Exc:- 4.2

11) $$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{4}, & 0 \le x < 2 \\ 1, & 2 \le x \end{cases}$$

a) $P(X \le 1) = F(1) = \frac{1}{4}$

b) $P(.5 \le x \le 1) = F(1) - F(0.5)$

$= \frac{1}{4} - \frac{0.5^2}{4} = \frac{1 - 0.5^2}{4} = 0.1875$

c) $P(X > 1.5) = 1 - P(X \le 1.5) = 1 - F(1.5) = 1 - \frac{(1.5)^2}{4} = 0.4375$

d) $F(\tilde{\mu}) = 0.5$

$$\frac{\tilde{\mu}^2}{4} = 0.5$$

$$\Rightarrow \tilde{\mu}^2 = 2$$

$$\Rightarrow \tilde{\mu} = \sqrt{2} = 1.414$$

→ This is the only case when we can find $\tilde{\mu}$, when there is a variable.

Ex:- 4.2

1) a) We know that, $f(x) = F'(x)$

$$f(x) = \frac{d}{dx}\left(\frac{x^2}{4}\right) = \frac{2x}{4} = \frac{x}{2}, \quad 0 \le x < 2$$

$$f(x) = \begin{cases} \frac{x}{2}, & 0 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$$

f) $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$ $+0+0$

$= \int_{0}^{2} x \times \frac{x}{2}\, dx = \frac{1}{2} \times \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{6} = \frac{4}{3} = 1.33$

g) $E(x^2) = \int_{0}^{2} x^2 \times \frac{x}{2}\, dx = \frac{1}{2} \times \left[\frac{x^4}{4}\right]_0^2 = \frac{16}{8} = 2$

$\therefore V(x) = E(x^2) - \{E(x)\}^2$

$= 2 - \frac{16}{9}$

$= 0.222$

$\therefore \sigma_x = \sqrt{0.222} = 0.4712$

h) $E(h(x)) = \int_{-\infty}^{\infty} h(x) f(x)\, dx$

$= \int_{0}^{2} x^2 \times \frac{x}{2}\, dx + 0 + 0$

$= 2$

PAGE No.
DATE / /

12) $F(x) = \begin{cases} 0, & x < -2 \\ \frac{1}{2} + \frac{3}{32}\left(4x - \frac{x^3}{3}\right), & -2 \leq x < 2 \\ 1, & 2 \leq x \end{cases}$

~~a) $P(X<0) = \int_{-\infty}^{0} f(x)\,dx$~~

~~$= \int_{-2}^{0} \left\{\frac{1}{2} + \frac{3}{32}\left(4x - \frac{x^3}{3}\right)\right\} dx$~~

~~$= 0.325$ [Calculated directly from calculator]~~

a) $P(X<0) = F(0)$

$= \frac{1}{2} + \frac{3}{32} \times 0 = \frac{1}{2} = 0.5$

b) $P(-1<X<1) = F(1) - F(-1)$

$= \frac{1}{2} + \frac{3}{32}\left(4 - \frac{1}{3}\right) - \left(\frac{1}{2} + \frac{3}{32}\left(-4 + \frac{1}{3}\right)\right)$

$= \frac{1}{2} + \frac{3}{32}\left(4 - \frac{1}{3}\right) - \frac{1}{2} + \frac{3}{32}\left(4 - \frac{1}{3}\right)$

$= 2 \times \frac{3}{32}\left(4 - \frac{1}{3}\right)$

$= 0.6875$

c) $P(X > 0.5) = 1 - P(X \leq 0.5)$

$= 1 - F(0.5)$

$= 1 - \left\{\frac{1}{2} + \frac{3}{32}\left(4 \times 0.5 - \frac{0.5^3}{3}\right)\right\}$

$= 0.316$

d) $f(x) = F'(x)$

$\therefore \frac{d}{dx}\left(\frac{1}{2} + \frac{3}{32}\left(4x - \frac{x^3}{3}\right)\right)$

$= 0 + \frac{3}{32}\frac{d}{dx}\left(4x - \frac{x^3}{3}\right)$

$= \frac{3}{32} \times \left(4 - \frac{1}{3} \times 3x^2\right)$

$= \frac{3}{32}\left(4 - x^2\right), \quad -2 \leq x < 2$

$\frac{d}{dx}(0) = 0, \quad x < -2$

$\frac{d}{dx}(1) = 0, \quad 2 \leq x$

PAGE No.

DATE / /

$$f(x) = \begin{cases} \frac{3}{32}(4-x^2), & -2 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$$

e) $F(\tilde{\mu}) = 0.5$

$\Rightarrow \frac{1}{2} + \frac{3}{32}\left(4\tilde{\mu} - \frac{\tilde{\mu}^3}{3}\right) = 0.5$

$\Rightarrow 4\tilde{\mu} - \frac{\tilde{\mu}^3}{3} = 0$

$\Rightarrow \tilde{\mu}\left(4 - \frac{\tilde{\mu}^2}{3}\right) = 0$

$\Rightarrow \tilde{\mu} = 0, \quad | \quad 4 - \frac{\tilde{\mu}^2}{3} = 0$

$\Rightarrow \tilde{\mu}^2 = 12$

$\Rightarrow \tilde{\mu} = \pm\sqrt{12}$

$\Rightarrow \tilde{\mu} = \pm 3.464$

So, $\tilde{\mu} = 0$ Hence, proved.

12) $f(x) = \begin{cases} \frac{k}{x^4}, & x > 1 \\ 0, & x \le 1 \end{cases}$

a) we know that, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_1^{\infty} \frac{k}{x^4}\,dx = 1$

x^{-4}

$\frac{x^{-3}}{-3}$

$\Rightarrow \left[\frac{-k}{3x^3}\right]_1^{\infty} = 1$

$\Rightarrow \left[0 + \frac{k}{3}\right] = 1$

$\Rightarrow k = 3$

b) $F(x) = \int_{-\infty}^{x} f(x)\,dx$

$= \int_1^x \frac{k}{x^4}\,dx$

$= 3 \times \left(-\frac{1}{3x^3}\right)_1^x = \left(\frac{-1}{x^3}\right)_1^x = 1 - \frac{1}{x^3}, \quad x > 1$

DATE / /

$$F(x) = \begin{cases} 1 - \frac{1}{x^3}, & x > 1 \\ 0, & x \le 1 \end{cases}$$

c) $P(2 < X < 3)$

$= F(3) - F(2)$

$= 1 - \frac{1}{27} - 1 + \frac{1}{8}$

$= 0.088$

$P(X > 2)$

$= 1 - P(X \le 2)$

$= 1 - F(2)$

$= 1 - 1 + \frac{1}{8}$

$= \frac{1}{8}$

$= 0.125$

d) $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$

$= \int_{1}^{\infty} \left(x \times \frac{3}{x^4} \right) dx$

$= 3\int_{1}^{\infty} \frac{dx}{x^3}$

$= \left[\frac{-3}{2x^2} \right]_1^{\infty}$

$= \left[0 + \frac{3}{2} \right]$

$= 1.5$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

$= \int_{1}^{\infty} \left(x^2 \times \frac{3}{x^4} \right) dx$

$= 3\int_{1}^{\infty} \frac{1}{x^2}\, dx$

$= \left[\frac{-3}{x} \right]_1^{\infty}$

$= \left[0 + 3 \right] = 3$

$$V(x) = E(x^2) - \{E(x)\}^2$$
$$= 3 - (1.5)^2$$
$$= 0.75$$
$$\sigma = \sqrt{V(x)} = \sqrt{0.75} = 0.866$$

e) $P(E(x) - \sigma_x < X < E(x) + \sigma_x)$

$$= P(1.5 - 0.866 < X < 1.5 + 0.866)$$
$$= P(0.634 < X < 2.366)$$
$$= F(2.366) - F(0.634)$$
$$= 1 - \frac{1}{2.366^3} \cancel{- 1 + \frac{1}{0.6}} - 0$$
$$= 0.924$$

$= 2$

15) $f(x) = \begin{cases} 90x^8(1-x), & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$

x is between 0 and 1

$F(x) = \int_{-\infty}^{x} f(x)\,dx$

$= 0 + \int_{0}^{x} 90x^8(1-x)\,dx$

$= 90\int_{0}^{x} (x^8 - x^9)\,dx$

$= 90 \times \left[\frac{x^9}{9} - \frac{x^{10}}{10}\right]_0^x$

$= 90 \times \left[\frac{x^9}{9} - \frac{x^{10}}{10} \cancel{- 0 + 0}\right] = 10x^9 - 9x^{10};$

$\cancel{= 90 \times \frac{1}{90} = 1}$

PAGE No
DATE / /

~~b) $F(0.5) = P(X \le 0.5) =$~~

$$F(x) = \begin{cases} 0, & x \le 0 \\ 10x^9 - 9x^{10}, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$$

b) $P(X \le 0.5) = F(0.5) = 10 \times (0.5)^9 - 9(0.5)^{10}$

$= 0.0107$

c) $P(0.25 < X \le 0.5) = F(0.5) - F(0.25)$

$= 0.0107 - \{10 \times (0.25)^9 - 9(0.25)^{10}\}$

$= 0.0107$

$P(0.25 \le X \le 0.5) = 0.0107$

d)

100 pth percentile $= F\{\eta(p)\}$ $\qquad 100 \times p = 75 \Rightarrow p$

$= \int_{-\infty}^{\eta(p)} f(y)\,dy$

$p = 0.75$ $\qquad 100 \times p = 75$
$\Rightarrow p = 0.75$

$0.75 = F\{\eta(p)\}$

$0.75 = \int_{-\infty}^{\eta(p)} f(x)\,dx$ $\qquad$ Considering $\eta(p)$ lying between 0 and 1

$\Rightarrow 0.75 = \int_0^{\eta(p)} 90x^8(1-x)\,dx$

$\Rightarrow 0.75 = \left[10x^9 - 9x^{10}\right]_0^{\eta(p)}$

$\boxed{\Rightarrow 0.75 = 10 \times \eta^9 - 9\eta^{10}}$ $\quad \{\eta = \eta(p)\}$

~~$\Rightarrow \eta^9(10 - 9\eta) - 0.75 = 0$~~ $\Rightarrow \eta = 0.9036$

e) $E(x) = \int_{-\infty}^{\infty} f(x)\,dx$

$= \int_0^1 90x^8(1-x)x\,dx = 90\int_0^1 (x^9 - x^{10})\,dx$

$$= 90 \times \left[\frac{x^{10}}{10} - \frac{x^{11}}{11}\right]_0^1 = 90 \times \left[\frac{1}{10} - \frac{1}{11}\right] = \frac{90}{110}$$

$$= 0.82$$

$$E(x^2) = \int_0^1 90x^{10}(1-x) = 90 \times \left[\frac{x^{11}}{11} - \frac{x^{12}}{12}\right]_0^1 = 0.682$$

$$V(x) = E(x^2) - \{E(x)\}^2 = 0.682 - (0.82)^2 = 0.0096$$

$$\sigma_x = \sqrt{0.0096} = 0.098$$

f) $P(X > E(x) + \sigma_x)$ $\nearrow$ $\pm$ for continuous only

$= P(X > 0.82 \pm 0.098)$

$= P(X > 0.918)$

$= 1 - P(X < 0.918)$

$= 1 - F(0.918)$

$= 1 - [10 \times (0.918)^9 - 9 \times (0.918)^{10}]$

$= 0.195$

$P(0.722 < X < 0.918)$

$= F(0.918) - F(0.722)$

$= 0.195 - [10 \times 0.722^9 - 9 \times 0.722^{10}]$

$= 0.0083$

19) $F(x) = \begin{cases} 0, & x \le 0 \\ \frac{x}{4}\left[1+\ln\left(\frac{4}{x}\right)\right], & 0<x\le 4 \\ 1, & x>4 \end{cases}$

a) $P(X\le 1) = F(1) = \frac{1}{4}\left[1+\ln 4\right] = 0.596$

b) $P(1\le X\le 3) = F(3)-F(1) = \frac{3}{4}\left[1+\ln\left(\frac{4}{3}\right)\right] - 0.596$

$= 0.37$

c) We know that, $f(x) = F'(x)$

$$\frac{d}{dx}\left(\frac{x}{4}\left(1+\ln\left(\frac{4}{x}\right)\right)\right)$$

$$= \frac{x}{4} \times \frac{d}{dx}\left(1+\ln\left(\frac{4}{x}\right)\right) + \left(1+\ln\left(\frac{4}{x}\right)\right) \times \frac{d}{dx}\left(\frac{x}{4}\right)$$

$$= \frac{x}{4}\left(0+ \frac{\cancel{x}}{\cancel{4}} \times \cancel{4} \times (-)\frac{1}{x^{\cancel{2}}}\right) + \left(1+\ln\left(\frac{4}{x}\right)\right) \times \frac{1}{4}$$

$$= -\frac{\cancel{x}}{4}\left(\frac{1}{\cancel{x}}\right) + \frac{1}{4} + \frac{1}{4}\ln\left(\frac{4}{x}\right)$$

$$= -\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\ln\left(\frac{4}{x}\right)$$

$= \frac{1}{4}\ln\left(\frac{4}{x}\right), \quad 0 < x \le 4$

$\frac{d}{dx}(0) = 0, \quad x \le 0$

$\frac{d}{dx}(1) = 0, \quad x > 4$

$$f(x) = \begin{cases} \frac{1}{4}\ln\left(\frac{4}{x}\right), & 0 < x \le 4 \\ 0, & \text{otherwise} \end{cases}$$

20) $$f(y) = \begin{cases} \frac{1}{25}y, & 0 \le y < 5 \\ \frac{2}{5} - \frac{1}{25}y, & 5 \le y < 10 \\ 0, & \text{otherwise} \end{cases}$$

$$F(y) = \int_{-\infty}^{y} f(y)\,dy$$

$$= \cancel{0+}\int_0^5 \frac{y}{25}\,dy + \int_5^{10}\left(\frac{2}{5} - \frac{1}{25}y\right)dy \cancel{+0}$$

When $0 \le y < 5$,

$$F(y) = \int_{-\infty}^{y} f(y)\,dy$$

$$= \int_0^y \frac{y}{25}\,dy$$

$$= \frac{1}{25} \times \left[\frac{y^2}{2}\right]_0^y$$

$$= \frac{1}{50} \times y^2$$

When $5 \le y \le 10$

$$F(y) = \int_5^y \left(\frac{2}{5} - \frac{y}{25}\right)dy + \int_0^5 \frac{y}{25}\,dy$$

$$= \frac{2}{5}[y]_5^y - \frac{1}{25} \times \left[\frac{y^2}{2}\right]_5^y + 0.5$$

$$= \frac{2}{5}[y-5] - \frac{1}{50} \times [y^2 - 25] + 0.5$$

$= \frac{2y}{5} - 2 - \frac{y^2}{50} + 1$

$= -\frac{y^2}{50} + \frac{2y}{5} - 1, \quad 5 \le y \le 10$

$$F(y) = \begin{cases} 0, & y < 0 \\ \frac{y^2}{50}, & 0 \le y < 5 \\ -\frac{y^2}{50} + \frac{2y}{5} - 1, & 5 \le y \le 10 \\ 1, & y > 10 \end{cases}$$

b) $p = \int_{-\infty}^{\eta(p)} f(y)\,dy$

when, $0 \le \eta(p) < 5$

$p = \int_{0}^{\eta(p)} \frac{y}{25}\,dy$

$\Rightarrow p = \left[\frac{1}{25} \times \frac{y^2}{2}\right]_0^{\eta(p)}$ Let us say, $\eta(p) = \eta$

$\Rightarrow p = \frac{\eta^2}{50} \Rightarrow \eta = \pm\sqrt{50p} \Rightarrow \eta = \sqrt{50p}$ as η cannot be negative $[\because 0 \le \eta < 5]$

when, $5 \le \eta \le 10$

$-\frac{\eta^2}{50} + \frac{2\eta}{5} - 1 = p$

$\Rightarrow \frac{\eta^2}{50} - \frac{2\eta}{5} + 1 + p = 0$

$\Rightarrow \eta = \dfrac{\frac{2}{5} \pm \sqrt{\frac{4}{25} - 4 \times \frac{1}{50} \times (1+p)}}{\frac{2}{50}}$

$\Rightarrow \eta = 10 \pm 25\sqrt{\frac{4}{25} - 4 \times \frac{1}{50}(1+p)}$

$\Rightarrow \eta = 10 \pm \sqrt{-50p + 50}$

$\because 5 \le \eta \le 10$

$\eta = 10 - \sqrt{50 - 50p}$

PAGE No.
DATE / /

c) $E(\cancel{x}^{Y}) = \int_{-\infty}^{\infty} y f(y) dy$

$= \int_{0}^{5} y \times \frac{y}{25} dy + \int_{5}^{10} y \times \left(\frac{2}{5} - \frac{y}{25}\right) dy$

$= 5$ (calculated directly from calculator)

~~V(Y) = E~~

$E(Y^2) = \int_{0}^{5} y^2 \times \frac{y}{25} dy + \int_{5}^{10} y^2 \times \left(\frac{2}{5} - \frac{y}{25}\right) dy$

$= 29.167$

$V(X) = E(X^2) - \{E(X)\}^2$

$= 29.167 - 25$

$= 4.167$

E(Y) for a single bus when it is uniformly distributed on (0,5)

$= \frac{0+5}{2} = 2.5$

σ for a single bus $= \frac{5-0}{\sqrt{12}} = \frac{5}{\sqrt{12}}$

$\sigma^2 = V(Y)$ for a single bus $= \frac{25}{12} = 2.083$

So, we ~~see th~~ observed that the expected waiting time and variance for a single bus is half that ~~of~~ for two buses

21) $f(r) = \begin{cases} \frac{3}{4}(1-(10-r)^2) & , 9 \le r \le 11 \\ 0 & , \text{otherwise} \end{cases}$

Area of the circle $= \pi r^2$

$E(\pi r^2) = \int_{-\infty}^{\infty} \pi r^2 f(r)\, dr$

$$E(\pi r^2) = \int_9^{11} \pi r^2 \frac{3}{4}(1-(10-r)^2)\, dr + 0 + 0$$

$= 314.29$

PAGE No.
DATE / /

22) a) $f(x) = \begin{cases} 2\left(1-\frac{1}{x^2}\right), & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{1}^{x} 2\left(1-\frac{1}{x^2}\right) dx$$

$$= 2\int_{1}^{x} \left(1 - x^{-2}\right) dx$$

$$= 2\left[[x]_1^x + [x^{-1}]_1^x \right]$$

$$= 2\left[x-1 + \left[\frac{1}{x} - 1\right]\right]$$

$$= 2\left[x+\frac{1}{x} - 2\right]$$

$$= 2\left(x+\frac{1}{x}\right) - 4$$

$$F(x) = \begin{cases} 0, & x \le 1 \\ 2\left(x+\frac{1}{x}\right); & 1 < x < 2 \\ 1; & x \ge 2 \end{cases}$$

b) 100 pth percentile $= \int_{-\infty}^{\eta(p)} f(y)\,dy$

$p = F(\eta(p))$

$$\Rightarrow p = \int_{1}^{\eta(p)} 2\left(1-\frac{1}{y^2}\right) dy$$

$$\Rightarrow p = 2\left[y + \frac{1}{y}\right]_1^{\eta(p)}$$

$\boxed{\eta(p) = \eta \text{ (say)}}$

$$\Rightarrow p = 2\left[\eta + \frac{1}{\eta} - 2\right]$$

$$\Rightarrow 2\times\left(\frac{\eta^2+1}{\eta}\right) - 4 = p$$

$$\Rightarrow 2\eta^2 + 2 - 4\eta = p\eta$$

$\Rightarrow 2\eta^2 - (4+p)\eta + 2 = 0$

$\Rightarrow \eta = \dfrac{4+p \pm \sqrt{(4+p)^2 - 16}}{4}$

$F(\tilde{\mu}) = 0.5 \rightarrow p$

So, $p = 0.5$, $\eta = \tilde{\mu}$

$\therefore \tilde{\mu} = \dfrac{4+0.5 \pm \sqrt{(4+0.5)^2 - 16}}{4}$

$\tilde{\mu} = 1.64, 0.6096$

c) $E(x) = \int_{-\infty}^{\infty} x f(x) = \int_1^2 x \times 2\left(1 - \frac{1}{x^2}\right) dx$

$= 1.614$

$V(x) = E(x^2) - \{E(x)\}^2$

$E(x^2) = \int_1^2 x^2 \times 2x\left(1 - \frac{1}{x^2}\right) dx = 2.67$

$V(x) = 2.67 - (1.614)^2$

$= 0.065$

d) $h(x) = 1.5 - X$ The amount left $= h(x)$

$E(h(X)) = E(1.5 - X) = 1.5 - E(X) = 1.5 - (1.614) = -ve$

$h(x) = \max\{1.5 - X, 0\} = 0$

23) $E(x) = 120°C$

$\sigma = 2°C$

$E(1.8X + 32) = 1.8E(X) + 32$

$E(x)$ in fahrenheit $= (1.8 \times 120 + 32) = 248°F$

σ in fahrenheit $=$ ~~$1.8 \times 2 + 32 = 35.6°F$~~ $|a|\sigma_x$

$= (1.8) \times 2$

$= 3.6$

$E(1.8X + 32)$

$1.8E(x) + 32$

$V(1.8X + 32)$

$= 1.8^2 \times 2$

22(d) $E(h(X)) = E(1.5 - X) = \int_{1}^{1.5} (1.5 - x)\, f(x)\, dx$

$$= \int_{1}^{1.5} (1.5 - x) * 2\left(1 - \frac{1}{x^2}\right) dx$$

$$= 0.061$$

we have read ~~unf~~ uniform distribution earlier.

2A) $f(x;k,\theta) = \begin{cases} \frac{k\cdot\theta^k}{x^{k+1}}, & x \geq \theta \\ 0, & x < \theta \end{cases}$

i) $E(X) = \int_{-\infty}^{\infty} x f(x;k,\theta)\, dx$

$= \int_{\theta}^{\infty} x \frac{k\cdot\theta^k}{x^{k+1}}\, dx$ $\quad x^{-k+1}$

$= k\cdot\theta^k \int_{\theta}^{\infty} \frac{dx}{x^k}$

$= \frac{k\cdot\theta^k}{1-k} \times \left[x^{-k+1}\right]_{\theta}^{\infty}$

PAGE No.
DATE / /

$$= \frac{-k\cdot\theta^k}{1-k} \times \left(\theta^{-k+1}\right)$$

$$= \frac{-k\cdot\theta}{1-k}$$

$$E(x) = \frac{k\cdot\theta}{k-1}$$

b) when $k=1$,

$$E(X) = \int_{\theta}^{\infty} \frac{x\cdot\theta}{x^2}\, dx$$

$$= \theta \int_{\theta}^{\infty} \frac{dx}{x}$$

$$= \theta \left[\ln|x|\right]_{\theta}^{\infty}$$

$$E(x) = \theta\left[\ln(\infty) - \ln\theta\right]$$

$\therefore$ $E(x)$ will be undefined in this case.

c) $$E(x^2) = \int_{\theta}^{\infty} x^2 \times \frac{k\cdot\theta^k}{x^{k+1}}\, dx$$

x^{2-k-1}

x^{1-k}

$$= k\cdot\theta^k \int_{\theta}^{\infty} x^{1-k}\, dx$$

$$= k\cdot\theta^k \times \left[\frac{x^{1-k+1}}{1-k+1}\right]_{\theta}^{\infty}$$

$$= \frac{k\cdot\theta^k}{2-k}\left[x^{2-k}\right]_{\theta}^{\infty}$$

Given that $k>2$

$k\cdot\theta^2$

$$= \frac{k\cdot\theta^k}{2-k}\left(0 - \theta^{2-k}\right)$$

$$= \frac{-k\cdot\theta^2}{2-k}$$

$$= \frac{k\cdot\theta^2}{k-2}$$

$$\therefore V(x) = E(x^2) - \{E(x)\}^2$$

$$= \frac{k\cdot\theta^2}{k-2} - \frac{k^2\theta^2}{(k-1)^2} = k\theta^2\left[\frac{1}{k-2} - \frac{k}{(k-1)^2}\right]$$

PAGE No.
DATE / /

$$= k\theta^2\left[\frac{(k-1)^2 - k(k-2)}{(k-2)(k-1)^2}\right]$$

$$= k\theta^2\left(\frac{\cancel{k^2} - \cancel{2k} + 1 - \cancel{k^2} + \cancel{2k}}{(k-2)(k-1)^2}\right)$$

$$= \cancel{k\theta^2}\left(\frac{k^2 - \cancel{3k} + \cancel{1}}{\cancel{(k-2)(k-1)^2}}\right)$$

$$= k\theta^2\left(\frac{1}{(k-2)(k-1)^2}\right)$$

Hence, proved.

d) $V(x)$ will be infinite if $k = 2$

e) $k > n$ for $E(X^n)$ to be finite

25) a)

PAGE No.

DATE / /

Ex: 4.2

25) a) $P(Y \leq 1.8\tilde{\mu}+32)$

$= P(1.8X+32 \leq 1.8\tilde{\mu}+32)$

$= P(X \leq \tilde{\mu})$

$= 0.5$ [$\because$ The median of X is $\tilde{\mu}$]

So, $1.8\tilde{\mu}+3$ is the median of Y.

b) The 90th percentile of Y equals $= 1.8\eta(0.9)+32$

where $\eta(0.9)$ is the 90th percentile of X.

$P(Y \leq 1.8\eta(0.9)+32)$

$= P(1.8X+32 \leq 1.8\eta(0.9)+32)$

$= P(X \leq \eta(0.9))$

$= 0.9$ [$\because$ $\eta(0.9)$ is the 90th percentile of X]

So, $1.8\eta(0.9)+32$ is the 90th percentile of Y.

c) $Y = aX+b$

Let, $\eta(p)$ is the 100 pth percentile of X.

Then the 100 pth percentile $= a\eta(p)+b$