interest are (*i*) phase difference between applied voltage and circuit current (*ii*) circuit impedance (*iii*) power consumed *etc*. To begin with, we shall study these characteristics for simple a.c. circuits containing one circuit element only (*R* or *L* or *C*) and extend our discussion to the combination of these circuit elements in the later chapters.

15.22. A.C. CIRCUIT CONTAINING RESISTANCE ONLY

Consider a circuit containing a pure resistance of $R\Omega$ connected across an alternating voltage source. Let the alternating voltage be given by the equation:

$$v = V_m \sin \omega t \quad ...(i)$$

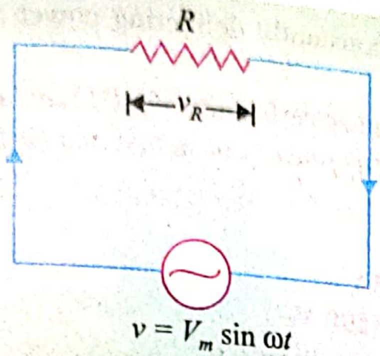


Fig. 15.25

Fig. 15.26

As a result of this voltage, an alternating current *i* will flow in the circuit. The applied voltage has to overcome the drop in the resistance only *i.e.*,

$$v = iR$$

or
$$i = \frac{v}{R}$$

Substituting the value of *v*, we get,

$$i = \frac{V_m}{R} \sin \omega t \quad ...(ii)$$

The value of *i* will be maximum (*i.e.*, I_m) when $\sin \omega t = 1$.

$\therefore$
$$I_m = V_m/R$$

$\therefore$ Eq. (*ii*) becomes,
$$i = I_m \sin \omega t \quad ...(iii)$$

It is clear from eqs. (*i*) and (*iii*) that the applied voltage and circuit current are in phase with each other. This fact is also shown by the phasor diagram in Fig. 15.26 and wave diagram in Fig. 15.27. Note that r.m.s. values have been used in drawing the phasor diagram.

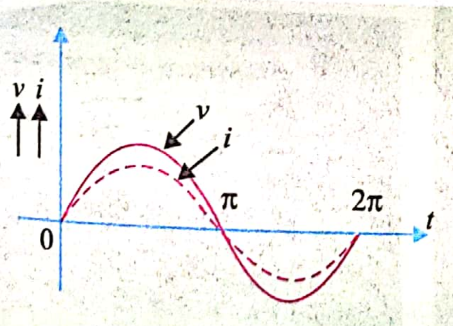


Fig. 15.27

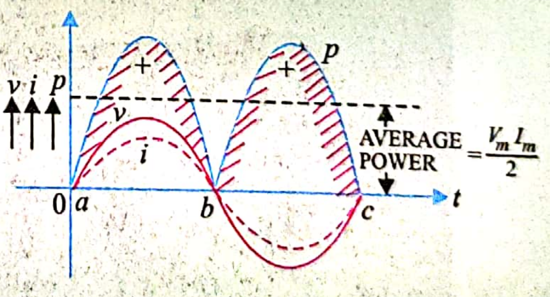


Fig. 15.28

Power

Instantaneous power, $p = vi = (V_m \sin \omega t)\ (I_m \sin \omega t) = V_m I_m \sin^2 \omega t$

$$= V_m I_m \frac{(1 - \cos 2\omega t)}{2}$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

Thus power consists of two parts *viz.* a constant part $(V_m I_m/2)$ and a fluctuating part $(V_m I_m/2)$ cos 2 ωt. Since power is a scalar quantity, average power over a complete cycle is to be considered. For a complete cycle, the average value of $(V_m I_m/2)$ cos 2ωt is zero.

∴ Power consumed, $$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or $$P = V I$$

where V = r.m.s. value of the applied voltage

I = r.m.s. value of the circuit current

Fig. 15.28 shows the power curve for a pure resistive circuit. It is clear that power is always positive. This means that the voltage source is constantly delivering power to the circuit which is consumed by the circuit.

Example 15.16. *An a.c. circuit consists of a pure resistance of 10 Ω and is connected across an a.c. supply of 230 V, 50Hz. Calculate (i) current (ii) power consumed and (iii) equations for voltage and current.*

Solution.

(*i*) Current, $I = V/R = 230/10 =$ **23A**

(*ii*) Power, $P = VI = 230 \times 23 =$ **5290 W**

(*iii*) Now, $V_m = \sqrt{2}V = \sqrt{2} \times 230 = 325.27$ volts

$I_m = \sqrt{2}I = \sqrt{2} \times 23 = 32.52$ A

$\omega = 2\pi f = 2\pi \times 50 = 314 \text{ rad } s^{-1}$

∴ Equations of voltage and current are:

$$e = 325.27 \sin 314t \; ; \; i = 32.52 \sin 314t$$

Example 15.17. *In a pure resistive circuit, the instantaneous voltage and current are given by*

$$v = 250 \sin 314t \; ; \; i = 10 \sin 314t.$$

Determine (i) the peak power and (ii) average power.

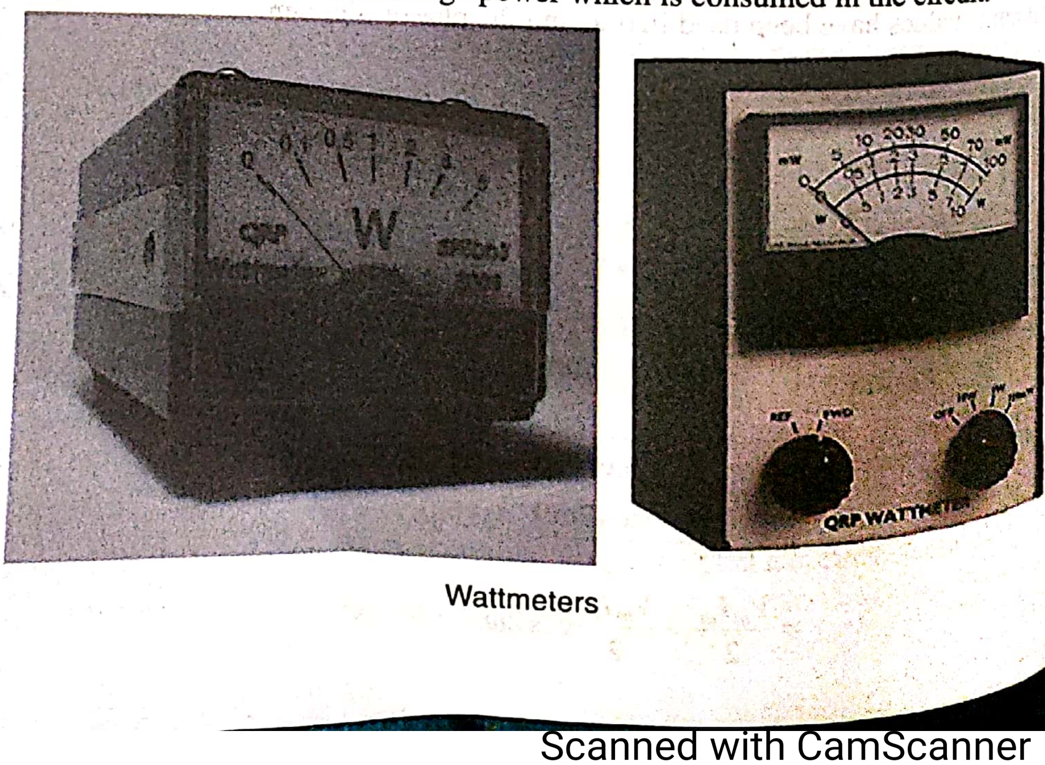
Solution.

In a pure resistive a.c. circuit,

(*i*) Peak power $= V_m I_m = 250 \times 10 =$ **2500W**

(*ii*) Average power, $P = \frac{V_m I_m}{2} = \frac{2500}{2} =$ **1250 W**

The reader may note that it is the average power which is consumed in the circuit.



Wattmeters

15.23. A.C. CIRCUIT CONTAINING INDUCTANCE ONLY

When an alternating current flows through a pure *inductive coil, a back e.m.f. (= $L\,di/dt$) is induced due to the inductance of the coil. This back e.m.f. at every instant opposes the change in current through the coil. Since there is no ohmic drop, the applied voltage has to overcome the back e.m.f. only.

Applied alternating voltage = Back e.m.f.

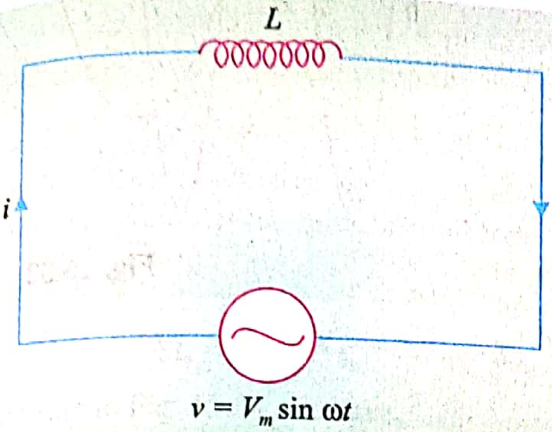


Fig. 15.29

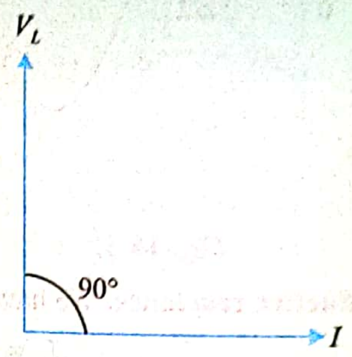


Fig. 15.30

Consider an alternating voltage applied to a pure inductance of L henry as shown in Fig. 15.29. Let the equation of the applied alternating voltage be:

$$v = V_m \sin \omega t \qquad ...(i)$$

Clearly,
$$V_m \sin \omega t = L\frac{di}{dt}$$

or
$$di = \frac{V_m}{L} \sin \omega t \, dt$$

Integrating both sides, we get,

$$i = \frac{V_m}{L}\int \sin \omega t \, dt$$

$$= \frac{V_m}{\omega L}(-\cos \omega t)$$

or
$$i = \frac{V_m}{\omega L} \sin(\omega t - \pi/2) \qquad ...(ii)$$

The value of i will be maximum (*i.e.*, I_m) when $\sin(\omega t - \pi/2)$ is unity.

$\therefore$
$$I_m = V_m/\omega L$$

$\therefore$ Eq. (*ii*) becomes, $i = I_m \sin(\omega t - \pi/2)$ where $I_m = V_m/\omega L$ ...(*iii*)

It is clear from eqs. (*i*) and (*iii*) that current lags behind the voltage by $\pi/2$ radians or 90°. *Hence in a pure inductance, current lags behind the voltage by* 90°. This fact is also shown in the phasor diagram in Fig. 15.30 and wave diagram in Fig. 15.31.

* Any circuit that is capable of producing flux has inductance as was pointed out in chapter 9. When alternating current flows through such a circuit, there is change in flux linking it and hence back e.m.f. (= Ldi/dt) is induced in the circuit. This e.m.f. opposes the applied voltage at every instant.

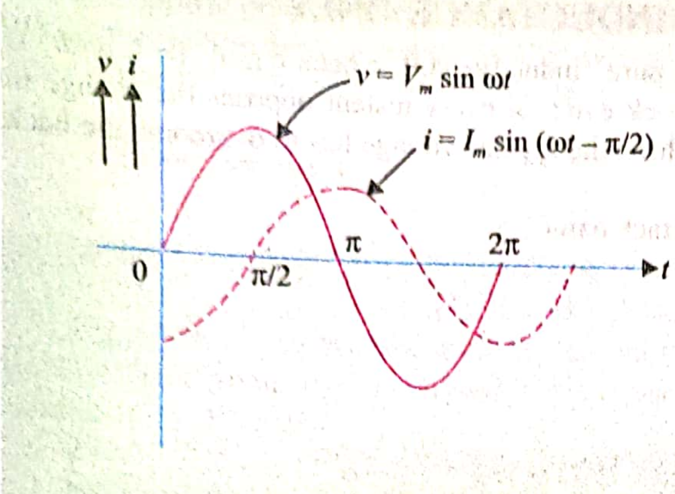


Fig. 15.31

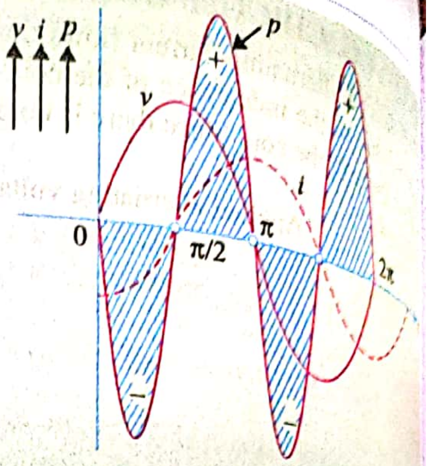


Fig. 15.32

Inductive reactance. We have seen above that :

$$I_m = V_m / \omega L$$

or $$^{*}\frac{V_m}{I_m} = \omega L$$

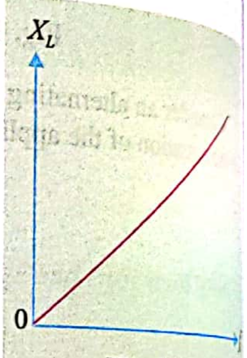


Fig. 15.33

Clearly, the opposition offered by inductance to current flow is ωL. The quantity ωL is called *inductive reactance* X_L of the coil and is measured in Ω. Note that $X_L (= \omega L = 2\pi fL)$ will be in Ω if L is in henry and f in Hz. Since $X_L = 2\pi fL$, $X_L \propto f$. Therefore graph between X_L and f is a straight line passing through the origin as shown in Fig. 15.33.

Power

Instantaneous power, $p = vi = V_m \sin \omega t \times I_m \sin(\omega t - \pi/2)$

$$= -V_m I_m \sin \omega t \cos \omega t = -\frac{V_m I_m}{2} \sin 2\omega t$$

∴ Average power, P = Average of p over one cycle

$$= \frac{1}{2\pi} \int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t) = 0$$

Hence, power absorbed in pure inductance is zero.

Fig. 15.32 shows the power curve for a pure inductive circuit. An examination of power curve over one cycle shows that positive power is equal to the negative power. Hence the resultant power over one cycle is zero *i.e.*, pure inductance consumes no power. The electric power merely flows from the source to the coil and back again.

Example15.18. *A pure inductive coil allows a current of 10A to flow from a 230V, 50Hz supply. Find (i) inductive reactance (ii) inductance of the coil (iii) power absorbed. Write down the equations for voltage and current.*

Solution.

(*i*) Circuit current, $I = V/X_L$

∴ Inductive reactance, $X_L = V/I = 230/10 = 23\Omega$

(*ii*) Now, $X_L = 2\pi fL$

* If V and I are the r.m.s. values, $\frac{V_m}{I_m} = \frac{V}{I} = \omega L$.

$$L = \frac{X_L}{2\pi f} = \frac{23}{2\pi \times 50} = \mathbf{0.073\,H}$$

∴ (iii) Power absorbed = **Zero**

$V_m = 230 \times \sqrt{2} = 325.27\,\text{V}$; $I_m = 10 \times \sqrt{2} = 14.14\,\text{A}$; $\omega = 2\pi \times 50 = 314\,\text{rad s}^{-1}$

Since in a pure inductive circuit, current lags behind the voltage by $\pi/2$ radians, the equations are:

$v = 325.27 \sin 314t$; $i = 14.14 \sin (314t - \pi/2)$

For the circuit shown in Fig. 15.34, determine L and total inductive reactance

$2\pi f \quad 2\pi \times 250$

. A.C. CIRCUIT CONTAINING CAPACITANCE ONLY

hen an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in ection and then in the other as the voltage reverses. The result is that electrons move to and fro he circuit, connecting the plates, thus constituting alternating current.

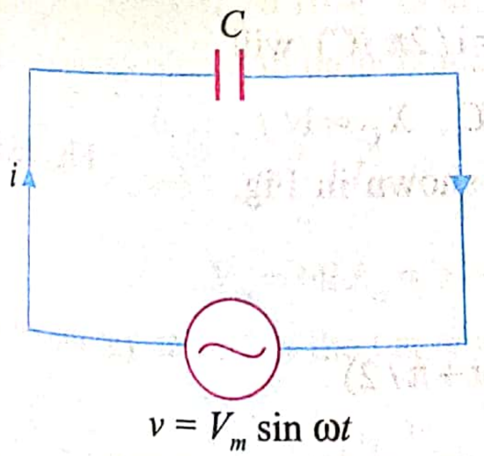


Fig. 15.35

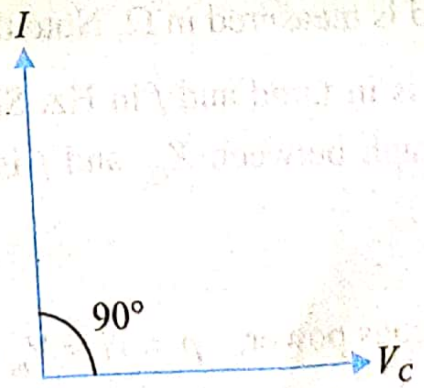


Fig. 15.36

Consider an alternating voltage applied to a capacitor of capacitance C farad as shown in Fig. 15.35. Let the equation of the applied alternating voltage be:

$$v = V_m \sin \omega t$$

As a result of this alternating voltage, alternating current will flow through the circuit. Let at any instant i be the current and q be the charge on the plates.

Charge on capacitor, $q = Cv = CV_m \sin \omega t$

$\therefore$ Circuit current,
$$i = \frac{d}{dt}(q) = \frac{d}{dt}(CV_m \sin \omega t) = \omega CV_m \cos \omega t$$

or
$$i = \omega CV_m \sin(\omega t + \pi/2)$$

The value of i will be maximum (*i.e.*, I_m) when $\sin(\omega t + \pi/2)$ is unity.

$\therefore$
$$I_m = \omega C V_m$$

Substituting the value $\omega CV_m = I_m$ in eq. (*ii*), we get,

$$i = I_m \sin(\omega t + \pi/2)$$

It is clear from eqs. (*i*) and (*iii*) that current leads the voltage by $\pi/2$ radians or 90°. Hence in a *pure capacitance, current leads the voltage by* 90°. This fact is also shown in the phasor diagram in Fig. 15.36 and wave diagram in Fig. 15.37.

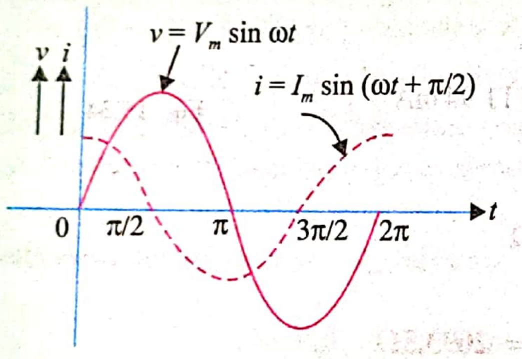


Fig. 15.37

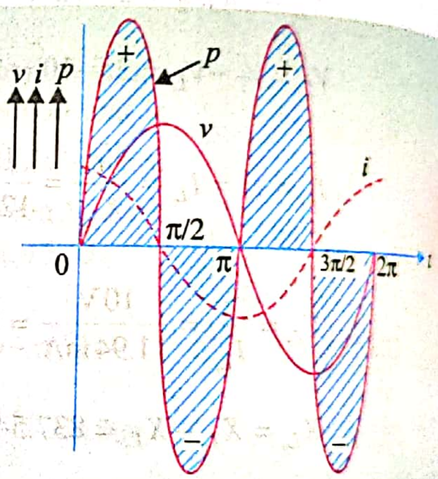


Fig. 15.38

Capacitive reactance. We have seen above that:

$$I_m = \omega CV_m$$

or
$$^{*}\frac{V_m}{I_m} = \frac{1}{\omega C}$$

Clearly, the opposition offered by capacitance to current flow is $1/\omega C$. The quantity $1/\omega C$ is called *capacitive reactance* X_C of the capacitor and is measured in Ω. Note that $X_C (=1/\omega C = 1/2\pi fC)$ will be in Ω if C is in farad and f in Hz. Since $X_C = 1/2\pi fC$, $X_C \propto 1/f$. Therefore, graph between X_C and f is a hyperbola as shown in Fig. 15.39.

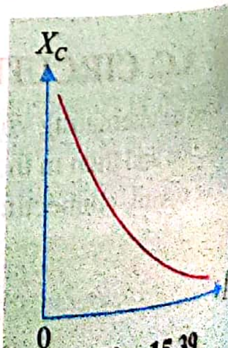


Fig. 15.39

Power

Instantaneous power, $p = vi = V_m \sin \omega t \times I_m \sin(\omega t + \pi/2)$

* If V and I are the r.m.s. values, $\frac{V_m}{I_m} = \frac{V}{I} = \frac{1}{\omega C}$.

$$= V_m I_m \sin \omega t \cos \omega t = \frac{V_m I_m}{2} \sin 2\omega t$$

Average power, P = Average of p over one cycle

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t) = 0$$

Hence, power absorbed in a pure capacitance is zero.

Fig. 15.38 shows the power curve for a pure capacitive circuit. The power curve is similar to that of pure inductor because now current leads the voltage by 90°. It is clear that positive power is equal to the negative power over one cycle. Hence net power absorbed in a pure capacitor is zero.

Example 15.20. *A 318μF capacitor is connected across a 230V, 50Hz system. Determine the capacitive reactance (ii) r.m.s. value of current and (iii) equations for voltage and current.*

Solution.

(i) Capacitive reactance, $X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 50 \times 318} = 10\Omega$

(ii) R.M.S. value of current, $I = V/X_C = 230/10 = 23A$

(iii) $V_m = 230 \times \sqrt{2} = 325.27$ volts; $I_m = \sqrt{2} \times 23 = 32.53A$; $\omega = 2\pi \times 50 = 314 \text{ rad s}^{-1}$

∴ Equations for voltage and current are : $v = 325.27 \sin 314t$; $i = 32.53 \sin (314t + \pi/2)$

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