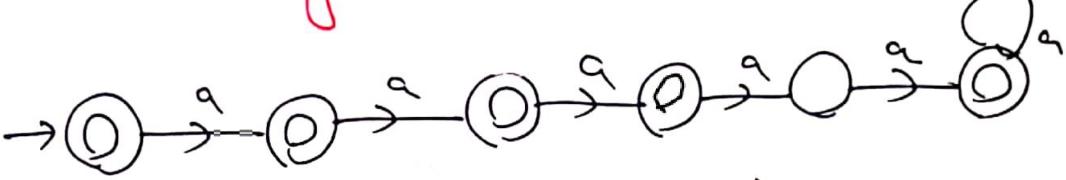
2020 Mid Sem Solution

By ~~HPGer~~

1) a)

Show that language $L = \{a^n : n \geq 0, n \neq 4\}$ is regular.



Since Finite automata exists for 'L'

So, 'L' is regular.

Note: $\boxed{\text{using pumping lemma, proof is wrong..}}$

Bcoz, pumping lemma is used to prove the language is not regular.

b) Prove or disprove that the following pairs of regular expressions define the same language over $\Sigma = \{a, b\}$.

$(ab+a)^* ab$ and $(a a^* b)^*$

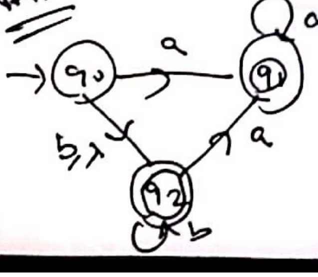
Minimum string from $(ab+a)^* ab = ab$

Minimum string from $(aa^*b)^* = \lambda$

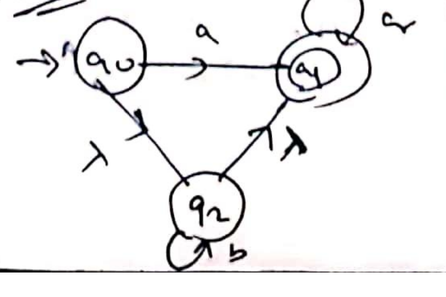
So, $(ab+a)^* ab$ & $(aa^*b)^*$ are not generating same set of strings. So, both are not same.

c) $L = \{a^n : n \geq 1\} \cup \{b^k a^m : m \geq 0, k \geq 0\}$ Design NFA with 3 states

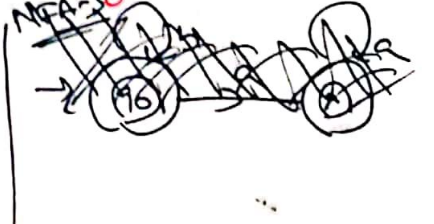
NFA-1



NFA-2



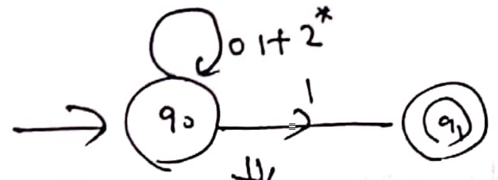
~~NFA-3~~



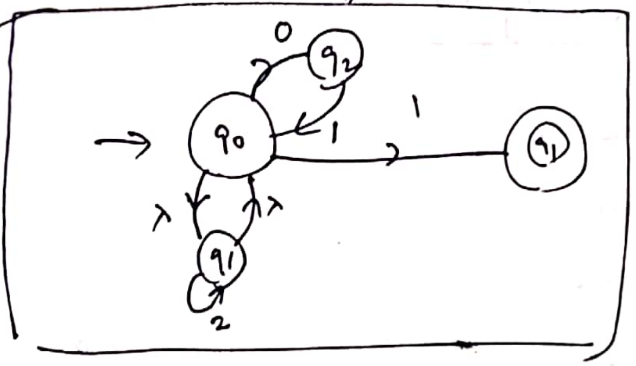
d) Construct an NFA equivalent to Regular expression

$r = (01+2^*)^* 1.$

~~NFA-1~~



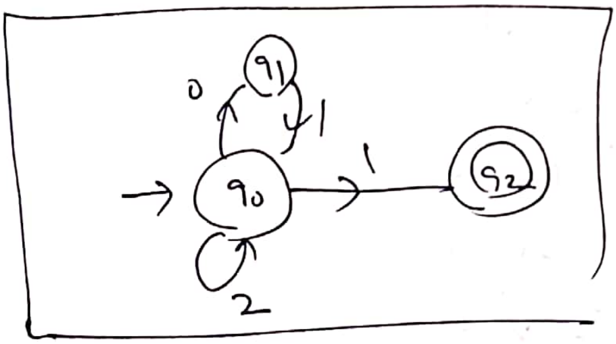
NFA-1



NFA-2

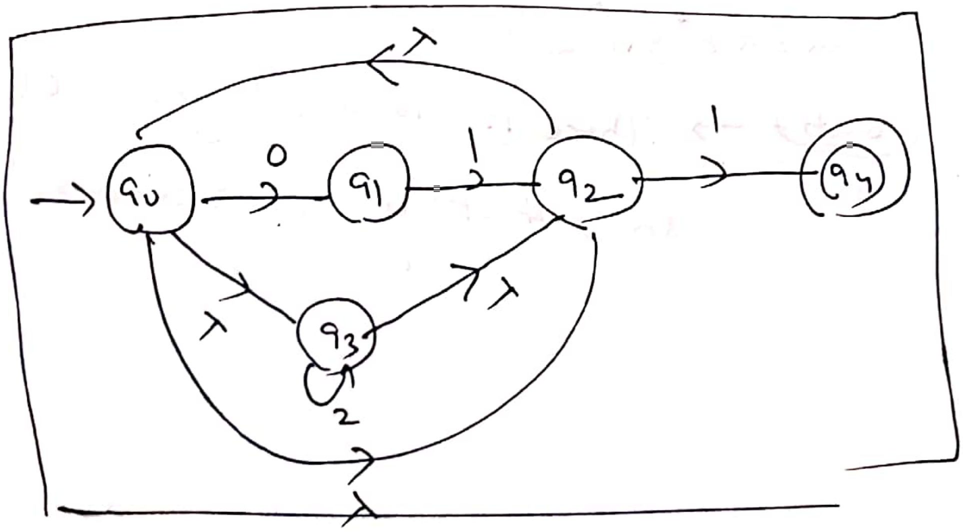
$r = (01+2^*)^* 1 = \underline{(01+2)^* 1}$

$(\text{Bcoz } (a+b^*)^* = (a+b)^*)$



NFA-3

Book Method →



e) Find the shortest string not in language over $\Sigma = \{a, b\}$ of RE:

$a^* b^* (ba)^* a^*$.

Ans:

Shortest string in language.

$\lambda = a^0 b^0 (ba)^0 a^0$

$a = a^1 b^0 (ba)^0 a^0$

$b = a^0 b^1 (ba)^0 a^0$

$aa = a^2 b^0 (ba)^0 a^0$

$ab = a^1 b^1 (ba)^0 a^0$

$ba = a^0 b^1 (ba)^0 a^1$

$bb = a^0 b^2 (ba)^0 a^0$

$aaa = a^3 b^0 (ba)^0 a^0$

$aab = a^2 b^1 (ba)^0 a^0$

$aba = a^1 b^1 (ba)^0 a^1$

$abb = a^1 b^2 (ba)^0 a^0$

$baa = a^0 b^1 (ba)^0 a^2$

bab → There is no way to generate bab

So shortest string not in language is bab

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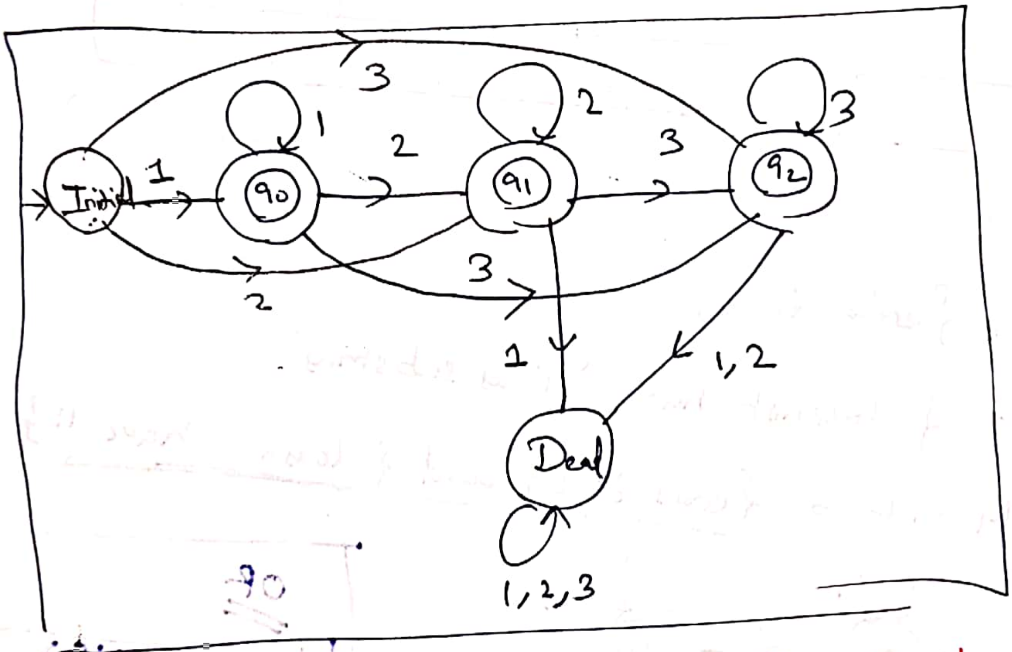
2) a) Design a DFA to accept all strings ω over $\Sigma = \{1,2,3\}$ s.t the digits in ω appear in non-decreasing order.

For ex: It accepts 1123, but not 1232, (2.5 marks)

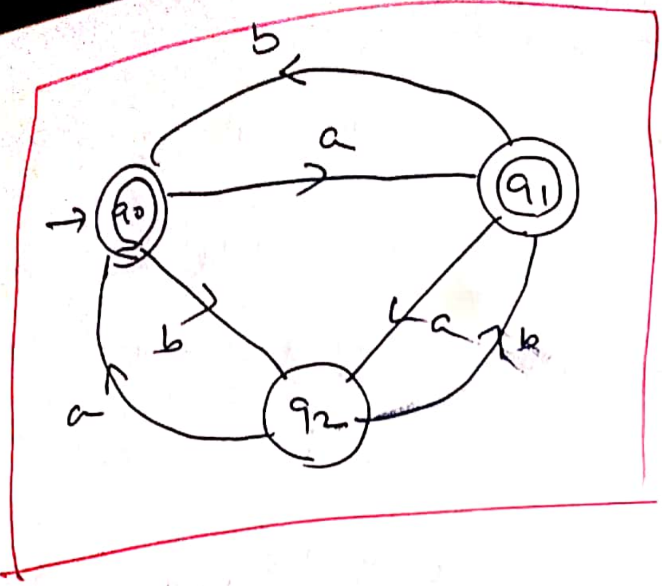


2) b) Design a DFA for the following language on $\Sigma = \{a,b\}$ $L = \{w : n_a(w) + 2n_b(w) \bmod 3 < 2\}$

$\{w : (n_a(w) + 2n_b(w)) \bmod 3 < 2\}$ (2.5 marks)

As 'mod 3' we require only '3' states, q_0, q_1, q_2

String	$n_a(w)$	$n_b(w)$	$2n_b(w)$	$n_a(w) + 2n_b(w)$	mod 3	output state
a	1	0	0	1+0=1	1%3	1 = q_1
b	0	1	2	0+2=2	2%3	2 = q_2
aa	2	0	0	2+0=2	2%3	2 = q_2
ab	1	1	2	1+2=3	3%3	0 = q_0
aaa	3	0	0	3+0=3	3%3	0 = q_0
aab	2	1	2	2+2=4	4%3	1 = q_1



As $\underline{<2}$ so, q_0 & q_1 are final.

3) a) Design an NFA that accepts strings over {0, 1} that ends in '0' but doesnot have '11' as substring.

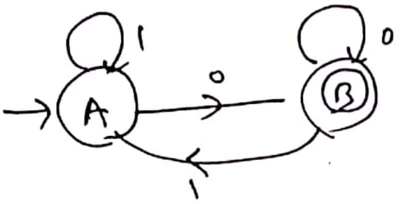
Convert NFA to DFA :

L_1 = {ends in 0}

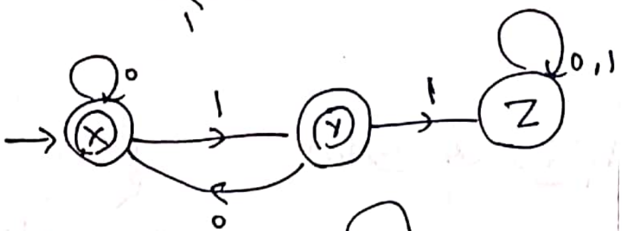
L_2 = {doesnot have '11' as substring}

$L = L_1 \cap L_2$ = {ends in 0} and {doesnot have 11}

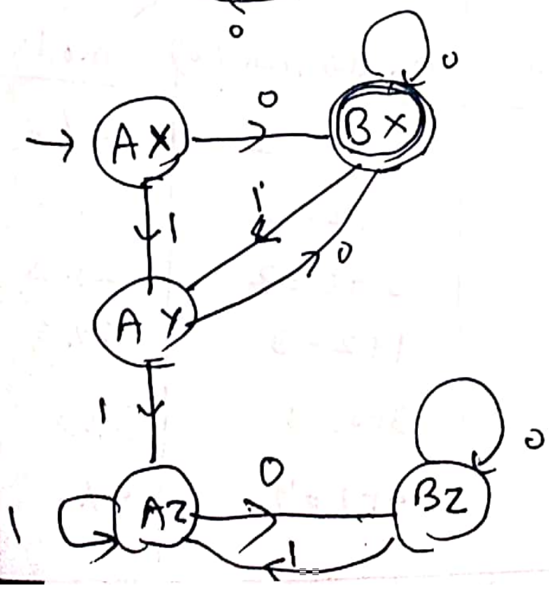
FA for L_1



FA for L_2

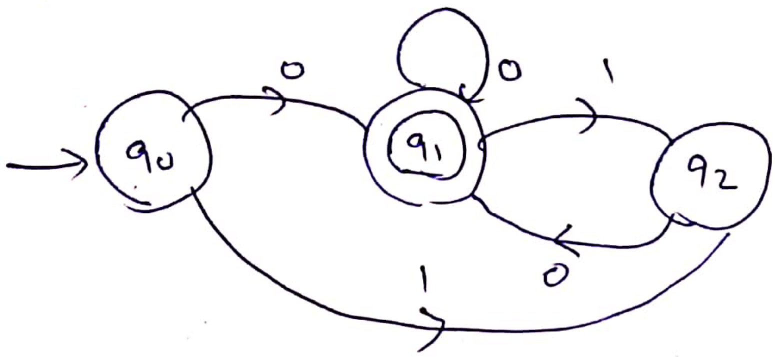


$L_1 \cap L_2$

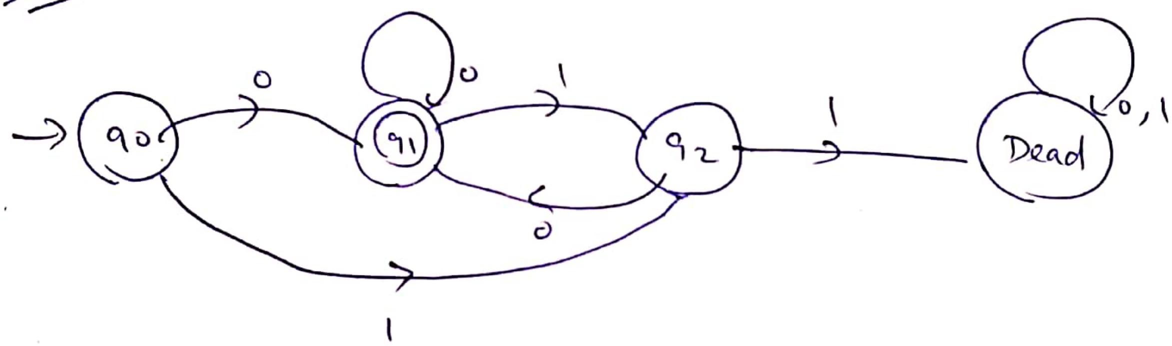


3(a)
→ or

NFA:



DFA:



3) b) Find the minimum state DFA equivalent to following DFA.

Q/Σ	0	1
→ q_0	q_1	q_3
q_1	q_0	q_3
q_2	q_1	q_4
* q_3	q_5	q_5
q_4	q_3	q_3
* q_5	q_5	q_5

Remove Unreachable state:

Remove 'q_2'

	0	1
→ q_0	q_1	q_3
q_1	q_0	q_3
* q_3	q_5	q_5
q_4	q_3	q_3
* q_5	q_5	q_5

Again remove 'q_4' bcoz 'q_4' is also unreachable

	0	1
→ q_0	q_1	q_3
q_1	q_0	q_3
* q_3	q_5	q_5
* q_5	q_5	q_5

0-equivalence:-

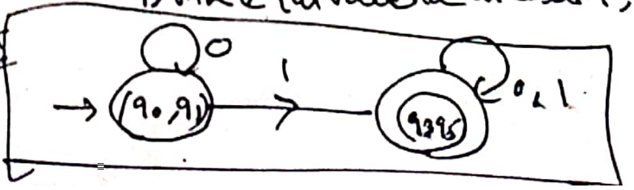
$[q_0, q_1]$ $[q_3, q_5]$

1-equivalence

$[q_0, q_1]$ $[q_3, q_5]$

Both equivalence are same, so stop

MINIMIZED DFA



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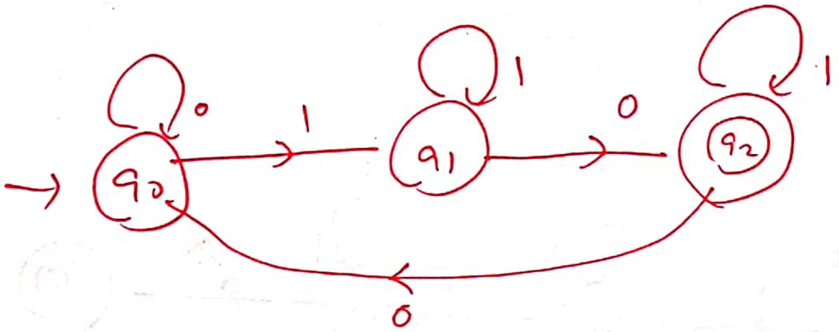
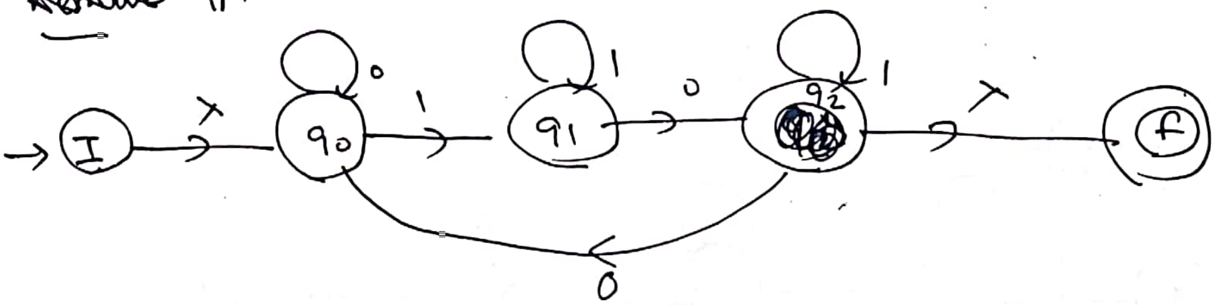
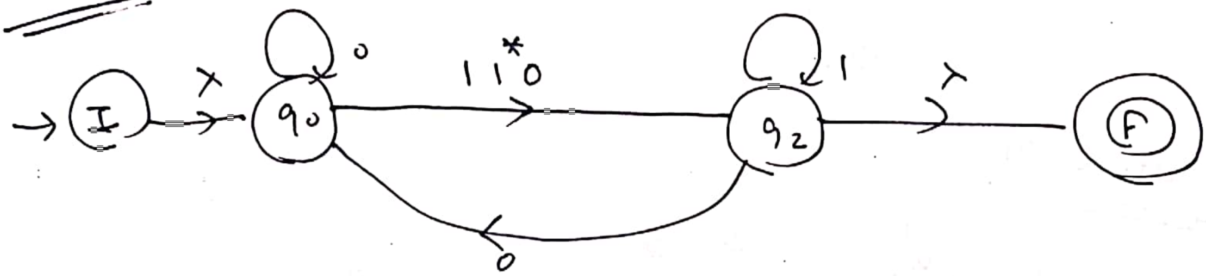
4) a)



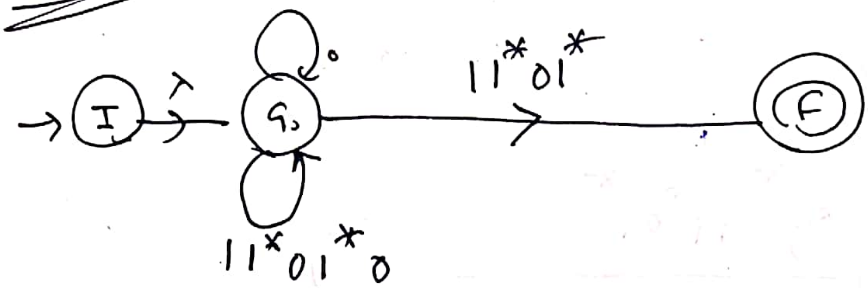
~~Remove q_f~~



Remove q_1

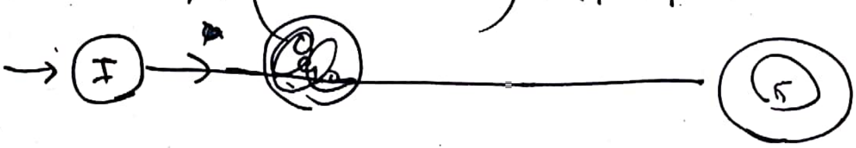


Remove q_2

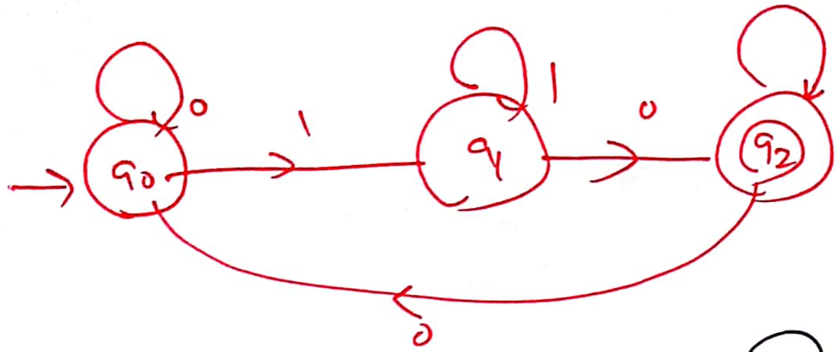


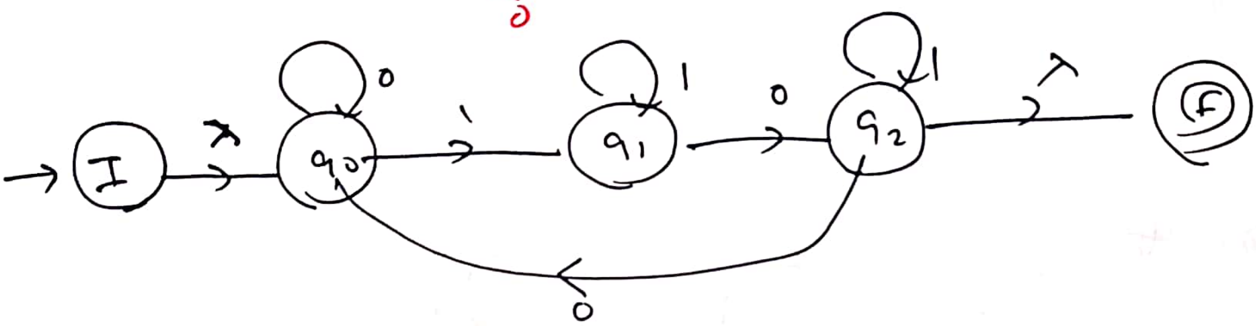
Remove q_0

$$\lambda \cdot (0 + 11^*01^*0)^* \cdot 11^*01^*$$

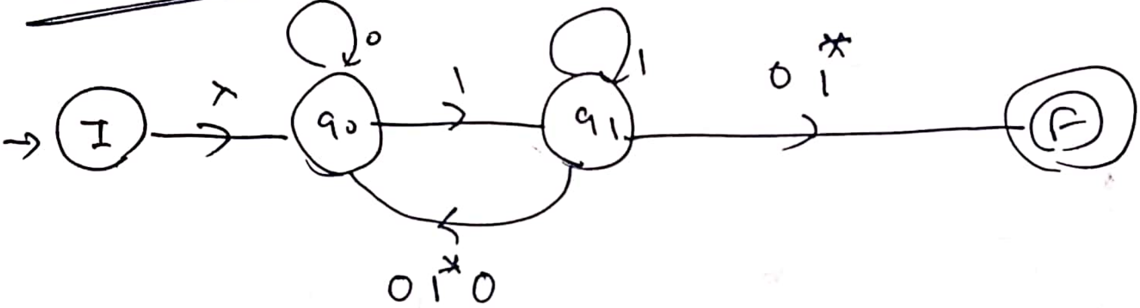


OR

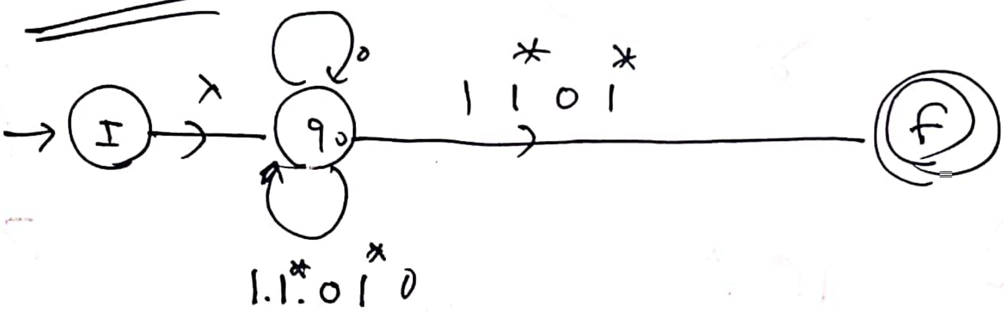




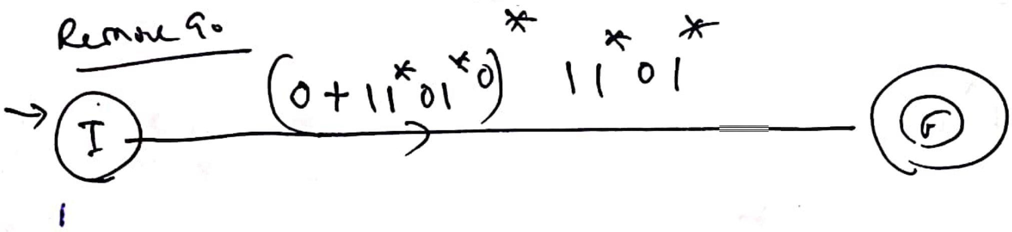
Remove q_2



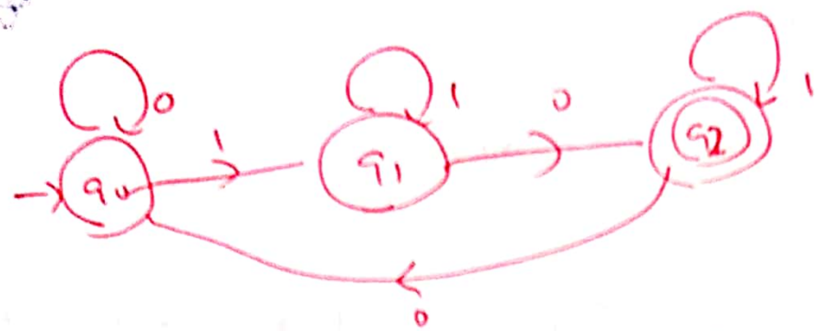
Remove q_1



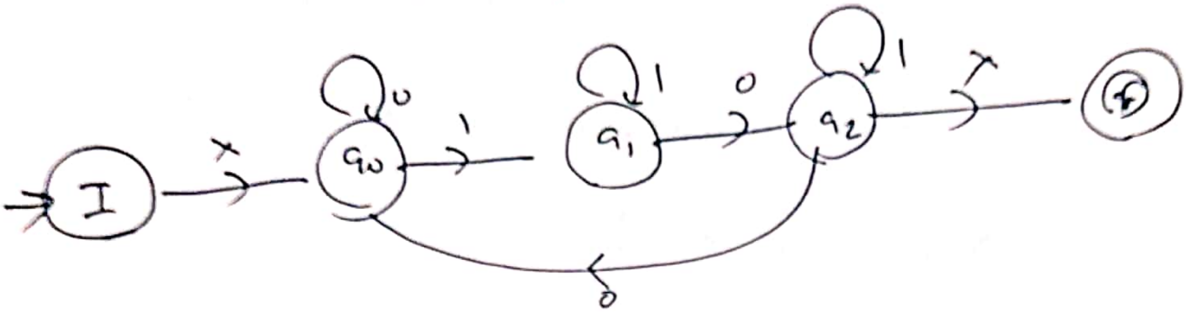
Remove q_0



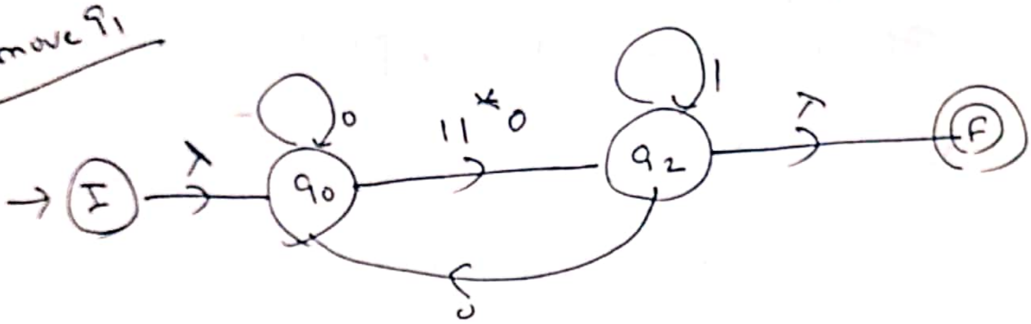
4) a)



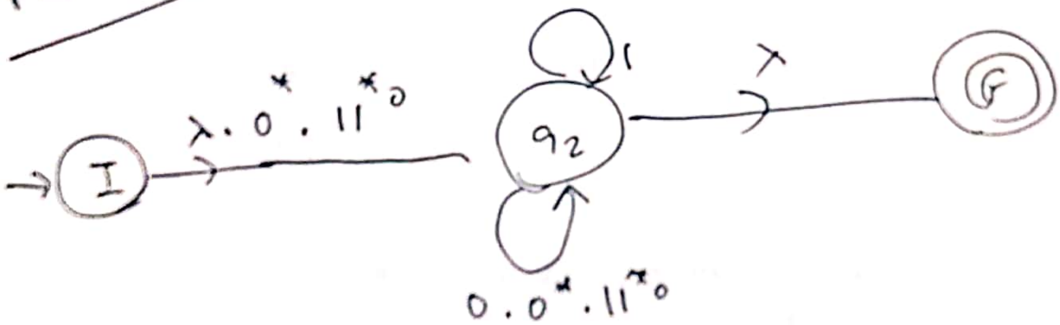
OR

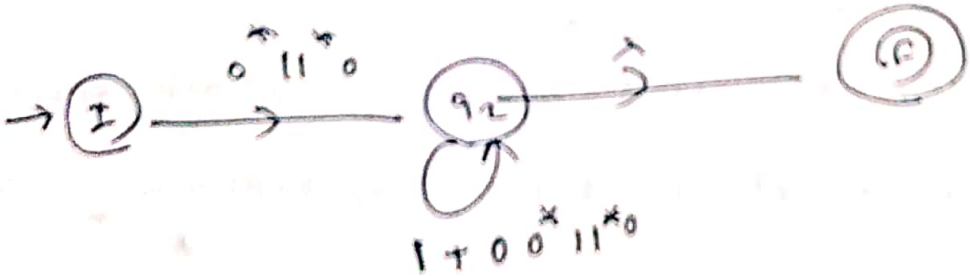


Remove q1

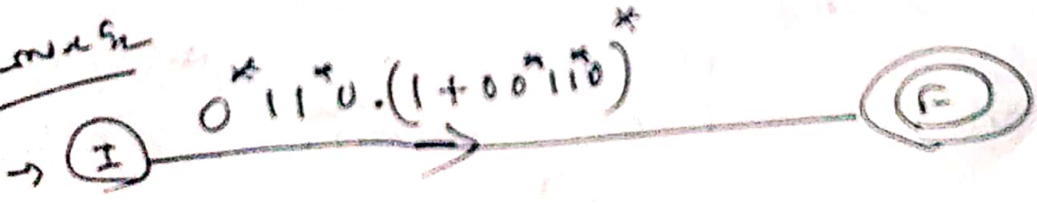


Remove q0





Remove q2



4) b) Write Regular expression for the following languages

i) $L = \{b^m a b^n \mid m>0, n>0\}$

RE-1

$$b^+ a b^+$$

RE-2

$$b.b^* a b.b^*$$

ii) Any string of length multiple of 5 over {0,1,2}

$$[(0+1+2)^5]^*$$

iii) Any string where 1st symbol is 0 & 3rd symbol from right is 0 over {0,1}

$$\underbrace{0(0+1)^2}_{\text{Minimum string of length 3}} + \underbrace{0(0+1)^*0(0+1)^2}_{\text{length} \geq 4}$$

5> a> If L_1 & L_2 are two regular languages, then prove that $L_1 - L_2$ is also regular. (2 marks)

Proof:

$$L_1 - L_2 = L_1 \cap \overline{L_2}$$

As L_2 is regular, $\overline{L_2}$ is also regular bcoz, Regular language is closed under complementation, and because of regular language is closed under intersection, so, $L_1 \cap \overline{L_2}$ is a regular. — (Proved)

5> b> State Pumping lemma for regular languages. Show that language $L = \{a^n b^{2n} \mid n \geq 1\}$ is not regular. (3 marks)

(1 mark)

Statement:

Let L be an infinite regular language. Then $\exists$ some +ve integer 'p' such that any $w \in L$ with $|w| \geq p$ can be decomposed as,

$$w = xyz$$

with $|xy| \leq p$

and $|y| \geq 1$

Such that $w_i = xy^i z \in L$ for all $i = 0, 1, 2, \ldots$

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Proof !

$L = \{a^n b^{2n} \mid n \geq 1\}$

(2 marks)

→ Assume 'L' is not regular.

→ Let 'p' be the pumping length.

→ Assume $\omega = a^p b^{2p}$

$|\omega| = |a^p b^{2p}| = 3p \geq p$ ✓

→ Divide ω into xyz.

$a^p b^{2p}$

a^{p-1} (x) a (y) b^{2p} (z)

S.t. $|xy| = |a^{p-1} a| \leq p$

$= p \leq p$ ✓

$|y| = |a| = 1 \geq 1$ ✓

Let,

→ Choose $i = 2$,

$\omega_2 = xy^2z = a^{p-1} a^2 b^{2p} = a^{p+1} b^{2p} \notin L$.

Hence, it is a contradiction to the assumption of L to be regular.

So, $L = \{a^n b^{2n} \mid n \geq 1\}$ is not regular.

— (Proved)