SPRING END SEMESTER EXAMINATION-2016

2nd Semester B.Tech & B.Tech Dual Degree

MATHEMATICS-II

MA-1002

(Regular-2015 & Back-2014 Admitted Batches)

Time: 3 Hours **Full Marks: 60**

Answer any SIX questions including Question No.1 which is compulsory.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. Answer all the following questions. [2 × 10

a) Determine the radius of convergence.

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{3^m (m+1)^2} (x+1)^{2m}$$

b) Evaluate

$\int_{-1}^{1} \{P_{10}(x)\}^2 \, dx$, where $P_{10}(x)$ is Legendre polynomial of degree ten.

c) Show that $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$.

d) Represent the function $f(t) = t^2 \quad (0 < t < 3)$, using unit step function and find its Laplace transform.

e) Find the convolution of e^{at} and e^{bt} $(a \neq b)$.

f) Find the directional derivative of $f(x,y,z) = xyz$ at $P:(-1,1,3)$ in the direction of $\vec{a} = [1, -2, 2]$.

g) Given a curve C: $\vec{r}(t) = [t, t^2, 0]$, find a tangent vector $r'(t)$, a unit tangent vector $u'(t)$, and the tangent of C at P(2,4,0).

h) Evaluate $\oint_C \frac{\partial w}{\partial n} ds$ counterclockwise over the boundary curve C of the triangle with vertices (0, 0), (2, 0), (2, 1) where $w = \sinh x$.

i) Find the Fourier cosine transform of

$$f(x) = \begin{cases} -1 , & 0 < x < 1 \\ 1 , & 1 < x < 2 \\ 0 , & x > 2 \end{cases}$$

j) Express Beta function $\beta(5/2, 3/2)$ in the form of Gamma function.

2. a) Solve the initial value problem by using Laplace Transform. [4

$$y'' + y = \begin{cases} t , & 0 < t < 1 \\ 0 , & t > 1 \end{cases} ; \; y(0) = y'(0) = 0$$

b) Find $L^{-1}\left\{\cot^{-1}\frac{s}{\omega}\right\}$. [4

3. a) Using Laplace Transform solve the integral equation [4

$$y(t)+\int_0^t e^{2(t-\tau)}y(\tau)\,d\tau = t^2 - t - \frac{1}{2} + \frac{e^{2t}}{2}$$

b) Prove the Bonnet's recursion [4

$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$ where $n=1,2,\cdots$

4. a) Find a solution by Frobenius method [4

$$xy'' + 2y' - xy = 0$$

b) Prove that [4

$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$, where $J_{1/2}$ is the Bessel function of first kind of order ½.

5. a) Is the flow of velocity $\vec{V}$ of a steady fluid flow irrotational? Incompressible? Find the streamlines. [4

$$\vec{V} = [y, -x, 0]$$

b) Calculate $\int_C F(r)\cdot dr$ where $F = [x^2, y^2, 0]$, C: the semicircle from (2, 0) to (-2, 0), $y \geq 0$. [4

6. a) Show that the differential form under the integral sign is exact in space and evaluate the integral. [4

$$\int_{(0,0,0)}^{(0,1,2)} e^{x^2+y^2-2z}(xdx + ydy - dz)$$

b) Evaluate by changing the order of integration. [4

$$\int_0^1 \int_x^{2x} xye^{x^2-y^2}\, dydx$$

7. a) Using Green's theorem, evaluate [4

$\int_C F(r)\,.\,dr$, Counterclockwise around the boundary curve C of the region R, where $F=[-y^3, x^3]$, C: the circle $x^2+y^2=25$.

b) Find the Fourier series of the function $f(x)=x^2 \quad (-1<x<1)$ of period $P=2$ and hence show that $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots=\frac{\pi^2}{6}$. [4

8. a) Is the given function even or odd? Find its Fourier series. [4

$$f(x)=\begin{cases} x\,, & -\frac{\pi}{2}<x<\frac{\pi}{2} \\ \pi-x\,, & \frac{\pi}{2}<x<\frac{3\pi}{2} \end{cases}$$

b) Show that the given integral represents the indicated function. [4

$$\int_0^\infty \frac{\cos xw + w\sin xw}{1+w^2}\,dw=\begin{cases} 0, & x<0 \\ \frac{\pi}{2}, & x=0 \\ \pi e^{-x}, & x>0 \end{cases}$$

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