Semester:	•••
Programme:	
Branch/Specialization:	

Full Marks: 50



## **SPRING END SEMESTER EXAMINATION-2023**

4<sup>th</sup> Semester, B.Tech (Programme)

## SUBJECT-DMS MA 2013

(For 2021-22 Admitted Batches)

Time: 3 Hours

Answer any SIX questions.

Question paper consists of four SECTIONS i.e. A, B, C and D.

Section A is compulsory.

Attempt minimum one question each from Sections B, C, D.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

	SECTION-A (Learning levels 1 and 2)		Learning levels as per Bloom's taxonomy	Course Outcomes (CO)	
1.		Answer the following questions.	[1×10]		
	(a)	Write the negation of the statement $p \land (q \rightarrow r)$ .		LL 1	CO1
	(b)	State the rule of inference used in this argument: "If it rains, I shall not go to school", "if I don't go to school, I won't need to do homework", Therefore if it rains, I won't need to do homework.		LL 2	CO1
	(c)	Write the logical translation of the statement "None of my friends are perfect".		LL 2	CO1
	(d)	Let $n(A) = m$ , $n(B) = n$ , then what is the total number of nonempty relations that can be defined from A to B?		LL 2	CO2
	(e)	A relation R on a set of real numbers defined as $R = \{(a, b): a \le b^2\}$ . Is it reflexive, symmetric, and transitive?		LL 2	CO3
	(f)	Find the maximal and minimal elements of the poset ({2,4,5,10,12,}, I).		LL 1	CO3
	(g)	Write the recurrence relation capturing the optimal execution time of the tower of Hanoi problem with $n$ disk.		LL 1	CO4
	(h)	Let $A = \{1,2,3,\dots\infty\}$ and a binary operation " + " is defined by $a + b = ab, \forall a, b \in A$ . Is $(A, +)$ a semigroup, monoid or group?		LL 2	CO5
	(i)	Let $G = \{1, -1, i, -i\}$ is group under multiplication, then find the inverses of 'i' and '-i'.		LL 2	CO5
	(j)	What is an integral domain? Give a suitable example.		LL 1	CO6
		SECTION-B (Learning levels 1,2, and 3)		Learning levels as per Bloom's	Course Outcomes (CO)

				taxonomy	
2.	(a)	Show that $\sim (p \leftrightarrow q)$ and $p \leftrightarrow \sim q$ are logically equivalent.	[4]	LL 2	CO 1
	(b)	Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with initial conditions $a_0 = 2$ , $a_1 = 5$ , $a_2 = 15$ .	[4]	LL 2	CO 4
3.	(a)	Using Principle of inclusion and exclusion find the sum of all integers from 1 to 100 that are multiple of 2 or 3.	[4]	LL 2	CO2
	(b)	Shoe that residue classes modulo 5 is a group with respect to addition.	[4]	LL 2	CO 5
		SECTION-C (Learning Levels 3 and 4)		Learning levels as per Bloom's taxonomy	Course Outcomes (CO)
4.	(a)	Check the validity of the argument: "All clear explanations are satisfactory"; "Some excuses are unsatisfactory". "Hence some excuses are not clear explanation".	[4]	LL 3	CO 1
	(b)	Find the transitive closure of the relation using Warshall's algorithm. $R = \{(a, b), (b, d), (d, a), (d, c)\}$ on the set $A = \{a, b, c, d\}$ .	[4]	LL 4	CO 2
5.	(a)	Prove by induction that for each natural number $n$ , the number $2^{2n} - 1$ is a multiple of 3.	[4]	LL 3	CO 1
	(b)	Define a Ring, zero divisor in a ring. Show that $Z_{10}$ is ring and find it's zero divisors.	[4]	LL 4	CO 6
6.	(a)	Using strong mathematical induction, prove that if $n$ is an integer greater than 1, then it is either a prime or can be written as the product of primes.	[4]	LL 3	CO 1
	(b)	Let R be a relation on Q defined by	[4]	LL 4	CO3
		$R = \{(a, b)   2(a - b) \in Z\}$			
		Show that $R$ is an equivalence relation and find the equivalence classes of $[0]$ and $\left[\frac{1}{4}\right]$ .			
		SECTION-D (Learning levels 4,5,6)		Learning levels as per Bloom's taxonomy	Course Outcomes (CO)
7.	(a)	Use truth table and an explanation to prove that Modus ponens rule of inference is a valid form of an argument.	[4]	LL 5	CO1
	(b)	Show that $(P(S), \subseteq)$ is a POSET and construct the Hasse diagram where $P(S)$ is the power set of $S = \{1,2,3\}$ .	[4]	LL 6	CO 3
8.	(a)	Solve the recurrence relation $a_n = 3a_{n-1} + 10a_{n-2} + 7.5^n$ with $a_0 = 4$ , $a_1 = 3$ .	[4]	LL 5	CO 4

(b)	If G is a group such that $(a.b)^2 = a^2.b^2$ for all $a, b \in G$ , then show that G must be abelian group.	[4]	LL 6	CO 5
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Signature of Paper Setter/Moderator:Dr Akshaya Kumar Panda
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