Relational Algebra

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Query Language

Language in which user requests information from the database are:

- Procedural language
- Nonprocedural language

The categories of different languages are:

- SQL
- Relational Algebra
- Relational Calculus
  - Tuple Relational Calculus
  - Domain Relational Calculus

Relational Algebra

Relational algebra is a procedural language for manipulating relations. Relational algebra operations manipulate relations. That is, these operations use one or two existing relations to create a new relation

- Fundamental operators
  - Unary: SELECT, PROJECT, RENAME
  - Binary: UNION, SET DIFFERENCE, CARTESIAN PRODUCT
- Secondary operators
  - INTERSECTION, NATURAL JOIN, DIVISION, and ASSIGNMENT

Relational Algebra...

The relational schemas used for different operations are:

- **Customer(cust_name, cust_street, cust_city)**
  - used to store customer details
- **Branch(branch_name, branch_city, assets)**
  - used to store branch details
- **Account(acc_no, branch_name, balance)**
  - stores the account details
- **Loan(loan_no, branch_name, amount)**
  - stores the loan details
- **Depositor(cust_name, acc_no)**
  - stores the details about the customers' account
- **Borrower(cust_name, loan_no)**
  - used to store the details about the customers' loan

SELECT Operator(σ)

SELECT Operator(σ)

SELECT operation is used to create a relation from another relation by selecting only those tuples or rows from the original relation that satisfy a specified condition. It is denoted by **sigma (σ) symbol.** The predicate appears as a subscript to σ

The argument relation is in parenthesis after the σ. The result is a relation that has the same attributes as the relation specified in <relation-name>. The general syntax of select operator is:

$$\sigma_{<selection-condition>}\ \textbf{(<relation name>)}$$

Query: Find the details of the loans taken from 'Bhubaneswar Main' branch.

$$\sigma_{branch\_name='BhubaneswarMain'}\ (Loan)$$

- The operators used in selection predicate may be: =, /=, <, ≤, >, ≥ .
- Different predicates can be combined into a larger predicate by using the connectors like: AND(V), OR(W), NOT(¬)

SELECT Operator(σ)...

Loan

loan_no	branch_name	amount
L201	Bhubaneswar Main	50,000,000.00
L202	Bhubaneswar Main	5,000,000.00
L203	Mumbai Main	100,000,000.00
L204	Juhu	60,000,000.00

$\sigma_{branch\_name='BhubaneswarMain'}$ (Loan)

loan_no	branch_name	amount
L201	Bhubaneswar Main	50,000,000.00
L202	Bhubaneswar Main	5,000,000.00

$\sigma_{branch\_name='BhubaneswarMain' AND amount > 10,000,000}$ (Loan)

loan_no	branch_name	amount
L201	Bhubaneswar Main	50,000,000.00

PROJECT Operator(π)

PROJECT Operator(π)

PROJECT operation can be thought of as eliminating unwanted columns. It eliminates the duplicate rows. It is denoted by pie(π) symbol. The attributes needed to be appeared in the resultant relation appear as subscript to π.

The argument relation follows in parenthesis. The general syntax of project operator is:

π <attribute-list> **(<relation name>)**

Query: Find the loan numbers and respective loan amounts.

$\pi_{loan\_no,amount}$ (Loan)

PROJECT Operator(π)...

Loan

loan_no	branch_name	amount
L201	Bhubaneswar Main	50,000,000.00
L202	Bhubaneswar Main	5,000,000.00
L203	Mumbai Main	100,000,000.00
L204	Juhu	60,000,000.00

$\pi_{loan\_no,amount}$ (Loan)

loan_no	amount
L201	50,000,000.00
L202	5,000,000.00
L203	100,000,000.00
L204	60,000,000.00

Composition of Relational Operators

Composition of Relational Operators

Relational algebra operators can be composed together into a relational algebra expression to answer the complex queries

Q:Find the name of the customers who live in Bhubaneswar

Customer

cust_name	cust_street	cust_city
Rishi	India Gate	New Delhi
Sarthak	M. G. Road	Bangalore
Manas	Shastri Nagar	Bhubaneswar
Ramesh	M. G. Road	Bhubaneswar
Mahesh	Juhu	Mumbai

$\Pi_{cust\_name}\ (\sigma_{cust\_city='Bhubaneswar'}\ (\text{Customer}))$

cust_name
Manas
Ramesh

RENAME Operator(ρ)

RENAME Operator(ρ)

The results of relational algebra expressions do not have a name that can be used to refer them. It is useful to be able to give them names; the rename operator is used for this purpose. It is denoted by rho(ρ) symbol.

The general syntax of rename operator is: ρ_X **(E)**

Assume E is a relational-algebra expression with arity n. The second form of rename operation is: $\rho_{X(b_1,b_2,\ldots b_n)}$ **(E)**

π_{cust_name} ($\sigma_{cust\_city='Bhubaneswar'}$ (Customer)) can be written as:

1. $\rho_{Customer\_Bhubaneswar}$ ($\sigma_{cust\_city='Bhubaneswar'}$ (Customer))

2. π_{cust_name} (Customer_Bhubaneswar)

RENAME Operator(ρ)...

The different forms of the rename operation for renaming the relation are:

a. ρ_{S} **(R)**

b. $\rho_{S(b_1,b_2,\ldots b_n)}$ **(R)**

c. $\rho_{(b_1,b_2,\ldots b_n)}$ **(R)**

For example, the attributes of Customer (cust_name, cust_street, cust_city) can be renamed as:

$\rho_{(name,street,city)}$ **(Customer)**

Union Compatibility

Union Compatibility

To perform the set operations such as UNION, DIFFERENCE and INTERSECTION, the relations need to be union compatible for the result to be a valid relation

Two relations $R_1(a_1,a_2,\ldots a_n)$ and $R_2(b_1,b_2,\ldots b_m)$ are union compatible iff:

- n = m, i.e. both relations have same arity
- $\text{dom}(a_i) = \text{dom}(b_i)$ for $1 \leq i \leq n$

UNION Operator(∪)

UNION Operator(∪)

The union operation is used to combine data from two relations. It is denoted by union(∪) symbol. The union of two relations $R_1(a_1,a_2,\ldots a_n)$ and $R_2(b_1,b_2,\ldots b_n)$ is a relation $R_3(c_1,c_2,\ldots c_n)$ such that:

dom(c_i) = dom(a_i) ∪ dom(b_i), 1 ≤ i ≤ n

$R_1 \cup R_2$ is a relation that includes all tuples that are either present in R_1 or R_2 or in both without duplicate tuples

Depositor

cust_name	acc_no
Manas	A101
Ramesh	A102
Rishi	A103
Mahesh	A104
Mahesh	A105

Borrower

cust_name	loan_no
Ramesh	L201
Ramesh	L202
Mahesh	L203
Rishi	L204

π_{cust_name} (Depositor) ∪ π_{cust_name} (Borrower)

cust_name
Manas
Ramesh
Rishi
Mahesh

DIFFERENCE Operator(-)

DIFFERENCE Operator(-)

The difference operation is used to identify the rows that are in one relation and not in another. It is denoted as (-) symbol. The difference of two relations $R_1(a_1,a_2,\ldots a_n)$ and $R_2(b_1,b_2,\ldots b_n)$ is a relation R_3 $(c_1,c_2,\ldots c_n)$ such that:

dom(c_i) = dom(a_i) - dom(b_i), 1 $\leq$ i $\leq$ n

R_1 - R_2 is a relation that includes all tuples that are in R_1, but not in R_2

Depositor

cust_name	acc_no
Manas	A101
Ramesh	A102
Rishi	A103
Mahesh	A104
Mahesh	A105

Borrower

cust_name	loan_no
Ramesh	L201
Ramesh	L202
Mahesh	L203
Rishi	L204

π_{cust_name} (Depositor) - π_{cust_name} (Borrower)

cust_name
Manas

Cartesian Product Operator(×)

Cartesian Product Operator(×)

The Cartesian product of two relations $R_1(a_1,a_2,\ldots a_n)$ with cardinality **i** and $R_2(b_1,b_2,\ldots b_m)$ with cardinality **j** is a relation R_3 with

- degree **k = n + m**,
- cardinality **i*j** and
- attributes ($a_1,a_2,\ldots a_n$, $b_1,b_2,\ldots b_m$))

$R_1 \times R_2$ is a relation that includes all the possible combinations of tuples from R_1 and R_2 . The Cartesian product is used to combine information from any two relations.

It is not a useful operation by itself; but is used in conjuction with other operations

Cartesian Product Operator(×)...

Borrower		Loan		
cust_name	loan_no	loan_no	branch_name	amount
Ramesh	L201	L201	Bhubaneswar Main	50,000,000.00
Ramesh	L202	L202	Bhubaneswar Main	5,000,000.00
Mahesh	L203	L203	Mumbai Main	100,000,000.00
Rishi	L204	L204	Juhu	60,000,000.00

Borrower × Loan

cust_name	Borrower.loan_no	Loan.loan_no	branch_name	amount
Ramesh	L201	L201	Bhubaneswar Main	50,000,000.00
Ramesh	L201	L202	Bhubaneswar Main	5,000,000.00
Ramesh	L201	L203	Mumbai Main	100,000,000.00
Ramesh	L201	L204	Juhu	60,000,000.00
Ramesh	L202	L201	Bhubaneswar Main	50,000,000.00
Ramesh	L202	L202	Bhubaneswar Main	5,000,000.00
Ramesh	L202	L203	Mumbai Main	100,000,000.00
Ramesh	L202	L204	Juhu	60,000,000.00
Mahesh	L203	L201	Bhubaneswar Main	50,000,000.00
Mahesh	L203	L202	Bhubaneswar Main	5,000,000.00
Mahesh	L203	L203	Mumbai Main	100,000,000.00
Mahesh	L203	L204	Juhu	60,000,000.00
Rishi	L204	L201	Bhubaneswar Main	50,000,000.00
Rishi	L204	L202	Bhubaneswar Main	5,000,000.00
Rishi	L204	L203	Mumbai Main	100,000,000.00
Rishi	L204	L204	Juhu	60,000,000.00

Cartesian Product Operator(×)...

Query: Find out the customer and their loan details taken from Bhubaneswar Main branch.

Ans: σ *branch_name='BhubaneswarMain'ANDBorrower.loan_no=Loan.loan_no*
(Borrower ×Loan)

cust_name	Borrower.loan_no	Loan.loan_no	branch_name	amount
Ramesh	L201	L201	Bhubaneswar Main	50,000,000.00
Ramesh	L202	L202	Bhubaneswar Main	5,000,000.00

Intersection Operator(∩)

Intersection Operator(∩)

The intersection operation is used to identify the rows that are common to two relations. It is denoted by (∩) symbol. The intersection of two relations $R_1(a_1,a_2,\ldots a_n)$ and $R_2(b_1,b_2,\ldots b_n)$ is a relation R_3 $(c_1,c_2,\ldots c_n)$ such that:

dom(c_i) = dom(a_i) ∩ dom(b_i), 1 ≤ i ≤ n

$R_1 \cap R_2$ is a relation that includes all tuples that are present in both R_1 and R_2

The intersection operation can be rewritten by a pair of set difference operations as **R ∩ S = R - (R - S)**

Depositor

cust_name	acc_no
Manas	A101
Ramesh	A102
Rishi	A103
Mahesh	A104
Mahesh	A105

Borrower

cust_name	loan_no
Ramesh	L201
Ramesh	L202
Mahesh	L203
Rishi	L204

π_{cust_name} (Depositor) ∩ π_{cust_name} (Borrower)

cust_name
Ramesh
Mahesh
Rishi

JOIN Operator(⋈)

JOIN Operator(⋈)

The join is a binary operation that is used to combine certain selections and a Cartesian product into one operation. It is denoted by join (⋈) symbol.

The join operation forms a Cartesian product of its two arguments, performs a selection forcing equality on those attributes that appear in both relations, and finally removes the duplicate attributes

Query: Find the names of customers who have a loan at the bank, along with the loan number and the loan amount.
Ans:This query can be solved by using the PROJECT, SELECT and CARTESIAN PRODUCT operators as:

$\pi_{cust\_name, Loan.loan\_no, amount}\ (\sigma_{Borrower.loan\_no = Loan.loan\_no}$ (Borrower × Loan))

This same expression can be simplified by using the JOIN as:

$\pi_{cust\_name, loan\_no, amount}$ (Borrower ⋈ Loan))

JOIN Operator(un)...

Borrower

cust_name	loan_no	loan_no	branch_name	amount
Ramesh	L201	L201	Bhubaneswar Main	50,000,000.00
Ramesh	L202	L202	Bhubaneswar Main	5,000,000.00
Mahesh	L203	L203	Mumbai Main	100,000,000.00
Rishi	L204	L204	Juhu	60,000,000.00

$\Pi_{cust\_name,loan\_no,amount}$ (Borrower ⧆ Loan))

cust_name	loan_no	amount
Ramesh	L201	50,000,000.00
Ramesh	L202	5,000,000.00
Mahesh	L203	100,000,000.00
Rishi	L204	60,000,000.00

Division Operator(÷)

Division Operator(÷)

The division operation creates a new relation by selecting the rows in one relation that match every row in another relation. The division operation requires that we look at an entire relation at once. It is denoted by division (÷) symbol

Let A, B, C are three relations and we desire B ÷ C to give A as the result. This operation is possible iff:

- The columns of C must be a subset of the columns of B. The columns of A are all and only those columns of B that are not columns of C
- A row is placed in A if and only if it is associated with B and with every row of C

The division operation is the reverse of the Cartesian product operation as: B = (B × C) ÷ C

Division operator is suited to queries that include the phrase every or all as part of the condition

Division Operator(÷)...

Depositor

cust_name	acc_no
Manas	A101
Ramesh	A102
Rishi	A103
Mahesh	A104
Mahesh	A105

Account

acc_no	branch_name	balance
A101	Bhubaneswar Main	100,000.00
A102	Shastri Nagar	50,000.00
A103	India Gate	5,000,000.00
A104	Juhu	600,000.00
A105	Mumbai Main	10,000,000.00

Branch

branch_name	branch_city	assets
Bhubaneswar Main	Bhubaneswar	Gold
Shastri Nagar	Bhubaneswar	Mines
India Gate	New Delhi	Gold
Juhu	Mumbai	Sea Shore
Mumbai Main	Mumbai	Movie

Query: Find all the customers who have an account at all the branches located in Mumbai

$\Pi_{cust\_name, branch\_name}$ (Depositor ⋈ Account) ÷ Π_{branch_name} ($\sigma_{branch\_city = 'Mumbai'}$ (Branch))

cust_name
Mahesh

Assignment Operator(←)

Assignment Operator(←)

It works like assignment in a programming language. In relational algebra, the assignment operator gives a name to a relation. It is denoted by (←) symbol
Assignment must always be made to a temporary relation variable. The result of the right of the ← symbol is assigned to the relation variable on the left of the ← symbol
With the assignment operator, a query can be written as a sequential program consisting of:

- a series of assignment,
- followed by an expression whose value is displayed as a result of the query

$\Pi_{cust\_name, branch\_name}$ (Depositor ⋈ Account) ÷ Π_{branch_name} ($\sigma_{branch\_city = 'Mumbai'}$ (Branch))
can be simplified as:
Temp1 ← $\Pi_{cust\_name, branch\_name}$ (Depositor ⋈ Account)
Temp2 ← Π_{branch_name} ($\sigma_{branch\_city = 'Mumbai'}$ (Branch))
Result = Temp1 ÷ Temp2