because the three line currents are displaced 120° from one another. When one is positive, the other two might both be negative or one positive and one negative. Thus, at any one instant, current flows from the alternator through one of the lines to the load and returns through the other two lines. Or else current flows from the alternator through two of the lines and returns by means of the third. It may be noted that arrows placed alongside currents (or voltages) in the diagram indicate the directions of currents (or voltages) when they are assumed to be positive and *not* their actual directions at a particular instant.

19.9. VOLTAGES AND CURRENTS IN BALANCED Δ CONNECTION

We shall now investigate the characteristics of a balanced Δ-connection.

(i) **Line voltage and phase voltage.** Since the system is balanced, the three phase voltages are equal in magnitude (say each equal to V_{ph}, the phase voltage) but displaced 120° from one another. An examination of Fig. 19.19 shows that only one phase winding is included between any pair of lines. Hence in Δ connection, the line voltage is equal to the phase voltage, *i.e.*

$$V_L = V_{ph}$$

Since the phase sequence is *RYB*, the line voltage V_{RY} is 120° ahead of V_{YB} and 240° ahead of V_{BR}. Incidentally, these are also the phase voltages.

(ii) **Line current and phase current.** Since the system is balanced, the three phase currents I_R, I_Y and I_B are equal in magnitude (say each equal to I_{ph}, the phase current) but displaced 120° from one another as shown in the phasor diagram in Fig. 19.20. An examination of the circuit diagram in Fig. 19.19 shows that current in any line is equal to the *phasor difference* of the currents in the two phases attached to that line. Thus :

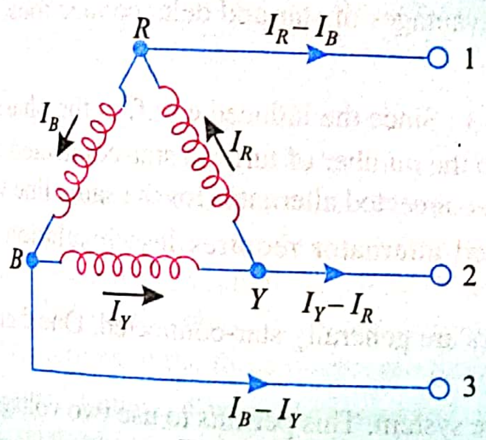

Fig. 19.19

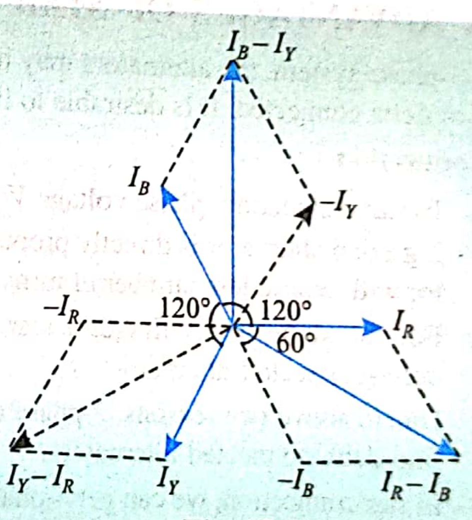

Fig. 19.20

Current in line 1, $\quad I_1{}^* = I_R - I_B$ ... *phasor difference*

Current in line 2, $\quad I_2 = I_Y - I_R$ ... *phasor difference*

Current in line 3, $\quad I_3 = I_B - I_Y$ —do—

The current I_1 in line 1 is the phasor difference of I_R and I_B. To subtract I_B from I_R, reverse the phasor I_B and find its phasor sum with I_R as shown in Fig. 19.20. The two phasors I_R and $-I_B$ are equal in magnitude (= I_{ph}) and are 60°apart.

* Consider the line current I_1 in line 1 connected to the common point *R* of red and blue phase windings. It is clear that I_1 is equal to phasor difference of I_R and I_B since positive direction for I_R is towards point *R* and for I_B it is away from point *R*.

$\therefore$
$$I_1 = 2\,I_{ph}\cos(60°/2) = 2\,I_{ph}\cos 30° = \sqrt{3}\,I_{ph}$$

Similarly,
$$I_2 = I_Y - I_R$$
$$= \sqrt{3}\,I_{ph} \quad \text{... phasor di}$$

and
$$I_3 = I_B - I_Y$$
$$= \sqrt{3}\,I_{ph} \quad \text{... phasor di}$$

The three line currents I_1, I_2 and I_3 are equal in magnitude; each being equal to $\sqrt{3}\,I_{ph}$. Hence in a balanced Δ connection :

(a) Line current, $I_L = \sqrt{3}\,I_{ph}$

(b) All the line currents are equal in magnitude ($= \sqrt{3}\,I_{ph}$) but displaced 120° from one a[nother] as seen from Fig. 19.20.

(c) Line currents are 30° behind the respective phase currents.

(iii) Power

Total power,
$$P = 3 \times \text{Power per phase}$$
$$= 3\,V_{ph} I_{ph} \cos\phi$$

For Δ connection,
$$V_{ph} = V_L;\ I_{ph} = I_L / \sqrt{3}$$

$\therefore$
$$P = 3 \times V_L \times (I_L / \sqrt{3}) \times \cos\phi$$
$$= \sqrt{3}\,V_L I_L \cos\phi$$

where $\cos\phi$ is the power factor of each phase. Either of relations (i) and (ii) can be used to dete[rmine] the power.

19.10. ADVANTAGES OF STAR AND DELTA CONNECTED SYSTEMS

In 3-phase system, the alternators may be star or delta connected. Likewise, 3-phase loads [may] be star or delta connected. It is desirable to list the advantages of star and delta connections.

Star Connection

(i) In star connection, phase voltage $V_{ph} = V_L/\sqrt{3}$. Since the induced e.m.f. in the phase w[ind]ing of an alternator is directly proportional to the number of turns, a star-connected altern[a]tor will require less number of turns than a Δ-connected alternator for the same line volta[ge].

(ii) For the same line voltage, a star-connected alternator requires less insulation th[an] delta-connected alternator.

Due to above two reasons, 3-phase alternators are generally star-connected. One can h[ardly] find delta-connected alternators.

(iii) In star connection, we can get 3-phase, 4-wire system. This permits to use two voltages *v*[iz.] phase voltages as well as line voltages. Remember that in star connection, $V_L = \sqrt{3}\,E_{ph}$. Single phase loads (*e.g.*, lights, etc.) can be connected between any one line and the neu[tral] wire while the 3-phase loads (*e.g.*, 3-phase motors) can be put across the three lines. Su[ch] flexibility is not available in Δ connection.

(iv) In star connection, the neutral point can be earthed. Such a measure offers many advantag[es]. For example, in case of line to earth fault, the insulators have to bear $1/\sqrt{3}$ (*i.e.*, 57.7%) times the line voltage. Moreover, earthing of neutral permits to use protective devices (e.g. relays) to protect the system in case of ground faults.

Delta Connection

(i) This type of connection is most suitable for rotary convertors.

(ii) Most of the 3-phase loads are Δ-connected rather than Y-connected. One reason for this, at least for the case of an unbalanced load, is the flexibility with which loads may be added or removed on a single phase. This is difficult (*or* impossible) to do with a Y-connected 3-wire load.

(iii) Most of 3-phase induction motors are delta-connected.

19.11. HOW TO APPLY LOAD?

A natural question arises how to apply load to a 3-phase star-connected supply? One can apply *single phase loads (*i.e.*, loads connected between any line terminal and neutral wire) if the neutral wire is accessible. If the neutral wire is not accessible, the load shall be, quite logically, a 3-phase load. A ** 3-phase load may be star-connected or delta-connected.

(i) For a 3-phase, 3-wire, star-connected supply (*i.e.*, neutral wire not available), there are only three lines. Hence one can connect a 3-phase load (star or delta load) as shown in Fig. 19.21. Most of the 3-phase loads (*e.g.*, 3-phase motors) are **balanced**, *i.e.* all the three branches have identical impedances—each impedance has same magnitude and power factor. In that case, the three line currents (I_R, I_Y, I_B)are equal in magnitude but 120° apart in phase. Also voltages across the branch impedances are equal in magnitude but 120° apart in phase. Note that problems on balanced loads can be solved by considering one phase only; the conditions in the other two phases being similar. *In this chapter, a 3-phase load means 3-phase balanced load unless stated otherwise.*

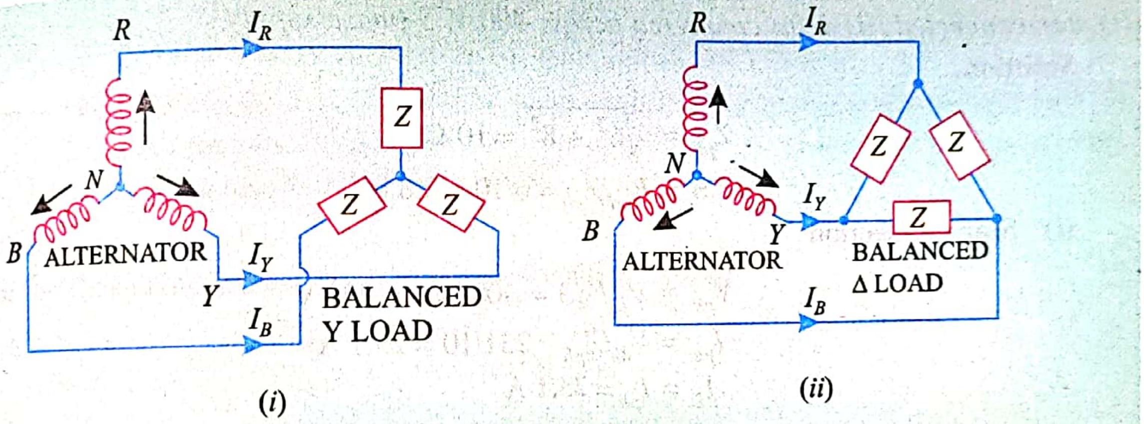

Fig. 19.21

If the branch impedances of the 3-phase load are not identical, it is called *unblanced 3-phase load.* In such a case, line or phase currents are different and are displaced from one another by unequal angle. Problems on unabalanced 3-phase loads are difficult to handle because conditions in the three phases are different.

(ii) For a 3-phase, 4-wire star connected supply, both single phase and 3-phase loads can be connected [See Fig. 19.22]. A single phase load can be connected between any line and the neutral wire while a 3-phase load can be applied across the three lines. The current I_N in the neutral wire will be the phasor sum of the three line currents *i.e.*

$$I_N = I_R + I_Y + I_B \quad \text{...phasor sum}$$

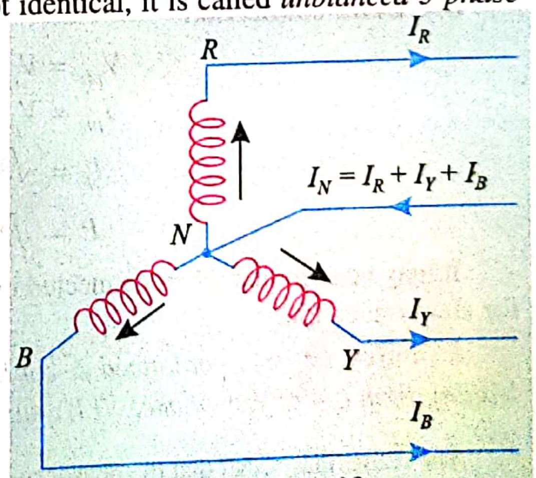

Fig. 19.22

* So called because voltage across the load is that due to one (single) phase only *i.e.* voltage across the load is equal to the phase voltage of the supply.

** Just as the three phases of an alternator can be connected in star or in delta, so the three impedances can also be connected in star or delta. Such a load is called a 3-phase load.

If the loads are balanced (*i.e.*, each of the three phases has the same load), then the three line currents will be equal in magnitude but 120° apart in phase. Consequently, their phasor sum is *zero and the neutral wire carries no current. It may be noted that except as a rare coincidence, 3-phase, 4-wire supply has **unbalanaced loads.

Example19.8. *A balanced 3-phase, Δ -connected load has per phase impedance of (25 + j40) Ω. If 400V, 3-phase supply is connected to this load, find (i) phase current (ii) line current (iii) power supplied to the load.*

Solution. Fig. 19.23 shows the circuit diagram.

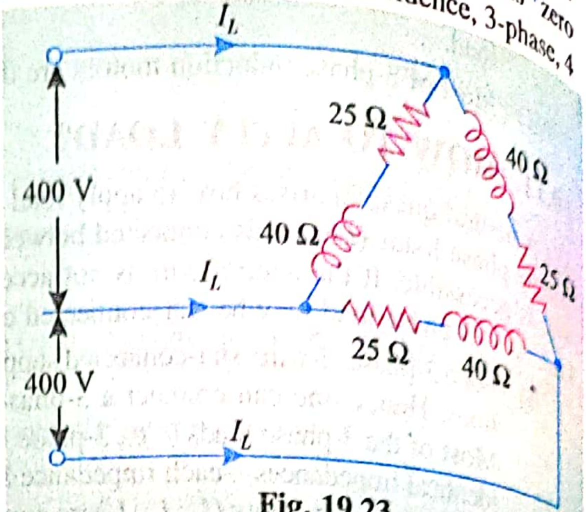

Fig. 19.23

$$Z_{ph} = \sqrt{25^2 + 40^2} = 47.17\ \Omega$$

(i) $I_{ph} = V_{ph}/Z_{ph} = 400/47.17 = \mathbf{8.48\ A}$

(ii) $I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 8.48 = \mathbf{14.7\ A}$

(iii) $\cos\phi = R_{ph}/Z_{ph} = 25/47.17 = 0.53\ lag$

$$P = \sqrt{3}\ V_L\ I_L\ \cos\phi = \sqrt{3} \times 400 \times 14.7 \times 0.53 = \mathbf{5397.76\ W}$$

Example19.9. *A balanced 3-phase load consists of three coils, each of resistance 6 Ω, and inductive reactance of 8 Ω. Determine the line current and power absorbed when the coils are; (i) star-connected (ii) delta-connected across 400 V, 3-phase supply.*

Solution.

$$Z_{ph} = \sqrt{6^2 + 8^2} = 10\ \Omega$$

$$\cos\phi = R_{ph}/Z_{ph} = 6/10 = 0.6 \text{ lag}$$

(*i*) **Star connection**

$$V_{ph} = V_L/\sqrt{3} = 400/\sqrt{3} = 231\ \text{V}$$

$$I_{ph} = V_{ph}/Z_{ph} = 231/10 = 23.1\ \text{A}$$

$$\therefore \quad I_L = I_{ph} = \mathbf{23.1\ A}$$

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 23.1 \times 0.6 = \mathbf{9602.5\ W}$$

(*ii*) **Delta connection**

$$V_{ph} = V_L = 400\ \text{V}$$

$$I_{ph} = V_{ph}/Z_{ph} = 400/10 = 40\ \text{A}$$

$$I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 40 = \mathbf{69.28\ A}$$

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 69.28 \times 0.6 = \mathbf{28799\ W}$$

It may be seen that when connected in Δ, the line current and power drawn are *three times* as that for star connection.

Example 19.10. *A balanced Δ-connected load takes a line current of 18 A at a p.f. of 0.85 leading from a 400 V, 3-phase, 50 Hz supply. Calculate the resistance of each leg of the load.*

* The reader may recall that the phasor sum of the three phasors equal in magnitude but displaced 120° from one another is always zero (See Art. 19.3). The phasors may be line currents or line voltages or phase currents or phase voltages.

** No doubt 3-phase loads (*e.g.*, 3-phase motors) connected to this supply are balanced loads but when we add single phase loads (*e.g.*, lights, fans, *etc.*), the balance is lost. It is because it is rarely possible that single phase loads on all the three phases have the same magnitude and power factor.

Solution.

$$V_{ph} = V_L = 400 \text{ V}; \; I_L = 18 \text{ A}; \; \cos\phi = 0.85 \text{ lead}$$
$$I_{ph} = I_L/\sqrt{3} = 18/\sqrt{3} = 10.39 \text{ A}$$
$$Z_{ph} = V_{ph}/I_{ph} = 400/10.39 = 38.5\ \Omega$$
$$\therefore \quad R_{ph} = Z_{ph}\cos\phi = 38.5 \times 0.85 = \mathbf{32.72\ \Omega}$$

Example 19.11. *Three similar resistors are connected in star across 400V, 3-phase supply. The line current is 5A. Calculate the value of each resistance. To what value should the line voltage be changed to obtain the same line current with the resistors connected in delta ?*

Solution.

Star connection

$$V_{ph} = V_L/\sqrt{3} = 400/\sqrt{3} = 231 \text{ V}$$
$$I_{ph} = I_L = 5 \text{ A}$$
$$\therefore \quad R_{ph} = V_{ph}/I_{ph} = 231/5 = \mathbf{46.2\ \Omega}$$

Delta connection

$$I_L = 5 \text{ A} \qquad \text{... given}$$
$$I_{ph} = I_L/\sqrt{3} = 5/\sqrt{3} = 2.88 \text{ A}$$
$$V_L = V_{ph} = I_{ph}R_{ph} = 2.88 \times 46.2 = \mathbf{133\ V}$$

It may be seen that line voltage required is one-third that of star value.

Example 19.12. *Three 40 Ω non-inductive resistances are connected in delta across 400 V, 3-phase lines. Calculate the power taken from the mains. If one of the resistances is disconnected, what would be the power taken from the mains ?*

Solution.

When the three resistances are delta connected [Fig. 19.24].

$$V_{ph} = V_L = 400 \text{ V}; \; R = 40\ \Omega$$
$$I_{ph} = V_{ph}/R = 400/40 = 10 \text{ A}$$
$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 10 = 17.32 \text{ A}$$
$$P = \sqrt{3}\, V_L I_L \cos\phi$$
$$= \sqrt{3} \times 400 \times 17.32 \times 1 = \mathbf{12000\ W}$$

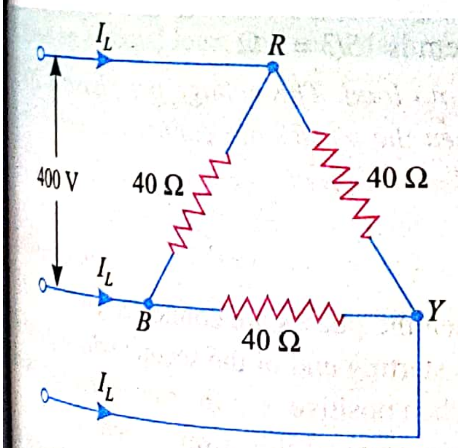

Fig. 19.24

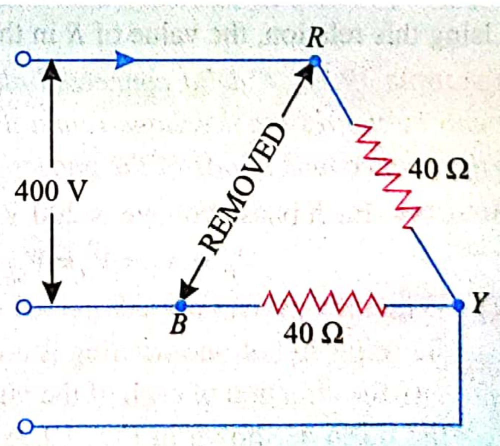

Fig. 19.25

When one resistor is removed. Fig. 19.25 shows the circuit with one resistor removed. In this case, each of the two resistors acts independently as if 400V were applied to 40 Ω.

Current in each resistor, $I = 400/40 = 10$ A

Power consumed in the two $= 2I^2R = 2 \times (10)^2 \times 40 = 8000$ W

Hence by disconnecting one resistor, the power consumption is reduced by one-third.

Example 19.13. *Three identical resistances, each of 15 Ω, are connected in delta across 400 V, 3-phase supply. What value of resistance in each leg of balanced star-connected load would take the same line current ?*

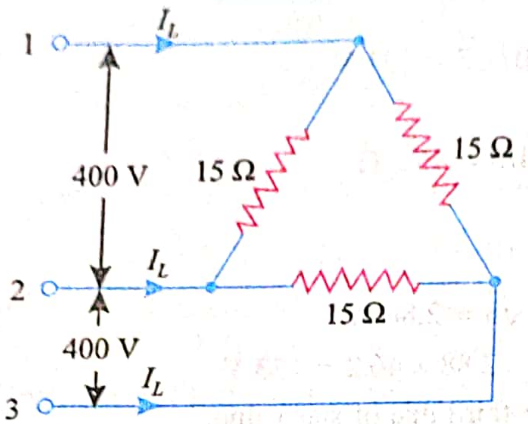

Fig. 19.26

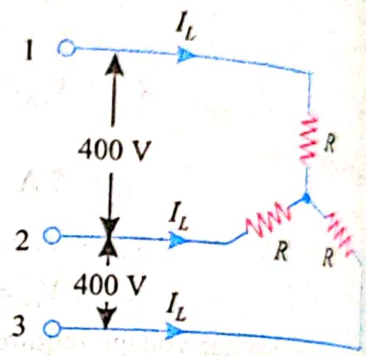

Fig. 19.27

Solution. Let R ohms be the required resistance in each leg of the star-connection (See Fig. 19.27). Since the two circuits have the same line voltage and line current, the resistance between any two corresponding terminals of the two circuits is the same. Considering the terminals 1 and 3,

For delta connection, $\quad R_{13} = 15 \parallel (15 + 15) = \dfrac{15 \times 30}{15 + 30} = 10\ \Omega$

For star connection, $\quad R_{13} = R + R = 2R$

$\therefore \quad 2R = 10 \quad$ or $\quad R = 10/2 = 5\ \Omega$

Note. Thus, Δ-connected impedances can be replaced by the equivalent Y-connected impedances using the following relation:

$$Z_Y = Z_\Delta/3$$

Using this relation, the value of R in the above problem is $15/3 = 5\ \Omega$.