Optimization Techniques

Operations Research = operations + Research

Operations: The activities carried out in an organization

Research: The process of observation & testing characterized by the scientific method. It involves situation, Problem statement, Model construction, validation, experimentation and solution.

Operation Research: is a quantitative approach to decision making based on scientific methods of problem solving

The word operations research was coined by McClosky & Trefthen of U.K. in 1940. This concept came into existence ~~During world war II~~ in a military context.

During world war-II, the U.K. military management called on the scientists of different disciplines and organised them into teams to solve the strategical & tactical problems related to land/air defence. They were called with an objective to formulate the specific plans for the military commands to arrive at a decision with optimal utilization of military resources & efforts and implement them effectively.

The plan was a great success ~~and~~ which attracted scientists of various displines to apply the same in Manufacturing, service industries, Logistics, Transportation, Health care, situation with complexity, situations with uncertainty etc.

The optimization models are vastly used in computer science especially in software engineering & Network theory domain. Some important applications of operations research in computer science are:

a) Simulation b) Resource allocation
c) Data Mining d) Network routing
e) pattern recognition f) queuing theory etc.

Some operations research model are

- Linear programming
- Integer programming
- Non-Linear programming
- Dynamic programming
- Network programming etc.

optimization Techniques are the set of powerful tools to solve the complex real-world problems. The application domain has grown manifold in recent years covering almost all engineering and science disciplines. Many new algorithms are designed by taking inspiration from different natural phenomena. One of the most popular algorithm amongst them is Genetical algorithm.

<u>Linear programming problem</u>: deals with the optimization (maximization/minimization) of functions (known as objective functions) of variables (known as decision variables) subject to a set of linear equalities and/or inequalities (known as constraints) satisfying the non-negativity restrictions.

<u>Formulation of Linear programming problems (LPP)</u>:

The procedure for mathematical formulation of a LPP involves the following steps.

<u>Step-1</u>: To identify & to write the decision variables of the problem.

<u>Step-2</u>: To formulate the objective function to be optimized (maximized/minimized) as a linear function of decision variables.

<u>Step-3</u>: To formulate the constraints of the problem such as resource limitation, market constraints, time constraints, ~~market~~ interrelations between the variables etc. as linear inequalities and/or equalities of the decision variables.

<u>Step-4</u>: To add the non negativity constraints so that the -ve values of the decision variables do not have any valid physical interpretation.

General formulation of LPP:

The general formulation of the LPP can be stated as follows:

$$\max.\ (\text{or } \min.)\ z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \quad \text{(objective function)}$$

subject to

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \ (\leq = \geq)\ b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \ (\leq = \geq)\ b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \ (\leq = \geq)\ b_m \end{array}\right\} \text{(constraints)}$$

satisfying

$$x_1 \geq 0,\ x_2 \geq 0,\ \cdots,\ x_n \geq 0 \quad \text{(Non negative restrictions)}$$

Example: Reddy Mikks produces both interior & exterior paints from two raw materials, M_1 and M_2. The following table provides the basic data of the problem:

Raw Material	Tons of raw Material per ton of Exterior Paint	Tons of raw Material per ton of Interior Paint	Maximum daily availability (tons)
M_1	6	4	24
M_2	1	2	6
Profit per ton ($1000)	5	4	

The daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton. Also, the maximum daily demand for interior paint is 2 tons. Reddy Mikks wants to determine the optimum product mix of interior and exterior paints that maximizes the total daily profit. (Formulate the problem as a LPP)

Ans: Let x_1 be the tons of exterior paint produced daily and x_2 be the tons of interior paint produced daily.

From the given basic data of the problem, it is clear that

Profit from exterior paint $= 5x_1$ (thousand) dollars and

Profit from interior paint $= 4x_2$ (thousand) dollars.

Let z be the total daily profit (in thousand dollars) and the objective of Reddy Mikks is to maximize the total daily profit of both paints, the objective function is

$$\max. \ z = 5x_1 + 4x_2$$

The daily usage of raw material M_1 is 6 tons/ton of exterior paint and 4 tons/ton of interior paint, where the maximum availabity of M_1 is 24 tons. Thus,

$$6x_1 + 4x_2 \leq 24 \quad \text{(Resource limitation)}$$

Similarly, the daily usage of raw material M_2 is 1 ton/ton of exterior paint and 2 tons/ton of interior paint, where the maximum availability of M_2 is 6 tons. Thus,

$$x_1 + 2x_2 \leq 6 \quad \text{(Resource limitation)}$$

Since, the daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton, the daily production of interior paint can not exceed that of exterior paint by more than 1 ton. Thus,

$$x_2 - x_1 \leq 1 \quad \text{(Market constraint)}$$

Again since the maximum daily demand for interior paint is 2 tons, so

$$x_2 \leq 2 \quad \text{(Demand constraint)}$$

As the number of tons of exterior/interior paints to be produced can never be negative, so

$$x_1, x_2 \geq 0. \quad \text{(Non -ve restrictions)}$$

Thus, the LPP is

$$\max. \ z = 5x_1 + 4x_2$$

subject to

$$6x_1 + 4x_2 \leq 24$$
$$x_1 + 2x_2 \leq 6$$
$$x_2 - x_1 \leq 1$$
$$x_2 \leq 2$$
$$x_1, x_2 \geq 0.$$

Example:

A company produces two products A and B. The sales volume for A is atleast 80% of the total sales of both A and B. However, the company cannot sell more than 110 units of A per day. Both the products use one raw material, of which the maximum daily availability is 300 lb. The usage rates of the raw material are 2lb per unit A, and 4lb per unit of B. The profit units for A and B are \$40 and \$90, respectively. Formulate a LPP to determine the optimal product mix for the company.

Ans: Let

x_1 = number of units of product A

and

x_2 = number of units of product B.

As the company makes a profit of \$40/unit of product A and \$90/unit of product B and the company's objective is to maximize its profit, the objective function is

$$\max.\ Z = 40x_1 + 90x_2$$

The sales volume for A is atleast 80% of the total sales of both A and B. Thus,

$$x_1 \geq \frac{80}{100}(x_1 + x_2) \qquad \text{Market constraint}$$

$$\Rightarrow -0.2x_1 + 0.8x_2 \leq 0 \qquad (\cancel{\text{resource limitation}})$$

Since, the company cannot sell more than 110 units of A/day, so,

$$x_1 \leq 110 \qquad (\text{Market constraint})$$

usage rates of raw material are 2lb/unit of A and 4lb/unit of B, where the maximum availability is 300 lb. so,

$$2x_1 + 4x_2 \leq 300 \qquad (\text{resource limitation})$$

Also, the no. of units of product A and B cannot be -ve, so

$$x_1, x_2 \geq 0. \qquad (\text{Non -ve restrictions})$$

Thus, the LPP is

$$\max.\ Z = 40x_1 + 90x_2$$

subject to

$$\begin{aligned} -0.2x_1 + 0.8x_2 &\leq 0 \\ x_1 \quad\quad &\leq 110 \\ 2x_1 + 4x_2 &\leq 300 \\ x_1, x_2 &\geq 0 \end{aligned}$$