22)

$p(x,y)$	0	5	10	15
0	0.02	0.06	0.02	0.1
5	0.04	0.15	0.2	0.1
10	0.01	0.15	0.14	0.01

(column heading: y)

a) $E(X+Y) = \sum_x \sum_y (x+y) \cdot p(x,y)$

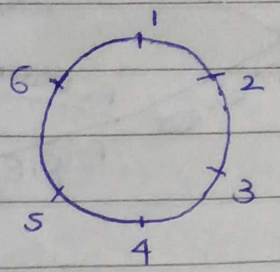
$= (0+0)\cdot p(0,0) + (0+5)\,p(0,5) + (0+10)\cdot p(0,10) + (0+15)\,p(0,15)$
$+ (5+0)\cdot p(5,0) + (5+5)\,p(5,5) + (5+10)\cdot p(5,10) + (5+15)\,p(5,15)$
$+ (10+0)\cdot p(10,0) + (10+5)\,p(10,5) + (10+10)\,p(10,10) + (10+15)\cdot p(10,15)$

$= 5\times 0.06 + 10\times 0.02 + 15\times 0.1$
$+ 5\times 0.04 + 10\times 0.15 + 15\times 0.2 + 20\times 0.1$
$+ 10\times 0.01 + 15\times 0.15 + 20\times 0.14 + 25\times 0.01$
$= 14.1$

b) $E(\max(X,Y)) = \sum_x \sum_y \max(0,0)\cdot p(0,0) + p(0,5)\cdot p(0,5) + \ldots$

$= 5\times 0.06 + 10\times 0.02 + 15\times 0.1$
$+ 5\times 0.04 + 5\times 0.15 + 10\times 0.2 + 15\times 0.1$
$+ 10\times 0.01 + 10\times 0.15 + 10\times 0.14 + 15\times 0.01$
$= 9.6$

24)



$g(x,y)$ represents the no. of individuals including A and B who handles the message.

A, B cannot have the same seat number

$g(x,y)$	1	2	3	4	5	6
1	x	2	3	4	3	2
2	2	x	2	3	4	3
3	3	2	x	2	3	4
4	4	3	2	x	2	3
5	3	4	3	2	x	2
6	2	3	4	3	2	x

$p(x,y) = \frac{1}{6 \times 5} = \frac{1}{30}$

$g(x,y) = 6 \times (2+3+4+3+2) = 84$

$\therefore E(g(x,y)) = \sum\sum g(x,y)\, p(x,y)$

$= \frac{84}{30}$

$= 2.8$

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Ex: 5.2

27) $X \to$ arrival time of Annie
$Y \to$ arrival time of Alvie.
X & Y are independent.

$$f_X(x) = \begin{cases} 3x^2, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} 2y, & 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

If Annie arrives first then $h(X,Y)$ or the waiting time
$= Y - X : X \le Y$
or $X - Y < 0$

If Alvie arrives first then $h(X,Y)$ or the waiting time
$= X - Y; \ X - Y \geq 0$

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Similarly,

$$h(x,y) = \begin{cases} Y - X; & X - Y < 0 \\ X - Y; & X - Y \geq 0 \end{cases}$$

$$= \begin{cases} -(X-Y); & X - Y < 0 \\ X - Y; & X - Y \geq 0 \end{cases}$$

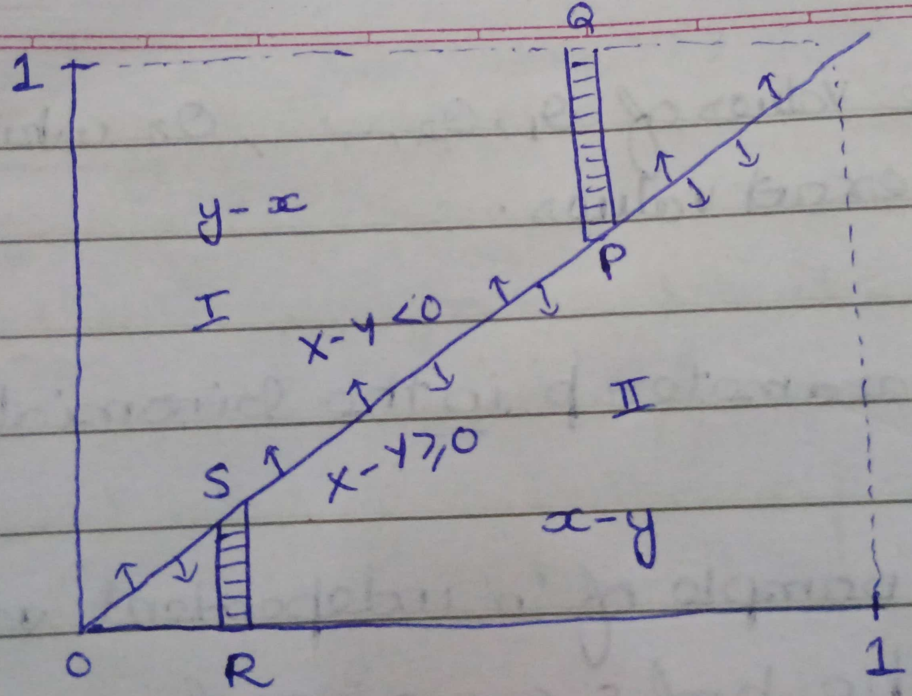
$$= |X - Y|$$

$$E(h(X,Y)) = E(|X-Y|)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |x-y| \, f(x,y) \, dy \, dx$$

Since, X and Y are independent

$$f(x,y) = f_X(x) \cdot f_Y(y)$$

$$\Rightarrow f(x,y) = \begin{cases} 6x^2 y; & 0 \le x < 1, \ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$


1
Q
y - x
I
P
x - y < 0
II
S
x - y ≥ 0
x - y
O
R
1

$\therefore E\{|X-Y|\} = \iint_{I} |X-Y|\, 6x^2y\, dy\, dx + \iint_{II} |X-Y|\, 6x^2y\, dy\, dx$

In region I, $X-Y<0$

constant limits for 'X' are

$\boxed{x=0}$ and $\boxed{x=1}$

Variable limits for 'Y' are

$\boxed{y=x}$ and $\boxed{y=1}$

In region II, $X-Y>0$

constant limits for 'X' are

$\boxed{x=0}$ and $\boxed{x=1}$

Variable limits for 'Y' are

$\boxed{Y=0}$ and $\boxed{y=x}$

$$E(|X-Y|) = \underbrace{\int_0^1\int_x^1 (Y-x)\,6x^2y\,dy\,dx}_{I_1\ (\text{say})} + \underbrace{\int_0^1\int_0^x (x-y)\,6x^2y\,dy\,dx}_{I_2\ (\text{say})}$$

$$I_1 = \int_0^1\int_x^1 (6x^2y^2 - 6x^3y)\,dy\,dx$$

$$= \int_0^1 \left\{2x^2y^3 - 3x^3y^2\right\}_x^1 dx$$

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$$= \int_0^1 2x^2(1-x^3) - 3x^3(1-x^2)\, dx$$

$$= \frac{1}{12} = 0.083$$

$$I_2 = \int_0^1 \{3x^3y^2 - 2x^2y^3\}_0^x\, dy$$

$$= \int_0^1 \{3x^3(0-x^2) - 2x^2(0-y^3)\}\, dy$$

$$= \int_0^1 3x^5 - 2x^5\, dx = \frac{1}{6} \approx 0.17$$

$$E\{h(x,y)\} = 0.08 + 0.17 = 0.25$$

28) Q) Show that $E(XY) = E(X) \cdot E(Y)$; if x and y are continuous and independent

$$f(x,y) = f_X(x) \cdot f_Y(y)$$

Proof:- We know

$$E(x) = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx$$

$$E(X) \cdot E(Y) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx \cdot \int_{-\infty}^{\infty} y\, f_Y(y)\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_X(x) \cdot f_Y(y)\, dy\, dx$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\, dy\, dx$$

$$= E(X,Y)$$

33) we know that,

$E(XY) = E(X) \cdot E(Y)$ when X and Y are independent

we also know that,

$$cov(X,Y) = E(XY) - E(X) \cdot E(Y)$$
$$= E(X) \cdot E(Y) - E(X) \cdot E(Y)$$
$$= 0$$

$$corr(X,Y) = \frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y} = 0$$

$\therefore$ $cov(X,Y) = corr(X,Y) = 0$ when X, Y are independent

Hence, proved.

35) a) We know that, $cov(X,Y) = E(X \cdot Y) - E(X) \cdot E(Y)$

$$\therefore cov(aX+b, cY+d) = E([aX+b][cY+d]) - E(aX+b) \cdot E(cY+d)$$

$$= E(acXY + adX + bcY + bd) - \{(aE(X)+b) \cdot (cE(Y)+d)\}$$
$$= ac\,E(XY) + \cancel{ad\,E(X)} + \cancel{bc\,E(Y)} + \cancel{bd}\,E(1) - \{ac\,E(X) \cdot E(Y) + \cancel{ad\,E(X)} + \cancel{bc\,E(Y)} + \cancel{bd}\}$$

$\because [E(1) = 1]$

$$= ac\{E(XY) - E(X) \cdot E(Y)\}$$
$$= ac\, cov(X,Y)$$

Hence, proved

35) b) $\text{corr}(aX+b, CY+d) = \dfrac{\text{cov}(aX+b, CY+d)}{\sigma_{ax+b}\,\sigma_{cy+d}}$

$= \dfrac{ac\,\text{cov}(X,Y)}{|a||c|\,\sigma_x\,\sigma_y}$ $\left[\because \text{cov}(aX+b, CY+d) = ac\,\text{cov}(X,Y)\right]$

when a and c have same sign

$\text{corr}(aX+b, CY+d) = \text{corr}(X,Y)$

Hence, proved.

35) c) If a and c have opposite signs,

$$\text{corr}(aX+b) = -\text{corr}(X,Y)$$

36)

If $a \neq 0$, $corr(X, aX+b) = \pm 1$, here, $Y = aX+b$

prove this.

$$corr(X, aX+b) = \frac{cov(X, aX+b)}{\sigma_X \sigma_{aX+b}}$$

$$= \frac{E((x-\mu_x)\cdot(aX+b-\mu_{aX+b}))}{\sigma_X \cdot \sigma_{aX+b}}$$

$$= \frac{E(((x-\mu_x)\cdot(ax+b-a\mu_x-b))}{\sigma_X \cdot \sigma_{ax+b}}$$

$\sigma_{ax+b} =$

$$= \frac{a \cdot E((X-\mu_x)^2)}{\sigma_x \cdot |a| \sigma_x}$$

$$= \frac{a \cdot \sigma_x^2}{|a| \sigma_x^2}$$

$$= \frac{a}{|a|} = \frac{a}{\pm a}$$

$$= \pm 1$$

36) If $a > 0$, $\text{corr}(X, aX+b) = 1$

If $a < 0$, $\text{corr}(X, aX+b) = -1$