AFL 2023 END SEM SOLUTION

SECTION-A

1 (a) Design a Context Free Grammar for the language $L=\{a^nb^{2n} : n \geq 1\}$ 1 mark

$S \rightarrow aSbb \mid abb$

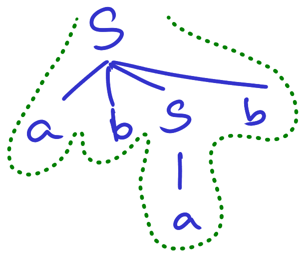
(b) Check whether the following grammar is ambiguous or not: 1 mark

$S \rightarrow a|abSb|aAb$
$A \rightarrow bS|aAAb$
$B \rightarrow b|\lambda$

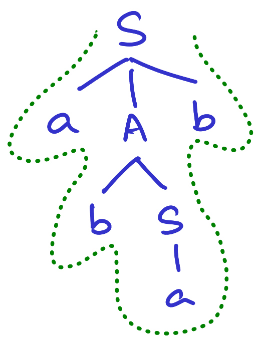
Yes, the given grammar is ambiguous.

Bcoz, for the string 'abab', we can generate more than 1 derivation trees.

Derivation tree-1



Derivation tree-2



(c) Design a Context Free Grammar for the following language: 1 mark
$L = \{w : w \in \{0, 1\}^* \text{ and } w \text{ is a palindrome string}\}$

$S \rightarrow \lambda \mid 0 \mid 1 \mid 0S0 \mid 1S1$

(d) Distinguish between CNF and GNF.

CNF	GNF
→ A CFG is in CNF if all productions are of the form; $\boxed{A \rightarrow BC \text{ or } A \rightarrow a \text{ where } A, B, C \in V \text{ and } a \in T}$	→ A CFG is in GNF if all productions are of the form; $\boxed{A \rightarrow ax \text{ where } a \in T \text{ and } x \in V^*}$

CNF	GNF
→ The no. of steps required to derive a string of length 'n' is $(2n-1)$.	→ The no. of steps required to derive a string of length 'n' is (n).

(e) All linear grammars are not regular grammar but all regular grammars are linear. True or False, Justify.

1 mark

(TRUE)

→ A regular grammar is either right linear or left linear i.e. atmost one variable appears on the RHS of productions with restriction on position of the variable.

→ A linear grammar is a grammar in which atmost one variable appears on the RHS of productions without restriction on position of the variable.

→ Example:

$S \to A$
$A \to aB \mid \lambda$
$B \to Ab$

→ This grammar is a linear grammar but not regular grammar.

(f) State the Pumping Lemma for Regular Languages.

1 mark

Statement:

Let L be an infinite regular language.

Then there exists some +ve integer 'p' such that any string 'w' $\in L$ with $\boxed{|w| \geq p}$ can be decomposed as;

$w = xyz$

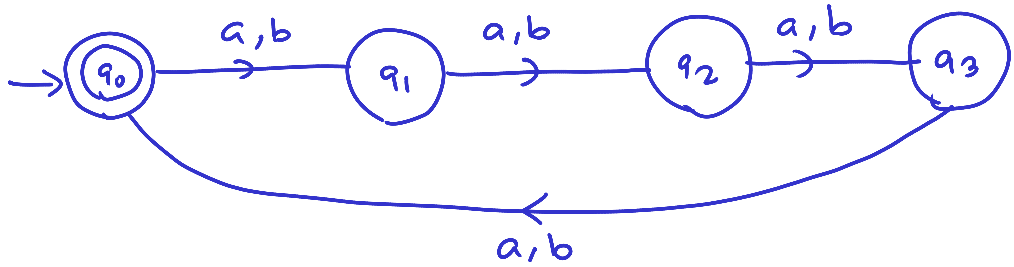
with $|xy| \leq p$

and $|y| \geq 1$

such that $\boxed{w_i = xy^iz \in L}$ for all $i = 0,1,2 \ldots$

(g) Design a finite automata for the following language:
$L = \{w : w \in \{a, b\}^*$ and length of w is a multiple of four$\}$

1 mark



(h) Find a regular expression for the following language:
$L = \{a^p b^q : p + q \text{ is even}\}$

1 mark

$(aa)^*(bb)^* + a(aa)^* b(bb)^*$

(i) Star closure of every regular language is infinite. State True or False with justification.

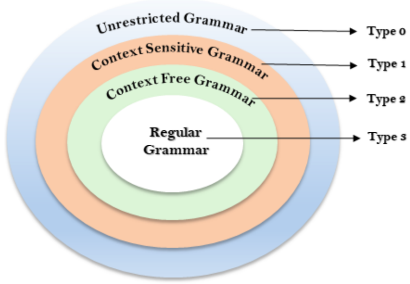
1 mark

(FALSE) We know, ϕ is a regular language.

star-closure of ϕ is 'λ' which is finite.

(j) What is Chomsky's Language Hierarchy?

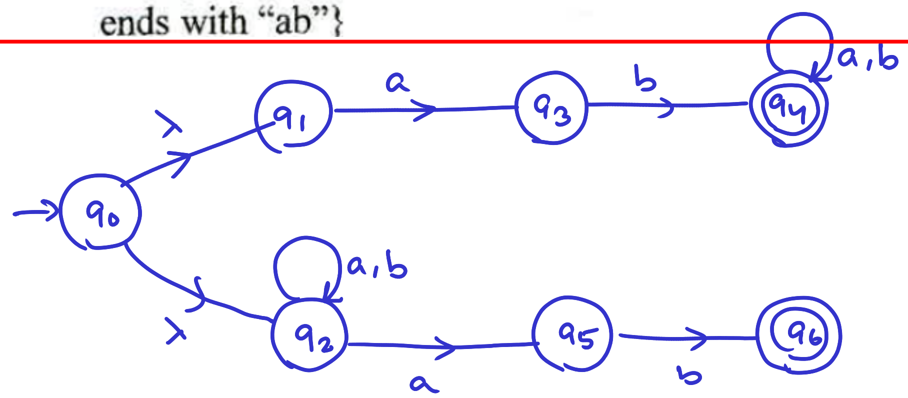
1 mark



SECTION-B

2. (a) Design a NFA for the following language and also find a regular expression for the same language: $L = \{w : w \in \{a, b\}^*$ and either w starts with "ab" or ends with "ab"$\}$ [4]

NFA:



Any other possible NFA can be accepted.

RE:

$$ab(a+b)^* + (a+b)^* ab$$

(b) Design Context Free Grammars for the following languages: [2+2]

i) $L=\{0^p1^q2^r : \text{where } p, q, r >= 0 \text{ and } q = p + r\}$

ii) $L= \{0^i1^j: i<=j+4\}$

i)

$S \to AB$

$A \to 0A1 \mid \lambda$

$B \to 1B2 \mid \lambda$

ii)

$S \to A \mid B \mid 0S1$

$A \to \lambda \mid 0 \mid 00 \mid 000 \mid 0000$

$B \to 1 \mid 1B$

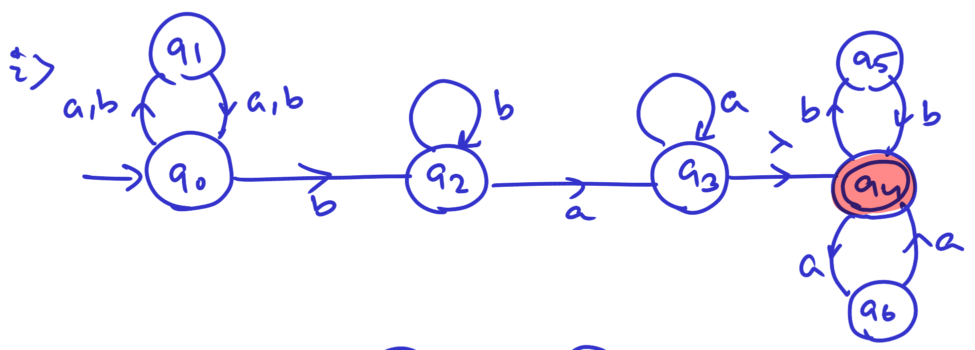
3. (a) Design a DFA or NFA for the following languages: [4]

i) $L=L((aa+bb+ab+ba)^*bb^*aa^*(bb+aa)^*)$

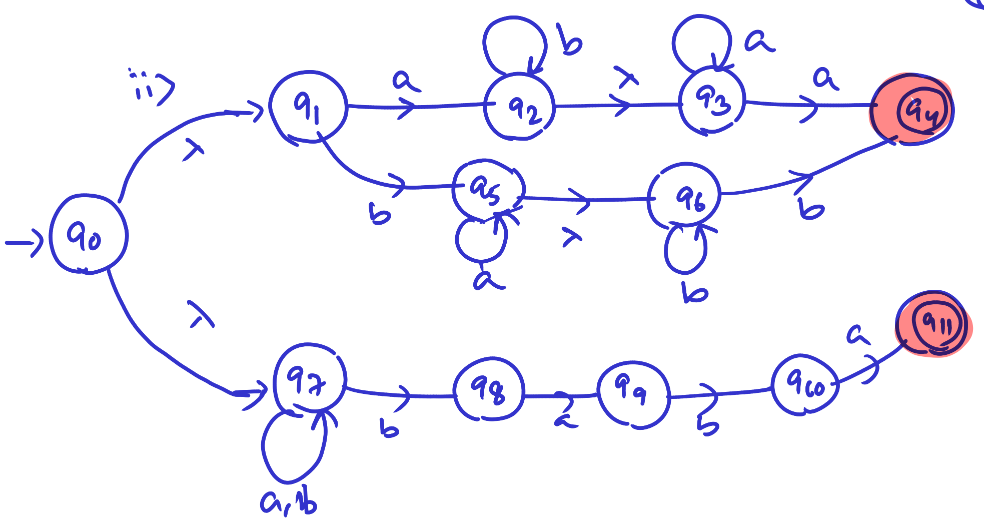
ii) $L=L(ab^*a^*a+ba^*b^*b) \cup L((a+b)^*baba)$

NFA

i)



ii)



(b) Convert the following grammar into Greibach Normal Form: [4]

$S \rightarrow AB|aB$
$A \rightarrow aab|\lambda$
$B \rightarrow bbA|a$

Eliminate λ-production:

Nullable variable = $\{A\}$

$S \rightarrow AB|B|aB|a$
$A \rightarrow aab$
$B \rightarrow bbA|a|bb$

Eliminate Unit-production:

$S \rightarrow AB|bbA|a|bb|aB|a$
$A \rightarrow aab$
$B \rightarrow bbA|a|bb$

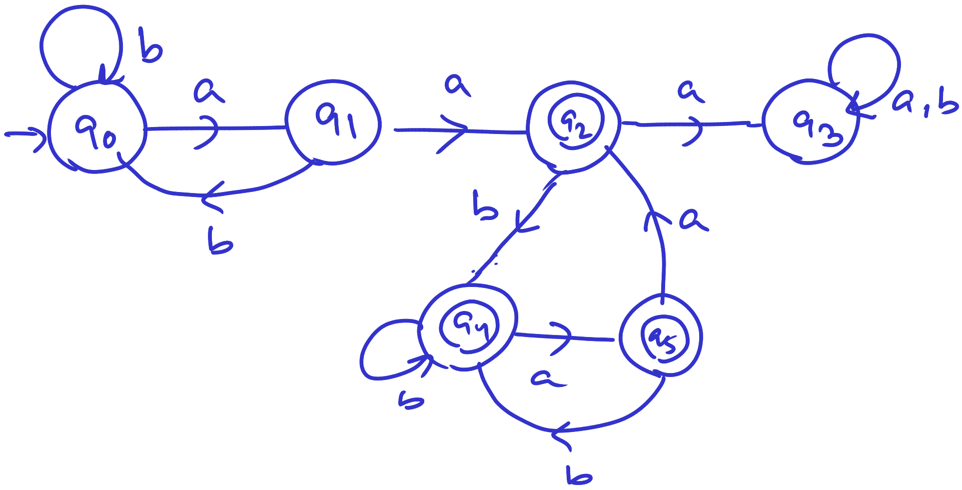
GNF

$S \rightarrow aabB|bbaab|a|bb|abbA|a$
$A \rightarrow aab$
$B \rightarrow bbA|a|bb$

$\Downarrow$

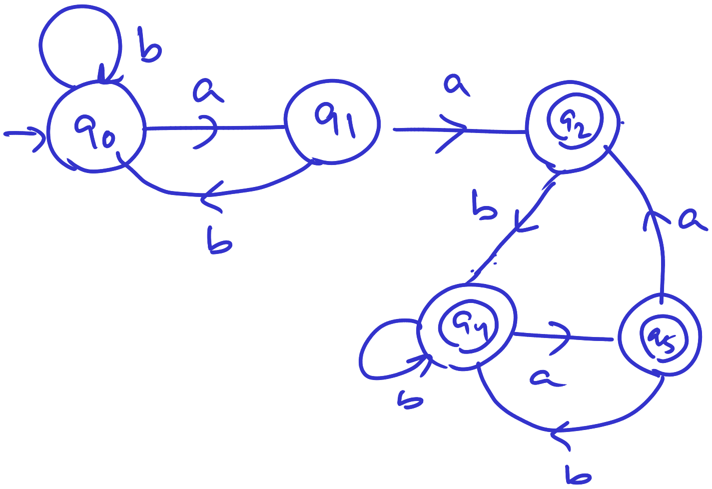
$S \rightarrow aV_1V_2B|bV_2V_1V_1V_2|a|bV_2|aV_2V_2A|a$
$A \rightarrow aV_1V_2$
$B \rightarrow bV_2A|a|bV_2$
$V_1 \rightarrow a$
$V_2 \rightarrow b$

4. (a) Considering the set of strings on symbol {a, b}, construct a DFA for all string containing "aa" but not "aaa". Then find out its corresponding regular expression. [2+2]

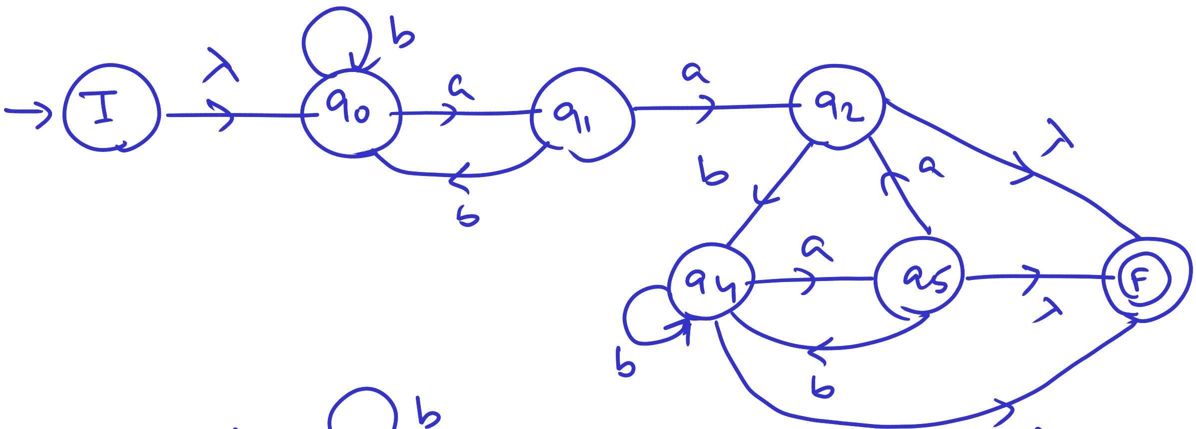


Removing dead state:

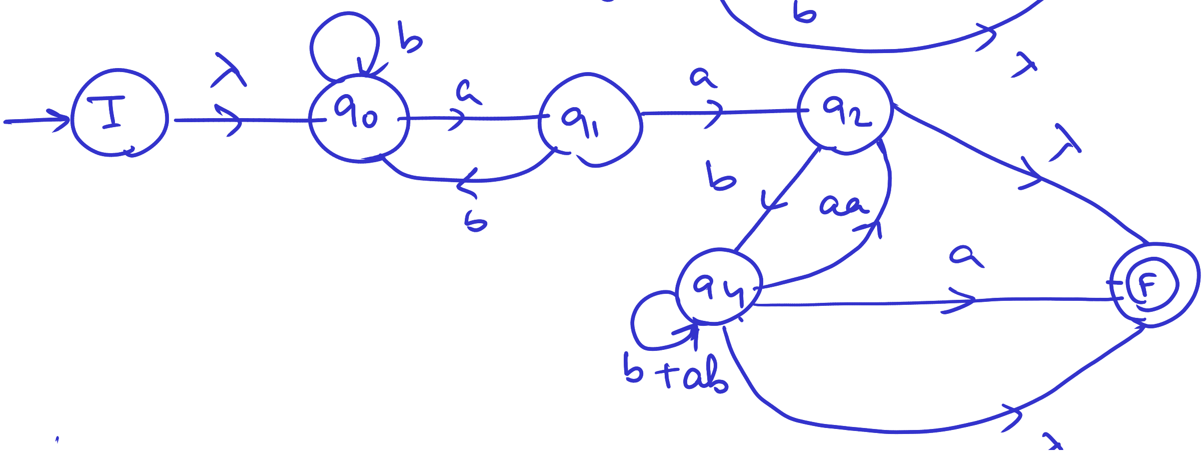
Step-1:



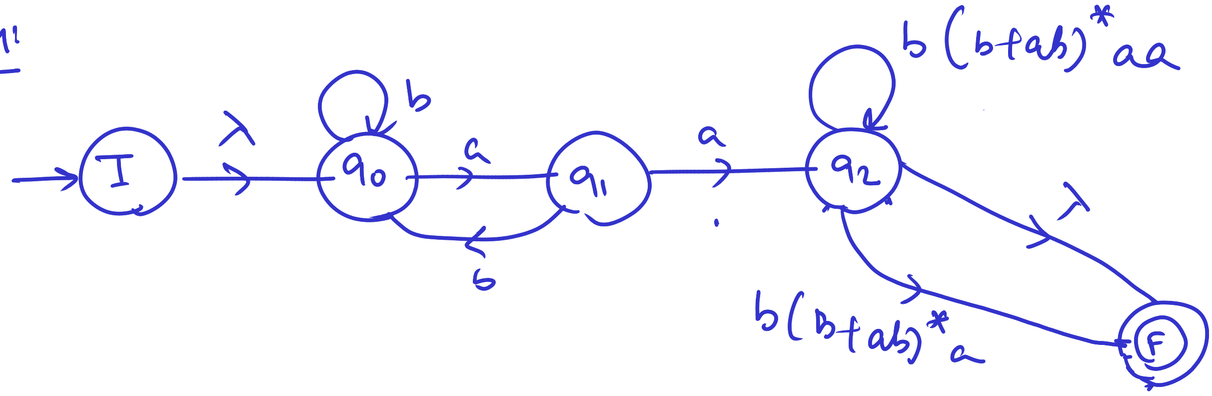
Step-2:



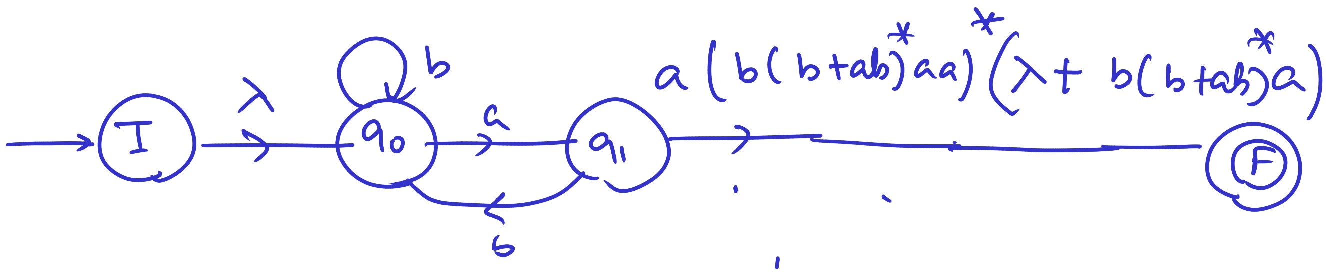
Remove q5:



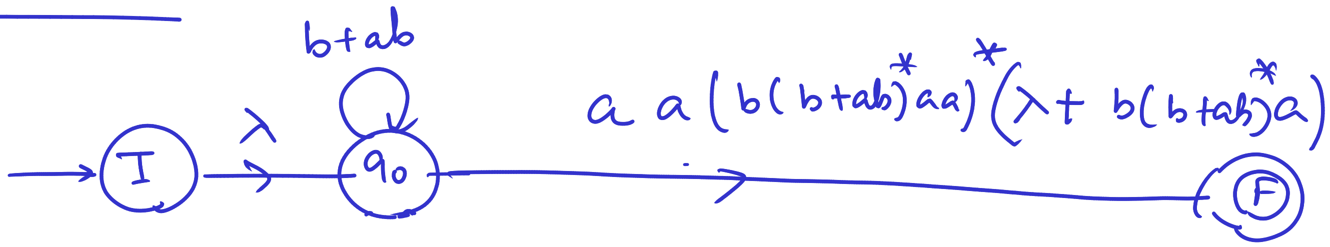
Remove q_4:



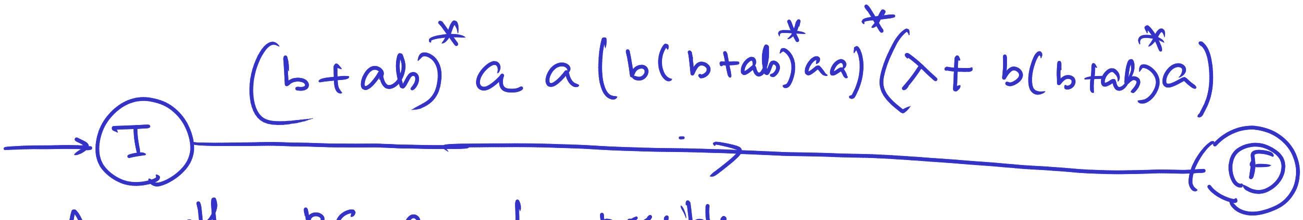
Remove q_2:



Remove q_1:



Remove q_0:

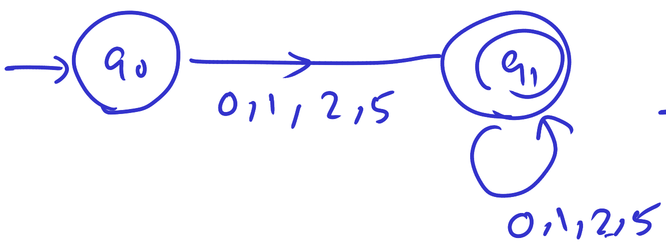


Any other REs can be possible.

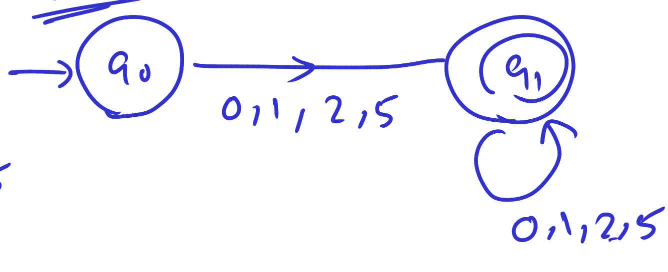
(b) Design an NFA which will accept all string generated by the symbols present in your roll number. Convert the NFA into its corresponding DFA. [4]

For e.g. Roll no = {2105001} Σ = {0,1,2,5}

NFA



DFA



SECTION-C

5. (a) Consider a language, L= {ww : w∈ {a, b}*}. Show that L is not a Context Free Language, using Pumping Lemma. [4]

Solution:

Consider the language

$$L = \{ww : w \in \{a, b\}^*\}.$$

Although this language appears to be very similar to the context-free language of Example 5.1, it is not context free.

Take the string

$$a^m b^m a^m b^m.$$

There are many ways in which the adversary can now pick vxy, but for all of them we have a winning countermove. For example, for the choice in Figure 8.2, we can use $i = 0$ to get a string of the form $a^k b^j a^m b^m$, $k < m$ or $j < m$, which is not in L. For other choices by the adversary, similar arguments can be made. We conclude that L is not context free.

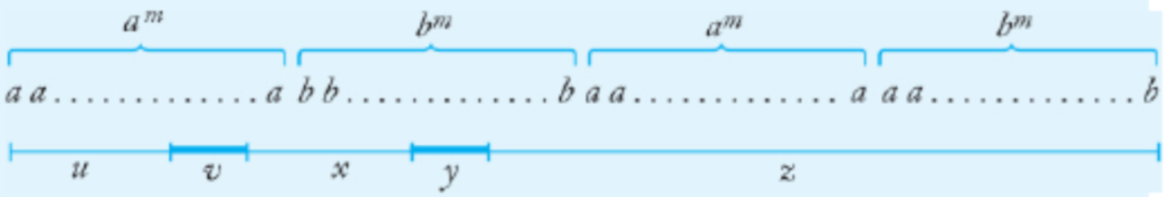
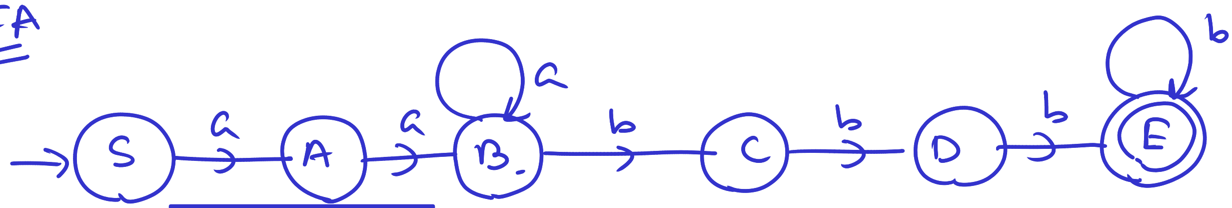


FIGURE 8.2

(b) Find a left-linear and a right-linear grammar for the language
L = {$a^p b^q$ | p >=2 and q >=3}. [4]

FA

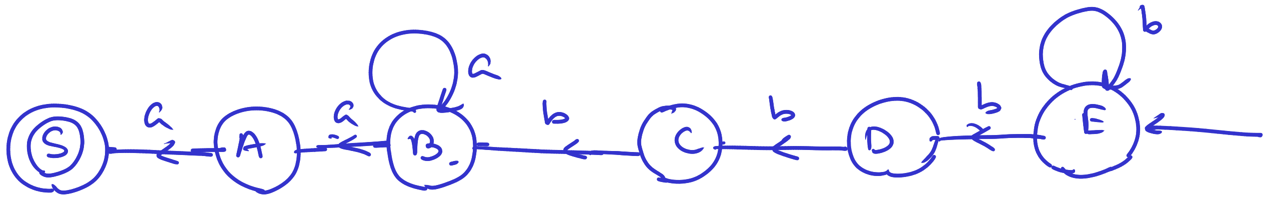


RLG

$S \rightarrow aA$
$A \rightarrow aB$
$B \rightarrow aB \mid bC$
$C \rightarrow bD$
$D \rightarrow bE$
$E \rightarrow \lambda \mid bE$

— Ans-1

FA^R



RLG of FAR

$E \to bE \mid bD$
$D \to bC$
$C \to bB$
$B \to aB \mid aA$
$A \to aS$
$S \to \lambda$

$\Rightarrow$

LLG

$E \to Eb \mid Db$
$D \to Cb$
$C \to Bb$
$B \to Ba \mid Aa$
$A \to Sa$
$S \to \lambda$

$\to$ Ans-2

6. (a) Convert the following Context Free Grammar to Chomsky Normal Form. [5]
$S \to AABC$
$A \to aAb \mid \lambda$
$B \to aB \mid a$
$C \to aBa \mid bCb \mid \lambda$

Remove λ-productions:

Nullable variable = $\{A, C\}$

$S \to AABC \mid ABC \mid AAB \mid B$
$A \to aAb \mid ab$
$B \to aB \mid a$
$C \to aBa \mid bCb \mid bb$

Remove Unit-productions:

$S \to AABC \mid ABC \mid AAB \mid aB \mid a$
$A \to aAb \mid ab$
$B \to aB \mid a$
$C \to aBa \mid bCb \mid bb$

CNF:

$$
\begin{aligned}
S &\to V_1V_2 \mid AV_2 \mid V_1B \mid V_3B \mid a \\
A &\to V_3AV_4 \mid V_3V_4 \\
B &\to V_3B \mid a \\
C &\to V_3BV_3 \mid V_4CV_4 \mid V_4V_4 \\
V_1 &\to AA \\
V_2 &\to BC \\
V_3 &\to a \\
V_4 &\to b
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
S &\to V_1V_2 \mid AV_2 \mid V_1B \mid V_3B \mid a \\
A &\to V_5V_4 \mid V_3V_4 \\
B &\to V_3B \mid a \\
C &\to V_6V_3 \mid V_7V_4 \mid V_4V_4 \\
V_1 &\to AA \\
V_2 &\to BC \\
V_3 &\to a \\
V_4 &\to b \\
V_5 &\to V_3A \\
V_6 &\to V_3B \\
V_7 &\to V_4C
\end{aligned}
$$

(b) Prove that Context Free Languages are closed under Concatenation operation. [3]

Let us assume two CFLs L_1 and L_2 with their corresponding grammars $G_1 = \{V_1, T_1, P_1, S_1\}$ and $G_2 = \{V_2, T_2, P_2, S_2\}$ respectively such that $V_1 \cap V_2 = \phi$

Let L be the new language generated by concatenation of L_1 and L_2 with its corresponding grammar $G = \{V, T, P, S\}$. Thus L generates the L_1L_2 language as $S \to S_1 S_2$ ($\because$ Concatenation)

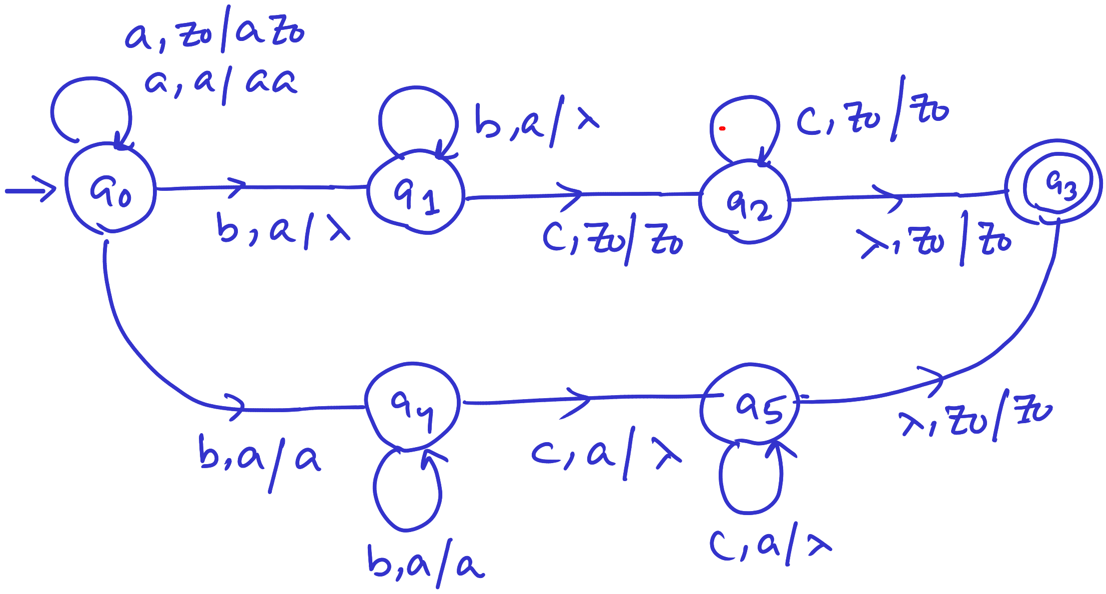
Grammar for the L can be derived from L_1 and L_2 as follows::

$G = \{V_1 \cup V_2 \cup \{S\}, T_1 \cup T_2, P_1 \cup P_2 \cup \{S \to S_1 S_2\}, S\}$.

L is a CFL as the new production $S \to S_1 S_2$ does not violate the rule of being context free.

SECTION-D

7\. (a) Construct a PDA or NPDA for the following language: [4]
$L=\{a^i b^j c^k : \text{where } i = j \text{ or } i = k\}$



(b) Construct a NPDA for the given CFG, G = ({S, M, N}, {0,1}, S, P) where the production rules P are given below: [4]

$S \to MN1 | 0$
$M \to 00M | N$
$N \to 1M1$

Remove Unit productions:

$S \to MN1 | 0$
$M \to 00M | 1M1$
$N \to 1M1$

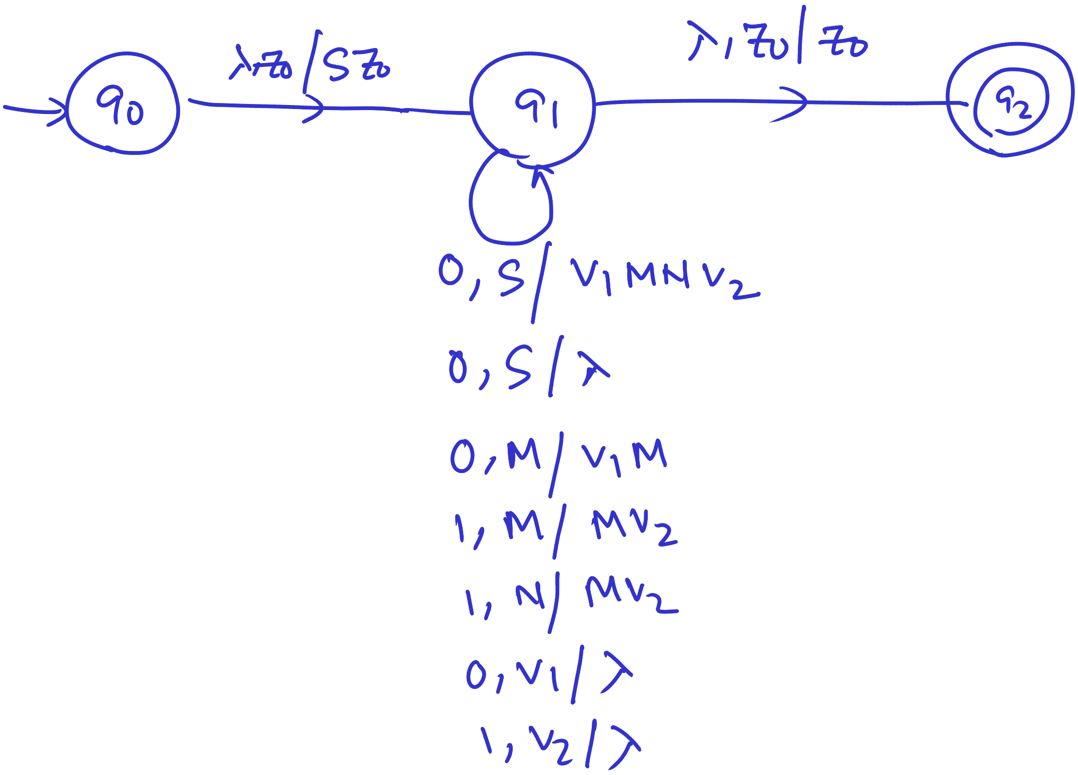
GNF:

$S \to 00MN1 | 0$
$M \to 00M | 1M1$
$N \to 1M1$

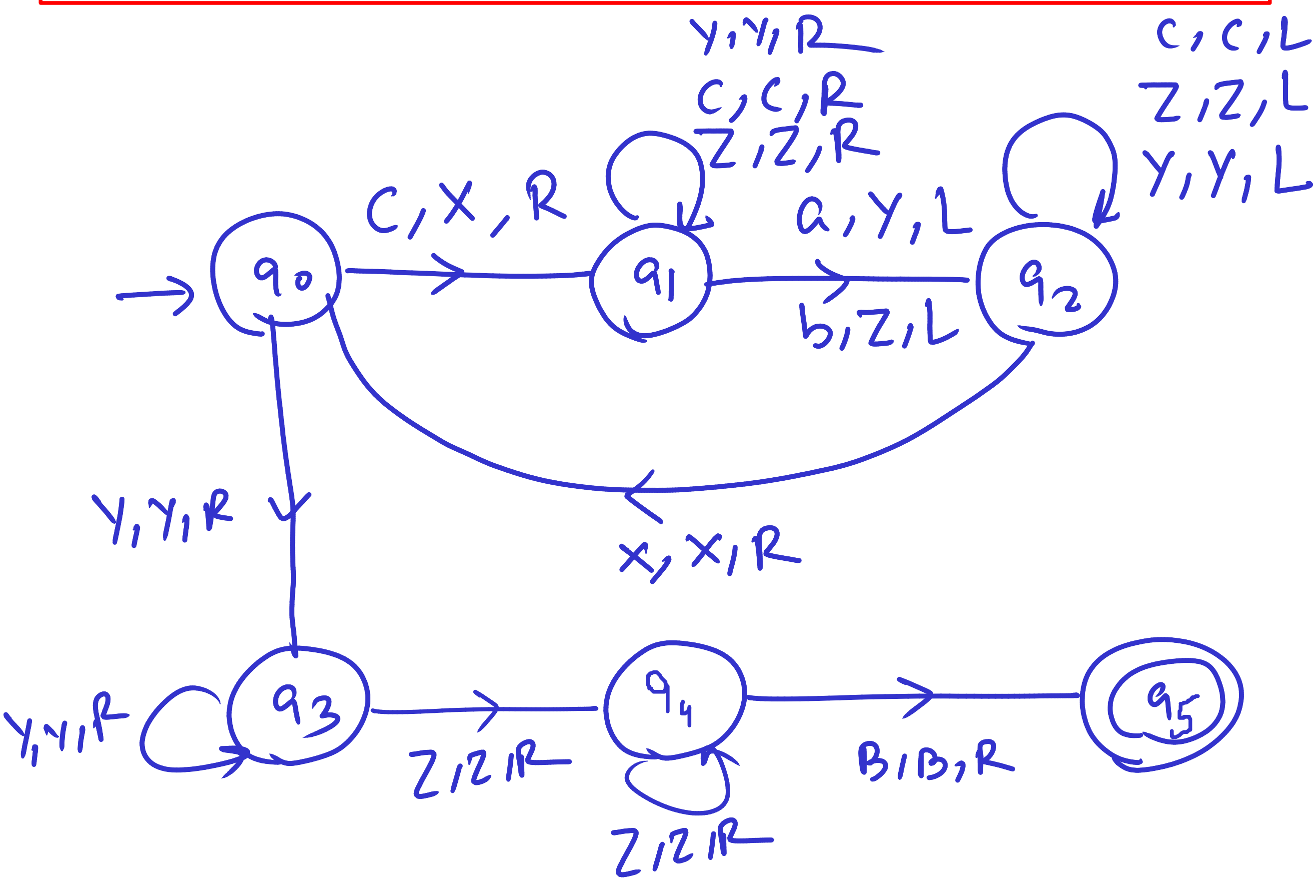
$\to$

$S \to 0V_1MNV_2 | 0$
$M \to 0V_1M | 1MV_2$
$N \to 1MV_2$
$V_1 \to 0$
$V_2 \to 1$

NPDA



8. (a) Design a Turing machine for the following language: [5]
$L = \{ c^{m+n}a^m b^n : n, m \geq 1 \}$



(b) Write the Instantaneous Descriptions (IDs) for the input string "cccaab" to the above designed TM. [3]

*****

$$q_0 cccaab \vdash X q_1 ccaab$$
$$\vdash X c q_1 caab$$
$$\vdash X c c q_1 aab$$
$$\vdash X c q_2 c Y ab \qquad X c \underset{\uparrow q_2}{c} \overset{Y}{\cancel{a}} ab$$
$$\vdash X q_2 c c Y ab$$
$$\vdash q_2 X c c Y ab$$
$$\vdash X q_0 c c Y ab$$
$$\vdash X X q_1 c Y ab$$
$$\vdash X X c q_1 Y ab$$
$$\vdash X X c Y q_1 ab$$
$$\vdash X X c q_2 Y Y b$$
$$\vdash X X q_2 c Y Y b$$
$$\vdash X q_2 X c Y Y b$$
$$\vdash X X q_0 c Y Y b$$
$$\vdash X X X q_1 Y Y b$$
$$\vdash X X X Y q_1 Y b$$
$$\vdash X X X Y Y q_1 b$$
$$\vdash X X X Y q_2 Y Z$$

$\vdash X X X q_2 Y Y Z$

$\vdash X X q_2 X Y Y Z$

$\vdash X X X q_0 Y Y Z$

$\vdash X X X Y q_3 Y Z$

$\vdash X X X Y Y q_3 Z$

$\vdash X X X Y Y Z q_3 B$

$\vdash X X X Y Y Z B q_4 B$