

Deterministic Finite Automata

And Regular Languages

Finite Automata

The *finite state machine* or *finite automaton* is the simplest computational model of limited memory computers.

Example: an automatic door opener at a supermarket decides when to open or close the door, depending on the input provided by its sensors.

Example: Door Opener

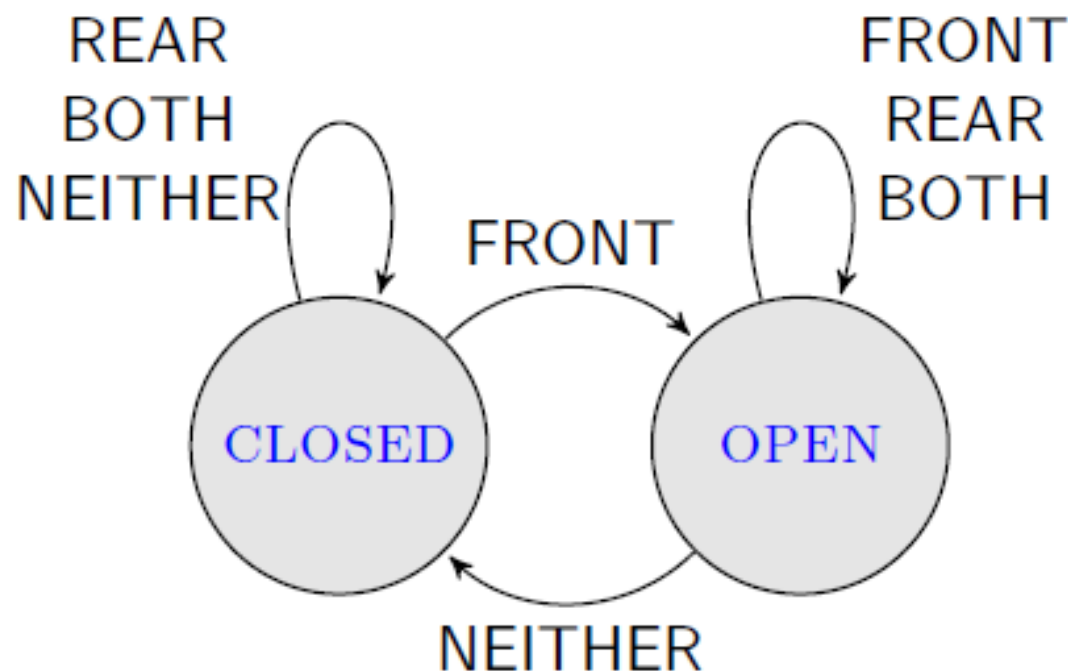
states: closed, open

input conditions: front, rear, both, neither

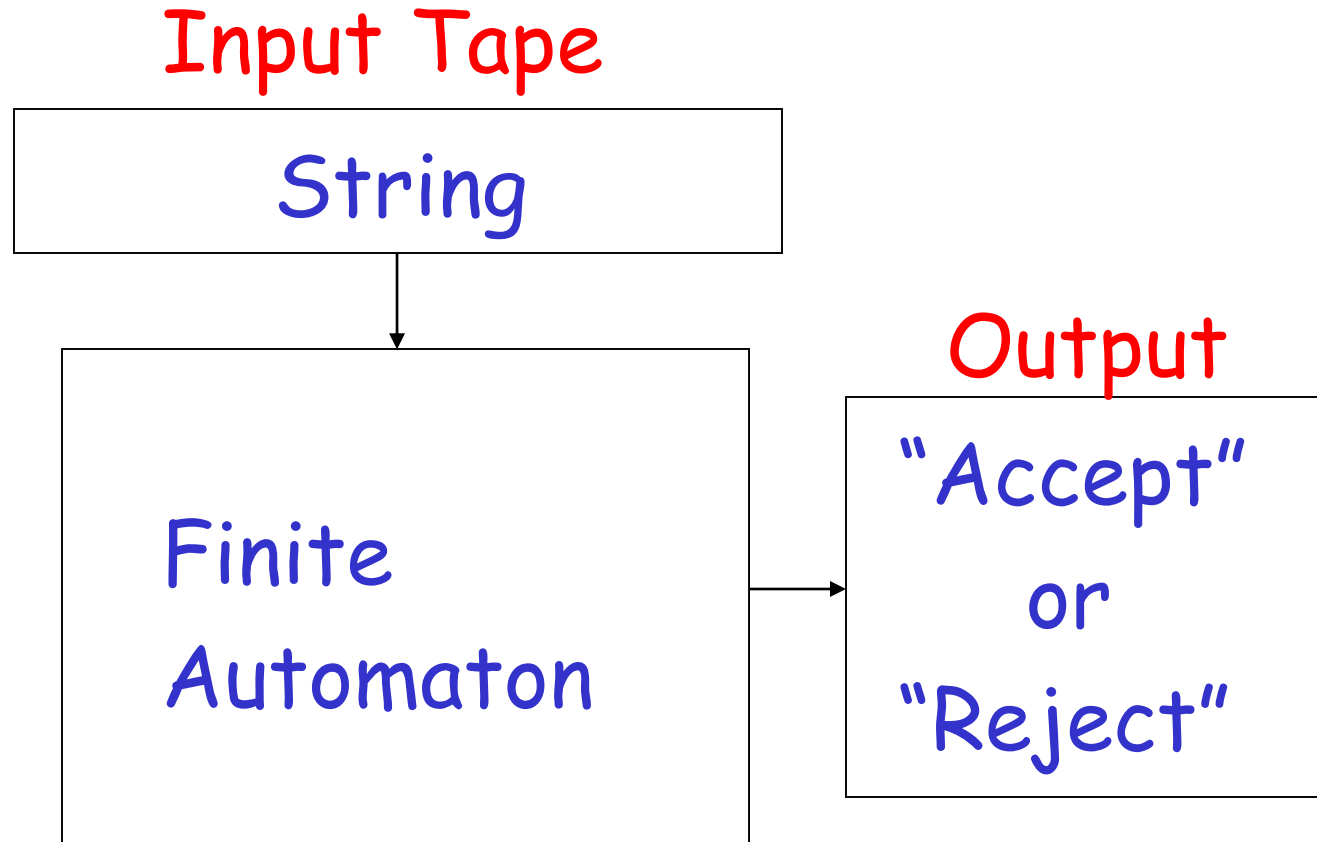
nonloop transitions:

closed \rightarrow open on front

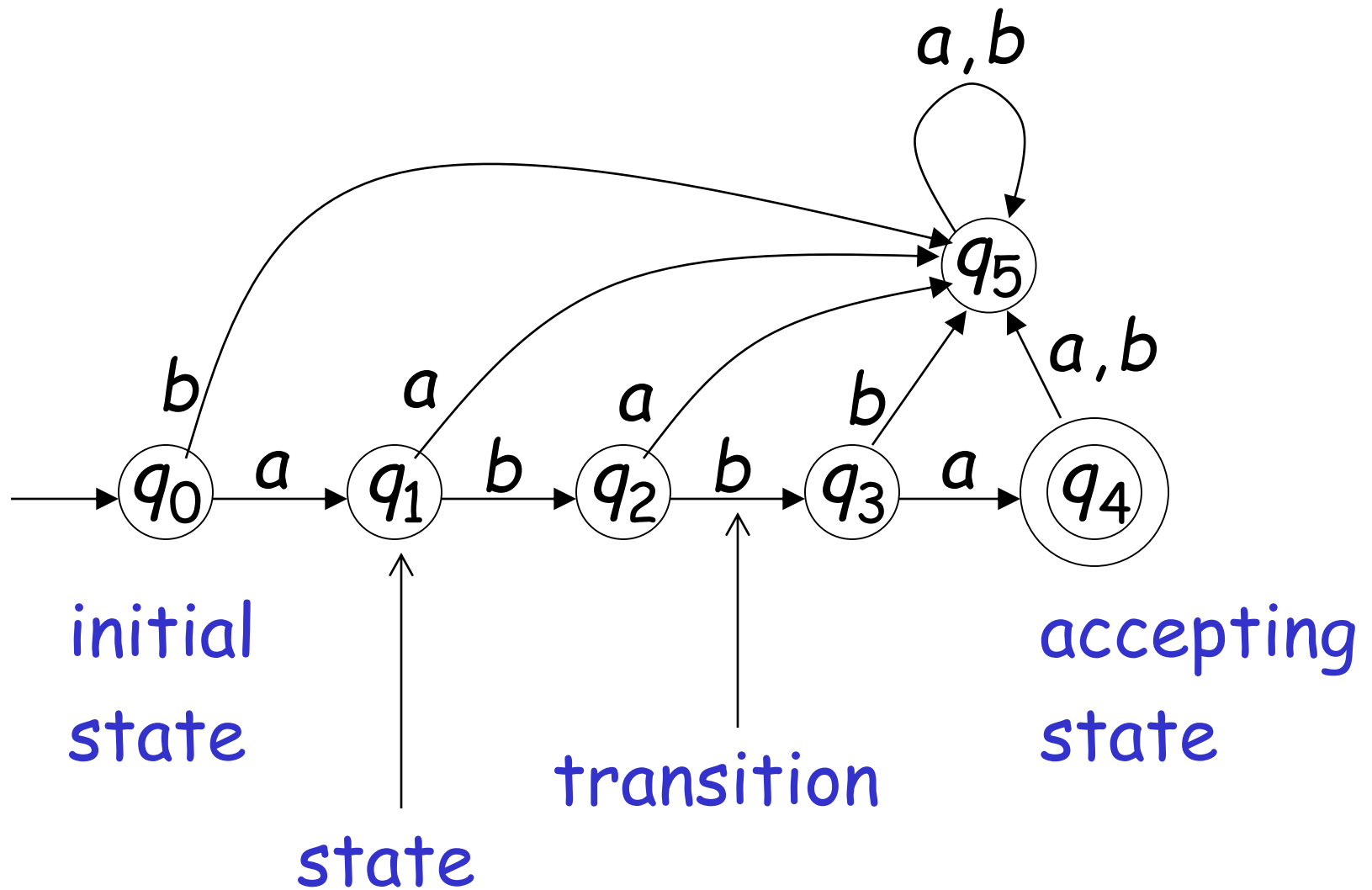
open \rightarrow closed on neither



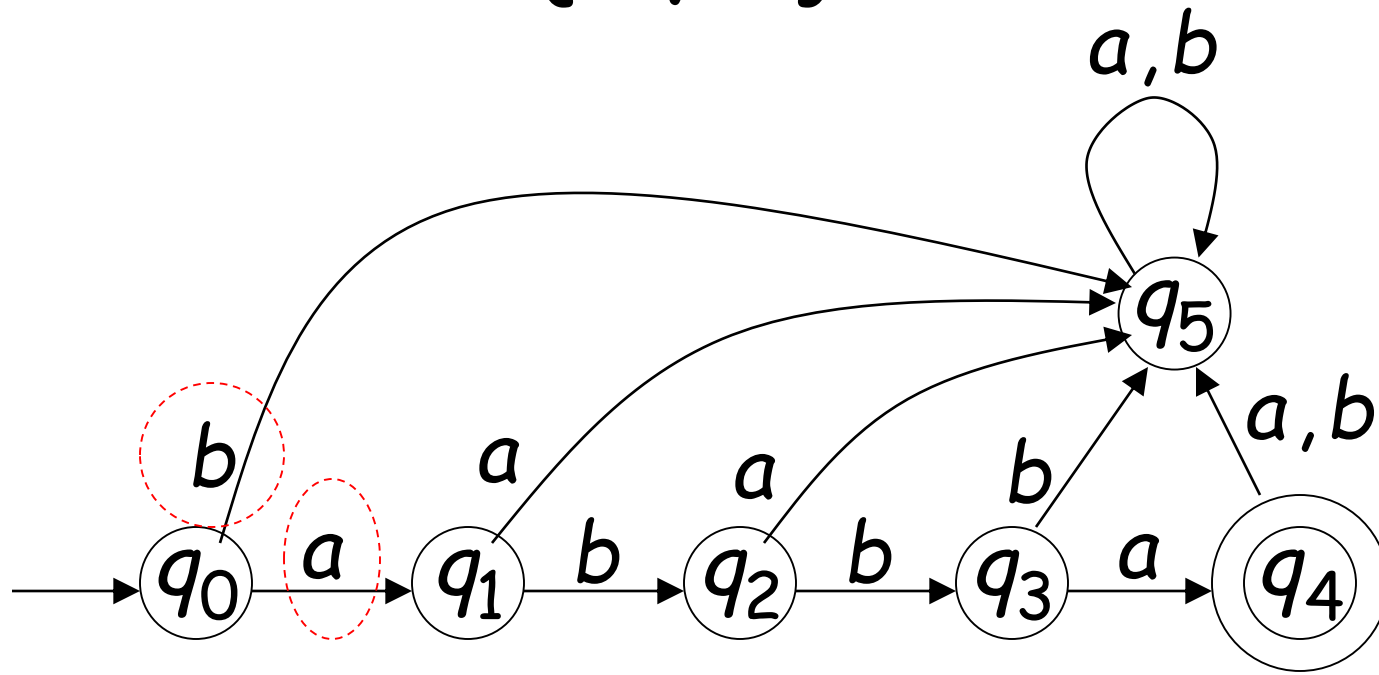
Deterministic Finite Automaton (DFA)



Transition Graph



Alphabet $\Sigma = \{a, b\}$



For every state, there is a transition for every symbol in the alphabet

Initial Configuration

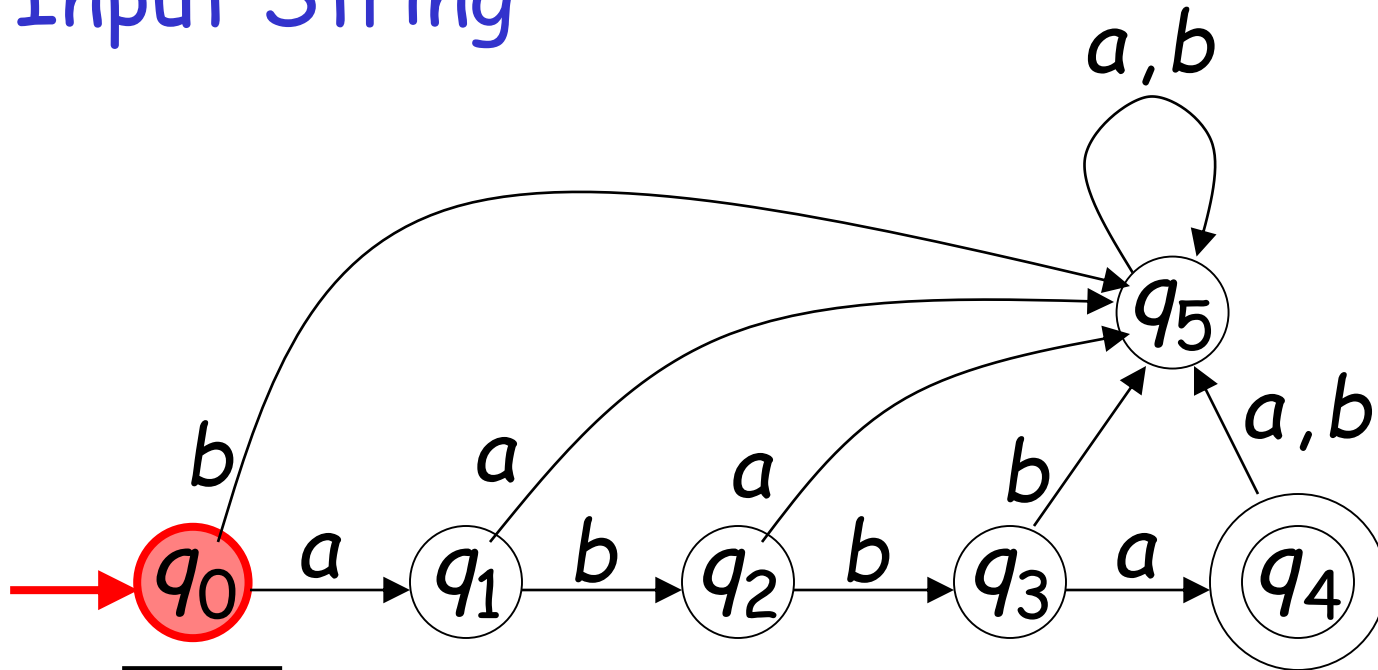
head



Input Tape

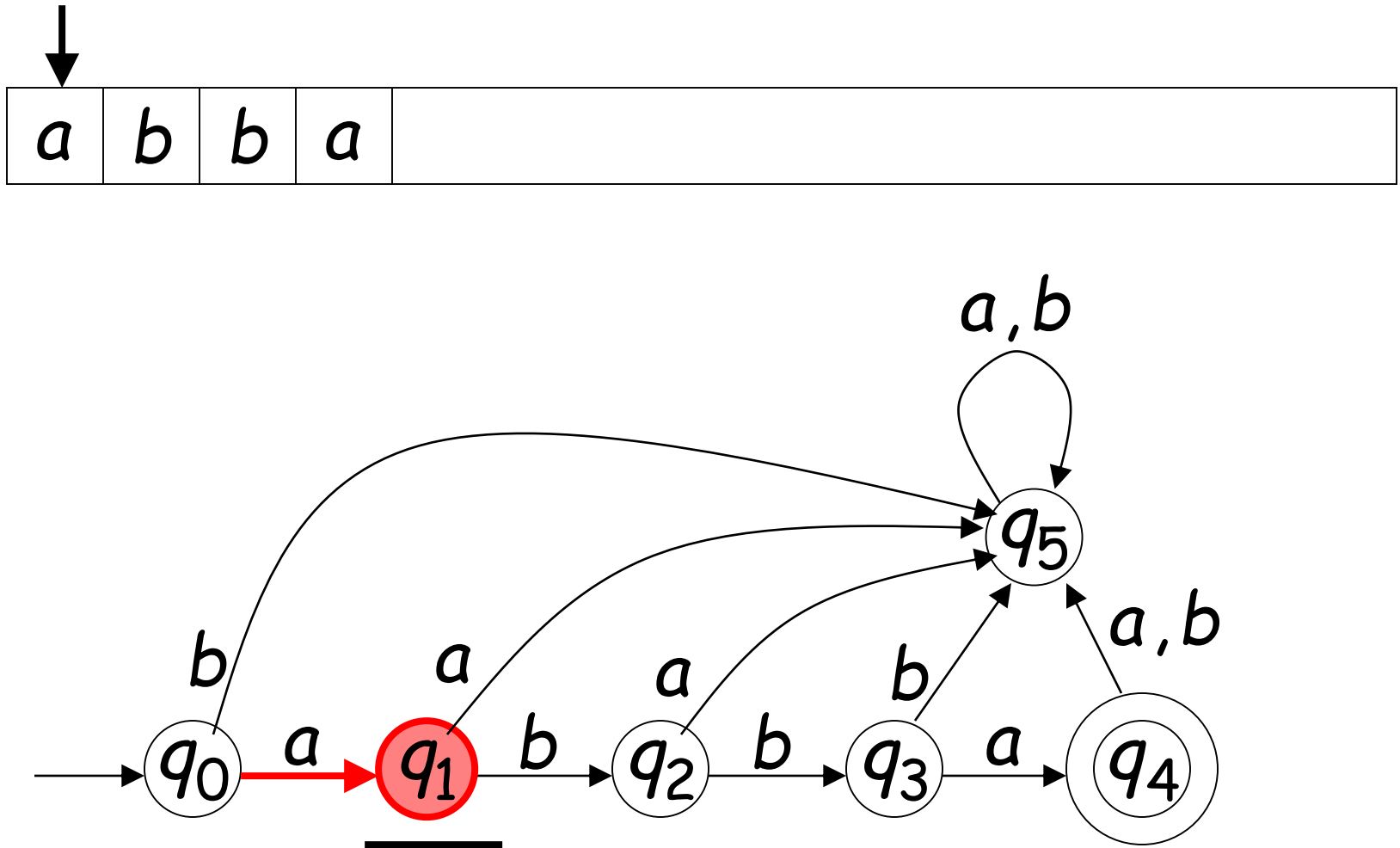


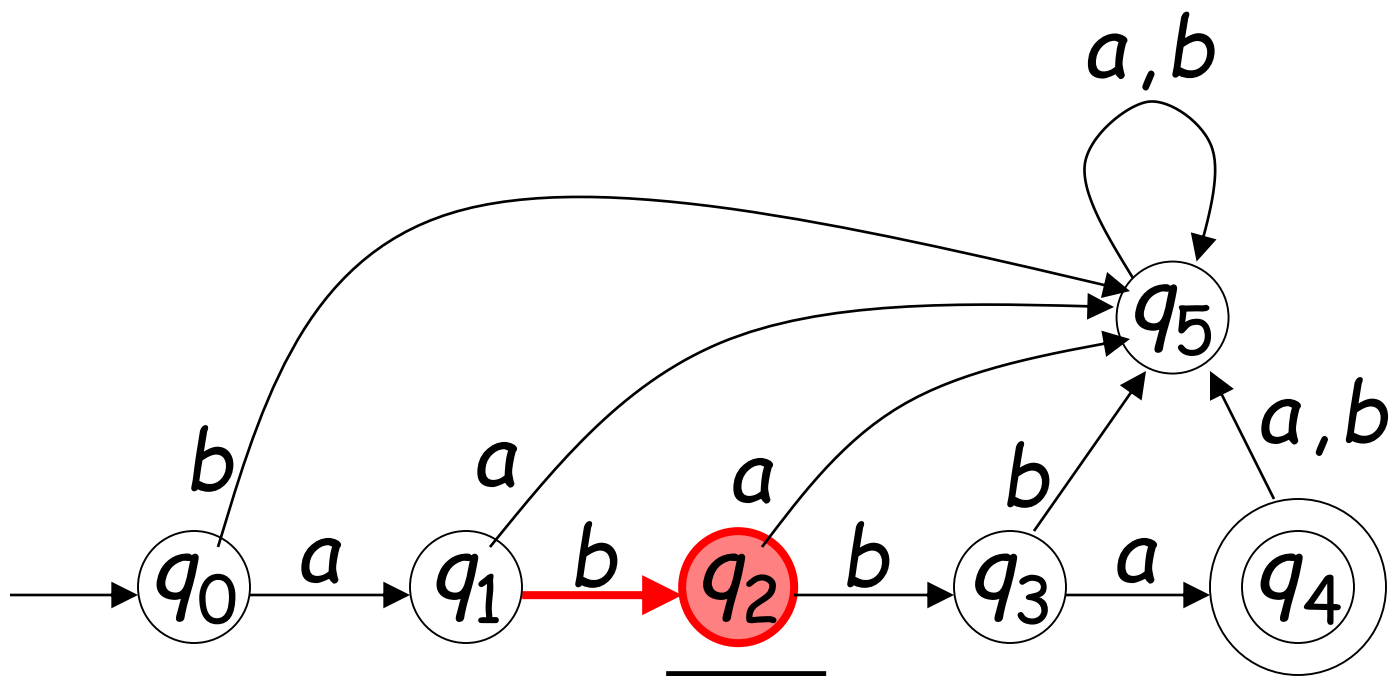
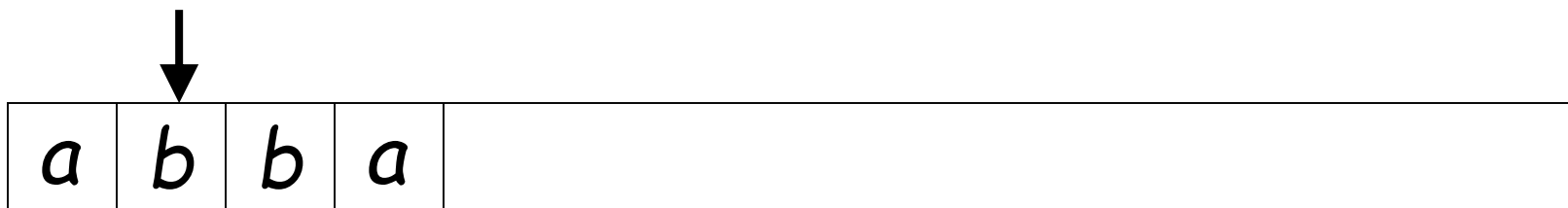
Input String

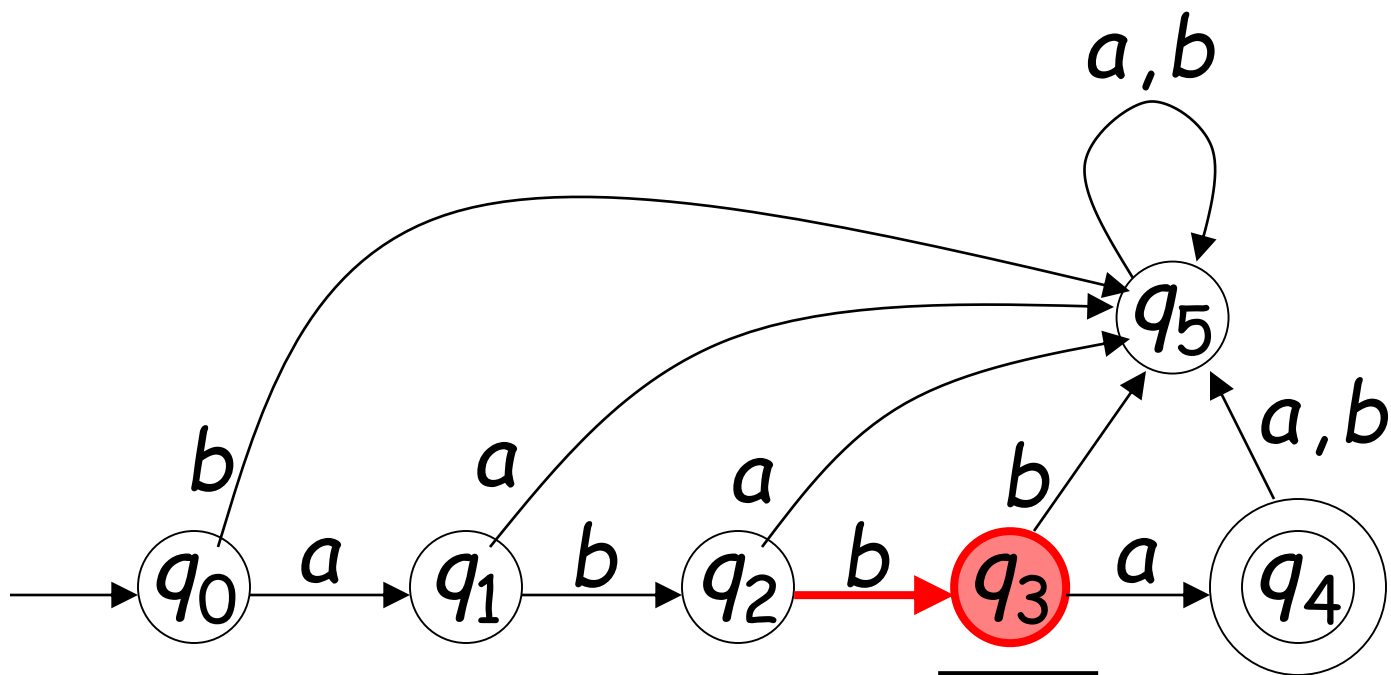
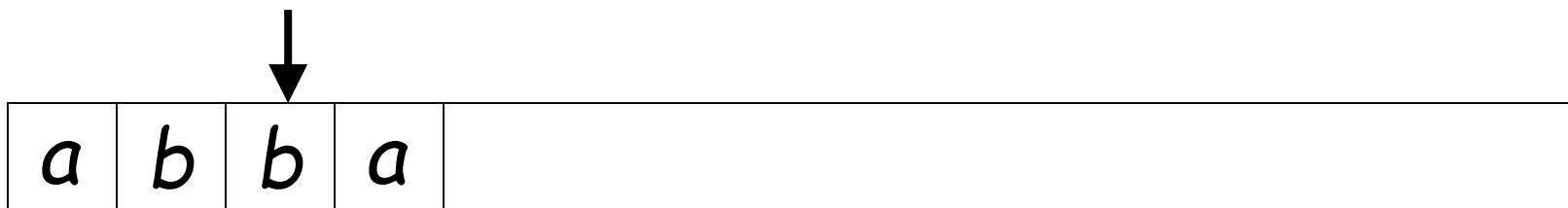


Initial state

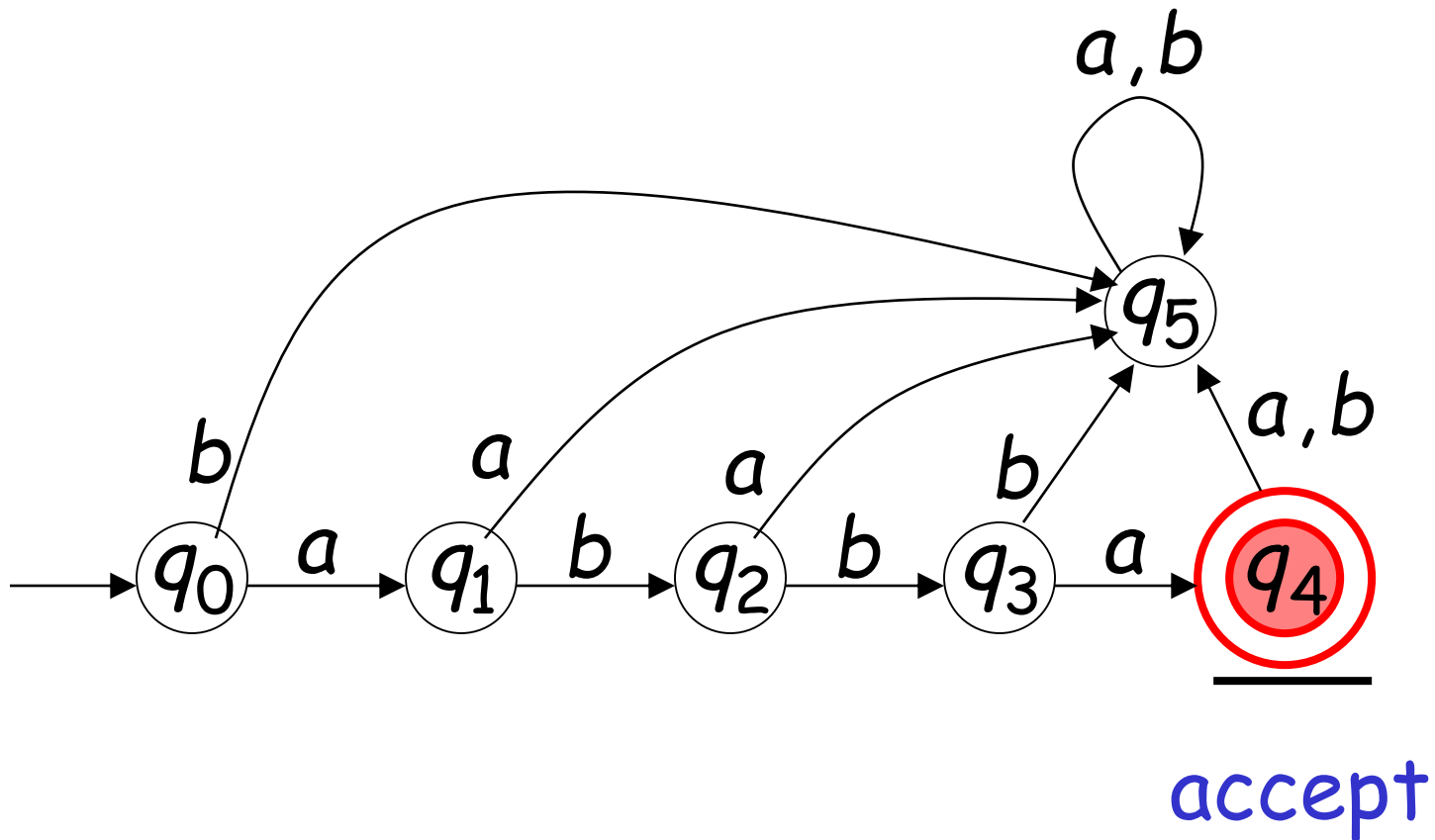
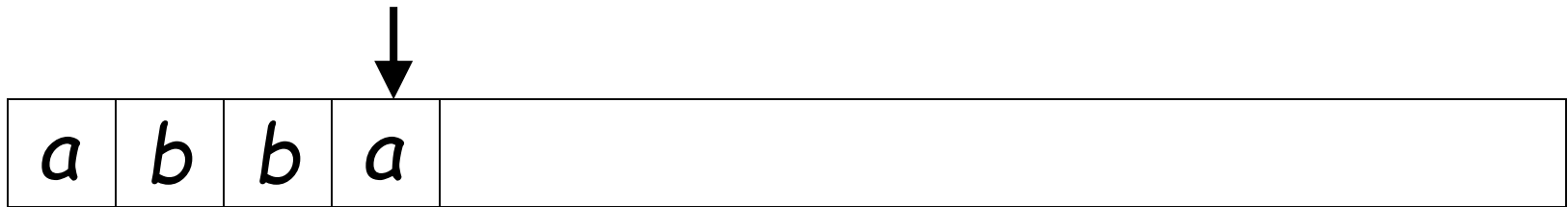
Scanning the Input







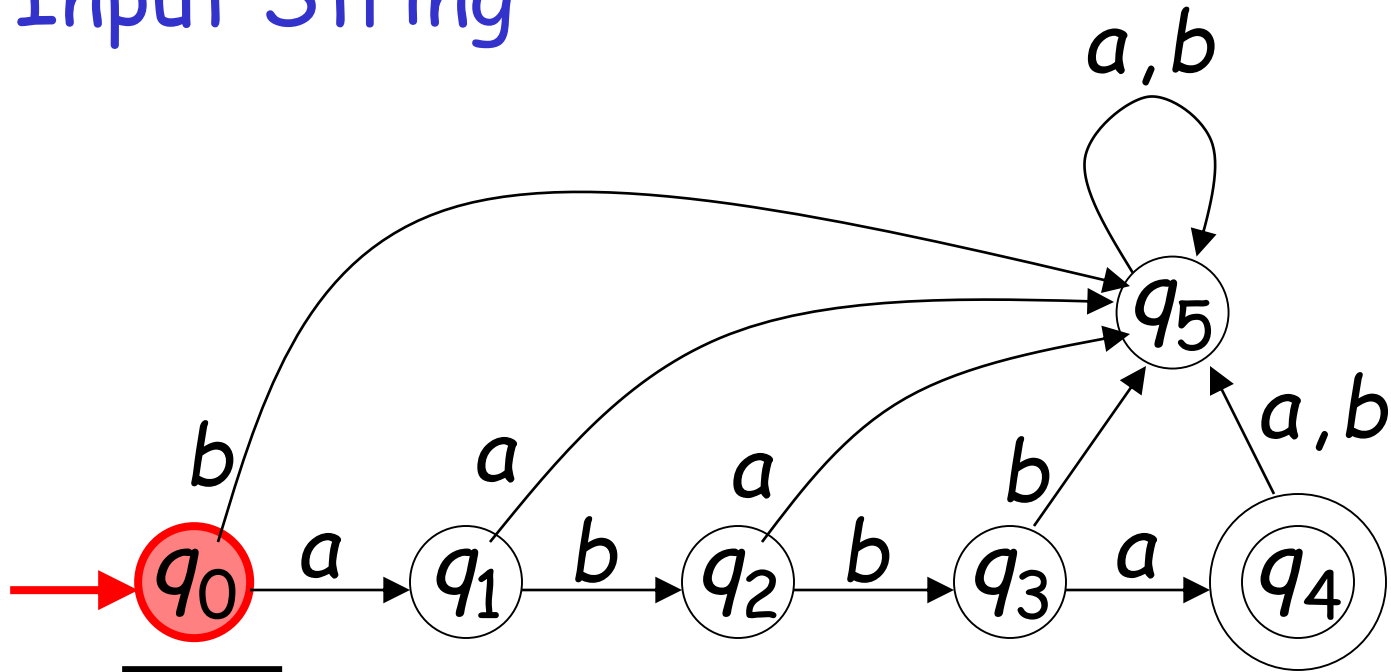
Input finished

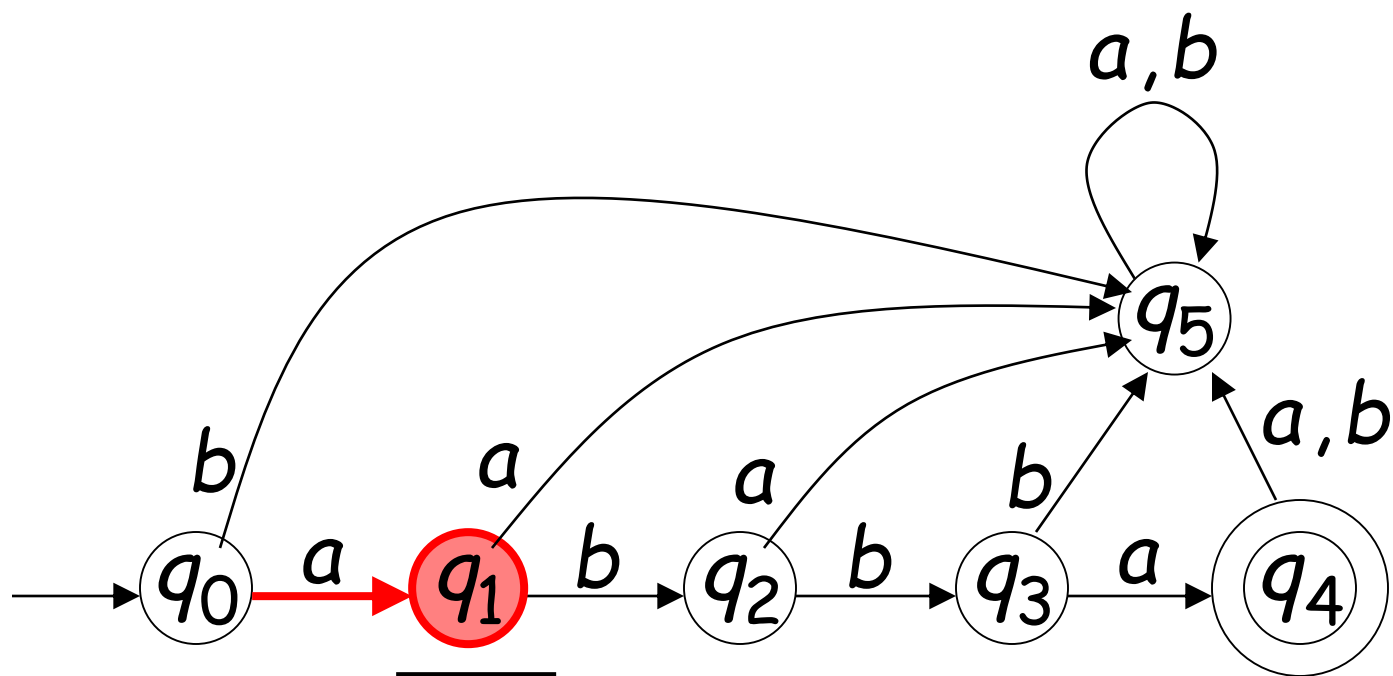
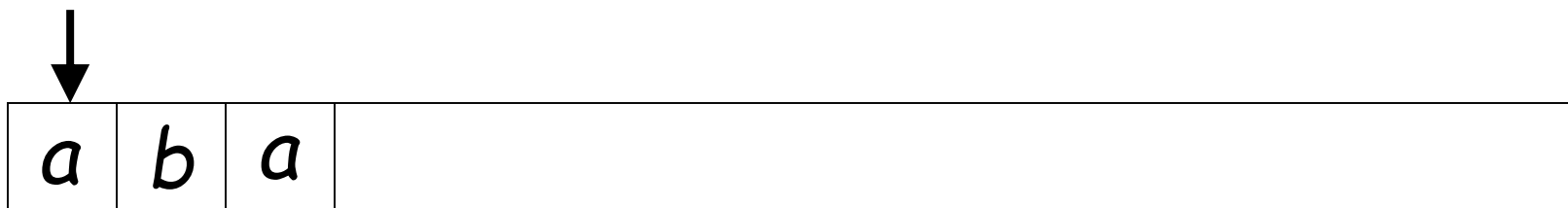


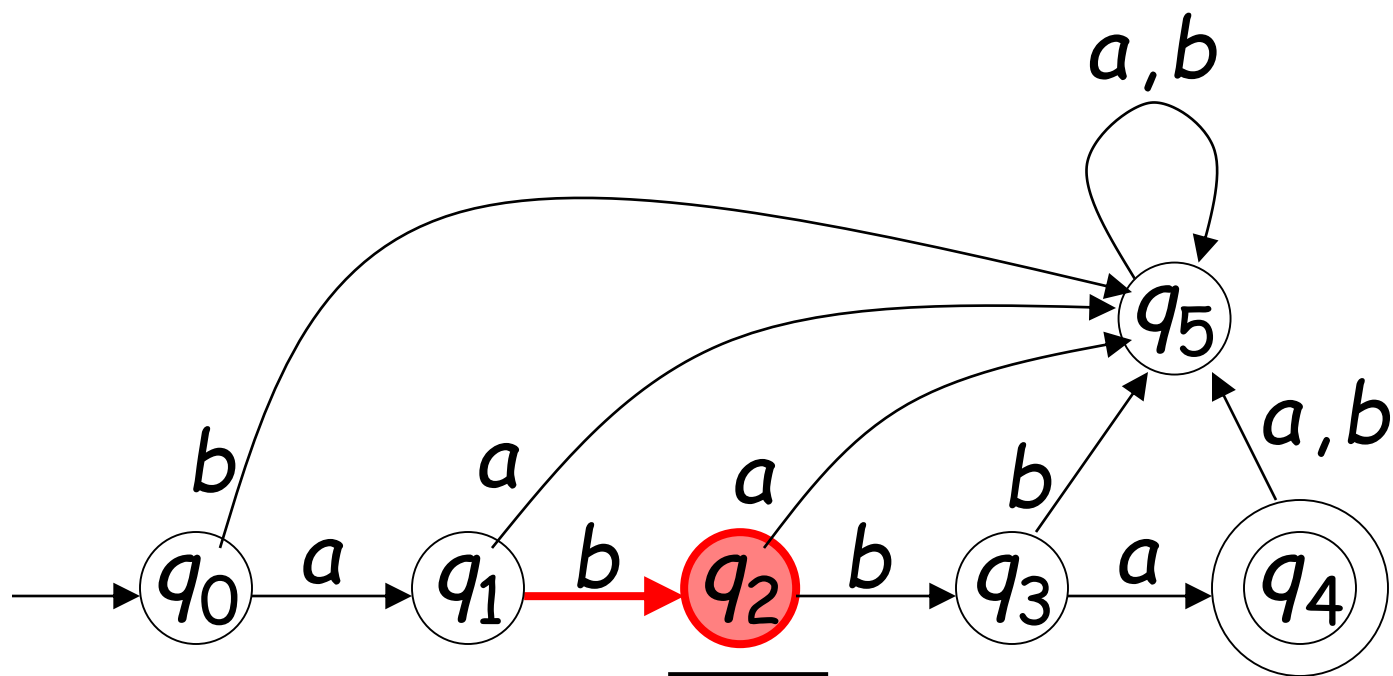
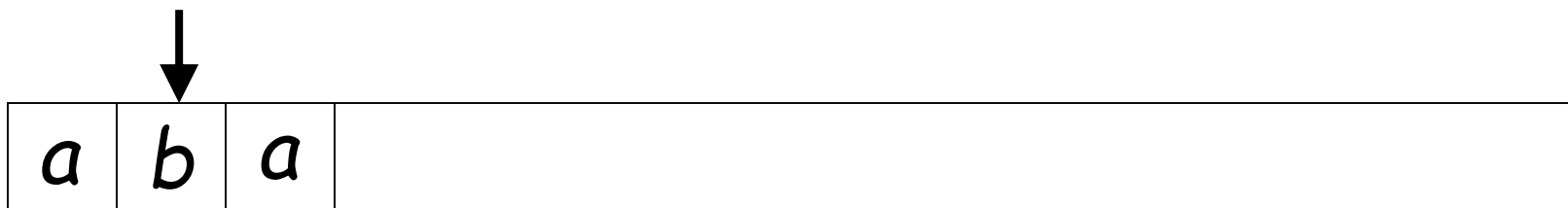
A Rejection Case



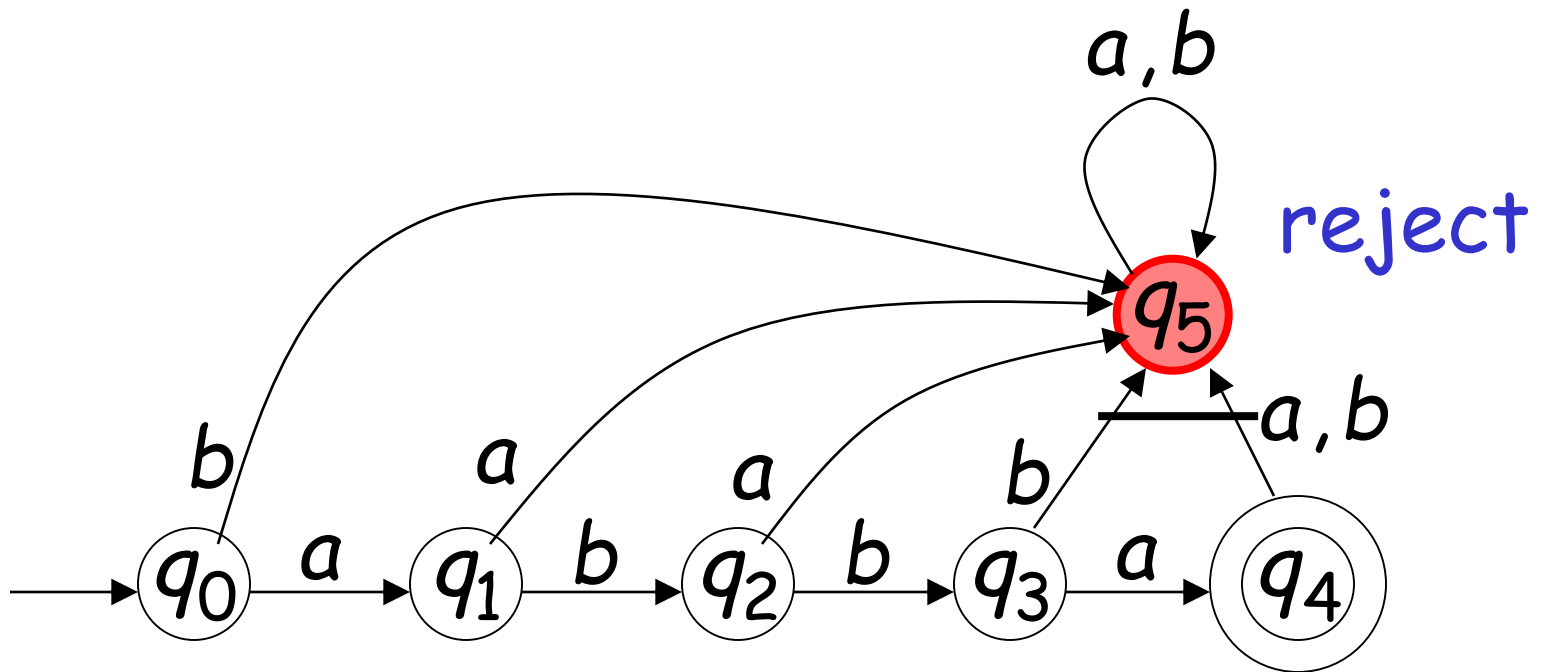
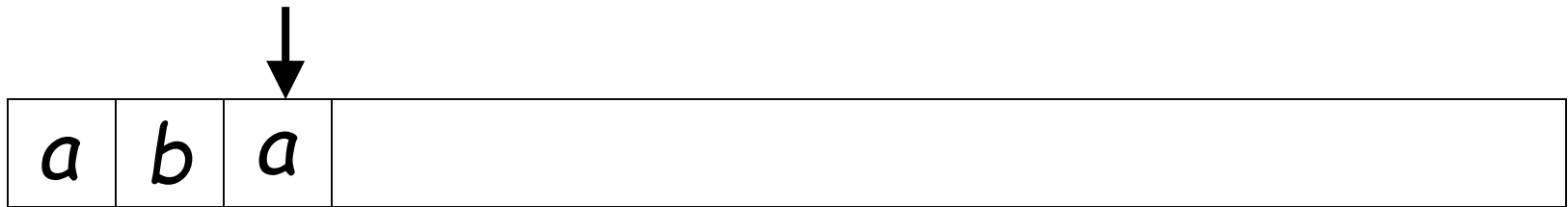
Input String







Input finished



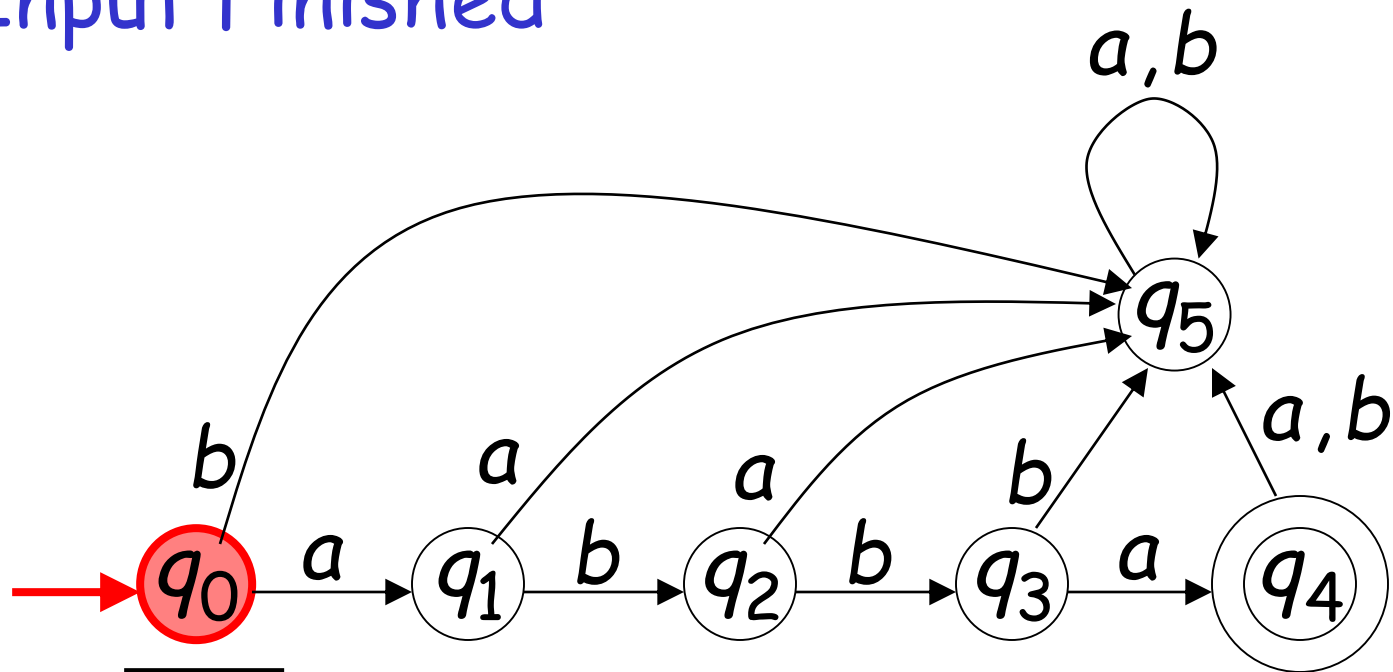
Another Rejection Case



Tape is empty

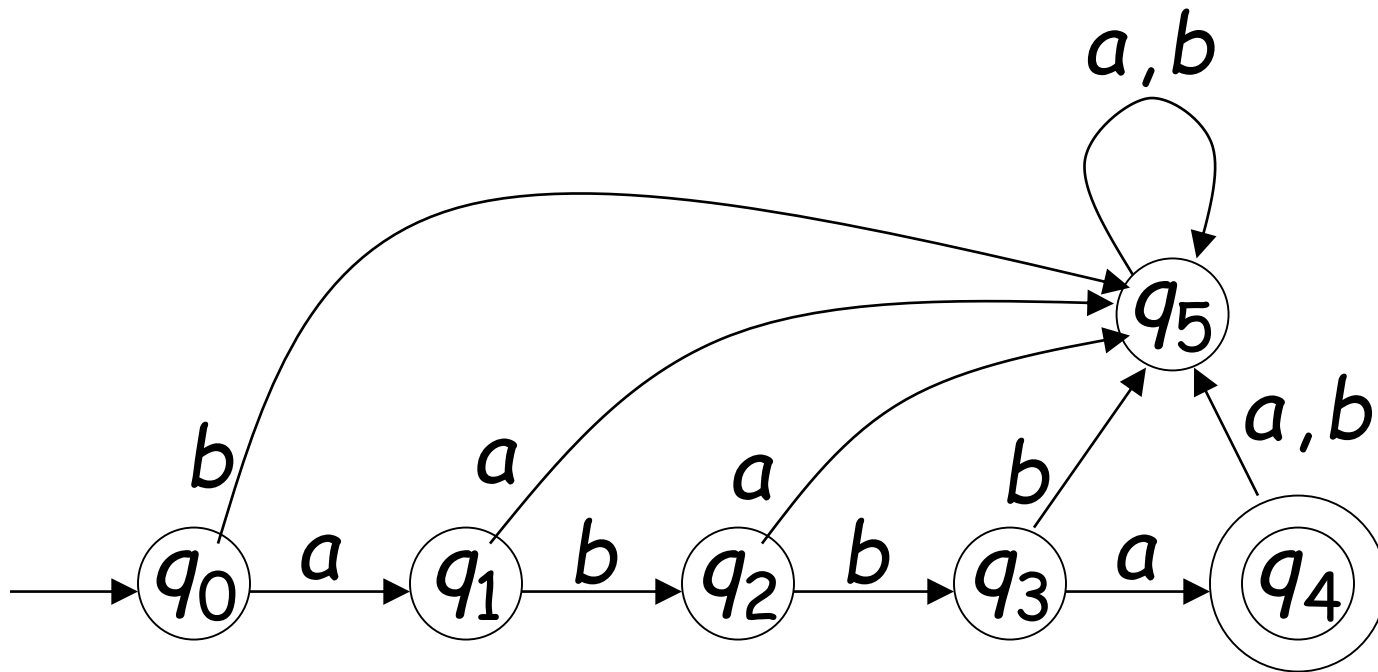
(λ)

Input Finished



reject

Language Accepted: $L = \{abba\}$



To accept a string:

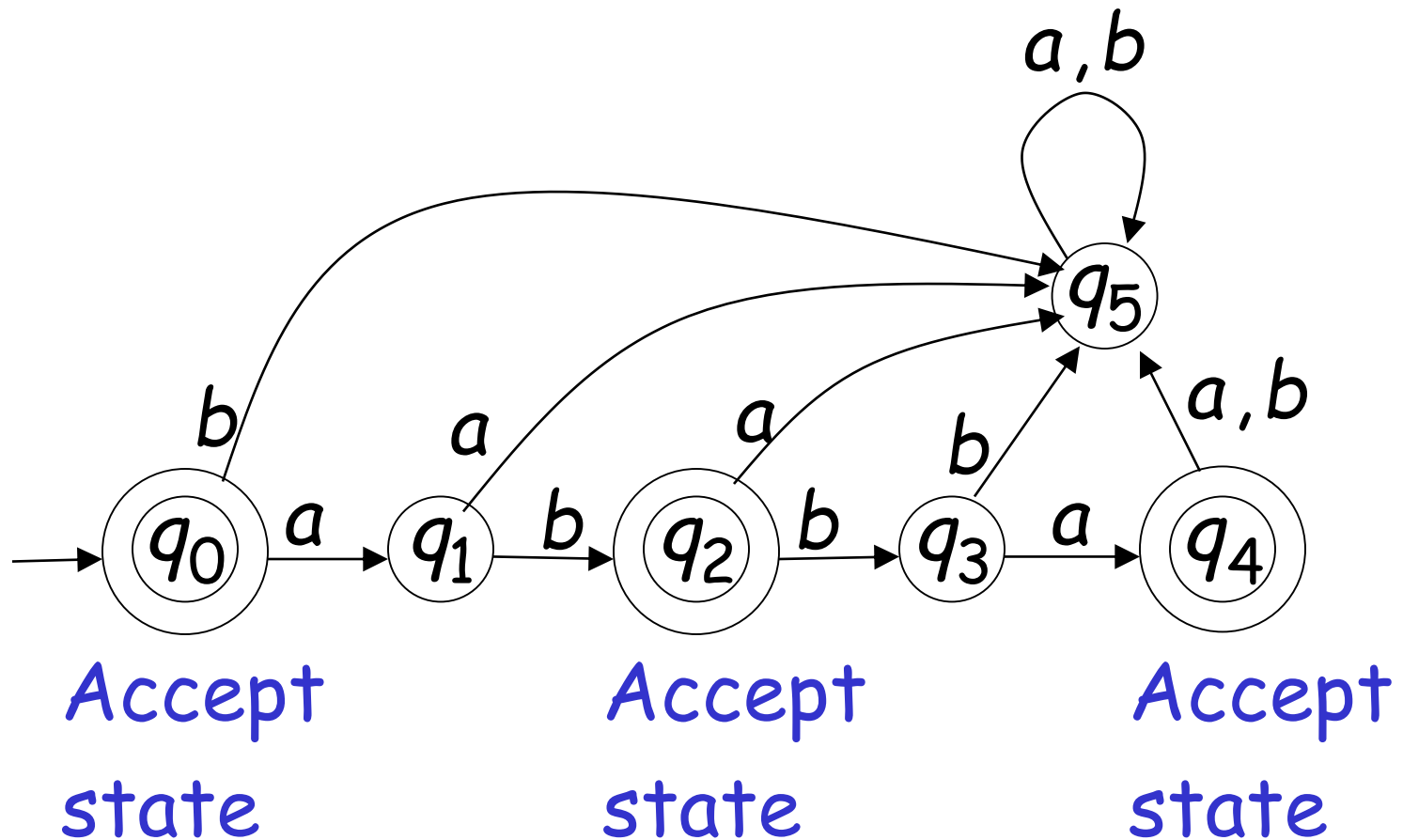
all the input string is scanned
and the last state is accepting

To reject a string:

all the input string is scanned
and the last state is non-accepting

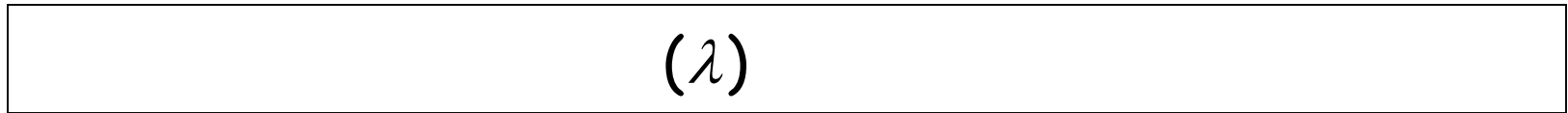
Another Example

$$L = \{\lambda, ab, abba\}$$

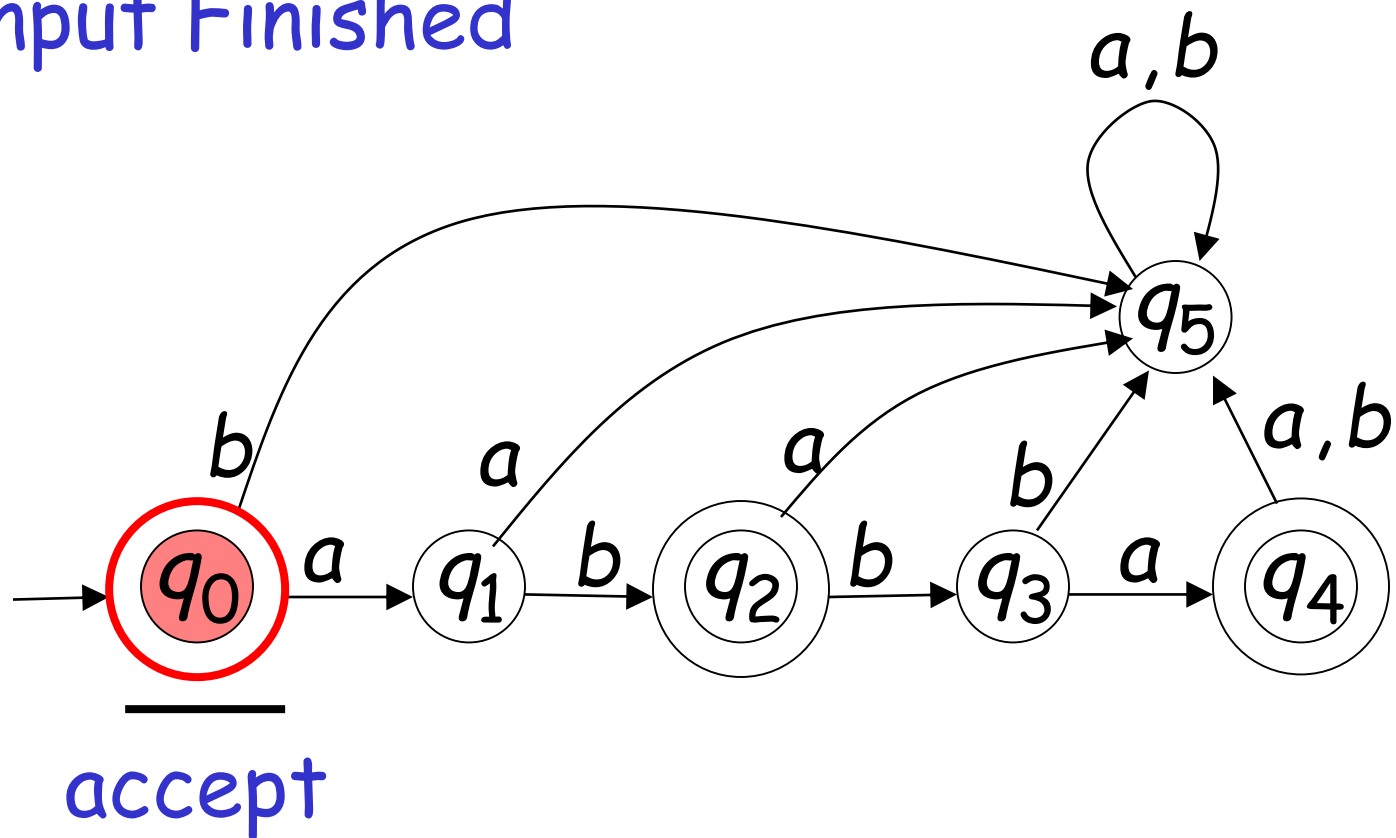




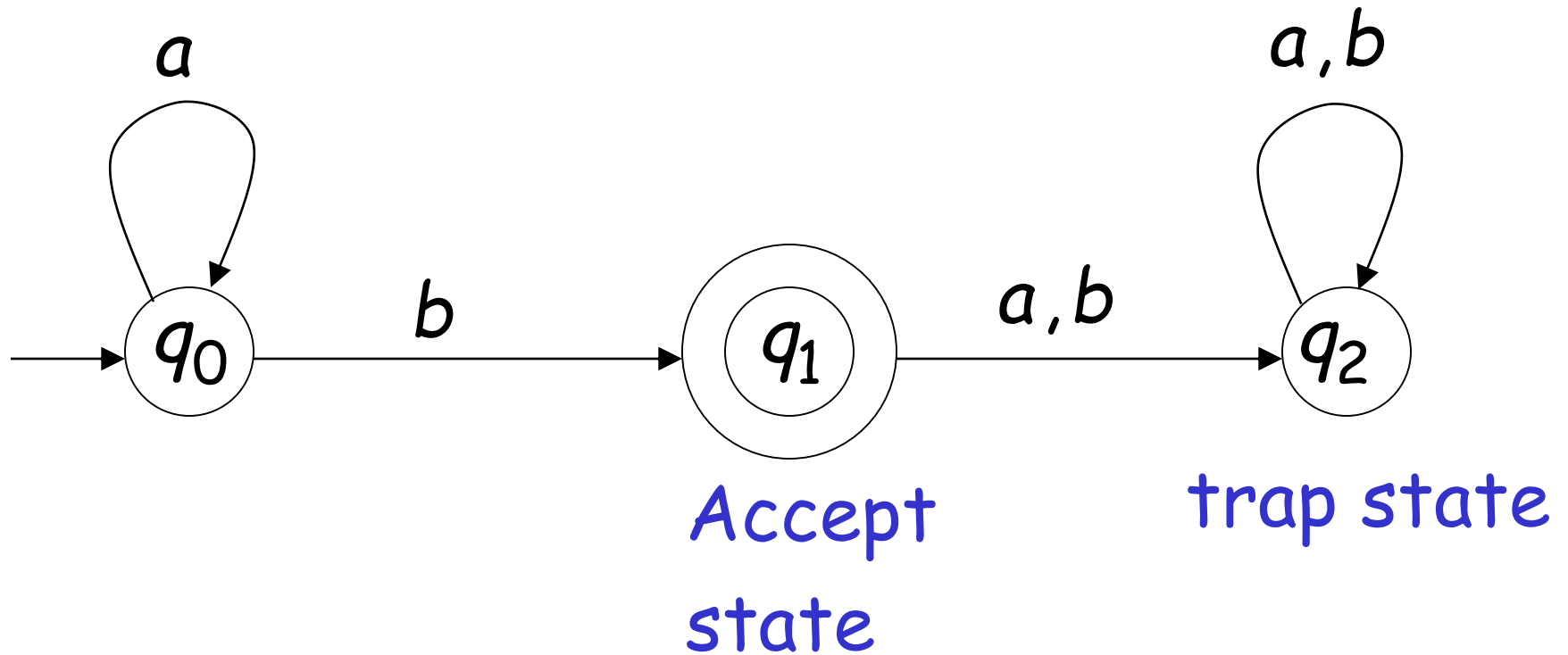
Empty Tape

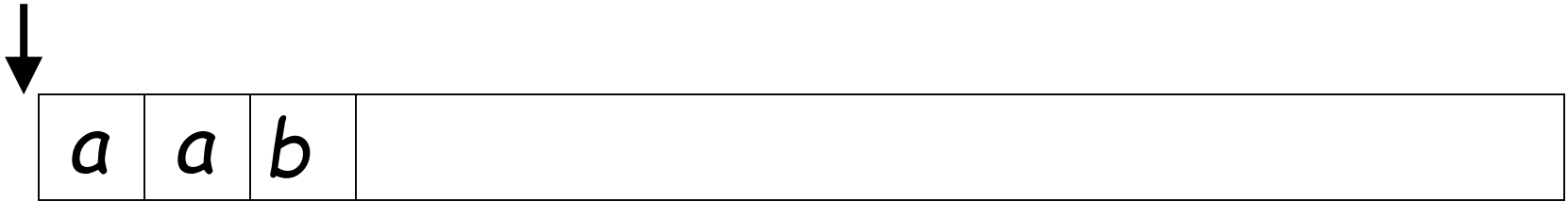


Input Finished

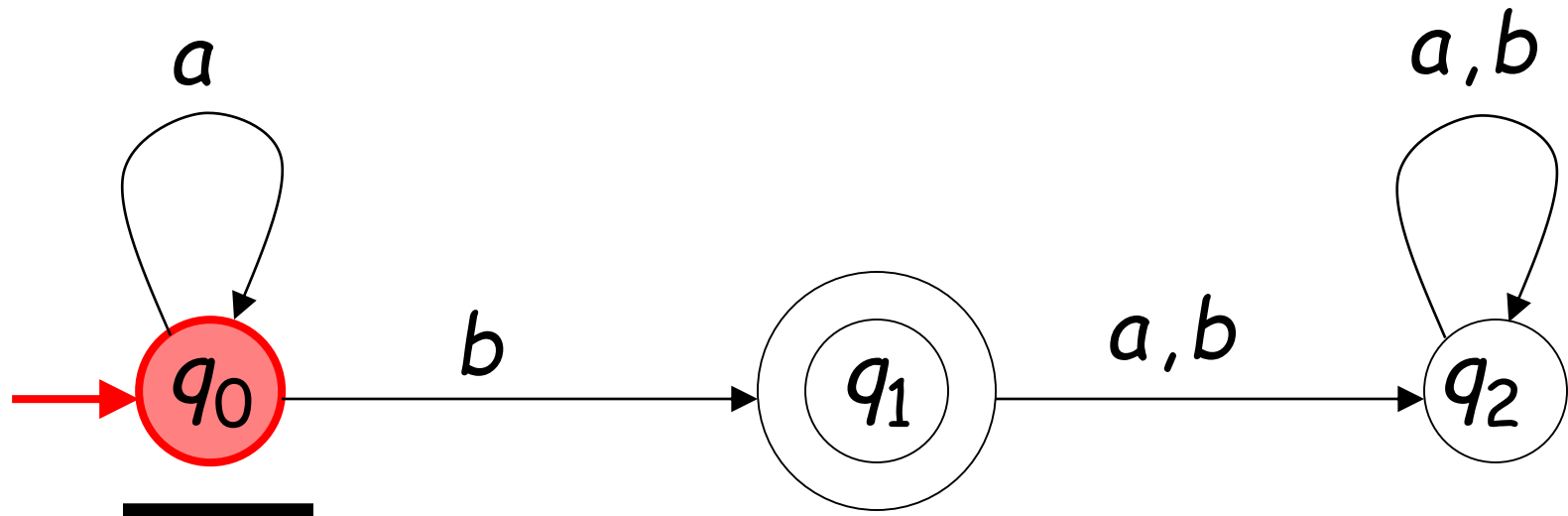


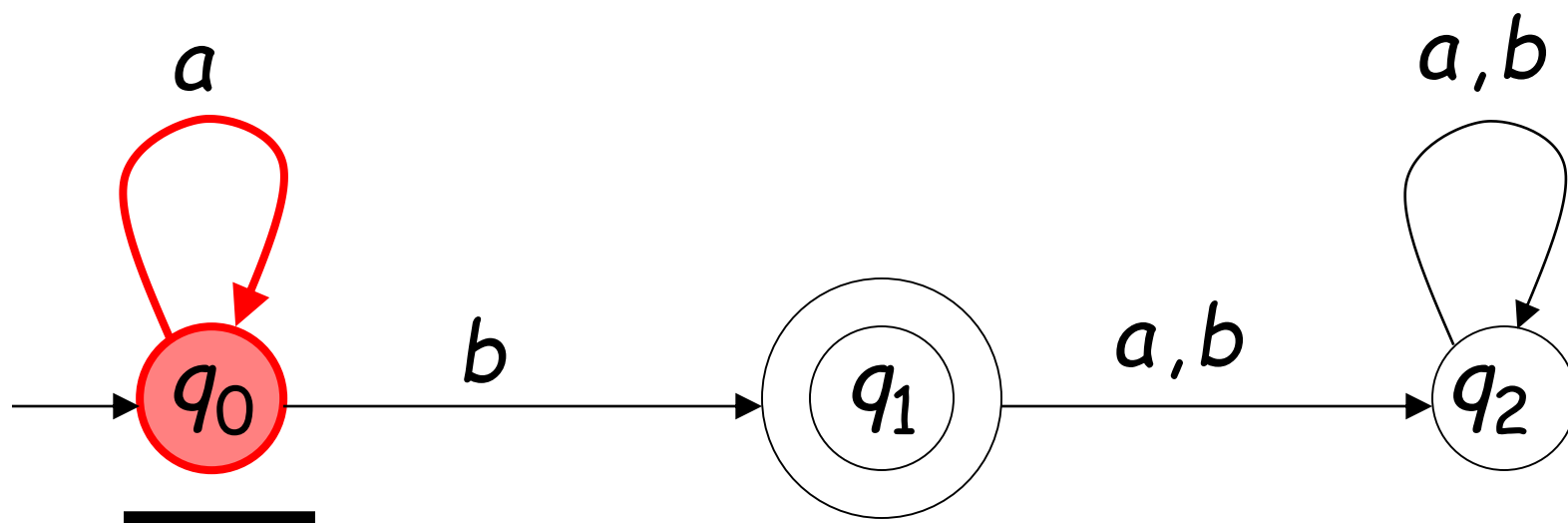
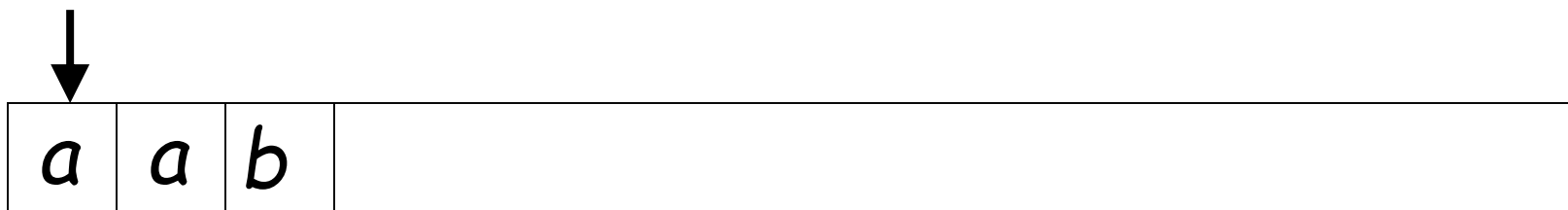
Another Example

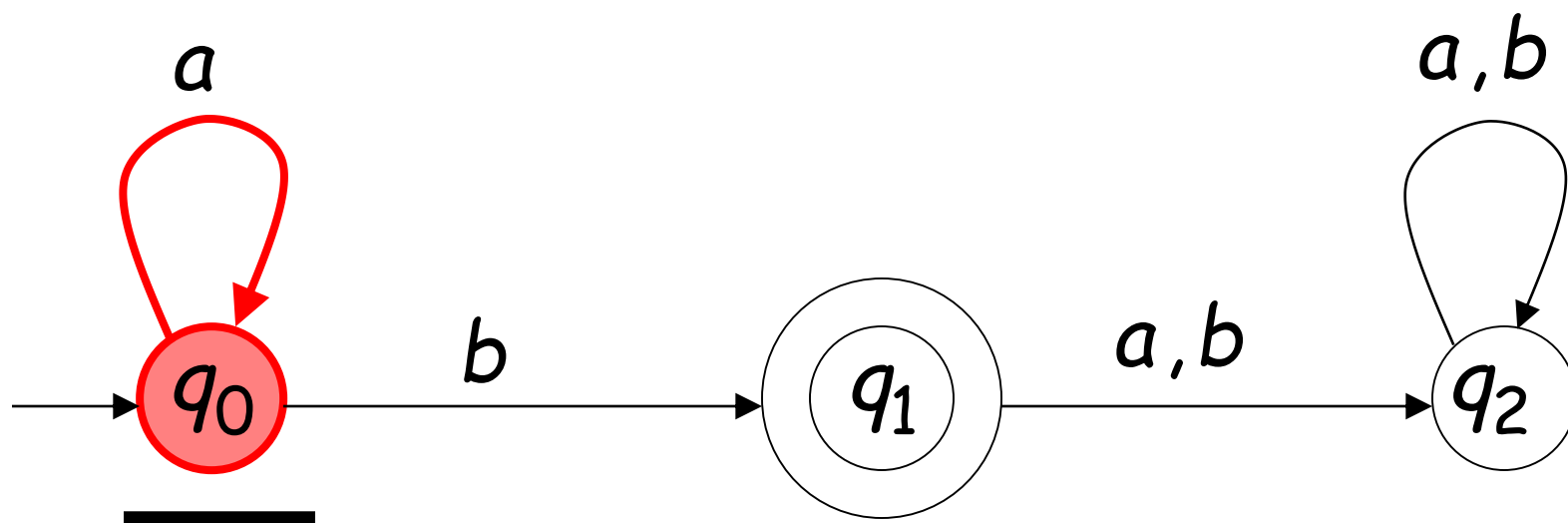
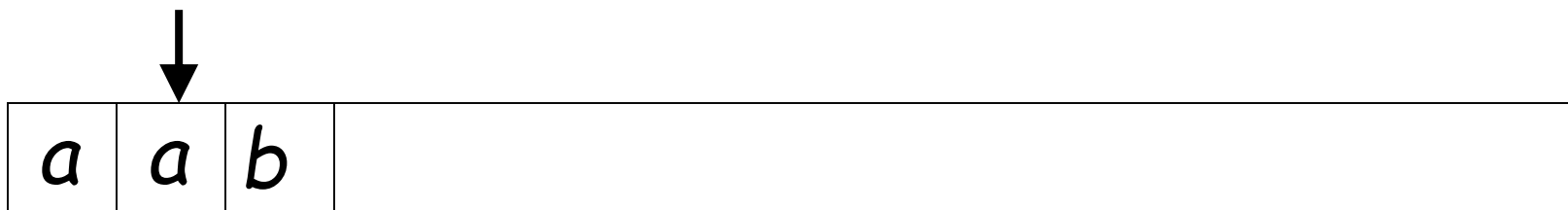




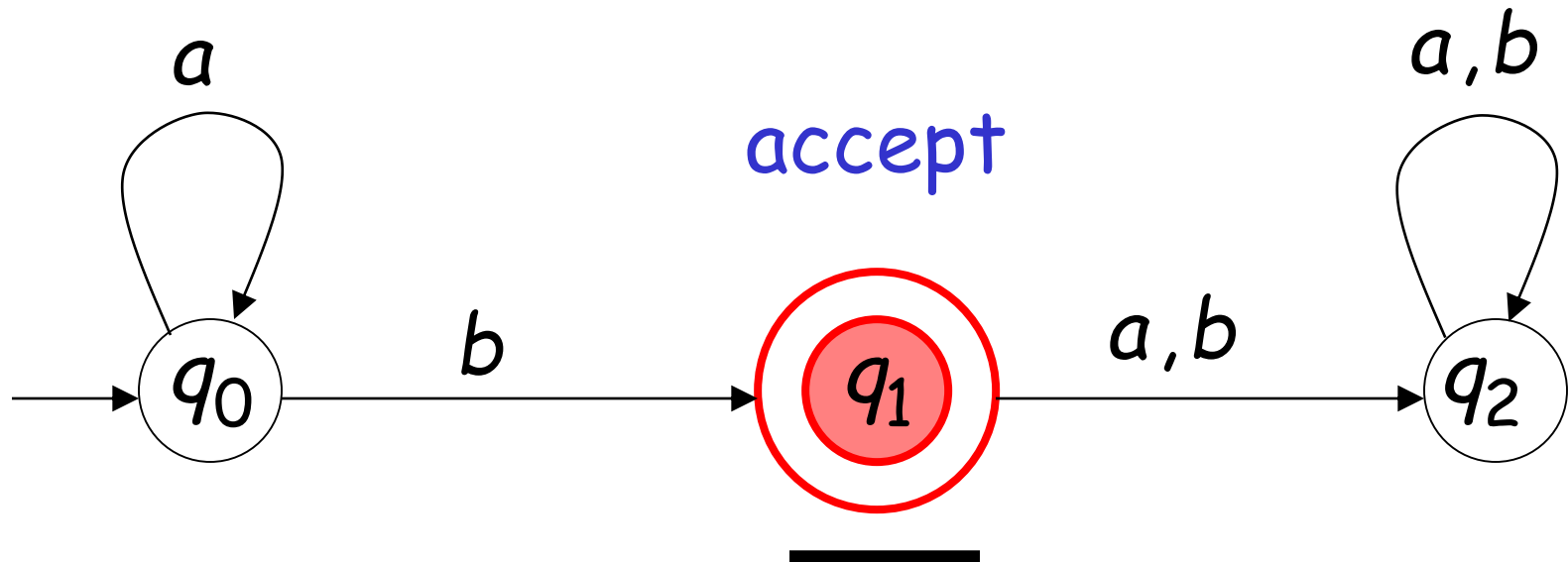
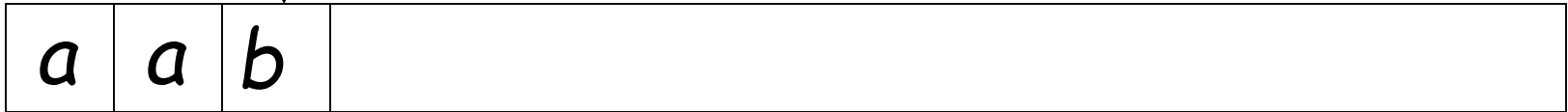
Input String



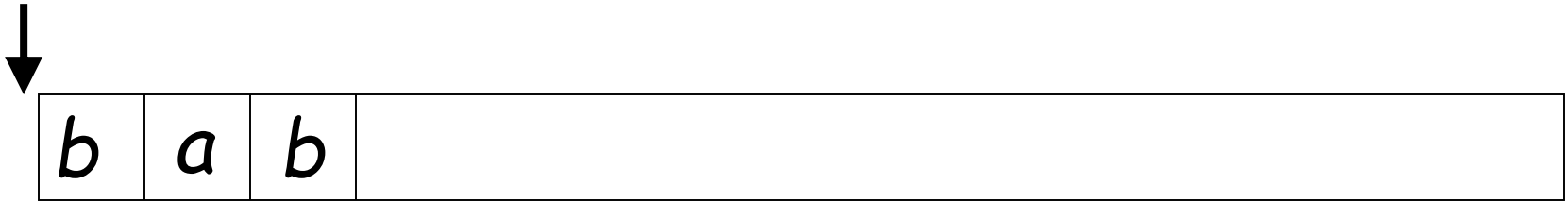




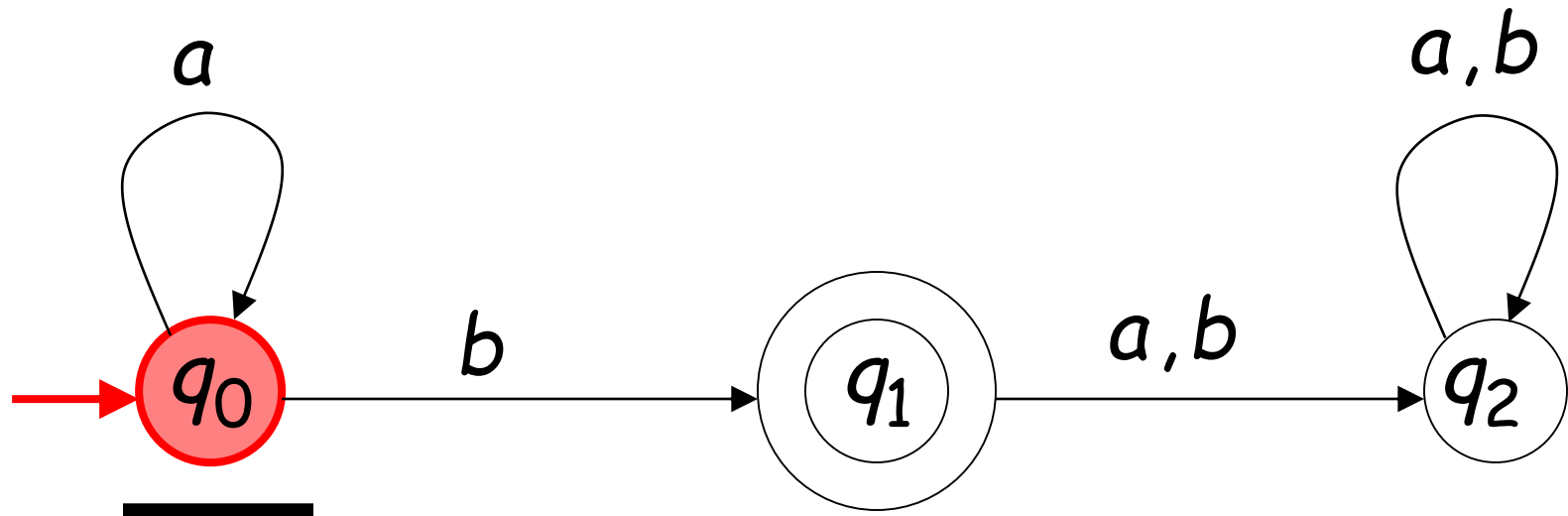
Input finished

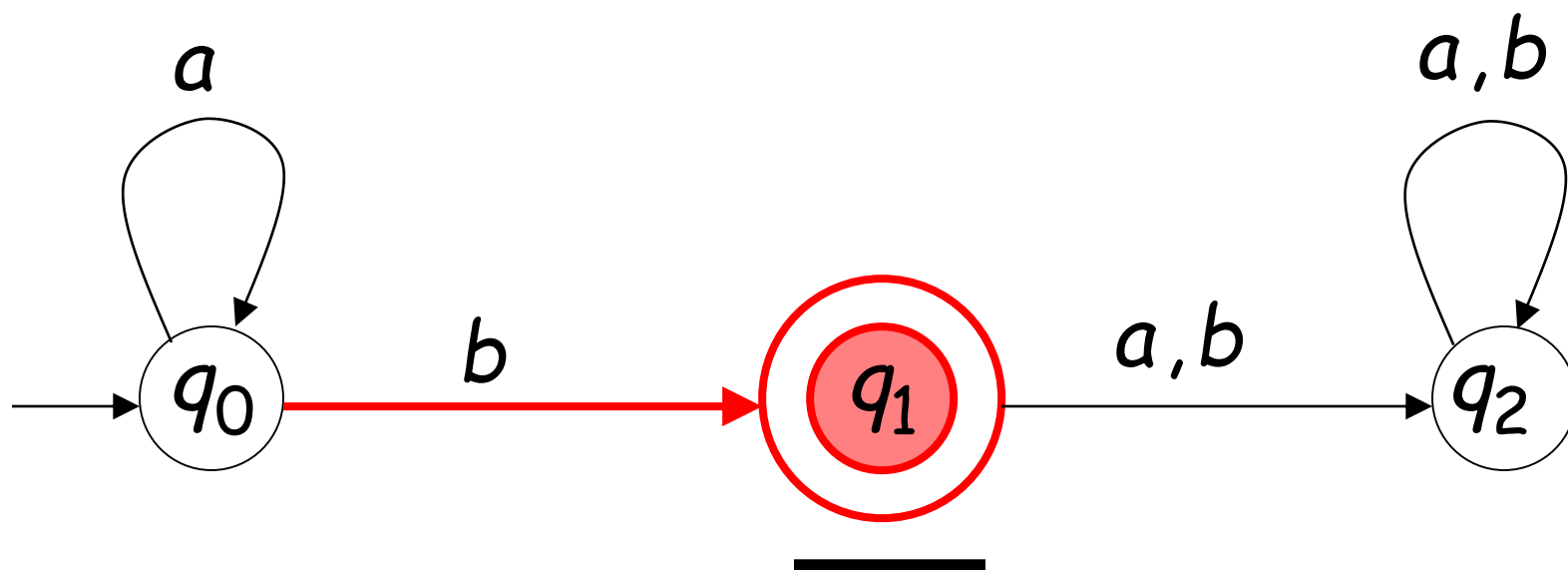
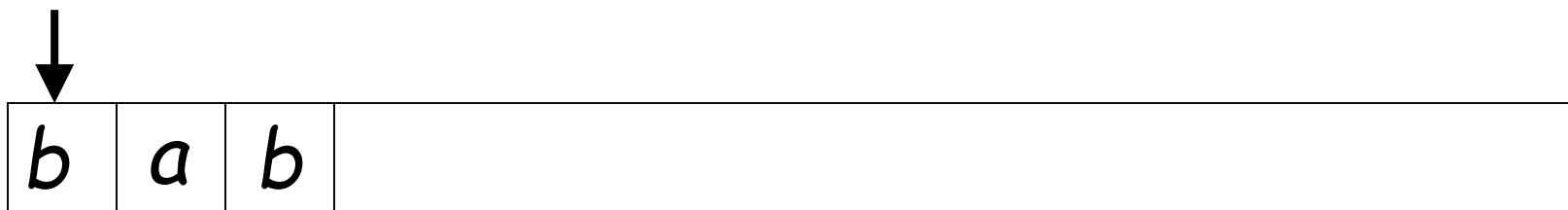


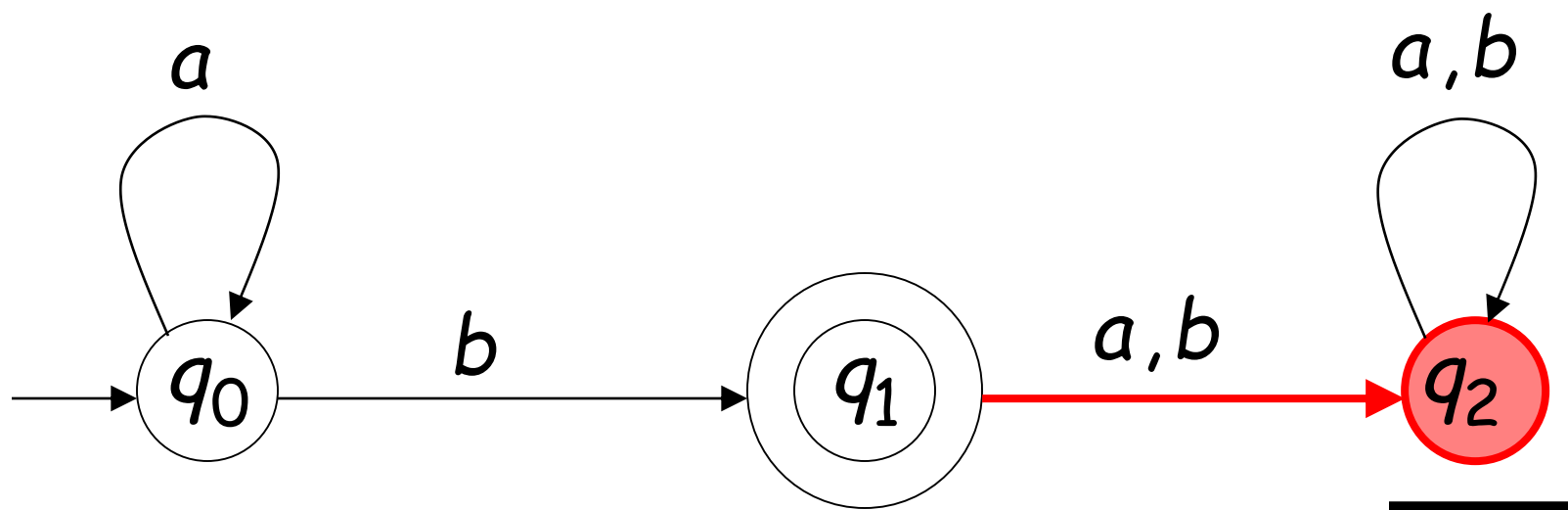
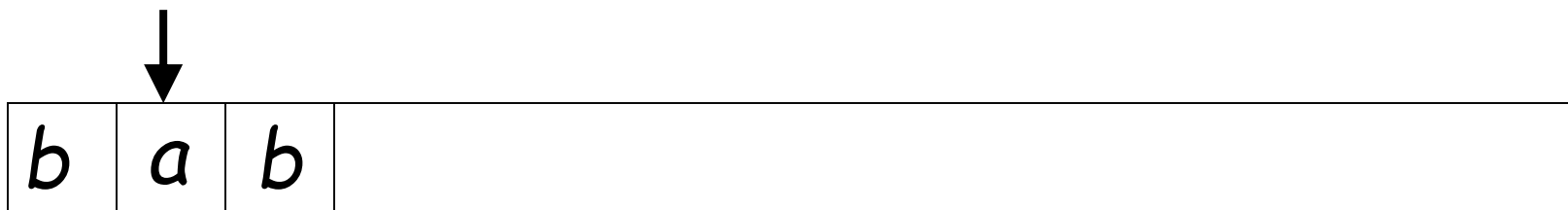
A rejection case



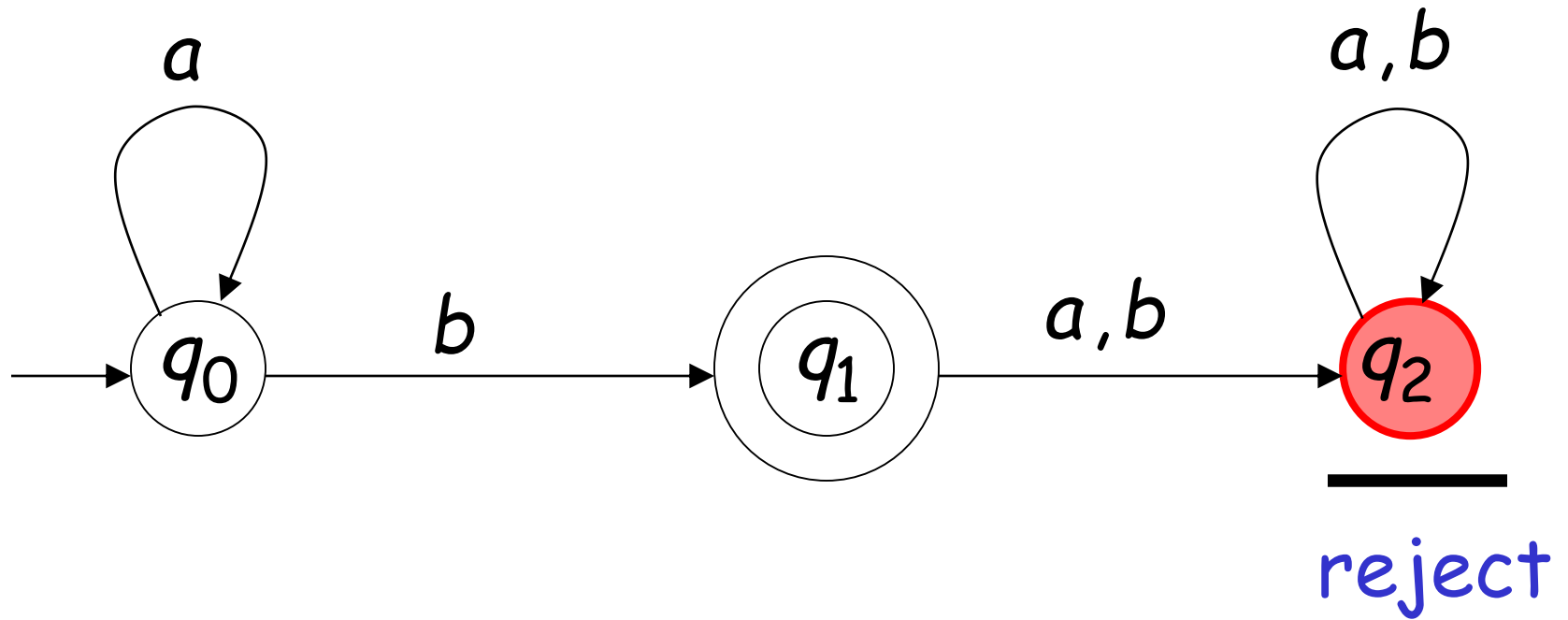
Input String



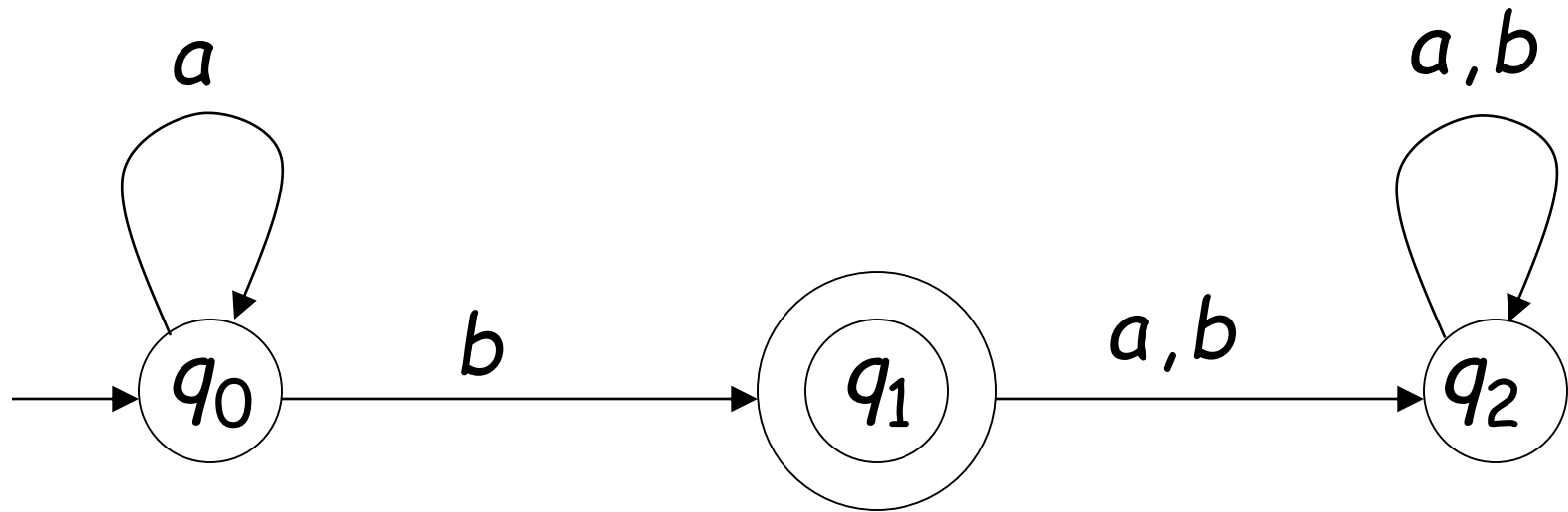




Input finished

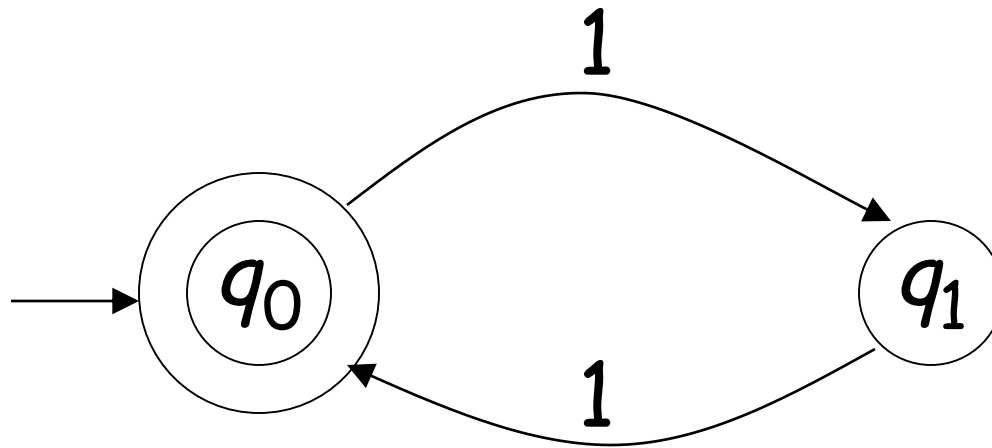


Language Accepted: $L = \{a^n b : n \geq 0\}$



Another Example

Alphabet: $\Sigma = \{1\}$



Language Accepted:

$$\begin{aligned} \text{EVEN} &= \{x : x \in \Sigma^* \text{ and } x \text{ is even}\} \\ &= \{\lambda, 11, 1111, 111111, \dots\} \end{aligned}$$

Formal Definition

Deterministic Finite Automaton (DFA)

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : set of states

Σ : input alphabet $\lambda \notin \Sigma$

δ : transition function

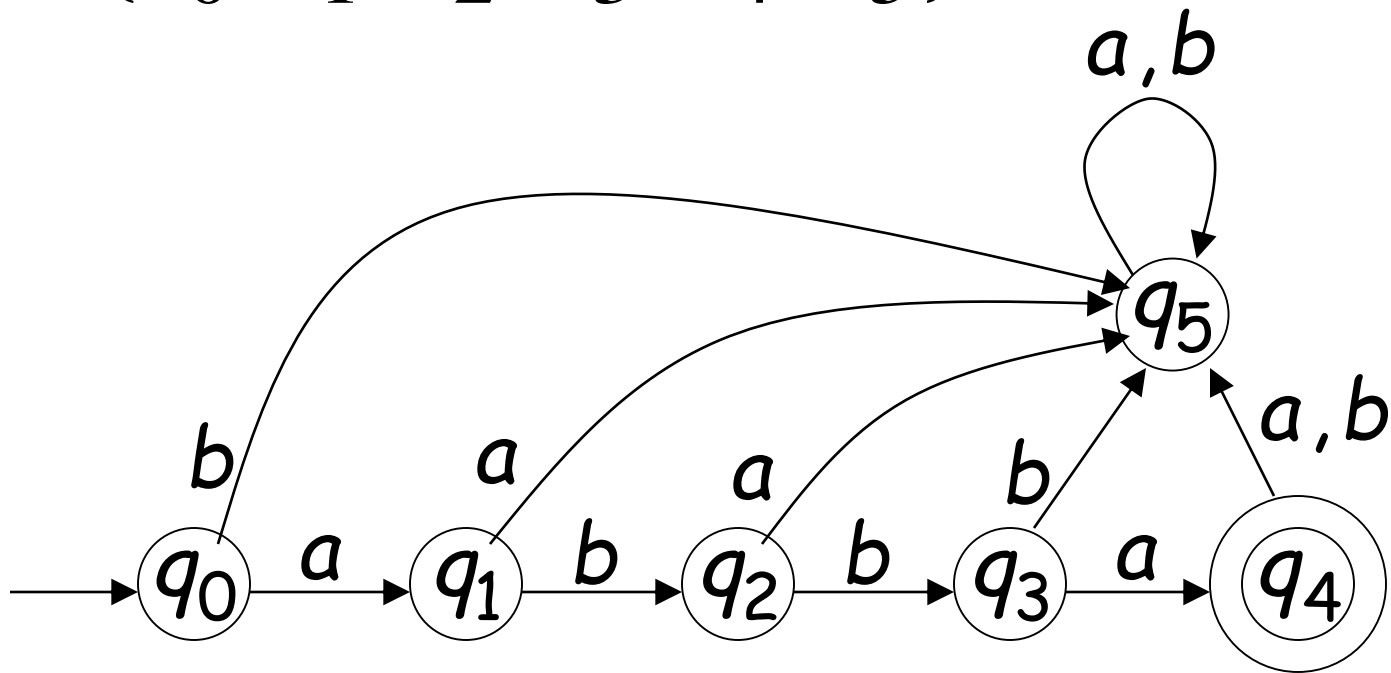
q_0 : initial state

F : set of accepting states

Set of States Q

Example

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

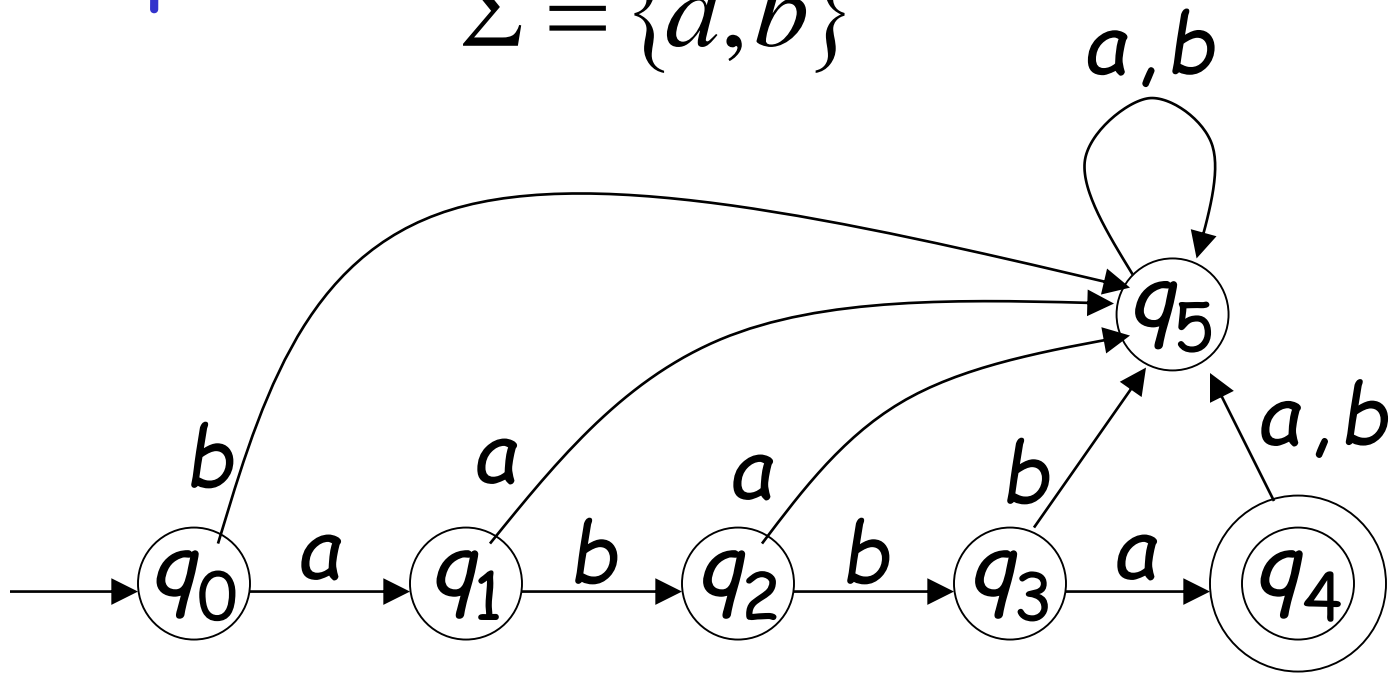


Input Alphabet Σ

$\lambda \notin \Sigma$: the input alphabet never contains λ

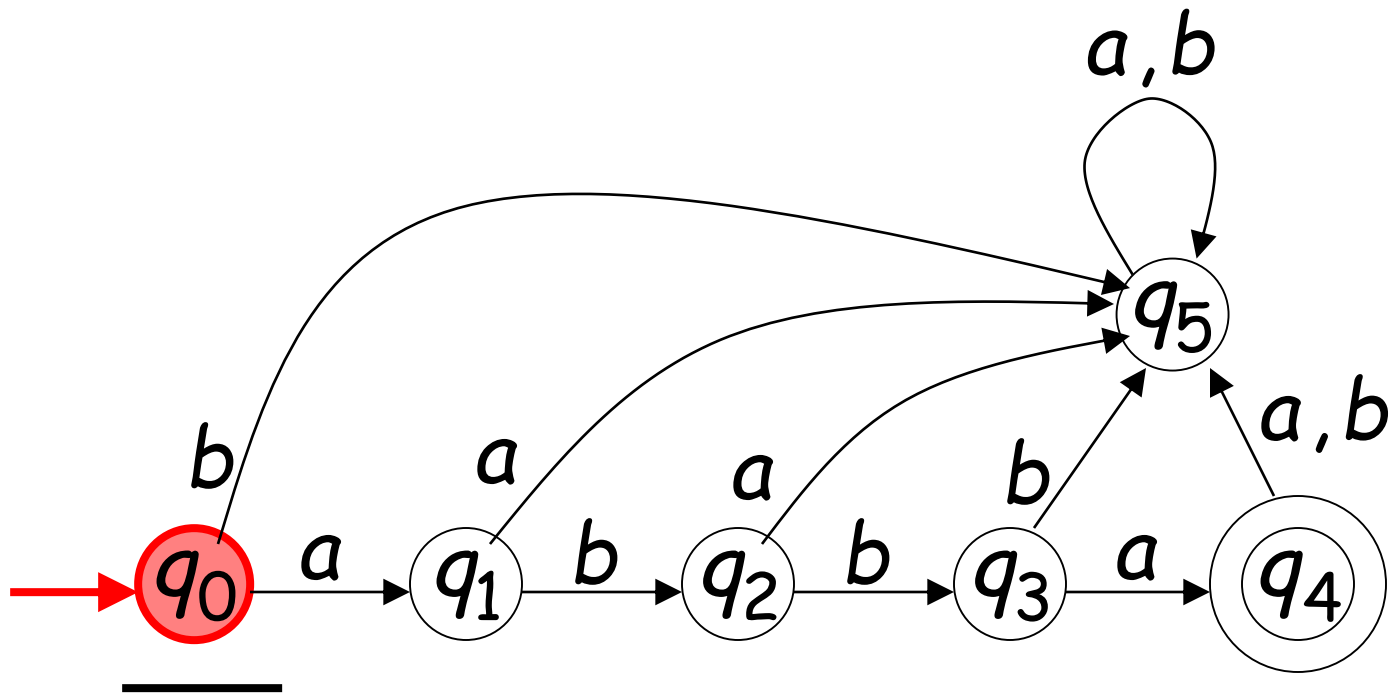
Example

$$\Sigma = \{a, b\}$$



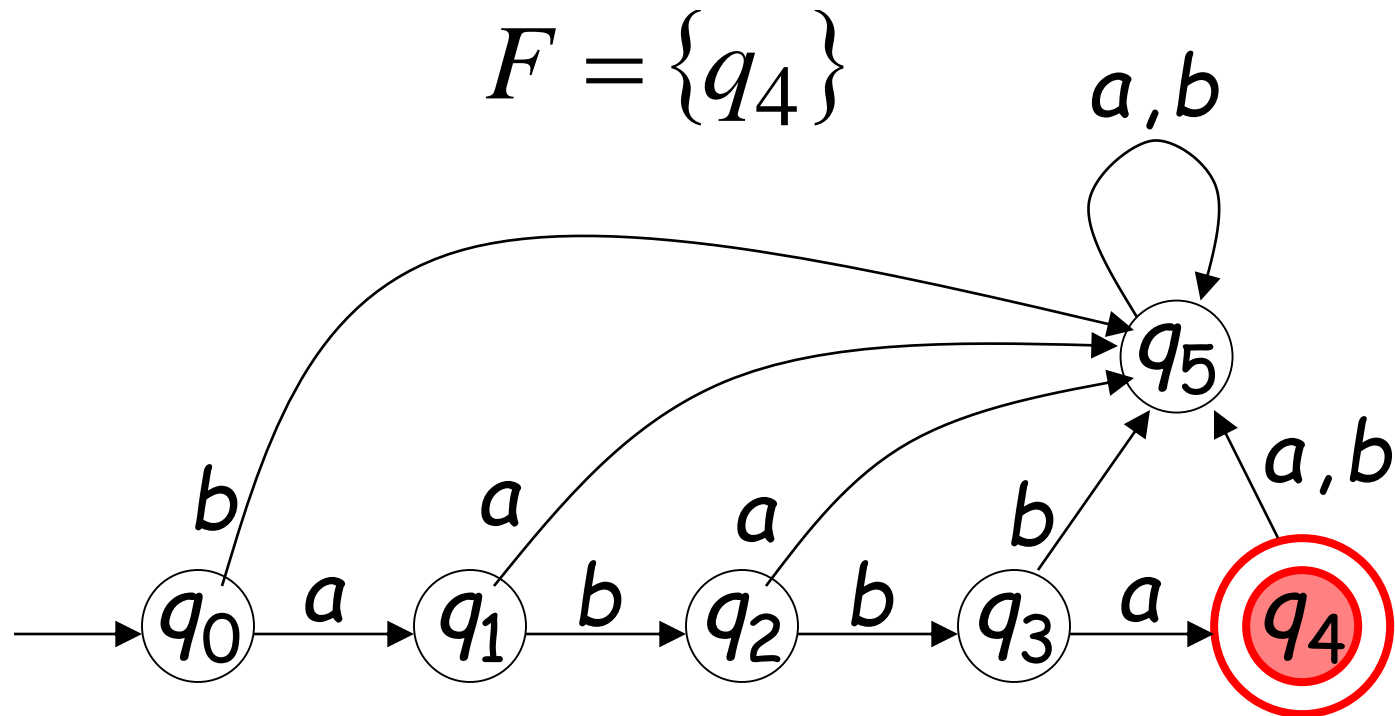
Initial State q_0

Example



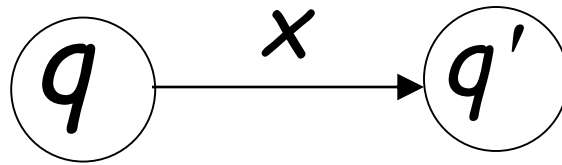
Set of Accepting States $F \subseteq Q$

Example



Transition Function $\delta : Q \times \Sigma \rightarrow Q$

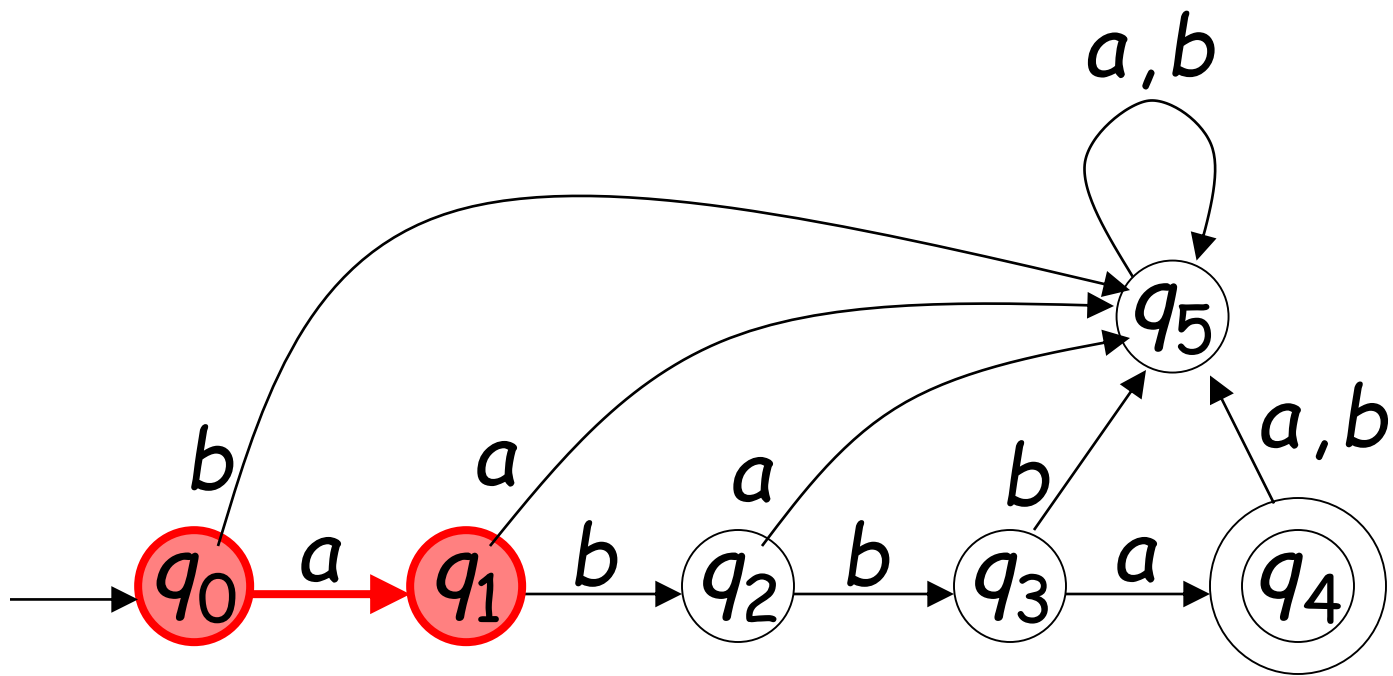
$$\delta(q, x) = q'$$



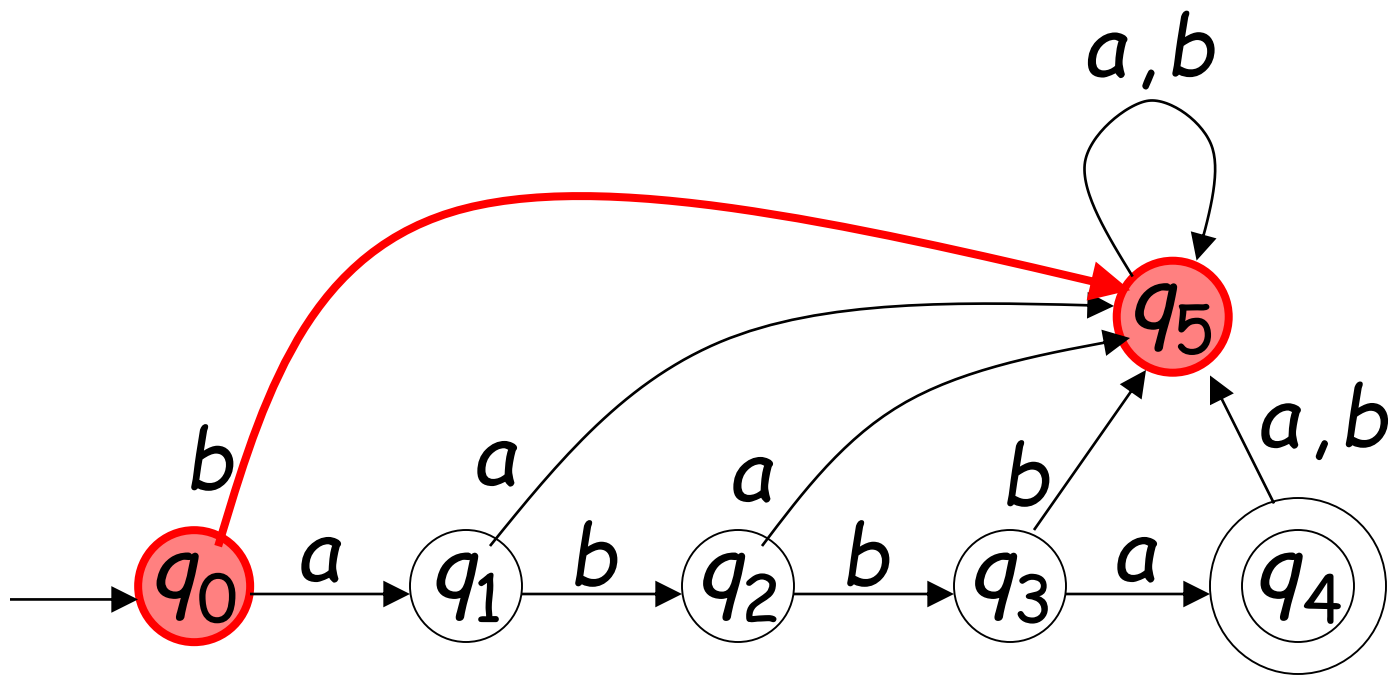
Describes the result of a transition
from state q with symbol x

Example:

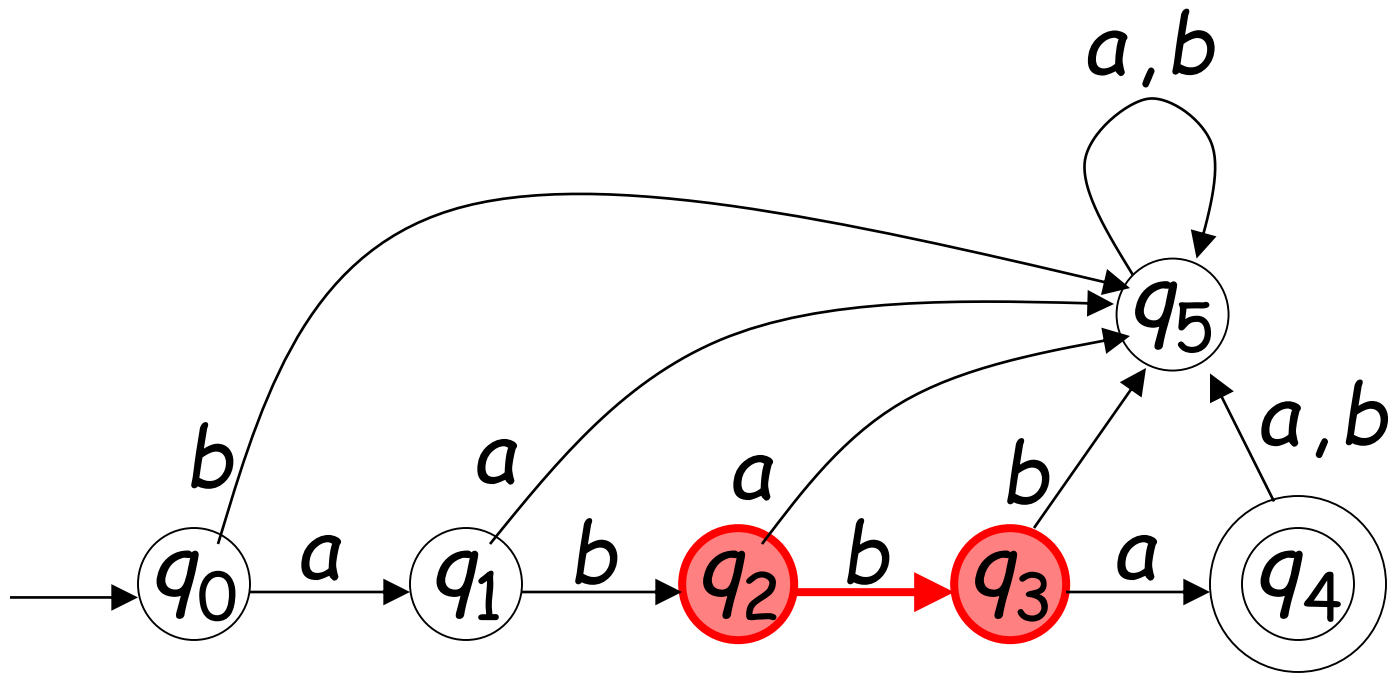
$$\delta(q_0, a) = q_1$$



$$\delta(q_0, b) = q_5$$



$$\delta(q_2, b) = q_3$$

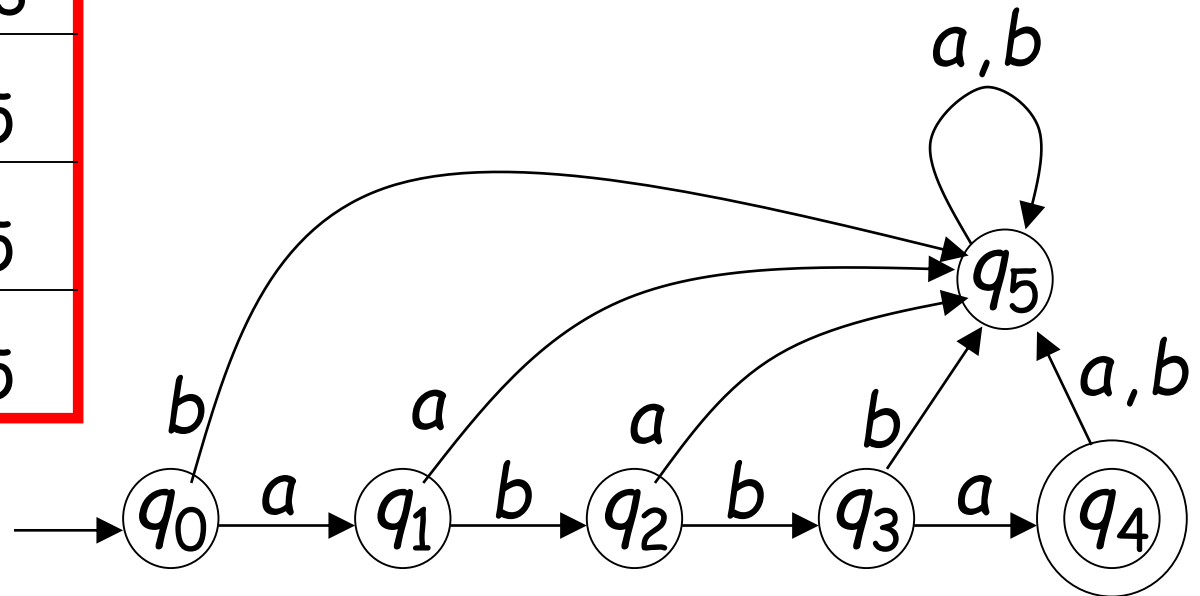


Transition Table for δ

symbols

δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5	q_5	q_5

states



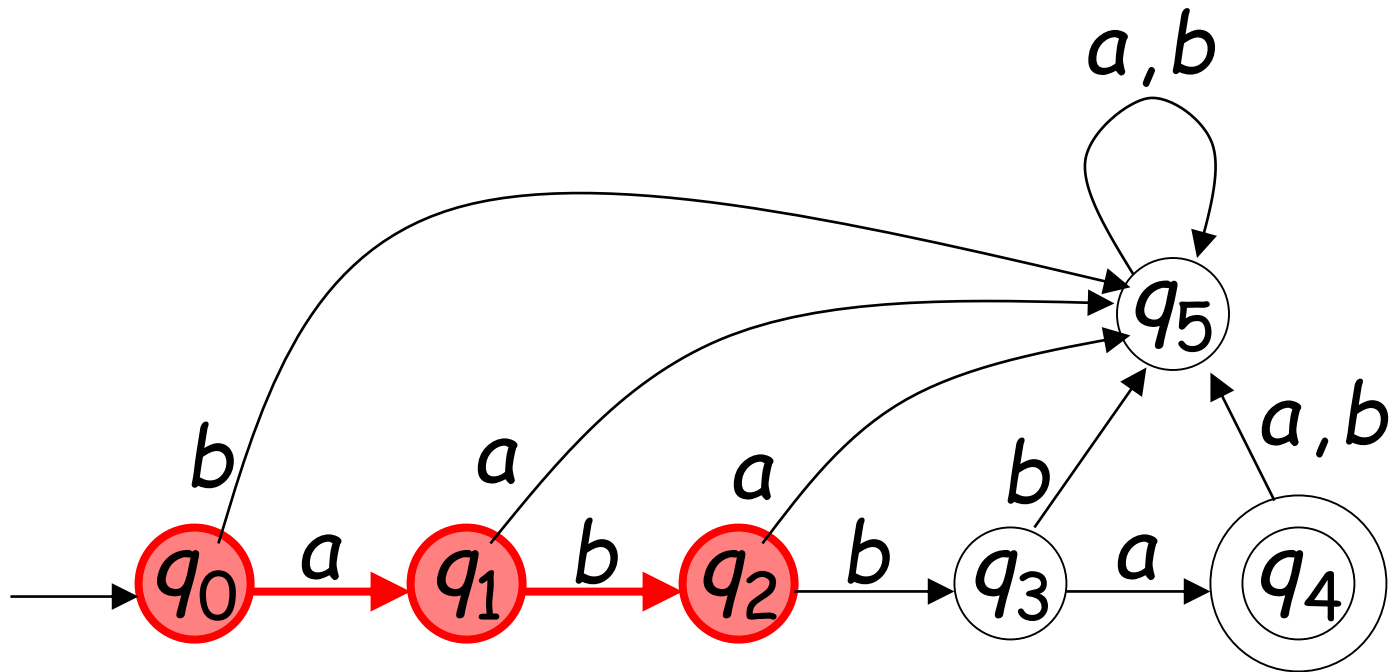
Extended Transition Function

$$\delta^* : Q \times \Sigma^* \rightarrow Q$$

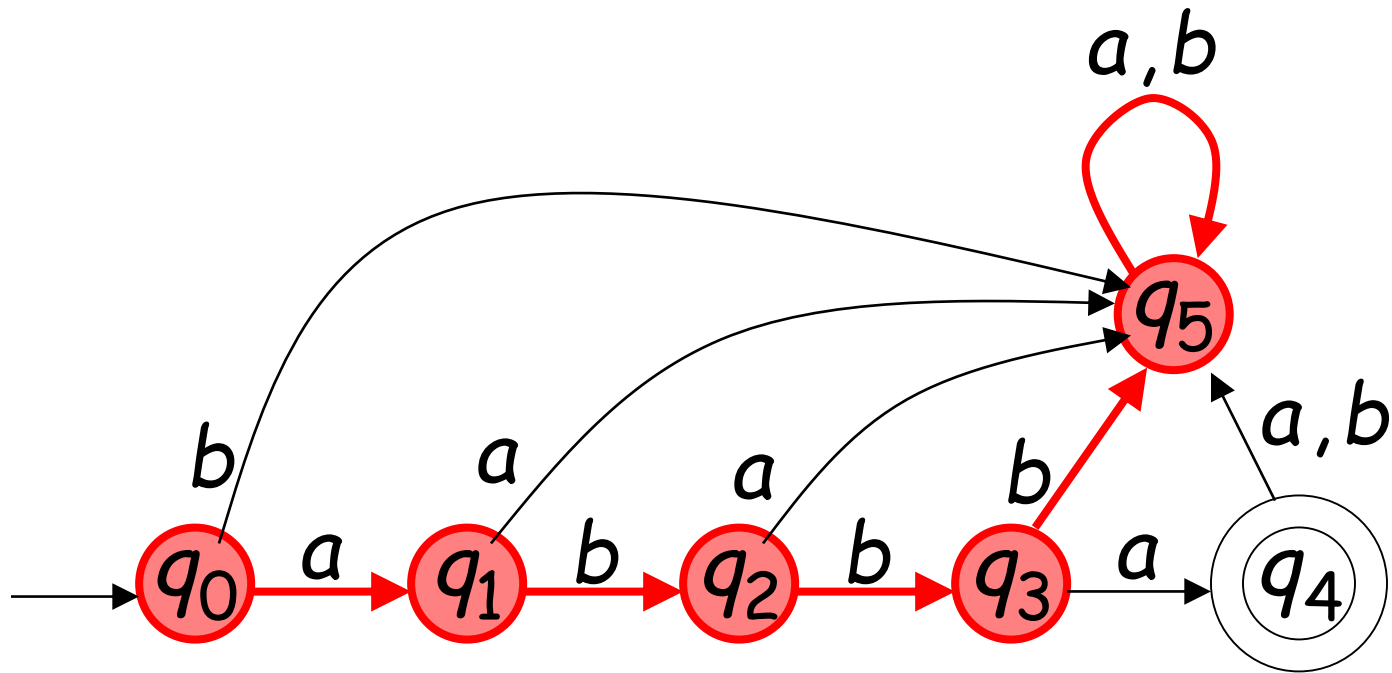
$$\delta^*(q, w) = q'$$

Describes the resulting state
after scanning string w from state q

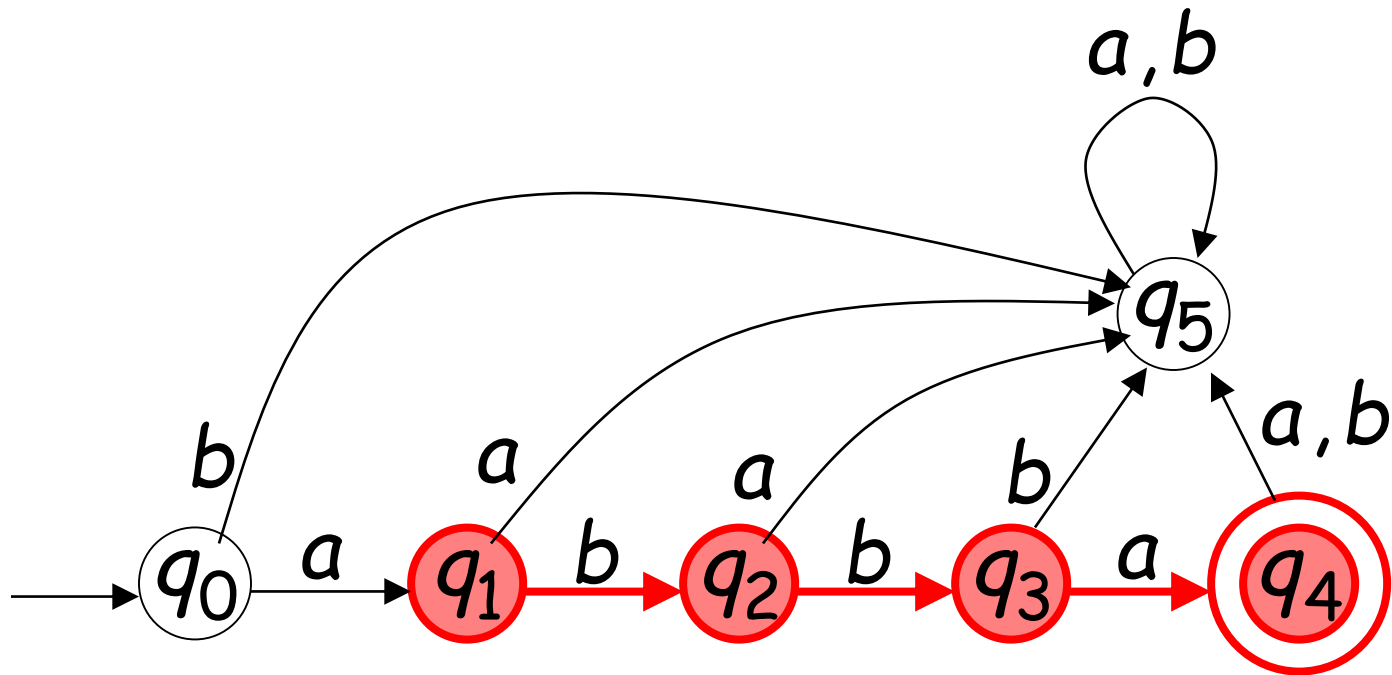
Example: $\delta^*(q_0, ab) = q_2$



$$\delta^*(q_0, abbbaa) = q_5$$



$$\delta^*(q_1, bba) = q_4$$



Special case:

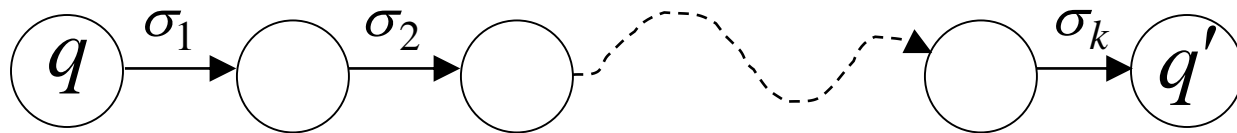
for any state q

$$\delta^*(q, \lambda) = q$$

In general: $\delta^*(q, w) = q'$

implies that there is a walk of transitions

$$w = \sigma_1 \sigma_2 \cdots \sigma_k$$



states may be repeated



Language Accepted by DFA

Language of DFA M :

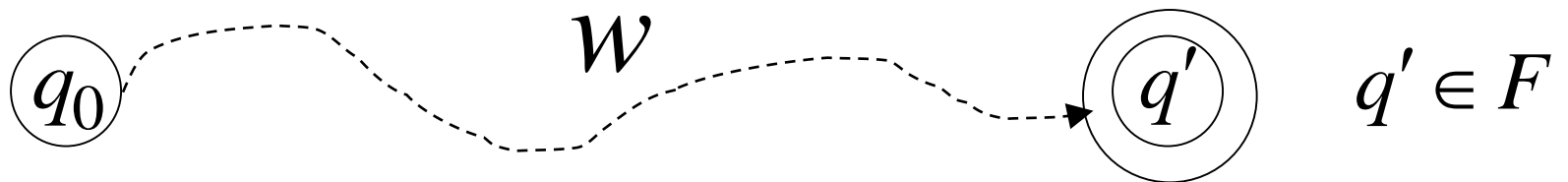
it is denoted as $L(M)$ and contains all the strings accepted by M

We say that a language L' is accepted (or recognized) by DFA M if $L(M) = L'$

For a DFA $M = (Q, \Sigma, \delta, q_0, F)$

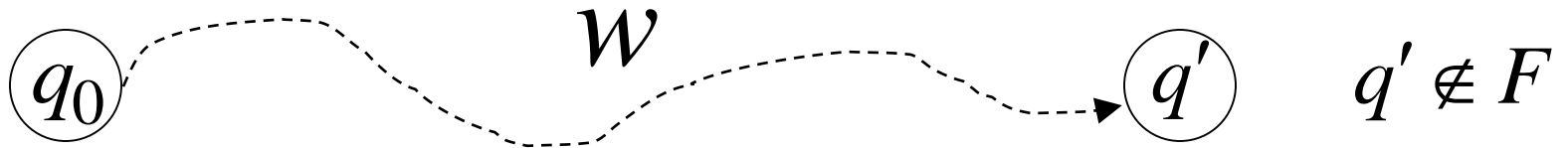
Language accepted by M :

$$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}$$



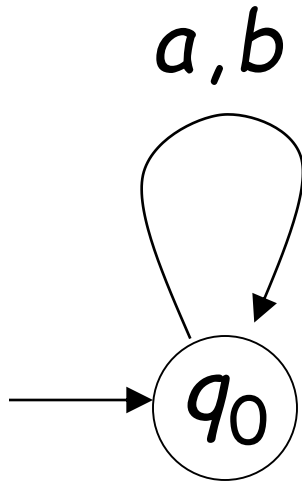
Language rejected by M :

$$\overline{L(M)} = \{w \in \Sigma^* : \delta^*(q_0, w) \notin F\}$$



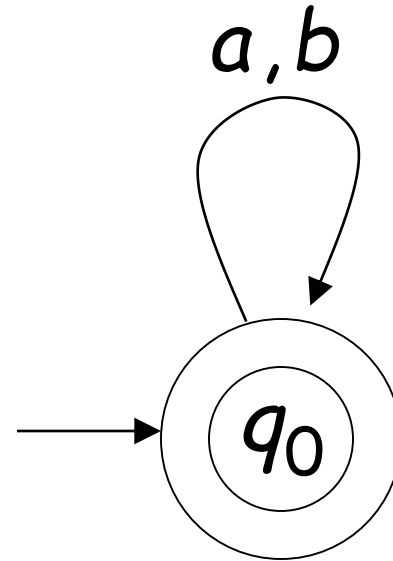
More DFA Examples

$$\Sigma = \{a, b\}$$



$$L(M) = \{ \}$$

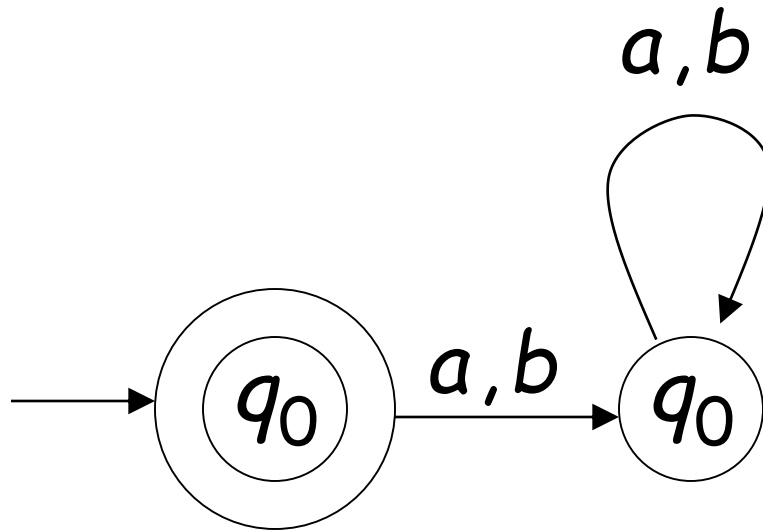
Empty language



$$L(M) = \Sigma^*$$

All strings

$$\Sigma = \{a, b\}$$

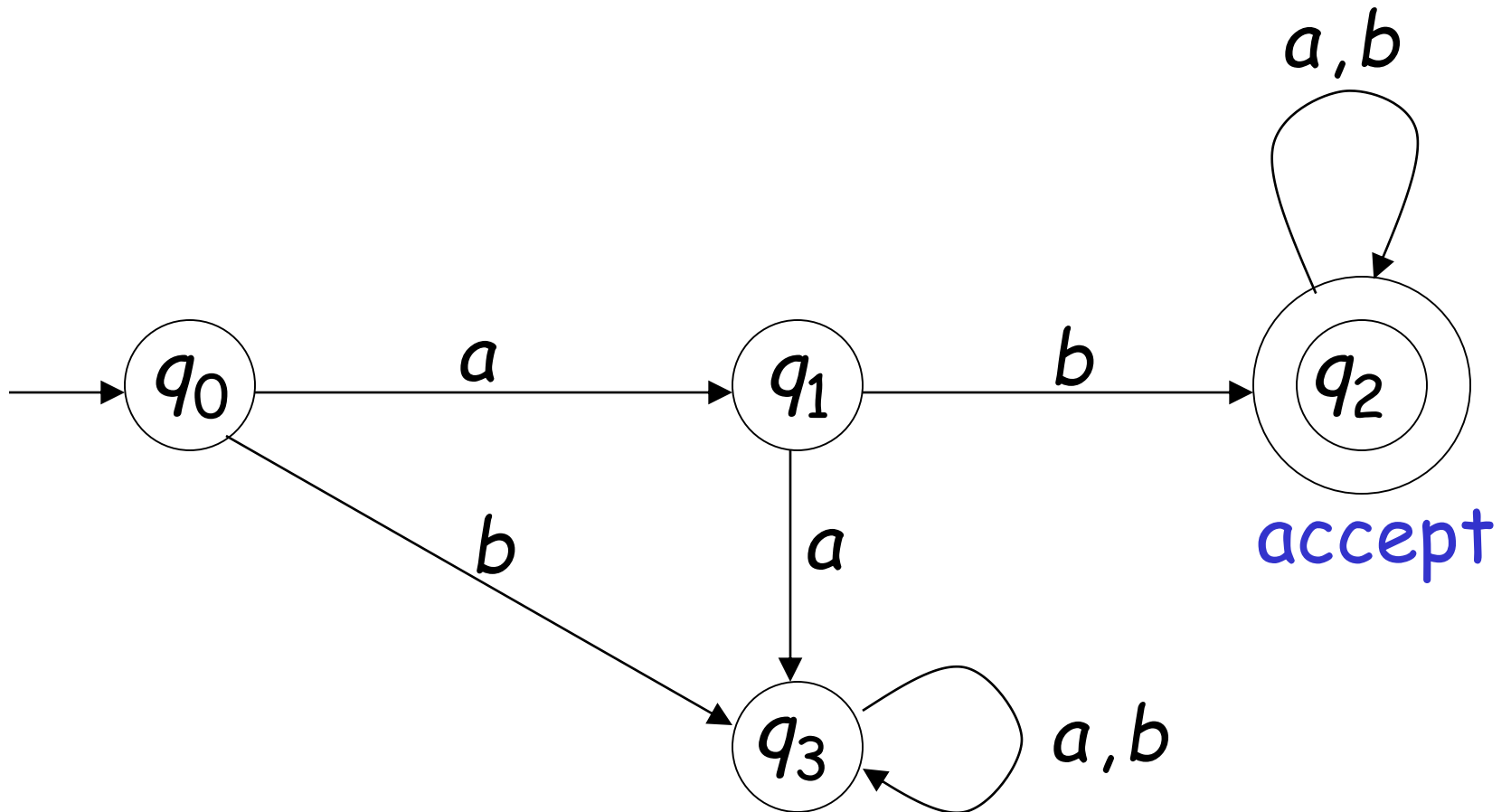


$$L(M) = \{\lambda\}$$

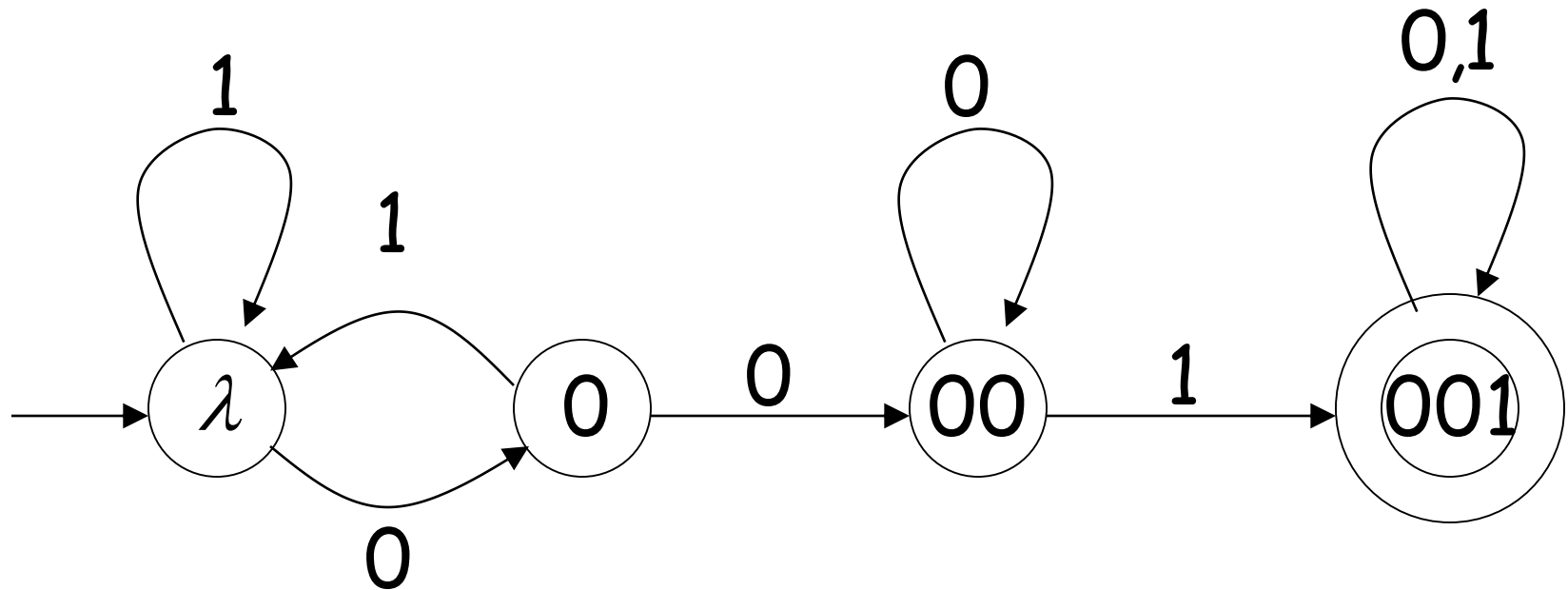
Language of the empty string

$$\Sigma = \{a, b\}$$

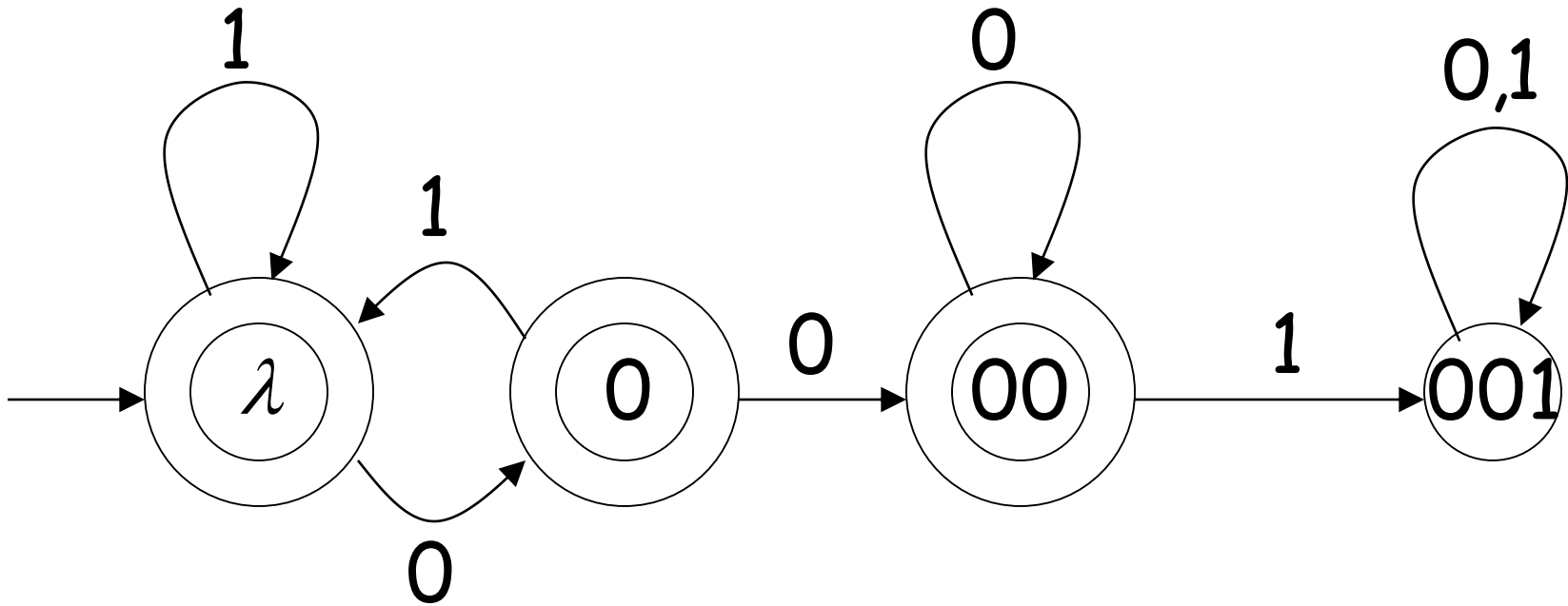
$L(M) = \{ \text{all strings with prefix } ab \}$



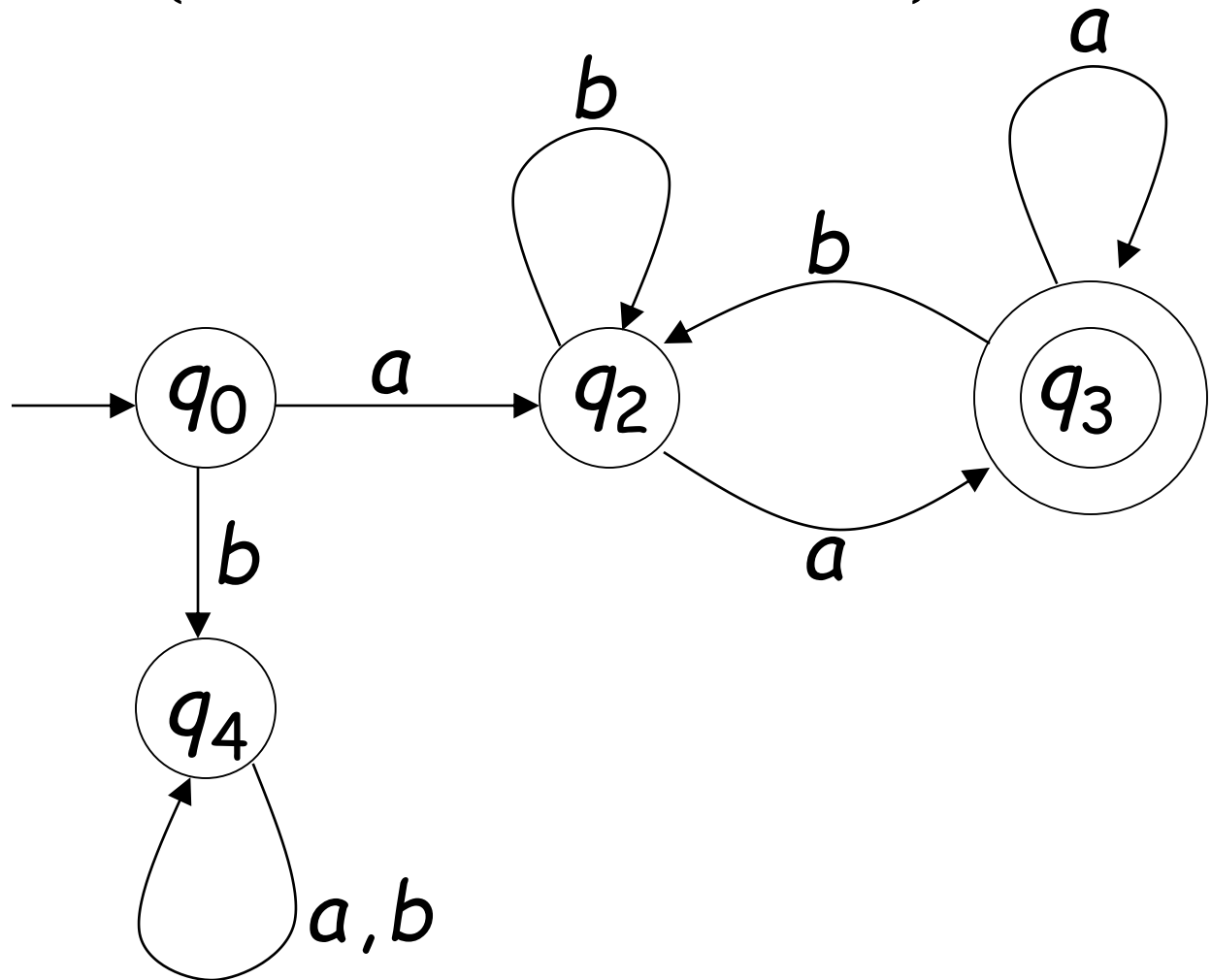
$L(M) = \{ \text{all binary strings containing} \\ \text{substring } 001 \}$



$L(M) = \{ \text{all binary strings without} \\ \text{substring } 001 \}$



$$L(M) = \{awa : w \in \{a,b\}^*\}$$



Regular Languages

Definition:

A language L is **regular** if there is a DFA M that accepts it ($L(M) = L$)

The languages accepted by all DFAs form the family of **regular languages**

Example regular languages:

$\{abba\}$ $\{\lambda, ab, abba\}$

$\{a^n b : n \geq 0\}$ $\{awa : w \in \{a,b\}^*\}$

$\{\text{all strings in } \{a,b\}^* \text{ with prefix } ab\}$

$\{\text{all binary strings without substring } 001\}$

$\{x : x \in \{1\}^* \text{ and } x \text{ is even}\}$

$\{\}$ $\{\lambda\}$ $\{a,b\}^*$

There exist automata that accept these languages (see previous slides).

There exist languages which are not Regular:

$$L = \{a^n b^n : n \geq 0\}$$

$$\text{ADDITION} = \{x + y = z : x = 1^n, y = 1^m, z = 1^k, \\ n + m = k\}$$

There is no DFA that accepts these languages
(we will prove this in a later class)