

Theory of Computation  
Mid-Semester Examination  
School of Computer Engineering  
KIIT University, Bhubaneswar.

Time: 2 hours

Full Marks: 50

(Answer any five questions including question number 1)

1. Write true or false, and justify your answer briefly. [10]
- (i) Let  $D$  be a DFA that accepts a string of length 3, then  $D$  has at least 4 states.
  - (ii) Let  $L$  and  $M$  be languages such that both  $L$  and  $LM$  are regular. Then  $M$  is also regular.
  - (iii) The regular expressions  $(0 + 1 + 00)^*$  and  $(1^*0^* + 11)^*$  are equivalent.
  - (iv) There exists an infinite regular language  $L$  such that  $L^* = L^+$ .
  - (v) If both  $L$  and  $M$  are non-regular languages, then  $L \cup M$  is also non-regular.
2. (a) Let  $L = \{ulw : u, w \in \{0,1\}^*, |u| \geq 2 \text{ and } |w| \leq 2\}$ . [5]
- (i) Write a regular expression for the language  $L$ .
  - (ii) Design an NFA that accepts  $L$ .
  - (iii) Design a DFA that accepts  $L$ .
- (b) Convert the following NFA to a DFA. [5]

$\delta$	$\lambda$	$a$	$b$
$\rightarrow q_0$	$q_1$	$\Phi$	$q_2$
$q_1$	$\{q_2, q_3\}$	$\{q_0, q_4\}$	$\Phi$
$q_2$	$\Phi$	$\Phi$	$q_4$
$q_3$	$\Phi$	$q_4$	$\Phi$
$* q_4$	$q_3$	$\Phi$	$\Phi$

3. (a) Write regular expressions for the following languages over the alphabet  $\{a, b\}$ . [5]
- (i) The set of all strings starting with  $aa$  and ending with  $aa$ .
  - (ii) The set of all strings not containing the substring  $ab$ .
  - (iii) The set of all strings containing even number of  $b$ 's.
  - (iv)  $\{a^m b^n : m + n \text{ is odd}\}$ .
  - (v) The set of all strings of length at least 50 and at most 90.

- (b) Let  $\Sigma = \{a, b\}$ , and  $L$  be the language of all strings that contain neither  $aa$  nor  $bb$  as a substring. [5]
- Design an *NFA* that accepts  $L$ .
  - Design a *DFA* that accepts  $L$ .
  - Write a regular expression for the language  $L$ .
4. (a) State the pumping lemma for regular languages. [5]  
 Prove that the language  $L = \{a^n : n \text{ is a prime number}\}$  is not regular.
- (b) (i) Design an *NFA* with four states that accepts the language of the regular expression  $(01 + 011 + 0111)^*$ . [5]  
 (ii) What languages do the regular expressions  $\Phi^*$  and  $a\Phi$  denote?
5. (a) Design a *DFA* that accepts the language  $L = \{w : cab \text{ is not a substring of } w\}$  over the alphabet  $\{a, b, c\}$ . Convert your *DFA* to a regular expression using the state-elimination method. [5]
- (b) Convert the following regular expressions to *NFA*. [5]
- $(a + b)^*ba$
  - $bb(a + b)^*a$
6. (a) Consider the following *DFA*: [5]

$\delta$	$a$	$b$
$\rightarrow p_0$	$p_1$	$p_5$
$p_1$	$p_0$	$p_2$
$* p_2$	$p_1$	$p_3$
$p_3$	$p_3$	$p_1$
$p_4$	$p_3$	$p_4$
$* p_5$	$p_4$	$p_3$

- Compute the blocks of indistinguishable states.
  - Construct the equivalent minimal *DFA*.
- (b) (i) What do you mean by indistinguishable pair of states in a *DFA*. Illustrate with an example. [5]
- (ii) Is the distinguishability relation of states of a *DFA* transitive?