Autumn Midsemester Examination-2019

Discrete Mathematics [MA-2013]

Full Marks: 20 **Time: 1.5 hours**

Answer any four questions including question no.1 which is compulsory.

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1. Answer all the following questions. [1 × 5]

   (a) What is the inverse of " If it is below freezing then it is snowing" ?

   (b) Write the negation of the statement "Some birds can not fly".

   (c) Let, p : Today is Monday, q : It is raining, r : It is hot. Write the proposition $\neg(p \vee q) \longleftrightarrow r$ as an English sentence.

   (d) Which rule of inference does the tautology $[(p \longrightarrow q) \wedge \neg q] \longrightarrow \neg p$ represent ?

   (e) Write the reflexive closure and symmetric closure of a relation $R = \{(a, b) : a > b\}$ on the set real numbers.

2. [2.5 × 2]

   (a) Show that, $[\neg p \wedge (p \vee q)] \longrightarrow q$ is a tautology.

   (b) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent without using truth table.

3. [2.5 × 2]

   (a) Show that the following argument is valid using rules of inference.
   Hypotheses: $\neg p \longleftrightarrow q$, $q \longrightarrow r$, $\neg r$ and Conclusion: p

   (b) Derive the relevant conclusion(s) from the following hypotheses using rules of inference.
   "All insects have six legs", "Dragonflies are insects", "Spiders do not have six legs"

4. [2.5 × 2]

   (a) Among the first 500 positive integers, determine the number of integers which are neither divisible by 3, 7 nor 11.

   (b) Let $R = \{(1,1),(1,2),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ be a relation defined on $A = \{1, 2, 3\}$. Is it reflexive ? symmetric ? antisymmetric ? transitive ?

5. [2.5 × 2]

   (a) Prove that $\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2$, $\forall\, n \in \mathbb{Z}^+$ using mathematical induction.

   (b) Let $a_0 = 1$, $a_n = a_{n-1} + a_{n-2} + \ldots + a_0 + 1$, prove that $a_n = 2^n$, $n \geq 1$ using method of strong induction.

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