

Joint probability distributions and Random samples

Chapter 5

Let X and Y be two discrete r.v's defined on the sample space S of an experiment. The joint probability mass function $p(x, y)$ is defined for each pair of numbers (x, y) by

$$p(x, y) = P(X=x \text{ and } Y=y).$$

It must satisfy the following conditions;

i) $p(x, y) \geq 0$

ii) $\sum_x \sum_y p(x, y) = 1.$

The (marginal) probability mass function of X , denoted by $p_x(x)$, is given by

$$p_x(x) = \sum_{y: p(x, y) > 0} p(x, y) \text{ for each possible value of } x.$$

Similarly, the marginal probability mass function of Y , denoted by $p_y(y)$, is given by

$$p_y(y) = \sum_{x: p(x, y) > 0} p(x, y) \text{ for each possible value } y.$$

E.g. A service station has both self-service and

full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let

X denote the number of hoses being used on the self-service island at a particular time, and let

Y denote the number of hoses on the full-service

island in use at that time. The joint pmf of X and Y appears in the accompanying tabulation.

x	$p(x,y)$		
	0	1	2
0	0.1	0.04	0.02
1	0.08	0.2	0.06
2	0.06	0.14	0.3

a) Find $P(X=1 \text{ and } Y=1) = p(1,1) = 0.2$

b) Find $P(X \leq 1 \text{ and } Y \leq 1)$

" $P(X \leq 1 \text{ and } Y \leq 1) = P((X=0, Y=0), (X=0, Y=1), (X=1, Y=0), (X=1, Y=1))$

" $p(0,0) + p(0,1) + p(1,0) + p(1,1)$

" $0.1 + 0.04 + 0.08 + 0.2 = 0.42$

c) Word description of the event $(X \neq 0 \text{ and } Y \neq 0)$ and compute the probability of this event.

$(X \neq 0 \text{ and } Y \neq 0) =$ At least one of the hoses is used on both islands.

$P(X \neq 0 \text{ and } Y \neq 0) = p(1,1) + p(1,2) + p(2,1) + p(2,2)$

" $= 0.2 + 0.06 + 0.14 + 0.3$
" $= 0.7$

d) Compute the marginal pmf of X and of Y .

Using $p_X(x)$, what is $P(X \leq 1)$?

$p_X(x) = \sum_{y \in \{0,1,2\}} p(x,y) = p(x,0) + p(x,1) + p(x,2)$

" $p_X(0) = 0.1 + 0.04 + 0.02 = 0.16$

$$p(x,1) = 0.08 + 0.2 + 0.06 = 0.34$$

$$p(x,2) = 0.06 + 0.14 + 0.3 = 0.5$$

$$\therefore p_x(x) = 0.16 + 0.34 + 0.5 = 1.0$$

$$\begin{aligned} p_y(y) &= \sum_{x \in \{0,1,2\}} p(x,y) = p(0,y) + p(1,y) + p(2,y) \\ &= 0.24 + 0.38 + 0.38 \end{aligned}$$

$$P(X \leq 1) = p_x(0) + p_x(1) = 0.16 + 0.34 = 0.5.$$

Defn Two random variables X and Y are said to be Independent if for every pair of x and y values

$$p(x,y) = p_x(x) \cdot p_y(y)$$

when X and Y are discrete.

If X and Y not satisfied the above condition for all (x,y) , then X and Y are said to be dependent.

e) Are X and Y independent rv's.

$$\text{We have, } p_x(2) = 0.5, \quad p_y(2) = 0.38$$

$$p_x(2) \times p_y(2) = 0.5 \times 0.38 = 0.19.$$

$$p(2,2) = 0.3.$$

$$\therefore p_x(2) \cdot p_y(2) \neq p(2,2).$$

\therefore Therefore, X and Y are dependent rvs.

Selected questions.

1, 2, 3, 6, 9, 11, 12, 19.