# Waves and Interference

**Notes for**

**B.Tech Physics Course**

**(PH-1007)**

**2020-21**

**Department of Physics**

**School of Applied Sciences**

**Kalinga Institute of Industrial Technology**

**Bhubaneswar, India**

**Contents**

## 1. Introduction to waves

A wave is basically a disturbance or variation that transfers energy progressively from one point to another in a medium that may take the form of elastic deformation or variation of pressure, electric or magnetic field intensity, electric potential or temperature.

Fig.1. Ripples on the surface of water on throwing a stone

For example – a stone dropped on the surface of water produces ripples on its surface (Fig. 1). In such a wave, the particles on the surface of water vibrate up and down from their mean/equilibrium position and the waves travel outward from the point of origin. Some other examples include sound waves, light waves, waves on a slinky etc.

Watch video:

https://www.youtube.com/watch?v=KWzyQKcJBYg&list=RDCMUCiX8pAYWBppIbtUZTfGnRJw&start_radio=1&t=193

### 1.1 Characteristics of wave motion

Some of the important characteristics of wave motion are listed below:

1. It is a disturbance produced in a medium due to the repeated periodic motion of the particles of the medium.
2. In the wave motion, wave travels in the forward direction while particles of the medium vibrate about their mean positions (Fig. 2(a)).

3. There is a regular phase change between the particles of the medium.
4. The velocity of wave is different from the velocity of the particle. The velocity of the wave is uniform while the velocity of particle is different at different positions.

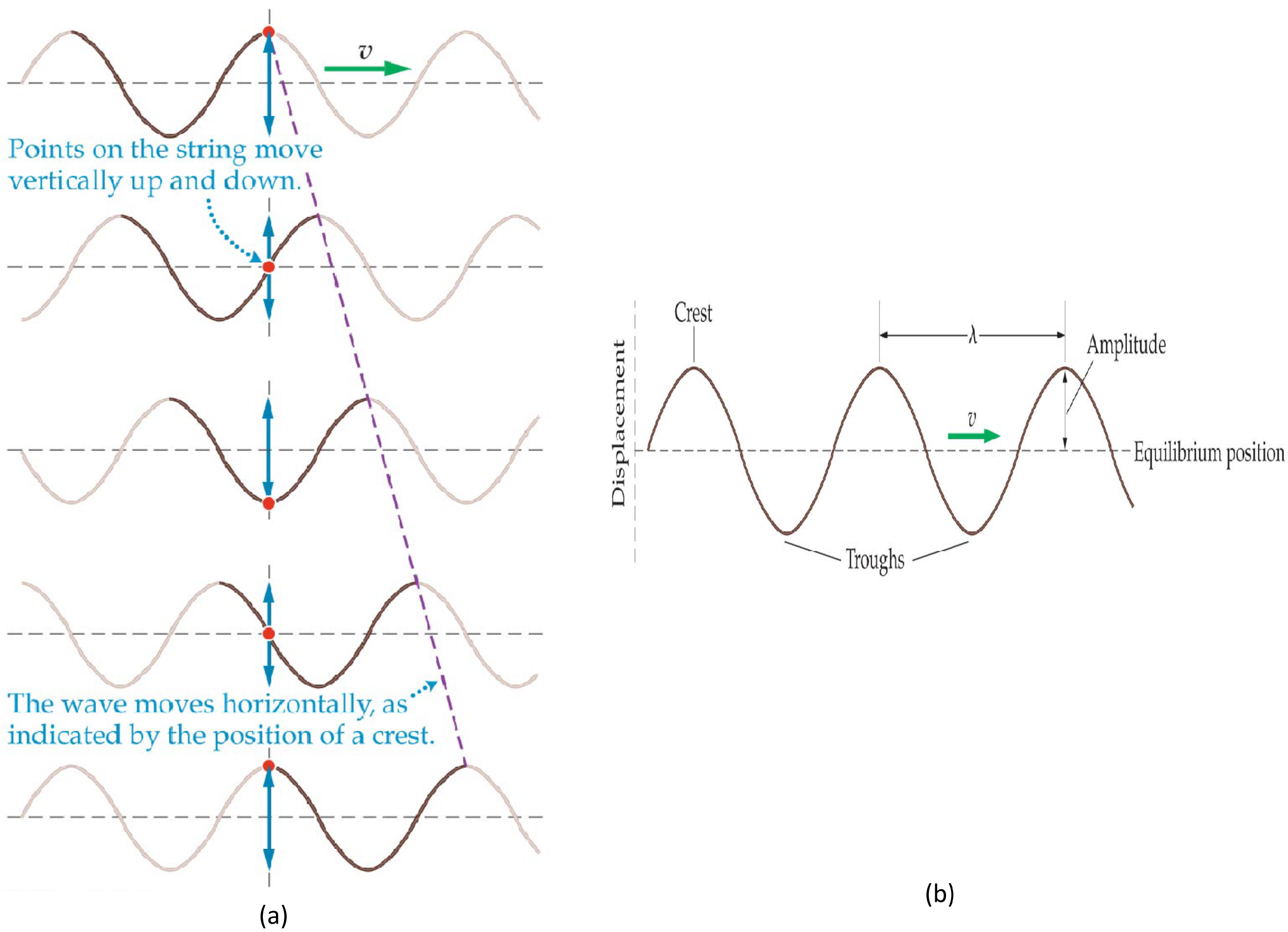



Fig. 2 (a) Motion of a wave (b) Different attributes of waves

## 1.2 Types of waves

There are basically two important types of waves:

A. Longitudinal waves-
   - These are the disturbances in a medium in which the particles vibrate about their mean position in a direction parallel to the direction of wave propagation.
   - These waves are made up of compressions and rarefactions (Fig. 3).

➢ They can travel through solids, liquids and gases.

➢ Sound waves are longitudinal in nature.

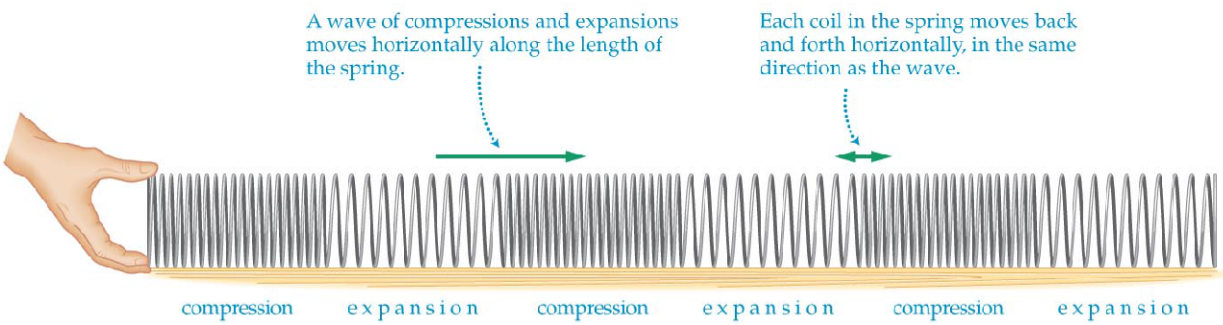



Fig. 3 Compressions and rarefactions in longitudinal waves

B. Transverse waves

➢ These are disturbances in a medium in which particles vibrate about their mean position in a direction perpendicular to the direction of wave propagation.

➢ These waves are made up of crests and troughs (Fig. 4).

➢ They can travel mostly in solids and liquids but not gases.

➢ Light waves are transverse in nature.

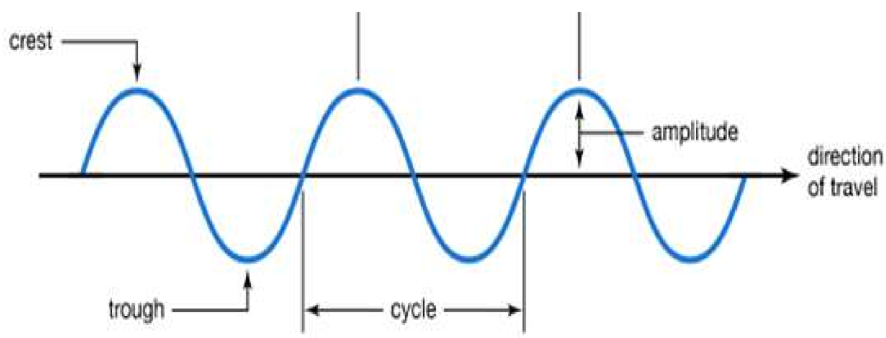



Fig. 4 Crests and troughs in transverse waves

Watch the video for types of waves: https://www.youtube.com/watch?v=RVyHkV3wIyk

## 1.3 The wave equation

Consider the displacement of a particle at a point 'P' showing simple harmonic oscillation is given by,

$y = asin\omega t$……………………………………………………………………………...(1)

Let there be another particle at 'Q' at a distance 'x' from 'P' and the wave is travelling with a velocity 'v' from P to Q, then the displacement of the particle at 'Q' may be given as

$y = asin(\omega t - \varphi)$…………………………………………………………………………(2)

where $\varphi$ is the phase difference between the particles 'P' and 'Q'.

$\varphi = \frac{2\pi}{\lambda} \times path\ difference = \frac{2\pi x}{\lambda}$ ……………………………………………………...(3)

By definition of angular frequency,

$\omega = \frac{2\pi}{T} = 2\pi f = \frac{2\pi v}{\lambda}$ ………………………………………………………………….(4)

Substituting equations (3) and (4) in equation (2), we get

$y = asin\left(\frac{2\pi}{\lambda}t - \frac{2\pi}{\lambda}x\right)$……………………………………………………………………(5)

Or, $\boxed{y = asin\frac{2\pi}{\lambda}(vt - x)}$ …………………………………………………………………(6)

Equation (6) represents the equation of displacement of a particle when the wave is travelling in +x direction.

If the wave is travelling in –x direction such that the particle at Q is at a distance x in the negative direction, then the equation of displacement will be

$\boxed{y = asin\frac{2\pi}{\lambda}(vt + x)}$ ……………………………………………………………………(7)

## 1.4 Differential equation of wave motion

Considering equation (6),

$y = asin\frac{2\pi}{\lambda}(vt - x)$, and differentiating it wrt time, we get

$\frac{dy}{dt} = \frac{2\pi av}{\lambda}cos\frac{2\pi}{\lambda}(vt - x)$……………………………………………………………………(8)

Differentiating equation (8) again wrt time,

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 a v^2}{\lambda^2} sin\frac{2\pi}{\lambda}(vt - x) \qquad \text{.....(9)}$$

Again double differentiating equation (6) wrt distance x,

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} cos\frac{2\pi}{\lambda}(vt - x) \qquad \text{.....(10)}$$

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2 a}{\lambda^2} sin\frac{2\pi}{\lambda}(vt - x) \qquad \text{.....(11)}$$

Comparing equations (9) and (11), we get,

$$\boxed{\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}} \qquad \text{.....(12)}$$

Equation (12) represents the general differential wave equation in one dimension where $v$ represents the **wave velocity**.

## 2. The concept of interference of light

Fig. 5 Beautiful colors on the neck and belly of Humming bird and; brilliant colors in the peacock feathers

The beautiful colors observed in the feathers of humming bird are not due to pigment (Fig. 5). The iridescence that makes brilliant colors often appear in the neck and belly is due to the interference effects caused by the structure in the feathers. The colors may also vary with the viewing angle. Similarly the bright colors of peacock feathers are also due to interference (Fig. 5). The structures in the peacock feathers split and recombine the visible light so that interference occurs for certain colors. Others examples of interference are colors on a soap bubble, thin oil film on the surface of water etc.

By now we are familiar with the fact that interference usually occurs due to the interaction of light waves. So let us see how the waves interact and what the principle of superposition is.

## 2.1 Interacting waves and the principle of superposition

When two or more waves interact with each other, they can combine to give a resultant wave following the principle of superposition. The **principle of superposition** states that

"The resultant displacement of a particle of a medium when acted upon by two or more waves simultaneously is the algebraic sum of the displacements of the same particle due to individual waves in the absence of others." Fig. 6 and 7 show constructive and destructive interference.

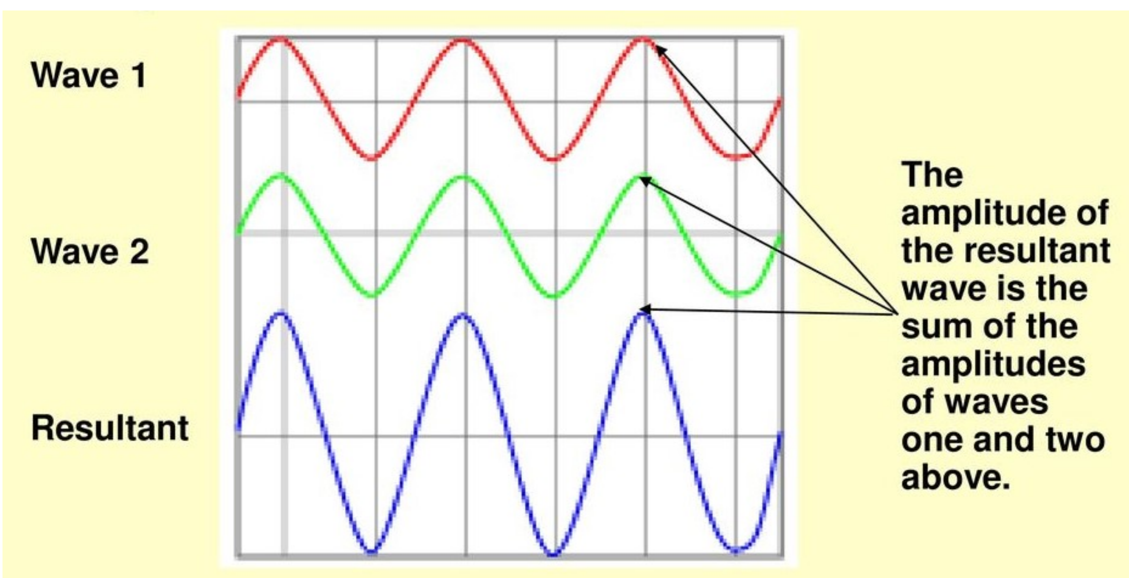



Fig. 6. Constructive Superposition

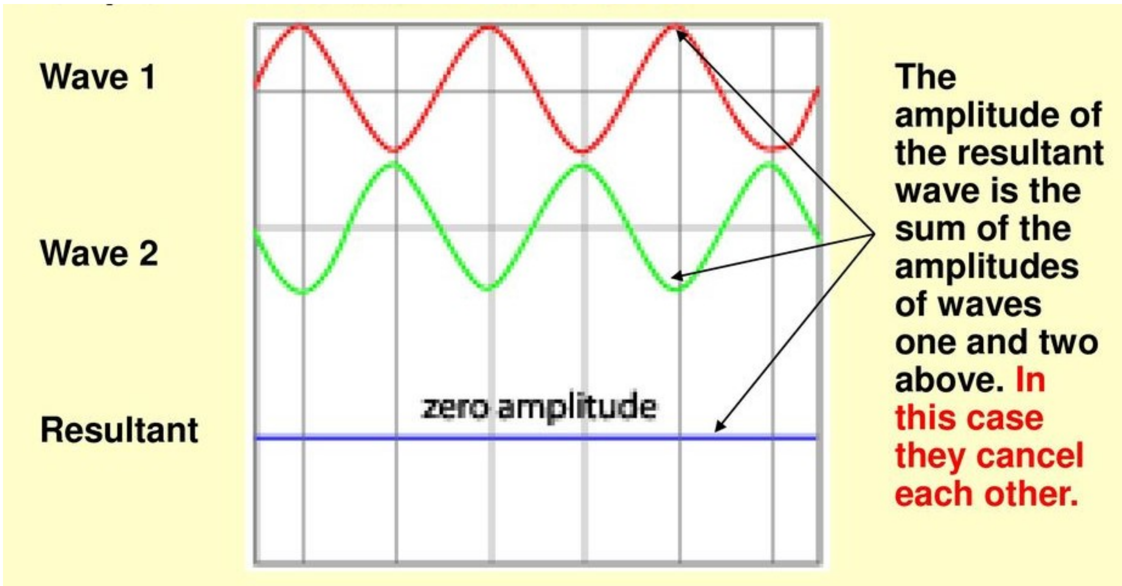



Fig. 7 Destructive Superposition

There can be different types of superposition such as interference, beats and standing waves etc.

## 2.2 Interference

When two or more waves of the same frequency travel in approximately the same direction and maintain a phase difference that remains constant with time, then the resultant intensity of light is not distributed uniformly in space. This non-uniform distribution of light intensity due to superposition of two or more waves is called **interference**. Due to interference, alternate regions of bright and dark bands are formed which correspond to maxima and minima. When the amplitude of the resultant wave at a given position or time is greater than that of the either individual wave, it is called **constructive interference** while in **destructive interference** resultant amplitude is less than that of either individual wave.

## 2.3 Coherent Sources of light

Have you ever thought why don't we observe interference effects when two light bulbs are placed side by side? It is because the light waves from one bulb are emitted independently of those from the other bulb. The emissions from the two bulbs do not maintain a constant phase relationship with each other over time. Light waves from an ordinary source such as a light bulb/

sun/ glowing candle etc undergo random phase changes in time intervals less than a nanosecond. Therefore, the conditions of constructive and destructive interference or some intermediate state are maintained only for such short time intervals. Because our eye cannot follow such rapid changes, no interference effects are observed. Such light sources are said to be **incoherent**.

So, two or more sources are said to be **coherent** if their relative phases do not change with time. This is possible if either the phase of each source remains constant in time or changes by the same amount so that the phase difference between remains constant over time.

## 2.4 Practical realization of coherent sources

Although ordinary sources of light are incoherent, it is always possible to obtain effective coherent sources from an individual source by the following arrangement:

- A narrow beam of light originating from a given source can be split in to two or more components by reflection, refraction or other suitable processes.
- The resulting components are allowed to travel different optical path lengths so that an optical path difference is introduced between them.
- The components finally superpose in a region to produce interference.

Most of the experimental arrangements for the study of interference, design of interferometers and even the interference phenomena usually observed in nature are based on this type of effective coherent sources.

## 2.5 Types of interference based on the production of effective coherent sources

Depending on the way in which the original light wave splits into its components for the realization of coherent sources, interference can be classified as:

**Division of wave front**

Wave front from the original source is split in to two or more parts by suitable arrangements. This is usually achieved by reflection, refraction or other methods. One part of the wave front travels along one path while the other part travels another optical path. A path difference is introduced between the two parts of the wave front before they superpose. For example, in Young's double slit experiment, two coherent light sources are produced from a single monochromatic source by placing a barrier with two openings (in the form of slits). The light emerging from the two slits is coherent because a single source produces original light beam and two slits serve only to separate the original light beam in to two parts (Fig. 8). Other examples are Fresnel's biprism, Lloyd's single mirror and bimirror.

**Division of amplitude**

Interference effects observed commonly in thin films such as thin layers of oil on water or the thin surface of a soap bubble are usually due to division of amplitude. The amplitude (or intensity) of the incident light undergoing multiple reflections and refractions on the upper and lower surfaces of the thin film is divided. Subsequently, a path difference is introduced between the reflected and refracted parts of light beam as they travel through different path lengths inside the film as shown in Fig. 8 and superpose to form the interference pattern. Examples of such type of interference are Newton's rings experiment, Michelson's interferometer.

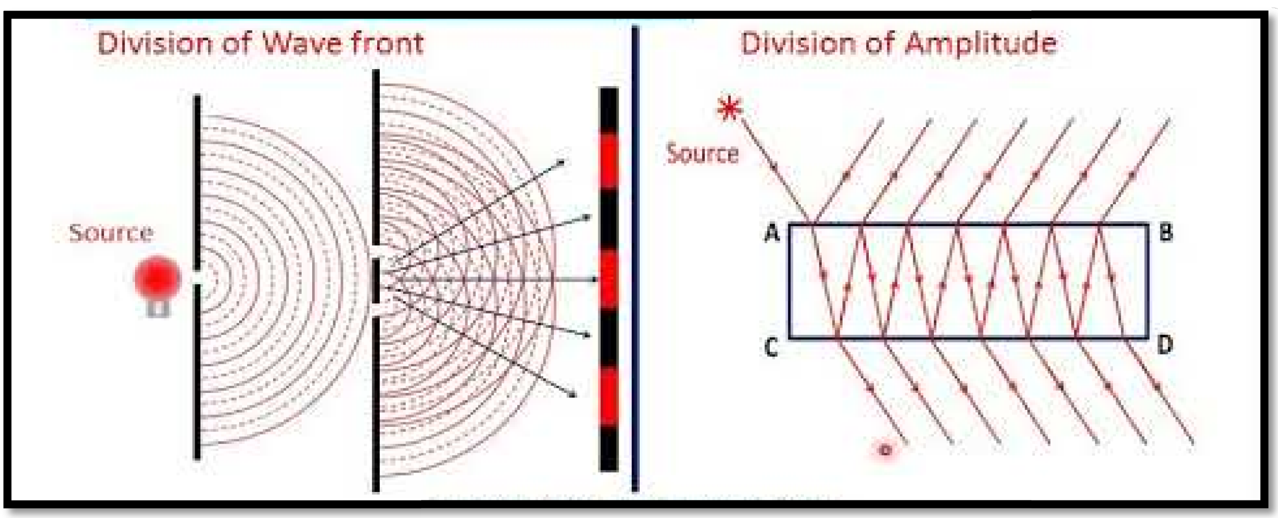



Fig. 8 A schematic comparison between interference by division of wave front and division of amplitude

## 2.6 Conditions of interference

The conditions of interference may be broadly categorized in to three groups:

1. For sustained interference:
    - The interfering waves must be coherent.
    - Frequency of the waves must be same.
    - The waves must be in the same state of polarization.
2. For clear observation of fringes:
    - The distance between the sources and the screen must be large.
    - The distance between the sources must be small.
    - The background must be black.
3. For better contrast of fringes
    - The amplitude of the waves must be equal or nearly equal (though this is not a necessary condition of interference).
    - The light source must be monochromatic.
    - A narrow source of light may be used.

## 4. Phase Change Due to Reflection (Qualitative)

Young's method of producing two coherent light sources involves illuminating a pair of slits using a single source. Another simple arrangement for producing an interference pattern is known as Lloyd's mirror (Fig. 15).

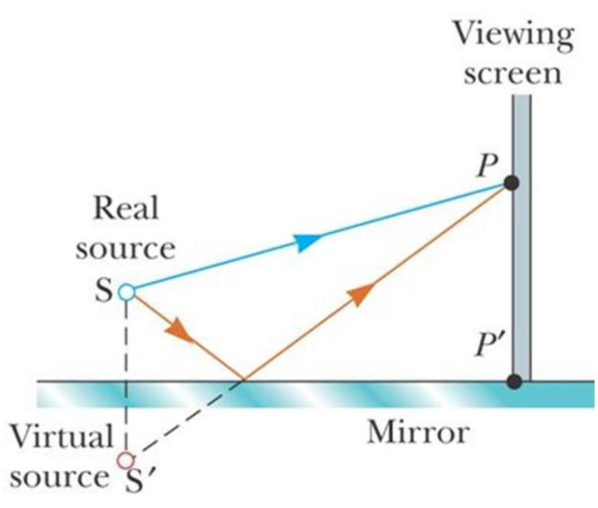



Fig. 15 Lloyd's mirror - An interference pattern is produced at a point P on the screen as a result of the combination of the direct ray (blue) and the reflected ray (brown). The reflected ray undergoes a phase change of 180°.

A point light source is placed at point S close to a mirror, and the screen is placed at some distance away and perpendicular to the mirror. Light waves can reach the point P on the screen either directly from S to P or by the path involving reflection from the mirror. The reflected ray can be treated as a ray originating from a virtual source at point S′. As a result, this arrangement can be thought of as a double slit source with the distance between the points S and S′ comparable to length 'd' as in Fig. 15. Hence at observation points far away from the source (L >> d), the waves from S and S′ can be expected to form interference pattern just like the one seen from two real coherent sources and an interference pattern can indeed be observed. However, the positions of bright and dark fringes are reversed relative to the pattern obtained from two real coherent sources (Young's experiment). This can only occur if the coherent sources at points S and S′ differ by phase 180°.

To illustrate this further, let us consider the point P′, the point where the mirror intersects the screen. This point is equidistant from points S and S′. If the path difference alone were responsible for the phase difference, it would have resulted in a bright fringe at point P′ (since the path difference is zero for this point), corresponding to the central fringe of the two-slit pattern. Instead a dark fringe is observed at point P′. From this, it may be concluded that a 180° phase change must have been produced by the reflection from the mirror. In general, **an electromagnetic wave undergoes a phase change of 180° upon reflection from a medium that has a higher refractive index than the one in which the wave is travelling**.

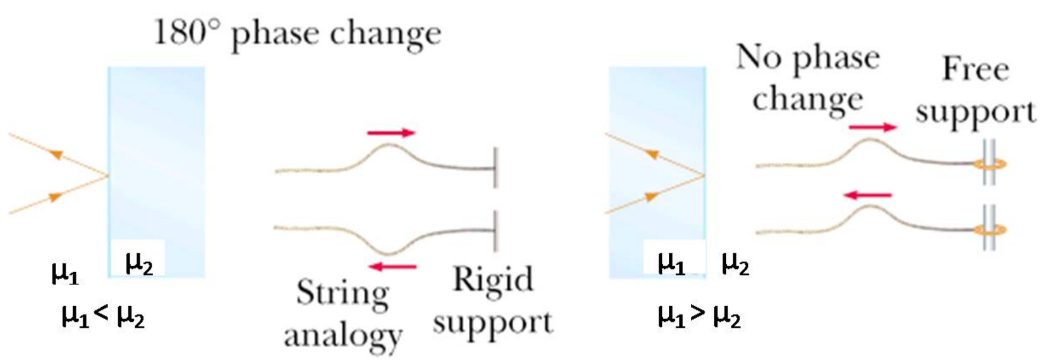
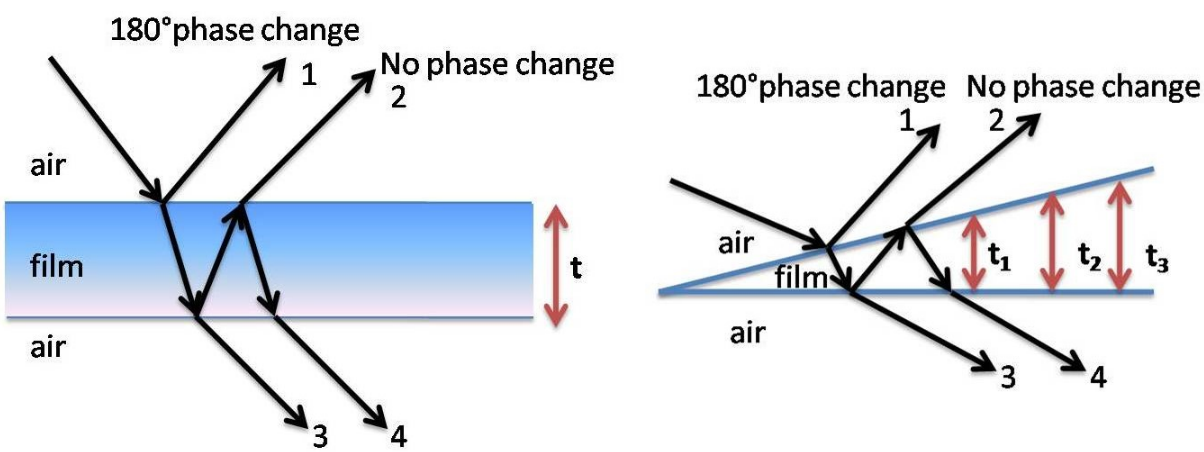



Fig. 16 String analogy with phase change on reflection

This condition is very analogous to the reflection of transverse wave pulse on a stretched string. The reflected pulse on the string undergoes a phase change of 180° when reflected from the boundary of a rigid support but no phase change occurs when the pulse is reflected from the boundary of free support (Fig. 16). Similarly, an electromagnetic wave undergoes a phase change of 180° when reflected from the boundary backed by an optically denser medium (medium having higher refractive index), but no phase change occurs when the wave is reflected from the boundary backed by an optically rarer medium (medium having lower refractive index). This rule can be derived from the Maxwell's electromagnetic equations but that treatment is beyond the scope of this content.

## 5. Interference in Thin Films (Division of Amplitude)

Interference fringes observed in thin films such as oil film on the surface of water or a soap bubble are due to division of amplitude as discussed in section 2.5. **Optical thin films may be defined as layers of material whose thickness is of the order of wavelength of visible light and may be categorized in to parallel thin film and wedge shaped thin film** (Fig. 17). Here we will discuss how the phenomenon of interference takes place in these films qualitatively.




(a) (b)

Fig. 17 Schematic diagram showing the difference between (a) parallel and (b) wedge shaped thin film

Consider a film of uniform thickness 't' and index of refraction 'μ' ad shown in Fig. 17 (a). Assume that the light rays travelling in air are nearly normal to the two surfaces of the film. To determine whether the reflected rays interfere constructively or destructively, the following points may be noted:

- A light wave travelling from a medium of refractive index $\mu_1$ towards a medium of refractive index $\mu_2$ undergoes a phase reversal of 180° upon reflection when $\mu_2 > \mu_1$ while undergoes no phase change when $\mu_2 < \mu_1$.
- Wavelength of light $\lambda_\mu$ in a medium of refractive index μ is given by

  $\lambda_\mu = \frac{\lambda}{\mu}$.........................................................................................(23)

  where λ is the wavelength of the light in free space.

Let us apply these rules to the film in Fig. 17 (a), where $\mu_{film} > \mu_{air}$. Reflected ray 1 which is reflected from the upper surface (A), undergoes a phase change of 180° with respect to the incident wave. Reflected ray 2 which is reflected from the lower surface of the film (B) undergoes no phase change since it is reflected from the surface of a medium (air) having lower refractive index. Therefore ray 1 is 180° out of phase with ray 2 which is equivalent to a phase difference of λ/2. Here, it may be marked that ray 2 travels an extra distance of '$2\mu_{film}t$' before the light waves combine in the air above the surface A. (Please note: we are considering the light rays which are very close to the normal to the surface. If the rays are not close to the normal, then the path difference would have been even larger.)

In general, the rays 1 and 2 recombine in phase and the result is **constructive interference** if the effective path difference $(2\mu_{film}t + \frac{\lambda}{2})$ is an even multiple of λ/2. So the condition of constructive interference in thin films is

$2\mu_{film}t + \frac{\lambda}{2} = 2n\frac{\lambda}{2}$ (n = 0, 1, 2, 3, 4…….) ……………………………………………………(24)

$\boxed{2\mu_{film}t = (2n+1)\frac{\lambda}{2} \text{ (n = 0, 1, 2, 3, 4…..)}}$ …………………………………………………...(25)

However, rays 1 and 2 will recombine to form **destructive interference pattern** if the effective path difference $(2\mu_{film}t + \frac{\lambda}{2})$ is an odd multiple of λ/2. Hence, the general equation for destructive interference in thin films can be presented as

$2\mu_{film}t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$ (n = 0, 1, 2, 3, 4…….) ……………………………………………(24)

$2\mu_{film}t = 2n\frac{\lambda}{2}$ (n = 0, 1, 2, 3, 4…..) ……………………………………………….............(25)

$\boxed{2\mu_{film}t = n\lambda \text{ (n = 0, 1, 2, 3, 4…..)}}$ …………………………………………………...............(26)

It may be noted that the foregoing conditions for constructive and destructive interference are valid when the medium above the top surface of the film is same as the medium below the bottom surface or if there are different media above and below the film, the refractive index of both is less than $\mu_{film}$.

Rays 3 and 4 in Fig. 17 (a) lead to interference effects in the light transmitted through the thin film. Similar kind of analysis can be made to study the fringe pattern in transmitted light.

**Please note:** Full interference effect in a thin film requires an analysis of infinite number of reflections back and forth between the top and bottom surfaces of the film. Here we only focus on a single reflection from the bottom of the film which provides the largest contribution to the interference effect.

## 5.1 Newton's Rings

Wedge shaped thin films can be obtained by placing a plano-convex lens on top of a flat glass plate. With this arrangement the thickness of the air film enclosed between the glass surfaces varies from zero at the point of contact to some value't'. If the radius of curvature of the lens is much larger and if the system is viewed from top, a pattern of dark and bright circular fringes are seen which are called as **Newton's Rings**.

### 5.1.1 Experimental Set up

L is a plano-convex lens of large radius of curvature. This lens with its convex surface is placed on a plane glass plate (P) and makes contact at a point 'O'. Light from an extended monochromatic source such as sodium lamp falls on a glass plate 'G' held at an angle 45° with the vertical. The glass plate 'G' reflects normally a part of the incident light towards the air film enclosed by the lens 'L' and the glass plate 'P'. Then, a part of the incident light is reflected by the curved surface of the lens 'L' and another part is transmitted which is again reflected back from the plane surface of the glass plate 'P'. These two reflected rays interfere and give rise to an interference pattern in the form of circular rings. These rings are localised in the air film and can be seen with the help of travelling microscope placed on the top of it. The experimental set up is displayed in Fig. 18.

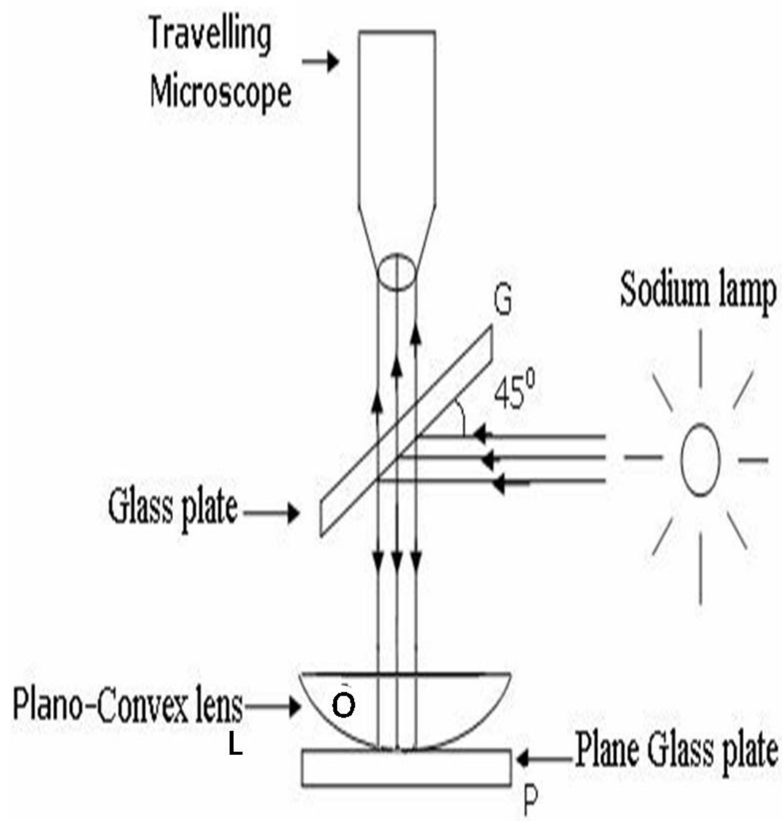



Fig. 18 Experimental set up of Newton's Rings

## 5.1.2 Formation of Newton's Rings (Reflected Light)

Newton's rings are formed due to the interference of the light waves reflected from the top and the bottom surfaces of the air film formed between the lower surface of the plano convex lens and the upper surface of the glass plate (Fig. 19).

AB is a monochromatic ray of light which falls on an air film. A part of it is reflected at C (glass air boundary) which goes out into the air in the form of ray 1 without any phase reversal. The other ray is refracted along CD at the point D; it is again reflected and goes out into the air in the form of ray 2 with a phase reversal of $\pi$. The reflected rays 1 and 2 are in the position to interfere as they have been derived from the same ray AB and hence fulfil the condition of interference.

As the rings are observed in reflected light, the effective path difference between them (considering normal incidence of light) will be:

$\Delta = 2\mu_{film} t cos\alpha + \frac{\lambda}{2}$........................................................................................(27)

For air film, $\mu_{film}=1$

The radius of curvature 'R' of the lower surface of the lens is very large, α is very very small and can be neglected. Hence the effective path difference takes the form,

$\boxed{\Delta = 2t + \frac{\lambda}{2}}$........................................................................................(28)

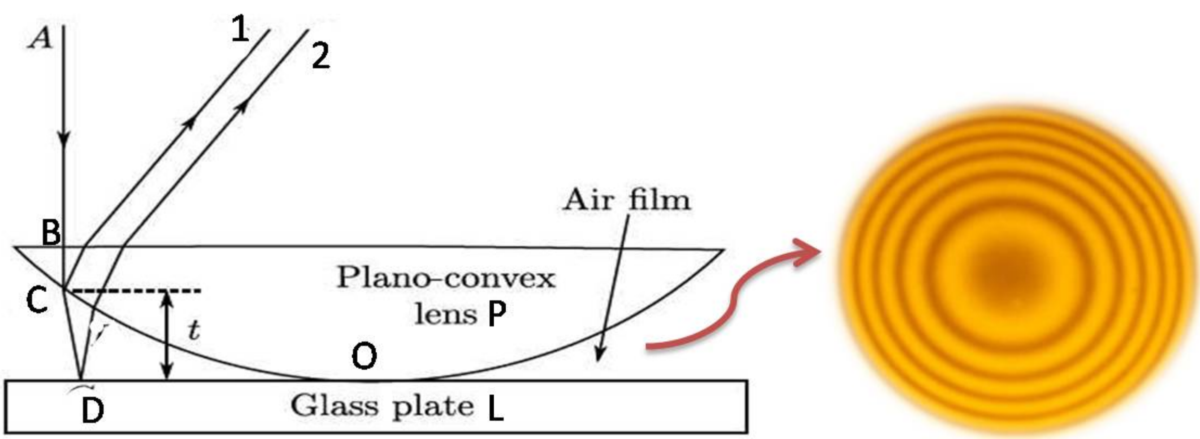



Fig. 19 Schematic diagram showing the formation of Newton's rings

At the **centre of the fringe system**, thickness of the film 't' is zero, so

$\boxed{\Delta = \frac{\lambda}{2}}$........................................................................................(29)

This is the condition for minimum intensity and hence the **central spot must be ideally dark**.

**Condition for maxima:**

The effective path difference must be an even multiple of $\frac{\lambda}{2}$.

$2t + \frac{\lambda}{2} = 2n\frac{\lambda}{2}, (n = 0,1,2, \ldots \ldots)$

$$\boxed{2t = (2n-1)\frac{\lambda}{2}, (n = 1,2,3, \ldots\ldots)} \ldots\ldots\ldots\ldots \tag{30}$$

The above equation represents the **condition for maxima**.

**Condition for minima:**

The effective path difference must be an odd multiple of $\frac{\lambda}{2}$.

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}, (n = 0,1,2, \ldots\ldots)$$

$$2t = 2n\frac{\lambda}{2}, (n = 0,1,2,3, \ldots\ldots)$$

$$\boxed{2t = n\lambda, (n = 0,1,2,3, \ldots\ldots)} \ldots\ldots\ldots\ldots \tag{31}$$

The above equation represents the **condition for minima**.

### 5.1.3 Diameter of Newton's Rings

Let LOL' represent the section of the lens which placed on the glass plate 'P'; 'C' is the centre of curvature and 'R' is the radius of curvature for the curved surface LOL'. Let 'r' be the radius of the Newton's Ring which is formed corresponding to a film of thickness't' as shown in Fig. 20.

From the property of circles,

$$QM \times MN = OM \times MD$$

$$r \times r = t \times (2R - t)$$

$$r^2 = 2Rt - t^2$$

Here, $t << R$, so $t^2 <<< R$.

Hence, neglecting $t^2$ in the previous equation, we get

$$r^2 = 2Rt$$

$$\boxed{t = \frac{r^2}{2R}} \ldots\ldots\ldots\ldots \tag{32}$$

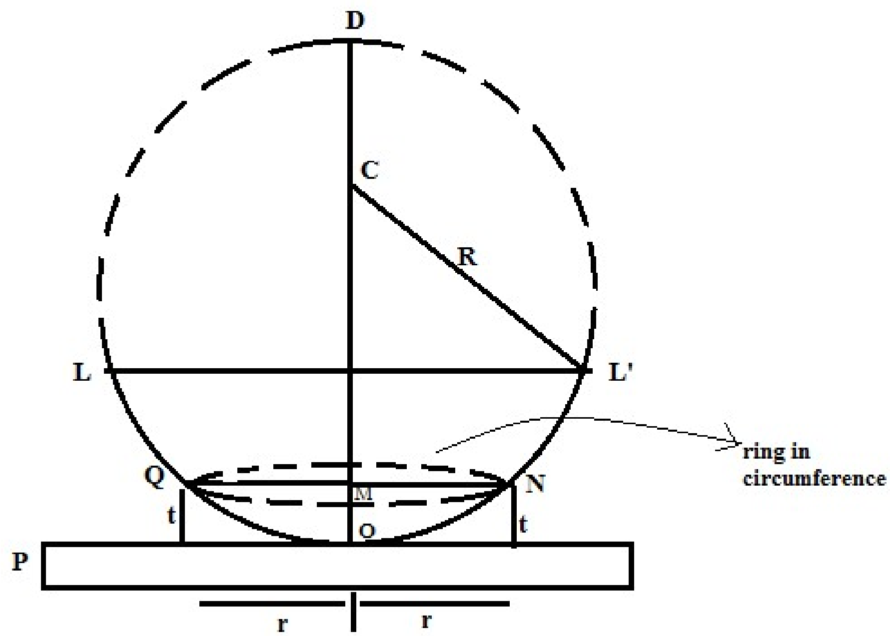



Fig. 20 Schematic diagram for the diameter of Newton's Rings

For **dark rings**,   (Condition for minima from equation (31))

$2t = n\lambda, (n = 0,1,2,3, \ldots\ldots)$

Substituting the value of t from equation (32) in equation (31), we get

$2\frac{r^2}{2R} = n\lambda$

$r^2 = Rn\lambda$

Considering dependence of 'r' on 'n' values, r in the above equation can be replaced more appropriately by $r_n$.

$r_n^2 = Rn\lambda$

If '$D_n$' is the diameter of $n^{th}$ dark ring, then,

$\frac{{D_n}^2}{4} = Rn\lambda$

$\boxed{{D_n}^2 = 4n\lambda R, (n = 0,1,2, \ldots\ldots)}$ ……………………………………………………………….(33)

$$\boxed{D_n = \sqrt{4n\lambda R}, (n = 0, 1, 2, 3, \ldots \ldots ..)} \ldots\ldots(34)$$

$$\boxed{D_n \propto \sqrt{n}, (n = 0, 1,2, 3, \ldots \ldots .)} \ldots\ldots(35)$$

Thus the **diameters of the dark rings are proportional to the square root of any natural number**.

For **bright rings**, (Condition for maxima from equation (30))

$$2t = (2n-1)\frac{\lambda}{2}, (n = 1, 2, 3, \ldots \ldots ..)$$

Substituting the value of t from equation (32) in equation (30), we get

$$2\frac{r^2}{2R} = (2n-1)n\frac{\lambda}{2}, (n = 1, 2, 3, \ldots \ldots .)$$

$$r^2 = R(2n-1)\frac{\lambda}{2}, (n = 1, 2, 3, \ldots \ldots \ldots .)$$

Considering dependence of 'r' on 'n' values, r in the above equation can be replaced more appropriately by $r_n$.

$$r_n^2 = R(2n-1)\frac{\lambda}{2}, (n = 1, 2, 3, \ldots \ldots \ldots .)$$

If '$D_n$' is the diameter of $n^{th}$ bright ring, then,

$$\frac{{D_n}^2}{4} = R(2n-1)\frac{\lambda}{2}, (n = 1, 2, 3, \ldots \ldots \ldots ..)$$

$$\boxed{{D_n}^2 = 4R(2n-1)\frac{\lambda}{2}, (n = 1, 2, 3, \ldots \ldots \ldots)} \ldots\ldots(36)$$

$$\boxed{D_n = \sqrt{2\lambda R(2n-1)}, (n = 1, 2, 3, \ldots \ldots \ldots .)} \ldots\ldots(37)$$

Thus, the **diameters of the bright rings are proportional to the square root of odd natural numbers**.

## 5.1.4 Applications of Newton's Rings Experiment

### 5.1.4.1 Determination of wavelength of a monochromatic source

To determine the wavelength of a monochromatic source of light using Newton's Rings method, the experimental arrangement is shown Fig. 18 and discussed in section 5.1.1. Referring to the experimental set up, if we consider $D_n$ and $D_{n+p}$ are the diameters of $n^{th}$ and $(n+p)^{th}$ dark rings formed the air film enclosed between the lower surface of the plano-convex lens and upper surface of the glass plate,

For $n^{th}$ dark ring,

$D_n{}^2 = 4n\lambda R, (n = 0,1,2, \dots \dots)$ (from equation (33))……………………………………….(38)

and for $(n+p)^{th}$ dark ring,

$D_{n+p}{}^2 = 4(n+p)\lambda R, (n = 0,1,2, \dots \dots)$…………………………………………………...(39)

From equations (38) and (39)

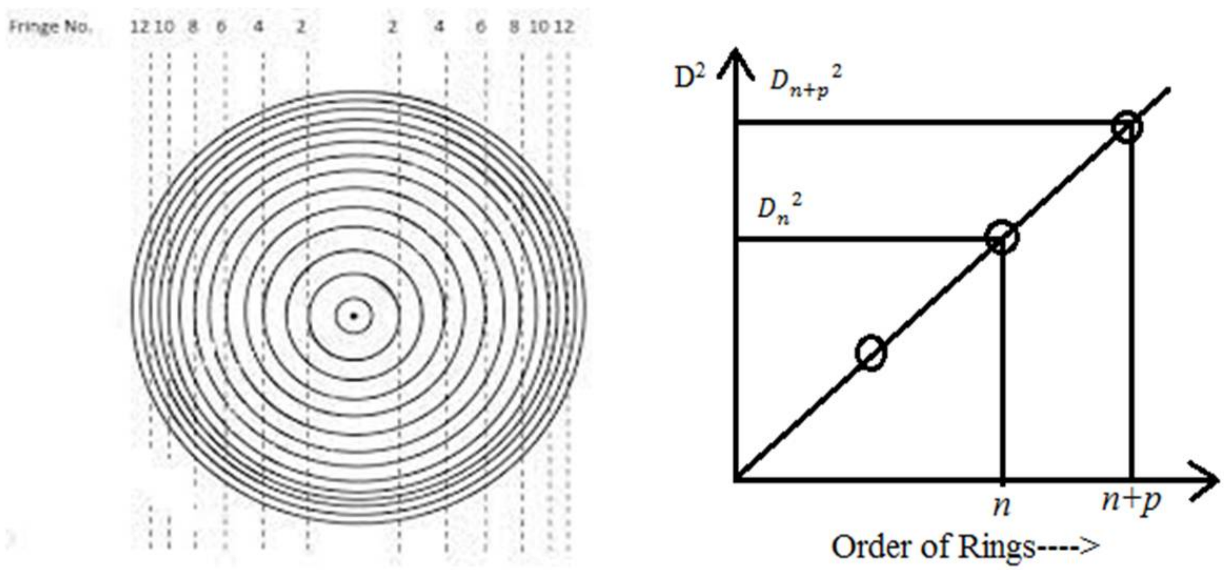
$D_{n+p}^2 - D_n^2 = 4p\lambda R$……………………………………………………………………….(40)

$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$………………………………………………………………………….(41)

After setting up the experiment, the rings are viewed through the eyepiece of the travelling microscope which can move in horizontal direction. The crosswire of the eyepiece is set on the periphery of extreme $(n+15)^{th}$ distinct dark ring (n is any order of the ring except the central dark fringe) making it tangential. The microscopic position is marked on the horizontal scale. Then the crosswire is slowly moved towards the centre and microscopic position is noted for the next $((n+14)^{th})$ dark ring (Fig. 21(a)). The procedure is repeated till the crosswire reaches the $(n+15)^{th}$ distinct dark ring on the extreme right of the central fringe. The diameters of all the rings are evaluated from these readings. A straight line graph is plotted between the square of the diameters to the order of the corresponding rings. The slope of the plotted graph (Fig. 21(b)) gives the slope as $\frac{D_{n+p}^2 - D_n^2}{(n+p)-n}$.

Or, slope of the graph is $\frac{D_{n+p}^2 - D_n^2}{p}$. Substituting this in equation (41), we can find out the wavelength of the monochromatic source of light as

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} = \frac{1}{4R} \times slope} \quad \text{.....(42)}$$




(a) (b)

Fig. 21 (a) Schematic diagram showing the position of crosswire made tangential to different order rings and (b) $D^2$ vs order of rings

### 5.1.4.2 Determination of refractive index of a liquid

First of all the experiment is performed when there is an air film between the plano-convex lens and the glass plate and the diameters of the $n^{th}$ and $(n+p)^{th}$ dark rings are determined with the help of a travelling microscope.

If $D_n$ and $D_{n+p}$ are the diameters are the diameters of nth and $(n+p)^{th}$ dark rings in air film, then

$$D_n^2 = 4n\lambda R \quad \text{.....(42)}$$

$$D_{n+p}^2 = 4(n+p)\lambda R \quad \text{.....(43)}$$

From equation (42) and (43),

$D_{n+p}^2 - D_n^2 = 4p\lambda R$..................................................................................................................(44)

Next, the air film is replaced by the liquid whose refractive index is to be determined (without disturbing the set up) and the again the diameters of the $n^{th}$ and $(n+p)^{th}$ dark rings are estimated using the travelling microscope.

If $D_n'$ and $D_{n+p}'$ are the diameters of the $n^{th}$ and $(n+p)^{th}$ dark rings in the liquid film, then

$D_n'^2 = \frac{4n}{\mu_{film}}$..................................................................................................................(45)

Going back to equation (27),

$\Delta = 2\mu_{film} t cos\alpha + \frac{\lambda}{2}$..........................................................(27) (in presence of a liquid film)

The radius of curvature 'R' of the lower surface of the lens is very large, α is very very small and hence cos α can be neglected.

Equation (27) takes the form,

$\Delta = 2\mu_{film} t + \frac{\lambda}{2}$ ........................................................................(X)

Therefore the condition of minima now becomes,

$2\mu_{film} t = n\lambda$..........................................................................(Y)

Substituting equation (32) in equation (Y) and solving we arrive at

$D_n'^2 = \frac{4n\lambda R}{\mu_{film}}$........................................................................(45)

$D_{n+p}'^2 = \frac{4(n+p)\lambda R}{\mu_{film}}$..................................................................................................................(46)

From equation (45) and (46), we get,

$D_{n+p}'^2 - D_n'^2 = \frac{4p\lambda R}{\mu_{film}}$..................................................................................................................(47)

Again from equations (44) and (47), the value of the **refractive index of the liquid film** is found to be

$$\boxed{\mu_{film} = \frac{[D_{n+p}^2 - D_n^2]}{[D'^2_{n+p} - D'^2_n]}} \quad ..........(48)$$

## 5.2 Michelson's Interferometer

The interferometer, invented by the American physicist A. A. Michelson (1852 – 1931), splits a light beam into two parts and then recombines the parts to form an interference pattern. The device can be used to measure wavelengths or other lengths with great precision because a large and precisely measurable displacement of one of the mirrors is related to an exactly countable number of wavelengths of light. One of the breakthroughs in research was achieved with detection of gravitational waves by LIGO detector in USA which is a laser interferometer. This ground breaking discovery won Nobel Prize in 2017. A schematic diagram of the interferometer is shown in Fig. 22.

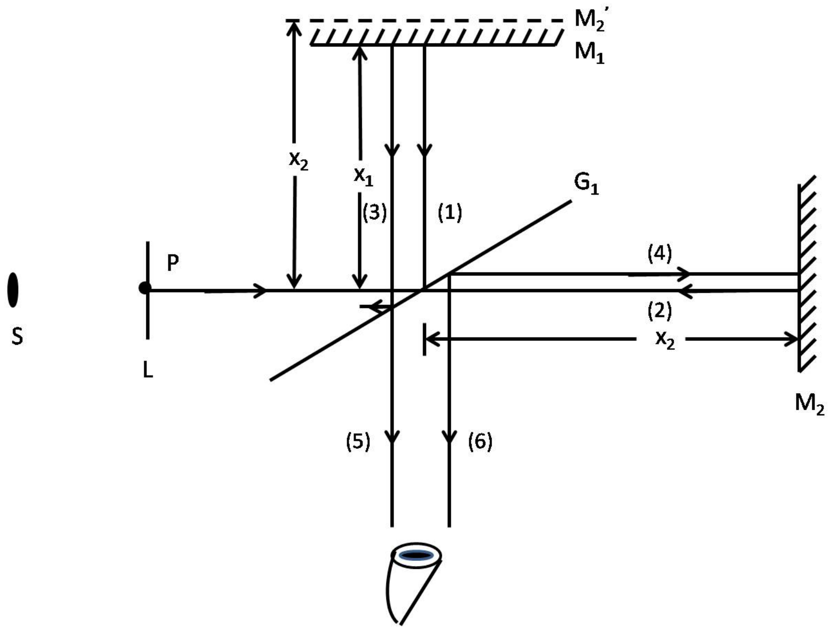



Fig. 22 Schematic ray diagram of a Michelson Interferometer

Light from a source S (which may be a sodium lamp) placed in the focal plane of a converging lens L is incident on the interferometer. Such an arrangement to ensure an extended source of almost uniform intensity is formed. $G_1$ (partially silvered) is a beam splitter, i.e., a beam incident on $G_1$ gets partially reflected and partially transmitted. $M_1$ and $M_2$ are good quality plane mirrors having very high reflectivity. One of the mirrors (usually $M_2$) is fixed and other (usually $M_1$) is capable of moving away or towards the glass plate $G_1$ along an accurately machined track by means of a screw. In the normal adjustment of the interferometer, the mirrors $M_1$ and $M_2$ are perpendicular to each other and $G_1$ is at 45° to the mirror.

Waves emanating from a point P get partially reflected and partially transmitted by the beam splitter $G_1$, and the two resulting beams are made to interfere in the following manner. The reflected wave [shown as (1) in Fig. 22] undergoes a further reaction at $M_1$ and this reflected wave gets (partially) transmitted through $G_1$; this is shown as (5) in the figure. The transmitted wave [shown as (2) in Fig. 22] gets reflected by $G_1$ and results in the wave shown as (6) in the figure. Waves (5) and (6) interfere in a manner exactly similar to that shown in Fig. 23.

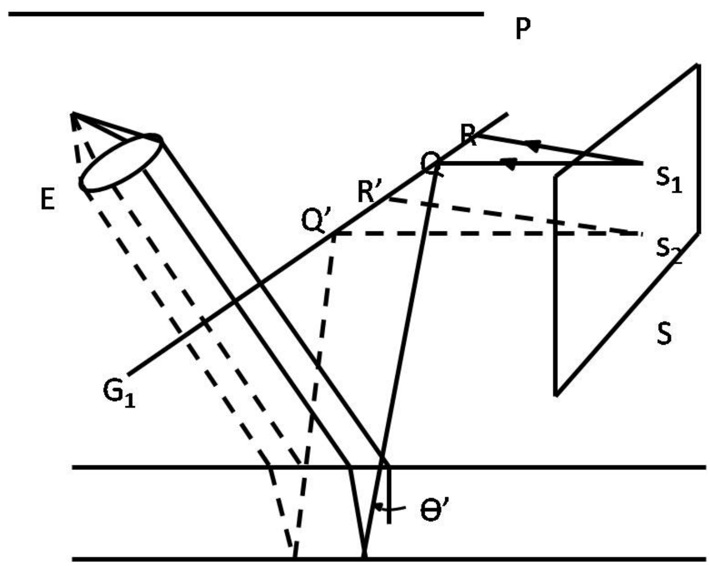



Fig. 23 Light emanating from an extended source illuminates a thin film. G represents the partially reflecting plate and P represents the photographic plate. The eye E is focused at infinity

This can easily be seen from the fact that if $x_1$ and $x_2$ are the distances of the mirrors $M_1$ and $M_2$ from the plate $G_1$, then to the eye the waves emanating from the point P will appear to get reflected by two parallel mirrors [$M_1$ and $M_2'$ – see Fig. 22] separated by a distance ($x_1 \sim x_2$). As discussed earlier, if we use an extended source, then no definite interference pattern will be obtained on a photographic plate placed at the position of the eye.

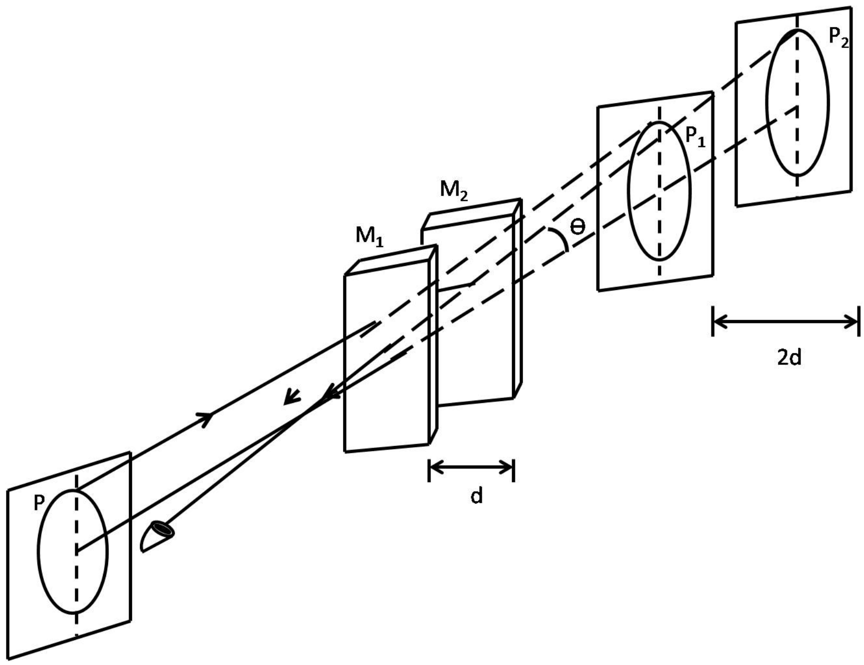



Fig. 24 A schematic of the formation of circular fringes

Instead, if we have a camera focused for infinity, then on the focal plane we will obtain circular fringes, each circle corresponding to a definite value of $\theta$ (see Figs. 2 and 3); the circular fringes will look like the ones shown in Fig.4. Now, if the beam splitter is just a simple glass plate, the beam reflected from the mirror $M_2$ will undergo an abrupt phase change of $\pi$ (when getting reflected by the beam will traverse will be $2(x_1 \sim x_2)$, the condition for destructive interference will be

$$2\,d\,cos\theta = 2m\frac{\lambda}{2} \qquad \text{......(49)}$$

where m=0, 1, 2, 3, …..

$$d = x_1 \sim x_2 \qquad \text{......(50)}$$

And the angle $\theta$ represents the angle that the rays make with the axis (which is normal to the mirrors as shown in Fig. 24). Similarly, the condition for a bright ring would be

$$2d\ \cos\theta = (2m+1)\frac{\lambda}{2} \qquad \text{......(51)}$$

For example, for $\lambda = 6\times10^{-5}$ cm if d=0.3 mm, the angles at which the dark rings will occur will be

$\theta = \cos^{-1}(\frac{m}{1000}) = 0°, 2.56°, 3.62°, 4.44°, 5.13°, 5.73°, 6.28°, \ldots ..$

Corresponding to m= 1000, 999, 998, 997, 996, 995, ….

Thus the central dark ring in Fig. 25 (a) corresponds to m=1000, the first dark ring corresponds to m=999, etc. If we now reduce the separation between the two mirrors so that d=0.15 mm, the angles at which the dark rings will be see Fig. 25 (b)]

$\theta = \cos^{-1}\left(\frac{m}{500}\right) = 0°, 3.62°, 5.13°, 6.28°, 7.25°, ....$

Where the angle now correspond to m=500, 499, 498, 497, 496, 495, … thus as we start reducing the value of d, the fringes will appear to collapse at the centre and the fringes become less closely placed. It may be noted that if d is now slightly decreased, say from 0.15 mm to 1.14985 mm,

$2d = 499.5\lambda$

the dark central spot in Fig. 25 (b) (corresponding to m=50) would disappear and the central fringe will become bright. Thus, as d decreases, the fringe pattern tends to collapse towards the centre. (Conversely, if d is increased, the fringe pattern will expand.) Indeed, if N fringes collapse to the centre as the mirror $M_1$ moves by a distance $d_0$, then we must have

$2d = m\lambda$

$2(d - d_0) = (m - N)\lambda$ ……………………………………………………………………(52)

where we have put $\theta' = 0$ because we are looking at the central fringe. Thus,

$\lambda = \frac{2d_0}{N}$ ……………………………………………………………………………………(53)

This provides us with a method for the measurement of the wavelength. For example, in a typical experiment, if one finds 1000 fringes collapse to the center as the mirror is moved through a distance of $2.90\times10^{-2}$ cm, then

$\lambda = 5800$ Å

The above method was used by Michelson for the standardization of the meter. He had found that the red cadmium line ($\lambda$= 6438.4696 Å) is one of the ideal monochromatic sources and as such this wavelength was used as a reference for the standardization of the meter. In fact, he defined the meter by the following relation:

1 meter = 1553164.13 red cadmium wavelengths,

The accuracy is almost one part in $10^9$.

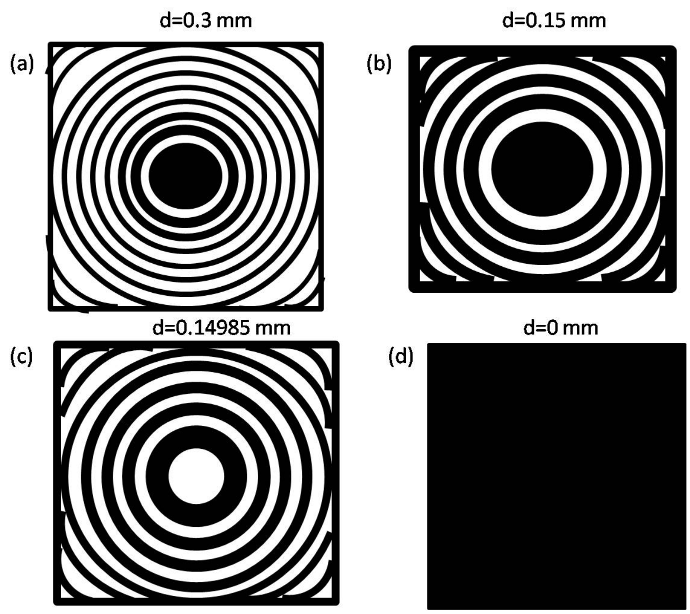



Fig. 25 Computer generated interference pattern produced by a Michelson interferometer

In an actual Michelson interferometer, the beam splitter $G_1$ consists of a plate (which may be about ½ cm thick), the back surface of which is partially silvered and the reflections occur at the back surface as shown in the Fig. 26.It is immediately obvious that he beam (5) traverse the glass plate thrice and in order to compensate for this additional path, one introduces a 'compensating plate' $G_2$ which is exactly of the same thickness as $G_1$. The compensating plate is not really necessary for a monochromatic source because the additional path 2(n-1)t introduced by $G_1$ can be compensated by moving the mirror $M_1$ by a distance (n-1)t where n is the refractive index of the material of the glass plate $G_1$.

However, for a white light source it is not possible to simultaneously satisfy the zero path difference condition for all wavelengths, since the refractive index depends on wavelength. For example, for λ=6560 Å and 4861 Å, the refractive index of crown glass is 1.5244 and 1.5330 respectively. If we are using a 0.5 cm thick crown glass plate as $G_1$, then $M_1$ should be moved by 0.2622 cm for λ=6560 Å, the difference between the two positions corresponding to over hundred wavelengths. Thus, if we have a continuous range of wavelengths from 4861 Å to 6560 Å, the path difference between any pair of interfering rays (see Fig. 22) will vary so rapidly with wavelength that we would observe only a uniform white light illumination. However, in the presence of the compensating plate $G_2$, one would observe a few colored fringes around the point corresponding to zero path difference.

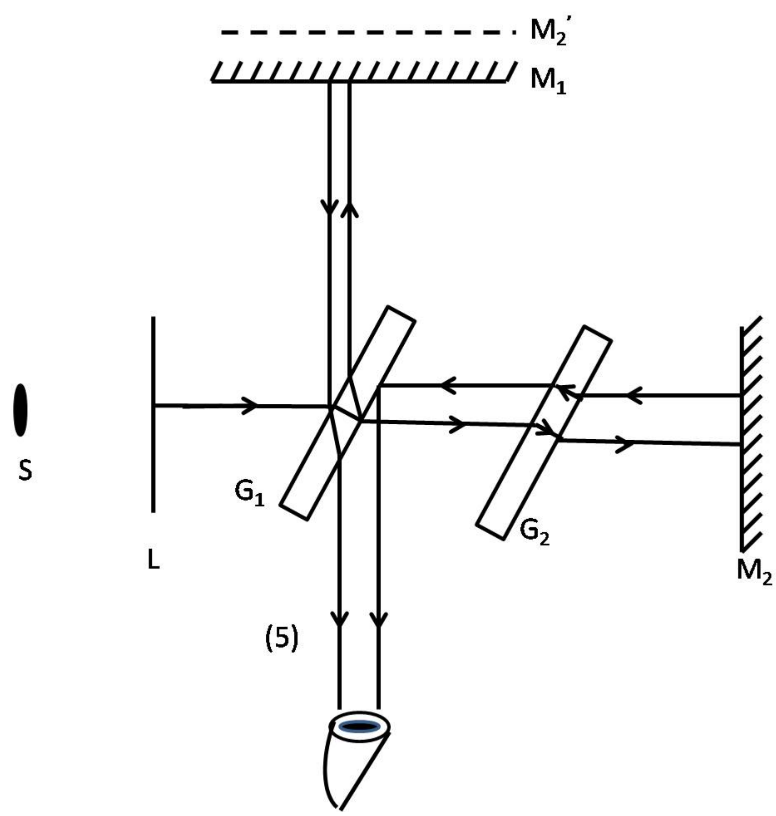



Fig. 26 In an actual interferometer there is also a compensating plate $G_2$

The other type of

Apart from circular fringes formed when the $M_1$ and $M_2'$ are parallel to each other, curved and straight fringes can also be created using different orientations of the $M_1$ and $M_2'$.

(i) When $M_1$ and $M_2'$ are parallel to each other, the film enclosed within them is a parallel thin film. Such condition gives rises to circular fringes are known to be fringes of equal inclination.

(ii) When $M_1$ and $M_2'$ are inclined to each other, the film enclosed is a wedge shaped film. Curved fringes, also known as fringes of equal thickness are formed as shown in Fig. 27.

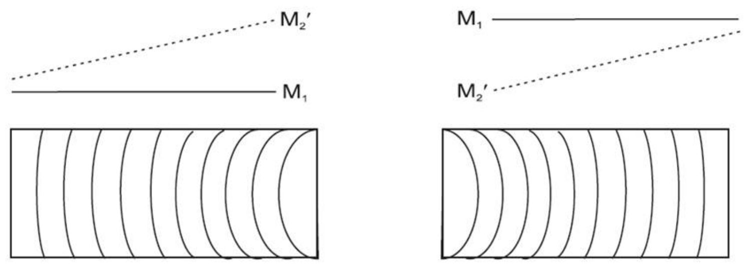



Fig. 27 Curved fringes

(iii) When $M_1$ and $M_2'$ intersect, straight line fringes are obtained around the point of intersection. The path difference along the line of intersection is zero and therefore is same for all wavelengths. Straight line fringes are shown in Fig. 28 below:

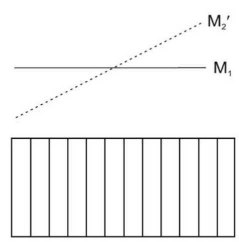
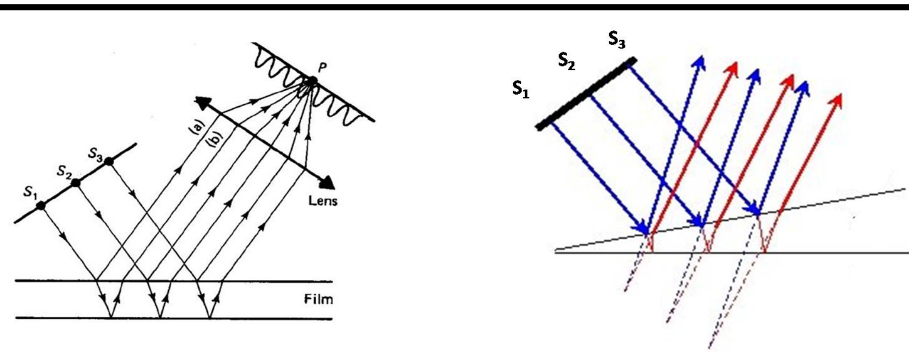



Fig. 28 Straight line fringes




(a) (b)

Fig. 29 Fringes of (a) equal inclination and (b) equal thickness