$= 0.937$

59) X = The time between two successive arrivals at the drive-up window of a local bank

$\lambda = 1$

a) $E(x) = \frac{1}{\lambda} = 1$

b) $\sigma = \frac{1}{\lambda} = 1$

c) $P(X \leq 4) = F(4;1) = 1 - e^{-\lambda x} = 1 - e^{-4} = 0.982$

d) $P(2 \leq X \leq 5) = F(5;1) - F(2;1) = 1 - e^{-5} - (1 - e^{-2})$

$= e^{-2} - e^{-5}$

$= 0.13$

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Ex: 4.4

60) $\lambda = 0.01386$

X: The distance (in m) that an animal moves from its birth site to the first territorial vacancy it encounters.

$P(X \le x) = F(x)$

a) $P(X \le 100) = F(100; 0.01386) \rightarrow F(x; \mu)$

$= 1 - e^{-100 \times 0.01386}$

$= 0.25$

$P(100 < X < 200) = F(200; 0.01386) - F(100; 0.01386)$

$= e^{-100(0.01386)} - e^{-200(0.01386)}$

$= 0.19$

b) ~~$P(X > \mu + 2\sigma) = P(X > \frac{1}{\lambda} + \frac{2}{\lambda})$~~

$\mu = \frac{1}{\lambda} = 72.15$

$\sigma = 72.15$

~~$P(|X - \mu| < 2\sigma) = P(\mu - 2\sigma < X < \mu + 2\sigma)$~~

~~$= P(-72.15 < X < 216.45)$~~

~~$= e^{72.15(0.01386)} - e^{-216.45(0.01386)}$~~

~~$=$~~

$P(X - \mu > 2\sigma) = P(X > \mu + 2\sigma)$

$= P(X > 216.45) = 1 - P(X \le 216.45)$

$= 1 - (1 - e^{-216.45(0.01386)})$

$= e^{-216.45 \times 0.01386} = 0.0498$

c) • $F(\tilde{\mu}) = 0.5$ (Holds for any distribution under rv) $\rightarrow$ Median

• $F(\tilde{\mu}; \lambda) = 1 - e^{-\lambda \tilde{\mu}} = 0.5$

$\Rightarrow 0.5 = e^{-0.01386 \tilde{\mu}}$

$\Rightarrow -0.01386 \tilde{\mu} = \ln 0.5$

$\Rightarrow \tilde{\mu} = -\frac{\ln(0.5)}{0.01386}$

$\Rightarrow \tilde{\mu} = 50.01$

Ex:- 4.4

6(a) $P(X \le 200) = F(200; 0.01386) = 1 - e^{-0.01386 \times 200}$

$= 0.937$

61) $E(X) = 2.725$

$\Rightarrow \frac{1}{\lambda} = 2.725$

$\Rightarrow \lambda = 0.367$

a) $P(X \geq 2) = 1 - P(X<2) = 1 - F(2; 0.367)$

$= 1 - (1 - e^{-0.367 \times 2})$

$= e^{-0.367 \times 2}$

$= 0.48$

$P(X \leq 3) = F(3; 0.367) = 1 - e^{-0.367 \times 3} = 0.667$

$P(2 \leq X \leq 3) = F(3; 0.367) - F(2; 0.367) = (1 - e^{-0.367 \times 3}) - (1 - e^{-0.367 \times 2})$

$= e^{-0.367 \times 2} - e^{-0.367 \times 3}$

$= 0.147$

b) $P(X > \mu + 2\sigma)$

$= P(X > 2.725 + 2 \times 2.725)$

$= 1 - P(X \leq 8.175)$

$= 1 - F(8.175; 0.367)$

$= 1 - (1 - e^{-0.367 \times 8.175})$

$= e^{-0.367 \times 8.175}$

$= 0.05$

$P(X < \mu - \sigma)$

$= P(X < 2.725 - 2.725)$

$= P(X < 0)$

$\cancel{= F(5.45, 0.367)} = 0$ [By definition]

$\cancel{= 1 - e^{-5.45 \times 0.367}}$

$\cancel{= 0.865}$

62) $f(x) = \left\{ 0.5\lambda e^{-\lambda|x|}, \; -\infty < x < \infty \right\}$

~~a)~~ $\cancel{\sigma = \frac{1}{\lambda} = 40.9}$

$\cancel{\Rightarrow \lambda = 0.0244}$

$\cancel{\mu = 40.9 = \frac{1}{\lambda}}$

a) $f(x) = \begin{cases} 0.5\lambda e^{-\lambda x}, & x \geq 0 \\ 0.5\lambda e^{\lambda x}, & x < 0 \end{cases}$

$E(x) = \int_{-\infty}^{\infty} x f(x) \, dx$

$= \int_{-\infty}^{0} x \, 0.5\lambda e^{\lambda x} dx + \int_{0}^{\infty} x \, 0.5\lambda e^{-\lambda x} dx$

$\cancel{= 0.5\lambda \left[\left[\frac{e^{\lambda x}}{\lambda} \right]_{-\infty}^{0} + \left[\frac{e^{-\lambda x}}{-\lambda} \right]_{0}^{\infty} \right]}$

$\cancel{= 0.5\lambda \left[\frac{1}{\lambda} - \frac{1}{\lambda} \right]}$

$= 0$

$$\cancel{E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx}$$

$$\cancel{= \int_{-\infty}^{0} x^2}$$

$$= 0.5\lambda\left[\int_{-\infty}^{0} x e^{\lambda x}dx + \int_{0}^{\infty} x e^{-\lambda x}dx\right]$$

$$= 0.5\lambda\left[x\int_{-\infty}^{0} e^{\lambda x}dx - \int_{-\infty}^{0}\left(\int e^{\lambda x}dx\right)dx + x\int_{0}^{\infty} e^{-\lambda x}dx - \int_{0}^{\infty}\left(\int e^{-\lambda x}dx\right)dx\right]$$

$$= 0.5\lambda\left[\left(\frac{xe^{\lambda x}}{\lambda} - \frac{e^{\lambda x}}{\lambda^2}\right)_{-\infty}^{0} + \left(\frac{xe^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2}\right)_{0}^{\infty}\right]$$

$$= 0.5\lambda\left[\frac{-1}{\lambda^2} + \frac{1}{\lambda^2}\right]$$

$$= 0$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \underbrace{\int_{-\infty}^{0} x^2 0.5\lambda e^{\lambda x}dx}_{\mathrm{I}} + \underbrace{\int_{0}^{\infty} x^2 0.5\lambda e^{-\lambda x}dx}_{\mathrm{II}}$$

$$\int_{-\infty}^{0} x^2 0.5\lambda e^{\lambda x}dx = 0.5\lambda\left[x^2\int_{-\infty}^{0} e^{\lambda x}dx - 2\int_{-\infty}^{0}\left(x\int e^{\lambda x}dx\right)dx\right]$$

$$= 0.5\lambda\left[x^2\frac{e^{\lambda x}}{\lambda} - \frac{2}{\lambda}\int_{-\infty} xe^{\lambda x}dx\right]^{0}$$

$$= 0.5\lambda\left[x^2\frac{e^{\lambda x}}{\lambda} - \frac{2}{\lambda}\left(\frac{xe^{\lambda x}}{\lambda} - \frac{e^{\lambda x}}{\lambda^2}\right)\right]_{-\infty}^{0}$$

$$= 0.5\lambda\left[\frac{2}{\lambda^3}\right] = \frac{1}{\lambda^2}$$

$$\int_{0}^{\infty} x^2 0.5\lambda e^{-\lambda x}dx = 0.5\lambda\left[x^2\int_{0}^{\infty} e^{-\lambda x}dx - 2\int_{0}^{\infty}\left(x\int e^{-\lambda x}dx\right)dx\right]$$

$$= 0.5\lambda\left[\frac{x^2e^{-\lambda x}}{-\lambda} + \frac{2}{\lambda}\int_{0}^{\infty} xe^{-\lambda x}dx\right]$$

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$$= 0.5\lambda\left[\frac{x^2e^{-\lambda x}}{-\lambda} + \frac{2}{\lambda}\left[\frac{xe^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2}\right]\right]_0^\infty$$

$$= 0.5\lambda\left[\frac{2}{\lambda^3}\right]$$

$$= \frac{1}{\lambda^2}$$

$$\therefore E(x^2) = \frac{2}{\lambda^2}$$

$$\therefore V(x) = E(x^2) - \{E(x)\}^2$$

$$= \frac{2}{\lambda^2} - 0$$

$$= \frac{2}{\lambda^2}$$

$$\therefore \sigma = \frac{\sqrt{2}}{\lambda} = 40.9$$

$$\Rightarrow \lambda = 0.0345\!77$$

b) $P(\mu - \sigma < X < \mu + \sigma)$

$$= P(-40.9 < X < 40.9)$$

$$= \int_{-40.9}^{40.9} 0.5\lambda e^{-\lambda|x|}\,dx$$

$$= 0.5\lambda\left[\int_{-40.9}^{0} e^{\lambda x}\,dx + \int_{0}^{40.9} e^{-\lambda x}\,dx\right]$$

$$= 0.5\lambda\left[\left[\frac{e^{\lambda x}}{\lambda}\right]_{-40.9}^{0} + \left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^{40.9}\right]$$

$$= 0.5\lambda\left[\frac{1}{\lambda} - \frac{e^{-\lambda\times 40.9}}{\lambda} - \frac{e^{-\lambda\times 40.9}}{\lambda} + \frac{1}{\lambda}\right]$$

$$= 0.5\lambda\left[\frac{2}{\lambda} - \frac{2e^{-\lambda\times 40.9}}{\lambda}\right]$$

$$= 0.5\left[2 - 2e^{-0.034577\times 40.9}\right]$$

$$= 1 - e^{-0.034577\times 40.9} = 0.757$$

X: The no. of minutes in duration.

63) b) $h_1(x) = 10x$

$$h_2(x) = \begin{cases} 99, & x \le 20 \\ 99 + (x-20)\times 10, & x > 20 \end{cases}$$

$$E(h_1(x)) = 10\,E(h_1(x)) = 10\mu = \frac{10}{\lambda}$$

$$E(h_2(x)) = 99 + \int_{20}^{\infty} (x-20)\times 10\,\lambda e^{-\lambda x}\,dx$$

$$= 99 + 10\lambda \int_{20}^{\infty} (x-20)e^{-\lambda x}\,dx$$

$$= 99 + 10\lambda \left[\int_{20}^{\infty} x e^{-\lambda x}\,dx - 20\int_{20}^{\infty} e^{-\lambda x}\,dx \right]$$

$$= 99 + 10\lambda \left[x\int_{20}^{\infty} e^{-\lambda x}\,dx - \int_{20}^{\infty}\left(\int e^{-\lambda x}\,dx\right)dx + \frac{20e^{-\lambda x}}{\lambda} \right]$$

$$= 99 + 10\lambda \left[\frac{x e^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2} + \frac{20e^{-\lambda x}}{\lambda} \right]_{20}^{\infty}$$

$$= 99 + 10\lambda \left[\cancel{\frac{20e^{-\lambda \times 20}}{\lambda}} + \frac{e^{-\lambda\times 20}}{\lambda^2} - \cancel{\frac{20e^{-\lambda\times 20}}{\lambda}} \right]$$

$$= 99 + \frac{10e^{-\lambda\times 20}}{\lambda}$$

$$E(h_2(x)) = 99 + 10\mu e^{-20/\mu}$$

when $\mu = 10$,

$$E(h_1(x)) = 10\times 10 = 100¢ = 1\$$$

$$E(h_2(x)) = 99¢ + 10\times 10\,e^{-20/10} = 112.53¢ = 1.13\$$$

when $\mu = 15$,

$$E(h_1(x)) = 10\times 15 = 150¢ = 1.5\$$$

$$E(h_2(x)) = 99 + 10\times 15\,e^{-20/15} = 138.54¢ = 1.38\$$$

The first plan is better if the expected call length is lower and the second plan is better if the expected call length is somewhat higher.

a) If the customer's calls are typically short the first plan is better and if the customer's calls are long, the second plan makes more sense.

Eq) $F(\eta(p)) = \int_{-\infty}^{\eta(p)} f(y)\,dy = p$ → 100pth percentile

$\eta = \eta(p)$

$p = F\{\eta ; \lambda\} = 1 - e^{-\eta\lambda}$

We know, $F(\tilde{\mu}; \lambda) = 0.5$

$\Rightarrow 1 - e^{-\tilde{\mu}\lambda} = 0.5$

$\Rightarrow e^{-\tilde{\mu}\lambda} = 0.5$

$\Rightarrow -\tilde{\mu}\lambda = \ln 0.5$

$\Rightarrow \tilde{\mu} = \frac{-\ln 0.5}{\lambda}$

$\Rightarrow \tilde{\mu} = \frac{0.693}{\lambda}$