



Engineering

Economics :

Economics

It is a social science which studies human behaviour.

* Definition given by Adam Smith (father of Economics)

He defined Economics as an enquiry into the nature and causes of wealth of the nation.

* Definition given by Robbins.

He defined Economics as a science which studies Human behaviour as a relationship between ends and scarce means which have alternative uses.

Engineering Economics

* Definition given by L. Crampt (father of E. Economics)

He defined E. Economics as that branch of Economics which deals with method to make one that can enable one to make economic decisions towards evaluation of projects and engineering alternatives.

Nature of Engineering

Economics :

Four central problems

- ① what to produce?
- ② How to Produce?

- (3) For whom to produce?
- (4) Economic growth problems.

Scope of Engineering Economics :

* Demand :

It refers to the effective desire to have something backed up by the ability and willingness to pay for it.

* Demand Schedule :

It refers to the tabular representation of different quantities of a commodity demanded at different prices at a point of given point of time.

Price of x (in Rs) quantity demand
for x (in units)

10	2
8	4
6	6
4	8

where x is any commodity.



Types of Demand Schedule:

* Individual Demand Schedule.

* Market Demand Schedule.

Individual demand schedule.

It refers to the tabular representation of different quantities of a commodity demanded at different prices by an individual consumer at a given point of time.

Price of pen (in Rs)	Quantity demanded from pen. (in units)
50	5.
30	7.
20	8
10	10.

Individual Market demand Schedule:

It refers to the tabular representation of different quantities of a commodity demanded at different prices in the market at a given point of time.

Price of x (in Rs)	Quantity D by 1 (in units)	Quantity D by 2 (in units)	Quantity D by 3 (in units)	Market Demand
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40

1

2

3

6

30

2

3

4

9

20

3

4

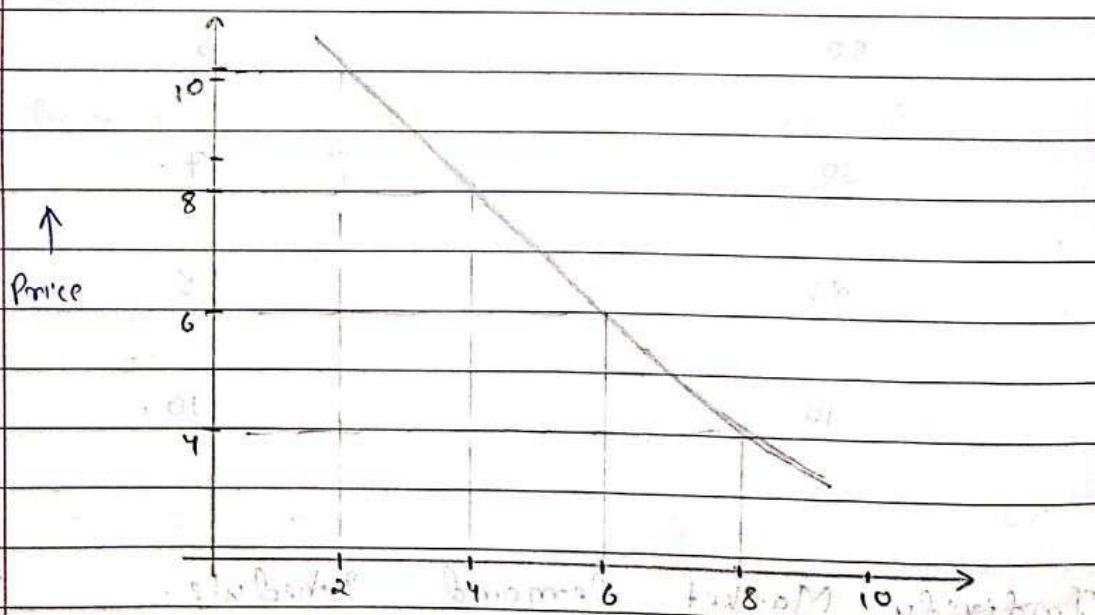
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14

Demand curve.

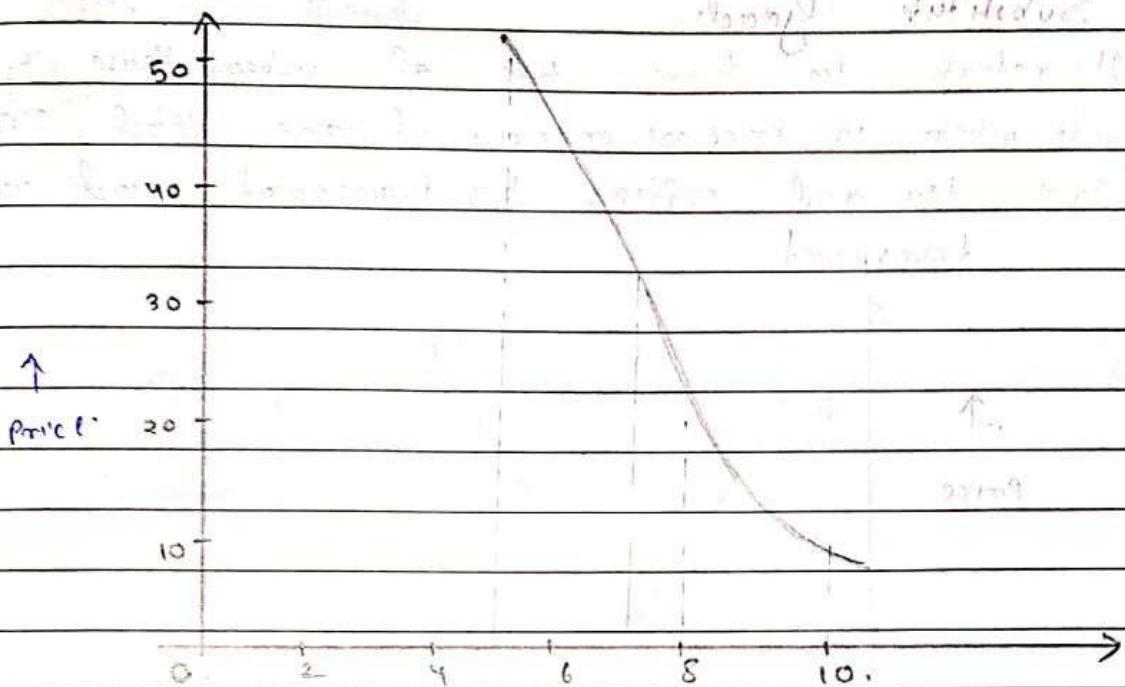
It refers to the graphical representation of demand schedule.

(Demand curve)



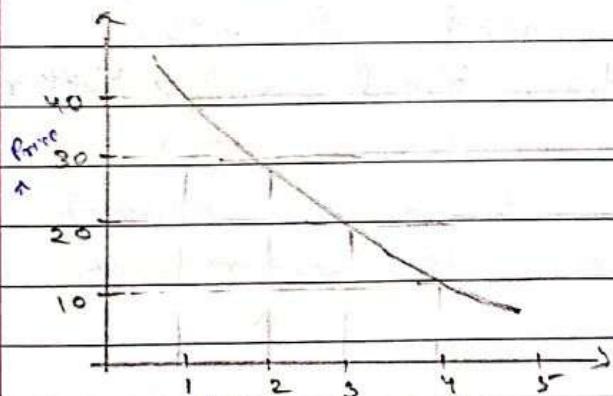
Quantity demand \rightarrow

Individual Demand Curve.



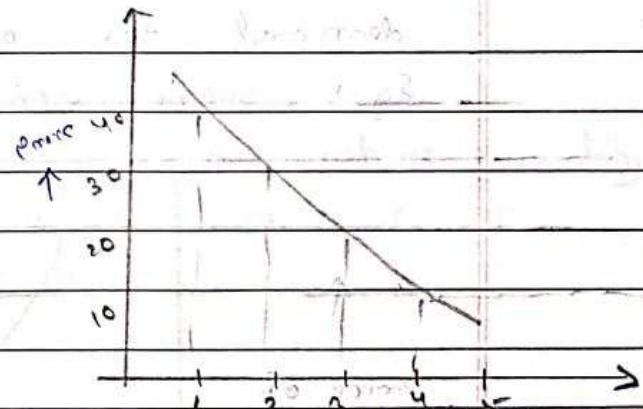
Quantity demand. \rightarrow

Individual demand
curve for cons. 1

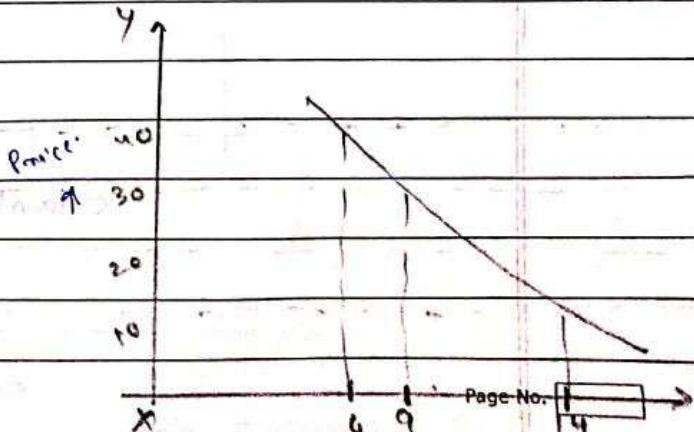
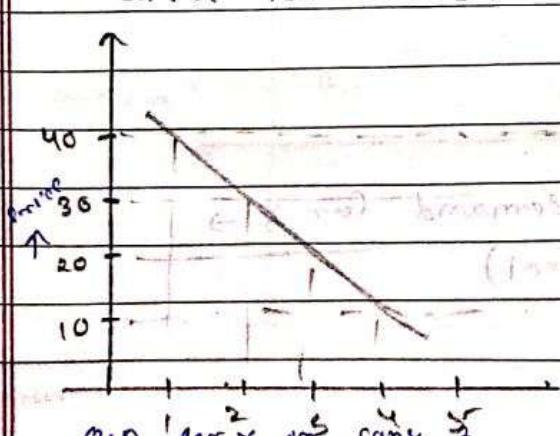


individual demand
curve for cons 3.

Individual demand
curve for cons. 2.

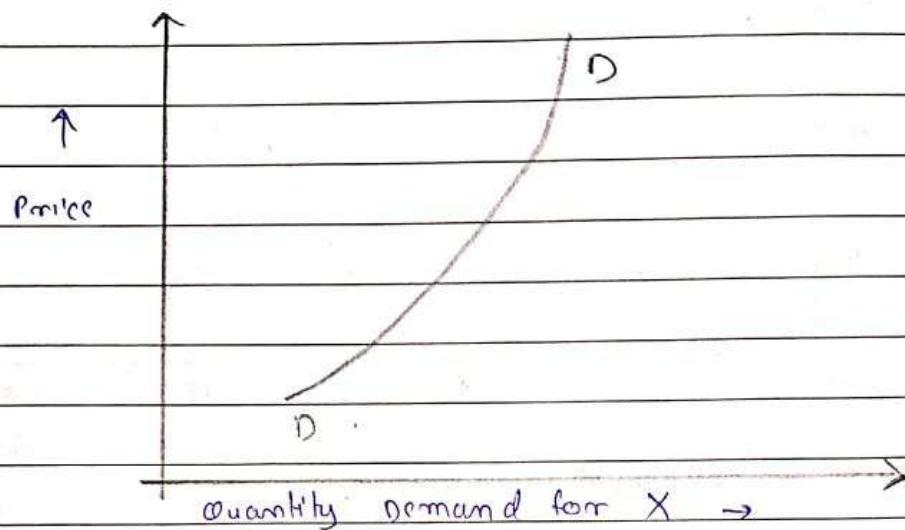


market demand curve.



Substitute Goods.

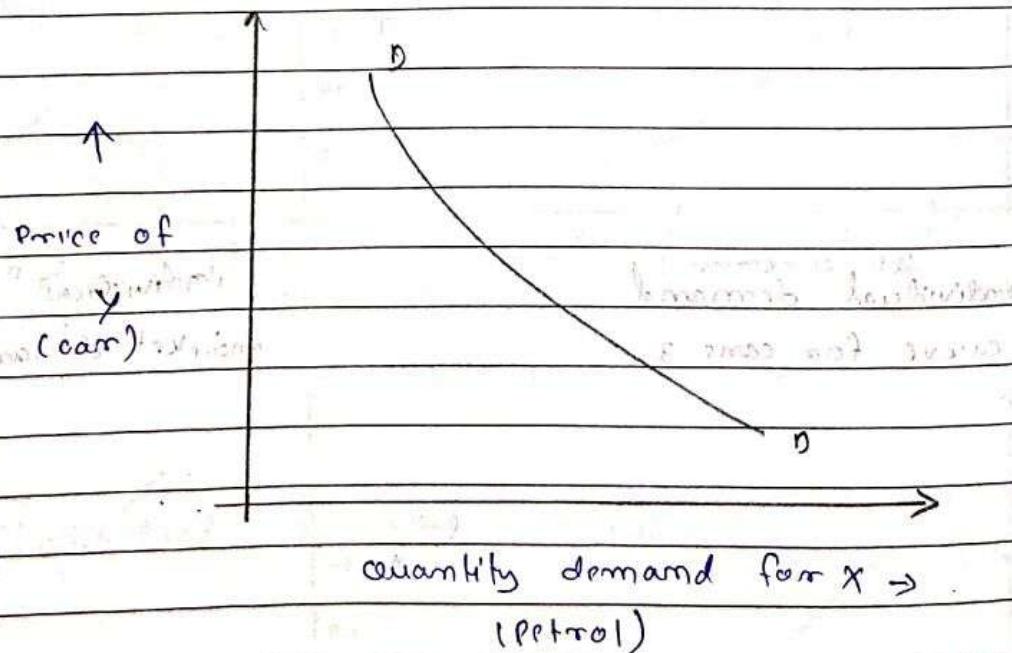
It refers to those type of where those type of goods where the increase in price of one good ^{causes the} for other good to increase. Eg → tea and coffee, bus transport and railway transport.



Complementary goods.

It refers to those type of goods where with the rise in price of one good, quantity demand for other good - decreases.

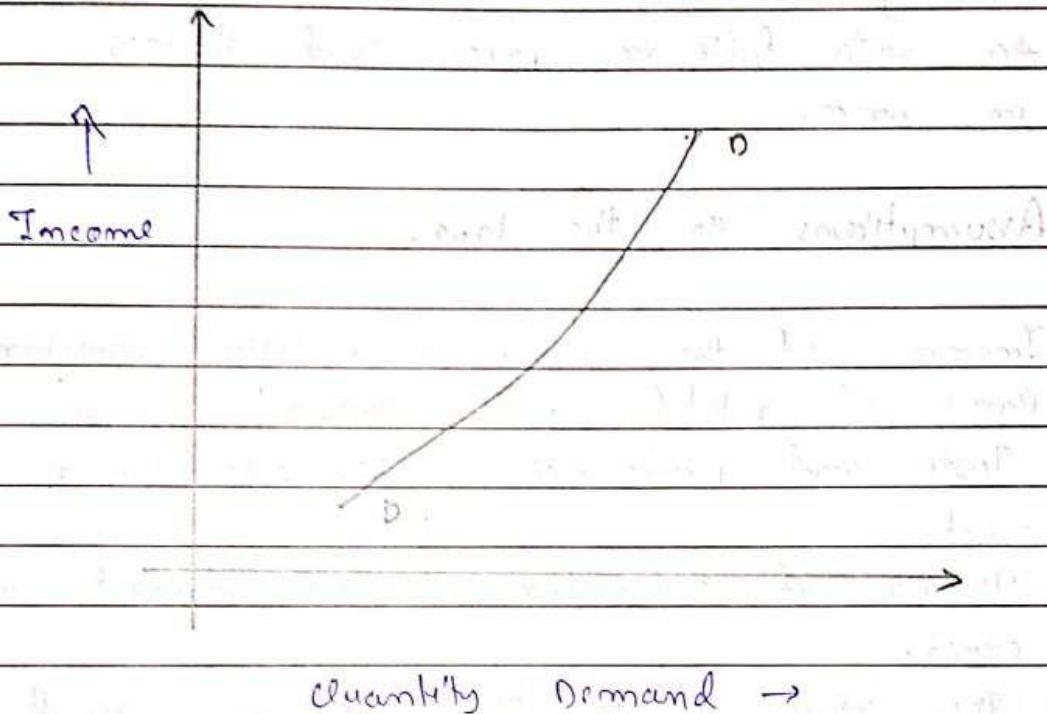
Eg → sugar and tea, bread and butter.





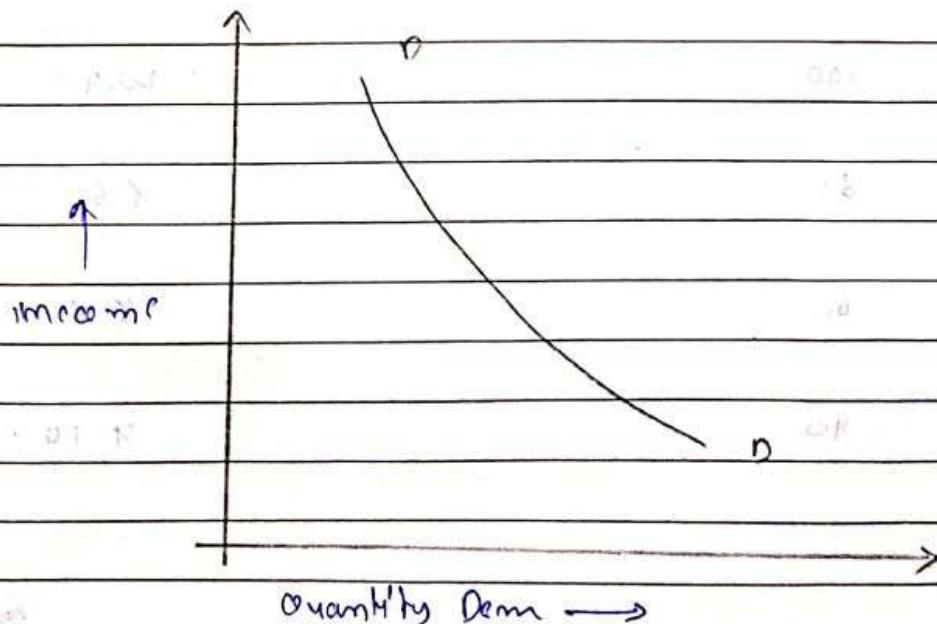
Normal Goods.

It refers to those type of goods whose quantity demand increases with increase in the income of consumer.



Inferior good.

It refers to those type of goods whose quantity demand decreases with increase in the income of consumer.





Law of Demand.

Definition given by Marshall

The defined law of demand as other things remaining constant (*ceteris paribus*) quantity demand for a commodity increases with fall in price and decreases with rise in price.

Assumptions of the law.

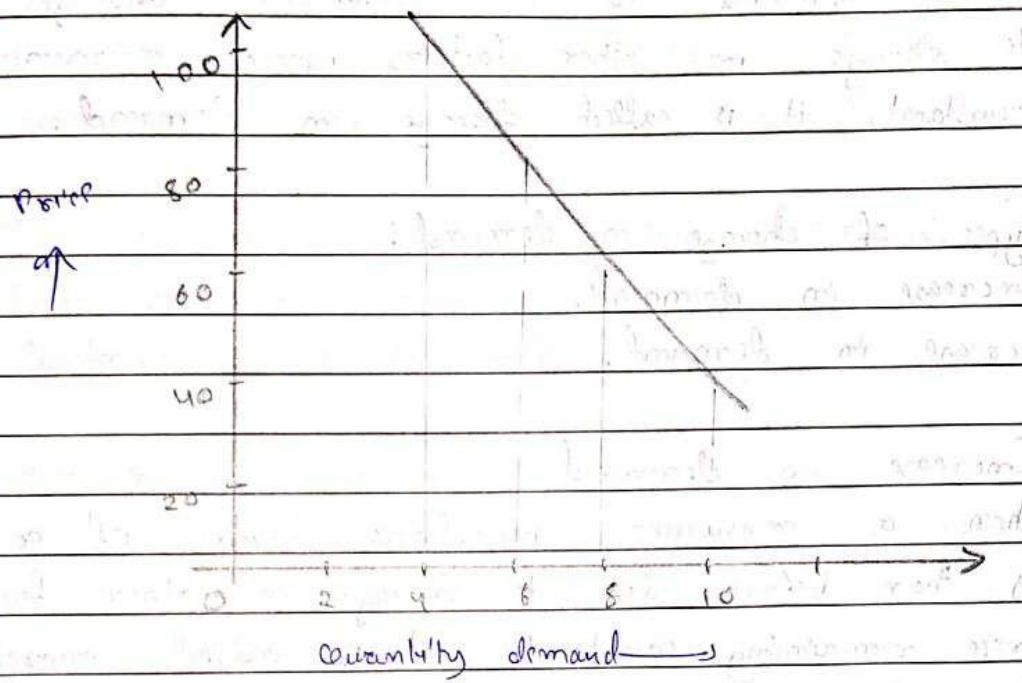
- ① Income of the consumer remains constant.
- ② Prices of related good doesn't change.
- ③ Taste and preference of the consumer remains const.
- ④ Number of consumer in the market remains const.
- ⑤ The good should be a normal good.

Demand schedule

Price of x (in Rs)	Quantity demand for x (in units)
100	104
80	86
60	68
40	40



Demand Curve.



Limitations of law of demand / Exceptions to the law of demand.

- ① Giffen good. (named after Robert Giffen)
- ② Veblen good. (named after Thorstein Veblen)
- ③ Speculation.
- ④ War & Energy.
- ⑤ Other factors.



Change in Demand :

When demand for a commodity changes due to change in other factors price remaining constant, it is called change in demand.

Types of change in demand:

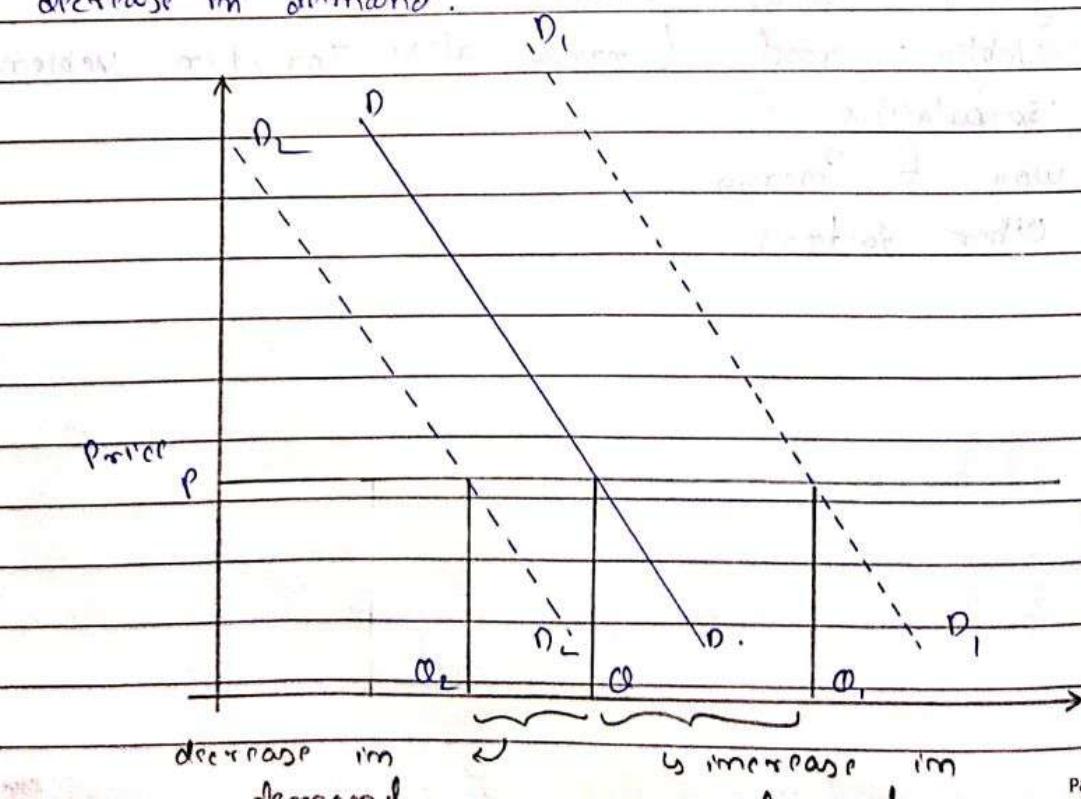
- ① Increase in demand.
- ② Decrease in demand.

Increase in demand

When a consumer purchases more of a commodity than before due to change in other factors price remaining constant, it is called increase in demand.

Decrease in demand

When a consumer purchases less than of a commodity than before due to change in other factors price remaining constant, it is called decrease in demand.





Change in Quantity demand: when demand for a commodity changes due to change in its price other factors remaining constant, it is called change in Quantity Demand.

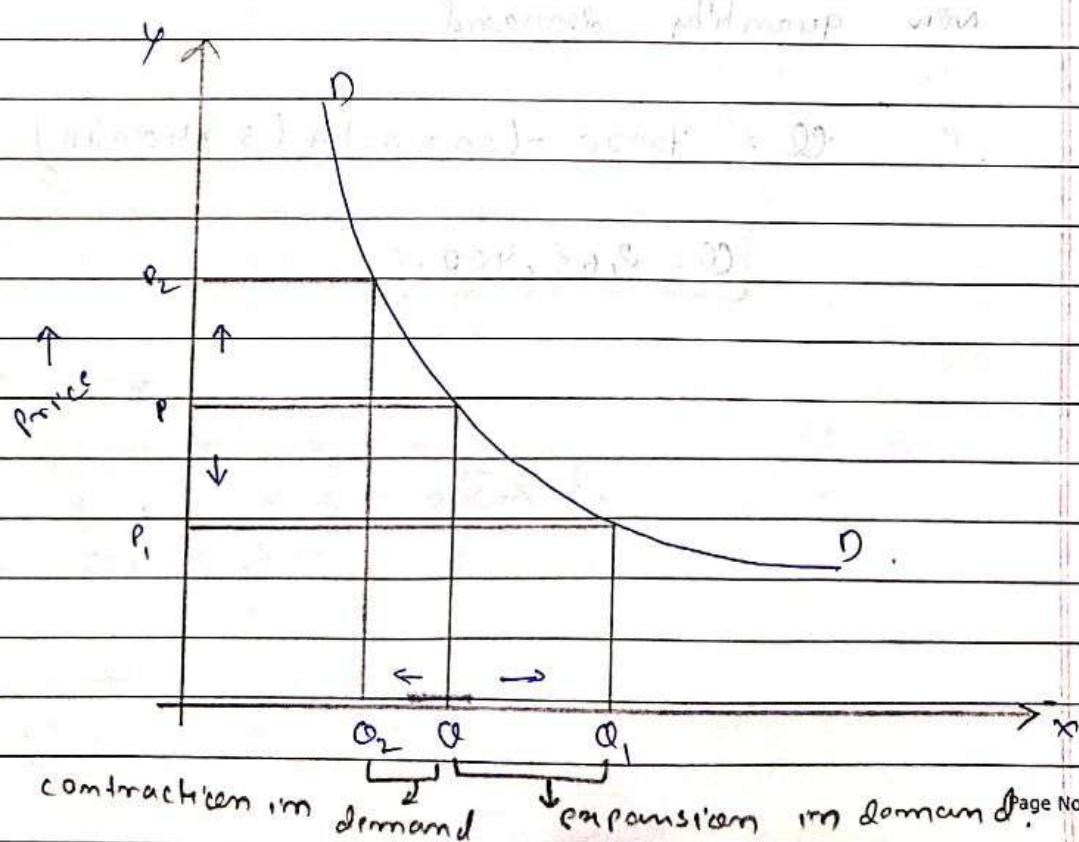
Types of change in Quantity Demand.

- ① Expansion in demand.
- ② Contraction in demand.

Expansion in demand: when demand for a commodity increases due to a fall in its price other factors remaining constant, it is called expansion in demand.

Contraction in demand:

when quantity demand for a commodity decreases due to a rise in its price other factors remaining constant, it is called contraction in demand.



contraction in demand expansion in demand. Page No. []



- (Q) From the following demand function find out quantity demand for a commodity whose price is 100/- and income of the consumer is 40,000.

$$Q = 70,000 - 20P + 5Y$$

Find out new quantity demand if income remaining constant, price declines to 80 rs/unit and also mention what type of change this change in demand imply.

Sol:

$$Q = 70,000 - 20P + 5Y$$

$$\therefore Q = 70,000 - (20 \times 100) + (5 \times 40000)$$

$$\therefore Q = 2,68,000$$

New quantity demand

$$Q = 70000 - (20 \times 80) + (5 \times 40000)$$

$$Q = 2,68,400$$



Demand Forecasting

- (Q) From the following information forecast sales for the year 1996 and 1997.

Years	Sales (in ₹ x 1000)
1991	20
1992	35
1993	42
1994	56
1995	68

Soln:

Given,

$$n = 5$$

∴ The common year is 1993.

Years	Sales	X	X^2	X^4
1991	20	-2	-4	16
1992	35	-1	1	1
1993	42	0	0	0
1994	56	1	1	1
1995	68	2	4	16

$$\sum xy = 117$$

Linear equation is.

$$y = a + bx$$

To solve:

①

$$\sum y = N a + b \sum x$$

$$\Rightarrow 221 = 5a + 0$$

$$\therefore a = 44.2$$

②

$$\sum x^2 = a \sum x + b \sum x^4$$



$$\therefore 11.7 = (44.2 \times 0) + (b \times 10)$$

$$\therefore b = 11.7.$$

$$\therefore Y = 44.2 + 11.7x$$

* For year 1996.

$$x = 3.$$

$$\Rightarrow Y = 44.2 + (11.7 \times 3).$$

$$\Rightarrow Y = 79.3.$$

* For year 1997.

$$x = 4.$$

$$\therefore Y = 44.2 + (11.7 \times 4).$$

$$\therefore Y = 91.$$

(1) For the following time series forecast the sales for year 2007 & 2009.

	Y	sales (in $\times 1000 \$$)
1 year.		
2001	30	
2002	49	
2003	58	
2004	62	
2005	80	
2006	95	
	$\sum Y = 374$	

SOL:

Hence,

$$N = 6.$$

Date 24/7/19



Common year = 2003.5

X	XY	X^2
-2.5	-7.5	6.25
-1.5	-7.5	2.25
-0.5	-2.5	0.25
+0.5	3.5	0.25
+1.5	12.0	2.25
+2.5	23.75	6.25
$\sum X = 0$		
$\sum XY = 21.0$		

Linear equation:

$$Y = a + bx$$

For a

$$\sum Y = N \cdot a + b \sum X$$

$$21.374 = 6 \cdot a + 0$$

$$\therefore a = 62.33$$

For b

$$\sum XY = a \sum X + b \sum X^2$$

$$21.211 = b \cdot (17.5)$$

$$\therefore b = 12.06$$

$$\text{So, } Y = 62.33 + (12.06 \cdot X)$$

For year 2007

$$X = 3.5$$

$$\Rightarrow Y = 62.33 + (12.06 \times 3.5)$$

$$\Rightarrow Y = 104.54$$

For year 2009

$$X = 5.5$$

$$Y = 62.33 + (12.06 \times 5.5)$$

$$\Rightarrow Y = 128.66$$



Elasticity of Demand:

It refers to the degree of responsiveness of quantity demand of a commodity in response to change in its price.

Types of Elasticity of Demand:

- ① Price elasticity of demand
- ② Income elasticity of demand.
- ③ Cross elasticity of demand.

Price elasticity of demand.

It refers to the degree of responsiveness of quantity demand for a commodity in response to a change in its price.

$$E(p) = \frac{\text{Proportionate change in Quantity Demand}}{\text{Proportionate change in price}}$$

Or,

$$\frac{\text{Percentage change in Quantity Demand}}{\text{Percentage change in price}}$$

$$\Rightarrow \frac{\text{Change in Q.D}}{\text{Original Q.D}} \times 100$$

$$\frac{\text{Change in price}}{\text{Original price}} \times 100$$



Q → Quantity of Demand
Δ → Change

P → Price

$e_p \rightarrow$ price elasticity of demand

E → elasticity of demand.

$$= \frac{\Delta Q}{Q} \times 100.$$

$$\Rightarrow \frac{\Delta Q}{Q} \times \frac{P}{\Delta P}.$$

$$\Rightarrow E(e_p) = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}.$$

or

$$E(e_p) = \frac{dQ}{dP} \times \frac{P}{Q}$$

Eg:

$$\therefore e_p = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} = \frac{15 - 20}{-5} \times \frac{20}{60}.$$

$$\times 20 \quad \times 20 \\ \times 20 \quad \times 20 \quad \boxed{-1}$$

∴ 2.



Q) Find elasticity of a commodity if Q.D from a the commodity decreases from 40 to 35 units due to a rise in its price from 20 to 27 Rs per unit.

Sol:

Given,

<u>Q.D.</u>	P.
40	20
35	27

$$\therefore E(p) = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

$$\therefore \frac{-5}{7} \times \frac{20}{40}$$

$$\Rightarrow \left(-0.357 \right)$$

$$\Rightarrow \boxed{0.357}$$

Q) From the following demand function find out price elasticity of demand if price of the commodity is 500 rs per unit.

$$Q_x = 90,000 - 25 P_x$$

where,

 $Q_x \rightarrow Q.D. of X$ $P_x \rightarrow P. of X$ $X \rightarrow \text{any commodity.}$

so 1st:

Given,

$$P = 500 \text{ Rs.}$$

$$Q_x = 90,000 - 25 P_x$$

$$e_p = \frac{dQ_x}{dP_x} \times \frac{P_x}{Q_x}$$

$$e_{P_x} = -25 \times \frac{500}{(90,000 - (25 \times 500))}$$

$$\Rightarrow e_{P_x} = -25 \times$$

$$\Rightarrow e_{P_x} = |-0.16|$$

$$\Rightarrow e_{P_x} = 0.16$$



Income Elasticity of Demand:

It refers to the degree of responsiveness of quantity demand for a commodity in response to a change in income of the consumer.

Suppose $y \rightarrow$ income of the consumer

$e_y = \frac{\text{Percentage change in } Q.D}{\text{Percentage change in income}}$

Proportionate change in Q.D

Proportionate change in income.



$\Rightarrow \frac{\text{change in Q.D}}{\text{change in income}} \times 100$

$$e_y = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q} \quad \text{or} \quad e_y = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$$

where,

$\Delta Q \rightarrow$ change in Q.D.

$\Delta Y \rightarrow$ change in income.

$Y \rightarrow$ original income.

$Q \rightarrow$ original quantity demand.

$e_y \rightarrow$ income elasticity of demand.

Q. Find out income elasticity of a commodity whose Q.D increases from 1000 to 1500 units due to a rise in income of the consumer from 35,000 Rs to 40,000 Rs per month.

Y	Q.
35,000.	1000.
40,000.	1500

$$e_y = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$$

$$e_y = \frac{500}{5000} \times \frac{35000}{10000}$$

$$e_y = 0.1 \times 3.5$$

3.5



Q. From the following demand function find out income elasticity of demand if income of the consumer is 70,000 Rs per month.

$$Q = 90,000 + 7Y$$

$Q \rightarrow$ quantity demanded

$Y \rightarrow$ income of consumer / month.

Sol:

$$Q = 90,000 + 7Y$$

$$\epsilon_Y = \frac{dQ}{dY} \times \frac{Y}{Q}$$

$$\therefore \epsilon_Y = +7 \times \frac{70,000}{(90000 - (7 \times 70000))}$$

$$\therefore \epsilon_Y = 0.225 \boxed{0.84}$$

(Q) From the following demand function. Find out price elasticity of demand & income elasticity of demand if price of the commodity is 200 Rs/ unit and income of the consumer is 50,000 per month.

$$Q = 1,00,000 - 20P + 10Y$$

$P \rightarrow$ price of commodity,

$Y \rightarrow$ income of consumer

$Q \rightarrow$ Quantity Demanded



Sol:

$$e_p = \frac{dQ}{dP} \times \frac{P}{Q}$$

$$\Rightarrow e_p = -20 \times \frac{200}{(1,00,000 - (20 \times 200) + (10 \times 50,000))}$$

$$= -6.7114 \times 10^{-3}$$

$$\Rightarrow 6.711 \times 10^{-3}$$

$$\Rightarrow 0.007$$

$$e_y = \frac{dQ}{dy} \times \frac{y}{Q}$$

$$\Rightarrow e_y = -10 \times \frac{50,000}{596,000}$$

$$\Rightarrow e_y = 0.838$$



Cross Elasticity of Demand :

If two goods X and Y are so related that quo quantity demand for good X depends on price of good Y then cross elasticity of demand is defined as the degree of responsiveness of quantity demand for good X in response to a change in price of good Y.

$$E_{\text{cross}} = \frac{\text{Percentage change in Q.D for } X}{\text{Percentage change in price of } Y}$$

$$E_{\text{cross}} = \frac{\Delta Q_x}{\Delta P_y} \times \frac{P_y}{Q_x}$$

$$E_{\text{cross}} = \frac{dQ_x}{dP_y} \times \frac{P_y}{Q_x}$$

where,

$\Delta Q_x \rightarrow$ change in Q.D of X

$\Delta P_y \rightarrow$ change in price of Y.

$P_y \rightarrow$ original price of Y.

$Q_x \rightarrow$ original quantity of demand of X.

- (Q) Find out cross elasticity of demand b/w Tea and coffee if Q.D for coffee increases from 3000 to 4000 units due to a rise in price of tea from 300 to 400 Rs per 250 gms pack.



Sol:

Given,

Q. D (coffee) Price (tea)

3000.

300.

4000.

450.

$$\text{Cross} = \frac{\Delta Q_c}{\Delta P_t} \times \frac{P_t}{Q_c}$$

$$\Rightarrow \text{Cross} = \frac{1000}{150} \times \frac{300}{3000}$$

$$\Rightarrow \text{Cross} = 0.667$$

- (Q) From the following demand functions, find out cross, if price of tea is 1000 rs / 500 gms. pack.

$$Q_c = 30,000 - 35 P_t$$

 $Q_c \rightarrow Q \cdot D$ for coffee.

 $P_t \rightarrow$ price of tea.

Sol:

$$\text{Cross} = \frac{dQ_c}{dP_t} \times \frac{P_t}{Q_c}$$

$$\text{Cross} = -35 \times 1000$$

$$\Rightarrow \text{Cross} = 0.538$$



Q)	Price of good X	Q.D for. good Y	Income.
	200	80	20,000 .
	300	70	40,000 .
	400	65	50,000 .
	500	40	55,000 .

From the above table find out

- Cross elasticity of demand b/w good X and good Y. if price of the commodity X increases from 200 to 400 ~~Rs. 10/- per unit~~.
- Mention good X and good Y are what type of goods on the basis of above question.
- Find out income elasticity of demand for good Y if income of the consumer will increase from 40,000 Rs. to 55,000 Rs. per month.
- Mention good Y is what type of good on the basis of above question (iii).

Sol:

$$\text{Cross} = \frac{\Delta Q_{\text{D},Y}}{\Delta P_X} \times \frac{P_X}{Q_{\text{D},Y}}$$



$$\text{2) Loss} \neq \frac{10}{100} *$$

$$\Rightarrow \text{Loss} = -15 \times \frac{200}{200} = \frac{300}{80}$$

$$\Rightarrow \text{Loss} = -0.1875$$

iv)

Since, loss is -ve so good X and good Y are complementary goods.

vii)

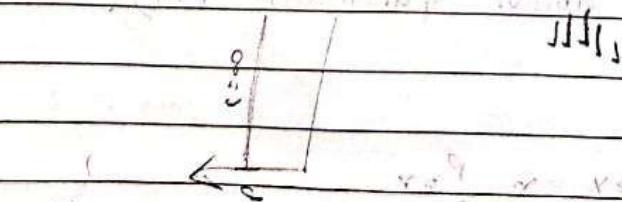
$$e_I = \frac{\Delta C_Y}{\Delta I} \times \frac{I}{C_Y}$$

$$\Rightarrow e_I = \frac{-30}{15,000} \times \frac{40,000}{70}$$

$$\Rightarrow e_I = -1.142$$

iv)

The demand of product Y decreases with increase in income so it is an inferior good.



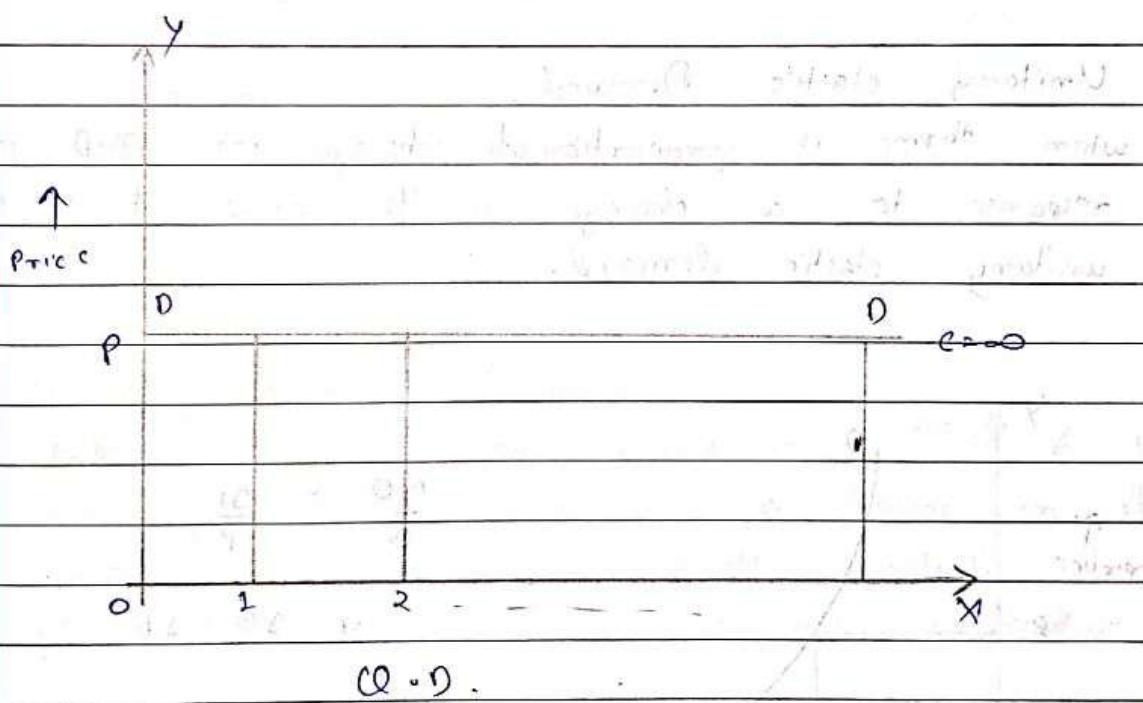


Degrees of Elasticity of Demand :-

- ① Perfectly elastic Demand.
- ② Relatively / More elastic Demand
- ③ Unitary Elastic Demand.
- ④ Relatively inelastic / less elastic Demand.
- ⑤ Perfectly inelastic demand.

① Perfectly elastic demand :

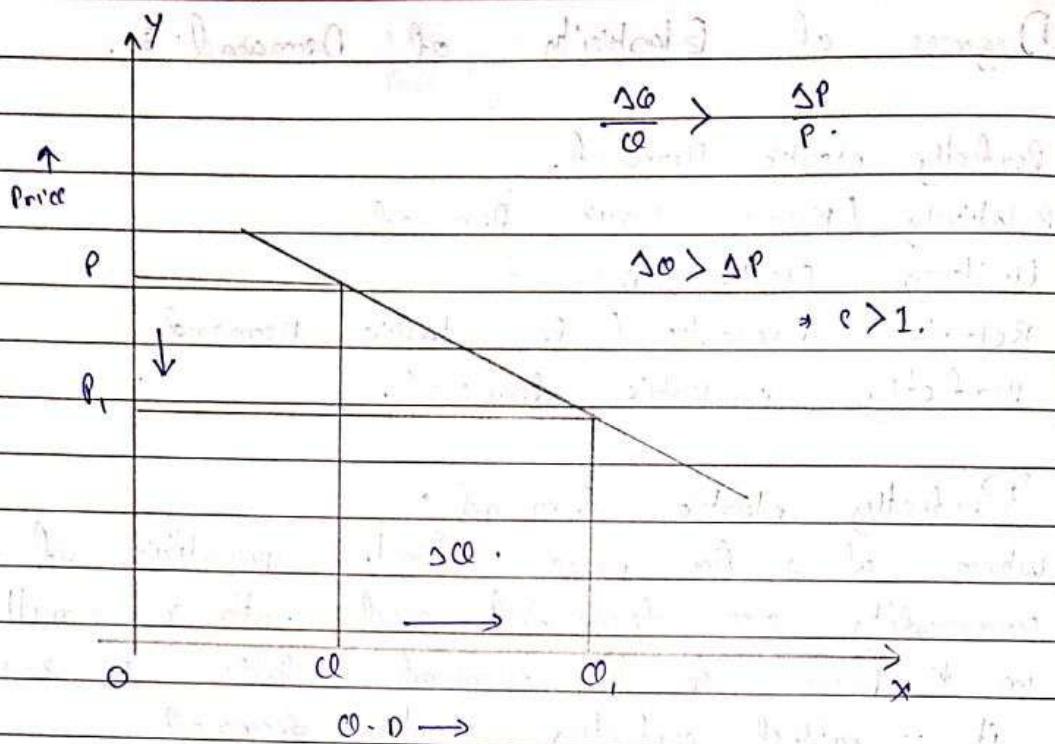
When at a fix price infinite quantities of a commodity are demanded and with a small increase in the price quantity demand falls to zero, it is called perfectly elastic demand.



② Relatively elastic demand.

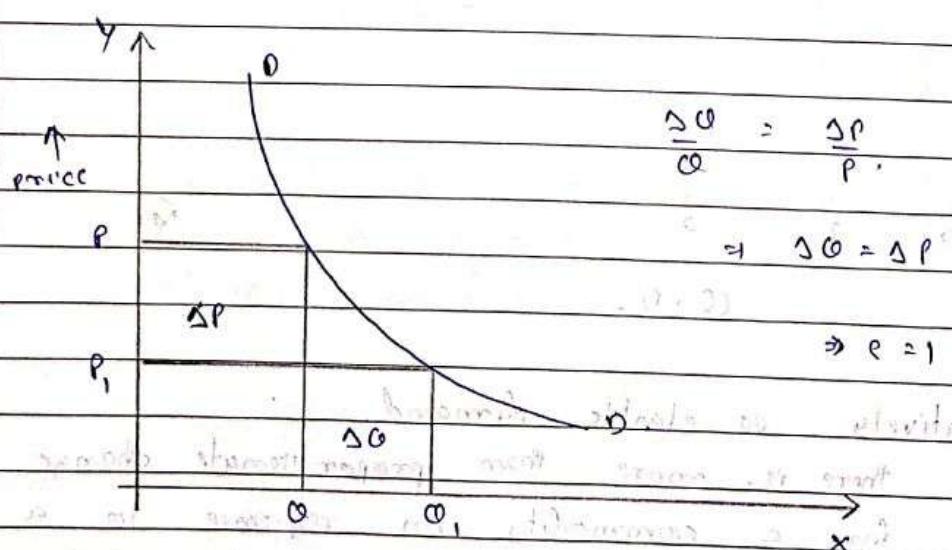
When there is more than proportionate change in Q and for a commodity in response to a change in its price it is called relatively elastic demand.

Eg: All type of luxuriously goods.



Unitary elastic Demand.

When there is proportionate change in Q.D in response to a change in its price it is called unitary elastic demand.

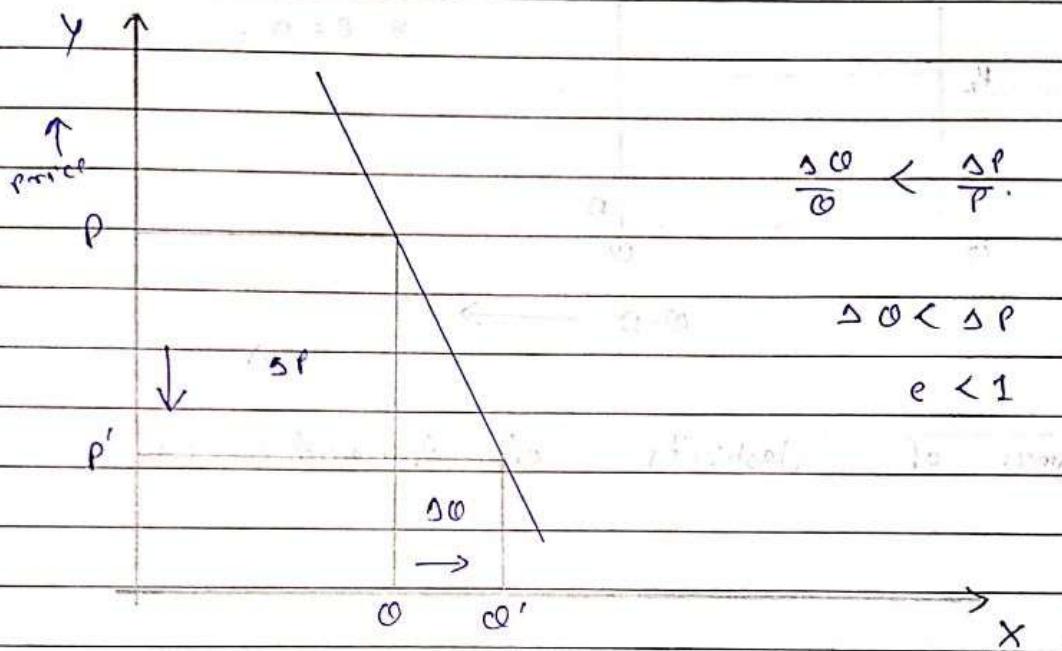




Relative inelastic demand.

When there is less than proportionate change in Q.D for a commodity in response to change in its price is called relative inelastic demand.

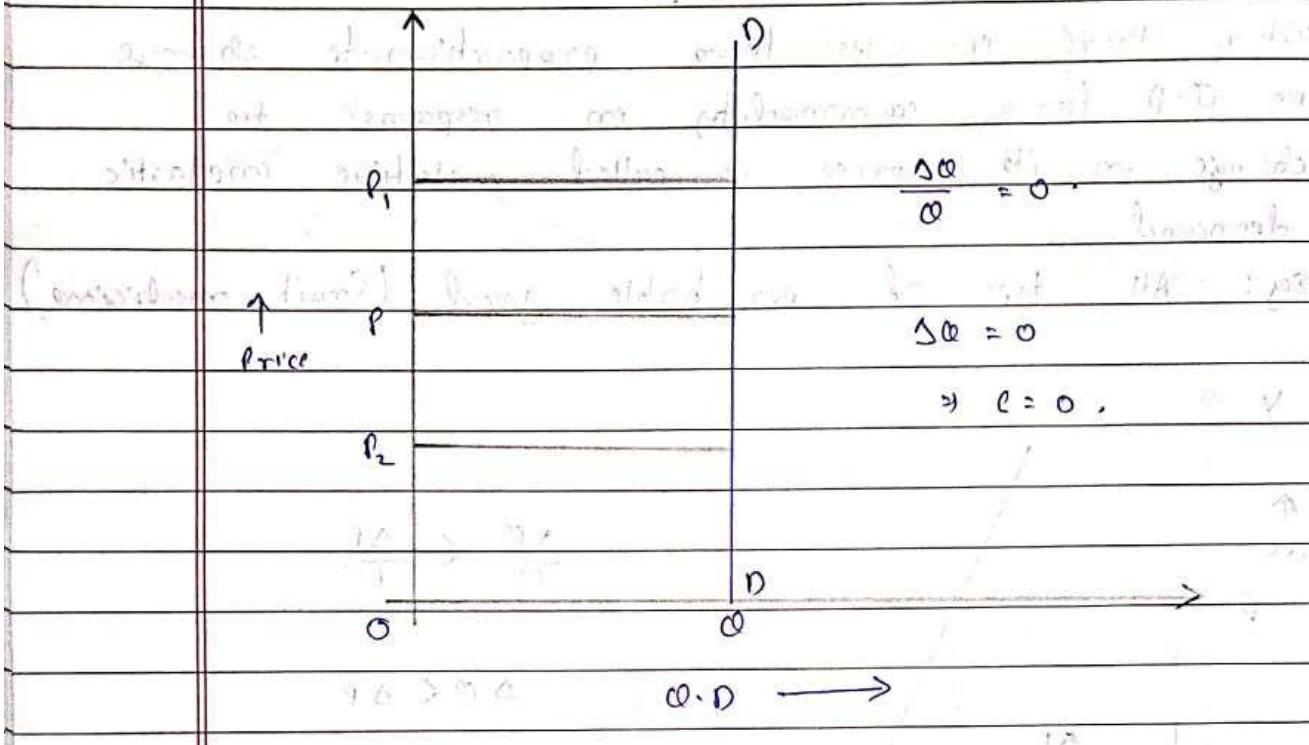
Eg: All type of perishable good. (Fruit, vegetables)



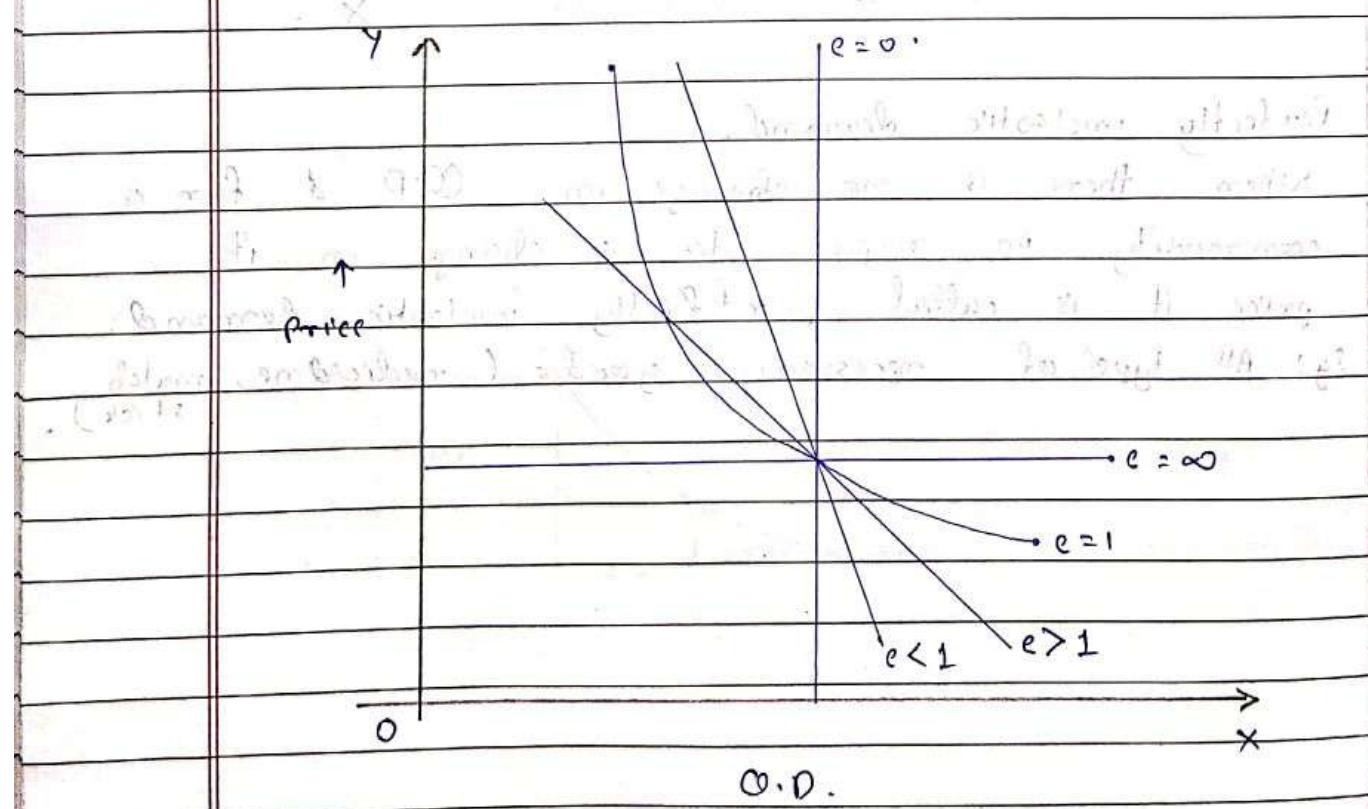
Perfectly inelastic demand.

When there is no change in Q.D & for a commodity in response to a change in its price it is called perfectly inelastic demand.

Eg: All type of necessary goods (medicine, match stick).

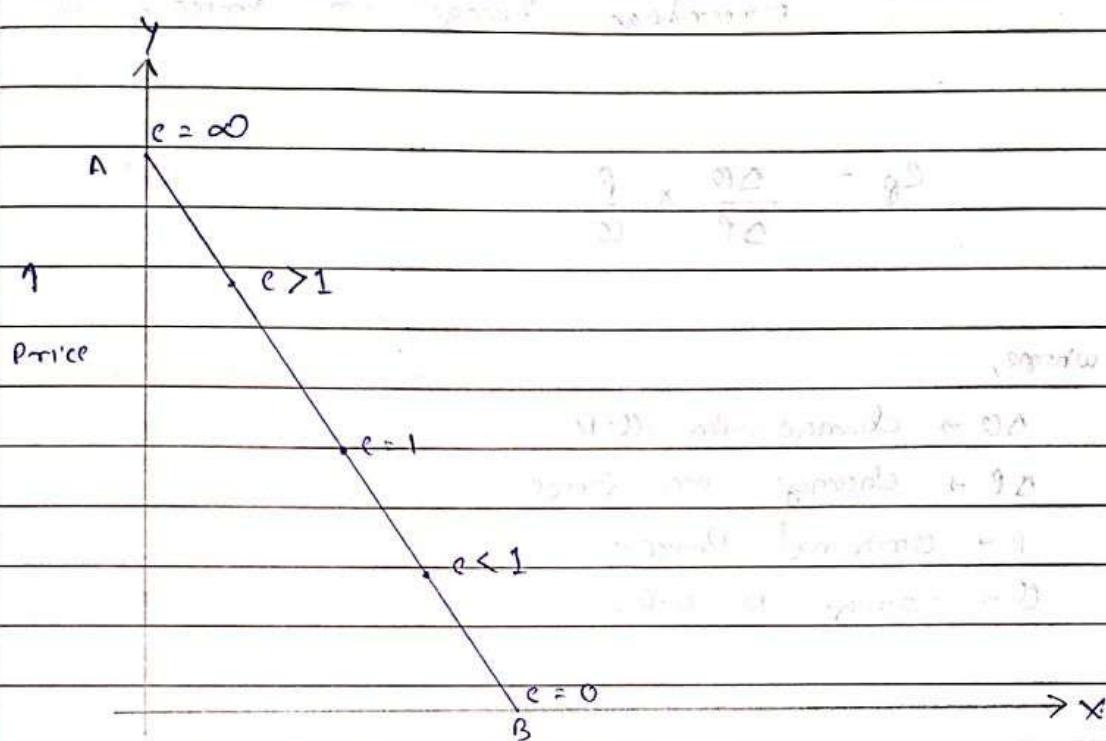


Degrees of elasticity of demand: in a





Show all the degrees of elasticity on a straight line Demand curve.



Measurement of elasticity of demand.

- ① Percentage Method
- ② Arc / Mid point method.
- ③ Total expenditure method.

Percentage method.

It refers to that method where elasticity of demand can be measured on the basis of percentage change in Q.D for a commodity in response to a percentage change in its price.



$$\epsilon_p = \frac{\text{Percentage change in Q.D}}{\text{Percentage change in Price}}$$

$$\epsilon_p = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

where,

$\Delta Q \rightarrow$ change in Q.D

$\Delta P \rightarrow$ change in Price.

P → original Price.

Q → orig. Dr Q.D.

- Q) find out elasticity of demand for a commodity whose Q.D increased from 1000 to 1500 units due to a fall in its price from 40 to 32 rs / unit on the basis of percentage method.

Sol:

<u>Q</u>	<u>P</u>
1000	40
1500	32

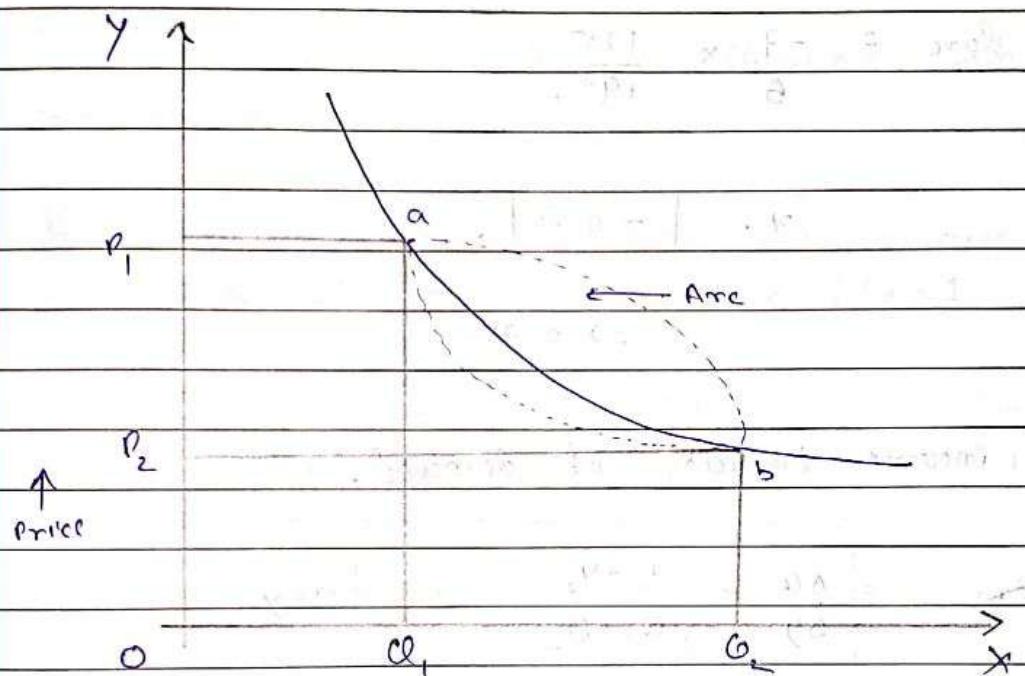
$$\Rightarrow \epsilon_p = \frac{500}{-8} \times \frac{40}{1000} = -2.5$$

$$\Rightarrow (-2.5) = -2.5$$



Arc / Mid-point method.

It refers to that method where elasticity of demand can be measured b/w two separate points.



$\text{Q.D.} \rightarrow .$

$$\text{Earc} = \left(\frac{\text{change in Q.D.}}{\text{original Q.D. + New Q.D.}} \right) / 2 \times 100$$

$$\left(\frac{\text{change in price}}{\text{original price + New price}} \right) / 2 \times 100$$

$$\rightarrow \frac{\Delta Q}{(Q_1 + Q_2)/2} \times 100$$

$$\text{Earc} = \frac{\Delta Q}{\Delta P} \times \frac{P_1 + P_2}{Q_1 + Q_2}$$

$$\frac{\Delta P}{(P_1 + P_2)/2} \times 100$$



Q) Find e_{arc} for a commodity whose O.D. declines from 100 to 93 units due to a rise in its price. from 50 to 55 rs / unit.

SOL:

$$e_{arc} = -\frac{7}{5} \times \frac{105}{193}$$

$$\Rightarrow (-0.76),$$

$$\Rightarrow 0.76.$$

* Arc income elasticity of demand.

$$e_y = \frac{\Delta y \times y_1 + y_2}{\Delta y \times y_1 + y_2} = \text{Carry}$$

Total Expenditure Method:

It refers to that method where elasticity of demand can be measured on the basis of change in total expenditure due to a change in O.D. of a commodity in response to change in its price.

$$T.E = P \times Q,$$

where,

P = Price of the commodity per unit.

Q = No. of units of the commodity purchased.



Three Possibilities :-

- ① If total expenditure increases with a fall in price or decreases with an rise in price then $E_p > 1$
- ② If total expenditure remains constant with rise or fall in price. ($E_p = 1$)
- ③ If total expenditure increases with a rise in price or decreases with a fall in price ($E_p < 1$).

Price	Quantity Demand.	Total Expenditure	Price Elasticity.
500	30	15000	$E_p > 1$
475	40	19000	$E_p > 1$
450	50	2250	$E_p > 1$
425	60	2550	$E_p > 1$
400	75	3000	$E_p = 1$
375	80	3000	$E_p < 1$
350	83	2925	$E_p < 1$
325	87	2827.5	$E_p < 1$

- (Q) If with a fall in price from 100 to 80 Rs per unit quantity demand for a commodity increases from 8 to 10 units. Find elasticity of demand with the help of total expenditure method.

Sol:

Given,

Price • commodity.

100.	8	$E_p = ?$
80.	10.	



Determinants of Elasticity of Demand or factors affecting elasticity of demand.

- ① Availability of substitutes \rightarrow no substitute, $e = 0$
- ② Nature of the good \rightarrow perishable $e < 1$, luxurious $e > 1$
- ③ Postponement of consumption \rightarrow normal
- ④ Alternative uses \rightarrow luxurious $e > 1$
- ⑤ Time period \rightarrow short period $e = 0$
- ⑥ Habit \rightarrow long period $e > 1$

Supply :

* Stock and supply.

It refers to total amount of production whereas supply refers to the actual amount offered for sale in the market.

* Supply Schedule.

It refers to the tabular representation of different quantities of a commodity supplied to the market at different prices.

Price of the commodity supplied

\times

5

10

10

15

15

18

20

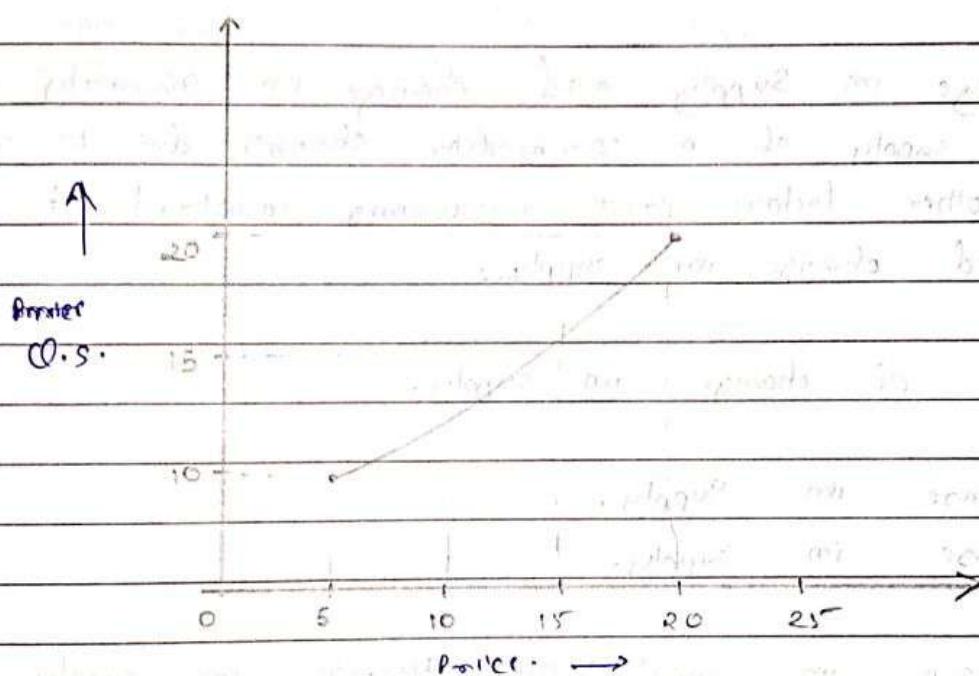
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Types of Supply Schedule :

- ① Individual supply schedule.
- ② Market supply schedule.

Supply curve :



Law of Supply.

It is defined as other things remain constant a quantity supply of commodity increases with increase in price and decreases with decrease in price.

Assumptions :

- ① Techniques of production remain constant.
- ② Price of input remain constant.
- ③ Price of other products remain constant.
- ④ Motive of the producer does not change.
- ⑤ Number of producers in the market remain constant.



Limitation of law of supply.

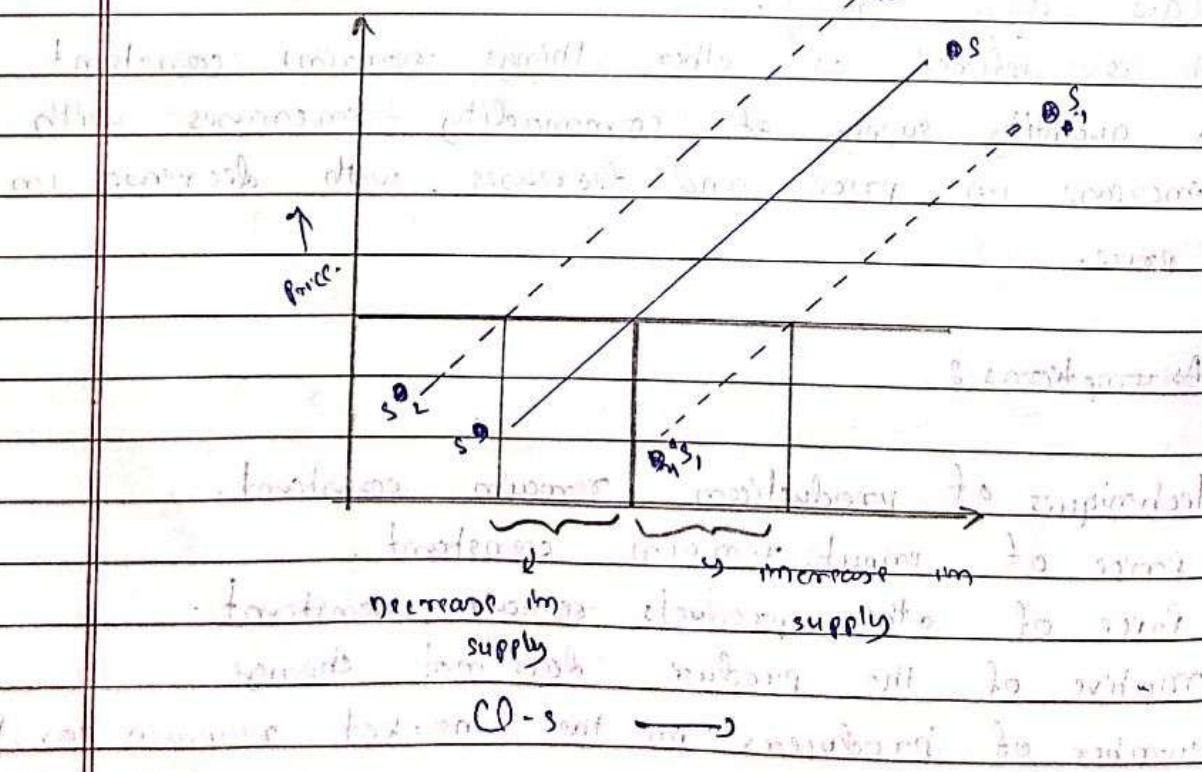
- ① In case of old claims or rare paintings.
- ② In case of old stocks.
- ③ If price of labour will increase.
- ④ A business person at the time of closing his/her business.

Change in Supply and change in quantity supply when supply of a commodity changes due to change in other factors price remaining constant it is called change in supply.

Types of change in supply.

- ① Increase in Supply.
- ② Decrease in Supply.

Increase in supply & decrease in supply.



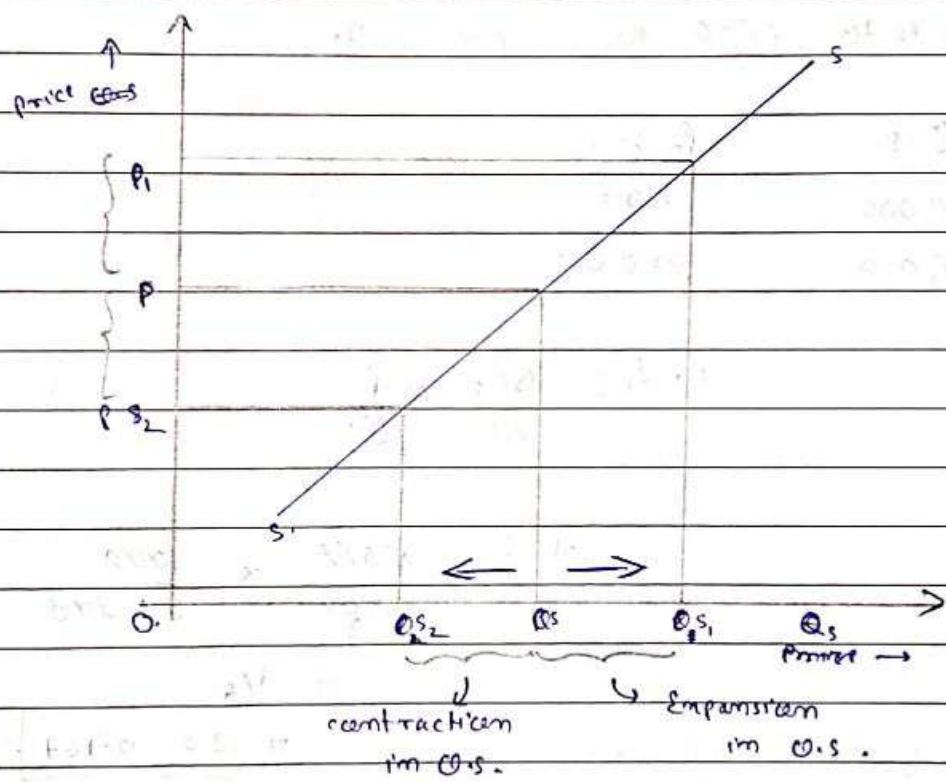


Change in Quantity Supply.

When supply of a commodity changes due to a change in its price other factors remaining constant, it is called change in quantity supply.

Types of change in Quantity Supply.

- ① ^{term} Expansion in Quantity supply.
- ② Contraction in Quantity supply.



Elasticity of Supply:

It refers to the degree of responsiveness of quantity supplied of a commodity in response to change in price.

$$Q_s = \frac{\text{Percentage change in Q.S.}}{\text{Percentage change in Price.}}$$



$$e_s = \frac{\Delta Q_s}{\Delta P} \times \frac{P}{Q}$$

or

$$e_s = \frac{dQ_s}{dP} \times \frac{P}{Q}$$

- (1) Find out elasticity of supply of a commodity whose O.S. increases from 20,000 to 25,000 units due to a rise in its price from 1000 Rs to 2500 Rs per unit.

O.S	Price
20,000	1000
25,000	2500

$$\Rightarrow e_s = \frac{\Delta Q_s}{\Delta P} \times \frac{P}{Q}$$

$$\Rightarrow e_s = \frac{5000}{5000} \times \frac{1000}{2500}$$

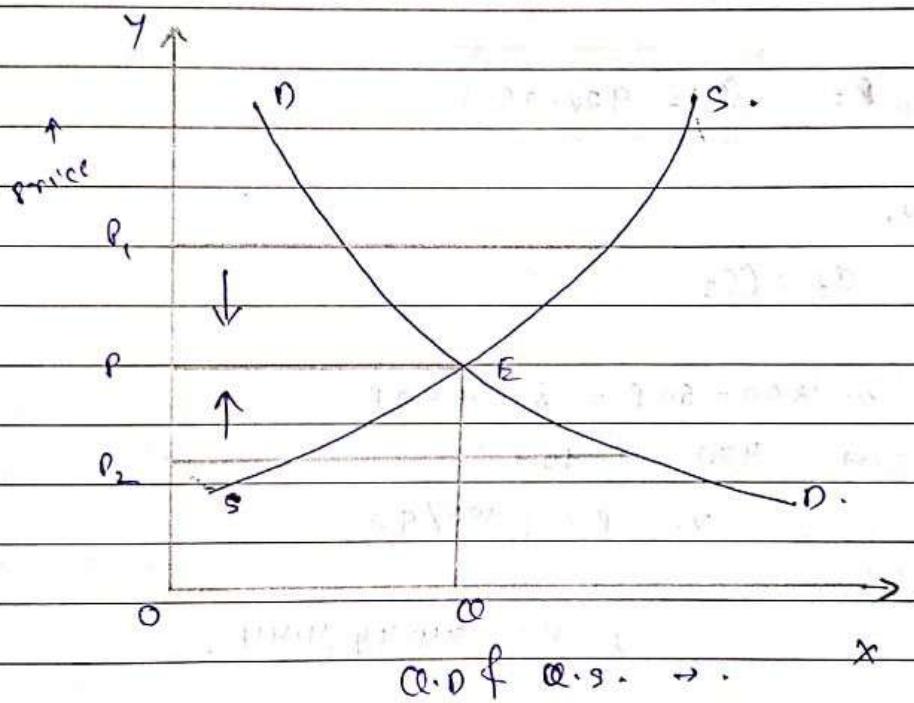
$$\Rightarrow \frac{1}{6}$$

$$\Rightarrow e = 0.167$$



Degrees of Elasticity of Demand, Supply

- ① Perfectly elastic ~~of Demand, Supply~~
- ② Relatively / more elastic ~~of Demand, Supply~~
- ③ Unitary elasticity ~~of Supply~~
- ④ Relatively inelastic supply.
- ⑤ Perfectly inelastic supply.



From the following Demand and supply functions find out eq^u price and eq^u quantity.

$$Q_d = 1000 - 30 P,$$

$$Q_s = 800 + 40 P.$$

Find out new eq^u price and quantity, if supply remaining constant G.D increases to .

$$Q_d = 1200 - 50 P.$$

Date / /



Sol^u:

At equilibrium,

$$O_d = O_s$$

$$\Rightarrow 1000 - 50P = 800 + 40P$$

$$\Rightarrow 200 = 90P$$

$$\Rightarrow P = \frac{200}{90}$$

$$P = 2.222\ldots$$

now, $P =$

$$O_d = 922.22$$

now,

$$O_d = O_s$$

$$\Rightarrow 1200 - 50P = 800 + 40P$$

$$\Rightarrow 400 = 90P$$

$$\Rightarrow P = 4.444\ldots$$

$$\Rightarrow P = 4.44$$

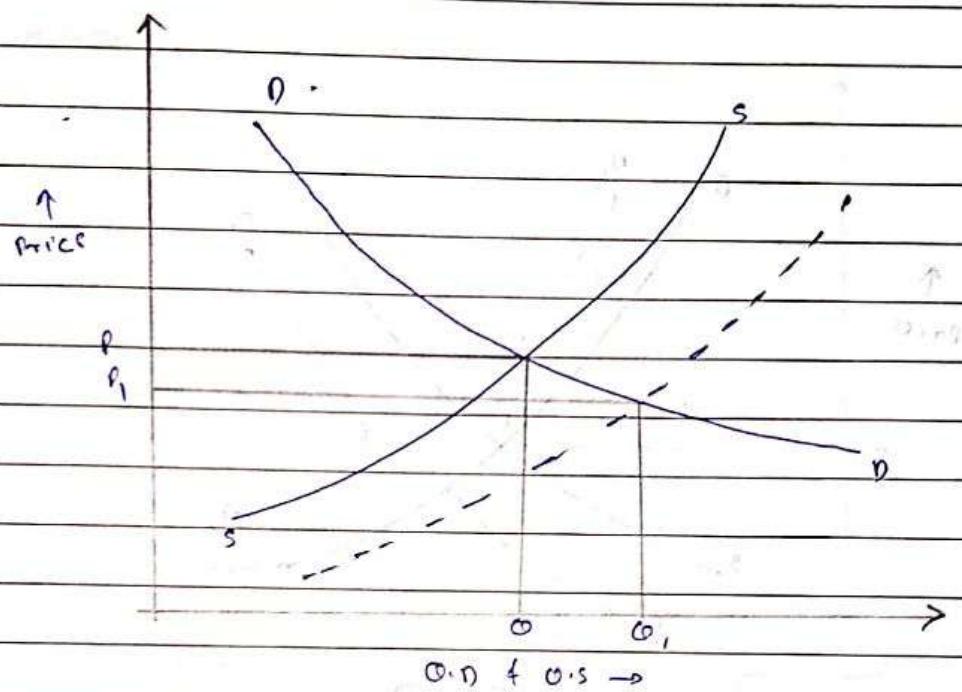
$$O_s = 1200 - (50 \times 4.44)$$

$$\Rightarrow 1200 - 222$$

$$\Rightarrow O_s = 978$$



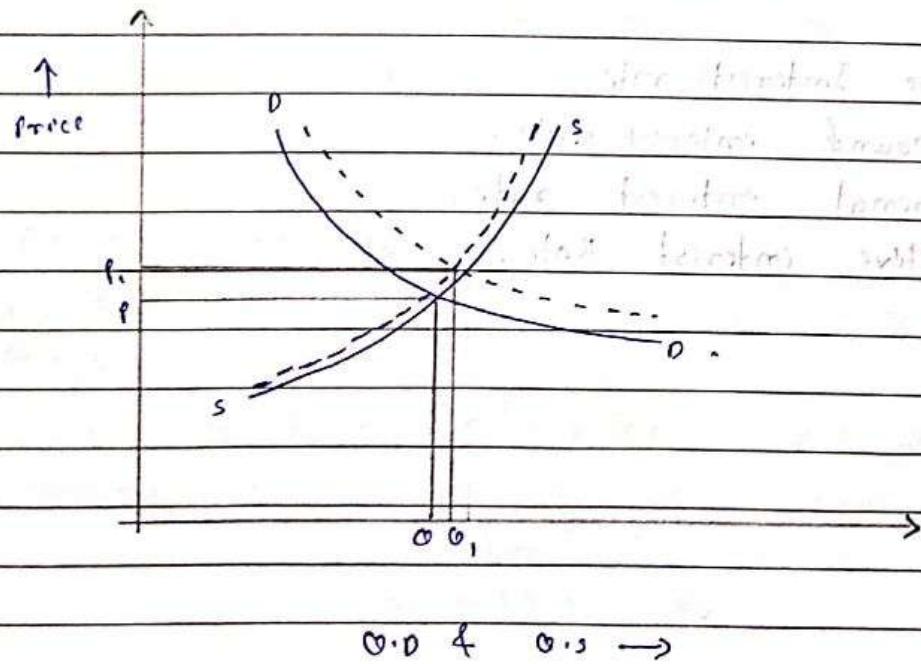
What happens to equilibrium price and quantity if demand remaining constant and supply



what happens to eqⁿ price and quantity if increase in demand is more than decrease in supply.

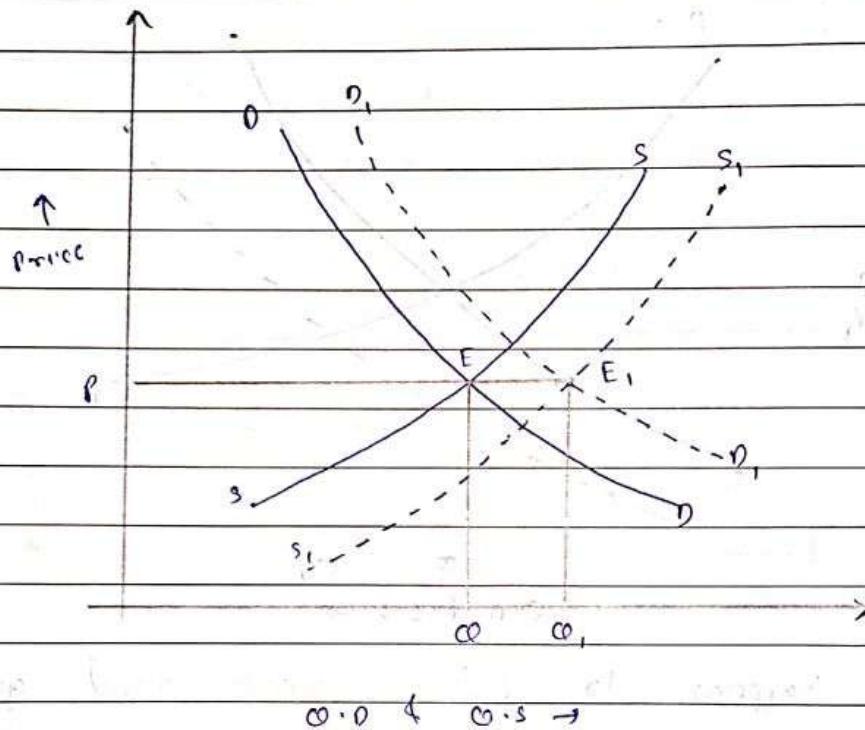


Both quantity and price increases.



Date / /

what happens to eq^{uilibrium} price & quantity if both demand and supply increases by the same amount.



Interest Rates :

- ① Simple Interest rate
- ② Compound interest rate.
- ③ Nominal interest rate.
- ④ Effective interest rate.



Simple Interest Rate:

It refers to that type of interest rate where principal amount remains same for various years.

$$I = P \times i \times N.$$

$I \rightarrow$ simple interest rate.

$P \rightarrow$ Principal.

$N \rightarrow$ number of years.

$i \rightarrow$ interest rate.

* For future value.

$$F = P + I.$$

$$\Rightarrow F = P + P \cdot i \cdot N.$$

$$\Rightarrow F = P(1 + iN).$$

Q) Find out future value of $\text{₹}4,00,000$ which is deposited in a bank at 7% interest rate for 3 yrs with the help of simple interest rate.

Soln:

$$P = 4,00,000, \text{ Rs.}$$

$$i = 7\%, \text{ per annum}$$

$$N = 3, \text{ years}$$

$$\Rightarrow F = P + (P \cdot i \cdot N)$$

$$\Rightarrow F = P(1 + i \cdot N), \text{ formula}$$

$$\Rightarrow 4,84,000 \text{ Rs.}$$



Compound Interest Rate:

It refers to that type of interest rate where principal amount keeps on changing every year.

$$F_m = P(1+i)^m.$$

P → Principal amount.

i → interest rate.

m → number of years.

- (Q) Find out future value of 5. Lakh Rs after 7 years at 8% interest rate, compound annual.

Sol: $P = 5,00,000.$

$m = 7.$

$i = 0.08.$

$$\Rightarrow F_m = 5,00,000 \times (1 + 0.08)^7.$$

$$\Rightarrow F_m = 8,56,912.1.$$

* Nominal Interest Rate:

It refers to that type of interest rate where interest rate is calculated several times in a year i.e. quarterly, monthly, semi-annually, half yearly and each day to calculate find out compound interest rate.



Quarterly :

$$F_4 = P \left(1 + \frac{i}{4} \right)^{4N}$$

Monthly :

$$F_12 = P \left(1 + \frac{i}{12} \right)^{12N}$$

Semi annualy / Half - yearly.

$$F_2 = P \left(1 + \frac{i}{2} \right)^{2N}$$

Each day :

$$F_{365} = P \left(1 + \frac{i}{365} \right)^{365N}$$

- (Q) Find out future value of \$,00,000 rs at 7% p.a interest rate if the compounding is F monthly. for 6 years.

SOL:

$$F_{12} = P \left(1 + \frac{i}{12} \right)^{12N}$$

$$\Rightarrow F_{12} = \$,00,000 \times \left(1 + \frac{7\%}{12} \right)^{12 \times 6}$$

$\rightarrow F$.

Effective Interest Rate.

It refers to the ratio of interest charged for 1 year to the principal amount.

$$i_{\text{eff}} = \frac{F - P}{P}$$

- (Q) Find out effective interest rate of some account of money amounting to lakh Rs at 8% interest after 1 year, if the compounding is quarterly.

Soln: $P = 10,00,000$.

$i = 0.08$.

$$F_4 = P \left(1 + \frac{i}{4} \right)^{4N}$$

$$\therefore F_4 = 10,00,000 \times \left(1 + \frac{0.08}{4} \right)^{4 \times 1}$$

$$\therefore F_4 = 1082432.16 \text{ Rs.}$$

$$i_{\text{eff}} = \frac{F_4 - P}{P}$$

If principal amount is not given:-

$$i_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1.$$

$r \rightarrow$ nominal interest rate
 $m \rightarrow$ number of times interest rate is calculated in a year.

- (Q) If a credit plan charges 30% interest rate, find out effective interest rate if the compounding is half-yearly.

Soln: $i = 0.3.$

$$i_{\text{eff}} = \left(1 + \frac{0.3}{2}\right)^2 - 1.$$

$$\therefore i_{\text{eff}} = 0.3225.$$

\therefore effective interest rate = 32.25%

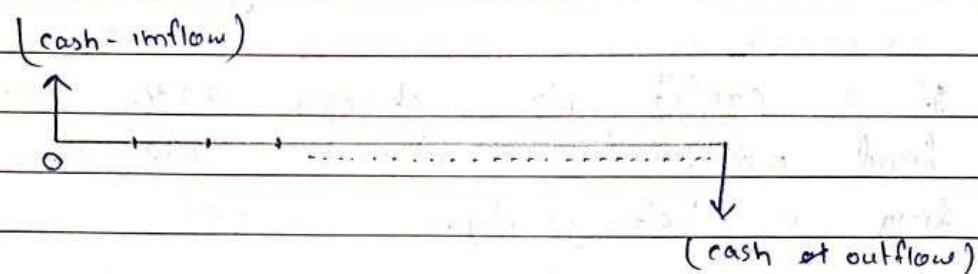


Time Value of Money

It refers to the purchasing power of money at a particular time period.

Cash flow diagrams.

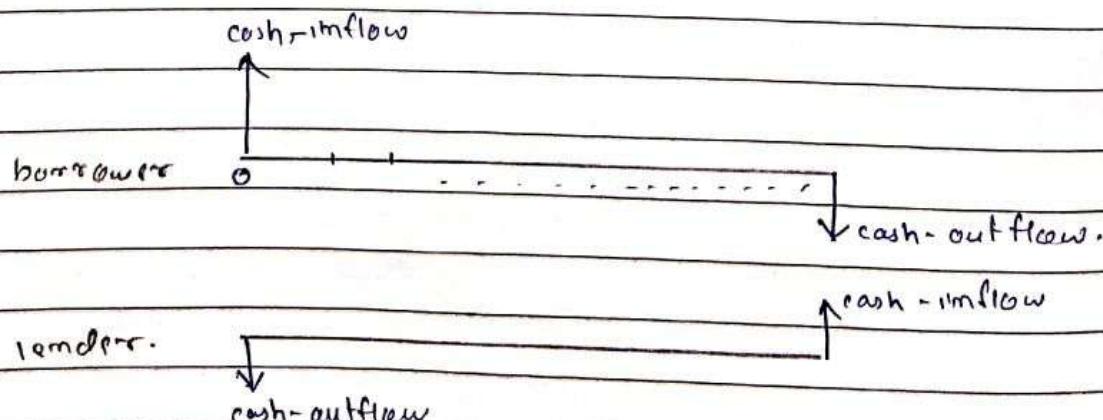
It refers to the graphical representation of cash inflow and cash outflows along the timeline.



① Seven Types of cash flow diagrams:

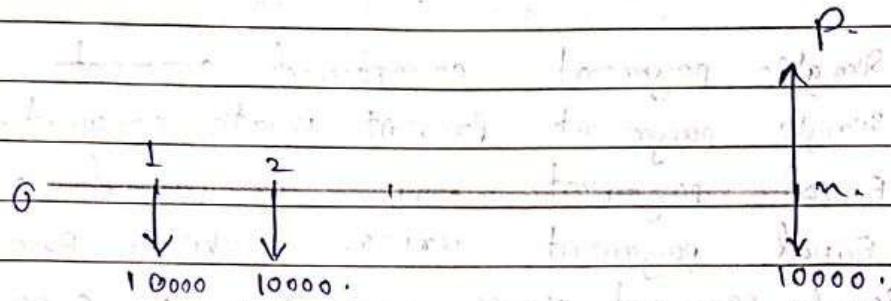
- ① Single payment cashflow.
- ② Uniform payment cashflow.
- ③ Linear - gradient series.
- ④ Geometric gradient series.
- ⑤ Irregular payment series.

① Single payment cashflow.



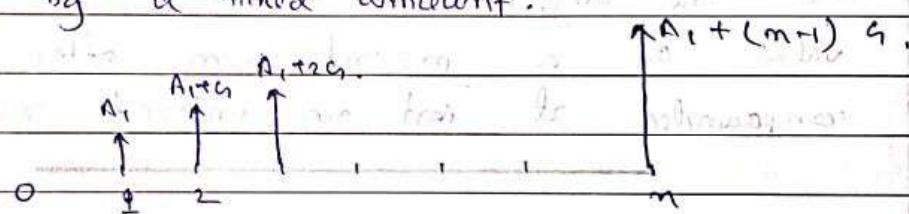


③ Uniform payment series:



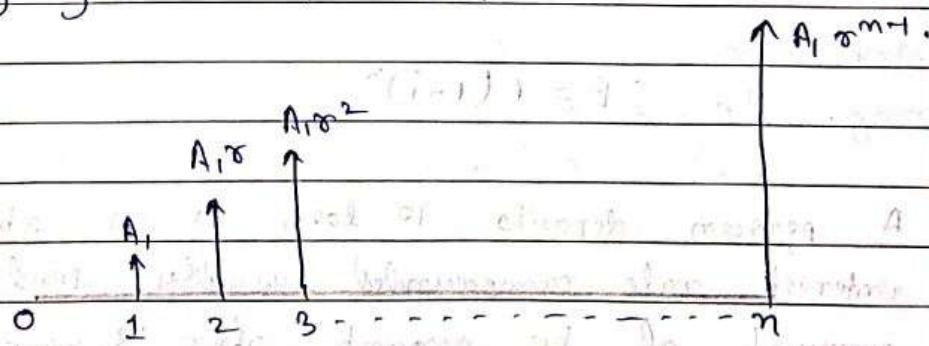
Linear gradient series:

It refers to series of cash flows increasing or decreasing by a fixed amount.

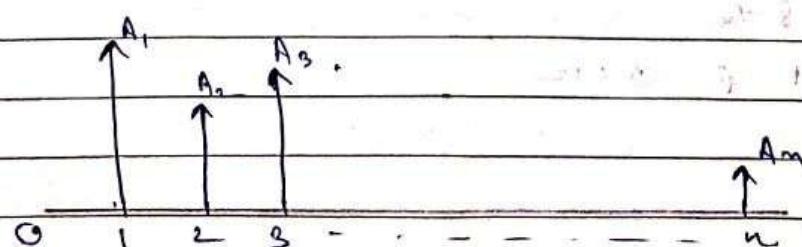


Geometric gradient series:

It refers to series of cash flows increasing or decreasing by a fixed rate.



Irregular payment series:



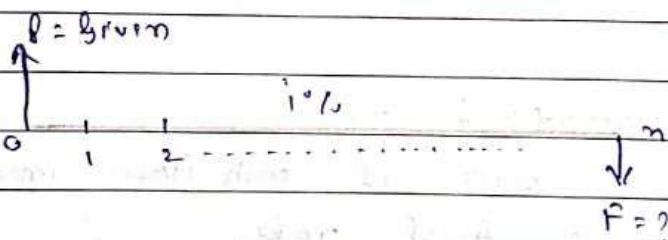


Types of compound amount series.

- ① Single payment compound amount.
- ② Single payment present worth amount.
- ③ Equal payment series compound amount.
- ④ Equal payment series sinking fund.
- ⑤ Equal payment series present worth amount.
- ⑥ Equal ann. payment series capital recovery amount.
- ⑦ Linear gradient series annual equivalent amount.

Single payment compound amount.

Here the objective is to find out the future value of a present sum after n^{th} year compounded at int. an interest rate i .



$$F = P(1+i)^n$$

- (Q) A person deposits 10 lakh Rs in a bank at 8% interest rate compounded annually, find out maturity amount of his account after 12 years.

Ans. $P = 10,00,000 \text{ Rs.}$

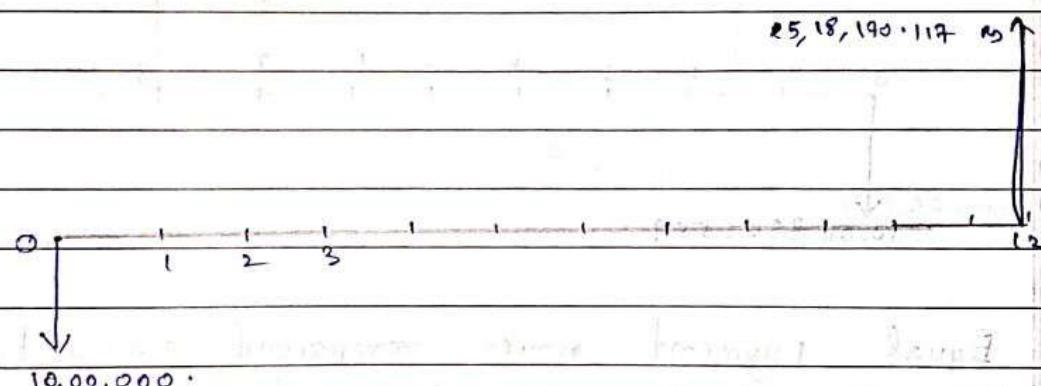
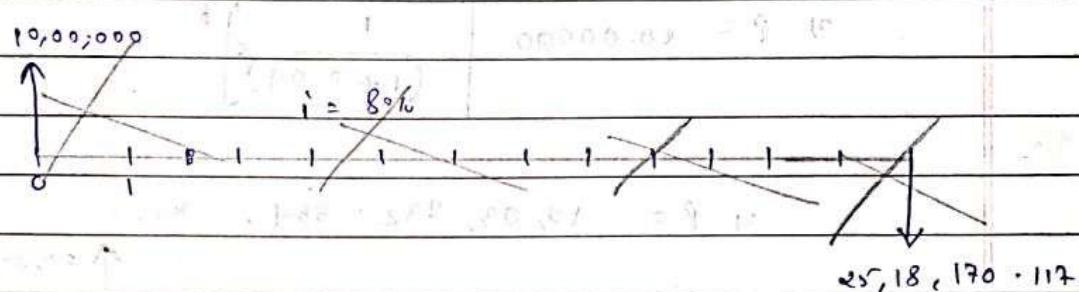
$i = 8\%$

$n = 12$



$$F = 10,00,000 \left(1 + 0.08 \right)^{12}$$

$$\Rightarrow F = 25,18,170.117 \text{ Rs.}$$



Simple payment present worth amount.

Here the objective is to find out the present value of the future sum after n th year compounded at an interest rate i .

$$P = P \left[\frac{1}{(1+i)^n} \right]$$

- (Q) A person needs 20 lakh rupees after 8 yrs. Find out how much money has he to deposit now if the interest rate 9% compounded annually.

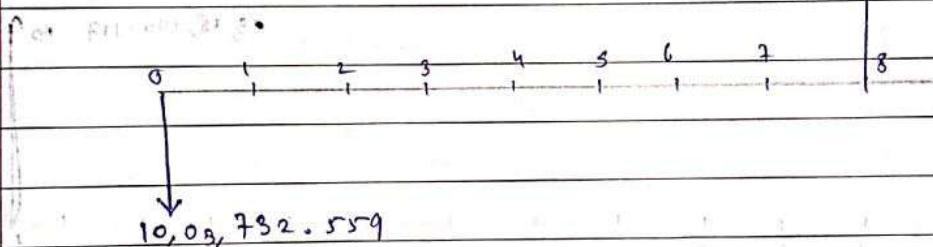
Sol:

$$P = F \left[\frac{1}{(1+i)^n} \right], \quad i = 0.09, n = 8$$

$$\therefore P = 20,00000 \left[\frac{1}{(1+0.09)^8} \right]$$

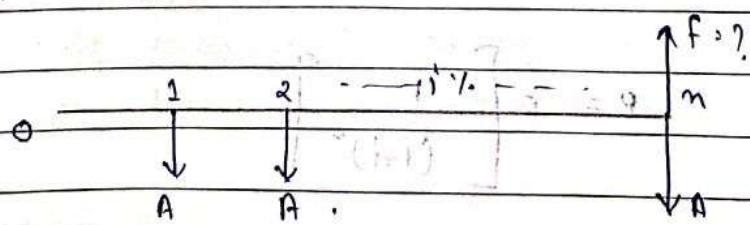
$$\therefore P = 10,03,732.559. \text{ Rs.}$$

For payment at 8 years, if amount is 20,00,000.



Equal payment series compound amount.

Here the objective is to find out future value of n equal payments i.e. to be made at the end of every year compounded till the end of n^{th} year compounded at an interest rate i .



$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$



(v) If a person invests equal amount of Rs. 20,000 at the end of every year till the end of 12th year at 9% interest rate compounded annually. Find out the future value of his account.

Ans: Given,

$$A = 20,000 \text{ Rs.}$$

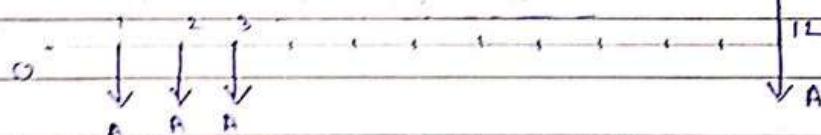
$$n = 12$$

$$i = 0.09.$$

$$F = 20,000 \times \left[\frac{(1+0.09)^{12} - 1}{0.09} \right]$$

$$\therefore F = 402,814.9 \text{ Rs.}$$

402,814.9.



$$A = 20,000 \text{ Rs.}$$

Equal payment series sinking fund

Here the objective is to find out n equal payments A, that is to be collected at the end of every year till the end of the nth year to realise a future sum compounded at an interest rate i.

$$A = F \left[\frac{i}{(1+i)^n - 1} \right]$$



(Q) A person needs 20 lakh rupees after 14 years. Find out how much equal amount of money a person should invest at the end of each year starting from end of the next year, if the i is 11% compounded annually.

Soln: Given,

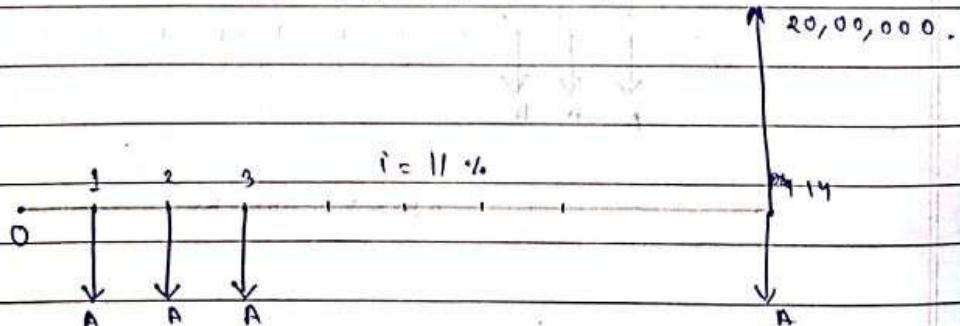
$$F = 20 \text{ lakh.}$$

$$n = 14.$$

$$i = 0.11.$$

$$\therefore A = \frac{20,00,000}{\left[\frac{0.11}{(1+0.11)^{14} - 1} \right]}$$

$$\Rightarrow A = 66456.4 \text{ Rs.}$$



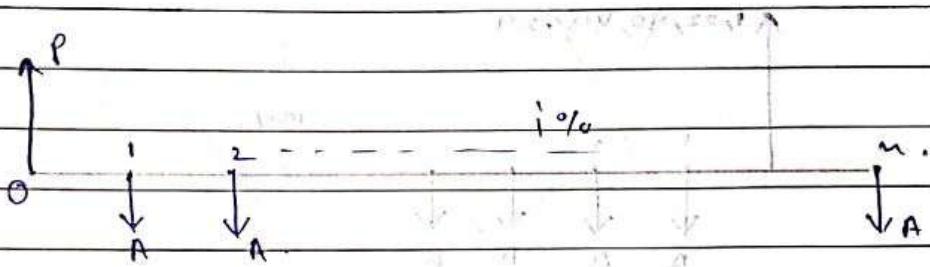
$$A = 66456.4 \text{ Rs.}$$

$$\frac{1 - (1+i)^{-n}}{i} = A$$



Equal present payment series present worth amount!

Here the objective is to find out present value of n equal payments. that is to be made at the end of every year till the end of n^{th} year compounded at an interest rate i .



$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

- (Q) A company wants to setup a reserve which will help it to have an annual equivalent amount of 17 lakh rupees. for the next 23 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 18% compounded annually. Find the single payment that must be made as the reserve amount now.

SOLY: Given,

$$A = 17,00,000.$$

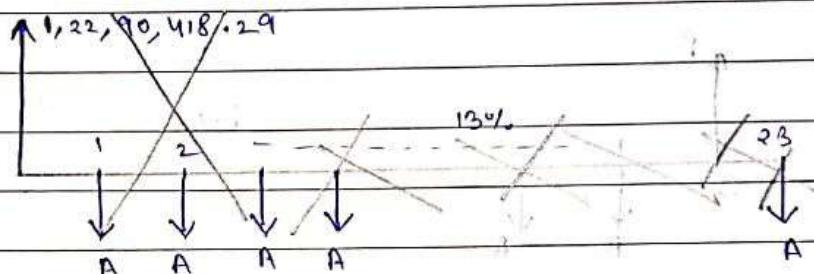
$$n = 23.$$

$$i = 18\% = 0.18.$$

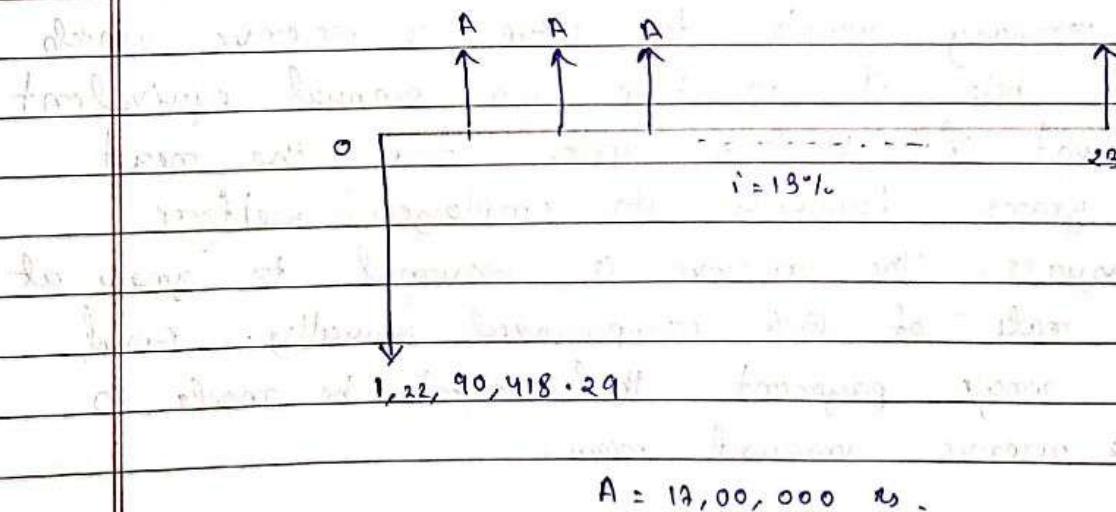


$$\Rightarrow P = 17,00,000 \times \left[\frac{(1+0.13)^{23} - 1}{0.13(1+0.13)^m} \right]$$

$$\text{Therefore } \Rightarrow P = 1,22,90,418.29 \text{ Rs.}$$



$$\Rightarrow A = 17,00,000.$$



$$A = 17,00,000 \text{ Rs.}$$



Equal payment series capital recovery amount.

Here the objective is to find out n equal payments that is to be recovered at the end of every year till the end of the nth year for the loan that is sanctioned (now compounded) at an interest rate i.

$$\therefore A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

- (Q) A company has taken a loan of 50 lakh rupees at 12% interest rate compounded annually. Find out how much installment amount the company has to pay at the end of every year starting from the end of the next year. If number of installment is 22.

Given,

$$P = 50,00,000.$$

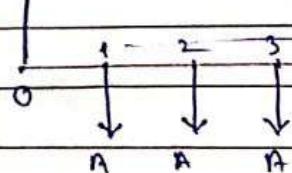
$$i = 12\%.$$

$$n = 22$$

$$\therefore A = 50,00,000 \times \left[\frac{0.12 \times (1 + 0.12)^{22}}{(1 + 0.12)^{22} - 1} \right]$$

$$\therefore A = 6,54,052.544 \text{ Rs.}$$

$\uparrow 50,00,000.$



$$i = 12\%.$$

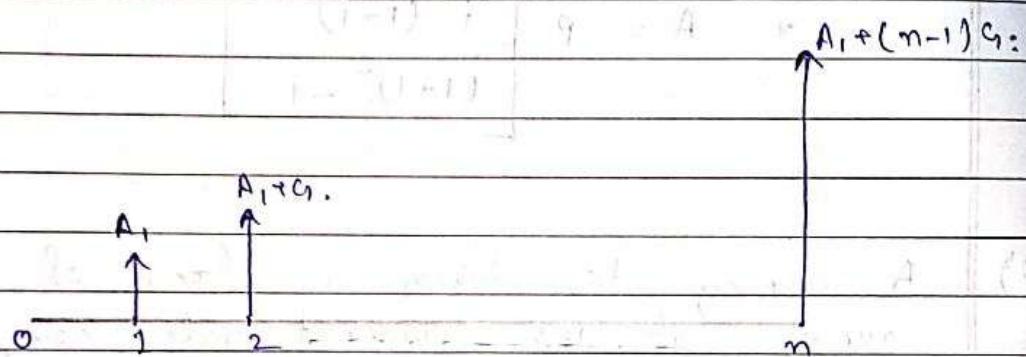
$\downarrow 22$

$$A = 6,54,052.544 \text{ Rs.}$$



Linear gradient Series annual equivalent amount.

Here the objective is to find out annual equivalent amount from a series of equal amount (A_1) starting in the first year with a fixed amount (G). increasing or decreasing thereafter at the end of every year following the first year till the end of the n^{th} year compounded at an interest rate i .



* Increasing Series :

$$A = A_1 + G \cdot \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1} \right]$$

Decreasing series :

$$A = A_1 - G \cdot \left[\frac{1}{i} \left(\frac{1}{(1+i)^n} - 1 \right) \right]$$



A person is planning for his retirement. He has 15 more yrs of service. He would like to deposit 10% of his salary which is 5000 Rs at the end of first year with an annual increase of 1000 rs. for the next 14 years with an interest rate of 10% compounded annually. Find the total amount at the end of the 15th year of above series.

Soln:

Given,

$$A_1 = 5000 \text{ Rs.}$$

$$n = 15$$

$$i = 10\%$$

$$C_1 = 1000 \text{ Rs.}$$

$$\text{Now, } A = A_1 + C_1 \left[\frac{1 - \frac{m}{i}}{(1+i)^m - 1} \right]$$

$$\therefore A = 5000 + 1000 \left[\frac{1 - \frac{15}{0.1}}{(1+0.1)^{15} - 1} \right]$$

$$\therefore A = 10,298.933 \text{ Rs.}$$

Now,

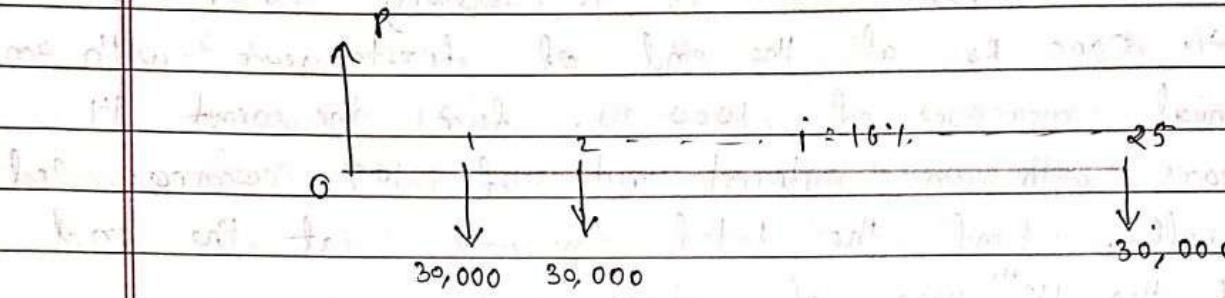
$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$\therefore F = 10,298.933 \times \left[\frac{(1+0.1)^{15} - 1}{0.1} \right]$$

$$\therefore F = 16,454.89 \text{ Rs.}$$



- (Q) From the following cash flow diagram find out the present value.



Soln: Given,

$$A = 30,000$$

$$i = 16\% = 0.16$$

$$n = 25$$

$$\therefore P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\Rightarrow P = 30,000 \times \left[\frac{(1.16)^{25} - 1}{0.16 \times (1.16)^{25}} \right]$$

$$\therefore P = 1,82,912.759 \text{ Rs.}$$



Revenue.

It refers to the income earned by a producer by selling different units of output at different prices.

Types of Revenue.

- ① Total Revenue (TR).
- ② Marginal Revenue (MR)
- ③ Average Revenue (AR).

Total Revenue :

It refers to the total income earned by a producer by selling different units of output at different prices.

$$TR = P \times Q. \quad \text{---(1)}$$

P → selling price per unit of the commodity.

Q → Number of units of output sold.

Marginal Revenue :

It refers to the net addition to the total revenue by selling one extra unit of output.

$$MR = \frac{d(TR)}{dQ}.$$

Average Revenue :

It refers to the total income earned per unit of output sold.

$$AR = \frac{TR}{Q} \Rightarrow TR = AR \times Q \quad \text{---(2)}$$

Comparing (i) and (ii), answer A

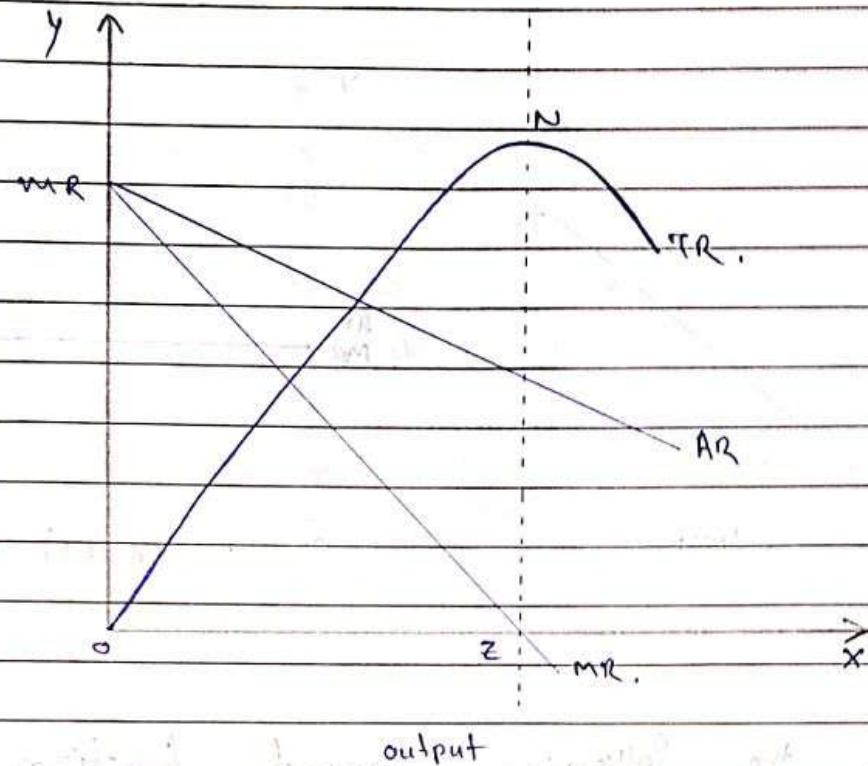
From the following table find out total revenue and marginal revenue. (i) Answer Total (i)

units of output sold (Q)	Average Revenue (P)
1	16
2	15
3	14
4	13
5	12
6	11
7	10
8	9
9	8
10	7

SOLY:

TR when MR = 0

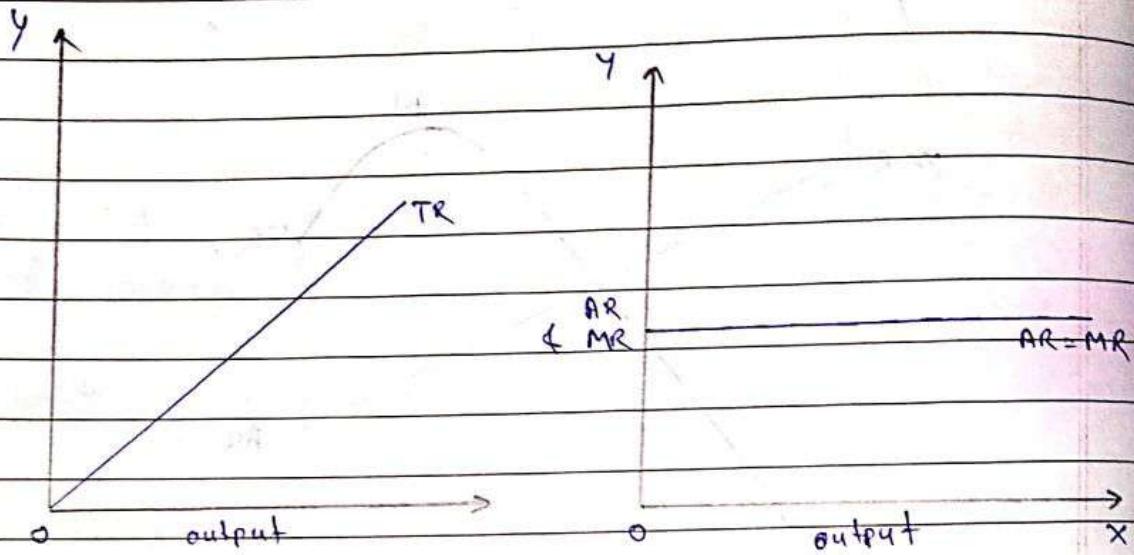
16.	16.	TR is maximum when MR is zero,
30.	14	
42.	12	
52	10.	
60	8	
66.	6	
70.	4	
72	2	
72.	0.	
70.	-2	



- (Q) Find the out from the following table for TR and MR.

Q	AR	TR	MR
1	40	40	40
2	40	80	40
3	40	120	40
4	40	160	40
5	40	200	40
6	40	240	40
7	40	280	40.

when AR remains constant, then $AR = MR$.



Q) From the following demand function find out.

- i) TR and MR.
- ii) Price and quantity, if MR = 0.
- iii) Price and quantity if TR is maximum.

soltz:

$$Q = 60,000 - 20P$$

soltz:

$$i) TR = 20P = 60,000 - 0$$

$$\Rightarrow P = 3000 - \frac{Q}{20}$$

now

$$TR = PQ = 3000Q - Q^2$$

$$MR = \frac{d(TR)}{dQ} = 3000 - \frac{Q}{10}$$



ii) If $MR = 0$

$$\therefore 3000 - \frac{Q}{10} = 0$$

$\therefore Q = 30,000$

$$P = 1500$$

iii) TR is maximum when MR is 0.

Relationship between Revenue and Elasticity.

To prove: $MR = AR \left(1 - \frac{1}{e} \right)$.

L.H.S.: $MR = \frac{d(TR)}{dQ}$.

$$\Rightarrow \frac{d(PQ)}{dQ}$$

R.H.S. = $P \left(1 - \frac{1}{e} \right)$.

$$\Rightarrow \frac{dP \cdot Q + P \cdot 1}{dQ}$$

$\therefore TR = \frac{Q}{P} \left\{ 1 - \frac{1}{\left(\frac{Q \cdot P}{dP} + \frac{P}{Q} \right)} \right\}$

$$\Rightarrow P \left\{ 1 + \frac{dP \cdot Q}{dQ \cdot P} \right\}$$

$\therefore TR = \frac{Q}{P} \left\{ 1 - \frac{dP \cdot Q}{P \cdot dQ} \right\}$.

$$\Rightarrow AR \left\{ 1 + \frac{1}{(-e)} \right\}$$

Date / /

$$\Rightarrow MR = AR \left\{ 1 - \frac{1}{e} \right\}.$$

- (Q) Find out MR if price of a commodity is 11 Rs/unit and coefficient of elasticity is 3.

Sol:

Given,

$$AR = 11 \text{ Rs/unit}.$$

$$\epsilon = 3.$$

$$MR = 11 \left\{ 1 - \frac{1}{3} \right\}.$$

$$\therefore MR = 7.33.$$

(Ans)

(1-1) $\times 11 = 22.2$

$$\left\{ 1 - \frac{1}{3} \right\} = \left\{ \frac{2}{3} \right\}$$

$$\left\{ \frac{2}{3} \times 11 \right\} = 7.33$$



Utility

It refers to want satisfying capacity of a commodity.

Types of utility.

- (+) Positive utility
- (1) Total utility (TU)
- (2) Marginal utility (MU).

Total utility (TU)

It refers to the total satisfaction that a consumer gets by consuming various units of a commodity.

Marginal Utility (MU).

It refers to the net addition to the total utility by consuming one extra unit of a commodity.

$$MU_m = TU_m - TU_{m-1}$$

In derivative form:

$$MU_m = \frac{d(TU)}{dm}$$

where m is a commodity.

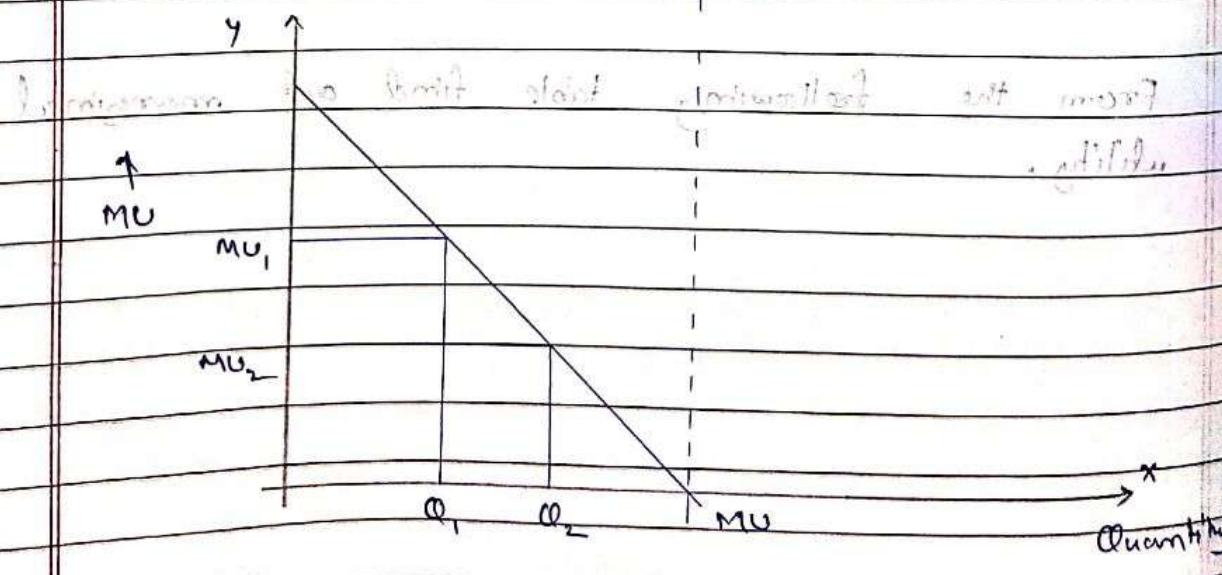
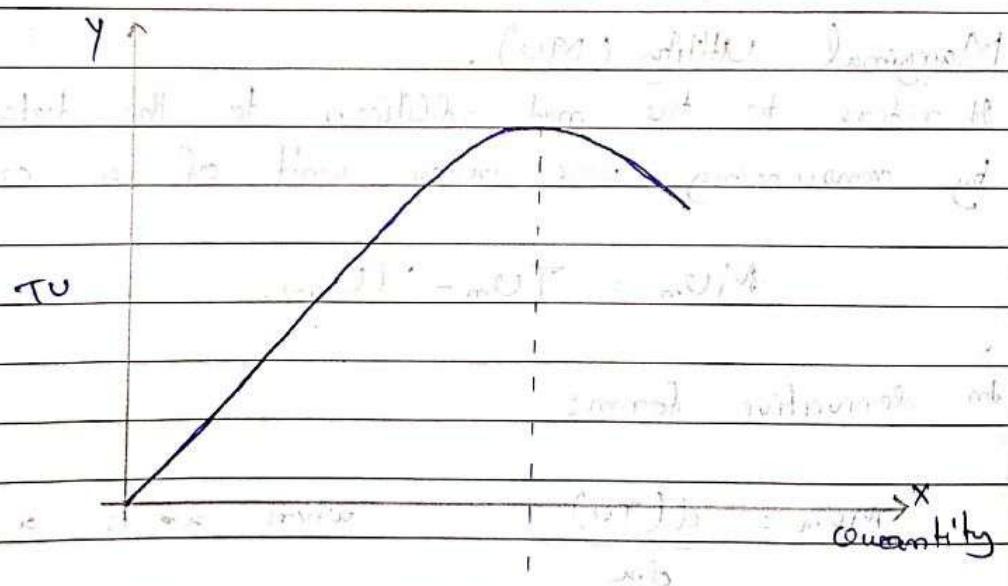
- (Q) From the following table find out marginal utility.

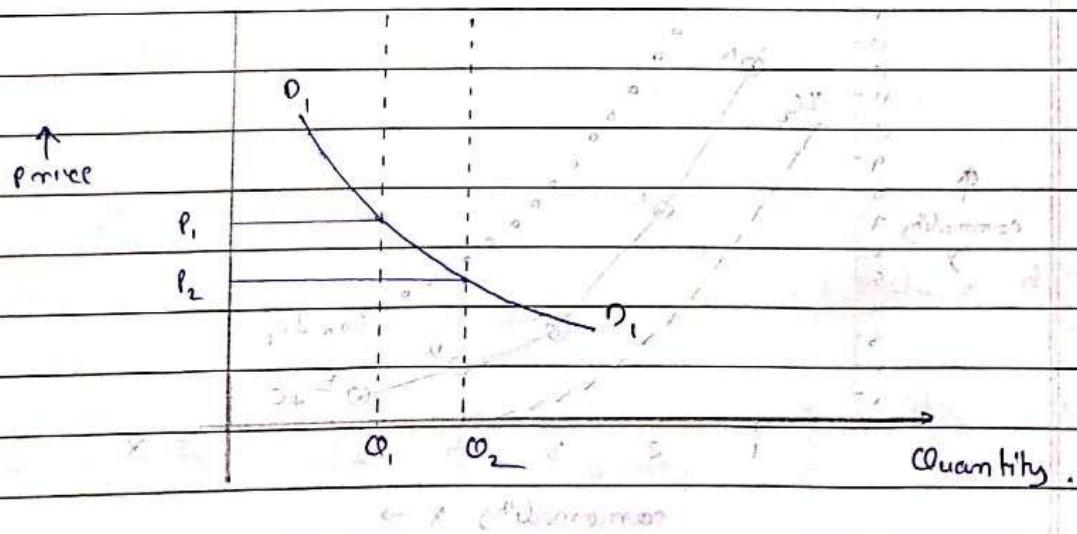
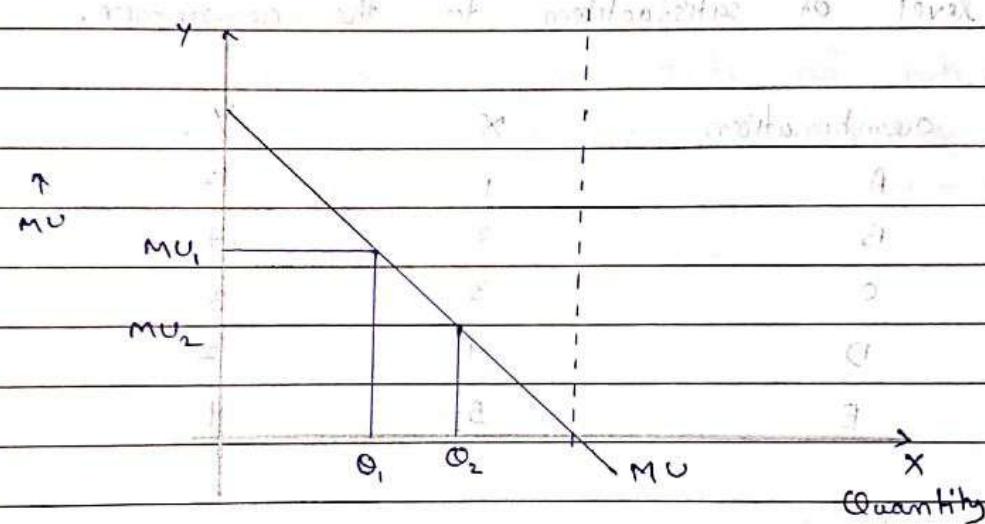
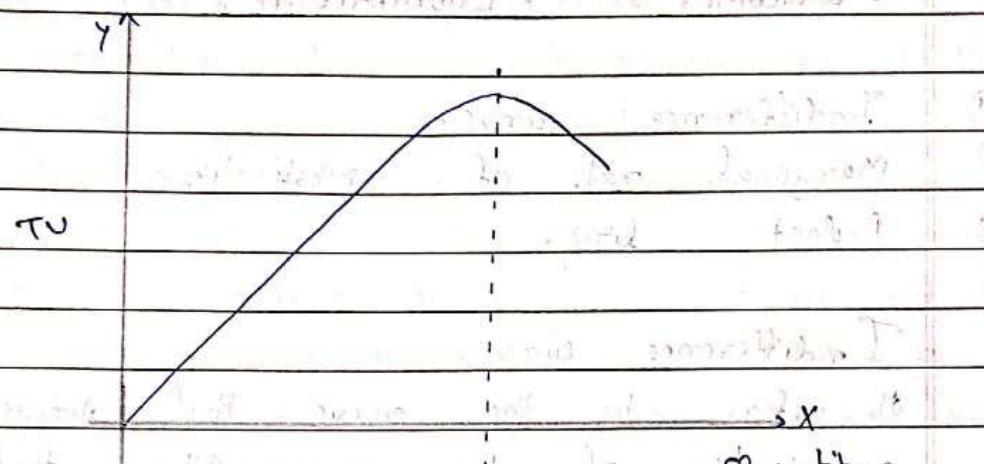
Quantity TU MU

1	10	10
2	18	8
3	24	6
4	28	4
5	30	(at) 2 additional units
6	30	0 (no additional gain)
7	28	-2

(or) Total Utility

Total utility will be maximum when the marginal utility will be zero.





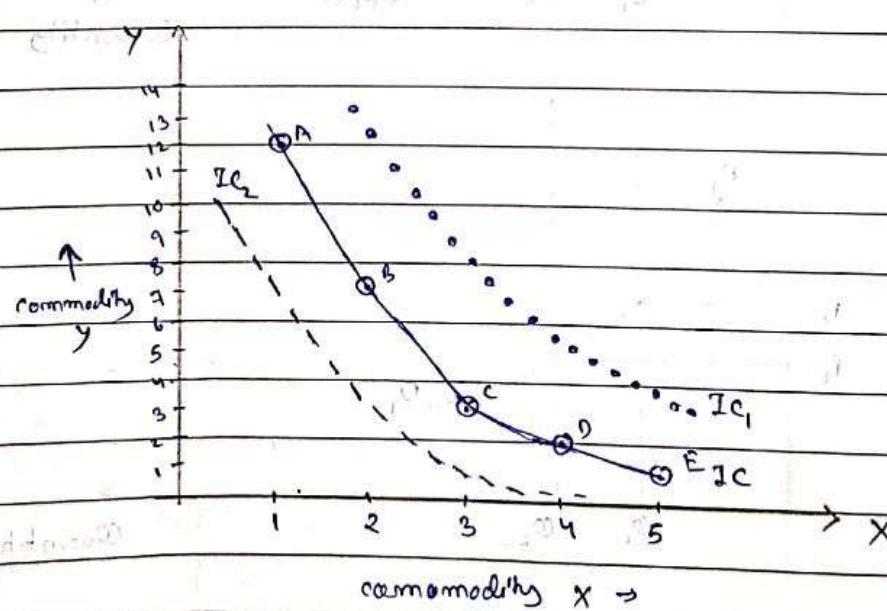
Consumer's Equilibrium :

- ① Indifference curve.
- ② Marginal rate of substitution.
- ③ Budget line.

Indifference curve.

It refers to the curve that shows various combinations of two commodities that give equal level of satisfaction to the consumer.

combination	X	Y
A	1	12
B	2	7
C	3	3
D	4	2
E	5	1



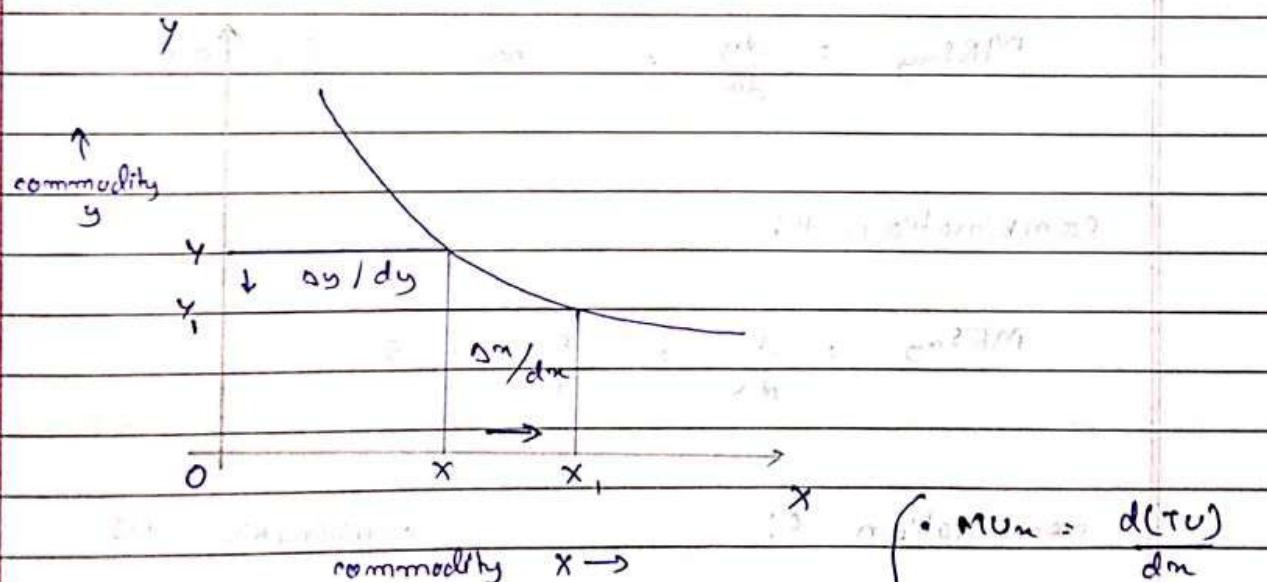


Marginal rate of substitution:

It refers to the rate at which number of units of one commodity substituted to have one more unit of other commodity.

$MRS_{xy} \rightarrow$ It refers to the rate at which number of units of commodity y substituted to have one more unit of commodity x.

$MRS_{yx} \rightarrow$ It refers to the rate at which number of units of commodity x substituted to have one more unit of commodity y.



* slope of the indifference curve.

$$MRS_{xy} = \frac{dy}{dx} = \frac{MUm}{MU_y}$$

$$MU_y = \frac{d(TU)}{dy}$$

$$\Rightarrow dy = \frac{d(TU)}{MU_y}$$



$$MRS_{xy} = \frac{dy}{dx} = \frac{MU_y}{MU_x}$$

From the following table find out MRS_{xy} .

combination. x y MRS_{xy}

A	1	12	-
B	2	7	5
C	3	3	4
D	4	2	2
E	5	1	5

Sol:

combination A:

$$MRS_{xy} = \frac{dy}{dx} = \text{no. mo. dy from}$$

combination B:

$$MRS_{xy} = \frac{dy}{dx} = \frac{5}{1} = 5$$

combination C:

$$MRS_{xy} = \frac{4}{1} = 4$$

combination D:

$$MRS_{xy} = \frac{1}{1} = 1$$

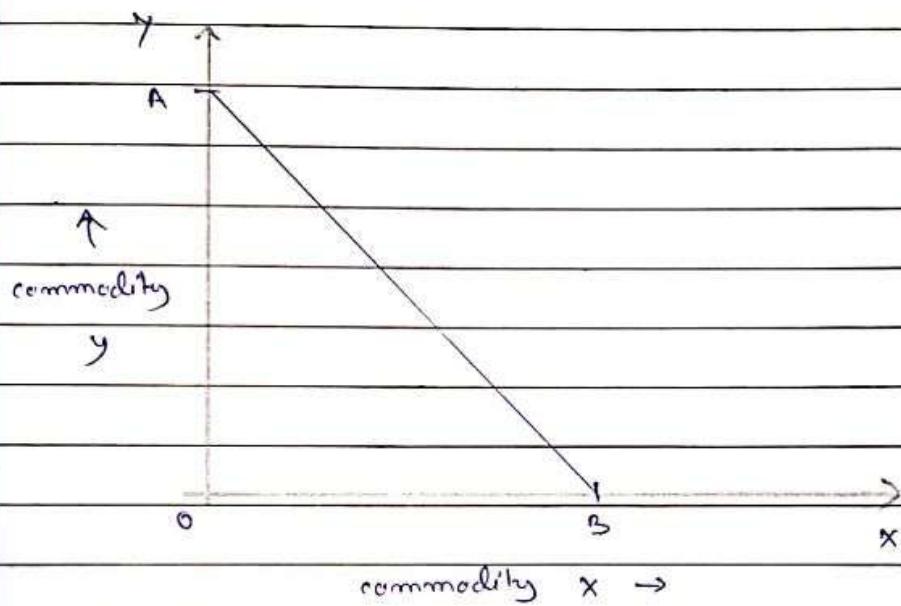
combination E:

$$MRS_{xy} = \frac{1}{1} = 1$$



Budget line :

It refers to the line that shows various combinations of two commodities that a consumer can purchase with a given level of income.



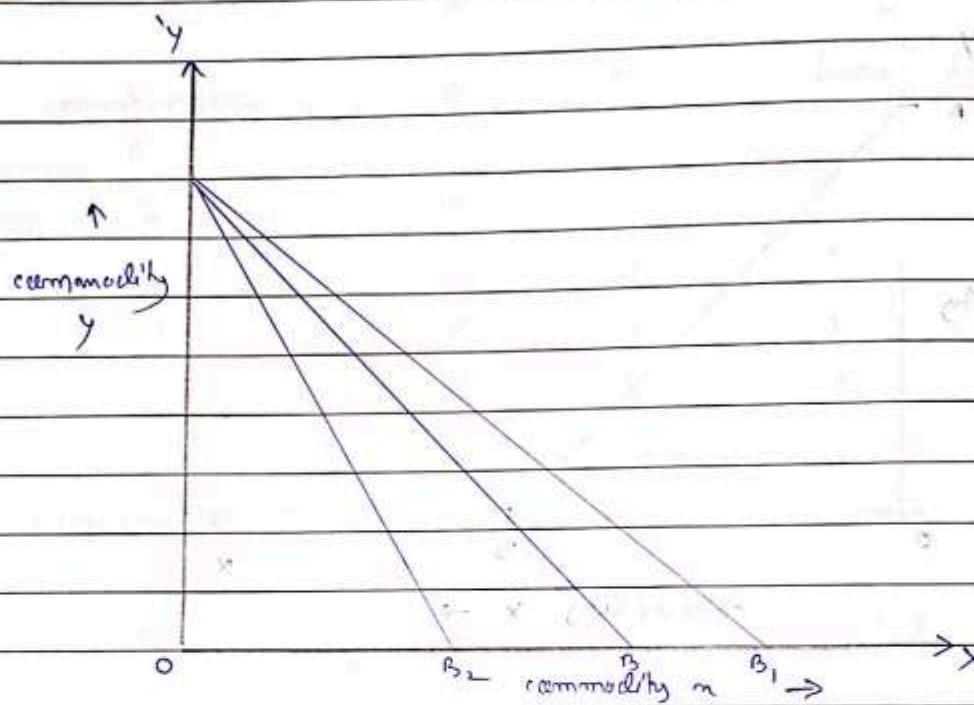
* Equations of the budget line :

$$M = P_x \cdot q_x + P_y \cdot q_y.$$

* Slope of the budget line :

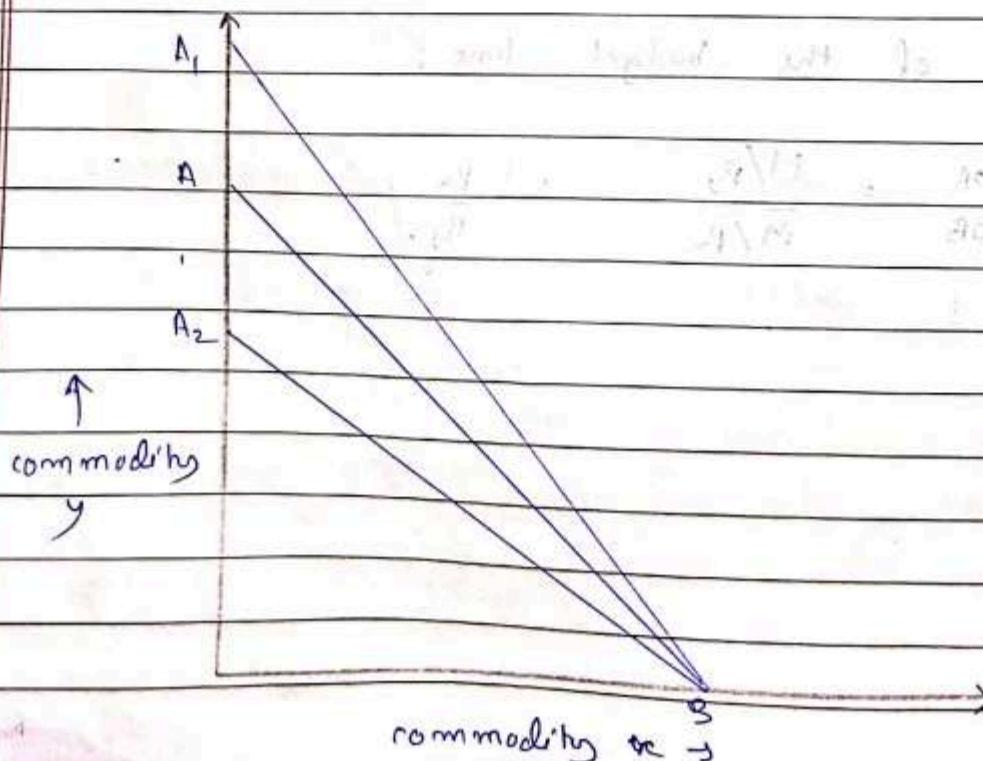
$$\frac{OA}{OB} = \frac{M/P_y}{M/P_x} = \frac{P_x}{P_y}.$$

Shift in the budget line.
If price of commodity x changes that of commodity y and income remaining constant.



Shift in budget line.

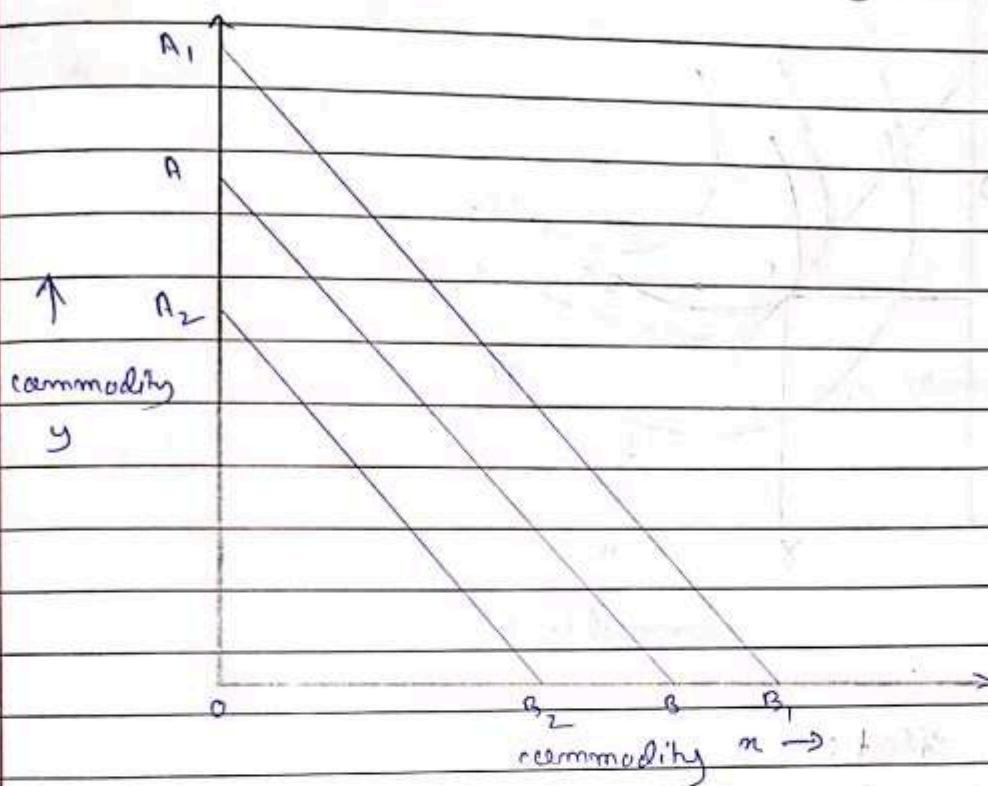
Shift of price of commodity y changes that of commodity x and income remaining constant.





Shift in the budget line.

If income of the consumer changes price of both the commodities remaining constant.



Conditions

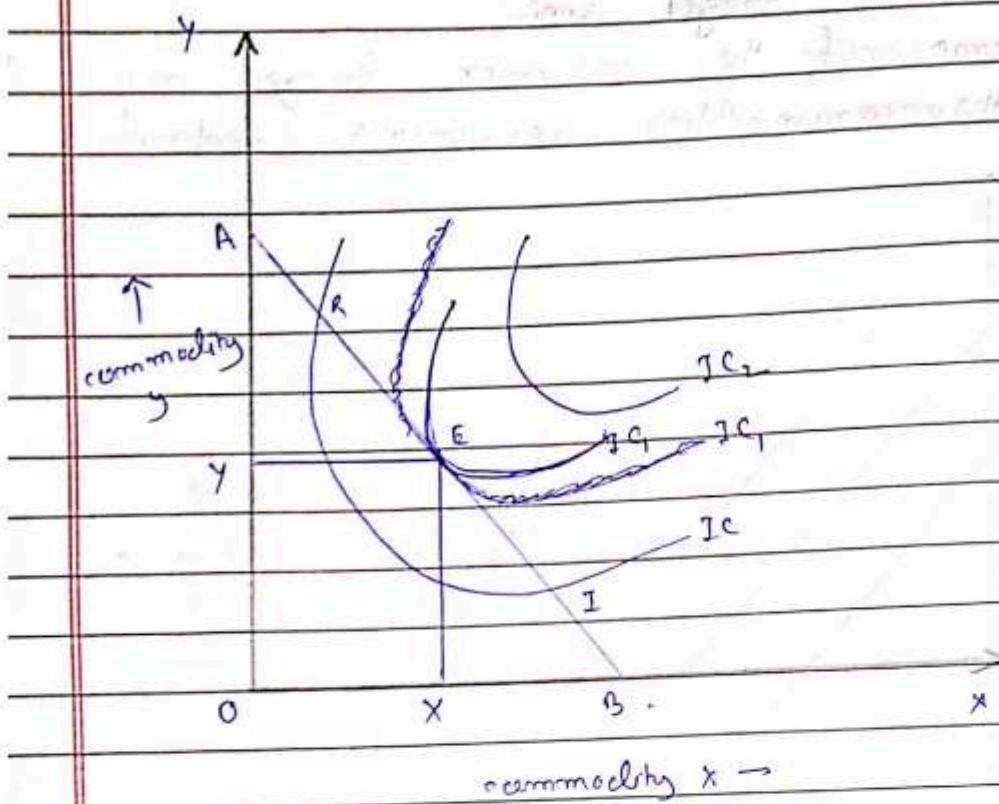
conditions of a consumer to be in equilibrium

① Slope of the indifference curve should be equal to slope of budget line.

② Indifference curve should be convex at equilibrium point.

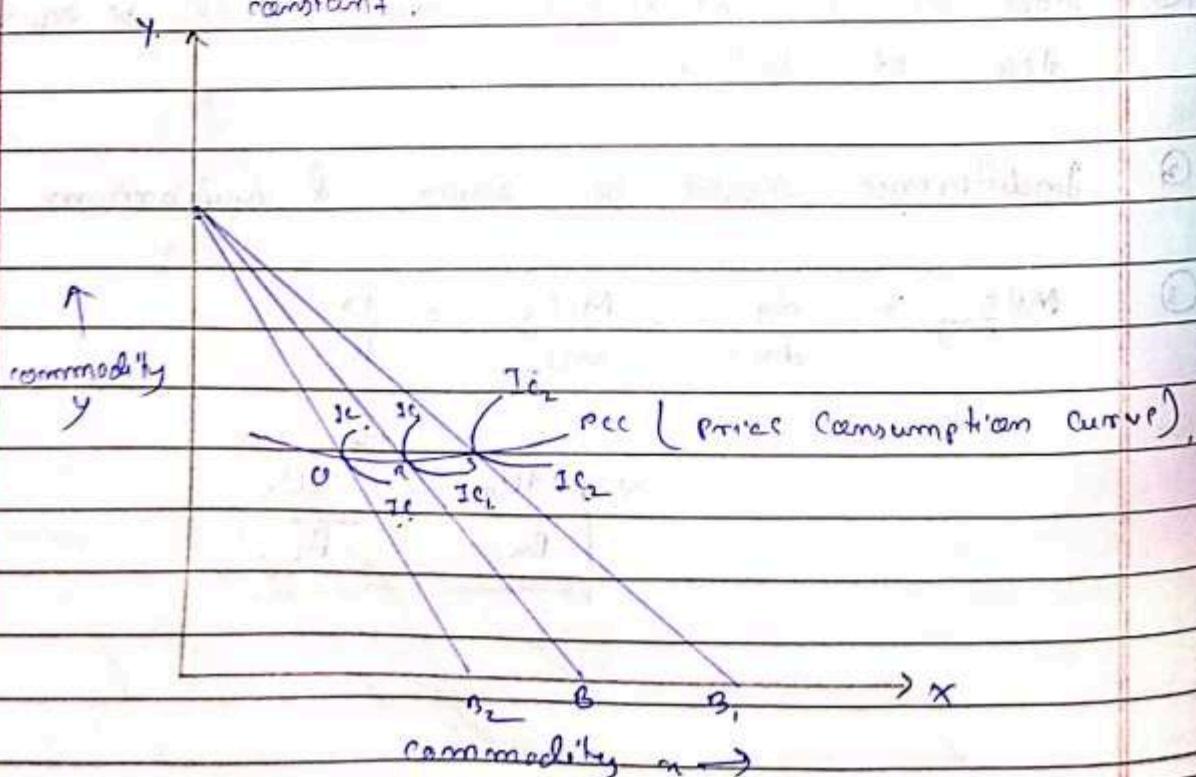
$$\text{MRS}_{xy} = \frac{dy}{dx} = \frac{MU_m}{MU_y} = \frac{P_m}{P_y}$$

$$\Rightarrow \left| \frac{MU_m}{P_m} = \frac{MU_y}{P_y} \right|$$



Price Effect:

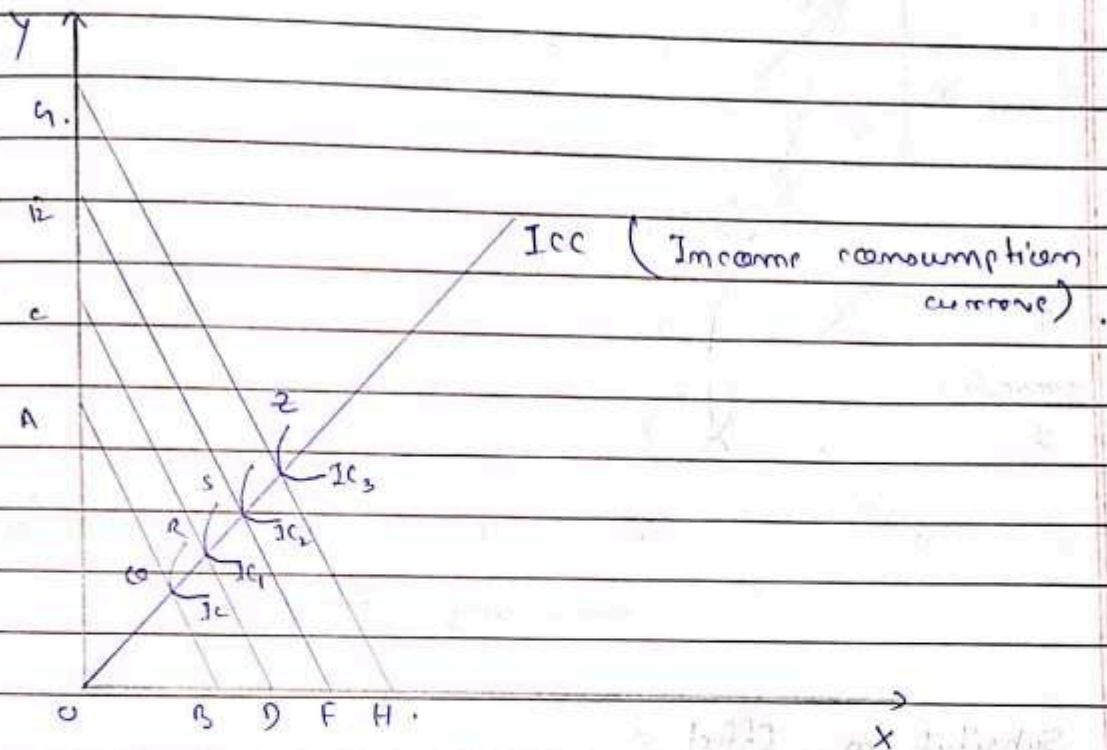
It refers to effect of change in price of one commodity on purchase of other commodities, price of commodity and income remaining constant.



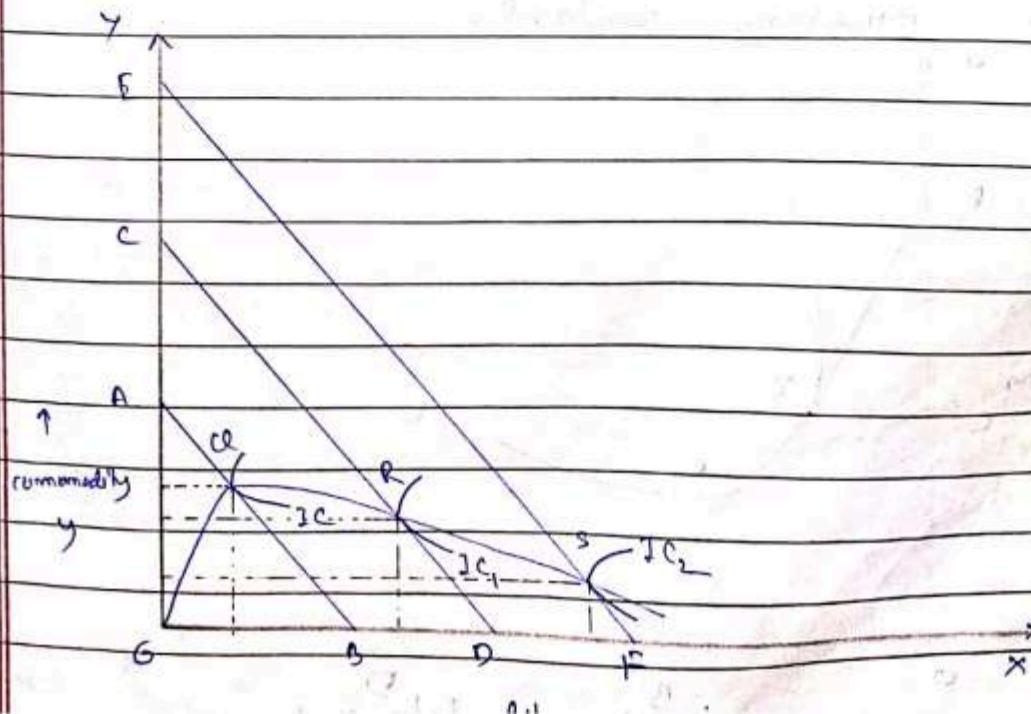


Income Effect

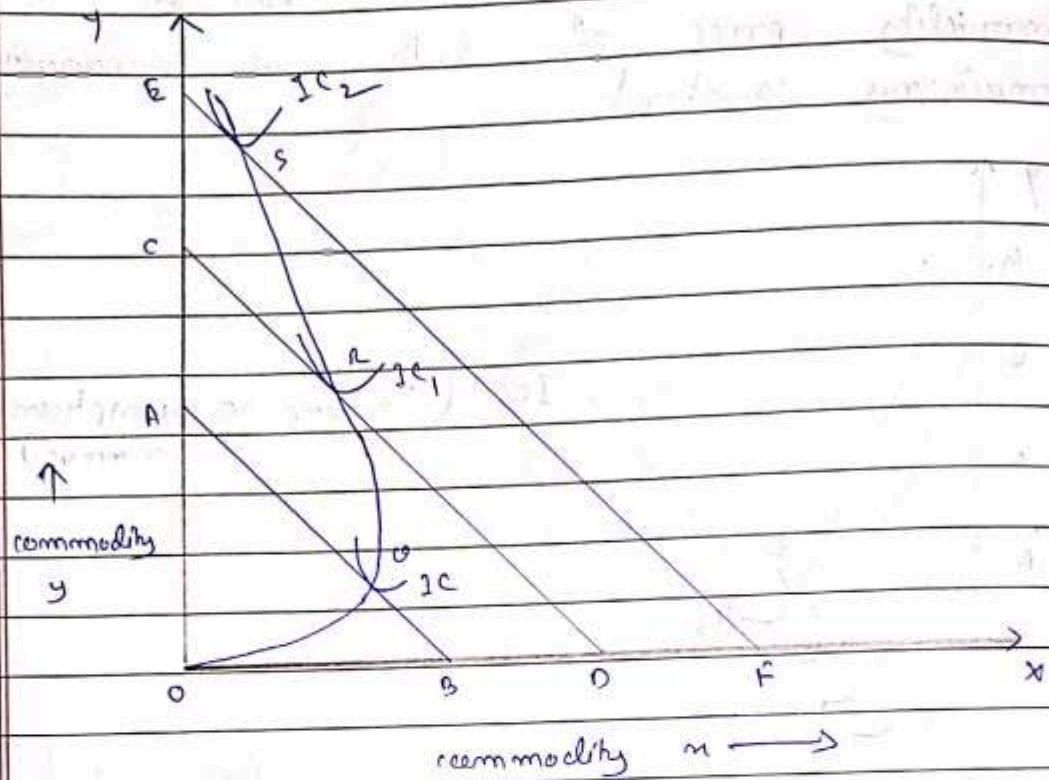
If refers to the effect of change in income for of consumer can purchase of commodity price of both the commodity remaining constant.



Income consumption curve if commodity y is inferior good.

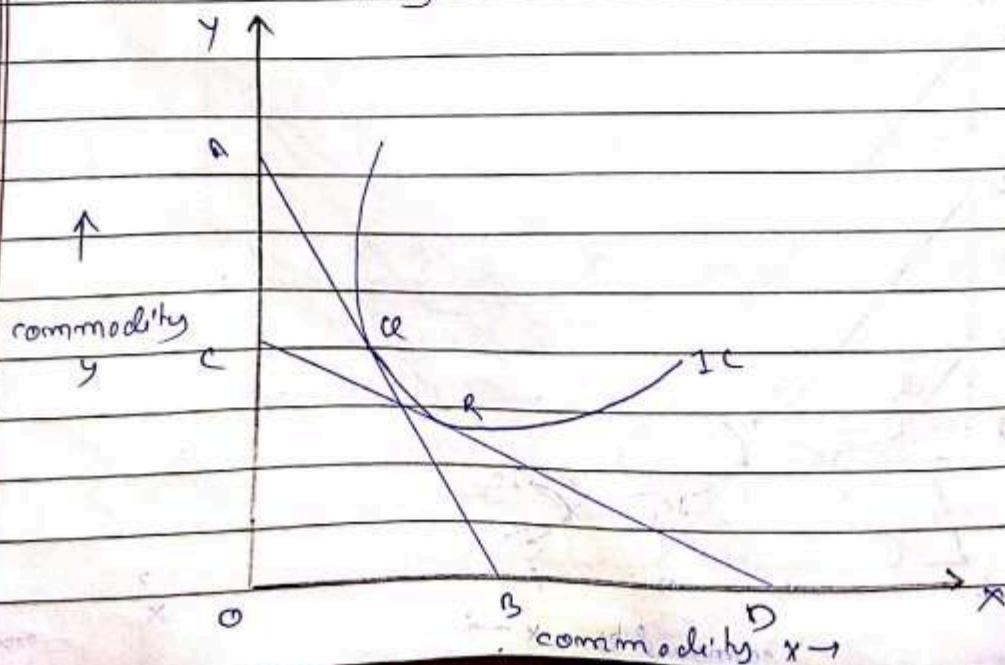


I income consumption curve if commodity m is inferior good.



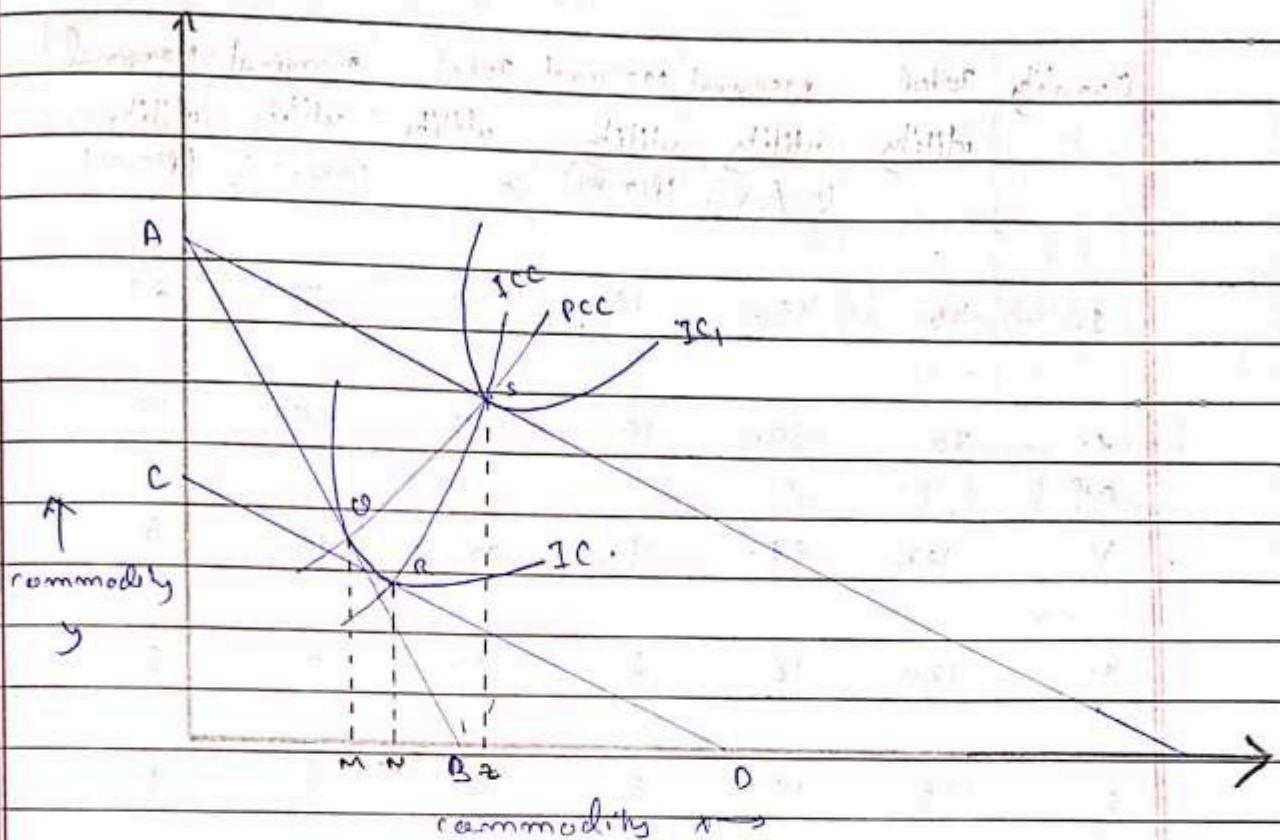
Substitution Effect :

It refers to the effect of fall in price of one commodity and increase in price of other commodity on purchase of commodities income remaining constant.





Income Effect, Substitution Effect and Price effect.



$MZ = \text{Price Effect (PE)}$.

$MN = \text{Substitution Effect (SE)}$

$NZ = \text{Income Effect (IE)}$.

$$\Rightarrow MZ = MN + NZ$$

$$\Rightarrow PE = SE + IE$$

From the following table.

If price of commodity x is 3 Rs/unit and commodity y is 2 Rs/unit.

- If income of consumer is 10 Rs, how many units of both the commodity should be purchased.
- If the income of the consumer increases for

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18 Qs. Find out the optimal consumption bundle.

Quantity	Total utility	marginal utility (MU/ $\mu_1 \times 3$)	Marginal utility (P/ μ_1)	Total utility	marginal utility (MU/ μ_2), (P/ μ_2)	Marginal utility (P/ μ_2)
1.	45	45.	15.	40	40	20
2.	75	30.	10.	60	20	10
3.	102	27.	9	72	12	6
4.	120	18	6	82	10	5
5	135	15	5	90	8	4
6.	144.99.	9.99	3.33.	92	2	1

★ Optimal

$$M = (P_1 \cdot q_1) + (P_2 \cdot q_2)$$

$$3 \cdot 10 = 3 \cdot 0 + P_2 \cdot 0$$

$$\therefore q_{\mu_1} = q_{\mu_2} = 12, \text{ i.e., } 12 \text{ units}$$

At 2 unit.

$$\text{If } M_U_{\mu_1} = M_U_{\mu_2}, \text{ i.e., } 10 = 10 \text{ or } \frac{P_1}{P_2} = 1$$

So, 2 is optimal condition.



now, $M = 18$

$$18 = 3 \textcircled{O} + 2 \textcircled{O}$$

$$\therefore P_m = 4, P_y = 3, \text{ or } P_m = 2, P_y = 6.$$

At unit y of m and unit \textcircled{O} of y .

$$\frac{M_{U_m}}{M_{U_y}} = \frac{6}{6} = 1$$

\therefore It is the optimal condition.

Money value of all the final goods and services produced within the domestic territory of a country during a given year.

→ GDP

→ GNP

→ NNP

→ NNP

→ GNP_{mp}

→ GNP_{fc}

→ NNP_{fc}.

GDP

Money value of all the final goods and services produced within the domestic territory of a country during a given year.

$$GDP = C + I + G + (X - M)$$

↓ ↓ ↓ ↓
 consumption government expenditures export import
 investment

Date _____

GNP (Gross National Product).

$$GNP = GDP + NFA$$

(Net factor income stream abroad.)

NDP (Net Domestic Product).

$$GDP - \text{Depreciation}.$$

NNP (Net National Product).

$$GNP - \text{Depreciation}.$$

GNP_{MP} (Gross National Product at Market Price).

$$GDP + NFA.$$

GNP_{FC} (Gross National Product at Factor Cost).

$$GNP_{MP} - \text{Net indirect taxes}.$$

$$\Rightarrow GNP_{MP} - \text{tax} - \text{subsidy}.$$

NNP_{FC} (Net National Product at factor cost).

$$NNP_{FC} = GNP_{FC} - \text{Depreciation}.$$

$$= GNP_{MP} - \text{Net indirect taxes} - \text{Depreciation}$$



Q From the following data GNP_{MP} and NNP_{FC}.

- * Net Indirect taxes \rightarrow 15
- * NFIA = 50.
- * Consumption of Fixed capital = 10.
(depreciation)
- * GDP_{MP} = 200.

Sol: $GNP_{MP} = GDP_{MP} + NFIA$.

$$\Rightarrow GNP_{MP} = 200 + 50 \\ = 250, \text{ crores.}$$

$$NNP_{FC} = GNP_{FC} - \text{Depreciation.}$$

$$\Rightarrow GNP_{MP} - \text{Net Indirect taxes} - \text{Depreciation} \\ \text{1. } 250 - 15 - 10 \\ \Rightarrow 225 \text{ crores.}$$

Q From the following inform find out
GNP_{FC} and NNP_{FC}
Indirect taxes \rightarrow 10000
Subsidy \rightarrow 1500.
depreciation \rightarrow 1700.
NFIA \rightarrow 3000.
NDP_{MP} \rightarrow 76,000.

Sol: $GNP_{FC} = GNP_{MP} - \text{Net indirect taxes}$
 $= GDP_{MP} + NFIA - (\text{Indirect taxes} - \text{subsidy})$
 $\Rightarrow NDP_{MP} + \text{Depreciation} + NFIA - IT - \text{subsidy}$
 $\Rightarrow 76,000 + 1700 + 3000 - (10000 - 1500)$
 $\Rightarrow 69,200 \text{ crores.}$

Date ___ / ___ / ___

$$\text{NNP}_{Fc} = \text{GNP}_{Fc} - \text{Depreciation}$$
$$= 69,200 - 1700$$
$$= 67,500 \text{ crores}$$

at 10% will be 67.5 million
(approx)

10% of 67.5 million = 6.75 million

10% of 6.75 million = 0.675 million

10% of 0.675 million = 0.0675 million

10% of 0.0675 million = 0.00675 million

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10% of 0.0000000000000000000000675 million = 0.00000000000000000000000675 million

salvage value is money received after scrapping a used product.

Date 04/09/2019



Comparison of Alternatives:

- ① Present Worth method,
- ② Future Worth method,
- ③ Annual worth method,
- ④ Rate of return method,
- ⑤ Cost Benefit Analysis.

Present Worth method

In case of one project.

$$* \text{NPW (i\%)} = PW(B) - PW(C).$$

If $NPW > 0$, project will be selected.

If $NPW < 0$, project will be rejected.

If $NPW = 0$, project may or may not be selected.

* In case of mutually exclusive project.

Methods:

- ① Revenue Based method,
- ② Cost Based method.

Revenue Based method,

It refers to that method where all types of benefit like profit, revenue, income or earning and salvage value will be assigned with +ve sign and all type of cost i.e. payment or investment will be assigned with -ve sign.



Cost Based Method.

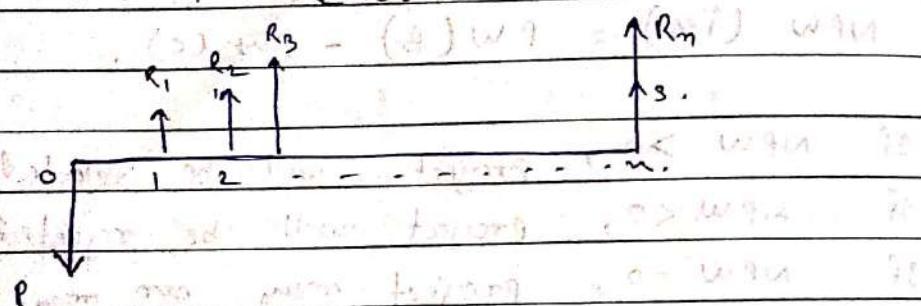
It refers to that method where all types of benefit will be assigned with +ve sign and cost will be assigned with -ve sign.

Comparison of Alternatives.

(1) Present worth method

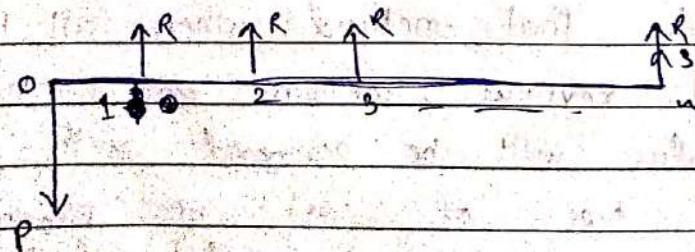
(2) Revenue based method.

Present worth method. ① Different series.



$$\text{NPW}(i\%) = -P + R_1 \left[\frac{1}{(1+i)^1} \right] + R_2 \left[\frac{1}{(1+i)^2} \right] + R_3 \left[\frac{1}{(1+i)^3} \right] + \dots + R_m \left[\frac{1}{(1+i)^m} \right] + S \left[\frac{1}{(1+i)^n} \right].$$

② Equal Payment series.



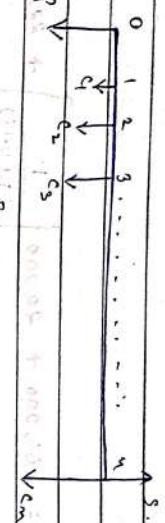
$$\text{NPW}(i\%) = -P + R \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] + S \left[\frac{(1+i)^{n-3} - 1}{i(1+i)^n} \right].$$

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Cost Based Method:

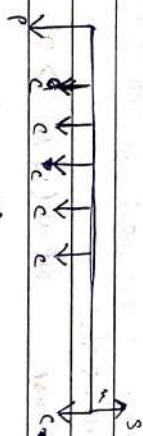
* Different series :



$$NPW(i\%) = P + C_1 \left[\frac{1}{(1+i)^1} \right] + C_2 \left[\frac{1}{(1+i)^2} \right] + \dots + C_n \left[\frac{1}{(1+i)^n} \right] - S \left[\frac{1}{(1+i)^m} \right].$$

$$NPW(i\%) = P + C_1 \left[\frac{1}{(1+i)^1} \right] + C_2 \left[\frac{1}{(1+i)^2} \right] + \dots + C_n \left[\frac{1}{(1+i)^n} \right] - S \left[\frac{1}{(1+i)^m} \right].$$

* Equal payment series: $S = C_1 + C_2 + \dots + C_n$



$$NPW(i\%) = P + C \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] - S \left[\frac{1}{(1+i)^m} \right].$$

(ii) From the following table find out project

will be selected or rejected if $i = 18\%$. compounded annually.

Timeline of Year cash flows

Year 0 - 65,000

1 20,000

2 22,000

3 30,000

4 36,000

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so,

$$NPW(i\%) = -P + R_1 \left[\frac{1}{(1+i)^1} \right] + R_2 \left[\frac{1+i}{(1+i)^2} \right] + \dots$$

so,

$$NPW(i\%) = -65,000 + 28,000 \left[\frac{1}{(1+0.18)^1} \right] + 22,000 \left[\frac{1}{(1+0.18)^2} \right]$$

$$+ 30,000 \left[\frac{1}{(1+0.18)^3} \right] + 36,000 \left[\frac{1}{(1+0.18)^4} \right]$$

$$\text{or } NPW(1\%) = -65000 + 16949.152 + 15800.057$$

$$+ 18258.926 + 18568.399$$

$$\therefore 4,576.53$$

selected because $NPW(18\%) > 0$.

- (c) From the following table find out the project is financially feasible or not on the basis of present worth method if $i = 20\%$, compounded annually.

End of year	cash flows
0	-40,00,000
1	5,00,000
2	5,00,000
3	5,00,000
4	5,00,000
5	"
6	"



7

8

9

10

11

12

13

14

15

Solve!

$$NPW(i\%) = -P + R \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$1 - 40,00,000 + 5,00,000 \left[\frac{(1+0.2)^{15} - 1}{0.2 (1+0.2)^{15}} \right]$$

$$= -166,226.34679$$

It is not feasible because,

$$NPW(20\%) < 0$$

- (c) From the following table which technology will be selected from Present Worth Method

if $i = 16\%$ compounded annually.

Date / /



Technology	Initial Outlay	Annual Income	Life in yrs.
1	10,00,000	5,00,000	15 yrs
2	18,00,000	7,00,000	15 yrs
3	16,00,000	6,00,000	15 yrs.

* revenue based method

Sol: Technology 1:

NPW (16%).

$$= -10,00,000 + 5,00,000 \left[\frac{(1+0.16)^{15} - 1}{0.16 (1+0.16)^{15}} \right]$$

$$\Rightarrow 17,87,728.081.$$

Technology 2:

NPW (16%).

$$= -18,00,000 + 7,00,000 \left[\frac{(1+0.16)^{15} - 1}{0.16 (1+0.16)^{15}} \right]$$

$$\Rightarrow 21,02,819.314$$

Technology 3:

NPW (16%).

$$= -16,00,000 + 6,00,000 \left[\frac{(1+0.16)^{15} - 1}{0.16 (1+0.16)^{15}} \right]$$

$$\Rightarrow 12,43,273.698.$$

So, Tech 2 will be selected because it has greatest NPW.

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from the following table. Find which machine will be selected by Present worth method, if $i = 17\%$. compounded annually.

Machine	Initial cost	Service life in yrs	Annual op. & maint. cost	salvage value
A	6,00,000	20 yrs.	35,000	15,000
B	7,00,000	20 yrs.	40,000	12,000

Soln:

Machine A :

$$NPW(17\%) = 6,00,000 + 35,000 \left[\frac{(1+0.17)^{20} - 1}{0.17(1+0.17)^{20}} \right] - 15,000 \times \left[\frac{1}{(1+0.17)^{20}} \right]$$

$$\Rightarrow NPW(17\%) = 7,96,322.664$$

Machine B :

$$NPW(17\%) = 7,00,000 + 40,000 \left[\frac{(1+0.17)^{20} - 1}{0.17(1+0.17)^{20}} \right] - 12,000 \times \left[\frac{1}{(1+0.17)^{20}} \right]$$

$$NPW(17\%) = 9,24,591.339$$

Machine A will be selected because cost on machine

A is less.



Future Worth Method.

- * In case of one project:

$$NFW(i\%) = FW(B) - FW(C).$$

If $NFW > 0$, project will be selected

If $NFW < 0$, project will be rejected

If $NFW = 0$, project may or may not be selected.

- * In case of mutually exclusive projects:

Revenue Based Method:

- * Different series:

$$NFW(i\%) = -P(1+i)^m + R_1(1+i)^{m-1} + R_2(1+i)^{m-2} + \dots + R_m + S$$

- * Equal payment series.

$$NFW(i\%) = -P(1+i)^m + R \left[\frac{(1+i)^m - 1}{i} \right] + S.$$

Cost Based Method:

- * Different series.

$$NFW(i\%) = P(1+i)^m + C_1(1+i)^{m-1} + C_2(1+i)^{m-2} + \dots + C_m - S.$$



* Equal payment series.

$$NFW(i\%) = P(1+i)^m + C \left[\frac{(1+i)^m - 1}{i} \right] \rightarrow$$

- (Q) From the following table find out which alternative will be selected on the basis of future worth method, if $i = 13\%$ compounded annually.

Particulars	Alternative A	Alternative B
initial cost	4,00,000	6,00,000
uniform annual benefit	64,000	96,000
useful life (yrs)	15 yrs	15 yrs.

Sols: Alternative A:

$$NFW(13\%) = -4,00,000 (1+0.13)^{15} + 64,000 \left[\frac{(1.13)^{15} - 1}{0.13} \right]$$

$$\Rightarrow NPW(13\%) = 85004.573.$$

Alternative B:

$$NFW(13\%) = -6,00,000 (1.13)^{15} + 96,000 \left[\frac{(1.13)^{15} - 1}{0.13} \right]$$

$$\therefore NPW(13\%) = 127514.36.$$

Alternative B will be selected because $NFW(13\%)$ for B > A.
(Annual benefit is more).

Date: / /



- (Q) From the following table find out which machine will be selected on the basis of future worth method if $i = 11\%$. compounded annually.

Particulars	Machine 1	Machine 2	Machine 3
Initial investment	80,00,000	70,00,000	90,00,000
Life (years)	17 yrs.	17 yrs	17 yrs
Annual op. & f. maintenance cost	8,00,000	9,00,000	8,50,000
Salvage value	5,00,000	4,00,000	3,00,000

$i = 11\%:$ Machine 1:

$$\text{NFW}(11\%) = 80,00,000 (1.11)^{-17} + 8,00,000 \left[\frac{(1.11)^{17} - 1}{0.11} \right] - 5,00,000 \\ = 8,236,415.91$$

Machine 2:

$$\text{NFW}(11\%) = 70,00,000 (1.11)^{-17} + 9,00,000 \left[\frac{(1.11)^{17} - 1}{0.11} \right] - 4,00,000 \\ = 8,09,16,409.48$$

Date _____



- (Q) From the following table find out which machine will be selected on the basis of future worth method if $i = 11\%$, compounded annually.

Particulars	Machine 1	Machine 2	Machine 3
Initial investment	80,00,000	70,00,000	90,00,000
Life (years)	17 yrs	17 yrs	17 yrs
Annual op. & f.	8,00,000	9,00,000	8,50,000
Maintenance cost	000/-	000/-	000/-
Salvage value	5,00,000	4,00,000	3,00,000

SOL: Machine 1:

$$NFW(11\%) = 80,00,000 (1.11)^{-17} + 8,00,000 \left[\frac{(1.11)^{17} - 1}{0.11} \right] - 5,00,000.$$

$$= 82361415.91$$

Machine 2:

$$NFW(11\%) = 70,00,000 (1.11)^{17} + 9,00,000 \left[\frac{(1.11)^{17} - 1}{0.11} \right] - 4,00,000$$

$$= 8,09,16,407.48.$$

Date / /



Machine B:

$$NFW(11\%) = 90,00,000 (1.11)^{18} + 8,50,000 \left[\frac{(1.11)^{18} - 1}{0.11} \right] -$$

7,00,000

$$= 9,01,81,550.76.$$

Machine B cost

Machine 2 will be selected because cost will be less

Q From the following table find out which alternative will be selected on the basis of future worth method if $i = 12\%$ compounded annually?

Particulars	Alternative 1	Alternative 2
First cost	15,00,000	20,00,000
Annual property taxes	70,000	90,000
Annual income	5,00,000	7,00,000
Life (years)	15 yrs	15 yrs.
Net annual income	4,30,000	6,10,000

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$$NFW(12\%) = -15,00,000 (1.12)^{15} + 4,30,000 \frac{(1.12)^{15} - 1}{0.12}$$

$$NFW(12\%) = 78,19,928.665$$

$$NFW(12\%) = -20,00,000 (1.12)^{15} + 6,10,000 \frac{(1.12)^{15} - 1}{0.12}$$

$$NFW(12\%) = 1,17,93,449.42$$

Alternative 2 will be selected.

- (i) From the following table, find out which machine will be selected on the basis of FWM, if $i=16\%$, compounded annually.

Particulars	Machine A	Machine B
Initial cost	5,00,000	3,00,000
Life (yrs)	8 yrs	7 yrs.
Salvage value	1,00,000	2,00,000
Annual maintenance cost	30,000	0

Date / /



selst.

machine A:

$$NFW(16\%) = 5,00,000 (1.16)^7 + 30,000 \left[\frac{(1.16)^7 - 1}{0.16} \right] - 1,00,000$$

$$\Rightarrow NFW(16\%) = 16,55,926.067.$$

machine B:

$$NFW(16\%) = 7,00,000 (1.16)^7 - 2,00,000.$$

$$\Rightarrow 17,78,353.814$$

Machine A will be selected.

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Annual Worth Method / Annual Equivalent Method

- * In case of same project:

$$NAW(i\%) = AW(B) - AW(c).$$

If $NAW > 0$, project will be selected.

If $NAW < 0$, project will be rejected.

If $NAW = 0$, project will be may or may not be selected

- * In case of mutually exclusive projects.

- * Different series.

- * Equal payment series: (Revenue based method)

$$NAW(i\%) = -P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] + R + S \left[\frac{i}{(1+i)^m - 1} \right]$$

- * Cost based method.

$$NAW(i\%) = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] + C - S \left[\frac{i}{(1+i)^m - 1} \right]$$

- * Different series:

$$NAW(i\%) = NPW \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

same for both revenue and cost based method.



Q) From the following table find out which technology will be selected on the basis of annual equivalent method if $i = 18\%$. compounded annually.

Particulars	Technology A	Technology B
Initial cost	₹, 00, 000	₹, 00, 000
End of Year	Cash inflows year end	Cash inflows year end
1	10,000	15,000
2	20,000	30,000
3	30,000	0
4	45,000	0

Soln: Technology A is the new investment.

$$NAW(i\%) = \left\{ -50,00,000 + 10,000 \left[\frac{1}{(1+18)^1} \right] + 20,000 \left[\frac{1}{(1+18)^2} \right] \right.$$

$$+ 30,000 \left[\frac{1}{(1+18)^3} \right] + 45,000 \left[\frac{1}{(1+18)^4} \right]$$

$$\times \left[\frac{0.18 \cdot (1+18)^4}{(1+18)^4 - 1} \right]$$

$$\Rightarrow -4,35,692 \cdot 30 \times \left[\frac{0.18 \cdot (1+18)^4}{(1+18)^4 - 1} \right]$$

$$\therefore -1,61,963 \cdot 676$$

Date / /

Technology B:

$$NAW(18\%) = \left\{ -3,00,000 + 15,000 \left[\frac{1}{1.18^1} \right] + 30,000 \left[\frac{1}{1.18^2} \right] \right\}$$

$$\left[\frac{0.18 \times (1.18)^2}{(1.18)^2 - 1} \right]$$

$$\Rightarrow -425,220.18$$

Technology A will be selected

$$\therefore NAW_A(18\%) > NAW_B(18\%)$$

Q. Present the following

which machine will be selected on the basis of AEM, if $i = 20\%$ compounded annually.

Machine	Down payment	Yearly equal installment	No. of installments
1	5,00,000	2,00,000	15
2	4,00,000	3,80,000	15
3.	6,00,000	1,50,000	15

Machine 1:

$$NAW(20\%) = 5,00,000 \left[\frac{0.20 \times (1.20)^{15}}{(1.20)^{15} - 1} \right] + 2,00,000$$

$$\Rightarrow NAW(20\%) = 3,06,941.059$$

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Machine 2 :

$$NAW(20\%) = 4,00,000 \left[\frac{0.20 \cdot (1.20)^{15}}{(1.20)^{15} - 1} \right] + 3,00,000.$$

$$\Rightarrow NAW(20\%) = 3,85,552.847$$

Machine 3 :

$$NAW(20\%) = 6,00,000 \left[\frac{0.20 \cdot (1.20)^{15}}{(1.20)^{15} - 1} \right] + 1,50,000.$$

$$\Rightarrow NAW(20\%) = 2,78,329.271.$$

Machine B will be ~~not~~ selected, because cost is min^m.

- Q) From the following table find out which alternative will be selected on the basis of AWM if $i = 25\%$. compounded annually and life of both the alternatives is 5 yrs

Particulars	Alternative A	Alternative B
Investment	1,50,000	1,75,000.
Annual equal return	1025,60,000	70,000.
Salvage value	15,000	35,000.



Alternative A:

$$NAW(25\%) = -1,200,000 \left[\frac{(1.05)^5 \cdot (0.25)}{(1.05)^5 - 1} \right] + 60,000$$

$$+ 18,000 \left[\frac{0.25}{(1.05)^5 - 1} \right]$$

$$NAW(25\%) = 60,50,69$$

Alternative B:

$$NAW(25\%) = -125,000 \left[\frac{(1.05)^5 \cdot (0.25)}{(1.05)^5 - 1} \right] + 70,000.$$

$$+ 35,000 \left[\frac{0.25}{(1.05)^5 - 1} \right]$$

$$NAW(25\%) = 9,191,456$$

As return from alternative B is more so B will be selected.

- (c) From the following table find out which machine will be selected on the basis of annual equivalent method if $i = 22\%$ compounded annually.

Particulars	Machine A	Machine B
Initial cost	3,00,000	6,00,000

Date _____

Life in
yrs

10 yrs

10 yrs.

Salvage
value

2,00,000

3,00,000

Annual

30,000

0.

maintenance

Soln: Machine A:

$$NAW(22\%) = 3,00,000 \left[\frac{(1.22)^{10} \cdot (0.22)}{(1.22)^{10} - 1} \right] + 30,000$$

- 2,00,000

0.22

 $\square (1.22)^{10} - 1$

$$\Rightarrow NAW_A(22\%) = 99,489.498.$$

Machine B:

$$NAW(22\%) = 6,00,000 \left[\frac{(1.22)^{10} \cdot (0.22)}{(1.22)^{10} - 1} \right]$$

$$3,00,000 \left[\frac{0.22}{(1.22)^{10} - 1} \right]$$

$$\Rightarrow NAW_B(22\%) = 1,42,468.494.$$

$NAW_A(22\%) > NAW_B(22\%)$ machine A is selected because.

$$NAW_A(22\%) < NAW_B(22\%)$$

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Rate of Return Method :-

Types of Return.

① Minimum acceptable rate of return / minimum attractive rate of return. (MARR).

② Net Present Value (NPV)

③ Internal Rate of Return (IRR)

Minimum Acceptable Rate of Return

It refers to the lower limit of project acceptability beyond which if rate of return falls project will be rejected.

Net Present value : (NPV).

It refers to the addition of present values of all the future stream of benefits during the life span of a project.

Internal Rate of Return

It refers to the rate of return which equals the present value of benefit

If $NPV \neq 0$.

$IRR = \frac{\text{Interest rate of the last positive result} - \text{(the difference between the two interest rates)}}{\text{r} \times \text{value}}$



$$\text{IRR} = \left(\frac{\text{Interest rate}}{\text{of the last two results}} \right) + \left(\frac{\text{Difference b/w the two interest rates}}{\text{FVE value - salvage value}} \right) \times \frac{\text{FVE value} - \text{salvage value}}{\text{FVE value} - (\text{NPV value})}$$

* Selection or rejection of the project:

If $\text{IRR} > \text{MARR}$, project will be selected.

If $\text{IRR} < \text{MARR}$, project will be rejected.

If $\text{IRR} = \text{MARR}$, project may or may not be selected

(Q) From the following table find fo out the investor should go with the new bus. or not on the basis of RRM.

If MARR is 10%.

	Year	Cash Flows
0	Initial Investment	-1,00,000
1	Annual Income	30,000
2	Annual Income	30,000
3	Annual Income	30,000
4	Annual Income	30,000
5	Annual Income	30,000

$$\text{NPW} = -P + R \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \rightarrow 0$$

$$\Rightarrow -100,000 + 30,000 \left[\frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} \right] = 13723.60$$

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$$\Rightarrow NPW(12\%) = -1,00,000 + 30,000 \left[\frac{(1+0.12)^5 - 1}{0.12(1+0.12)^5} \right]$$

$$\Rightarrow NPW(12\%) = 8143.286.$$

$$NPW(15\%) = 564.65.$$

$$NPW(17\%) = -1771.19.$$

$$\therefore IRR = 15\% + 2\% \times \left[\frac{564.65 - 0}{564.65 + 1771.19} \right].$$

$$\Rightarrow IRR = 15.48\%.$$

The investor may go in with the new business because $IRR > MARR$.

- Q A company is trying to diversify its bus. in a new product line. the life of the project is 10 yrs with no salvage value at the end of its life, the initial outlay of the project is 20,00,000 Rs annual cash inflow is 3,50,000. Rs. Find the rate of return for the new business.

Soln: Taking MARR as 10%.

$$NPW(10\%) = -P + R \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right].$$

$$\Rightarrow -20,00,000 + 3,50,000 \left[\frac{(1+10\%)^{10} - 1}{0.1(1+10\%)^{10}} \right].$$



$$NPW(10\%) = 150598.487 \text{ Rs.}$$

$$NPW(11\%) = 61231.2039 \text{ Rs.}$$

$$NPW(12\%) = -22421.94 \text{ Rs.}$$

$$\therefore IRR = 11\% + 1\% \times \left[\frac{61231.2039 - 0}{61231.2039 + 22421.94} \right]$$

$$= 11.738\%$$

$$= 11.7\%$$

Q) From the following table find out rate of return for all the alternatives and find out which alternative which will be selected on the basis of RRM if MARR is 12%.

Particulars.	Alternative A1	Alternative A2	Alternative AB
Investment	1,50,000	2,10,000	2,55,000
Annual net income	48,570	58,200	69,000
n (in yrs).	5 yrs	5 yrs	5 yrs

So, Alternative A1.

$$By NPW(12\%) = -1,50,000 + 48,570 \left[\frac{(1+0.12)^5 - 1}{0.12 \times (1+0.12)^5} \right]$$

$$\therefore NPW(12\%) = 14209.651.$$

$$\begin{aligned} IRR_A_1 &= 15.8\% \\ IRR_A_2 &= 12\% \\ IRR_A_3 &= 11\% \end{aligned}$$



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$$NPW(13\%) = 10280.228$$

$$NPW(14\%) = 6445.0499$$

$$NPW(15\%) = 2757.707$$

$$NPW(16\%) = -790.438$$

$$IRR = 15\% + 1\% \times \frac{2757.707}{2757.707 + 790.438}$$

$$IRR = 15.77\%$$

Alternative A2:

$$NPW(12\%) = -2,10,000 + 58,260 \left[\frac{(1+12)^5 - 1}{0.12 \times (1+12)^5} \right]$$

$$NPW(13\%) = -5086.106$$

$$IRR = 12\% + 1\% \times \frac{14.261 + 0}{14.261 + 5086.106}$$

Alternative A3:

$$NPW(12\%) = -2,55,000 + 69,000 \left[\frac{(1+12)^5 - 1}{0.12 \times (1+12)^5} \right]$$

$$-6290.44$$



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$$NPW(11\%) = 16.894$$

$$\therefore IRR = 11\% + \frac{16.894}{16.894 + 6270.44} \times 1\%$$

$$\Rightarrow IRR = 11.00\%$$

Alternative A2 will be selected because it has
more ~~rate of return~~ ^{rate of} 15.77%.

Benefit Cost Analysis:



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Benefit Cost Analysis

- * Selection or rejection of the project:

Benefit cost ratio:

$$\frac{B}{C} = \frac{\text{Present value of Benefit}}{\text{Present value of cost}} = \frac{PW(B)}{PW(C)}$$

If $\frac{B}{C} > 1$, project will be selected.

If $\frac{B}{C} < 1$, project will be rejected.

If $\frac{B}{C} = 1$ then project may or may not be selected.

- From the following table find out which project will be selected on the basis of B/C ratio.

Project.	Present worth of Benefit	Present worth of cost
1	60,00,000	40,00,000
2	80,00,000	20,00,000
3	90,00,000	35,00,000

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$$\left(\frac{B}{C}\right)_1 = \left[\frac{PW(B)}{PW(C)} \right] = 1.5$$

$$\left(\frac{B}{C}\right)_2 = \left[\frac{PW(B)}{PW(C)} \right] = 4.$$

$$\left(\frac{B}{C}\right)_3 = \left[\frac{PW(B)}{PW(C)} \right] = 2.57$$

Project 2 will be selected (\because B/C ratio is more).

Q) In a particular locality of a state, the vehicle users take around about most time to reach certain places because of presence of a minor. This results in excessive travel time and increased fuel cost. So the state government is planning to construct a bridge across the minor. The estimated initial investment for constructing the bridge is 40,00,000. The estimated life of the bridge is 15 yrs. The annual maintenance and op. cost is 1,50,000. The value of fuel savings due to the construction of the bridge is 6,00,000 per first year and it increases by 50,000 rs every year thereafter till the end of the life of the bridge. Check whether the project is justified based on B/C ratio by assuming an interest rate of 12% compounded annually.

Initial investment = 40,00,000

Life = 15 yrs, R = 1,50,000.

Benefit : 6,00,000.



Sol:

$$\star \text{ PW}(A) = -40,00,000 + 1,50,000 \left[\frac{(1+0.12)^{15} - 1}{0.12(1.12)^{15}} \right]$$

$$\Rightarrow \text{Cost} = 41,800,061.81. 5021629.693$$

$$\star A_i = 6,00,000$$

$$c_i = 50,000.$$

$$A(B) = 6,00,000 + 50,000 \left[\frac{1 - \frac{1}{(1.12)^{15}}}{0.12} \right]$$

$$\Rightarrow A(B) = 849015.169$$

$$\star \text{ PW}(B) = A(B) \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\Rightarrow 849015.169 \left[\frac{(1+0.12)^{15} - 1}{0.12 \times (1+0.12)^{15}} \right]$$

$$\Rightarrow 5782527.266$$

$$\frac{B}{C} = \frac{5782527.266}{5021629.693}$$

∴ Project is justified because $B/C > 1$

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state government is planning a hydro electric project in addition of to the production of electric power. This project will provide flood control, irrigation and navigation benefits. The estimated benefits and cost are given as follows.

initial cost \rightarrow 8 crore.

Annual power sales \rightarrow 60,00,000, Rs.

Annual flood control savings \rightarrow 30,00,000.

Annual irrigation benefits \rightarrow 50,00,000.

Annual navigation benefits \rightarrow 20,00,000.

Annual operating & maintenance cost \rightarrow 30,00,000.

life of project \rightarrow 50 yrs.

$i = 15\%$.
Check whether the government should implement the project or not on the basis of benefit & cost analysis.

$$\text{soln: } PW(C) = 8,00,00,000 + (30,00,000 - 30,00,000) \times$$

$$\left[\frac{(1+0.15)^{50} - 1}{0.15 (1+0.15)^{50}} \right]$$

$$\therefore PW(C) = +9965088 \text{ Rs. } 99981543.98.$$

$$PW(B) - PW(C) = A = 1,60,00,000.$$

$$PW(B) = 1,60,00,000 \left[\frac{(1+0.15)^{50} - 1}{0.15 (1+0.15)^{50}} \right]$$

$$= 106568234.6$$

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$$\frac{PW(B)}{PW(C)} = \frac{106568234.6}{99981543.98}$$

$$= 1.06$$

\therefore Project must be implemented

$$\because PW(B) / PW(C) \text{ ratio} > 1.$$

- (Q) Two mutually exclusive projects are being considered for investment, project A, requires an initial outlay of 30,00,000 with net receipts estimated as 9,00,000 Rs per year over the next 5 years. The initial outlay for the project A₂ is 60,00,000 and net receipts have been estimated as 15,00,000 Rs per year for the next 7 yrs. Using B/C ratio. find out which project will be selected if $i = 10\%$ compounded annually.

SOL: Project A1:

$$PW(C) = 30,00,000.$$

$$A = 9,00,000.$$

$$PW(B) = 9,00,000 \times \left[\frac{(1.1)^5 - 1}{(1.1)^5 \times 0.1} \right]$$

$$\therefore PW(B) = 8411708.092$$

$$\therefore B/C = \frac{8411708.092}{30,00,000} = 1.137$$

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Project A₂:

$$PW(C) = 60,00,000.$$

$$PW(B) = 15,00,000 \left[\frac{(1+1)^7 - 1}{(1+1)^7 \times 0.1} \right]$$

$$= 7302628.227$$

$$B/C = 7302628.227.$$

$$60,00,000$$

$$\approx 1.214.$$

Project A₂ will be selected

$$(B/C)_{A_1} < (B/C)_{A_2}.$$

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Pay-Back Period Method :

- * In case of equal return.

$$P = \frac{I}{C}$$

I → Initial investment.
C → yearly equal cash inflows.
Pay back
years.

- Q) Suppose an investor has invested ₹ 50,00,000 in a project for which he is getting equal return of ₹ 1,00,000 at the end of every year. Find out how many yrs it will take to the investor to get back his money, which he has initially invested with the help of pay-back period method.

Sol: $P = I/C$

$$\Rightarrow P = \frac{50,00,000}{1,00,000} =$$

$$25$$

∴ 25 years.

- * In case of different amount of return.

Remaining amount
next cash inflow × 12.



One investor has invested ₹ 10,00,000/- on a project, the investor wants to purchase a machine for which there are two machines available in the market find out which machine will be selected on the basis of pay back period method.

cash inflow from machine -1 cash inflow from machine -2

End of year	machine -1	machine -2
1	1,00,000	2,00,000
2	3,00,000	4,00,000
3	5,00,000	4,50,000
4	7,00,000	5,60,000
5	8,00,000	7,00,000

* for machine -1.

$$\text{number of months} = \frac{1,00,000}{7,00,000} \times 12 = 1.7$$

2 months

Total time = 3 years 2 months.

* for machine -2

$$\text{number of months} = \frac{4,00,000}{14,50,000} \times 12 = 10.66.$$

11 months.

Total time = 2 yrs 11 months

Machine 2 will be selected because it requires less time.



Date / /

A government is planning for which estimated benefits and costs are given as follows.

Particulars	Project A	Project B	Project C
	Initial cost	15,00,000	25,00,000
Annual operating and maintenance cost	20,00,000	25,00,000	32,00,000
Annual power sales	1,20,00,000	1,20,00,000	1,80,00,000
Flood control saving	25,00,000	35,00,000	50,00,000
Irrigation benefits	35,00,000	45,00,000	60,00,000
Recreational benefits	10,00,000	20,00,000	35,00,000

Find out which project will be selected on the basis of B/C ratios if $i = 9\% \text{ compounded annually}$.

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Project A:

$$PW(A) = 15,00,00,000 + \frac{4500,20,00,000}{\left[\frac{(1.09)^{50} - 1}{0.09 (1.09)^{50}} \right]}$$

$$\therefore 171923365.8.$$

$$PW(B) = 1,70,00,000 \left[\frac{(1.09)^{50} - 1}{0.09 (1.09)^{50}} \right]$$

$$\therefore PW(B) = 186348609.3.$$

$$\therefore B/C = \frac{186348609.3}{171923365.8} = 1.08.$$

Project B:

$$PW(C) = 25,00,00,000 + 25,00,000 \left[\frac{(1.09)^{50} - 1}{0.09 (1.09)^{50}} \right]$$

$$\therefore 277404207.3.$$

$$PW(B) = 2,20,00,000 \left[\frac{(1.09)^{50} - 1}{0.09 (1.09)^{50}} \right]$$

$$\therefore 241157023.8.$$

$$\therefore B/C = \frac{241157023.8}{277404207.3} = 0.869.$$

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Project C:

$$PW(C) = 40,00,00,000 + 35,00,000 \left[\frac{(1.09)^{50} - 1}{0.09(1.09)^{50}} \right]$$

$$\therefore PW(C) = 438365890.2$$

$$PW(B) = 3,25,00,000 \left[\frac{(1.09)^{50} - 1}{0.09(1.09)^{50}} \right]$$

$$\therefore PW(B) = 356254094.3 \quad \therefore B/C = 0.812$$

∴ Project A is selected

- (Q) From the following table find out a company should select which alternative on the basis of rate of return method if MARR = 12%.

Alternative	Initial investment	Yearly revenue
1	5,00,000	8,00,000
2	8,00,000	270,000

m=5

Sol: Alternative 1:

$$NPW(12\%) = -5,00,000 + 180,000 \left[\frac{(1.12)^5 - 1}{0.12(1.12)^5} \right]$$

$$\therefore NPW(12\%) = 112811.95$$

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$$NPW(13\%) = 97929.314$$

$$NPW(14\%) = 83623.704$$

$$NPW(15\%) = 69806.36$$

$$NPW(16\%) = 31619.04$$

$$NPW(20\%) = 8404.063$$

$$NPW(21\%) = -2582.662$$

$$IRR = 20\% + 1\% \times \frac{8404.063}{8404.063 + 2582.662} \\ = 20.76\%$$

Alternative 2:

$$NPW(12\%) = -8,00,000 + 2,70,000 \left[\frac{(1.12)^5 - 1}{0.12 (1.12)^5} \right]$$

$$NPW(13\%) = 173289.57$$

$$NPW(14\%) = 149652.44$$

$$NPW(20\%) = 7465.297$$

$$NPW(21\%) = -9984.229$$

$$IRR = 20\% + \left(\frac{7465.297}{7465.297 + 9984.229} \right) = 20.4278\%$$

∴ Alternative 1 will be selected because IRR > Past New

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(Q)

From the following table find out which route will be selected on the basis of annual worth method if $i = 15\%$ compounded annually.

Particulars	route around the lake	route through the lake
Length	15 km	5 km
First cost	₹ 1,50,000	= ₹ 1,50,000.
Useful life	15 yrs	15 yrs.
Maintenance cost	6,000 $\text{km}^{-1} \text{yr}^{-1}$	12,000 $\text{km}^{-1} \text{yr}^{-1}$
Salvage value	90,000 km^{-1}	1,50,000 km^{-1}
Yearly power loss	15,000 km^{-1}	15,000 km^{-1}

Sol: Route around the lake

$$P = 1,50,000$$

$$C = 6,000 \text{ km}^{-1} \text{yr}^{-1}$$

$$\therefore C_{\text{total}} = 6,000 \times 15 = 90,000$$

$$S = 90,000 \text{ km}^{-1}$$

$$S_{\text{total}} = 90,000 \times 15 = ₹ 1,35,00,000.$$

$$\text{Yearly power loss} = 15,000 \text{ km}^{-1}$$

$$= 1,50,000 \times 15 = ₹ 2,25,000$$

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$$\therefore \text{NAW (15\%)} = 1,50,000 \left[\frac{0.15 \times (1.15)^{15}}{(1.15)^{15} - 1} \right] + (90,000 + 2,25,000)$$

$$= 1,50,000 \left[\frac{0.15}{1.15^{15} - 1} \right].$$

$$\approx \text{NAW (15\%)} = 312,299.536$$

* Route under the lake.

$$P = 7,50,000$$

$$\begin{aligned} C_{\text{total}} &= 12,000 \times 5 \\ &= 60,000 \end{aligned}$$

$$S = 1,50,000 \times 5.$$

$$\approx 7,50,000$$

$$\text{Loss} = 75,000$$

$$\therefore \text{NAW (15\%)} = 7,50,000 \left[\frac{0.15 \times 1.15^{15}}{1.15^{15} - 1} \right] + (60,000 + 75,000)$$

$$= 7,50,000 \left[\frac{0.15}{1.15^{15} - 1} \right]$$

$$\text{NAW (15\%)} = 247,500$$

Route under the lake will be selected because the cost is less.



Production

It refers to the process of physical transformation of inputs into output.

Production Function:

It refers to the functional relationship between inputs and output.

$$Q = f(N, L, K)$$

$$Q = f(n_1, n_2, n_3, \dots, n_m)$$

If there are n numbers of inputs.

* Factors of Production:

Fixed factor.

Variable factor.

Fixed factor:

It refers to those factors which can't be changed or remain fixed during the process of production.

Eg: Plant size, big machines and land.

Variable factor:

It refers to those factors which can be changed during the process of production.

Eg: Labour and raw materials.



* Production time period :

Short period of production. (Short run)

Long period of production (long run)

Short period.

It refers to that period of production where fixed factor can't be changed but variable factor can be changed.

Long period.

It refers to that period of production where nothing is called fixed, all the factors are variable factors.

* Three Concepts of Production:

- Total Product (TP)
- Marginal Product (MP)
- Average Product (AP).

Total Product:

It refers to the total amount of output produced with a fixed amount of variable factors.

Marginal Product:

It refers to the net addition to the total product by employing one more unit of input.

$$MP_m = TP_m - TP_{m-1}$$

of Labour is input

$$MP_L = \frac{d(TP)}{dL} = dQ/dL$$



- If capital (K) is input

$$MP_K = \frac{d(TP)}{dK} = \frac{dQ}{dK}$$

Average Product :

It refers to the total amount of output produced per unit of a variable factor.

- If L is the input

$$AP_L = \frac{TP}{L} = \frac{Q}{L}$$

- If K is the input

$$AP_K = \frac{TP}{K} = \frac{Q}{K}$$

~~Theories of Production Function:~~

- Law of variable production (theory of short run)
- Law of return to scale (e.g. of long run).

Law of variable Prodⁿ

It is a short run production function which discusses the relationship b/w output and one variable input.

It refers to the prodⁿ function where as equal increments of one input

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are added, the inputs of other productive services being held constant, the resulting increment of product beyond a certain point will decrease i.e. marginal product will diminish (given by Cr. Stigler).

Assumptions of the theory:

- * The state of technology remain constant.
- * There must be some inputs whose quantities can be kept as fixed.

Unit of Labour used	Total Product	Marginal Product	Average Product.
1	80	80	80
2	170	90	85
3	270	100	90
4	368	98	92
5	430	62	86
6	503	73	83.83
7	503	0	71.85
8	495	-8	61.9
9	480	-15	53.33
10	430	-50	43.

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Units of labour.

TP

MP

AP

Stage & stage of operation.

because TP increases at an increasing rate

1 80 80 80 stage I

2 170 90 85 Increasing return

3 290 100 90

4 368 98 92 stage II

TP increases

5 430 62 86

at A 6 480 50 80 Diminishing

diminishing rate 7 503 23 71.8 return

8 503 0 62.8

9 490 -13 54.4 stage III

10. 485 -5 48.5 negative

Increasing total product leads to positive return.

Producer will want to produce in stage II

because total capacity is utilised.

- Q) From the following table find out Marginal product and AP and also find stages of operation

Units of

TP

MP

AP.

labour used (L)

1

30

30

83.0

2

80

50

40

3

140

60

46.67

4

190

50

47.5

5

190

0

38

6

180

-10

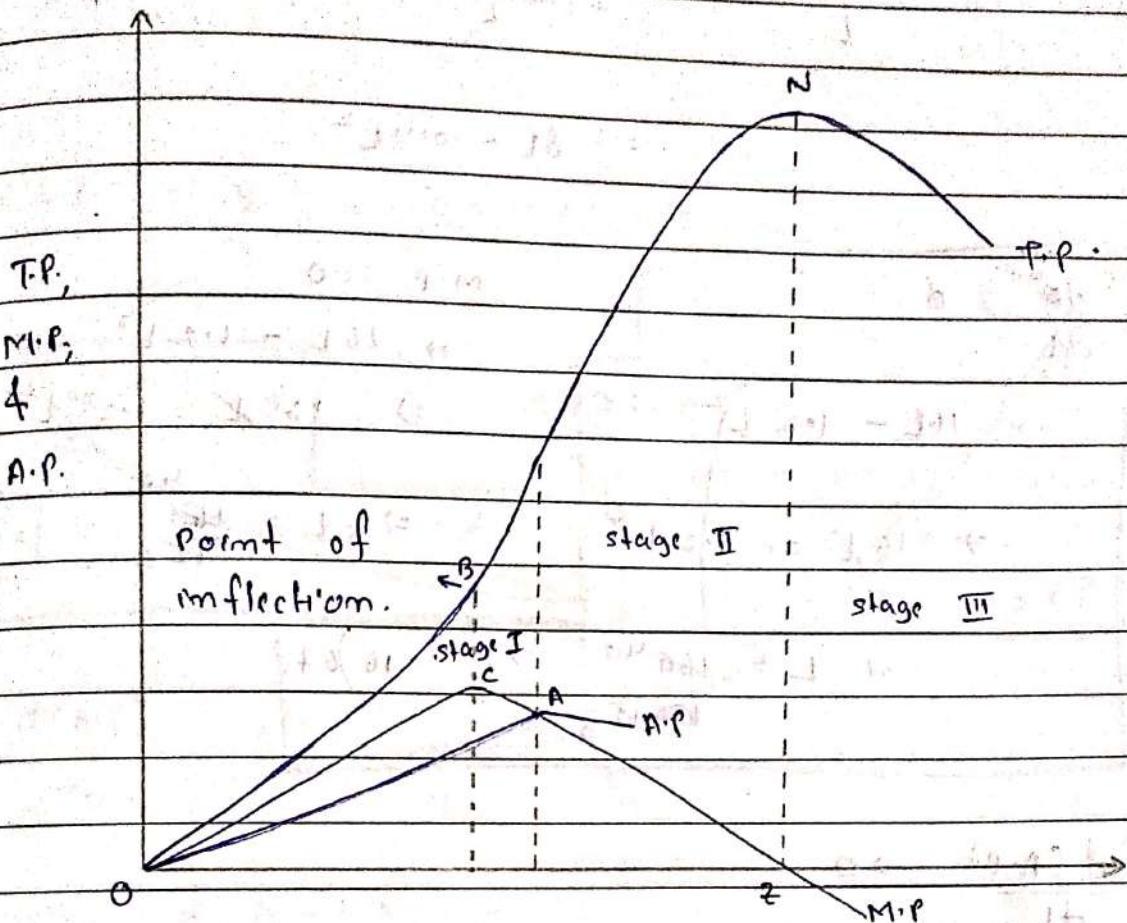
30

stage

II R



Diagrammatic Representation :-



Quantity of variable factors \rightarrow

- (1) From the following short run production function find out

- (1) Marginal product function.
- (2) Average product function.
- (3) Value of L at which output will be maximum.
- (4) Value of L at which average product is maximum

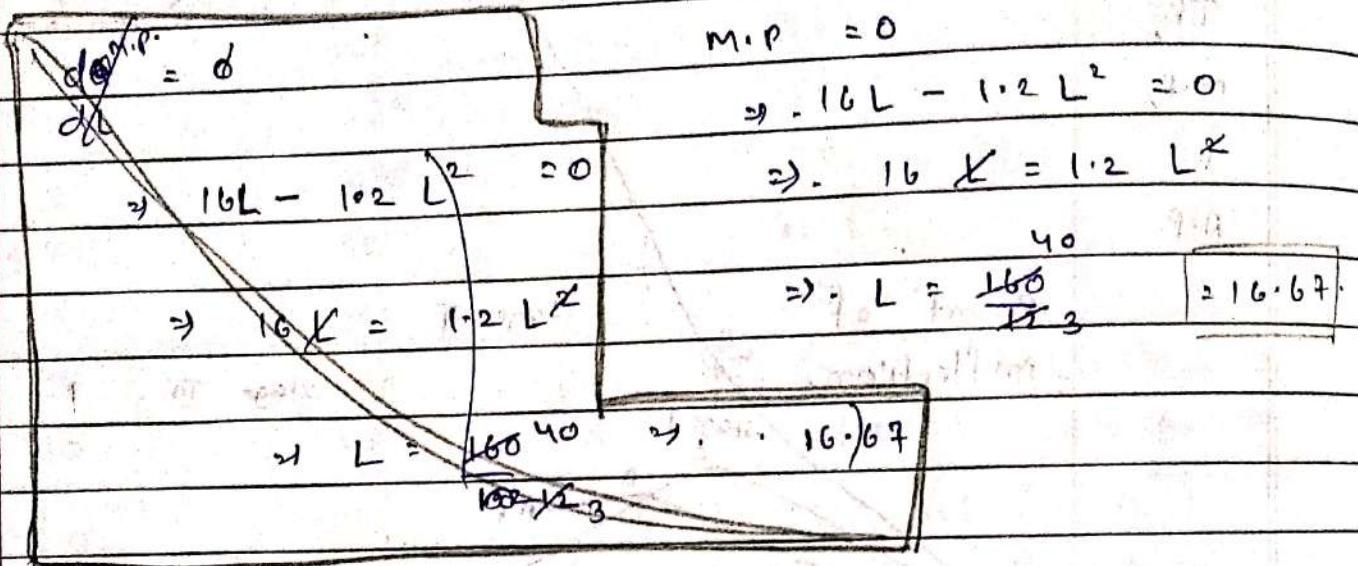
$$Q = 8L^2 - 0.4L^3$$

Soln:- $M.P. = \frac{dQ}{dL} = 16L - 1.2L^2$

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$$A.P. = \frac{Q}{L} = \frac{8L^2 - 0.4L^3}{L}$$

$$= 8L - 0.4L^2$$



$$\frac{d(A.P.)}{dL} = 0$$

$$\therefore 8 - 0.8L = 0.$$

$$\therefore 8 = 0.8L \Rightarrow L = 8/0.8 = 10$$

- (i) From the following short run production function find out :-
• M.P. function.
• A.P. function.
• Find x at which output will be maximum
• Find x at which A.P. is max.

$$Q = 6x^2 - 0.5x^3$$

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$$\frac{dQ}{dm} = 12m - 1.5m^2 - M \cdot P = 0$$

$$A \cdot P = Q = \frac{1}{m} (6m - 0.5m^2)$$

$$\frac{d(A \cdot P)}{dm} = 0 \Rightarrow 12m - 1.5m^2 = 0 \Rightarrow 12m = 1.5m^2 \Rightarrow m = 8$$

$$\frac{d(A \cdot P)}{dm} = 0 \Rightarrow 12m - 1.5m^2 = 0 \Rightarrow m = 8$$

$$\frac{d(A \cdot P)}{dm} = 0 \Rightarrow 12m - 1.5m^2 = 0 \Rightarrow m = 8$$

$$\frac{d(A \cdot P)}{dm} = 0 \Rightarrow 12m - 1.5m^2 = 0 \Rightarrow m = 8$$

$$\frac{d(A \cdot P)}{dm} = 0 \Rightarrow 12m - 1.5m^2 = 0 \Rightarrow m = 8$$

$$\therefore m = 6.$$

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Law of Returns to Scale :

It is a long-run homogeneous production function which shows the relationship between output and all the variable inputs.

Types of returns to scale :

- ① Increasing returns to scale.
- ② Constant returns to scale.
- ③ Decreasing returns to scale.

Increasing returns to scale :

When the rate of change in output is more than the rate of change in input or doubling of input results in more than doubling of output it is called increasing returns to scale.

Eg:

C	K	Q
20	30	200
40	60	600

Constant returns to scale.

When the rate of change in output is equal to the rate of change in input or doubling of input results in doubling of output it is called constant returns to scale.

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C	K	Q
20	30	200
40	60	400

* Decreasing returns to scale.

When the rate of change in output is less than the rate of change in input or doubling of input results in less than doubling of output, it is called decreasing returns to scale.

Eg:

C	K	Q
20	30	200
40	60	300

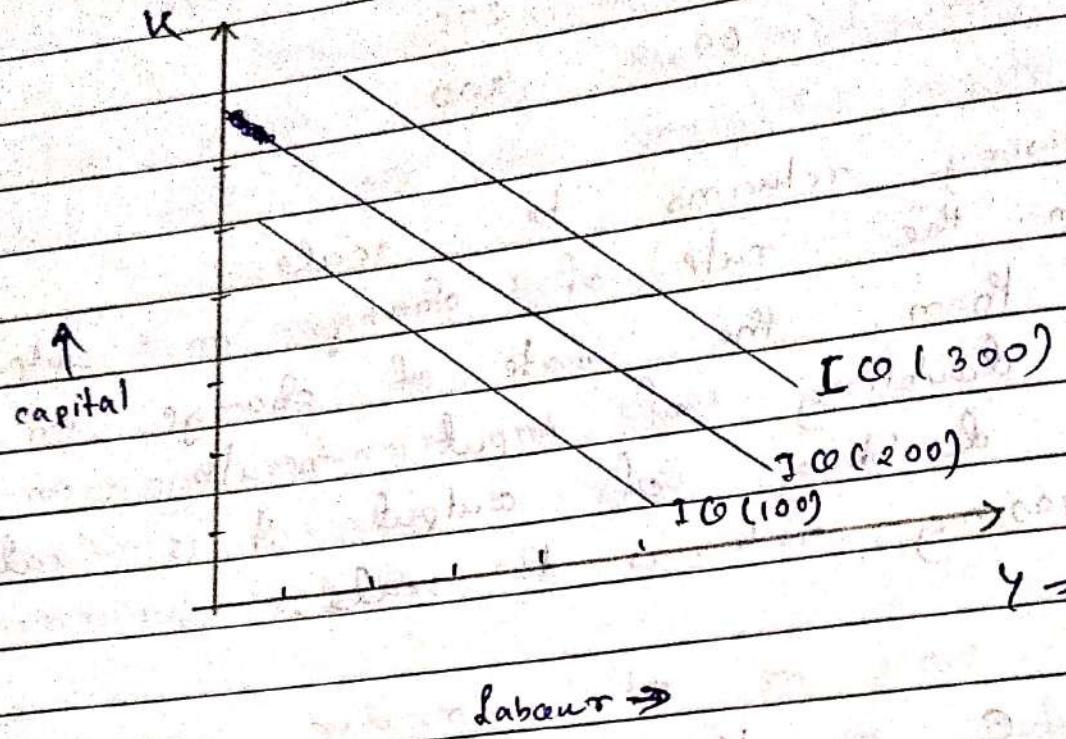
* Isoquant

It refers to those curves which shows various combinations of two factors producing same level of output.

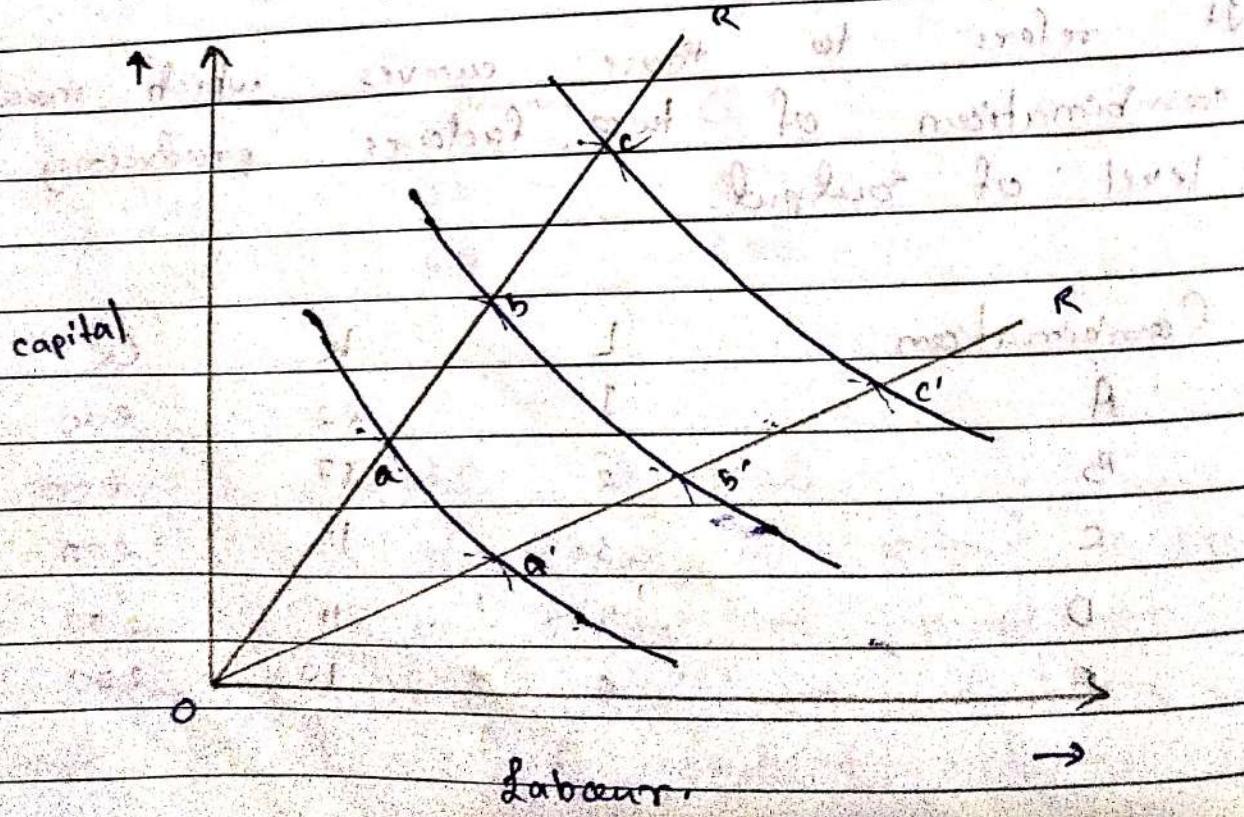
Combinations

	L	K	Q
A	1	22	200
B	2	17	200
C	3	13	200
D	4	11	200
E	5	10	200

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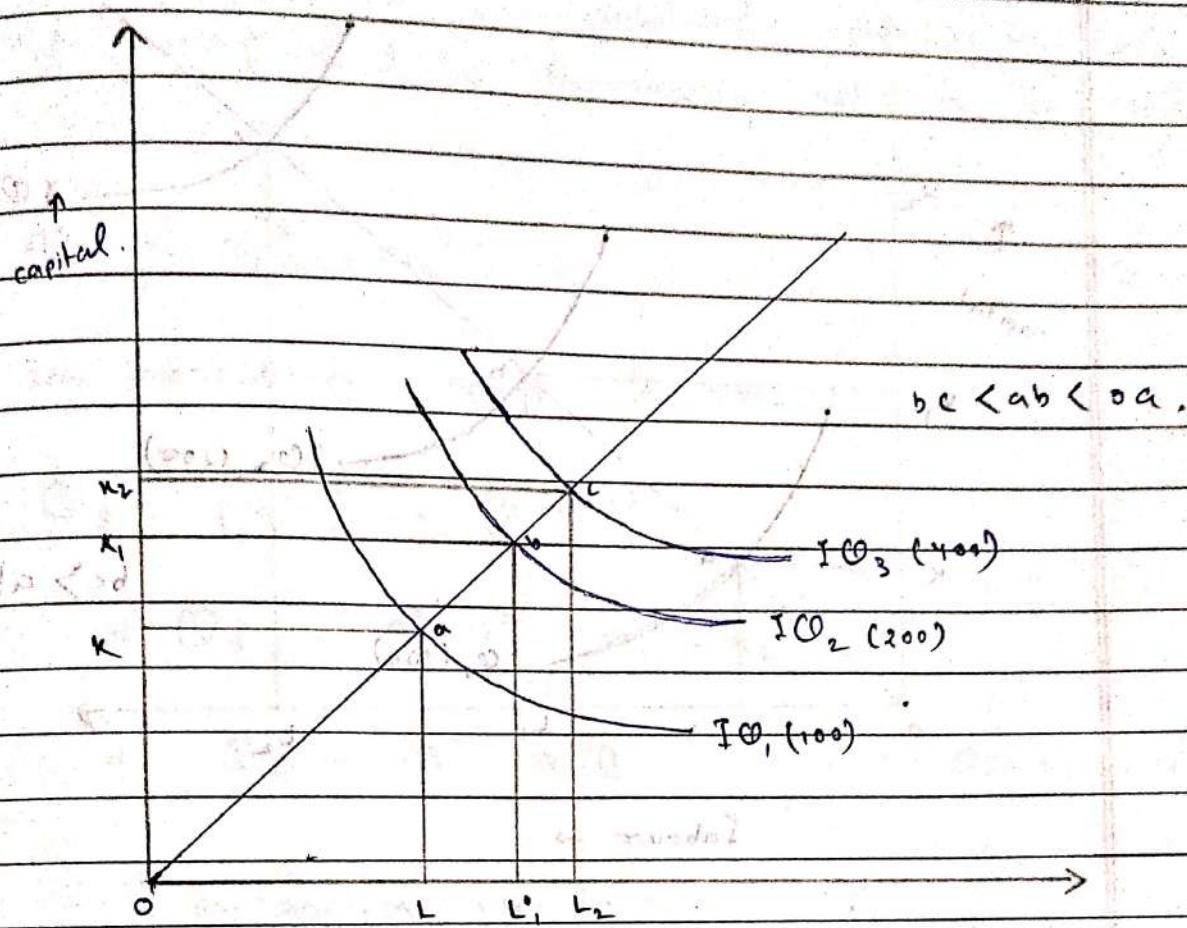


Isocline :
It refers to the locus of points on various isoquants.

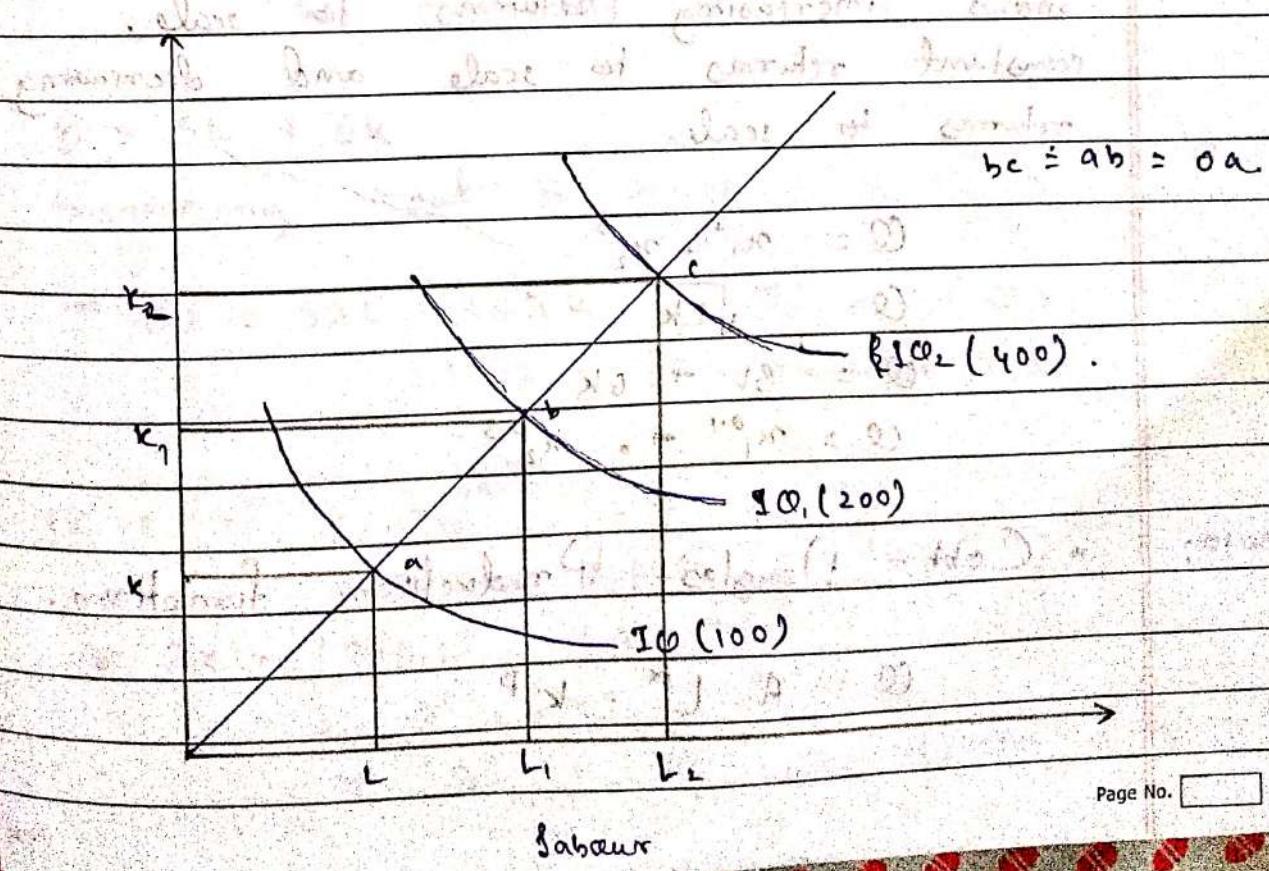




Increasing returns to scale:



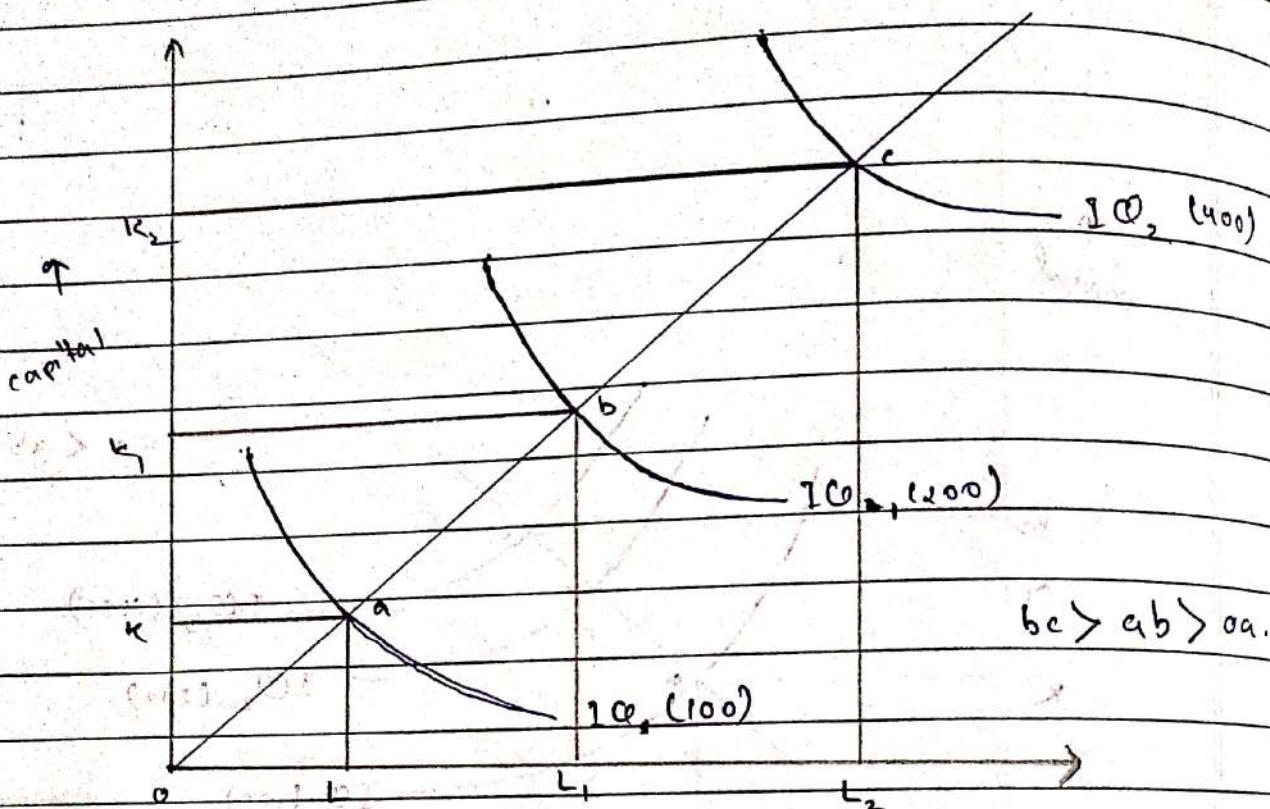
Constant returns to scale:



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Decreasing returns to scale:



Labour →

- (a) From the following production function find out the production functions that shows increasing returns to scale, constant returns to scale and decreasing returns to scale.

$$Q = n_1^2 \cdot n_2^3$$

$$Q = \sqrt{LK}$$

$$Q = BL + GK$$

$$Q = n_1^{0.1} \oplus n_2^{0.3}$$

Sol:

* Cobb-Douglas Production function.

$$Q = A \cdot L^\alpha \cdot K^\beta$$



If $\alpha + \beta > 1$, Increasing returns to scale.

If $\alpha + \beta = 1$, constant returns to scale.

If $\alpha + \beta < 1$, Decreasing returns to scale.

$$Q = \alpha x_1^2 \cdot x_2^3$$

∴ Increasing inputs \Rightarrow time

$$Q_* = (\lambda x_1)^2 \cdot (\lambda x_2)^3$$

$$\therefore Q_* = \lambda^2 \cdot \lambda^3 \cdot x_1^2 \cdot x_2^3$$

$$\therefore Q_* = \lambda^5 \cdot Q \quad \left\{ \because Q = x_1^2 \cdot x_2^3 \right.$$

∴ Increasing

$$Q = \sqrt{Lk} = L^{1/2} \cdot k^{1/2} \Rightarrow \text{that } \alpha + \beta = 1$$

so, constant r.s.

$$Q = SL + GK$$

Increasing input \Rightarrow time

$$\therefore Q_* = 3\lambda L + 3G\lambda K \Rightarrow Q_* = 3(Q)$$

∴ $Q_* = 3Q$ (constant r.s.)

$$Q = x_1^{0.1} \cdot x_2^{0.3}$$

∴ Increasing input \Rightarrow time.

$$\therefore Q_* = (\lambda x_1)^{0.1} \cdot (\lambda x_2)^{0.3}$$

$$\therefore \lambda^{0.4} \times (x_1^{0.1} \cdot x_2^{0.3})$$

$$\therefore \lambda^{0.4} \times (Q)$$

Decreasing r.s.

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* Properties of Cobb-Douglas Production function :-

1. The ratios of marginal product and average product gives its exponents.
 $(MP_L/AP_L = \alpha) + (MP_K/AP_K) = \beta$.
2. Elasticity of output is equal to the respective exponents.

Proof:

$$Q = A \cdot L^\alpha \cdot K^\beta$$

$$\frac{dQ}{dL} = A \cdot \alpha \cdot L^{\alpha-1} \cdot K^\beta$$

$$\Rightarrow \frac{dQ}{dL} = \alpha \cdot \frac{A L^\alpha \cdot K^\beta}{L}$$

$$\Rightarrow \frac{dQ}{dL} = \alpha \cdot \frac{Q}{L}$$

$$\Rightarrow \frac{dQ}{dL} = \alpha \cdot AP_L$$

$$\therefore MP_L = \alpha \cdot AP_L$$

$$\Rightarrow \frac{MP_L}{AP_L} = \alpha$$



$$Q = A L^\alpha K^\beta$$

$$\frac{dQ}{dK} = \beta A L^\alpha K^{\beta-1}$$

$$\Rightarrow MP_K = \beta \cdot \frac{Q}{K} : \quad \left\{ Q = A L^\alpha K^\beta \right.$$

$$\Rightarrow MP_K = \beta \cdot AP_K$$

$$\Rightarrow \frac{MP_K}{AP_K} = \beta$$

Remark:

Elasticity of Output.

$$= \frac{\text{Proportionate change in output}}{\text{Proportionate change in input.}}$$

$$E_L = \frac{dQ}{dL} \times \frac{L}{Q}$$

$$= MP_L \times \frac{1}{AP_L} = \frac{MP_L}{AP_L}$$

$$\boxed{E_L = \alpha.}$$

$$E_K = \frac{dQ}{dK} \times \frac{K}{Q}$$

$$= MP_K \times \frac{1}{AP_K}$$

$$\Rightarrow \frac{MP_K}{AP_K} = \beta$$

$$\therefore \beta_K = \beta$$

- (Q) From the following production function, find out
 - M.F. function (short run prodn funⁿ) if
 the fixed quantity of capital is 10000 units.

$$O = L^{0.75} K^{0.25}$$

$$\alpha = 0.75, \beta = 0.25$$

Marginal production function.

$$MP_L = \frac{dO_L}{dL} = 0.75 L^{-0.25} K^{0.75}$$

$$MP_K = \frac{dO_K}{dK} = 0.25 \cdot L^{0.75} K^{-0.75}$$

$$\Rightarrow O = L^{0.75} K^{0.25}$$

$$= L^{0.75} (10000)^{0.25}$$

$$= 10 L^{0.75}$$

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From the following prodⁿ funⁿ find out
M.P. funⁿ and A.P.ⁿ funⁿ and also find out
elasticity of output w.r.t input.

$$Q = 1.50 L^{0.75} K^{0.25}$$

$$\star M.P_L = 1.50 \times 0.75 \times L^{0.75 - 0.25} K^{0.25} \\ = 1.125 L^{-0.25} K^{0.25}$$

$$M.P_K = 0.375 L^{0.75} K^{-0.75}$$

$$\star A.P_L = \frac{M.P_L}{A.P_L} = \alpha \quad \text{or} \quad AP_L = \frac{M.P_L}{\alpha}$$

$$\star \therefore A.P_L = \frac{M.P_L}{\alpha} = \frac{1.125 L^{-0.25} K^{0.25}}{0.75} \\ = 1.5 L^{-0.25} K^{0.25}$$

$$A.P_K = \frac{\beta M.P_K}{\alpha} = \frac{0.375 L^{0.75} K^{-0.75}}{0.25} \\ = 1.5 L^{0.75} K^{-0.75}$$

$$\star E_L = \alpha = 0.75 : \frac{dQ/dL \times L/Q}{A.P_L} = \frac{M.P_L}{A.P_L}$$

$$E_K = \beta = 0.25 : \frac{dQ/dK \times K/Q}{A.P_K} = \frac{M.P_K}{A.P_K}$$

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Short run production function with two variable input.

Producer's Equilibrium:

- Output maximisation.
- Cost minimisation.

① Iso-quant / Iso-product curves,

② Marginal Rate of Technical substitution.

Iso-quant / Iso-product curves.

It refers to those curves which shows various comb. of two factors producing same level of output.

Marginal Rate of Technical Substitution

It refers to the rate at which number of units of one factors substituted to have one more unit of another factor.

MRTS_{LK}

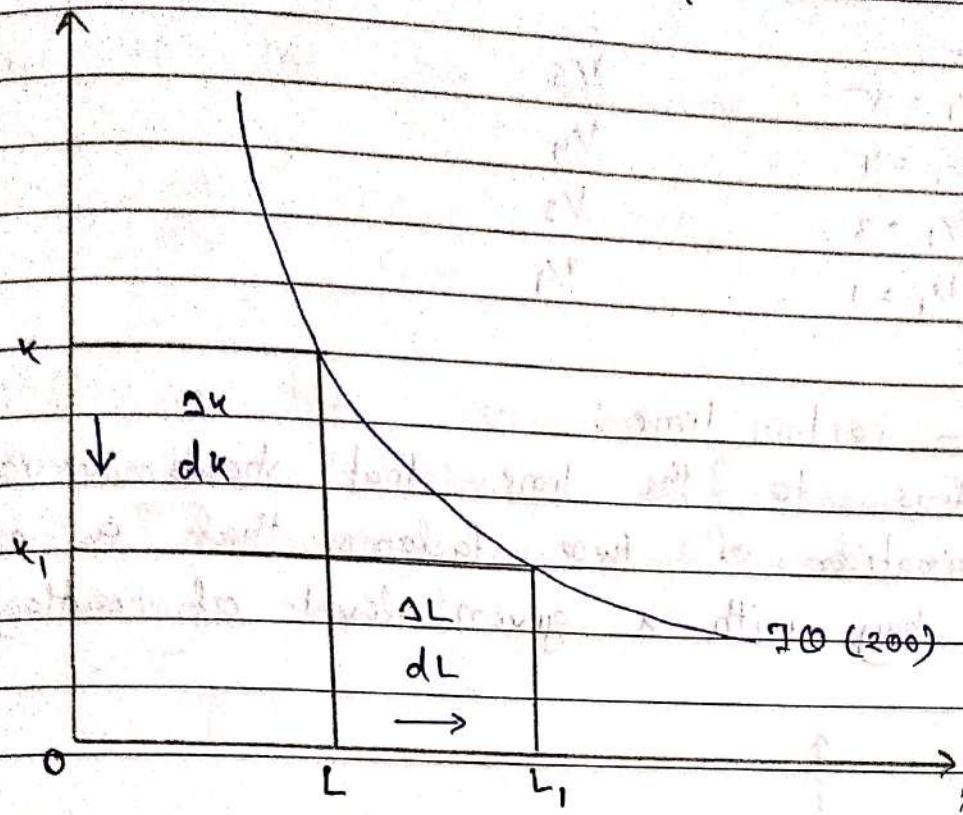
It refers to the rate at which number of units of capital substituted to have one more unit of labour.

MRTS_{KL}

It refers to the rate at which the number of units of labour substituted to have one more unit of labour-capital.



Slope of the isoquant.



$$MRTS_{LK} \approx \frac{dK}{dL} = \frac{MP_L}{MP_K}$$

$$MRTS_{KL} = \frac{dL}{dK} = \frac{MP_K}{MP_L}$$

- (Q) From the following table find out $MRTS_{LK}$ and $MRTS_{KL}$.

Combination.	L	K	Q	$MRTS_{LK}$
A	1	25	200	25
B	2	20	200	10
C	3	16	200	16/3
D	4	13	200	13/4
E	5	12	200	12/5

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MRTS_{LK}

MRTS_{KL}

$$5v_1 = 5$$

$$4v_1 = 4$$

$$3v_1 = 3$$

$$1v_1 = 1$$

$$1v_5$$

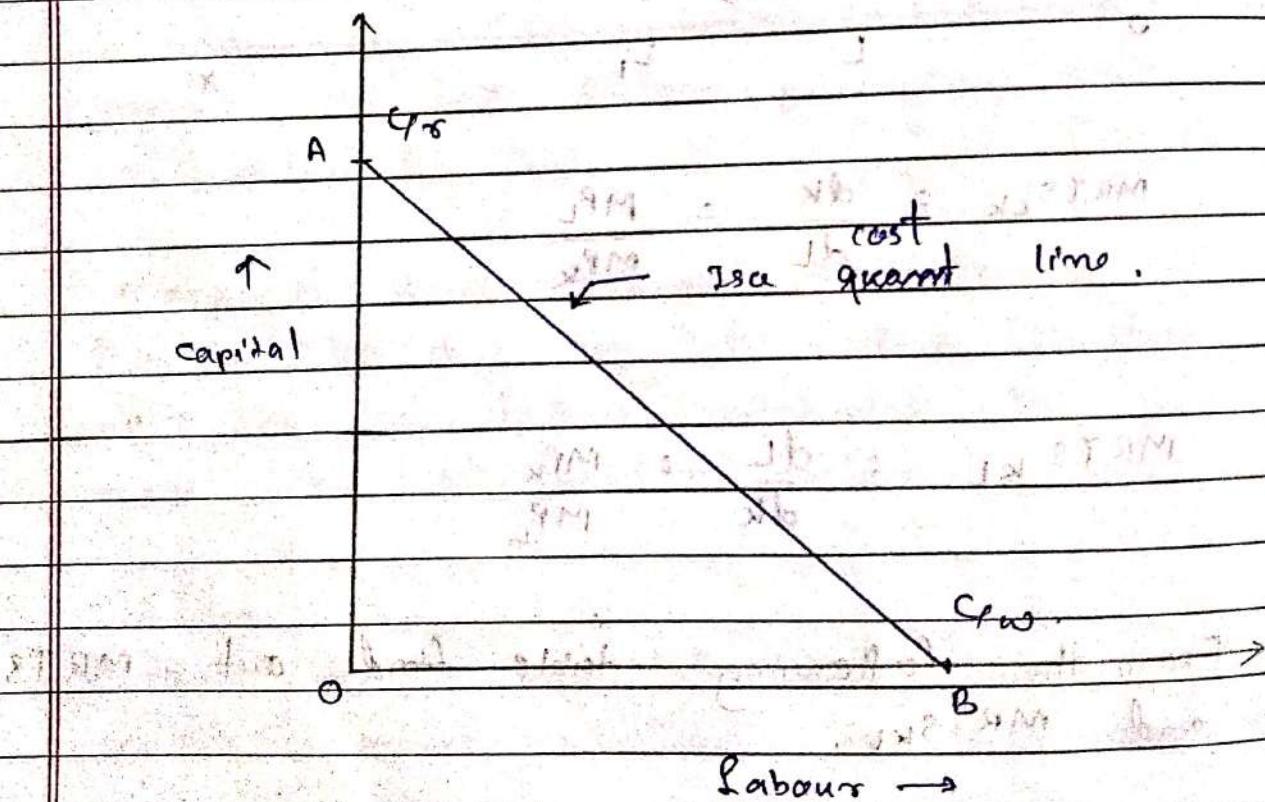
$$1v_4$$

$$1v_3$$

$$1v_1$$

Iso-cost line:

It refers to the line that shows various combinations of two factors that a producer can buy with a given level of outlay.



Equation of the iso-cost line:

$$C = WL + rK$$

L = No. of units of labour used,

w = wages of the labour.

r = No. of units of capital used

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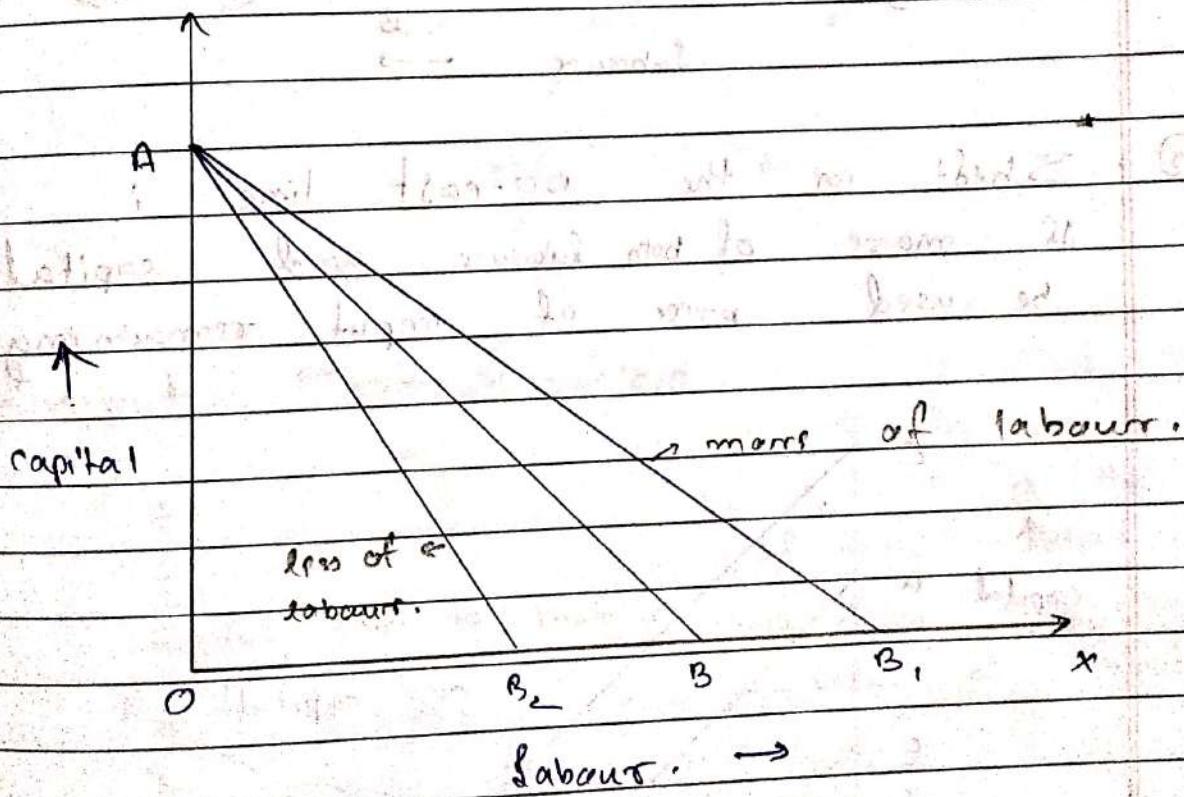
r = Price of capital.

slope of the iso-cost line.

$$\frac{OA}{OB} = \frac{C/r}{C/w} = w/r.$$

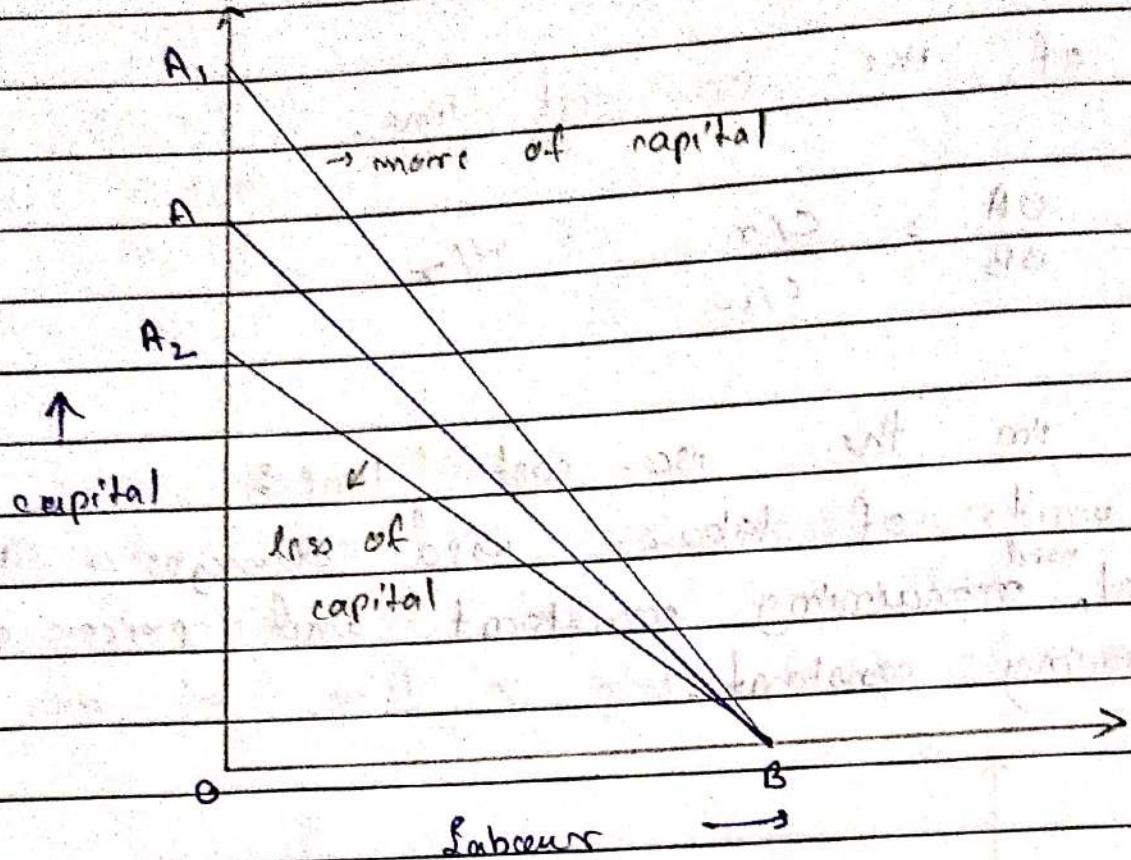
Shift in the iso-cost line:

If units of labour used changes that of capital, remaining constant and price of input remaining constant.

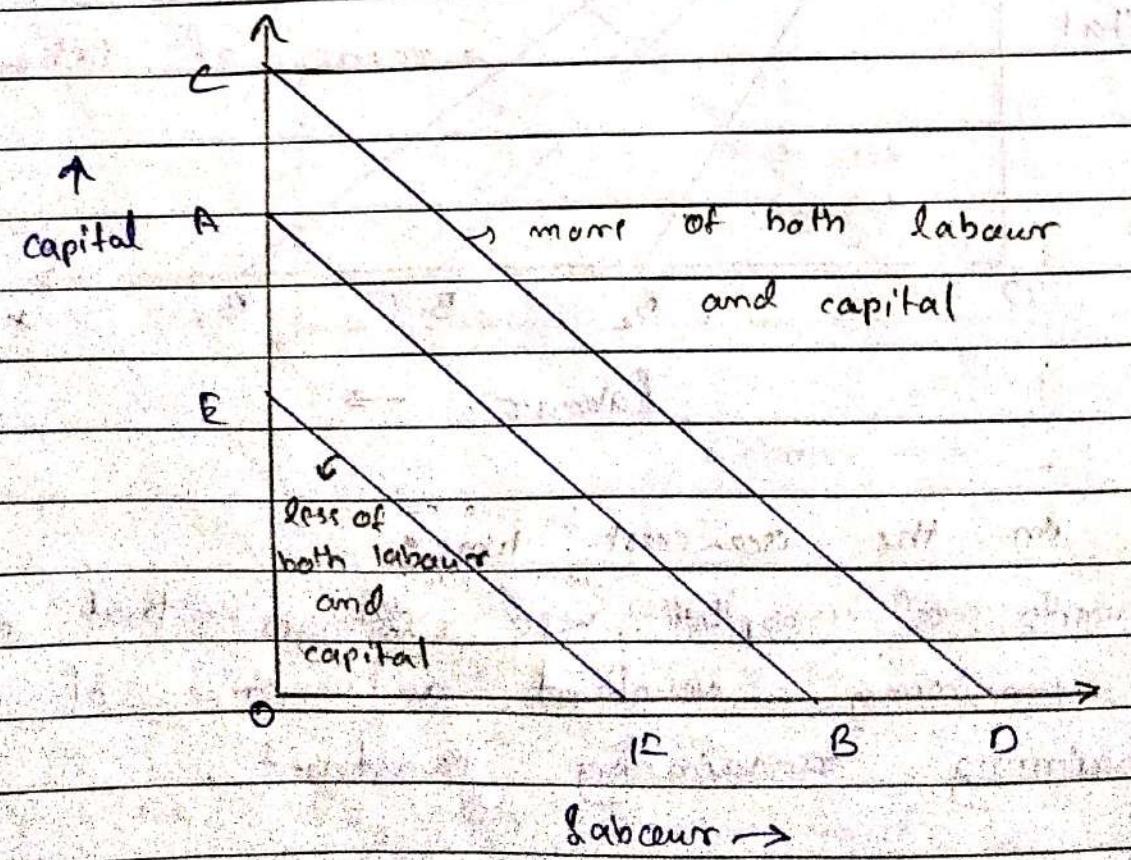


Shift in the iso-cost line:

If units of capital used changes that of labour used remaining constant and price of input remaining remaining constant.



- ③ Shift in the iso-cost line :
 If more of both labour and capital will be used price of input remaining constant



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Conditions favor the producer to be in equilibrium.



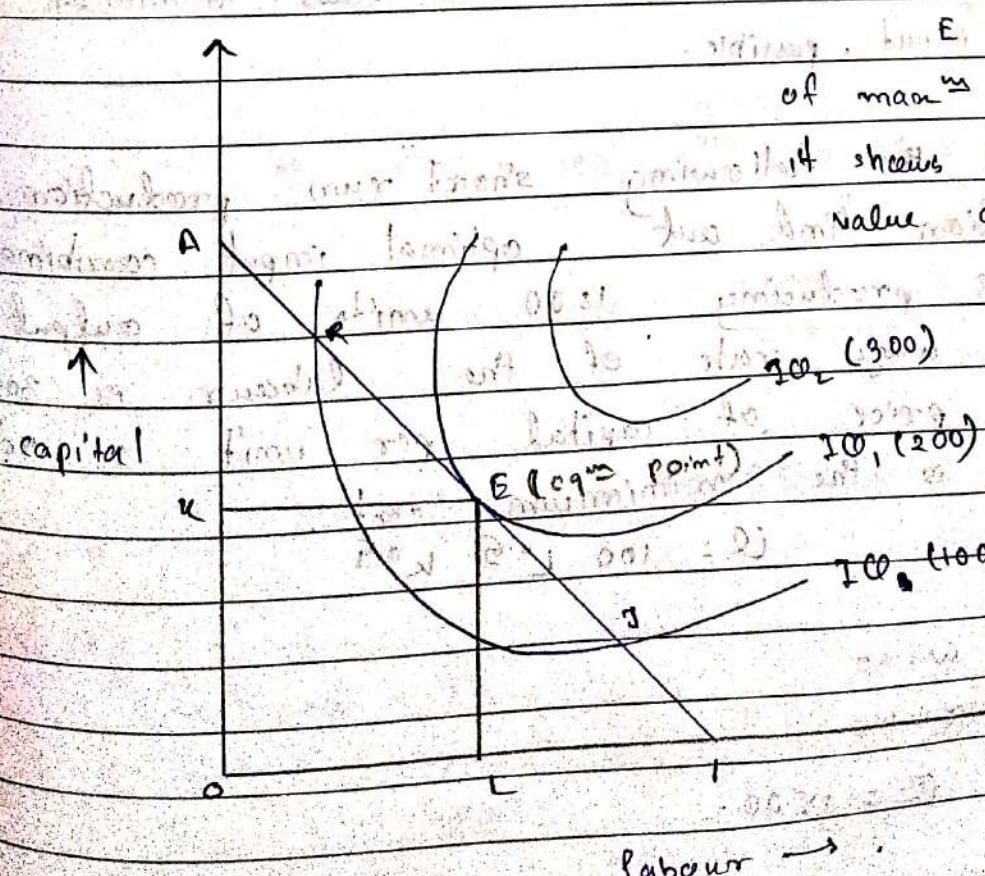
- (1) The slope of the isoquant should be equal to the slope of iso-cost line.
- (2) The isoquant should be convex at equilibrium point.

Condition 1:

$$MRTS_{LK} = \frac{dk}{dL} = \frac{MP_L}{MP_K} = \frac{w}{r}$$

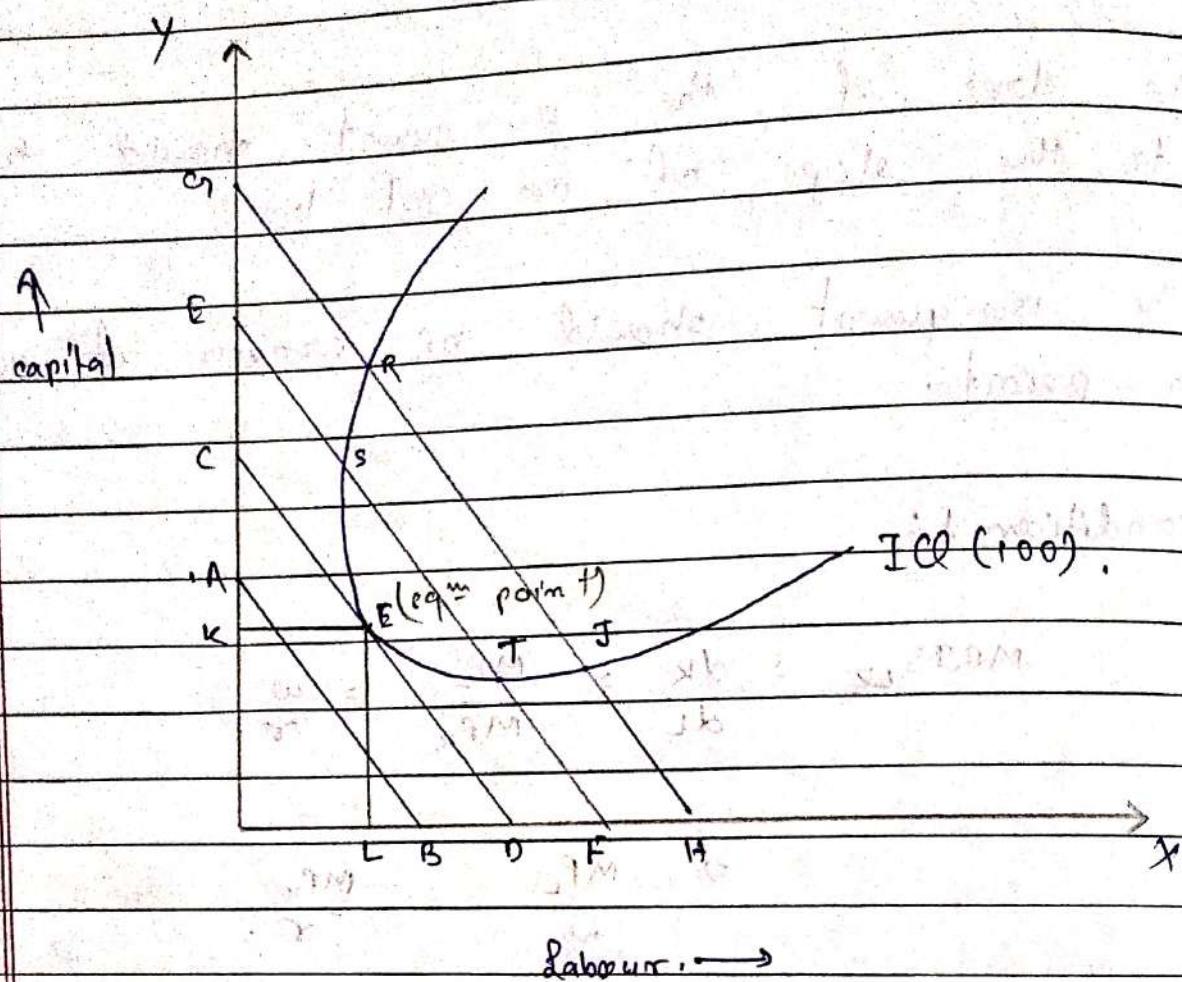
$$2) \frac{MP_L}{w} = \frac{MP_K}{r}$$

Output Maximization.



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Cost Minimisation:



- * E is the point of minimum cost because it lies on the ICL which shows minimum cost of input, possible.

- (Q) From the following short run production function find out optimal input combination for producing 1500 units of output if the wage rate of the labour is 30Rs and price of capital per unit is 40Rs what is the minimum cost.

$$C = 100 L^{0.5} K^{0.5}$$

Soln:

Given,

$$w = 30$$

$$r = 40$$

$$C = 1500$$

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App

$$MP_L = \frac{d\alpha}{dL} = 50 L^{-0.5} K^{0.5}$$

$$MP_K = \frac{d\alpha}{dK} = 50 L^{0.5} K^{-0.5}$$

now, $\frac{MP_L}{MP_K} = \frac{w}{r}$

$$\therefore \frac{50 L^{-0.5} K^{0.5}}{50 L^{0.5} K^{-0.5}} = \frac{w}{r} = \frac{3}{4}$$

$$\therefore L^{-1} \cdot K^1 = 3/4$$

$$\therefore \frac{K}{L} = \frac{3}{4}$$

now, so, $1500 = 100 \times L^{0.5} \times \left(\frac{3L}{4}\right)^{0.5}$

$$\therefore 1500 = 100 \times L^{0.5} \times 0.866 \times L^{0.5}$$

$$\therefore L = \frac{1500}{100 \times 0.866} \approx 17.32 \approx 17.$$

$$K = 17.32 \times \frac{3}{4} = 12.99 \approx 13.$$

so,

$$C = wL + rk$$

$$\therefore C = (30 \times 17.32) + (40 \times 13)$$

$$\therefore C = 1030.$$



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For output max^m

From the following prod^m fun^m find out
the quantity of labour and capital that
the company should use in order to
maximise output and also find max.
output

$$w = 30$$

$$r =$$

$$C = 1800$$

- * Fixed cost can never be zero.
- * Variable cost can be zero.

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Cost

It refers to the amount of money incurred on a thing.

Cost of Production.

It refers to the expenditures incurred on factors of production to produce the output.

Types of cost:

(1) Fixed cost.

(2) Variable cost.

(3) Marginal cost.

(4) Sunk cost.

Fixed cost:

It refers to those cost which remains fixed or remain independent of the level of output which doesn't change with level of output.

Variable cost:

It refers to those cost - which changes in direct proportion to the level of output.

Fixed cost: (e.g. → cost of machine installation).

Output

100

200

300

50

Cost of machine.

10,000

10,000

10,000

10,000.

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Variable cost:

Output	cost of raw material.
100	10,000
200	20,000
300	30,000
400	40,000
500	50,000
600	60,000
700	70,000
800	80,000
900	90,000
1000	100,000

Marginal cost:

It refers to the net addition to the total cost from producing one extra unit of output.

Output	Total cost.
20	200
30	300

$$MC_m = Tc_m - Tc_{m-1}$$

$$MC = \frac{d(Tc)}{dQ}$$

Sunk Cost:

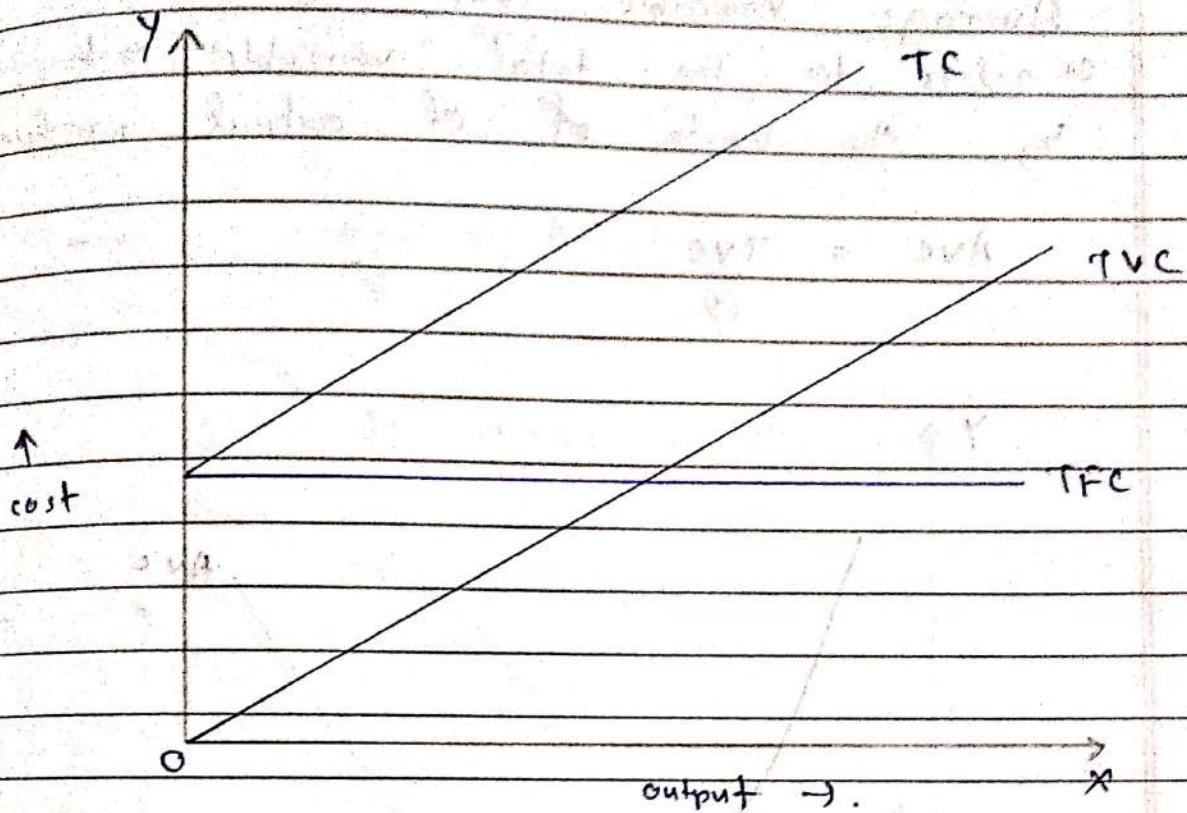
It refers to those cost which can't be recovered.

$$\star \text{Total cost} = TFC + TVC$$

TFC → Total fixed cost

TVC → Total variable cost.

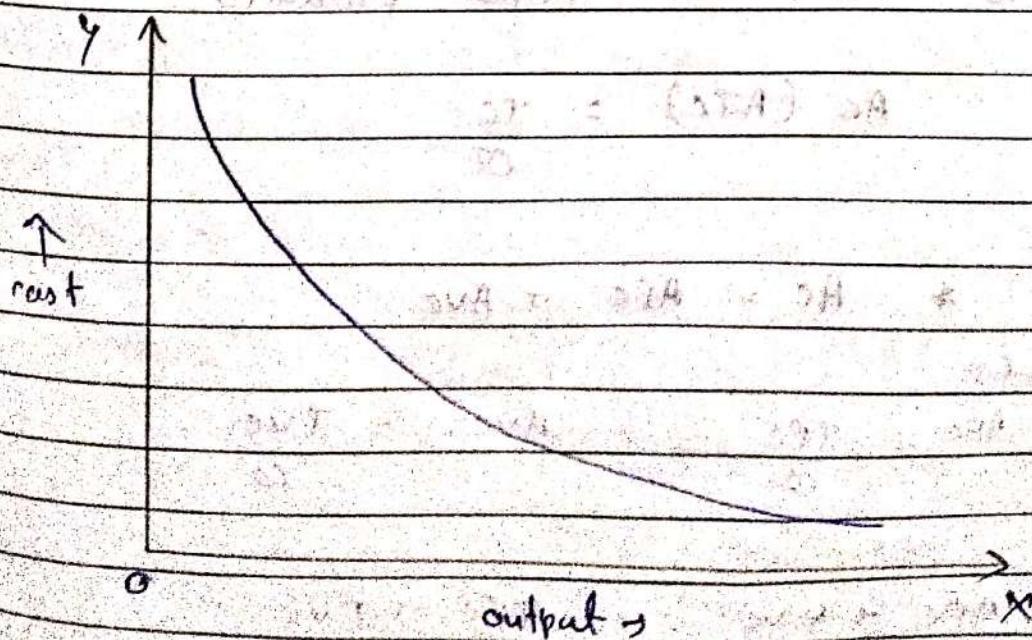
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① Average Fixed cost (AFC).

It refers to the total fixed cost divided by unity of output produced.

$$AFC = \frac{TFC}{Q}$$



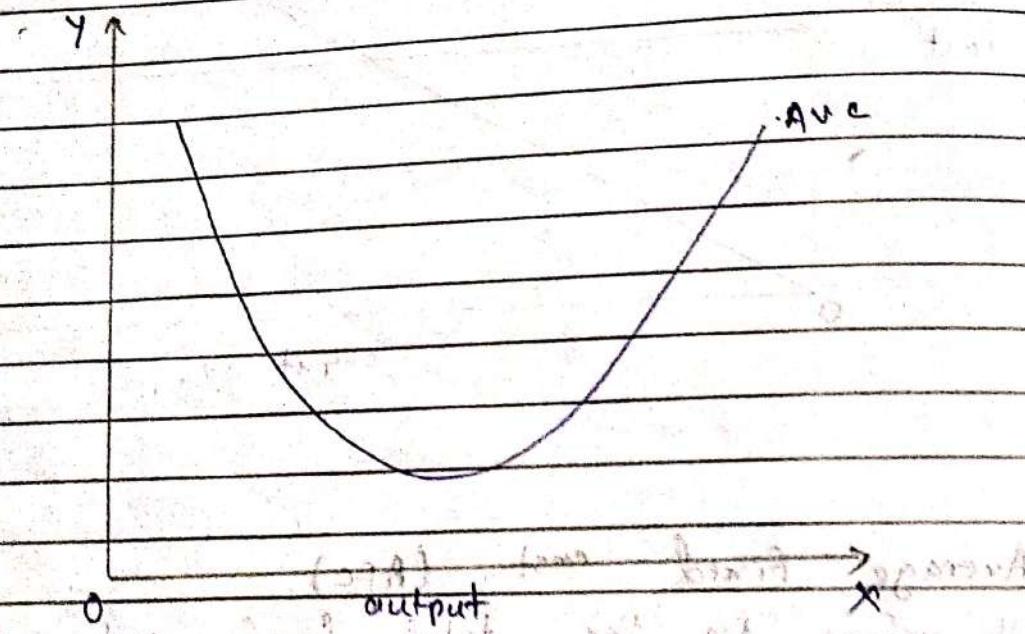
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Average Variable cost.

It refers to the total variable cost divided by the units of output produced

$$AVC = \frac{TVC}{Q}$$



Average Total cost.

It refers to total cost / units divided by the units of output produced.

$$AC (ATC) = \frac{TC}{Q}$$

* $AC = AFC + AVC$

Present:

$$AFC = \frac{TPC}{Q}$$

$$AVC = \frac{TVC}{Q}$$

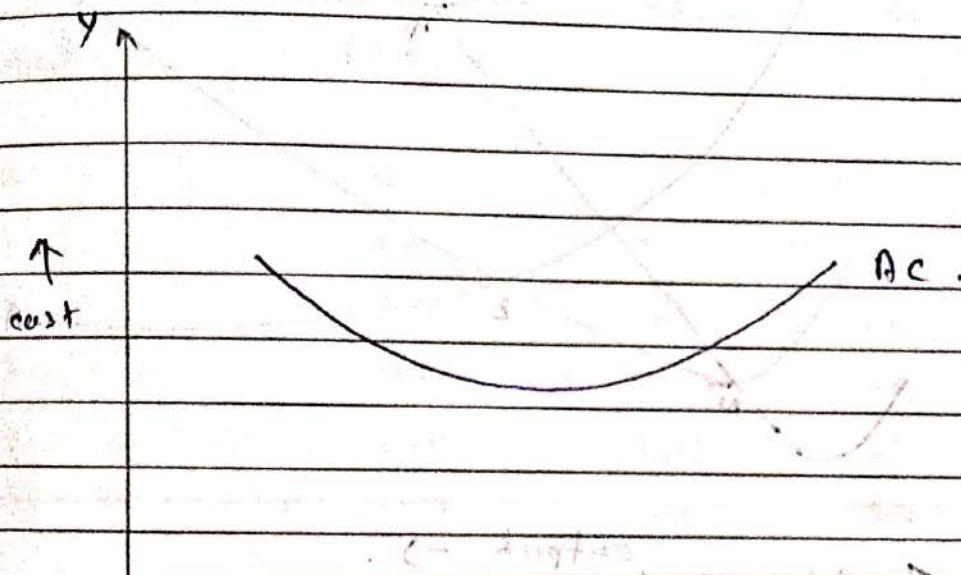
$$ATC = AFC + AVC = \frac{TPC}{Q} + \frac{TVC}{Q}$$



∴ after 2) $AFC + AVC = \frac{TC}{Q}$

whr $\frac{TC}{Q} = AC (ATC)$

∴ $AFC + AVC = AC.$



O (OVA & SM) output. \rightarrow min. of AC

Proof

$$MC_m = TVC_m - TVC_{m-1}$$

Proof :

$$MC_m = TC_m - TC_{m-1}$$

$$\Rightarrow MC_m = TFC_m + TVC_m - TFC_{m-1} - TVC_{m-1}$$

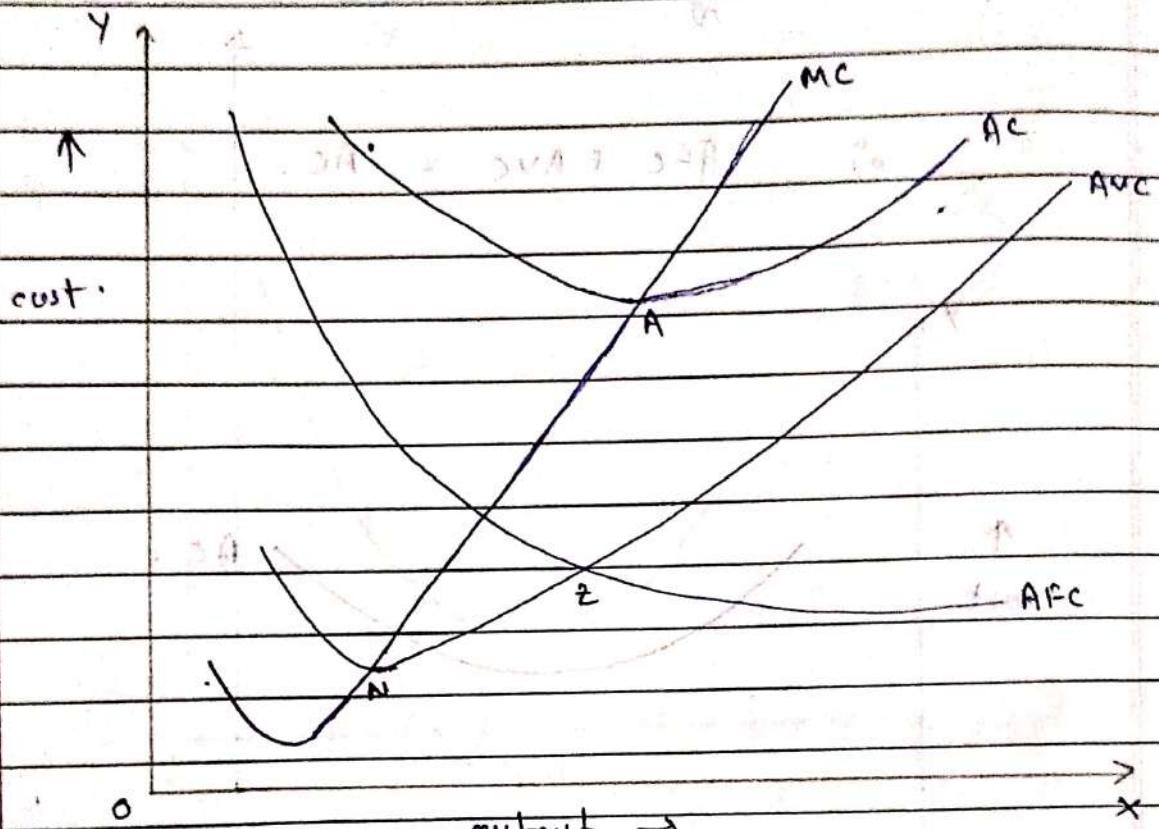
($\because TFC_m$ remains constant)

$$\therefore MC_m = TVC_m - TVC_{m-1}$$



So,

$$MC = \frac{d(TVC)}{dQ}$$



Relationship b/w MC & AVC

- * If MC is below AVC. ($MC < AVC$)
AVC decreases.

- * If MC is above AVC
 $MC > AVC$, AVC is increasing.

- * If $MC = AVC$
AVC is constant

Relationship b/w MC & AC.

- * If $MC < AC$, AC is decreasing.

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* If $MC > AC$, AC is increasing.

* If $AR = MC = AC$, AC is constant \rightarrow minimum.

* Relationship b/w AVC , AFC , AC

Q) From the following table find out AVC , AFC , AC , MC if the fixed cost is ₹ 50.

units of output	Total variable cost	AFC	AVC	AC	MC
0	0	—	—	—	—
1	20	50	20	70	—
2	40	25	20	45	25
3	60	16.67	20	46.67	16.67
4	80	12.5	20	42.5	12.5
5	100	10	20	40	10

TC	TC	MC
50	50	—
70	70	20
90	90	20
110	110	50
130	130	30
150	150	30

Q) From the following cost function find out MC function, AVC function, AC function and also find out at what level of output AVC & MC is minimum. If fixed cost is ₹ 200.



$$TVC = 300Q - 12Q^2 + 0.25Q^3$$

$$TC = 300Q - 12Q^2 + 0.25Q^3 + 200.$$

Sol: $\star MC = \frac{d(TC)}{dQ}$

$$\Rightarrow MC = 300 - 24Q + 0.75Q^2$$

$\star AVC = \frac{TVC}{Q}$

$$\Rightarrow AVC = 300 - 12Q + 0.25Q^2$$

$\star AC = \frac{TC}{Q}$

$$= 300 - 12Q + 0.25Q^2 + 200$$

$\star \frac{d(MC)}{dQ} = 0$ $\star d(MC) = 0$

$$\Rightarrow -24 + 0.15Q = 0$$

$$\Rightarrow Q = \frac{24}{0.15} = \frac{240}{1.5} = 16.$$

$$\therefore Q = 16,$$

$\star \frac{d(AVC)}{dQ} = 0$

$$\Rightarrow -12 + 0.5Q = 0$$

$$\therefore Q = 24.$$

Date: 1/1/

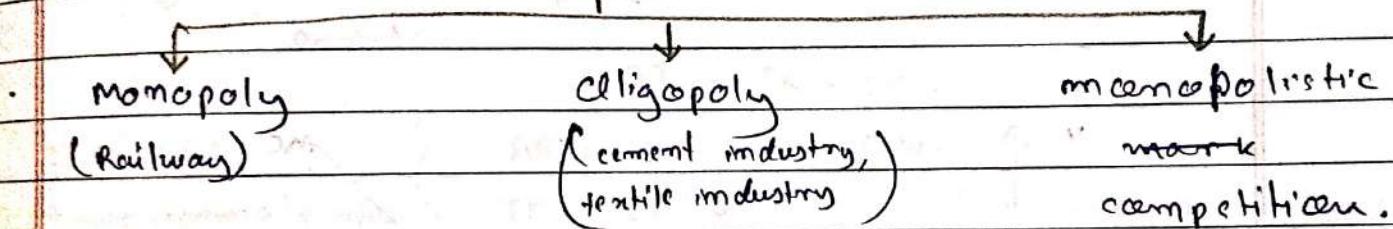
Market.

It refers to the place where not only goods are purchased or sold but there is a contact b/w buyers and sellers who agreed at a common price.

Types of market:

→ Perfect competition / perfectly competitive market.

→ Imperfect competition



Perfectly competitive market (not possible in real world)

It refers to that type of market where.

① There are large number of buyers and sellers.

② There is free entry and free exit.

* Uniform Price.

③ Buyers and sellers are producing and selling homogeneous products.

④ Buyers and sellers have knowledge about the actual price of the product prevailing in the market.

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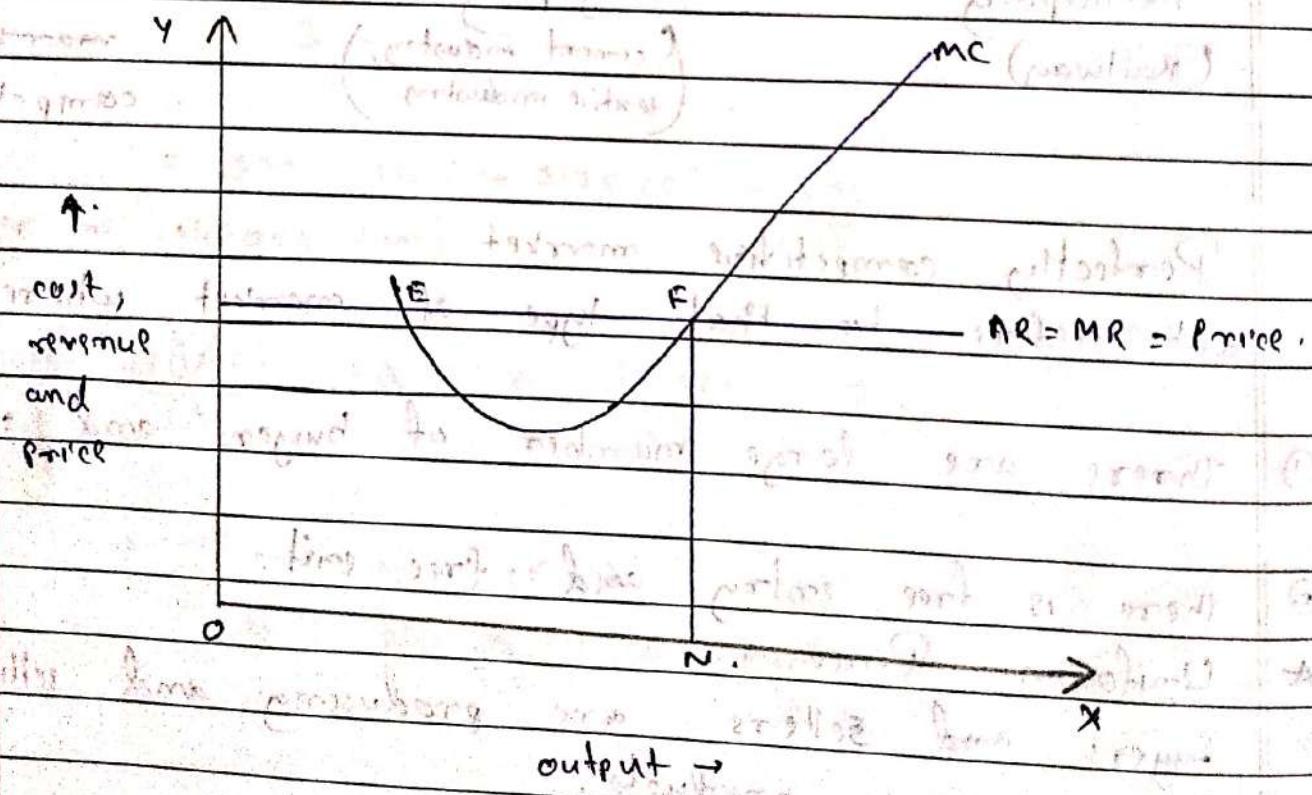


Equilibrium of the firms under perfectly competitive market.

- (1) Short run equilibrium of the firm.
- (2) Long-run eqⁿ of the firm.

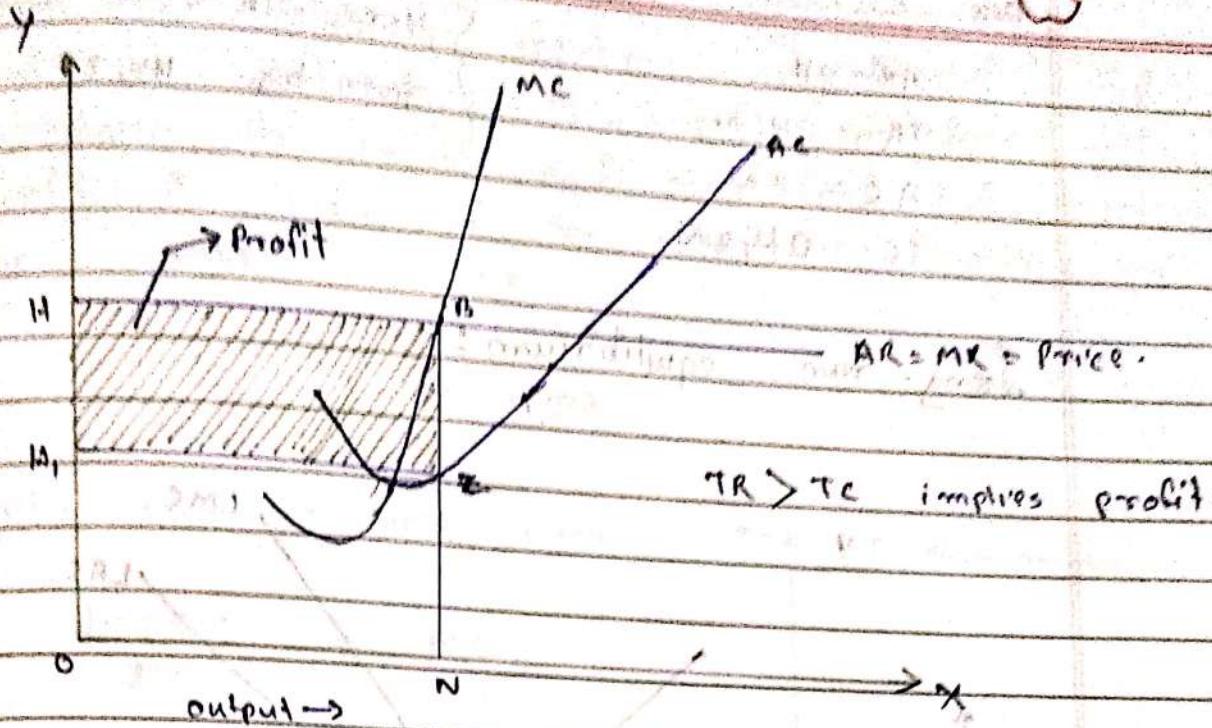
Condition of for the firms to be in eqⁿ in the short run under perfectly competitive market.

- (1) $MR = MC$
- (2) MC must be increasing at equilibrium point.



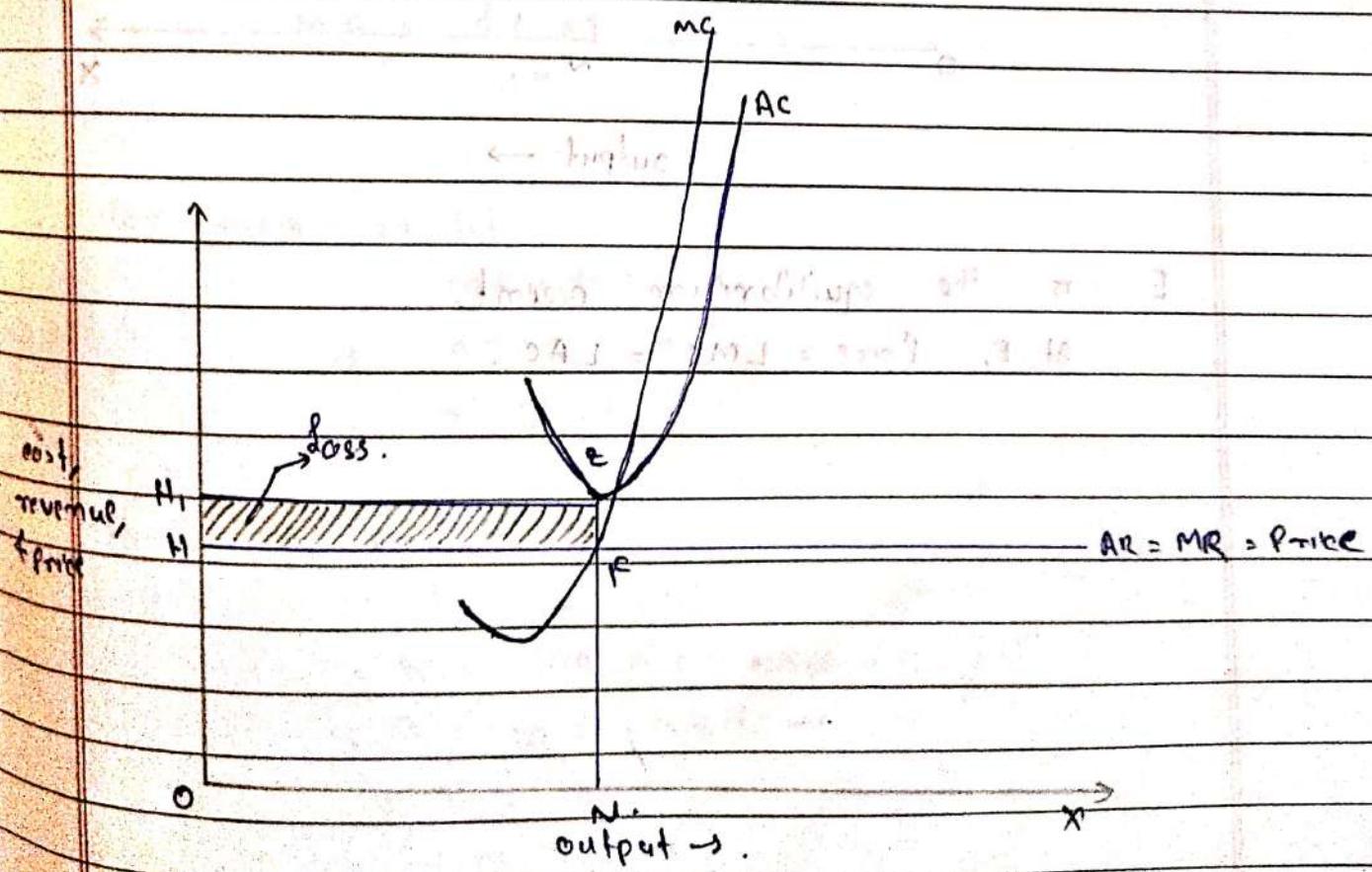
F shows the eqⁿ point because it satisfies both the condition, i.e. $MR = MC$ & MC is increasing.

Date / /



$$\Pi = TR - TC.$$

- if $AR > AC$
- \rightarrow super normal profit
- if $AR = AC$
- \rightarrow normal profit.
- $\left. \begin{array}{l} AR : OH \text{ or } BN, \\ TR : OHBN \\ TC : OH_1, BN. \end{array} \right\}$ (i.e. $AR \times BN = OH \times BN$)



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$$AR = OH$$

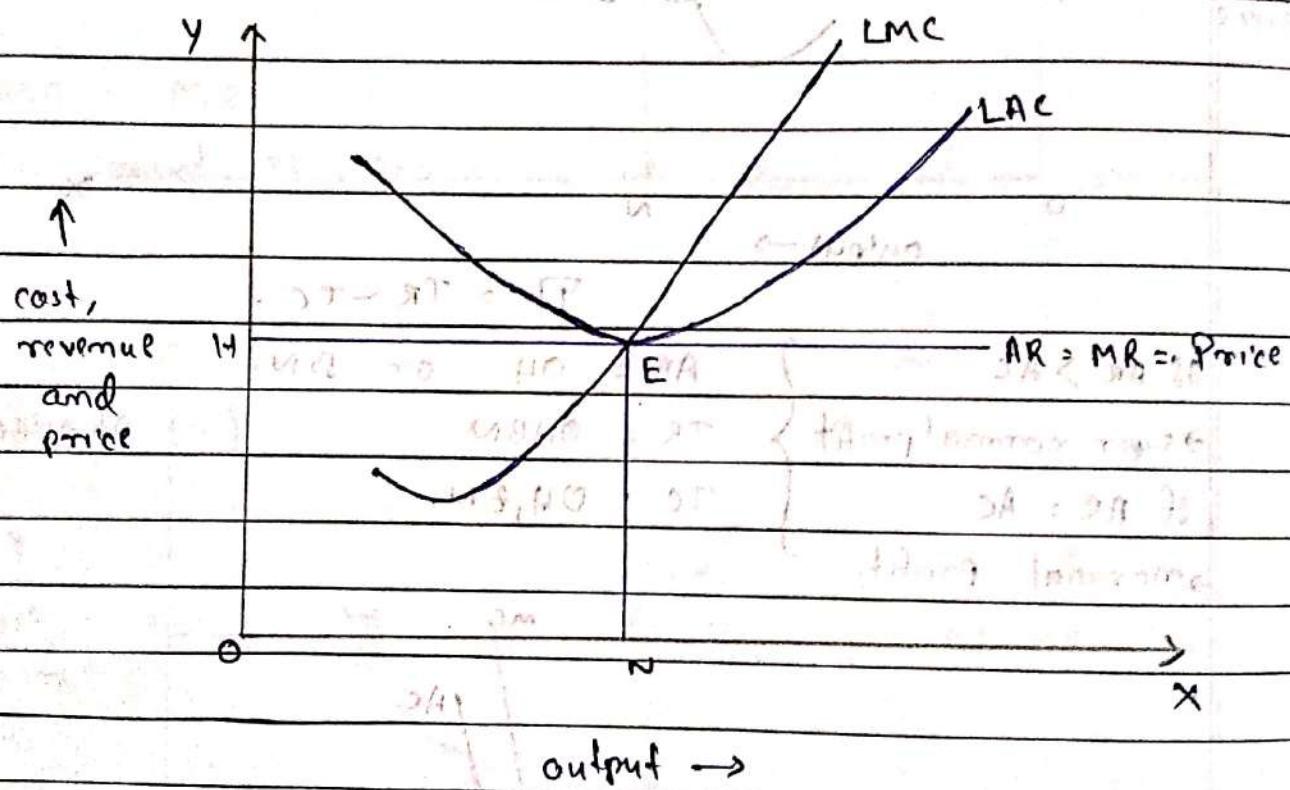
$$TR = OH \cdot FN.$$

$$AC = ZN.$$

$$TC = OH, ZN.$$

Hence, $TR < TC$ so, loss.
given by OH, ZF

Long-run equilibrium:



E is the equilibrium point,

At E, $\text{Price} = LMC = LAC$.



- (c) From the following short-run cost function find out at what level of output, the firm will maximise its profit and also find out maximum profit if price of the product prevailing in the market is 10 Rs.

$$TC = 4 + 8Q + Q^2$$

Soln: Profit will be max. when TC is minimum.

$$\frac{d^2(TC)}{dQ^2} = MC$$

$$\therefore MC = 8 + 2Q.$$

$$\star MR = TR = 10 \times Q$$

$$\therefore MR = \frac{d(TR)}{dQ} = 10.$$

For max. profit,

$$MR = MC$$

$$\Rightarrow 8 + 2Q = 10.$$

$$\Rightarrow Q = 1.$$

For max. profit:

$$\Pi = TR - TC.$$

$$\therefore \Pi = 10Q - (4 + 8Q + Q^2)$$

$$\therefore \Pi = 10 - (4 + 8 + 1)$$

$$\therefore \Pi = 10 - 13.$$

$$\therefore \Pi = -3.$$

$$\therefore \text{Profit} = -3.$$

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- (Q) From the following cost function and revenue function find out max. profit of the firm under perfectly competitive market in the short run.

$$TR = 12Q$$

$$TC = 2000 + 4Q + 0.02Q^2$$

Sol: $\frac{d}{dQ} (TR) = 12$

$$\frac{d}{dQ} (TC) = 4 + 0.04Q$$

At equilibrium,

$$MR = MC$$

$$\Rightarrow 12 = 4 + 0.04Q$$

$$\Rightarrow Q = \frac{12 - 4}{0.04}$$

$$\Rightarrow Q = \frac{8}{0.04} = 200$$

now,

$$\Pi = TR - TC$$

$$\Rightarrow 12Q - (2000 + 4Q + 0.02Q^2)$$

$$\Rightarrow (12 \times 200) - (2000 + 800 + 800)$$

$$\Rightarrow 2400 - 3600$$

$$\Rightarrow \Pi = -1200$$



Shutting Down Point.

conditions for the short shut down of a firm perfectly competitive market in short run.

$$\text{Price} = \text{AVC} = \text{MC}$$

- (Q) From the following total variable cost function in the short run under perfectly competitive market, find out the price at which the firm should shut down its production.

$$TVC = 150Q - 20Q^2 + 10^3$$

Soln: $\text{AVC} = \frac{TVC}{Q} = 150 - 20Q + 10^2$

$$\text{MC} \Rightarrow$$

* Minimum AVC

$$\frac{d(\text{AVC})}{dQ} = 0.$$

$$\Rightarrow -20 + 20 = 0.$$

$$\Rightarrow Q = 10.$$

\therefore Price is $(\text{AVC})_{\min}$

$$\therefore \text{AVC} = 150 - 20Q + 10^2$$

$$= 150 - (20 \times 10) + 100$$

$$= 150 - 200 + 100$$

$$\Rightarrow 50.$$

So, Price at which firm is shut down is 50.

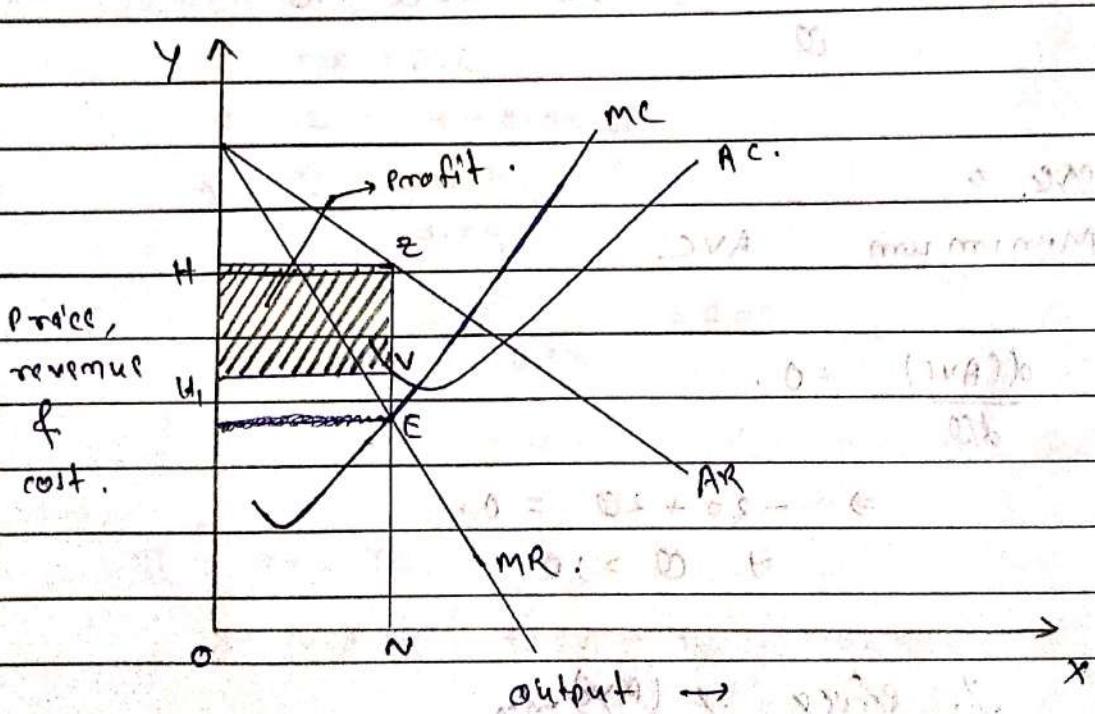


Monoopoly:

It refers to that type of market where there is a single seller and large number of buyers and there is restriction for the new firms enter into the industry and firms are producing selling the goods which have no close substitute.

Condition for the monoopolist to be in equilibrium

- ① $MR = MC$.
- ② MC must cut MR from below.

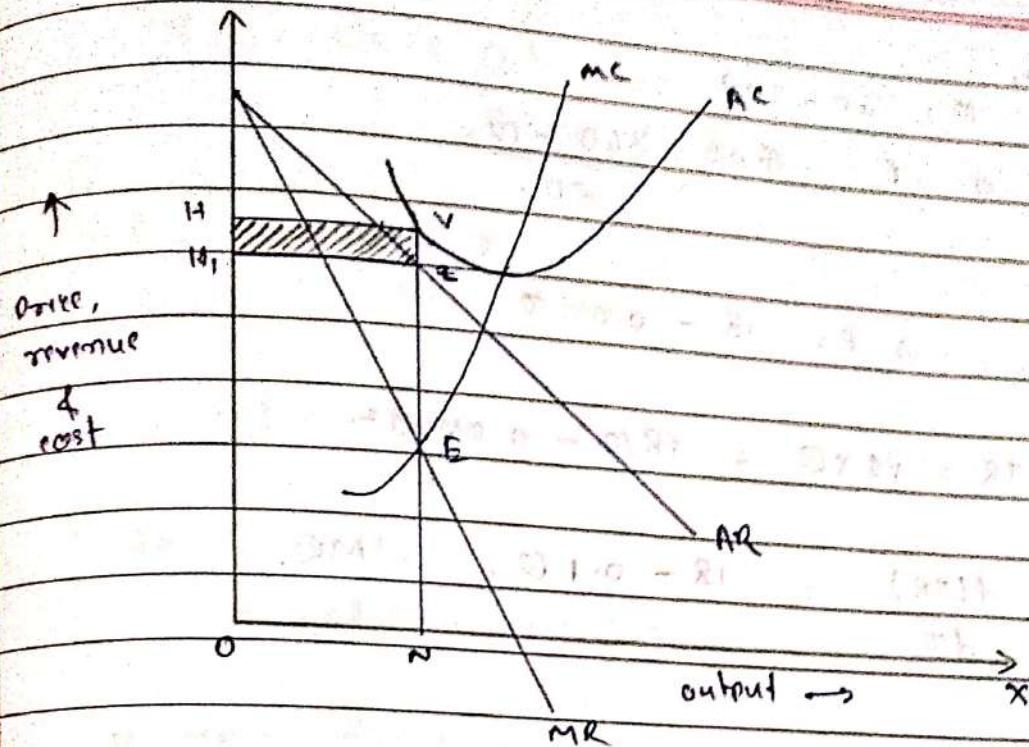


$$AR = OH$$

$$\therefore TR = OH \times N - (OH \times ON)$$

$$AC = OH,$$

$$\therefore TC = OH \times N - (OH \times ON).$$



$$AR = OH_1$$

$$TR = OH_1 \times ZN \quad (OH_1 \times BN)$$

$$AC = OH$$

$$TC = OH \times N \quad (OH \times BN)$$

- Q) From the following short run cost function and demand function find out max. profit of the monopolist - in the short run.

$$\begin{aligned} TC &= Q \cdot 6Q + 0.05 Q^2 \\ Q &= 360 - 20P \end{aligned} \quad \left. \begin{array}{l} P \rightarrow \text{Price} \\ Q \rightarrow \text{quantity} \\ \text{Demand} \end{array} \right.$$

$$MC = \frac{d(TC)}{dQ}$$

$$2) MC = 6 + 0.1Q$$

$$\text{now, } TR = P \times Q = 360P - 20P^2$$

Date / /

now,

$$0 > 360 - 20P$$

$$\Rightarrow P = \frac{360 - CQ}{20}$$

$$\Rightarrow P = 18 - 0.05CQ$$

$$\therefore TR = P \times CQ = 18CQ - 0.05CQ^2$$

$$\therefore \frac{d(TR)}{dCQ} = 18 - 0.1CQ = MR$$

For eq^u:

$$MR = MC$$

$$\Rightarrow 18 - 0.1CQ = 6 + 0.1CQ \quad (MC = 6)$$

$$\Rightarrow 0.2CQ = 12$$

$$\Rightarrow CQ = 60$$

(Ans) (Ans) (Ans) (Ans)

Now, for max profit

$$\Pi = TR - TC$$

$$\therefore \Pi = \left(18CQ - \frac{CQ^2}{20}\right) - 6CQ - 0.05CQ^2$$

$$\therefore \Pi = 360$$

- (Q) From the following cost and demand fun^u
find the eq^u price and quantity for
the monopolist and also find out
max profit.



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$$\text{Q} = 400 - 20P$$

$$TC = 50Q + \frac{Q^2}{50}$$

$$= 50 + \frac{Q^2}{50}$$

$$P = \frac{400 - Q}{20}$$

$$\therefore P = 20 - 0.05Q$$

$$\therefore TR = P \times Q$$

$$= 20Q - 0.05Q^2$$

$$\therefore MR = \frac{d(TR)}{dQ} = 20 - 0.1Q$$

$$\star MC = \frac{d(TC)}{dQ} = 50 + \frac{Q}{25}$$

For eqm. i.e. In equilibrium

$$\therefore MR = MC \Rightarrow 20 - 0.1Q = 5 + \frac{Q}{25}$$

$$\therefore 20 - 0.1Q = 5 + \frac{Q}{25}$$

$$\therefore 15 = 0.1Q + 0.04Q$$

$$\therefore 15 = 0.14Q \Rightarrow Q = 107.14$$

$$\therefore Q = 107.14$$

$$\therefore \text{Prof. } \pi = TR - TC =$$

$$= (20Q - 0.05Q^2 - 50 - \frac{Q^2}{50})$$

$$\therefore \pi = 803.57$$



Depreciation.

It refers to the loss in the value of any asset due to its constant use.

Methods:

- (1) Straight line method.
- (2) Declining Balance method.
- (3) Sum-of-the-year digit method.
- (4) Sinking fund method.

Straight line method:

It refers to that method of depreciation where a fixed rate of depreciation is charged directly on the initial value of asset on various years.

(1) Depreciation amount per year (D) = $\frac{I - S}{N}$.

$I \rightarrow$ initial value of the asset.

$S \rightarrow$ Salvage value / reduced value of asset.

$N \rightarrow$ Number of year life of the asset.

(2) Rate of depreciation (d)

$$= \frac{D}{I} \times 100$$

(3) Book value (B_t) = $I - (t \times D)$ {or $B_{t-1} - D_t$ }

$t \rightarrow$ the number of years the asset has been used.



(Q) An equipment has been purchased at 80,000 rs if estimated salvage value of the equipment at the end of the life of the equipment 10th year life of equipment is 1000. find out depreciation amount and book value of the asset after 5th year. with the help of straight line method.

$$I = 80,000.$$

$$S = 1000.$$

$$N = 5$$

$$D = \frac{I - S}{N}$$

$$\approx D = \frac{80,000 - 1000}{10} = 7900 \text{ Rs.}$$

* Book value = $I - (t \times D)$

$$\Rightarrow 80,000 - (5 \times 7900).$$

$$\approx 40500. \text{ Rs.}$$

R Rate of depreciation

(Q) If initial value of a machine is 50,000 rs with estimated salvage value of 5000 rs at the end of its service life of 8 years find out depreciation amount and book value of various eg every year.



Date / /

End of D. Bt.

year.

0	800	50,000
1	5625	80,000, 44375
2	5625	44375, 38750
3	5625	38750, 33125
4	5625	33125, 27500
5	5625	27500, 21875
6	5625	21875, 16250
7	5625	16250, 11250
8	5625	11250, 6000

Given,

$$I = 50,000$$

$$S = 5000$$

$$N = 8$$

$$(a \times I) - S = \text{value needed}$$

$$D = 50,000 - 5000 = 45,000$$

$$D = (0.08 \times 8) = 0.000,96$$

Declining Balance method.

It refers to that method of depreciation where a fixed rate of depreciation is charged on the declining year for various years.

①

$$D_t = k \times B_{t-1}$$

k = fixed percentage.



$$B_t = B_{t-1} - D_t.$$

- Q) If a machine has been purchased at 90,000 rs. find out its depreciation amount and book value for every year till the end of the 7th year of the life of the machine if the fixed rate of depreciation is 30%. using declining balance method.

Solu: End of the D_t. B_t
year.

0	-	90,000.
1	27000.	63,000
2	18900	44,100.
3	13230	30870.
4	9261.	21609
5	12.12164827	15126.3
6	4537.89	10588.41
7	3176.523.	7411.887-

* Sum of the year digit method of depreciation:

① Sum of the year $\frac{N(N+1)}{2}$

② D_t = Rate (I-s)

Rate = $\frac{\text{Rank}}{\text{sum of the year}}$

③ B_t = B_{t-1} - D_t.



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- (Q) If an asset has been purchased at ₹ 2,00,000 with estimated salvage value of ₹ 40,000 at the end of its service life of 10 yrs. Find out depreciation amount and book value at the end of various years with the help of sum of the year digit method.

Soln: $I = 2,00,000 \text{ Rs.}$ sum of the year = $10 \times 11 / 2 = 55$
 $S = 40,000.$ $\text{Rate } D_t = \frac{10}{55} (I-S).$
 $N = 10.$

End of the D_t B_t .
 year.

rank.	0	10	200000.	su 14:
1	10	1829090.90	200000.170909.1	
2	9	26181.81	1868181.44727.29	
3	8	23292.727	1805478.121454.563	
4	7	20363.636	101090.927	
5	6	17454.545	83636.382	
6	5	14545.454	69090.928	
7	4	11636.363	53454.565	
8	3	8727.272	48327.293	
9	2	5818.181	42909.112	
10.	1	2909.090,	40000.022	

Date 6/10/19.



Sinking Fund Method.

$$① A = (I - S) \left[\frac{i}{(1+i)^n - 1} \right]$$

$$② D_t = A \cdot (1+i)^{t-1}$$

$$③ B_t = B_{t-1} - D_t$$

Q) If $I = 100000$, $S = 20,000$, $m = 4$ yrs. Find out depreciation amount and book value for various years if $i = 12\%$. compounded annually with the help of sinking fund method.

Sol: $I = 1,00,000$.

$S = 20,000$ and $i = 0.12$

$m = 4$.

* 2). $A = I(I-S) \left[\frac{i}{(1+i)^n - 1} \right]$

2) $A = (100000 - 20000) \times \left[\frac{0.12}{(1.12)^4 - 1} \right]$

∴ $A = 16738.7549$ Rs.

∴ $D_t = A (1+i)^{t-1}$

Date _____

End of A D+ B+

Year

0	16738.754		1,00,000.
1	16738.754	16738.754.	83201.246.
2	16938.754	18747.404	64513.842
3	16938.754	20997.093	43516.749
4	16938.754	23516.744	20,000.

Break - Even Analysis

It refers to that analysis which discusses the behaviour of total revenue and total cost as the level of output changes.

Break - even point (BEP)

It refers to that point where $TR = TC$. It is also called point of no profit.

$$TR = TC$$

so, $\Pi = 0$

$$\{ \begin{aligned} \Pi &= TR - TC \\ \Pi &= 0, \end{aligned} \}$$

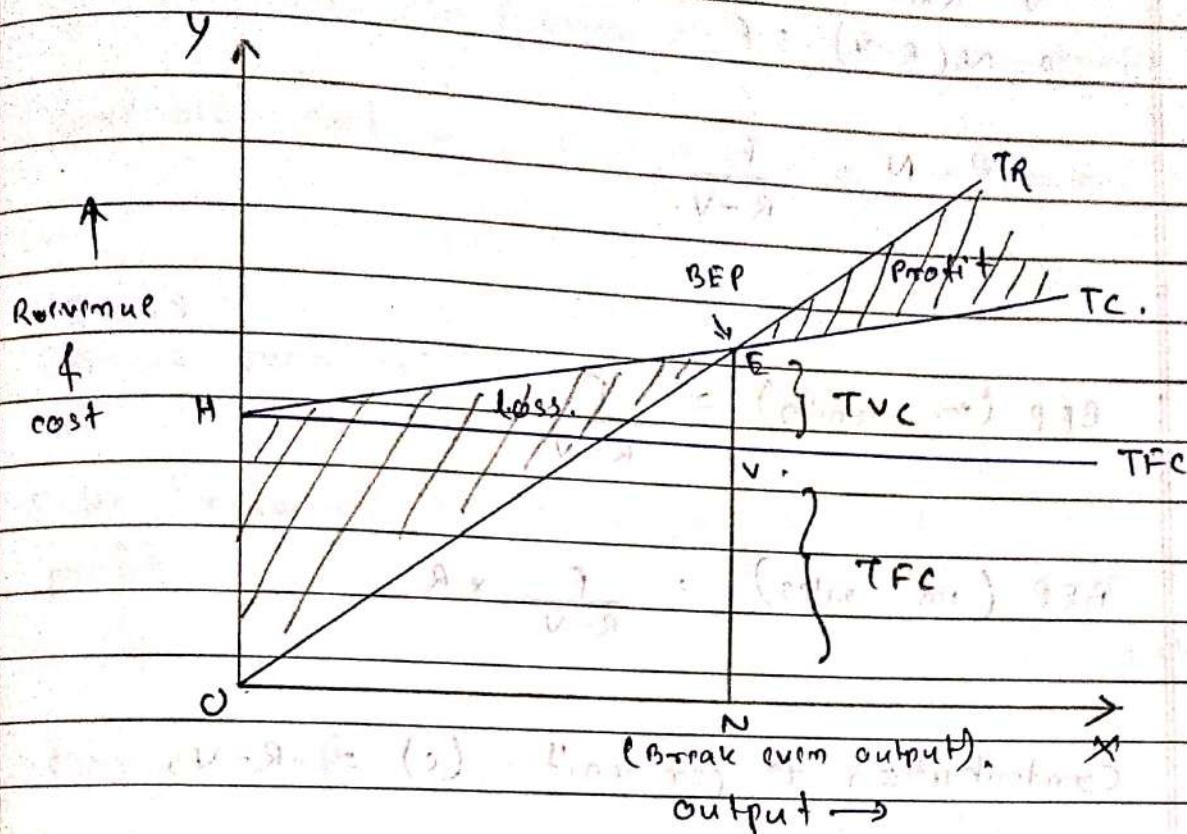
Methods of calculating BEPA

① Graphical method.

② Algebraic method.



① Graphical method:



Algebraic method:

R → selling price per unit.

N → number of units of output sold.

V → Variable cost per unit.

TR → Total

F → Total Fixed cost.

At BEP,

$$TR = TC \quad \text{--- (1)}$$

- $TR = P \times Q$

- $TR = R \times N$

- $TC = TFC + TVC$

- $TC = NF + VN$

From (1)

• $RN = F + VN$

$$\Rightarrow RN - VN = F \\ \Rightarrow N(R-V) = F$$

$$\Rightarrow N = \frac{F}{R-V}$$

$$\textcircled{1} \quad \text{BEP (in units)} = \frac{F}{R-V}$$

$$\textcircled{2} \quad \text{BEP (in sales)} = \frac{F}{R-V} \times R$$

$$\textcircled{3} \quad \text{Contribution to per unit (c)} = R-V$$

$$\textcircled{4} \quad \text{Profit} = \text{Total contribution} - F$$

$$\textcircled{5} \quad \text{Total contribution} = F+P$$

$$\textcircled{6} \quad \text{Units required to have a targeted profit} = \frac{F+P \rightarrow \text{targeted profit}}{R-V}$$

$$\textcircled{7} \quad \text{Sales required to have a targeted profit}$$

$$= \frac{F+P}{R-V} \times R$$

$$\textcircled{8} \quad \text{Margin of safety} = \text{Actual sales} - \text{BES}$$

$$\textcircled{9} \quad \text{P/V ratio} = \frac{\text{Contribution per unit}}{\text{Selling price per unit}}$$



(10) Fixed cost = $(\text{sales} \times \text{P/V ratio}) - \text{Profit}$.

(11) Variable cost = $\left(1 - \frac{P}{V} \text{ ratio}\right) \times \text{sales}$.

(12) BE_S (Break even sales) = $\frac{F}{\text{P/V ratio}}$

(13) Sales required to have a targeted profit = $\frac{F + P}{\text{P/V ratio}}$

(14) Margin of safety = $\frac{\text{Profit}}{\text{P/V ratio}}$.

(15) P/V ratio = $\frac{\text{change in profit}}{\text{change in sales}} \times 100$.

(Q) From the following information calculate.

BEOP, P/V ratio.

Selling price / unit = 80 rupees

variable cost / unit = 20 rupees

Total Fixed cost = 20,000 rupees

Date / /

Soln: Given, $R = 30 \text{ Rs.}$
 $V = 20 \text{ Rs.}$

$$P = 20,000 \text{ Rs.}$$

* BEP (in sales) = $\frac{F}{R-V} \times R = \frac{20,000}{(30-20)} \times 30 = 60,000 \text{ Rs.}$

* BEP (in units) = $\frac{F}{R-V} = \frac{20,000}{10} = 2000 \text{ units}$

* P/V ratio = $\frac{R-V}{R} \times 100 = \frac{10}{30} \times 100 = 33.33\%$

(Q)

P/V ratio, fixed cost, variable cost in 2010,
BEP (in sales), sales required to earn a
profit of 20,000 Rs.

Year	Sales	Cost
2010	1,20,000	1,10,000
2011	1,40,000	1,27,000

Given,

$$P = 20,000 \text{ Rs.} \quad \text{Profit} = 9,000 \text{ Rs.}$$

$$13,000 \text{ Rs.}$$

$\text{P/V ratio} = \frac{\text{change in profit}}{\text{change in sales}} \times 100$

$$\Rightarrow \text{P/V ratio} = \frac{4000}{\frac{20000}{5}} \times 100$$

$$\Rightarrow \text{P/V ratio} = 20\%$$

* Fixed cost = (sales \times P/V ratio) - profit

$$= (1,20,000 \times 0.2) - 9,000$$

* Variable cost = $(1 - \text{P/V ratio}) \text{ sales}$

$$\Rightarrow (1 - 0.2) \times 1,20,000$$

$$= 96,000 \text{ Rs.}$$

$$\text{BEP (sale)} = \frac{F}{\text{P/V ratio}} = \frac{15,000}{0.2}$$

$$= 75,000 \text{ Rs.}$$

* Sales to earn a profit of 20,000

$$= 15,000 + 20,000 \\ \times 0.2$$

$$= 1,78,000 \text{ Rs.}$$

Date _____ / _____ / _____

① Topic name, & roll no., name, submitted to.

② Certificate.

③ Acknowledgement.

④ Index.

⑤ Introduction.

Topic. (photos). (10 pages - write up + diagram)

⑥ Conclusion.

Q) From the following information find out
i) P/V ratio: ii) BEP iii) Profit when the output
is 50,000 units.

$$F = 1,20,000 \quad , \quad V = 0.8X, R = 7.1$$

$$\text{SOL: i) P/V ratio} = \frac{R-V}{R} \times 100 = \frac{7.1-0.8}{7.1} \times 100 = 57.143\%$$

$$\text{ii) BEP (units)} = \frac{F}{R-V} = \frac{1,20,000}{7.1-0.8} = 30,000 \text{ units}$$

$$\text{iii) BEP (sales)} = \text{BEP (units)} \times \frac{R}{R-V} = 30,000 \times 7 = 210,000 \text{ Rs}$$

iv) Given, units required to have a target profit = 50,000

$$50,000 = \frac{F+P}{R-V}$$

$$\Rightarrow P = 50,000 \times (7.1) - 1,20,000$$

$$\Rightarrow P = 80,000 \text{ Rs.}$$



Inflation

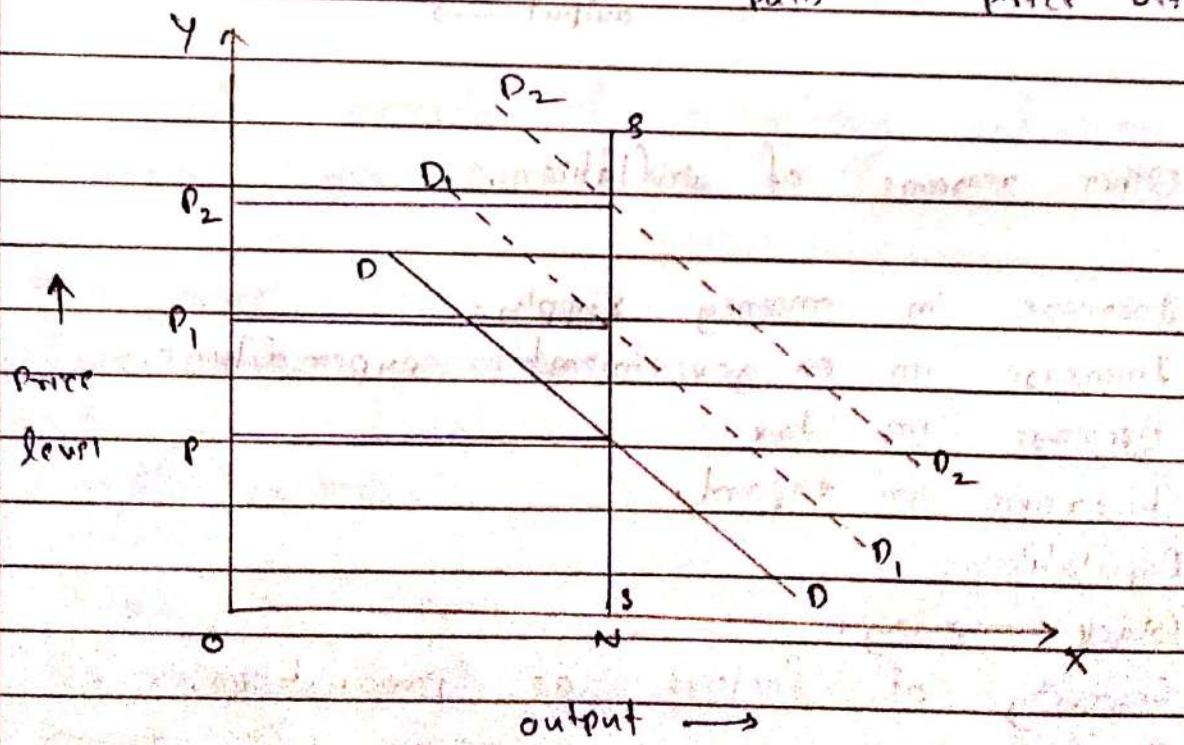
It refers to a substantial and continuous rise in price level which reduces purchasing power of money.

Causes of inflation:

- ① Demand - Pull inflation.
- ② Cost - Push inflation.

Demand - Pull inflation.

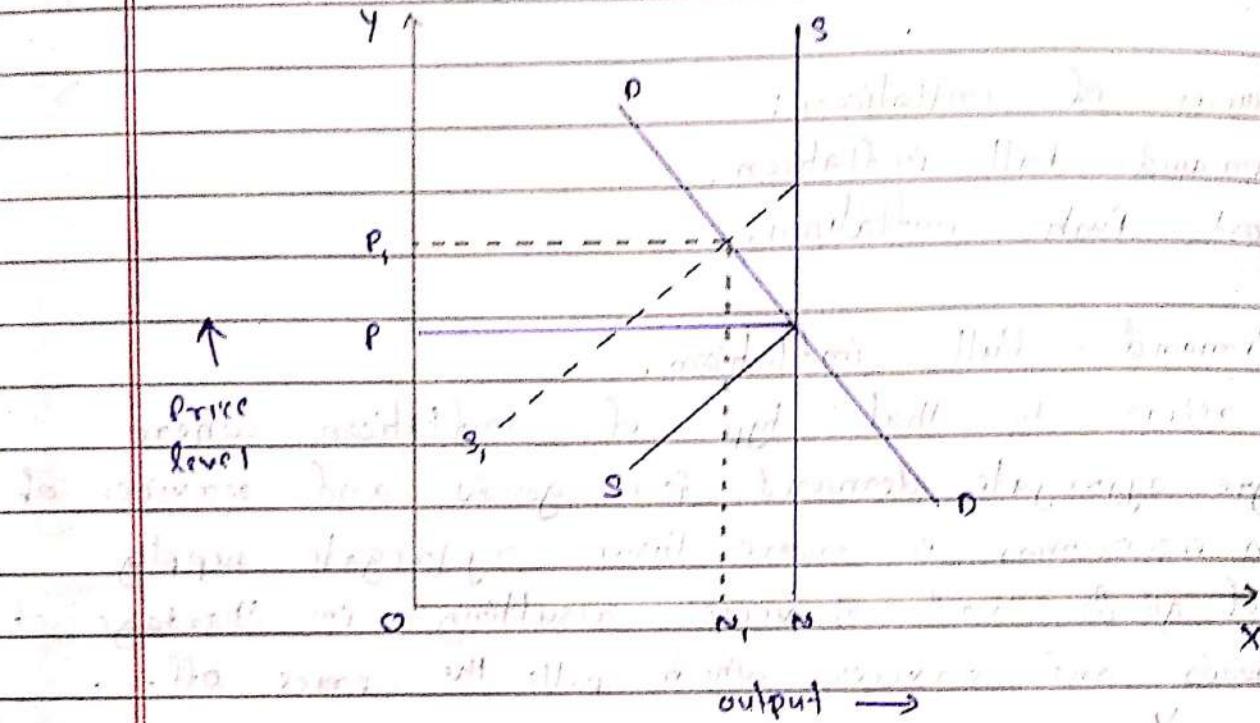
It refers to that type of inflation where aggregate demand for goods and services of an economy is more than aggregate supply of goods and services resulting in shortage of goods and services which pulls the price off.



Cost - Push inflation.

It refers to that type of inflation which results from increase in the cost of production because of which instead of increasing the price,

producers reduces the quantity supplied to the market.



Other reasons of inflation.

- ① Increase in money supply.
- ② Increase in government expenditure.
- ③ Decrease in tax.
- ④ Increase in export.
- ⑤ Population.
- ⑥ Black money.
- ⑦ Scarcity of factors of production.
- ⑧ Trade union.



Factors of controlling inflation.

- ① Monetary Policy.
- ② Fiscal Policy.

Monetary policy.

- ① Increase in bank rate.
- ② sale and purchase of government securities.
- ③ Increase in cash reserve ratio (C.R.R.).
- ④ Credit control.

Fiscal Policy:

- ① Increase in (Tax. \times 100,00,000)
- ② Decrease in government expenditure.
- ③ Increase in public borrowing.

Quantitative methods of controlling inflation by Reserve Bank of India

- ① Bank rate.
 - ② Open market operation.
 - ③ CRR.
 - ④ Credit control.
- (Q) From the following information find out P/V ratio, fixed cost, BEP, variable cost, Margin of safety.

Particulars	1996	2000	2007
Sales	1,50,00,000	2,00,00,000	2,00,00,000
Profit	30,00,000	50,00,000	50,00,000

SOLY!

* P/V ratio = $\frac{\text{change in profit} \times 100}{\text{change in sales}}$

$$= \frac{20,00,000}{50,00,000} \times 100$$

* Fixed cost = (sales \times P/V ratio) - Profit.

$$= (1,50,00,000 \times 0.4) - 30,00,000$$

$$= 30,00,000$$

* Break Even Point = $\frac{\text{Fixed cost}}{\text{P/V ratio}}$ $= \frac{30,00,000}{0.4}$

$$= 75,00,000$$

* Margin of Safety = $\frac{\text{Profit}}{\text{P/V ratio}}$

$$= \frac{30,00,000}{0.4}$$

* Variable cost MOS₂₀₀₇ = $\frac{50,00,000}{0.4}$

$$= 1,25,00,000$$

Date ___ / ___ / ___



10, 8

$$\star V_{2006} = (1 - \text{P/V ratio}) \times \text{sales}$$

$$= (1 - 0.4) \times 1,50,00,000$$

$$= 90,00,000 \text{ ru.}$$

$$\star V_{2007} = (1 - 0.4) \times 2,00,00,000$$

$$= 1,20,00,000 \text{ ru.}$$