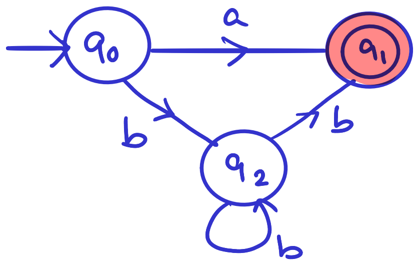
AFL 2023 Mid Sem Solution

1) (a) Design an NFA over $\Sigma = \{a,b\}$ without λ-transitions and with a single final state that accepts the set :

$\{a\} \cup \{b^n : n \geq 2\}$.

1 mark



(b) Let L1 and L2 be two languages over the same alphabet Σ. Given that L1 and L1.L2 both are regular. Prove or disprove L2 must be regular.

1 mark

Disprove

Let $L_1 = \phi$ (regular)

$L_2 = \{a^n b^n : n \geq 1\}$ (non-regular)

$L_1 . L_2 = \phi . \{a^n b^n : n \geq 1\}$

$= \phi$ (regular).

Hence, L_2 must not be regular.

1 mark

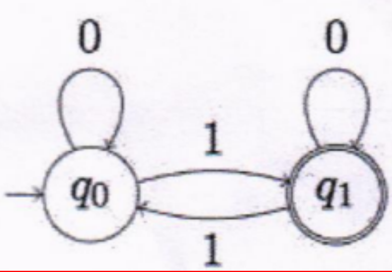
(c) What is the length of the shortest string not in the language denoted by the regular expression (ba+a)*(b+ba)*

$L = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, aaaa, aaab, aaba, aabb, abaa, abab, abba, abbb, baaa, baab, baba, babb, bbaa\}$

Not generated

Length of shortest string not in language = 4.

(d) Find a regular expression for the following DFA :



1 mark

Regular Expression-1 : $0^* 1 (0 + 10^* 1)^*$

Regular expression-2 : $(0 + 10^* 1)^* 10^*$

(e) Two regular expressions over the same alphabet are called equivalent if they generate the same language. Find out whether the followingtwo regular expressions are equivalent:

$(ab^*a+ba^*b)^*$ and $(ab^*a)^* + (ba^*b)^*$.

1 mark

Not equivalent.

For instance, $aabb \in \mathcal{L}(ab^*a+ba^*b)^*$.

while $aabb \notin \mathcal{L}((ab^*a)^*+(ba^*b)^*)$.

2. (a) Design a DFA for the following regular language over $\Sigma = \{0, 1\}$.

L={The set of all strings, interpretated as binary representation of integers, are $\underbrace{\text{divisible by 2}}_{L_1}$ but $\underbrace{\text{not divisible by 3}}_{L_2}$}.

4 marks

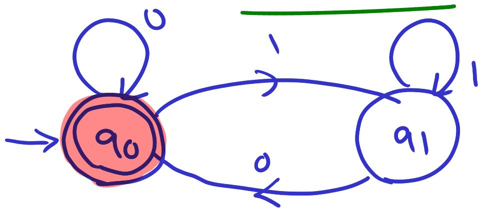
Let L_1 = set of all strings are divisible by 2

L_2 = set of all strings are not divisible by 3.

$L_1 \cap L_2$ = set of all strings that are divisible by 2 but not divisible by 3.

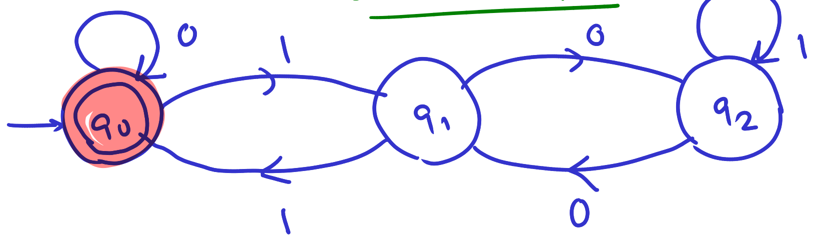
DFA for L_1

Divisible by 2

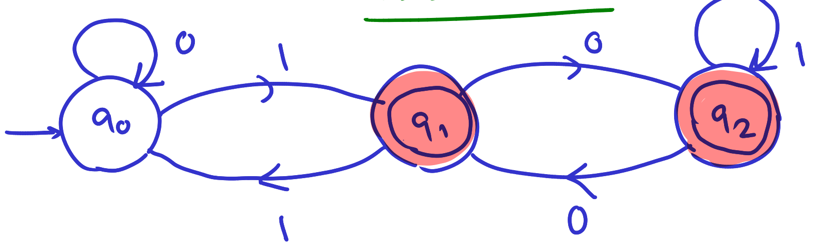


DFA for L_2

Divisible by 3

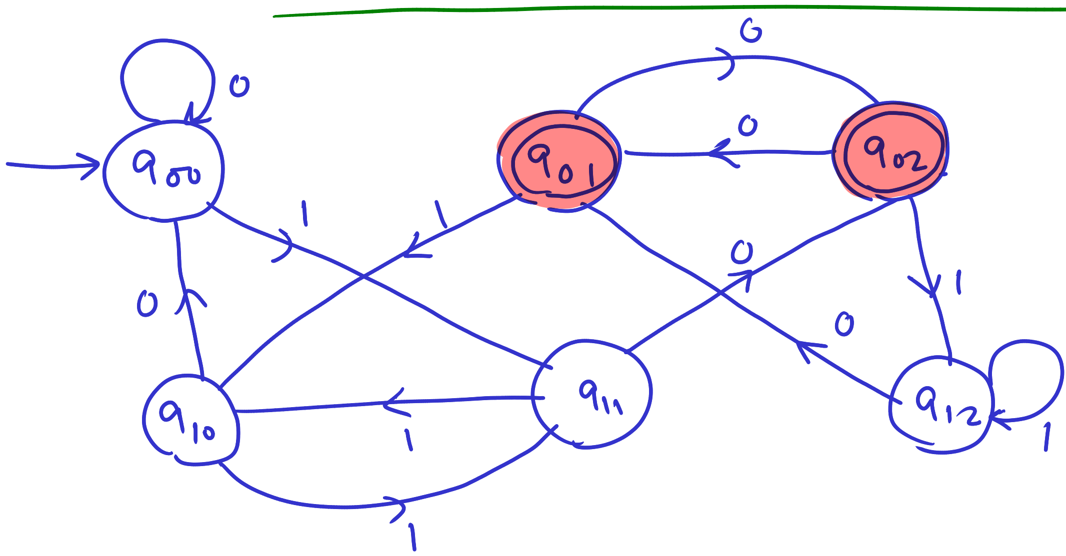


Not divisible by 3



DFA for $L_1 \cap L_2$

Divisible by 2 but not divisible by 3



(b) Write down the statement of Pumping Lemma for regular languages.

1 mark

Statement:

Let L be an infinite regular language.

Then there exists some +ve integer 'p' such that any string 'w' $\in$ L with $\boxed{|w| \geq p}$ can be decomposed as;

$$w = xyz$$

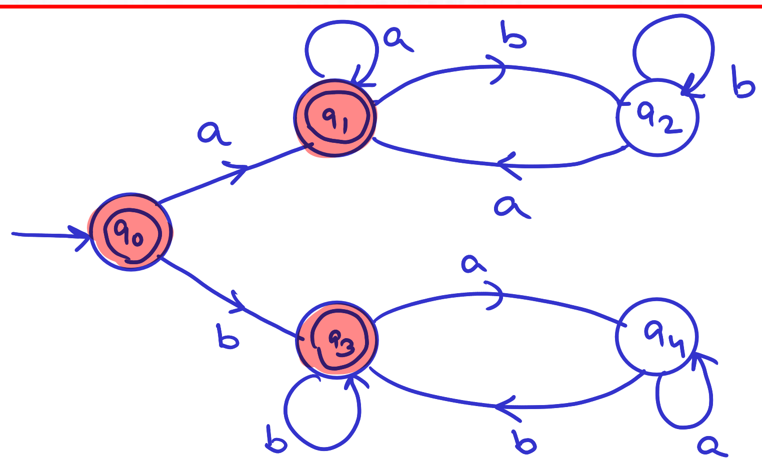
with $|xy| \leq p$

and $|y| \geq 1$

such that $\boxed{w_i = xy^iz \in L}$ for all $i = 0, 1, 2 \ldots$

3.(a) Design a DFA for the language L={w ϵ (a+b)* | w contains equal number of occurrences of the substrings 'ab' and 'ba'}. For example, the string 'ababa' is in the language, whereas 'bbaba' is not in the language.

3 marks



(b) Write down regular expressions for the following languages:

i) L = {w ∈ {a, b}* : w starts with 'ab' but does not end with 'ab'}

0.5 mark

$ab(a+b)^*(aa+ba+bb) + aba + abb$

ii) L={ w ∈ {a, b}*: $n_b(w)$ mod 5 > 0}

0.5 mark

$(a^*ba^*ba^*ba^*ba^*ba^*)^*(a^*ba^* + a^*ba^*ba^* + a^*ba^*ba^*ba^* + a^*ba^*ba^*ba^*ba^*)$

iii) L={ w ∈ {a, b}* : every 'a' in w is immediately preceded and followed by 'b'}

0.5 mark

$(b+bab)^*$

iv) L={ w ∈ {a, b}* : |w| mod 3 ≠ 0}

0.5 mark

$((a+b)^3)^*((a+b)+(a+b)^2)$

4. (a) Consider the DFA {{q0, q1, q2, q3, q4, q5, q6, q7, q8}, {a,b}, δ, q0, {q4, q5, q8}}

Initial state

Final states.

3 marks

δ	a	b
q0	q1	q2
q1	q3	q8
q2	q4	q3
q3	q5	q3
q4	q4	q6
q5	q8	q6
q6	q7	q4
q7	q6	q5
q8	q8	q7

Minimize the above DFA and show the indistinguishable states.

Minimization of DFA by state equivalence method.

0-equivalence: $\underbrace{\{q_0, q_1, q_2, q_3, q_6, q_7\}}_{\text{Non-final}} \underbrace{\{q_4, q_5, q_8\}}_{\text{final}}$

contd..

1 - equivalence: $\{q_0\}$ $\{q_4, q_5, q_8\}$

$\{q_1, q_6, q_7\}$

$\{q_2, q_3\}$

2 - equivalence: $\{q_0\}$ $\{q_4, q_5, q_8\}$

$\{q_1\}$

$\{q_6, q_7\}$

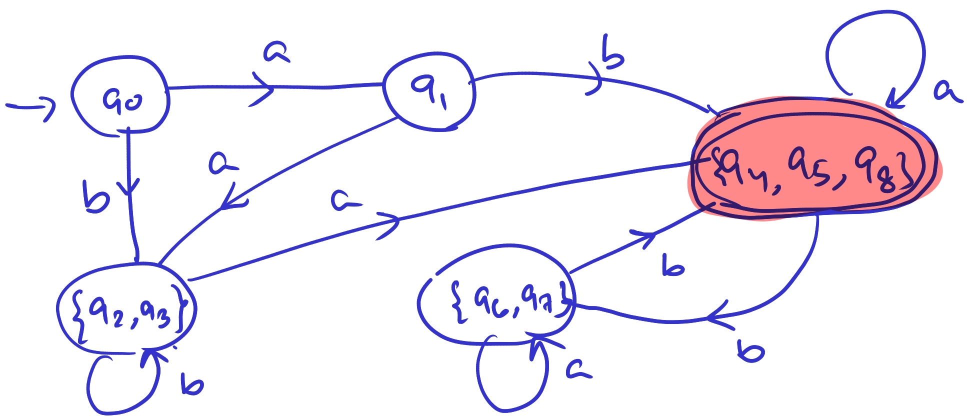
$\{q_2, q_3\}$

3 - equivalence: $\{q_0\}$

$\{q_1\}$ $\{q_4, q_5, q_8\}$

$\{q_6, q_7\}$

$\{q_2, q_3\}$



Indistinguishable states are: (q_2, q_3),

(q_6, q_7)

(q_4, q_5, q_8)

Contd..

(b) Convert the NFA defined by:

$\delta(q0, a) = \{q0, q1\}$

$\delta(q1, b) = \{q1, q2\}$

$\delta(q2, a) = \{q2\}$

$\delta(q0, \lambda) = \{q2\}$

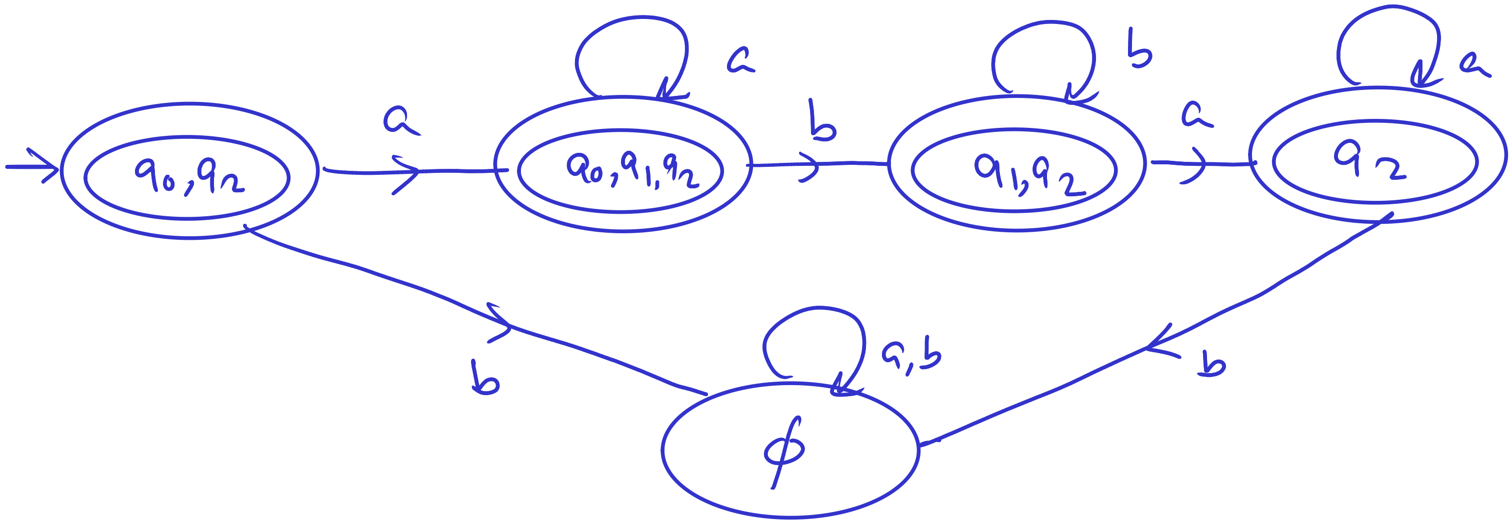
with initial state {q0} and final state {q2} into an equivalent DFA.

2 marks

δ	a	b	λ	λ^*
$\rightarrow q_0$	$\{q_0, q_1\}$	–	$\{q_2\}$	$\{q_0, q_2\}$
q_1	–	$\{q_1, q_2\}$	–	$\{q_1\}$
$* q_2$	$\{q_2\}$	–	–	$\{q_2\}$

- Initial state of DFA $= \lambda^*$(Initial state of NFA)

δ	a	b
$*\rightarrow \{q_0, q_2\}$	$\{q_0, q_1, q_2\}$	ϕ
$* \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
ϕ	ϕ	ϕ
$* \{q_1, q_2\}$	$\{q_2\}$	$\{q_1, q_2\}$
$* \{q_2\}$	$\{q_2\}$	ϕ



5. (a) Prove that $L = \{ww^R \mid w \in \Sigma^*\}$ is not regular. **3 marks**

→ Assume that given L be regular.

→ Let 'p' be a pumping length.

→ Choose string $w = \underbrace{a^p b}_{w}\underbrace{b a^p}_{w^r}$

$|w| = |a^p b b a^p| = 2p+2 \geq p$ ✓

→ Decompose 'w' into 3 parts.

$w \to x \; y \; z$

$x = a^{p-1}, \quad y = a, \quad z = bba^p$

i) $|xy| = |a^{p-1}a| = p \leq p$

ii) $|y| = |a| = 1 \geq 1$

→ Choose 'i' $i = 2$

$xy^iz = xy^2z$
$= a^{p-1}a^2 bba^p = a^{p+1}bba^p \notin L.$

Hence, it is a contradiction to our assumption of L being regular. (Proved)

(b) Show that regular languages are closed under intersection. **2 marks**

Sol^n-1

Non-constructive Proof / Proof by shorter argument.

$\overline{A \cap B} = \bar{A} \cup \bar{B}$ (DeMorgan's law).

$\Rightarrow \overline{\overline{A \cap B}} = \overline{\bar{A} \cup \bar{B}}$ (Complementing both sides)

$\Rightarrow A \cap B = \overline{\bar{A} \cup \bar{B}}$

$L_1 \cap L_2 = \overline{\bar{L_1} \cup \bar{L_2}}$

$= \overline{\overline{\text{Regular}} \cup \overline{\text{Regular}}}$

$= \overline{\text{Regular} \cup \text{Regular}}$ (Closed under Complement & union)

$= \overline{\text{Regular}} = \text{Regular}$ (Proved)

Sol^n-2

Constructive Proof:

Let $L_1 = L(M_1)$ be a language accepted by DFA machine M_1.

$L_2 = L(M_2)$ be " " " " machine M_2.

where, $M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$

$M_2 = (R, \Sigma, \delta_2, r_0, F_2)$

We can construct from M_1 and M_2, a combined automaton $\hat{M} = (\hat{Q}, \Sigma, \hat{\delta}, (q_0, r_0), \hat{F})$,

where state set $\hat{Q} = Q \times R$ consists of pairs (q_i, r_j) and transition function

$$\hat{\delta}((q_i, r_j), a) = (q_k, r_l)$$

whenever, $\delta_1(q_i, a) = q_k$

$\delta_2(r_j, a) = r_l$

$$\boxed{\hat{F} = q_i \in F_1 \text{ and } r_j \in F_2}$$

Then, it is simple matter to show that $w \in L_1 \cap L_2$ iff it is accepted b $\hat{M}$. Consequently, $L_1 \cap L_2$ is regular.

(Proved)

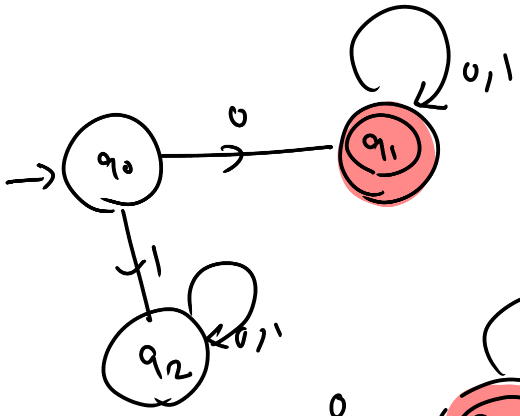
Contd..

Example

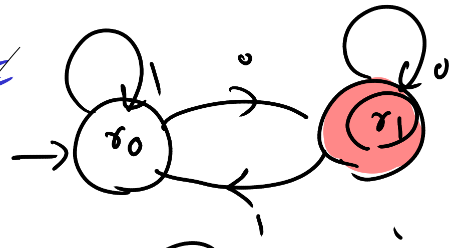
Let $L_1 = \{0w : w \in \{0,1\}^*\}$ → starts with 0

$L_2 = \{w0 : w \in \{0,1\}^*\}$ → ends with 0

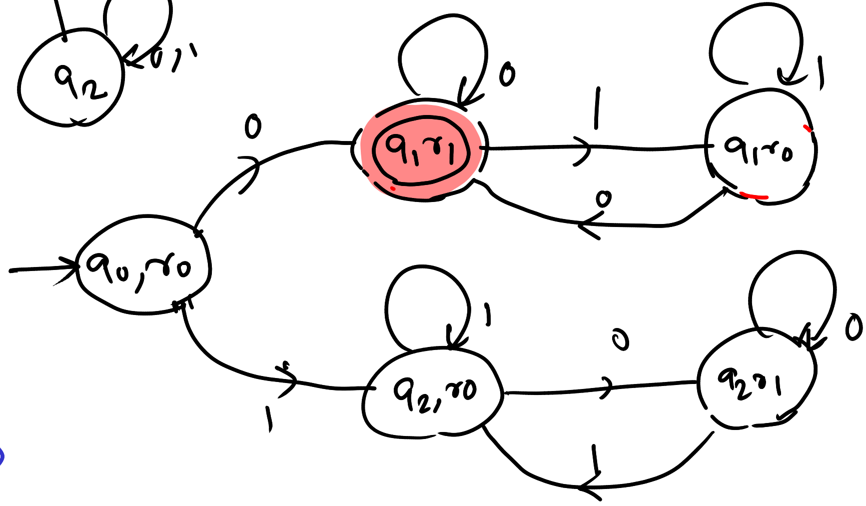
M_1



M_2



$\hat{M}$



$R = \{r_0, r_1\}$

$Q = \{q_0, q_1, q_2\}$

$F_1 = \{q_1\}$

$F_2 = \{r_1\}$

$F = q_i \in F_1$ and $r_j \in F_2$

$= \{q_1, r_1\}$

$L = L_1 \cap L_2 = \{$starts with 0 and ends with 0$\}$