Probability:

2.1-2.2

Experiment:→

An experiment is a process of measurement or observation in a laboratory in a factory, on the street, in nature, etc.

Random experiment:→

An experiment E is called a random experiment if

i) all possible outcomes of E are Known in advance.

ii) it is impossible to predict which outcome will occur at a particular trial.

iii) E can be repeated, under identical conditions for infinite number of times.

Example:

i) The experiment of tossing a coin is an example of random experiment. Here the possible outcomes are $\{H, T\}$ but it is impossible to predict which outcome, namely "H or T" will occur at a particular toss of the coin.

ii) throwing a die

iii) Inspecting a light bulb.

Trial:

A trial is a single performance of an experiment. Its result is called an outcome or a sample point.

Sample Space (S):

The set of all possible outcomes of an experiment is called sample space. It is denoted by S.

Events:

An event is any collection (subset) of outcomes contained in the sample space S. An event is simple if it consists of exactly one outcome and compound if it consists of more than one outcomes.

Example:

i) Rolling a dice, $S = \{1, 2, 3, 4, 5, 6\}$, events are

$A =$ odd Nos. $= \{1, 3, 5\}$, $B =$ even Nos. $= \{2, 4, 6\}$

ii) Inspecting a light bulb, $S = \{$Defective, Non-defective$\}$

iii) Asking for opinion about a new car model,

$S = \{$Like, Dislike, undecided$\}$

Classical definition of Probability:

If the sample space S of an experiment consists of finitely many outcomes (points) that are equally likely then the probability $P(A)$ of an event A is

$$P(A) = \frac{\text{number of points in } A}{\text{number of points in } S} = \frac{m}{n}$$

Clearly $P(S) = 1$.

Example:

In rolling a dice once, what is the probability of A of obtaining 5 or 6? The probability of B : "even number".

Ans: $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{5, 6\}$ $\quad B = \{2, 4, 6\}$

$\therefore P(A) = \frac{2}{6} = \frac{1}{3}$ $\quad P(B) = \frac{3}{6} = \frac{1}{2}$.

Mutually exclusive events:

A and B are said to be mutually exclusive events, if $A \cap B = \phi$, ϕ = null event, the event consisting no outcomes.

Similarly, $A_1, A_2, A_3, \ldots$ are said to be mutually exclusive or pairwise disjoint if no two events have any outcomes in common.

i.e. $A_i \cap A_j = \phi \quad (i \neq j)$.

Axioms (Basic properties) of probability: (General definition)

Given a sample space S, with each event A of S there is associated a number P(A), called the probability of A, such that the following axioms of probability are satisfied

i) For any event A in S, $0 \leq P(A) \leq 1$.

ii) $P(S) = 1$.

iii) For mutually exclusive events A and B ie for $A \cap B = \phi$
$P(A \cup B) = P(A) + P(B)$
and for mutually exclusive events $A_1, A_2, A_3, \ldots$
~~$P(A_1 \cup A_2)$~~ $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$
$= \sum_{i=1}^{\infty} P(A_i)$

Complementary events:

The complement of an event A, denoted by A' or A^c or $\bar{A}$, is the set of all outcomes in S that are not contained in A.

Note:- $P(\phi) = 0$, where ϕ is the null event, the event containing no outcomes.

$P(A \cup B) = P(A + B)$

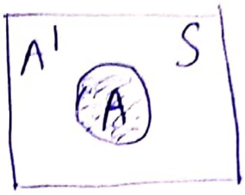
$P(A \cap B) = P(AB)$

More probability properties:

1) For any event A, $P(A) + P(A') = 1$ from which $P(A') = 1 - P(A)$, where A' = complement of A.

proof:

$A \cup A' = S$ and $A \cap A' = \phi$



$\Rightarrow P(A \cup A') = P(S)$

$\Rightarrow P(A) + P(A') = 1$ [$\because P(S) = 1$ and A, B are mutually exclusive]

$\Rightarrow P(A') = 1 - P(A)$. (Proved)

Example:

Five coins are tossed simultaneously. Find the probability of the event A, A: at least one head turns up.

Ans: Since each coin can turn up heads or tails, the sample space consists of $2^5 = 32$ outcomes.

Then the event A' (= No head turn up) consist of only one outcome

$\therefore P(A') = \frac{1}{32}$ $\quad \therefore P(A) = 1 - P(A') = 1 - 1/32$

$= \frac{31}{32}$. Ans.

2) For finite mutually exclusive events $A_1, A_2, \ldots, A_n$ in a sample space S, $P(A_1 + A_2 + \cdots + A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$.

Proof; By induction you can proove this.

3) For any event A, $P(A) \leq 1$.

proof; We know that $P(A) + P(A') = 1$

$\Rightarrow P(A) \leq 1$ [$\because P(A') \geq 0$]

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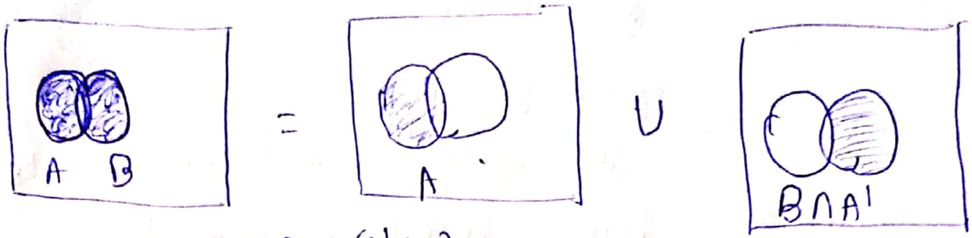
4) For any two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof:



We can see $\quad B = (A \cap B) \cup (A' \cap B)$

$P(B) = P(A \cap B) + P(A' \cap B) \quad \rightarrow (1)$ [∵ $A \cap B$ and $A' \cap B$ are mutually exclusive]

Also, $P(A \cup B) = P(A) + P(B \cap A')$

$= P(A) + \{P(B) - P(A \cap B)\}$ [by (1)]

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Proved)

Note: For any three events A, B and C,

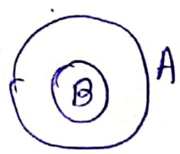
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

5) If $B \subseteq A$, then $P(B) \le P(A)$.

Proof:

Since $\quad B \subseteq A$

$\Rightarrow A = B \cup (A - B)$



$\Rightarrow P(A) = P(B) + P(A-B)$ [∵ A and B−A are mutually exclusive events]

$\Rightarrow P(A-B) = P(A) - P(B)$

$\Rightarrow P(A) - P(B) \geq 0$ [∵ $P(A-B) \geq 0$]

$\Rightarrow P(A) \geq P(B)$

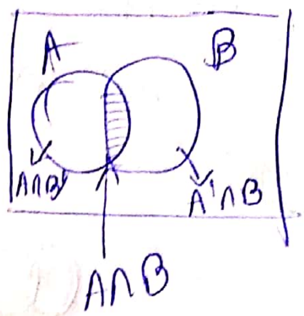
$\Rightarrow P(B) \le P(A)$ (proved)

(6) $P(A \cap B') = P(A) - P(A \cap B)$

Proof: $A = (A \cap B) \cup (A \cap B')$

$\Rightarrow P(A) = P(A \cap B) + P(A \cap B')$

[$\because$ $A \cap B$ and $A \cap B'$ are mutually exclusive events]



$\Rightarrow P(A \cap B') = P(A) - P(A \cap B)$ (proved)