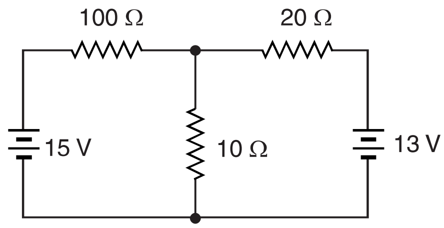

Fig. 3.15

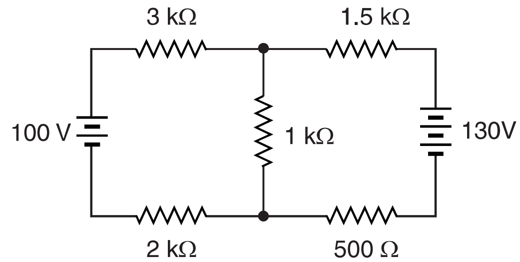

Fig. 3.16

2. Using mesh analysis, find the voltage drop across the 1 kΩ resistor in Fig. 3.16. **[50 V]**
3. Using mesh analysis, find the currents in 50 Ω, 250 Ω and 100 Ω resistors in the circuit shown in Fig. 3.17. **[I(50 Ω) = 0·171 A → ; I(250 Ω) = 0·237 A ← ; I(100 Ω) = 0·408 A ↓]**

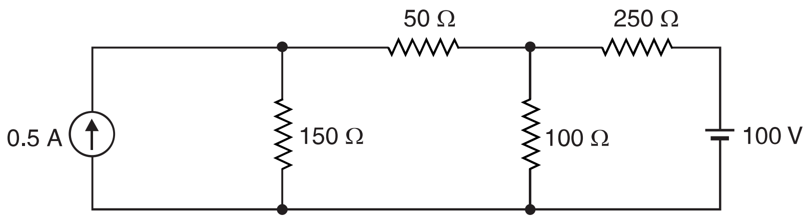

Fig. 3.17

4. For the network shown in Fig. 3.18, find the mesh currents I_1, I_2 and I_3. **[5 A, 1 A, 0·5 A]**

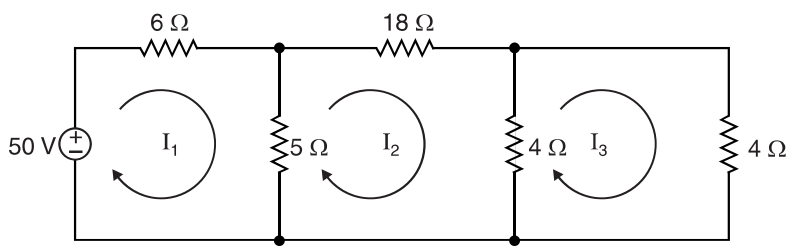

Fig. 3.18

5. In the network shown in Fig. 3.19, find the magnitude and direction of current in the various branches by mesh current method. **[*FAB* = 4 A ; *BF* = 3 A ; *BC* = 1 A ; *EC* = 2 A ; *CDE* = 3 A]**

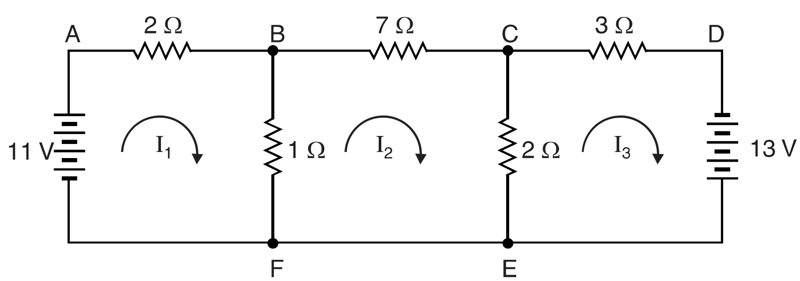

Fig. 3.19

3.6. Nodal Analysis

Consider the circuit shown in Fig. 3.20. The branch currents in the circuit can be found by Kirchhoff's laws or Maxwell's mesh current method. There is another method, called *nodal analysis* for determining branch currents in a circuit. In this method, one of the nodes (Remember a node is a point in a network where two or more circuit elements meet) is taken as the *reference node*. The potentials of all the points in the circuit are measured w.r.t. this reference node. In Fig. 3.20, A, B, C and D are four nodes and the node D has been taken as the *reference node. The fixed-voltage nodes are called *dependent nodes*. Thus in Fig. 3.20, A and C are fixed nodes because $V_A = E_1 =$ 120 V and $V_C = 65$ V. The voltage from D to B is V_B and its magnitude depends upon the parameters of circuit elements and the currents through these elements. Therefore, node B is called *independent node*. Once we calculate the potential at the independent node (or nodes), each branch current can be determined because the voltage across each resistor will then be known.

Hence **nodal analysis** *essentially aims at choosing a reference node in the network and then finding the unknown voltages at the independent nodes w.r.t. reference node*. For a circuit containing N nodes, there will be N–1 node voltages, some of which may be known if voltage sources are present.

Circuit analysis. The circuit shown in Fig. 3.20 has only one independent node B. Therefore, if we find the voltage V_B at the independent node B, we can determine all branch currents in the circuit. We can express each current in terms of e.m.f.s, resistances (or conductances) and the voltage V_B at node B. Note that we have taken point D as the reference node.

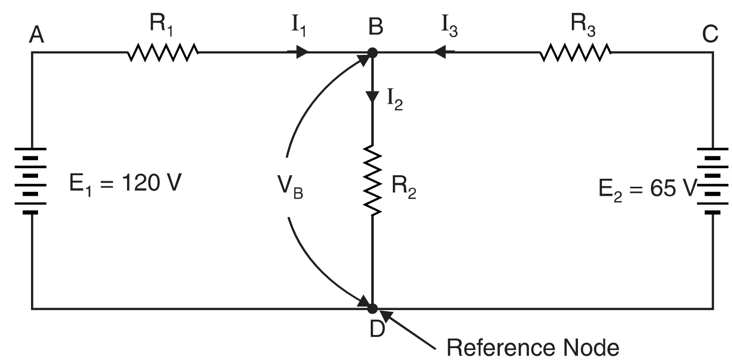

Fig. 3.20

The voltage V_B can be found by applying **Kirchhoff's current law at node B.

$$I_1 + I_3 = I_2 \quad \text{...(i)}$$

In mesh $ABDA$, the voltage drop across R_1 is $E_1 - V_B$.

$$\therefore \quad I_1 = \frac{E_1 - V_B}{R_1}$$

In mesh $CBDC$, the voltage drop across R_3 is $E_2 - V_B$.

$$\therefore \quad I_3 = \frac{E_2 - V_B}{R_3}$$

Also $$I_2 = \frac{V_B}{R_2}$$

Putting the values of I_1, I_2 and I_3 in eq. (i), we get,

$$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_2} \quad \text{...(ii)}$$

All quantities except V_B are known. Hence V_B can be found out. Once V_B is known, all branch currents can be calculated. It may be seen that nodal analysis requires only one equation [eq. (ii)] for determining the branch currents in this circuit. However, Kirchhoff's or Maxwell's solution would have needed two equations.

* An obvious choice would be ground or common, if such a point exists.

** Since the circuit unknowns are voltages, the describing equations are obtained by applying KCL at the nodes.

Notes.

(*i*) We can mark the directions of currents at will. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed.

(*ii*) We can also express the currents in terms of conductances.

$$I_1 = \frac{E_1 - V_B}{R_1} = (E_1 - V_B)G_1 \;;\; I_2 = \frac{V_B}{R_2} = V_B\,G_2 \;;\; I_3 = \frac{E_2 - V_B}{R_3} = (E_2 - V_B)G_3$$

3.7. Nodal Analysis with Two Independent Nodes

Fig. 3.21 shows a network with two independent nodes B and C. We take node D (or E) as the reference node. We shall use Kirchhoff's current law for nodes B and C to find V_B and V_C. Once the values of V_B and V_C are known, we can find all the branch currents in the network.

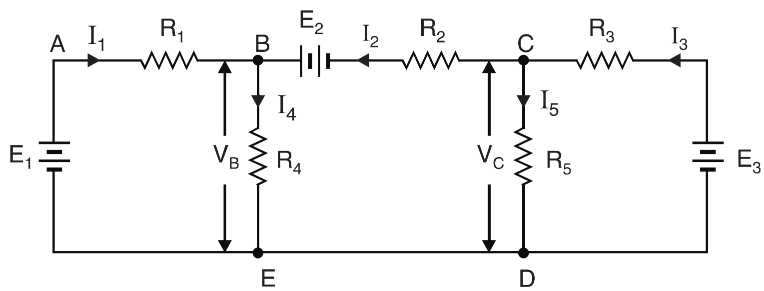

Fig. 3.21

Each current can be expressed in terms of e.m.f.s, resistances (or conductances), V_B and V_C.

$$E_1 = V_B + I_1R_1 \qquad \therefore\; I_1 = \frac{E_1 - V_B}{R_1}$$

$$E_3 = V_C + I_3R_3 \qquad \therefore\; I_3 = \frac{E_3 - V_C}{R_3}$$

$$E_2{}^* = V_B - V_C + I_2R_2 \qquad \therefore\; I_2 = \frac{E_2 - V_B + V_C}{R_2}$$

Similarly, $$I_4 = \frac{V_B}{R_4} \;;\; I_5 = \frac{V_C}{R_5}$$

At node B. $$I_1 + I_2 = I_4$$

or $$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B + V_C}{R_2} = \frac{V_B}{R_4} \qquad ...(i)$$

At node C. $$I_2 + I_5 = I_3$$

or $$\frac{E_2 - V_B + V_C}{R_2} + \frac{V_C}{R_5} = \frac{E_3 - V_C}{R_3} \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we can find V_B and V_C since all other quantities are known. Once we know the values of V_B and V_C, we can find all the branch currents in the network.

Note. We can also express currents in terms of conductances as under :

$I_1 = (E_1 - V_B)\,G_1 \;;\; I_2 = (E_2 - V_B + V_C)\,G_2$

$I_3 = (E_3 - V_C)\,G_3 \;;\; I_4 = V_B\,G_4 \;;\; I_5 = V_C\,G_5$

* As we go from C to B, we have,

$$V_C - I_2R_2 + E_2 = V_B$$

$$\therefore \quad E_2 = V_B - V_C + I_2R_2$$

Example 3.9. *Find the currents in the various branches of the circuit shown in Fig. 3.22 by nodal analysis.*

Solution. Mark the currents in the various branches as shown in Fig. 3.22. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed. Take point *E* (or *F*) as the reference node. We shall find the voltages at nodes *B* and *C*.

At node B. $I_2 + I_3 = I_1$

or $\frac{V_B}{10} + \frac{{}^*V_B - V_C}{15} = \frac{100 - V_B}{20}$

or $13V_B - 4V_C = 300$ ...(*i*)

At node C. $I_4 + I_5 = I_3$

or $\frac{V_C}{10} + \frac{V_C + 80}{10} = \frac{V_B - V_C}{15}$

or $V_B - 4V_C = 120$ ...(*ii*)

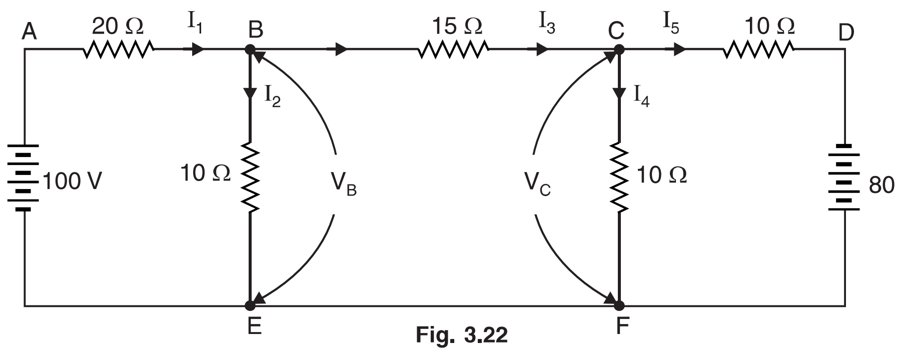

Fig. 3.22

Subtracting eq. (*ii*) from eq. (*i*), we get, $12V_B = 180$ $\therefore$ $V_B = 180/12 = 15$ V

Putting $V_B = 15$ volts in eq. (*i*), we get, $V_C = -26.25$ volts.

By determinant method

$$13V_B - 4V_C = 300$$
$$V_B - 4V_C = 120$$

$$\therefore \quad V_B = \frac{\begin{vmatrix} 300 & -4 \\ 120 & -4 \end{vmatrix}}{\begin{vmatrix} 13 & -4 \\ 1 & -4 \end{vmatrix}} = \frac{(300 \times -4) - (120 \times -4)}{(13 \times -4) - (1 \times -4)} = \frac{-720}{-48} = 15 \text{ V}$$

$$\text{and} \quad V_C = \frac{\begin{vmatrix} 13 & 300 \\ 1 & 120 \end{vmatrix}}{\text{Denominator}} = \frac{(13 \times 120) - (1 \times 300)}{-48} = \frac{1260}{-48} = -26{\cdot}25 \text{ V}$$

$\therefore$ Current $I_1 = \frac{100 - V_B}{20} = \frac{100 - 15}{20} =$ **4·25 A**

Current $I_2 = V_B/10 = 15/10 =$ **1·5 A**

Current $I_3 = \frac{V_B - V_C}{15} = \frac{15 - (-26.25)}{15} =$ **2·75 A**

* Note that the current I_3 is assumed to flow from *B* to *C*. Therefore, with this assumption, $V_B > V_C$.

$\therefore$ Power loss in the circuit $= I_1^2 \times 1 + I_2^2 \times 1 + I_3^2 \times 0{\cdot}5 + I_4^2 \times 2 + I_5^2 \times 1$

$= (5{\cdot}75)^2 \times 1 + (9{\cdot}25)^2 \times 1 + (3{\cdot}5)^2 \times 0{\cdot}5 + (5{\cdot}5)^2 \times 2 + (9)^2 \times 1$

$=$ **266·25 W**

Example 3.12. *Using nodal analysis, find node-pair voltages V_B and V_C and branch currents in the circuit shown in Fig. 3.25. Use conductance method.*

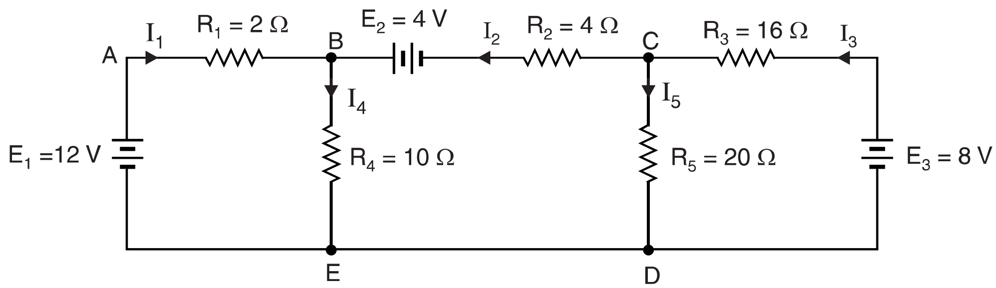

Fig. 3.25

Solution. Mark the currents in the various branches as shown in Fig. 3.25. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed. Take point D (or E) as the reference node. We shall find the voltages at nodes B and C and hence the branch currents.

$$G_1 = \frac{1}{R_1} = \frac{1}{2} = 0{\cdot}5 \text{ S} \;;\; G_2 = \frac{1}{R_2} = \frac{1}{4} = 0{\cdot}25 \text{ S} \;;\; G_3 = \frac{1}{R_3} = \frac{1}{16} = 0{\cdot}0625 \text{ S} \;;$$

$$G_4 = \frac{1}{R_4} = \frac{1}{10} = 0.1 \text{ S} \;;\quad G_5 = \frac{1}{R_5} = \frac{1}{20} = 0{\cdot}05 \text{ S}$$

At node *B*. $I_1 + I_2 = I_4$

or $(E_1 - V_B)G_1 + (E_2 - V_B + V_C)G_2 = V_B G_4$

or $E_1G_1 + E_2G_2 = V_B(G_1 + G_2 + G_4) - V_C G_2$

or $(12 \times 0.5) + (4 \times 0.25) = V_B(0{\cdot}5 + 0{\cdot}25 + 0{\cdot}1) - V_C \times 0{\cdot}25$

or $7 = 0{\cdot}85\ V_B - 0{\cdot}25\ V_C$ ...(*i*)

At node *C*. $I_3 = I_2 + I_5$

or $(E_3 - V_C)G_3 = (E_2 - V_B + V_C)G_2 + V_C \times G_5$

or $E_3G_3 - E_2G_2 = -V_BG_2 + V_C(G_2 + G_3 + G_5)$

or $(8 \times 0.0625) - (4 \times 0.25) = -V_B(0{\cdot}25) + V_C(0{\cdot}25 + 0{\cdot}0625 + 0{\cdot}05)$

or $-0{\cdot}5 = -0{\cdot}25\ V_B + 0{\cdot}362\ V_C$ ...(*ii*)

From equations (*i*) and (*ii*), we get, $V_B =$ **9.82 V** ; $V_C =$ **5·4V**

$\therefore$ $I_1 = (E_1 - V_B)G_1 = (12 - 9{\cdot}82) \times 0{\cdot}5 =$ **1·09 A**

$I_2 = (E_2 - V_B + V_C)G_2 = (4 - 9{\cdot}82 + 5{\cdot}4) \times 0{\cdot}25 =$ **–0·105A**

$I_3 = (E_3 - V_C)G_3 = (8 - 5{\cdot}4) \times 0{\cdot}0625 =$ **0·162A**

$I_4 = V_BG_4 = 9{\cdot}82 \times 0{\cdot}1 =$ **0·982A**

$I_5 = V_CG_5 = 5{\cdot}4 \times 0.05 =$ **0·27A**

The negative sign for I_2 means that the actual direction of this current is opposite to that shown in Fig. 3.25.

Example 3.14. *Find the current I in Fig. 3.27 (i) by changing the two voltage sources into their equivalent current sources and then using nodal method. All resistances are in ohms.*

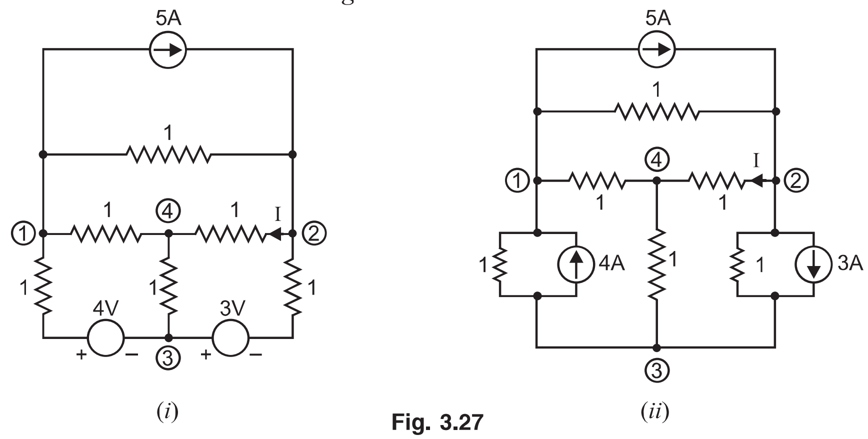

Fig. 3.27

Solution. Since we are to find I, it would be convenient to take node 4 as the reference node. The two voltage sources are converted into their equivalent current sources as shown in Fig. 3.27. (*ii*). We shall apply *KCL* at nodes 1, 2 and 3 in Fig. 3.27 (*ii*) to obtain the required solution.

At node 1. Applying *KCL*, we have,

$$\frac{V_3 - V_1}{1} + 4 = \frac{V_1}{1} + \frac{V_1 - V_2}{1} + 5$$

or $$3V_1 - V_2 - V_3 = -1 \qquad ...(i)$$

At node 2. Applying *KCL*, we have,

$$5 + \frac{V_1 - V_2}{1} = \frac{V_2}{1} + \frac{V_2 - V_3}{1} + 3$$

or $$V_1 - 3V_2 + V_3 = -2 \qquad ...(ii)$$

At node 3. Applying *KCL*, we have,

$$\frac{V_2 - V_3}{1} + 3 - \frac{V_3}{1} = \frac{V_3 - V_1}{1} + 4$$

or $$V_1 + V_2 - 3V_3 = 1 \qquad ...(iii)$$

From eqs. (*i*), (*ii*) and (*iii*), we get, $V_2 = 0.5$ V.

$$\therefore \quad \text{Current } I = \frac{V_2 - 0}{1} = \frac{0.5 - 0}{1} = \mathbf{0.5A}$$

Example 3.15. *Use nodal analysis to find the voltage across and current through 4 Ω resistor in Fig. 3.28 (i).*

Solution. We must first convert the 2V voltage source to an equivalent current source. The value of the equivalent current source is $I = 2\text{V}/2\Omega = 1$ A. The circuit then becomes as shown in Fig. 3.28 (*ii*).

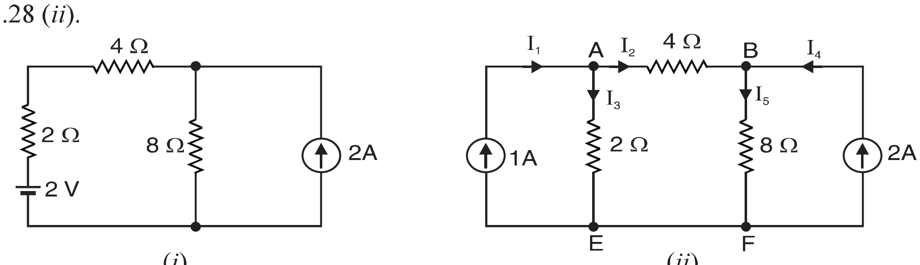

Fig. 3.28

Mark the currents in the various branches as shown in Fig. 3.28 (*ii*). Take point *E* (or *F*) as the reference node. We shall calculate the voltages at nodes *A* and *B*.

At node A. $I_1 = I_2 + I_3$

or $1 = \dfrac{^*V_A - V_B}{4} + \dfrac{V_A}{2}$

or $3V_A - V_B = 4$ ...(*i*)

At node B. $I_2 + I_4 = I_5$

or $\dfrac{V_A - V_B}{4} + 2 = \dfrac{V_B}{8}$

or $2V_A - 3V_B = -16$ ...(*ii*)

Solving equations (*i*) and (*ii*), we find $V_A = 4$V and $V_B = 8$V. Note that $V_B > V_A$, contrary to our initial assumption. Therefore, actual direction of current is from node *B* to node *A*.

By determinant method

$$3V_A - V_B = 4$$
$$2V_A - 3V_B = -16$$

$$\therefore \quad V_A = \frac{\begin{vmatrix} 4 & -1 \\ -16 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix}} = \frac{(-12)-(16)}{(-9)-(-2)} = \frac{-28}{-7} = 4\text{V}$$

$$V_B = \frac{\begin{vmatrix} 3 & 4 \\ 2 & -16 \end{vmatrix}}{\text{Denominator}} = \frac{(-48)-(8)}{-7} = \frac{-56}{-7} = 8\text{V}$$

Voltage across 4Ω resistor = $V_B - V_A = 8 - 4 =$ **4V**

Current through 4Ω resistor = $\dfrac{4\text{V}}{4\Omega}$ = **1A**

We can also find the currents in other resistors.

$$I_3 = \frac{V_A}{2} = \frac{4}{2} = 2\text{A}$$

$$I_5 = \frac{V_B}{8} = \frac{8}{8} = 1\text{A}$$

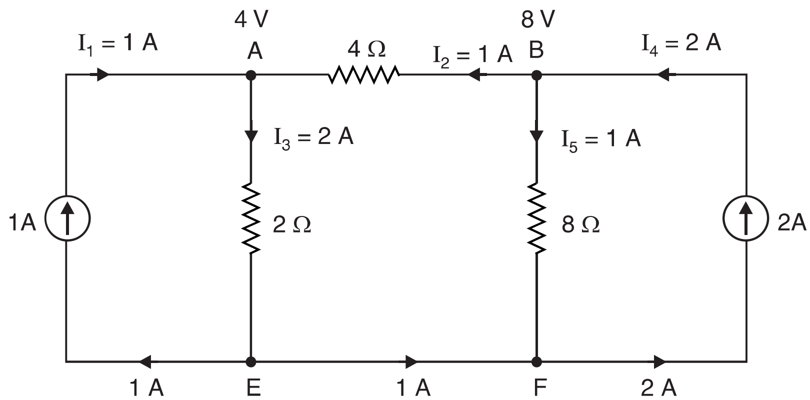

Fig. 3.29

* We assume that $V_A > V_B$. On solving the circuit, we shall see whether this assumption is correct or not.