$$\text{Maximum power} = \frac{V^2}{2X_C} = \frac{100^2}{2\times 1} = 5000 \text{ W}$$

16.10. R-L-C SERIES CIRCUIT

This is a general series a.c. circuit. Fig. 16.23 shows R, L and C connected in series across a supply voltage V (r.m.s.). The resulting circuit current is I (r.m.s.).

$\therefore$ Voltage across R, $V_R = IR$ ... V_R is in phase with I

Voltage across L, $V_L = IX_L$ ... where V_L leads I by 90°

Voltage across C, $V_C = IX_C$ ... where V_C lags I by 90°

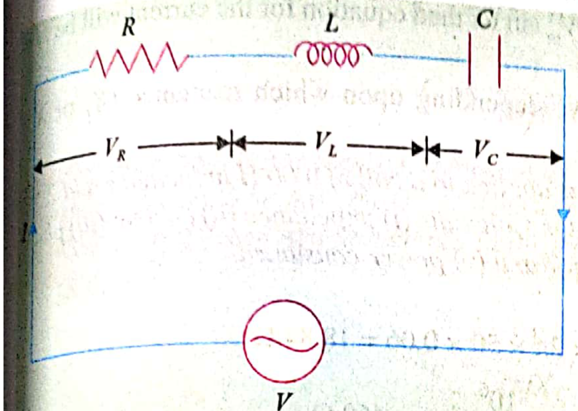

Fig. 16.23

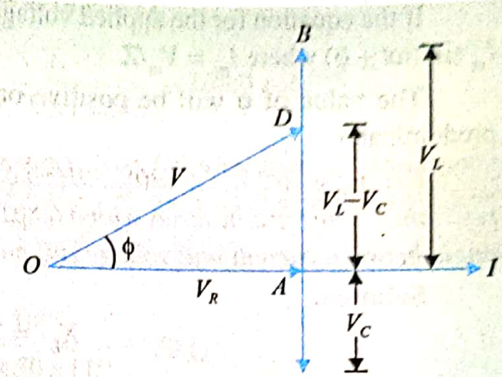

Fig. 16.24

As before, the phasor diagram is drawn taking current as the reference phasor. In the phasor diagram (See Fig. 16.24), OA represents V_R, AB represents V_L and AC represents V_C. It follows that the circuit can either be effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant. For the case considered, $V_L > V_C$ so that net voltage drop across L-C combination is $V_L - V_C$ and is represented by AD. Therefore, the applied voltage V is the phasor sum of V_R and $V_L - V_C$ and is represented by OD.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The quantity $\sqrt{R^2 + (X_L - X_C)^2}$ offers opposition to current flow and is called *impedance* of the circuit.

Circuit power factor, $$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \quad ...(i)$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad ...(ii)$$

Power consumed, $P = VI\cos\phi = {}^*I^2 R$

* $P = VI\cos\phi = (IZ)\,I \times \frac{R}{Z} = I^2 R$

This is expected because there is no power loss in L or C.

Three cases of R-L-C series circuit. We have seen that the impedance of a R-L-C series circuit is given by :

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

(i) When $X_L - X_C$ is positive (*i.e.* $X_L > X_C$), the phase angle ϕ is positive and the circuit is inductive.

(ii) When $X_L - X_C$ is negative (*i.e.* $X_C > X_L$), the phase angle ϕ is negative and the circuit is capacitive.

(iii) When $X_L - X_C$ is zero (*i.e.* $X_L = X_C$), the phase angle ϕ is zero and the circuit is purely resistive.

If the equation for the applied voltage is $v = V_m \sin \omega t$ then equation for the current will be : $i = I_m \sin(\omega t \pm \phi)$ where $I_m = V_m/Z$.

The value of ϕ will be positive or negative depending upon which reactance (X_L or X_C) predominates.

Example 16.24. *A 230V, 50Hz a.c. supply is applied to a coil of 0.06 H inductance and 2.5Ω resistance connected in series with a 6.8μF capacitor. Calculate (i) impedance (ii) current (iii) phase angle between current and voltage (iv) power factor and (v) power consumed.*

Solution.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.06 = 18.84\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 6.8} = 468\ \Omega$$

$$X = X_L - X_C = 18.84 - 468 = -449.16\ \Omega$$

(i) $$Z = \sqrt{R^2 + X^2} = \sqrt{(2.5)^2 + (-449.16)^2} = \mathbf{449.2\ \Omega}$$

(ii) $$I = V/Z = 230/449.2 = \mathbf{0.512\ A}$$

(iii) $$\phi = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{-449.16}{2.5} = \mathbf{-89.7°}$$

The negative sign with ϕ shows that current is leading the voltage.

(iv) $$p.f. = \cos\phi = R/Z = 2.5/449.2 = \mathbf{0.0056}\ \textit{lead}$$

(v) $$P = VI\cos\phi = 230 \times 0.512 \times 0.0056 = \mathbf{0.66\ W}$$

Example 16.25. *A resistance R, an inductance L = 0.01H and a capacitance C are connected in series. When an alternating voltage v = 400 sin (3000t –20°) is applied to the series combination, the current flowing is 10 $\sqrt{2}$ sin (3000t – 65°). Find the values of R and C.*

Solution.

$$\phi = 65° - 20° = 45°\ \textit{lag i.e.}\ \text{circuit is inductive.}$$

$$X_L = \omega L = 3000 \times 0.01 = 30\Omega$$

$$\tan 45° = X/R \quad \therefore \quad X = R$$

$$Z = V_m/I_m = 400/10\sqrt{2} = 28.3\Omega$$

$$Z^2 = R^2 + X^2 = R^2 + R^2 = 2R^2$$

$\therefore$ $$R = Z/\sqrt{2} = 28.2/\sqrt{2} = \mathbf{20\Omega}$$

Now $$X = X_L - X_C$$

$\therefore$ $$X_C = X_L - X = 30 - 20 = 10\Omega$$

$$\therefore \quad C = \frac{1}{\omega X_C} = \frac{1}{3000 \times 10} = 33.3 \times 10^{-6}\,F$$

Example 16.26. *A coil of resistance 8 Ω and inductance 0.03H is connected to an a.c. supply 240 V, 50 Hz. Calculate the value of capacitance which when connected in series with the above coil causes no change in the value of current and power taken from the supply.*

Solution.

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.03 = 9.424\ \Omega$$

To maintain the same current and power, the impedance of the circuit should not change. This is possible if the series capacitor has a capacitive reactance equal to twice the inductive reactance *i.e.*

$$X_C = 2X_L = 2 \times 9.424 = 18.848\ \Omega$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 18.848} = 168.9 \times 10^{-6}\,F$$

Example 16.27. *A coil of p.f. 0.8 is connected in series with a 110 μF capacitor. The supply frequency is 50 Hz. The p.d. across the coil is found to be equal to the p.d. across the capacitor. Calculate the resistance and inductance of the coil.*

Solution.

$$X_C = \frac{1}{2\pi f X_C} = \frac{10^6}{2\pi \times 50 \times 110} = 29\ \Omega$$

Now $\quad I Z_{coil} = IX_C$

$\therefore \quad Z_{coil} = X_C = 29\ \Omega$

For the coil, $\quad \cos\phi = R/Z_{coil}$

$\therefore \quad R = Z_{coil}\cos\phi = 29 \times 0.8 = 23.2\ \Omega$

Reactance of coil, $\quad X_L = Z_{coil}\sin\phi = 29 \times 0.6 = 17.4\ \Omega$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{17.4}{2\pi \times 50} = 0.055\ H$$

Example 16.28. *A resistance of 1 ohm, an inductance of 1 henry and a capacitance of 1 farad are connected in series across a voltage and the line current is 1A. Find the energy consumed in one hour.*

Solution. Energy is only consumed in R since L or C does not consume any energy.

Energy consumed $= I^2Rt = (1)^2 \times 1 \times 3600 = 3600$ J

TUTORIAL PROBLEMS

1. A circuit is made up of 5Ω resistance, 0.2 H inductance and 60 μFcapacitance in series. If the circuit current is 20A at a supply frequency of 50Hz, find the applied voltage. **[219 V]**
2. A series circuit consists of a non-inductive resistor of 10Ω, an inductor having a reactance of 50Ω and a capacitor having a reactance of 30Ω. It is connected to a 230V a.c. supply. Calculate (*i*) current (*ii*) the power consumed and (*iii*) the power factor.
   [(*i*) 10.3A (*ii*) 1060W (*iii*) 0.447 lag]
3. A coil of resistance 20Ω is in series with an inductance of 0.04H. A supply of 230V, 50Hz is applied to the combination. Determine the capacitance which when connected in series with the coil causes no change in the magnitude and power taken from the supply.
   [67.5 μF]