AUTUMN END SEMESTER EXAMINATION-2014

3rd Semester B.Tech / B.Tech Dual Degree

DISCRETE MATHEMATICS MA-2003

(Regular-2013 Admitted Batch)

Full Marks: 60 **Time: 3 Hours**

Answer any SIX questions including Question No.1 which is compulsory.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. Answer the following: [2×10

a) Find the converse and inverse of the following statement:-

"Sum of two Odd Integers is Even"

b) What are the negation of the statements:

(i) $\forall x(x^2 > x)$ and (ii) $\exists x(x^2 = 2)$

c) Let R be the relation $\{(a, b) : a \neq b\}$ on the set of integers. What is the reflexive closure of R?

d) Let A={1, 2, 3, 4, 5, 6} be any set. Find the adjacency matrix of the relation:

$R = \{(a,b) \mid a \text{ divides } b, \forall a, b \in A\}$

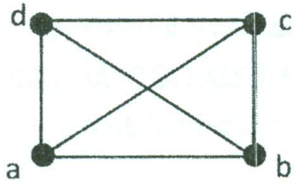
Draw its Hasse diagram.

e) Find the generating function of the following numeric functions:

(i) $a_n = 7.2^n, n \geq 0$ and (ii) $a_n = 2^n + 3^n, n \geq 0$

f) Using Havel-Hakimi theorem, check whether the following degree sequence 1, 2, 3, 4, 5, 6 is graphic or not.

g) Define Eulerian graph. Does the following graph is Eulerian?



h) Define Integral domain with example.

i) Prove that a group in which every element in its own inverse is abelian.

j) Let * be the binary operation defined on the set of natural number N as

$\forall a, b \in N; a * b = \max(a, b)$. Is it a monoid? If yes, find its identity element.

2. a) Construct the truth table for [4

$[(p \to q) \wedge (q \to r)] \to (p \to r)$. Is it tautology?

b) Use mathematical induction to prove that $4^{n+1} + 5^{2n-1}$ is divisible by 21 whenever n is a positive integer. [4

3. a) Use generating function to solve the recurrence relation $a_n = 3a_{n-1} + 4^{n-1}$ with the initial condition $a_0 = 1$. [4

b) Solve the recurrence relation $a_n - 3a_{n-1} + 2a_{n-2} = 2^n$, for $n \geq 2$, $a_0 = 6.\ a_1 = 11$. [4

4. a) State and prove Lagrange's theorem. [4

b) Prove that S_3 is a non-abelian group of order 6 under the composition of two mappings, where S_3 is the set of all permutations defined on $A = \{1,2,3\}$. [4

5. a) Prove that every connected acyclic graph is a Tree. [4

b) Define the following: [4

(i) Bipertite graph

(ii) Connected graph

(iii) Eulerian graph and

(iv) Hamiltonian graph

6. a) Let $A = \{1,2,3,4\}$ and R be a relation on A defined by $R = \{(1,1),(1,2),(2,4),(3,2),(4,3)\}$. Find the transitive closure of R. [4

b) How many positive integers not exceeding 1000 are divisible by 7 or 11? [4

7. a) Express the following Boolean function f(x, y, z) in CNF & DNF. [4

Input			Output
x	y	z	f(x, y, z)
0	0	0	1
1	0	0	0
1	0	1	1
0	1	1	1
1	1	1	0
1	1	0	1
0	1	0	0
0	0	1	0

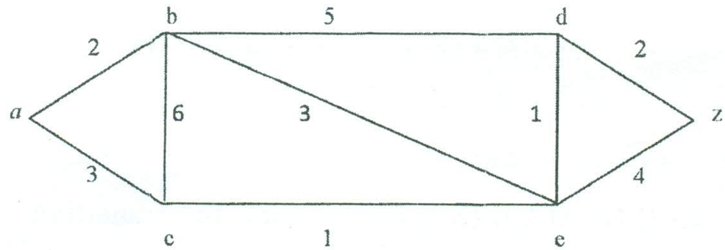
b) Let $(G,*)$ and $(\overline{G},0)$ be two groups. [4

$f : G \to \overline{G}$ be a homomorphism. Then prove that –

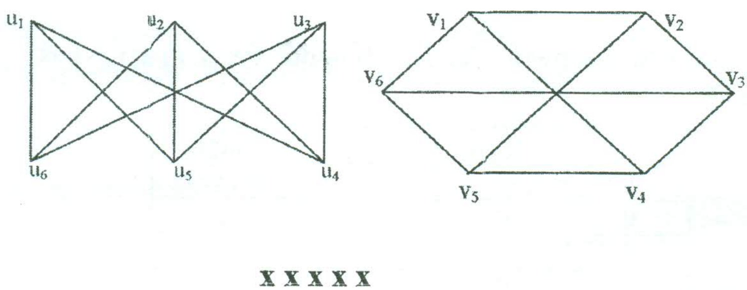
(i) $f(e) = \overline{e} : e \;\&\; \overline{e}$ are the Indentity elements of $G \;\&\; \overline{G}$ respectively.

(ii) $f(x^{-1}) = [f(x)]^{-1},\ \forall x \in G$

8. a) Use *Dijkshra's* algorith to find the length of a shortest path from vertex '*a*' to vertex '*z*' by in the following graph: [4



b) Show that the graphs given below are isomorphic. [4



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