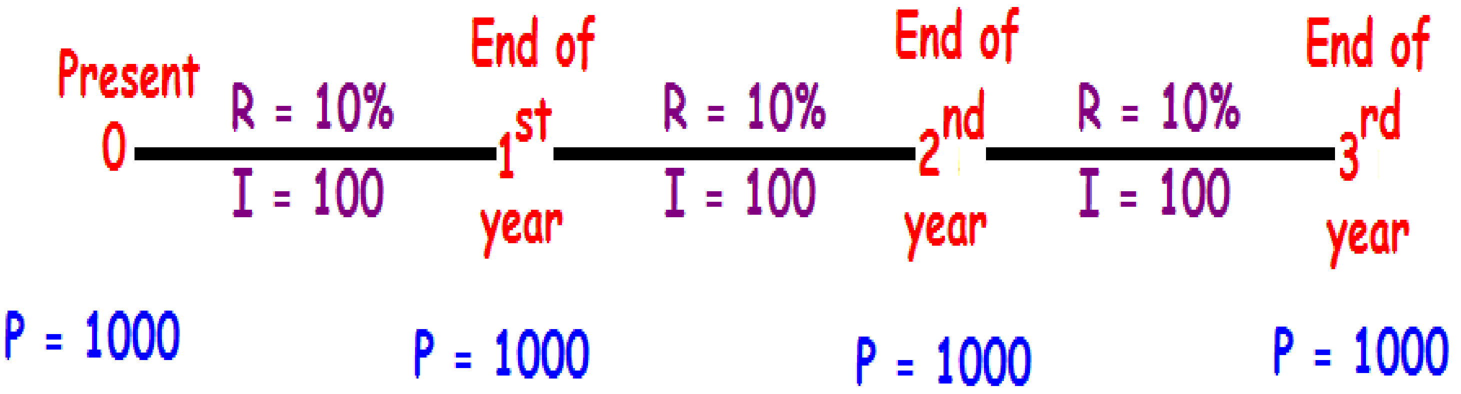
INTEREST FORMULAS AND ITS APPLICATION

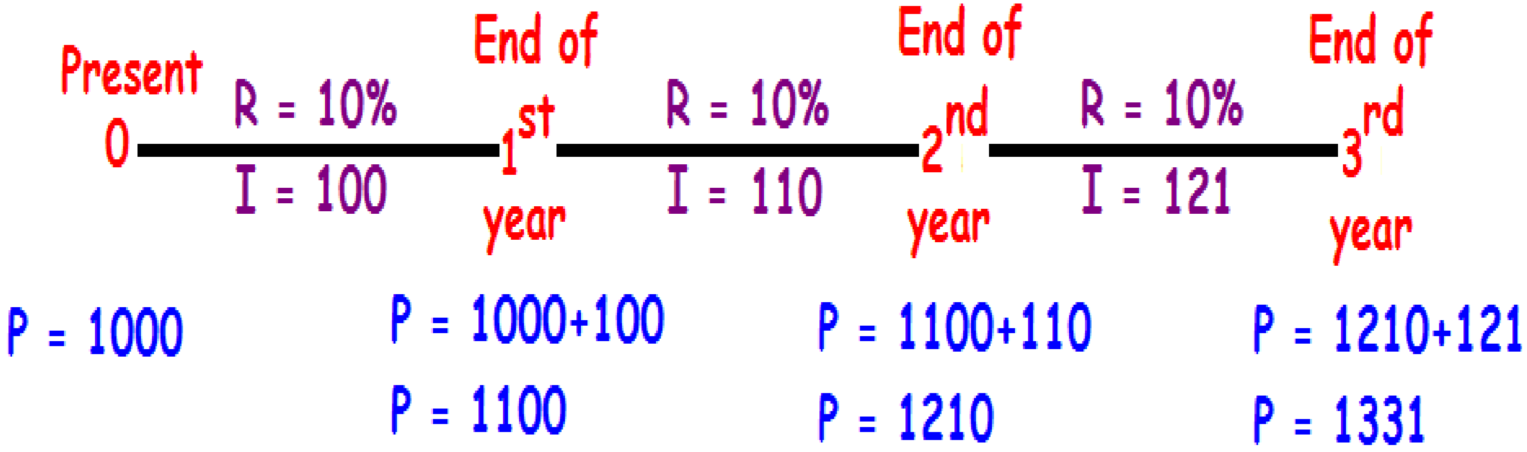
Simple Interest and Compound Interest

- What is the difference between simple interest and compound interest?
  - <u>Simple interest</u>: Interest is earned only on the principal amount.
  - <u>Compound interest</u>: Interest is earned on both the principal and accumulated interest of prior periods.

SIMPLE INTEREST :



COMPOUND INTEREST :



Difference between

SIMPLE INTEREST
&
COMPOUND INTEREST

Interest

Money

BANK

Borrower  Lender

Money + Interest %

Simple Interest

$SI = P \times R \times T$

$P = 1,00,000$
$R = 9\%$
$T = 2$ years

$SI = 1,00,000 \times \frac{9}{100} \times 2 \Rightarrow 18,000$

Total amount to be paid = P + SI => 1,00,000 + 18,000
1,18,000

Compound Interest

2 or more elements

Principal & Rate of interests

$P = 1,00,000$
$R = 9\%$
$T = 2$ years

$CI = P \times (1+R)^n$

$CI = 1,00,000 \times \left(1 + \frac{9}{100}\right)^2$

$CI = 1,00,000 \times (1.09)^2$

$CI = 1,00,000 \times 1.09 \times 1.09$

CI = 1,18,810

NOTATIONS FOR INTEREST FORMULAS

P = principal amount

n = No. of interest periods

i = interest rate (It may be compounded monthly, quarterly, semiannually or annually)

F = future amount at the end of year n

A = equal amount deposited at the end of every interest period

G = uniform amount which will be added/subtracted period after period to/ from the amount of deposit A1 at the end of period 1

Single-Payment Compound Amount

- Notation:
  - i = interest rate per compounding period
  - n = number of compounding periods
  - P = a present sum of money
  - F = a future sum of money

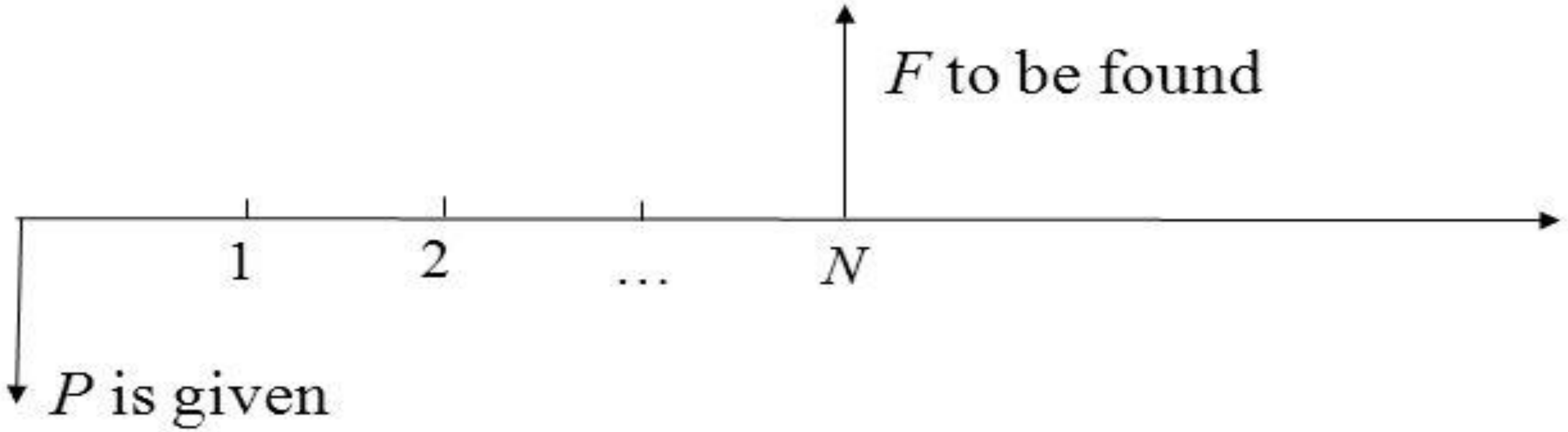
Single Payment Compound Amount Formula

$$F = P(1+i)^n \qquad \text{(Eq. 3-3)}$$

$$F = P(F/P, i, n) \qquad \text{(Eq. 3-4)}$$

The above notation is read as **Find F, given P, at *i*, over n**

Given P, to find F



- Formula: $F=P(1+i)^N$
- The term $(1+i)^N$, denoted by $(F/P, i\%, N)$, is called single payment compound amount factor
- Values of $(F/P, i\%, N)$ can be found in the appendix of the book

A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

Solution

$$P = \text{Rs. } 20{,}000$$
$$i = 18\% \text{ compounded annually}$$
$$n = 10 \text{ years}$$
$$F = P(1 + i)^n = P(F/P, i, n)$$
$$= 20{,}000\ (F/P,\ 18\%,\ 10)$$
$$= 20{,}000 \times 5.234 = \text{Rs. } 1{,}04{,}680$$

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

Single payment present worth factor (SPPWF):

- The single payment present worth factor is used to determine the present worth of a known future worth (F) at the end of "n" years at a given interest rate 'i' per interest period.
- The present worth (P), future worth (F) and the total interest period n years are shown in Fig. 1.8.

$$P = F\left[\frac{1}{(1+i)^n}\right] \qquad 8$$

- The factor $1/(1+i)^n$ in above equation is known as ***single payment present worth factor (SPPWF).***
- Thus if future worth (F) at the end of n years is known, the present worth (P) at interest rate of i (per year) can be calculated by multiplying the future worth with the single payment present worth factor.

A person wishes to have a future sum of Rs. 1,00,000 for his son’s education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

F = Rs. 1,00,000

i = 15%, compounded annually

n = 10 years

$P = F/(1 + i)n = F(P/F, i, n)$

= 1,00,000 (P/F, 15%, 10)

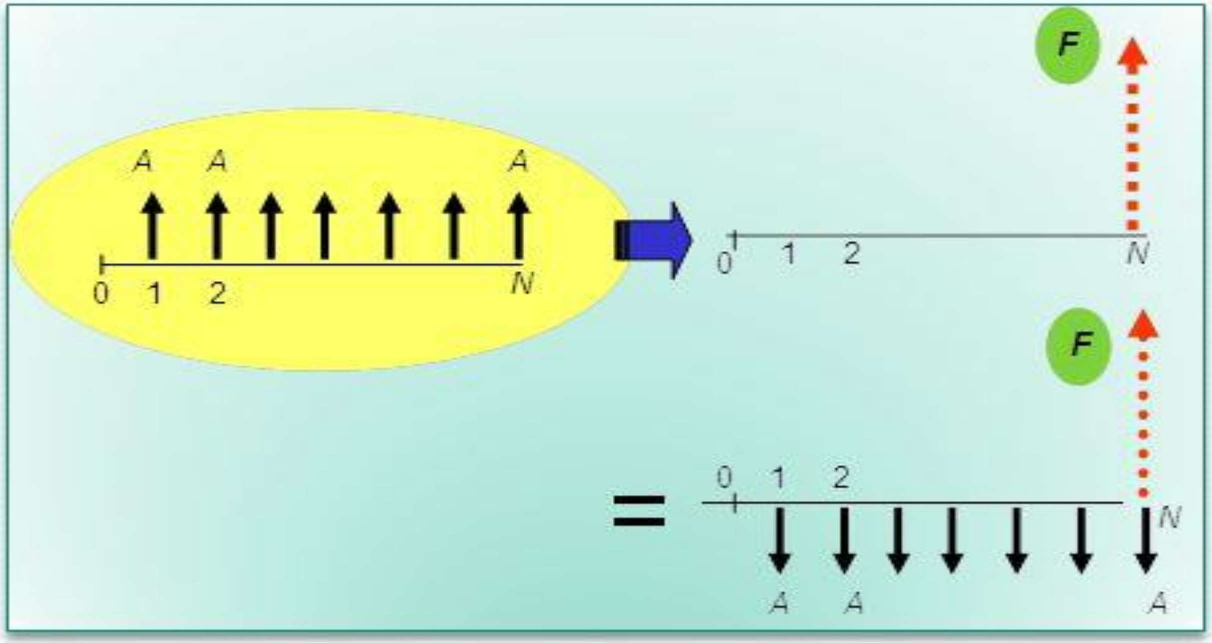
= 1,00,000 0.2472

= Rs. 24,720

The person has to invest Rs. 24,720 now so that he will get a sum of

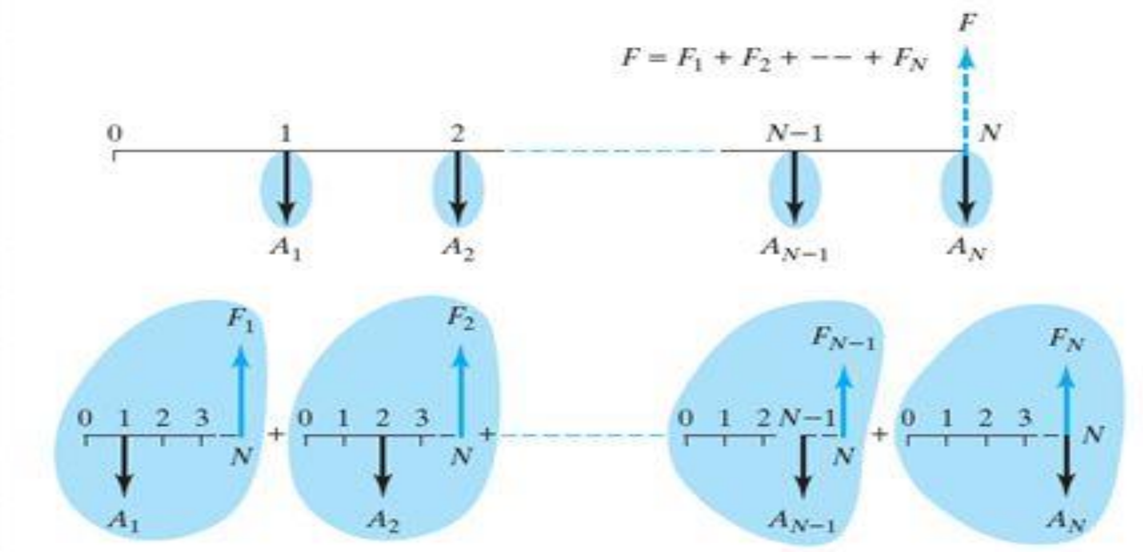
Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

Equal-Payment Series Compound Amount Factor



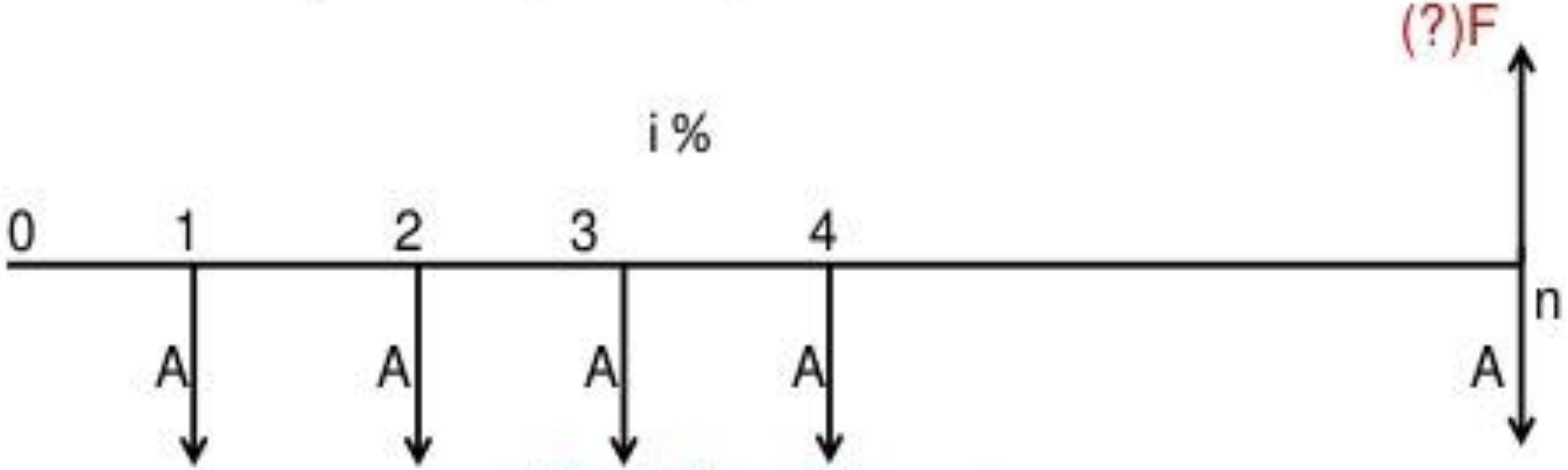
- **Formula**

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A, i, N)$$



Equal Payment Series Compound Amount

- Formula $F = A\frac{(1+i)^n - 1}{i} = A\,(F/A, i, n)$



Cash Flow Diagram

- A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs.10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20 % interest rate, compounded annually. Find the maturity value of his account when he is 60years old.

A = Rs. 10,000
n = 25 years
i = 20%
F = ?

The corresponding cash flow diagram is shown in Fig. 3.5.

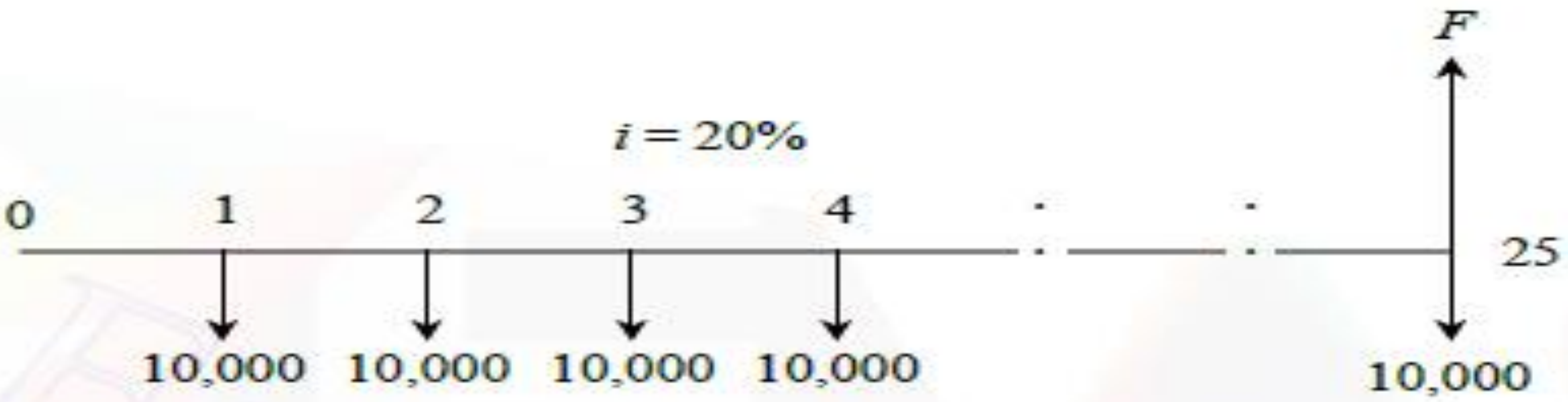


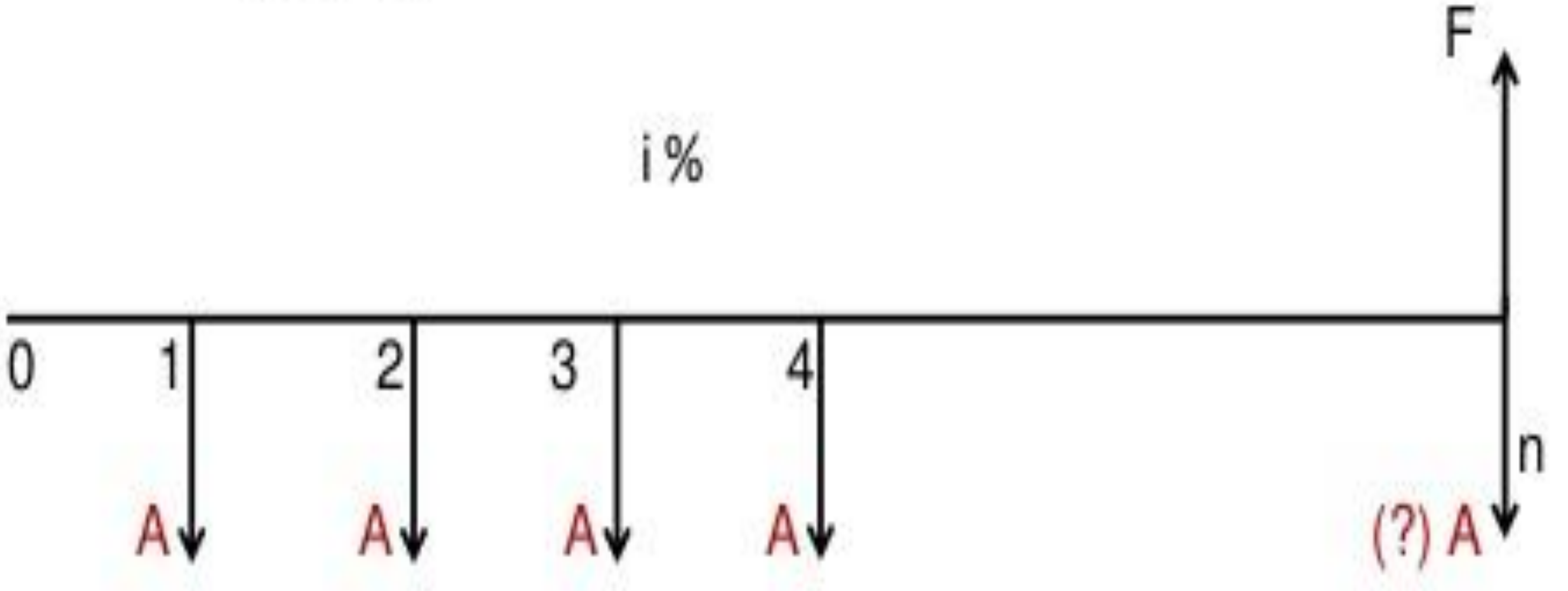
Fig. 3.5 Cash flow diagram of equal-payment series compound amount.

$$F = A\,\frac{(1+i)^n - 1}{i}$$
$$= A(F/A,\ i,\ n)$$
$$= 10{,}000(F/A,\ 20\%,\ 25)$$
$$= 10{,}000 \times 471.981$$
$$= \text{Rs. } 47{,}19{,}810$$

The future sum of the annual equal payments after 25 years is equal to Rs. 47,19,810.

Equal Payment Series Sinking Fund

- Formula $A = F \frac{i}{(1+i)^n - 1} = F\ (A/F, i, n)$



Cash Flow Diagram

EXAMPLE 3.4 A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

$A = Fi/[(1 + i)n - 1]$

$= F(A/F, i, n)$

$= 5,00,000(A/F, 18\%, 15)$

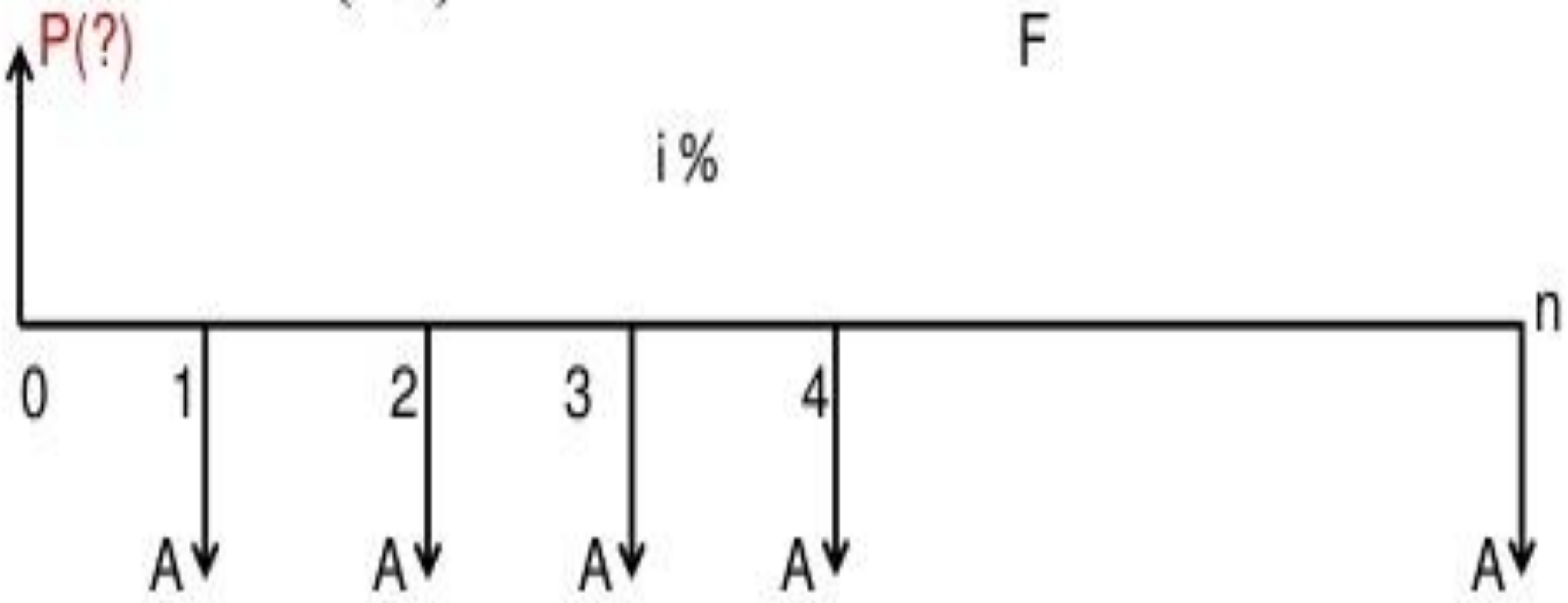
$= 5,00,000 \ \ 0.0164$

$=$ Rs. 8,200

The annual equal amount which must be deposited for 15 years is Rs. 8,200.

Equal Payment Series Present Worth Amount

- Formula $P = A\ \dfrac{(1+i)^n - 1}{i(1+i)^n} = A\,(P/A, i\ , n)$



Cash Flow Diagram

EXAMPLE 3.5 A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n)$$

$$= 10{,}00{,}000 \times (P/A, 15\%, 20)$$

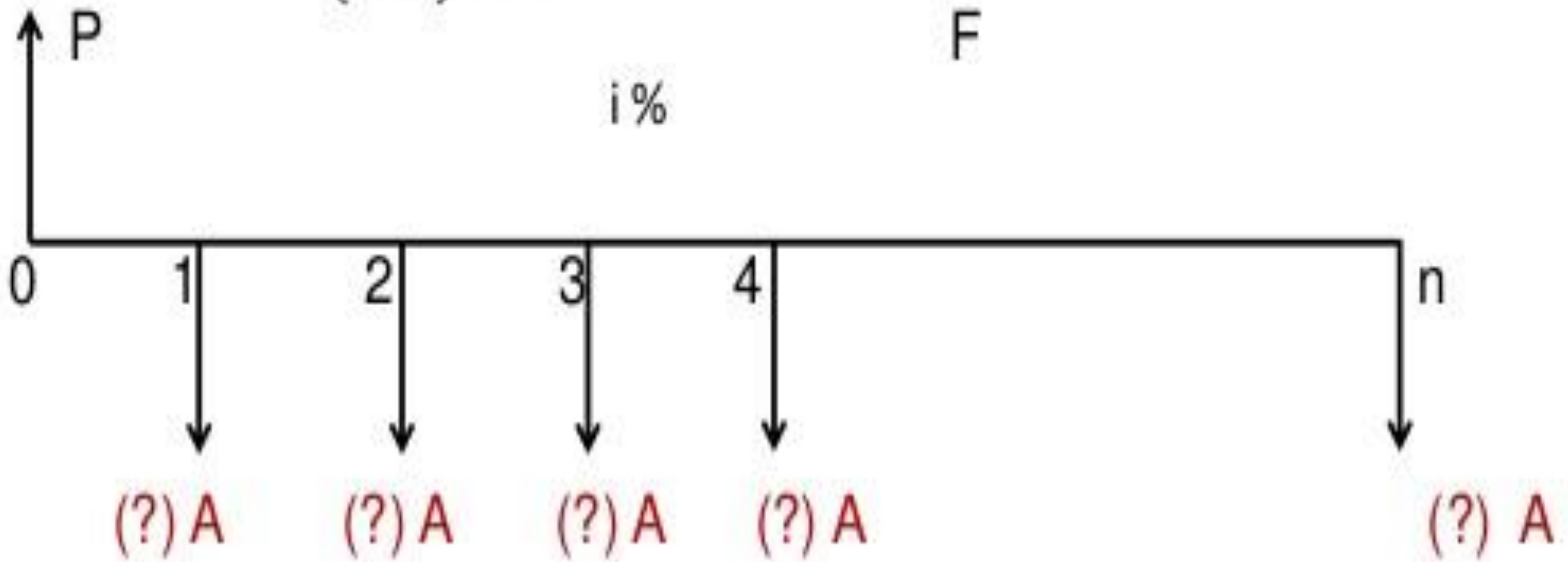
$$= 10{,}00{,}000 \times 6.2593$$

$$= \text{Rs. } 62{,}59{,}300$$

The amount of reserve which must be set-up now is equal to Rs. 62,59,300.

Equal Payment Series Capital Recovery Amount

- Formula $A = P \dfrac{i(1+i)^n}{(1+i)^n - 1} = P\,(A/P, i, n)$



Cash Flow Diagram

A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n)$$

$$= 10,00,000 \times (A/P,\ 18\%,\ 15)$$

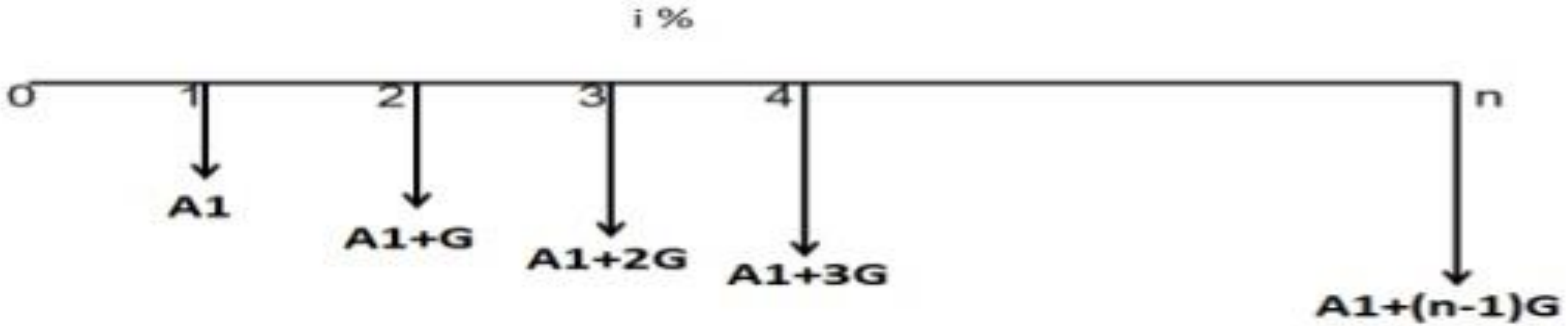
$$= 10,00,000 \times (0.1964)$$

$$= \text{Rs. } 1,96,400$$

The annual equivalent installment to be paid by the company to the bank is Rs. 1,96,400.

Uniform Gradient Series Annual Equivalent Amount

- Formula $A = A_1 + G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i} = A_1 + G(A/G, i, n)$



Two cases

Annual Increase $= A1 + G$, $A = A1 + G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i} = A_1 + G(A/G, i, n)$

Annual Decrease $= A1 - G$, $A = A1 - G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i} = A_1 + G(A/G, i, n)$

Step1= Find the Annual Equivalent Amount

Step2= Find the Future Worth, $F = A\,\frac{(1+i)^n - 1}{i} = A\,(F/A, i, n)$

3.3.7 Uniform Gradient Series Annual Equivalent Amount

The objective of this mode of investment is to find the annual equivalent amount of a series with an amount $A1$ at the end of the first year and with an equal increment (G) at the end of each of the following $n - 1$ years with an interest rate i compounded annually.

The corresponding cash flow diagram is shown in Fig. 3.12.

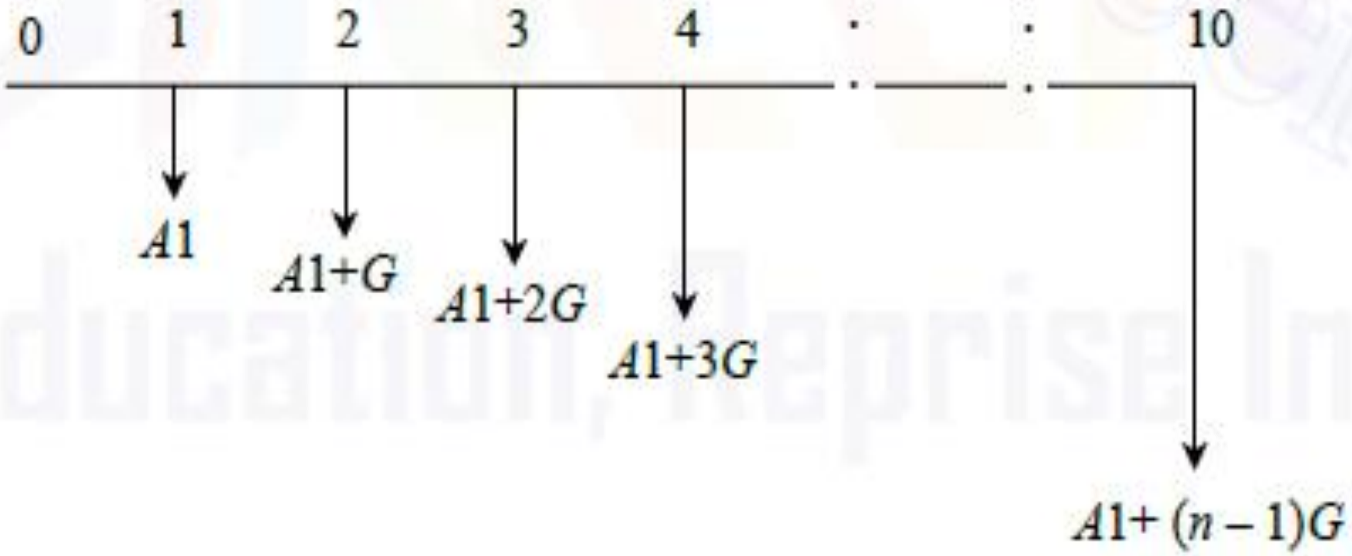


Fig. 3.12 Cash flow diagram of uniform gradient series annual equivalent amount.

Uniform Gradient Series Annual Equivalent Amount

- A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his Salary, which is Rs.4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of Rs.500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10^{th} year of the above series.

- A1=Rs.4,000, G=Rs.500, i=15%, n=10years, A=?, F=?

$$A = A1 + G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i} = A_1 + G(A/G, i, n)$$

$$A = 4{,}000 + 500\,(A/G, 15\%, 10) = Rs.5691.60$$

$$\text{Future Worth } F = F = A\,\frac{(1+i)^n - 1}{i} = A\,(F/A, i, n)$$

$$F = 5691.60(F/A, 15\%, 10)$$

$$F = 5691.60(20.304) = Rs.1{,}15{,}562.25$$

Nominal Versus Effective Interest Rates

Nominal Interest Rate:
Interest rate quoted based on an annual period

Effective Interest Rate:
Actual interest earned or paid in a year or some other time period

Effective Annual Interest Rate (Yield)

- **Formula**

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1$$

r = nominal interest rate per year
i_a = effective annual interest rate
M = number of interest periods per year

Monthly=12
Quarterly=4
Semi annually = 2

- **Example**
  - 18% compounded monthly

$$i_a = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 19.56\%$$

- **What it really means**
  - 1.5% per month for 12 months
  - 19.56% compounded once per year

A person invests a sum of Rs. 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.

METHOD 2

No. of interest periods per year, $C = 4$

Effective interest rate, $R = (1 + i/C)^C - 1$

$= (1 + 12\%/4)^4 - 1$

$= 12.55\%$, compounded annually.

$$F = P(1 + R)^n = 5{,}000(1 + 0.1255)^{10}$$

$= \text{Rs. } 16{,}308.91$