Chapter 8

ORTHOGRAPHIC PROJECTION

8-0. INTRODUCTION

Practical solid geometry or *descriptive geometry* deals with the representation of points, lines, planes and solids on a flat surface (such as a sheet of paper), in such a manner that their relative positions and true forms can be accurately determined.

8-1. PRINCIPLE OF PROJECTION

If straight lines are drawn from various points on the contour of an object to meet a plane, the object is said to be projected on that plane. The figure formed by joining, in correct sequence, the points at which these lines meet the plane, is called the *projection* of the object. The lines from the object to the plane are called *projectors*.

8-2. METHODS OF PROJECTION

In engineering drawing following *four* methods of projection are commonly used, they are:

(1) Orthographic projection
(2) Isometric projection
(3) Oblique projection
(4) Perspective projection.

In the above methods (2), (3) and (4) represent the object by a pictorial view as eyes see it. In these methods of projection a three dimensional object is represented on a projection plane by one view only. While in the orthographic projection an object is represented by two or three views on the mutual perpendicular projection planes. Each projection view represents two dimensions of an object. For the complete description of the three dimensional object at least *two* or *three* views are required.

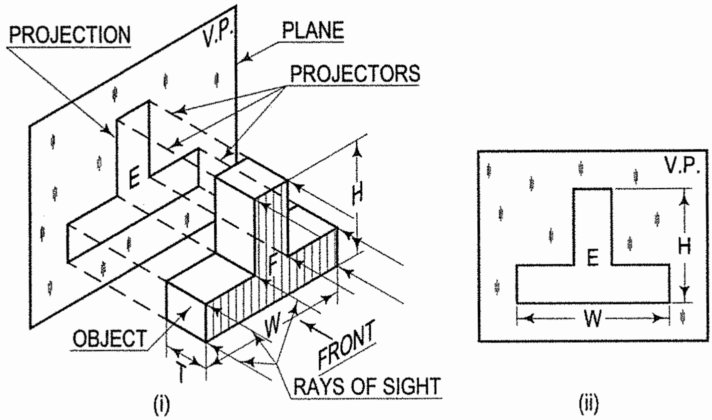
8-3. ORTHOGRAPHIC PROJECTION

When the projectors are parallel to each other and also perpendicular to the plane, the projection is called *orthographic projection*.

Step 1: Imagine that a person looks at the block [fig. 8-1(i)] from a theoretically infinite distance, so that the rays of sight from his eyes are parallel to one another and perpendicular to the front surface *F*. The view of this block will be the shaded figure, showing the front surface of the object in its true shape and proportion.

Step 2: If these rays of sight are extended further to meet perpendicularly a vertical plane (marked V.P.) set up behind the block.

Step 3: The points at which they meet the plane are joined in proper sequence, the resulting figure (marked *E*) will also be exactly similar to the front surface and this is known as *an elevation or front-view*. This figure is the projection of the block. *The lines from the block to the plane are the projectors.* As the projectors are perpendicular to the plane on which the projection is obtained, it is the orthographic projection. The projection is shown separately in fig. 8-1(ii). It shows only two dimensions of the block viz. the height *H* and the width *W*. It does not show the thickness. Thus, we find that only one projection is insufficient for complete description of the block.



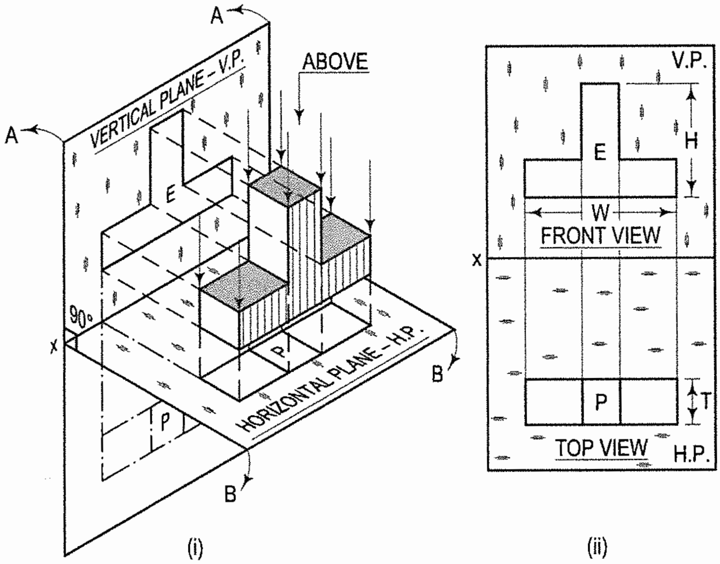


FIG. 8-2

Let us further assume that another plane marked H.P. (horizontal plane) [fig. 8-2(i)] is hinged at right angles to the first plane, so that the block is in front of the V.P. and above the H.P. The projection on the H.P. (figure *P*) shows the top surfaces of the block. If a person looks at the block from above, he will obtain the same view as the figure *P* and is known as a *plan* or *top-view*. It shows the width *W* and the thickness *T* of the block. It however does not show the height of the block.

One of the planes is now rotated or turned around on the hinges so that it lies in extension of the other plane. This can be done in *two* ways:

(i) by turning the V.P. in direction of arrows *A*

(ii) by turning the H.P. in direction of arrows *B*.

The H.P. when turned and brought in line with the V.P. is shown by dashed lines. The two projections can now be drawn on a flat sheet of paper, in correct relationship with each other, as shown in fig. 8-2(ii).

When studied together, they supply all information regarding the shape and the size of the block. Any solid may thus be represented by means of orthographic projections or orthographic views.

8-4. PLANES OF PROJECTION

The two planes employed for the purpose of orthographic projections are called *reference planes or principal planes of projection*. They intersect each other at right angles. The *vertical plane* of projection (in front of the observer) is usually denoted by the letters V.P. It is often called the *frontal plane* and denoted by the letters F.P.

The other plane is the *horizontal plane* of projection known as the H.P. The line in which they intersect is termed the *reference line* and is denoted by the letters *xy*. The projection on the V.P. is called the *front view* or the *elevation* of the object. The projection on the H.P. is called the *top view* or the *plan*.

8-5. FOUR QUADRANTS

When the planes of projection are extended beyond the line of intersection, they form four quadrants or dihedral angles which may be numbered as in fig. 8-3. The object may be situated in any one of the quadrants, its position relative to the planes being described as "*above or below the H.P.*" and "*in front of or behind the V.P.*"

The planes are assumed to be transparent. The projections are obtained by drawing perpendiculars from the object to the planes, i.e. by looking from the front and from above. They are then shown on a flat surface by rotating one of the planes as already explained. *It should be remembered that the first and the third quadrants are always opened out while rotating the planes*. The positions of the views with respect to the reference line will change according to the quadrant in which the object may be situated. This has been explained in detail in the next chapter.

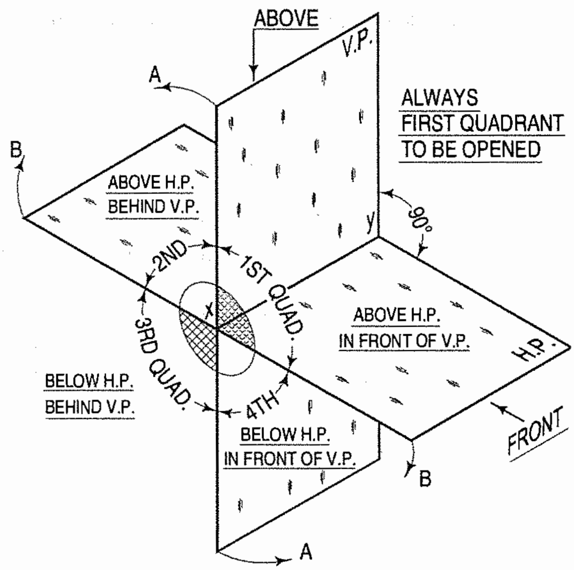


FIG. 8-3

8-6. FIRST-ANGLE PROJECTION

We have assumed the object to be situated in front of the V.P. and above the H.P. i.e. in the first quadrant and then projected it on these planes. This method of projection is known as *first-angle projection method*. The object lies between the observer and the plane of projection. In this method, when the views are drawn in their relative positions, the top view comes below the front view. In other words, the view seen from above is placed on the other side of (i.e. below) the front view. Each projection shows the view of that surface (of the object) which is remote from the plane on which it is projected and which is nearest to the observer.

TABLE 8-1
DIFFERENCE BETWEEN FIRST-ANGLE PROJECTION METHOD AND THIRD-ANGLE PROJECTION METHOD

No.	First-angle projection method	Third-angle projection method
1.	The object is kept in the *first quadrant*.	The object is assumed to be kept in the *third quadrant*.
2.	The object lies between the observer and the plane of projection.	The plane of projection lies between the observer and the object.
3.	The plane of projection is assumed to be non-transparent.	The plane of projection is assumed to be *transparent*.
4.	In this method, when the views are drawn in their relative positions, the *plan* comes *below* the *elevation*, the view of the object as observed from the *left-side* is drawn to the *right of elevation*.	In this method, when the views are drawn in their relative positions, the *plan*, comes *above* the elevation, *left hand side view* is drawn to the *left hand side of the elevation*.
5.	This method of projection is now recommended by the "Bureau of Indian Standards' from 1991.	This method of projection is used in U.S.A. and also in other countries.

8-7. THIRD-ANGLE PROJECTION

In this method of projection, the object is assumed to be situated in the third quadrant [fig. 8-4(i)]. The planes of projection are assumed to be transparent. They lie between the object and the observer. When the observer views the object from the front, the rays of sight intersect the V.P. The figure formed by joining the points of intersection in correct sequence is the front view of the object. The top view is obtained in a similar manner by looking from above. When the two planes are brought in line with each other, the views will be seen as shown in fig. 8-4(ii). The top view in this case comes above the front view.

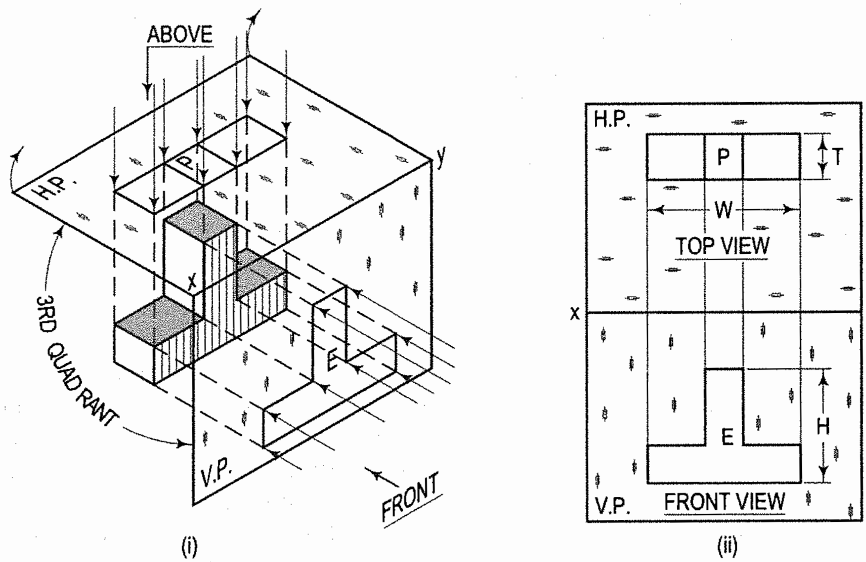


FIG. 8-4

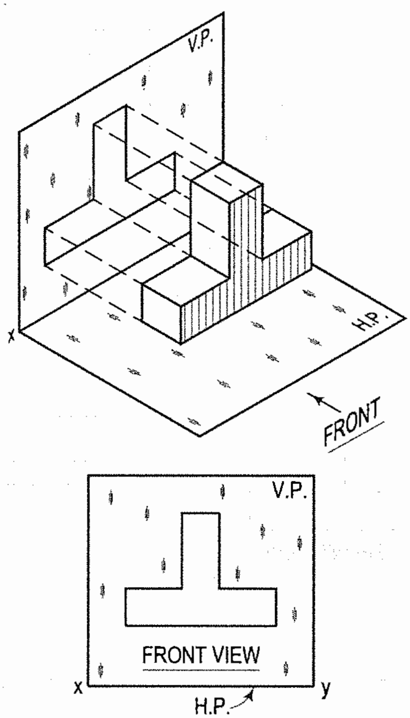
In other words, the view seen from above the object is placed on the same side of (i.e. above) the front view.

Each projection shows the view of that surface (of the object) which is nearest to the plane on which it is projected.

On comparison, it is quite evident that the views obtained by the two methods of projection are completely identical in shape, size and all other details. The difference lies in their relative positions only.

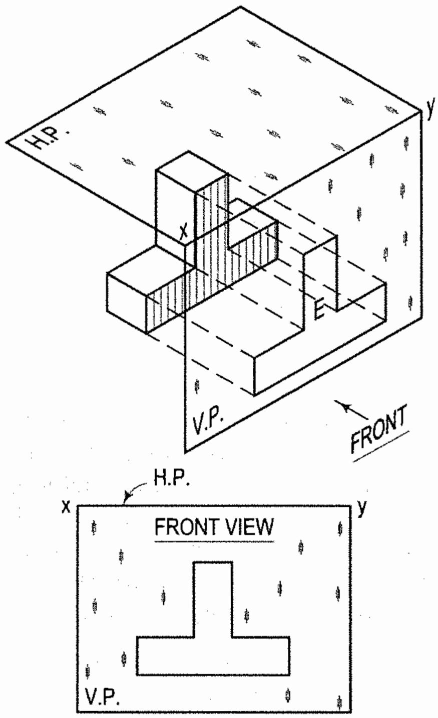
8-8. REFERENCE LINE

Studying the projections independently, it can be seen that while considering the front view (fig. 8-5 and fig. 8-6), which is the view as seen from the front, the H.P. coincides with the line *xy*. In other words, *xy* represents the H.P.



FIRST-ANGLE PROJECTION

FIG. 8-5

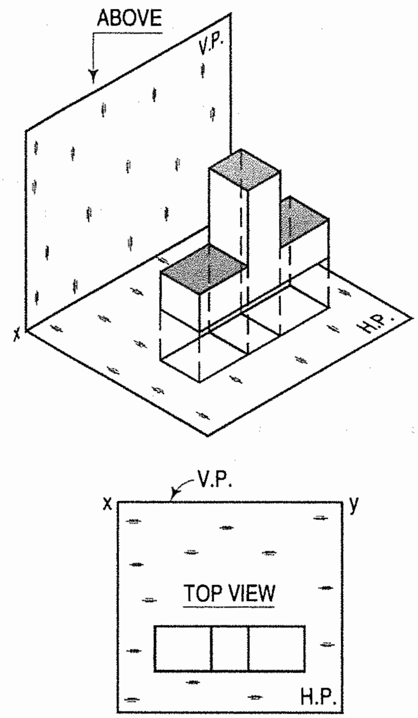


THIRD-ANGLE PROJECTION

FIG. 8-6

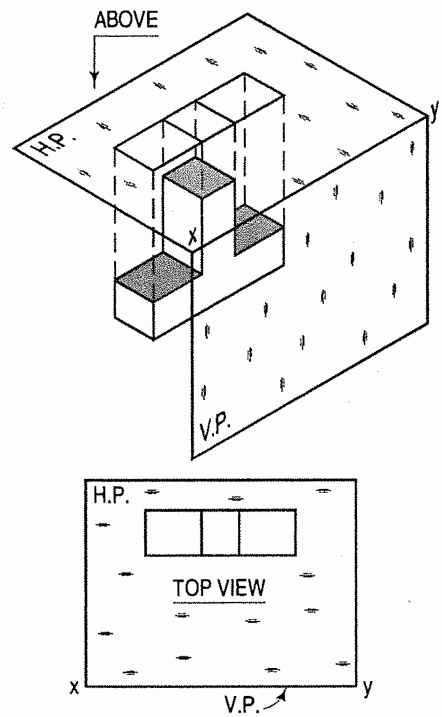
Similarly, while considering the top view (fig. 8-7 and fig 8-8), which is the view obtained by looking from above, the same line *xy* represents the V.P. Hence, when the two projections are drawn in correct relationship with each other (fig. 8-9), *xy* represents both the H.P. and the V.P. This line *xy* is called the *reference line*. The squares or rectangles for individual planes are thus unnecessary and are therefore discarded.

Further, in first-angle projection method, the H.P. is always assumed to be so placed as to coincide with the ground on or above which the object is situated. Hence, in this method, the line *xy* is also the line for the ground.



FIRST-ANGLE PROJECTION

FIG. 8-7



THIRD-ANGLE PROJECTION

FIG. 8-8

In third-angle projection method, the H.P. is assumed to be placed above the object. The object may be situated on or above the ground. Hence, in this method, the line *xy* does not represent the ground. The line for the ground, denoted by letters GL, may be drawn parallel to *xy* and below the front view [fig. 8-9(ii)].

In brief, when an object is situated on the ground, in first-angle projection method, the bottom of its front view will coincide with xy; in third-angle projection method, it will coincide with GL, while xy will be above the front view and parallel to Ground line.

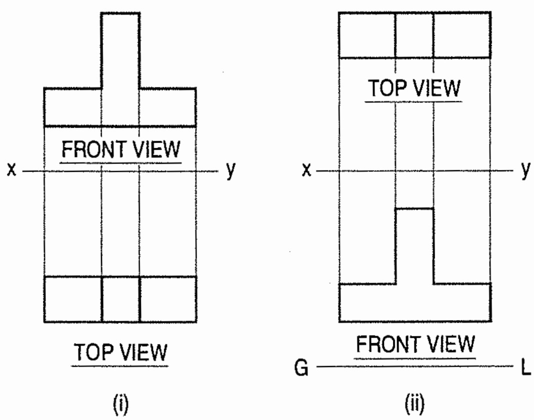


FIG. 8-9

Symbols for methods of projection: For every drawing it is absolutely essential to indicate the method of projection adopted. This is done by means of a symbolic figure drawn within the title block on the drawing sheet.

The symbolic figure for the first-angle projection method is shown in fig. 8-10, while that for the third-angle projection method is shown in fig. 8-11 which are self expanatory. These symbolic figures are actually the projections of a frustum of cone of convenient dimensions according to the size of drawing.

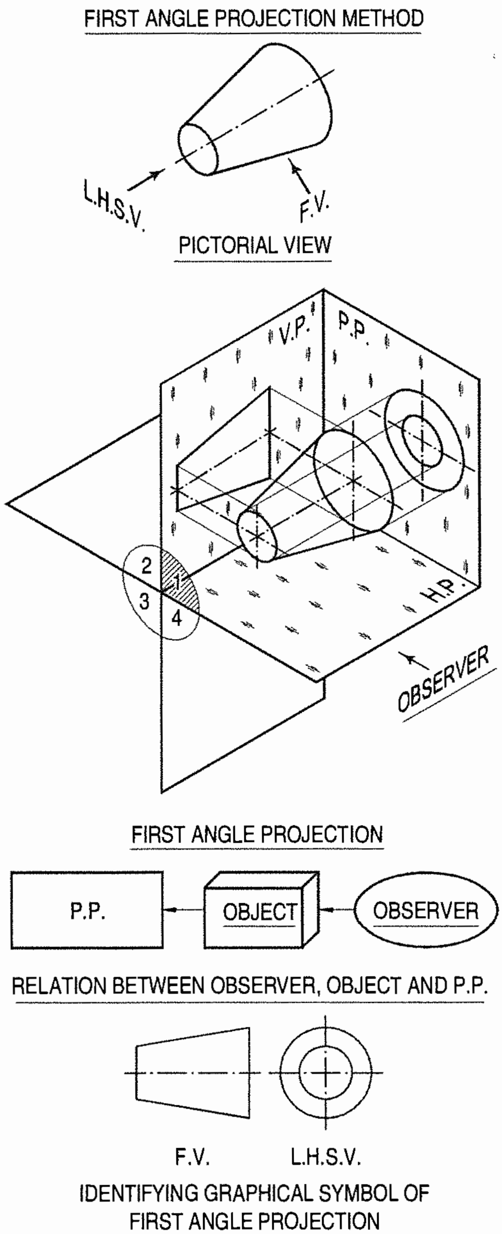


FIG. 8-10

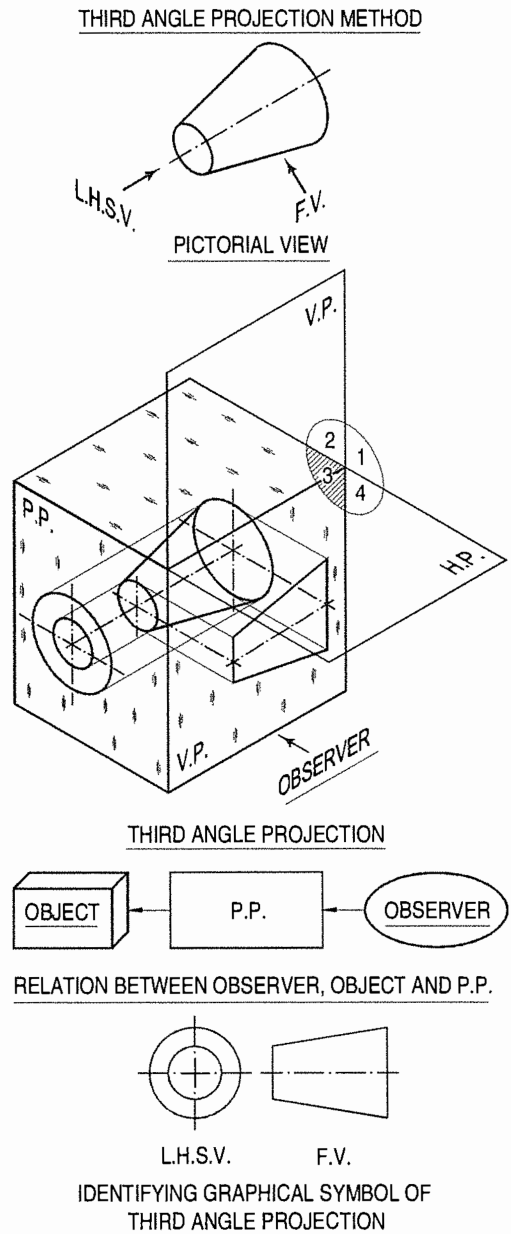


FIG. 8-11

Six views of an Object: There are *three* important elements of this projection system, namely

(a) an object
(b) plane of projection
(c) an observer.

Very often, two views are not sufficient to describe an object completely. The planes of projection being imaginary, following six views are obtained:

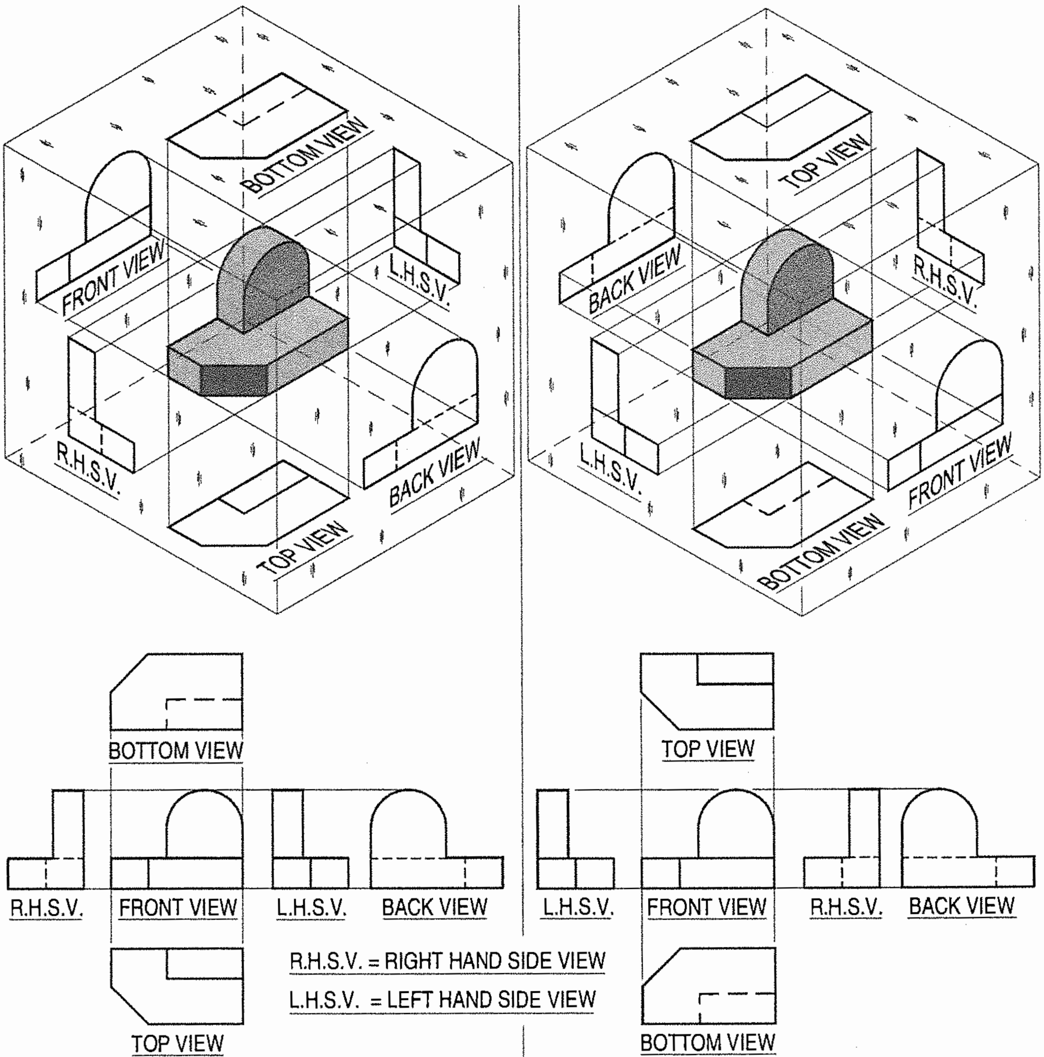
(1) Front view (4) Right hand side view

(2) Top view (5) Back view

(3) Left hand side view (6) Bottom view

These projections are shown projected on the respective planes, placed by the methods of first-angle projection and third-angle projection as shown in fig. 8-12 and fig. 8-13 respectively.

Ordinarily, two views — the front view and top view are shown. Two other views i.e. L.H.S.V. or R.H.S.V. may be required to describe an object completely. Only in exceptional cases, when an object is of a very complex nature, five or six views may be found necessary.



FIRST ANGLE PROJECTION

FIG. 8-12

THIRD ANGLE PROJECTION

FIG. 8-13

8-9. B.I.S. CODE OF PRACTICE

The method of first-angle projection is the British standard practice. The third-angle projection is the standard practice followed in America and in the continent of Europe.

In our country, the first-angle projection method was formerly in use. The Indian Standards Institution (I.S.I.) now Bureau of Indian Standards (B.I.S.), in its earlier versions of Indian Standard (IS:696) 'Code of Practice for General Engineering Drawing' published in 1955 and revised in 1960 had recommended the use of third-angle projection method.

In the second revised version of this standard published in December 1973, the committee responsible for its preparation left the option of selecting first-angle or third-angle projection method to the users.

The committee again reviewed the position and *finally recommended revised SP:46-1988 and SP:46-2003 for implementation of first-angle method of projection in our country*, by replacing earlier IS:696 drawing standard.

Persons engaged in engineering profession may come across drawings from industries and organizations following any one method. It is therefore necessary for them to be perfectly conversant with both the methods.

In this book, the method of first-angle projection has been generally followed. Third-angle projection method is also adequately treated in the form of illustrative problems and set exercises.

Conventions employed: In this book, actual points, ends of lines, corners of solids etc., in space are denoted by capital letters *A*, *B*, *C* etc. Their *top views* are marked by corresponding small letters *a*, *b*, *c*, etc., their *front views* by small letters with dashes *a*', *b*', *c*', their side views by a_1, b_1, c_1, and their auxiliary views by a'_1, b'_1, c'_1, etc. In pictorial views, the projectors from the points in space to the planes are shown by dashed lines.

The lines from the projections to the reference line *xy* (which are also called projectors, though they are the projections of the projectors) are shown as dash and dot lines. In orthographic views, the projectors and other construction lines are shown continuous, but thinner than the lines for actual projections.

8-10. TYPICAL PROBLEMS

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 18 for the following problem.

Problem 8-1. (fig. 8-14): *A L-shaped solid object having dimentions of length (L), width (W) and height (H) in the fig. 8-14. Assuming that this object is lying in the first quadrant. Draw it's front view, top view and side view.*

When the given object is in the first quadrant, its front view appears in the imaginary vertical plane V.P. behind it while its top view appears in an imaginary horizontal plane, H.P. below. The side view appears to the right or left of the front view depending on from which side the object is being viewed.

(i) Mark the visible corners of the given block as shown

Drawing front orthogonal view:

Assume that you are viewing the object in the direction of the arrow towards the imaginary V.P. What you will see is a rectangle of height H and width W on V.P. This will be the front view. To draw this view:

(ii) Draw a reference line xy, which represents the intersecting line of the planes V.P. and H.P. Draw a rectangle as shown W and H, above xy make sure that the width is parallel to the line xy. The rectangle is the front orthogonal view of the object.

(iii) Draw a line parallel to and thickness of h to the line 1-2. The rectangle 1-2-4-3 is the front view of the horizontal L-shaped stem of the object.

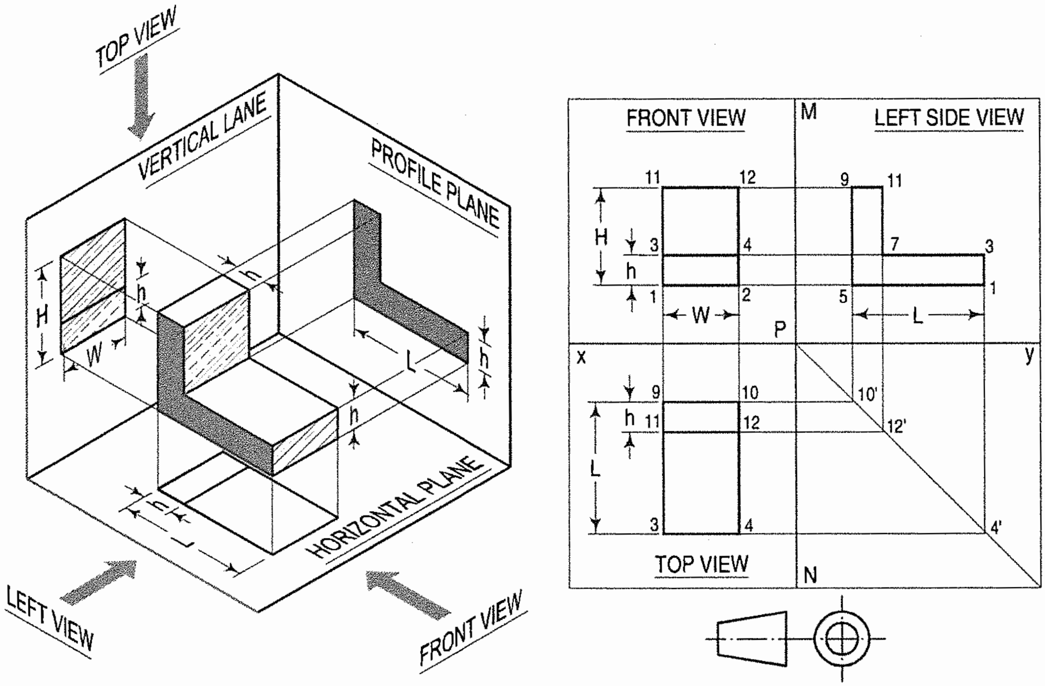


FIG. 8-14

Drawing top orthogonal view:

Now, if you look the object from the top, you will see a rectangle of Length L and width W on the horizontal plane. This is the top view of plan of the object. To draw this view:

(iv) Draw vertical projectors from 1 and 2 and extend them beyond the line xy. Draw a line 9-10 below and parallel to the reference line xy. Draw the lines 9-3 and 10-4 equal to the length L of the object. Join line 3-4. The rectangle 9-10-3-4 is the top view of the object.

(v) Draw a line 11-12 parallel to and below 9-10 of thickness h. The rectangle 9-11-12-10 is the view of the vertical stem of the object.

Drawing the side orthogonal view:

Now if you look at the object from the left side, what you will see is an L-shaped image having a length of L and height of H on the auxiliary plane, AP. This view appears adjacent and to the right of front view. To draw this view:

(vi) Draw a reference vertical line *MN* at right angles to *xy* cutting it at *P*. From *P* draw a construction line at 45° in the fourth quadrant.

(vii) Project lines from the points 10, 12 and 4 of the top view to meet this inclined line at 10', 12' and 4'.

(viii) Project lines from points 2, 4 and 12 from the front view parallel to line *xy*. From points 10', 12' and 4' project lines vertically upwards to meet these horizontal projections.

(ix) Join points 5-1-3-7-11-9. This will be the side view of the object.

(x) Finally draw the symbol of first angle projection at the right bottom corner of the drawing.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 19 for the following problem.

Problem 8-2. (fig. 8-15 and fig. 8-16): *A pictorial view of a machine bracket is shown in the figure. Draw using the first angle projection method front view, top view and right end side view.*

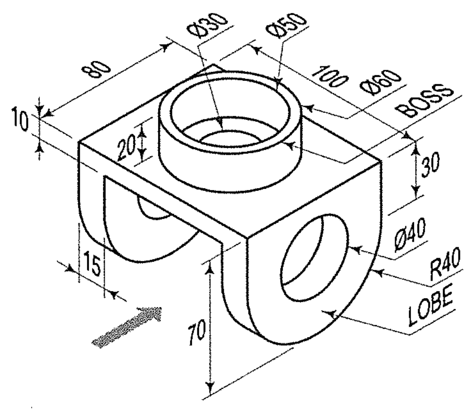


FIG. 8-15

Assume that you are viewing the object in the direction of the arrow towards the imaginary V.P. What will you see? It is a rectangle of height 70 mm and width 100 mm on V.P. This will be the front view. To draw this view:

(i) Draw a Reference line *xy*, which represents the intersecting line of the planes V.P. and H.P.

(ii) Draw rectangular block of size 100 × 70 and the thickness is 15 mm each on the two parallel portions of the slot.

(iii) Now on the top of this rectangular block draw a rectangle of size 60 mm × 20 mm. This rectangle represents a boss of ø60 on the rectangular block.

(iv) Draw a hole of ø50 in the above boss. Note carefully that the lines are dashed lines as they are invisible from the front side. The ø30 hole is in the rectangular block.

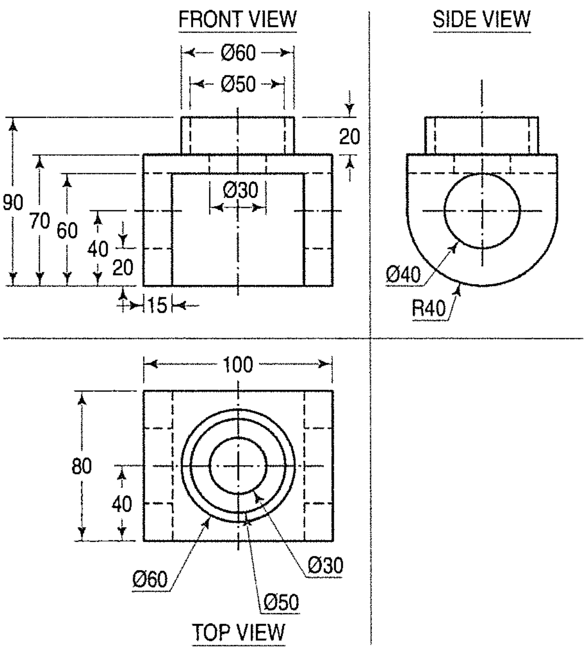


FIG. 8-16

(v) Draw the two holes on the parallel edges of the rectangular block. These holes are represented by dotted lines in the front view as they are hidden. This completes the front view construction.

(vi) To draw the top view, project all the details from front view.

(vii) Draw a vertical centre line form the centre of the rectangular block.

(viii) The top view of the rectangular block appears as a rectangle of size 100 × 80.

(ix) Draw the holes in the centre of the rectangle. These holes are drawn taking the common centre point on vertical centre line.

(x) The two holes of ø40 on the lobes are not visible, so they are projected as dashed lines. This completes the top view.

(xi) To draw the side view, draw projectors from all the points in front view and top view to side view. Join the intersection points. This completes the side view.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 20 for the following problem.

Problem 8-3. *Fig. 8-17 shows the pictorial view of the object. Draw the front view, top view and left hand side view.*

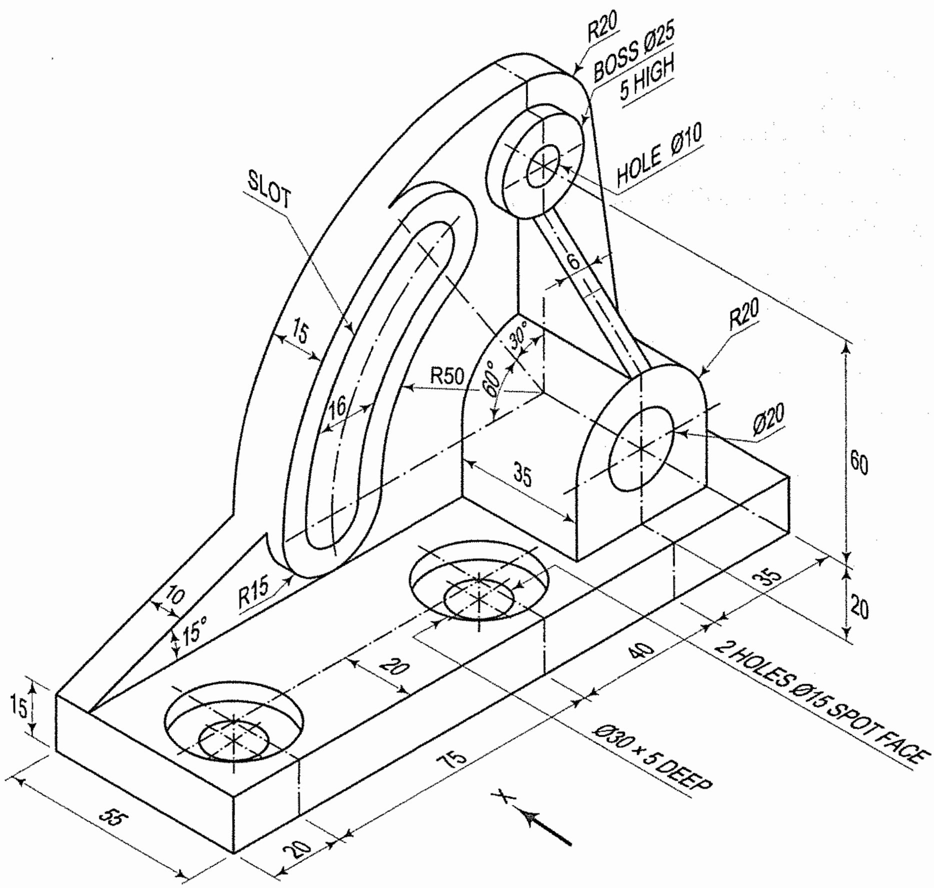


FIG. 8-17

(i) To draw F.V. in the direction of *X*, L.H.S.V. and T.V.

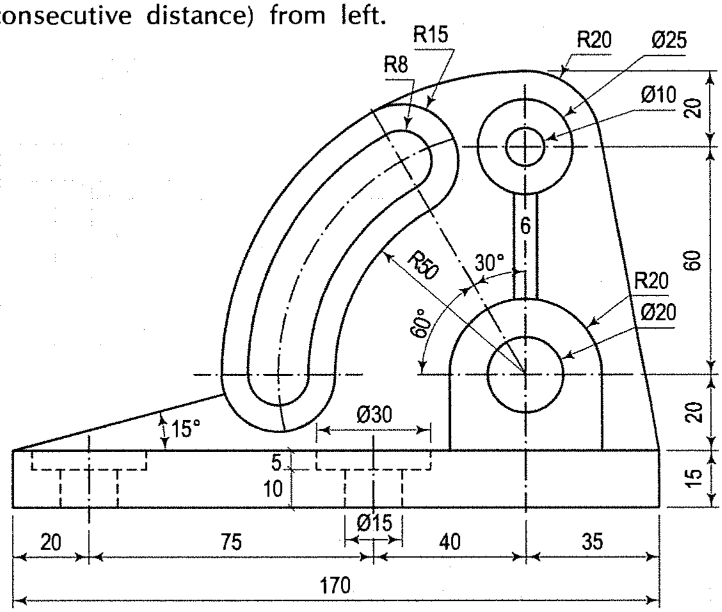
(ii) As the L.H.S.V. is to be drawn, fix the position of *xy* at centre of page and x_1y_1 to the left side as L.H.S.V. will come to the right of F.V.

(iii) Draw the rectangles for the F.V. (170 × 115), project it down to locate T.V. (170 × 55) and take the projections of F.V. to left to draw L.H.S.V., complete the rectangle of L.H.S.V. by taking the projections from T.V.

(iv) First we will start with F.V. (fig. 8-18).

(a) Draw the rectangle for base plate 170 × 15

(b) Draw vertical centre lines at 20 mm and 75 mm and 40 mm (consecutive distance) from left.



Front view

FIG. 8-18

(c) Draw hidden lines for 5 mm depth for two ø30 spot face holes, draw dotted lines for 2 holes of ø15 (Inside these holes).

(d) Draw horizontal centre line at 35 mm from base, in semi-circle.

(e) Referring third centre line and this horizontal line draw the circle of 10 mm radius and semicircle of 20 mm radius.

(f) Draw vertical lines from end of this semi-circle. Till it touches to base.

(g) Draw another horizontal centre line at 60 mm above previous horizontal centre line.

(h) With this as centre, draw circles of ø10 and ø25.

(i) Also draw the arc of radius 20 mm, join this to base with inclined line to represent rib.

(j) Draw two lines at 3 mm to both the sides, vertical centre lines of the circles to represent the rib.

(k) From third centre point, draw centre line at 30°.

(l) Draw arcs of 50 mm, 73 mm, 57 mm, 80 mm.

(m) Draw an arc of centre line pattern of 65 mm radius. (Draw centre line for an arc)

(n) Taking intersection of this as reference and horizontal centre line draw semi-circle of radius 15 mm and 8 mm.

(o) Also draw same arcs, where this line intersects with 30° inclined line.

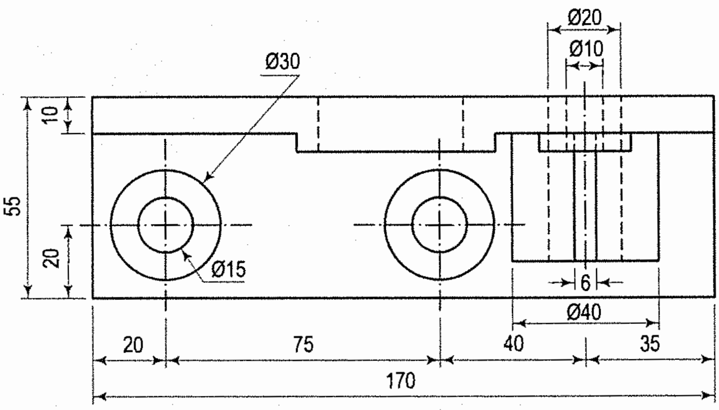
(p) Erase unwanted arcs if any.

(q) Draw inclined line at 15° from left end of block.

(r) Show the required dimensions.

(s) Now project all the required line to draw T.V.

(v) Now draw the top view (fig. 8-19).



Top view
FIG. 8-19

(a) Complete the rectangle of the top view (170 × 55)

(b) Project the centre lines. (Vertical at 20, 75 and 40 consecutive distances from left and at 20 above base)

(c) Draw the two circles of ø30 and ø15 to represent spot holes.

(d) Draw projection of inclined rib 10 thick and circular slot 15 thick.

(e) Draw hidden lines for inner circular slot.

(f) Draw rectangle for semicircular projection from F.V. projections

(g) Draw hidden lines for hole of ø20 in it.

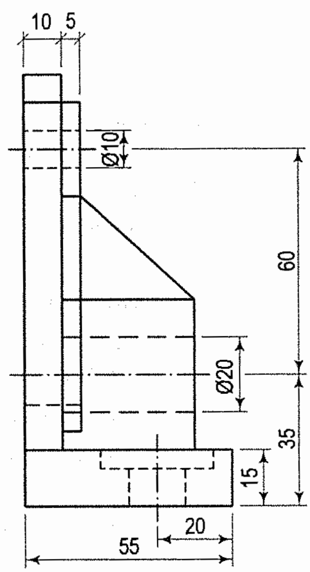
(h) Show the projections of bosse holes.

(i) Draw hidden lines for hole of ø10.

(j) Show the required dimensions.

(vi) Take projections from F.V. and T.V. to complete L.H.S.V. (fig. 8-20).

(a) Complete the base rectangle of 55 mm × 15 mm. To represent the base.



L.H.S.V.
FIG. 8-20

(b) Complete the rectangle at left side in *L* shape of 10 mm thick.
(c) Show the projections of projected semi-circle.
(d) Show hidden lines of hole in it.
(e) Show projection of projected slot and show hidden lines in it.
(f) Draw projection of boss and show hole in it by dotted line.
(g) Show inclined line for the rib.
(h) Draw front view, top view and L.H.S.V. in their relative position.

EXERCISES 8

1. Define orthographic projection. Describe briefly the method of obtaining an orthographic projection of an object.
2. Write short notes on:
   Reference planes; Reference line; Projector; Front view; Ground line.
3. Sketch neatly the symbols used for indicating the method of projection adopted in a drawing. State where this symbol is drawn on a drawing sheet.
4. Explain briefly how the reference line represents both the principal planes of projection.
5. Explain clearly the difference between the first-angle projection method and the third-angle projection method.
6. Fill-up the blanks in the following with appropriate words selected from the list of words given below:
   (a) In ________ projection, the ________ are perpendicular to the ________ of projection.
   (b) In first-angle projection method,
      (i) the ________ comes between the ________ and the ________.
      (ii) the ________ view is always ________ the ________ view.
   (c) In third-angle projection method,
      (i) the ________ comes between the ________ and the ________.
      (ii) the ________ view is always ________ the ________ view.

List of words for Exercise (6):

1. Above	5. Object	9. Projectors
2. Below	6. Orthographic	10. Plane
3. Front	7. Observer	11. Side
4. Left	8. Right	12. Top.

Answer to Exercise (6):

(a) – 6, 9 and 10,
(b) (i) – 5, 7 and 10,
(b) (ii) – 12, 2 and 3,
(c) (i) – 10, 5 and 7,
(c) (ii) – 12, 1 and 3.

7. Why second and fourth quadrants are not used in practice?
8. What is the convention of representing first-angle projection method?

9. The pictorial view of different types of objects are shown in fig. 8-21. Sketch, looking from arrow, elevation, plan and end-view using first-angle projection method.

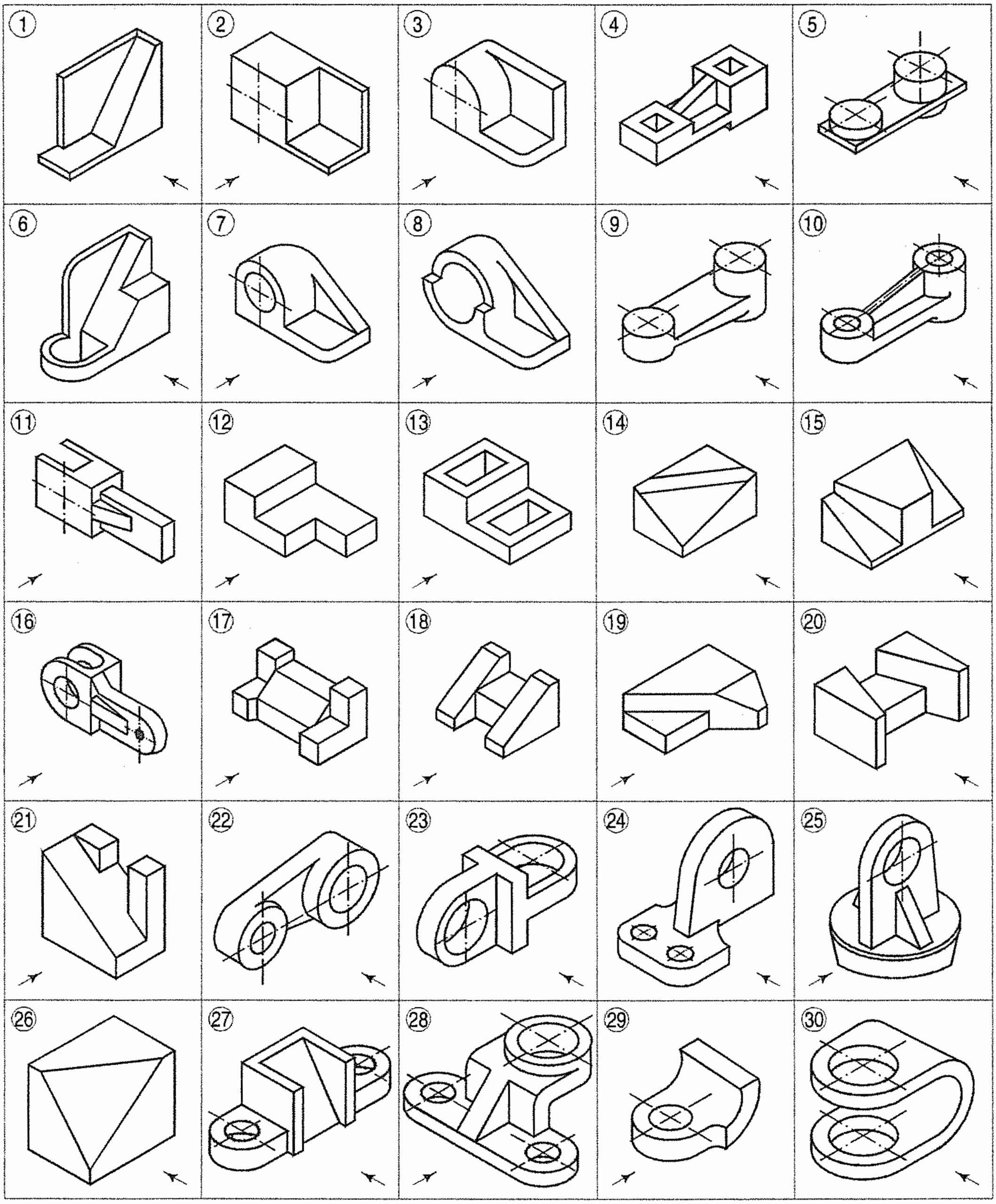


FIG. 8-21

[Draw orthographic projections of each object and then compare your answer with the solution given in fig. 8-26.]

10. What dimensions of an object are given by
    (i) Front view or elevation?
    (ii) Plan or top view?
    (iii) Left-hand side view and right-hand side view?

11. Fig. 8-22 and fig. 8-23 show the orthographic projections of the objects in the first-angle projection method.

    Draw them in the third-angle projection method.

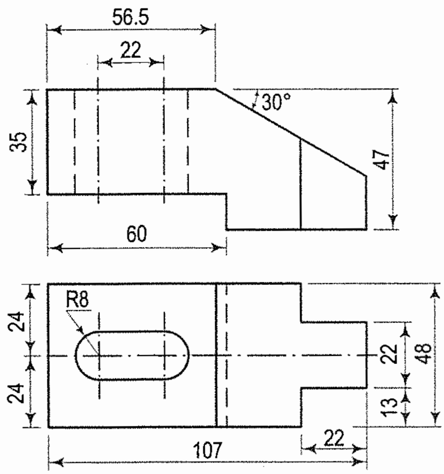


FIG. 8-22

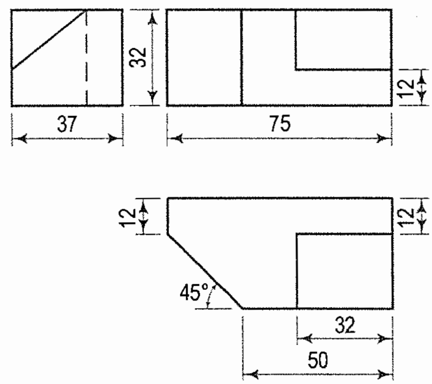


FIG. 8-23

12. Fig. 8-24 and fig. 8-25 show the orthographic projections of the objects in the third-angle projection method.

    Redraw them in the first-angle projection method.

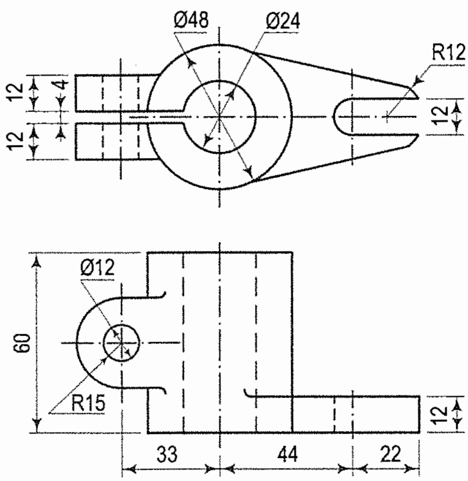


FIG. 8-24

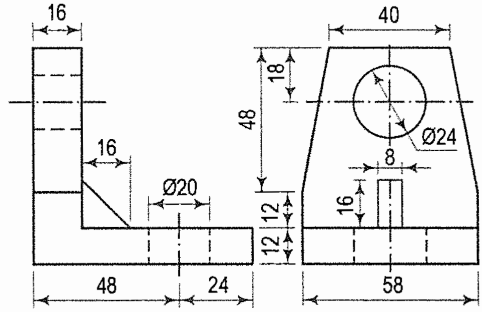
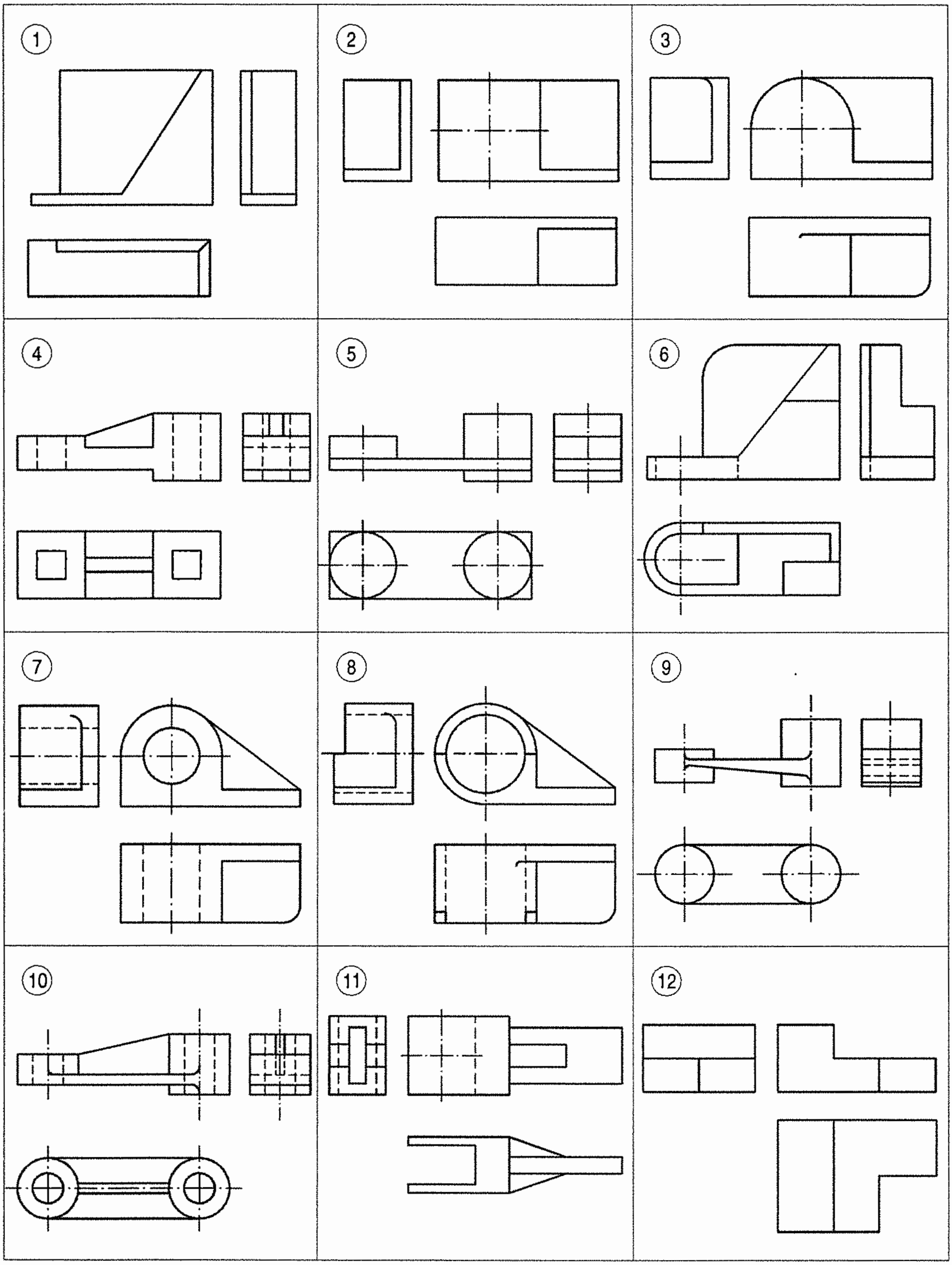
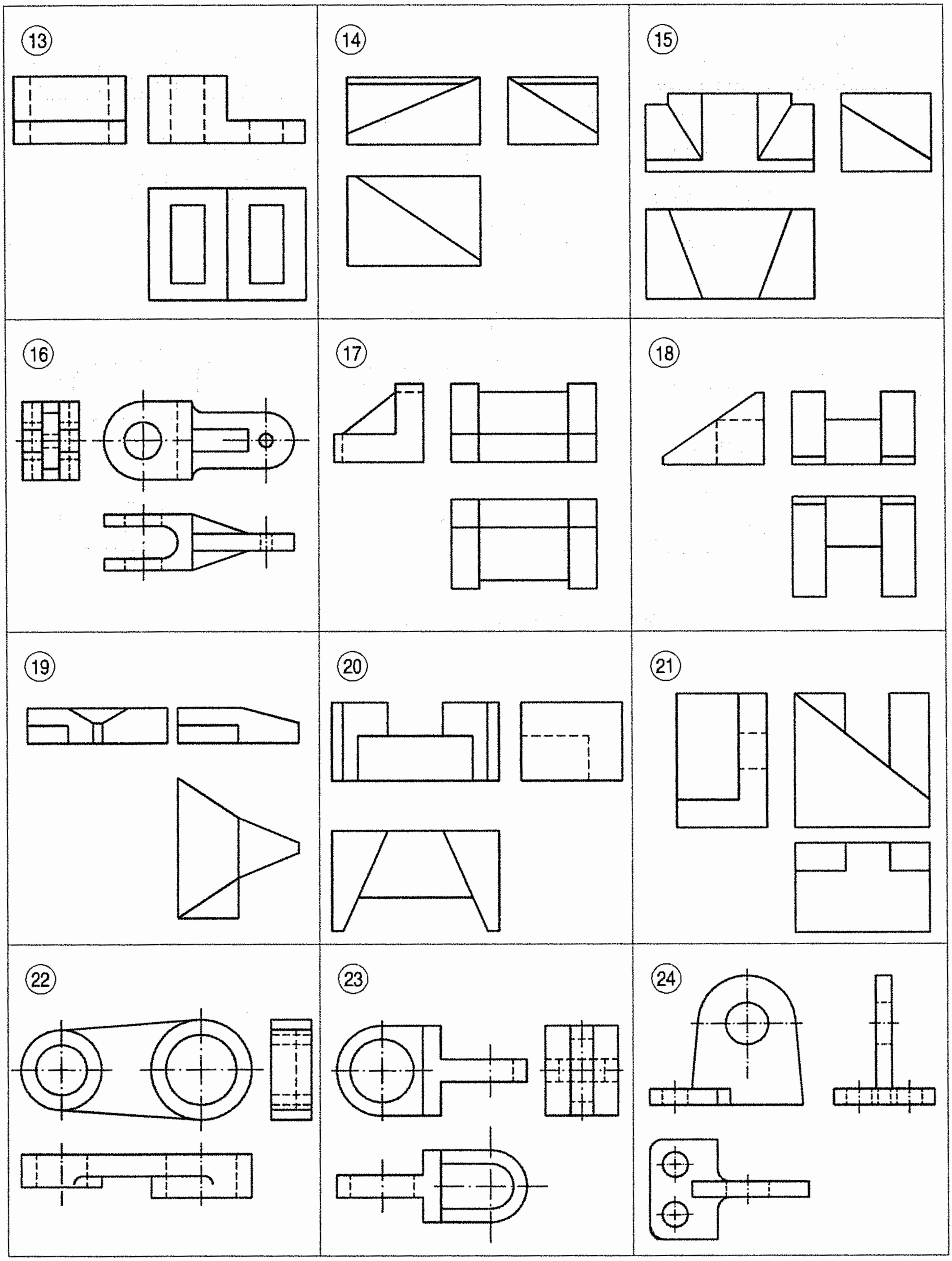


FIG. 8-25

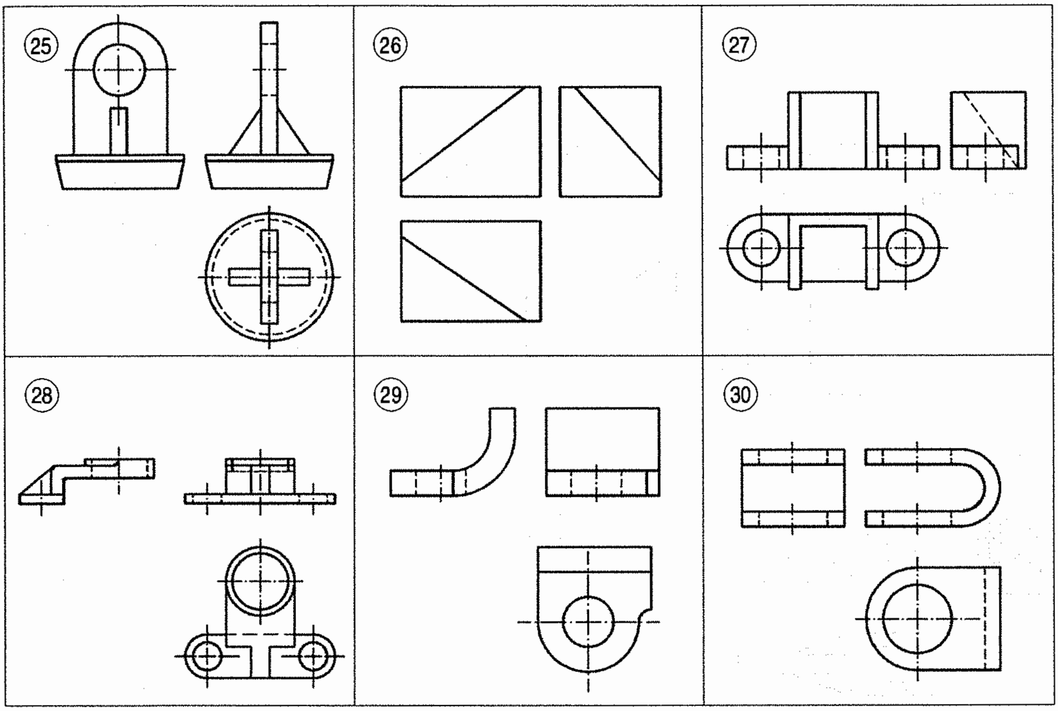
13. What are the important elements of the projection system? Draw any simple object and obtain its six views, projected on imaginary planes by the methods of first angle proection and third angle projection.



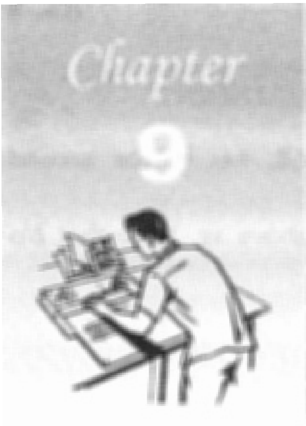
[Answer to Exercise (9), fig. 8-21] (Continued...)
FIG. 8-26



[Answer to Exercise (9), fig. 8-21] (Continued...)
FIG. 8-26



[Answer to Exercise (9), fig. 8-21]
FIG. 8-26



PROJECTIONS OF POINTS

9-0. INTRODUCTION

A point may be situated, in space, in any one of the four quadrants formed by the two principal planes of projection or may lie in any one or both of them. Its projections are obtained by extending projectors perpendicular to the planes.

One of the planes is then rotated so that the first and third quadrants are opened out. The projections are shown on a flat surface in their respective positions either above or below or in *xy*.

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 21 for the projections of points.

9-1. A POINT IS SITUATED IN THE FIRST QUADRANT

The pictorial view [fig. 9-1(i)] shows a point *A* situated above the H.P. and in front of the V.P., i.e. in the first quadrant. *a'* is its front view and *a* the top view. After rotation of the plane, these projections will be seen as shown in fig. 9-1(ii).

The front view *a'* is above *xy* and the top view *a* below it. The line joining *a'* and *a* (which also is called a projector), intersects *xy* at right angles at a point *o*. It is quite evident from the pictorial view that *a'o* = *Aa*, i.e. the distance of the front view from *xy* = the distance of *A* from the H.P. viz. *h*. Similarly, *ao* = *Aa'*, i.e. the distance of the top view from *xy* = the distance of *A* from the V.P. viz. *d*.

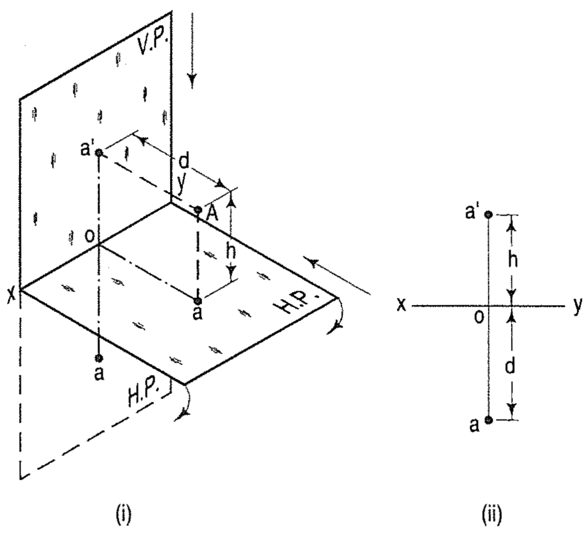


FIG. 9-1

9-2. A POINT IS SITUATED IN THE SECOND QUADRANT

A point *B* (fig. 9-2) is above the H.P. and behind the V.P., i.e. in the second quadrant. *b*' is the front view and *b* the top view.

When the planes are rotated, both the views are seen above *xy*. Note that *b*'*o* = *Bb* and *bo* = *Bb*'.

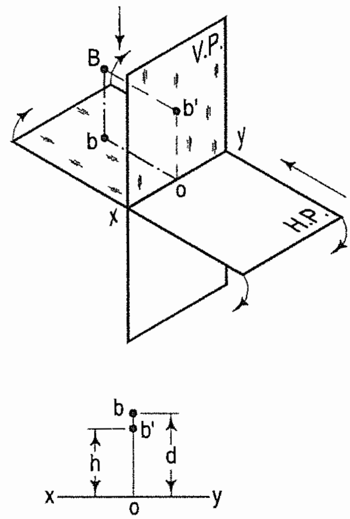


FIG. 9-2

9-3. A POINT IS SITUATED IN THE THIRD QUADRANT

A point *C* (fig. 9-3) is below the H.P. and behind the V.P., i.e. in the third quadrant. Its front view *c*' is below *xy* and the top view *c* above *xy*. Also *c*'*o* = *Cc* and *co* = *Cc*'.

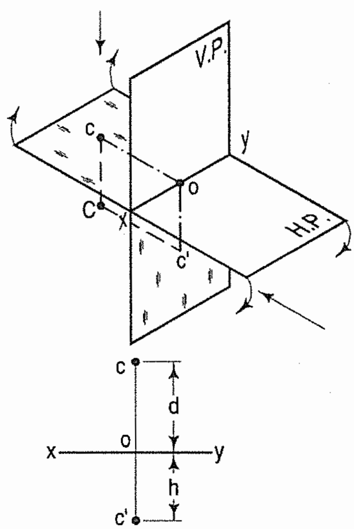


FIG. 9-3

9-4. A POINT IS SITUATED IN THE FOURTH QUADRANT

A point *E* (fig. 9-4) is below the H.P. and in front of the V.P., i.e. in the fourth quadrant. Both its projections are below *xy*, and *e'o* = *Ee* and *eo* = *Ee'*.

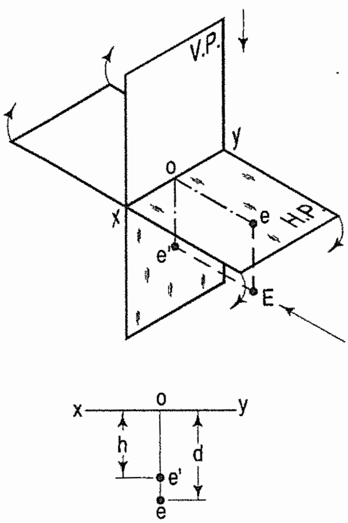


FIG. 9-4

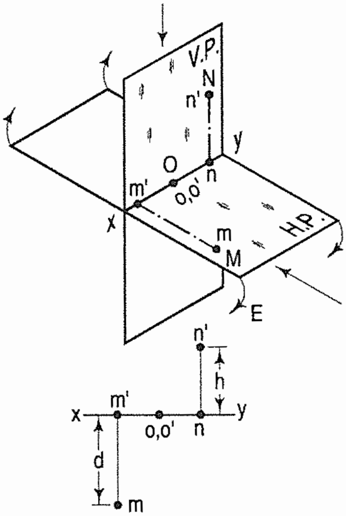


FIG. 9-5

Referring to fig. 9-5, we see that,

(i) A point *M* is in the H.P. and in front of the V.P. Its front view *m'* is in *xy* and the top view *m* below it.

(ii) A point *N* is in the V.P. and above the H.P. Its top view *n* is in *xy* and the front view *n'* above it.

(iii) A point *O* is in both the H.P. and the V.P. Its projection *o* and *o'* coincide with each other in *xy*.

9-5. GENERAL CONCLUSIONS

(i) The line joining the top view and the front view of a point is always perpendicular to *xy*. It is called a *projector*.

(ii) When a point is above the H.P., its front view is above *xy*; when it is below the H.P., the front view is below *xy*. The distance of a point from the H.P. is shown by the length of the projector from its front view to *xy*, e.g. *a'o*, *b'o* etc.

(iii) When a point is in front of the V.P., its top view is below *xy*; when it is behind the V.P., the top view is above *xy*. The distance of a point from the V.P. is shown by the length of the projector from its top view to *xy*, e.g. *ao*, *bo* etc.

(iv) When a point is in a reference plane, its projection on the other reference plane is in *xy*.

Problem 9-1. (fig. 9-1): *A point A is 25 mm above the H.P. and 30 mm in front of the V.P. Draw its projections.*

(i) Draw the reference line *xy* [fig. 9-1(ii)].

(ii) Through any point o in it, draw a perpendicular.

As the point is above the H.P. and in front of the V.P. its front view will be above xy and the top view below xy.

(iii) On the perpendicular, mark a point a' above xy, such that $a'o = 25$ mm. Similarly, mark a point a below xy, so that $ao = 30$ mm. a' and a are the required projections.

Problem 9-2. (fig. 9-6): *A point A is 20 mm below the H.P. and 30 mm behind the V.P. Draw its projections.*

As the point is below the H.P. and behind the V.P., its front view will be below xy and the top view above xy.

Draw the projections as explained in problem 9-1 and as shown in fig. 9-6.

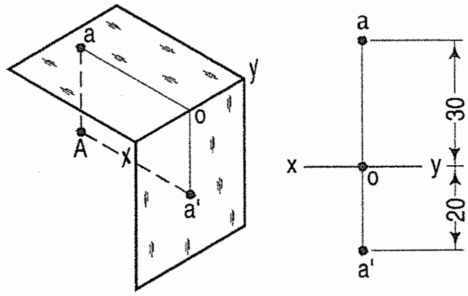


FIG. 9-6

Problem 9-3. (fig. 9-7): *A point P is in the first quadrant. Its shortest distance from the intersection point of H.P., V.P. and Auxiliary vertical plane, perpendicular to the H.P. and V.P. is 70 mm and it is equidistant from principal planes (H.P. and V.P.). Draw the projections of the point and determine its distance from the H.P. and V.P.*

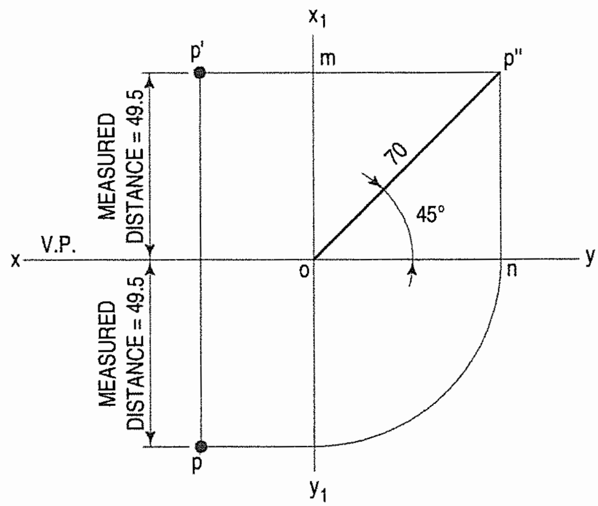


FIG. 9-7

Note: O represents intersection of H.P., V.P. and A.V.P.

(i) Draw xy and $x_1\ y_1$ perpendicular reference lines.

(ii) O represents intersection of H.P., V.P. and A.V.P.

(iii) Draw from O a line inclined at 45° of 70 mm length.

(iv) Project from P'' on xy line and $x_1\ y_1$. The projections are n and m respectively as shown in figure. From O draw arc intersecting $x_1\ y_1$.

(v) Draw a parallel line at convinent distance from $x_1\ y_1$. Extend $P''m$ to intersect a parallel line at p' and p as shown.

(vi) Measure distance from xy line, which is nearly 49.4974 mm say 49.5 mm.

Projections on auxiliary plane: Sometime projections of object on the principal (H.P. and V.P.) are insufficient. In such situation, another projection plane perpendicular to the principal planes is taken. This plane is known as auxiliary plane. The projection on the auxiliary plane is known as side view or side elevation. Refer fig. 9-8.

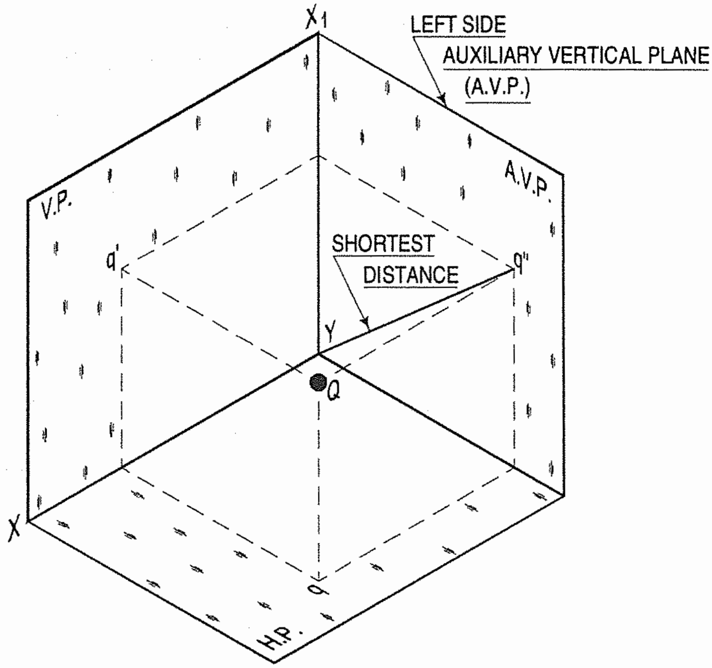


FIG. 9-8

The A.V.P. can be also taken right side also. For more details on projection on auxiliary plane, refer chapter 11.

EXERCISES 9

1. Draw the projections of the following points on the same ground line, keeping the projectors 25 mm apart.

   A, in the H.P. and 20 mm behind the V.P.

   B, 40 mm above the H.P. and 25 mm in front of the V.P.

   C, in the V.P. and 40 mm above the H.P.

   D, 25 mm below the H.P. and 25 mm behind the V.P.

   E, 15 mm above the H.P. and 50 mm behind the V.P.

   F, 40 mm below the H.P. and 25 mm in front of the V.P.

   G, in both the H.P. and the V.P.

2. A point *P* is 50 mm from both the reference planes. Draw its projections in all possible positions.

3. State the quadrants in which the following points are situated:

   (a) A point *P*; its top view is 40 mm above *xy*; the front view, 20 mm below the top view.

   (b) A point *Q*, its projections coincide with each other 40 mm below *xy*.

4. A point P is 15 mm above the H.P. and 20 mm in front of the V.P. Another point Q is 25 mm behind the V.P. and 40 mm below the H.P. Draw projections of P and Q keeping the distance between their projectors equal to 90 mm. Draw straight lines joining (i) their top views and (ii) their front views.

5. Projections of various points are given in fig. 9-9. State the position of each point with respect to the planes of projection, giving the distances in centimetres.

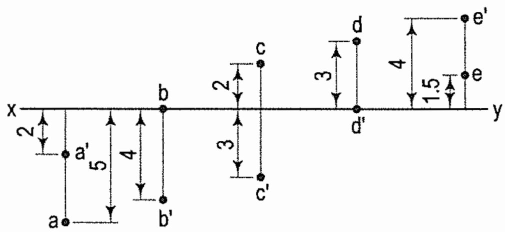


FIG. 9-9

6. Two points A and B are in the H.P. The point A is 30 mm in front of the V.P., while B is behind the V.P. The distance between their projectors is 75 mm and the line joining their top views makes an angle of 45° with xy. Find the distance of the point B from the V.P.

7. A point P is 20 mm below H.P. and lies in the third quadrant. Its shortest distance from xy is 40 mm. Draw its projections.

8. A point A is situated in the first quadrant. Its shortest distance from the intersection point of H.P., V.P. and auxiliary plane is 60 mm and it is equidistant from the principal planes. Draw the projections of the point and determine its distance from the principal planes.

9. A point 30 mm above xy line is the plan-view of two points P and Q. The elevation of P is 45 mm above the H.P. while that of the point Q is 35 mm below the H.P. Draw the projections of the points and state their position with reference to the principal planes and the quadrant in which they lie.

10. A point Q is situated in first quadrant. It is 40 mm above H.P. and 30 mm in front of V.P. Draw its projections and find its shortest distance from the intersection of H.P., V.P. and auxiliary plane.

Chapter 10

PROJECTIONS OF STRAIGHT LINES

10-0. INTRODUCTION

A straight line is the shortest distance between two points. Hence, the projections of a straight line may be drawn by joining the respective projections of its ends which are points.

The position of a straight line may also be described with respect to the two reference planes. It may be:

1. Parallel to one or both the planes.
2. Contained by one or both the planes.
3. Perpendicular to one of the planes.
4. Inclined to one plane and parallel to the other.
5. Inclined to both the planes.
6. Projections of lines inclined to both the planes.
7. Line contained by a plane perpendicular to both the reference planes.
8. True length of a straight line and its inclinations with the reference planes.
9. Traces of a line.
10. Methods of determining traces of a line.
11. Traces of a line, the projections of which are perpendicular to xy.
12. Positions of traces of a line.

10-1. LINE PARALLEL TO ONE OR BOTH THE PLANES (FIG. 10-1)

(a) Line *AB* is parallel to the H.P.

a and *b* are the top views of the ends *A* and *B* respectively. It can be clearly seen that the figure *ABba* is a rectangle. Hence, the top view *ab* is equal to *AB*.

a'b' is the front view of *AB* and is parallel to *xy*.

(b) Line *CD* is parallel to the V.P.

The line *c'd'* is the front view and is equal to *CD*; the top view *cd* is parallel to *xy*.

(c) Line *EF* is parallel to the H.P. and the V.P.

ef is the top view and *e'f'* is the front view; both are equal to *EF* and parallel to *xy*.

Hence, when a line is parallel to a plane, its projection on that plane is equal to its true length; while its projection on the other plane is parallel to the reference line.

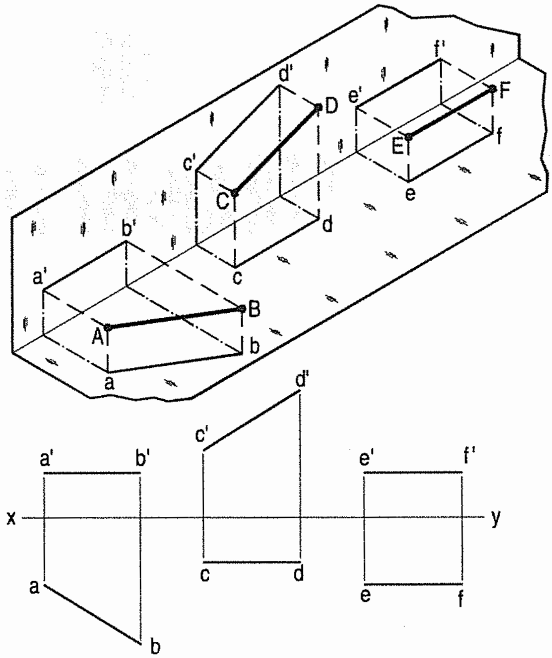


FIG. 10-1

10-2. LINE CONTAINED BY ONE OR BOTH THE PLANES (FIG. 10-2)

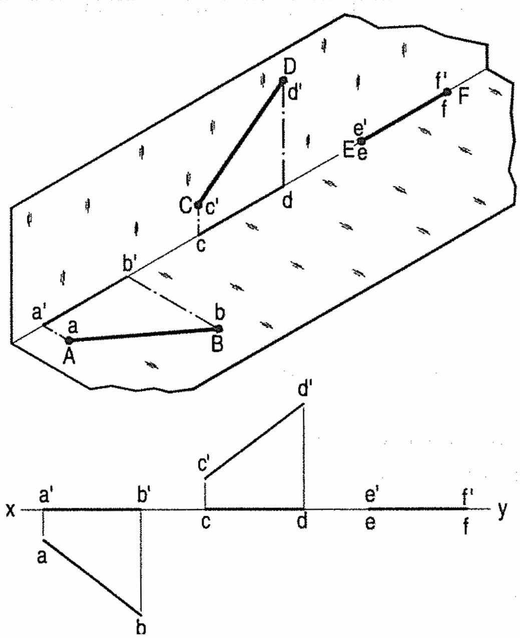


FIG. 10-2

Line *AB* is in the H.P. Its top view *ab* is equal to *AB*; its front view *a' b'* is in *xy*.

Line *CD* is in the V.P. Its front view *c'd'* is equal to *CD*; its top view *cd* is in *xy*.

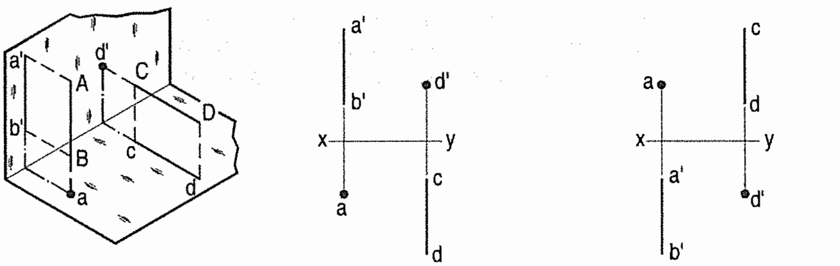
Line *EF* is in both the planes. Its front view *e' f'* and the top view *ef* coincide with each other in *xy*.

Hence, when a line is contained by a plane, its projection on that plane is equal to its true length; while its projection on the other plane is in the reference line.

10-3. LINE PERPENDICULAR TO ONE OF THE PLANES (FIG. 10-3)

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 22 for the line perpendicular to one of the planes.

When a line is perpendicular to one reference plane, it will be parallel to the other.



(FIRST-ANGLE PROJECTION) (THIRD-ANGLE PROJECTION)

FIG. 10-3

(a) Line *AB* is perpendicular to the H.P. The top views of its ends coincide in the point *a*. Hence, the top view of the line *AB* is the point *a*. Its front view *a' b'* is equal to *AB* and perpendicular to *xy*.

(b) Line *CD* is perpendicular to the V.P. The point *d'* is its front view and the line *cd* is the top view. *cd* is equal to *CD* and perpendicular to *xy*.

Hence, when a line is perpendicular to a plane its projection on that plane is a point; while its projection on the other plane is a line equal to its true length and perpendicular to the reference line.

In *first-angle projection method*, when top views of two or more points coincide, the point which is comparatively *farther* away from *xy* in the front view will be visible; and when their front views coincide, that which is *farther* away from *xy* in the top view will be visible.

In *third-angle projection method*, it is just the reverse. When top views of two or more points coincide the point which is comparatively *nearer xy* in the front view will be visible; and when their front views coincide, the point which is *nearer xy* in the top view will be visible.

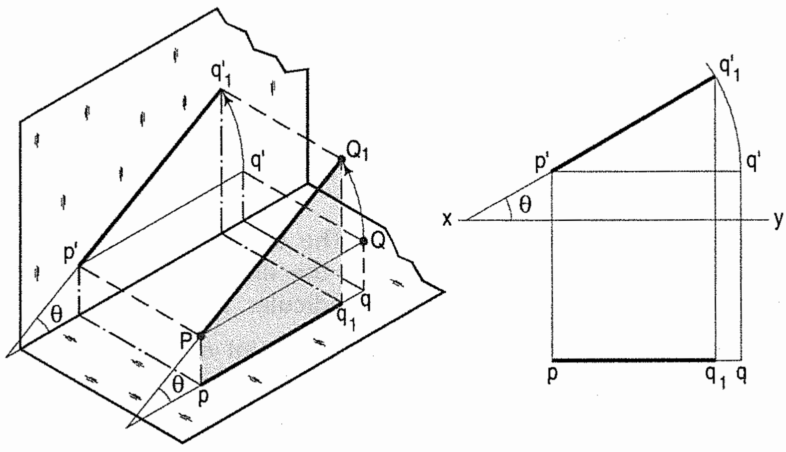
10-4. LINE INCLINED TO ONE PLANE AND PARALLEL TO THE OTHER

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 23 for the line inclined to one plane and parallel to the other.

The inclination of a line to a plane is the angle which the line makes with its projection on that plane.

(a) Line PQ_1 [fig. 10-4(i)] is inclined at an angle θ to the H.P. and is parallel to the V.P. The inclination is shown by the angle θ which PQ_1 makes with its own projection on the H.P., viz. the top view pq_1.

The projections [fig. 10-4(ii)] may be drawn by first assuming the line to be parallel to both the H.P. and the V.P. Its front view $p'q'$ and the top view pq will both be parallel to xy and equal to the true length. When the line is turned about the end P to the position PQ_1 so that it makes the angle θ with the H.P. while remaining parallel to the V.P., in the front view the point q' will move along an arc drawn with p' as centre and $p'q'$ as radius to a point q'_1 so that $p'q'_1$ makes the angle θ with xy. In the top view, q will move towards p along pq to a point q_1 on the projector through q'_1. $p'q'_1$ and pq_1 are the front view and the top view respectively of the line PQ_1.



(i) (ii)

FIG. 10-4

(b) Line RS_1 [fig. 10-5(i)] is inclined at an angle ø to the V.P. and is parallel to the H.P. The inclination is shown by the angle ø which RS_1 makes with its projection on the V.P., viz. the front view $r's'_1$. Assuming the line to be parallel to both the H.P. and the V.P., its projections $r's'$ and rs are drawn parallel to xy and equal to its true length [fig. 10-5(ii)].

When the line is turned about its end R to the position RS_1 so that it makes the angle ø with the V.P. while remaining parallel to the H.P., in the top view the point s will move along an arc drawn with r as centre and rs as radius to a point s_1 so that rs_1 makes the angle ϕ with xy. In the front view, the point s' will move towards r' along the line $r's'$ to a point s'_1 on the projector through s_1. rs_1 and $r's'_1$ are the projections of the line RS_1.

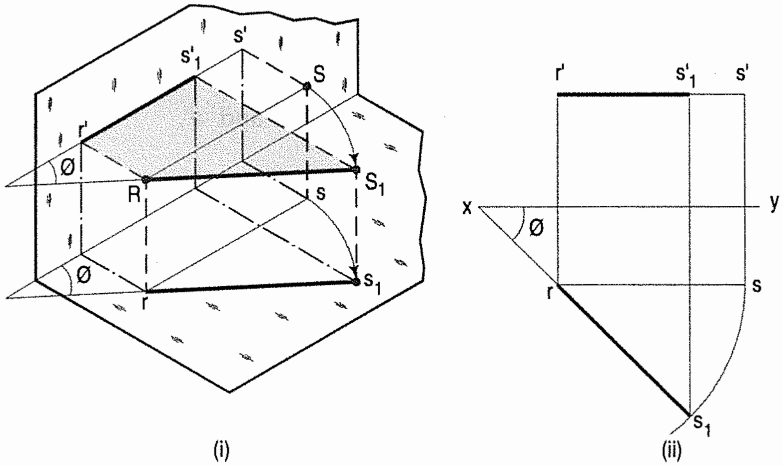


FIG. 10-5

Therefore, when the line is inclined to the H.P. and parallel to the V.P., its top view is shorter than its true length, but parallel to xy; its front view is equal to its true length and is inclined to xy at its true inclination with the H.P. And when the line is inclined to the V.P. and parallel to the H.P., its front view is shorter than its true length but parallel to xy; its top view is equal to its true length and is inclined to xy at its true inclination with the V.P.

Hence, when a line is inclined to one plane and parallel to the other, its projection on the plane to which it is inclined, is a line shorter than its true length but parallel to the reference line. Its projection on the plane to which it is parallel, is a line equal to its true length and inclined to the reference line at its true inclination.

In other words, the inclination of a line with the H.P. is seen in the front view and that with the V.P. is seen in the top view.

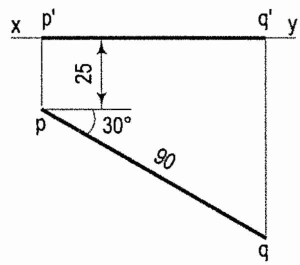


FIG. 10-6

Problem 10-1. (fig. 10-6): *A line PQ, 90 mm long, is in the H.P. and makes an angle of 30° with the V.P. Its end P is 25 mm in front of the V.P. Draw its projections.*

As the line is in the H.P., its top view will show the true length and the true inclination with the V.P. Its front view will be in xy.

(i) Mark a point p, the top view 25 mm below xy. Draw a line pq equal to 90 mm and inclined at 30° to xy.

(ii) Project p to p' and q to q' on xy.

pq and $p'q'$ are the required top view and front view respectively.

Problem 10-2. (fig. 10-7): *The length of the top view of a line parallel to the V.P. and inclined at 45° to the H.P. is 50 mm. One end of the line is 12 mm above the H.P. and 25 mm in front of the V.P. Draw the projections of the line and determine its true length.*

As the line is parallel to the V.P., its top view will be parallel to *xy* and the front view will show its true length and the true inclination with the H.P.

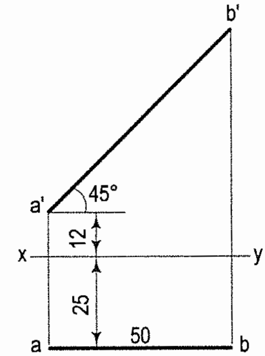


FIG. 10-7

(i) Mark *a*, the top view, 25 mm below *xy* and *a'*, the front view, 12 mm above *xy*.

(ii) Draw the top view *ab* 50 mm long and parallel to *xy* and draw a projector through *b*.

(iii) From *a'* draw a line making 45° angle with *xy* and cutting the projector through *b* at *b'*. Then *a'b'* is the front view and also the true length of the line.

Problem 10-3. (fig. 10-8): *The front view of a 75 mm long line measures 55 mm. The line is parallel to the H.P. and one of its ends is in the V.P. and 25 mm above the H.P. Draw the projections of the line and determine its inclination with the V.P.*

As the line is parallel to the H.P., its front view will be parallel to *xy*.

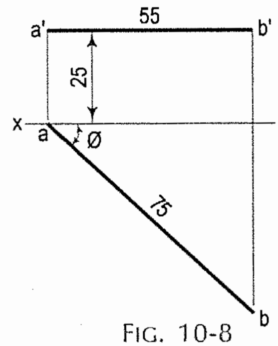


FIG. 10-8

(i) Mark *a*, the top view of one end in *xy*, and *a'*, its front view, 25 mm above *xy*.

(ii) Draw the front view *a'b'*, 55 mm long and parallel to *xy*. With *a* as centre and radius equal to 75 mm, draw an arc cutting the projector through *b'* at *b*.

Join *a* with *b*. *ab* is the top view of the line. Its inclination with *xy*, viz. ø is the inclination of the line with the V.P.

EXERCISES 10(a)

1. Draw the projections of a 75 mm long straight line, in the following positions:
   - (a) (i) Parallel to both the H.P. and the V.P. and 25 mm from each.
     - (ii) Parallel to and 30 mm above the H.P. and in the V.P.
     - (iii) Parallel to and 40 mm in front of the V.P. and in the H.P.
   - (b) (i) Perpendicular to the H.P., 20 mm in front of the V.P. and its one end 15 mm above the H.P.
     - (ii) Perpendicular to the V.P., 25 mm above the H.P. and its one end in the V.P.
     - (iii) Perpendicular to the H.P., in the V.P. and its one end in the H.P.
   - (c) (i) Inclined at 45° to the V.P., in the H.P. and its one end in the V.P.
     - (ii) Inclined at 30° to the H.P. and its one end 20 mm above it; parallel to and 30 mm in front of the V.P.
     - (iii) Inclined at 60° to the V.P. and its one end 15 mm in front of it; parallel to and 25 mm above the H.P.

2. A 100 mm long line is parallel to and 40 mm above the H.P. Its two ends are 25 mm and 50 mm in front of the V.P. respectively. Draw its projections and find its inclination with the V.P.
3. A 90 mm long line is parallel to and 25 mm in front of the V.P. Its one end is in the H.P. while the other is 50 mm above the H.P. Draw its projections and find its inclination with the H.P.
4. The top view of a 75 mm long line measures 55 mm. The line is in the V.P., its one end being 25 mm above the H.P. Draw its projections.
5. The front view of a line, inclined at 30° to the V.P is 65 mm long. Draw the projections of the line, when it is parallel to and 40 mm above the H.P., its one end being 30 mm in front of the V.P.
6. A vertical line *AB*, 75 mm long, has its end *A* in the H.P. and 25 mm in front of the V.P. A line *AC*, 100 mm long, is in the H.P. and parallel to the V.P. Draw the projections of the line joining *B* and *C*, and determine its inclination with the H.P.
7. Two pegs fixed on a wall are 4.5 metres apart. The distance between the pegs measured parallel to the floor is 3.6 metres. If one peg is 1.5 metres above the floor, find the height of the second peg and the inclination of the line joining the two pegs, with the floor.
8. Draw the projections of the lines in Exercises 1 to 6, assuming them to be in the third quadrant, taking the given positions to be below the H.P. instead of above the H.P., and behind the V.P., instead of in front of the V.P.

10-5. LINE INCLINED TO BOTH THE PLANES

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 24 for the line inclined to both the planes.

(a) A line *AB* (fig. 10-9) is inclined at θ to the H.P. and is parallel to the V.P. The end *A* is in the H.P. *AB* is shown as the hypotenuse of a right-angled triangle, making the angle θ with the base.

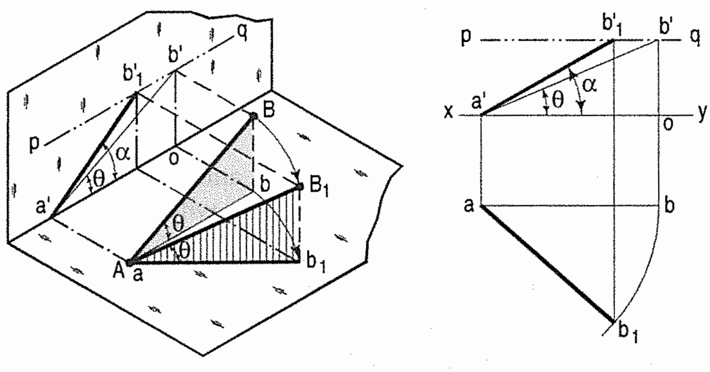


FIG. 10-9

The top view *ab* is shorter than *AB* and parallel to *xy*. The front view *a'b'* is equal to *AB* and makes the angle θ with *xy*.

Keeping the end *A* fixed and the angle θ with the H.P. constant, if the end *B* is moved to any position, say B_1, the line becomes inclined to the V.P. also.

In the top view, *b* will move along an arc, drawn with *a* as centre and *ab* as radius, to a position b_1. The new top view ab_1 is equal to *ab* but shorter than *AB*.

In the front view, *b'* will move to a point b'_1 keeping its distance from *xy* constant and equal to *b'o*; i.e. it will move along the line *pq*, drawn through *b'* and parallel to *xy*. This line *pq* is the locus or path of the end *B* in the front view. b'_1 will lie on the projector through b_1. The new front view $a'b'_1$ is shorter than *a'b'* (i.e. *AB*) and makes an angle α with *xy*. α is greater than θ.

Thus, it can be seen that as long as the inclination θ of *AB* with the H.P. is constant, even when it is inclined to the V.P.

(i) its length in the top view, viz. *ab* remains constant; and

(ii) the distance between the paths of its ends in the front view, viz. *b'o* remains constant.

(b) The same line *AB* (fig. 10-10) is inclined at ø to the V.P. and is parallel to the H.P. Its end *A* is in the V.P. *AB* is shown as the hypotenuse of a right-angled triangle making the angle ø with the base.

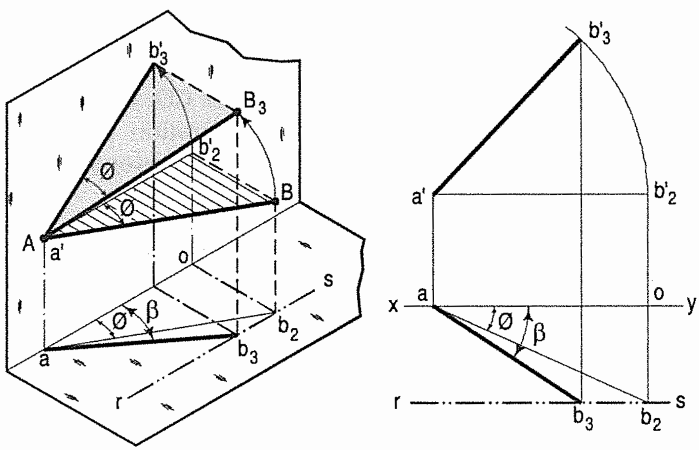


FIG. 10-10

The front view $a'b'_2$ is shorter than *AB* and parallel to *xy*. The top view ab_2 is equal to *AB* and makes an angle ø with *xy*.

Keeping the end *A* fixed and the angle ø with the V.P. constant, if *B* is moved to any position, say B_3, the line will become inclined to the H.P. also.

In the front view, b'_2, will move along the arc, drawn with a' as centre and $a'b'_2$ as radius, to a position b'_3. The new front view $a'b'_3$ is equal to $a'b'_2$ but is shorter than *AB*.

In the top view, b_2 will move to a point b_3 along the line *rs*, drawn through b_2 and parallel to *xy*, thus keeping its distance from the path of *a*, viz. b_2o constant. *rs* is the locus or path of the end *B* in the top view. The point b_3 lies on the projector through b'_3. The new top view ab_3 is shorter than ab_2 (i.e. *AB*) and makes an angle β with *xy*. β is greater than $\varnothing$.

Here also we find that, as long as the inclination of *AB* with the V.P. does not change, even when it becomes inclined to the H.P.

(i) its length in the front view, viz. $a'b'_2$ remains constant; and

(ii) the distance between the paths of its ends in the top view, viz. b_2o remains constant.

Hence, when a line is inclined to both the planes, its projections are shorter than the true length and inclined to xy at angles greater than the true inclinations. These angles viz. α *and* β *are called apparent angles of inclination.*

10-6. PROJECTIONS OF LINES INCLINED TO BOTH THE PLANES

From Art. 10-5(a) above, we find that as long as the inclination of *AB* with the H.P. is constant

(i) its length in the top view, viz. *ab* remains constant, and

(ii) in the front view, the distance between the loci of its ends, viz. *b'o* remains constant.

In other words if

(i) its length in the top view is equal to *ab*, and

(ii) the distance between the paths of its ends in the front view is equal to *b'o*, the inclination of *AB* with the H.P. will be equal to θ.

Similarly, from Art. 10-5(b) above, we find that as long as the inclination of *AB* with the V.P. is constant

(i) its length in the front view, viz. $a'b'_2$ remains constant, and

(ii) in the top view, the distance between the loci of its ends, viz. b_2o remains constant.

The reverse of this is also true, viz.

(i) if its length in the front view is equal to $a'b'_2$, and

(ii) the distance between the paths of its ends in the top view is equal to b_2o, the inclination of *AB* with the V.P. will be equal to $\varnothing$.

Combining the above two findings, we conclude that when *AB* is inclined at θ to the H.P. and at $\varnothing$ to the V.P.

(i) its lengths in the top view and the front view will be equal to ab_2 and $a'b'_2$ respectively, and

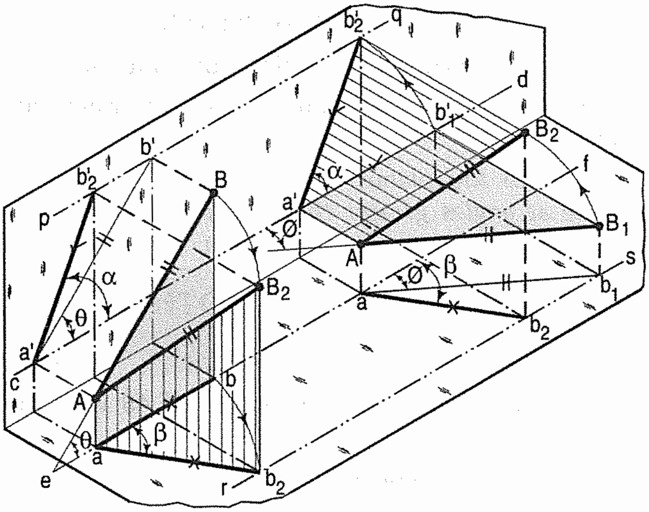
(ii) the distances between the paths of its ends in the front view and the top view will be equal to b'_2o and b_2o respectively.

The two lengths when arranged with their ends in their respective paths and in projections with each other will be the projections of the line *AB*, as illustrated in problem 10-4.

Problem 10-4. *Given the line AB, its inclinations θ with the H.P. and ø with the V.P. and the position of one end A. To draw its projections.*

Mark the front view *a'* and the top view *a* according to the given position of *A* (fig. 10-12).

Let us first determine the lengths of *AB* in the top view and the front view and the paths of its ends in the front view and the top view.

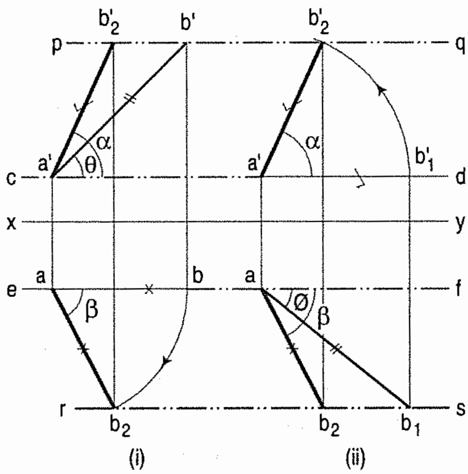


(i) (ii)

FIG. 10-11

(1) Assume *AB* to be parallel to the V.P. and inclined at θ to the H.P. *AB* is shown in the pictorial view as a side of the trapezoid *ABba* [fig. 10-11(i)]. Draw the front view *a'b'* equal to *AB* [fig. 10-12(i)] and inclined at θ to *xy*. Project the top view *ab* parallel to *xy*. Through *a'* and *b'*, draw lines *cd* and *pq* respectively parallel to *xy*. *ab* is the length of *AB* in the top view and, *cd* and *pq* are the paths of *A* and *B* respectively in the front view.

(2) Again, assume AB_1 (equal to *AB*) to be parallel to the H.P. and inclined at ø to the V.P. In the pictorial view [fig. 10-11(ii)], AB_1 is shown as a side of the trapezoid $AB_1b'_1a'$. Draw the top view ab_1 equal to *AB* [fig. 10-12(ii)] and inclined at ø to *xy*. Project the front view $a'b'_1$ parallel to *xy*. Through *a* and b_1, draw lines *ef* and *rs* respectively parallel to *xy*. $a'b'_1$ is the length of *AB* in the front view and, *ef* and *rs* are the paths of *A* and *B* respectively in the top view.



(i) (ii)

FIG. 10-12

We may now arrange

(i) *ab* (the length in the top view) between its paths *ef* and *rs*, and

(ii) $a'b'_1$ (the length in the front view) between the paths *cd* and *pq*, keeping them in projection with each other, in one of the following *two* ways:

(a) In case (1) [fig. 10-11(i)], if the side *Bb* is turned about *Aa*, so that *b* comes on the path *rs*, the line *AB* will become inclined at ø to the V.P. Therefore, with *a* as centre [fig. 10-12(i)] and radius equal to *ab*, draw an arc cutting *rs* at a point b_2. Project b_2 to b'_2 on the path *pq*.

Draw lines joining a with b_2, and a' with b'_2. ab_2 and $a'b'_2$ are the required projections. Check that $a'b'_2 = a'b'_1$.

(b) Similarly, in case (2) [fig. 10-11(ii)], if the side $B_1b'_1$ is turned about Aa' till b_1' is on the path pq, the line AB_1 will become inclined at θ to the H.P.

Hence, with a' as centre [fig. 10-12(ii)] and radius equal to $a'b'_1$, draw an arc cutting pq at a point b'_2. Project b'_2 to b_2 in the top view on the path rs.

Draw lines joining a with b_2, and a' with b'_2. ab_2 and $a'b'_2$ are the required projections. Check that $ab_2 = ab$.

Fig. 10-13 shows (in pictorial and orthographic views) the projections obtained with both the above steps combined in one figure and as described below.

First, determine

(i) the length ab in the top view and the path pq in the front view and

(ii) the length $a'b'_1$ in the front view and the path rs in the top view.

Then, with a as centre and radius equal to ab, draw an arc cutting rs at a point b_2. With a' as centre and radius equal to $a'b'_1$, draw an arc cutting pq at a point b'_2.

Draw lines joining a with b_2 and a' with b'_2. ab_2 and $a'b'_2$ are the required projections. Check that b_2 and b'_2 lie on the same projector.

It is quite evident from the figure that the apparent angles of inclination α and β are greater than the true inclinations θ and ϕ respectively.

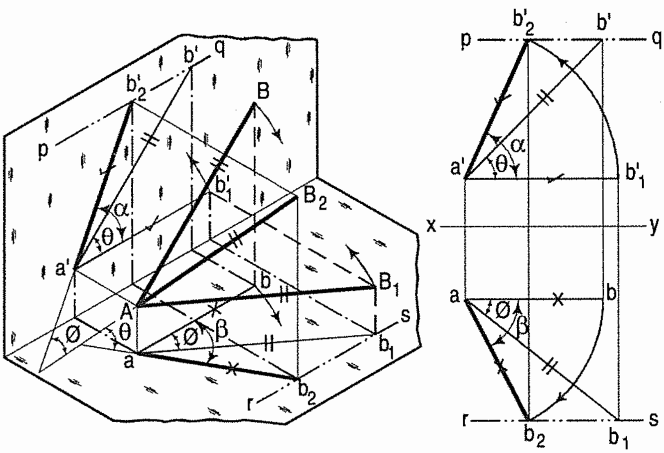


FIG. 10-13

10-7. LINE CONTAINED BY A PLANE PERPENDICULAR TO BOTH THE REFERENCE PLANES

As the two reference planes are at right angles to each other, the sum total of the inclinations of a line with the two planes, viz. θ and ϕ can never be more than 90°. When $\theta + \phi = 90°$, the line will be contained by a third plane called the profile plane, perpendicular to both the H.P. and the V.P.

A line EF (fig. 10-14), is inclined at θ to the H.P. and at ϕ [equal to $(90° - \theta)$] to the V.P. The line is thus contained by the profile plane marked P.P.

The front view $e'f'$ and the top view ef are both perpendicular to xy and shorter than EF.

Therefore, when a line is inclined to both the reference planes and contained by a plane perpendicular to them, i.e. when the sum of its inclinations with the H.P. and the V.P. is 90°, its projections are perpendicular to xy and shorter than the true length.

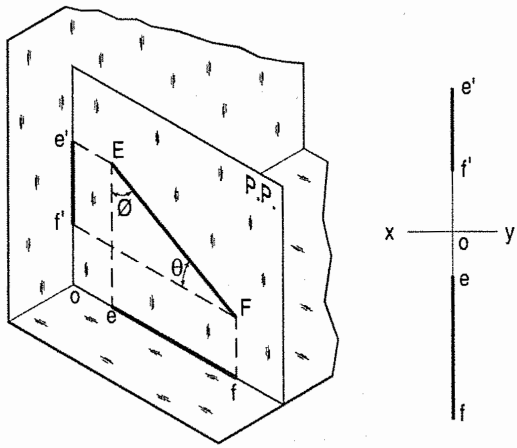


FIG. 10-14

10-8. TRUE LENGTH OF A STRAIGHT LINE AND ITS INCLINATIONS WITH THE REFERENCE PLANES

When projections of a line are given, its true length and inclinations with the planes are determined by the application of the following rule:

When a line is parallel to a plane, its projection on that plane will show its true length and the true inclination with the other plane.

The line may be made parallel to a plane, and its true length obtained by any one of the following *three* methods:

Method I:

Making each view parallel to the reference line and projecting the other view from it. This is the exact reversal of the processes adopted in Art. 10-5 for obtaining the projections.

Method II:

Rotating the line about its projections till it lies in the H.P. or in the V.P.

Method III:

Projecting the views on auxiliary planes parallel to each view.

(This method will be dealt with in chapter 11).

The following problem shows the application of the first two methods and problem 10-29 and problem 10-31 show application of third method.

Problem 10-5. *The top view ab and the front view a'b' of a line AB are given. To determine its true length and the inclinations with the H.P. and the V.P.*

Method I:

Fig. 10-15(i) shows AB the line, $a'b'$ its front view and ab its top view. If the trapezoid $ABba$ is turned about Aa as axis, so that AB becomes parallel to the V.P., in the top view, b will move along an arc drawn with centre a and radius equal to ab, to b_1, so that ab_1 is parallel to xy. In the front view, b' will move along its locus pq, to a point b'_1 on the projector through b_1.

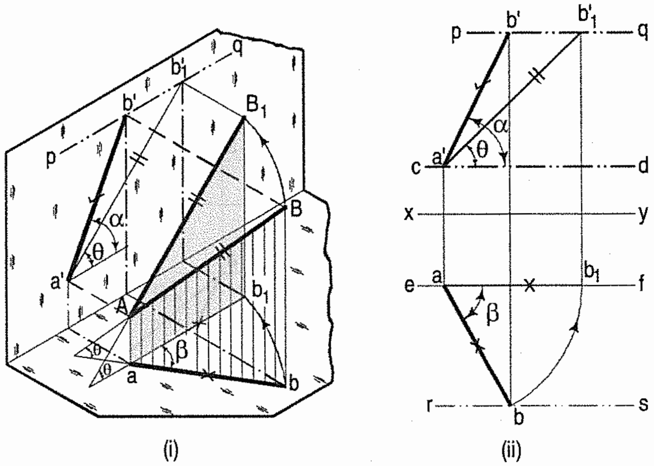


FIG. 10-15

(i) Therefore, with centre *a* and radius equal to *ab* [fig. 10-15(ii)], draw an arc to cut *ef* at b_1.

(ii) Draw a projector through b_1 to cut *pq* (the path of *b*') at b'_1.

(iii) Draw the line $a'b'_1$ which is the true length of *AB*. The angle θ, which it makes with *xy* is the inclination of *AB* with the H.P.

Again, in fig. 10-16(i) *AB* is shown as a side of a trapezoid *ABb'a'*. If the trapezoid is turned about *Aa'* as axis so that *AB* is parallel to the H.P., the new top view will show its true length and true inclination with the V.P.

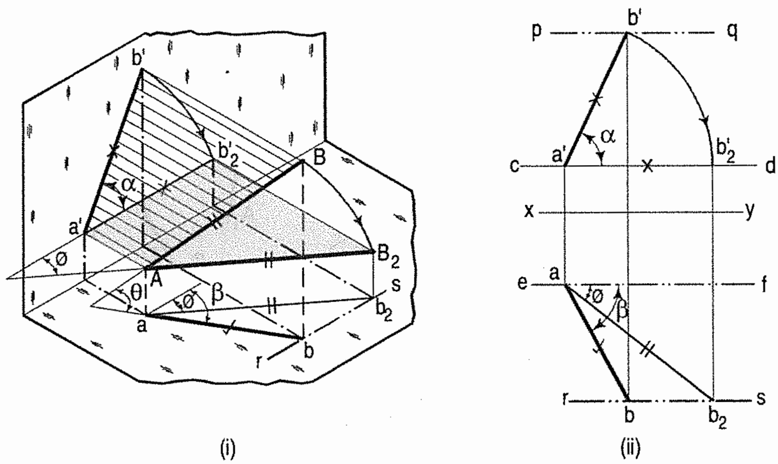


FIG. 10-16

(i) With *a*' as centre and radius equal to *a'b*' [fig. 10-16(ii)], draw an arc to cut *cd* at b'_2.

(ii) Draw a projector through b'_2 to cut *rs* (the path of *b*) at b_2.

(iii) Draw the line ab_2, which is the true length of *AB*. The angle ø which it makes with *xy* is the inclination of *AB* with the V.P.

Fig. 10-17(i) shows the above two steps combined in one figure.

The same results will be obtained by keeping the end *B* fixed and turning the end *A* [fig. 10-17(ii)], as explained below.

(i) With centre *b* and radius equal to *ba*, draw an arc cutting *rs* at a_1 (thus making *ba* parallel to *xy*).

(ii) Project a_1 to a'_1 on *cd* (the path of *a'*) a'_1b' is the true length and θ is the true inclination of *AB* with the H.P.

(iii) Similarly, with centre *b'* and radius equal to *b'a'*, draw an arc cutting *pq* at a_2'.

(iv) Project a_2' to a_2 on *ef* (the path of *a*). a_2b is the true length and ø is the true inclination of *AB* with the V.P.

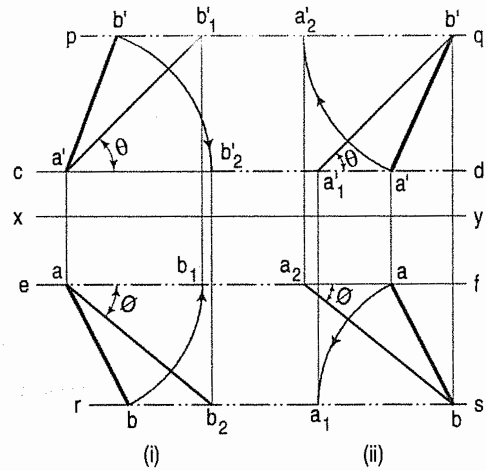


FIG. 10-17

Method II:

Referring to the pictorial view in fig. 10-18(i) we find that *AB* is the line, *ab* its top view and *a'b'* its front view.

In the trapezoid *ABb'a'* (i) *a'A* and *b'B* are both perpendicular to *a'b'* and are respectively equal to ao_1 and bo_2 (the distances of *a* and *b* from *xy* in the top view), and (ii) the angle between *AB* and *a'b'* is the angle of inclination ø of *AB* with the V.P.

Assume that this trapezoid is rotated about *a'b'*, till it lies in the V.P.

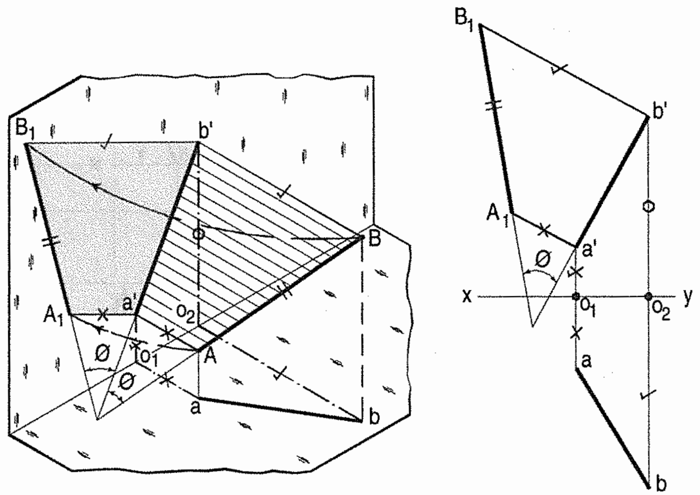


FIG. 10-18

In the orthographic view [fig. 10-18(ii)], this trapezoid is obtained by drawing perpendiculars to *a'b'*, viz. $a'A_1$ (equal to ao_1) and $b'B_1$ (equal to bo_2) and then joining A_1 with B_1. The line A_1B_1 is the true length of *AB* and its inclination ø with *a' b'* is the inclination of *AB* with the V.P.

Similarly, in trapezoid *ABba* in fig. 10-19(i), *AB* is the line and *ab* its top view. *Aa* and *Bb* are both perpendicular to *ab* and are respectively equal to $a'o_1$ and $b'o_2$ (the distances of a' and b' from *xy* in the front view). The angle θ between *AB* and *ab* is the inclination of *AB* with the H.P.

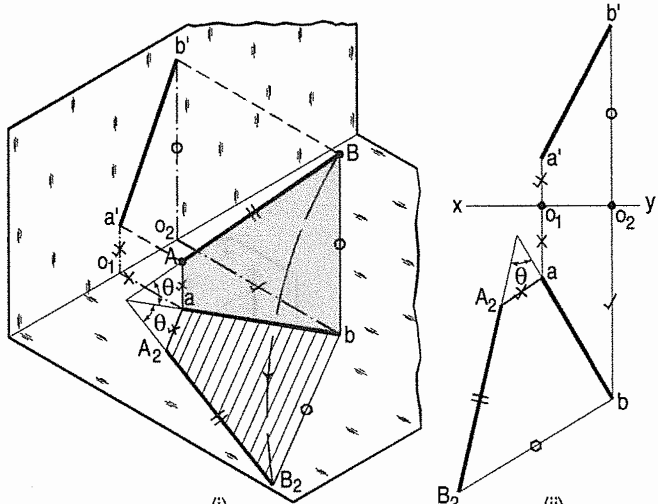


FIG. 10-19

This figure may now be assumed to be rotated about *ab* as axis, so that it lies in the H.P.

In the orthographic view [fig. 10-19(ii)], this trapezoid is obtained by erecting perpendiculars to *ab*, viz. aA_2 equal to $a'o_1$ and bB_2 equal to $b'o_2$ and joining A_2 with B_2. The line A_2B_2 is the true length of *AB* and its inclination θ with *ab* is the inclination of *AB* with the H.P.

Note: The perpendiculars on *ab* or *a'b'* can also be drawn on its other side assuming the trapezoid to be rotated in the opposite direction.

10-9. TRACES OF A LINE

When a line is inclined to a plane, it will meet that plane, produced if necessary. The point in which the line or line-produced meets the plane is called its *trace*.

The point of intersection of the line with the H.P. is called the *horizontal trace*, usually denoted as H.T. and that with the V.P. is called the *vertical trace* or V.T.

Refer to fig. 10-20.

(i) A line *AB* is parallel to the H.P. and the V.P. It has no trace.

(ii) A line *CD* is inclined to the H.P. and parallel to the V.P. It has only the H.T. but no V.T.

(iii) A line *EF* is inclined to the V.P. and parallel to the H.P. It has only the V.T. but no H.T.

Thus, when a line is parallel to a plane it has no trace upon that plane.

Refer to fig. 10-21.

(i) A line *PQ* is perpendicular to the H.P. Its H.T. coincides with its top view which is a point. It has no V.T.

(ii) A line *RS* is perpendicular to the V.P. Its V.T. coincides with its front view which is a point. It has no H.T.

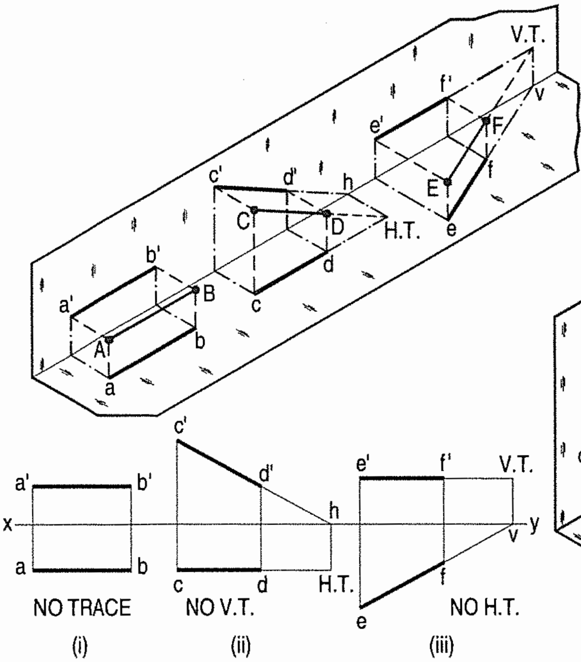


FIG. 10-20

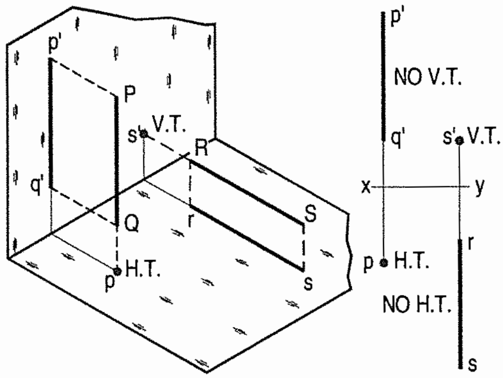


FIG. 10-21

Hence, when a line is perpendicular to a plane, its trace on that plane coincides with its projection on that plane. It has no trace on the other plane.

Refer to fig. 10-22.

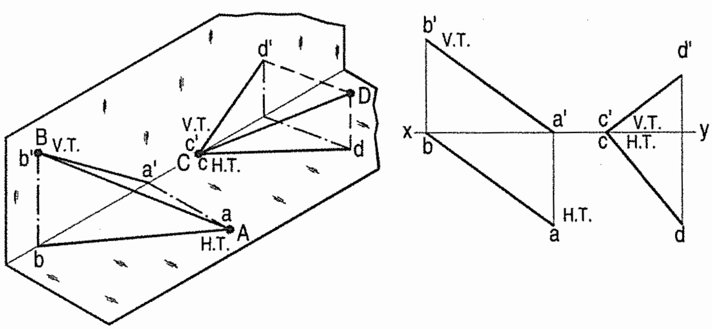


FIG. 10-22

(i) A line *AB* has its end *A* in the H.P. and the end *B* in the V.P. Its H.T. coincides with *a* the top view of *A* and the V.T. coincides with *b*' the front view of *B*.

(ii) A line *CD* has its end *C* in both the H.P. and the V.P. Its H.T. and V.T. coincide with *c* and *c*' (the projections of *C*) in *xy*.

Hence, when a line has an end in a plane, its trace upon that plane coincides with the projection of that end on that plane.

10-10. METHODS OF DETERMINING TRACES OF A LINE

Method I:

Fig. 10-23(i) shows a line *AB* inclined to both the reference planes. Its end *A* is in the H.P. and *B* is in the V.P.

a'b' and *ab* are the front view and the top view respectively [fig. 10-23(ii)].

The H.T. of the line is on the projector through *a'* and coincides with *a*. The V.T. is on the projector through *b* and coincides with *b'*.

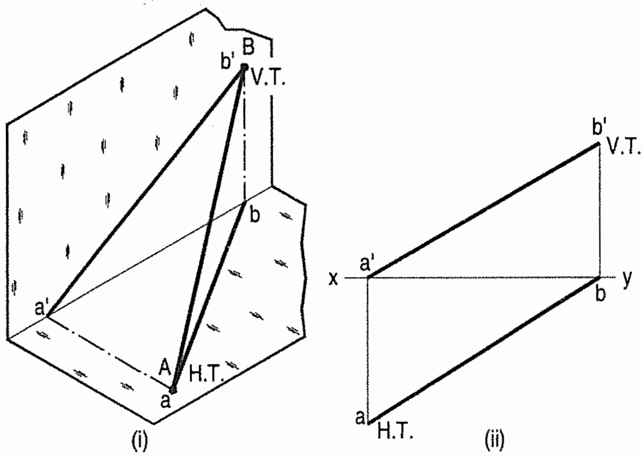


FIG. 10-23

Let us now assume that *AB* is shortened from both its ends, its inclination with the planes remaining constant. The H.T. and V.T. of the new line *CD* are still the same as can be seen clearly in fig. 10-24(i).

c'd' and *cd* are the projections of *CD* [fig. 10-24(ii)]. Its traces may be determined as described below.

(i) Produce the front view *c'd'* to meet *xy* at a point *h*.

(ii) Through *h*, draw a projector to meet the top view *cd*-produced, at the H.T. of the line.

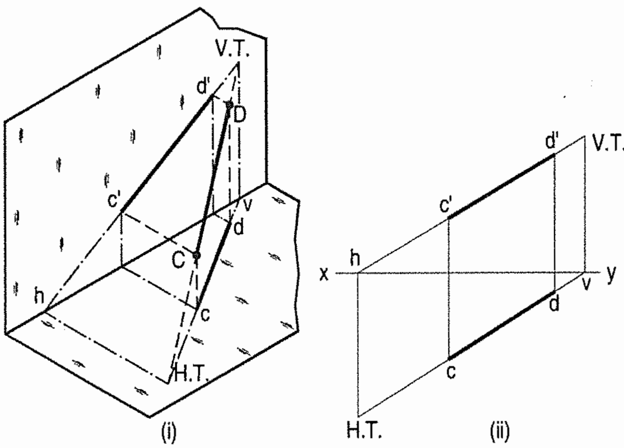


FIG. 10-24

(iii) Similarly, produce the top view *cd* to meet *xy* at a point *v*.

(iv) Through *v*, draw a projector to meet the front view *c'd'*-produced, at the V.T. of the line.

Method II:

c'd' and *cd* are the projections of the line *CD* [fig. 10-25(ii)]. Determine the true length C_1D_1 from the front view *c'd'* by trapezoid method. The point of intersection between *c'd'*-produced and C_1D_1-produced is the V.T. of the line.

Similarly, determine the true length C_2D_2 from the top view *cd*. Produce them to intersect at the H.T. of the line.

The above is quite evident from the pictorial view shown in fig. 10-25(i).

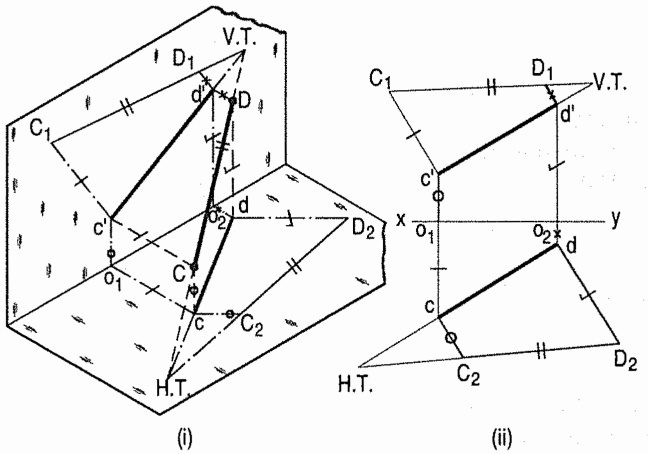


FIG. 10-25

10-11. TRACES OF A LINE, THE PROJECTIONS OF WHICH ARE PERPENDICULAR TO *xy*

When the projections of a line are perpendicular to *xy*, i.e. when the sum of its inclinations with the two principal planes of projection is 90°, it is not possible to find the traces by the first method. Method II must, therefore, be adopted as shown in fig. 10-26.

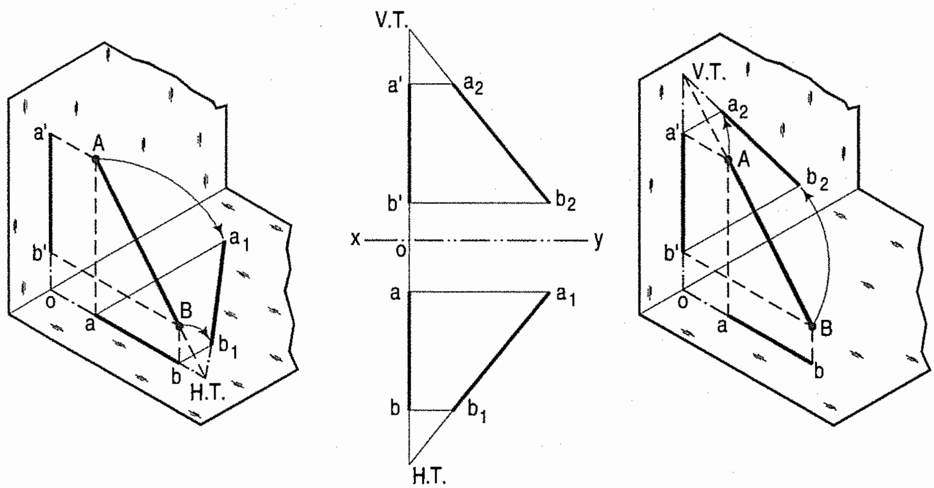


FIG. 10-26

10-12. POSITIONS OF TRACES OF A LINE

Although the line may be situated in the third quadrant, its both traces may be above or below *xy*, as shown in problem 10-6 and in fig. 10-27 and fig. 10-28. When a line intersects a plane, its traces on that plane will be contained by its projection on that plane as shown in problem 10-7.

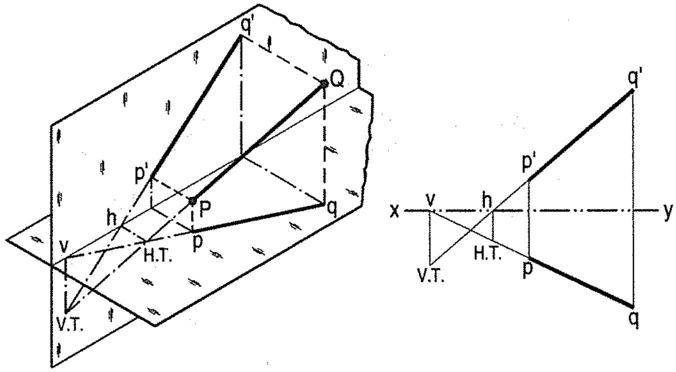


FIG. 10-27

Problem 10-6. *Projections of a line PQ are given. Determine the positions of its traces.*

Let *pq* and *p'q'* be the projections of *PQ* (fig. 10-27 and fig. 10-28).

(i) Produce the top view *pq* to meet *xy* at *v*. Draw a projector through *v* to meet the front view *p'q'*-produced at the V.T.

(ii) Through *h*, the point of intersection between *p'q'*-produced and *xy*, draw a projector to meet the top view *pq*-produced at the H.T.

Note that in fig. 10-27, the traces are below *xy* while in fig. 10-28 they are above it.

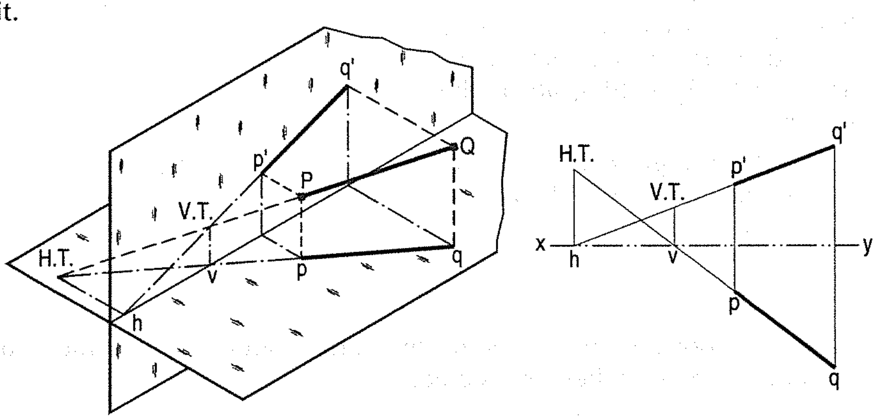


FIG. 10-28

Problem 10-7. *A point A is 50 mm below the H.P. and 12 mm behind the V.P. A point B is 10 mm above the H.P. and 25 mm in front of the V.P. The distance between the projectors of A and B is 40 mm. Determine the traces of the line joining A and B.*

Draw the projections *ab* and *a'b'* of the line *AB*.

Method I: (fig. 10-29):

(i) Through *v*, the point of intersection between *ab* and *xy*, draw a projector to meet *a'b'* at the V.T. of the line.

(ii) Similarly, through *h*, the point of intersection between *a'b'* and *xy*, draw a projector to cut *ab* at the H.T. of the line.

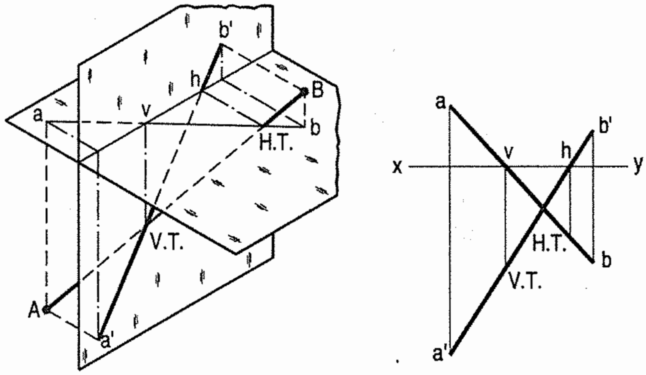


FIG. 10-29

Method II: (fig. 10-30):

At the ends a' and b', draw perpendiculars to $a'b'$, viz. $a'A_1$ equal to ao_1 and $b'B_1$ equal to bo_2 on its opposite sides (as a and b are on opposite sides of xy).

Draw the line A_1B_1 intersecting $a'b'$ at the V.T. of the line.

Similarly, at the ends a and b, draw perpendiculars to ab, viz. aA_2 equal to $a'o_1$ and bB_2 equal to $b'o_2$, on its opposite sides (as a' and b' are on opposite sides of xy). Join A_2 with B_2 cutting ab at the H.T. of the line.

Note that $A_1B_1 = A_2B_2 = AB$ and that θ and $\varnothing$ are the inclinations of AB with the H.P. and the V.P. respectively.

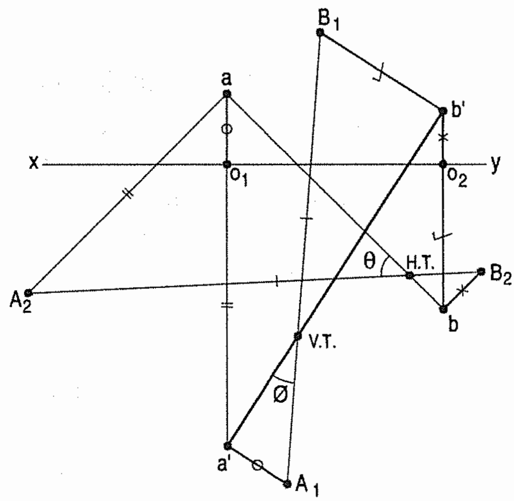


FIG. 10-30

10-13. ADDITIONAL ILLUSTRATIVE PROBLEMS

In the following problems, the ends of the lines should be assumed to be in the first quadrant, unless otherwise stated.

Problem 10-8. (fig. 10-31): *A line AB, 50 mm long, has its end A in both the H.P. and the V.P. It is inclined at 30° to the H.P. and at 45° to the V.P. Draw its projections.*

As the end A is in both the planes, its top view and the front view will coincide in xy.

(i) Assuming AB to be parallel to the V.P. and inclined at θ (equal to 30°) to the H.P., draw its front view ab' (equal to AB) and project the top view ab.

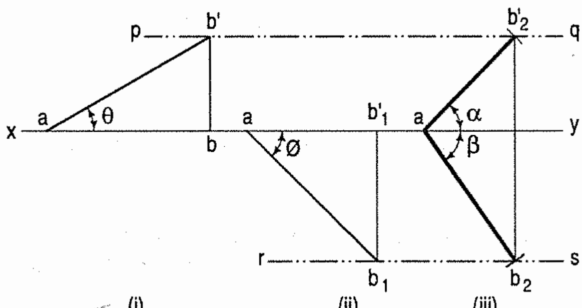


FIG. 10-31

(ii) Again assuming *AB* to be parallel to the H.P. and inclined at ø (equal to 45°) to the V.P., draw its top view ab_1 (equal to *AB*). Project the front view ab'_1.

ab and ab'_1 are the lengths of *AB* in the top view and the front view respectively, and *pq* and *rs* are the loci of the end *B* in the front view and the top view respectively.

(iii) With *a* as centre and radius equal to ab'_1, draw an arc cutting *pq* in b'_2. With the same centre and radius equal to *ab*, draw an arc cutting *rs* in b_2.

Draw lines joining *a* with b'_2 and b_2. ab'_2 and ab_2 are the required projections.

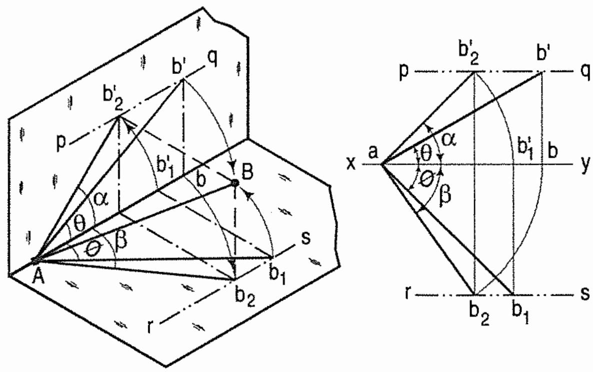


FIG. 10-32

Fig. 10-32 shows in pictorial and orthographic views, the solution obtained with all the above steps combined in one figure only.

Problem 10-9. (fig. 10-33): *A line PQ 75 mm long, has its end P in the V.P. and the end Q in the H.P. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.*

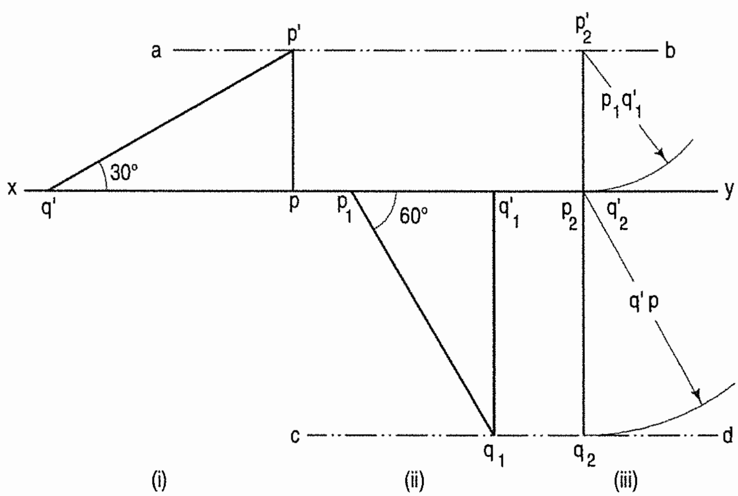


FIG. 10-33

The top view of *P* and the front view of *Q* will be in *xy*. As shown in the previous problem, determine

(i) the length of PQ in the top view, viz. $q'p$ and the path ab of the end P in the front view;

(ii) the length $p_1q'_1$ in the front view and the path cd of the end Q in the top view.

(iii) Mark any point p_2 (the top view of P) in xy and project its front view p'_2 in ab.

(iv) With p'_2 as centre and radius equal to $p_1q'_1$, draw an arc cutting xy in q'_2. It coincides with p_2.

(v) With p_2 as centre and radius equal to $q'p$, draw an arc cutting cd in q_2. p_2q_2 and $p'_2q'_2$ are the required projections. They lie in a line perpendicular to xy because the sum of the two inclinations is equal to 90°.

Problem 10-10. (fig. 10-34): *A line PQ 100 mm long, is inclined at 30° to the H.P. and at 45° to the V.P. Its mid-point is in the V.P. and 20 mm above the H.P. Draw its projections, if its end P is in the third quadrant and Q in the first quadrant.*

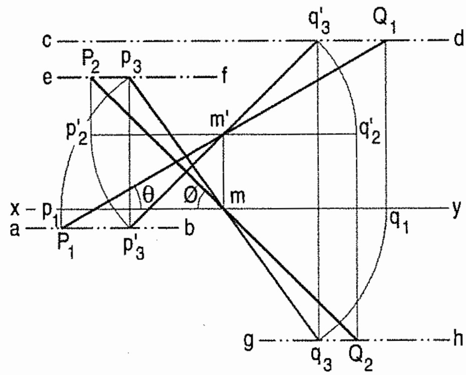


FIG. 10-34

The front view and the top view of P will be below and above xy respectively, while those of Q will be above and below xy respectively.

(i) Mark m, the top view of the mid-point in xy and project its front view m', 20 mm above xy.

(ii) Through m', draw a line making an angle θ (equal to 30°) with xy and with the same point as centre and radius equal to $\frac{1}{2}$ PQ, cut it at P_1 below xy and at Q_1 above xy. Project P_1Q_1 to p_1 q_1 on xy. p_1q_1 is the length of PQ in the top view. ab and cd are the paths of P and Q respectively in the front view.

(iii) Similarly, through m, draw a line making angle ø (equal to 45°) with xy and cut it with the same radius at P_2 above xy and at Q_2 below it.

(iv) Project P_2Q_2 to $p'_2q'_2$ on the horizontal line through m'. $p'_2q'_2$ is the length of PQ in the front view and ef and gh are the paths of P and Q respectively in the top view.

(v) With m as centre and radius equal to mp_1 or mq_1, draw arcs cutting ef at p_3 and gh at q_3. With m' as centre and radius equal to $m'p'_2$ or $m'q'_2$, draw arcs cutting ab at p'_3 and cd at q'_3. p_3q_3 and $p'_3q'_3$ are the required projections.

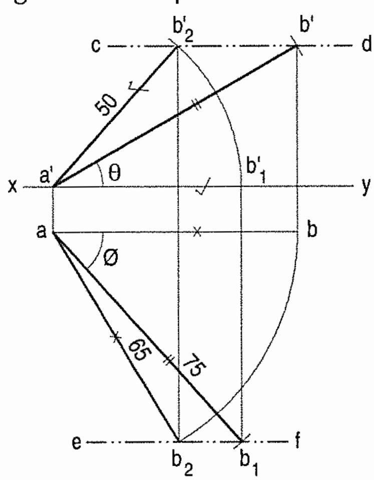


FIG. 10-35

Problem 10-11. (fig. 10-35): *The top view of a 75 mm long line AB measures 65 mm, while the length of its front view is 50 mm. Its one end A is in the H.P. and 12 mm in front of the V.P. Draw the projections of AB and determine its inclinations with the H.P. and the V.P.*

(i) Mark the front view a' and the top view a of the given end A.

(ii) Assuming *AB* to be parallel to the V.P., draw a line *ab* equal to 65 mm and parallel to *xy*. With *a'* as centre and radius equal to 75 mm, draw an arc cutting the projector through *b* at *b'*. The line *cd* through *b'* and parallel to *xy*, is the locus of *B* in the front view and θ is the inclination of *AB* with the H.P.

(iii) Similarly, draw a line $a'b'_1$ in *xy* and equal to 50 mm. With *a* as centre and radius equal to *AB*, draw an arc cutting the projector through b'_1 at b_1. *ef* is the locus of *B* in the top view and ø is the inclination of *AB* with the V.P.

(iv) With *a'* as centre and radius equal to $a'b'_1$, draw an arc cutting *cd* in b'_2. With *a* as centre and radius equal to *ab*, draw an arc cutting *ef* in b_2. $a'b'_2$ and ab_2 are the required projections.

Problem 10-12. (fig. 10-36): *A line AB, 65 mm long, has its end A 20 mm above the H.P. and 25 mm in front of the V.P. The end B is 40 mm above the H.P. and 65 mm in front of the V.P. Draw the projections of AB and show its inclinations with the H.P. and the V.P.*

(i) As per given positions, draw the loci *cd* and *gh* of the end *A*, and *ef* and *jk* of the end *B* in the front view and the top view respectively.

(ii) Mark any point *a* (the top view of *A*) in *gh* and project it to *a'* on *cd*. With *a'* as centre and radius equal to 65 mm, draw an arc cutting *ef* in *b'*. Join *a'* with *b'*. θ, the inclination of *a'b'* with *xy*, is the inclination of *AB* with the H.P. Project *b'* to *b* on *gh*. *ab* is the length of *AB* in the top view.

(iii) With *a* as centre and radius equal to 65 mm, draw an arc cutting *jk* in b_1. Join *a* with b_1. ø, the inclination of ab_1 with *xy*, is the inclination of *AB* with the V.P. Project b_1 to b'_1 on *cd*. $a'b'_1$ is the length of *AB* in the front view.

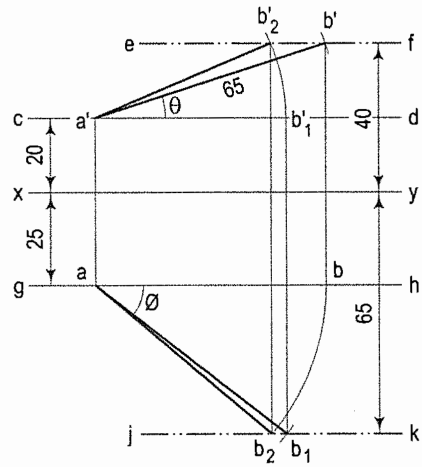


FIG. 10-36

Arrange *ab* and $a'b'_1$ between their respective paths as shown. $a'b'_2$ and ab_2 are the required projections of *AB*.

Problem 10-13. (fig. 10-37 and fig. 10-38): *The projectors of the ends of a line AB are 50 mm apart. The end A is 20 mm above the H.P. and 30 mm in front of the V.P. The end B is 10 mm below the H.P. and 40 mm behind the V.P. Determine the true length and traces of AB, and its inclinations with the two planes.*

Draw two projectors 50 mm apart. On one projector, mark the top view *a* and the front view *a'* of the end *A*. On the other, mark the top view *b* and the front view *b'* of the end *B*, as per given distances. *ab* and *a'b'* are the projections of *AB*.

Determine the true length, traces and inclinations by any one of the following two methods:

Method I:

By making the line parallel to a plane (fig. 10-37):

(i) Keeping *a* fixed, turn *ab* to a position ab_1, thus making it parallel to *xy*. Project b_1 to b'_1 on the locus of *b*'. $a'b'_1$ is the true length of *AB* and θ is its true inclination with the H.P.

(ii) Similarly, turn *a'b'* to the position a'_1b' and project a'_1 to a_1 on the path of *a* (because the end *a* has been moved). a_1b is the true length of *AB* and ø is its inclination with the V.P.

Traces:

(i) Through *v* the point of intersection of the top view *ab* with *xy*, draw a projector to cut *a'b'* at the V.T.

(ii) Through *h* the point of intersection of the front view *a'b'* with *xy*, draw a projector to cut *ab* at the H.T. of the line.

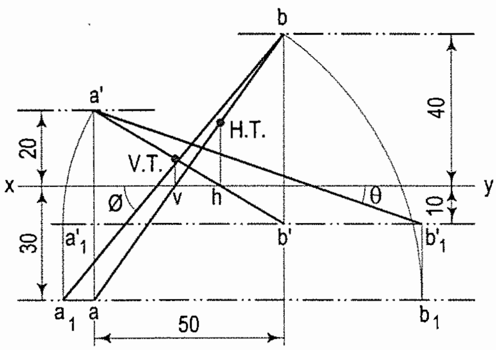


FIG. 10-37

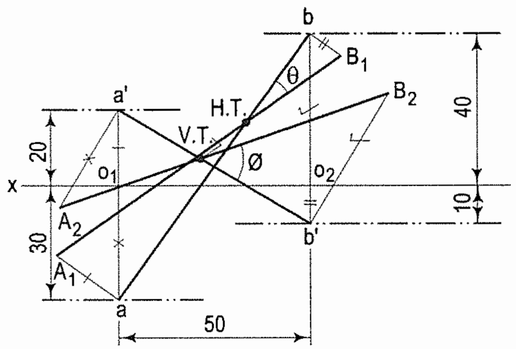


FIG. 10-38

Method II:

By rotating the line about its projections till it lies in H.P. or V.P. (fig. 10-38):

(i) At the ends *a* and *b* of the top view *ab*, draw perpendiculars to *ab*, viz. aA_1 equal to $a'o_1$ and bB_1 equal to $b'o_2$, on opposite sides of it (because *a*' and *b*' are on opposite sides of *xy*). A_1B_1 is the true length of *AB*. θ (its inclination with *ab*) is the inclination of *AB* with the H.P. and the point at which A_1B_1 intersects *ab* is the H.T. of *AB*.

(ii) Similarly, at the ends *a*' and *b*' of the front view *a'b'*, draw perpendiculars to *a'b'*, viz. $a'A_2$ equal to ao_1 and $b'B_2$ equal to bo_2, on opposite sides of it. A_2B_2 is the true length of *AB*. ø (its inclination with *a'b'*) is the inclination of *AB* with the V.P. and the point at which A_2B_2 intersects *a'b'* is the V.T. of *AB*.

Problem 10-14. (fig. 10-39): *A line AB, 90 mm long, is inclined at 45° to the H.P. and its top view makes an angle of 60° with the V.P. The end A is in the H.P. and 12 mm in front of the V.P. Draw its front view and find its true inclination with the V.P.*

(i) Mark *a* and *a*', the projections of the end *A*.

(ii) Assuming AB to be parallel to the V.P. and inclined at 45° to the H.P., draw its front view $a'b'$ equal to AB and making an angle of 45° with xy. Project b' to b so that ab the top view is parallel to xy. Keeping the end a fixed, turn the top view ab to a position ab_1 so that it makes an angle of 60° with xy. Project b_1 to b'_1 on the locus of b'. Join a' with b'_1. $a'b'_1$ is the front view of AB.

(iii) To find the true inclination with the V.P., draw an arc with a as centre and radius equal to AB, cutting the locus of b_1 in b_2. Join a with b_2. ø is the true inclination of AB with the V.P.

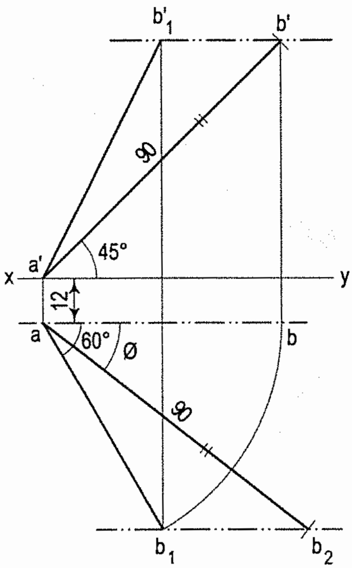


FIG. 10-39

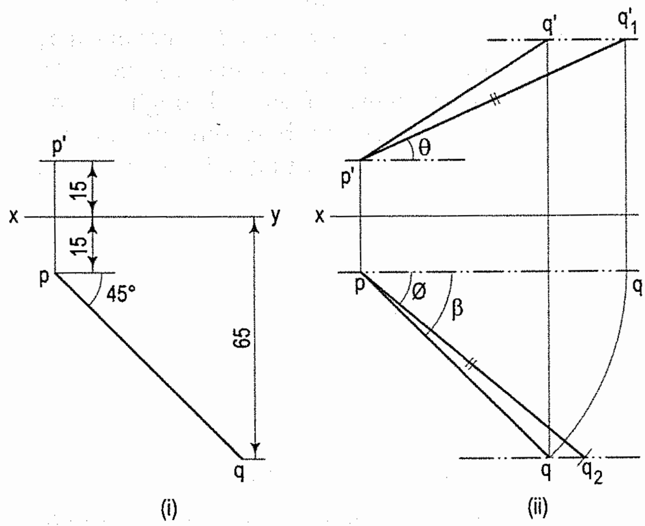


FIG. 10-40

Problem 10-15. (fig. 10-40): *Incomplete projections of a line PQ, inclined at 30° to the H.P. are given in fig. 10-40(i). Complete the projections and determine the true length of PQ and its inclination with the V.P.*

(i) Turn the top view pq [fig. 10-40(ii)] to a position pq_1, so that it is parallel to xy. Through p', draw a line making an angle θ (equal to 30°) with xy and cutting the projector through q_1 at q'_1. $p'q'_1$ is the front view of PQ.

(ii) Through q'_1, draw a line parallel to xy and cutting the projector through q at q'. $p'q'$ is the front view of PQ.

(iii) With p as centre and radius equal to $p'\ q'_1$, draw an arc cutting the locus of q at q_2.

(iv) Join p with q_2. ø is the inclination of PQ with the V.P.

Problem 10-16. (fig. 10-41): *The end A of a line AB is 25 mm behind the V.P. and is below the H.P. The end B is 12 mm in front of the V.P. and is above the H.P. The distance between the projectors is 65 mm. The line is inclined at 40° to the H.P. and its H.T. is 20 mm behind the V.P. Draw the projections of the line and determine its true length and the V.T.*

Draw the top view ab and mark the H.T. on it, 20 mm above xy.

We have seen that the line representing the true length obtained by the trapezoid method, intersects the top view or the top view-produced, at the H.T. at an angle equal to the true inclination of the line with the V.P.

(i) Hence, at the ends *a* and *b*, draw perpendiculars to *ab* on its opposite sides (as one end is below the H.P. and the other end above it). Through the H.T., draw a line making angle θ (equal to 40°) with *ab* and cutting the perpendiculars at A_1 and B_1, as shown. A_1B_1 is the true length of *AB*. aA_1 and bB_1 are the distances of the ends *A* and *B* respectively from the H.P.

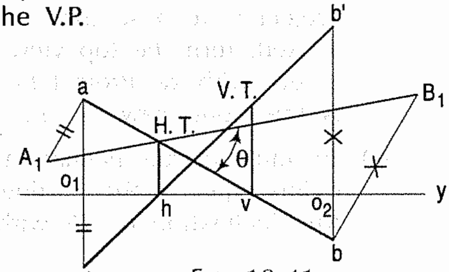


FIG. 10-41

(ii) Project *a* and *b* to *a'* and *b'*, making $a'o_1$ equal to aA_1 and $b'o_2$ equal to bB_1. *a'b'* is the front view of *AB*. Through *v*, the point of intersection between *ab* and *xy*, draw a projector cutting *a'b'* at the V.T. of the line.

Problem 10-17. (fig. 10-42): *A line AB, 90 mm long, is inclined at 30° to the H.P. Its end A is 12 mm above the H.P. and 20 mm in front of the V.P. Its front view measures 65 mm. Draw the top view of AB and determine its inclination with the V.P.*

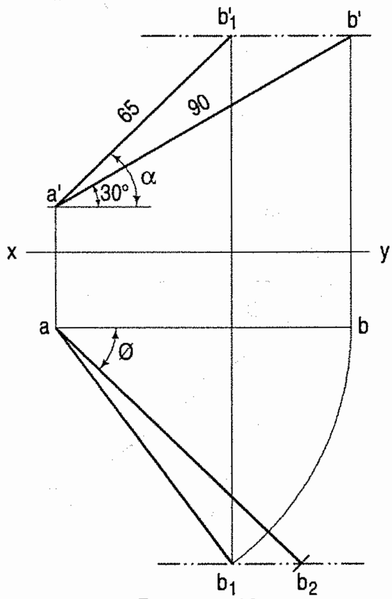


FIG. 10-42

(i) Mark *a* and *a'* the projections of the end *A*. Through *a'*, draw a line *ab'* 90 mm long and making an angle of 30° with *xy*.

(ii) With *a'* as centre and radius equal to 65 mm, draw an arc cutting the path of *b'* at b'_1. $a'b'_1$ is the front view of *AB*.

(iii) Project *b'* to *b*, so that *ab* is parallel to *xy*. *ab* is the length of *AB* in the top view.

(iv) With *a* as centre and radius equal to *ab*, draw an arc cutting the projector through b'_1 at b_1. Join *a* with b_1. ab_1 is the required top view.

Determine ø as described in problem 10-14.

Problem 10-18. (fig. 10-43): *The ends of a line PQ are on the same projector. The end P is 30 mm below the H.P. and 12 mm behind the V.P. The end Q is 55 mm above the H.P. and 45 mm in front of the V.P. Determine the true length and traces of PQ and its inclinations with the two planes.*

Note: When the ends of a line are on the same projector or sum of angles of inclinations of a line with *xy* is 90°, use Method II only.

Draw the projections *pq* and *p' q'* as per given positions of the ends *P* and *Q*. They will partly coincide with each other.

(i) At the ends *p* and *q* of the top view *pq*, erect perpendiculars, viz. pP_1 equal to *p'o*, and qQ_1 equal to *q'o* and on opposite sides of *pq*. P_1Q_1 is the true length of *PQ*. θ is the inclination of *PQ* with the H.P. and the point of intersection between P_1Q_1 and *pq* is the H.T. of *PQ*.

(ii) Similarly, draw perpendiculars to $p'q'$. viz. $p'P_2$ equal to po and $q'Q_2$ equal to qo and on opposite sides of $p'q'$. P_2Q_2 is the true length. ø is the true inclination of PQ with the V.P. and the point where P_2Q_2 cuts $p'q'$ is the V.T. of PQ.

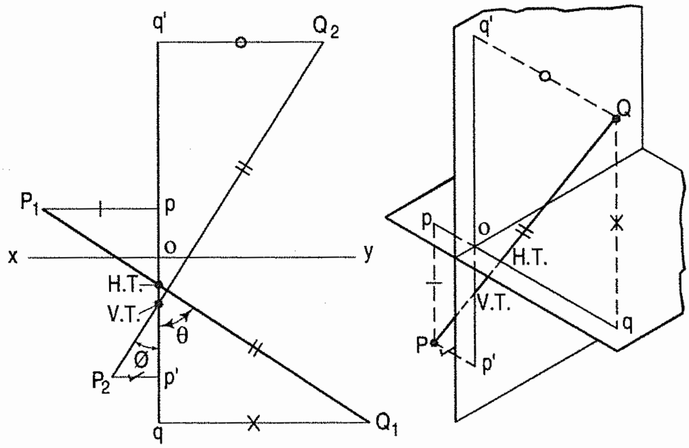


FIG. 10-43

Problem 10-19. (fig. 10-44): *A line AB, inclined at 40° to the V.P., has its ends 50 mm and 20 mm above the H.P. The length of its front view is 65 mm and its V.T. is 10 mm above the H.P. Determine the true length of AB, its inclination with the H.P. and its H.T.*

(i) Draw the front view $a'b'$ as per given positions of A and B and the given length.

(ii) Draw a line parallel to and 10 mm above xy. This line will contain the V.T. Produce $a'b'$ to cut this line at the V.T. Draw a projector through V.T. to v on xy.

(iii) Assuming a' V.T. to be the front view of a line which makes 40° angle with the V.P. and whose one end v is in the V.P., let us determine its true length.

(iv) Keeping V.T. fixed, turn the end a' to a'_1 so that the line becomes parallel to xy. Through v, draw a line making an angle of 40° with xy and cutting the projector through a'_1 at a_1. The line through a_1, drawn parallel to xy, is the locus of A in the top view. Project a' to a on this line. av is the top view of the line, whose front view is a' V.T. and whose true length is equal to a_1v.

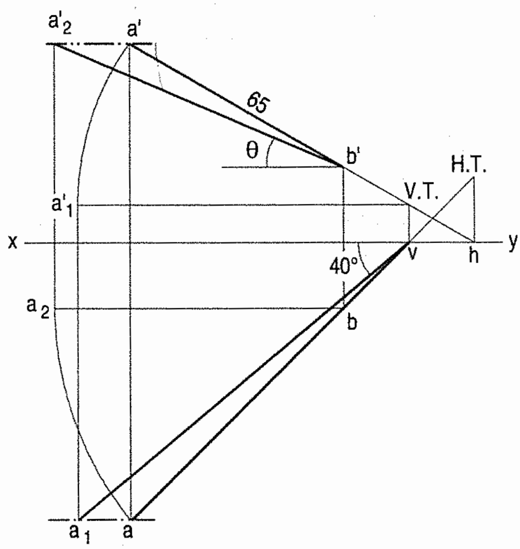


FIG. 10-44

(v) But $a'b'$ is the given front view of AB. Therefore, project b' to b on av. ab is the top view of AB. Obtain the inclination θ with the H.P. by making the top view ab parallel to xy, as shown.

Produce $a'b'$ to meet xy at h. Draw a projector through h to cut ab-produced, at the H.T. of the line.

Problem 10-20. (fig. 10-45): *The front view $a'b'$ and the H.T. of a line AB, inclined at 23° to the H.P. are given in fig. 10-45(i). Determine the true length of AB, its inclination with the V.P. and its V.T.*

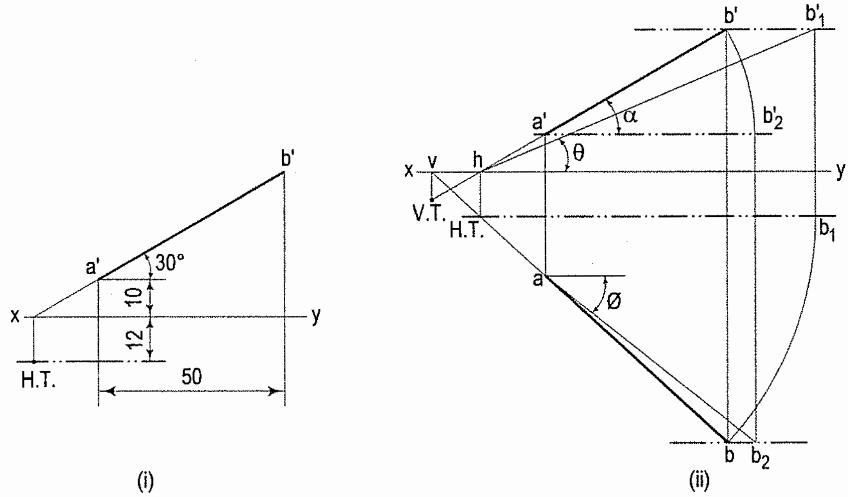


FIG. 10-45

Consider that hb' is the front view of a line inclined at 23° to the H.P. and the top view of whose one end is in H.T.

(i) Through h [fig. 10-45(ii)], draw a line making an angle of $\theta = 23°$ with xy and cutting the locus of B in the front view in b'_1. hb'_1 is the true length of the line whose length in the top view is H.T. b_1.

(ii) With H.T. as centre and radius equal to H.T. b_1, draw an arc cutting the projector through b' at b. H.T. b is the top view and hb' is the front view of a line which contains AB.

(iii) Therefore, through a', draw a projector cutting H.T. b at a. ab is the top view of AB.

Obtain the true length ab_2 (of AB) and its inclination ø with the V.P. by making a' b' parallel to xy.

(iv) Produce ba to meet xy in v. Draw a projector through v to cut $b'a'$-produced, at the V.T. of the line.

Problem 10-21. (fig. 10-46): *A tripod stand rests on the floor. One of its legs is 150 mm long and makes an angle of 70° with the floor. The other two legs are 163 mm and 175 mm long respectively. The upper ends of the legs are attached to the corners of a horizontal equilateral triangular frame of 50 mm side, one side of which is parallel to the V.P. In the top view, the legs appear as lines 120° apart, which if produced, would meet in a point. Draw the projections of the tripod and determine the angle which each of the other two legs makes with the floor. Assume the thickness of the frame and of the legs to be equal to that of the line.*

(i) At any point P on xy, draw a line PA, 150 mm long and making 70° angle with xy. h is the height of the tripod and PA_1 is the length of the leg in the top view.

(ii) Draw an equilateral triangle *abc* of 50 mm side with one side parallel to and below *xy*. Project the front view *a'b'c'* at the height *h* above *xy*. Determine the lengths of the other two legs in the top view as described below.

(iii) With *b'* as centre and radius equal to 163 mm, draw an arc cutting *xy* in q'_1. Similarly with *a'* and *c'* as centre and radius equal to 150 mm and 175 mm, draw an arc cutting *xy* in p'_1 and r'_1 respectively ap_1, bq_1 and cr_1 are the lengths of the three legs in the top view and α and β respectively are their inclinations with the floor (H.P.).

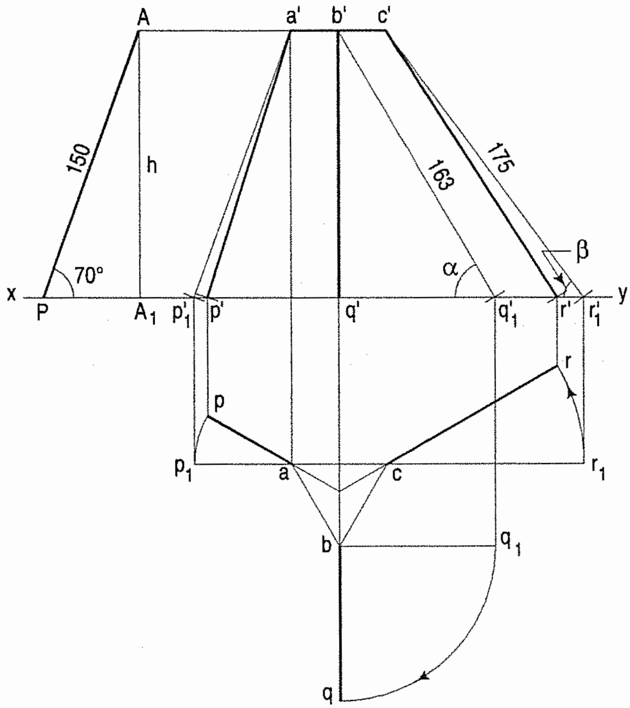


FIG. 10-46

(iv) The legs in the top view are to be inclined at 120° to each other and to meet at a point, if produced. Therefore, draw lines bisecting the angles of the triangle, making *ap* equal to PA_1, *bq* equal to bq_1 and *cr* equal to cr_1, thus completing the top view.

(v) Project *p*,*q* and *r* to *p'*,*q'* and *r'* respectively on *xy*. Complete the front view by drawing lines *a'p'*, *b'q'* and *c'r'*.

Problem 10-22. (fig. 10-47): *A straight road going uphill from a point A, due east to another point B, is 4 km long and has a slope of 15°. Another straight road from B, due 30° east of north, to a point C is also 4 km long but is on ground level. Determine the length and slope of the straight road joining the points A and C. Scale, 10 mm = 0.4 km.*

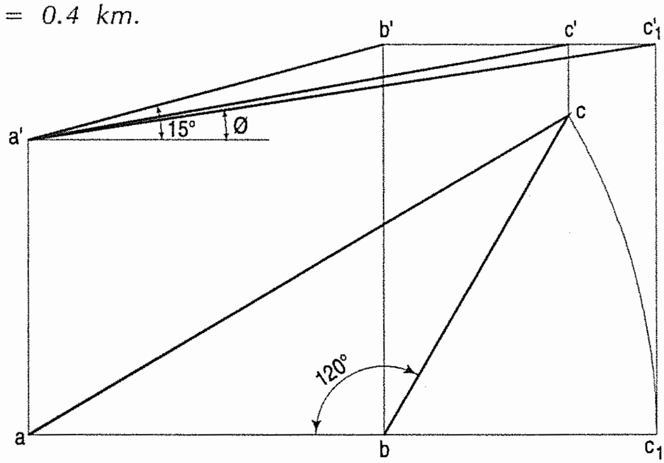


FIG. 10-47

(i) Mark any point a'. Draw a line $a'b'$, 100 mm long, to the right of a' and inclined upwards at 15° to the horizontal (to represent the road from A to B). Project its top view ab keeping it horizontal.

(ii) As the road from B to C is on ground level, the top view bc will be equal to 100 mm and inclined at (90° + 30°) i.e. 120° to ab.

(iii) From b, draw a line bc, 100 mm long and making 120° angle with ab. Project c to c' making $b'c'$ horizontal. $a'c'$ and ac are the front view and the top view respectively of the road from A to C,

Determine the true length $a'c'_1$ and the angle ø as shown, which are respectively the length and slope of the road from A to C.

Problem 10-23. (fig. 10-48): *Two lines AB and AC make an angle of 120° between them in their front view and top view. AB is parallel to both the H.P. and the V.P. Determine the real angle between AB and AC.*

Draw any line $b'a'$ parallel to and above xy, and another line $a'c'$ of any length making 120° angle with $b'a'$. Join b' with c'.

(i) Project the top view ba parallel to xy and the top view ac, making 120° angle with ba. Join b with c. $b'a'$ or ba is the true length of AB. Determine the true lengths of AC and BC, viz. $a'c'_1$ and $b'c'_2$, as shown.

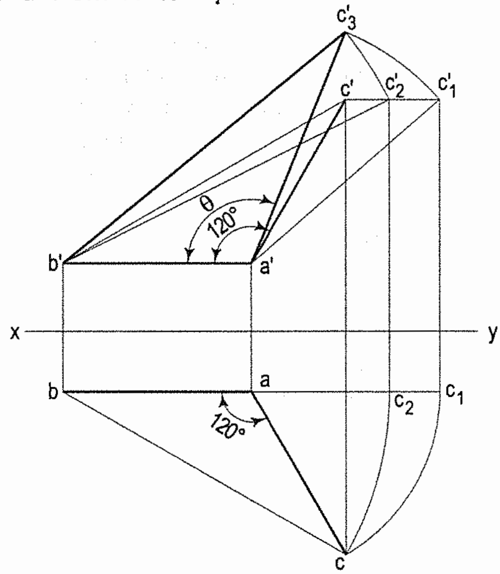


FIG. 10-48

(ii) Draw a triangle $a'b'c'_3$ making $a'c'_3$ equal to $a'c'_1$ and $b'c'_3$ equal to $b'c'_2$. $\angle b'a'c'_3$ is the real angle between AB and AC.

Problem 10-24. (fig. 10-49): *An object O is placed 1.2 m above the ground and in the centre of a room 4.2 m × 3.6 m × 3.6 m high. Determine graphically its distance from one of the corners between the roof and two adjacent walls. Scale, 10 mm = 0.5 m.*

(i) Draw the front view (of the room) $a'b'c'd'$ as seen from the front of, say 3.6 m wall. $a'b'$ is the width of the room and $a'd'$ is the height. The front view o' of the object will be seen 1.2 m above the mid-point of $a'b'$. c' and d' are the top corners of the room. $o'c'$ is the front view of the line joining the object with a top corner.

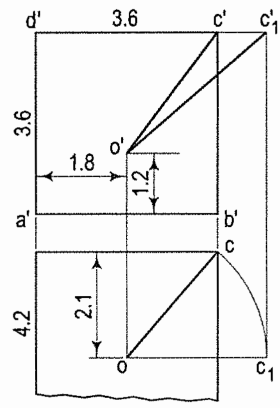


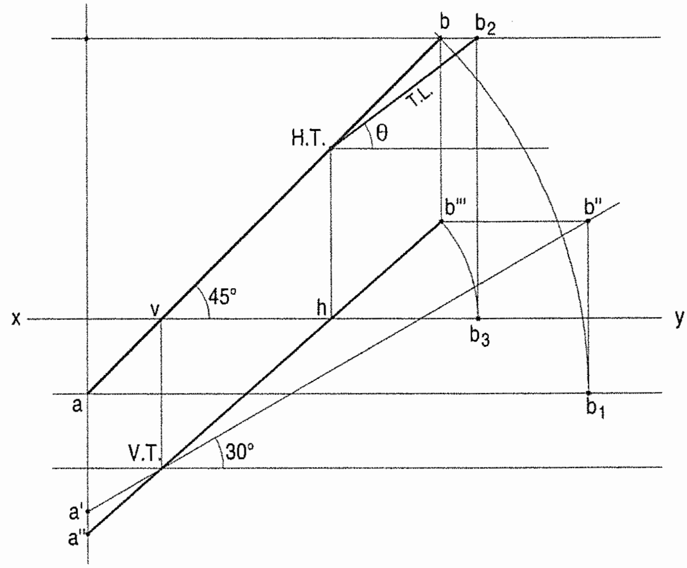
FIG. 10-49

(ii) Draw the top view of the room. It will be a rectangle of sides equal to 3.6 m and 4.2 m. The top view o of the object will be in the centre of the rectangle. oc is the top view of the line joining the object with the top corner.

Determine the true length $o'c'_1$, which will show the distance of the object from one of the top corners of the room.

Problem 10-25. *The straight line AB is inclined at 30° to H.P., while its top view at 45° to a line xy. The end A is 20 mm in front of the V.P. and it is below the H.P. The end B is 75 mm behind the V.P. and it is above the H.P. Draw the projections of the line when its V.T. is 40 mm below. Find the true length of the portion of the straight line which is in the second quadrant and locate its H.T.*

Refer to fig. 10-50.



T.L. = 50 mm. ∠V.P., θ = 37°

FIG. 10-50

(i) Mark the points a (top view of A) and b (top view of B) at the distances of 20 mm and 75 mm below and above xy respectively.

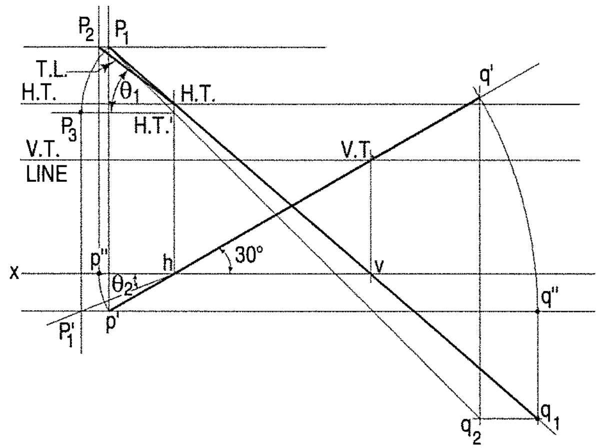
(ii) Through the point a, draw a line at 45° intersecting xy and the path of b at v and b respectively as shown.

(iii) Construct a line containing V.T. 40 mm below xy. Draw perpendicular from v to the line V.T.

(iv) With a as centre and radius equal to ab, draw an arc which intersects at b_1 a line drawn from a parallel to xy. From V.T. draw a line at 30° intersecting the projector of b_1 at b''. From b'', draw $b''b'''$ parallel to xy to intersect projector of b at b'''. Join V.T. b'''. Produce it to meet the projector from a at a''. $a''\ b'''$ is the required projection. $a''b'''$ intersects line xy at h. From h draw the perpendicular to meet ab. The intersection point represents H.T.

(v) With h as centre and radius equal to hb''', draw an arc intersecting at b_3. Draw projector from b_3 to cut the path of b at b_2. Join H.T. b_2. Measure the angle H.T. b_2 with xy. This is an angle made by the line with the V.P.

Problem 10-26. (fig. 10-51.): *The front view of a line PQ makes an angle of 30° with xy. The H.T. of the line is 45 mm behind the V.P. While its V.T. is 30 mm above the H.P. The end P of the line is 10 mm below the H.P. and the end Q is in the first quadrant. The line is 150 mm long. Draw the projections of the line and determine the true-length of the portion of the line which is in the second quadrant. Also find the angle of the line with the H.P. and V.P.*



T.L. = 25 mm. ∠V.P., θ_1 = 37° ∠V.P., θ_2 = 22°

FIG. 10-51

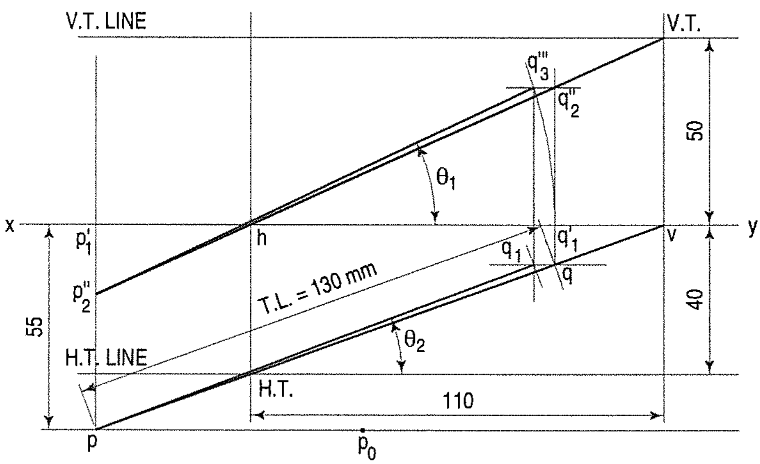
(i) Draw lines containing H.T. and V.T. at 45 mm and 30 mm above *xy* respectively. Mark point *p*' at 10 mm below line *xy*. Draw *p*'*q*' at 30° from *p*' intersecting *xy* at *h* and line V.T. at V.T. From *h*, draw perpendicular to line H.T. to locate H.T.

(ii) Draw perpendicular from V.T. to intersect *xy* at *v*. Join *v* H.T. and produce to intersect the projector of *p*' at p_1. Draw $p_1 q_1$ of 150 mm representing true length of the line in top view. From *p*', draw line parallel to *xy* representing front view of the line *PQ*. Draw projector from q_1 to cut the line drawn from *p*' at *q*''.

(iii) Keeping *p*' fixed, turn *p*'*q*'' such that it cuts the line drawn from *p*' at *q*'. From q_1 draw line parallel to *xy* which intersects the vertical projector drawn from *q*' at q_2. Join P_1q_2. This is the required projection.

(iv) Keeping *h* fixed, rotate *hp*' and make it parallel to *xy*. From *P*'' draw projector intersecting the path of p_1 at p_2. H.T. p_2 is true-length of the line. Similarly keeping H.T.' fixed, turn H.T.' p_1 to make it parallel to *xy* as shown. From P_3, draw projector to intersect the horizontal line drawn from $P'P'_1$. Measure angle xhp'_1. This is the required angle with H.P.

Problem 10-27. (fig. 10-52): *The end P of a line PQ 130 mm long, is 55 mm in front of the V.P. The H.T. of the line is 40 mm in front of the V.P. and the V.T. is 50 mm above the H.P. The distance between H.T. and V.T. is 110 mm. Draw the projections of the line PQ and determine its angles with the H.P. and the V.P.*



∠H.P., θ_1 = 25.5°; ∠V.P., θ_2 = 21°

FIG. 10-52

(i) Mark p below xy at a distance of 55 mm. Draw lines containing H.T. and V.T. at 40 mm below and 50 mm above xy respectively. Construct projectors through H.T. and V.T. 110 mm apart intersecting xy at h and v respectively.

(ii) Draw perpendiculars from h and v intersecting the lines containing H.T. and V.T. Join H.T. v and produce to cut the line drawn from point P as shown. Join h V.T. and extend further to intersect the projector drawn from P at P''_2.

(iii) Mark true length 130 mm on H.T.v. Let it be pq. Draw projector from q cutting xy at q'_1. With p'_1 as centre and radius equal to $p'_1q'_1$, draw an arc cutting a horizontal line drawn from q''_2 at q'''_3. Join pq_1 and $p''_2q'''_3$, are the required projections. $\theta_1 = 25.5°$ and $\theta_2 = 21°$ are the measured angles.

Problem 10-28. (fig. 10-53): *The distance between the end projectors of a line AB is 70 mm and the projectors through the traces are 110 mm apart. The end of a line is 10 mm above H.P. If the top view and the front view of the line make 30° and 60° with xy line respectively, draw the projections of the line and determine* (i) *the traces,* (ii) *the angles with the H.P. and the V.P.,* (iii) *the true length of the line. Assume that the line is in the first quadrant.*

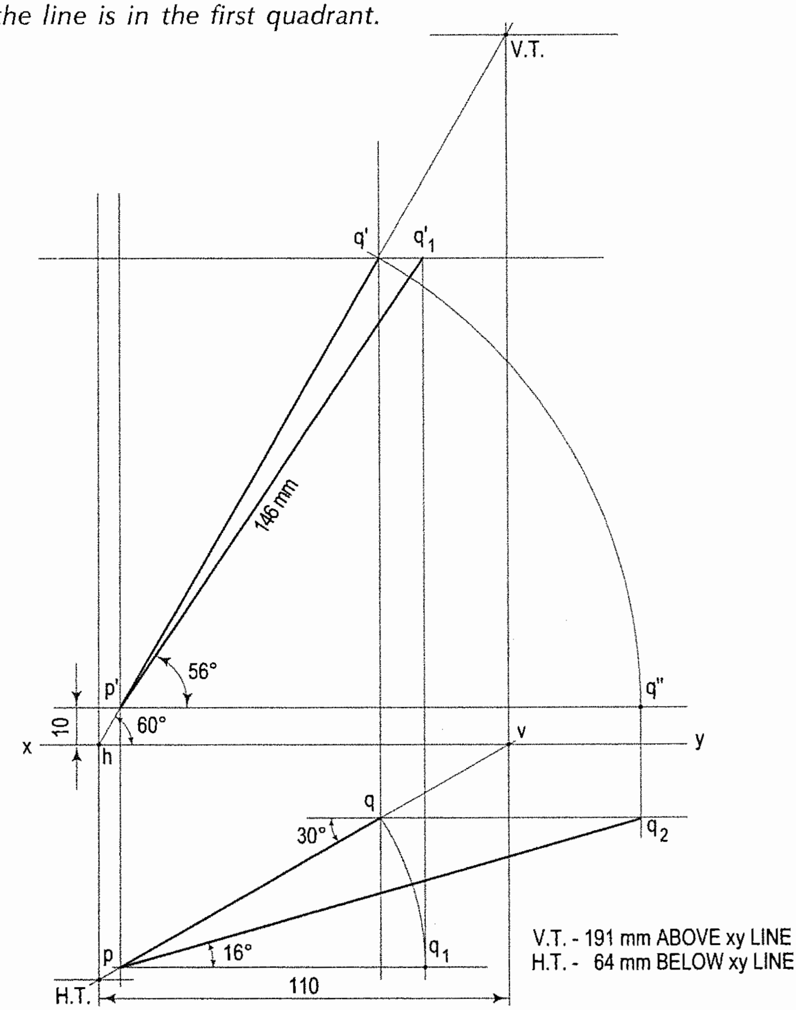


FIG. 10-53

(i) Draw two vertical lines 110 mm apart representing projectors through the traces of the line. Mark intersection of these projectors *h* and *v* on *xy* as shown.

(ii) From *v* and *h*, draw lines at 30° and 60°.

(iii) Mark *p*', 10 mm above *xy*. Draw two vertical projectors at 70 mm apart keeping equal distance from the projectors through traces. *p*'*q*' and *pq* represent front view and top view of the line as shown.

(iv) Keeping *p* fixed, turn *pq* to position pq_1. From q_1, draw a vertical projector intersecting the path of *q*' at q'_1. Join $p'q'_1$. This is the true length of the line. Measure angle $q''p'q'_1$ with the horizontal line as shown. This is the angle made by the line with the H.P. Similarly rotate *p*'*q*' making it parallel to *xy* as shown.

Draw a vertical projector from *q*'' to intersect the path of *q* at q_2. Measure the angle q_1pq_2 with the horizontal line. This is the angle made by the line with V.P.

Note: Problem 10-29 and problem 10-31 are solved by using auxiliary plane method.

Problem 10-29. *Two pipes PQ and RS seem to intersect at a' and a in front view and top view as shown in fig. 10-54. The point A is 400 mm above H.P. and 300 mm in front of a wall.*

Neglecting the thickness of the pipes, determine the clearance between the pipes.

Refer to fig. 10-54.

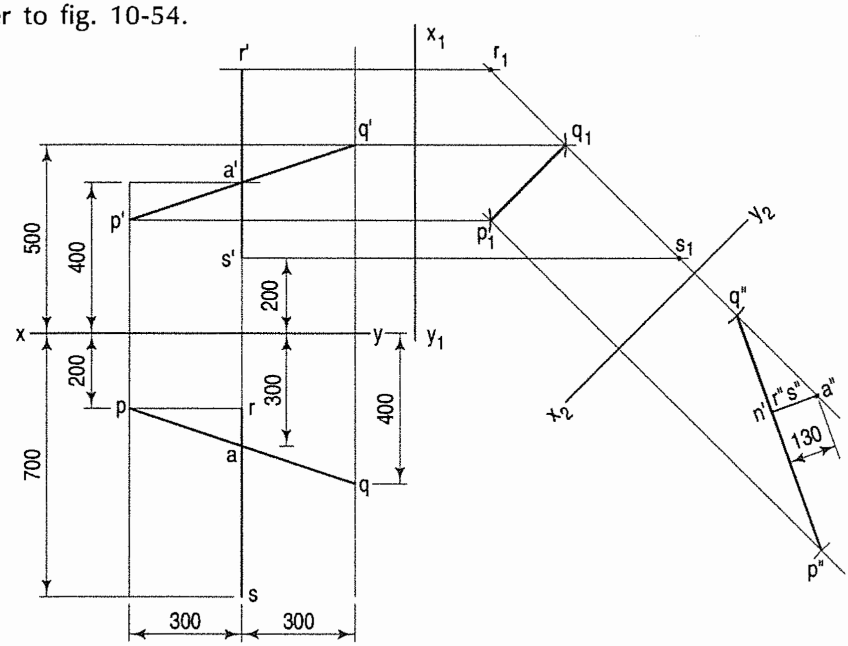


FIG. 10-54

(i) Draw the projections of pipes treating them as lines.

(ii) Draw x_1y_1 perpendicular to the line *xy* and project auxiliary top view. Note that the distances of p_1, q_1, r_1 and s_1 from x_1y_1 are equal to the distances of *p*, *q*, *r* and *s* from *xy*.

(iii) Mark an auxiliary reference line x_2y_2 perpendicular to the line r_1s_1 for obtaining the point view of the line. Draw an auxiliary front view as shown. Note that the distances of p'', q'', r'' and s'' from x_2y_2 are equal to the distances of p', q', r' and s' from x_1y_1.

(iv) a'' n' represents clearance between two pipes which is approximately 130 mm.

Problem 10-30. *The end projectors of line AB are 22 mm apart. A is 12 mm in front of the V.P. and 12 mm above the H.P. The point B 6 mm in front of the V.P. and 40 mm above the H.P. Locate the H.T. and the V.T. of the line and also determine its inclinations with the V.P. and the H.P.*

If the line *AB* is shifted to II, III and IV quadrants as shown in fig. 10-55 (assume that the distances of *A* and *B* from the projection-planes are same as the first quadrant), draw the projections of line and locate the traces.

The solution of first part of problem is shown in pictorial view. For the second part of the problem, the locations of the line in the respective quadrants are shown in fig. 10-55.

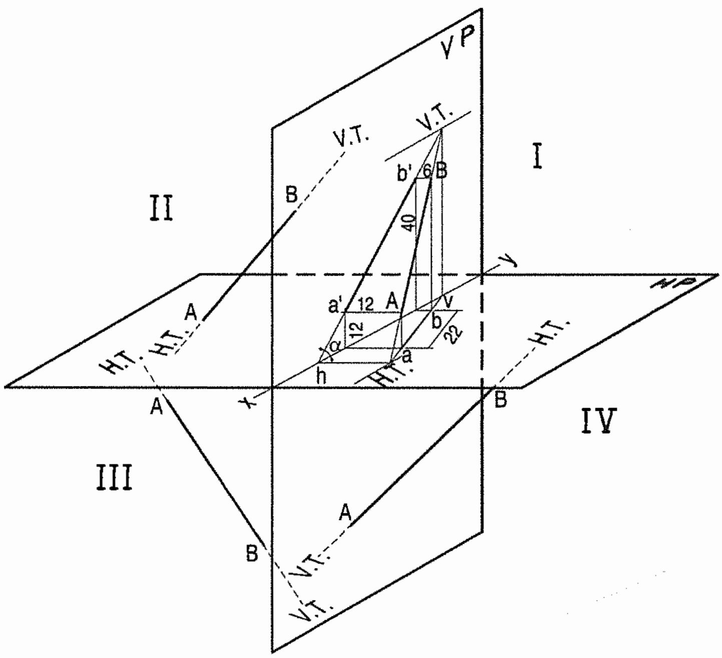


FIG. 10-55

Students are advised to draw the orthographic projections of the line for the respective quadrants.

Problem 10-31. *The end projectors of line PQ are 80 mm apart. The end P is 15 mm in front of the V.P. and 60 mm above the H.P. While Q is 50 mm in front of the V.P. and 10 mm above the H.P. A point R is situated on the projector at a distance of 31 mm from the projector through P measured towards the projector of Q. The point R is 70 mm in front of the V.P. and above the H.P. A perpendicular is drawn from R on PQ. Draw its projections.*

Refer to fig. 10-56.

(i) Draw two end projectors of the line *PQ* at 80 mm apart.

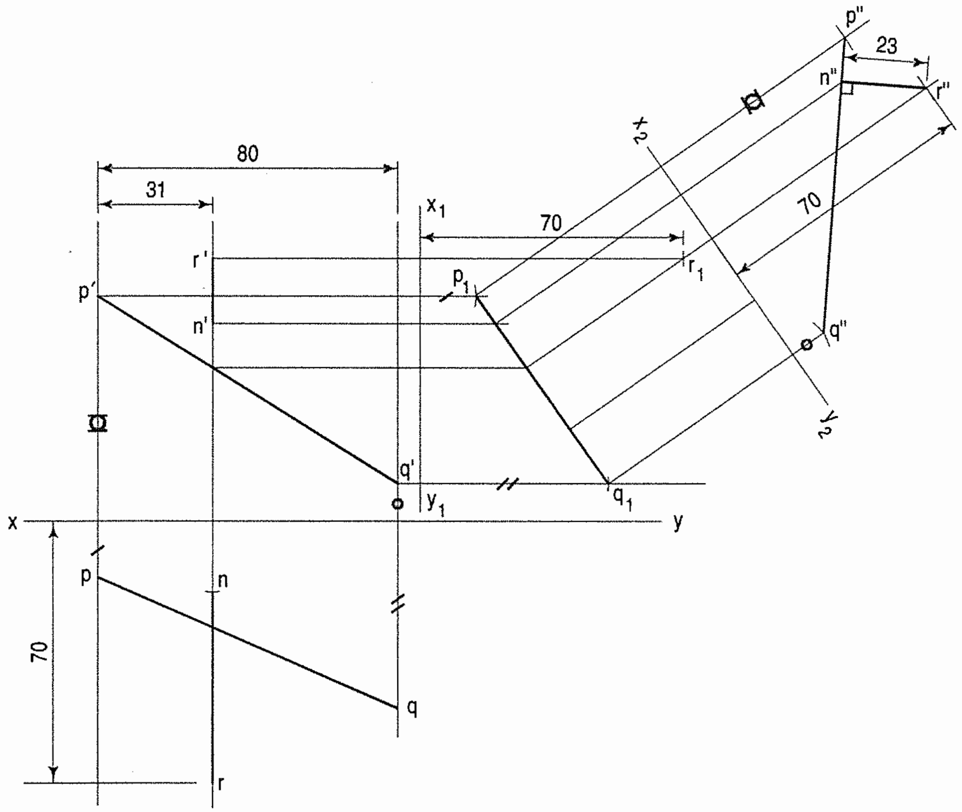
(ii) Mark the front view *p*' and the top view *p* at 60 mm and 15 mm from *xy* on the end projector of *P*. Similarly mark *q*' and *q* at 10 mm and 50 mm from *xy* on the end projector of *Q*.

(iii) Draw a vertical line at 31 mm away from *p'p* towards *q'q*. Mark the position of *R* in the top view *r* and the front view *r*' at 70 mm from *xy* as shown.

(iv) Draw x_1y_1 perpendicular to *xy* as shown. Draw projectors from *p*', *q*' and *r*' on x_1y_1. Transfer the distances 15 mm, 50 mm and 70 mm of *p*, *q* and *r* from *xy* to the new top view p_1, q_1 and r_1 from x_1y_1.

(v) Draw another reference line x_2y_2 for the new front view parallel to p_1q_1. Transfer the distances 60 mm, 10 mm and 70 mm of *p*', *q*' and *r*' from *xy* to the new front view *p*", *q*" and *r*" from x_2y_2.

(vi) From *r*", draw perpendicular to *p*"*q*" intersecting at *n*" as shown which is measured as 23 mm.



n"r" = 23 mm

FIG. 10-56

Problem 10-32. *The end projectors of a line PQ are 65 mm apart. P is 25 mm behind the V.P. and 30 mm below the H.P. The point Q is 40 mm above the H.P. and 15 mm in front of the V.P. Find the third point C in the H.P. and in front of the V.P. such that its distance from a point P is 45 mm and that from Q is 60 mm. Determine inclinations of PQ with the H.P. and the V.P.*

Refer to fig. 10-57.

(i) Draw the end projectors of the line PQ 65 mm apart. Mark the projection of ends P and Q according to given distances. $p'q'$ and pq are the front view and the top view respectively.

(ii) Mark the front view of point c in the line xy because it is lying in the H.P. Extend the projection line from c' to c on the top view pq.

(iii) Keeping c fixed, turn cp to cp_2 making parallel to xy.

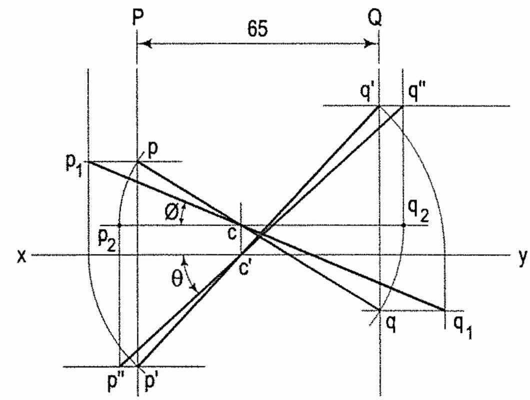


FIG. 10-57

(iv) Project p_2 to p''. Join $c'p''$. This is the true distance of the line cp. Similarly turn cq to cq_2 making it parallel to xy. Project q_2 to q'' join $c'q''$. This represents the true distance of the line cq.

(v) Measure angles θ and $\varnothing$ as shown.

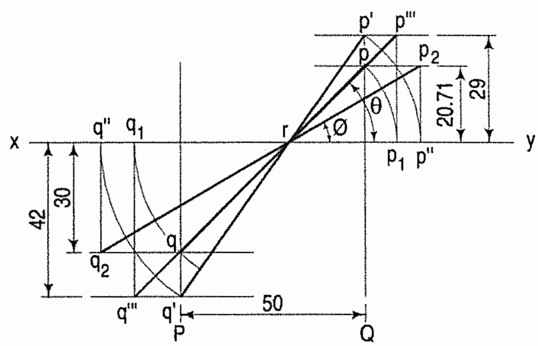
Problem 10-33. (fig. 10-58): *The distance between the end-projectors of a line PQ is 50 mm. A point P is 29 mm above H.P. and 20.71 mm behind V.P. While a point Q is 42 mm below H.P. and 30 mm in front of V.P.*

Draw the projections of the line and determine the true length and the true inclinations of line with H.P. and V.P.

(i) Draw the end-projectors 50 mm apart. Mark p' and p, the front view and the top view of the end P at 29 mm and 20.71 mm respectively. Similarly mark q and q' at 42 mm and 30 mm on the end-projector Q as shown. Join $p'q'$ and pq. They are intersecting xy at r. Mark paths of p', q', p and q parallel to xy.

(ii) With centre r and radius rp', draw an arc intersecting xy at p''. Through p'', draw projector cutting the path of p at p_2. Similarly with the same centre and radius rq', draw an arc intersecting xy at q''. Through q'', draw projector cutting the path of q at q_2. Join p_2q_2 which represents true length. Measure $\varnothing$ angle made by p_2q_2 with xy at r. This is an angle made by the line with V.P.

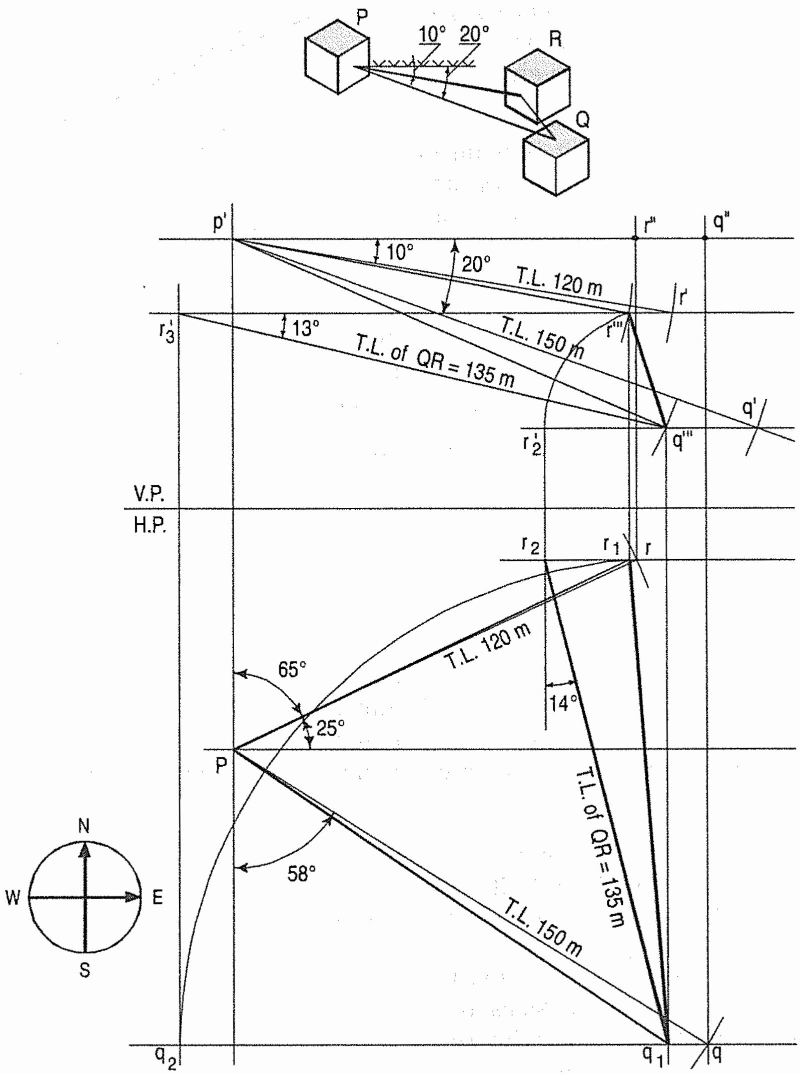
(iii) Similarly centre as r and radii rp and rq, draw arcs intersecting xy at p_1 and q_1 respectively. Through p_1 and q_1 draw projectors to intersect the paths of p' and q' at p''' and q'''. Join $p'''q'''$. Measure θ angle made by it with xy. This is an angle by line with H.P.



$\angle\theta = 45°$; $\angle\varnothing = 30°$,
true length = 100 mm

FIG. 10-58

Problem 10-34. (fig. 10-59): *Two pipes emerge from a common tank. The pipe PQ is 150 metre long and bears S 58° E on a downward slope of 20°. The pipe PR is 120 metre long and bears N 65° E on a downward slope of 10°. Determine the length of pipe required to connect Q and R. Take scale 1 mm = 1 m.*



T.L. of QR = 135 m. S 13 E at downward slope of 13°

FIG. 10-59

(i) Mark position of *P* in the top view and the front view as shown.

(ii) At *P*, draw a line *pq* 150 mm at 58° with the vertical measuring anti-clockwise as the angle is required to measure from south to east. Similarly draw a line *pr* 120 mm at 25° measured from east to north.

(iii) Draw the horizontal lines from *q* and *r*.

(iv) From point *p*', draw a line *p*'*q*' 150 mm and *p*'*r*' 120 mm at 20° and 10° with the horizontal lines. Draw horizontal lines from *r*' and *q*'.

(v) Draw a projector from q to intersect the horizontal line drawn from p' at q''. $p'q''$ is the front view of PQ. With p' as centre and radius equal to $p'q''$, draw an arc intersecting the path of q' at q'''. Join $p'q'''$ and draw a projector from q''' cutting the path of q at q_1. Then $p'q'''$ and pq_1 are required projection. Similarly obtain the projection of $p'r'''$ and pr_1 for the line PR as shown. Join $r'''q'''$ and r_1q_1. With centre q''' and a radius $q'''r'''$, draw an arc intersecting the path of q''' at r'_2. Draw a projector from r'_2 cutting the path of r at r_2. Join q_1r_2 and measure its true length and angle.

(vi) With centre q_1 and radius q_1r_1, draw an arc intersecting the path of q at q_2. Draw a projector from q_2 cutting the path of r' at r'_3. Join $q'''r'_3$ and measure its true length and angle.

(vii) QR = 135 m is the measured true length and S 13 E at downward slope of 13° is the measured angle.

Note: Depression or front view angles are seen in front view while bearing angles are seen in top view.

Problem 10-35. (fig. 10-60): *The projectors of two points P and Q are 70 mm apart. The point P is 25 mm behind the V.P. and 30 mm below the H.P. The point Q is 40 mm above the H.P. and 15 mm infront of the V.P. Find the third point S which is in the H.P. and infront of the V.P. such that its distance from point P is 90 mm and that from Q is 60 mm.*

(i) Draw xy line.

(ii) Draw the projectors of P and Q 70 mm apart.

(iii) Mark on the projector of the point p the front view and top view of the point p at 25 mm and 30 mm from xy respectively, say p' and p. Similarly on the projector of the point Q, mark the front view q' and the top view q for given distance from the xy.

(iv) Join $p'q'$ and pq. They are the front view and the top view of the line PQ.

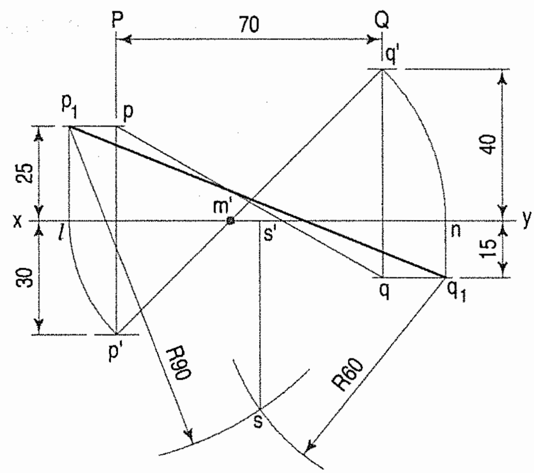


FIG. 10-60

(v) The front view $p'q'$ intersects xy line at m'. Taking m' as centre, $m'p'$ and $m'q'$ as radii rotate so that $p'q'$ becomes parallel to the xy line or intersect xy line at l and n.

(vi) Draw the projectors from l and n to intersect path of the point p and the point q at p_1 and q_1. Join p_1 and q_1, it represents true length of the line PQ.

(vii) Now p_1 as centre and 90 mm radius, draw the arc. Take q_1 as centre and 60 mm as radius, draw the another arc so that it intersects previous arc at s. From s draw the projector to intersect xy line at s', which is front view of s.

Problem 10-36. (fig. 10-61): *Two unequal lines PQ and PR meeting at P makes an angle of 130° between them in their front view and top view. Line PQ is parallel and 6 mm away from both the principal planes. Assume the front view length of PQ and PR 50 mm and 60 mm respectively. Determine real angle between them.*

(i) Mark reference line *xy*. Draw at convinent distance above and below two parallel lines to the *xy* representing the front view and top view of *PQ* as shown. At the point *P'* construct angle of 130° with the length of *p'q'* and *p'r'* of 50 mm and 60 mm respectively.

(ii) Draw projector from *r'* to intersect the line at angle of 130° at *p* in the top view of *pq*.

(iii) Make *pr* and *qr* parallel to *xy* line and project above *xy* to intersect the path of *r'* at *r''* and *r'''* respectively.

Join *p'r'''* and *q'r''*. They are true length of the line *PR* and the line *QR*.

(iv) Construct triangle with sides *PQ* = 50 mm, *PR* = *p'r'''* and *QR* = *q'r''* as shown. Measure angle ∠*QPR* equal and it is 120° approximately.

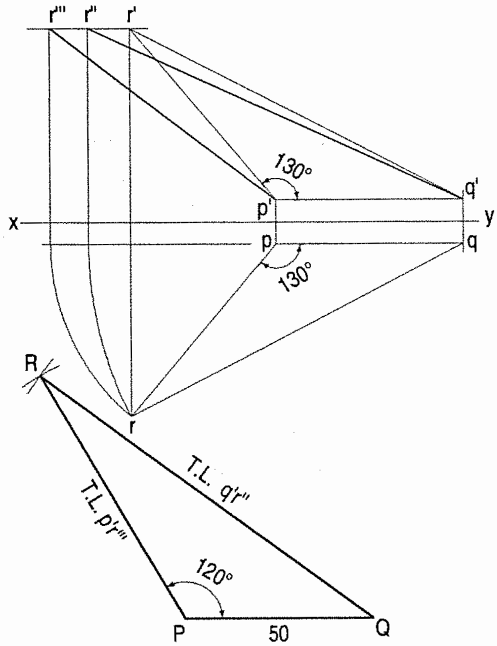


FIG. 10-61

Problem 10-37. (fig. 10-62): *The distance between end projectors of a straight line AB is 80 mm. The point A is 15 mm below the H.P. and 20 mm infront of the V.P. B is 60 mm behind the V.P. Draw projections of the line if it is inclined at 45° to V.P. Determine also true length and inclination with the H.P.*

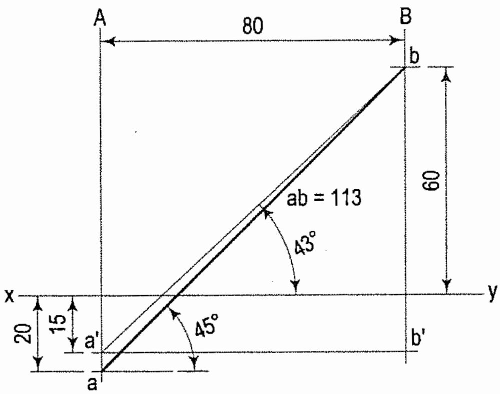


FIG. 10-62

(i) Draw *xy* line and two vertical parallel lines 80 mm apart showing the end projectors of the line *AB*.

(ii) Mark the point a' and the point a 15 mm and 20 mm below *xy* line on the projector of *A*.

(iii) Mark the point *b* above *xy* line at distance of 60 mm on the projector of *B*. Join the point *a* and the point *b*.

(iv) If we measure angle of *ab* with *xy* line it is 45° which is also inclination of the line *AB* with V.P.

(v) Therefore *ab* shows true length. From *a'* draw parallel line to *xy* to intersect the projector of *B* at *b'*. It is front view of the line *AB*. Measure angle of a'*b* with *xy* line. It is 43° with *xy* line.

Problem 10-38. (fig.10-63): *Two mangoes on a tree are respectively 2.0 m and 3.5 m above the ground and 1.5 m and 2.0 m away from 0.2 m thick compound wall, but on the opposite sides of it. The distance between the mangoes, measured along the ground and parallel to the wall is 2.7 m. Determine the real distance between the mangoes. Take scale 1 m = 10 mm.*

(i) Pictorial view is shown for understanding purpose.

(ii) Draw reference line (ground line) *xy*. Mark two parallel lines as end-projectors at 2.7 m (27 mm) apart.

(iii) Let *P* be mango behind the wall and *Q* be infront of the wall.

(iv) Mark *P'* and *P* along projector *P* to given distances. Similarly mark *q'* and *q* for given distances on the projector *Q*.

(v) Join the point *p* and the point *q*, the point *p'* and the point *q'*. Then *pq* and *p'q'* are projections of *PQ*.

(vi) Rotates *pq* taking *q* as centre, make it parallel to the ground line *xy*, intersecting at the poin P_1. Draw the projector from p_1. Draw line parallel to the ground line *xy* from *p'*, intersecting at *p''*.

(vii) Join *p''q'*. The line *p''q'* is true distance between mangoes *P* and *Q*. It is approximately 4.8 m.

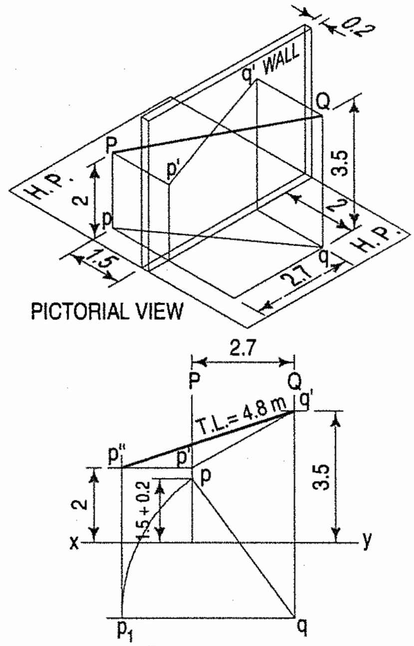


FIG. 10-63

Problem 10-39. (fig. 10-64): *The front view of straight line AB is 60 mm long and is inclined at 60° to the reference line xy. The end point A is 15 mm above H.P. and 20 mm infront of V.P. Draw the projections of a line AB if it is inclined at 45° to the V.P. and is situated in the first quadrant (Dihedral angle). Determine its true length, and inclination with the H.P.*

See fig. 10-64 which is self explanatory.

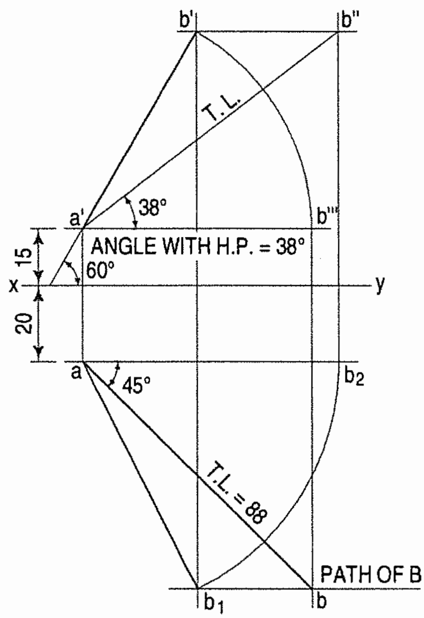


FIG. 10-64

Problem 10-40. (fig. 10-65): *A room 6 m × 5 m × 4 m high has a light-bracket above the centre of the longer wall and 1 m below the ceiling. The light bulb is 0.3 m away from the wall. The switch for the light is on an adjacent wall, 1.5 m above the floor and 1 m from the other longer wall. Determine graphically the shortest distance between the bulb and the switch.*

(i) Draw to scale 1 m = 10 mm front view and a top view of the room.

(ii) Mark mid point of longer wall (i.e. 6 m), say 6 mm.

(iii) Mark the point in the V.P. at distance 1 m (= 10 mm) from midpoint *m*. It is the front view of the light bulb, say *b*'.

(iv) From *xy* line at 0.3 m (i.e. 3 mm) on the projector passing through *b*' mark the point *b* (top view).

(v) Consider adjacent wall right side. Mark *s*' front view of the switch at 1.5 m (i.e. 15 mm) above *xy* line and on the same line mark *s* (top view of switch) at 4 m away from *xy*.

(vi) Join *b*'*s*' and *bs* which are the front view and the top view of the line.

(vii) *s* as centre and *bs* as radius, draw the arc to intersect a parallel line passing through *s* at b_1. From b_1, draw projector to intersect a parallel line to *xy* drawn from *b*' at *b*''. Join *s*'*b*''.

(viii) Line *s*'*b*'' shows shortest distance between the switch and the bulb. Here it is approximately 5 m.

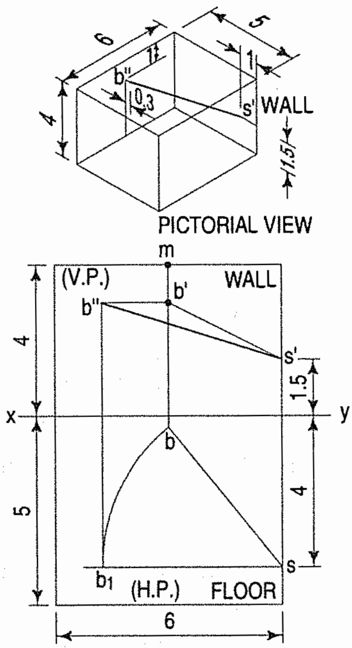
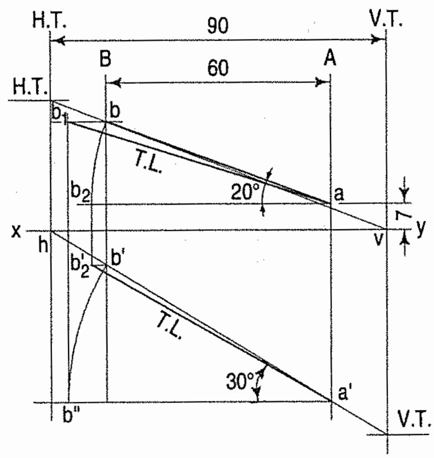


FIG. 10-65

Problem 10-41. (fig. 10-66): *The distance between end projectors of a line AB are 60 mm apart, while the projectors passing from H.T. and V.T. are 90 mm apart. The H.T. is 35 mm behind the V.P., the V.T. is 55 mm below the H.P. The point A is 7 mm behind the V.P. Find graphically true length of the line and inclinations with the H.P. and the V.P.*

See fig. 10-66 which is self-explanatory.



True length = 74;
Angle with H.P. = 30°;
Angle with V.P. = 20°
FIG. 10-66

Problem 10-42. (fig. 10-67): *A line CD is inclined at 30° to the H.P. and it is in the first quadrant. The end C is 15 mm above the H.P. while the end D is in the V.P. The mid point M of of the line is 40 mm above H.P. The distance between the end projectors of the line is 70 mm. Draw the projections of the line CD and the mid point M. Determine graphically the length of front view and top view and true length of the line. Also determine inclination of the line with the V.P.*

(i) Draw xy line.

(ii) Draw two projectors 70 mm, apart.

(iii) On the projector of *C*, mark *c*' at 15 mm above *xy* line.

(iv) Draw a line parallel to *xy* at 40 mm, to represent the path of mid-point *M*.

(v) From *c*' draw a 30° inclined line t cut the path of mid-point at *m*'. *c*'*m*' is half true length. With *m*' as centre and radius equal to *c*'*m*', draw an arc cutting the 30° inclined line at *d*'. *c*'*d*' is true length. From *d*', draw a line parallel to *xy*, to represent path of *D* in front view.

(vi) The path of D will intersect the projector of D at d''. Join $c'd''$. It is front view of CD.

(vii) With c' as centre and radius equal to $c'd''$, draw an arc cutting a line drawn parallel to xy from c' at d'''. Project d''' to cut a line drawn with c as centre and $c'd'$ (true length) as radius, at d_3. From d_3, draw a line parallel to xy, represssenting path of D in top view.

(viii) Project d'' to cut path of D in top view at d_4. Join cd_4. It is top view of CD.

(ix) The results are shown in fig. 10-67.

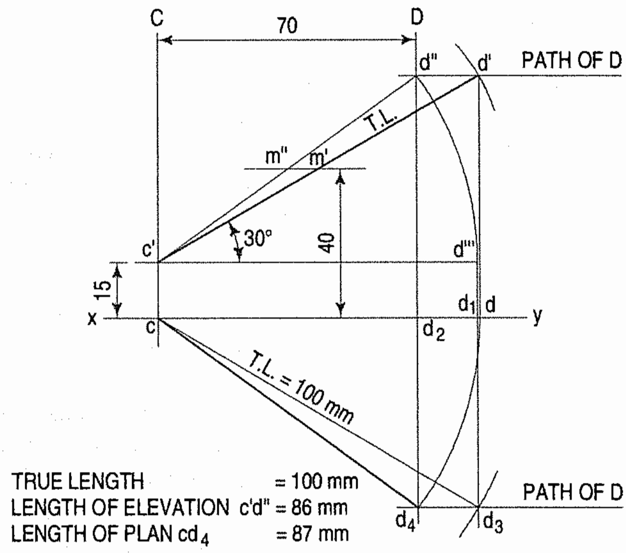


FIG. 10-67

EXERCISES 10(b)

1. A line AB, 75 mm long, is inclined at 45° to the H.P. and 30° to the V.P. Its end B is in the H.P. and 40 mm in front of the V.P. Draw its projections and determine its traces.
2. Draw the projections of a line AB, 90 mm long, its mid-point M being 50 mm above the H.P. and 40 mm in front of the V.P. The end A is 20 mm above the H.P. and 10 mm in front of the V.P. Show the traces and the inclinations of the line with the H.P. and the V.P.
3. The front view of a 125 mm long line PQ measures 75 mm and its top view measures 100 mm. Its end Q and the mid-point M are in the first quadrant, M being 20 mm from both the planes. Draw the projections of the line PQ.
4. A line AB, 75 mm long is in the second quadrant with the end A in the H.P. and the end B in the V.P. The line is inclined at 30° to the H.P. and at 45° to the V.P. Draw the projections of AB and determine its traces.
5. The end A of a line AB is in the H.P. and 25 mm behind the V.P. The end B is in the V.P. and 50 mm above the H.P. The distance between the end projectors is 75 mm. Draw the projections of AB and determine its true length, traces and inclinations with the two planes.
6. The top view of a 75 mm long line CD measures 50 mm. C is 50 mm in front of the V.P. and 15 mm below the H.P. D is 15 mm in front of the V.P. and is above the H.P. Draw the front view of CD and find its inclinations with the H.P. and the V.P. Show also its traces.
7. A line PQ, 100 mm long, is inclined at 45° to the H.P. and at 30° to the V.P. Its end P is in the second quadrant and Q is in the fourth quadrant. A point R on PQ, 40 mm from P is in both the planes. Draw the projections of PQ.
8. A line AB, 65 mm long, has its end A in the H.P. and 15 mm in front of the V.P. The end B is in the third quadrant. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.

9. The front view of a line *AB* measures 65 mm and makes an angle of 45° with *xy*. *A* is in the H.P. and the V.T. of the line is 15 mm below the H.P. The line is inclined at 30° to the V.P. Draw the projections of *AB* and find its true length and inclination with the H.P. Also locate its H.T.
10. A room is 4.8 m × 4.2 m × 3.6 m high. Determine graphically the distance between a top corner and the bottom corner diagonally opposite to it.
11. A line *AB* is in the first quadrant. Its end *A* and *B* are 20 mm and 60 mm in front of the V.P. respectively. The distance between the end projectors is 75 mm. The line is inclined at 30° to the H.P. and its H.T. is 10 mm above *xy*. Draw the projections of *AB* and determine its true length and the V.T.
12. Two oranges on a tree are respectively 1.8 m and 3 m above the ground, and 1.2 m and 2.1 m from a 0.3 m thick wall, but on the opposite sides of it. The distance between the oranges, measured along the ground and parallel to the wall is 2.7 m. Determine the real distance between the oranges.
13. Draw an isosceles triangle *abc* of base *ab* 40 mm and altitude 75 mm with *a* in *xy* and *ab* inclined at 45° to *xy*. The figure is the top view of a triangle whose corners *A*, *B* and *C* are respectively 75 mm, 25 mm and 50 mm above the H.P. Determine the true shape of the triangle and the inclination of the side *AB* with the two planes.
14. Three points *A*, *B* and *C* are 7.5 m above the ground level, on the ground level and 9 m below the ground level respectively. They are connected by roads with each other and are seen at angles of depression of 10°, 15° and 30° respectively from a point *O* on a hill 30 m above the ground level. A is due north-east, *B* is due north and *C* is due south-east of *O*. Find the lengths of the connecting roads.
15. A pipe-line from a point *A*, running due north-east has a downward gradient of 1 in 5. Another point *B* is 12 m away from and due east of *A* and on the same level. Find the length and slope of a pipe-line from *B* which runs due 15° east of north and meets the pipe-line from *A*.
16. The guy ropes of two poles 12 m apart, are attached to a point 15 m above the ground on the corner of a building. The points of attachment on the poles are 7.5 m and 4.5 m above the ground and the ropes make 45° and 30° respectively with the ground. Draw the projections and find the distances of the poles from the building and the lengths of the guy ropes.
17. A plate chimney, 18 m high 0.9 m diameter is supported by two sets of three guy wires each, as shown in fig. 10-68.

    One set is attached at 3 m from the top and anchored 6 m above the ground level. The other set is fixed to the chimney at its mid-height and anchored on the ground. Determine the length and slope with the ground, of one of the wires from each set.

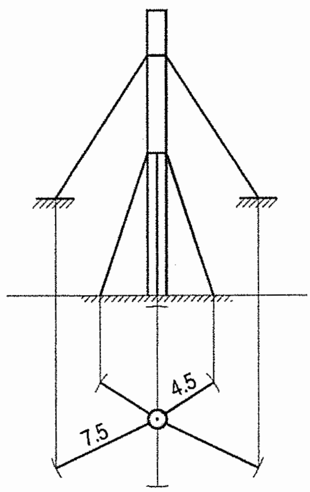


FIG. 10-68

18. The projectors drawn from the H.T. and the V.T. of a straight line *AB* are 80 mm apart while those drawn from its ends are 50 mm apart. The H.T. is 35 mm in front of the V.P., the V.T. is 55 mm above the H.P. and the end *A* is 10 mm above the H.P. Draw the projections of *AB* and determine its length and inclinations with the reference planes.

19. Three guy ropes *AB*, *CD* and *EF* are tied at points *A*, *C* and *E* on a vertical post 15 m long at heights of 14 m, 12 m and 10 m respectively from the ground. The lower ends of the ropes are tied to hooks at points *B*, *D* and *F* on the ground level. If the points *B*, *D* and *F* lie at the corners of an equilateral triangle of 9 m long sides and if the post is situated at the centre of this triangle, determine graphically the length of each rope and its inclination with the ground. Assume the thickness of the post and the ropes to be equal to that of a line.

20. A line *AB*, 80 mm long, makes an angle of 60° with the H.P. and lies in an auxiliary vertical plane (A.V.P.), which makes an angle of 45° with the V.P. Its end *A* is 10 mm away from both the H.P. and the V.P.

    Draw the projections of *AB* and determine (i) its true inclination with the V.P. and (ii) its traces.

21. A line *PQ* is 75 mm long and lies in an auxiliary inclined plane (A.I.P.) which makes an angle of 45° with the H.P. The front view of the line measures 55 mm and the end *P* is in the V.P. and 20 mm above the H.P.

    Draw the projections of *PQ* and find (i) its inclinations with both the planes and (ii) its traces.

22. A line *AB*, 80 mm long, makes an angle of 30° with the V.P. and lies in a plane perpendicular to both the H.P. and the V.P. Its end *A* is in the H.P. and the end *B* is in the V.P. Draw its projections and show its traces.

23. The front view of a line makes an angle of 30° with xy. The H.T. of the line is 45 mm in front of the V.P., while its V.T. is 30 mm below the H.P. One end of the line is 10 mm above the H.P. and the other end is 100 mm in front of the V.P.

    Draw the projections of the line and determine (i) its true length, and (ii) its inclinations with the H.P. and the V.P.

24. A room is 6 m × 5 m × 3.5 m high. An electric bracket light is above the centre of the longer wall and 1 m below the ceiling. The bulb is 0.3 m away from the wall. The switch for the light is on an adjacent wall, 1.5 m above the floor and 1 m away from the other longer wall. Find graphically the shortest distance between the bulb and the switch.

25. Three lines *oa*, *ob* and *oc* are respectively 25 mm, 45 mm and 65 mm long, each making 120° angles with the other two and the shortest line being vertical. The figure is the top view of the three rods *OA*, *OB* and *OC* whose ends *A*, *B* and *C* are on the ground, while *O* is 100 mm above it. Draw the front view and determine the length of each rod and its inclination with the ground.

26. The projectors of the ends of a line *PQ* are 90 mm apart. *P* is 20 mm above the H.P. while *Q* is 45 mm behind the V.P. The H.T. and the V.T. of the line coincide with each other on *xy*, between the two end projectors and 35 mm away from the projector of the end *P*. Draw the projections of *PQ* and determine its true length and inclinations with the two planes.

27. A person on the top of a tower 30 m high, which rises from a horizontal plane, observes the angles of depression (below the horizon) of two objects *H* and *K* on the plane to be 15° and 25°, the direction of *H* and *K* from the tower being due north and due west respectively. Draw the top view to a scale of 1 mm = 0.5 m showing the relative positions of the person and the two objects. Measure and state in metres the distance between *H* and *K*.

28. Two pegs A and B are fixed in each of the two adjacent side walls (of a rectangular room) which meet in a corner. Peg A is 1.5 m above the floor, 1.2 m from the side wall and is protruding 0.3 m from the wall. Peg B is 2 m above the floor, 1 m from the other side wall and is also protruding 0.3 m from the wall. Find the distance between the ends of the pegs.
29. Two objects A and B, 10 m above and 7 m below the ground level respectively, are observed from the top of a tower 35 m high from the ground. Both the objects make an angle of depression of 45° with the horizon. The horizontal distance between A and B is 20 m. Draw to scale 1:250, the projections of the objects and the tower and find (a) the true distance between A and B, and (b) the angle of depression of another object C situated on the ground midway between A and B.
30. A room measures 8 m long, 5 m wide and 4 m high. An electric point hangs in the centre of the ceiling and 1 m below it. A thin straight wire connects the point to a switch kept in one of the corners of the room and 2 m above the floor. Draw the projections of the wire, and find the length of the wire and its slope-angle with the floor.
31. A rectangular tank 4 m high is strengthened by four stay rods one at each corner, connecting the top corner to a point in the bottom 0.7 m and 1.2 m from the sides of the tank. Find graphically the length of the rod required and the angle it makes with the surface of the tank.
32. Three vertical poles AB, CD and EF are respectively 5, 8 and 12 metres long. Their ends B, D and F are on the ground and lie at the corners of an equilateral triangle of 10 metres long sides. Determine graphically the distance between the top ends of the poles, viz. AC, CE and EA.
33. The front view of a line AB measures 70 mm and makes an angle of 45° with xy. A is in the H.P. and the V.T. of the line is 15 mm below the H.P. The line is inclined at 30° to the V.P. Draw the projections of AB, and find its true length, inclination with the H.P. and its H.T.
34. A line AB measures 100 mm. The projectors through its V.T. and the end A are 40 mm apart. The point A is 30 mm below the H.P. and 20 mm behind the V.P. The V.T. is 10 mm above the H.P. Draw the projections of the line and determine its H.T. and inclinations with the H.P. and the V.P.
35. A horizontal wooden platform is 3.5 m long and 2 m wide. It is suspended from a hook by means of chains attached at its four corners. The hook is situated vertically above the centre of the platform and at a distance of 5 m above it. Determine graphically the length of each chain and the angle which it makes with the platform. Assume the thickness of the platform and the chain to be equal to that of a line. Scale: 10 mm = 0.5 m.
36. A picture frame 2 m wide and 1 m high is to be fixed on a wall railing by two straight wires attached to the top corners. The frame is to make an angle of 40° with the wall and the wires are to be fixed to a hook on the wall on the centre line of the frame and 1.5 m above the railing. Find the length of the wires and the angle between them.
37. The top view of line AB measures 60 mm and inclined to reference line at 60°. The end point A is 15 mm above the H.P. and 30 mm infront of the V.P. Draw the projections of the line when it is inclined at 45° to the H.P. and is situated in the first quadrant. Find true length and inclination of the line with the V.P. and traces.

Chapter 11

PROJECTIONS ON AUXILIARY PLANES

11-0. INTRODUCTION

Two views of an object, viz. the front view and the top view (projected on the principal planes of projection), are sometimes not sufficient to convey all the information regarding the object. Additional views, called *auxiliary views*, are therefore, projected on other planes known as *auxiliary planes*. These views are often found necessary in technical drawings. Auxiliary views may also be used for determining

(i) the true length of a line,

(ii) the point-view of a line,

(iii) the edge-view of a plane,

(iv) the true size and form of a plane etc.

They are thus very useful in finding solutions of problems in practical solid geometry.

This chapter deals with the following topics:

1. Types of auxiliary planes and views.
2. Projection of a point on an auxiliary plane.
3. Projections of lines and planes by the use of auxiliary planes.
4. To determine true length of a line.
5. To obtain point-view of a line and edge-view of a plane.
6. To determine true shape of a plane figure.

11-1. TYPES OF AUXILIARY PLANES AND VIEWS

Auxiliary planes are of *two* types:

(i) *auxiliary vertical plane* or A.V.P., and

(ii) *auxiliary inclined plane* or A.I.P.

(i) Auxiliary vertical plane is perpendicular to the H.P. and inclined to the V.P. Projection on an A.V.P. is called *auxiliary front view*.

(ii) Auxiliary inclined plane is perpendicular to the V.P. and inclined to the H.P. Projection on an A.I.P. is called *auxiliary top view*.

The orthographic views of the auxiliary projections are drawn by rotating the auxiliary plane about that principal plane to which it is perpendicular.

11-2. PROJECTION OF A POINT ON AN AUXILIARY PLANE

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 25 for the projection of a point on an auxiliary plane.

(1) Projection of a point on an auxiliary vertical plane:

A point A [fig. 11-1(i)] is situated in front of the V.P. and above the H.P. A.V.P. is the auxiliary vertical plane inclined at an angle α to the V.P. The H.P. and the A.V.P. meet at right angles in the line x_1y_1.

a' and a are respectively, the front view and the top view of the point A. a'_1 is the auxiliary front view obtained by drawing a projector Aa'_1 perpendicular to the A.V.P. It can be clearly seen that a'_1o_1 (the distance of the auxiliary front view a'_1 from x_1y_1) = $a'o$ (the distance of the front view a' from xy) = Aa (the distance of the point A from the H.P.).

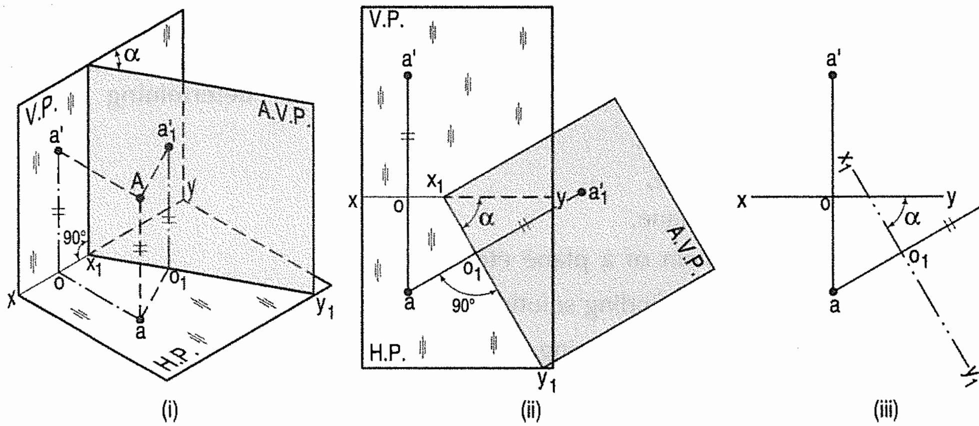


FIG. 11-1

Fig. 11-1(ii) shows the V.P. and the A.V.P. rotated about the H.P. to which they are perpendicular. The line of intersection x_1y_1 between the A.V.P. and the H.P., is inclined at the angles α to xy. The line joining the top view a with the auxiliary front view a'_1, is at right angles to x_1y_1 and intersects it at o_1. Note that $a'_1o_1 = a'o$.

To draw the orthographic views [fig. 11-1(iii)], start with the reference line xy and mark the front view a' and the top view a.

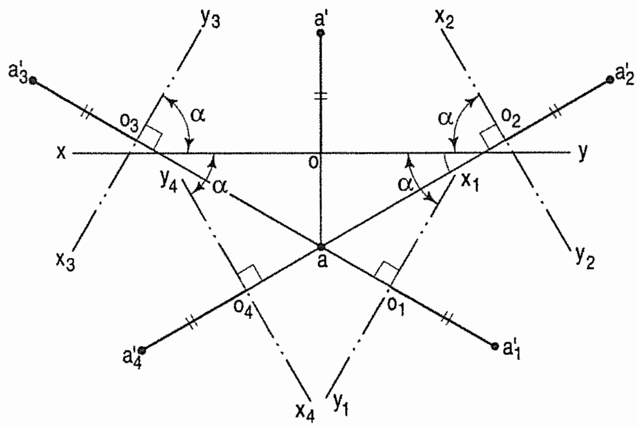


FIG. 11-2

Draw a new reference line x_1y_1, making the angle α with xy. Through the top view a, draw a projector aa'_1 perpendicular to and intersecting x_1y_1 at o_1 and such that $a'_1o_1 = a'o$. a'_1 is the required auxiliary front view.

The new reference line making the angle α with xy, can be drawn in four different positions, as shown in fig. 11-2 by lines x_1y_1, x_2y_2 etc. All the front views are projected from the top view a and their distances from their respective reference lines are equal, i.e. $a'_1o_1 = a'_2o_2..... = a'o$.

(2) Projection of a point on an auxiliary inclined plane:

A point P [fig. 11-3(i)] is situated above the H.P. and in front of the V.P. A.I.P. is an auxiliary inclined plane making an angle β with the H.P. It meets the V.P. at right angles and in a line x_1y_1.

p' and p are respectively the front view and the top view of the point P. p_1 is the auxiliary top view obtained by drawing the projector Pp_1 perpendicular to the A.I.P. It can be seen that p_1o_1 (the distance of the auxiliary top view p_1 from x_1y_1) = po (the distance of the top view p from xy) = Pp' (the distance of the point P from the V.P.).

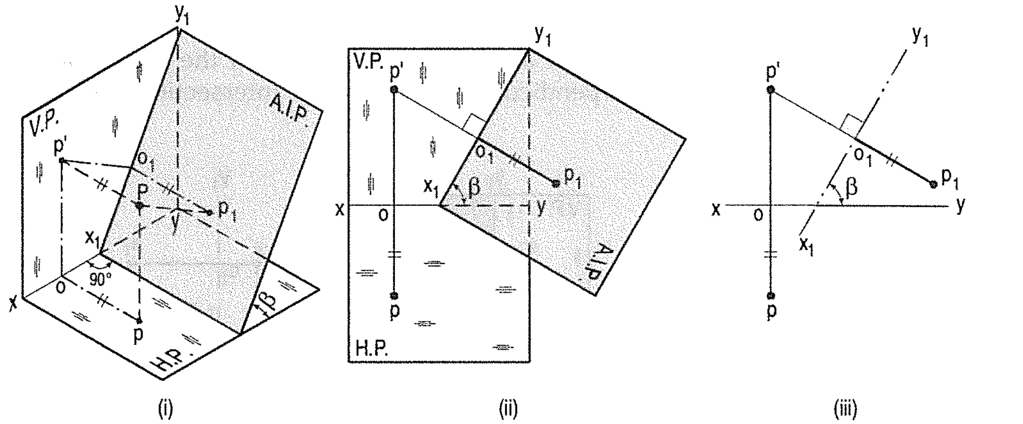


FIG. 11-3

The H.P. and the A.I.P. are then rotated about the V.P. to which they are perpendicular [fig. 11-3(ii)]. x_1y_1, the line of intersection between the V.P. and the A.I.P. makes the angle β with xy. The line joining the front view p' and the auxiliary top view p_1 is at right angles to x_1y_1 and intersects it at o_1. Note that $p_1o_1 = po$.

To draw the orthographic views [fig. 11-3(iii)], draw xy and mark p' and p. Draw x_1y_1 making the angle β with xy.

Through the front view p', draw a projector $p'p_1$ perpendicular to and intersecting x_1y_1 at o_1 and such that $p_1o_1 = po$. p_1 is the required auxiliary front view.

In this case also, the new reference line can be drawn in four different positions as shown in fig. 11-4 by lines x_1y_1, x_2y_2 etc., each inclined at β to xy. All the top views are projected from the front view p' and their distances from their respective reference lines are equal, i.e. $p_1o_1 = p_2o_2..... = po$.

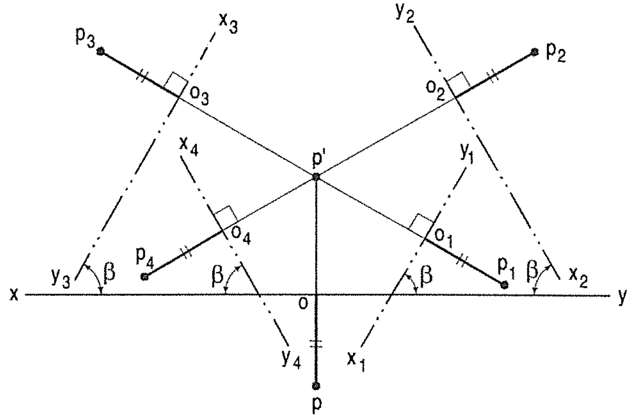


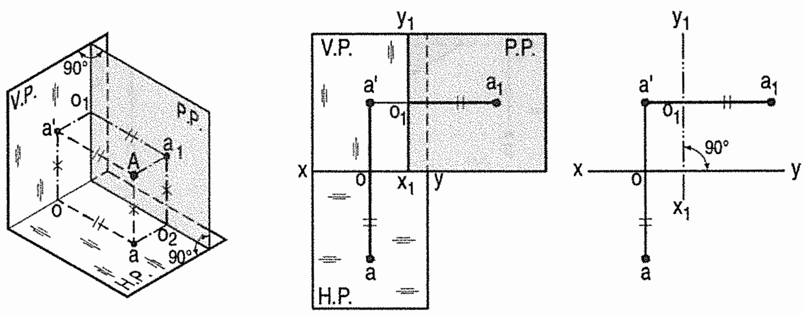
FIG. 11-4

If the inclination of the A.V.P. with the V.P. is increased so that $\alpha = 90°$, the A.V.P. will be perpendicular to both the planes. Similarly, if the inclination of the A.I.P. with the H.P. is increased so that $\beta = 90°$, it will also be perpendicular to both the H.P. and the V.P. *This plane is called the profile plane* (P.P.). It may be rotated about any one of the two principal planes. The view on this plane can, therefore, be projected from either the top view or the front view and named accordingly.

(3) Projection of a point on an auxiliary plane perpendicular to both the principal planes:

In fig. 11-5(i), *A* is a point. P.P. is an auxiliary plane perpendicular to the H.P. and the V.P. a_1 is the auxiliary view projected on the P.P. It can be seen that $a_1o_2 = a'o = Aa$ (the distance of *A* from the H.P.). Also $a_1o_1 = ao = Aa'$ (the distance of *A* from the V.P.).

Fig. 11-5(ii) shows the P.P. rotated about the V.P. a_1 lies on the projector drawn through the front view a' and perpendicular to the line of intersection between the V.P. and the P.P.

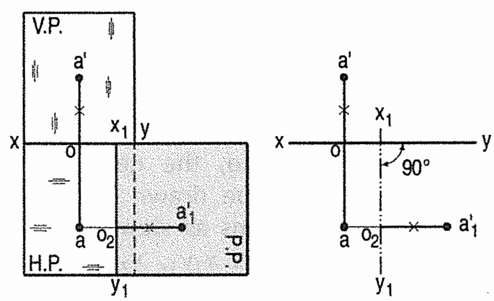


(i) (ii) (iii)

FIG. 11-5

It is thus projected from the front view and hence, called the auxiliary top view. In technical drawings, this view is generally termed as the *side view, end view* or *end front view*. Note that $a_1o_1 = ao$.

Note that, when seen from the left, the new reference line and the side view are placed to the right of the front view. When seen from the right, they would be placed to the left of the front view. Thus, the view seen from any side of the front view is placed on its other side.



(i) (ii)

FIG. 11-6

Fig. 11-6 shows the P.P. rotated about the H.P. The view on the P.P. now lies on the projector drawn through the top view *a*. Hence, it is called the auxiliary front view. In this case, $a'_1o_2 = a'o$.

The orthographic views [fig. 11-5(iii) and fig. 11-6(ii)] in both cases are self-explanatory.

General conclusions:

(i) *The auxiliary top view* of a point lies on a line drawn through the front view, perpendicular to the new reference line (x_1y_1) and at a distance from it, equal to the distance of the first top view from its own reference line (xy).

(ii) *The auxiliary front view* of a point lies on a line drawn through the top view, perpendicular to the new reference line (x_1y_1) and at a distance from it, equal to the distance of the first front view from its own reference line (xy).

(iii) The distances of all the front views of the same point (projected from the same top view) from their respective reference lines are equal.

(iv) The distances of all the top views of the same point (projected from the same front view) from their respective reference lines are equal.

Problem 11-1. (fig. 11-7 and fig. 11-8): *The projections of a line AB are given. Draw (i) an auxiliary front view of the line on an A.V.P. inclined at 60° to the V.P. and (ii) an auxiliary top view on an A.I.P. making an angle of 75° with the H.P.*

Let *ab* and *a'b'* be the given projections.

(i) Draw a new reference line x_1y_1, inclined at 60° to xy to represent the A.V.P. (fig. 11-7).

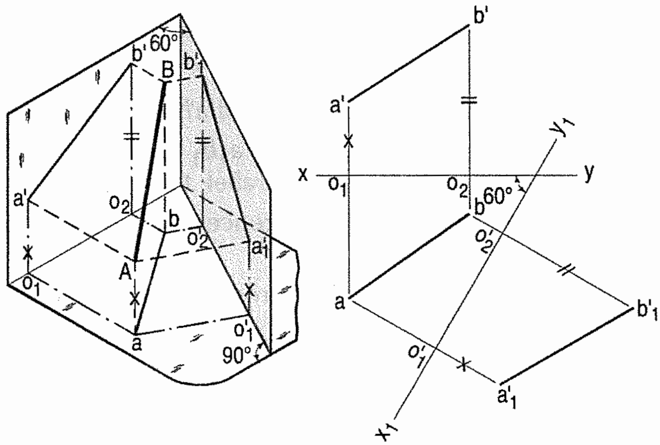


FIG. 11-7

(ii) Project the auxiliary front view $a'_1b'_1$ from the top view *ab*, by making $a'_1o'_1$ equal to $a'o_1$, and $b'_1o'_2$ equal to $b'o_2$.

(iii) Similarly, draw x_2y_2 for the A.I.P. inclined at 75° to xy (fig. 11-8).

(iv) Project the auxiliary top view a_2b_2 from the front view $a'b'$, making $a_2o''_1$ equal to ao_1 and $b_2o''_2$ equal to bo_2.

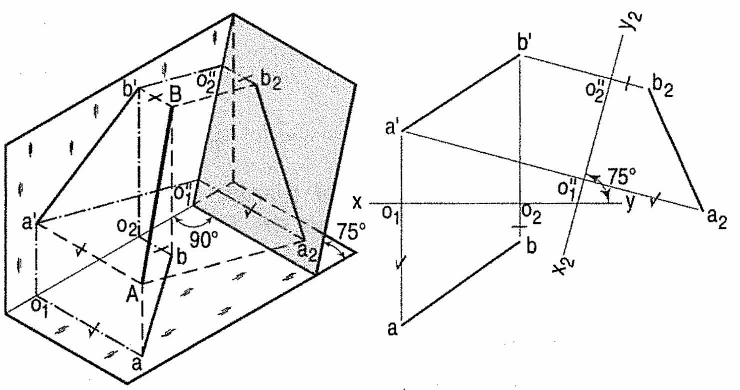


FIG. 11-8

11-3. PROJECTIONS OF LINES AND PLANES BY THE USE OF AUXILIARY PLANES

Projections of lines and planes at given inclinations to one or both the planes may also be obtained by the use of auxiliary planes. The method adopted is called the *alteration or change-of-reference-line method.*

The line, in its initial position, is assumed to be parallel to both the planes of projection. Then, instead of making the line inclined to one of the planes, an auxiliary plane inclined to the line is assumed, i.e. a new reference line is drawn and the view is projected on it.

In case of a plane, it is kept parallel to one of the planes of projection in the initial stage, the required views being obtained by projecting it on new reference lines.

Problem 11-2. (fig. 11-9): *A line AB, 50 mm long, is inclined at 30° to the H.P. and its top view makes an angle of 60° with the V.P. Draw its projections.*

(i) Draw the projections *ab* and *a'b'*, assuming *AB* to be parallel to both the planes.

(ii) Project a new top view a_1b_1 on a new reference line x_1y_1 inclined at 30° to *a'b'*. a_1b_1 is still parallel to x_1y_1.

(iii) Draw the another reference line x_2y_2 to represent an A.V.P. inclined at 60° to the top view a_1b_1.

(iv) Project the required front view $a'_1b'_1$ on x_2y_2. Note that $a_1o_1 = ao$, $a'_1o_2 = a'o_1$ etc.

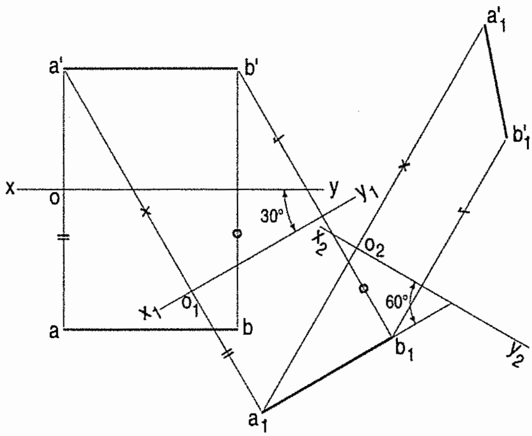


FIG. 11-9

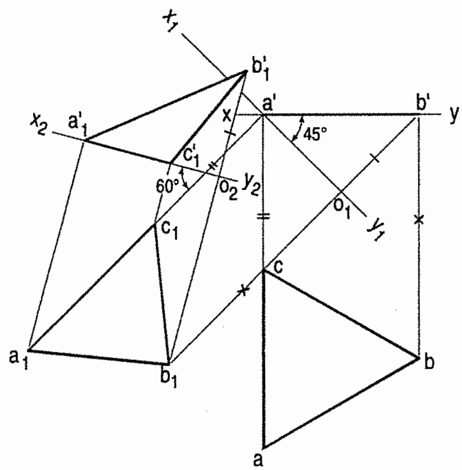


FIG. 11-10

Problem 11-3. (fig. 11-10): *An equilateral triangle of 40 mm long sides has an edge on the ground and inclined at 60° to the V.P. Its plane makes an angle of 45° with the H.P. Draw its projections.*

(i) Draw the top view *abc* and the front view *a'b'*, assuming the triangle to be lying on the ground and one side perpendicular to the V.P.

(ii) Draw x_1y_1 inclined at 45° to *a'b'* (the front view) and passing through *a'* (because the triangle has its edge on the ground).

(iii) Project the new top view $a_1b_1c_1$. Again draw a new reference line x_2y_2 inclined at 60° to the line a_1c_1 (which is to make 60° angle with the V.P.).

(iv) Project the final front view $a'_1b'_1c'_1$ on x_2y_2. Note that $a'_1c'_1$ is in x_2y_2.

11-4. TO DETERMINE TRUE LENGTH OF A LINE

We have seen that the true length of a line and its inclinations with the planes of projection can be determined by making each of its projections parallel to *xy* (chapter 10).

Instead of changing the position of the projection, that of the plane may be altered, i.e. a new reference line representing an auxiliary plane may be drawn parallel to the projection. The auxiliary projection on that reference line will show the true length and true inclination of the line with the other plane.

Problem 11-4. (fig. 11-11): *The projections of a line AB are given. To determine its true length and true inclinations with the reference planes.*

Let *ab* and *a'b'* be the given projections of *AB*.

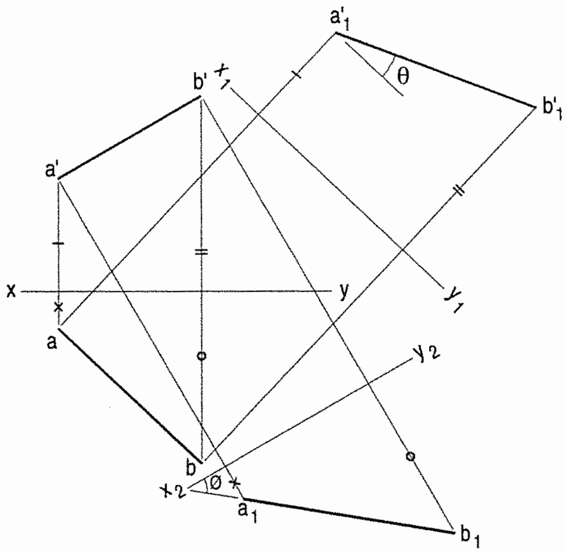


FIG. 11-11

(i) Draw a reference line x_1y_1 to represent an A.V.P. parallel to *ab*, the top view.

(ii) Project the auxiliary front view $a'_1b'_1$ which is the true length of *AB*. θ is its inclination with the H.P.

(iii) Similarly, draw x_2y_2 parallel to the front view *a'b'* and project the auxiliary top view a_1b_1. It is the true length of *AB* and ø is its true inclination with the V.P.

Problem 11-5. (fig. 11-12): *The projections of a line AB viz. ab and a'b' are on the same projector as shown in fig. 11-12(ii). Find the true length, inclinations with the H.P. and the V.P. and the traces of AB.*

(i) Draw a reference line x_1y_1 parallel to the projections. It will be perpendicular to *xy* and will represent an auxiliary plane at right angles to both the H.P. and the V.P. as shown in fig. 11-12(i).

(ii) Project the side view $a'_1b'_1$, which will be the true length of *AB*. θ and ø are its true inclinations with the H.P. and the V.P. respectively.

(iii) Produce $a'_1b'_1$ to meet xy at v and x_1y_1 at h. The H.T. will be on the top view ab-produced, so that oH.T. = o_1h. The V.T. is on the front view $a'b'$-produced, so that oV.T. = o_1v.

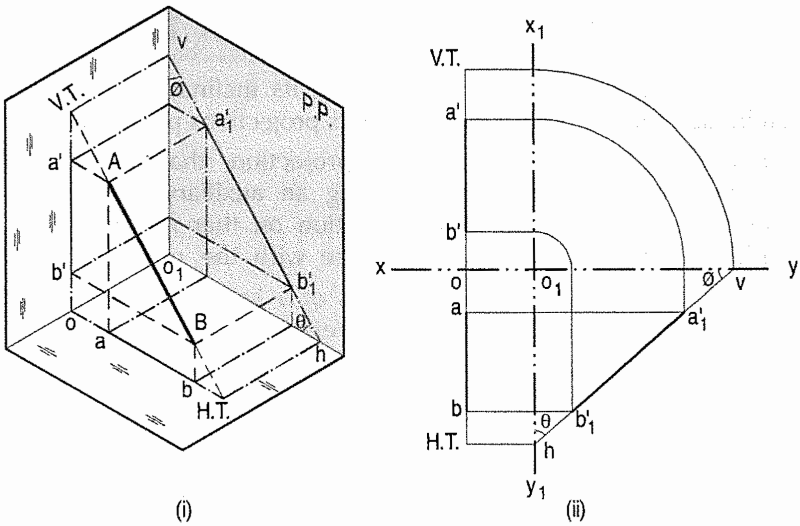


FIG. 11-12

The distances of the ends of the line from x_1y_1 are kept equal to their respective distances above xy by drawing lines inclined at 45° as shown.

11-5. TO OBTAIN POINT-VIEW OF A LINE AND EDGE-VIEW OF A PLANE

We have seen in chapter 10 that when a line is perpendicular to a reference plane, its projection on that plane is a point; while its projection on the other reference plane shows its true length. In other words, the projection of the view of a line showing its true length, on an auxiliary plane perpendicular to that view will be a point.

Similarly, a plane will be seen as a line, when it is projected on a plane, perpendicular to the true length of any one of its elements. The projection of the line-view or edge-view of a plane on an auxiliary plane parallel to it, will show the true shape and size of the plane.

Uses of the point-view of a line and edge-view of a plane on auxiliary planes are illustrated in problems 11-6 and 11-7 respectively.

Problem 11-6. (fig. 11-13): *The projections of a line PQ are given. Determine*

(i) *the distance of its mid-point from xy, and*

(ii) *the shortest distance of the line from xy.*

Let pq and $p'q'$ be the given projections of PQ.

(i) Draw a new reference line x_1y_1 perpendicular to xy and project the side view $p'_1q'_1$ on it.

When considering the side view, x_1y_1 is the edge-view of the V.P. and xy is that of the H.P. The point o is the side view or the point-view of the line of intersection of the V.P. and the H.P., i.e. x_1y_1 and xy. The line joining any point on $p'_1q'_1$, with o will show the shortest distance of that point from xy.

(ii) Find the mid-point a'_1 of $p'_1q'_1$ and join it with o. oa'_1 is the distance of the mid-point of PQ from xy.

(iii) From o, draw a line ob perpendicular to $p'_1q'_1$. ob is the shortest distance of PQ from xy. It will be perpendicular to both PQ and xy.

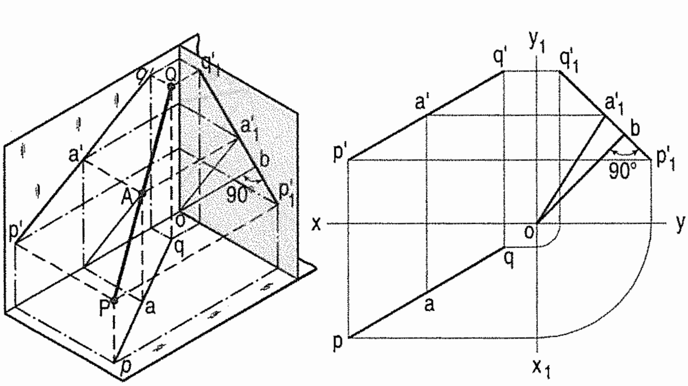


FIG. 11-13

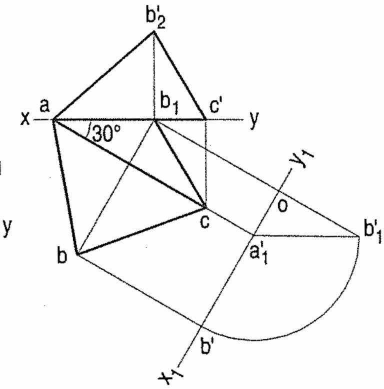


FIG. 11-14

Problem 11-7. (fig. 11-14): *An isosceles triangle ABC, base 60 mm and altitude 40 mm has its base AC in the H.P. and inclined at 30° to the V.P. The corners A and B are in the V.P. Draw its projections.*

Assume the triangle to be lying in the H.P. with the base AC inclined at 30° to the V.P. and A in the V.P. Its true shape will be seen in the top view.

(i) Therefore, in the top view, draw ac 60 mm long and inclined at 30° to xy and complete the triangle abc.

(ii) Project c to c' on xy. When the triangle is tilted about the edge AC, so that the corner B is in the V.P., in the top view the point b will move along a line perpendicular to ac, to a point b_1 in xy. The distance of the front view of B below xy may now be determined by means of an auxiliary plane.

(iii) Draw a reference line x_1y_1 perpendicular to ac.

(iv) Project an auxiliary front view of the triangle abc. It will be a line $b'a'_1$, showing the edge-view of the triangle. a'_1 is the point-view of the line ac. When b moves to b_1, b' will move along the arc drawn with a'_1 as centre and radius equal to a'_1b' to b'_1 on the line through b_1 drawn perpendicular to x_1y_1. ob'_1 is the required distance.

(v) Therefore, through b_1, draw a line perpendicular to xy and on it, mark a point b'_2 such that $b_1b'_2 = ob'_1$.

(vi) Join a and c' with b'_2. ab'_2c' and ab_1c are the required projections.

11-6. TO DETERMINE TRUE SHAPE OF A PLANE FIGURE

The true shape of any plane figure may be determined by means of its projections on auxiliary planes, as illustrated in problems 11-8 and 11-9.

CHAROTAR COGNIFRONT This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 26 for the following problem.

Problem 11-8. (fig. 11-15): *Projections of a pentagon resting on the ground on one of its sides are given. Determine the true shape of the pentagon.*

ae is the true length of the side because it is parallel to the H.P.

(i) Draw a reference line x_1y_1 perpendicular to *ae*. Project an auxiliary front view on it. It will give a point-view a'_1 and e'_1 of *ae* and an edge-view $a'_1c'_1$ of the pentagon.

(ii) Draw another reference line x_2y_2 parallel to $a'_1c'_1$ and project an auxiliary top view $a_1b_1c_1d_1e_1$, which will be the true shape of the pentagon.

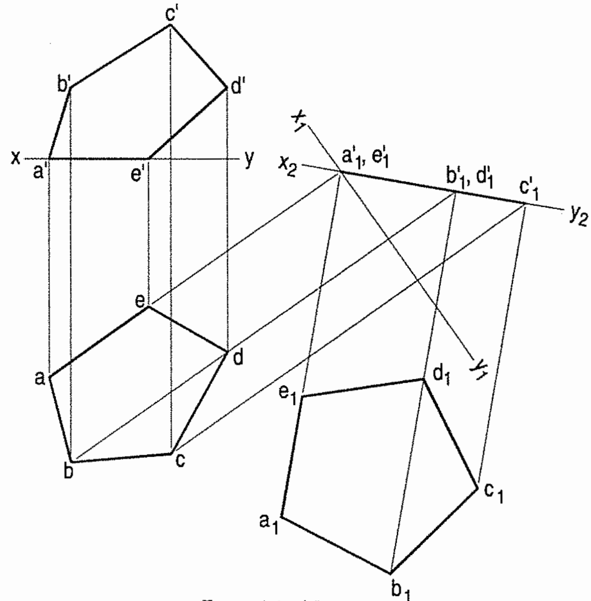


FIG. 11-15

Problem 11-9. *Fig. 11-16 shows the top view abcd and front view a'b'c'd' of a quadrilateral. Determine its true shape.*

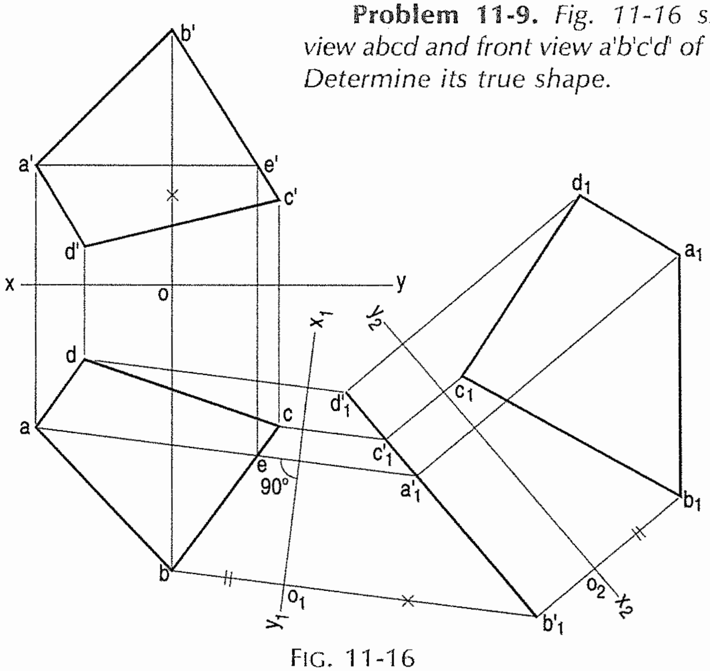


FIG. 11-16

(i) Through any corner, say a', draw a line parallel to xy and meeting $b'c'$ at e'.

(ii) Project e' to e on the line bc in the top view. ae is the true length of the element AE.

(iii) Draw a reference line x_1y_1 perpendicular to ae. Project a new front view from the top view. It is a line $d'_1b'_1$.

(iv) Again, draw another reference line x_2y_2 parallel to the line-view $d'_1b'_1$ and project on it a new top view $a_1b_1c_1d_1$ which will show the true shape of the quadrilateral.

Note that $b'_1o_1 = b'o$, $b'_1o_2 = bo_1$ etc.

Problem 11-10. (fig. 11-17): *The projections of a triangular plate PQR appear as under:*

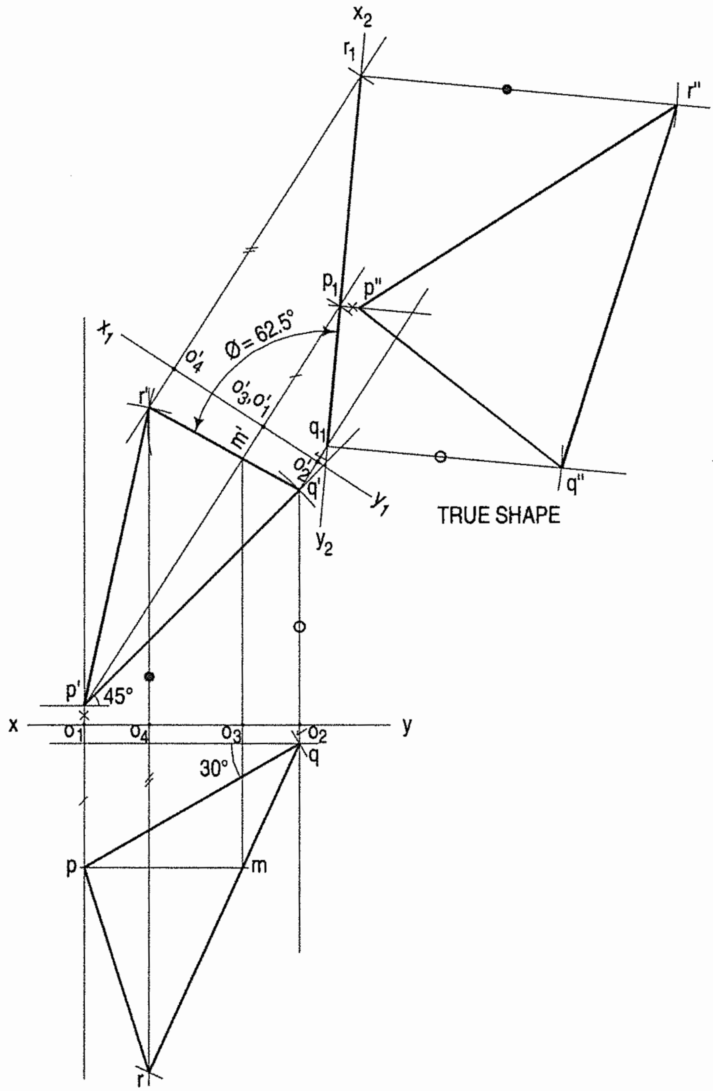


FIG. 11-17

(a) *Top view* : *pq makes 30° with xy.*

qr = 95 mm. The corner q is 5 mm in front of the V.P.

(b) *Front view* : *p'q' = 80 mm; q'r' = 45 mm and r'p' = 80 mm.*

p'q' makes an angle of 45° with xy. The corner P of the plate is 5 mm above the H.P.

Draw the projections of the plate and determine true shape of the triangular plater

(i) Mark *xy*. Draw top view and front view from the given dimensions.

(ii) Mark a line *pm* in the top view parallel to *xy*. Draw the projection of *pm* in the front view *p'm'* as shown.

(iii) Select x_1y_1 perpendicular to *p'm'* and project new top view taking the distances o_1p, o_4r and o_2q from the top view.

(iv) Select x_2y_2 and draw new front view as shown in fig. 11-17.

(v) Measure angle ø.

Problem 11-11. *A regular pentagon of 50 mm side is resting on one of its sides on the H.P. having that side parallel to and 25 mm in front of V.P. It is tilted about that side so that its highest corner rests in the V.P. Draw the projections of the pentagon.*

(i) Keep the plane of pentagon parallel to V.P. Draw the front view and top view as shown in fig. 11-18.

(ii) Project *a'b'c'd'e'* on the line parallel to and 25 mm behind x_1y_1. Taking *a"* as a center, tilt *a"d"* such that *d"* touches x_1y_1 at *d'''* Complete the projections as shown in fig. 11-18.

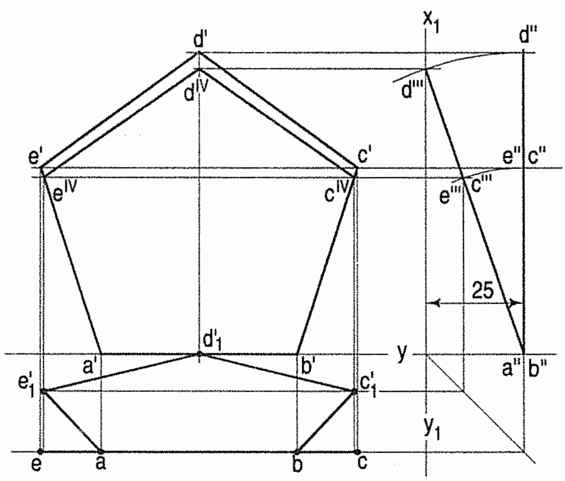


FIG. 11-18

Note that the points e''' and c''' should be transferred in the top view correctly as shown.

Problem 11-12. *A thin square plate with 60 mm side stands on one of its corners in H.P. and the opposite corner is raised so that one of its diagonals is twice that of the others. If, one of the diagonal is parallel to both the planes, draw its projection and determine an inclination of the plane of the plate with H.P.*

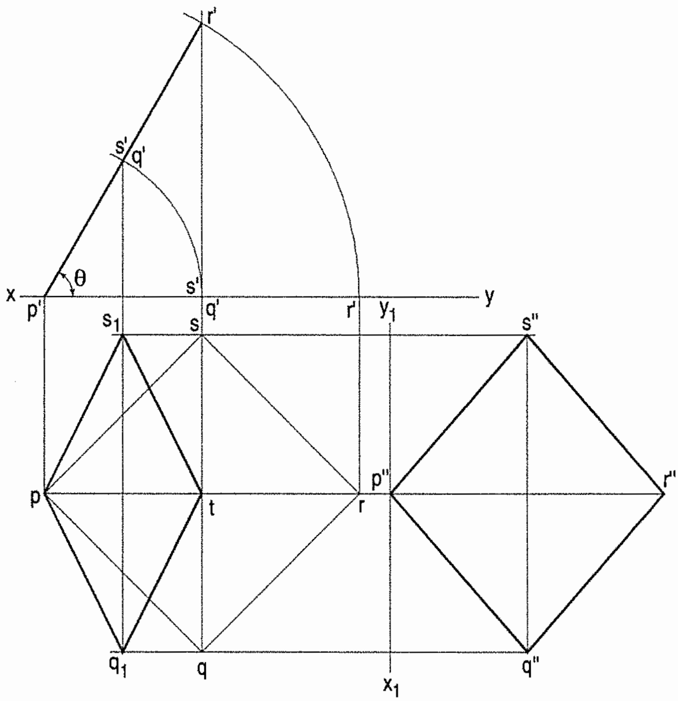


FIG. 11-19

(i) Assuming that the square plate is equally inclined with a vertical plane, draw its top view and front view as shown in fig. 11-19.

(ii) Raise r' so that the diagonal qs becomes twice the diagonal pt. Draw the top view p, q_1, t and s_1. Measure angle θ.

(iii) Draw new reference line x_1y_1 parallel to s_1q_1. Project new front view, keeping the same distances of points p', q', r' and s' from xy.

(iv) Measure angle θ.

EXERCISES 11

Solve the following exercises by applying the method of projections on auxiliary planes:

1. Determine the true length, inclinations with the H.P. and the V.P., and the traces of the line *PQ* in problem 10-15 of chapter 10.
2. Find the true length and the distance of the mid-point from *xy* of the line whose projections are given in problem 10-13 of chapter 10.
3. Draw the projections of the various plane figures as required in Exercises 1, 2 and 3 of chapter 12.
4. abc is an equilateral triangle of altitude 50 mm with *ab* in *xy* and *c* below it. *abc'* is an isosceles triangle of altitude 75 mm and c' is above *xy*. Determine the true shape of the triangle *ABC*, of which *abc* is the top view and *abc'* is the front view.

5. Determine the true shape of the figure, the top view of which is a regular pentagon of 35 mm sides, having one side inclined at 30° to *xy* and whose front view is a straight line making an angle of 45° to *xy*.

6. An equilateral triangle *ABC* of sides 75 mm long has its side *AB* in the V.P. and inclined at 60° to the H.P. Its plane makes an angle of 45° with the V.P. Draw its projections.

7. An isosceles triangle *PQR* having the base *PQ* 50 mm long and altitude 75 mm has its corners *P*, *Q*, and *R* 25 mm, 50 mm and 75 mm respectively above the ground. Draw its projections.

8. A thin regular pentagonal plate of 60 mm long edges has one of its edges in the H.P. and perpendicular to the V.P. while its farthest corner is 60 mm above the H.P. Draw the projections of the plate. Project another front view on an A.V.P. making an angle of 45° with the V.P.

9. A thin composite plate consists of a square of 70 mm long sides with an additional semi-circle constructed on *CD* as diameter. The side *AB* is vertical and the surface of the plate makes an angle of 45° with the V.P. Draw its projections. Project another top view on an A.I.P. making an angle of 30° with the side *AB*.

10. A 60° set-square of 125 mm longest side is so kept that the longest side is in the H.P. making an angle of 30° with the V.P. and the set-square itself inclined at 45° to the H.P. Draw the projections of the set-square.

11. A plane figure is composed of an equilateral triangle *ABC* and a semi-circle on *AC* as diameter. The length of the side *AB* is 50 mm and is parallel to the V.P. The corner *B* is 20 mm behind the V.P. and 15 mm below the H.P. The plane of the figure is inclined at 45° to the H.P. Draw the projections of the plane figure.

12. An equilateral triangle *ABC* having side length as 50 mm is suspended from a point *O* on the side *AB* 15 mm from *A* in such a way that the plane of the triangle makes an angle of 60° with the V.P. The point *O* is 20 mm below the H.P. and 40 mm behind the V.P. Draw the projections of the triangle.

13. A hexagonal plate of side 40 mm, is resting on a corner in V.P. with its surface making an angle of 30° with the V.P. The front view of the diagonal passing through that corner is inclined at 45° to the line *xy*. Draw the projections of the hexagonal plate.

14. A rectangular plate 50 mm × 70 mm stands on one of its shorter edges in H.P. and is raised about this edge so that the top view becomes a square of 50 mm. Determine an inclination of the plate with the horizontal plane.

Chapter 12

PROJECTIONS OF PLANES

12-0. INTRODUCTION

Plane figures or surfaces have only two dimensions, viz. length and breadth. They do not have thickness. A plane figure may be assumed to be contained by a plane, and its projections can be drawn, if the position of that plane with respect to the principal planes of projection is known.

In this chapter, we shall discuss the following topics:

1. Types of planes and their projections.
2. Traces of planes.

12-1. TYPES OF PLANES

Planes may be divided into two main types:

(1) Perpendicular planes.

(2) Oblique planes.

(1) Perpendicular planes: These planes can be divided into the following sub-types:

(i) Perpendicular to both the reference planes.

(ii) Perpendicular to one plane and parallel to the other.

(iii) Perpendicular to one plane and inclined to the other.

(i) ***Perpendicular to both the reference planes*** (fig. 12-1): A square *ABCD* is perpendicular to both the planes. Its H.T. and V.T. are in a straight line perpendicular to *xy*.

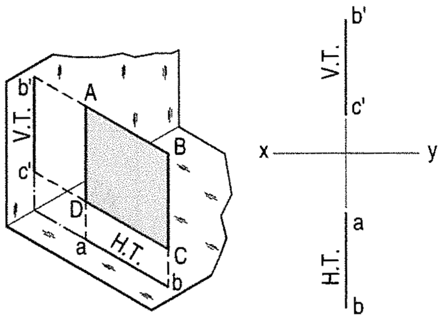


FIG. 12-1

The front view *b'c'* and the top view *ab* of the square are both lines coinciding with the V.T. and the H.T. respectively.

(ii) *Perpendicular to one plane and parallel to the other plane:*

(a) Plane, perpendicular to the H.P. and parallel to the V.P. [fig. 12-2(i)].

A triangle *PQR* is perpendicular to the H.P. and is parallel to the V.P. Its H.T. is parallel to *xy*. It has no V.T.

The front view *p'q'r'* shows the exact shape and size of the triangle. The top view *pqr* is a line parallel to *xy*. It coincides with the H.T.

(b) Plane, perpendicular to the V.P. and parallel to the H.P. [fig. 12-2(ii)].

A square *ABCD* is perpendicular to the V.P. and parallel to the H.P. Its V.T. is parallel to *xy*. It has no H.T.

The top view *abcd* shows the true shape and true size of the square. The front view *a'b'* is a line, parallel to *xy*. It coincides with the V.T.

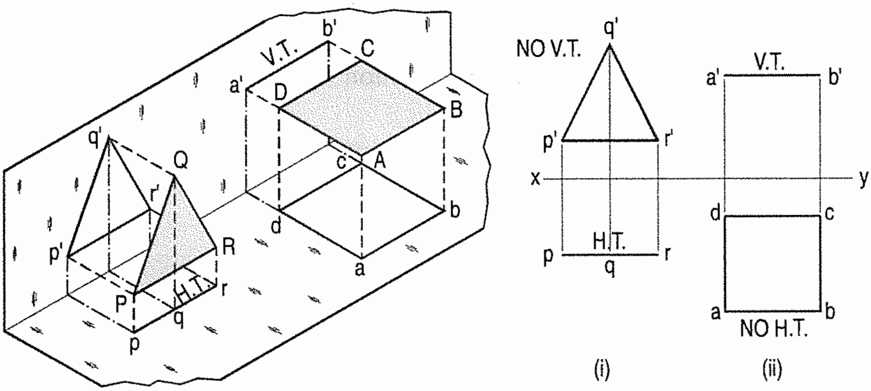


FIG. 12-2

(iii) *Perpendicular to one plane and inclined to the other plane:*

(a) Plane, perpendicular to the H.P. and inclined to the V.P. (fig. 12-3).

A square *ABCD* is perpendicular to the H.P. and inclined at an angle ø to the V.P. Its V.T. is perpendicular to *xy*. Its H.T. is inclined at ø to xy.

Its top view *ab* is a line linclined at ø to *xy*. The front view *a'b'c'd'* is smaller than *ABCD*.

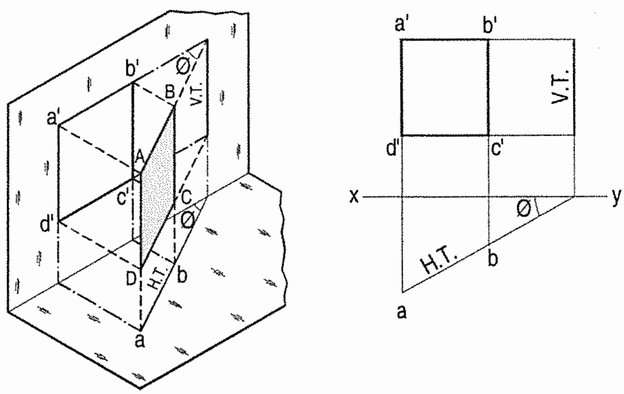


FIG. 12-3

(b) Plane, perpendicular to the V.P. and inclined to the H.P. (fig. 12-4).

A square *ABCD* is perpendicular to the V.P. and inclined at an angle θ to the H.P. Its H.T. is perpendicular to *xy*. Its V.T. makes the angle θ with *xy*. Its front view *a'b'* is a line inclined at θ to *xy*. The top view *abcd* is a rectangle which is smaller than the square *ABCD*.

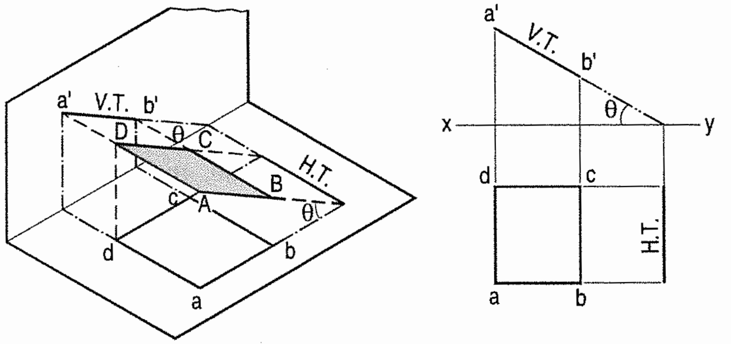


FIG. 12-4

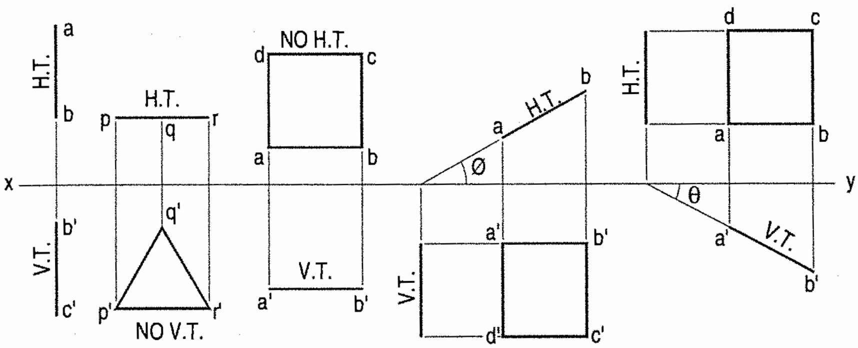


FIG. 12-5

Fig. 12-5 shows the projections and the traces of all these perpendicular planes by third-angle projection method.

(2) Oblique planes: Planes which are inclined to both the reference planes are called *oblique planes*. Representation of oblique planes by their traces is too advanced to be included in this book.

A few problems on the projections of plane figures inclined to both the reference planes are however, illustrated at the end of the chapter. They will prove to be of great use in dealing with the projections of solids.

12-2. TRACES OF PLANES

A plane, extended if necessary, will meet the reference planes in lines, unless it is parallel to any one of them.

These lines are called the *traces* of the plane. The line in which the plane meets the H.P. is called the *horizontal trace* or the H.T. of the plane. The line in which it meets the V.P. is called its *vertical trace* or the V.T. A plane is usually represented by its traces.

12-3. GENERAL CONCLUSIONS

(1) Traces:

(a) When a plane is perpendicular to both the reference planes, its traces lie on a straight line perpendicular to *xy*.

(b) When a plane is perpendicular to one of the reference planes, its trace upon the other plane is perpendicular to *xy* (except when it is parallel to the other plane).

(c) When a plane is parallel to a reference plane, it has no trace on that plane. Its trace on the other reference plane, to which it is perpendicular, is parallel to *xy*.

(d) When a plane is inclined to the H.P. and perpendicular to the V.P., its inclination is shown by the angle which its V.T. makes with *xy*. When it is inclined to the V.P. and perpendicular to the H.P., its inclination is shown by the angle which its H.T. makes with *xy*.

(e) When a plane has two traces, they, produced if necessary, intersect in *xy* (except when both are parallel to *xy* as in case of some oblique planes).

(2) Projections:

(a) When a plane is perpendicular to a reference plane, its projection on that plane is a straight line.

(b) When a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

(c) When a plane is perpendicular to one of the reference planes and inclined to the other, its inclination is shown by the angle which its projection on the plane to which it is perpendicular, makes with *xy*. Its projection on the plane to which it is inclined, is smaller than the plane itself.

Problem 12-1. *Show by means of traces, each of the following planes:*

(a) *Perpendicular to the H.P. and the V.P.*

(b) *Perpendicular to the H.P. and inclined at 30° to the V.P.*

(c) *Parallel to and 40 mm away from the V.P.*

(d) *Inclined at 45° to the H.P. and perpendicular to the V.P.*

(e) *Parallel to the H.P. and 25 mm away from it.*

Fig. 12-6 and fig. 12-7 show the various traces.

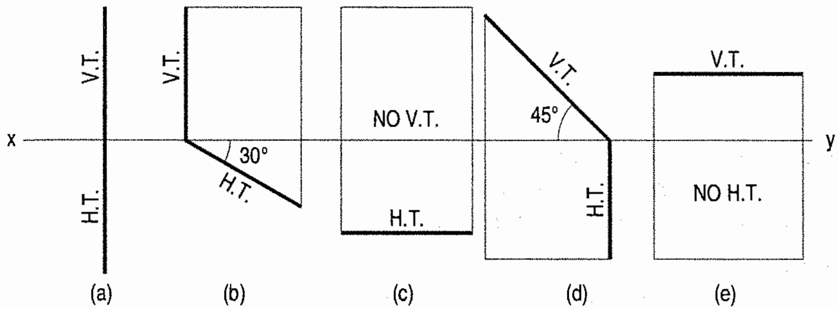
(a) The H.T. and the V.T. are in a line perpendicular to xy.

(b) The H.T. is inclined at 30° to *xy*; the V.T. is normal to *xy*; both the traces intersect in *xy*.

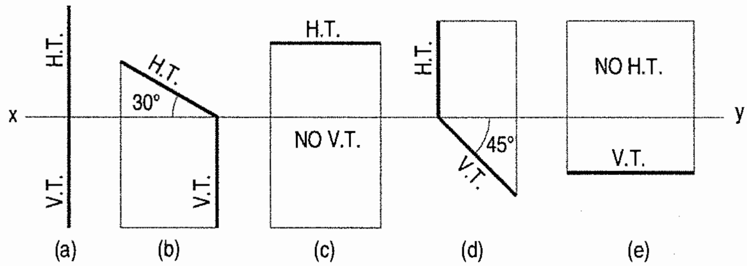
(c) The H.T. is parallel to and 40 mm away from *xy*. It has no V.T.

(d) The H.T. is perpendicular to *xy*; the V.T. makes 45° angle with *xy*; both intersect in *xy*.

(e) The V.T. is parallel to and 25 mm away from *xy*. It has no H.T.



(First-angle projection)
FIG. 12-6



(Third-angle projection)
FIG. 12-7

12-4. PROJECTIONS OF PLANES PARALLEL TO ONE OF THE REFERENCE PLANES

The projection of a plane on the reference plane parallel to it will show its true shape. Hence, beginning should be made by drawing that view. The other view which will be a line, should then be projected from it.

(1) When the plane is parallel to the H.P.: The top view should be drawn first and the front view projected from it.

Problem 12-2. (fig. 12-8): *An equilateral triangle of 50 mm side has its V.T. parallel to and 25 mm above xy. It has no H.T. Draw its projections when one of its sides is inclined at 45° to the V.P.*

As the V.T. is parallel to *xy* and as there is no H.T. the triangle is parallel to the H.P. Therefore, begin with the top view.

(i) Draw an equilateral triangle *abc* of 50 mm side, keeping one side, say *ac*, inclined at 45° to *xy*.

(ii) Project the front view, parallel to and 25 mm *above xy*, as shown.

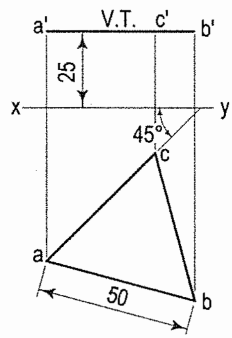


FIG. 12-8

(2) When the plane is parallel to the V.P.: Beginning should be made with the front view and the top view projected from it.

Problem 12-3. (fig. 12-9): *A square ABCD of 40 mm side has a corner on the H.P. and 20 mm in front of the V.P. All the sides of the square are equally inclined to the H.P. and parallel to the V.P. Draw its projections and show its traces.*

As all the sides are parallel to the V.P., the surface of the square also is parallel to it. The front view will show the true shape and position of the square.

(i) Draw a square *a'b'c'd'* in the front view with one corner in *xy* and all its sides inclined at 45° to *xy*.

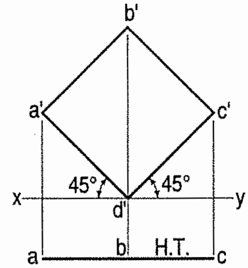


FIG. 12-9

(ii) Project the top view keeping the line *ac* parallel to *xy* and 30 mm below it. The top view is its H.T. It has no V.T.

12-5. PROJECTIONS OF PLANES INCLINED TO ONE REFERENCE PLANE AND PERPENDICULAR TO THE OTHER

When a plane is inclined to a reference plane, its projections may be obtained in two stages. In the initial stage, the plane is assumed to be parallel to that reference plane to which it has to be made inclined. It is then tilted to the required inclination in the second stage.

(1) Plane, inclined to the H.P. and perpendicular to the V.P.: When the plane is inclined to the H.P. and perpendicular to the V.P., in the initial stage, it is assumed to be parallel to the H.P. Its top view will show the true shape. The front view will be a line parallel to *xy*. The plane is then tilted so that it is inclined to the H.P. The new front view will be inclined to *xy* at the true inclination. In the top view the corners will move along their respective paths (parallel to *xy*).

Problem 12-4. (fig. 12-10): *A regular pentagon of 25 mm side has one side on the ground. Its plane is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections and show its traces.*

Assuming it to be parallel to the H.P.

(i) Draw the pentagon in the top view with one side perpendicular to *xy* [fig. 12-10(i)]. Project the front view. It will be the line *a'c'* contained by *xy*.

(ii) Tilt the front view about the point *a'*, so that it makes 45° angle with *xy*.

(iii) Project the new top view $ab_1c_1d_1e$ upwards from this front view and horizontally from the first top view. It will be more convenient if the front view is reproduced in the new position separately and the top view projected from it, as shown in fig. 12-10(ii). The V.T. coincides with the front view and the H.T. is perpendicular to *xy*, through the point of intersection between *xy* and the front view-produced.

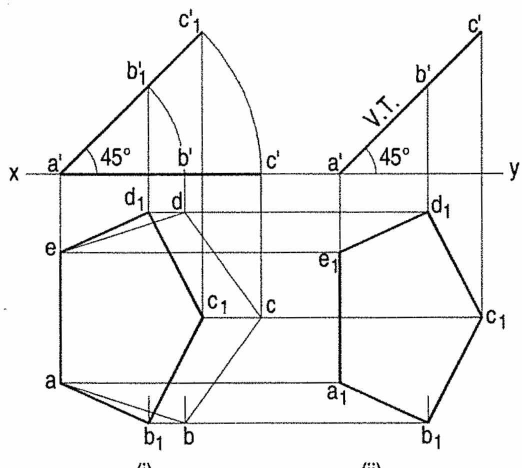


FIG. 12-10

(2) Plane, inclined to the V.P. and perpendicular to the H.P.: In the initial stage, the plane may be assumed to be parallel to the V.P. and then tilted to the required position in the next stage. The projections are drawn as illustrated in the next problem.

Problem 12-5. (fig. 12-11): *Draw the projections of a circle of 50 mm diameter, having its plane vertical and inclined at 30° to the V.P. Its centre is 30 mm above the H.P. and 20 mm in front of the V.P. Show also its traces.*

A circle has no corners to project one view from another. However, a number of points, say twelve, equal distances apart, may be marked on its circumference.

(i) Assuming the circle to be parallel to the V.P., draw its projections. The front view will be a circle [fig. 12-11(i)], having its centre 30 mm above *xy*. The top view will be a line, parallel to and 20 mm below *xy*.

(ii) Divide the circumference into twelve equal parts (with a 30°-60° set-square) and mark the points as shown. Project these points in the top view. The centre *O* will coincide with the point 4.

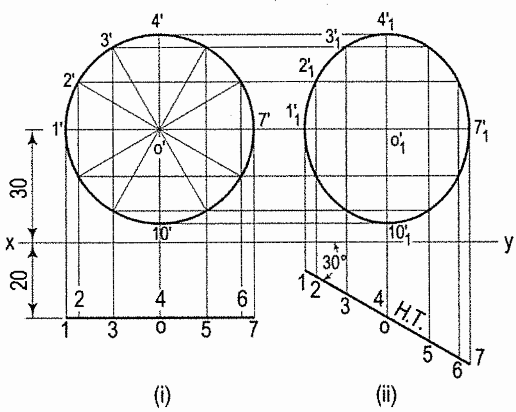


FIG. 12-11

(iii) When the circle is tilted, so as to make 30° angle with the V.P., its top view will become inclined at 30° to *xy*. In the front view all the points will move along their respective paths (parallel to *xy*). Reproduce the top view keeping the centre *o* at the same distance, viz. 20 mm from *xy* and inclined at 30° to *xy* [fig. 12-11(ii)].

(iv) For the final front view, project all the points upwards from this top view and horizontally from the first front view. Draw a freehand curve through the twelve points $1'_1$, $2'_1$ etc. This curve will be an ellipse.

12-6. PROJECTIONS OF OBLIQUE PLANES

When a plane has its surface inclined to one plane and an edge or a diameter or a diagonal parallel to that plane and inclined to the other plane, its projections are drawn in three stages.

(1) If the surface of the plane is inclined to the H.P. and an edge (or a diameter or a diagonal) is parallel to the H.P. and inclined to the V.P.,

  (i) in the initial position the plane is assumed to be parallel to the H.P. and an edge perpendicular to the V.P.

  (ii) It is then tilted so as to make the required angle with the H.P. As already explained, its front view in this position will be a line, while its top view will be smaller in size.

  (iii) In the final position, when the plane is turned to the required inclination with the V.P., only the position of the top view will change. Its shape and size will not be affected. In the final front view, the corresponding distances of all the corners from *xy* will remain the same as in the second front view.

If an edge is in the H.P. or on the ground, in the initial position, the plane is assumed to be lying in the H.P. or on the ground, with the edge perpendicular to the V.P. *If a corner is in the H.P. or on the ground, the line joining that corner with the centre of the plane is kept parallel to the V.P.*

(2) Similarly, if the surface of the plane is inclined to the V.P. and an edge (or a diameter or a diagonal) is parallel to the V.P. and inclined to the H.P.,

(i) in the initial position, the plane is assumed to be parallel to the V.P. and an edge perpendicular to the H.P.

(ii) It is then tilted so as to make the required angle with the V.P. Its top view in this position will be a line, while its front view will be smaller in size.

(iii) When the plane is turned to the required inclination with the H.P., only the position of the front view will change. Its shape and size will not be affected. In the final top view, the corresponding distances of all the corners from *xy* will remain the same as in the second top view.

If an edge is in the V.P., in the initial position, the plane is assumed to be lying in the V.P. with an edge perpendicular to the H.P. *If a corner is in the V.P., the line joining that corner with centre of the plane is kept parallel to the H.P.*

Problem 12-6. (fig. 12-12): A *square ABCD of 50 mm side has its corner A in the H.P., its diagonal AC inclined at 30° to the H.P. and the diagonal BD inclined at 45° to the V.P. and parallel to the H.P. Draw its projections.*

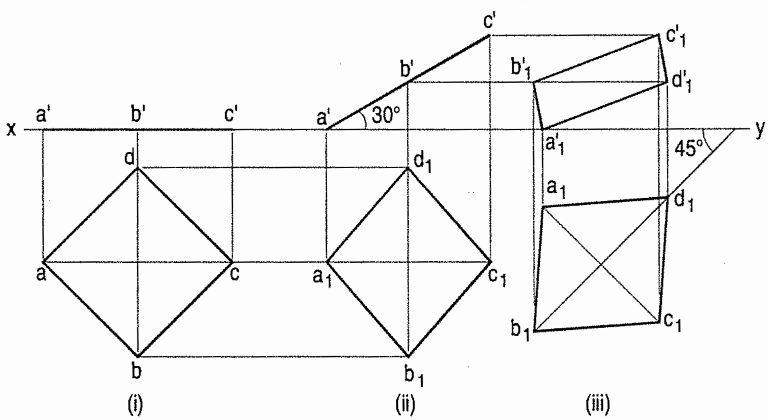


FIG. 12-12

In the initial stage, assume the square to be lying in the H.P. with *AC* parallel to the V.P.

(i) Draw the top view and the front view. When the square is tilted about the corner *A* so that *AC* makes 30° angle with the H.P., *BD* remains perpendicular to the V.P. and parallel to the H.P.

(ii) Draw the second front view with *a'c'* inclined at 30° to *xy*, keeping *a'* or *c'* in *xy*. Project the second top view. The square may now be turned so that *BD* makes 45° angle with the V.P. and remains parallel to the H.P. Only the position of the top view will change. Its shape and size will remain the same.

(iii) Reproduce the top view so that b_1d_1 is inclined at 45° to *xy*. Project the final front view upwards from this top view and horizontally from the second front view.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 27 for the following problem.

Problem 12-7. (Fig. 12-13): *A rectangular plane surface of size L × W is positioned in the first quadrant and is inclined at an angle of 60° with the H.P. and 30° with the V.P. Draw its projections.*

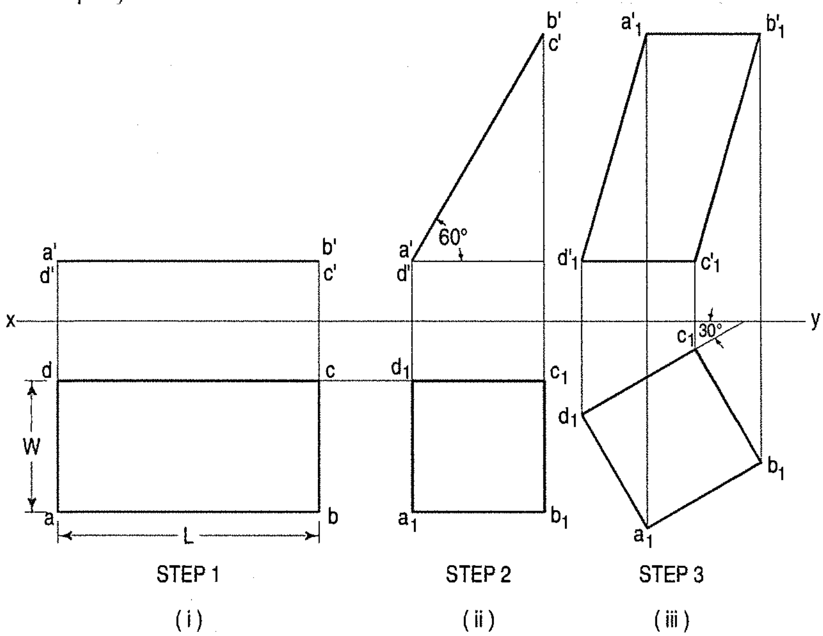


FIG. 12-13

(i) The plane is first assumed to be parallel to H.P. with its shorter edge perpendicular to V.P. In this position, true shape and size of the plane is given by its projection on H.P. The front view will be a true line parallel to the reference line *xy*.

(ii) Rotate the front view projection by 60° (the angle of inclination of plane with H.P.) as shown in Step 2 of fig. 12-13(ii). Draw vertical lines from the ends of line *a'd'* and *b'c'* to intersect horizontal lines drawn from the top view *abcd* (step 1) at points b_1, c_1, d_1 and a_1. Join $a_1b_1c_1d_1$ to obtain the top view of the plane in this inclined position.

(iii) Now rotate the edge d_1c_1 of the top view (step 2) by 30° (the angle of inclination of plane with V.P.) and reproduce it as shown in step 3 of the fig. 12-13(iii). Draw projections from a_1, b_1, c_1 and d_1 to intersect the horizontal projections from *a'd'* and *b'c'* to get the points a'_1, b'_1, c'_1 and d'_1. Join the lines $a'_1b'_1c'_1d'_1$ to obtain the final front view of the given plane surface.

Problem 12-8. (fig. 12-14): *Draw the projections of a regular hexagon of 25 mm side, having one of its sides in the H.P. and inclined at 60° to the V.P., and its surface making an angle of 45° with the H.P.*

(i) Draw the hexagon in the top view with one side perpendicular to *xy*. Project the front view *a'c'* in *xy*.

(ii) Draw *a'c'* inclined at 45° to *xy* keeping *a'* or *c'* in *xy* and project the second top view.

(iii) Reproduce this top view making a_1f_1 inclined at 60° to *xy* and project the final front view.

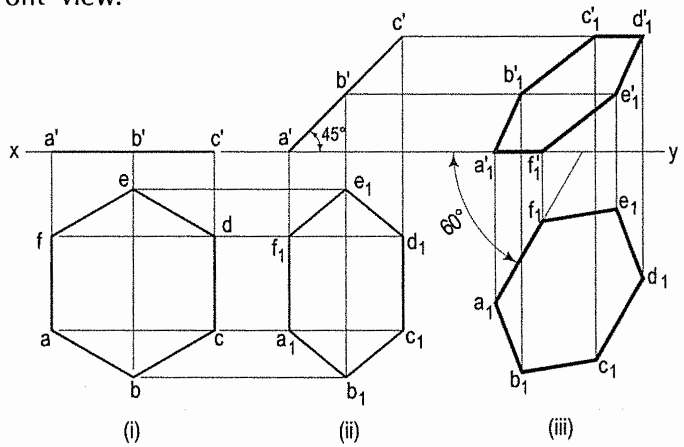


FIG. 12-14

Problem 12-9. (fig. 12-15): *Draw the projections of a circle of 50 mm diameter resting in the H.P. on a point A on the circumference, its plane inclined at 45° to the H.P. and*

(a) the top view of the diameter AB making 30° angle with the V.P.;

(b) the diameter AB making 30° angle with the V.P.

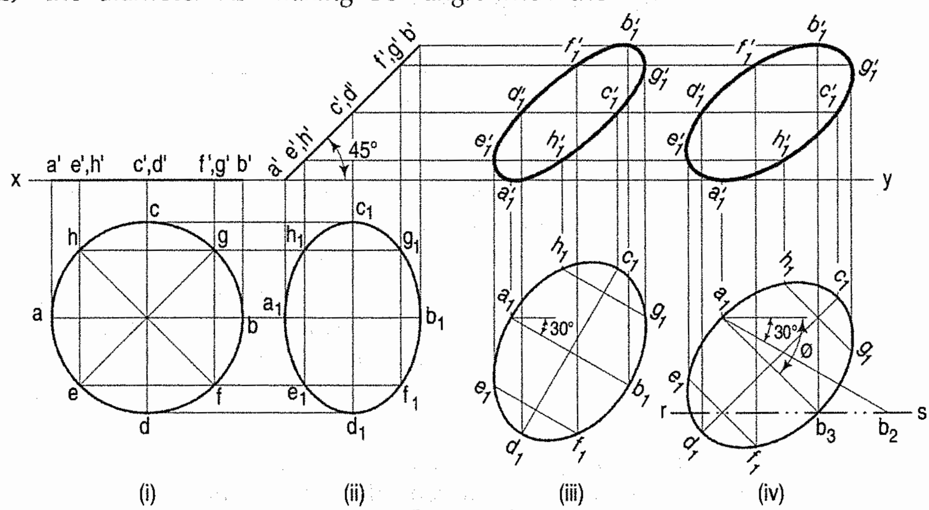


FIG. 12-15

Draw the projections of the circle with *A* in the H.P. and its plane inclined at 45° to the H.P. and perpendicular to the V.P. [fig. 12-15(i) and fig. 12-15(ii)].

(a) In the second top view, the line a_1b_1 is the top view of the diameter *AB*. Reproduce this top view so that a_1b_1 makes 30° angle with *xy* [fig. 12-15(iii)]. Project the required front view.

(b) If the diameter AB, which makes 45° angle with the H.P., is inclined at 30° to the V.P. also, its top view a_1b_1 will make an angle greater than 30° with xy. This apparent angle of inclination is determined as described below.

Draw any line a_1b_2 equal to AB and inclined at 30° to xy [fig. 12-15(iv)]. With a_1 as centre and radius equal to the top view of AB, viz. a_1b_1, draw an arc cutting rs (the path of B in the top view) at b_3. Draw the line joining a_1 with b_3, and around it, reproduce the second top view. Project the final front view. It is evident that a_1b_3 is inclined to xy at an angle ø which is greater than 30°.

Problem 12-10. (fig. 12-16): *A thin 30°-60° set-square has its longest edge in the V.P. and inclined at 30° to the H.P. Its surface makes an angle of 45° with the V.P. Draw its projections.*

In the initial stage, assume the set-square to be in the V.P. with its hypotenuse perpendicular to the H.P.

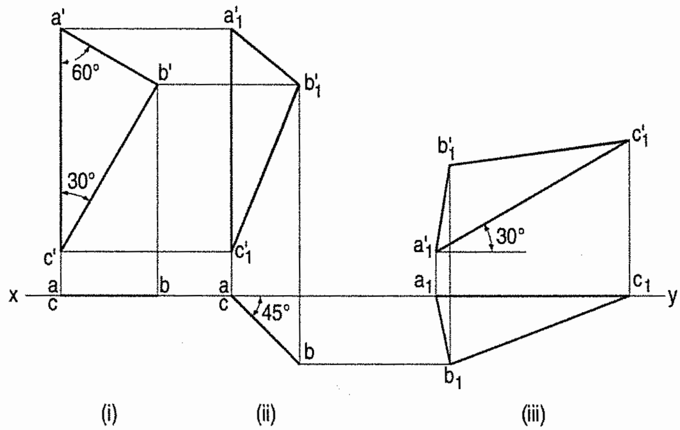


FIG. 12-16

(i) Draw the front view $a'b'c'$ and project the top view ac in xy.

(ii) Tilt ac around the end a so that it makes 45° angle with xy and project the front view $a'_1b'_1c'_1$.

(iii) Reproduce the second front view $a'_1b'_1c'_1$ so that $a'_1b'_1$ makes an angle of 30° with xy. Project the final top view $a_1b_1c_1$.

Problem 12-11. (fig. 12-17): *A thin rectangular plate of sides 60 mm × 30 mm has its shorter side in the V.P. and inclined at 30° to the H.P. Project its top view if its front view is a square of 30 mm long sides.*

As the front view of the plate is a square, its surface must be inclined to the V.P. Hence, assume the plate to be in the V.P. with its shorter edge perpendicular to the H.P.

(i) Draw the front view $a'b'c'd'$ and project the top view ab in xy [fig. 12-17(i)].

(ii) The line ab should be so inclined to xy that the front view becomes a square. Therefore, draw the square $a'_1b'_1c'_1d'_1$ of side equal to $a'd'$. With a as centre and radius equal to ab draw an arc cutting the projector through b'_1 at b. Then ab is the new top view.

(iii) Reproduce the second front view in such a way that $a'_1d'_1$ makes 30° angle with xy. Project the final top view as shown.

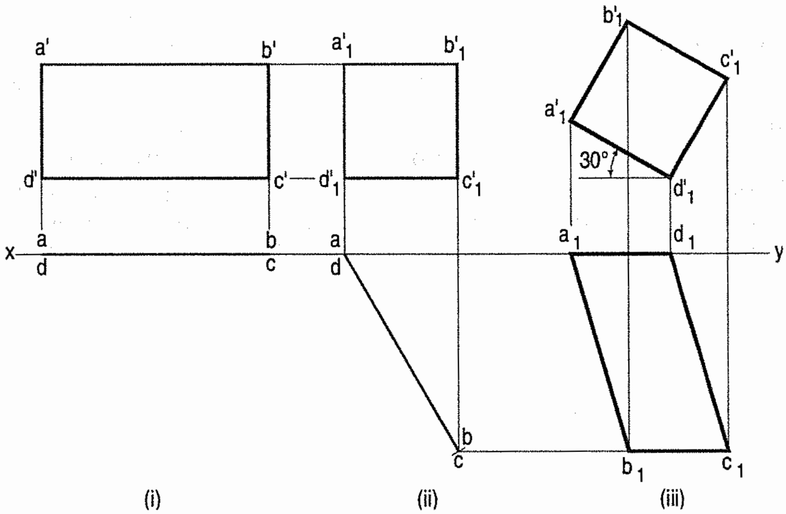
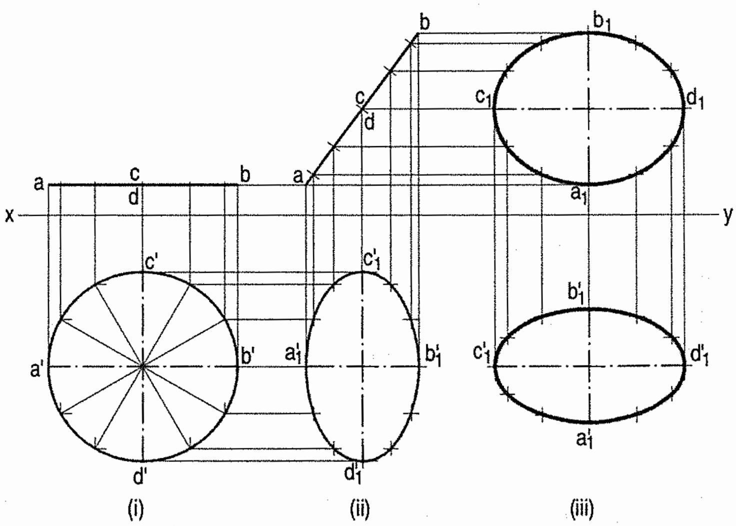


FIG. 12-17

Problem 12-12. (fig. 12-18): *A circular plate of negligible thickness and 50 mm diameter appears as an ellipse in the front view, having its major axis 50 mm long and minor axis 30 mm long. Draw its top view when the major axis of the ellipse is horizontal.*

As the plate is seen as an ellipse in the front view, its surface must be inclined to the V.P.



(Third-angle projection)

FIG. 12-18

(i) Therefore, assume it to be parallel to the V.P. and draw its front view and the top view.

(ii) Turn the line *ab* so that its length in the front view becomes 30 mm, and project the front view. It will be an ellipse whose major axis is vertical.

(iii) Reproduce this view so that the major axis $c'_1d'_1$ is horizontal, and project the required top view.

Problem 12-13. *Fig. 12-19 shows a thin plate of negligible thickness. It rests on its PQ edge with its plane perpendicular to V.P. and inclined 40° to the H.P. Draw its projections.*

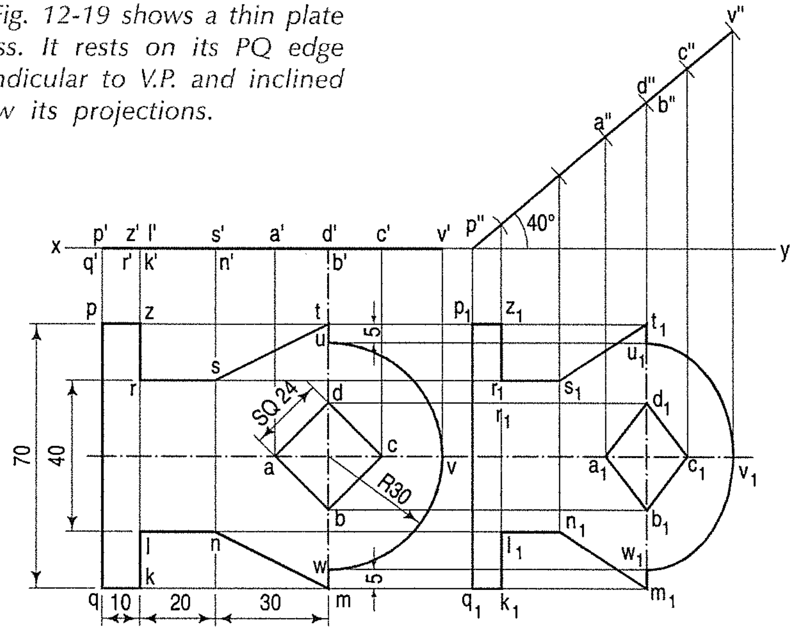


FIG. 12-19

(i) Keep the plane of plate in the H.P. and draw the projections as shown.

(ii) Tilt front view at *p*" making an angle of 40°.

(iii) Project *p*", *a*", *d*", *c*", *v*" etc. in the top view. Draw horizontal projectors intersecting previously drawn projectors from the front view. Join by smooth curve to complete the top view.

Problem 12-14. (fig. 12-20): *A pentagonal plate of 45 mm side has a circular hole of 40 mm diameter in its centre. The plane stands on one of its sides on the H.P. with its plane perpendicular to V.P. and 45° inclined to the H.P. Draw the projections.*

(i) Keep the plane of plate in the horizontal plane.

(ii) Draw top view and front view as shown.

(iii) Tilt the front view *a*" .....*d*" at *a*" making an angle of 45°. Draw the projectors from various points *a*" ....*d*".

(iv) Draw horizontal projectors from the top view *abcd* as shown. Join the intersection points and complete new top view a_1, b_1, c_1, d_1, e_1, as shown.

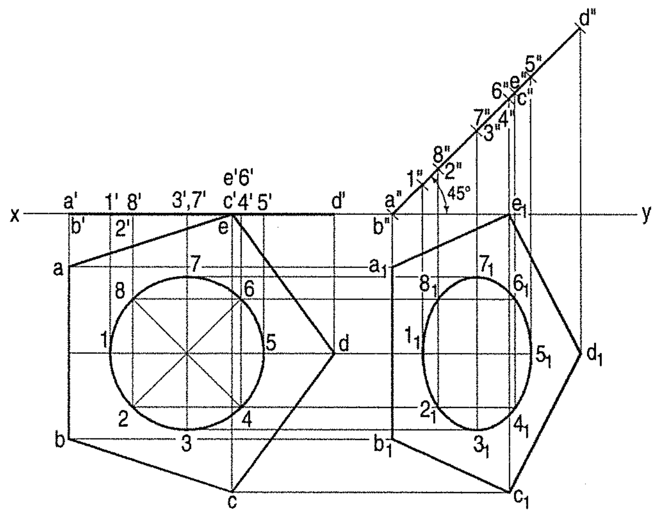


FIG. 12-20

Problem 12-15. (fig. 12-21): *A thin circular plate of 70 mm diameter is resting on its circumference such that its plane is inclined 60° to the H.P. and 30° to the V.P. Draw the projections of the plate.*

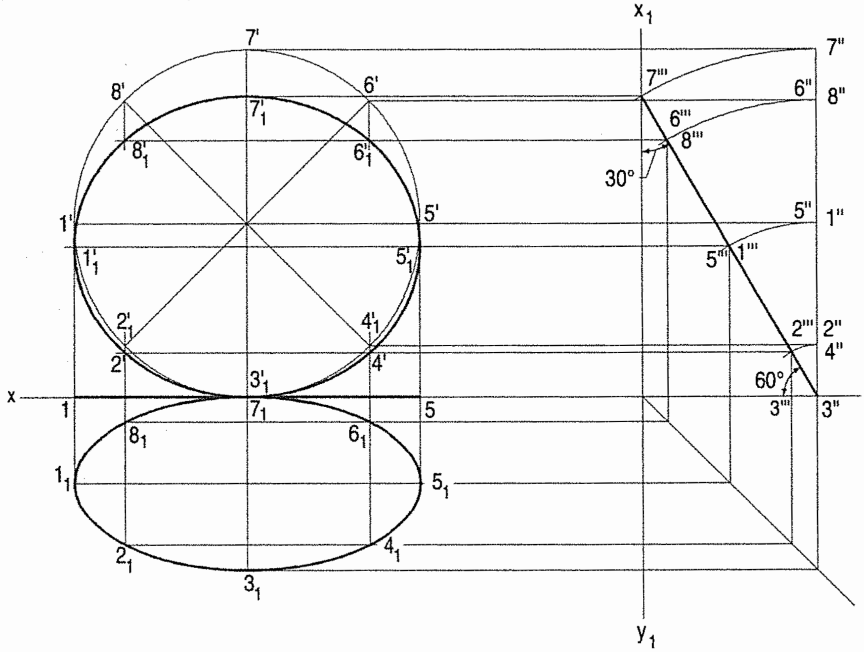


FIG. 12-21

(i) Draw the projection of the plate keeping its plane parallel to the V.P. as shown in fig. 12-21.

(ii) Mark a reference line x_1y_1 perpendicular to *xy* line to represent the auxiliary plane which is at right angle to both the H.P. and the V.P.

(iii) Divide the front view in eight parts and mark the points 1', 2' ....8'. Project these points on the side view as 1", 2" ....8".

(iv) Tilt the side view 3" 7" such that it touches the x_1y_1 line and also makes 60° with the *xy* line.

(v) Complete the projection as shown in fig. 12-21.

Problem 12-16. (fig. 12-22): *PQRS is a rhombus having diagonal PR = 60 mm and QS = 40 mm and they are perpendicular to each other. The plane of the rhombus is inclined with H.P. such that its top view appears to be square. The top view of PR makes 30° with the V.P. Draw its projections and determine inclination of the plane with the H.P.*

(i) Assume that the rhombus is lying in H.P. with its longest diagonal parallel to *xy* line.

(ii) Draw the plans of diagonals *PR* = 60 mm and *QS* = 40 mm (true length) perpendicular each other as shown.

(iii) Join points *p*, *q*, *r* and *s*. It is top view of the rhombus. Project the points *p*, *q*, *r* and *s* in the *xy* line. It is front view of the rhombus points *p'*, *q'*, *r'* and *s'* in the *xy* line as the plane of rhombus is perpendicular to the V.P.

(iv) *PR* and *QS* are lying in H.P. *pr* and *qs* are true length. As the plane of the rhombus is inclined to H.P., the top view of the rhombus is going to be a square. But diagonal *qs* does not change in the length as it is perpendicular to V.P.

(v) Draw the projectors from the points *p*, *q* and *r*, *s* parallel to the *xy*. From q_1 and s_1 draw square $p_1\ q_1\ r_1\ s_1$ such that $s_1\ q_1 = p_1\ r_1$ as shown in fig. 12-22.

(vi) Draw vertical projectors from p_1, q_1, r_1 and s_1.

(vii) Projector of p_1 intersects at p'' in xy line. Taking p'' as centre and the radius equal to 60 mm ($p'r'$), draw the arc to intersect the vertical projectors of r_1 at r''. Join $p''r''$. Measure angle of $p''r''$ with *xy* line.

(viii) Tilt diagonal p_1r_1 at 30° with xy and reproduce square $p_2q_2r_2s_2$. Draw vertical projectors from p_2, q_2, r_2 and s_2 to intersect the horizontal projectors from p'', q'', r'' and s'' at p''', q''', r''' and s'''. Join the points p''', q''', r''' and s''' as shown.

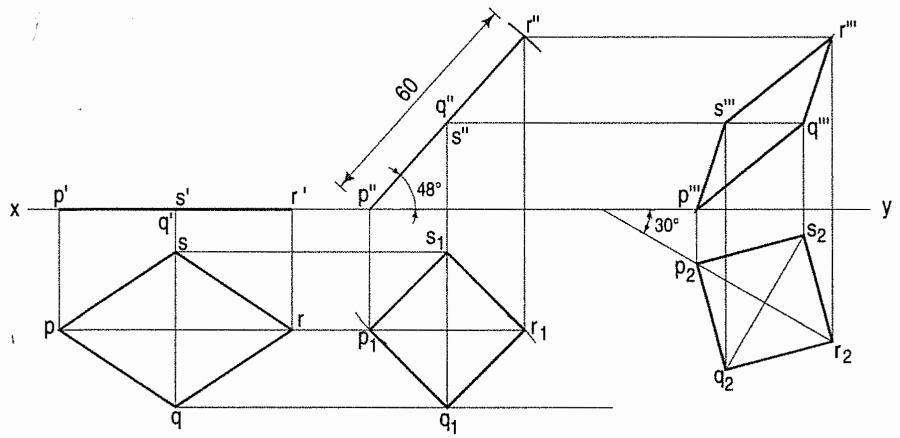


FIG. 12-22

EXERCISES 12

1. Draw an equilateral triangle of 75 mm side and inscribe a circle in it. Draw the projections of the figure, when its plane is vertical and inclined at 30° to the V.P. and one of the sides of the triangle is inclined at 45° to the H.P.

2. A regular hexagon of 40 mm side has a corner in the H.P. Its surface is inclined at 45° to the H.P. and the top view of the diagonal through the corner which is in the H.P. makes an angle of 60° with the V.P. Draw its projections.

3. Draw the projections of a regular pentagon of 40 mm side, having its surface inclined at 30° to the H.P. and a side parallel to the H.P. and inclined at an angle of 60° to the V.P.

4. Draw the projections of a rhombus having diagonals 125 mm and 50 mm long, the smaller diagonal of which is parallel to both the principal planes, while the other is inclined at 30° to the H.P.

5. Draw a regular hexagon of 40 mm side, with its two sides vertical. Draw a circle of 40 mm diameter in its centre. The figure represents a hexagonal plate with a hole in it and having its surface parallel to the V.P. Draw its projections when the surface is vertical and inclined at 30° to the V.P. Assume the thickness of the plate to be equal to that of a line.

6. Draw the projections of a circle of 75 mm diameter having the end *A* of the diameter *AB* in the H.P., the end *B* in the V.P., and the surface inclined at 30° to the H.P. and at 60° to the V.P.

7. A semi-circular plate of 80 mm diameter has its straight edge in the V.P. and inclined at 45° to the H.P. The surface of the plate makes an angle of 30° with the V.P. Draw its projections.

8. The top view of a plate, the surface of which is perpendicular to the V.P. and inclined at 60° to the H.P. is a circle of 60 mm diameter. Draw its three views.

9. A plate having shape of an isosceles triangle has base 50 mm long and altitude 70 mm. It is so placed that in the front view it is seen as an equilateral triangle of 50 mm sides and one side inclined at 45° to *xy*. Draw its top view.

10. Draw a rhombus of diagonals 100 mm and 60 mm long, with the longer diagonal horizontal. The figure is the top view of a square of 100 mm long diagonals, with a corner on the ground. Draw its front view and determine the angle which its surface makes with the ground.

11. A composite plate of negligible thickness is made-up of a rectangle 60 mm × 40 mm, and a semi-circle on its longer side. Draw its projections when the longer side is parallel to the H.P. and inclined at 45° to the V.P., the surface of the plate making 30° angle with the H.P.

12. A 60° set-square of 125 mm longest side is so kept that the longest side is in the H.P. making an angle of 30° with the V.P. and the set-square itself inclined at 45° to the H.P. Draw the projections of the set-square.

13. A plane figure is composed of an equilateral triangle *ABC* and a semi-circle on *AC* as diameter. The length of the side *AB* is 50 mm and is parallel to the V.P. The corner *B* is 20 mm behind the V.P. and 15 mm below the H.P. The plane of the figure is inclined at 45° to the H.P. Draw the projections of the plane figure.

14. An equilateral triangle *ABC* having side length as 50 mm is suspended from a point *O* on the side *AB* 15 mm from *A* in such a way that the plane of the triangle makes an angle of 60° with the V.P. The point *O* is 20 mm below the H.P. and 40 mm behind the V.P. Draw the projections of the triangle.

15. *PQRS* and *ABCD* are two square thin plates with their diagonals measuring 30 mm and 60 mm. They are touching the H.P. with their corners *P* and *A* respectively, and touching each other with their corresponding opposite corners *R* and *C*. If the plates are perpendicular to each other and perpendicular to V.P. also, draw their projections and determine the length of their sides.

Chapter 13

PROJECTIONS OF SOLIDS

13-0. INTRODUCTION

A solid has three dimensions, viz. length, breadth and thickness. To represent a solid on a flat surface having only length and breadth, at least two orthographic views are necessary. Sometimes, additional views projected on auxiliary planes become necessary to make the description of a solid complete.

This chapter deals with the following topics:

1. Types of solids.
2. Projections of solids in simple positions.
   (a) Axis perpendicular to the H.P.
   (b) Axis perpendicular to the V.P.
   (c) Axis parallel to both the H.P. and the V.P.
3. Projections of solids with axes inclined to one of the reference planes and parallel to the other.
   (a) Axis inclined to the V.P. and parallel to the H.P.
   (b) Axis inclined to the H.P. and parallel to the V.P.
4. Projections of solids with axes inclined to both the H.P. and the V.P.
5. Projections of spheres.

13-1. TYPES OF SOLIDS

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 28 for the types of solids.

Solids may be divided into *two* main groups:

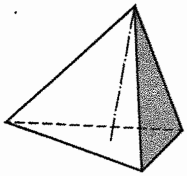
(1) Polyhedra
(2) Solids of revolution.

(1) Polyhedra: A polyhedron is defined as a solid bounded by planes called *faces*. When all the faces are equal and regular, the polyhedron is said to be regular.

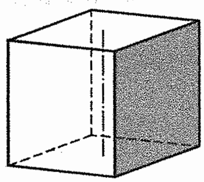
There are *seven* regular polyhedra which may be defined as stated below:

(i) *Tetrahedron* (fig. 13-1): It has four equal faces, each an equilateral triangle.
(ii) *Cube or hexahedron* (fig. 13-2): It has six faces, all equal squares.

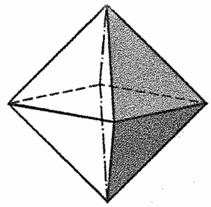
(iii) *Octahedron* (fig. 13-3): It has eight equal equilateral triangles as faces.



Tetrahedron
FIG. 13-1



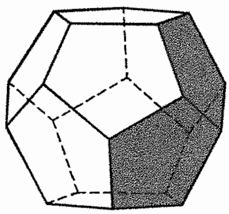
Cube
FIG. 13-2



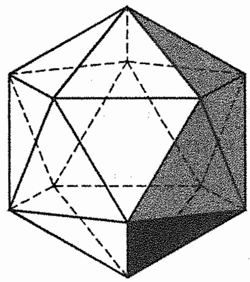
Octahedron
FIG. 13-3

(iv) *Dodecahedron* (fig. 13-4): It has twelve equal and regular pentagons as faces.

(v) *Icosahedron* (fig. 13-5): It has twenty faces, all equal equilateral triangles.



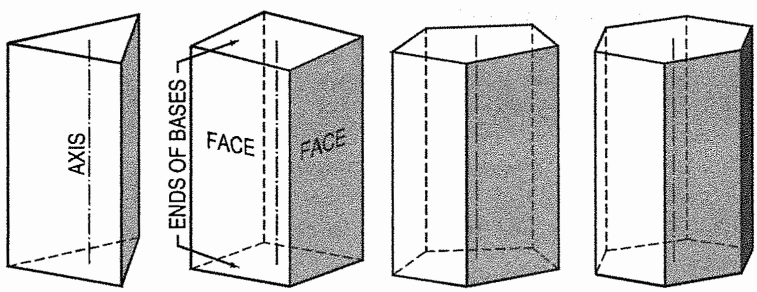
Dodecahedron
FIG. 13-4



Icosahedron
FIG. 13-5

(vi) *Prism:* This is a polyhedron having two equal and similar faces called its ends or bases, parallel to each other and joined by other faces which are parallelograms. The imaginary line joining the centres of the bases is called the axis.

A right and regular prism (fig. 13-6) has its axis perpendicular to the bases. All its faces are equal rectangles.



Triangular Square Pentagonal Hexagonal

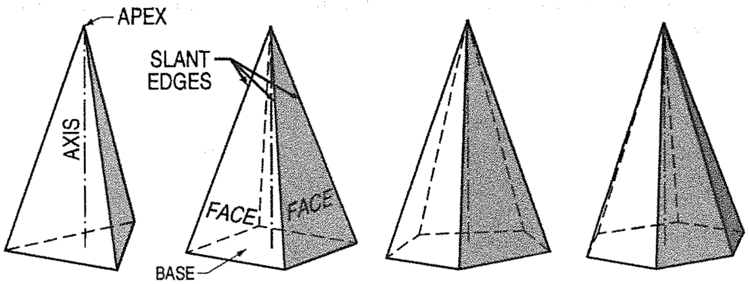
Prisms
FIG. 13-6

(vii) *Pyramid:* This is a polyhedron having a plane figure as a base and a number of triangular faces meeting at a point called the vertex or apex. The imaginary line joining the apex with the centre of the base is its axis.

A right and regular pyramid (fig. 13-7) has its axis perpendicular to the base which is a regular plane figure. Its faces are all equal isosceles triangles.

Oblique prisms and pyramids have their axes inclined to their bases.

Prisms and pyramids are named according to the shape of their bases, as triangular, square, pentagonal, hexagonal etc.



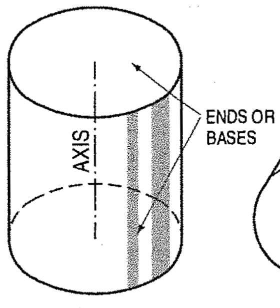
Pyramids
FIG. 13-7

(2) Solids of revolution:

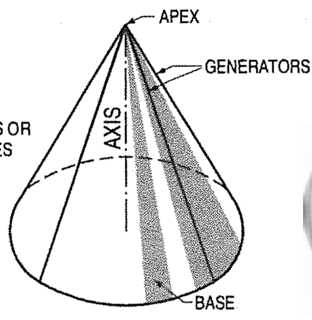
(i) *Cylinder* (fig. 13-8): A *right circular cylinder* is a solid generated by the revolution of a rectangle about one of its sides which remains fixed. It has two equal circular bases. The line joining the centres of the bases is the axis. It is perpendicular to the bases.

(ii) *Cone* (fig. 13-9): A *right circular cone* is a solid generated by the revolution of a right-angled triangle about one of its perpendicular sides which is fixed.

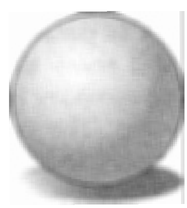
It has one circular base. Its axis joins the apex with the centre of the base to which it is perpendicular. Straight lines drawn from the apex to the circumference of the base-circle are all equal and are called *generators* of the cone. The length of the generator is the slant height of the cone.



Cylinder
FIG. 13-8



Cone
FIG. 13-9



Sphere
FIG. 13-10

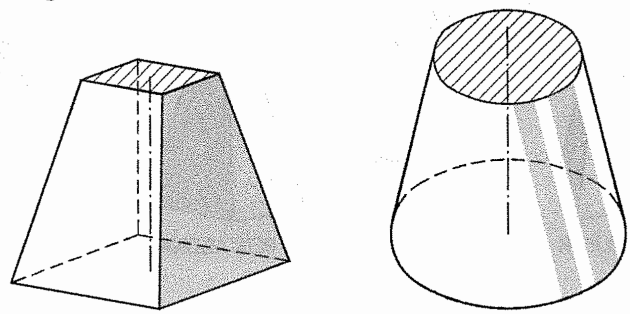
(iii) *Sphere* (fig. 13-10): A *sphere* is a solid generated by the revolution of a semi-circle about its diameter as the axis. The mid-point of the diameter is the centre of the sphere. All points on the surface of the sphere are equidistant from its centre.

Oblique cylinders and cones have their axes inclined to their bases.

(iv) *Frustum*: When a pyramid or a cone is cut by a plane parallel to its base, thus removing the top portion, the remaining portion is called its *frustum* (fig. 13-11).

(v) *Truncated*: When a solid is cut by a plane inclined to the base it is said to be *truncated*.

In this book mostly right and regular solids are dealt with. Hence, when a solid is named without any qualification, it should be understood as being right and regular.



Frustums
FIG. 13-11

13-2. PROJECTIONS OF SOLIDS IN SIMPLE POSITIONS

A solid in simple position may have its axis perpendicular to one reference plane or parallel to both. When the axis is perpendicular to one reference plane, it is parallel to the other. Also, when the axis of a solid is perpendicular to a plane, its base will be parallel to that plane. We have already seen that when a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

Therefore, the projection of a solid on the plane to which its axis is perpendicular, will show the true shape and size of its base.

Hence, when the axis is perpendicular to the ground, i.e. to the H.P., the top view should be drawn first and the front view projected from it.

When the axis is perpendicular to the V.P., beginning should be made with the front view. The top view should then be projected from it.

When the axis is parallel to both the H.P. and the V.P., neither the top view nor the front view will show the actual shape of the base. In this case, the projection of the solid on an auxiliary plane perpendicular to both the planes, viz. the side view must be drawn first. The front view and the top view are then projected from the side view. The projections in such cases may also be drawn in two stages.

(1) Axis perpendicular to the H.P.:

Problem 13-1. (fig. 13-12): *Draw the projections of a triangular prism, base 40 mm side and axis 50 mm long, resting on one of its bases on the H.P. with a vertical face perpendicular to the V.P.*

(i) As the axis is perpendicular to the ground i.e. the H.P. begin with the top view. It will be an equilateral triangle of sides 40 mm long, with one of its

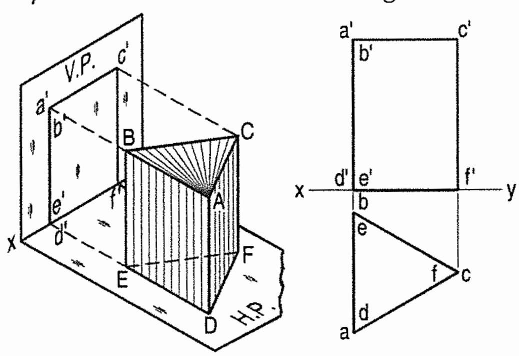


FIG. 13-12

sides perpendicular to xy. Name the corners as shown, thus completing the top view. The corners d, e and f are hidden and coincide with the top corners a, b and c respectively.

(ii) Project the front view, which will be a rectangle. Name the corners. The line $b'e'$ coincides with $a'd'$.

Problem 13-2. (fig. 13-13): *Draw the projections of a pentagonal pyramid, base 30 mm edge and axis 50 mm long, having its base on the H.P. and an edge of the base parallel to the V.P. Also draw its side view.*

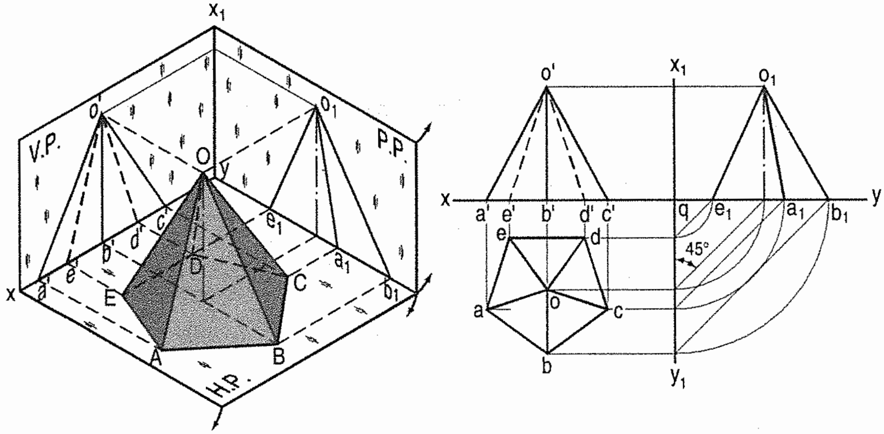


FIG. 13-13

(i) Assume the side DE which is nearer the V.P., to be parallel to the V.P. as shown in the pictorial view.

(ii) In the top view, draw a regular pentagon $abcde$ with ed parallel to and nearer xy. Locate its centre o and join it with the corners to indicate the slant edges.

(iii) Through o, project the axis in the front view and mark the apex o', 50 mm above xy. Project all the corners of the base on xy. Draw lines $o'a'$, $o'b'$ and $o'c'$ to show the visible edges. Show $o'd'$ and $o'e'$ for the hidden edges as dashed lines.

(iv) For the side view looking from the left, draw a new reference line x_1y_1 perpendicular to xy and to the right of the front view. Project the side view on it, horizontally from the front view as shown. The respective distances of all the points in the side view from x_1y_1, should be equal to their distances in the top view from xy. This is done systematically as explained below:

(v) From each point in the top view, draw horizontal lines upto x_1y_1. Then draw lines inclined at 45° to x_1y_1 (or xy) as shown. Or, with q, the point of intersection between xy and x_1y_1 as centre, draw quarter circles. Project up all the points to intersect the corresponding horizontal lines from the front view and complete the side view as shown in the figure. Lines o_1d_1 and o_1c_1 coincide with o_1e_1 and o_1a_1 respectively.

Problem 13-3. (fig. 13-14): *Draw the projections of (i) a cylinder, base 40 mm diameter and axis 50 mm long, and (ii) a cone, base 40 mm diameter and axis 50 mm long, resting on the H.P. on their respective bases.*

(i) Draw a circle of 40 mm diameter in the top view and project the front view which will be a rectangle [fig. 13-14(ii)].

(ii) Draw the top view [fig. 13-14(iii)]. Through the centre o, project the apex o', 50 mm above *xy*. Complete the triangle in the front view as shown.

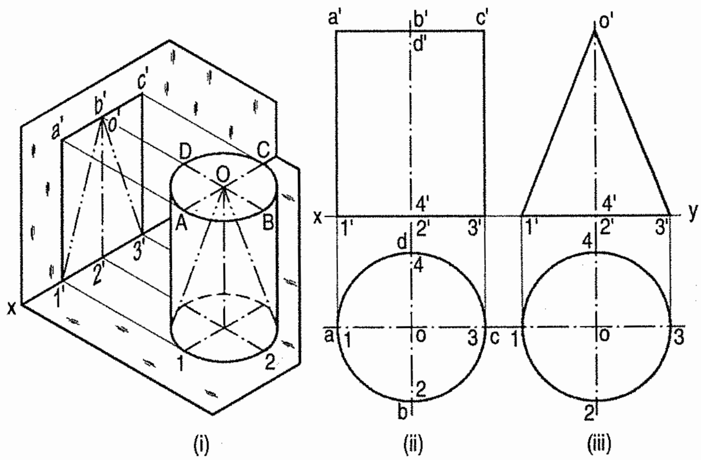


FIG. 13-14

In the pictorial view [fig. 13-14(i)], the cone is shown as contained by the cylinder.

Problem 13-4. (fig. 13-15): *A cube of 50 mm long edges is resting on the H.P. with its vertical faces equally inclined to the V.P. Draw its projections.*

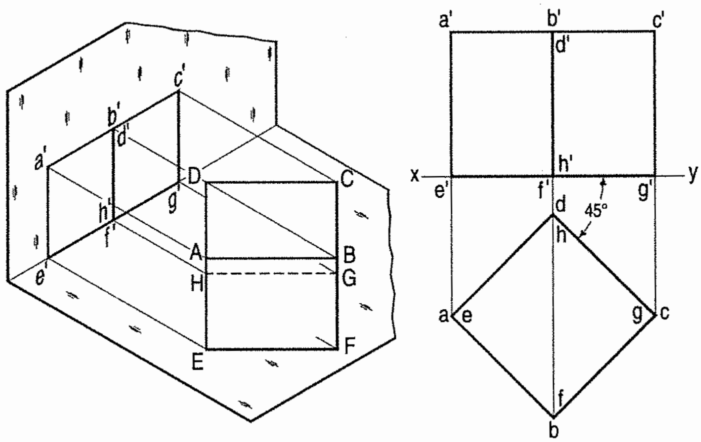


FIG. 13-15

Begin with the top view.

(i) Draw a square *abcd* with a side making 45° angle with *xy*.

(ii) Project up the front view. The line *d' h'* will coincide with *b' f'*.

Problem 13-5. (fig. 13-16): *Draw the projections of a hexagonal pyramid, base 30 mm side and axis 60 mm long, having its base on the H.P. and one of the edges of the base inclined at 45° to the V.P.*

(i) In the top view, draw a line *af* 30 mm long and inclined at 45° to *xy*. Construct a regular hexagon on *af*. Mark its centre *o* and complete the top view by drawing lines joining it with the corners.

(ii) Project up the front view as described in problem 13-2, showing the line *o'e'* and *o'f'* for hidden edges as dashed lines.

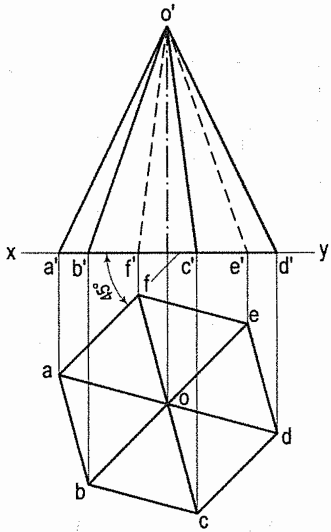


FIG. 13-16

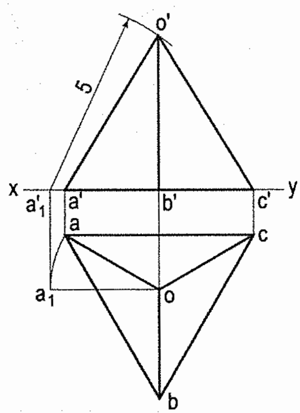


FIG. 13-17

Problem 13-6. (fig. 13-17): *A tetrahedron of 5 cm long edges is resting on the H.P. on one of its faces, with an edge of that face parallel to the V.P. Draw its projections and measure the distance of its apex from the ground.*

All the four faces of the tetrahedron are equal equilateral triangles of 5 cm side.

(i) Draw an equilateral triangle *abc* in the top view with one side, say *ac*, parallel to *xy*. Locate its centre *o* and join it with the corners.

(ii) In the front view, the corners *a'*, *b'* and *c'* will be in *xy*. The apex *o'* will lie on the projector through *o* so that its true distance from the corners of the base is equal to 5 cm.

(iii) To locate *o'*, make *oa* (or *ob* or *oc*) parallel to *xy*. Project a_1 to a'_1 on *xy*. With a'_1 as centre and radius equal to 5 cm cut the projector through *o* in *o'*. Draw lines *o'a'*, *o'b'* and *o'c'* to complete the front view. *o'b'* will be the distance of the apex from the ground.

(2) Axis perpendicular to the V.P.:

Problem 13-7. (fig. 13-18): *A hexagonal prism has one of its rectangular faces parallel to the H.P. Its axis is perpendicular to the V.P. and 3.5 cm above the ground.*

Draw its projections when the nearer end is 2 cm in front of the V.P. Side of base 2.5 cm long; axis 5 cm long.

(i) Begin with the front view. Construct a regular hexagon of 2.5 cm long sides with its centre 3.5 cm above *xy* and one side parallel to it.

(ii) Project down the top view, keeping the line for nearer end, viz. 1-4, 2 cm below *xy*.

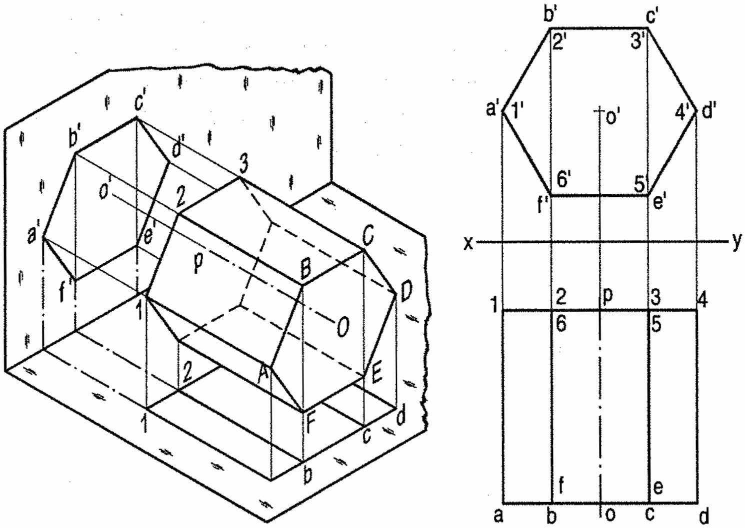


FIG. 13-18

Problem 13-8. (fig. 13-19): *A square pyramid, base 40 mm side and axis 65 mm long, has its base in the V.P. One edge of the base is inclined at 30° to the H.P. and a corner contained by that edge is on the H.P. Draw its projections.*

(i) Draw a square in the front view with the corner *d*' in *xy* and the side *d*'*c*' inclined at 30° to it. Locate the centre *o*' and join it with the corners of the square.

(ii) Project down all the corners in *xy* (because the base is in the V.P.). Mark the apex *o* on a projector through *o*'. Draw lines for the slant edges and complete the top view.

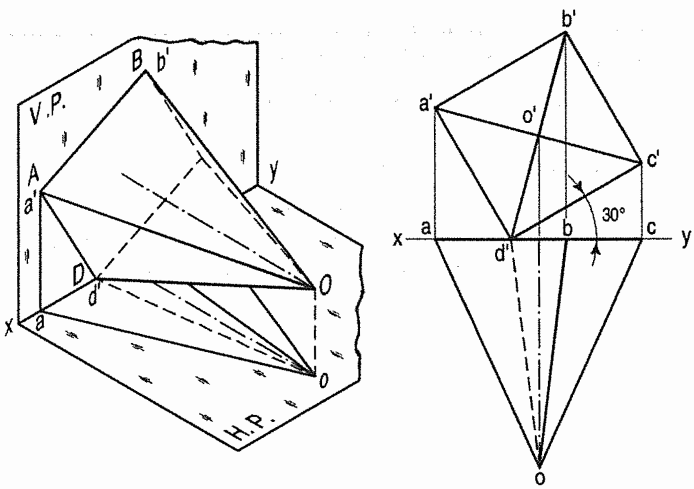


FIG. 13-19

(3) Axis parallel to both the H.P. and the V.P.:

Problem 13-9. (fig. 13-20): *A triangular prism, base 40 mm side and height 65 mm is resting on the H.P. on one of its rectangular faces with the axis parallel to the V.P. Draw its projections.*

As the axis is parallel to both the planes, begin with the side view.

(i) Draw an equilateral triangle representing the side view, with one side in *xy*.

(ii) Project the front view horizontally from this triangle.

(iii) Project down the top view from the front view and the side view, as shown.

This problem can also be solved in two stages as explained in the next article.

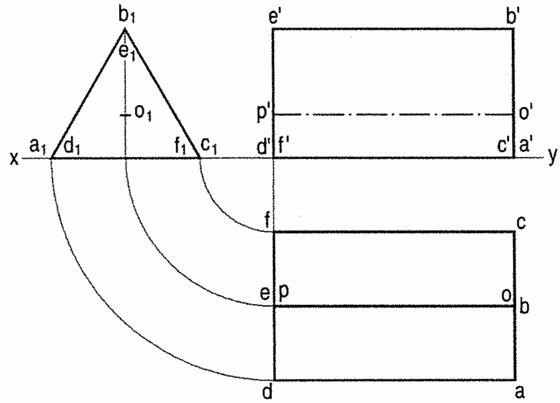


FIG. 13-20

EXERCISES 13(a)

Draw the projections of the following solids, situated in their respective positions, taking a side of the base 40 mm long or the diameter of the base 50 mm long and the axis 65 mm long.

1. A hexagonal pyramid, base on the H.P. and a side of the base parallel to and 25 mm in front of the V.P.
2. A square prism, base on the H.P., a side of the base inclined at 30° to the V.P. and the axis 50 mm in front of the V.P.
3. A triangular pyramid, base on the H.P. and an edge of the base inclined at 45° to the V.P.; the apex 40 mm in front of the V.P.
4. A cylinder, axis perpendicular to the V.P. and 40 mm above the H.P., one end 20 mm in front of the V.P.
5. A pentagonal prism, a rectangular face parallel to and 10 mm above the H.P., axis perpendicular to the V.P. and one base in the V.P.
6. A square pyramid, all edges of the base equally inclined to the H.P. and the axis parallel to and 50 mm away from both the H.P. and the V.P.
7. A cone, apex in the H.P. axis vertical and 40 mm in front of the V.P.
8. A pentagonal pyramid, base in the V.P. and an edge of the base in the H.P.

13-3. PROJECTIONS OF SOLIDS WITH AXES INCLINED TO ONE OF THE REFERENCE PLANES AND PARALLEL TO THE OTHER

When a solid has its axis inclined to one plane and parallel to the other, its projections are drawn in two stages.

(1) In the initial stage, the solid is assumed to be in simple position, i.e. *its axis perpendicular to one of the planes.*

If the axis is to be inclined to the ground, i.e. the H.P., it is assumed to be perpendicular to the H.P. in the initial stage. Similarly, if the axis is to be inclined to the V.P., it is kept perpendicular to the V.P. in the initial stage.

Moreover

(i) if the solid has an edge of its base parallel to the H.P. or in the H.P. or on the ground, that edge should be kept perpendicular to the V.P.; if the edge of the base is parallel to the V.P. or in the V.P., it should be kept perpendicular to the H.P.

(ii) If the solid has a corner of its base in the H.P. or on the ground, the sides of the base containing that corner should be kept equally inclined to the V.P.; if the corner is in the V.P., they should be kept equally inclined to the H.P.

(2) Having drawn the projections of the solid in its simple position, the final projections may be obtained by one of the following *two* methods:

(i) **Alteration of position:** The position of one of the views is altered as required and the other view projected from it.

(ii) **Alteration of reference line or auxiliary plane:** A new reference line is drawn according to the required conditions, to represent an auxiliary plane and the final view projected on it.

In the first method, the reproduction of a view accurately in the altered position is likely to take considerable time, specially, when the solid has curved surfaces or too many edges and corners. In such cases, it is easier and more convenient to adopt the second method. Sufficient care must however be taken in transferring the distances of various points from their respective reference lines.

After determining the positions of all the points for the corners in the final view, difficulty is often felt in completing the view correctly. The following sequence for joining the corners may be adopted:

(a) Draw the lines for the edges of the visible base. The base, which (compared to the other base) is further away from *xy* in one view, will be fully visible in the other view.

(b) Draw the lines for the longer edges. The lines which pass through the figure of the visible base should be dashed lines.

(c) Draw the lines for the edges of the other base.

It should always be remembered that, when two lines representing the edges cross each other, one of them must be hidden and should therefore be drawn as a dashed line.

13-3-1. AXIS INCLINED TO THE V.P. AND PARALLEL TO THE H.P.

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 29 for the following problem.

Problem 13-10. (fig. 13-21): *Draw the projections of a pentagonal prism, base 25 mm side and axis 50 mm long, resting on one of its rectangular faces on the H.P., with the axis inclined at 45° to the V.P.*

In the simple position, assume the prism to be on one of its faces on the ground with the axis perpendicular to the V.P.

Draw the pentagon in the front view with one side in xy and project the top view [fig. 13-21(i)].

The shape and size of the figure in the top view will not change, so long as the prism has its face on the H.P. The respective distances of all the corners in the front view from xy will also remain constant.

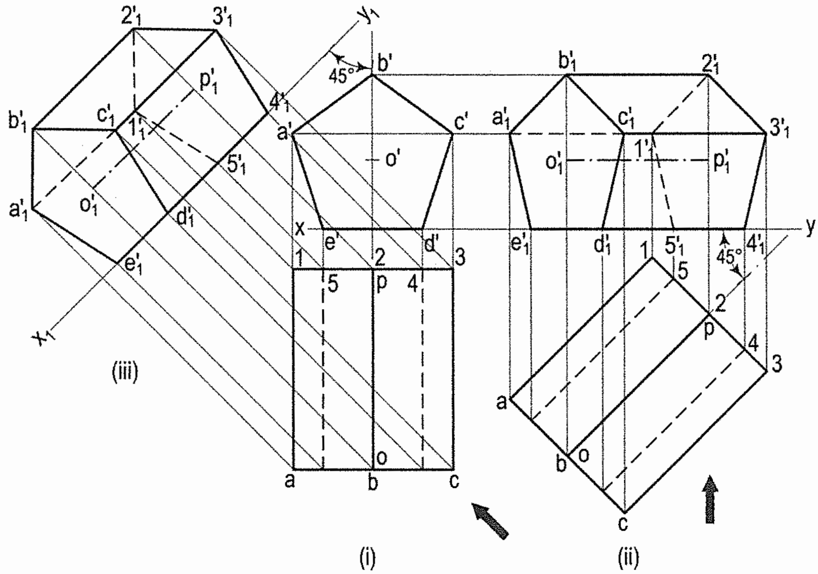


FIG. 13-21

Method I: [fig. 13-21(ii)]:

(i) Alter the position of the top view, i.e. reproduce it so that the axis is inclined at 45° to xy. Project all the points upwards from this top view and horizontally from the first front view, e.g. a vertical from a intersecting a horizontal from a' at a point a'_1.

(ii) Complete the pentagon $a'_1b'_1c'_1d'_1e'_1$ for the fully visible end of the prism. Next, draw the lines for the longer edges and finally, draw the lines for the edges of the other end. Note carefully that the lines $a'_11'_1$, $1'_12'_1$ and $1'_15'_1$ are dashed lines. $e'_15'_1$ is also hidden but it coincides with other visible lines.

Method II: [fig. 13-21(iii)]:

(i) Draw a new reference line x_1y_1, making 45° angle with the top view of the axis, to represent an auxiliary vertical plane.

(ii) Draw projectors from all the points in the top view perpendicular to x_1y_1 and on them, mark points keeping the distance of each point from x_1y_1 equal to its distance from xy in the front view. Join the points as already explained. The auxiliary front view and the top view are the required projections.

Problem 13-11. (fig. 13-22): *Draw the projections of a cylinder 75 mm diameter and 100 mm long, lying on the ground with its axis inclined at 30° to the V.P. and parallel to the ground.*

Adopt the same methods as in the previous problem. The ellipses for the ends should be joined by common tangents. Note that half of the ellipse for the hidden base will be drawn as dashed line.

Fig. 13-22(iii) shows the front view obtained by the method II.

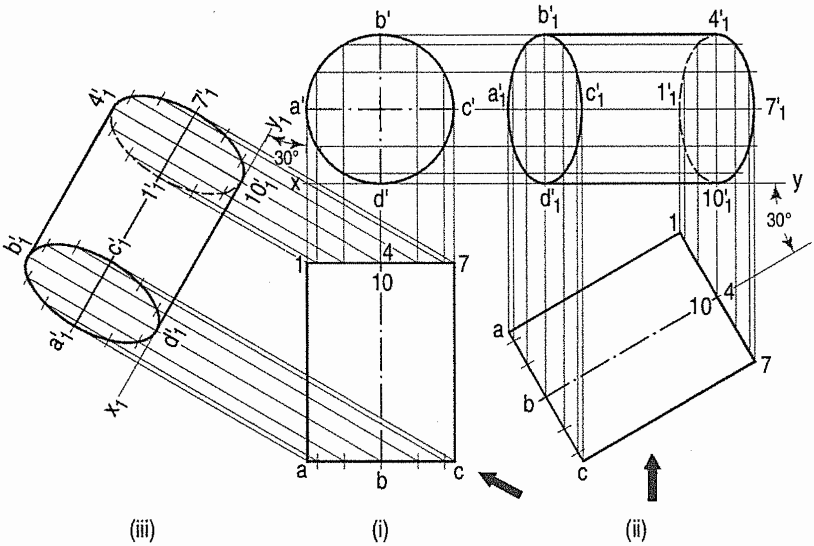


FIG. 13-22

13-3-2. AXIS INCLINED TO THE H.P. AND PARALLEL TO THE V.P.

Problem 13-12. (fig. 13-23): *A hexagonal pyramid, base 25 mm side and axis 50 mm long, has an edge of its base on the ground. Its axis is inclined at 30° to the ground and parallel to the V.P. Draw its projections.*

In the initial position assume the axis to be perpendicular to the H.P.

Draw the projections with the base in *xy* and its one edge perpendicular to the V.P. [fig. 13-23(i)].

If the pyramid is now tilted about the edge *AF* (or *CD*) the axis will become inclined to the H.P. but will remain parallel to the V.P. The distances of all the corners from the V.P. will remain constant.

The front view will not be affected except in its position in relation to *xy*. The new top view will have its corners at same distances from *xy*, as before.

Method I: [fig. 13-23(ii)]:

(i) Reproduce the front view so that the axis makes 30° angle with *xy* and the point *a'* remains in *xy*.

(ii) Project all the points vertically from this front view and horizontally from the first top view. Complete the new top view by drawing (a) lines joining the apex o'_1 with the corners of the base and (b) lines for the edges of the base.

The base will be partly hidden as shown by dashed line a_1b_1, e_1f_1 and f_1a_1. Similarly o_1f_1 and o_1a_1 are also dashed lines.

Method II: [fig. 13-23(iii)]:

(i) Through *a'* draw a new reference line x_1y_1 inclined at 30° to the axis, to represent an auxiliary inclined plane.

(ii) From the front view project the required top view on x_1y_1, keeping the distance of each point from x_1y_1 equal to the distance of its first top view from *xy*, viz. e_1q = *eb'* etc.

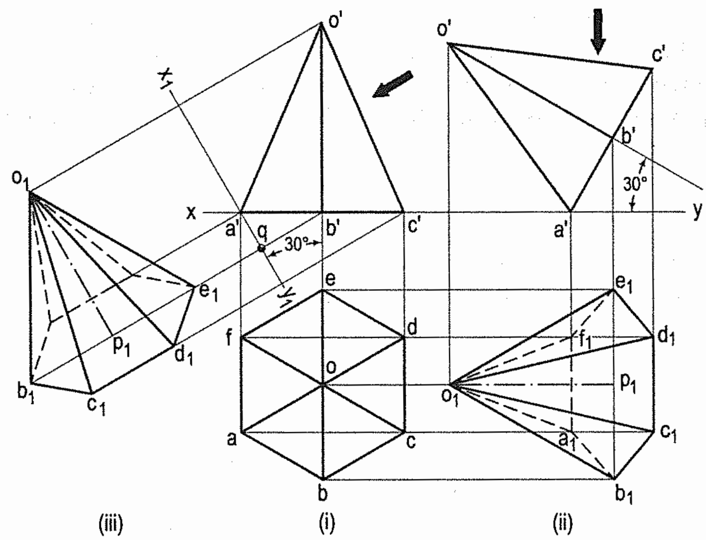


FIG. 13-23

Problem 13-13. (fig. 13-24): *Draw the projections of a cone, base 75 mm diameter and axis 100 mm long, lying on the H.P. on one of its generators with the axis parallel to the V.P.*

(i) Assuming the cone to be resting on its base on the ground, draw its projections.

(ii) Re-draw the front view so that the line *o*'7' (or *o*'1') is in *xy*. Project the required top view as shown. The lines from o_1 should be tangents to the ellipse.

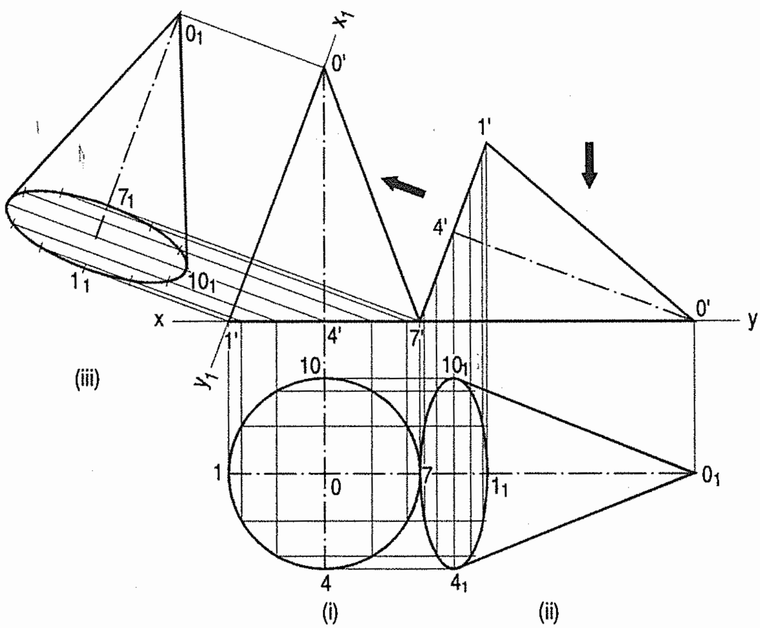


FIG. 13-24

The top view obtained by auxiliary-plane method is shown in fig. 13-24(iii). The new reference line x_1y_1 is so drawn as to contain the generator o'1' instead of *o*'7' (for sake of convenience). The cone is thus lying on the generator *o*'1'. Note that $1'1_1$ = 1'1, $o'o_1$ = 4'*o* etc. Also note that the base is fully visible in both the methods.

Problem 13-14. *The projections of a cylinder resting centrally on a hexagonal prism are given in fig. 13-25(i). Draw its auxiliary front view on a reference line inclined at 60° to xy.*

See fig. 13-25(ii) which is self-explanatory.

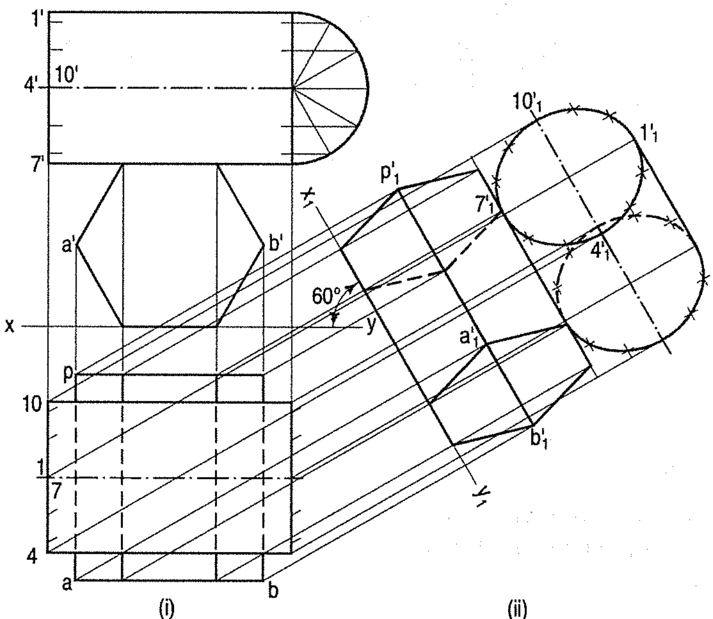


FIG. 13-25

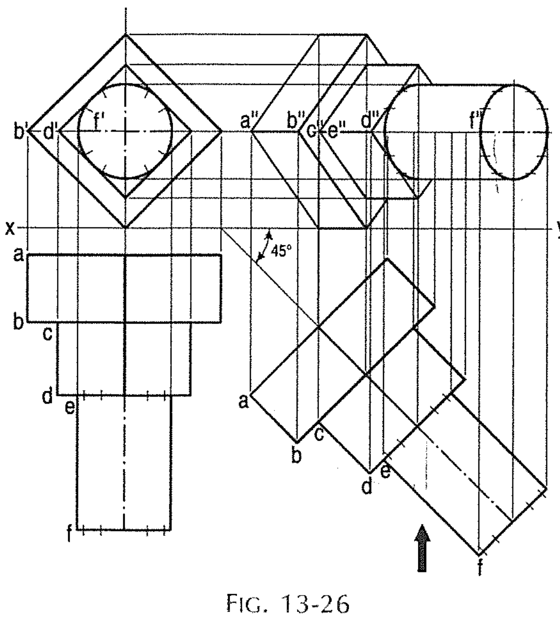


FIG. 13-26

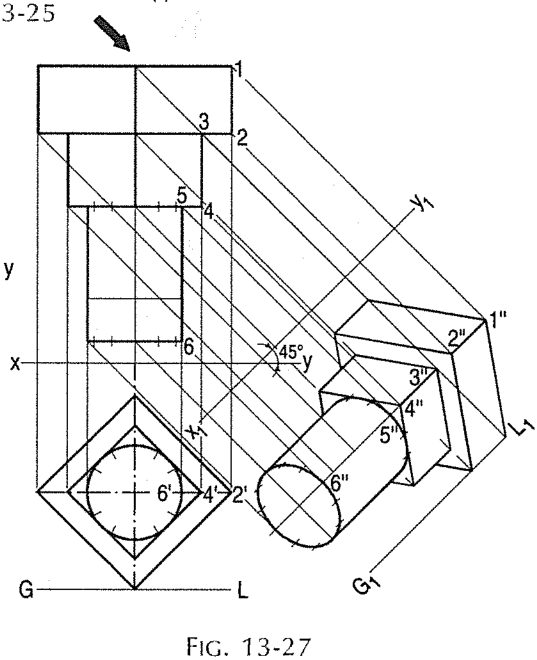


FIG. 13-27

Problem 13-15. (fig. 13-26 and fig. 13-27): *A square-headed bolt 25 mm diameter, 125 mm long and having a square neck has its axis parallel to the H.P. and inclined at 45° to the V.P.*

All the faces of the square head are equally inclined to the H.P. Draw its projections neglecting the threads and chamfer.

See fig. 13-26. The projections are obtained by the change-of-position method. The length of the bolt is taken shorter.

Fig. 13-27 shows the views in third-angle projection, obtained by the auxiliary-plane *method*.

Problem 13-16. (fig. 13-28): *A hexagonal prism, base 40 mm side and height 40 mm has a hole of 40 mm diameter drilled centrally through its ends. Draw its projections when it is resting on one of its corners on the H.P. with its axis inclined at 60° to the H.P. and two of its faces parallel to the V.P.*

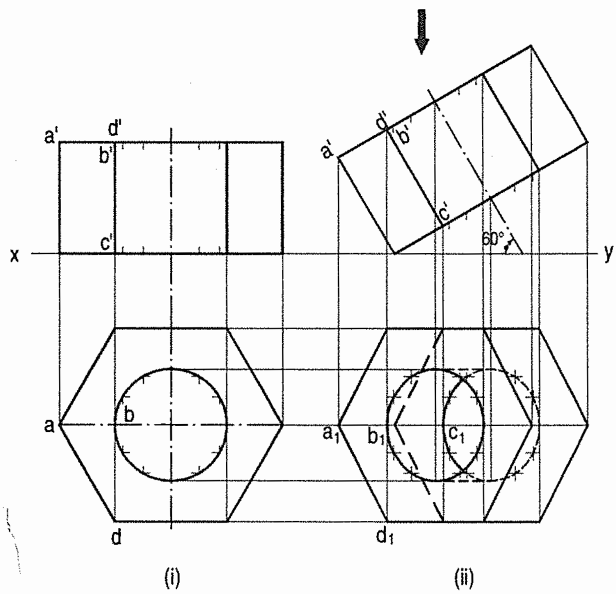


FIG. 13-28

(i) Begin with the top view and project up the front view assuming the axis to be vertical.

(ii) Tilt the front view, and project the required top view. Note that a part of the ellipse for the lower end of the hole will be visible.

Problem 13-17. (fig. 13-29): *The projections of a hopper made of tin sheet are given. Project another top view on an auxiliary inclined plane making 45° angle with the H.P.*

(i) Draw a new reference line x_1y_1 inclined at 45° to xy and project the required top view on it, from the front view.

(ii) Show carefully, the visible ellipses for the outer as well as the inner parts of the hopper rings.

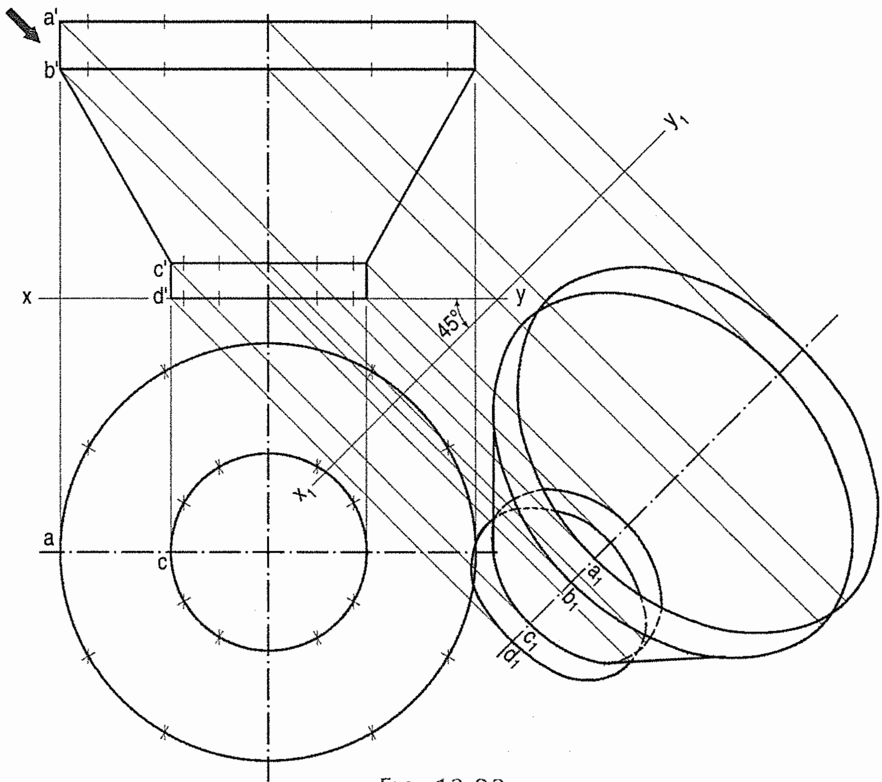


FIG. 13-29

13-4. PROJECTIONS OF SOLIDS WITH AXES INCLINED TO BOTH THE H.P. AND THE V.P.

The projections of a solid with its axis inclined to both the planes are drawn in three stages:

(i) Simple position

(ii) Axis inclined to one plane and parallel to the other

(iii) Final position.

The second and final positions may be obtained either by the alteration of the positions of the solid, i.e. the views, or by the alteration of reference lines.

Problem 13-18. *A square prism, base 40 mm side and height 65 mm, has its axis inclined at 45° to the H.P. and has an edge of its base, on the H.P and inclined at 30° to the V.P. Draw its projections.*

Method I: (fig. 13-30):

(i) Assuming the prism to be resting on its base on the ground with an edge of the base perpendicular to the V.P., draw its projections.

Assume the prism to be tilted about the edge which is perpendicular to the V.P., so that the axis makes 45° angle with the H.P.

(ii) Hence, change the position of the front view so that the axis is inclined at 45° to *xy* and *f'* (or *e'*) is in *xy*. Project the second top view.

Again, assume the prism to be turned so that the edge on which it rests, makes an angle of 30° with the V.P., keeping the inclination of the axis with the ground constant. The shape and size of the second top view will remain the same; only its position will change. In the front view, the distances of all the corners from xy will remain the same as in the second front view.

(iii) Therefore, reproduce the second top view making f_1g_1 inclined at 30° to xy. Project the final front view upwards from this top view and horizontally from the second front view, e.g. a vertical from a_1 and a horizontal from a' intersecting at a'_1. As the top end is further away from xy in the top view it will be fully visible in the front view. Complete the front view showing the hidden edges by dashed lines.

(iv) The second top view may be turned in the opposite direction as shown. In this position, the lower end of the prism, viz. $e'_1f'_1g'_1h'_1$ will be fully visible in the front view.

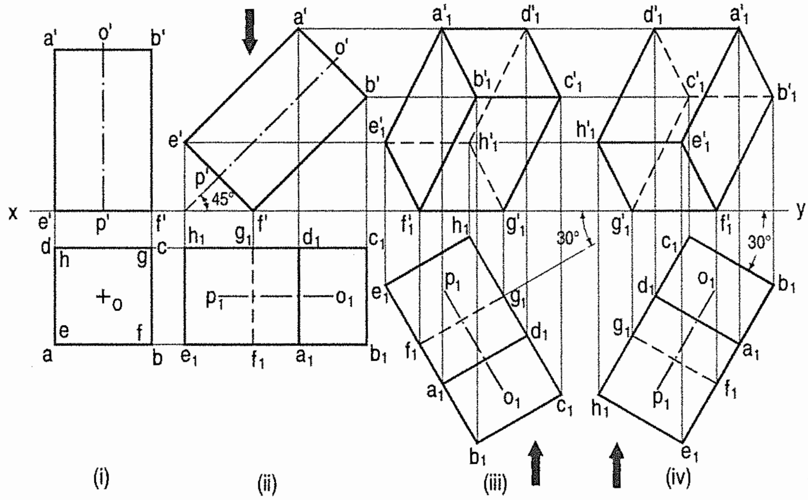


FIG. 13-30

Method II: (fig. 13-31):

(i) Draw the top view and the front view in simple position.

(ii) Through f', draw a new reference line x_1y_1 making 45° angle with the axis. On it, project the auxiliary top view.

(iii) Draw another reference line x_2y_2 inclined at 30° to the line f_1g_1. From the auxiliary top view, project the required front view, keeping the distance of each point from x_2y_2, equal to its distance (in the first front view) from x_1y_1 i.e. $a'_1q_1 = a'q$ etc. The problem is thus solved by change-of-reference line method only.

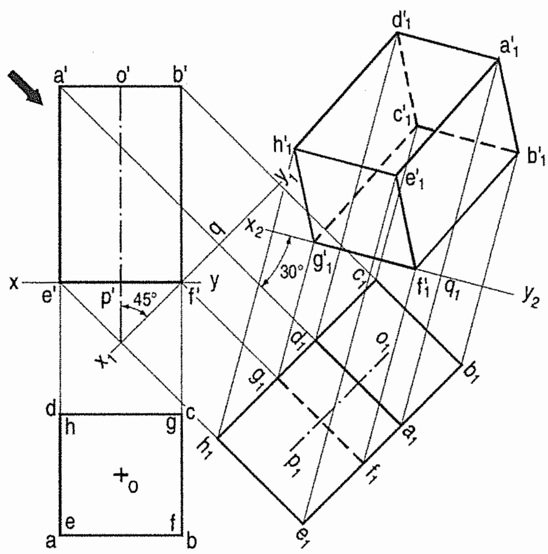


FIG. 13-31

Note: The new reference line satisfying the required conditions may be drawn in various positions, as explained in chapter 11.

Problem 13-19. (fig. 13-32): *Draw the projections of a cone, base 45 mm diameter and axis 50 mm long, when it is resting on the ground on a point on its base circle with (a) the axis making an angle of 30° with the H.P. and 45° with the V.P.; (b) the axis making an angle of 30° with the H.P. and its top view making 45° with the V.P.*

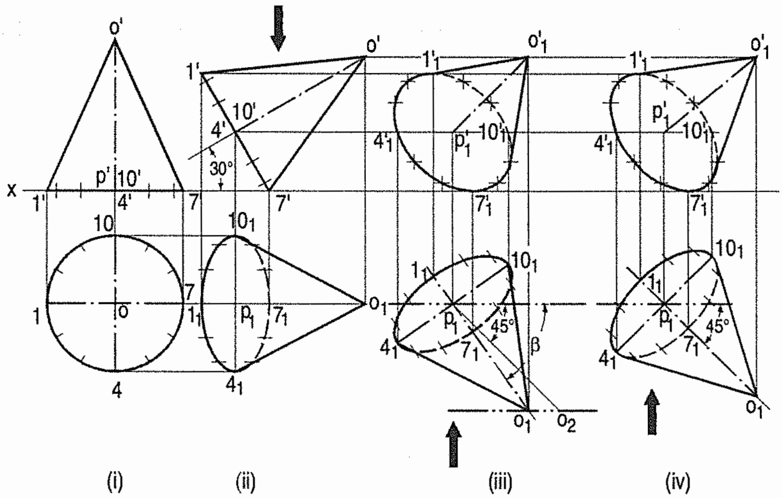


FIG. 13-32

(i) Draw the top view and the front view of the cone with the base on the ground.

(ii) Tilt the front view so that the axis makes 30° angle with *xy*. Project the second top view.

(a) In order that the axis may make an angle of 45° with the V.P., let us determine the apparent angle of inclination which the top view of the axis, viz. o_1p_1 should make with *xy* and which will be greater than 45°.

(iii) Mark any point p_1 below *xy*. Draw a line p_1o_2 equal to the true length of the axis, viz, *o'p'*, and inclined at 45° to *xy*. With p_1 as centre and radius equal to p_1o_1 (the length of the top view of the axis) draw an arc cutting the locus of o_2 at o_1. Then β is the apparent angle of inclination and is greater than 45°. Around p_1o_1 as axis, reproduce the second top view and project the final front view as shown.

Note that the base of the cone is not visible in the front view because it is nearer *xy* in the top view.

(b) When the top view of the axis is to make 45° angle with the V.P., it is evident that p_1o_1 should be inclined at 45° to *xy*. Hence, reproduce the top view accordingly and project the required front view [fig. 13-32(iv)].

Problem 13-20. (fig. 13-33): *A pentagonal pyramid, base 25 mm side and axis 50 mm long has one of its triangular faces in the V.P. and the edge of the base contained by that face makes an angle of 30° with the H.P. Draw its projections.*

(i) In the initial position, assume the pyramid as having its base in the V.P. and an edge of the base perpendicular to the H.P. The front view will have to be drawn first and the top view projected from it.

(ii) Change the position of the top view so that the line o1 (for the face o1 5) is in *xy*. Project the second front view.

(iii) Tilt this front view so that the line $1'_1 5'_1$ makes 30° angle with *xy*. Project the final top view. Note that the base is not visible in the top view as it is nearer *xy* in the front view.

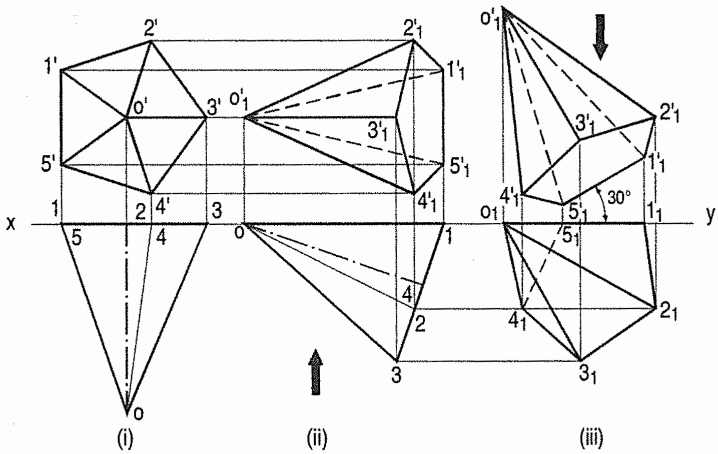


FIG. 13-33

Problem 13-21. (fig. 13-34): *A square pyramid, base 38 mm side and axis 50 mm long, is freely suspended from one of the corners of its base. Draw its projections, when the axis as a vertical plane makes an angle of 45° with the V.P. When a pyramid is suspended freely from a corner of its base, the imaginary line joining that corner with the centre of gravity of the pyramid will be vertical.*

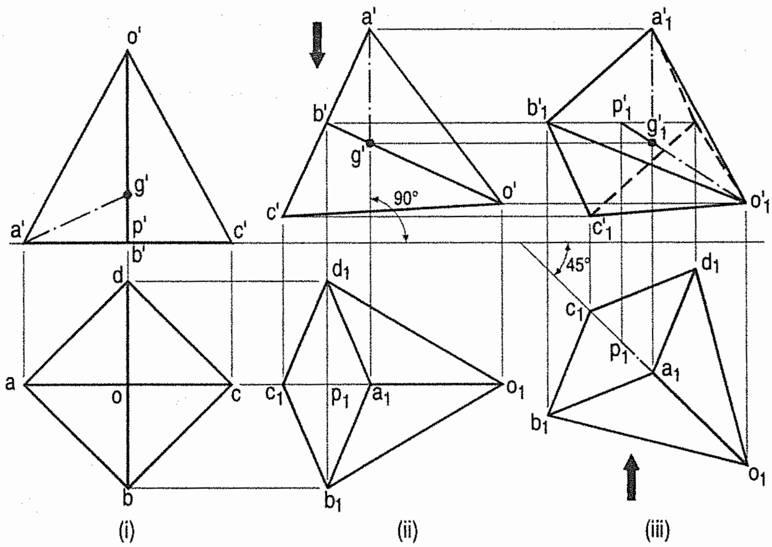


FIG. 13-34

The centre of gravity of a pyramid lies on its axis and at a distance equal to $\frac{1}{4}$ of the length of the axis from the base.

Assume the pyramid to be suspended from the corner *A* of the base.

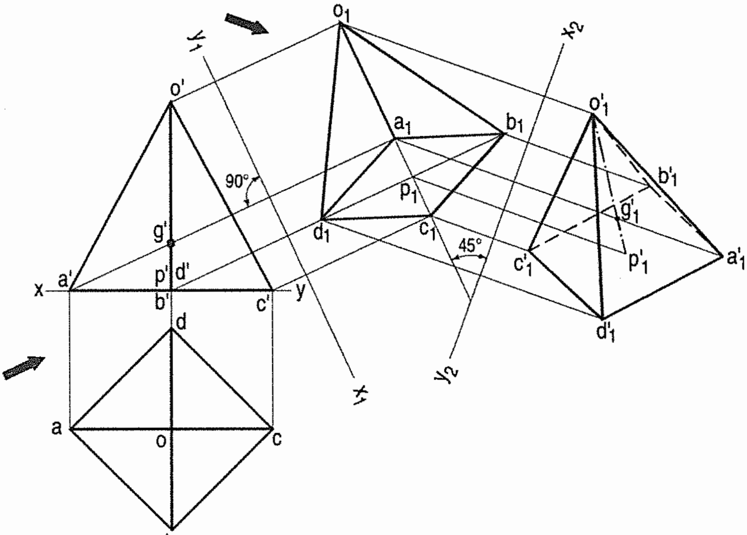


FIG. 13-35

In the initial position, the pyramid should be kept with its base on the ground and the line joining *A* with the centre of gravity *G*, parallel to the V.P. In the top view, *g* will coincide with *o* the top view of the axis.

(i) Draw a square *abcd* (in the top view) with *ag*, i.e. *ao* parallel to *xy*. Project the front view. Making *g'* at a distance equal to $\frac{1}{4}$ of the axis from *xy*. Join *a'* with *g'*.

(ii) Tilt the front view so that *a'g'* is perpendicular to *xy* and project the top view. The axis will still remain parallel to the V.P.

(iii) Reproduce this top view so that o_1p_1 (the top view of the axis) is inclined at 45° to *xy*. The axis as a vertical plane will thus be making 45° angle with the V.P. Project the final front view.

Fig. 13-35 shows the projections obtained by the change-of-reference-line method.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 30 for the following problem.

Problem 13-22. (fig. 13-36): *A hexagonal pyramid, base 25 mm side and axis 55 mm long, has one of its slant edges on the ground. A plane containing that edge and the axis is perpendicular to the H.P. and inclined at 45° to the V.P. Draw its projections when the apex is nearer the V.P. than the base.*

Assume the pyramid to be resting on the ground on its base with a slant edge parallel to the V.P.

(i) Draw the top view of the pyramid with a side of the hexagon parallel to *xy*. The lines *ao* and *do* for the slant edges will also be parallel to *xy*. Project the front view.

(ii) Tilt this front view so that *a'o'* or *d'o'* is in *xy*. Project the second top view.

(iii) Draw a new reference line x_1y_1 making 45° angle with o_1d_1 (the top view of the axis) and project the final front view.

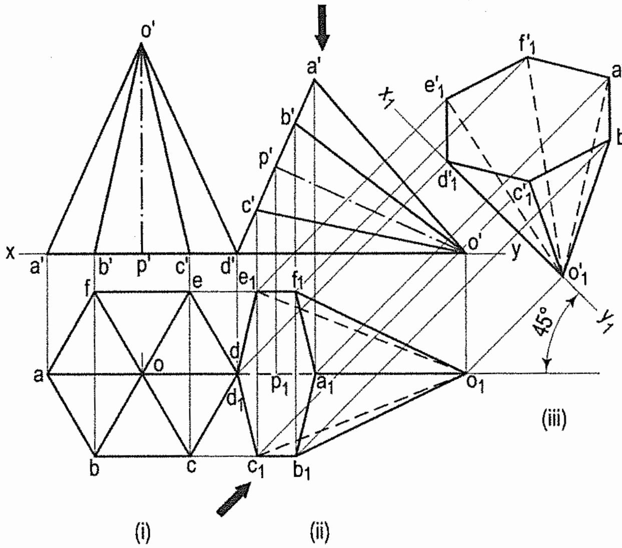


FIG. 13-36

The problem is thus solved by combination of the change-of-position and change-of-reference-line methods.

Problem 13-23. (fig. 13-37): *Draw the projections of a cube of 25 mm long edges resting on the H.P on one of its corners with a solid diagonal perpendicular to the V.P.*

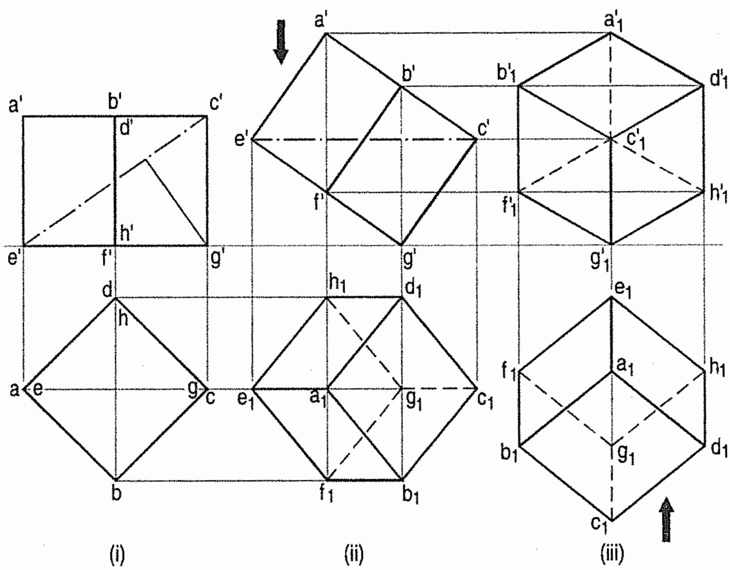


FIG. 13-37

Assume the cube to be resting on one of its faces on the H.P. with a solid diagonal parallel to the V.P.

(i) Draw a square *abcd* in the top view with its sides inclined at 45° to *xy*. The line *ac* representing the solid diagonals *AG* and *CE* is parallel to *xy*. Project the front view.

(ii) Tilt the front view about the corner g' so that the line $e'c'$ becomes parallel to xy. Project the second top view. The solid diagonal CE is now parallel to both the H.P. and the V.P.

(iii) Reproduce the second top view so that the top view of the solid diagonal, viz. e_1c_1 is perpendicular to xy. Project the required front view.

Problem 13-24. (fig. 13-38): *A triangular prism, base 40 mm side and axis 50 mm long, is lying on the H.P. on one of its rectangular faces with the axis perpendicular to the V.P. A cone, base 40 mm diameter and axis 50 mm long, is resting on the H.P. and is leaning centrally on a face of the prism, with its axis parallel to the V.P. Draw the projections of the solids and project another front view on a reference line making 60° angle with xy.*

It will first be necessary to draw the cone with its base on the H.P. to determine the length of its generator and to project the top view.

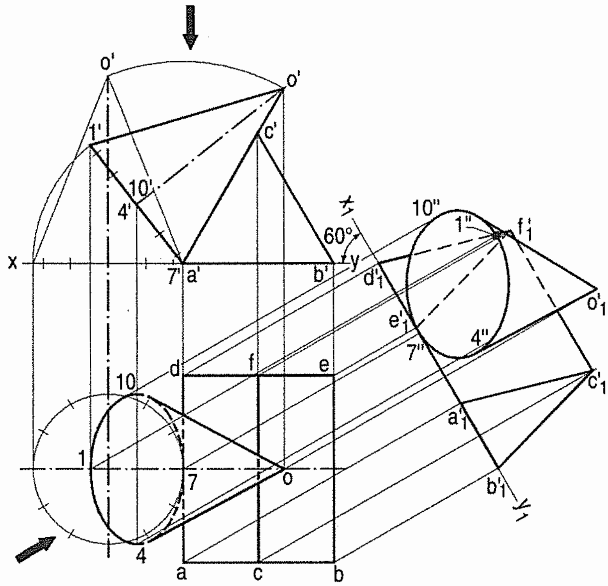


FIG. 13-38

Next, draw a triangle $a'b'c'$ for the prism and a triangle $o'1'7'$ for the cone as shown by the construction lines. Project the top view. Draw a reference line x_1y_1 and project the required front view as shown.

Problem 13-25. *A pentagonal prism is resting on one of the corners of its base on the H.P. The longer edge containing that corner is inclined at 45° to the H.P. The axis of the prism makes an angle of 30° to the V.P. Draw the projections of the solid.*

Also, draw the projections of the solid when the top view of axis is inclined at 30° to xy. Take the side of base 45 mm and height 70 mm.

(i) Assuming the prism to be resting on its base on the horizontal plane, draw its projections keeping one of the sides of its base perpendicular to xy.

(ii) Redraw the front view so that the edge $c'3'$ is inclined at 45° to xy. Project the required top view as shown in fig. 13-39(i).

(iii) Determine the apparent angle of inclination which the top view of the axis should make with xy when the axis makes an angle of 30° with the V.P.

(iv) Mark any point p_1 below xy. Draw a line p_1o_2 equal to the true length of the axis (70 mm) and inclined at 30° to xy. With p_1 as centre and radius equal to p_1o_1 (the length of the top view of the axis) draw an arc cutting the locus of o_2 at o_1. Then β is the required apparent angle of inclination. Considering p_1o_1 as axis, reproduce the second top view and project the final front view as shown in fig. 13-39(i).

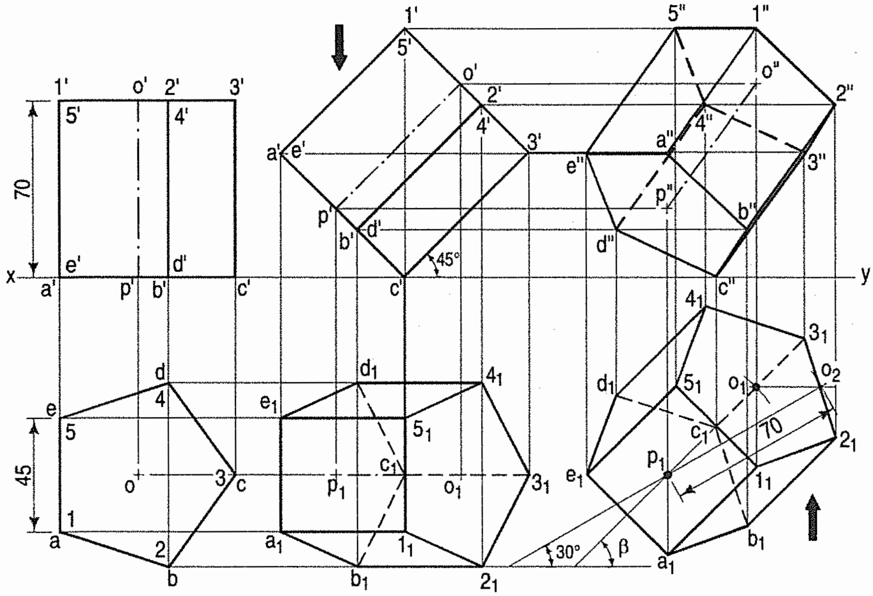


FIG. 13-39(i)

(v) When the top view of axis makes an angle of 30° with the V.P., it is evident that p_1o_1 is inclined at 30° to xy. Hence, reproduce the top view and the front view as shown in fig. 13-39(ii).

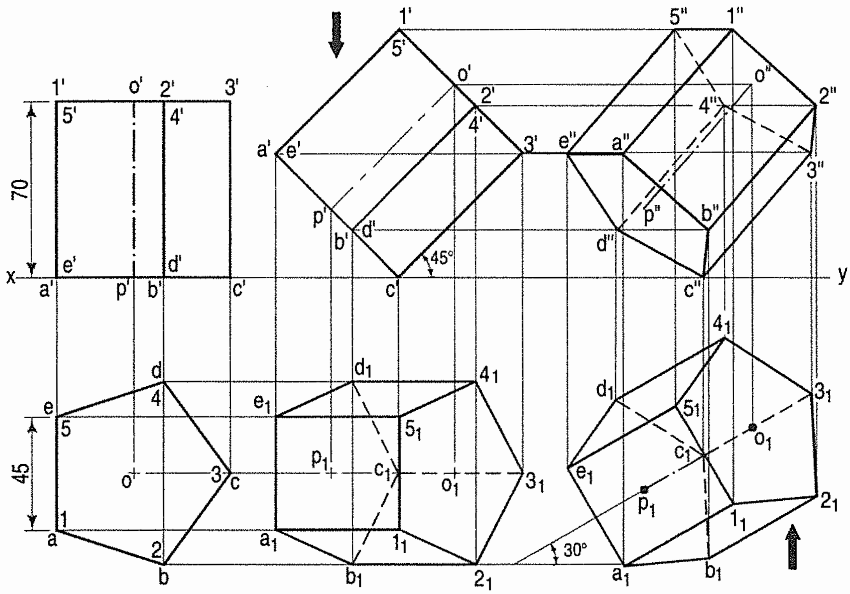


FIG. 13-39(ii)

Problem 13-26 (fig 13-40)**:** *A square prism, with the side of its base 40 mm and axis 70 mm long is lying on one of its base edges on the H.P. in such a way that this base edge makes an angle of 45° with the V.P. and the axis is inclined at 30° to the H.P. Draw the projections of the solid using the 'auxiliary plane method'.*

(i) In the initial position assume the axis of the prism to be perpendicular to the H.P. Draw the projections as shown.

(ii) Draw a new reference line x_1y_1 making an angle of 30° with the front view of the axis, to represent an auxiliary horizontal plane. Draw projectors from a', b' c', d' and 1', 2', 3', 4' perpendicular to x_1y_1 and on them, mark these points keeping the distance of each point from x_1y_1 equal to its distance from *xy* in the top view. Join the points as shown.

(iii) Draw another reference line x_2y_2 inclined at 45° to the line *a*1 (or *b*2). From the auxiliary top view, project the required new front view, keeping the distance of each point from x_2y_2, equal to its distance from x_1y_1, i.e. *q*'1" = *q*1' etc. Join the points as shown. Note that the view is obtained by observing the auxiliary top view from the top, along the projectors.

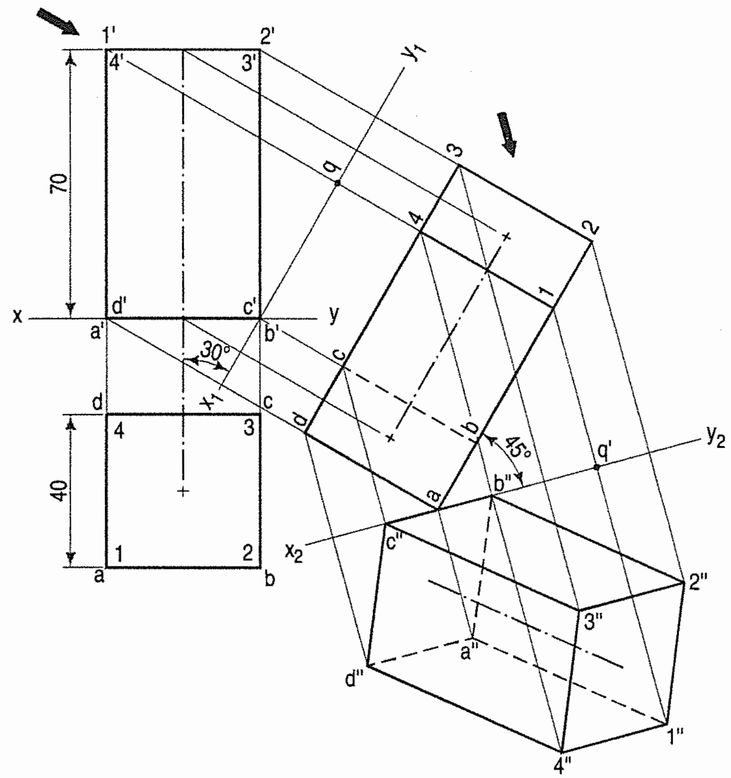


FIG. 13-40

Problem 13-27. *A hexagonal prism, with the side of the hexagon 30 mm and height of 70 mm is resting on the H.P. on one of the edges of its hexagonal base in such a way that, the edge is at 60° to the V.P. and the base is at 30° to the H.P. Draw to scale 1:1, the view from the front and the view from the top.*

Refer to fig. 13-41.

(i) Draw the top view and the front view in simple position keeping the axis perpendicular to the H.P.

(ii) Draw a new reference line x_1y_1 making 60° angle with the axis. On it, project the auxiliary top view.

(iii) Draw another reference line x_2y_2 inclined at 60° to the edge of base c_1d_1. From the auxiliary top view, project the required new front view, keeping the distance of each point from x_2y_2, equal to its distance from x_1y_1 i.e. $q3' = q'3''$ etc. Join the points as shown. It should be noted that the edge of base away from x_2y_2 will be observed as full lines and nearest lines from x_2y_2 will be dotted lines. i.e. $c''d''$, $d''e''$ and $e''f''$ are full lines while $f''a''$, $a''b''$ and $b''c''$ are dotted lines. Note that the view is drawn by observing the auxiliary top view from the top along the projectors.

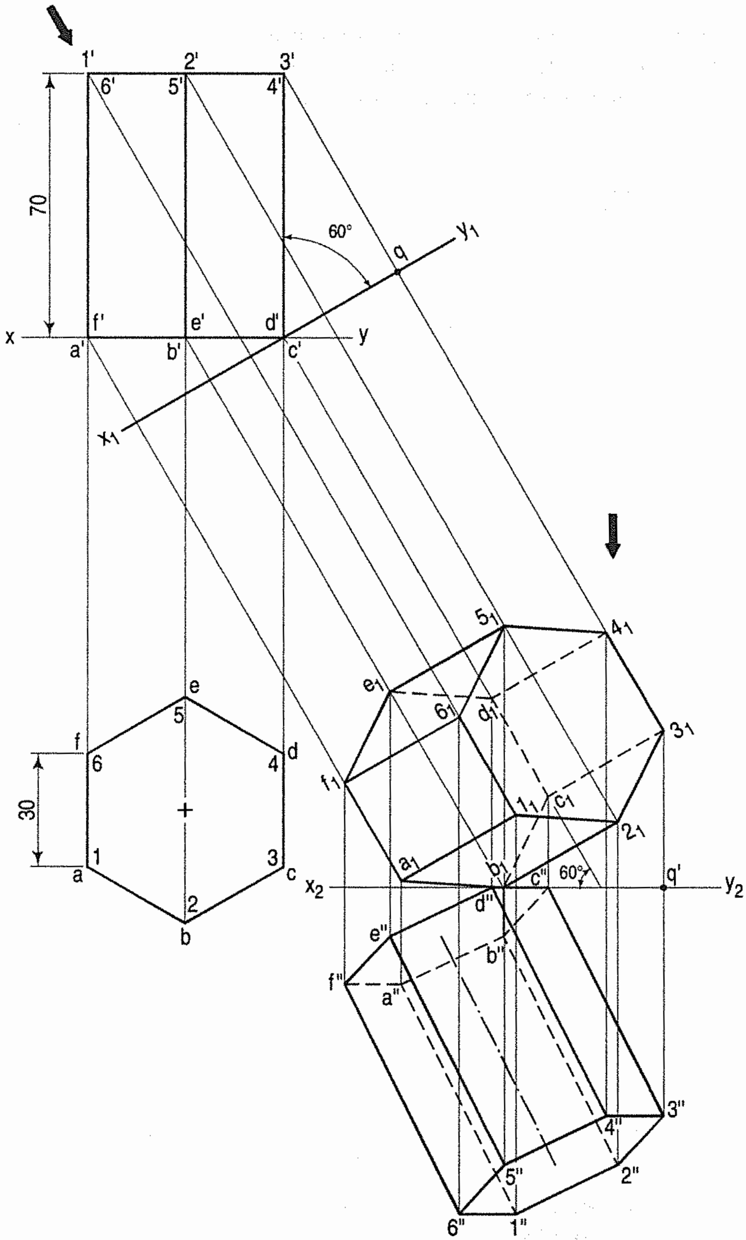


FIG. 13-41

Problem 13-28. *A regular pentagonal prism lies with its axis inclined at 60° to the H.P. and 30° to the V.P. The prism is 60 mm long and has a face width of 25 mm. The nearest corner is 10 mm away from the V.P. and the farthest shorter edge is 100 mm from the H.P. Draw the projections of the solid.*

(i) Draw initial position of the prism as shown in fig. 13-42.

(ii) With 4' as centre and radius equal to 100 mm, draw an arc. Mark tangent to the arc making 60° with the axis as shown. This is a new reference line x_1y_1. Project the required new top view.

(iii) Draw another reference line x_2y_2 inclined at 30° angle to the axis of new top view. Project the various points to obtain new front view as shown in fig. 13-42. Observe the auxiliary top view from the top along the projectors.

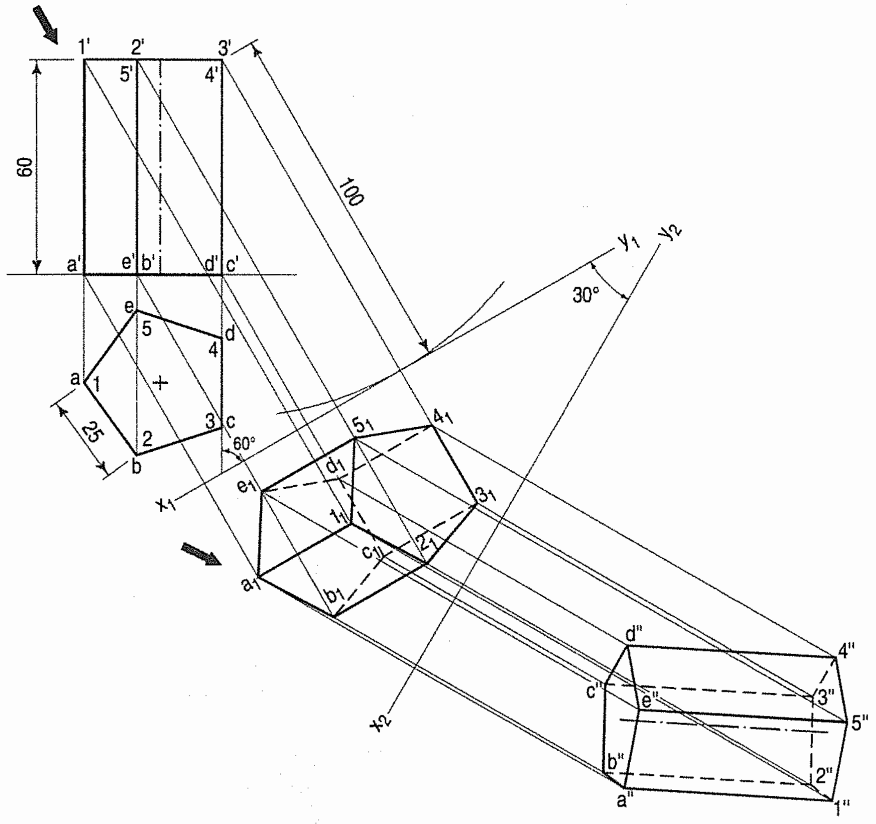


FIG. 13-42

Problem 13-29. *A square pyramid of 50 mm side of base and 50 mm length of axis is resting on one of its triangular faces on the H.P. having a slant edge containing that face parallel to the V.P. Draw the projections of the pyramid.*

(i) Assuming the axis of pyramid perpendicular to the H.P., draw the front view and the top view as shown in fig. 13-43.

(ii) Draw new reference line x_1y_1 coinciding with o'c' in the front view. Project new top view, keeping the distance of a_1, b_1...o_1 from x_1y_1 equal to the distance of *a*, *b*,...*o* from *xy*. Join these points.

(iii) Draw another reference line x_2y_2 parallel to the slant edge o_1c_1 or o_1b_1. Project new front view as shown. Observe auxiliary top view from the base *a, b, c, d, o* along projectors.

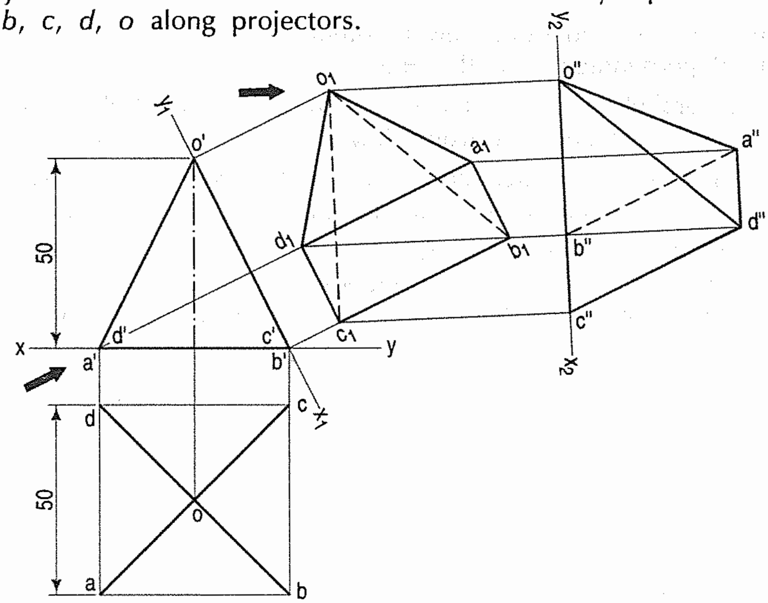


FIG. 13-43

Problem 13-30. *A regular pentagonal pyramid, base 30 mm side and height 80 mm rests on one edge of its base on the ground so that the highest point in the base is 30 mm above the ground. Draw its projection when the axis is parallel to the V.P.*

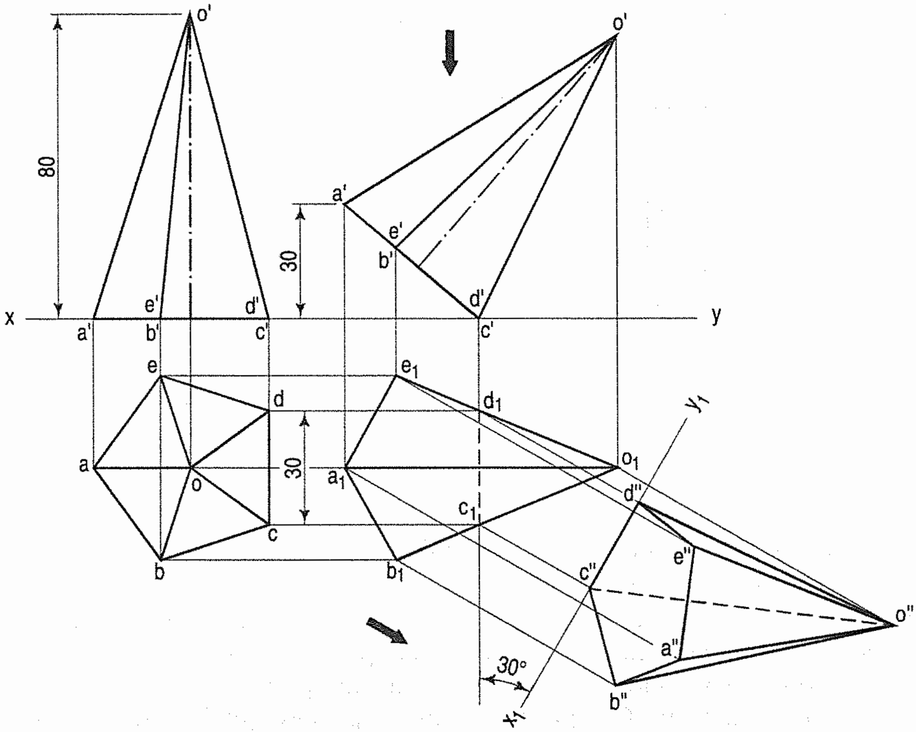


FIG. 13-44

Draw another front view on a reference line inclined at 30° to the edge on which it is resting so that the base is visible.

(i) Draw top view and front view in simple position assuming the axis of the pyramid perpendicular to the H.P.

(ii) Draw a parallel line at a distance of 30 mm from *xy*. Mark the point c' on the line *xy* and reproduce the front view as shown fig. 13-44.

(iii) Project points a', b', c' etc. and obtain new top view keeping distance of points a_1, b_1, c_1 etc. from *xy* equal to distance of *a*, *b*, *c*, etc. from the line *xy*.

(iv) Draw another reference line x_1y_1 making an angle of 30° with the side of base c_1d_1 and obtain a new front view as shown. Note that the base is visible. Observe from the base *a*, *b*, *c*, *d*, *e* along the projectors.

Problem 13-31. *A regular pentagonal pyramid with the sides of its base 30 mm and height 80 mm rests on an edge of the base. The base is tilted until its apex is 50 mm above the level of the edge of the base on which it rests. Draw the projection of the pyramid when the edge on which it rests, is parallel to the V.P. and the apex of the pyramid points towards V.P.*

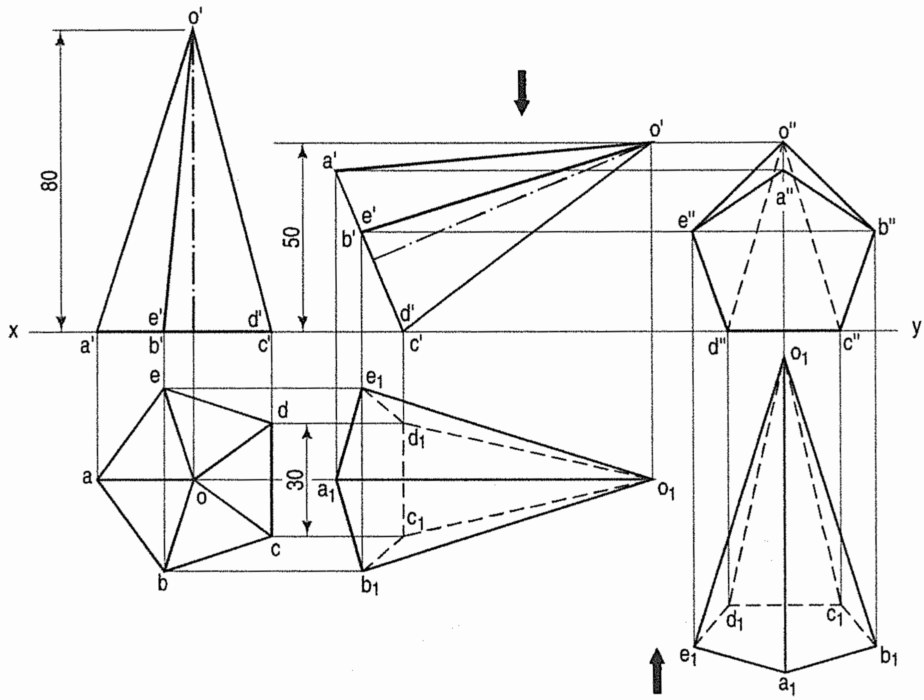


FIG. 13-45

(i) Draw top view and front view assuming the axis of the pyramid perpendicular to the H.P. as shown in fig. 13-45.

(ii) Draw a parallel line at a distance of 50 mm from *xy*. Reproduce the front view as shown. Draw projectors from points a', b', c' etc. vertically from the front view and horizontally from the points *a*, *b*, *c* etc. from the previous top view. Complete the new top view, joining the intersection of the projectors in the correct sequence as shown.

(iii) Redraw the top view keeping c_1d_1 parallel to xy. Project the points a_1, b_1, c_1 etc. vertically from the new top view and horizontal projectors from the points a', b', c' etc. of the front view. Join the intersection points of both the projectors in the correct sequence as shown.

Problem 13-32. *A right regular pentagonal pyramid, with the sides of the base 30 mm and height 65 mm rests on the edge of its base on the horizontal plane, the base being tilted until the vertex is at 60 mm above the H.P. Draw the projections of the pyramid when the edge on which it rests, is made parallel to FRP. Assuming the pyramid to be resting on its base on the horizontal plane, draw its projections keeping one of the sides of the base perpendicular to xy.*

Method I: Changing position of reference line [fig. 13-46(i)]:

(i) With o' as centre and radius equal to 60 mm, draw an arc. Draw the tangent to the arc passing through c' or d'. This is a new reference line x_1y_1. Project the required top view.

(ii) Draw another reference line x_2y_2 parallel to c_1d_1. Project new front view as shown. Observe auxiliary top view from the base a, b, c, d, e along the projectors.

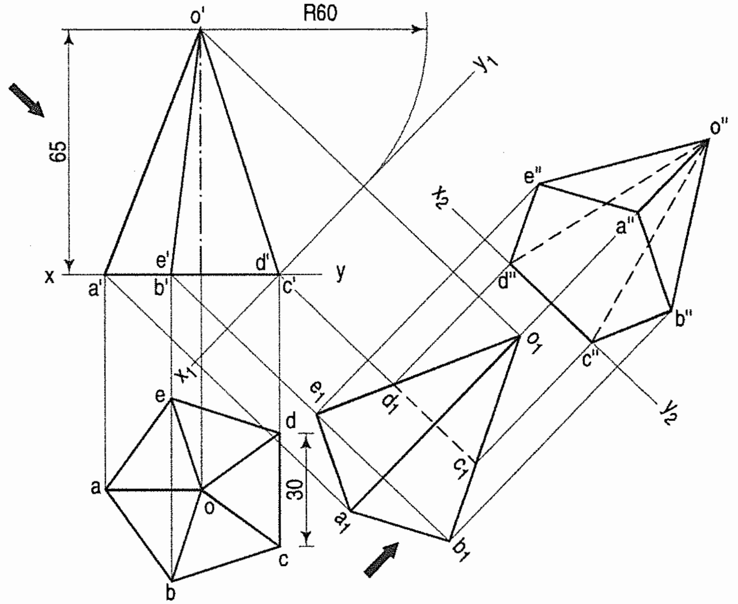


FIG. 13-46(i)

Method II: Changing positions of solid [(fig. 13-46(ii)]:

(i) Draw a line 60 mm parallel to xy. Mark point c' or d' on xy. With c' as centre and the radius equal to o'c', draw an arc cutting the above line at o'. With o' and c' as centre and radius equal to o'c' and a'c' draw an arc cutting each other at the point a'. Join a', o' and c' as shown. Project the required top view as shown.

(ii) Redraw the top view keeping side of base c'd' parallel to xy. Project new front view as shown.

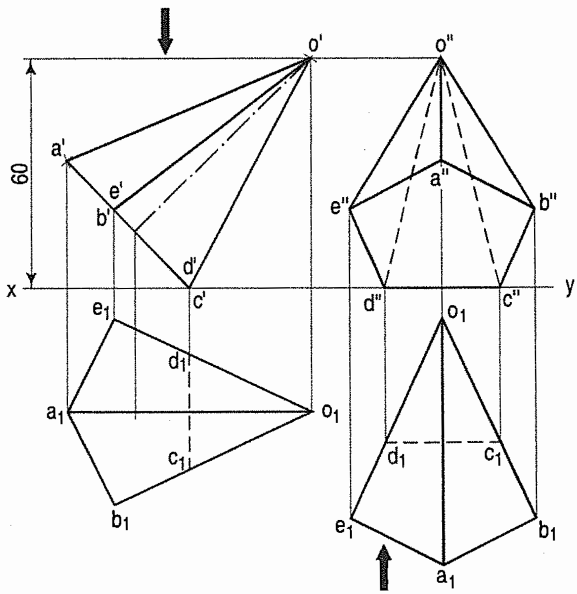


FIG. 13-46(ii)

Problem 13-33. *The front view, part top view and part auxiliary view of a casting are given in fig. 13-47(i). Project its side view.*

See fig. 13-47(ii).

The construction for the ellipse for 38 mm diameter circle has been shown in detail. Horizontal distances are taken from the auxiliary view. Other ellipses are drawn in the same manner.

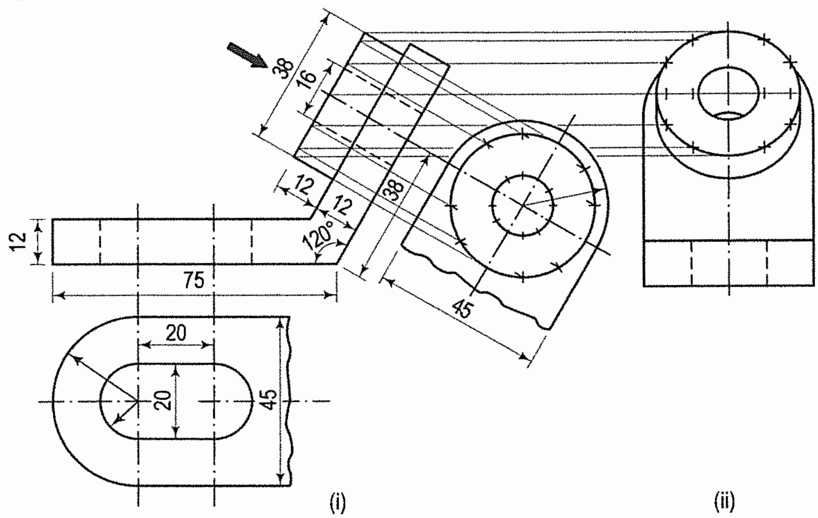


FIG. 13-47

13-5. PROJECTIONS OF SPHERES

The projection of a sphere in any position on any plane is always a circle whose diameter is equal to the diameter of the sphere (fig. 13-48). This circle represents the contour of the sphere.

A flat circular surface is formed when a sphere is cut by a plane. A hemisphere (i.e. a sphere cut by a plane passing through its centre) has a flat circular face of diameter equal to that of the sphere.

When it is placed on the ground on its flat face, its front view is a semi-circle, while its top view is a circle [fig. 13-49(i)].

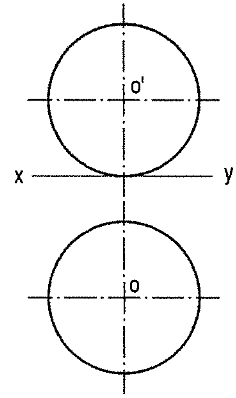


FIG. 13-48

When the flat face is inclined to the H.P. or the ground and is perpendicular to the V.P. it is seen as an ellipse (partly hidden) in the top view [fig. 13-49(ii)], while the contour of the hemisphere is shown by the arc of the circle drawn with radius equal to that of the sphere.

Fig. 13-50 shows the projections of a sphere, a small portion of which is cut off by a plane. Its flat face is perpendicular to the H.P. and inclined to the V.P. An ellipse is seen in the front view within the circle for the sphere.

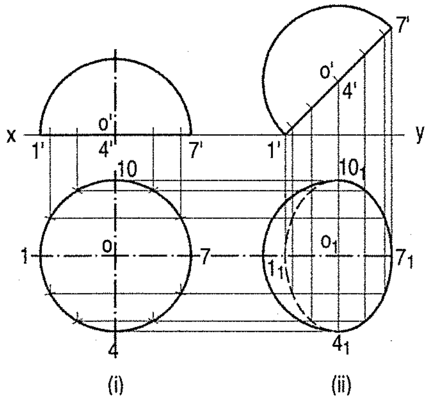


FIG. 13-49

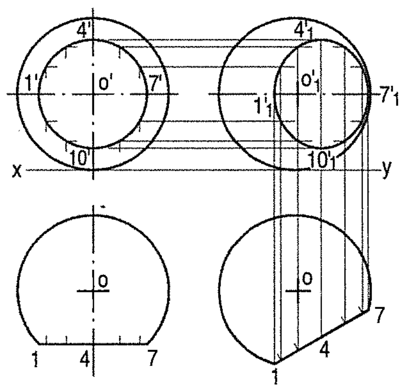


FIG. 13-50

When the flat face of a cut sphere is perpendicular to the V.P. and inclined to the H.P., its projections can be drawn as described in problem 13-34.

Problem 13-34. (fig. 13-51): *A brass flower-vase is spherical in shape with flat, circular top 35 cm diameter and bottom 25 cm diameter and parallel to each other. The greatest diameter is 40 cm. Draw the projections of the vase when its axis is parallel to the V.P. and makes an angle of 60° with the ground.*

(i) Draw the front view of the vase resting on its bottom with its axis vertical. Project the top view.

(ii) Tilt the front view so that the axis makes 60° angle with *xy* and project the top view. Note that a part of the ellipse for the bottom is also visible.

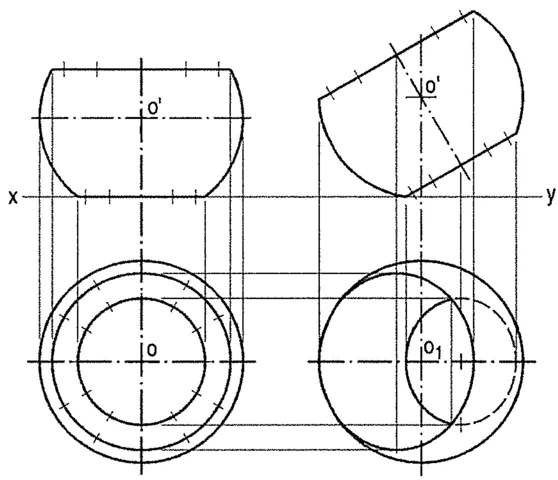


FIG. 13-51

(1) Spheres in contact with each other: Projections of two equal spheres resting on the ground and in contact with each other, with the line joining their centres parallel to the V.P., are shown in fig. 13-52.

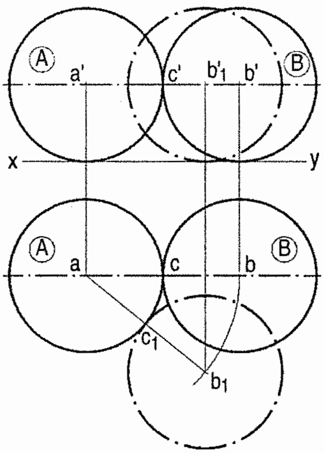


FIG. 13-52

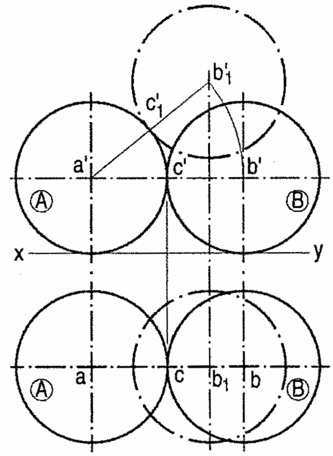


FIG. 13-53

As the spheres are equal in size, the line joining their centres is parallel to the ground also. Hence, both ab and $a'b'$ show the true length of that line (i.e. equal to the sum of the two radii or the diameter of the spheres). The point of contact between the two spheres is also visible in each view.

If the position of one of the spheres, say sphere *B*, is changed so that the line joining their centres is inclined to the V.P., in the front view, the centre b' will move along the line $a'b'$ to b'_1. The true length of the line joining the centres and the point of contact are now seen in the top view only.

When the sphere *B* is so moved that it remains in contact with the sphere *A* and the line joining their centres is parallel to the V.P., but inclined to the ground (fig. 13-53), the true length of that line and the point of contact are visible in the front view only.

Problem 13-35. (fig. 13-54): *Three equal spheres of 38 mm diameter are resting on the ground so that each touches the other two and the line joining the centres of two of them is parallel to the V.P.*

A fourth sphere of 50 mm diameter is placed on top of the three spheres so as to form a pile. Draw three views of the arrangement and find the distance of the centre of the fourth sphere above the ground.

As the spheres are resting on the ground and are equal in size, the lines joining their centres will be parallel to the ground. In the top view, the centres will lie at the corners of an equilateral triangle of sides equal to the sum of the two radii, i.e. 40 mm.

Draw (in the top view) an equilateral triangle *abc* of 40 mm long sides with one side, say *ab*, parallel to *xy*. At its corners, draw three circles of 40 mm diameter. Project the front view. The centres will lie on a line parallel to and 20 mm above *xy*.

When the fourth sphere is placed on top, its centre *d* in the top view will be in the centre of the triangle. In the front view, it will lie on a projector through *d*.

The true distance between the centre of the top sphere and that of any one of the bottom spheres will be equal to the sum of the two radii, viz. 20 mm + 25 mm, i.e. 45 mm.

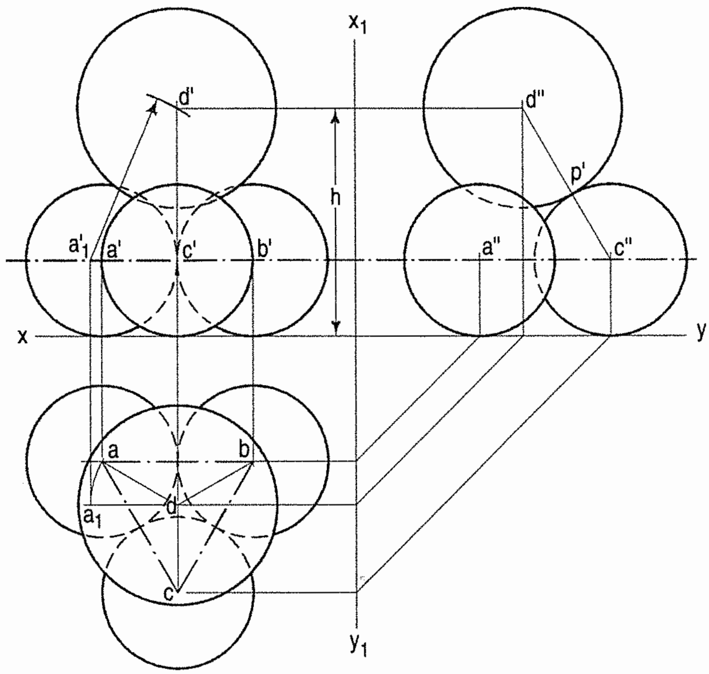


FIG. 13-54

But as none of the lines *da*, *db* or *dc* is parallel to *xy*, their front views will not show their true lengths. Therefore, to locate the position of the centre of the top sphere in the front view,

(i) make one of the lines, say *da*, parallel to *xy*;

(ii) project a_1 to a'_1 on the path of *a'* and

(iii) with a'_1 as centre and radius equal to 45 mm, draw an arc cutting the projector through *d* at the required point *d'*. With *d'* as centre and radius equal to 25 mm, draw the required circle which will be partly hidden as shown. *h* is the distance of the centre of the sphere from the ground.

(iv) Project the side view. As *c'd'* is parallel to the new reference line, *c''d''* will be equal to 45 mm and the point of contact *p'* between the spheres having centres *c* and *d* will be visible.

(2) Unequal spheres: When two unequal spheres are on the ground and are in contact with each other, their point of contact and the true length of the line joining their centres will be seen in the front view if that line is parallel to the V.P. In the top view, the length of the line will be shorter but will remain constant even when it is inclined to the V.P.

Problem 13-36. (fig. 13-55): *Three spheres A, B and C of 75 mm, 50 mm and 30 mm diameters respectively, rest on the ground each touching the other two. Draw their projections and show the three points of contact when the line joining the centres of the spheres A and B is parallel to the V.P.*

(i) With centre *a'* and radius equal to 37.5 mm, draw a circle of sphere *A*, mark *a'* at 37.5 mm above *xy* in front view. With *a'b'* equal to 62.5 mm, mark point *b'* 25 mm above *xy*. With *b'* as centre and radius equal to 25 mm, draw a circle of sphere *B* in front view.

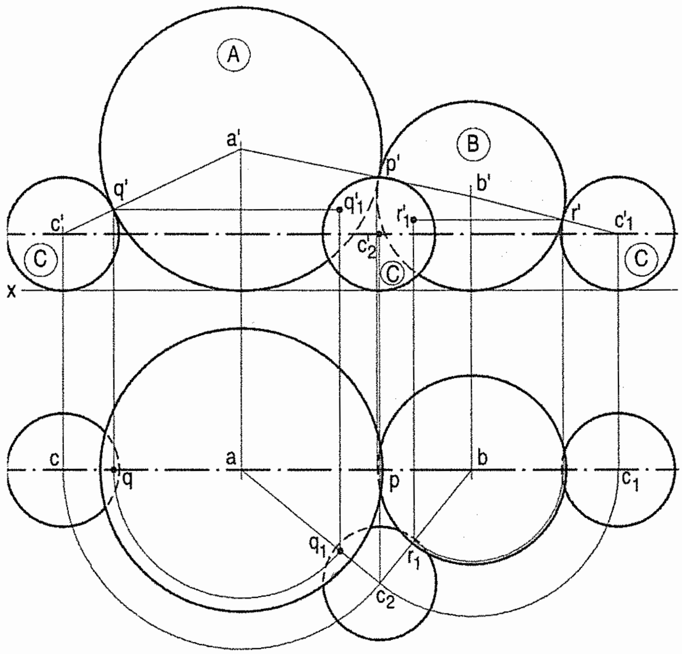


FIG. 13-55

(ii) Project the centres and obtain points *a* and *b* on a line parallel to *xy* in top view. With *a* as centre and radius equal to 37.5 mm of sphere *A*, and with *b* as centre and radius equal to 25 mm of sphere *B*, draw circles in the top view.

(iii) Similarly, draw the views of sphere *C* in contact with spheres *A* and *B*.

(iv) With *a* as centre and radius equal to *ac*, and with *b* as centre and radius equal to bc_1, draw arcs intersecting each other at c_2. With c_2 centre draw top view of the sphere *C*.

(v) Draw the projector through c_2 to cut the path of c' at c'_2. Then c'_2 is the required centre of the sphere *C* in the front view. *p*, q_1 and r_1, and *p'*, q'_1 and r'_1 are the points of contact in the top view and the front view respectively.

Problem 13-37. (fig. 13-56): *A square prism, base 20 mm side and axis 50 mm long, is resting on its base on the ground with two faces perpendicular to the V.P. Determine the radius of four equal spheres resting on the ground, each touching a face of the prism and other two spheres. Draw the projections of the arrangement.*

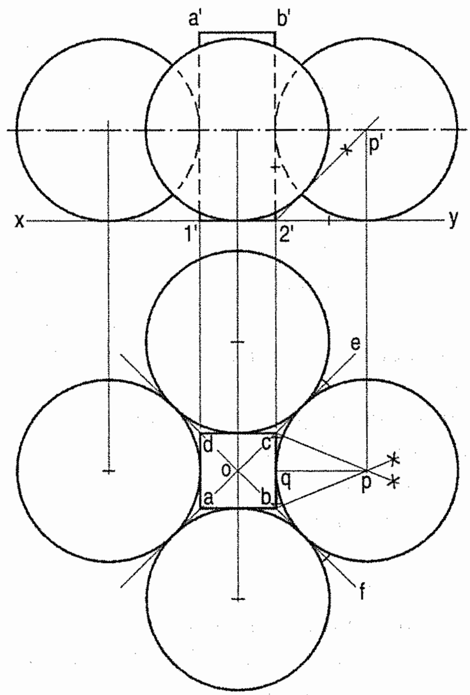


FIG. 13-56

(i) Draw the front and top views of the prism. In the top view, draw diagonals of the square (intersecting each other at *o*) and produce them on both sides.

(ii) Draw the bisectors of angles *bce* and *cbf* intersecting each other at *p*. From *p*, draw a perpendicular *pq* to *bc*. Then *pq* is the required radius of the sphere and *p* is the centre of the circle for the sphere.

(iii) Obtain the other three centres in the same manner. Or, with *o* as centre and radius equal to *op*, draw a circle to cut the centre lines through *o* at the required centres. Draw the four circles.

(iv) Draw a bisector of angle *b'2'y* intersecting the projector through *p* at *p'*. Then *p'* is the centre of the sphere in the front view. The centres for the other circles will lie on the horizontal line through *p'*. Project their exact positions from the top view and draw the circles.

Problem 13-38. (fig. 13-57): *Six equal spheres are resting on the ground, each touching other two spheres and a triangular face of a hexagonal pyramid resting on its base on the ground.*

Draw the projections of the solids when a side of the base of the pyramid is perpendicular to the V.P.

Determine the diameter of each sphere. Base of the pyramid 20 mm side; axis 50 mm long.

(i) Draw the projections of the pyramid in the required position. Assuming the solid to be a prism, locate the positions of the centre of one sphere (viz. *p* and *p'*) in the two views.

(ii) Draw a line joining *p'* with *a'* (the centre of the base) which coincides with 2'. The centre of the required sphere will lie on this line. Draw a bisector of angle o'3'y cutting *a'p'* at *c'*. Draw a line *c'q'* perpendicular to *xy*.

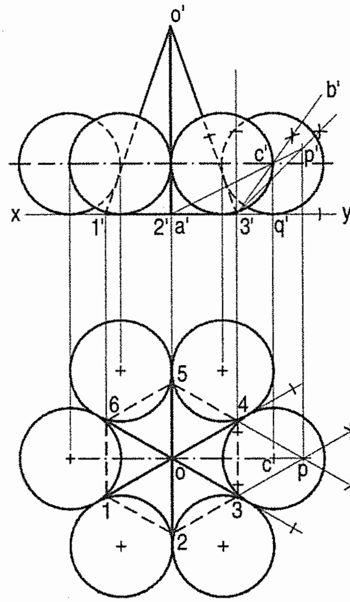


FIG. 13-57

(iii) With c' as centre and radius *c'q'*, draw one of the required circles. Project *c'* to *c* on *op* in the top view. Then *c* is the centre of the circle in the top view. Other centres may be located in the top view as shown and projected down in the front view.

(iv) Draw the six circles in the top view and four in the front view as shown in the fig. 13-57.

Problem 13-39. *The projections of a paper-weight with a spherical knob are given in fig. 13-58(i). Draw the two views and project another top view when its flat base makes an angle of 60° with the H.P.*

See fig. 13-58(ii) which is self-explanatory.

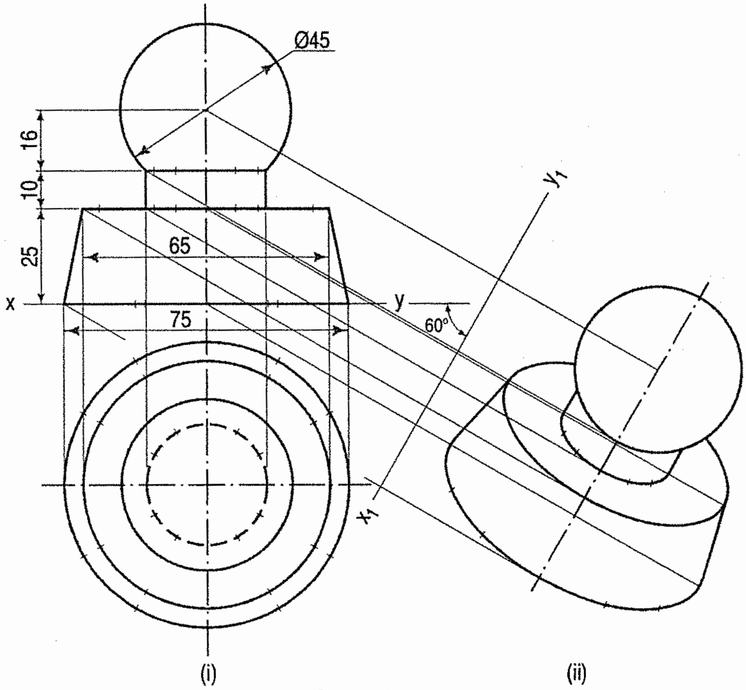


FIG. 13-58

Problem 13-40. (fig. 13-59): *A vertical hexagonal prism of base side 20 mm and thickness 15 mm has one side of hexagon perpendicular to the V.P. A right cone of 34 mm diameter and height 40 mm is placed on the top face of prism such that the base of cone touches top surface of prism while the axes of both coincide. Draw the front view and top view of the combined object. Draw also projections when axes of combined solid is inclined at 35° with auxiliary plane.*

(i) Draw the top view of hexagonal prism keeping one of sides perpendicular to *xy*. (i.e. *ab* or *ed*). Project above *xy* line and draw the front view of prism of height 15 mm.

(ii) Inscribe circle in the top view touching sides of the prism. Project it in the front view and mark the height of the cone as shown.

(iii) Draw auxiliary x_1y_1 inclined at 35° with the axes of the combined solids.

(iv) Draw the projectors from the various points of combined solids in the front view.

(v) Taking distance of various points from the top view of combined solids from *xy* and mark same distances along the respective projectors.

(vi) Complete auxiliary top view as shown.

Problem 13-41. (fig. 13-60): *A vertical cylindrical disc of thickness 10 mm and diameter 50 mm is resting on the ground. A vertical frustum of pentagonal pyramid, having bottom of 20 mm sides, top face of 40 mm sides with 60 mm height is resting on the top surface of the disc so that axes of the both solids coincide. Take one of sides of the base of pentagon is perpendicular to V.P. Draw the projections of combined solid when the axis of combined solids is inclined to 30° with the H.P.*

(i) Draw the top view of frustum of pyramid (pentagon) keeping one of the sides perpendicular to *xy* as shown.

(ii) Project the front view marking height of cylindrical disc and frustum of the pyramid 10 mm and 60 mm respectively.

(iii) Draw a line at angle of 30° with *xy*. (As axis is inclined with H.P., its inclination observed in the front view).

(iv) Reproduce the front view considering inclined line as axes of the combined solids.

(v) Draw the vertical projectors from various points of the front view.

(vi) Draw horizontal projectors to intersect respective vertical projectors. Obtain the auxiliary top view as shown.

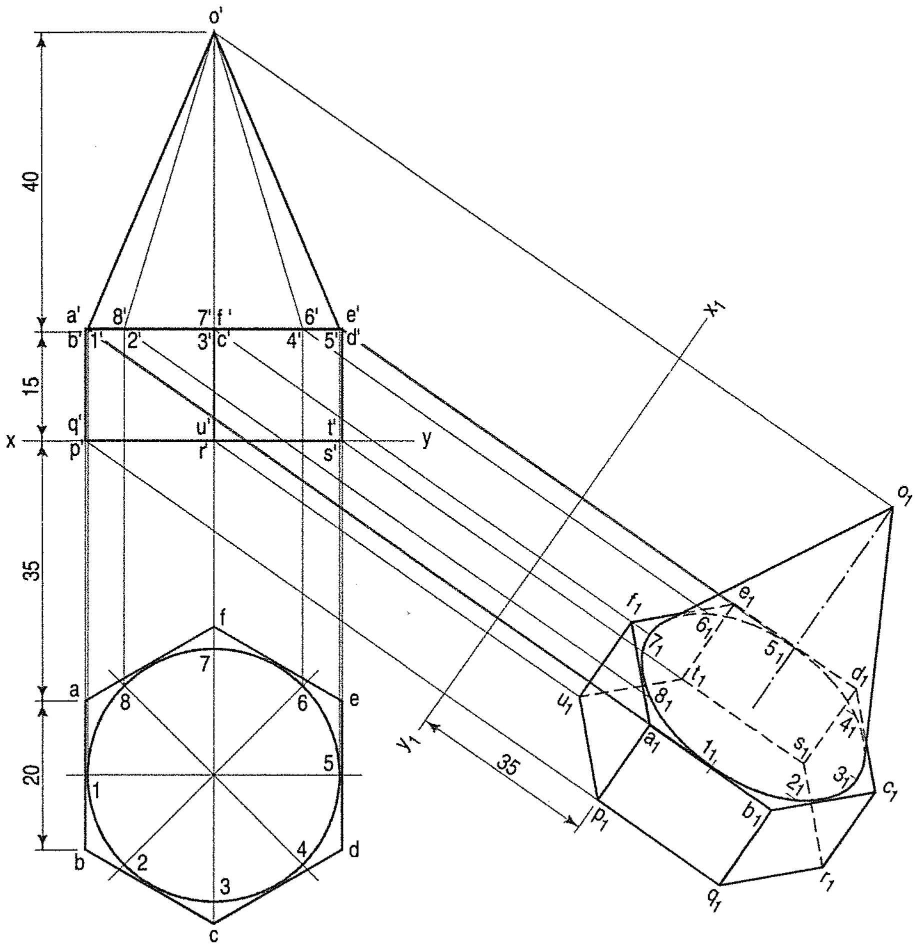


FIG. 13-59

Problem 13-42. (fig. 13-61): *A right hexagonal prism of side 25 mm and 20 mm thick with one side of the base is perpendicular to the V.P. resting on the ground. A vertical frustum of square pyramid of base 20 mm sides and top face side 30 mm and height 50 mm is resting on the prism such that one side of square makes 45° with the V.P. Assume that axes of both solids are coinciding. Draw the projections of the combined solids when top corner of the square pyramid is 70 mm above the ground (H.P.). Determine angle of combined solids with the H.P.*

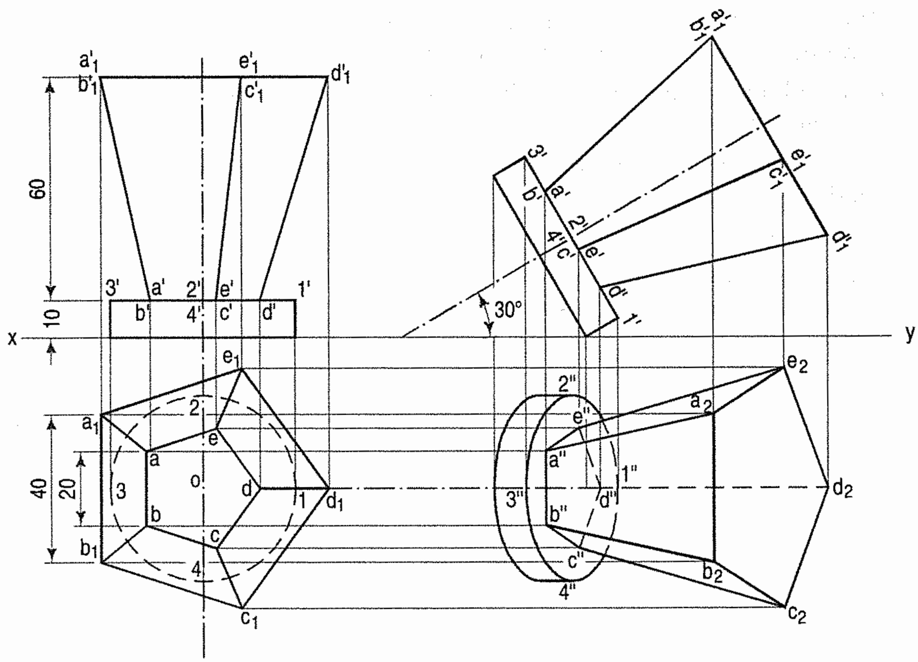


FIG. 13-60

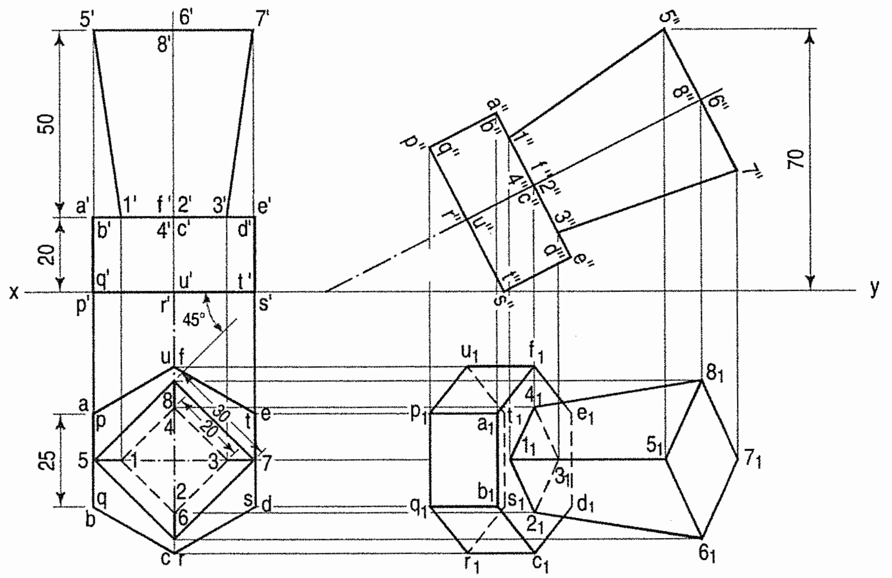


FIG. 13-61

(i) Draw the top view and front view as shown in figure. Keep one side of the hexagonal perpendicular to the *xy*.

(ii) Project the front view as shown.

(iii) Draw a parallel line at distance 70 mm away from *xy*. Reproduce the front view of the combined solids as shown.

(iv) Draw the projectors from the new front view.

(v) Draw from the top view horizontal projectors to intersect respective projectors drawn from the new front view.

(vi) Complete the top view as shown.

EXERCISES 13(b)

1. A rectangular block 75 mm × 50 mm × 25 mm thick has a hole of 30 mm diameter drilled centrally through its largest faces. Draw the projections when the block has its 50 mm long edge parallel to the H.P. and perpendicular to the V.P. and has the axis of the hole inclined at 60° to the H.P.
2. Draw the projections of a square pyramid having one of its triangular faces in the V.P. and the axis parallel to and 40 mm above the H.P. Base 30 mm side; axis 75 mm long.
3. A cylindrical block, 75 mm diameter and 25 mm thick, has a hexagonal hole of 25 mm side, cut centrally through its flat faces. Draw three views of the block when it has its flat faces vertical and inclined at 30° to the V.P. and two faces of the hole parallel to the H.P.
4. Draw three views of an earthen flower pot, 25 cm diameter at the top, 15 cm diameter at the bottom, 30 cm high and 2.5 cm thick, when its axis makes an angle of 30° with the vertical.
5. A tetrahedron of 75 mm long edges has one edge parallel to the H.P. and inclined at 45° to the V.P. while a face containing that edge is vertical. Draw its projections.
6. A hexagonal prism, base 30 mm side and axis 75 mm long, has an edge of the base parallel to the H.P. and inclined at 45° to the V.P. Its axis makes an angle of 60° with the H.P. Draw its projections.
7. A pentagonal prism is resting on a corner of its base on the ground with a longer edge containing that corner inclined at 45° to the H.P. and the vertical plane containing that edge and the axis inclined at 30° to the V.P. Draw its projections. Base 40 mm side; height 65 mm.
8. Draw three views of a cone, base 50 mm diameter and axis 75 mm long, having one of its generators in the V.P. and inclined at 30° to the H.P., the apex being in the H.P.
9. A square pyramid, base 40 mm side and axis 90 mm long, has a triangular face on the ground and the vertical plane containing the axis makes an angle of 45° with the V.P. Draw its projections.
10. A frustum of a pentagonal pyramid, base 50 mm side, top 25 mm side and axis 75 mm long, is placed on its base on the ground with an edge of the base perpendicular to the V.P. Draw its projections. Project another top view on a reference line parallel to the line which shows the true length of the slant edge. From this top view, project a front view on an auxiliary vertical plane inclined at 45° to the top view of the axis.
11. Draw the projections of a cone, base 50 mm diameter and axis 75 mm long, lying on a generator on the ground with the top view of the axis making an angle of 45° with the V.P.
12. The front view, incomplete top view and incomplete auxiliary top view of a casting are given in fig. 13-47. Draw all the three views completely in the third-angle projection.

13. A line sketch (in two views) of a shed with a curved roof is given in fig. 13-62. Draw its front view on an auxiliary vertical plane inclined at 60° to the V.P. All dimensions are in metres. Scale, 10 mm = 0.5 m.

14. The front view of a hexagonal pyramid [base 25 mm side] having one of its triangular faces resting centrally on a triangular face of a square pyramid [base 50 mm side and axis 50 mm long] is given in fig. 13-63. The plane containing the two axes is parallel to the V.P. Draw the top view of the solids. From this top view, project a front view on a reference line x_1y_1 inclined at 30° to xy; (ii) from the given front view, project another top view on a reference line x_2y_2 inclined at 45° to xy.

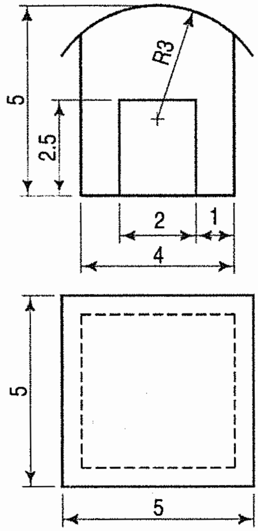


FIG. 13-62

15. A cube of 50 mm long edges is resting on the ground with its vertical faces equally inclined to the V.P. A hexagonal pyramid, base 25 mm side and axis 50 mm long, is placed centrally on top of the cube so that their axes are in a straight line and two edges of its base parallel to the V.P. Draw the front and top views of the solids. Project another top view on an A.I.P. making an angle of 45° with the H.P. From this top view project another front view on an auxiliary vertical plane inclined at 30° to the top view of the combined axis.

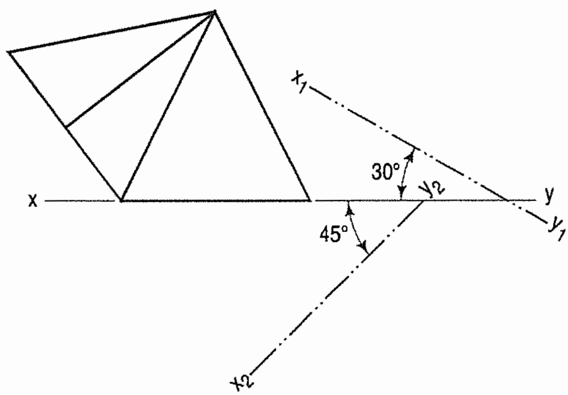


FIG. 13-63

16. Four equal spheres of 25 mm diameter are resting on the ground, each touching the other two spheres, so that a line joining the centres of two touching spheres is inclined at 30° to the V.P. A fifth sphere of 30 mm diameter is placed centrally on top of the four spheres, thus forming a pile. Draw the projections of the spheres and measure the height of the centre of the top sphere above the ground.

17. Three spheres of 25 mm, 50 mm and 75 mm diameter respectively are resting on the ground so that each touches the other two. Draw their projections when the top view of the line joining centres of any two of them is perpendicular to the V.P.

18. Three equal cones, base 50 mm diameter and axis 75 mm long, are placed on the ground on their bases, each touching the other two. A sphere of 40 mm diameter is placed centrally between them. Draw three views of the arrangement and determine the height of the centre of the sphere above the ground.

19. Five equal spheres are resting on the ground each touching the other two spheres and a vertical face of a pentagonal prism of 25 mm side. Determine the diameter of the spheres and draw the projections when a side of the base of the prism is perpendicular to the V.P.

20. Four equal spheres are resting on the ground, each touching the other two spheres and a triangular face of a square pyramid, having base 25 mm side and axis 50 mm long. Draw their projections and find the diameter of the spheres.

21. One of the body diagonals of a cube of 45 mm edge is parallel to the H.P. and inclined at 45° to the V.P. Draw the front view and the top view of the cube.

22. A pentagonal pyramid, base 40 mm side and height 75 mm rests on one edge of its base on the ground so that the highest point in the base is 25 mm above the ground. Draw its projections when the axis is parallel to the V.P. Draw another front view on a reference line inclined at 30° to the edge on which it is resting, and so that the base is visible.

23. A thin lamp shade in the form of a frustum of a cone has its larger end 200 mm diameter, smaller end 75 mm diameter and height 150 mm. Draw its three views when it is lying on its side on the ground and the axis parallel to the V.P.

24. A bucket made of tin sheet has its top 200 mm diameter and bottom 125 mm diameter with a circular ring 40 mm wide attached at the bottom. The total height of the bucket is 250 mm. Draw its projections when its axis makes an angle of 60° with the vertical.

25. A hexagonal pyramid, side of the base 25 mm long and height 70 mm, has one of its triangular faces perpendicular to the H.P. and inclined at 45° to the V.P. The base-side of this triangular face is parallel to the H.P. Draw its projections.

26. A pentagonal pyramid has an edge of the base in the V.P. and inclined at 30° to the H.P., while the triangular face containing that edge makes an angle of 45° with the V.P. Draw three views of the pyramid. Length of the side of the base is 30 mm, while that of the axis is 80 mm.

27. A square pyramid, base 40 mm side and axis 75 mm long is placed on the ground on one of its slant edges, so that the vertical plane passing through that edge and the axis makes an angle of 30° with the V.P. Draw its three views.

28. A hexagonal prism, side of base 40 mm and height 50 mm is lying on the ground on one of its bases with a vertical face perpendicular to the V.P. A tetrahedron is placed on the prism so that the corners of one of its faces coincide with the alternate corners of the top surface of the prism. Draw the projections of the solids. Project another top view on an auxiliary inclined plane making 45° with the H.P.

29. A square duct is in the form of a frustum of a square pyramid. The sides of top and bottom are 150 mm and 100 mm respectively and its length is 150 mm. It is situated in such a way that its axis is parallel to the H.P. and lies in a plane inclined at 60° to the V.P. Draw the projections of the duct, assuming the thickness of the duct-sheet to be negligible.

30. A pentagonal pyramid, base 30 mm edge and axis 75 mm long, stands upon a circular block, 75 mm diameter and 25 mm thick, so that their axes are in a straight line. Draw the projections of the solids when the base of the block is inclined at 30° to the ground, an edge of the base of the pyramid being parallel to the V.P.

31. The body diagonal of a cube is 75 mm long. The cube has a central 25 mm square hole. The faces of the hole make 45° with the side faces of the cube. Draw the projections of the cube when a body diagonal is perpendicular to the H.P.

32. A bucket, 300 mm diameter at the top and 225 mm diameter at the bottom has a circular ring 225 mm diameter and 50 mm wide attached at the bottom. The total height of the bucket is 300 mm. Draw the projections of the bucket when its axis is inclined at 60° to the H.P. and as a vertical plane makes an angle of 45° with the V.P. Assume the thickness of the plate of the bucket to be equal to that of a line.

33. The vertex-angle of the cone just touching the edges of a vertical hexagonal pyramid 125 mm in height is 45°. Draw the projections of the pyramid on a 45° inclined plane when the former is truncated by a plane making 45° with the axis and bisecting the axis.

34. A knob of a machine handle consists of 15 mm diameter × 150 mm long cylindrical portion and 40 mm diameter spherical portion. The centre of the sphere lies on the axis of the cylindrical portion. Draw the projections if its axis is inclined at 45° to the horizontal plane.

35. Six equal spheres rest on the ground in contact with each other and also with the slanting faces of a regular upright hexagonal pyramid, 25 mm edge of base and 125 mm length of axis. Draw the projections and find the diameter of the sphere.

36. A cylinder, 100 mm diameter and 150 mm long, has a rectangular slot 50 mm × 30 mm cut through it. The axis of the slot bisects the axis of the cylinder at right angles and the 50 mm side of the slot makes an angle of 60° with the base of the cylinder. Draw three views of the cylinder.

37. A very thin glass shade for a table lamp is the portion of a sphere 125 mm diameter included between two parallel planes at 15 mm and 55 mm from the centre, making the height 70 mm. If the axis of the shade is inclined at 30° to the vertical, obtain the projections of the shade.

38. A cone frustum, base 75 mm diameter, top 35 mm diameter and height 65 mm has a hole of 30 mm diameter drilled through it so that the axis of the hole coincides with that of the cone. It is resting on its base on the ground and is cut by a section plane perpendicular to the V.P., parallel to an end generator and passing through the top end of the axis. Draw sectional top view and sectional side view of the frustum.

39. Three vertical poles *AB*, *CD* and *EF* are respectively 5, 8 and 12 metres long. Their ends *B*, *D* and *F* are on the ground and lie at the corners of an equilateral triangle of 10 metres long sides. Determine graphically the distance between the top ends of the poles, viz. *AC*, *CE* and *EA*.

40. Two cylinders of 80 mm diameter each meet each other at right angles. The axis of one of the cylinders is parallel to both the reference planes and is 40 mm in front of the axis of the other cylinder. Draw three views of the cylinders showing lines of intersection in them. Take any suitable lengths of the cylinders.

41. A tetrahedron of side 40 mm rests on the top face of a hexagonal prism of base and height 25 mm such that their apex coincide. Draw the projections when the combination rests with one of the sides of the prism on the H.P., is perpendicular to the V.P., and the axis is inclined at 30° to the H.P.

42. A pentagonal pyramid, base 30 mm side and axis 70 mm long, has one of its slant edges in the H.P. and inclined at 30° to the V.P. Draw the projections of the solid when the apex is towards the observer.

Chapter 14

SECTIONS OF SOLIDS

14-0. INTRODUCTION

Invisible features of an object are shown by dotted lines in their projected views. But when such features are too many, these lines make the views more complicated and difficult to interpret. In such cases, it is customary to imagine the object as being cut through or *sectioned* by planes. The part of the object between the cutting plane and the observer is assumed to be removed and the view is then shown *in section*.

The imaginary plane is called a section *plane* or a *cutting plane*. The surface produced by cutting the object by the section plane is called the *section*. It is indicated by thin section lines uniformly spaced and inclined at 45°.

The projection of the section along with the remaining portion of the object is called a *sectional view*. Sometimes, only the word section is also used to denote a sectional view.

(1) Section planes: Section planes are generally perpendicular planes. They may be perpendicular to one of the reference planes and either perpendicular, parallel or inclined to the other plane. They are usually described by their traces. It is important to remember that the projection of a section plane, on the plane to which it is perpendicular, is a straight line. This line will be parallel, perpendicular or inclined to *xy*, depending upon the section plane being parallel, per-pendicular or inclined respectively to the other reference plane.

As per latest B.I.S. convention (SP: 46-2003), the cutting-plane line should be drawn as shown in fig. 3-2 which is reproduced here in fig. 14-1 for ready reference.

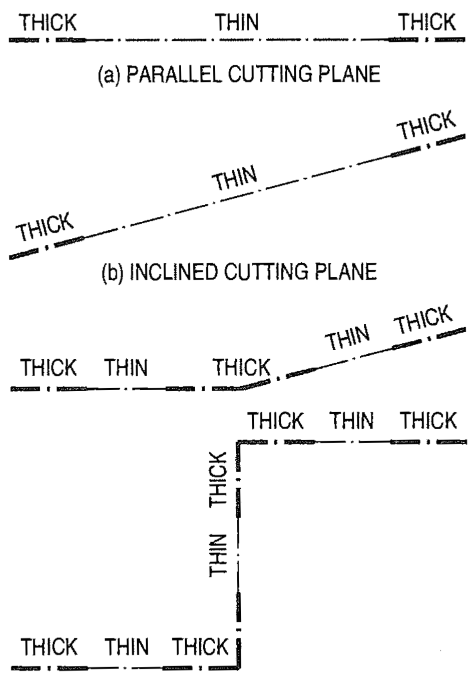


FIG. 14-1

(2) Sections: The projection of the section on the reference plane to which the section plane is perpendicular, will be a straight line coinciding with the trace of the section plane on it. Its projection on the other plane to which it is inclined is called *apparent section*. This is obtained by

(i) projecting on the other plane, the points at which the trace of the section plane intersects the edges of the solid and

(ii) drawing lines joining these points in proper sequence.

(3) True shape of a section: The projection of the section on a plane parallel to the section plane will show the true shape of the section. Thus, when the section plane is parallel to the H.P. or the ground, the true shape of the section will be seen in *sectional top view*. When it is parallel to the V.P., the true shape will be visible in the *sectional front view*.

But when the section plane is inclined, the section has to be projected on an auxiliary plane parallel to the section plane, to obtain its true shape. When the section plane is perpendicular to both the reference planes, the sectional side view will show the true shape of the section. In this chapter sections of different solids are explained in stages by means of typical problems as follows:

1. Sections of prisms
2. Sections of pyramids
3. Sections of cylinders
4. Sections of cones
5. Sections of spheres.

14-1. SECTIONS OF PRISMS

These are illustrated according to the position of the section plane with reference to the principal planes as follows:

(1) Section plane parallel to the V.P.

(2) Section plane parallel to the H.P.

(3) Section plane perpendicular to the H.P. and inclined to the V.P.

(4) Section plane perpendicular to the V.P. and inclined to the H.P.

(1) Section plane parallel to the V.P.

Problem 14-1. (fig. 14-2): *A cube of 35 mm long edges is resting on the H.P. on one of its faces with a vertical face inclined at 30° to the V.P. It is cut by a section plane parallel to the V.P. and 9 mm away from the axis and further away from the V.P. Draw its sectional front view and the top view.*

In fig. 14-2(i), the section plane is assumed to be transparent and the cube is shown with the cut-portion removed. It can be seen that four edges of the cube are cut and hence, the section is a figure having four sides.

Draw the projections of the whole cube in the required position [fig. 14-2(ii)].

As the section plane is parallel to the V.P., it is perpendicular to the H.P.; hence, the section will be seen as a line in the top view coinciding with the H.T. of the section plane.

(i) Draw a line H.T. in the top view (to represent the section plane) parallel to *xy* and 9 mm from *o*.

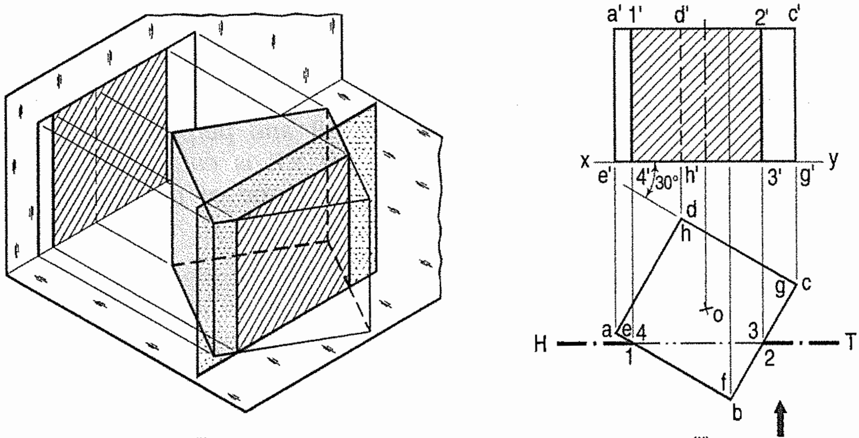
(ii) Name the points at which the edges are cut, viz. *ab* at 1, *bc* at 2, *gf* at 3 and *fe* at 4.

(iii) Project these points on the corresponding edges in the front view and join them in proper order.

As the section plane is parallel to the V.P., figure 1' 2' 3' 4' in the front view, shows the true shape of the section.

Show the views by dark but thin lines, leaving the lines for the cut-portion fainter.

(iv) Draw section lines in the rectangle for the section.



(i) (ii)

FIG. 14-2

(2) Section plane parallel to the H.P.

Problem 14-2. (fig. 14-3): *A triangular prism, base 30 mm side and axis 50 mm long, is lying on the H.P. on one of its rectangular faces with its axis inclined at 30° to the V.P. It is cut by a horizontal section plane, at a distance of 12 mm above the ground. Draw its front view and sectional top view.*

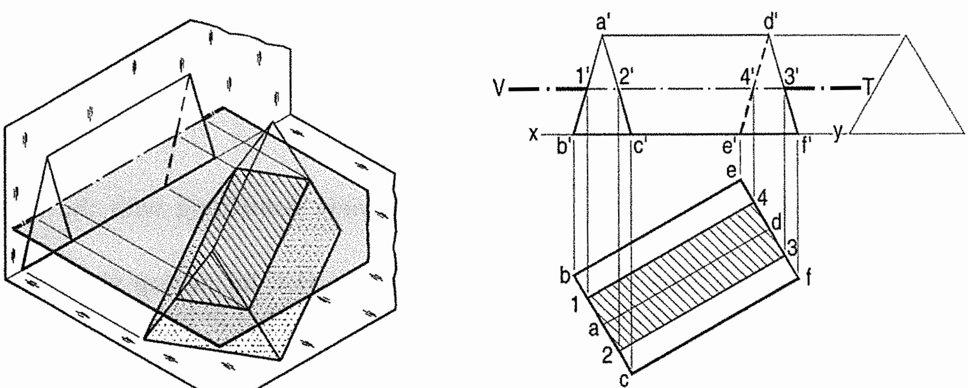


FIG. 14-3

Draw the projections of the prism in the required position.

As the section plane is horizontal, i.e. parallel to the H.P., it is perpendicular to the V.P. Hence, the section will be seen as a line in the front view, coinciding with the V.T. of the section plane.

(i) Therefore, draw a line V.T. in the front view to represent the section plane, parallel to *xy* and 12 mm above it.

(ii) Name in correct sequence, points at which the edges are cut viz. *a'b'* at 1', *a'c'* at 2', *d'f'* at 3' and *d'e'* at 4'.

(iii) Project these points on the corresponding lines in the top view and complete the sectional top view by joining them in proper order.

As the section plane is parallel to the H.P., the figure 1 2 3 4 (in the top view) is the true shape of the section.

(3) Section plane perpendicular to the H.P. and inclined to the V.P.

Problem 14-3. (fig. 14-4): *A cube in the same position as in problem 14-1, is cut by a section plane, inclined at 60° to the V.P. and perpendicular to the H.P., so that the face which makes 60° angle with the V.P. is cut in two equal halves. Draw the sectional front view, top view and true shape of the section.*

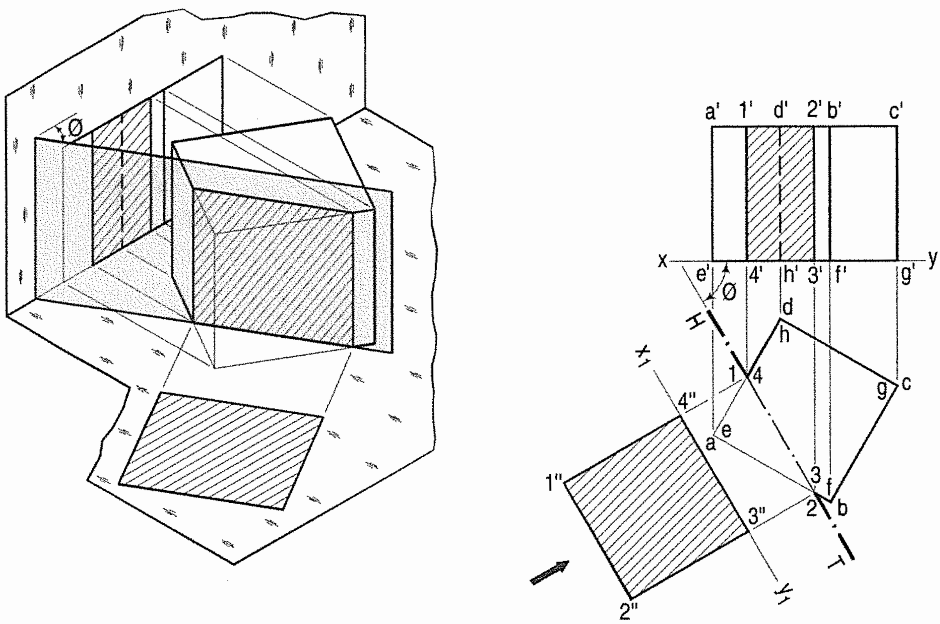


FIG. 14-4

The section will be seen as a line in the top view coinciding with the H.T. of the section plane.

(i) Draw the projections of the cube. Draw a line H.T. in the top view inclined at 60° to *xy* and cutting the line *ad* (or *bc*) at its mid-point.

(ii) Name the corners at which the four edges are cut and project them in the front view. As the section plane is inclined to the V.P., the front view of the section viz. 1' 2' 3' 4' does not reveal its true shape. Only the vertical lines show true lengths, while the true lengths of the horizontal lines are seen in the top view.

The true shape of the section will be seen when it is projected on an auxiliary vertical plane, parallel to the section plane.

(iii) Therefore, draw a new reference line x_1y_1 parallel to the H.T. and project the section on it. The distances of the points from x_1y_1 should be taken equal to their corresponding distances from xy in the front view. Thus 4" and 3" will be on x_1y_1. 1" 4" and 2" 3" will be equal to 1' 4' and 2' 3' respectively. Complete the rectangle 1" 2" 3" 4" which is the true shape of the section and draw section lines in it.

(4) Section plane perpendicular to the V.P. and inclined to the H.P.

Problem 14-4. (fig. 14-5): *A cube in the same position as in problem 14-1 is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and passing through the top end of the axis. (i) Draw its front view, sectional top view and true shape of the section. (ii) Project another top view on an auxiliary plane, parallel to the section plane.*

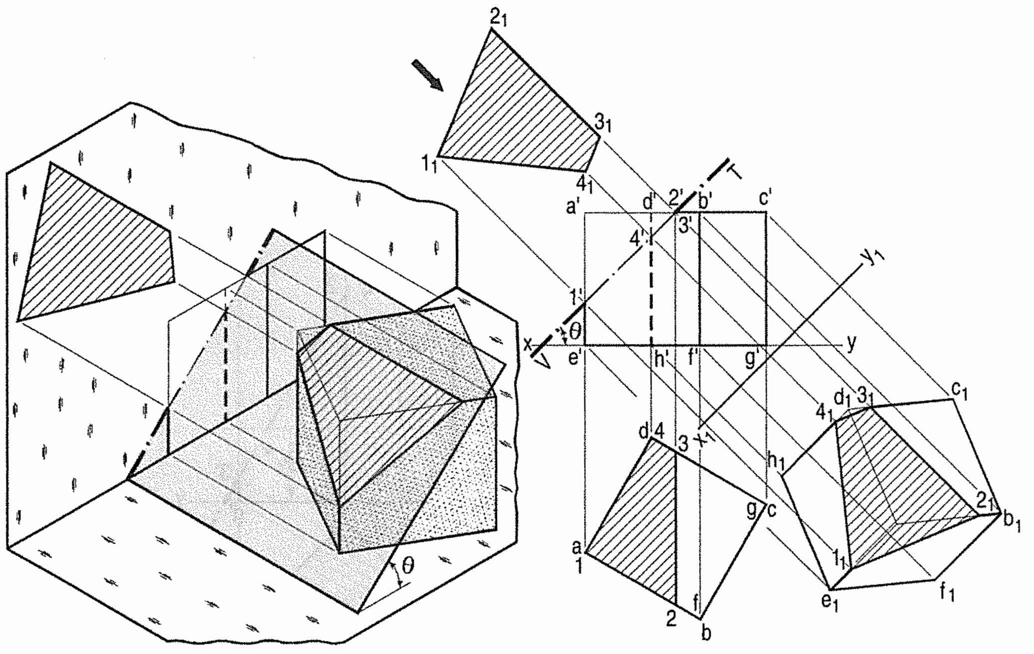


FIG. 14-5

The section will be seen as a line in the front view.

(i) Draw a line V.T. in the front view, inclined at 45° to xy and passing through the top end of the axis. It cuts four edges, viz. a' e' at 1', a' b' at 2', $c'd'$ at 3' and $d'h'$ at 4'.

(ii) Project the top view of the section, viz. the figure 1 2 3 4. It does not show the true shape of the section, as the section plane is inclined to the H.P.

To determine the true shape, an auxiliary top view of the section should be projected on an A.I.P. parallel to the section plane.

(iii) Assuming the new reference line for the A.I.P. to coincide with the V.T., project the true shape of the section as shown by quadrilateral 1_1 2_1 3_1 4_1.

The distances of all the points from the V.T. should be taken equal to their corresponding distances from xy in the top view, e.g. $1_1 1' = e'1$, $4_1 4' = h'4$ etc.

(iv) To project an auxiliary sectional top view of the cube, draw a new reference line $x_1 y_1$, parallel to the V.T. The whole cube may first be projected and the points for the section may then be projected on the corresponding lines for the edges. Join these points in correct sequence and obtain the required top view.

(v) Draw section lines in the cut-surface, in the views where it is seen. Keep the lines for the removed edges thin and fainter.

Additional problems on sections of prisms:

Problem 14-5. (fig. 14-6): *A square prism, base 40 mm side, axis 80 mm long, has its base on the H.P. and its faces equally inclined to the V.P. It is cut by a plane, perpendicular to the V.P., inclined at 60° to the H.P. and passing through a point on the axis, 55 mm above the H.P. Draw its front view, sectional top view and another top view on an A.I.P. parallel to the section plane.*

The problem is similar to problem 14-4 and needs no further explanation. The true shape of the section is seen in the auxiliary top view.

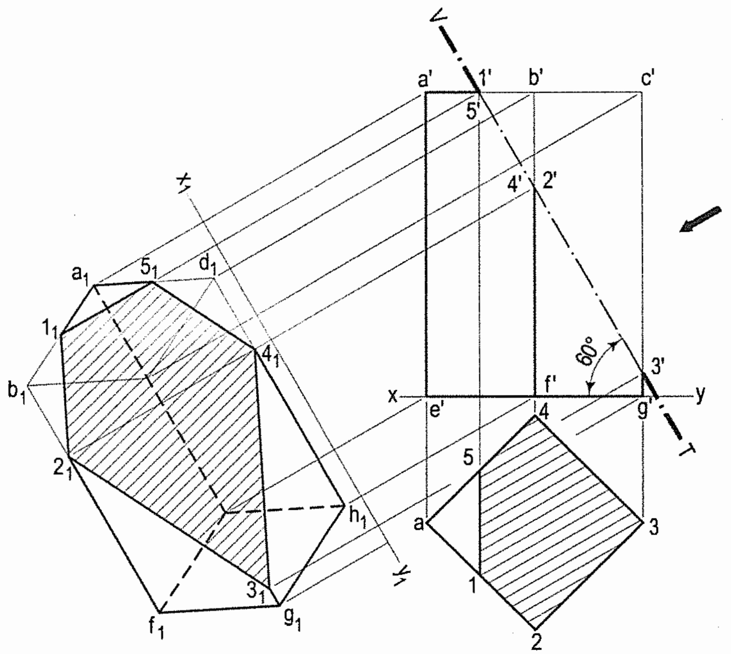


FIG. 14-6

Problem 14-6. (fig. 14-7): *A hexagonal prism, has a face on the H.P. and the axis parallel to the V.P. It is cut by a vertical section plane, the H.T. of which makes an angle of 45° with xy and which cuts the axis at a point 20 mm from one of its ends. Draw its sectional front view and the true shape of the section. Side of base 25 mm long; height 65 mm.*

(i) Draw the front view and the top view of the prism and show the H.T. of the section plane in the top view. Name in proper sequence, the points at which the lines are cut.

(ii) Project them on the corresponding lines in the front view. The positions of points 4 and 5 cannot be located directly. Hence, project them on the first top view to 4_1 on *ef* and 5_1 on *ed*. From this top view, obtain their positions $4'_1$ and $5'_1$ on the corresponding lines in the first front view. As the two front views are identical, these points can now be transferred to the second front view by making e'4' equal to $e'4'_1$ and e'5' equal to $e'5'_1$. 4' and 5' are the projections of points 4 and 5 respectively. Complete the sectional front view as shown.

(iii) Obtain the true shape of the section on x_1y_1 as explained in problem 14-3, making o"1" equal to o'1', etc.

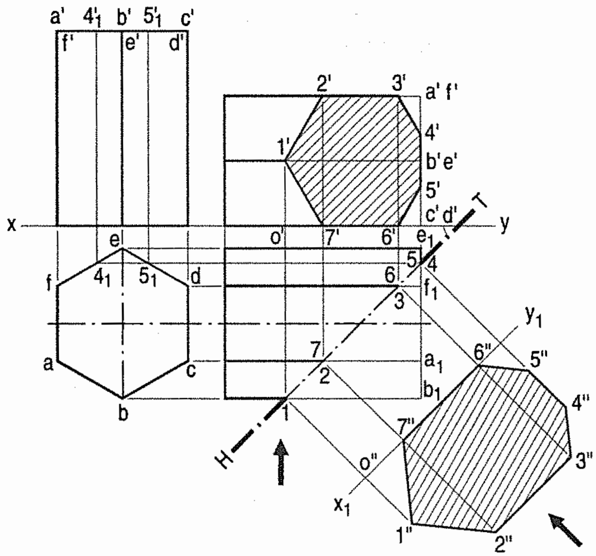


FIG. 14-7

Problem 14-7. (fig. 14-8): *A pentagonal prism, base 28 mm side and height 65 mm has an edge of its base on the H.P. and the axis parallel to the V.P. and inclined at 60° to the H.P. A section plane, having its H.T. perpendicular to xy, and the V.T. inclined at 60° to xy and passing through the highest corner, cuts the prism. Draw the sectional top view and true shape of the section.*

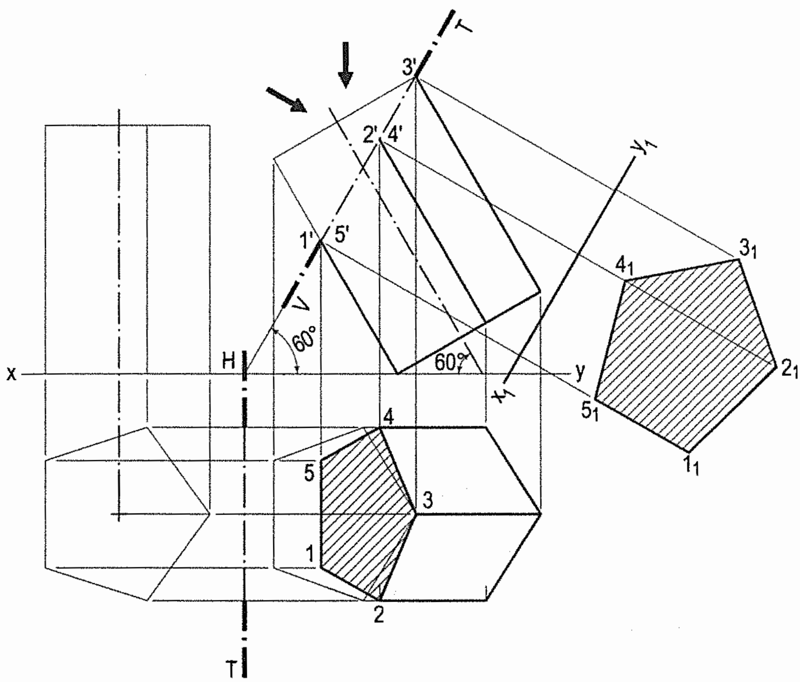


FIG. 14-8

(i) Draw the projections of the prism in the required position.

(ii) Draw the line V.T. passing through the highest corner 3' and inclined at 60° to *xy*. A perpendicular to *xy* through *V* will be the H.T. of the section plane.

(iii) Project the sectional top view and the true shape of the section, as shown in the figure.

Problem 14-8. (fig. 14-9): *A hollow square prism, base 40 mm side (outside), height 65 mm and thickness 8 mm is resting on its base on the H.P. with a vertical face inclined at 30° to the V.P. A section plane, inclined at 30° to the H.P., perpendicular to the V.P. and passing through the axis at a point 12 mm from its top end, cuts the prism. Draw its sectional top view, sectional side view and true shape of the section.*

(i) Draw the projections of the prism in the given position, showing the hidden edges by dashed lines.

(ii) Draw a line V.T. for the cutting plane and mark points at which the inside and outside edges are cut.

(iii) Project the sectional top view, true shape of the section and the sectional side view as shown.

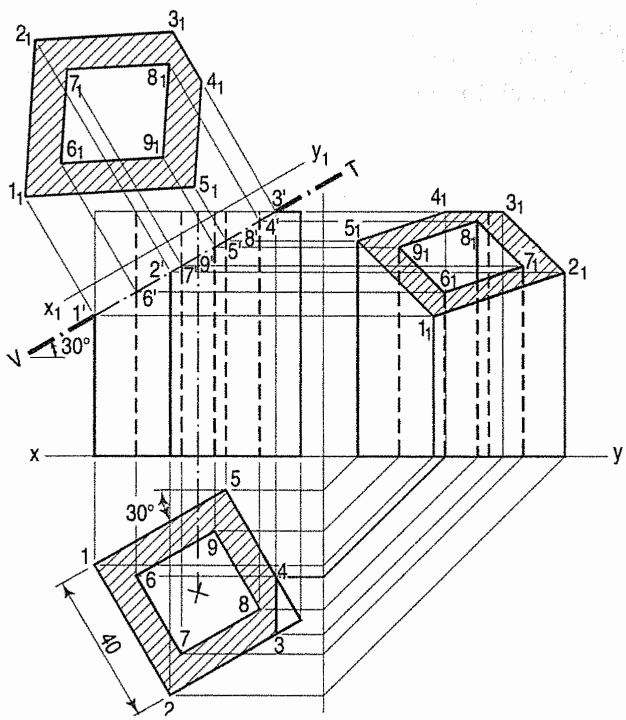


FIG. 14-9

14-2. SECTIONS OF PYRAMIDS

The following cases are discussed in details.

(1) Section plane parallel to the base of the pyramid.

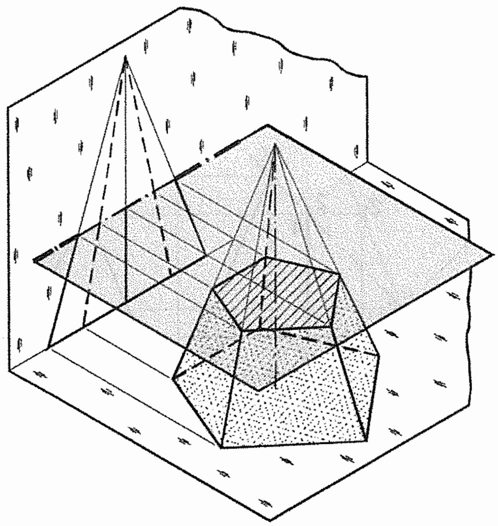
(2) Section plane parallel to the V.P.

(3) Section plane perpendicular to the V.P. and inclined to the H.P.

(4) Section plane perpendicular to the H.P. and inclined to the V.P.

(1) Section plane parallel to the base of the pyramid.

Problem 14-9. (fig. 14-10): *A pentagonal pyramid, base 30 mm side and axis 65 mm long, has its base horizontal and an edge of the base parallel to the V.P. A horizontal section plane cuts it at a distance of 25 mm above the base. Draw its front view and sectional top view.*



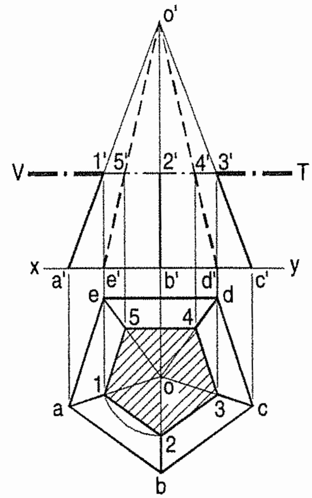


FIG. 14-10

(i) Draw the projections of the pyramid in the required position and show a line V.T. for the section plane, parallel to and 25 mm above the base.

All the five slant edges are cut.

(ii) Project the points at which they are cut, on the corresponding edges in the top view. The point 2' cannot be projected directly as the line *ob* is perpendicular to *xy*. But it is quite evident from the projections of other points that the lines of the section in the top view, viz. 3-4, 4-5 and 5-1 are parallel to the edges of the base in their respective faces and that the points 1, 3, 4 and 5 are equidistant from o.

(iii) Hence, line 1-2 also will be parallel to *ab* and o2 will be equal to o1, o3 etc. Therefore, with *o* as centre and radius o1, draw an arc cutting *ob* at a point 2 which will be the projection of 2′. Complete the sectional top view in which the true shape of the section, viz. the pentagon 1, 2, 3, 4 and 5 is also seen.

(iv) Hence, when a pyramid is cut by a plane parallel to its base, the true shape of the section will be a figure, similar to the base; the sides of the section will be parallel to the edges of the base in the respective faces and the corners of the section will be equidistant from the axis.

(2) Section plane parallel to the V.P.

Problem 14-10. (fig. 14-11): *A triangular pyramid, having base 40 mm side and axis 50 mm long, is lying on the H.P. on one of its faces, with the axis parallel to the V.P. A section plane, parallel to the V.P. cuts the pyramid at a distance of 6 mm from the axis. Draw its sectional front view and the top view.*

(i) Draw the projections of the pyramid in the required position and show a line H.T. (for the cutting plane) in the top view parallel to *xy* and 6 mm from the axis.

(ii) Project points 1, 2 and 3 (at which the edges are cut) on corresponding edges in the front view and join them. Figure 1' 2' 3' shows the true shape of the section.

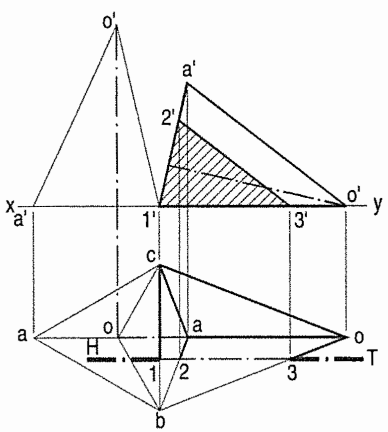


FIG. 14-11

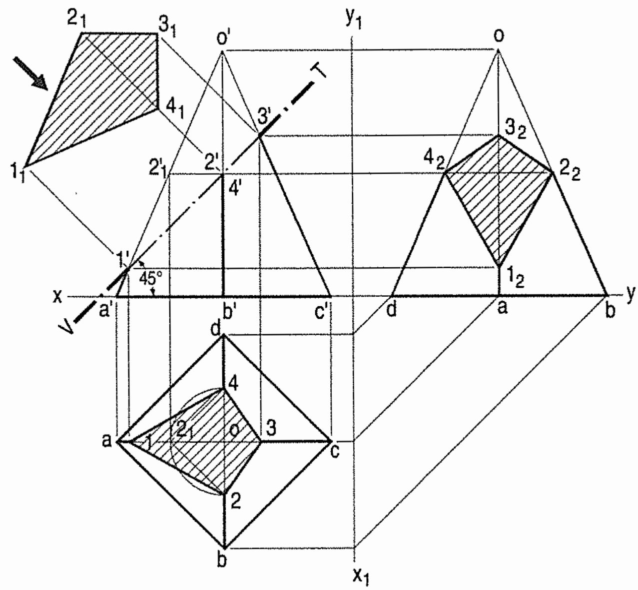


FIG. 14-12

(3) Section plane perpendicular to the V.P. and inclined to the H.P.

Problem 14-11. (fig. 14-12): *A square pyramid, base 40 mm side and axis 65 mm long, has its base on the H.P. and all the edges of the base equally inclined to the V.P. It is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and bisecting the axis. Draw its sectional top view, sectional side view and true shape of the section.*

(i) Draw the projections of the pyramid in the required position. The section plane will be seen as a line in the front view. Hence, draw a line V.T. through the mid-point of the axis and inclined at 45° to *xy*. Name in correct sequence the points at which the four edges are cut and project them in the top view. Here also, points 2' and 4' cannot be projected directly.

Hence, assume a horizontal section through 2' and draw a line parallel to the base, cutting *o' a'* at $2'_1$. Project $2'_1$ to 2_1 on *oa* in the top view. From 2_1 draw a line parallel to *ab* and cutting *ob* at a point 2. Or, with *o* as centre and radius o 2_1, draw an arc cutting *ob* at 2 and *ob* at 4. Complete the section 1 2 3 4 by joining the points and draw section lines in it.

(ii) Assuming the V.T. to be the new reference line, draw the true shape of the section. Project the side view from the two views. The removed portion of the pyramid may be shown by thin and faint lines.

(4) Section plane perpendicular to the H.P. and inclined to the V.P.

Problem 14-12. (fig. 14-13): *A pentagonal pyramid has its base on the H.P. and the edge of the base nearer the V.P., parallel to it. A vertical section plane, inclined at 45° to the V.P., cuts the pyramid at a distance of 6 mm from the axis. Draw the top view, sectional front view and the auxiliary front view on an A.V.P. parallel to the section plane. Base of the pyramid 30 mm side; axis 50 mm long.*

The section plane will be seen as a line in the top view. It is to be at a distance of 6 mm from the axis.

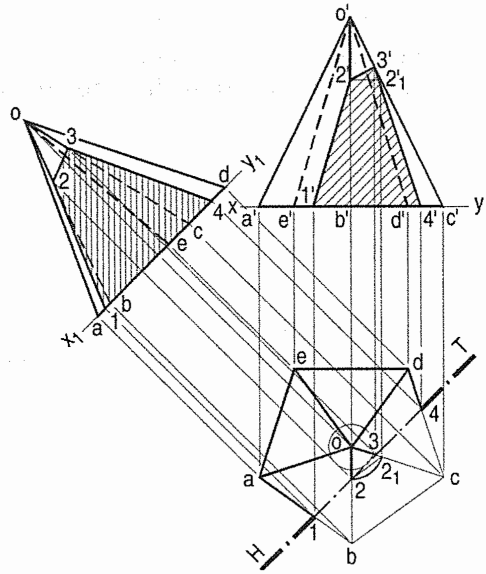


FIG. 14-13

(i) Hence, draw a circle with *o* as centre and radius equal to 6 mm.

(ii) Draw a line H.T., tangent to this circle and inclined at 45° to *xy*. It can be drawn in four different positions, of which any one may be selected.

(iii) Project points 1, 2 etc. from the top view to the corresponding edges in the front view. Here again, point 2 cannot be projected directly. The process shown in problem 14-11 must be reversed. With centre *o* and radius *o*2 draw an arc cutting any one of the slant edges, say *oc* at 2_1. Project 2_1 to $2'_1$ on o'c'.

(iv) Through $2'_1$, draw a line parallel to the base, cutting *o'b'* at 2'. Then 2' is the required point. Complete the view. It will show the apparent section.

(v) Draw a reference line x_1y_1 parallel to the H.T. and project an auxiliary sectional front view which will show the true shape of the section also.

Additional problems on sections of pyramids:

Problem 14-13. (fig. 14-14): *A hexagonal pyramid, base 30 mm side and axis 65 mm long, is resting on its base on the H.P. with two edges parallel to the V.P. It is cut by a section plane, perpendicular to the V.P. inclined at 45° to the H.P. and intersecting the axis at a point 25 mm above the base. Draw the front view, sectional top view, sectional side view and true shape of the section.*

This problem is similar to problem 14-11. In this case, the base is also cut and hence, the section is a heptagon. Care must be taken to name the points in proper sequence.

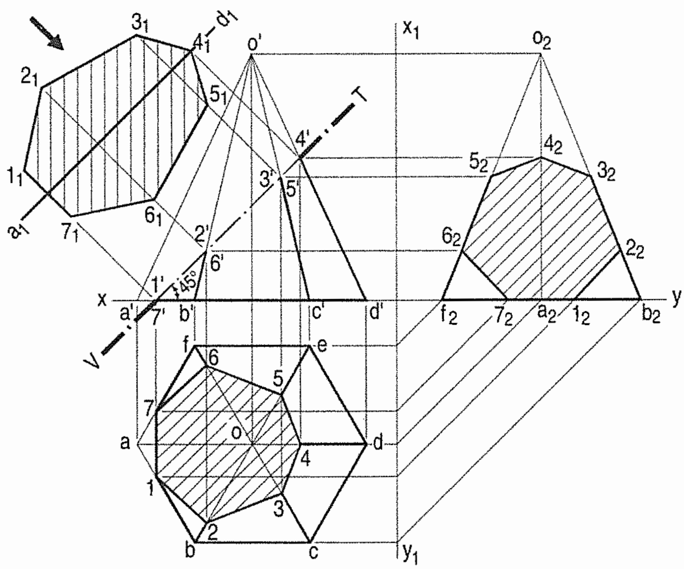


FIG. 14-14

The true shape may be drawn on the V.T. as a new reference line or around the centre line a_1d_1, drawn parallel to the V.T. as shown.

The distances of the points 1_1, 2_1 etc. from $a_1\,d_1$ are taken equal to the distances of points 1, 2 etc. from the line *ad* (which is parallel to *xy*).

Problem 14-14. (fig. 14-15): *A pentagonal pyramid, base 30 mm side and axis 60 mm long, is lying on one of its triangular faces on the H.P. with the axis parallel to the V.P. A vertical section plane, whose H.T. bisects the top view of the axis and makes an angle of 30° with the reference line, cuts the pyramid, removing its top part. Draw the top view, sectional front view, true shape of the section and development of the surface of the remaining portion of the pyramid.*

(i) Draw the H.T. of the section plane and name the points at which the edges are cut, in correct sequence, i.e. mark the visible edges first and then the hidden edges.

(ii) Project the sectional front view which will show the apparent section.

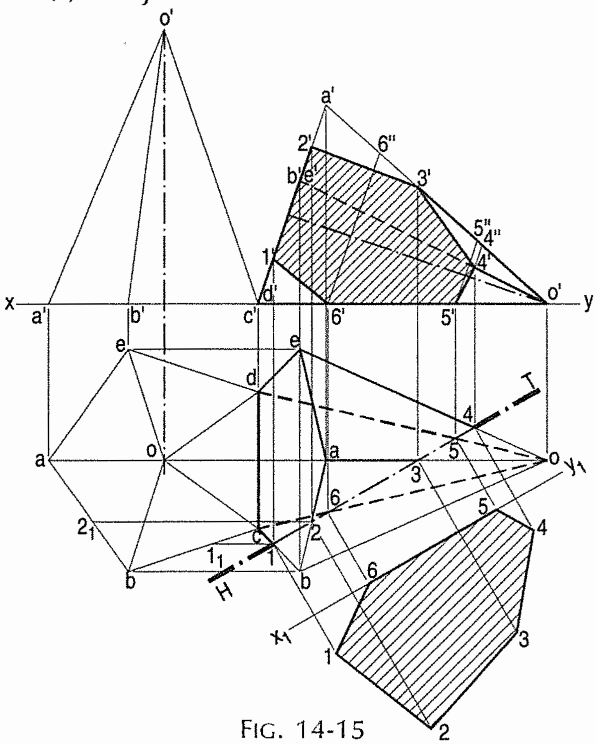


FIG. 14-15

(iii) Obtain the true shape of the section on x_1y_1 as a new reference line drawn parallel to the H.T.

Development (fig. 14-16): The line *o'a'* shows the true length of the slant edge.

(i) With any point *O* as centre and radius *o'a'*, draw an arc and construct the development of the whole pyramid. Mark points on it, taking the positions of 1 and 2 from the first top view and those of other points by projecting them on the true-length-line *o'a'*.

(ii) Draw lines joining these points and complete the development as shown in the figure.

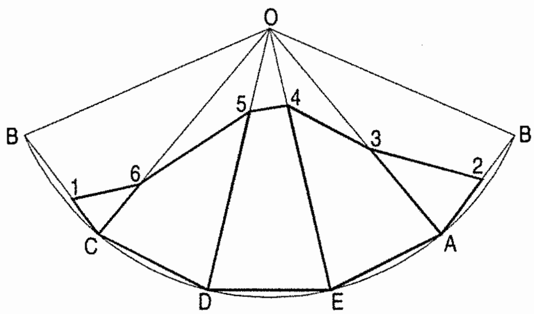


FIG. 14-16

Problem 14-15. (fig. 14-17): *A hexagonal pyramid, base 30 mm side and axis 60 mm long, has a triangular face on the H.P. and the axis parallel to the V.P. It is cut by a horizontal section plane which bisects the axis. Draw the front view and sectional top view and develop the surface of the cut-pyramid.*

The V.T. cuts six edges. The sectional top view shows the true shape of the section also.

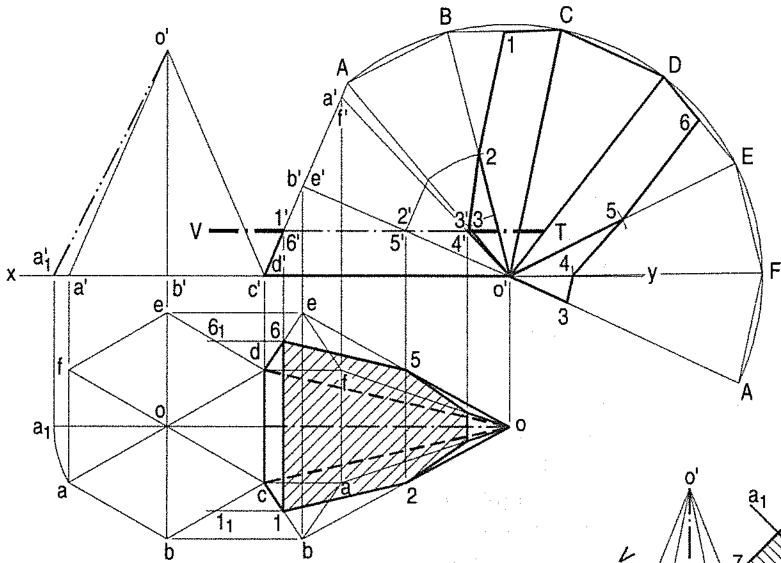


FIG. 14-17

Development: None of the edges shows the true length of the slant edge.

(i) Hence, determine the true length $o'a'_1$ and draw the development of the whole pyramid.

(ii) Locate positions of the points 1 and 6 by projecting them on the first top view and positions of other points by drawing lines through them, parallel to the base and upto the true length line *o'A*.

(iii) Mark these points on the development and complete it as shown.

Problem 14-16. (fig. 14-18): *A hexagonal pyramid, base 30 mm side and axis 75 mm long, resting on its base on the H.P. with two of its edges parallel to the V.P. is cut by two section planes, both perpendicular to the V.P. The horizontal section plane cuts the axis at a point 35 mm from the apex. The other plane which makes an angle of 45° with the H.P., also intersects the axis at the same point. Draw the front view, sectional top view, true shape of the section and development of the surface of the remaining part of the pyramid.*

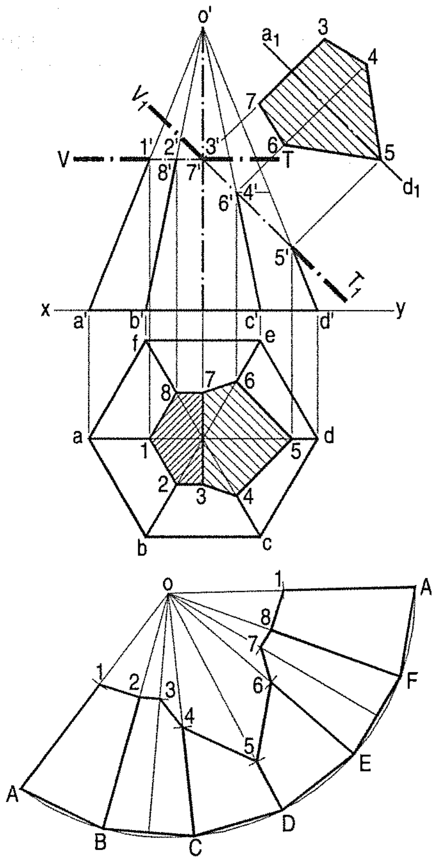


FIG. 14-18

(i) Draw lines V.T. and V_1T_1 for the two section planes. The top view will show the true shape of the horizontal section, the sides of which are parallel to the respective sides of the base. The true shape of the other section may be obtained on V_1T_1 as the reference line or around a_1d_1.

(ii) Draw the development with *o' a'* or *o' d'* as radius and locate the points on it, as shown in the figure.

14-3. SECTIONS OF CYLINDERS

We shall now learn the following three cases. They are

(1) Section plane parallel to the base
(2) Section plane parallel to the axis
(3) Section plane inclined to the base.

(1) Section plane parallel to the base: When a cylinder is cut by a section plane parallel to the base, the true shape of the section is a circle of the same diameter.

(2) Section plane parallel to the axis: When a cylinder is cut by a section plane parallel to the axis, the true shape of the section is a rectangle, the sides of which are respectively equal to the length of the axis and the length of the section plane within the cylinder (fig. 14-19). When the section plane contains the axis, the rectangle will be of the maximum size.

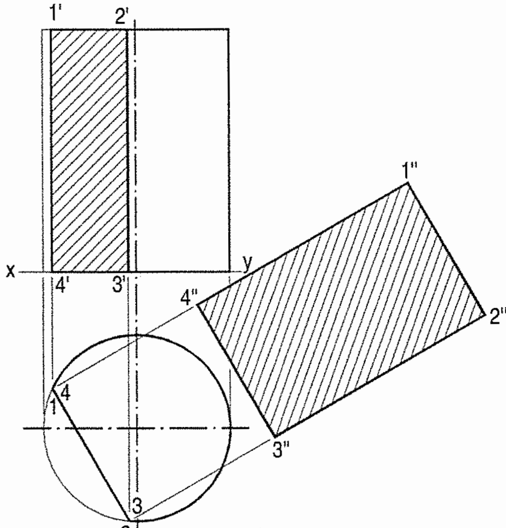


FIG. 14-19

(3) Section plane inclined to the base:

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 31 for the following problem.

Problem 14-17. (fig. 14-20): *A cylinder of 40 mm diameter, 60 mm height and having its axis vertical, is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and intersecting the axis 32 mm above the base. Draw its front view, sectional top view, sectional side view and true shape of the section.*

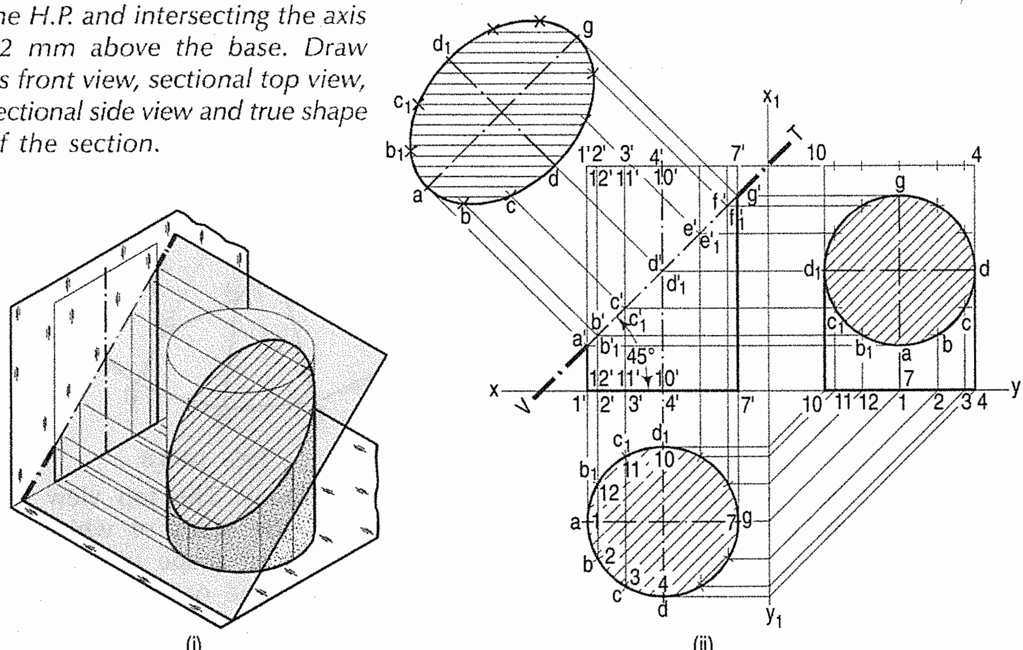


FIG. 14-20

As the cylinder has no edges, a number of lines representing the generators may be assumed on its curved surface by dividing the base-circle into, say 12 equal parts.

(i) Name the points at which these lines are cut by the V.T. In the top view, these points lie on the circle and hence, the same circle is the top view of the section. The width of the section at any point, say c', will be equal to the length of the chord cc_1 in the top view.

(ii) The true shape of the section may be drawn around the centre line *ag* drawn parallel to V.T. as shown. It is an ellipse the major axis of which is equal to the length of the section plane viz. *a'g'*, and the minor axis equal to the diameter of the cylinder viz. dd_1.

(iii) Project the sectional side view as shown. The section will be seen as a circle because the section plane makes 45° angle with *xy*.

Additional problems on sections of cylinders:

Problem 14-18. (fig. 14-21): *A cylinder 50 mm diameter and 60 mm long, is resting on its base on the ground. It is cut by a section plane perpendicular to the V.P., the V.T. of which cuts the axis at a point 40 mm from the base and makes an angle of 45° with the H.P. Draw its front view, sectional top view and another sectional top view on an A.I.P. parallel to the section plane.*

In this case, the top end of the cylinder is also cut. Hence, the true shape of the section is a part of an ellipse as shown in the auxiliary top view.

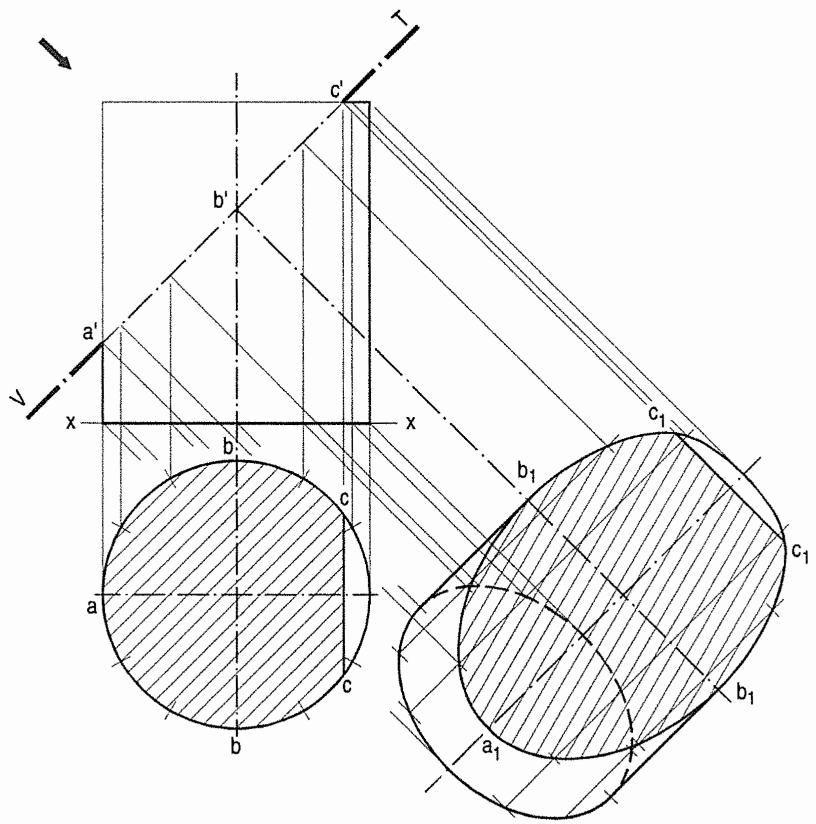


FIG. 14-21

Problem 14-19. (fig. 14-22): *A cylinder, 55 mm diameter and 65 mm long, has its axis parallel to both the H.P. and the V.P. It is cut by a vertical section plane inclined at 30° to the V.P., so that the axis is cut at a point 30 mm from one of its ends and both the bases of the cylinder are partly cut. Draw its sectional front view and true shape of the section.*

Draw the projections of the cylinder and a line H.T. for the section plane. Project the points at which the bases and the lines are cut. The points on the bases cannot be projected directly. Therefore, project them

(i) To the first top view i.e. a to a_1 and e to e_1.

(ii) Then to the first front view, i.e. a_1 to a'_1 and e_1 to e'_1.

(iii) Finally, transfer them to the second front view to a' and e' each, at two places as shown.

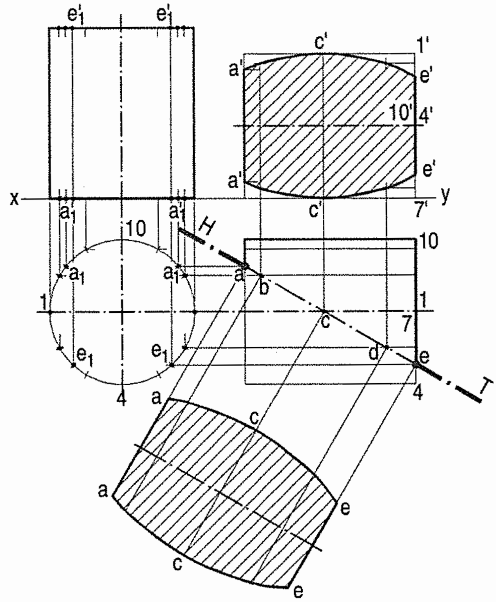


FIG. 14-22

(iv) Draw the true shape of the section either on a new reference line or symmetrically around the centre line and making *aa* equal to *a'a'*, *cc* equal to *c'c'* etc.

Problem 14-20. (fig. 14-23): *A hollow cylinder, 50 mm outside diameter, axis 70 mm long and thickness 8 mm has its axis parallel to the V.P. and inclined at 30° to the vertical. It is cut in two equal halves by a horizontal section plane. Draw its sectional top view.*

The figure is self-explanatory. Note that a part of the ellipse for the inside bottom will also be visible.

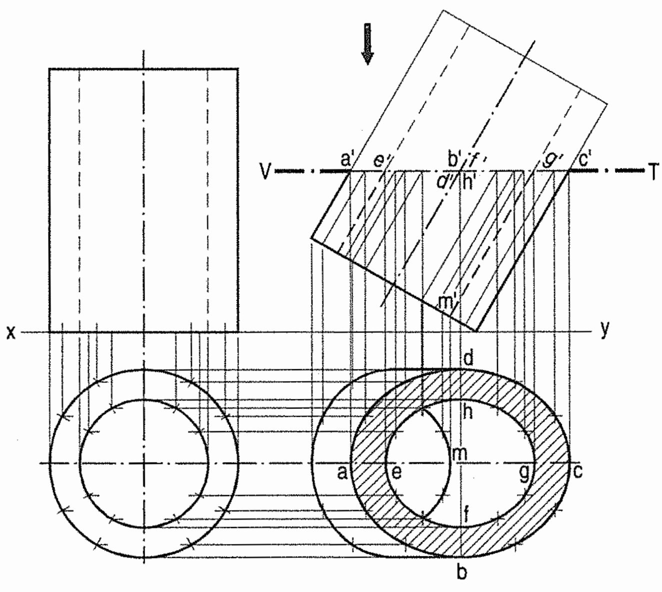


FIG. 14-23

14-4. SECTIONS OF CONES

This is discussed in details as follows:

(1) Section plane parallel to the base of the cone.

(2) Section plane passing through the apex of the cone.

(3) Section plane inclined to the base of the cone at an angle smaller than the angle of inclination of the generators with the base.

(4) Section plane parallel to a generator of the cone.

(5) Section plane inclined to the base of the cone at an angle greater than the angle of inclination of the generators with the base.

(1) Section plane parallel to the base of the cone:

The cone resting on the H.P. on its base [fig. 14-24(i)] is cut by a section plane parallel to the base. The true shape of the section is shown by the circle in the top view, whose diameter is equal to the length of the section viz. a'a'. The width of the section at any point, say *b'*, is equal to the length of the chord bb_1.

Problem 14-21. [fig. 14-24(ii)]: *To locate the position in the top view of any given point p' in the front view of the above cone.*

Method I:

(i) Through *p'*, draw a line *r'r'* parallel to the base.

(ii) With *o* as centre and diameter equal to *r'r'*, draw a circle in the top view.

(iii) Project *p'* to points *p* and p_1 on this circle. *p* is the top view of *p'*. p_1 is the top view of another point p'_1 on the back side of the cone and coinciding with *p'*. The chord pp_1 shows the width of the horizontal section of the cone at the point *p'*. This method may be called the *circle method*.

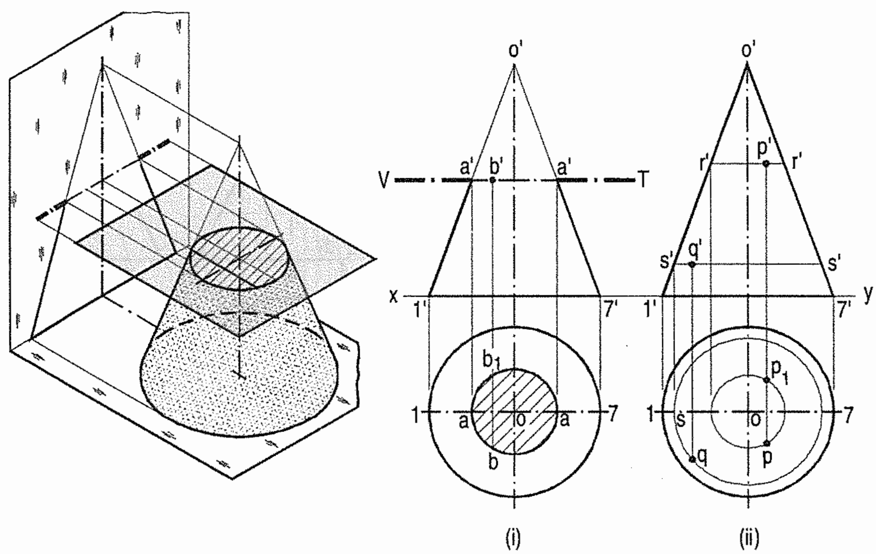


FIG. 14-24

Method II:

When the position of a point in the top view say *q* is given, its front view *q'* can be determined by reversing the above process.

(i) With centre *o* and radius *oq*, draw a circle cutting the horizontal centre line at *s*.

(ii) Through *s*, draw a projector cutting the slant side *o'1'* at *s'*.

(iii) Draw the line *s's'* parallel to the base, intersecting a projector through *q* at the required point *q'*.

(2) Section plane passing through the apex of the cone:

Problem 14-22. [fig. 14-25(i)]: *A cone, diameter of base 50 mm and axis 50 mm long is resting on its base on the H.P. It is cut by a section plane perpendicular to the V.P., inclined at 75° to the H.P. and passing through the apex. Draw its front view, sectional top view and true shape of the section.*

Draw the projections of the cone and on it, show the line V.T. for the section plane.

Mark a number of points *a'*, *b'* etc. on the V.T. and project them to points *a*, *b* etc. in the top view by the circle method. It will be found that these points lie on a straight line through *o*.

Thus, *od* is the top view of the line or generator *o'd'* and triangle odd_1 is the top view of the section. The width of the section at any point *b'* on the section is the line bb_1, obtained by projecting *b'* on this triangle. This method is called the generator method.

Project the true shape of the section. It is an isosceles triangle, the base of which is equal to the length of the chord on the base-circle and the altitude is equal to the length of the section plane within the cone.

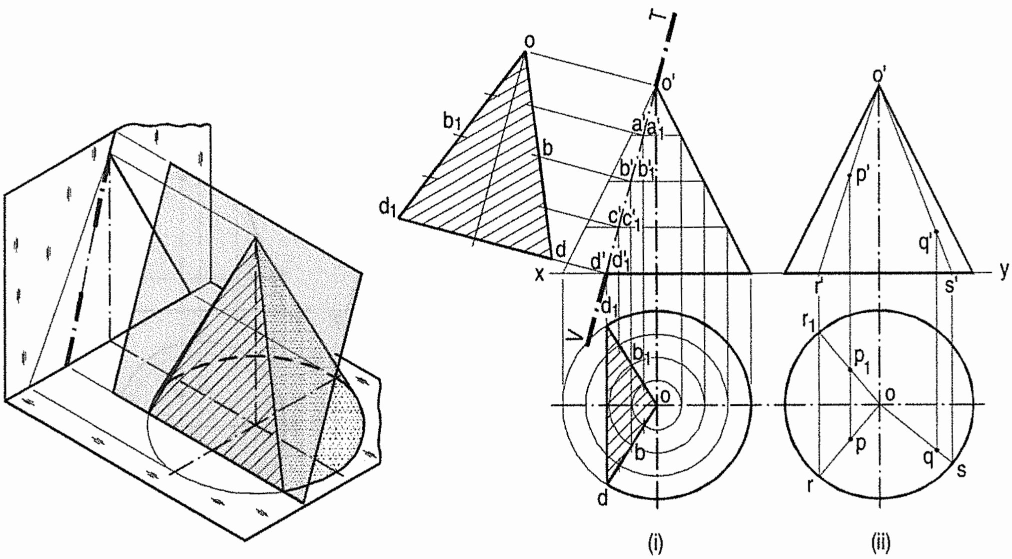


FIG. 14-25

Problem 14-23. [fig. 14-25(ii)]: *To determine by generator method, the position in the top view of a given point p' in the front view of the above cone.*

Draw the line $o'p'$ and produce it to cut the base at r'. Project r' to points r and r_1 on the base-circle in the top view. Draw lines or and or_1. Thus, or is the top view of the generator $o'r'$, and or_1 that of the generator (at the back) which coincides with $o'r'$. Project p' to p and p_1 on or and or_1 respectively. Thus, p is the top view of p', and p_1 is the top view of another point on the other side of the cone and coinciding with p'. The line pp_1 is the width of the horizon-section of the cone at p'.

The position in the front view of any point in the top view, say q, may be determined by reversing the process. Draw the line oq and produce it to cut the base-circle at s. Project s to s' on the base in the front view. Join o' with s'. Through, q, draw a projector to cut $o's'$ at the required point q'.

Sectional views of cones may be obtained by applying any one of the above two methods for locating the positions of points. The generator method is more suitable particularly when the cone is in inclined positions.

(3) Section plane inclined to the base of the cone at an angle smaller than the angle of inclination of the generators with the base:

Problem 14-24. *A cone, base 75 mm diameter and axis 80 mm long is resting on its base on the H.P. It is cut by a section plane perpendicular to the V.P., inclined at 45° to the H.P. and cutting the axis at a point 35 mm from the apex. Draw its front view, sectional top view, sectional side view and true shape of the section.*

Draw a line V.T. in the required position in the front view of the cone. The positions of points on this line and the width of section at each point can be determined by one of the methods explained in problem 14-21 and problem 14-23 and as described below.

(i) ***Generator method*** [fig. 14-26(i) and fig. 14-26(ii)]:

(a) Divide the base-circle into a number of equal parts, say 12. Draw lines (i.e. generators) joining these points with o. Project these points on the line representing the base in the front view.

(b) Draw lines o' 2', o' 3' etc. cutting the line for the section at points b', c' etc. Project these points on the corresponding lines in the top view. For example, point b' on o' 2', also represents point b'_1 on o'-12' which coincides with o'-12'. Therefore, project b' to b on o 2 and to b_1 on o'-12'. b and b_1 are the points on the section (in the top view).

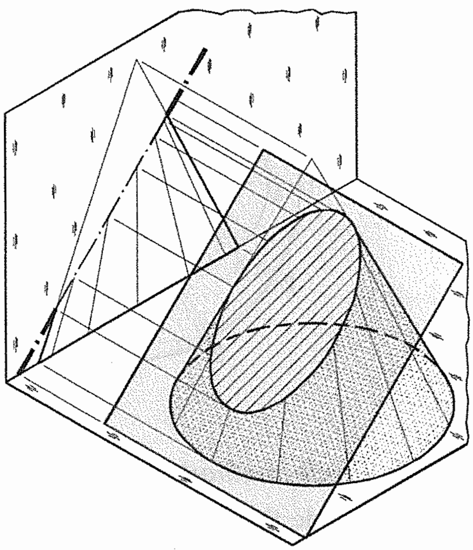


FIG. 14-26(i)

(c) Similarly, obtain other points. Point d' cannot be projected directly. Hence, the same method as in case of pyramids should be employed to determine the positions d and d_1, as shown. In addition to these, two more points for the maximum width of the section at its centre should also be obtained. Mark m', the mid-point of the section and obtain the points m and m_1. Draw a smooth curve through these points.

(d) The true shape of the section may be obtained on the V.T. as a new reference line or symmetrically around the centre line ag, drawn parallel to the V.T. as shown. It is an ellipse whose major axis is equal to the length of the section and minor axis equal to the width of the section at its centre.

Draw the sectional side view by projecting the points on corresponding generators, as shown.

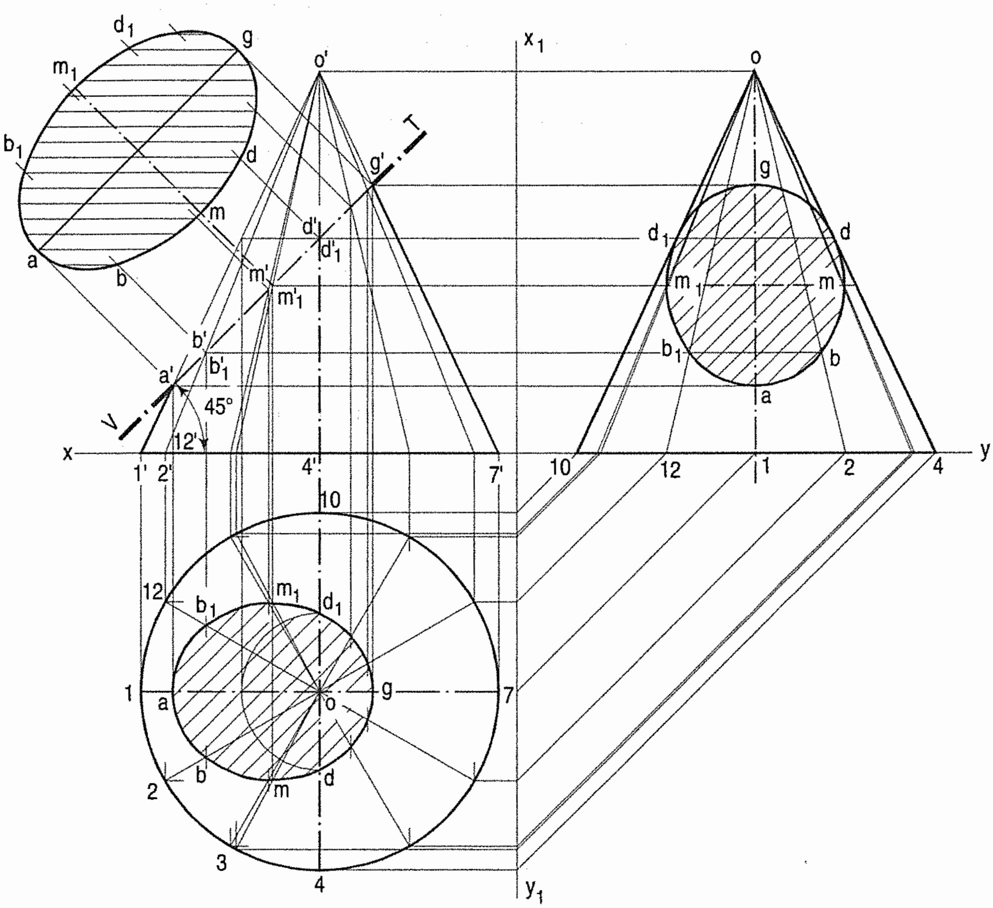


FIG. 14-26(ii)

(ii) ***Circle method*** (fig. 14-27):

(a) Divide the line of section into a number of equal parts. Determine the width of section at, and the position of each division-point in the top view by the circle method. For example, through c', draw a line $c''c''$ parallel to the base.

(b) With o as centre and radius equal to half of $c''c''$, draw an arc. Project c' to c and c_1 on this arc. Then c and c_1 are the required points. The straight line joining c and c_1 will be the width of the section at c'.

(c) Similarly, obtain all other points and draw a smooth curve through them. This curve will show the apparent section. The maximum width of the section will be at the mid-point e'. It is shown in the top view by the length of the chord joining e and e_1.

(d) Draw a reference line x_1 y_1 parallel to the V.T. and project the true shape of the section. In the figure, the auxiliary sectional top view of the truncated cone is shown. It shows the true shape of the section.

The sectional side view (not shown in the figure) may be obtained by projecting all the division-points horizontally and then marking the width of the section at each point, symmetrically around the axis of the cone.

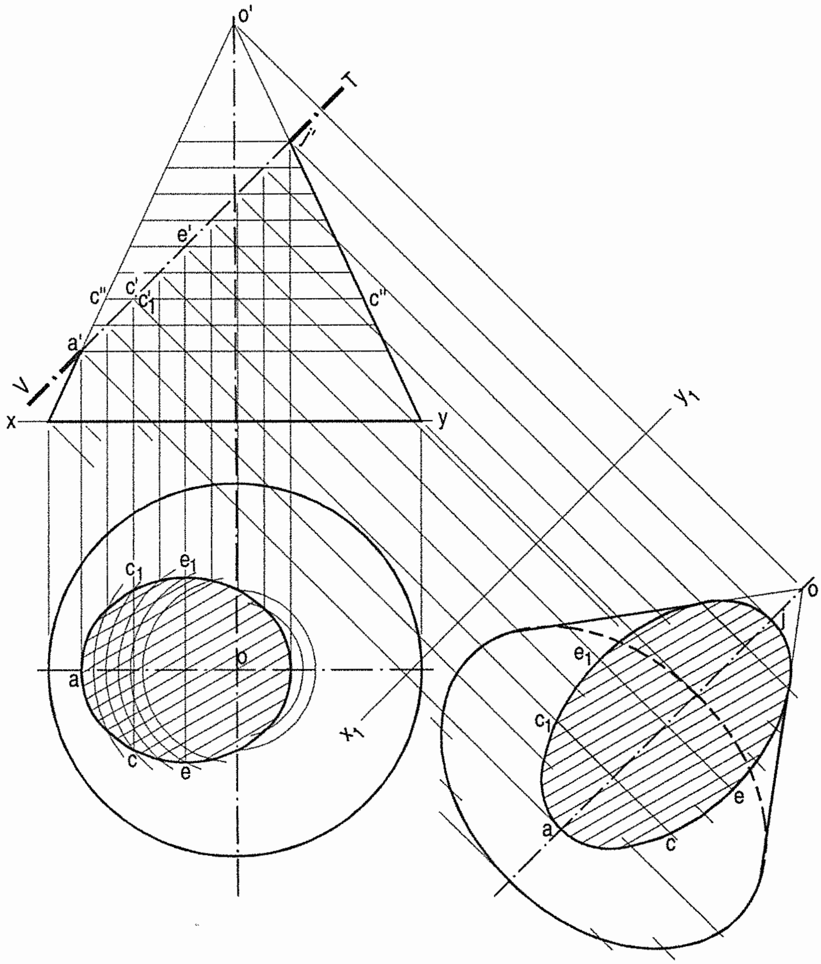


FIG. 14-27

(4) Section plane parallel to a generator of the cone:

Problem 14-25. (fig. 14-28): *The cone in same position as in problem 14-24, is cut by a section plane perpendicular to the V.P. and parallel to and 12 mm away from one of its end generators. Draw its front view, sectional top view and true shape of the section.*

(i) Draw a line V.T. (for the section plane) parallel to and 12 mm away from the generator $o'1'$.

(ii) Draw the twelve generators in the top view and project them to the front view. All the generators except $o'1'$, $o'2'$ and o'-$12'$ are cut by the section plane. Project the points at which they are cut, to the corresponding generators in the top view. The width of the section at the point where the base is cut will be the chord aa_1. Draw a curve through $a...f...a_1$. The figure enclosed between aa_1 and the curve is the apparent section.

(iii) Obtain the true shape of the section as explained in the previous problem. It will be a parabola.

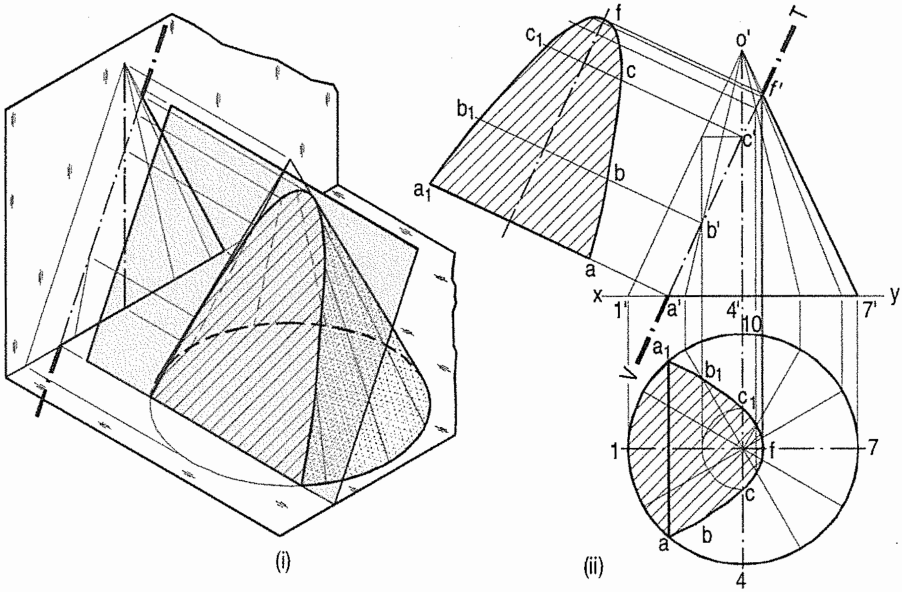


FIG. 14-28

(5) Section plane inclined to the base of the cone at an angle greater than the angle of inclination of the generators with the base:

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 32 for the following problem.

Problem 14-26. [fig. 14-29(i) and fig. 14-29(ii)]: *A cone, base 45 mm diameter and axis 55 mm long is resting on the H.P. on its base. It is cut by a section plane, perpendicular to both the H.P. and the V.P. and 6 mm away from the axis. Draw its front view, top view and sectional side view.*

The section will be seen as a line, perpendicular to xy, in both the front view and the top view. The side view will show the true shape of the section. The width of the section at any point, say c', will be equal to cc_1 obtained by the circle method [fig. 14-29(i)].

(i) Draw the side view of the cone.

(ii) Project the points (on the section) in the side view taking the widths from the top view. For example, through c' draw a horizontal line. Mark on it points c'' and c''_1 equidistant from and on both sides of the axis so that $c''\,c''_1 = cc_1$.

(iii) Draw a curve through the points thus obtained. It will be a hyperbola.

Fig. 14-29(ii) shows the views obtained by the generator method.

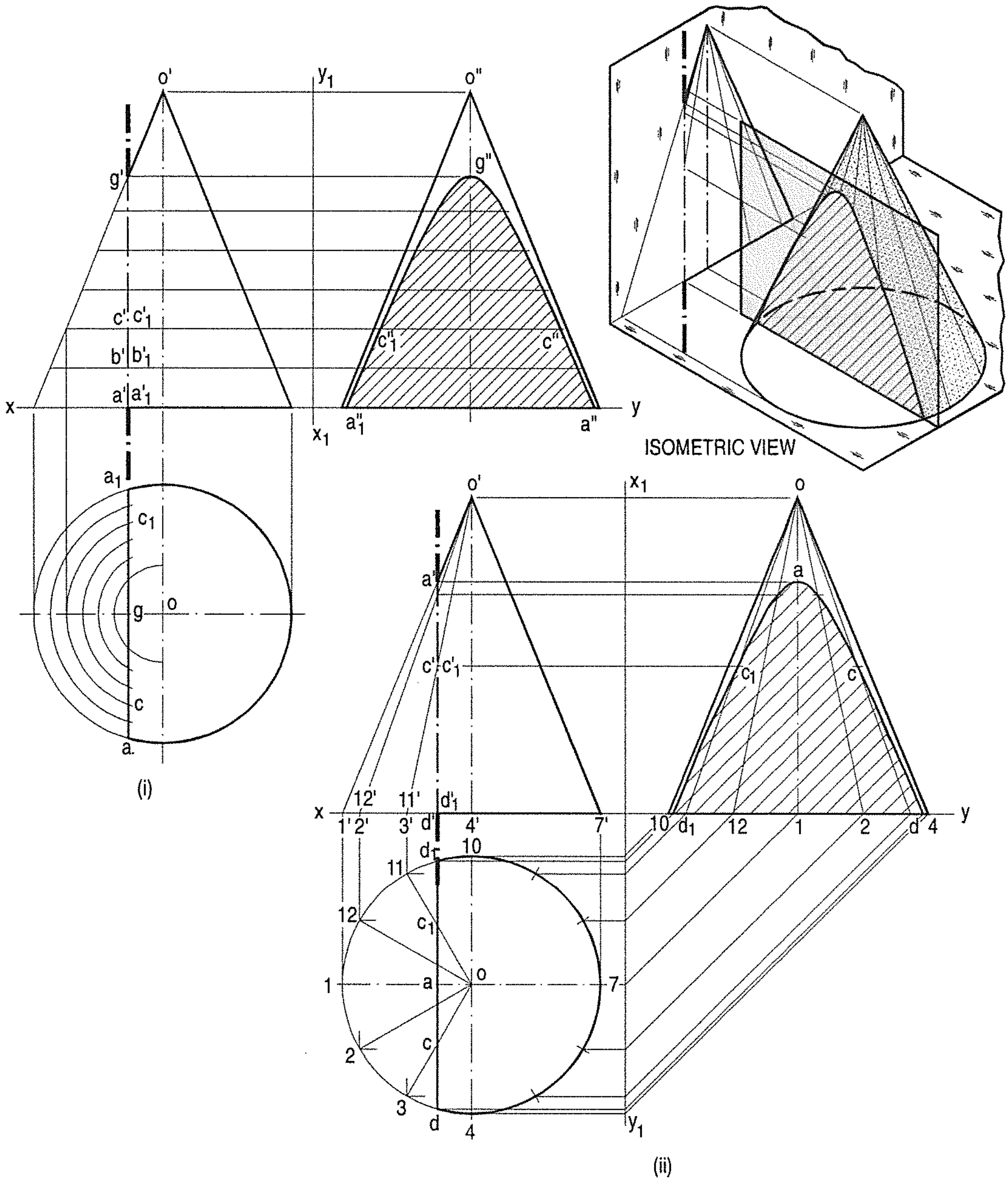


FIG. 14-29

Additional problems on sections of cones:

Problem 14-27. (fig. 14-30): *A cone, diameter of base 50 mm and axis 65 mm long, is lying on the H.P. on one of its generators with the axis parallel to the V.P. It is cut by a horizontal section plane 12 mm above the ground. Draw its front view, sectional top view, and development of its surface.*

Use the generator method and project the points in the top view. The curve will show the true shape of the section viz. a parabola.

For development, the true lengths of the cut-generators are obtained by drawing lines parallel to the base. Positions of points *a* and a_1 are determined by projecting them on the base-circle in the first top view.

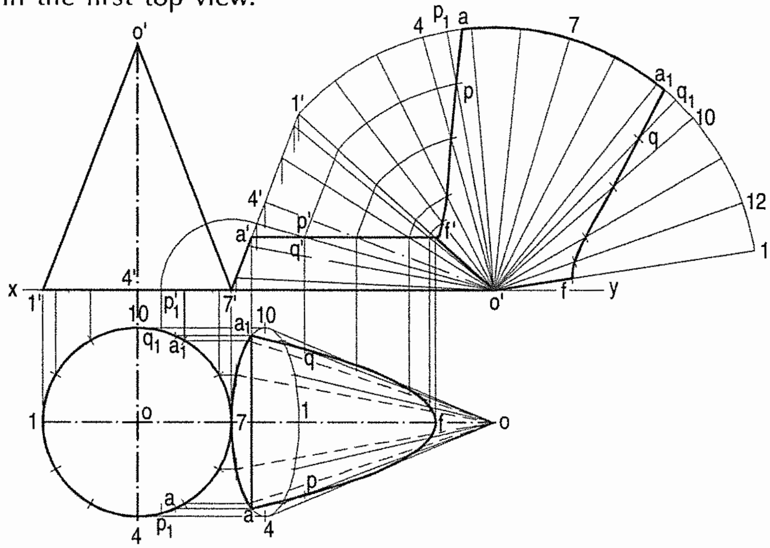


FIG. 14-30

Problem 14-28. (fig. 14-31): *A cone, base 70 mm diameter, axis 75 mm long and resting on its base on the H.P., is cut by a vertical section plane, the H.T. of which makes an angle of 60° with the reference line and is 12 mm away from the top view of the axis. (i) Draw the sectional front view and the true shape of the section. (ii) Also draw the sectional front view and the top view when the same section plane is parallel to the V.P.*

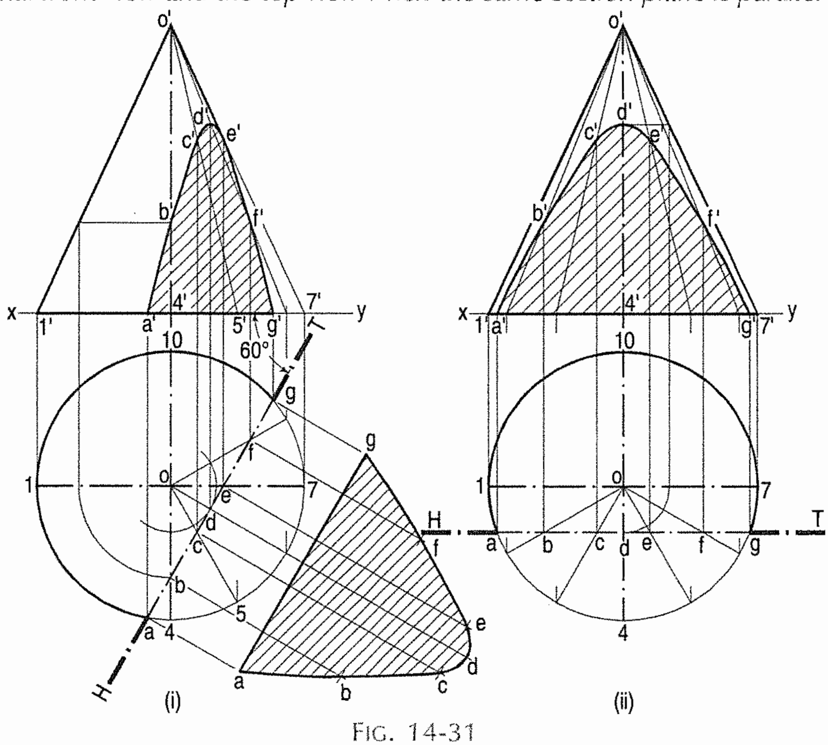


FIG. 14-31

(i) Draw a circle with centre o and radius equal to 12 mm.

(ii) Draw a line for the section plane, tangent to this circle and inclined at 60° to xy [fig. 14-31(i)].

(iii) Project the front view by the generator method as shown. Note how the point f' is obtained.

Fig. 14-31(ii) shows the two views when the section plane is parallel to the V.P.

Problem 14-29. (fig. 14-32): *A cone, base 60 mm diameter and axis 60 mm long is lying on the H.P. on one of its generators with the axis parallel to the V.P. A vertical section plane parallel to the generator which is tangent to the ellipse (for the base) in the top view, cuts the cone bisecting the axis and removing a portion containing the apex. Draw its sectional front view and true shape of the section.*

(i) Name in correct sequence the points at which the base and the generators are cut and project them in the front view.

(ii) Project the true shape of the section on the new reference line $x_1 y_1$ drawn parallel to the H.T.

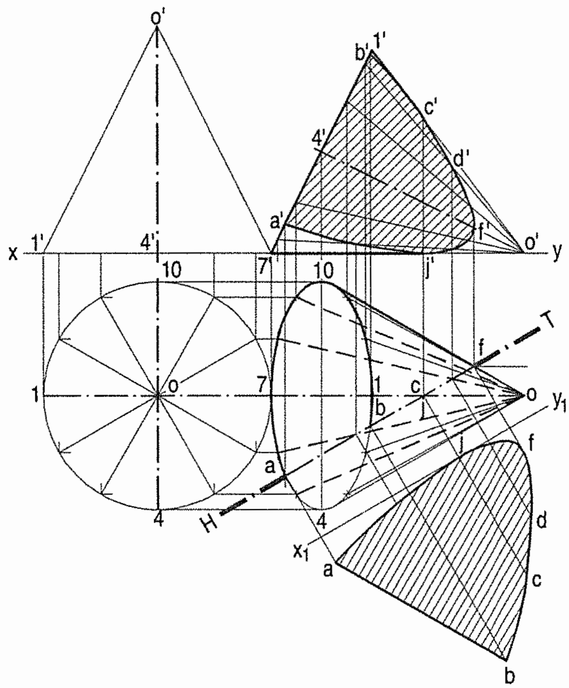


FIG. 14-32

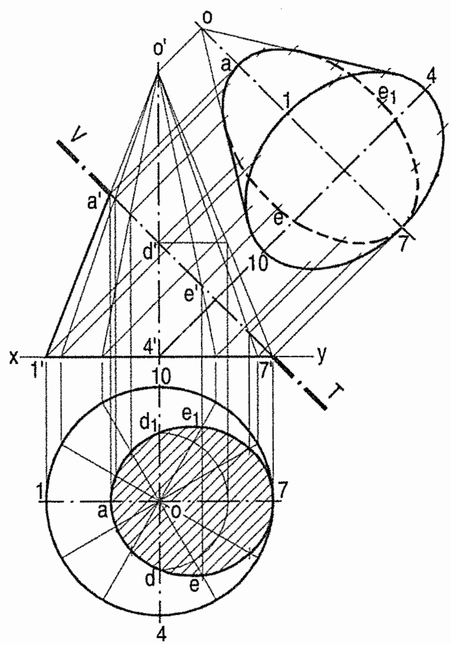


FIG. 14-33

Problem 14-30. (fig. 14-33): *A cone, base 60 mm diameter and axis 75 mm long, is resting on the H.P. on its base. It is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and intersecting the axis 30 mm above the base. Draw its front view and sectional top view. Also draw its top view when it is lying on the ground on its cut-surface with the axis parallel to the V.P.*

See the figure which is self-explanatory. Note that, when the cone is tilted so as to lie on the cut-surface, its base is fully visible, while the section is hidden in the top view.

14-5. SECTIONS OF SPHERES

These are discussed in details as under.

(1) Section plane parallel to the H.P.

(2) Section plane parallel to the V.P.

(3) Section plane perpendicular to the V.P. and inclined to the H.P.

(4) Section plane perpendicular to the H.P. and inclined to the V.P.

(1) Section plane parallel to the H.P.: When a sphere is cut by a plane, the true shape of the section is always a circle.

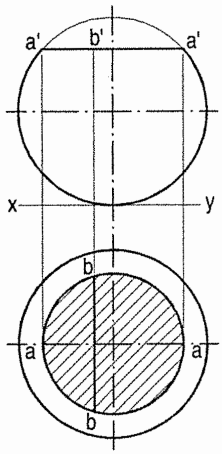


FIG. 14-34

The sphere in fig. 14-34 is cut by a horizontal section plane. The true shape of the section (seen in the top view) is a circle of diameter *a'a'*. The width of the section at any point say *b'*, is equal to the length of the chord *bb*.

(2) Section plane parallel to the V.P.: When the sphere is cut by a section plane parallel to the V.P. (fig. 14-35), the true shape of the section, seen in the front view, is a circle of diameter *cc*. The width of section at any point *d* is equal to the length of the chord *d'd'*.

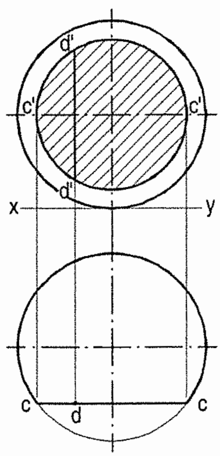


FIG. 14-35

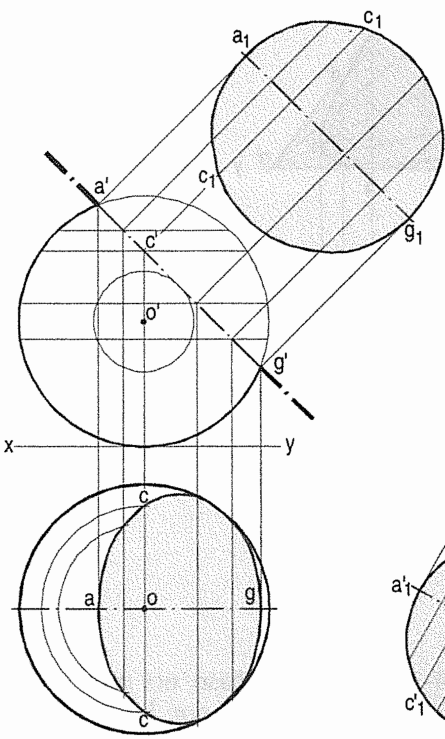


FIG. 14-36

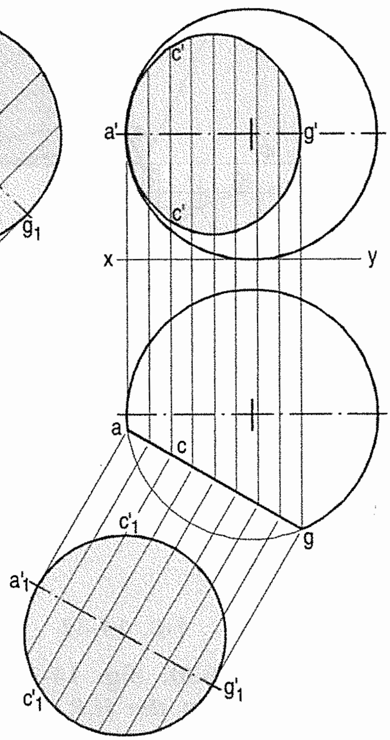


FIG. 14-37

(3) Section plane perpendicular to the V.P. and inclined to the H.P.:

Problem 14-31. (fig. 14-36): *A sphere of 50 mm diameter is cut by a section plane perpendicular to the V.P., inclined at 45° to the H.P. and at a distance of 10 mm from its centre. Draw the sectional top view and true shape of the section.*

Draw a line (for the section plane) inclined at 45° to xy and tangent to the circle of 10 mm radius drawn with o' as centre. Mark a number of points on this line.

Method I:

(i) Find the width of section at each point in the top view as shown in fig. 14-34. For example, the chord cc is the width of section at the point c'.

(ii) Draw a curve through the points thus obtained. It will be an ellipse. The true shape of the section will be a circle of diameter $a'g'$.

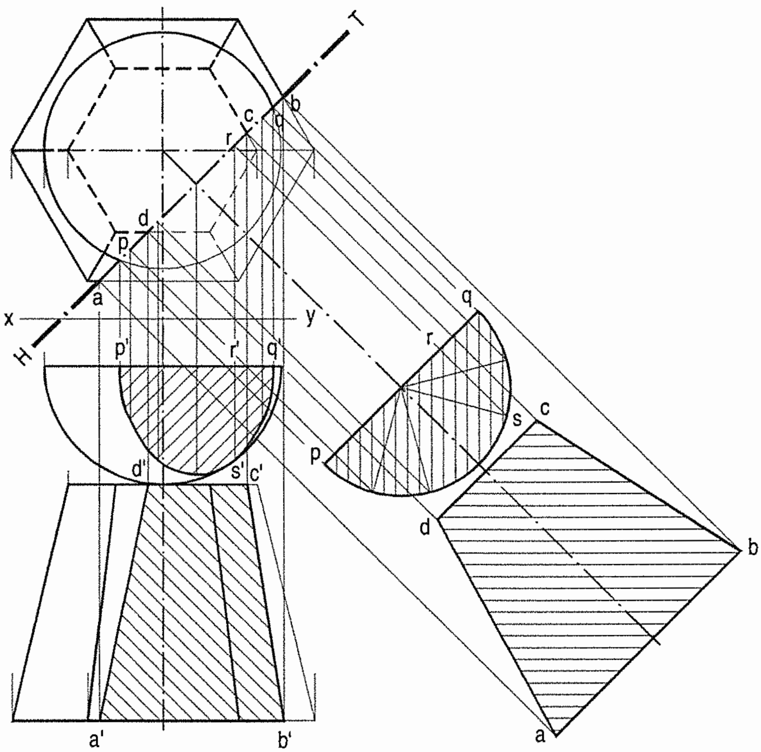
Method II:

It is known that the true shape of the section is a circle of diameter equal to $a'g'$. The width of section at any point say c' is equal to the chord c_1c_1 on this circle. Therefore, project c' to points c in the top view so that $cc = c_1c_1$. Similarly, obtain other points and draw the ellipse through them.

Fig. 14-37 shows the sectional front view and true shape of the section when the section plane is vertical and inclined to the V.P.

(4) Section plane perpendicular to the H.P. and inclined to the V.P.:

Problem 14-32. (fig. 14-38): *The projections of a hemisphere 50 mm diameter, placed centrally on the top of a frustum of a hexagonal pyramid, base 32 mm side, top 20 mm side and axis 50 mm long are given. Draw the sectional front view when the vertical section plane H.T. inclined at 45° to the V.P. and 10 mm from the axis, cuts them. Also draw the true shapes of the sections of both the solids.*



(Third-angle projection)

FIG. 14-38

The widths of the section of the sphere at various points are obtained from the semi-circle drawn in the top view.

14-6. TYPICAL PROBLEMS OF SECTIONS OF SOLIDS

Problem 14-33. *A solid composed of a half-cone and a half-hexagonal pyramid is shown in fig. 14-39. It is cut by a section plane, which makes an angle of 30° with the base, is perpendicular to the V.P. and contains an edge of the base of the pyramid. Draw its sectional top view, true shape of the section and development of the surface of the remaining portion. Base of cone 60 mm diameter; axis 70 mm long.*

(i) Draw a line V.T. inclined at 30° to the base and passing through a'.

(ii) Project the sectional top view. Note how points b and b_1 are obtained. The true shape of the section will be partly elliptical.

(iii) Draw the development of the half-cone and half-pyramid and show the lines for the section in it.

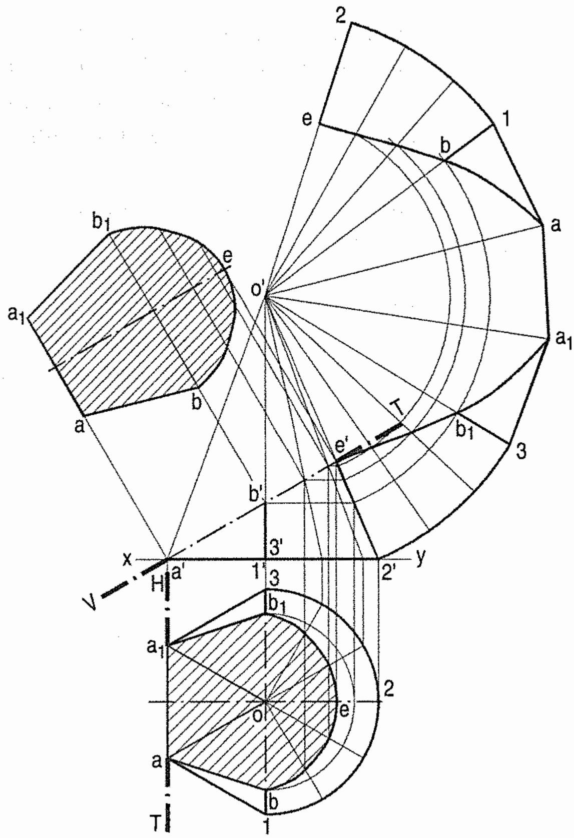


FIG. 14-39

Problem 14-34. (fig. 14-40): *A cylinder, base 50 mm diameter and axis 75 mm long, has a square hole of 25 mm side cut through it so that the axis of the hole coincides with that of the cylinder. The cylinder is lying on the H.P. with the axis perpendicular to the V.P. and the faces of the hole equally inclined to the H.P.*

A vertical section plane, inclined at 60° to the V.P. cuts the cylinder in two equal halves. Project the front view of the cylinder on an A.V.P. parallel to the section plane.

(i) Assuming the cylinder to be whole, draw its auxiliary front view.

(ii) Project the points at which the generators of the cylinder and the edges of the hole are cut. The section of the cylinder will be a part of an ellipse. Join the points at which the edges of the hole are cut. The back edges

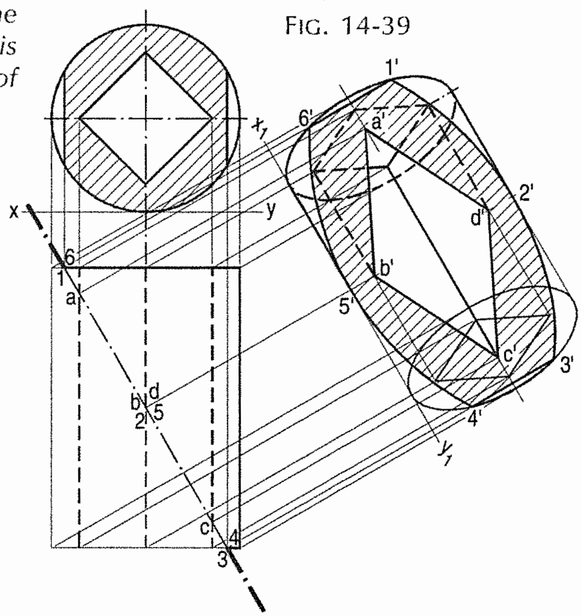


FIG. 14-40

of the hole will be visible within the section and hence, must be shown as full lines.

(iii) Complete the view by showing the section and the remaining portion of the cylinder with dark lines.

Problem 14-35. (fig. 14-41): *A square prism axis 110 mm long is resting on its base in the H.P. The edges of the base are equally inclined to V.P.*

The prism is cut by an A.I.P. passing through the mid-point of the axis in such a way that the true shape of the section is rhombus having diagonals of 100 mm and 50 mm. Draw the projections and determine the inclination of A.I.P. with the H.P.

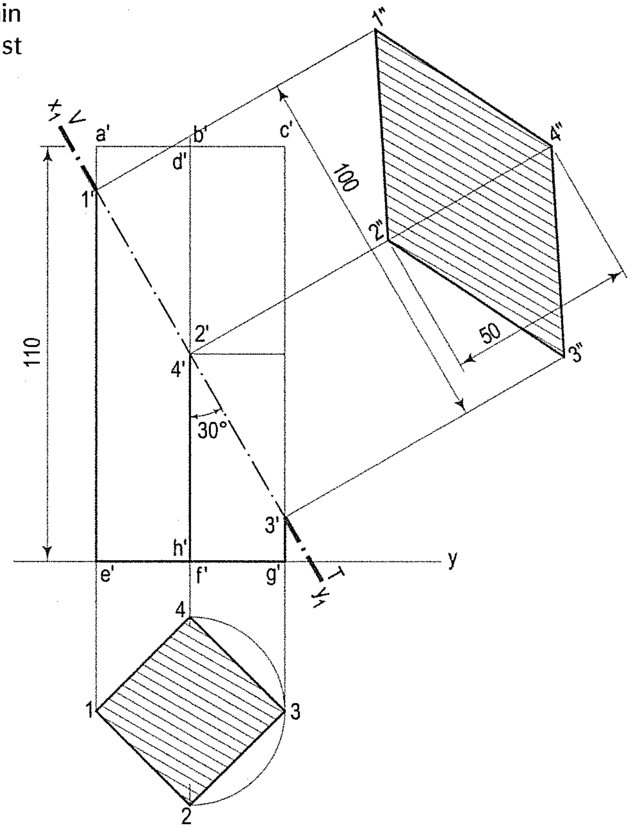


FIG. 14-41

(i) Draw the top view and the front view as shown.

(ii) Mark the mid-point of the axis in the front view.

(iii) With the mid-point of the axis as centre and radius equal to 50 mm (half of the longer diagonal i.e. 100 mm), draw an arc cutting the two opposite vertical sides of the prism. Project the points and complete the true shape as shown.

Problem 14-36. (fig. 14-42): *A vertical cylinder 50 mm diameter is cut by an A.V.P. making 30° to the V.P. in such a way that the true shape of the section is a rectangle of 40 mm × 80 mm sides. Draw the projections and the true shape of the section.*

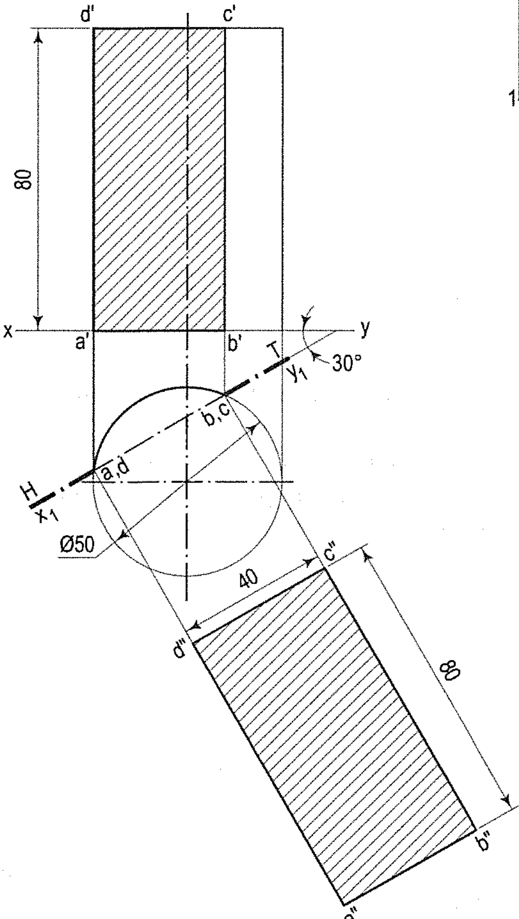


FIG. 14-42

(i) Draw a top view and an front view of the cylinder.

(ii) Draw x_1y_1 at 30° to xy in such a way that the chord length in the top view is 40 mm. Project points 1, 2, 3 and 4 and draw the rectangle of 40 mm × 80 mm as shown.

Problem 14-37. (fig. 14-43): *A hexagonal pyramid, base 30 mm side and axis 70 mm long is resting on its slant edge of the face on the horizontal plane.*

A section plane, perpendicular to the V.P., inclined to the H.P. passes through the highest corner of the base and intersecting the axis at 25 mm from the base. Draw the projections of the solid and determine the inclination of the section plane with the H.P.

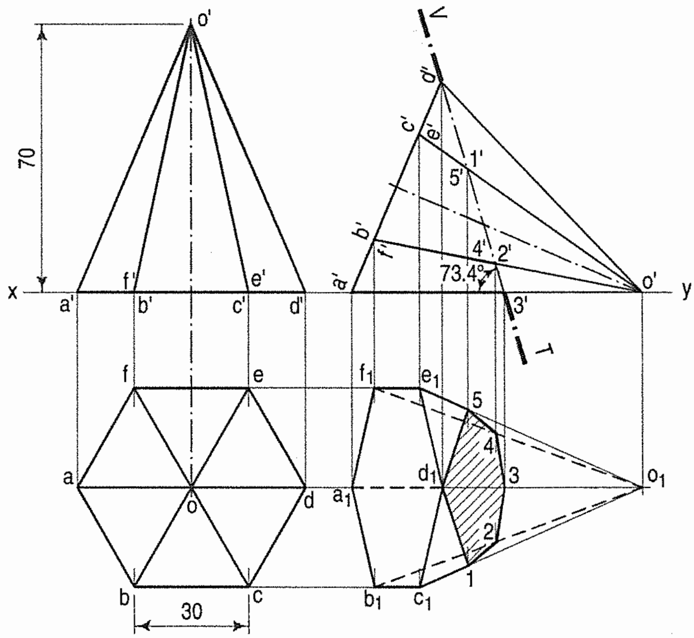


FIG. 14-43

(i) Draw the top view and the front view keeping one of the sides of the base parallel to xy.

(ii) With a' and o', as centres and radii equal to $a'd'$ and $o'd'$ draw arcs intersecting each other at point d'. Draw a section plane passing through d' and point 25 mm away from the base along the axis as shown.

(iii) Measure the angle by V.T. with xy.

Problem 14-38. (fig. 14-44): *A pentagonal pyramid, base side 30 mm, length of axis 80 mm is resting on a base edge on the H.P. with a triangular face containing that edge being perpendicular to the V.P. and inclined to the H.P. at 60°. It is cut by a horizontal section plane whose V.T. passes through the mid-point of the axis. Draw the front view, sectional top view and add a profile view.*

(i) Draw the top view and the front view keeping one of the sides of the base perpendicular to xy.

(ii) Tilt the front view on the points c', d' as shown.

(iii) Draw a line parallel to xy and passing through the mid-point of the axis representing V.T. of the section plane.

(iv) Complete the projection as shown.

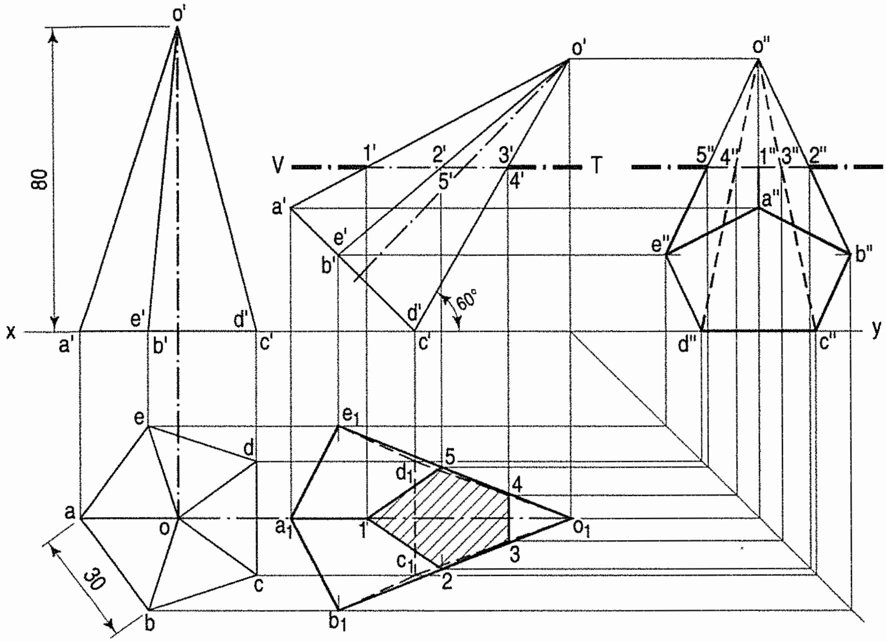


FIG. 14-44

Problem 14-39. (fig. 14-45): *A cone, diameter of the base 60 mm and axis 70 mm long is resting on its base on the H.P. It is cut by an A.I.P. so that the true shape of the section is an isosceles triangle having 50 mm base. Draw the top view, the front view and the true shape of the section.*

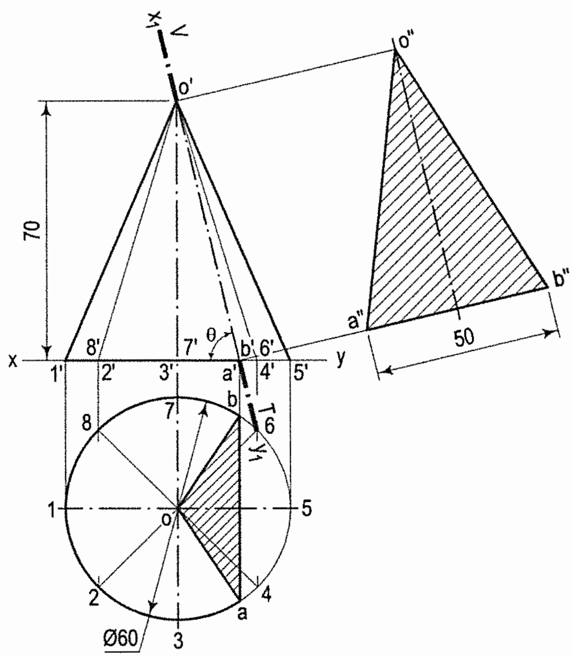


FIG. 14-45

When a section plane passes through an apex of a cone and cuts the base of the cone, the true-shape of section is a triangle.

(i) Draw a top view and an front view as shown.

(ii) Mark chord *ab* of 50 mm (the base of triangle) in the top view. Project points *a* and *b* in the front view intersecting base at *a*' or *b*'. Join points *a*' and *o*'.

This represents V.T. of the section plane.

(iii) Considering line V.T. as new line $x_1 y_1$, draw the projectors from *o*', *a*' and *b*'.

(iv) Construct the true shape of triangle as shown.

Problem 14-40. (fig. 14-46): *A square pyramid of 60 mm side of base and 70 mm length of axis is resting on its base on the H.P., having a side of base perpendicular to the V.P. It is cut by two cutting planes. One is parallel to its extreme right face and 10 mm away from it. While the other is parallel to the extreme left face.*

Both the cutting planes intersect each other on the axis of the pyramid. Draw the sectional top view, front view and the left hand side view.

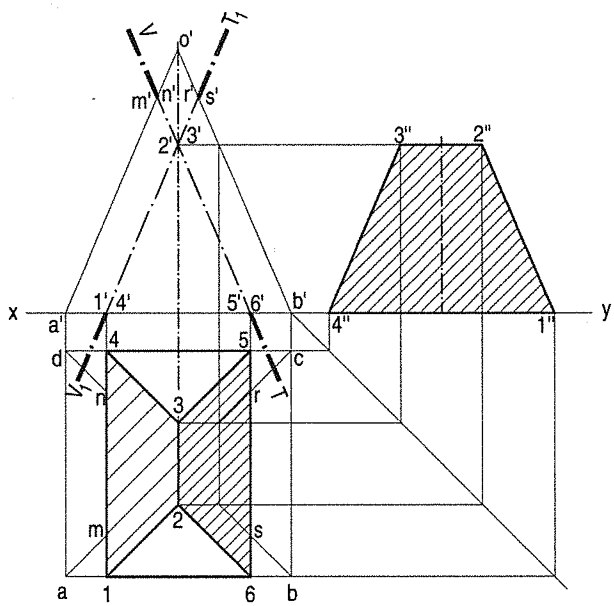


FIG. 14-46

(i) Draw the top view and the front view as shown.

(ii) Draw lines V.T. and V_1T_1 representing section planes in the front view intersecting at 2' and 3'.

(iii) Complete the projections. Note that the intersection points 2' and 3' are transferred on the slant edge *o'b'* and then projected in the top view.

Problem 14-41. (fig. 14-47): *A cylinder of diameter 50 mm and axial height 90 mm having 34 mm square hole centrally along the axis, rests on a point on the circular edge of the base remaining on the H.P. The axis of the cylinder is parallel to the V.P. and inclined at 30° to the H.P. and the rectangular faces of the square hole remain equally inclined to the V.P. A section plane perpendicular to the V.P. and inclined to the horizontal plane passing through the mid-point of the axis, such that the apparent section in the top view is a circle of 50 mm diameter cuts the cylinder. Draw the front view, section plane, true shape of the section and find the inclination of the section plane with H.P.*

Fig. 14-47 shows the projections of the solid. Note that the generators 4 and 6 cut the apparent section at the points *s*, *q*, s_1 and q_1. These points are transferred in the front view as shown. The required section plane must pass through the midpoint of the axis, and *s*, *q* (s_1 and q_1).

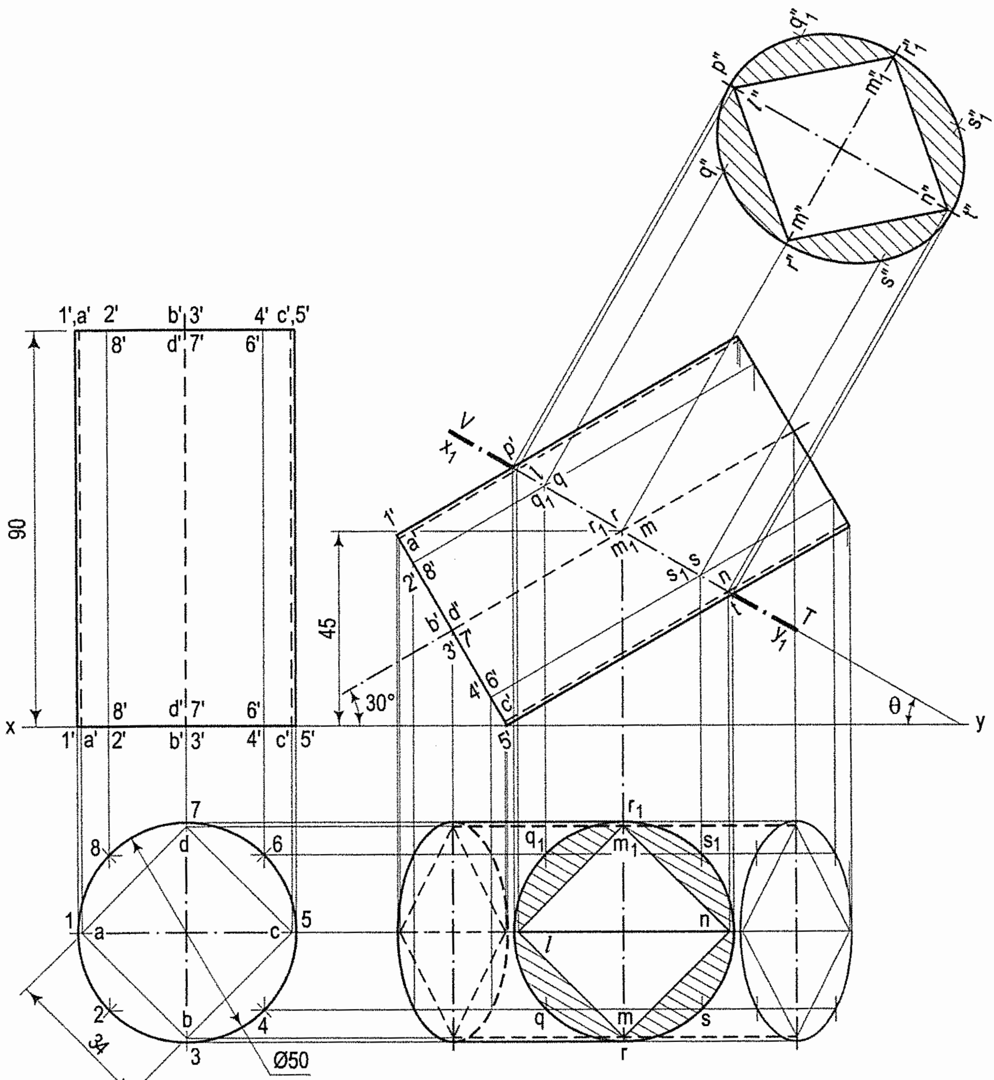


FIG. 14-47

Problem 14-42. (fig. 14-48): *A cylindrical disc of 46 mm diameter is resting on the H.P. and has height of 20 mm. A hexagonal prism of side 23 mm and height of 30 mm is resting on the disc such that their axes are in one line and its faces are equally inclined with the V.P.. It is cut by the auxiliary plane, offset 10 mm infront of the centre of the disc and is inclined at 40° with xy. Draw the projection of combined solids and obtain true shape of the section.*

(i) Draw the projections of the combined solids with given positions.

(ii) Draw an offset circle of 10 mm radius and mark a cutting plane inclined at 40°, touching this circle.

(iii) Project various points in the front view as shown.

(iv) Draw x_1y_1 parallel to the cutting plane.

(v) Obtain true shape as shown.

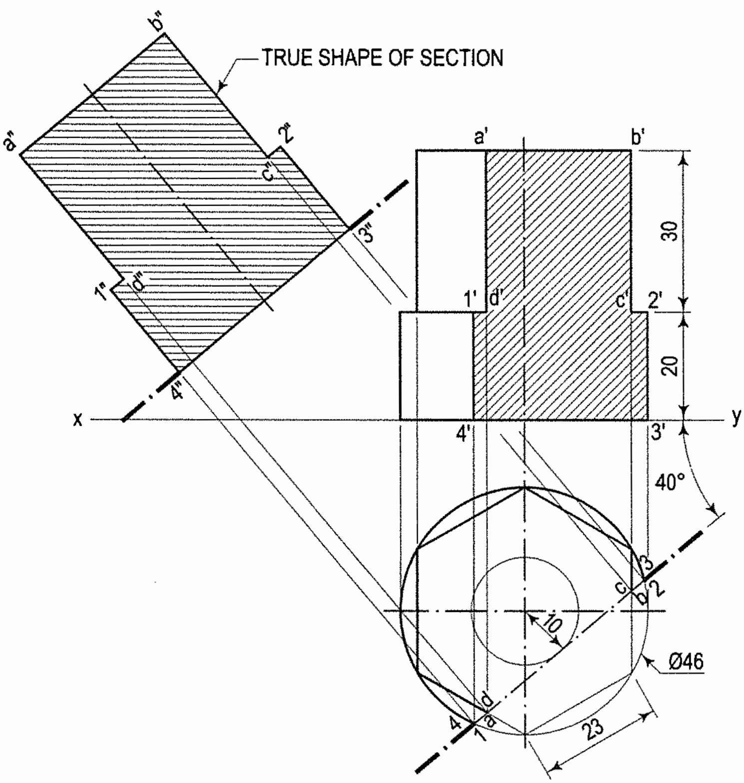


FIG. 14-48

Problem 14-43. (fig. 14-49): *A square prism of 40 mm side is resting on H.P. and has height of 30 mm. Its faces are equally inclined with the V.P. A frustum of cone having base diameter 40 mm and top 30 mm diameter with height 30 mm is kept on the prism such that axes of the both solids are coinciding. A sectional plane cuts the combined axes and inclined at 55° with H.P. passes through left corners of the prism. Draw the front view and section top view. Draw also true-shape of the section.*

(i) Draw the top view and front view of combined solid as shown.

(ii) In front view, draw a section plane at angle of 55° with *xy* passing through the corner of square prism. Mark points 1', 2', 3', 4', 5' and 6' as shown.

(iii) Project these points in the top view and draw section line as shown in fig. 14-49.

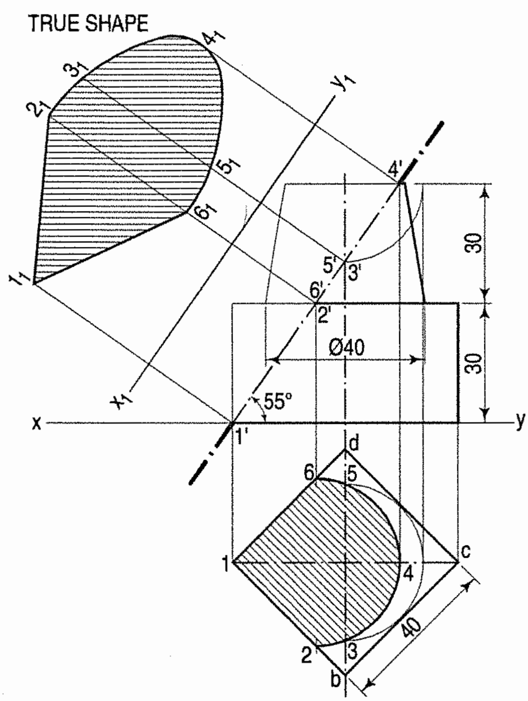


FIG. 14-49

(iv) Draw a new reference line x_1y_1 parallel to the section plane and project the section on it.

(v) The distances of the points from x_1y_1 should be taken equal to their corresponding distances from *xy* in the top view of combined solids.

Problem 14-44. (fig. 14-50): *A horizontal frustum of square pyramid having front square of 20 mm side and back square of 30 mm at length of 60 mm has axis perpendicular to the V.P. and the side of square is inclined to 45° with H.P. It is cut by section plane making 40° with V.P. and passing through a point on axis 30 mm from the surface of large square side. Draw projections of the frustum.*

(i) Mark point *b*' and construct the square a' b' c' d' with sides equally inclined to the *xy*. This is a front view of the frustum of square pyramid.

(ii) Project the top view such that the axis remains perpendicular to *xy* as shown in fig. 14-50.

(iii) Draw a section plane at distance of 20 mm along axis from large square at angle of 40° with *xy*.

(iv) Mark the points *p*, *q*, *r*, *s* and project them in the front view. Draw section lines in the front view for the section and complete the view.

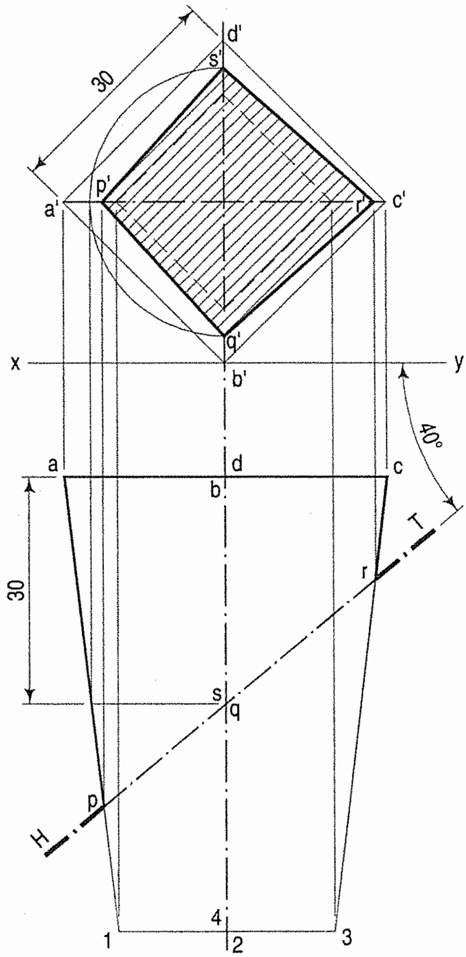


FIG. 14-50

EXERCISES 14

1. A cube of 50 mm long edges is resting on the H.P. with a vertical face inclined at 30° to the V.P. It is cut by a section plane, perpendicular to the V.P., inclined at 30° to the H.P. and passing through a point on the axis, 38 mm above the H.P. Draw the sectional top view, true shape of the section and development of the surface of the remaining portion of the cube.

2. A hexagonal prism, side of base 35 mm and height 75 mm is resting on one of its corners on the H.P. with a longer edge containing that corner inclined at 60° to the H.P. and a rectangular face parallel to the V.P. A horizontal section plane cuts the prism in two equal halves.
   (i) Draw the front view and sectional top view of the cut prism.
   (ii) Draw another top view on an auxiliary inclined plane which makes an angle of 45° with the H.P.

3. A pentagonal prism, side of base 50 mm and length 100 mm has a rectangular face on the H.P. and the axis parallel to the V.P. It is cut by a vertical section

plane, the H.T. of which makes an angle of 30° with *xy* and bisects the axis. Draw the sectional front view, top view and true shape of the section. Develop the surface of the remaining half of the prism.

4. A hollow square prism, base 50 mm side (outside), length 75 mm and thickness 9 mm is lying on the H.P. on one of its rectangular faces, with the axis inclined at 30° to the V.P. A section plane, parallel to the V.P. cuts the prism, intersecting the axis at a point 25 mm from one of its ends. Draw the top view and sectional front view of the prism.
5. A cylinder, 65 mm diameter and 90 mm long, has its axis parallel to the H.P. and inclined at 30° to the V.P. It is cut by a vertical section plane in such a way that the true shape of the section is an ellipse having the major axis 75 mm long. Draw its sectional front view and true shape of the section.
6. A cube of 65 mm long edges has its vertical faces equally inclined to the V.P. It is cut by a section plane, perpendicular to the V.P., so that the true shape of the section is a regular hexagon. Determine the inclination of the cutting plane with the H.P. and draw the sectional top view and true shape of the section.
7. A vertical hollow cylinder, outside diameter 60 mm, length 85 mm and thickness 9 mm is cut by two section planes which are normal to the V.P. and which intersect each other at the top end of the axis. The planes cut the cylinder on opposite sides of the axis and are inclined at 30° and 45° respectively to it. Draw the front view, sectional top view and auxiliary sectional top views on planes parallel to the respective section planes.
8. A square pyramid, base 50 mm side and axis 75 mm long, is resting on the H.P. on one of its triangular faces, the top view of the axis making an angle of 30° with the V.P. It is cut by a horizontal section plane, the V.T. of which intersects the axis at a point 6 mm from the base. Draw the front view, sectional top view and the development of the sectioned pyramid.
9. A pentagonal pyramid, base 30 mm side and axis 75 mm long, has its base horizontal and an edge of the base parallel to the V.P. It is cut by a section plane, perpendicular to the V.P., inclined at 60° to the H.P. and bisecting the axis. Draw the front view and the top view when the pyramid is tilted so that it lies on its cut-face on the ground with the axis parallel to the V.P. Show the shape of the section by dotted lines. Develop the surface of the truncated pyramid.
10. A tetrahedron of 65 mm long edges is lying on the H.P. on one of its faces, with an edge perpendicular to the V.P. It is cut by a section plane which is perpendicular to the V.P. so that the true shape of the section is an isosceles triangle of base 50 mm long and altitude 40 mm. Find the inclination of the section plane with the H.P. and draw the front view, sectional top view and the true shape of the section.
11. A hexagonal pyramid, base 50 mm side and axis 100 mm long, is lying on the H.P. on one of its triangular faces with the axis parallel to the V.P. A vertical section plane the H.T. of which makes an angle of 30° with the reference line, passes through the centre of the base and cuts the pyramid, the apex being retained. Draw the top view, sectional front view, true shape of the section and the development of the surface of the cut-pyramid.
12. A cone, base 75 mm diameter and axis 75 mm long, has its axis parallel to the V.P. and inclined at 45° to the H.P. A horizontal section plane cuts the

cone through the mid-point of the axis. Draw the front view, sectional top view and an auxiliary top view on a plane parallel to the axis.

13. A cone, base 65 mm diameter and axis 75 mm long, is lying on the H.P. on one of its generators with the axis parallel to the V.P. A section plane which is parallel to the V.P. cuts the cone 6 mm away from the axis. Draw the sectional front view and development of the surface of the remaining portion of the cone.
14. The cone in above problem 13 is cut by a horizontal section plane passing through the centre of the base. Draw the sectional top view and another top view on an auxiliary plane parallel to the axis of the cone.
15. A hemisphere of 65 mm diameter, lying on the H.P. on its flat face, is cut by a vertical section plane inclined to the V.P. so that the semi-ellipse seen in the front view has its minor axis 45 mm long and half major axis 25 mm long. Draw the top view, sectional front view and true shape of the section.
16. The top view of a cylinder 75 mm diameter, 125 mm long, placed on top of the frustum of a cone, base 100 mm diameter, top 50 mm diameter and axis 125 mm long is shown in fig. 14-51. Both the solids are cut by a vertical section plane, the H.T. of which is 12 mm from the axis of the frustum and makes 30° angle with *xy*. Draw the sectional front view and true shape of the sections.

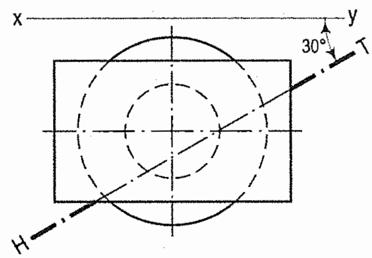


FIG. 14-51

17. A sphere of 75 mm diameter is cut by a section plane, perpendicular to the V.P. and inclined at 30° to the H.P. in such a way that the true shape of the section is a circle of 50 mm diameter. Draw its front view, sectional top view and sectional side view.
18. A frustum of a cone, base 75 mm diameter, top 50 mm diameter and axis 75 mm long, has a hole of 30 mm diameter drilled centrally through its flat faces. It is resting on its base on the H.P. and is cut by a section plane, the V.T. of which makes an angle of 60° with *xy* and bisects the axis. Draw its sectional top view and an auxiliary top view on a reference line parallel to the V.T., showing clearly the shape of the section.
19. A hexagonal prism, side of the base 25 mm long and axis 65 mm long is resting on an edge of the base on the H.P., its axis being inclined at 60° to the H.P. and parallel to the V.P. A section plane, inclined at 45° to the V.P. and normal to the H.P., cuts the prism and passes through a point on the axis at a distance of 20 mm from the top end of the axis. Draw its sectional front view and true shape of the section.
20. A pentagonal pyramid, edge of base 25 mm long and height 50 mm is resting on the H.P. on a corner of its base in such a way that the slant edge containing that corner makes an angle of 60° with the H.P. and is parallel to the V.P. It is cut by a section plane making an angle of 30° with the V.P., perpendicular to the H.P. and passing through a point on the axis at a distance of 6 mm from its base. Draw its sectional front view and true shape of the section.
21. The distance between the opposite parallel faces of a 50 mm thick hexagonal block is 75 mm. The block has one of its rectangular faces parallel to the H.P. and its axis makes an angle of 30° with the V.P. It is cut by a section plane making

an angle of 30° with the H.P., normal to the V.P. and bisecting the axis. Draw its sectional top view and another top view on a plane parallel to the section.

22. *PQR* is an isosceles triangle having base *PR* horizontal and 50 mm long, and altitude 50 mm. A point *A* is taken on *PR* at a distance of 15 mm from *P* and a straight line *AB* is drawn parallel to *PQ* cutting *QR* at *B*. If *AB* is regarded as the V.T. of an inclined plane perpendicular to the V.P., cutting a cone of which *PQR* is the front view, draw the sectional top view, sectional side view and true shape of the section.
23. A cone of 55 mm diameter and 75 mm height is resting on the H.P. on one of its generators in such a way that, the generator is parallel to the V.P. It is cut by a plane parallel to the V.P. and inclined at 90° to the H.P. and passing through a point 15 mm in front of its axis. Draw the sectional front view and the top view of the cone.
24. The true section of a vertical square prism cut by an inclined plane is a rectangle of 75 mm × 40 mm. The plane cuts one of the side faces at a height of 40 mm from the base. Draw three views of the cut prism when it rests on the cut face on the H.P. with its axis remaining parallel to the V.P.
25. An equilateral triangular prism, base 50 mm side and height 100 mm is standing on the H.P. on its triangular face with one of the sides of that face inclined at 90° to the V.P. It is cut by an inclined plane in such a way that the true shape of the section is a trapezium of 50 mm and 12 mm parallel sides. Draw the projections and true shape of the section and find the angle which the cutting plane makes with the H.P.
26. A horizontal cylinder, 30 mm diameter and length 60 mm, is placed centrally on the top of a frustum of a cone, diameter of the base 45 mm, diameter of the top 25 mm and height 45 mm. Draw a sectional front view of the two solids on a vertical plane, distance 12 mm from the axis of the cone and making an angle of 60° with the axis of the cylinder.
27. A cone, base 75 mm diameter and axis 100 mm long, has its base on the H.P. A section plane, parallel to one of the end generators and perpendicular to the V.P., cuts the cone intersecting the axis at a point 75 mm from the base. Draw the sectional top view and project another top view on a plane parallel to the section plane, showing the shape of the section clearly.
28. A solid is made up of a cylinder, 30 mm diameter and 75 mm long, which joins another cylinder, 75 mm diameter and 25 mm long, by a fillet of 20 mm radius, the axes of the two cylinders being in a straight line. Draw the top view of a horizontal section of the solid made by a plane parallel to and 15 mm above the axis.
29. A cone frustum, base 75 mm diameter, top 35 mm diameter and height 65 mm has a hole of 30 mm diameter drilled through it so that the axis of the hole coincides with that of the cone. It is resting on its base on the H.P. and is cut by a section plane perpendicular to the V.P., parallel to an end generator and passing through the top end of the axis. Draw sectional top view and sectional side view of the frustum.
30. A cube of 25 mm edge rests on one of its corners on the H.P. so that a solid diagonal is vertical and two of its faces are perpendicular to the V.P. A vertical section plane parallel to the V.P. cuts the cube at a distance of 8 mm from the solid diagonal and nearer to the V.P. Draw its sectional front view.

Chapter 15

DEVELOPMENT OF SURFACES

15-0. INTRODUCTION

Imagine that a solid is enclosed in a wrapper of thin material, such as paper. If this covering is opened out and laid on a flat plane, the flattened-out paper is the development of the solid. Thus, when surfaces of a solid are laid out on a plane, the figure obtained is called its development.

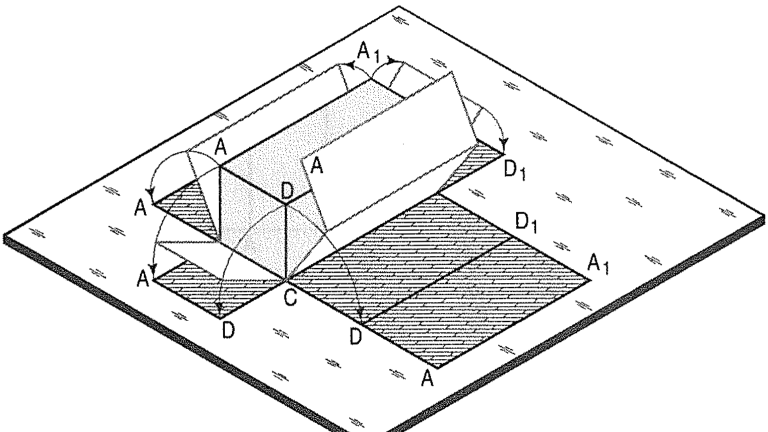


FIG. 15-1

Fig. 15-1 shows a square prism covered with paper in process of being opened out. Its development (fig. 15-2) consists of four equal rectangles for the faces and two similar squares for its ends. Each figure shows the true size and shape of the corresponding surface of the prism. The development of a solid, thus represents the actual shape of all its surfaces which, when bent or folded at the edges, would form the solid.

Hence, it is very important to note that every line on the development must be the true length of the corresponding edge on the surface.

The knowledge of development of surfaces is essential in many industries such as automobile, aircraft, ship building, packaging and sheet-metal work. In construction of boilers, bins, process-vessels, hoppers, funnels, chimneys etc., the plates are marked and cut according to the developments which, when folded, form the desired objects. The form of the sheet obtained by laying all the outer surfaces of the solid with suitable allowances for the joints is known as pattern.

Only the surfaces of polyhedra (such as prisms and pyramids) and single-curved surfaces (as of cones and cylinders) can be accurately developed. Warped and double-curved surfaces are undevelopable. These can however be approximately developed by dividing them up into a number of parts.

This chapter deals with the following topics:

1. Methods of development.
2. Developments of lateral surfaces of right solids.
3. Development of transition pieces.
4. Spheres (approximate method).

15-1. METHODS OF DEVELOPMENT

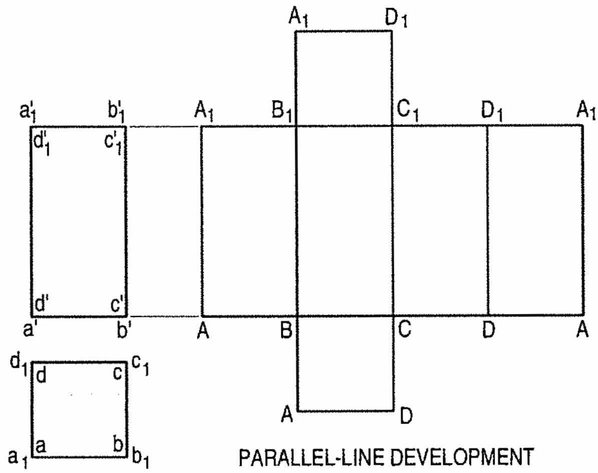
This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 33 for the methods of development of surfaces.

The following are the principal methods of development:

(1) **Parallel-line development:** It is employed in case of prisms and cylinders in which stretch-out-line principle is used. Lines *A-A* and A_1-A_1 in fig. 15-2 are called the stretch-out lines.

(2) **Radial-line development:** It is used for pyramids and cones in which the true length of the slant edge or the generator is used as radius.

(3) **Triangulation development:** This is used to develop transition pieces. This is simply a method of dividing a surface into a number of triangles and transferring them into the development.



PARALLEL-LINE DEVELOPMENT

FIG. 15-2

(4) **Approximate method:** It is used to develop objects of double curved or warped surfaces as sphere, paraboloid, ellipsoid, hyperboloid and helicoid.

15-2. DEVELOPMENTS OF LATERAL SURFACES OF RIGHT SOLIDS

The methods of drawing developments of surfaces of various solids are explained by means of the following typical problems. Only the lateral surfaces of the solids (except the cube) have been developed. The ends or bases have been omitted. They can be easily incorporated if required.

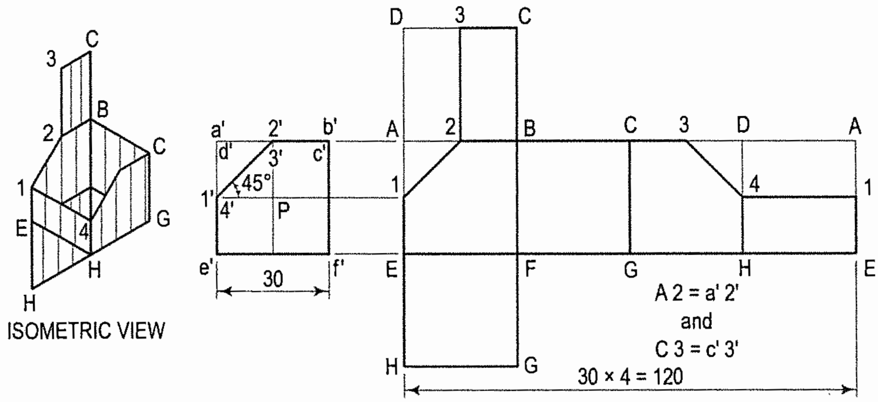
15-2-1. CUBE

The development of the surface of a cube consists of six equal squares, the length of the side of the squares being equal to the length of the edge of the cube.

Problem 15-1. *Draw the development of the surface of the part P of the cube, the front view of which is shown in fig. 15-3(i).*

Name all the corners of the cube and also the points at which the edges are cut.

(i) Draw the stretch-out lines *A-A* and *E-E* directly in line with the front view, and assuming the cube to be whole, draw four squares for the vertical faces, one square for the top and another for the bottom as shown in fig. 15-3(ii).



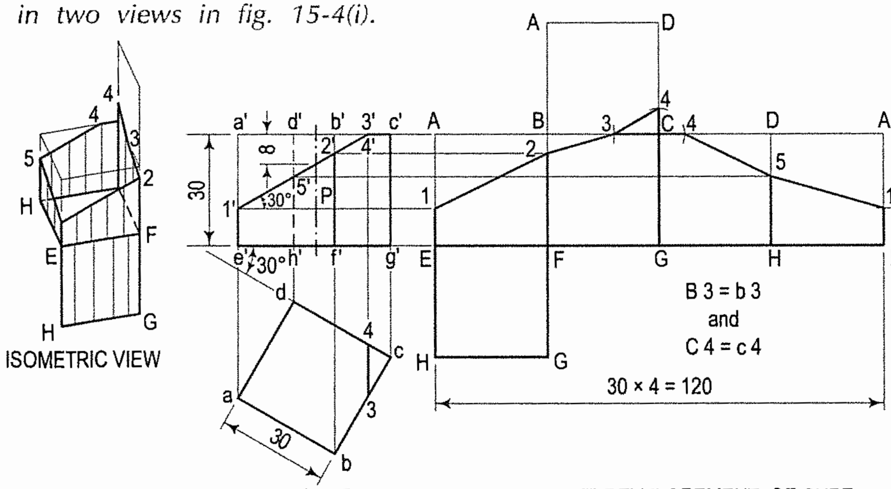
(i) CUBE (ii) DEVELOPEMENT OF CUBE

FIG. 15-3

(ii) Name all the corners. Draw a horizontal line through 1' to cut *AE* at 1 and *DH* at 4. a' b' is the true length of the edge. Hence, mark a point 2 on *AB* and 3 on *CD* such that *A* 2 = a' 2' amd *C* 3 = c' 3'. Mark the point 3 on *CD* in the top square also.

(iii) Draw lines 1-2, 2-3, 3-4 and 4-1, and complete the development as shown. Keep lines for the removed portion, viz. *A*1, *A*2, 3*D*, *D*4 and *DA* thin and fainter.

Problem 15-2. *Draw the development of the surface of the part P of the cube shown in two views in fig. 15-4(i).*



(i) CUBE FIG. 15-4 (ii) DEVELOPEMENTt OF CUBE

Name all the corners of the cube and also the points at which the edges are cut. Draw the development assuming the cube to be whole [fig. 15-4(ii)] as explained in problem 15-1.

(i) Draw horizontal lines through points 1', 2' and 5' to cut *AE* in 1, *BF* in 2 and *DH* in 5 respectively. Lines *b'c'* and *c'd'* do not show the true lengths of the edges. The sides of the square in the top view show the true length. Therefore, mark points 3 in *BC* and 4 in *CD* such that *B*3 = *b*3 and *C*4 = *c*4.

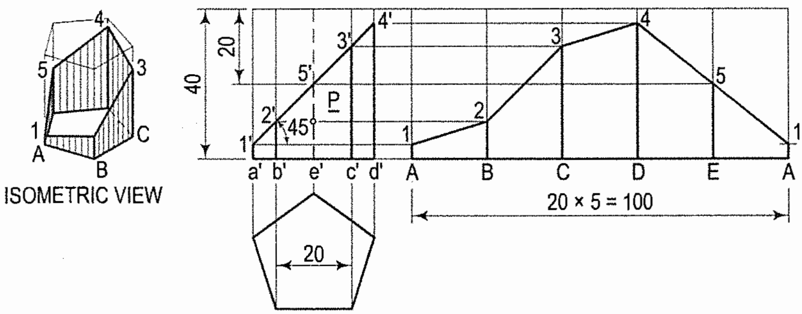
(ii) Draw lines joining 1, 2, 3 etc. in correct sequence and complete the required development. Keep the lines for the removed part fainter.

15-2-2. PRISMS

Development of the lateral surface of a prism consists of the same number of rectangles in contact as the number of the sides of the base of the prism. One side of the rectangle is equal to the length of the axis and the other side equal to the length of the side of the base.

CHAROTAR COGNIFRONT This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 34 for the following problem.

Problem 15-3. *Draw the development of the lateral surface of the part P of the pentagonal prism shown in fig. 15-5(i).*



(i) PENTAGONAL PRISM (ii) DEVELOPEMENT OF PRISM

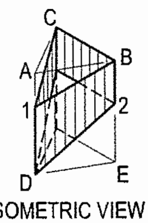
FIG. 15-5

Name the corners of the prism and the points at which the edges are cut.

(i) Draw the development assuming the prism to be whole [fig. 15-5(ii)]. It is made up of five equal rectangles.

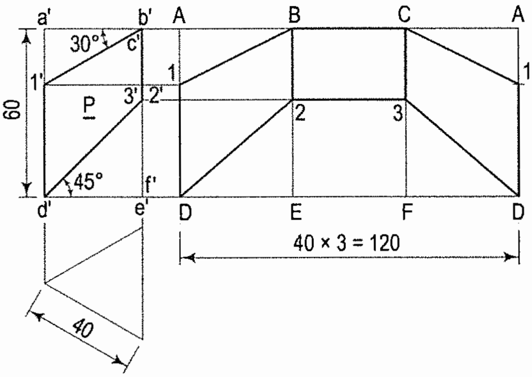
(ii) Draw horizontal lines through points 1', 2' etc. to cut the lines for the corresponding edges in the development at points 1, 2 etc.

(iii) Draw lines joining these points and complete the development as shown.



Problem 15-4. *Draw the development of the lateral surface of the part P of the triangular prism shown in fig. 15-6(i).*

Draw the development of the lateral surface of the whole prism [fig. 15-6(ii)] and obtain points 1, 2 and 3 on it. Draw lines 1*B*, *C*1, *D*2, 2-3 and 3*D*, and complete the development as shown.

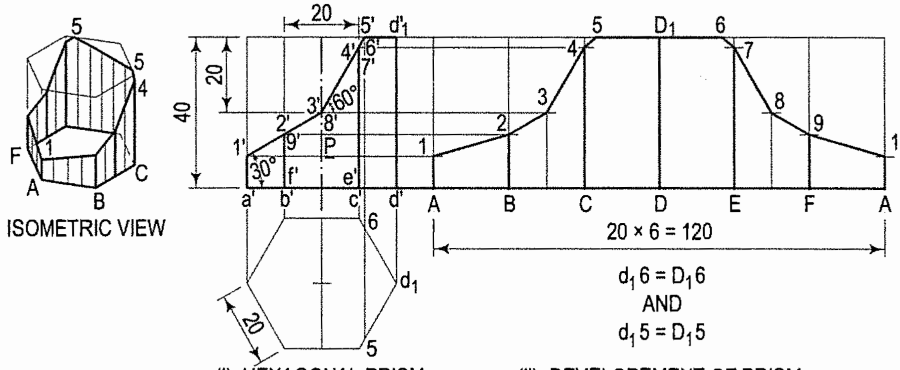


(i) TRIANGULAR PRISM (ii) DEVELOPEMENT OF PRISM

FIG. 15-6

Problem 15-5. *Draw the development of the lateral surface of the part P of the hexagonal prism shown in fig. 15-7(i).*

Name the points at which the edges are cut and draw the development assuming the prism to be whole [fig. 15-7(ii)].



(i) HEXAGONAL PRISM (ii) DEVELOPEMENT OF PRISM

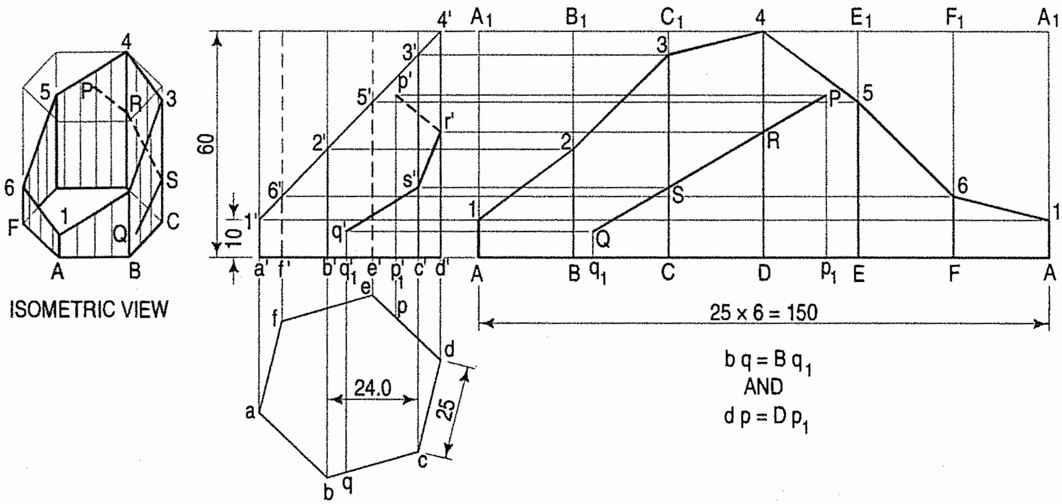
FIG. 15-7

(i) Obtain all the points except 5 and 6 by drawing horizontal lines. Note that points 3 and 8 lie on vertical lines drawn through the mid-points of *BC* and *EF*.

(ii) Mark points 5 and 6 such that $5D_1 = 5d_1$ and $D_1 6 = d_1 6$.

(iii) Draw lines joining points 1, 2, 3 etc. in correct sequence and complete the required development as shown.

Problem 15-6. *Draw the development of the lateral surface of the truncated prism shown in fig. 15-8(i). Also, draw the front view of the line joining the points P and Q (whose projections p, p' and q, q' are given) along the surface of the prism by the shortest distance.*



(i) TRUNCATED PRISM (ii) DEVELOPEMENT OF TRUNCATED PRISM

FIG. 15-8

Draw the required development [fig. 15-8(ii)] as explained in problem 15-3.

(i) Mark a point p_1 on DE and q_1 on BC such that $Dp_1 = dp$ and $Bq_1 = bq$. Draw verticals through points p_1 and q_1, and on them, obtain points P and Q by drawing horizontal lines through points p' and q'.

(ii) Draw a straight line joining P with Q. Then PQ shows the shortest distance between them. To draw this line in the front view, the process must be reversed. Let PQ cut the line $D4$ at R and $C3$ at S.

(iii) Draw horizontals through R and S cutting $d'4'$ at r' and $c'3'$ at s'. Draw lines $p'r'$, $r's'$ and $s'q'$ which show the front view of the line PQ. Note that $p'r'$ is a hidden line.

Problem 15-7. *The projections of a square prism with a hole drilled in it are given in fig. 15-9(i). Draw the development of the lateral surface of the prism.*

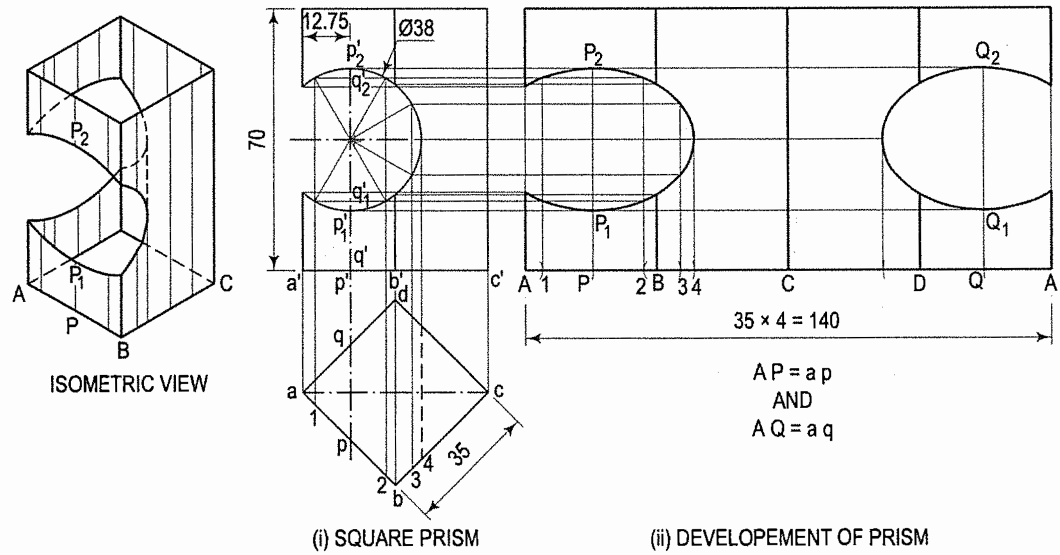


FIG. 15-9

(i) Mark a number of points on the circle for the hole. Draw the development of the whole prism [fig. 15-9(ii)] and locate the positions of these points on it.

(ii) For example, to locate points p'_1 and p'_2 and points q'_1 and q'_2 which coincide with them in the front view, draw a perpendicular through them cutting the base at p'. Project p' to p on ab and to q on ad. Mark points P and Q on AB and DA respectively such that $AP = ap$ and $AQ = aq$.

(iii) Draw verticals at points P and Q. Draw horizontal lines through p'_1 and p'_2 cutting these verticals at P_1, Q_1, P_2 and Q_2. Locate all points in the same manner and draw smooth curves through them, thus completing the development.

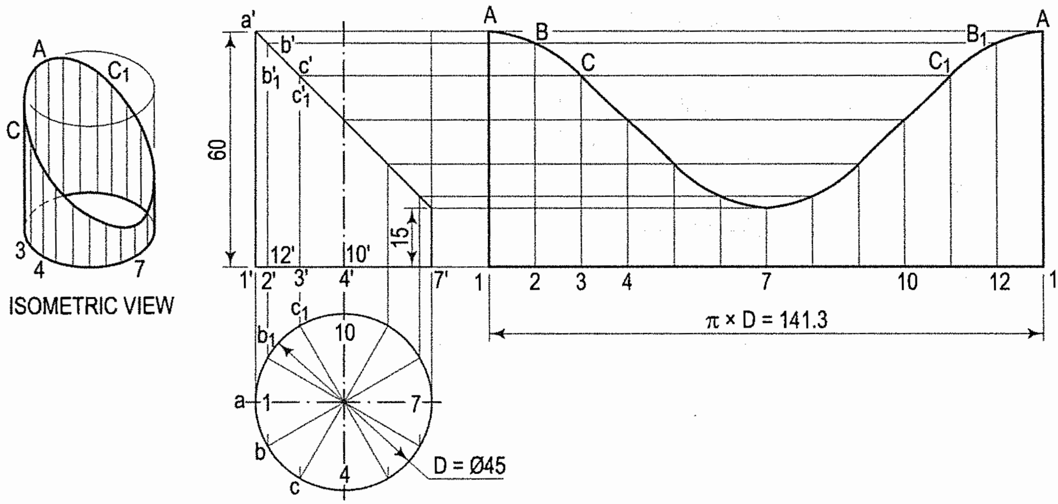
15-2-3. CYLINDERS

The development of the lateral surface of a cylinder is a rectangle having one side equal to the circumference of its base-circle and the other equal to its length.

Problem 15-8. *Develop the lateral surface of the truncated cylinder shown in fig. 15-10(i).*

(i) Divide the circle in the top view into twelve equal parts. Project the division points to the front view and draw the generators. Mark points a', b' and b'_1, c' and c'_1 etc. in which the generators are cut.

(ii) Draw the development of the lateral surface of the whole cylinder along with the generators [fig. 15-10(ii)]. The length of the line 1-1 is equal to $\pi \times D$ (circumference of the circle). This length can also be marked approximately by stepping off with a bow divider, twelve divisions, each equal to the chord-length *ab*. (The length thus obtained is about 1% shorter than the exact length; but this is permitted in drawing work.)

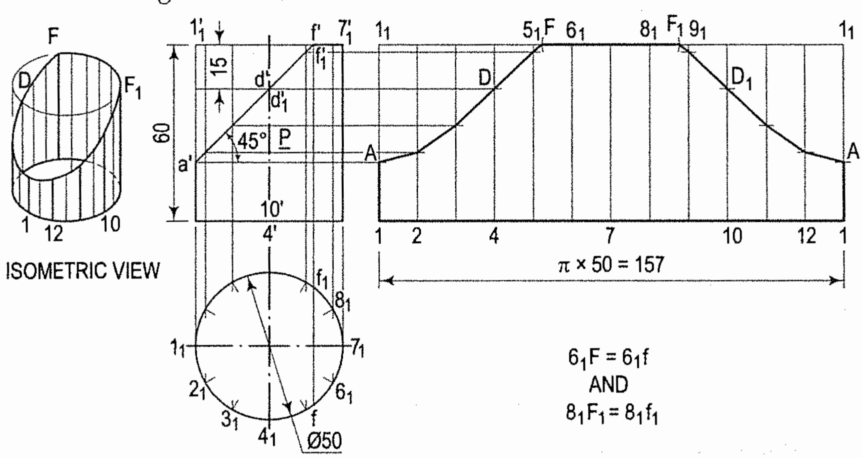


(i) TRUNCATED CYLINDER (ii) DEVELOPEMENT OF CYLINDER

FIG. 15-10

(iii) Draw horizontal lines through points a', b' and b'_1 etc. to cut the corresponding generators in points *A*, *B* and B_1 etc. Draw a smooth curve through the points thus obtained. The figure 1-*A*-*A*-1 is the required development.

Problem 15-9. *Draw the development of the lateral surface of the part P of the cylinder shown in fig. 15-11(i).*



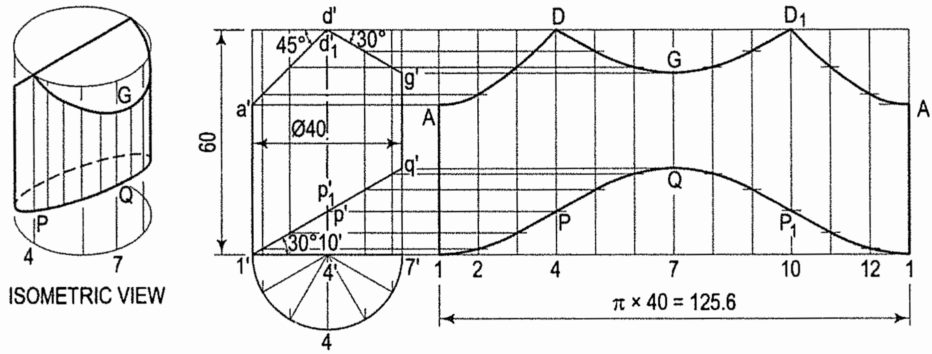
(i) CYLINDER (ii) DEVELOPEMENT OF CYLINDER

FIG. 15-11

Draw the development as explained in problem 15-8. Positions of the points at which the upper end of the cylinder is cut should be obtained from the top view. Mark these points, viz. F and F_1 on the line 1_1-1_1 between points 5_1 and 6_1 and between 8_1 and 9_1 in such a way that $6_1F = 6_1f$ and $8_1F_1 = 8_1f_1$. Draw curves FA and F_1A passing through these points and complete the required development as shown.

Problem 15-10. *Draw the development of the lateral surface of the cylinder cut by three planes as shown in fig. 15-12(i).*

See fig. 15-12(ii).

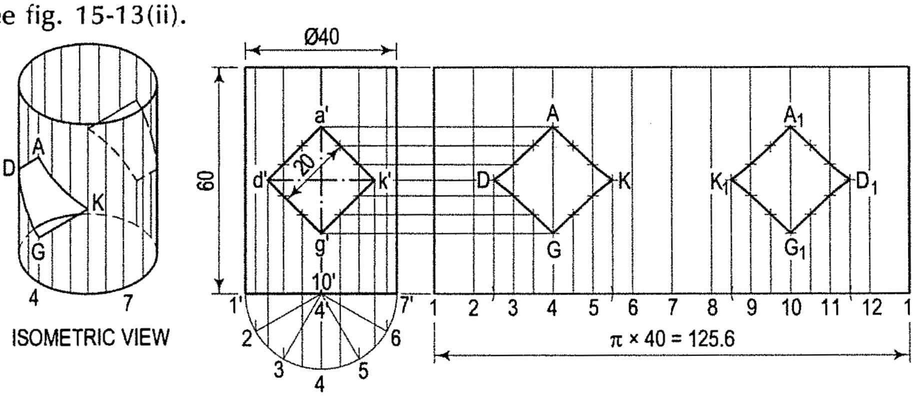


(i) CYLINDER CUT BY 3 PLANES (ii) DEVELOPEMENT OF CUT CYLINDER

FIG. 15-12

Problem 15-11. (fig. 15-13(ii): *Draw the development of the lateral surface of the cylinder having a square hole in it as shown in fig. 15-13(i).*

See fig. 15-13(ii).



(i) CYLINDER WITH SQUARE HOLE (ii) DEVELOPEMENT OF CYLINDER

FIG. 15-13

Problem 15-12. *Draw the development of the lateral surface of the cylinder cut as shown in fig. 15-14(i).*

See fig. 15-14(ii).

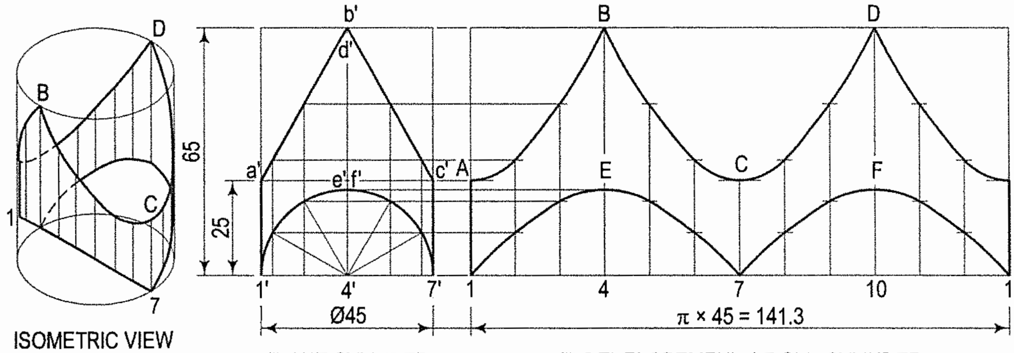
Problem 15-13. *Develop the surface of the three-piece cylindrical pipe elbow shown in fig. 15-15(i).*

The front view is drawn as shown below:

Draw a square *efgh* of sides equal to the diameter of the pipes (i.e. 50 mm). Bisect angles *ehf* and *ghf* by lines *Ph* and *Qh* to intersect *ef* and *fg* at *x* and *y* respectively. Then *xy* is the axis of the middle piece *B*.

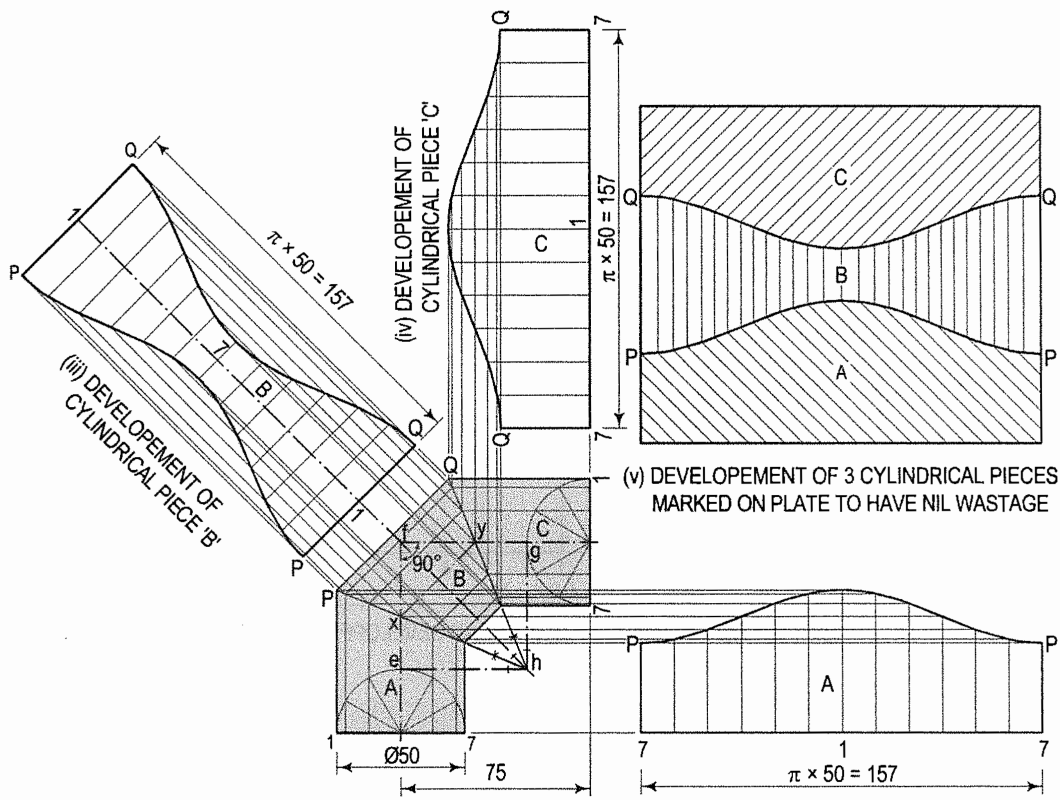
Parts *A* and *C* are similar and are in the form of cylinders truncated at one end only. The part *A* and care as is shown in fig 15-15(ii) and fig. 15-15(iv) are developed on a stretch-out line drawn directly in line with its base. The twelve divisions are obtained from the semi-circle drawn on the base as diameter. The part *B* is truncated at both the ends. It is developed on a stretch-out line drawn through *h* and at right angles to its axis *xy* as shown in fig 15-15(iii).

As the curves of the part *B* are exactly similar to those of *A* and *C*, the three developments may also be drawn combined to minimize wastage on plate as shown in fig. 15-15(v).



(i) CUT CYLINDER (ii) DEVELOPEMENT OF CUT CYLINDER

FIG. 15-14



(i) THREE PIECE CYLINDRICAL PIPE (ii) DEVELOPEMENT OF CYLINDRICAL PIECE 'A'

FIG. 15-15

Problem 15-14. *Three cylindrical pipes of 50 mm diameter form a Y-piece as shown in the front view in fig. 15-16(i). Draw the development of the surface of each pipe.*

(i) Draw a semi-circle on the base of pipe *A* as diameter and obtain twelve divisions. Draw the development [fig. 15-16(ii)] as in the previous problem.

(ii) Draw any convenient line at right angles to the axis of the pipe *B* [fig. 15-16(iii)]. On this line as a stretch-out line, draw the development as shown. Pipes *B* and *C* are similar and hence, their developments will also be similar.

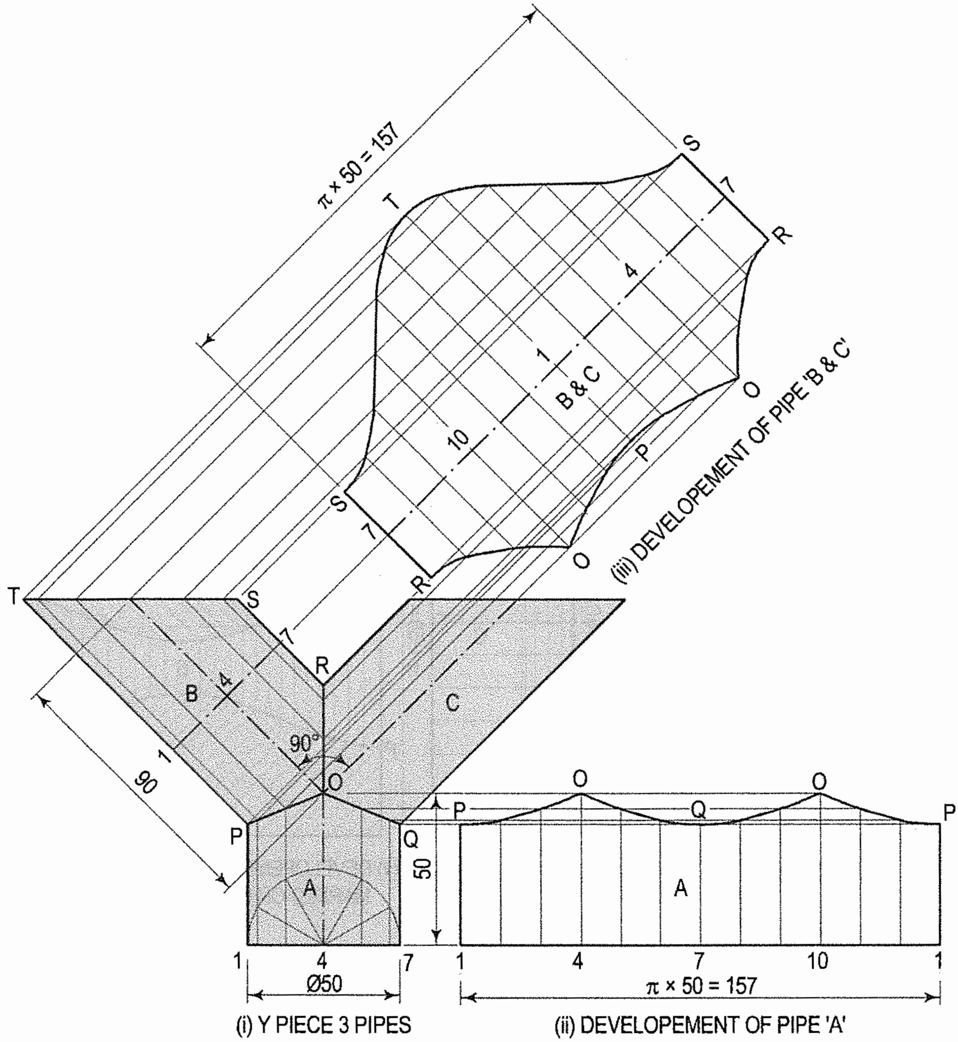


FIG. 15-16

15-2-4. PYRAMIDS

The development of the lateral surface of a pyramid consists of a number of equal isosceles triangles in contact. The base and the sides of each triangle are respectively equal to the edge of the base and the slant edge of the pyramid

Note: The true length of a slant edge of pyramid can be measured from the front view, if the top view of that edge is parallel to *xy*; and it can be measured from the top view, if the slant edge is parallel to *xy* in the front view.

Method of drawing the development of the lateral surface of a pyramid:

(i) With any point O as centre and radius equal to the true length of the slant edge of the pyramid, draw an arc of the circle. With radius equal to the true length of the side of the base, step-off (on this arc) the same number of divisions as the number of sides of the base.

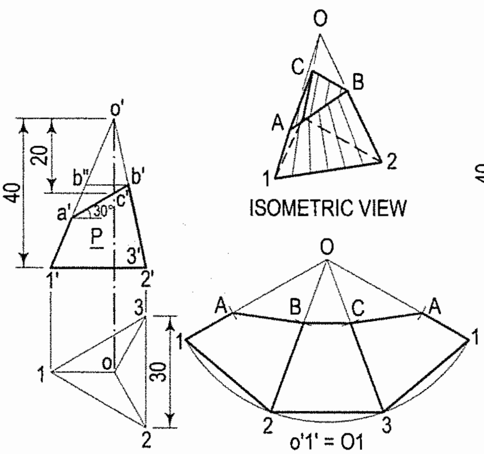
(ii) Draw lines joining the division-points with each other in correct sequence and also with the centre for the arc. The figure thus formed (excluding the arc) is the development of the lateral surface of the pyramid.

Problem 15-15. *Draw the development of the lateral surface of the part P of the triangular pyramid shown in fig. 15-17(i). The line o'1' in the front view is the true length of the slant edge because it is parallel to xy in the top view. The true length of the side of the base is seen in the top view.*

(i) Draw the development of the lateral surface of the whole pyramid [fig. 15-17(ii)] as explained above. On *O*1 mark a point *A* such that *OA* = *o'a'*. *o*'2' (with which *o*'3' coincides) is not the true length of the slant edge.

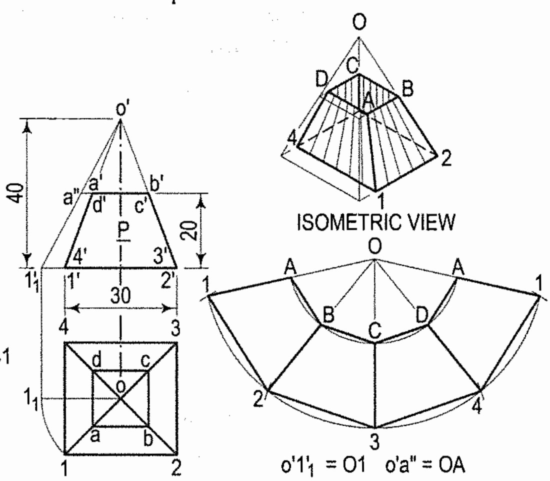
(ii) Hence, through *b*', draw a line parallel to the base and cutting *o*' *a*' at *b*''. *o'b''* is the true length of *o'b'* as well as *o'c'*. Mark a point *B* in *O*2 and *C* in *O*3 such that *OB* = *OC* = *o'b''*.

(iii) Draw lines *AB*, *BC* and *CA* and complete the required development as shown. Keep the arc and the lines for the removed part fainter.



(i) TRIANGULAR PYRAMID (ii) DEVELOPEMENT OF TRIANGULAR PYRAMID

FIG. 15-17



(i) SQUARE FRUSTUM PYRAMID (ii) DEVELOPEMENT OF PYRAMID

FIG. 15-18

Problem 15-16. *Draw the development of the lateral surface of the frustum of the square pyramid shown in fig. 15-18(i).*

(i) Determine the position of the apex. None of the lines in the front view shows the true length of the slant edge. Therefore, draw the top view and make any one line (for the slant edge) horizontal, i.e. parallel to *xy* and determine the true length $o'1'_1$. Through *a*', draw a line parallel to the base and obtain the true length *o'a''*.

(ii) With O as centre and radius $o'1'_1$, draw an arc and obtain the development of the lateral surface of the whole pyramid [fig. 15-18(ii)].

(iii) With centre O and radius o' a'', draw an arc cutting $O1$, $O2$ etc. at points A, B etc. respectively.

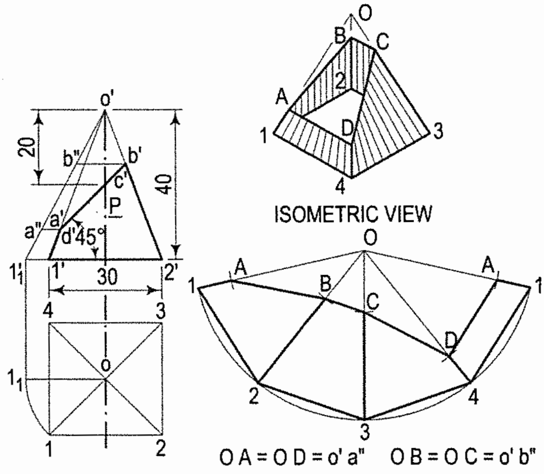
(iv) Draw lines AB, BC, CD and DA and complete the required development. Note that these lines are respectively parallel to lines 1-2, 2-3 etc.

Problem 15-17. *Draw the development of the lateral surface of the part P of the square pyramid shown in fig. 15-19(i).*

Draw the top view of the pyramid and determine the true length $o'1'_1$ of the slant edges as explained in problem 15-16. On this line, obtain the true lengths $o'a''$ and $o'b''$.

(i) Draw the development of the lateral surface of the whole pyramid [fig. 15-19(ii)].

(ii) Mark a point A in $O1$ and D in $O4$ such that $OA = OD = o'a''$. Similarly, mark B in $O2$ and C in $O3$ such that $OB = OC = o'b''$. Draw lines AB, BC etc. and complete the required development as shown.



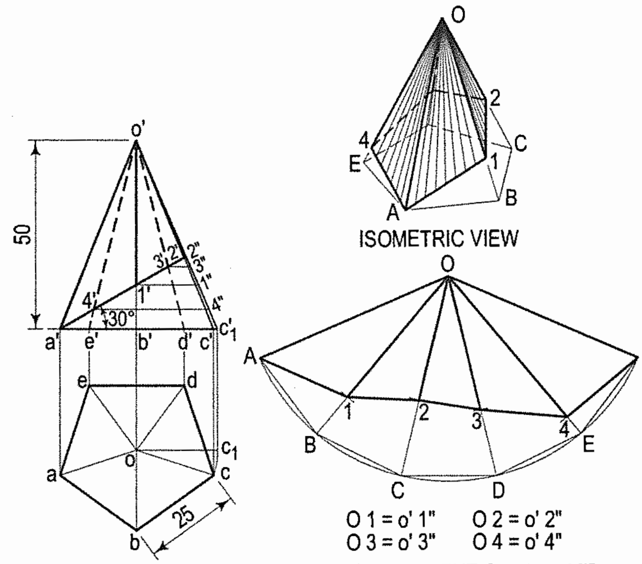
(i) SQUARE PYRAMID (ii) DEVELOPEMENT OF PYRAMID

FIG. 15-19

Problem 15-18. *Draw the development of the lateral surface of the pentagonal pyramid, the lower part of which is removed as shown in fig. 15-20(i).*

None of the lines in the front view shows the true length of the slant edges. Hence, draw the top view and determine the true length $o'c'_1$. Through points 1', 2' etc., draw lines parallel to the base and obtain the true lengths $o'1''$, $o'2''$ etc.

(i) Draw the development of the lateral surface of the whole pyramid [fig. 15-20(ii)]. Mark points 1, 2, 3 etc. on lines OB, OC, OD etc. such that $O1 = o'1''$, $O2 = o'2''$, $O3 = o'3''$ etc.

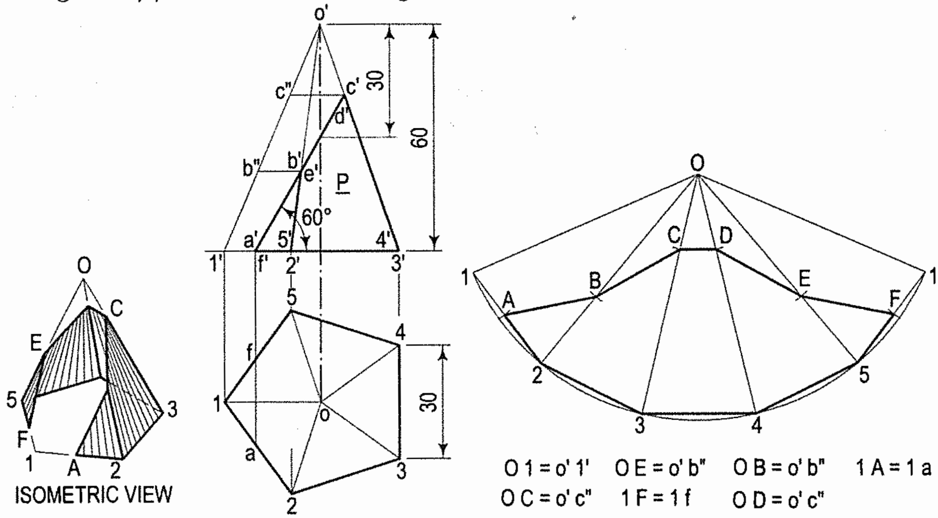


(i) PENTAGONAL PYRAMID (ii) DEVELOPEMENT OF PYRAMID

FIG. 15-20

(ii) Draw lines joining points A, 1, 2 etc. in correct sequence and complete the required development. Note that the lines for the lower part should be fainter.

Problem 15-19. *Draw the development of the lateral surface of the part P of the pentagonal pyramid shown in fig. 15-21(i).*



(i) PENTAGONAL PYRAMID　　(ii) DEVELOPEMENT OF PYRAMID

FIG. 15-21

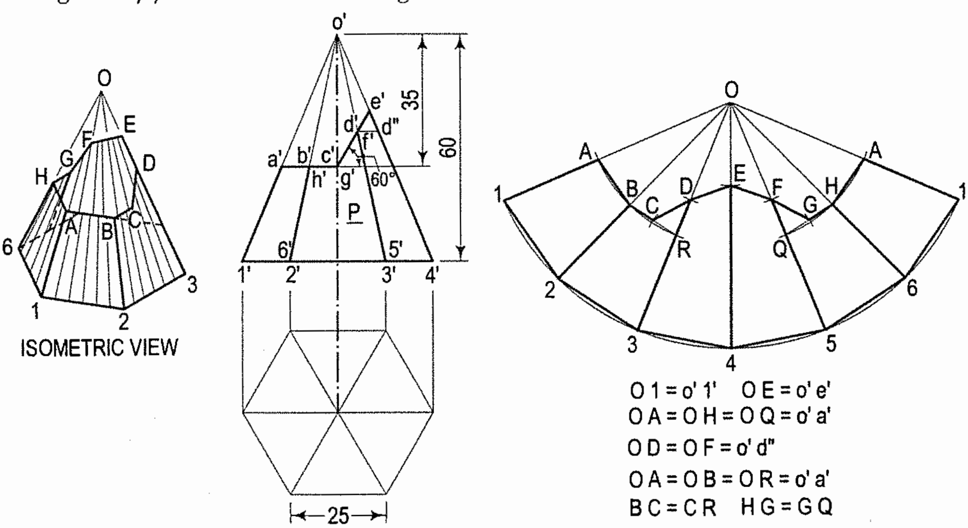
The line *o*'1' shows the true length of the slant edges. On it, obtain true lengths *o'b*" and *o'c*". Two edges of the base are also cut.

(i) Draw the top view and obtain the true lengths 1*a* and 1*f* as shown. Draw the development of the lateral surface of the whole pyramid.

(ii) Obtain points *B*, *C*, *D* and *E* as explained in problem 15-18. Mark a point *A* in 1-2 and *F* in 5-1 such that 1*A* = 1*a* and 1*F* = 1*f*.

(iii) Draw lines joining points *A*, *B*, *C* etc. and complete the required development.

Problem 15-20. *Draw the development of the lateral surface of the part P of the hexagonal pyramid shown in fig. 15-22(i).*



(i) HEXAGONAL PYRAMID　　(ii) DEVELOPEMENT OF PYRAMID

FIG. 15-22

Lines $o'1'$ and $o'4'$ show the true length of the slant edges. Draw the development of the lateral surface of the whole pyramid [fig. 15-22(ii)]. Obtain the development of the left-half of the pyramid as explained in problem 15-16 and that of the right-half as explained in problem 15-15.

Note that the points C and G are the mid-points of the lines BR and HQ respectively.

Problem 15-21. (fig. 15-23): *A frustum of a square pyramid has its base 50 mm side, top 25 mm side and height 75 mm. Draw the development of its lateral surface.*

Also, draw the projections of the frustum (when its axis is vertical and a side of its base is parallel to the V.P.), showing the line joining the mid-point of a top edge of one face with the mid-point of the bottom edge of the opposite face, by the shortest distance.

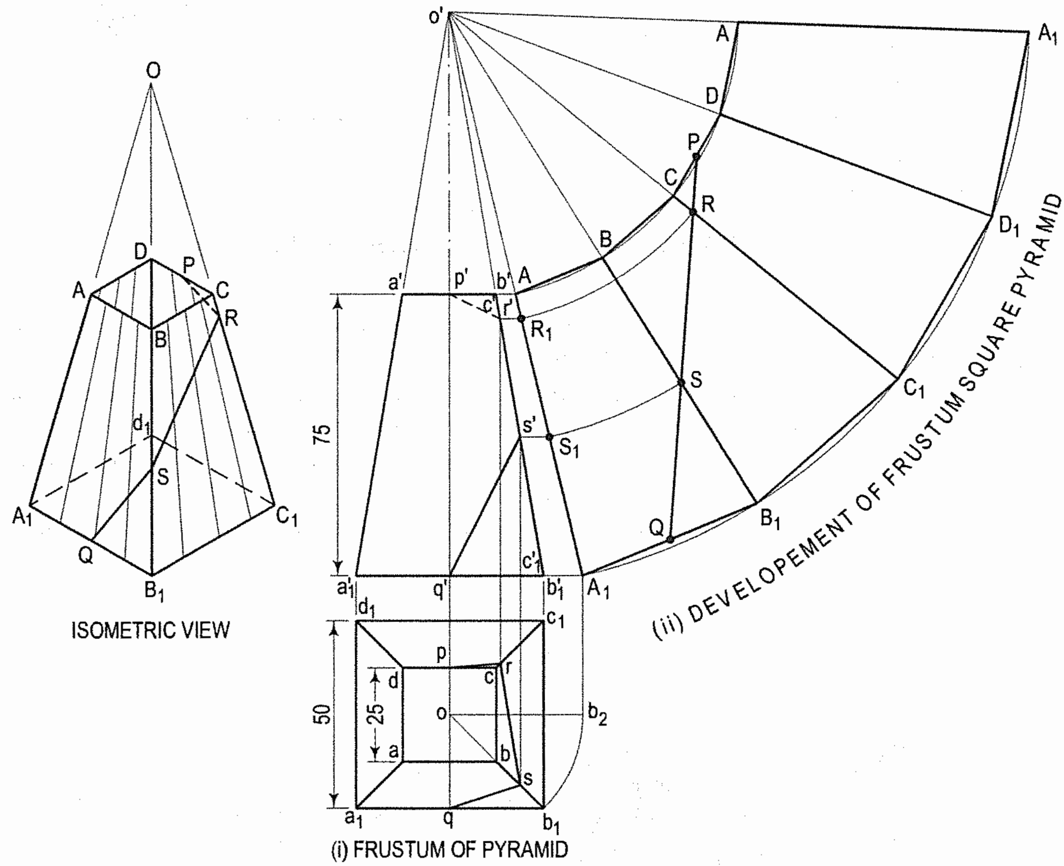


FIG. 15-23

Draw the development as explained in problem 15-16.

(i) Mark the mid-point P of CD and Q of A_1B_1. Draw a line joining P and Q and cutting CC_1 at R and BB_1 at S. Transfer these points to the front view and the top view. For example, with o' as centre and radius $o'R$, draw an arc cutting $o'A_1$ at R_1. Through R_1, draw a line parallel to the base and cutting $c'c'_1$ at r'. Project r' to r on cc_1 in the top view. r' and r are the projections of R.

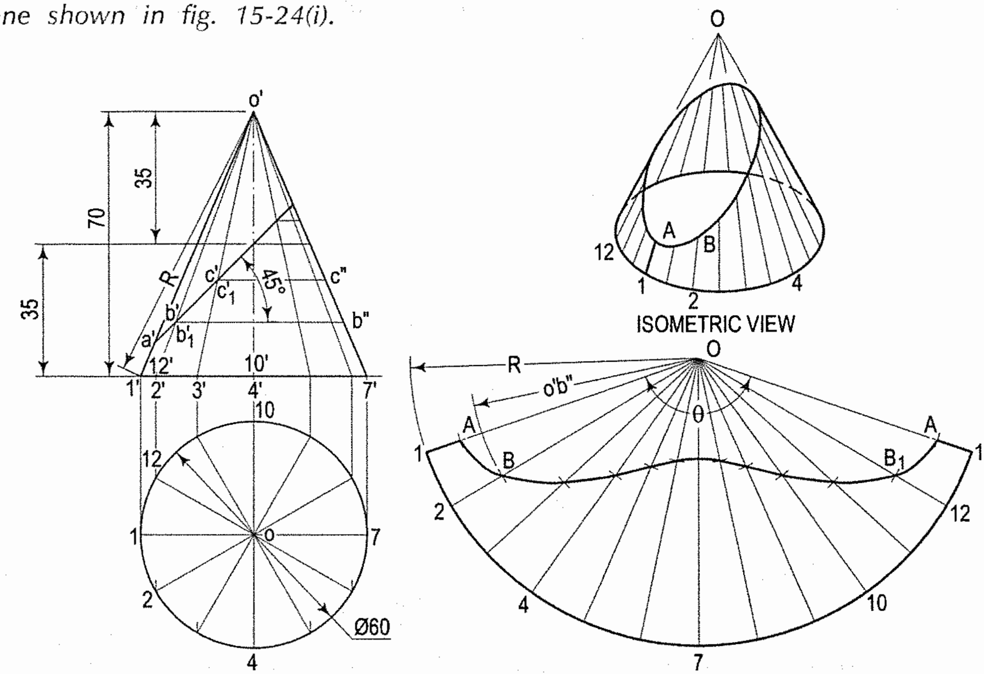
(ii) Similarly, obtain s' and s on $b'b'_1$ and bb_1 respectively. Draw lines pr, rs and sq which will show the top view of the line PQ. $p'r's'q'$ will be the path of the line PQ in the front view.

15-2-5. CONE

The development of the curved surface of a cone is a sector of a circle, the radius and the length of the arc of which are respectively equal to the slant height and the circumference of the base-circle of the cone.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 35 for the following problem.

Problem 15-22. *Draw the development of the lateral surface of the truncated cone shown in fig. 15-24(i).*



(i) TRUNCATED CONE (ii) DEVELOPEMENT OF TRUNCATED CONE

FIG. 15-24

Assuming the cone to be whole, let us draw its development.

(i) Draw the base-circle in the top view and divide it into twelve equal parts.

(ii) With any point O as centre and radius equal to $o'1'$ or $o'7'$, draw an arc of the circle [fig. 15-24(ii)]. The length of this arc should be equal to the circumference of the base circle. This can be determined in two ways.

(iii) Calculate the subtended angle θ by the formula,

$$\theta = 360^\circ \times \frac{\text{radius of the base circle}}{\text{slant height}}.$$

Cut-off the arc so that it subtends the angle θ at the centre and divide it into twelve equal parts.

(iv) Step-off with a bow-divider, twelve equal divisions on the arc, each equal to one of the divisions of the base-circle.

(This will give an approximate length of the circumference. Note that the base-circle should not be divided into less than twelve equal parts.)

(v) Join the division-points with O, thus completing the development of the whole cone with twelve generators shown in it [fig. 15-24(ii)].

(vi) The truncated portion of the cone may be deducted from this development by marking the positions of points at which generators are cut and then drawing a curve through them. For example, generators $o'2'$ and $o'12'$ in the front view are cut at points b' and b'_1 which coincide with each other. The true length of $o'\ b'$ may be obtained by drawing a line through b', parallel to the base and cutting $o'7'$ at b''. Then $o'b''$ is the true length of $o'b'$.

(vii) Mark points B and B_1 on generators $O2$ and O-12 respectively, such that $OB = OB_1 = o'\ b''$. Locate all points in the same way and draw a smooth curve through them. The figure enclosed this curve and the arc is the development of the truncated cone.

Problem 15-23. *Draw the development of the lateral surface of the part P of the cone shown in fig. 15-25(i).*

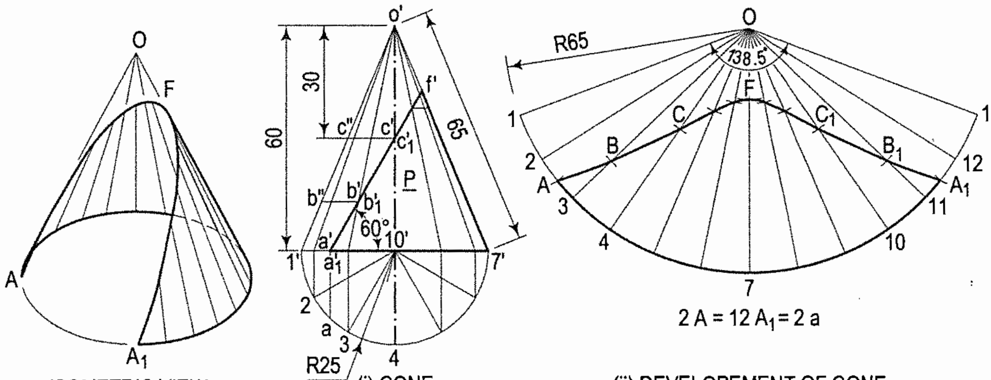


FIG. 15-25

Draw the development as explained in problem 15-22 [fig. 15-25(ii)]. For the points at which the base of the cone is cut, mark points A and A_1 on the arcs 2-3 and 11-12 respectively, such that $A2 = A_1$-$12 = a2$. Draw the curve passing through the points A, B, C etc. The figure enclosed between this curve and the arc A-A_1 is the required development.

Problem 15-24. *Draw the development of the lateral surface of the part P of the cone shown in fig. 15-26(i).*

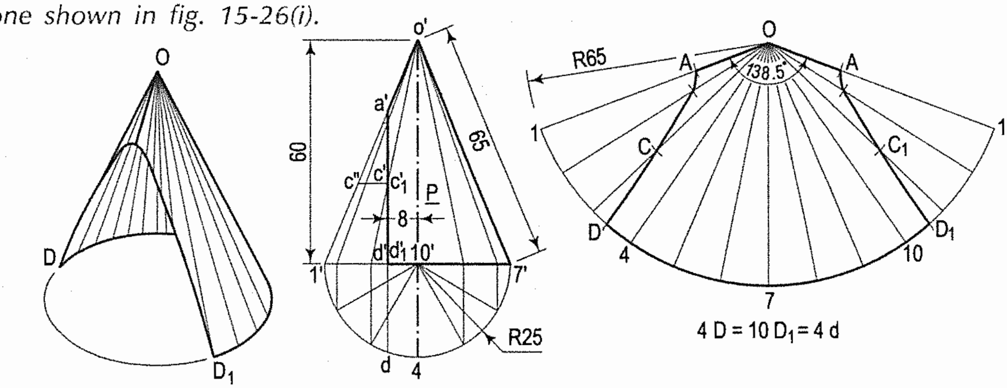


FIG. 15-26

Draw the development as explained in problem 15-23 and as shown in fig. 15-26(ii).

Problem 15-25. *Draw the development of the lateral surface of the part P of the cone shown in fig. 15-27(i).*

Draw the development of the lateral surface of the whole cone [fig. 15-27(ii)]. With O as centre and radius $o'q'$, draw the arc QQ cutting O_1 at Q.

Obtain the curve for the lower part as explained in problem 15-22. The figure enclosed between this curve and the arc QQ is the required development.

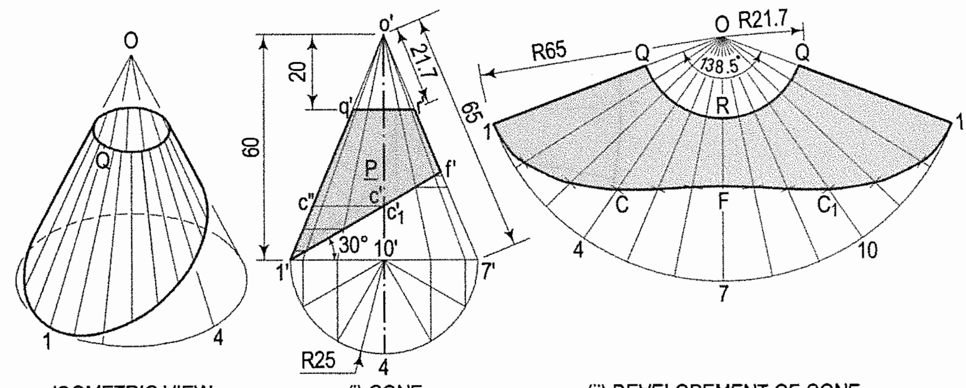


FIG. 15-27

Problem 15-26. *Draw the projections of a cone resting on the ground on its base and show on them, the shortest path by which a point P, starting from a point on the circumference of the base and moving around the cone will return to the same point. Base of cone 61 mm diameter; axis 75 mm long.*

(i) Draw the projections and the development of the surface of the cone showing all twelve generators (fig. 15-28). The development may be drawn attached to $o'1'$.

(ii) Assume that P starts from the point 1 (i.e. point 1' in the front view). Draw a straight line 1'1' on the development. This line shows the required shortest path.

To transfer this line to the front view the process adopted in problem 15-22 must be reversed. Let us take a point P_4 at which the path cuts the generator $o'4$. Mark a point P''_4 on $o'1'$ such that $o'P''_4 = o'P_4$. This can be done by drawing an arc with o' as centre and radius equal to $o'P_4$ cutting $o'1'$ at P''_4. Through P''_4, draw a line parallel to the base cutting $o'4'$ at P'_4. Then p'_4 is the position of the point p_4 in the front view.

Similarly, transfer all the points to the front view and draw the required curve through them. The curve at the back will coincide with the front curve.

(iii) Project these points to the top view on the respective generators. p'_4 and p'_{10} cannot be projected directly. Hence, project p''_4 to a point q on o_1. With o as centre and radius equal to oq, draw an arc cutting o_4 at p_4 and o-10 at p_{10}. Thus $op_4 = op_{10} = oq$. A curve drawn through the points thus obtained will show the path in the top view.

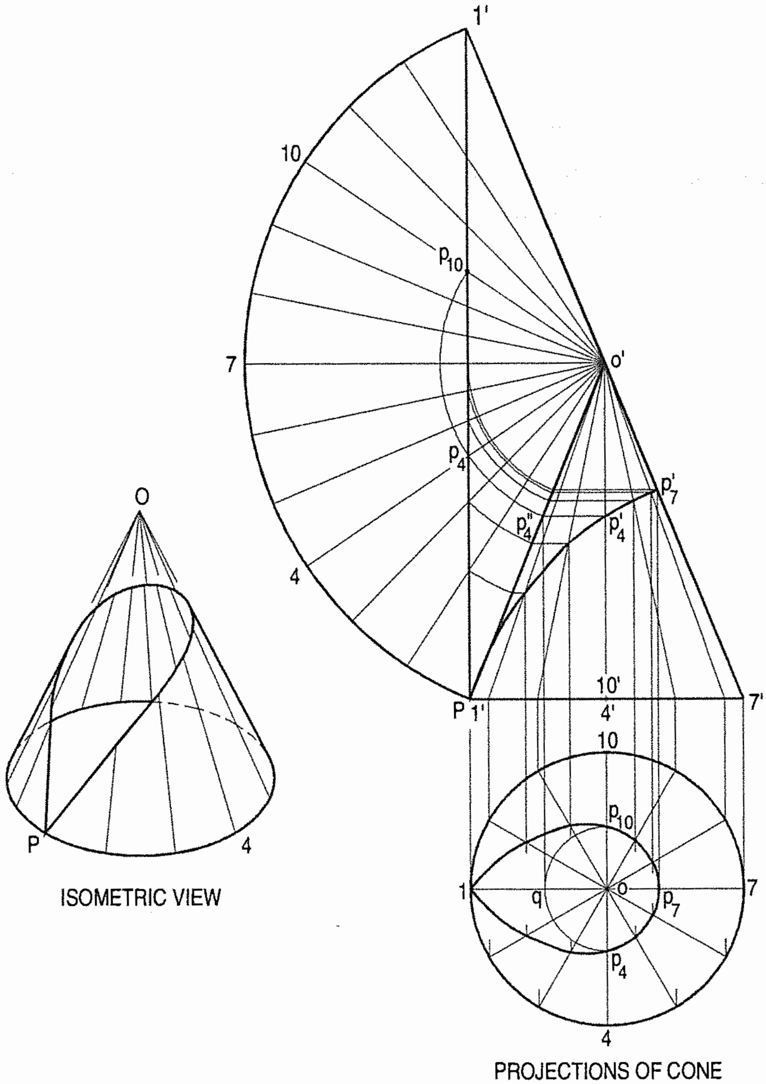


FIG. 15-28

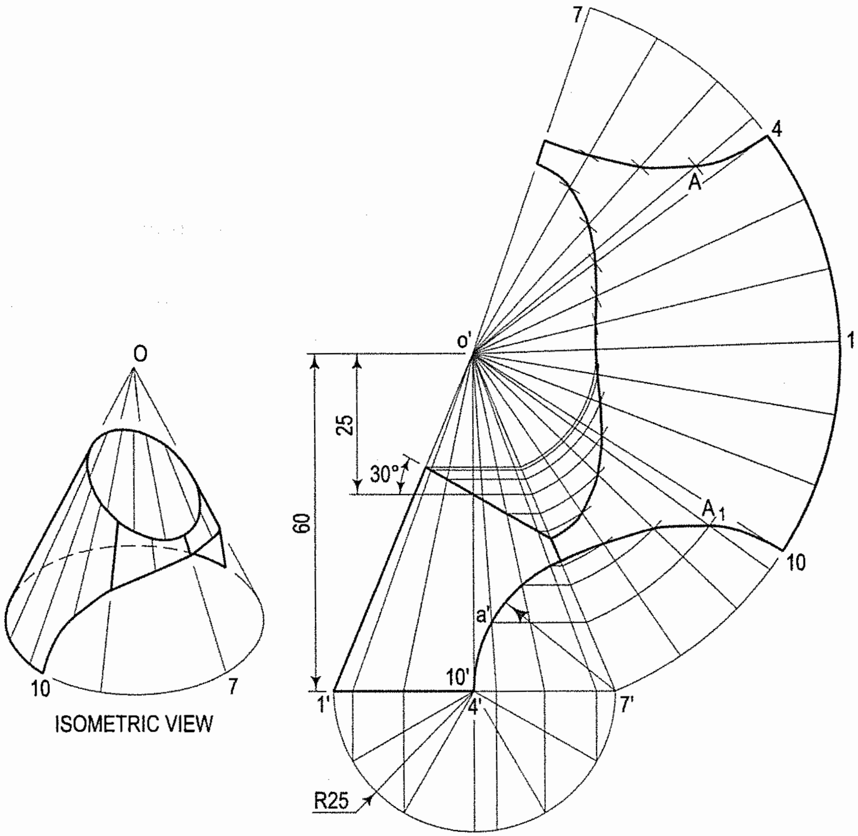
Problem 15-27. (fig. 15-29): *Draw the development of the lateral surface of the part of the cone, the front view of which is shown in fig. 15-29.*

(i) Draw a semi-circle on the base as a diameter and divide it into six equal parts for positions of generators.

(ii) Draw the development assuming the cone to be whole. Obtain points on the generators in the development as explained in problem 15-22. Additional points such as a' may also be marked to determine the correct shape of the curve.

(iii) Draw curves through these points and complete the development as shown.

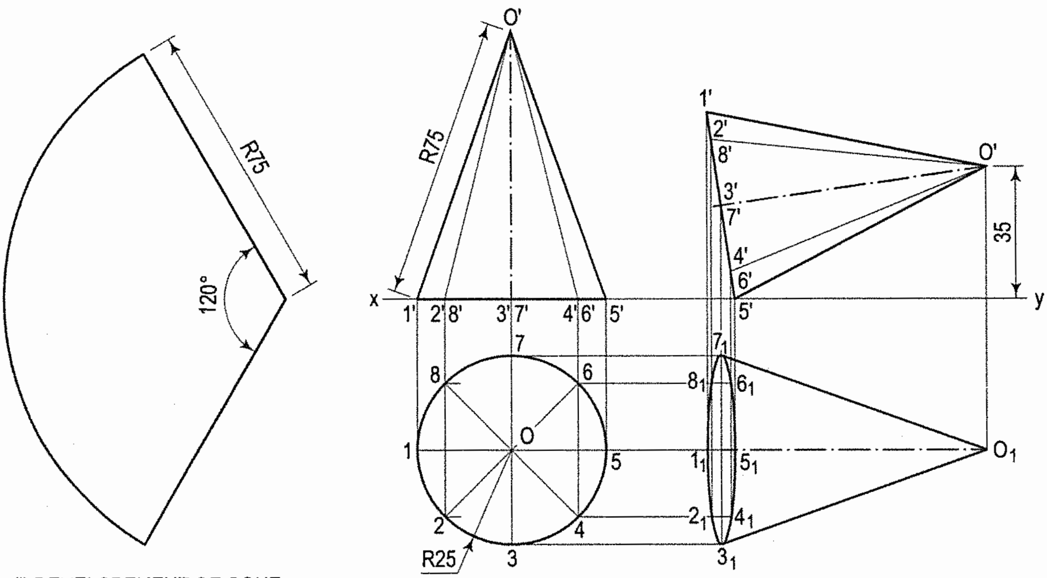
Problem 15-28. (fig. 15-30): *The development of the conical surface is a sector of the semi-circular plane. The radius of the sector is 75 mm and the angle subtended by the arc at the centre is 120°. Construct the cone and draw its projections when the apex is 35 mm above the H.P. and its axis is parallel to the V.P.*



ISOMETRIC VIEW

FRONT VIEW OF CONE

FIG. 15-29



(i) DEVELOPEMENT OF CONE

(ii) CONE AND PROJECTIONS GENERATED FROM DEVELOPEMENT OF CONE

FIG. 15-30

(i) Determine the radius of the base circle of the cone using following formula:

$$S = R.\,\theta = 2\,\pi\,r \quad \text{.......... (i)}$$

where S – The arc of circle
R – Radius of the arc
θ – Angle subtended by the arc at the centre
r – Radius of base circle of the cone.

Substitute $R = 75$ mm, $\theta = \dfrac{\pi \times 120}{180}$ radian, we obtain
$r = 25$ mm.

(ii) Draw the circle of radius of 25 mm representing the top view of the cone.
(iii) Project the front view with the height equal to 75 mm.
(iv) Tilt the front view such that the apex is 35 mm above xy line.
(v) Complete the projection as shown in fig. 15-30.

Problem 15-29. *Draw the shape of the sheet metal required for the funnel shown in fig. 15-31.*

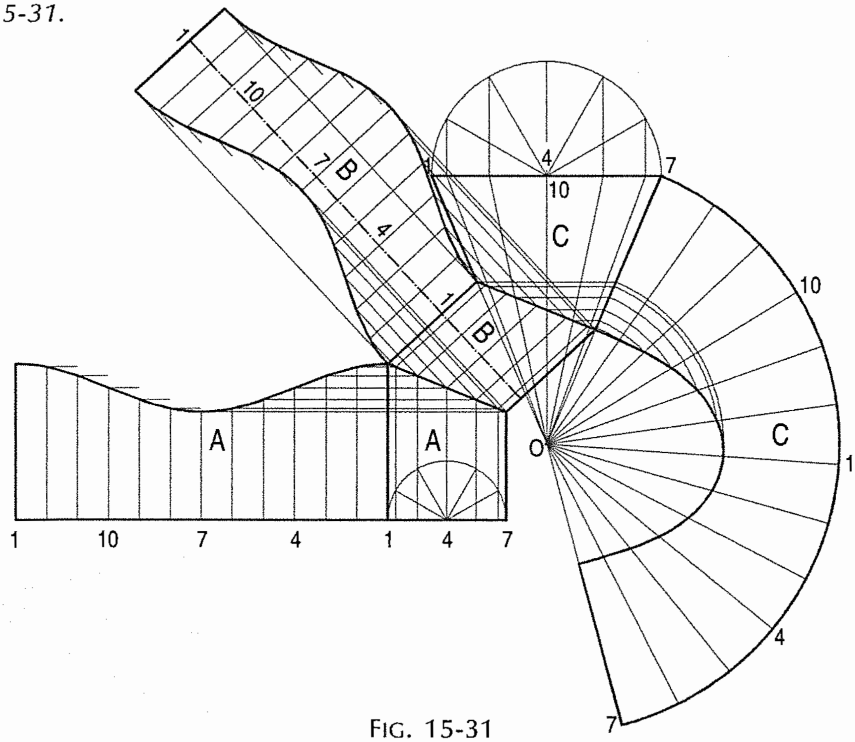


FIG. 15-31

The funnel consists of a conical part C and two cylindrical pipes A and B, all truncated. Draw the developments of the pipes as shown in problem 15-8, and that of the conical piece, as shown in problem 15-22. These are the required shapes.

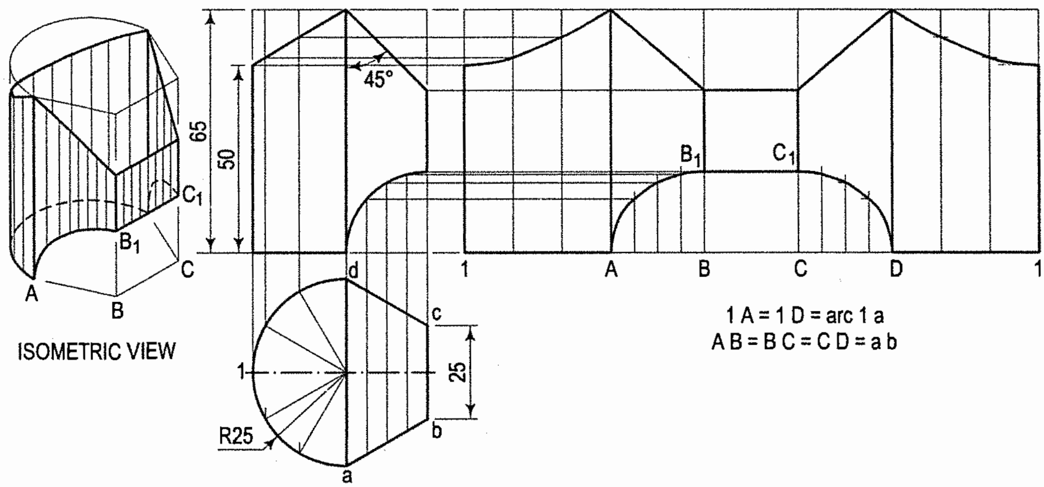
Problem 15-30. *The projections of a solid composed of a truncated half-cylinder and a cut half-prism are given in fig. 15-32(i). Draw the development of its lateral surface.*

Assuming the solid to be whole; draw the development of its surface [fig. 15-32(ii)].

Draw a stretch-out line and on it, step-off

(i) $1A$ equal to the arc $1a$,
(ii) AB, BC and CD, each equal to the edge ab of the base, and
(iii) $D1$ equal to the arc $d1$. Complete the rectangle.

Draw perpendiculars at *A*, *B* etc. and at other intermediate points. Locate on them, positions of points at which they are cut and draw the curves and straight lines as shown.



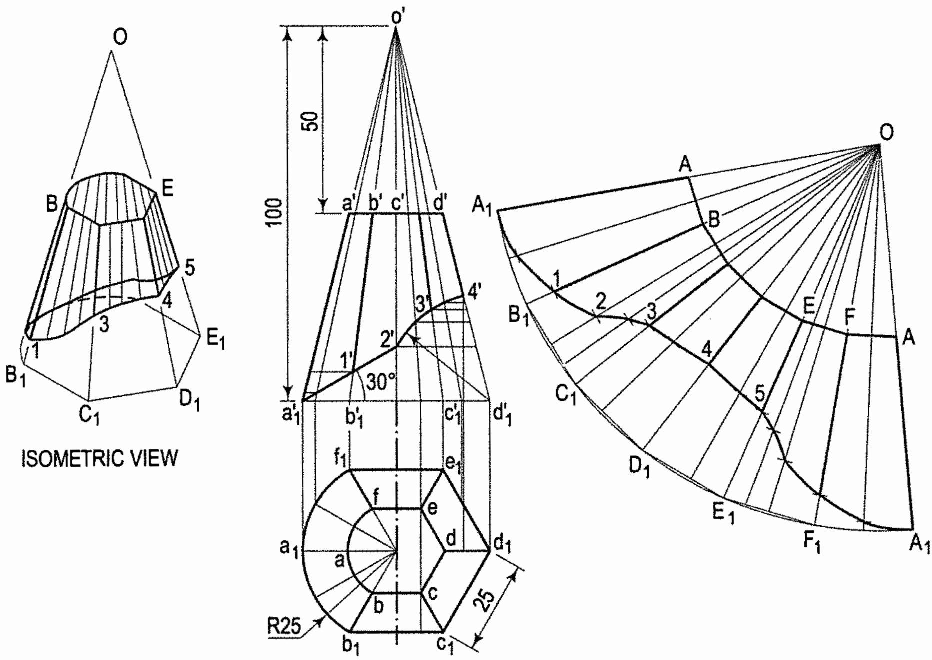
(i) TRUNCATED HALF CYLINDER AND CUT PRISM

(ii) DEVELOPEMENT OF TRUNCATED HALF CYLINDER AND CUT PRISM

FIG. 15-32

Problem 15-31. *Draw the development of the lateral surface of the solid shown in two views (drawn in first-angle projection) in fig. 15-33(i).*

The solid is made-up of portions of frusta of a cone and a hexagonal pyramid.



(i) FRUSTA OF CONE AND HEXAGONAL PYRAMID

(ii) DEVELOPEMENT OF FRUSTA OF CONE AND HEXAGONAL PYRAMID

FIG. 15-33

The slant height of the cone is equal to the slant edge of the pyramid.

(i) Therefore, with any point O as centre and radii $o'a'$ and $o'a'_1$, draw arcs [fig. 15-33(ii)].

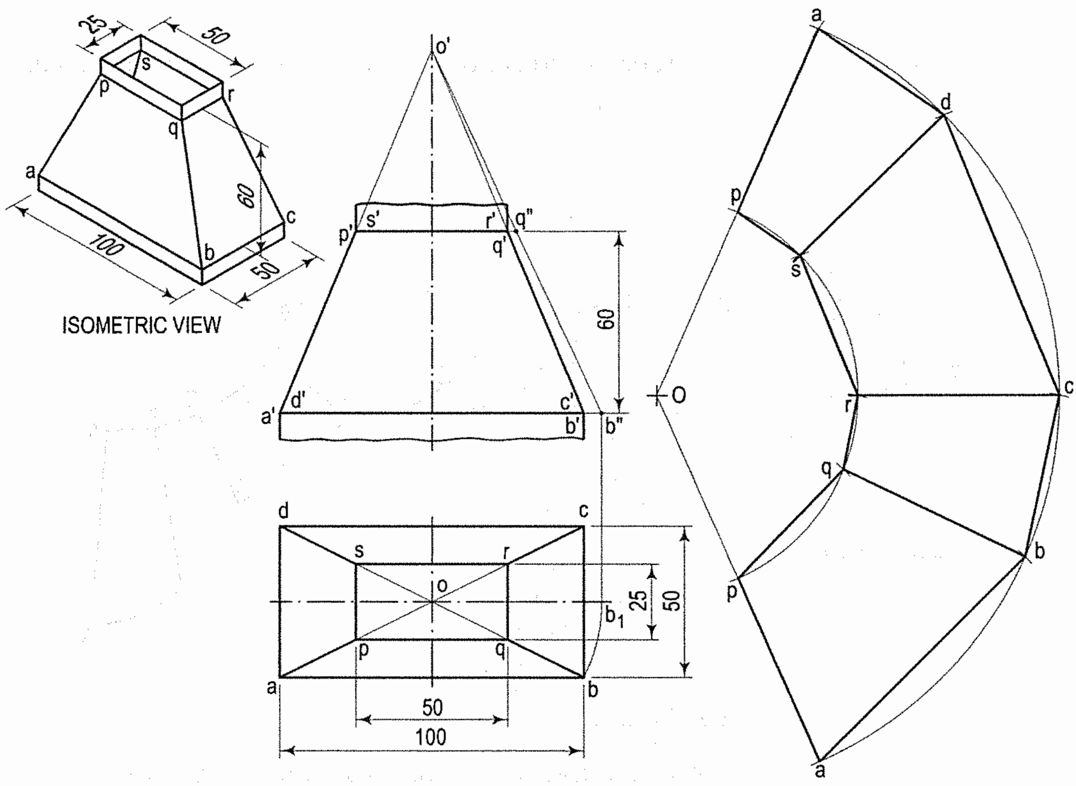
(ii) On the outer arc, step-off distances (i) A_1B_1 equal to the arc a_1b_1, (ii) B_1C_1, C_1D_1, D_1E_1 and E_1F_1, each equal to the side of the base, viz. b_1c_1 and (iii) F_1A_1 equal to the arc f_1a_1.

(iii) Join these points with O, cutting the inner arc at points A, B etc. Locate positions of various points as explained in problems 15-18 and 15-22 and complete the development as shown.

15-3. DEVELOPMENT OF TRANSITION PIECES

Pipes are used in many industries to convey hot or cold fluids. When two different sizes and shapes of pipes are joined using special pipe joint which is known as transition piece. In most cases, transition pieces are composed of plane surfaces and conical surfaces, the latter being developed by triangulation. The procedure of development of few transition pieces is illustrated in the following problems.

Problem 15-32. *In air-conditioning system a rectangular duct of 100 mm × 50 mm connects another rectangular duct of 50 mm × 25 mm through the transition piece as shown in fig. 15-34(i). Neglecting thickness of a metal sheet, develop the lateral surface of the transition piece as shown in fig. 15-34(ii).*



(i) RECTANGULAR DUCT (ii) DEVELOPEMENT OF DUCT

FIG. 15-34

The transition piece is a frustum of a rectangular pyramid.

(i) Determine the position of the apex of the pyramid by extending *a'p'* and *b'q'* as shown. None of the lines in the front view shows the true length of the slant edge. Therefore, draw the top view and make any slant line parallel to *xy* and determine its true length *o'b"*.

(ii) With *O* as centre and radius *o'b"*, draw an arc and obtain the development of a whole pyramid as shown.

(iii) With *O* as centre and radius *o'q"*, draw an arc cutting *oa*, *ob*, *oc* at points *p*, *q*, etc. respectively. Join them in sequence and complete the development as shown.

Problem 15-33. (fig. 15-35): *An air-conditioning duct of a square cross-section 70 mm × 70 mm connects a circular pipe of 40 mm diameter through the transition piece. Draw the projections and develop the lateral surface of the transition piece.*

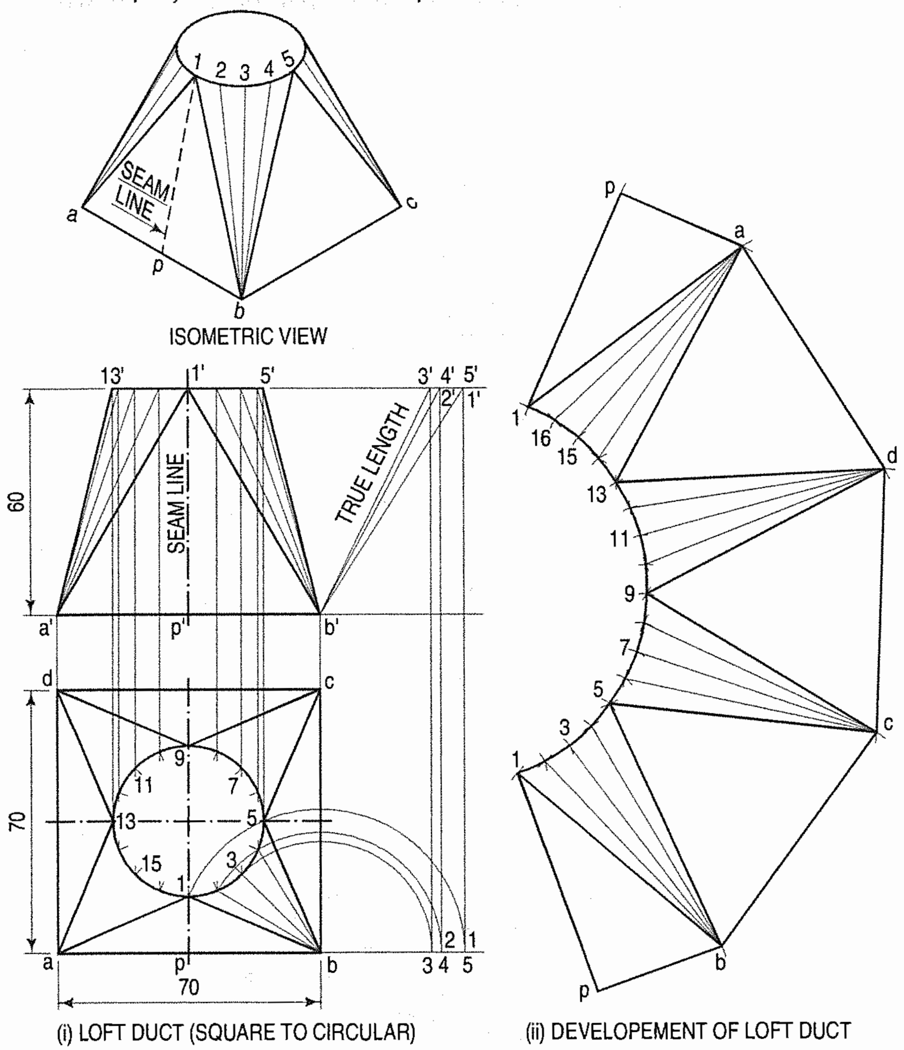


FIG. 15-35

(i) Draw the front view and the top view as shown in fig. 15-35(i).

(ii) Divide the top view of circle into some convenient divisions, say 16 parts as shown.

(iii) Note that the transition piece is composed of four isosceles triangles and four conical surfaces. The seam is along line 1-*P*.

(iv) Begin the development from the seam line 1-*P* (1'-*P*'). As shown in fig. 15-35(ii) draw the right angle triangle 1-*P*-*b*, whose base *pb* is equal to half the side *ab* and whose hypotenuse 1-*b* is equal to the true length 1'-*b*' of side 1-*b*.

(v) The conical surfaces are developed by the triangulation method as follows.

(vi) In the top view, join division of the circle 1, 2, 3 etc. with the corner *a*, *b*, *c* and *d*. Project them in the front view as shown. Obtain the true length of sides of each triangle as shown.

(vii) With *b* as centre and 2'*b*' (true length) radius draw an arc, cutting the arc drawn with 1' as centre and 1'2' as radius. Similarly, obtain the points 3', 4', 5' etc. Join them in the proper order as shown.

Problem 15-34. *A steam pipe bends at certain angle is connected by a transition piece as shown in fig. 15-36(i). Draw the development of lateral surface of the transition piece A.*

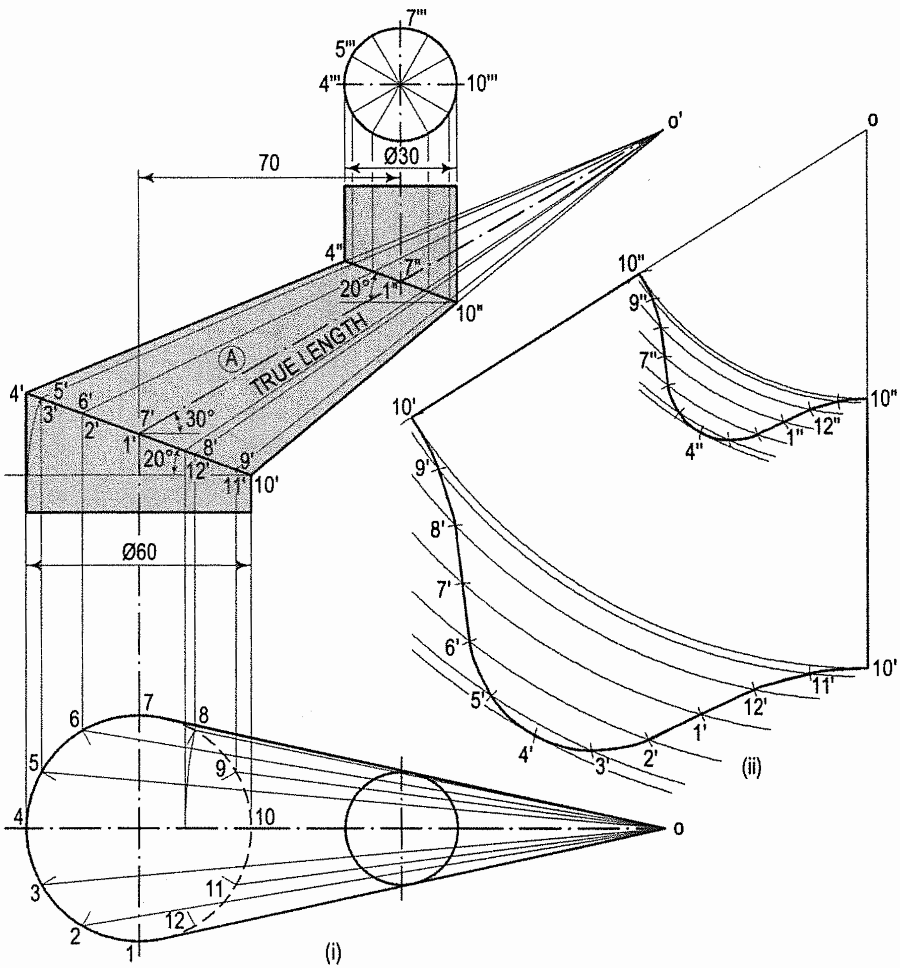


FIG. 15-36

(i) Draw the top view and the front view. The transition piece is a truncated cone. The apex of the cone is determined by extending extreme generators 4' 4" and 10' 10". The projections of the base and the top of truncated cone are inclined to *xy*, therefore in the top view their projections do not show the true shapes.

(The true-shape is an ellipse.) For approximate method of development, these projections in the top view can be taken as the true shape.

(ii) Draw the projections and the development of the cone showing all generators. Obtain the true length of generators.

(iii) With O as centre and radius O10', draw an arc of circle. Similarly draw the arcs of circle taking radii O9', O8' etc.

(iv) With 10' as centre and radius equal to 4-3 (a division of base circle in the top view), draw an arc cutting the previously drawn arc. Similarly obtain points 8', 7', 6' etc. Join them by smooth curve. Obtain the development of the top circle similar way by taking 4'''5''' radius (a division of top circle).

Problem 15-35. *The orthographic projections of an exhaust pipe required for an engine is shown in fig. 15-37. Draw development of the transition connector by triangulation method.*

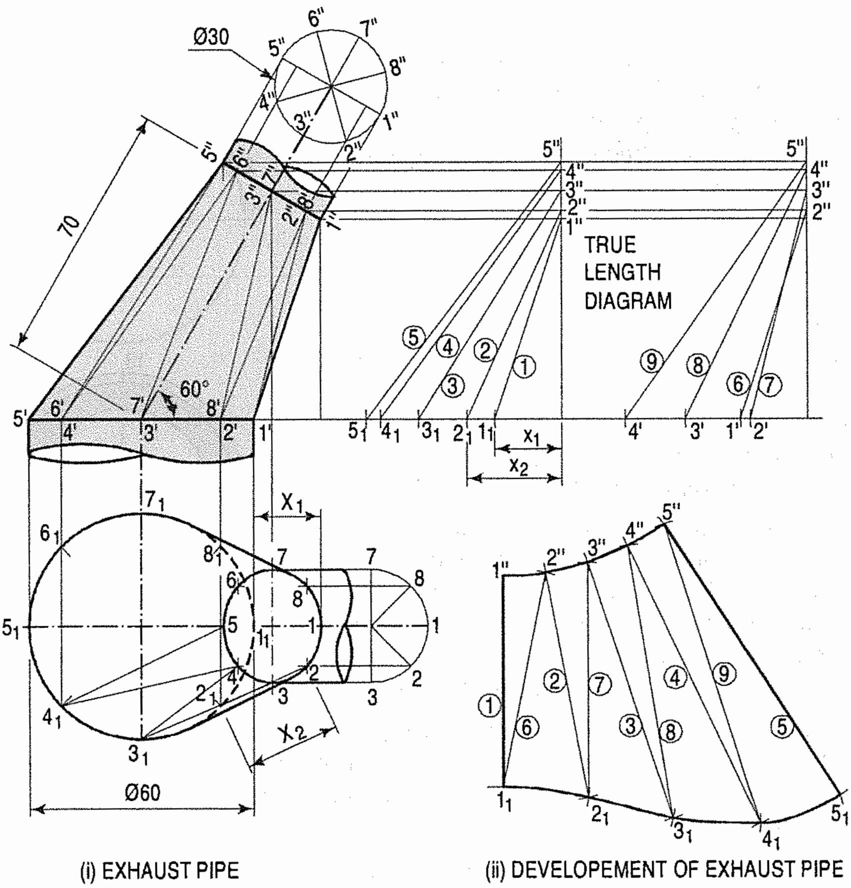


FIG. 15-37

(i) Draw the given projections. Divide the circle of base in the top view into eight equal parts, say 1_1, 2_1. . . etc. Project them in the front view 1', 2'. . . etc. as shown.

(ii) Draw an auxiliary view of the top pipe as shown and divide it into eight equal parts. Number them 1", 2" . . . etc.

(iii) Form the triangles on the lateral surface as shown.

(iv) Determine true length of each side of a triangle as shown.

(v) Mark the length of $1''1_1$. With 1 as centre and radius equal to 1'2" [line-(6)] draw an arc cutting the arc drawn with radius 1"2" (division of auxiliarly circle) and the centre as 1". A half development is shown.

15-4. SPHERES (APPROXIMATE METHOD)

The surface of a sphere can be approximately developed by dividing it into a number of parts. The divisions may be made in two different ways:

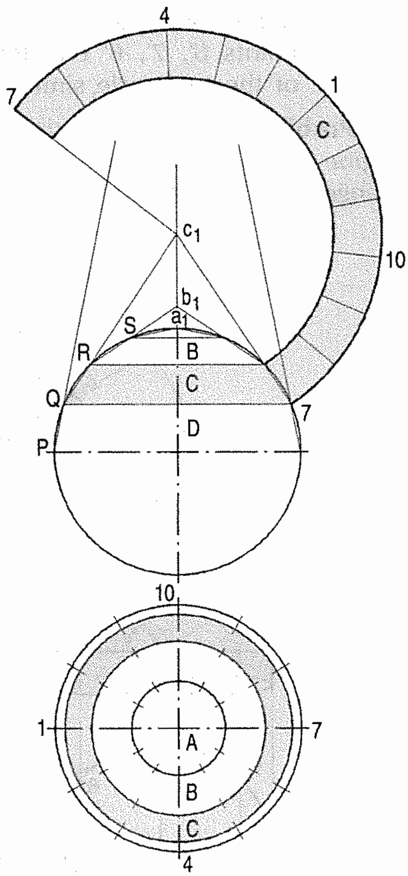
(i) in zones (ii) in lunes.

A *zone* is a portion of the sphere enclosed between two planes perpendicular to the axis. A *lune* is the portion between two planes which contain the axis of the sphere.

(1) Zone method: Fig. 15-38 shows the top half of a sphere divided into four zones of equal width. By joining the points *P*, *Q*, *R* etc. by straight lines, each zone becomes a cone frustum, except the upper-most zone which becomes a cone of small altitude.

Developments of these cone frusta and the upper cone will give the development of the half sphere. For example, take the zone *C*. It is a frustum of a cone whose vertex is at c_1. The surface of this frustum is shown developed in the front view. The length of the divisions on the arc is obtained from the top view. All the zones can be developed in the same manner.

(2) Lune method: A sphere may be divided into twelve lunes, one of which is shown in the front view in fig. 15-39. The semi-circle *qr* is the top view of the centre line of that lune. It is evident that the length of the lune is equal to the length of the arc *qr* and its maximum width is equal to *gh*.



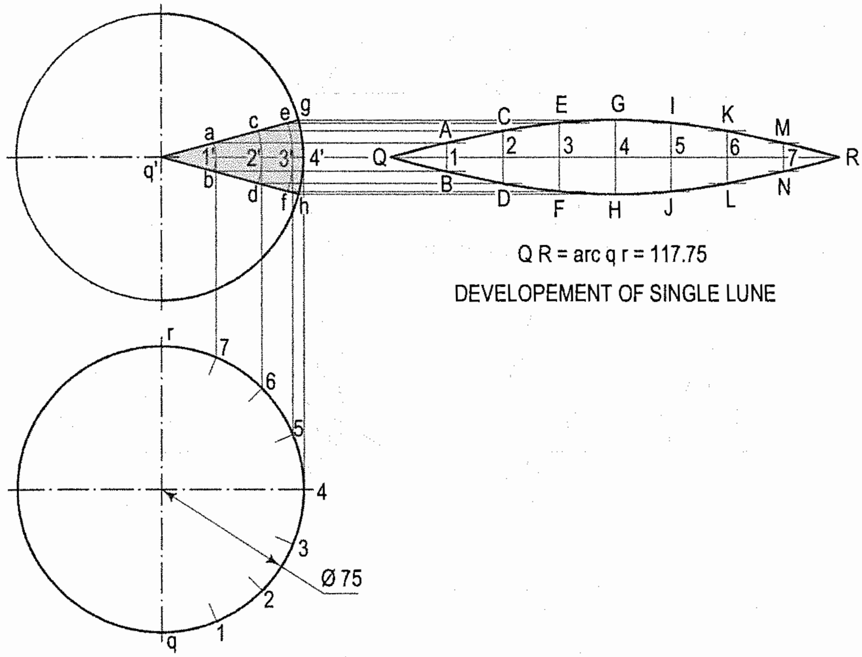
Zone method

FIG. 15-38

Divide the semi-circle into a number of equal parts say 8 and project the division-points on the front view to points 1', 2' etc. With *q*' as centre and radii equal to *q*'1', *q*'2' and *q*'3', draw arcs *ab*, *cd* and *ef* which will show the widths of the lune at points 1 and 7, 2 and 6, and 3 and 5 respectively.

Draw a line *QR* equal to the length of the arc *qr*. This may be obtained by stepping-off eight divisions, each equal to the chord-length *q*1.

Draw perpendiculars at each division-point and make *AB* and *MN* equal to *ab* at points 1 and 7, *CD* and *KL* equal to cd at points 2 and 6 etc. Draw smooth curves through points *Q*, *A*, *C* etc. The figure thus obtained will be the approximate development of one-twelfth of the surface of the sphere. Development of surfaces of some more solids cut by different planes, and solids with holes cut or drilled through them are treated in chapters 14 and 16.



Lune method
FIG. 15-39

EXERCISES 15

1. Draw the development of the lateral surface of the part *P* of each of the solids, the front views of which are shown in fig. 15-40 and described below.

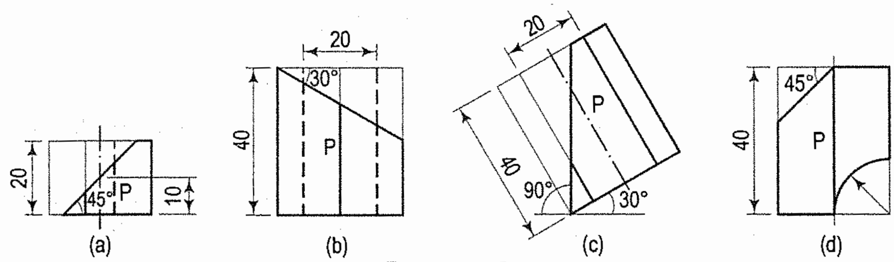


FIG. 15-40

(a) A cube, one vertical face inclined at 30° to the V.P.

(b) A pentagonal prism, a side of the base parallel to the V.P.

(c) A hexagonal prism, two faces parallel to the V.P.

(d) A square prism, length of the side of the base 20 mm and all faces equally inclined to the V.P.

2. Draw the development of the lateral surface of the part *P* of each of the cylinders, the front views of which are shown in fig. 15-41.

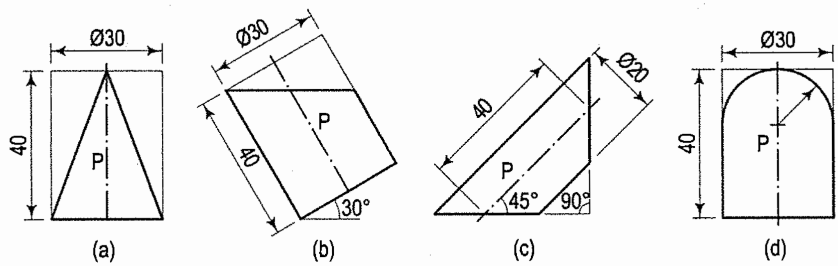


FIG. 15-41

3. Draw the development of the lateral surface of the part *P* of each of the pyramids, the front views of which are shown in fig. 15-42, and described below.

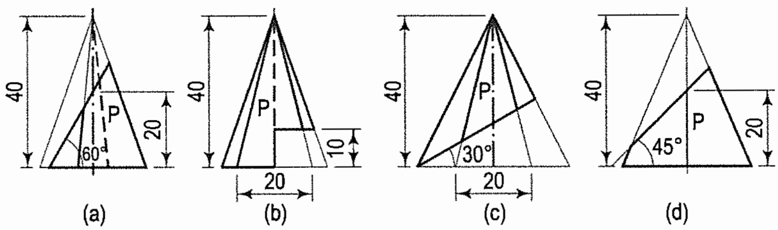


FIG. 15-42

(a) A square pyramid, side of the base 20 mm long and one side of the base inclined at 30° to the V.P.

(b) A pentagonal pyramid, one side of the base parallel to the V.P.

(c) A hexagonal pyramid, two sides of the base parallel to the V.P.

(d) A square pyramid, side of the base 20 mm long and all the sides of the base equally inclined to the V.P.

4. Draw the development of the lateral surface of the part *P* of each of the cones, front views of which are shown in fig. 15-43.

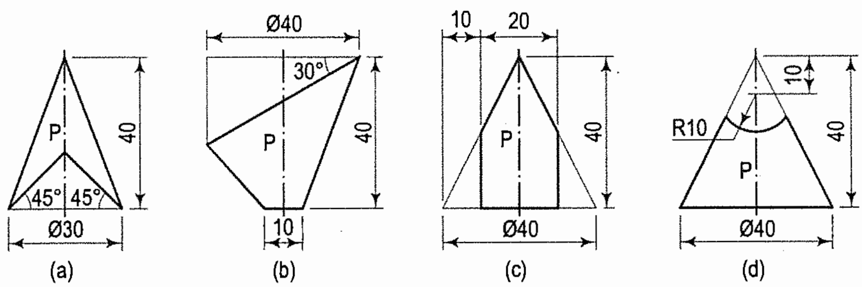


FIG. 15-43

Refer to fig. 15-44 for the following exercises and for dimensions, assume each square to be of 10 mm side.

5. Draw the development of the surfaces of the portions of the following prisms, front views of which are shown in the top row:

(a) A hexagonal prism having a face parallel to the V.P.
(b) A square prism, all faces equally inclined to the V.P.
(c) A pentagonal prism having a vertical face parallel to the V.P.
(d) A triangular prism having a vertical face parallel to the V.P.
(e) A hexagonal prism having two faces perpendicular to the V.P.

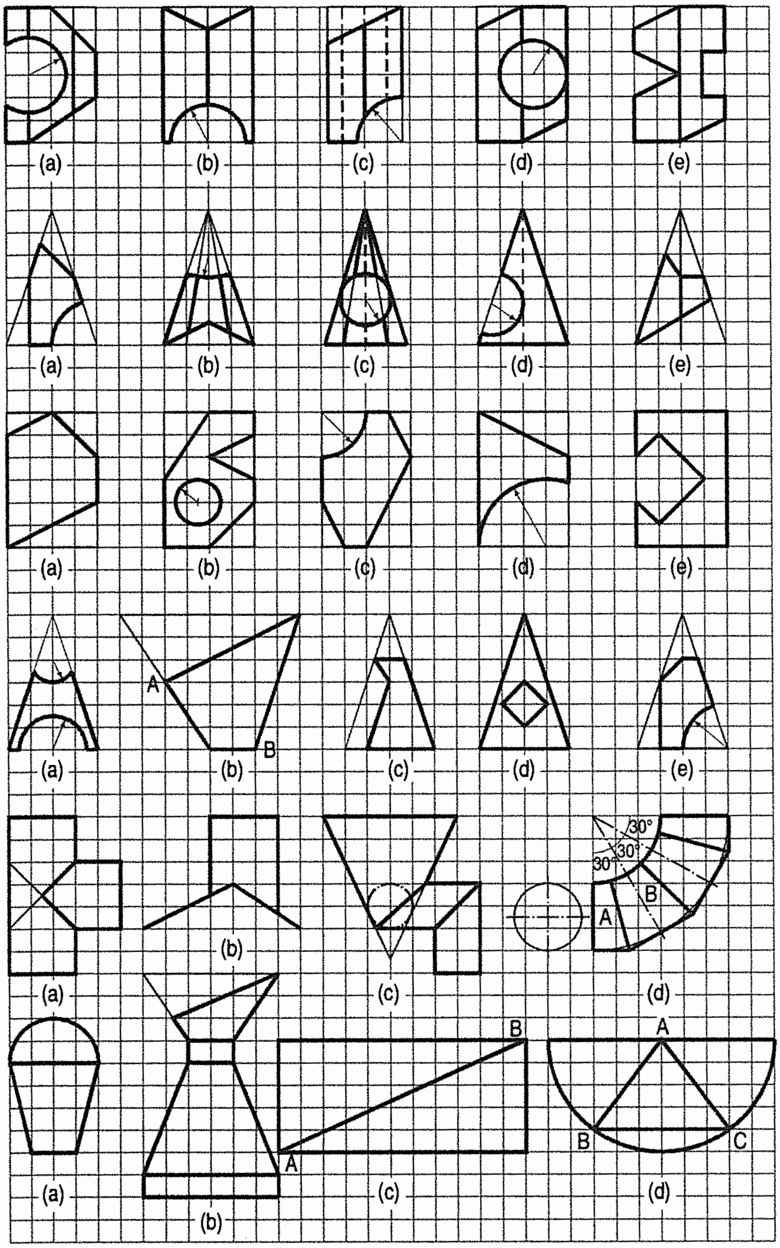


FIG. 15-44

6. Draw the development of the surfaces of the portions of the following pyramids, front views of which are shown in the second row:

(a) A square pyramid having a side of base perpendicular to the V.P.
(b) A hexagonal pyramid having a side of base parallel to the V.P.
(c) A pentagonal pyramid having a side of base parallel to the V.P.
(d) A triangular pyramid having a side of base parallel to the V.P.
(e) A square pyramid having all sides of the base equally inclined to the V.P.

7. Draw the development of the surfaces of the portions of the cylinders shown in the third row.

8. Draw the development of the surfaces of the portions of the cones shown in the fourth row.

9. Refer to the fifth row, and
   (i) Draw the development of the pipes forming a Tee shown at (a).
   (ii) Draw the development of the cylindrical steel chimney erected on a roof [shown at (b)], assuming the squares to be of 30 cm side.
   (iii) Draw the development of the three parts of the funnel shown at (c).
   (iv) Develop parts *A* and *B* of the transition piece shown at (d).

10. Refer to the last row, and
   (i) Develop the surface of the conical buoy with a hemispherical top shown at (a).
   (ii) Determine the shape of the tin sheet required to prepare the can shown at (b).
   (iii) The development of the surface of a cylinder is given at (c). Draw the front view of the cylinder showing the line *AB* in it.
   (iv) The development of the surface of a cone is shown at (d). Draw the projections of the cone showing lines *AB*, *BC* and *CA* in each view.

11. A pipe 40 mm diameter and 120 mm long (along the axis) is welded to the vertical side of a tank. Show the development of the pipe, if it makes an angle of 60° with the side to which it is welded, the other end of the pipe making an angle of 30° with its own axis. Neglect thickness of the pipe.

12. The inside of a hopper of a floor mill is to be lined with tin sheet. The top and bottom of the hopper are regular pentagons with each side equal to 450 mm and 300 mm respectively (internally). The height of the hopper is 450 mm. Draw the shape to which the tin sheet is to be cut so as to fit in the hopper. Scale, 1:10.

13. A 50 mm cylinderical pipe branches off at 90° from a 75 mm cylindrical main pipe as shown in fig. 15-45. Draw the developments of both the pipes at the joint. Assume suitable lengths for the main pipe as well as for the branch pipe.

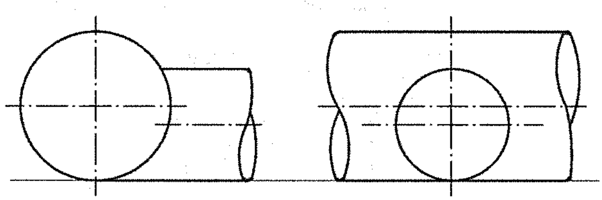


FIG. 15-45

14. A cone of 90 mm diameter of base and 90 mm height stands on its base on the ground. A semi-circular hole of 50 mm diameter is cut through the cone. The axis of the hole is horizontal and intersects the axis of the cone. It is 30 mm above the base of the cone. The flat surface of the hole contains the axis of the cone and is perpendicular to the V.P. Draw three views of the cone and also develop the surface of the cone.