16.8. R-C SERIES CIRCUIT

Fig. 16.15 shows a resistance of R ohms connected in series with a capacitor of C farad.

Let
V = r.m.s. value of the applied voltage
I = r.m.s. value of the circuit current
$V_R = IR$ .....where V_R is in phase with I
$V_C = IX_C$ .....where V_C lags I by 90°

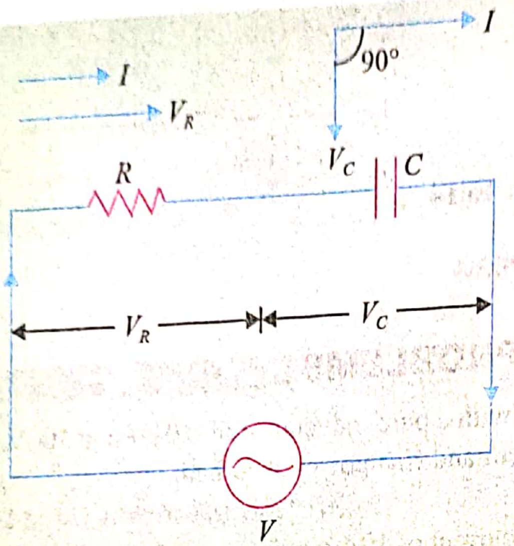


Fig. 16.15

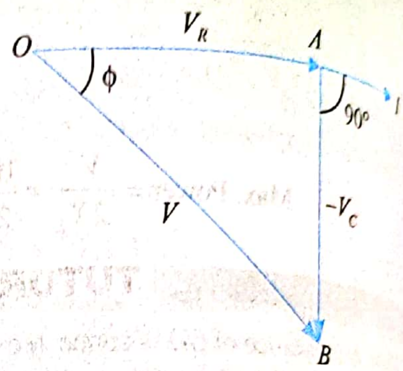


Fig. 16.16

The phasor diagram of the circuit is shown in Fig. 16.16. The supply voltage V is the phasor sum of V_R (= IR) and V_C (= IX_C) drops *i.e.*

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

$$\therefore \quad I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

The quantity $\sqrt{R^2 + X_C^2}$ offers opposition to current flow and is called *impedance* of the circuit.

$$\therefore \quad I = V/Z \quad \text{where } Z = \sqrt{R^2 + X_C^2}$$

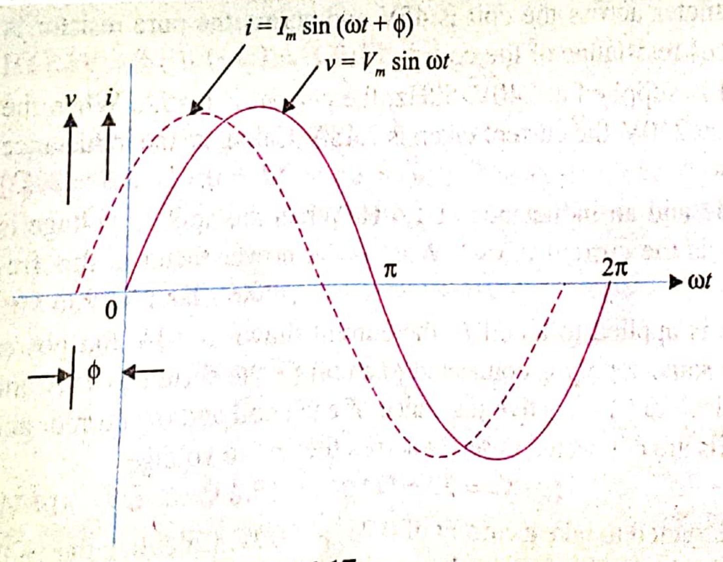
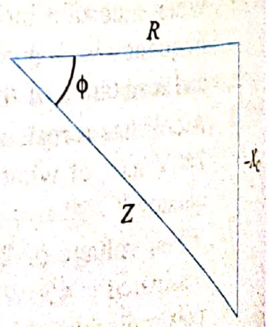


Fig. 16.17



IMPEDANCE TRIANGLE

Fig. 16.18

It is clear from the phasor diagram that *circuit current I leads the applied voltage V by φ°*

where :

$$\tan \phi = -\frac{V_C}{V_R} = -\frac{IX_C}{IR} = -\frac{X_C}{R}$$

Since current is taken as the reference phasor, negative phase angle implies that voltage lags behind the current. This is the same thing as current leads the voltage [See Fig. 16.17].

Power. The equations for voltage and current are :

$$v = V_m \sin \omega t$$

$$i = I_m \sin (\omega t + \phi)$$

Average power, P = Average of vi

$= VI \cos \phi$ [By same way as in Art. 16.1]

Alternatively

$$P = \text{Power in } R + \text{Power in } C$$

$$= I^2 R + 0$$

$$= IR \times I = IR \times \frac{V}{Z} = VI \times \frac{R}{Z} = VI \cos \phi$$

Example 16.17. *A capacitor of capacitance 79.5μF is connected in series with a non-inductive resistance of 30 Ω across 100 V, 50 Hz supply. Find (i) impedance (ii) current (iii) phase angle and (iv) equation for the instantaneous value of current.*

Solution.

(i) $$X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 50 \times 79.5} = 40\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = \mathbf{50\Omega}$$

(ii) $I = V/Z = 100/50 = \mathbf{2A}$

(iii) $\phi = \tan^{-1} X_C/R = \tan^{-1} 40/30 = 53°$ *lead*

(iv) $I_m = 2 \times \sqrt{2} = 2.828$ A; $\omega = 2\pi f = 2\pi \times 50 = 314$ rad s^{-1}

$\therefore i = 2.828 \sin (314\, t + 53°)$ **Ans.**

Example 16.18. *A capacitor and resistor are connected in series across a 120V, 50Hz supply. The circuit draws a current of 1.144 A. If power loss in the circuit is 130.8W, find the values of resistance and capacitance.*

Solution. Power loss occurs in resistance only as capacitor consumes no power.

$$P = I^2 R$$

$$\therefore \quad R = P/I^2 = 130.8/(1.144)^2 = \mathbf{100\Omega}$$

$$Z = V/I = 120/1.144 = 104.91\ \Omega$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(104.9)^2 - (100)^2} = 31.7\Omega$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 31.7} = \mathbf{100 \times 10^{-6} F}$$

Example 16.19. *A resistor R in series with a capacitor C is connected to 50Hz, 240V supply. Find the value of C so that R absorbs 300 W at 100 V. Find also the maximum charge and maximum energy stored in C.*

Solution.

R absorbs 300W when connected to 100V so that $R = V^2/P = 100^2/300 = 100/3\Omega$

Now,
$$I^2 R = 3000\text{W} \qquad \therefore I = \sqrt{300 \times 3/100} = 3\text{A}$$

$$Z = V/I = 240/3 = 80\Omega$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{80^2 - (100/3)^2} = 72.7\Omega$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 72.7} = \mathbf{43.7 \times 10^{-6}\,F}$$

$$Q_{max} = CV_{max} = (43.7 \times 10^{-6}) \times 240\sqrt{2} = \mathbf{0.0148C}$$

$$E = \frac{1}{2}CV_{max}^2 = \frac{1}{2} \times (43.7 \times 10^{-6}) \times (240 \times \sqrt{2})^2 = \mathbf{2.5\,J}$$

Example 16.20. *A 10 Ω resistor and 400 μF capacitor are connected in series to a sinusoidal supply. The circuit current is 5A. Calculate the supply frequency and phase angle between the current and voltage.*

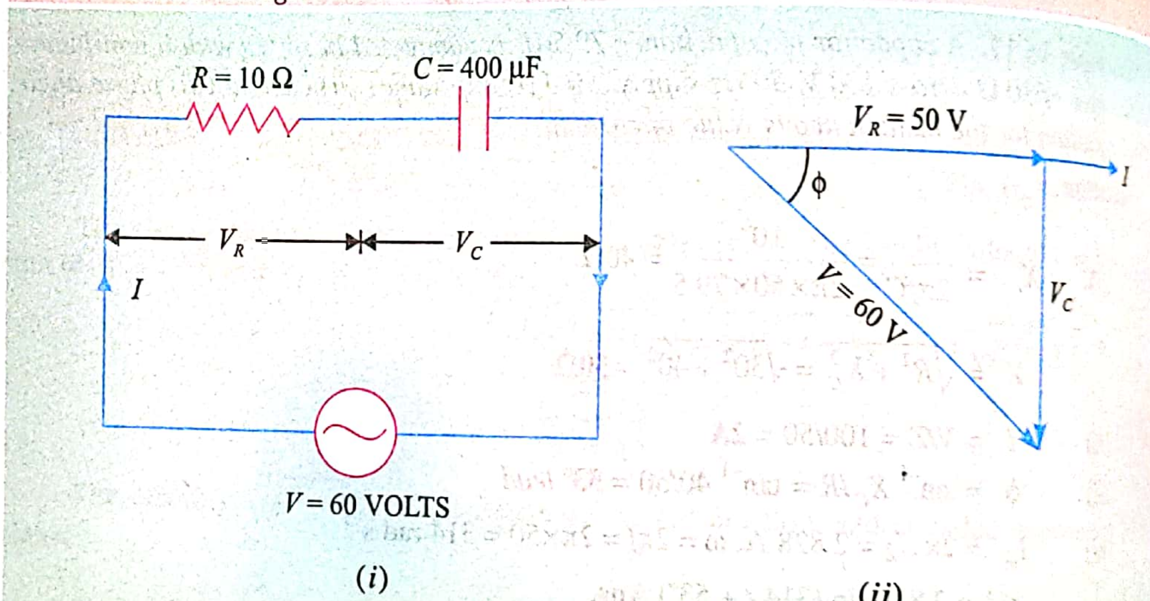


Fig. 16.19

Solution.

$$V_R = IR = 5 \times 10 = 50\text{V}$$

$$V_C = \sqrt{V^2 - V_R^2} = \sqrt{60^2 - 50^2} = 33.17\text{ V}$$

$$X_C = V_C/I = 33.17/5 = 6.634\Omega$$

$$f = \frac{1}{2\pi C X_C} = \frac{10^6}{2\pi \times 400 \times 6.634} = \mathbf{60\ Hz}$$

$$\phi = \tan^{-1} V_C/V_R = \tan^{-1} 33.17/50 = \mathbf{33.6°}\ \textit{lead}$$

Example 16.21. *Calculate the capacitance of a condenser to be connected in series with a 100V, 80W lamp to enable it to be used on a 200V, 50Hz supply. What is p.f. of the circuit?*

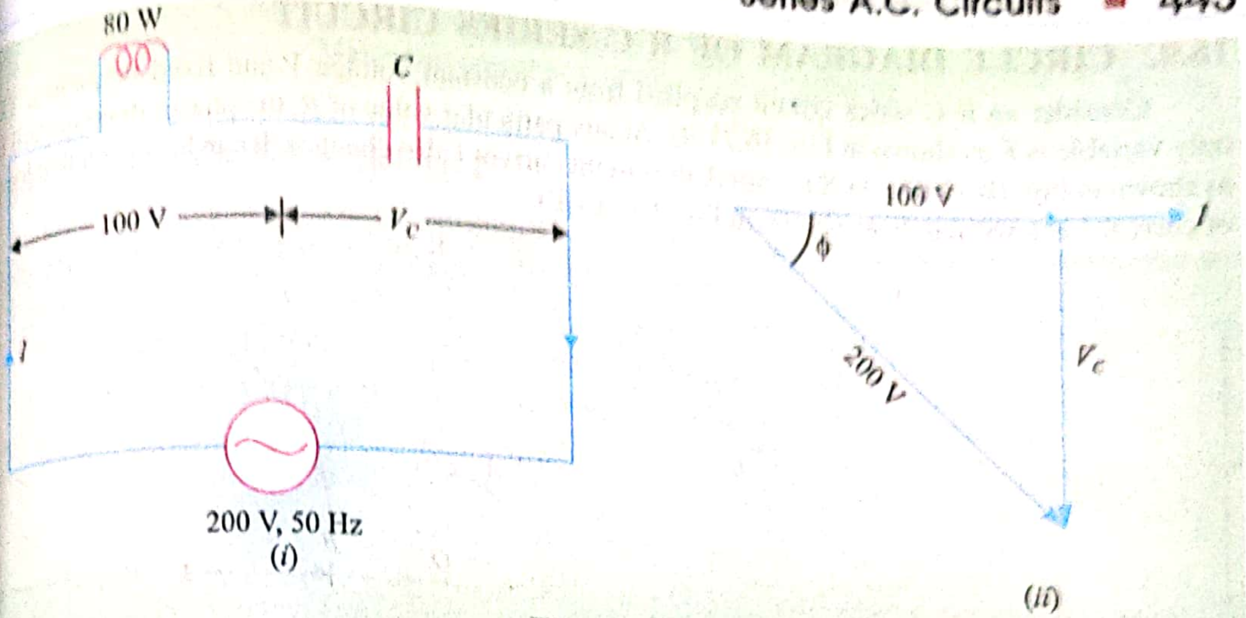


Fig. 16.20

Solution. The voltage across the bulb (pure resistance) should remain 100V for its proper operation. Fig. 16.20 (*i*) shows the required condenser of *C* farad in series with the lamp. The phasor diagram of the circuit is shown in Fig. 16.20 (*ii*).

Rated current of bulb, $I = 80/100 = 0.8\text{A}$

$$V_C = \sqrt{200^2 - 100^2} = 173.2\text{ V}$$

$$X_C = V_C/I = 173.2/0.8 = 216.5\ \Omega$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 216.5} = \mathbf{14.7 \times 10^{-6}\ F}$$

Circuit p.f. $= \cos\phi = V_R/V = 100/200 = \mathbf{0.5}$ lead

Example 16.22. *A load consisting of a capacitor in series with a resistor has an impedance of 50 Ω and power factor 0.707 leading. The load is connected in series with a 40 Ω resistor across an a.c. supply and the resulting current is 3A. Determine the supply voltage and the overall phase angle.*

Solution.

$$R_L = Z_L \cos\phi = 50 \cos 45° = 35.35\ \Omega$$

$$X_C = Z_L \sin\phi = 50 \sin 45° = 35.35\ \Omega$$

$$R_T = R + R_L = 40 + 35.35 = 75.35\ \Omega$$

$$Z = \sqrt{(75.35)^2 + (35.35)^2} = 83.23\Omega$$

$$V = IZ = 3 \times 83.23 = \mathbf{250V}$$

$$\phi = \tan^{-1} X_C/R_T = \tan^{-1} 35.35/75.35 = \mathbf{25.17°}\ \textit{lead}$$

TUTORIAL PROBLEMS

1. A circuit when connected to 200V, 50Hz mains takes a current of 10A, leading the voltage by one-twelfth of time period. Calculate (*i*) resistance (*ii*) capacitive reactance and (*iii*) capacitance of the circuit. **[(*i*) 17.32 Ω (*ii*) 10 Ω (*iii*) 318 × 10^{-6}F]**
2. A resistor *R* in series with a capacitance *C* is connected to a 50 Hz, 240V supply. Find the value of *C* so that *R* absorbs 300W at 100V. Find also maximum charge and the maximum stored energy in *C*. **[44μF; 0.0135C; 2.1J]**
3. A circuit has a fixed resistance of 2Ω and a capacitive reactance of 10Ω in series with a resistor *R* across 100V constant frequency mains. For what value of *R* is the power consumed in it a maximum? **[10.2 Ω]**