

## Exercises 2-1

Q.1 Four universities - 1, 2, 3 and 4 are participating in a holiday basketball tournament. In the 1st round, 1 will play 2 and 3 will play 4. Then the two winners will play for the championship, and the two losers will also play. One possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4 in the 1st round games, and then 1 beat 3 and 2 beats 4).

- Ⓐ List all outcomes in S.
- Ⓑ Let A denote the event that 1 wins the tournament. List the outcomes in A.
- Ⓒ Let B denotes the event that 2 gets into championship game. List outcomes in B.
- Ⓓ What are the outcomes in  $A \cup B$  and in  $A \cap B$ ? What are the outcomes in  $A^c$ .

Ⓐ Ⓑ Ⓒ Ⓓ

So 1)  
Q-1      1 3 2 4 → In 1st round 1 wins 2  
                        2 3 wins 4  
In 2nd round 1 wins 3

2) 1 will be champion if 2 2 wins 4  
outcomes are  
1 3 2 4  
1 3 4 2  
1 4 2 3  
1 4 3 2

2 will be the champion if the outcome  
are  
2 3 1 4  
2 3 4 1  
2 4 1 3  
2 4 3 1

3 will be the champion if the outcome  
are

3 1 4 2  
3 1 2 4  
3 2 4 1  
3 2 1 4

4 will be champion if the outcome  
are

4 1 3 2  
4 1 2 3  
4 2 1 3  
4 2 3 1

(a) Sample space is

$$S = \{ 1324, 1342, 1423, 1432, \\ 2314, 2341, 2413, 2431, \\ 3142, 3124, 3241, 3214, \\ 4132, 4123, 4231, 4213 \}$$

(b) List of outcomes of A denote 1 wins the tournament  
= { 1324, 1342, 1423, 1432 }

(c) List of outcomes of B denote 2 gets into the championship game

= B is in either 1st or 2nd position

$$= \{ 2314, 2341, 2413, 2431, \\ 3241, 3214, 4231, 4213 \}$$

(d) Outcomes of  $A \cup B$

$$= \{ 1324, 1342, 1423, 1432, \\ 2314, 2341, 2413, 2431, \\ 3241, 3214, 4231, 4213 \}$$

Outcomes of  $A \cap B = \emptyset$

Q ②

Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L) or go straight (S). Consider observing the direction for each of three successive vehicles.

- (a) List all outcomes in the event A that all three vehicles go in the same directions.
- (b) List all outcomes in the event B that all three vehicles take different directions.
- (c) List all outcomes in the event C that exactly two of three vehicles turn right.
- (d) List all outcomes in the event D that exactly two vehicles go in the same directions.
- (e) List outcomes in  $D'$ ,  $C \cap D$  and  $C \cup D$ .

Sol

Q-2

Directions are

R → right

L → left

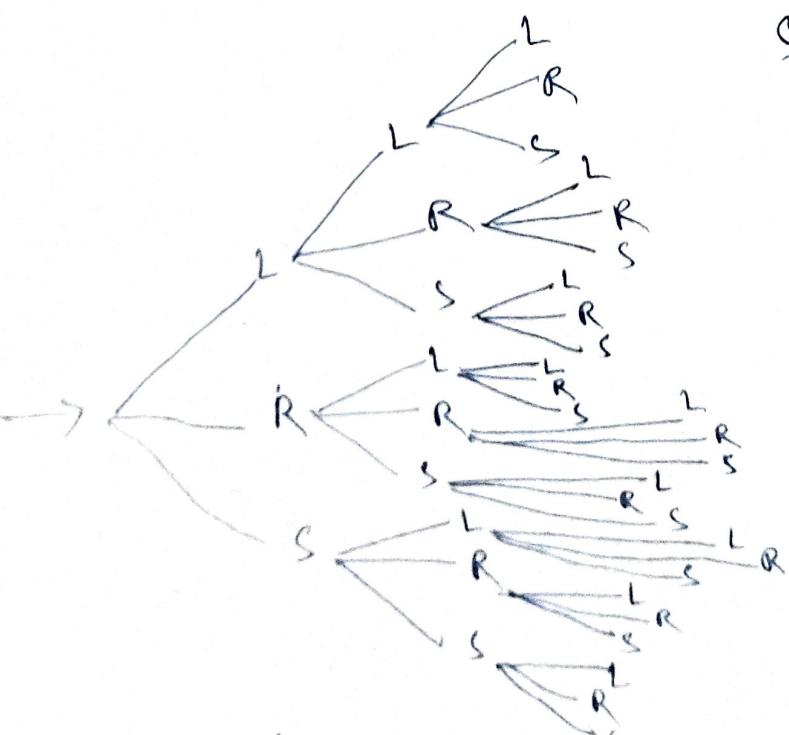
S → straight

Ⓐ A = ~~all three~~ vehicles go in the same direction

B = all three vehicles go in the different direction

C = exactly two vehicles ~~turn right~~  
~~go straight~~  
~~different directions~~

D = exactly two vehicles go in the same direction



Sample space S

= set of directions  
for three vehicles

= {LLL, LLR, LLS,  
LRL, LRR, LRS,  
RLL, RLQ, RLS,  
RRL, RRR, RRS,  
SSL, SLR, SLS,  
SRL, SRR, SRS,  
SSL, SSR, SSS}  
→ RSL, RSR, RSS}

Ⓐ A = {LLL, RRR, SSS}

Ⓑ B = {LRS, LSR, RLS, SLR, SRL, RSL}

$$\textcircled{c} \quad \text{Ans} = \{ \cancel{\text{LRR}}, \cancel{\text{RRL}}, \cancel{\text{RRS}}, \cancel{\text{RSR}}, \text{SRR}, \cancel{\text{RDS}} \\ \text{RLR} \}$$

$$C = \{ \text{LRR}, \text{RLR}, \text{RRL}, \text{RRS}, \text{RSR}, \text{SRR} \}$$

$$\textcircled{d} \quad D = \{ \text{LLR}, \text{LLS}, \text{LRl}, \text{LRR}, \text{LSL}, \text{LSS}, \\ \text{RLL}, \text{RLR}, \text{RRL}, \text{RRS}, \text{RSR}, \text{RSS}, \\ \text{SLL}, \text{SLS}, \text{SRR}, \text{SRS}, \text{SSL}, \text{SSR} \}$$

$$\textcircled{e} \quad D' = \{ \text{LLL}, \text{LRS}, \text{LSR}, \text{RLS}, \text{RRR}, \\ \text{RSL}, \text{SLR}, \text{SRL}, \text{SSS} \}$$

$$C \cup D = D \quad (\because C \subset D)$$

$$C \cap D = C \quad (\because C \subset D)$$

Q(3) Verify De Morgan's Laws

~~Ans~~  
~~Ans~~

$$(A \cup B)' = A' \cap B' \\ (A \cap B)' = A' \cup B'$$

$$\text{Sample space } S = \{1, 2, 3, 4\}$$

$$A = \{1, 2\}, B = \{2, 3\}$$

$$A' = \{3, 4\}, B' = \{1, 4\}$$

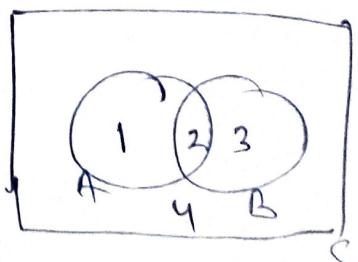
$$A \cup B = \{1, 2, 3\}, (A \cup B)' = \{4\}$$

$$A \cap B = \{2\}, (A \cap B)' = \{1, 3, 4\}$$

$$A' \cup B' = \{1, 3, 4\}, A' \cap B' = \{4\}$$

$$\text{Hence } (A \cup B)' = A' \cap B' = \{4\}$$

$$(A \cap B)' = A' \cup B' = \{4\}$$



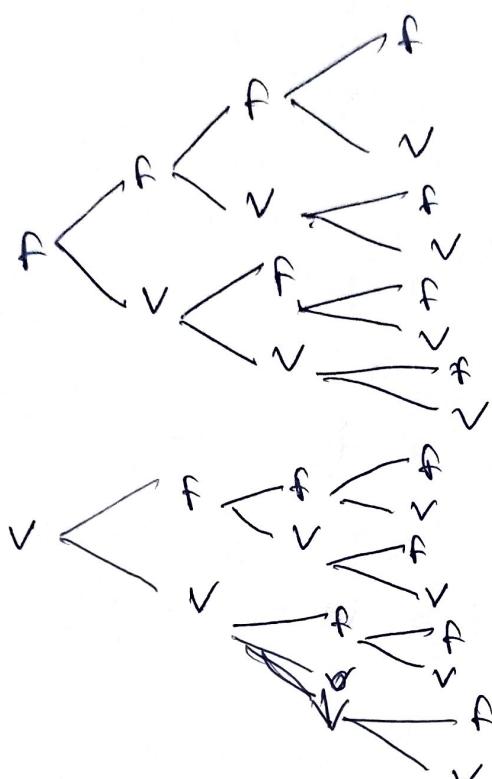
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Q(4) Each of a sample of four mortgages is classified as fixed rate (F) or variable-rate (V).

- (a) What are the 16 ( $= 2^4$ ) outcomes in S?
- (b) Which outcomes are in the event that exactly three of the selected mortgages are fixed rate?
- (c) Which outcomes are in the event that all four mortgages are of same type?
- (d) Which outcomes are in the event that at most one of four is a variable-rate mortgage?
- (e) What is the union of the events in parts (c) and (d), and what is the intersection of these two events?
- (f) What are the union and intersection of the two events in parts (b) & (c)?

Sol' @  $S = \{ \text{FFFF}, \text{FFFV}, \text{FFVF}, \text{FVFF}, \text{FFVV}, \text{FVFV}, \text{FVVF}, \text{FVVV}, \text{VVVV}, \text{VVVF}, \text{VVVF}, \text{VFVV}, \text{VVFF}, \text{VFVF}, \text{VFFF}, \text{VFVV} \}$

or  
soft @



$$S = \{ \text{ffff}, \text{fffv}, \text{ffvf}, \text{ffvv}, \text{fvff}, \text{fvfv}, \text{fvvf}, \text{fvvv}, \text{vfff}, \text{vvff}, \text{vfvf}, \text{vfvv}, \text{vvvf}, \text{vvvv} \}$$

(b)

$A =$  Exactly three of the selected mortgages are fixed rate

$$= \{ \text{ffffv}, \text{ffrvf}, \text{frvff}, \text{vffff} \}$$

(c)

$B =$  All mortgages are of same type

$$= \{ \text{ffff}, \text{vvvv} \}$$

(d)

$C =$  At most one of four is a valuable rate

$$= \{ \text{fffff}, \text{ffffv}, \text{ffvvf}, \text{fvvff}, \text{vffff} \}$$

(e)

$$B \cup C = \{ \text{ffff}, \text{vvvv}, \text{ffffv}, \text{ffvf}, \text{fvff}, \text{vfff} \}$$

$$B \cap C = \{ \text{ffff} \}$$

(f)

$$A \cup B = \{ \text{fffv}, \text{ffvf}, \text{fvff}, \text{vfff}, \text{ffff}, \text{vvvv} \}$$

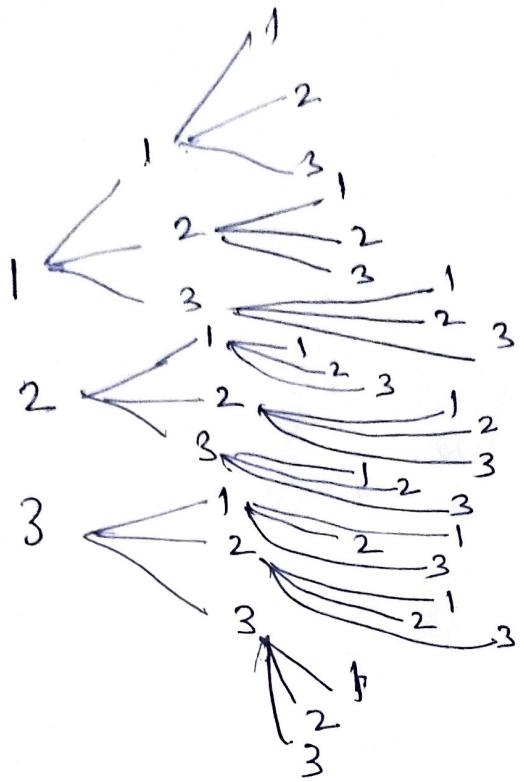
$$A \cap B = \emptyset$$

- Q(5) A family consisting of three persons A, B & C - goes to a medical clinic that always has a doctor at each station 1, 2 & 3. During a certain week, each member of the family visits the clinic once and is assigned ~~to~~ at random to a station. ~~The station~~  
The experiment consists of recording the station number for each member. One outcome is  $(1, 2, 1)$  for A to station 1, B to station 2 and C to station 1.
- a) List the 27 ( $= 3^3$ ) outcomes in the sample space  $\mathcal{S}$ .
  - b) List all the outcomes in the event that all three members go to same station.
  - c) List all outcomes in the event that all the members go to different stations
  - d) List all outcomes in the event that no one goes to station B.

Soln  
a) Sample space  $\mathcal{S} = \{(1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 3, 1), (1, 3, 2), (1, 3, 3), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 2, 1), (2, 2, 2), (2, 2, 3), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 1, 1), (3, 1, 2), (3, 1, 3), (3, 2, 1), (3, 2, 2), (3, 2, 3), (3, 3, 1), (3, 3, 2), (3, 3, 3)\}$

Sol Sample space  $S = \{(x, y, z) \mid A \rightarrow x, B \rightarrow y, C \rightarrow z\}$   
 $x, y, z \in \{1, 2, 3\}\}$

Ans



$$= \{(1,1,1), (1,1,2), (1,1,3), (1,2,1), (1,2,2), (1,2,3), (1,3,1), (1,3,2), (1,3,3), (2,1,1), (2,1,2), (2,1,3), (2,2,1), (2,2,2), (2,2,3), (2,3,1), (2,3,2), (2,3,3), (3,1,1), (3,1,2), (3,1,3), (3,2,1), (3,2,2), (3,2,3), (3,3,1), (3,3,2), (3,3,3)\}$$

(a) b

$A =$  All three members go to the same station

$$= \{(1,1,1), (2,2,2), (3,3,3)\}$$

(c)  $B =$  All members go to different stations

$$= \{(1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), (3,2,1)\}$$

(d)  $C =$  No one goes to station 2

$$= \{(1,1,1), (1,1,3), (1,3,1), (1,3,3), (3,1,1), (3,1,3), (3,3,1), (3,3,3)\}$$

Q(B) A college library has five copies of a certain text on reserve. Two copies (1 & 2) are first printings and the other three (3, 4 and 5) are 2nd printings.

A student examined those books in random order, stopping only when a 2nd printing had been selected. Other possible outcome is 5 and another is 213.

- ① List the outcomes in S.
- ② Let A denote the event that exactly one book must be examined. What outcomes are in A?
- ③ Let B be the event that book 5 is the one selected. What outcomes are in B?
- ④ Let C be the event that book 1 is not examined. What outcome are in C?  
Find unions & intersections A, B & C.  
The sample space is

Sol  
①

$$S = \{123, 124, 125, 213, 214, 215, 13, 14, 15, 23, 24, 25, 3, 4, 5\}$$

C : at most three books are selected & process stops when books from 2nd pt is selected

(b) A - exactly one book must be selected  
 $= \{3, 4, 5\}$

(c) B = Book 5 is selected  
 $= \{125, 215, 15, 25, 5\}$

(d) C = Book 1 is not examined  
 $= \{23, 24, 25, 3, 4, 5\}$

$$A \cup B = \{3, 4, 5, 15, 25, 125, 215\}$$

$$A \cap B = \{5\}$$

$$A \cup C = C \quad (\because A \subseteq C)$$

$$A \cap C = A \quad (\because A \subseteq C)$$

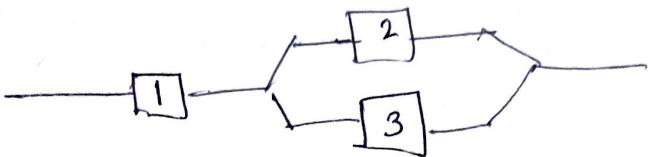
$$B \cup C = \{3, 4, 5, 15, 23, 24, 25, 125, 215\}$$

$$B \cap C = \{5, 25\}$$

$$A \cup B \cup C = \{3, 4, 5, 15, 23, 24, 25, 125, 215\}$$
$$= B \cup C$$

$$A \cap B \cap C = A \cap B = \{5\} \quad (\because A \cap C = A)$$

Q(9) Three components are connected to form a system as shown in the accompanying diagram.



Because the components <sup>in the 2-3 subsystem</sup> are connected in parallel that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must the 2-3 subsystem.

The experiment consists of determining the condition of each components

[S (success) for a functioning component and F (failure) for a nonfunctioning component]

(a) What outcomes are contained in the event A that exactly two out of the three components function?

(b) Which components are contained in the event B that at least two of the components function?

(c) which components are contained in the event  $C$  that the system functions?

(d) List outcomes in  $C$ ,  $A \cap C$ ,  $A \cap e$ , and  $B \cap e$

Sol: Let's denote the outcome SSF be component 1 functions, component 2 functions but component 3 does not.

(a) Event  $A$  = Exactly two out of three component function

$$= \{SSF, SFS, FSS\}$$

(b) Event  $B$  = At least two of components functions

$$= \{SSF, SFS, FSS, SSS\}$$

(c) Total system will function if component 1 will function and at least one of comp 2 & 3 will function. So the event  $e$  = the system functions

$$= \{SSS, SFS, SSF\}$$

(d) ~~stuck~~

d) the complement event

$$C' = \{ SFF, FSS, FSF \\ FFS, FFF \}$$

(System will not function if component 1 will not function or both 2-3 are not functioning)

<u>systems</u>		
1	2	3
S	S	S
S	S	F
S	F	S
F	F	F
F	S	S
F	S	F
F	F	S
F	F	F

$$A \cup C = \{ SSF, SFS, FSS, SSS \}$$

$$A \cap C = \{ SSF, SFS \}$$

$$B \cup C = \{ SSF, SFS, FSS, SSS \} \neq B$$

$$B \cap C = \{ SSF, SFS, SSS \} = C \quad (C \subset B)$$

Q.10 A small city has three automobile dealership: a GM dealer selling chevrolets and Buicks; a Ford seller selling Fords and Lincolns; and a Toyota dealer. If an experiment consists of observing the brand of the next car sold, then the events

$$A = \{\text{Chevrolet, Buick}\}$$

$$B = \{\text{Ford, Lincoln}\}$$

$$C = \{\text{Yaris, Camry, Fortuner}\}$$

$A, B, C$  are mutually exclusive

$A \cap B = \emptyset, A \cap C = \emptyset, B \cap C = \emptyset$ ; i.e.

$$A \cap B \cap C = \emptyset$$