Qn. Set Code-1

Semester: 3rd
Programme: B.Tech
Branch: CSE , IT, CSSE

AUTUMN END SEMESTER EXAMINATION-2022

3rd Semester B.Tech

DISCRETE MATHEMATICS

MA 2013

(For 2022 (L.E), 2021 & Previous Admitted Batches)

Time: 3 Hours Full Marks: 50

Answer any SIX questions.
Question paper consists of four SECTIONS i.e. A, B, C and D.
Section A is compulsory.
Attempt minimum one question each from Sections B, C, D.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A

1. Answer the following questions. [1 × 10]

(a) Translate the statement "Not all people are honest" into logical expression using predicate and quantifier.

(b) Which rule of inference is applied to the argument $[(\neg p \rightarrow \neg q) \wedge \neg p] \rightarrow \neg q \equiv T$?

(c) What is the converse of the conditional statement "To be a citizen of this country, it is sufficient that you were born in India" in the form of "If… , then…".

(d) List the ordered pairs in the equivalence relation R produced by the partitions $A_1 = \{a,b\}$, $A_2 = \{c\}$, $A_3 = \{d,e\}$ of $S = \{a,b,c,d,e\}$.

(e) How many positive integers not exceeding 1000 are divisible by 7 or 11?

(f) What is normal subgroup? Give an example.

(g) Solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 0$ for $n \geq 2$.

(h) Check whether the permutation function $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 1 & 3 & 2 & 4 & 7 & 6 \end{pmatrix}$ is even or odd ?

(i) Find the generator(s) of the group $(\{1,-1,i,-i\}, \times)$.

(j) Let $R = \{(a,b) \mid a < b\}$ be a relation defined on the set of real numbers. What are R^c and R^{-1}?

SECTION-B

2. (a) Prove that the order of any subgroup divides the order of the group. [4]

(b) Find all the permutations defined on the set {1, 2, 3, 4} and determine which of them are even and are odd. [4]

3. (a) Answer these questions for the poset ({2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72}, |). [4]

(i) Find the maximal element.

(ii) Find the minimal element.

(iii) Is there a greatest element? If yes, find.

(iv) Is there a least element? If yes, find.

(v) Find all upper bounds of {2, 9}.

(vi) Find the least upper bound of {2, 9}, if it exists.

(vii) Find all lower bounds of {60, 72}.

(viii) Find the greatest lower bound of {60, 72}, if it exists.

(b) Prove that 21 divides $4^{n+1} + 5^{2n-1}$ for any positive integer n by the method of induction. [4]

SECTION-C

4. (a) Prove that the relation $R=\{(a,b)\,|\,a\equiv b(\text{mod}5)\}$ is an equivalence relation. Find its disjoint equivalence classes. [4]

   (b) If Φ is homomorphism from the group $(G,*)$ to group $(\overline{G},\#)$, then show that [4]

   (i) $\Phi(e)=\overline{e}$ where e and $\overline{e}$ are identities of G and $\overline{G}$, respectively.

   (ii) $\Phi(a^{-1})=(\Phi(a))^{-1}$, $\forall a\in G$.

5. (a) Using generating functions, solve the recurrence relation $a_n-6a_{n-1}+9a_{n-2}=0$ with the initial condition $a_0=1$, $a_1=6$. [4]

   (b) Prove that $[\neg p\wedge(p\vee q)]\rightarrow q$ is a tautology by developing a series of equivalences. [4]

6. (a) Let $(\{a,b\},\#)$ be a semigroup where $a\#a=b$. Show that [4]

   (i) $a\#b=b\#a$

   (ii) $b\#b=b$

   (b) Use generating functions to solve the recurrence relation $a_n=3a_{n-1}+4^n$ with initial condition $a_0=1$. [4]

SECTION-D

7. (a) Find the general solution of the recurrence relation $a_n=4a_{n-1}-4a_{n-2}+n2^n$. [4]

   (b) Use Warshall's algorithm to find the transitive closure of the relation $\{(1,4),(2,1),(2,3),(3,1),(3,4),(4,3)\}$ on the set $\{1,2,3,4\}$. [4]

8. (a) Using the rules of inference show that the following argument is valid. [4]

If Ajaya learns Discrete Mathematics, then either Bijaya or Chinmay will learn Calculus. If Bijaya learns Calculus, then Ajaya will not learn Discrete Mathematics. If Deb learns Calculus, then Chinmay will not. Therefore, if Ajaya learn Discrete mathematics, Deb will not Learn Calculus.

(b) What is zero divisor in a ring? Find all the zero divisors in the ring $(Z_8, \oplus, \otimes)$. [4]

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