2.5 Independent Events and mutually independent events

The events A & B are independent

If $P(A\cap B) = P(A)\,P(B)$

i.e., prob. of A does not effect on the prob. of B & vice versa.

→ If A & B are independent, then

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)\,P(B)}{P(B)} = P(A)$$

and $P(B|A) = P(B)$

→ A & B are independent if $P(A|B) = P(A)$

→ If A & B are independent then A' & B' are independent

→ In general, the events $A_1, A_2, \ldots A_n$ are independent if

$$P(A_1\cap A_2\cap \cdots \cap A_k) = P(A_1)\,P(A_2)\cdots P(A_n)$$

for $k = 2, 3, \ldots n$

$$P(A_{i_1}\cap A_{i_2}\cap \cdots \cap A_{i_k}) = P(A_{i_1})\,P(A_{i_2})\cdots P(A_{i_k})$$

for any subset of indices $i_1, i_2, \ldots i_k$

i.e., P(intersection of the events) = product of prob. of events

Q. Prove that for any three events A, B & C

$$P(A \cap B \cap C) = P(A)\,P(B|A)\,P(C|A \cap B)$$

Proof

$$P(A \cap B \cap C) = P(C|A \cap B)\,P(A \cap B)$$

$$= P(C|A \cap B)\,P(B|A)\,P(A)$$

proved

Note: If A, B & C are independent then

$$P(B|A) = P(B)$$

$$P(C|A \cap C) = P(C)$$

$$\therefore P(A \cap B \cap C) = P(C)\,P(B)\,P(A)$$

Q. If A & B are mutually exclusive events and are independent with the event C then prove that $P(C|A \cup B) = P(C)$

Proof A & B are mutually exclusive, so

$$P(A \cup B) = P(A) + P(B)$$

A & B are mutually independent with C

$$\therefore P(A \cap C) = P(A)\,P(C)$$

$$P(B \cap C) = P(B)\,P(C)$$

Hence

$$\therefore P(C|A \cup B) = \frac{P(C \cap (A \cup B))}{P(A \cup B)}$$

$$= \frac{P[(C \cap A) \cup (C \cap B)]}{P(A) + P(B)} = \frac{P(C \cap A) + P(C \cap B)}{P(A) + P(B)}$$

$$= \frac{P(C)P(A) + P(C)P(B)}{P(A) + P(B)}$$

$$= \frac{P(C)[P(A) + P(B)]}{P(A) + P(B)}$$

$$= P(C) \text{ (proved)}$$