In the following problems, find the root as specified using the regula-falsi method (method of false position).

1. Find the positive root of $x^3 = 2x + 5$. (Do only four iterations).   (*A.U. Nov./Dec. 2006*)

2. Find an approximate root of $x \log_{10} x - 1.2 = 0$.

3. Solve the equation $x \tan x = -1$, starting with $a = 2.5$ and $b = 3$, correct to three decimal places.

4. Find the root of $xe^x = 3$, correct to two decimal places.

5. Find the smallest positive root of $x - e^{-x} = 0$, correct to three decimal places.

6. Find the smallest positive root of $x^4 - x - 10 = 0$, correct to three decimal places.

In the following problems, find the root as specified using the Newton-Raphson method.

7. Find the smallest positive root of $x^4 - x = 10$, correct to three decimal places.

8. Find the root between 0 and 1 of $x^3 = 6x - 4$, correct to two decimal places.

9. Find the real root of the equation $3x = \cos x + 1$.   (*A.U. Nov./Dec. 2006*)

10. Find a root of $x \log_{10} x - 1.2 = 0$, correct to three decimal places.

(*A.U. Nov./Dec. 2004*)

11. Find the root of $x = 2 \sin x$, near 1.9, correct to three decimal places.

12. (*i*) Write an iteration formula for finding $\sqrt{N}$ where N is a real number.

(*A.U. Nov./Dec. 2006, A.U. Nov./Dec. 2003*)

(*ii*) Hence, evaluate $\sqrt{142}$, correct to three decimal places.

13. (i) Write an iteration formula for finding the value of $1/N$, where N is a real number.

(ii) Hence, evaluate $1/26$, correct to four decimal places.

14. Find the root of the equation $\sin x = 1 + x^3$, which lies in the interval $(-2, -1)$, correct to three decimal places.

15. Find the approximate root of $xe^x = 3$, correct to three decimal places.

In the following problems, find the root as specified using the iteration method / method of successive approximations / fixed point iteration method.

16. Find the smallest positive root of $x^2 - 5x + 1 = 0$, correct to four decimal places.

17. Find the smallest positive root of $x^5 - 64x + 30 = 0$, correct to four decimal places.

18. Find the smallest negative root in magnitude of $3x^3 - x + 1 = 0$, correct to four decimal places.

19. Find the smallest positive root of $x = e^{-x}$, correct to two decimal places.

20. Find the real root of the equation $\cos x = 3x - 1$.   (*A.U. Nov. / Dec. 2006*)

21. The equation $x^2 + ax + b = 0$, has two real roots α and β. Show that the iteration method

(i) $x_{k+1} = -(ax_k + b)/x_k$, is convergent near $x = \alpha$, if $|\alpha| > |\beta|$,

(ii) $x_{k+1} = -b/(x_k + a)$, is convergent near $x = \alpha$, if $|\alpha| < |\beta|$.