Graphical Method:

Simple linear programming problems with two decision variables can be solved by graphical method. The procedure to solve such LPPs involves the following steps.

1. Consider each inequality constraint as equation.
2. Plot each equation on the graph as each will represent a straight line geometrically.
3. Mark the region. If the inequality constraint is corresponding to "$\leq$" sign then the region below the line lying in the 1st quadrant (due to non -ve restrictions) is to be shaded. Similarly, for the inequality constraint corresponding to "$\geq$" sign the region above the line in the 1st quadrant is to be shaded. The common region obtained is called the feasible region and the points lying in common region satisfies all the constraints.
4. Find the co-ordinates of the extreme points (corner points) of the feasible region and calculate the value of the objective function at the corner points. Select the one which gives the optimal solution.

There are LPPs which may have

i) a unique optimal solution
ii) an infinite number of optimal solution.
iii) an unbounded solution
iv) no solution

~~Each~~ The following examples will illustrate the above cases.

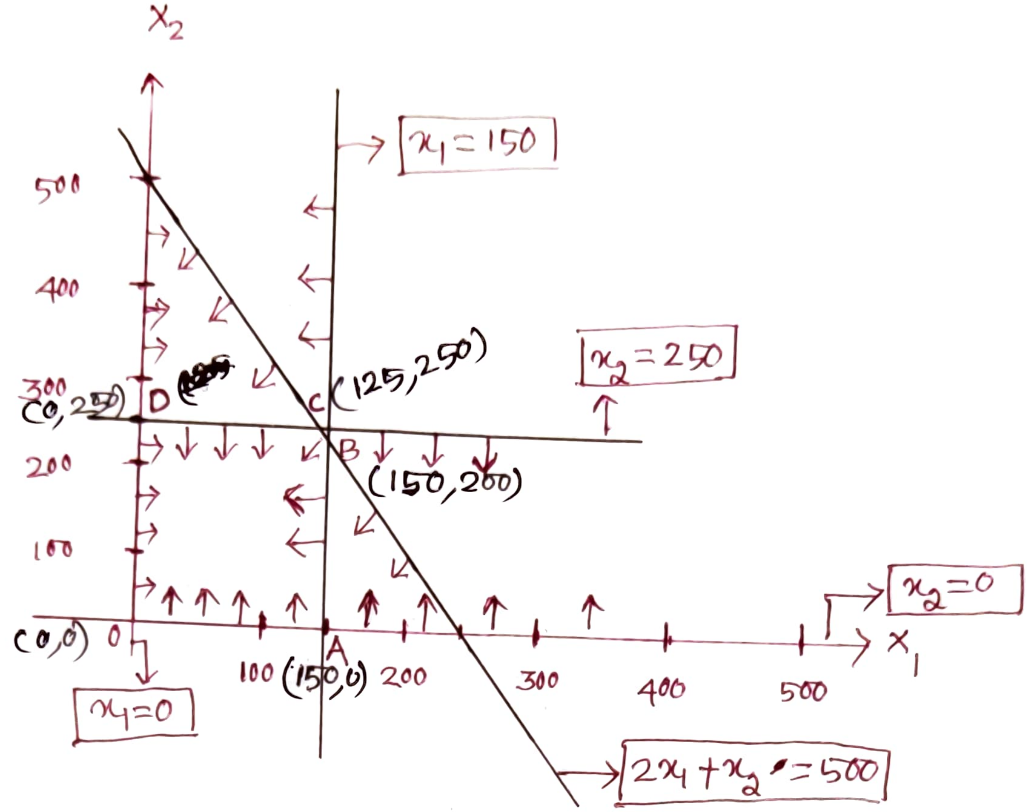
Example (A unique optimal solution)

$$\text{Max. } Z = 8x_1 + 5x_2$$

subject to

$$2x_1 + x_2 \leq 500$$
$$x_1 \leq 150$$
$$x_2 \leq 250$$
$$x_1, x_2 \geq 0$$

soln:



The feasible region is OABCD.

'B' is the point of intersection of the lines $2x_1 + x_2 = 500$ & $x_1 = 150$, from which we get $x_2 = 200$. Thus the coordinate of 'B' is (150, 200).

'C' is the point of intersection of the lines $2x_1 + x_2 = 500$ & $x_2 = 250$, from which we get $x_1 = 125$. Thus the coordinate of 'C' is (125, 250).

Corner Points	Value of $Z = 8x_1 + 5x_2$
O (0, 0)	→ 0
A (150, 0)	→ 1200
B (150, 200)	→ 2200
C (125, 250)	→ 2250 (Maximum).
D (0, 250)	→ 1250

Thus the optimal solution is

Max. $Z = 2250$ whenever $x_1 = 125$ & $x_2 = 250$.

Example (An infinite number of optimal solution)

Solve the following LPP by graphical method:

$$\text{Max. } Z = 100x_1 + 40x_2$$

subject to

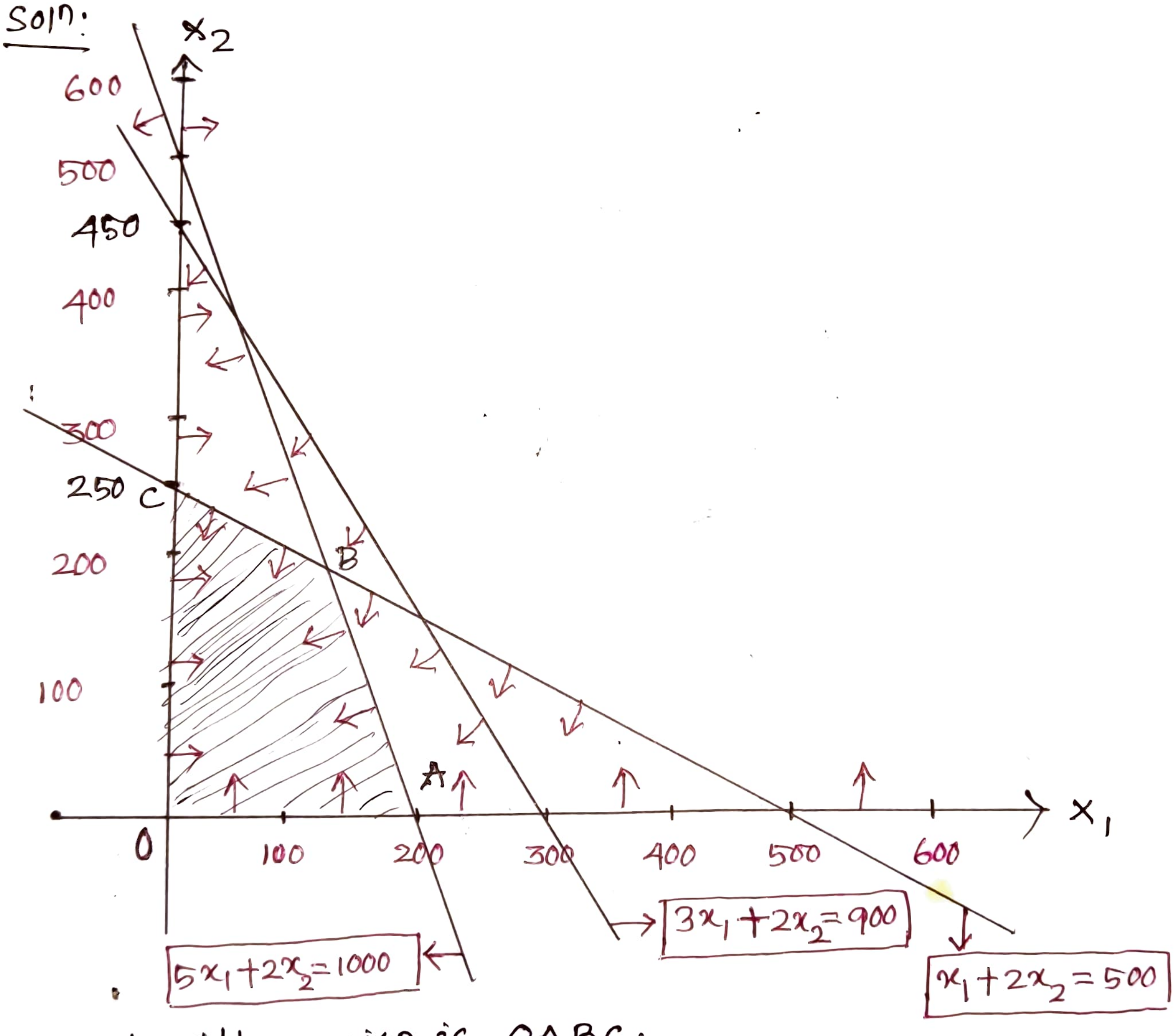
$$5x_1 + 2x_2 \leq 1000$$

$$3x_1 + 2x_2 \leq 900$$

$$x_1 + 2x_2 \leq 500$$

$$x_1, x_2 \geq 0$$

Soln:



The feasible region is OABC.

'B' is the point of intersection of 2 lines $5x_1+2x_2=1000$ and $x_1+2x_2=500$. Solving these two eqns. we get $x_1 = 125$ & $x_2 = 187.5$. Thus, 'B' has coordinate $(125, 187.5)$

Corner Points	Value of $Z = 100x_1 + 40x_2$
$O\ (0,0)$	$\longrightarrow 0$
$A\ (200,0)$	$\longrightarrow 20{,}000$ (maximum)
$B\ (125, 187.5)$	$\longrightarrow 20{,}000$
$C\ (0, 250)$	$\longrightarrow 10{,}000$

Clearly, the maximum value of Z occurs at two corner points A and B. Thus any pt. on the line joining A & B gives the same maximum value of Z. As there are infinite number of points between A & B, there are infinite number of optimal solutions for the LPP.

Example (An unbounded solution)

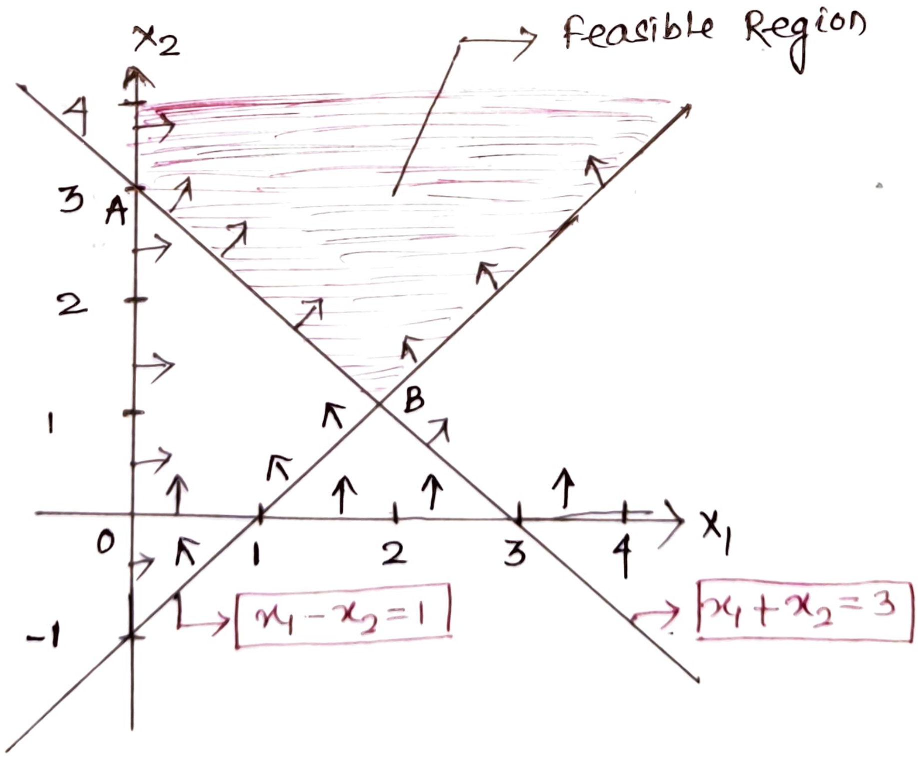
Use graphical method to solve the LPP

$$\max.\ Z = 3x_1 + 2x_2$$

subject to

$$x_1 - x_2 \leq 1$$
$$x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

Soln:



Here the feasible region is unbounded whose corner points are A & B. 'B' is the point of intersection of $x_1 - x_2 = 1$ & $x_1 + x_2 = 3$. solving these two equns. we get $x_1 = 2, x_2 = 1$. Thus, the coordinate of B is (2,1).

Corner Points	value of $Z = 3x_1 + 2x_2$
A (0,3) $\longrightarrow$	6
B (2,1) $\longrightarrow$	8 (Maximum)

But there exists points in the feasible region for which the objective function is more than 8. Infact, the maximum value of z occurs at infinity. Hence, the given LPP has an unbounded solution.

Example:- (No solution)

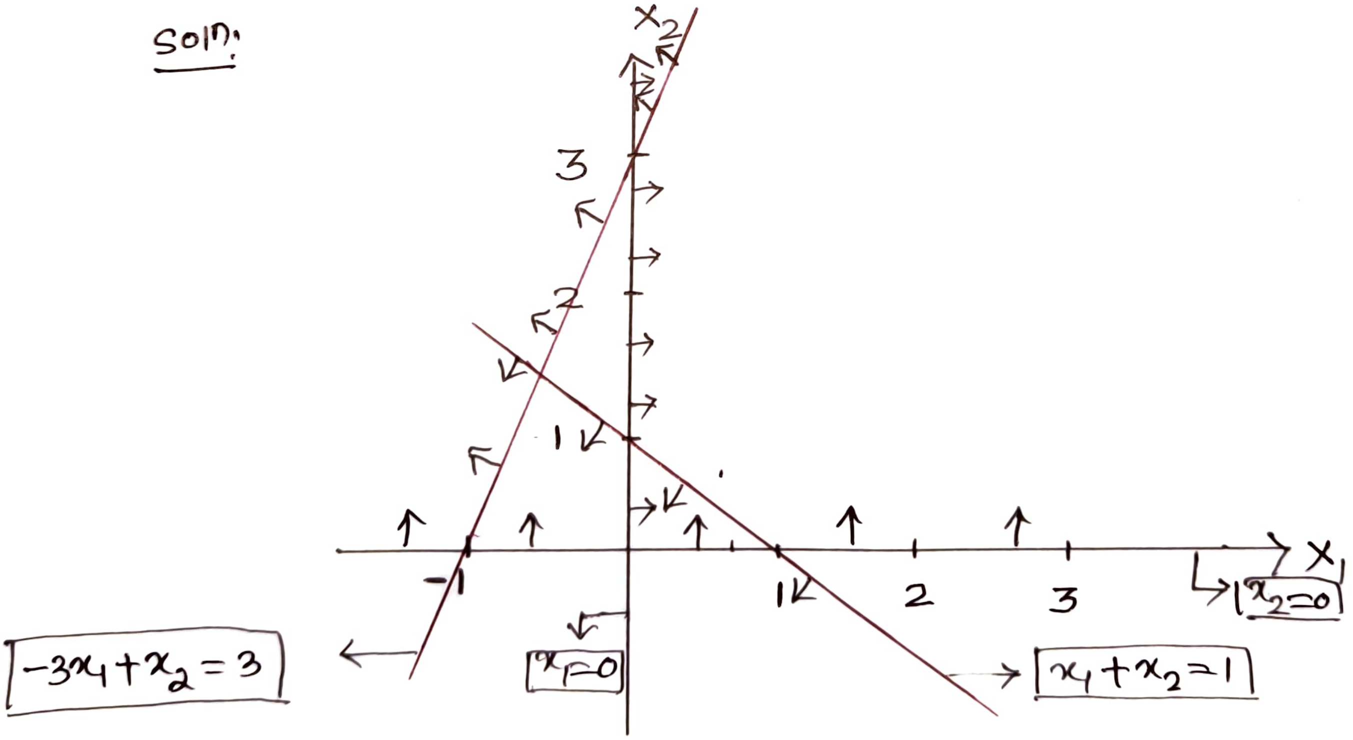
Use graphical method to solve the LPP

$$\max. Z = x_1 + x_2$$

subject to

$$x_1 + x_2 \leq 1$$
$$-3x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

Soln:



For this problem, we cannot find a feasible region. so, the given LPP has no solution.