Continious Random Variables and probability Distributions.

A discrete random variable is one whose possible values either constitute a finite set or else can be listed in a infinite sequence.

Note A random variable whose set of possible-values is an entire intervals of numbers is not discrete.

A continious random variable is a rv where the data can take infinitely many ~~times~~ values.

E.g. A rv measuring the time taken for something to be done is continious. (Since there are an infinite number of possible times that can be taken).

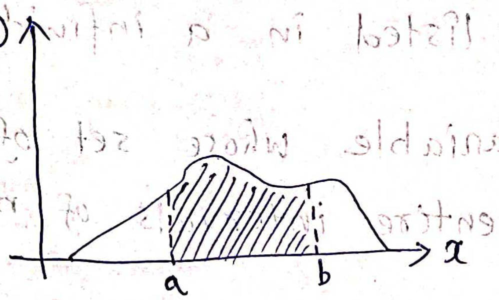
Def

Let X be a continious rv. Then a probability-distribution or probability density function (pdf) of X is a function $f(x)$ such that for any two numbers a and b with $a \leq b$, we have

$$P(a \leq X \leq b) = \int_a^b f(x)\, dx$$

The probability that X is in the interval $[a,b]$. can be calculated by integrating the pdf of the rv X.

The probability that X takes on a value in the interval [a,b] is the area above the this interval and under the graph of the density function.



$P(a \le X \le b)$ = the area under the density curve betn a and b.

A pdf of X must satisfy the following two conditions.

(a) $f(x) \geq 0$ for all x.

(b) $\int_{-\infty}^{\infty} f(x)\, dx = 1$
||
area under the entire graph of $f(x)$.

Eg. X = continious rv

Find 'k' if $f(x) = \begin{cases} 2k, & 0 < x < \frac{1}{2} \\ 0, & \text{otherwise.} \end{cases}$

Using the properties.

$$\int_0^{1/2} 2k\, dx = 2k\, x \Big|_0^{1/2}$$

$$= 2k\left(\tfrac{1}{2} - 0\right) = k = 1$$

$\Rightarrow k = 1$. //

Note probability at a single point in case of continious r.v is 0.

$$P(X=a) = \int_a^a f(x)\,dx = 0.$$

E.g. Let X be a continious r.v. with pdf

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax+3a, & 2 \le x \le 3 \\ 0, & x \ge 3 \end{cases}.$$

Find a) the value of 'a'.

b) $P(X \le 1.5)$

c) $P(X = 1.5)$.

Sol^n

(a) $$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx = 1$$

$$= a\frac{x^2}{2}\Big|_0^1 + ax\Big|_1^2 - a\frac{x^2}{2}\Big|_2^3 + 3ax\Big|_2^3$$

$$= a\left(\tfrac{1}{2}\right) + a - a\left(\tfrac{9}{2} - \tfrac{4}{2}\right) + 3a$$

$$= \frac{a}{2} + a - \frac{5a}{2} + 3a = \frac{a+2a-5a+6a}{2} = \frac{4a}{2} = 2a = 1$$

$$\Rightarrow a = \frac{1}{2}$$

(b) $$P(X \le 1.5) = \int_0^{1.5} f(x)\,dx$$

$$= \int_0^1 ax\,dx + \int_1^{1.5} a\,dx$$

$$= a\frac{x^2}{2}\Big|_0^1 + ax\Big|_1^{1.5}$$

$$= \frac{a}{2} + \frac{a}{2} = a. = \frac{1}{2}.$$

(c) $P(X = 1.5) = 0$.