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* Wave :- The disturbance that travelling through a medium or vaccum from one place to other by transferring the energy is called wave.

→ Perodic varation in space and mass (transfer of energy from one place to another
(x, y, z, t)

- when the wave is travelling through the medium it will experience some local oscillation but the particle in the medium do not travel with the wave.

⇒ Properties :-

① frequency ② Wavelength ③ Amplitude ④ Period ⑤ Speed

⇒ Characterstics :-

- Disturbance produced in a medium due to the re perodic motion of the particles of the medium

- In the wave motion wave travel in the forwa direction while particles of the medium vibrate about their mean position.

- There is regular phase change b/w the particles the medium

- The velocity of the wave is different from velocity of the particle.

- The velocity of the wave is uniform while the velocity of particle is different at different

* Types of Wave :-

Longitudnal Wave	Transverse Wave
Particle oscillates // and along to the direction of propagation of wave.	Particle oscilates ⊥r to the direction of propogation of wave
Ex- Sound, Sesmic wave	Ex- All electromagnetic wave, Waves on string. Repples in the pond.

* Oscillation :-

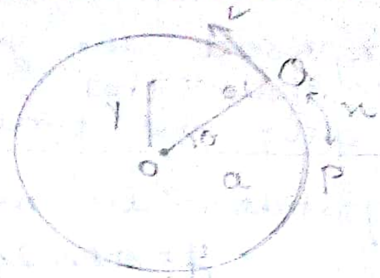
- It is a Kind of perodic motion.
- When a body moves in such a way that it retraces it's path after equal interval of time then it's motion is said to be perodic.
- If a body in perodic motion moves back and forth along the same path, it's motion is called oscillatory or vibratory.

Ex→ Simple pendulam, streched string, Prongs a tunning fork, Atoms in a solid. Electric & Magnetic field in a electro magnetic wave.

* Cause of oscillation

⇒ Body oscillates because of elasticity inertia.

* Wave Equation



Suppose a particle P covers distance x with velocity (v) from P to Q. The displacement of that particle is given by $\boxed{y = a\sin(\omega t - \phi)}$ —(1) where ϕ is the phase difference b/w the particles P and Q.

Since the particle is moving with angle 2π for a path difference of (λ) so we can write phase difference

180°

⇒ phase difference

$$\phi = \frac{2\pi}{\lambda} \times \text{path difference} \quad - \epsilon$$

⇒ Now, for the path difference (x) the phase difference is given as

$$\phi = \frac{2\pi x}{\lambda}$$

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi\nu}$$

$$V = \nu\lambda$$

$$\boxed{\nu = \frac{V}{\lambda}}$$

$$\boxed{\omega = \frac{2\pi V}{\lambda}} \quad -(3)$$

Now putting eqn (2) & (3) in (1) we get

$$y = a\sin\left(\frac{2\pi V}{\lambda}t - \frac{2\pi x}{\lambda}\right)$$

$$y = a\sin\frac{2\pi}{\lambda}(Vt - x) \quad -(4)$$

⇒ This represent the general solution ... which shows that a wave is perodic variation in space and time.

→ If particle is moving in clockwise direction $y = a\sin\frac{2\pi}{\lambda}(vt + x)$, or in anticlockwise,

then $y = a\sin\frac{2\pi}{\lambda}(vt - x)$.

$$y = a\sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$$

$$y = a\sin\left(2\pi\nu t - \frac{2\pi x}{\lambda}\right)$$

$$y = a\sin(\omega t - kx)$$

↑ 5 mark

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2 mark

* Differential Equation of wave equation

⇒ We know ~~the~~ general Equation/ Expression of the Wave is $y = a\sin\frac{2\pi}{\lambda}(vt - x)$ — (i)

Differentiating eqn. (i) w.r.t (t) we have

$$\frac{dy}{dt} = a\cos\frac{2\pi}{\lambda}(vt - x) \times \frac{2\pi}{\lambda}v \quad \text{— (2)}$$

Again diff. eqn (2) w.r.t (t)

$$\frac{d^2y}{dt^2} = -a\sin\frac{2\pi}{\lambda}(vt - x) \times \frac{2\pi}{\lambda}v \times \frac{2\pi}{\lambda}v$$

$$\frac{d^2y}{dt^2} = \cancel{a\sin} -a\frac{4\pi^2v^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt - x) \quad \text{— (3)}$$

Now, differentiation eqn (i) w.r.t x

$$\frac{dy}{dx} = -a\cos\frac{2\pi}{\lambda}(vt-x)\times\frac{2\pi}{\lambda} \quad \text{--- (4)}$$

$$\frac{d^2y}{dx^2} = -a\sin\frac{2\pi}{\lambda}(vt-x)\times\frac{4\pi^2}{\lambda^2} \quad \text{--- (5)}$$

→ Now, comparing eqn. (2) & (4) we have

$$\frac{dy}{dt} = (-v)\frac{dy}{dx}$$

⇒ Comparing eqn (3) & (5) we have

$$\frac{d^2y}{dt^2} = (+v^2)\frac{d^2y}{dx^2}$$

∴ This eqn represent diffrential eqn/form of wave motion

What is S.H.M → 1
Derive S.H.M eqn & solution
find total energy, frequency
Numerical — 1

✳ SIMPLE HARMONIC MOTION

→ S.H.M is a kind of perodic oscillatory motion in which the accleration of the body is proportional to it's displacement from the mean position of rest and is always directed towards the of rest.

ohms law →

Since the restoring force opposes the increase of of the body it is associated with a -ve sign.

In other words,

$$F = -kx, \text{ where } k \text{ is force constant}$$

The restoring force produces a velocity $\frac{dx}{dt}$ accleration $\frac{d^2x}{dt^2}$ in the body. if m is ... to N. 2

of motion,

$$F = ma$$
$$= m\frac{d^2x}{dt^2}$$

$\rightarrow$ The differential eqn. of motion can be written as

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0} \quad \text{—(i)} \qquad \text{where } \omega = \sqrt{\frac{k}{m}}$$

$\rightarrow$ The solution of eqn. (i) can be written as

$$\boxed{x = A_1 e^{i\omega t} + B_1 e^{-i\omega t}} \quad \text{—(2)} \qquad e^{i\theta} = \cos\theta + i\sin\theta$$

$$x = A_1[\cos\omega t + i\sin\omega t] + B_1[\cos\omega t - i\sin\omega t]$$

$$x = (A_1 + B_1)[\cos\omega t] + i(A_1 - B_1)[\sin\omega t]$$

$$\boxed{x = A_2\cos\omega t + B_2\sin\omega t} \quad \text{—(3)}$$

Now, Putting, $\left.\begin{array}{l} A_2 = A\cos\theta \\ B_2 = -A\sin\theta \end{array}\right]$ Putting in (3)

$$x = A\cos\theta\cos\omega t - A\sin\omega t\sin\theta$$

$$\boxed{x = A\cos(\omega t + \theta)} \quad \text{—(4)}$$

Again; Putting $\left.\begin{array}{l} A_2 = B\sin\delta \\ B_2 = B\cos\delta \end{array}\right]$ in eqn (3)

$$\rightarrow x = A\cos\omega t \sin\delta + B\sin\omega t\cos\delta$$

$$\boxed{x = B\sin(\omega t + \delta)} \quad (5)$$

* Properties of S.H.M

1. Amplitude (A)
2. Angular frequency (ω)
3. Phase constant [θ, δ]

θ → initial phase.

$\theta + \omega t$ – After time t, (instantaneous phase)

* Period of oscillation (2M)

$$\Rightarrow T = \frac{2\pi}{\omega}, \quad f = \frac{1}{T}$$

$$\omega = \sqrt{\frac{K}{m}} \qquad = \frac{1}{2\pi}\sqrt{K/m}$$

$$\boxed{T = 2\pi\sqrt{m/K}}$$

* Total Energy of S.H.M

$x = A\cos(\omega t + \theta)$ – x = wave function

$$\Rightarrow \text{Total Energy} = K.E + P.E$$

$$F = -kx$$

$$F = -\frac{dV}{dx} \rightarrow \text{(Potential)}$$

$$V = -\int F\,dx = \int kx\,dx = \frac{1}{2}kx^2$$

$$\because K.E = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$$

$$\frac{dx}{dt} = -A\omega\sin(\omega t + \theta)$$

$$KE = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + \theta)$$

Now,

$$TE = K.E + P.E$$

$$= \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + \theta) + \frac{1}{2} R A^2 \cos^2(\omega t + \theta)$$

$$\Rightarrow \omega = \sqrt{\frac{R}{m}}$$

$$\boxed{R = m\omega^2}$$

$$T.\ Energy = \frac{1}{2} m A^2 \omega^2 \left[\sin^2(\omega t + \theta) + \cos^2(\omega t + \theta)\right]$$

$$\checkmark\ \boxed{T.E = \frac{1}{2} m A^2 \omega^2}$$

* DAMPED OSCILLATION

→ Oscillation which takes place in presence of dissapitive force is known as damped oscillation. (1M)

• Damping force depends upon the velocity of the oscillating body. and for small velocities the damping force is directly ∝ (proportional) to the velocities.

⇒ Equation of motion for damped oscillations may be written as

$$m\frac{d^2x}{dt^2} = -kx - \mu\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x - \frac{\mu}{m}\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} + \frac{\mu}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$\boxed{\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0}$$

Where, $-kx$ = restoring force = F

$-\mu\frac{dx}{dt}$ = damping force = F

$\frac{\mu}{m} = 2b$

$\omega_0 = \sqrt{\frac{k}{m}}$

μ = damping constant / resistive constant

b = damping coffecient

ω_0 = natural frequency of the Undamped oscillation

Numerical

What is dam Oscillation ?, Set up, differ Eq. Find out For

* Solution

Suppose a trial solution is given by $x = Ae^{\alpha t}$ ①

$$x = Ae^{\alpha t}$$

$$\frac{dx}{dt} = A\alpha e^{\alpha t}$$

$$\frac{d^2x}{dt^2} = A\alpha^2 e^{\alpha t} \quad \text{— (2)}$$

→ Put eqn(2) in eqn.

$$A\alpha^2 e^{\alpha t} + 2bA\alpha e^{\alpha t} + w_0^2 A e^{\alpha t} = 0$$

$$\underbrace{Ae^{\alpha t}}_{\text{zero}}\underbrace{[\alpha^2 + 2b\alpha + w_0^2]}_{\text{zero}} = 0$$

It is soln so it can't be zero

$$\Rightarrow \alpha^2 + 2b\alpha + \omega_0^2 = 0$$

$$\alpha = -b \pm \sqrt{b^2 - \omega_0^2}$$

Now,

the general soln can be written as

$$x = A_1 e^{(-b + \sqrt{b^2 - w_0^2})t} + A_2 e^{(-b - \sqrt{b^2 - w_0^2})t}$$

or

$$x = e^{-bt}\left[A_1 e^{\sqrt{b^2 - w_0^2}\,t} + A_2 e^{\sqrt{b^2 - w_0^2}\,\cdot t}\right]$$

① Natural frequency

② b (Resistive force)

solution represents three distinct cases depending upon the relative magnitude of b and w_0.

① Case-1

$b^2 > \omega_0^2$

→ In this case the damping force is large compared to restoring force

$[b^2 - \omega_0^2 = \text{positive}]$

→ So, $\sqrt{b^2-\omega_0^2}$ = positive and real quantity.

→ and both the powers $(-b+\sqrt{b^2-\omega_0^2})t$, $(-b-\sqrt{b^2-\omega_0^2})t$ are negative

→ so both the terms decreases exponentially to zero. and there is no oscillation.

→ This type of motion is called over damped or dead beat.

→ Ex → Motion of a single pendullum inside a highly viscous liquid.

② Case -2

$b^2 = \omega_0^2$: Critical damping

→ In this condition

$x = (A_1 + A_2)e^{-bt}$

$x = Ae^{-bt}$

$x = Ae^{-bt}$ differential 2nd order has 2 constant

→ So order of diffrential eqn is two, so we are taking two constant

$x = (B+C)e^{-bt}$

→ The rate of decrese of displacement x in case of critical damping is much faster than that of over damped case.

This type of behaviour is shown by voltmeter ammeter where the pointer comes to rest at the correct values of current and voltage without undergoing any oscillation.

(iii) Case-3 $b^2 < \omega_0^2$ (Underdamping condition)

$\Rightarrow$ When $b^2 < \omega_0^2$ Then

$$\sqrt{b^2-\omega_0^2} = i\sqrt{\omega_0^2-b^2}$$
$$= i\beta \text{ (say)}, \quad \beta = \sqrt{\omega_0^2-b^2} \quad (1)$$

Now,
$$x = e^{-bt}(A_1 e^{i\beta t} + A_2 e^{-i\beta t})$$
$$= e^{-bt}\left[(A_1+A_2)\cos\beta t + i(A_1-A_2)\sin\beta t\right] \quad (2)$$

Now, by putting $A_1+A_2 = A\cos\phi$
$i(A_1-A_2) = A\sin\phi$

$$x = e^{-bt}\left[A\cos\beta t\cos\phi + A\sin\beta t\sin\phi\right]$$
$$x = Ae^{-bt}\left[\cos(\beta t - \phi)\right]$$
$$x = A e^{-bt}\left[\cos(\sqrt{\omega_0^2-b^2}\,t - \phi)\right] \quad (4)$$

Compare with SHM eq.
Amplitude decrese $Ae^{-bt} \rightarrow$ [logarithm decrement]
$\hookrightarrow$ Rate at which ampli decrese

- Amplitude $= Ae^{-bt}$
- Time period $= T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{\omega_0^2-b^2}}$

$\Rightarrow$ Constant A and ϕ may be determined by applying initial condition.

Suppose $t=0$, motion starts from mean position of rest ($x=0$) with an initial velocity (V_0).

[IMP] [When particle is oscillating under damped what are two initial cond... This value of A & ϕ you study find]

$x = Ae^{-bt}\cos(\sqrt{\omega_0^2 - b^2}\,t - \phi)$ — (i)

① At $t = 0$, $x = 0$

$0 = A\cos\phi$

$\boxed{\phi = \pi/2}$

② At $t = 0$, $\frac{dx}{dt} = V_0$, ? so $\phi = \frac{\pi}{2}$

$\frac{dx}{dt} = -Abe^{-bt}(\cos\sqrt{\omega_0^2 - b^2}\,t - \phi) - Ae^{-bt}\sqrt{\omega_0^2 - b^2}\,\sin(\sqrt{\omega_0^2 - b^2}\,t - \phi)$

$V_0 = +Ae^0\sqrt{\omega_0^2 - b^2}\,\sin\phi$

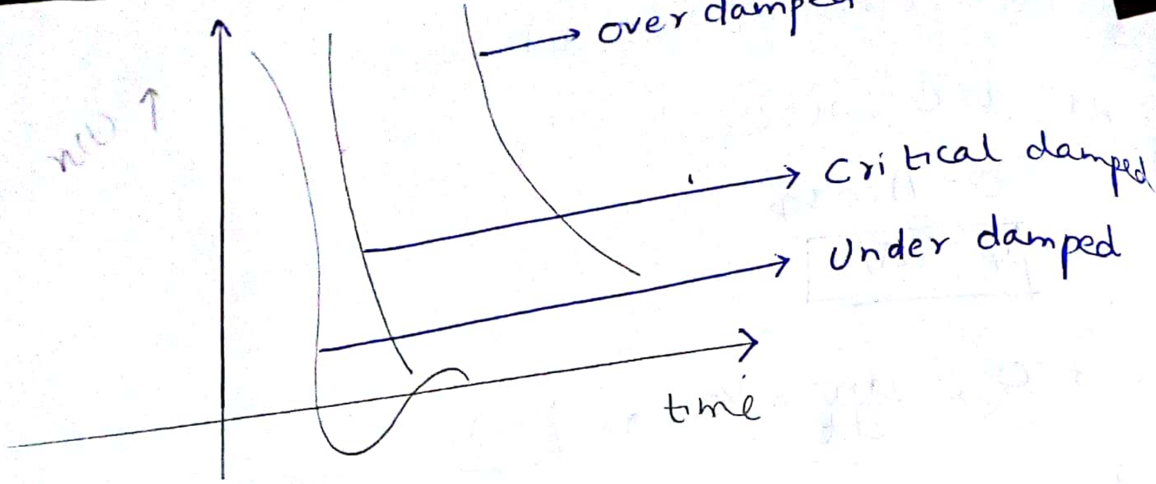
$V_0 = A\sqrt{\omega_0^2 - b^2}$

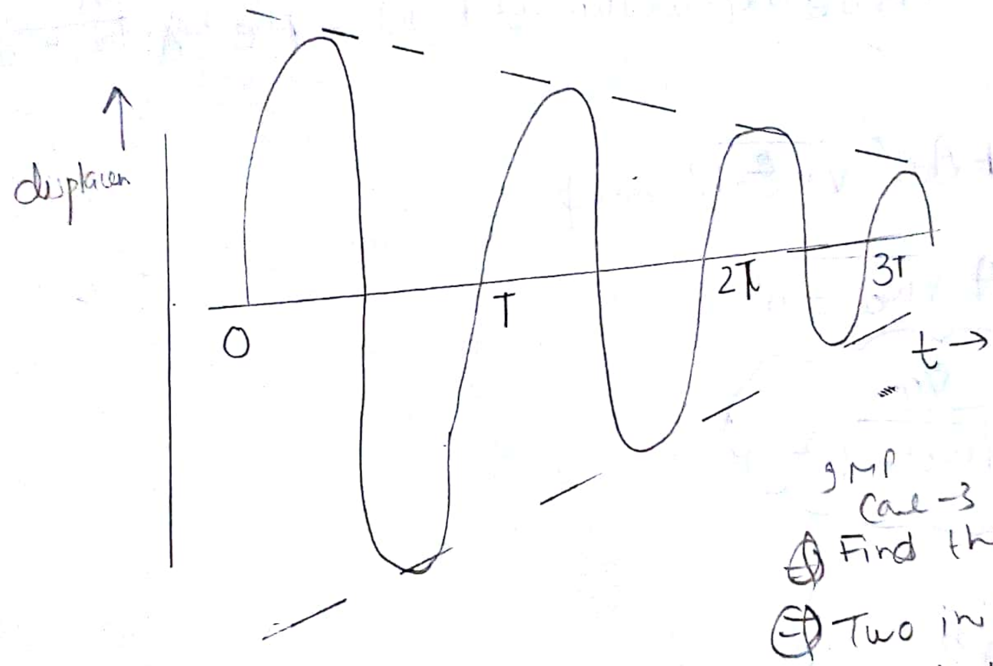
$A = \frac{V_0}{\sqrt{\omega_0^2 - b^2}}$

∴ Put in eqn. (i)

$x = \frac{V_0}{\sqrt{\omega_0^2 - b^2}}\,e^{-bt}\cos(\sqrt{\omega_0^2 - b^2}\,.t - \frac{\pi}{2})$

$x = \frac{V_0}{\sqrt{\omega_0^2 - b^2}}\,e^{-bt}\sin(\sqrt{\omega_0^2 - b^2}\,t)$





Case 3

6 M →

8 M + IN

IMP
Case-3

① Find the Amplitude

② Two initial condition ...
find out amplitude
under damping

① dissipative power

① relaxation time

① logarithmic dec...

* Dissipative (loss) power

⇒ The average power dissipated during one cycle may be expressed as (diss. Power is defined as) rate of loss of energy

(4M) →

Average energy. $\langle P(t)\rangle = \frac{d}{dt}\langle E\rangle$

(4M)

$x = \frac{v_0}{\sqrt{\omega^2 - b^2}}$

(2M)

$$\langle E\rangle = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2$$

In damped

$\ldots e^{-bt} \ldots \frac{1}{2}m\omega^2 A^2 \to \frac{1}{2}m\omega^2 A^2 e^{-2bt} = E_0$

where $E = \frac{1}{2} m \omega^2 A^2$

$\langle P(t) \rangle = \frac{d}{dt} E_0 e^{-2bt}$

Average ...

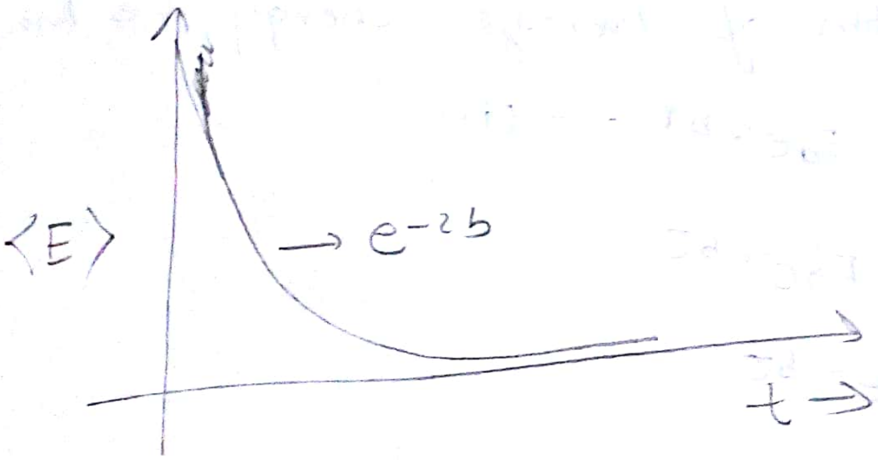
$= -2bE_0 e^{-2bt}$

$= -2b\langle E \rangle$ → Rate of loss of energy per unit cycle

$$\boxed{P\langle t \rangle = 2b\langle E \rangle}$$

(energy decay)

*Plot graph b/w loss of energy vs time



$\frac{1}{e} = 0.368$

* Forg

⇒ An c

is c

* Relaxation Time

⇒ Relaxation time is defined as time taken for total mechanical energy to decay to $\left(\frac{1}{e}\right)$ of it's original value.

Let (τ) be the relaxation time then at

$\Rightarrow t = \tau$

$\Rightarrow E = \frac{1}{e} E_0$

$A = Be^{-bt}$

$E = e^{-2bt}$

From the defination of Average Energy we know that

$$\langle E \rangle = E_0 e^{-2bt} \quad \text{— (i)}$$

Now,

$$\frac{1}{e} E_0 = E_0 e^{-2b\tau}$$

$$e^{-1} = e^{-2b\tau}$$

$$1 = 2b\tau$$

$$\tau = \frac{1}{2b} \quad \text{— (ii)}$$

Q. Relaxation time,

Put in eqn. ①

$$\langle E \rangle = E_0 e^{-t/\tau}$$

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* Forced Oscillation

⇒ An oscillation subjected to external perodic force is called forced Osillation or driven oscillation.

→ Let the driving force is represented by

$$F(t) = F_0 \cos\omega t$$

Now, Eqn. of motion can be written as

$$m\frac{d^2x}{dt^2} + \mu\frac{dx}{dt} + kx = F_0 \cos\omega t$$

$$\frac{d^2x}{dt^2} + \frac{\mu}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\cos\omega t$$

→ external frequency

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = f_0 \cos\omega t \quad \text{— (i)}$$

⇒ This is an inhomogeneous (Linear second order differential equation of motion for forced oscillation)

where $\frac{\mu}{m} = 2b$

$\sqrt{\frac{k}{m}} = \omega$

$\frac{F_0}{m} = f_0$

Ex → Vibration produced in telephone transmitter

⇒ The general soln is of the form

$$x(t) = x_c(t) + x_p(t)$$

$$x_c = A_1 e^{-bt} \cdot \cos(\sqrt{\omega_0^2 - b^2}\, t - \phi)$$ → complementary sol

$$x_p = A_2 \cos(\omega t - \delta)$$ — Particular integral

→ Complementry soln (x_c) daze away exponentially with time. and is a transient soln.
(transition period)

After sufficient long time general soln reduces to particular integral

$$x = x_p(t) = A_2 \cos(\omega t - \delta) \quad \text{— (2)}$$

$$\frac{dx}{dt} = -A_2 \omega \sin(\omega t - \delta)$$

$$\frac{d^2x}{dt^2} = -A_2 \omega^2 \cos(\omega t - \delta)$$

Now, Putting value in eqn (1)

$$-A_2 \omega^2 \cos(\omega t - \delta) + 2b(-A_2 \omega \sin(\omega t - \delta)) + \omega_0^2 (A_2 \cos(\omega t - \delta))$$

$$= f_0 \cos(\omega t)$$

$$= f_0 \cos(\omega t - \delta + \delta)$$

$$= f_0 \left[\cos(\omega t - \delta)\cos\delta - \sin(\omega t - \delta)\sin\delta\right]$$

$$(\omega_0^2 - \omega^2) A_2 \cos(\omega t - \delta) - 2b A_2 \omega \sin(\omega t - \delta) = f_0 \cos(\omega t - \delta)\cos\delta - f_0 \sin(\omega t - \delta)\sin\delta \quad \text{— (4)}$$

Now, comparing coefficient of $\cos(\omega t - \delta)$ and $\sin(\omega t - \delta)$ on both side of eqn we have

$$A_2(\omega_0^2 - \omega^2) = f_0 \cos\delta \quad \text{— (4.1)}$$

$$2b A_2 \omega = f_0 \sin\delta \quad \text{— (4.2)}$$

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→ Squaring and adding these two eqn 4.(1) & 4.2 we have

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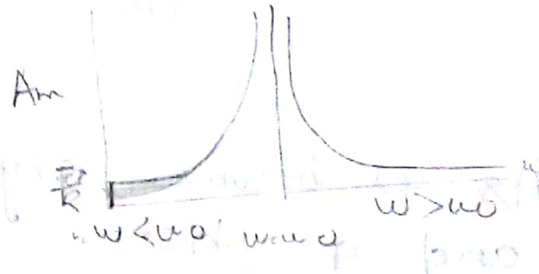
$$A_2^2(\omega_0^2-\omega^2)^2 + 4b^2\omega^2A_2^2 = f_0^2$$

$$A_2^2\left[(\omega_0^2-\omega^2)^2 + 4b^2\omega^2\right] = f_0^2$$

$$A_2 = \frac{f_0}{\left[(\omega_0^2-\omega^2)^2 + 4b^2\omega^2\right]^{1/2}}$$

⇒ Dividing eqn 4.2 by 4.1

$$\tan\delta = \frac{2b\omega}{(\omega_0^2-\omega^2)}$$



$$\delta = \tan^{-1}\left[\frac{2b\omega}{\omega_0^2-\omega^2}\right]$$

→ $x(t) = A_2\cos(\omega t-\delta)$

⇒ $$x(t) = \frac{f_0}{\left[(\omega_0^2-\omega^2)^2 + 4b^2\omega^2\right]^{1/2}}\cos(\omega t-\delta)$$

Case-1 $\omega \ll \omega_0$ (frequency of driving is low)

this case Amplitude of oscillation become

$$\Rightarrow A_2 = \frac{f_0}{\omega_0^2} = \frac{F_0/m}{K/m} = \frac{F_0}{K}$$

weak damping $4b^2\omega^2 \to 0$

$$\rightarrow \boxed{A_2 = \frac{F_0}{K}} \leftarrow$$

addition to $\omega \ll \omega_0$ damping is small then litude is given by $= \frac{F_0}{K}$

This shows that amplitude is almost constant for very small ω and as ω increases amplitude goes on increasing and a phase difference is introduced.

* Case - 2 $(\omega > \omega_0)$ low

⇒ So amplitude of oscillation becomes

$$A_2 = \frac{f_0}{\omega^2}$$

$\delta = \tan^{-1}\left(\frac{-2b}{\omega}\right) \rightarrow$ 2nd quad.

As ω becomes very large the amplitude reduces and a phase difference tends to π.

* Case - 3 $(\omega = \omega_0) \rightarrow$ what is resonance & condition of resonance when A becomes max.

⇒ In this amplitude becomes

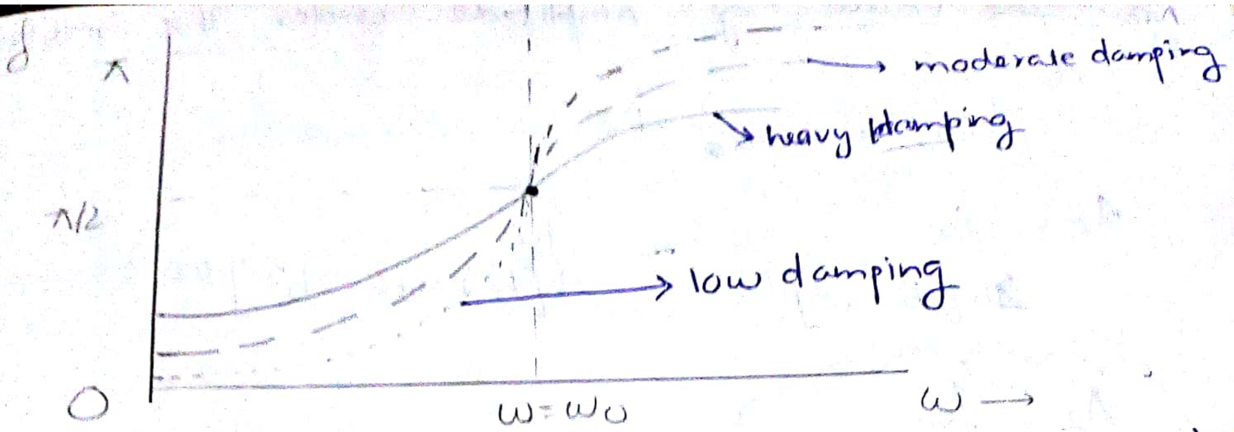
$$A_2 = \frac{f_0}{2b\omega} \Rightarrow \frac{f_0}{2b\omega_0}$$

$$\delta = \tan^{-1}(\infty) = \frac{\pi}{2}$$

The phase angle delta increases from 0 angle to π through $\frac{\pi}{2}$ at resonance $(\omega = \omega_0)$

Case of driving oscillation

draw phase diagram



Resonance :=

→ The amplitude of driven oscillator in steady state is given by $A_2 = \dfrac{f_0}{[(\omega_0^2 - \omega^2)^2 + 4b^2\omega^2]^{1/2}}$

- Amplitude of oscillation becomes large when the forcing frequency and the natural frequency of vibratory system are same
- Such kind of motion when a body oscillates with it's natural frequency under the influence of a perodic force having same frequency is called resonance.

→ Now Find the condition of resonance under which amplitude become maximum. i.e whe

$$\frac{d}{d\omega}[(\omega_0^2 - \omega^2)^2 + 4b^2\omega^2] = 0$$

$$2(\omega_0^2 - \omega^2)(-2\omega) + 8b^2\omega = 0$$

$$\omega_0^2 - \omega^2 = 2b^2$$

$$\omega^2 = \omega_0^2 - 2b^2$$

$$\omega = \sqrt{\omega_0^2 - 2b^2}$$

Condition for amplitude resonance

Now the value of amplitude ... is

$$A_2 = \frac{f_0}{[(2b^2)^2 + 4b^2\omega_0^2]^{1/2}}$$

$$A_2 = \frac{f_0}{2b[b^2+\omega^2]^{1/2}} = \frac{f_0}{2b[b^2+\omega_0^2-2b^2]^{1/2}}$$

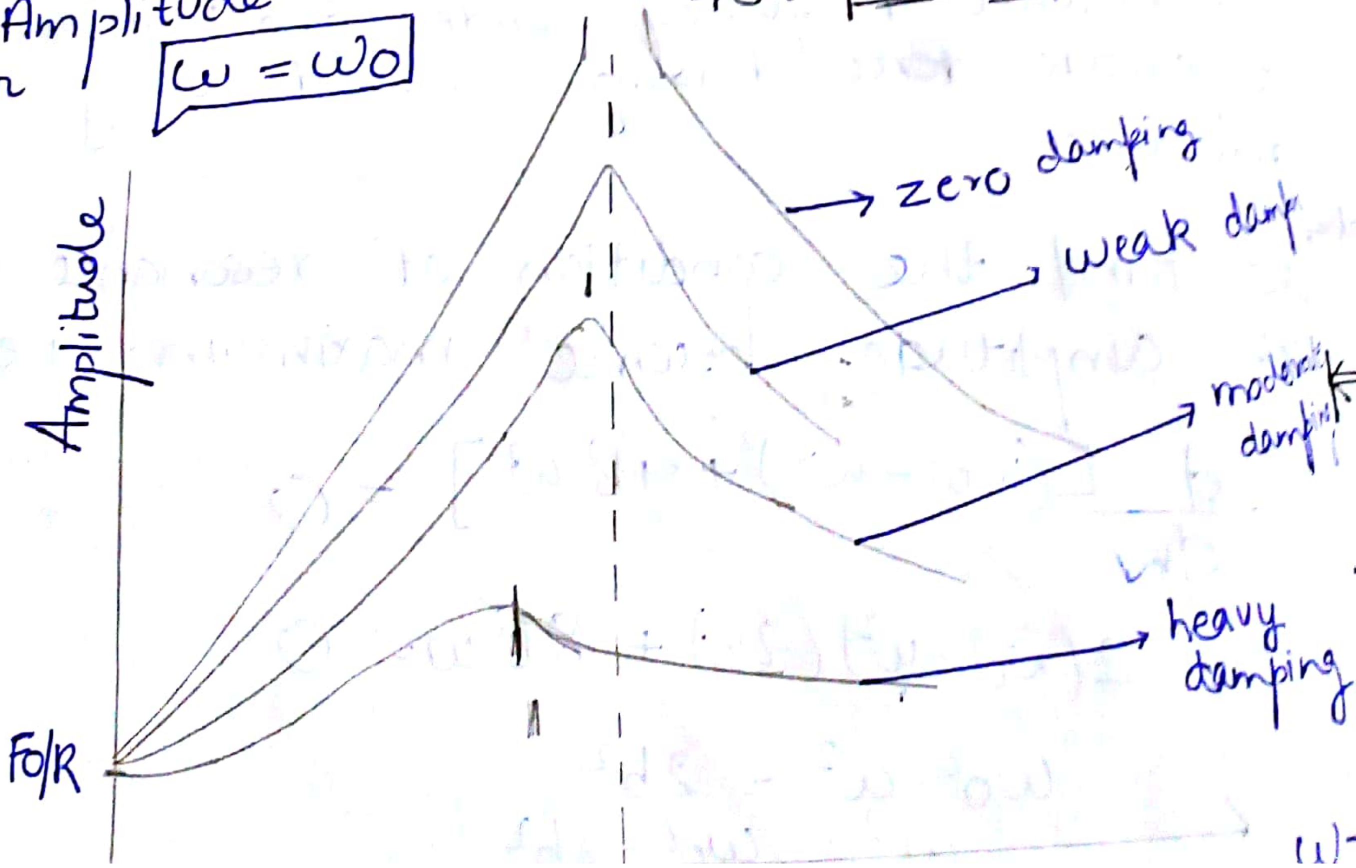
$$\boxed{A_2 = \frac{f_0}{2b[\omega_0^2-b^2]^{1/2}}}$$

In case of weak damping (b is very small) the amplitude of osillation become

$$\boxed{A_2 = \frac{f_0}{2b\omega_0}}$$

The Amplitude of resonance always occur at near $\boxed{\omega = \omega_0}$

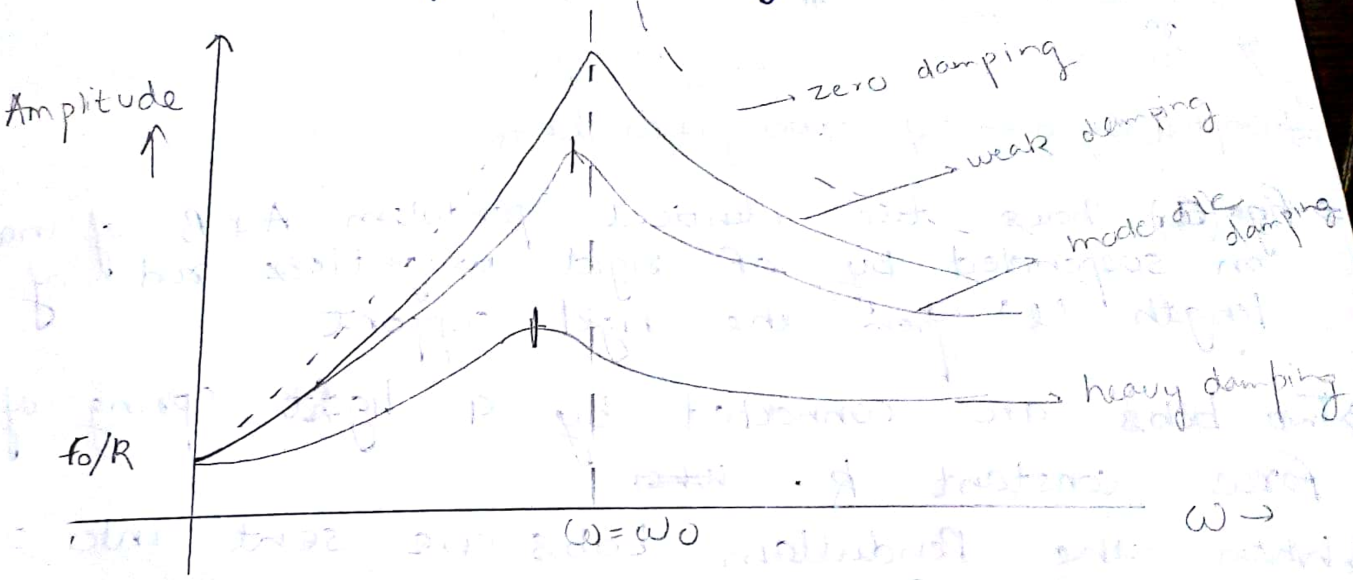
*Sharpness of Resonance



condition

- The amplitude of driven oscillator is max. at resonance frequency and decreses rapidly as the frequency increases or decreses from resonant frequency.
- How rapidly the amplitude decreses on either of resonant frequency is represented by sharpness of resonance.

⇒ The sharpness of resonance may therefore defined as the rapidity with which the amplitude falls as the frequency ω of driving force differs from the resonance value ω_0. The faster the fall the surfer is the resonance.

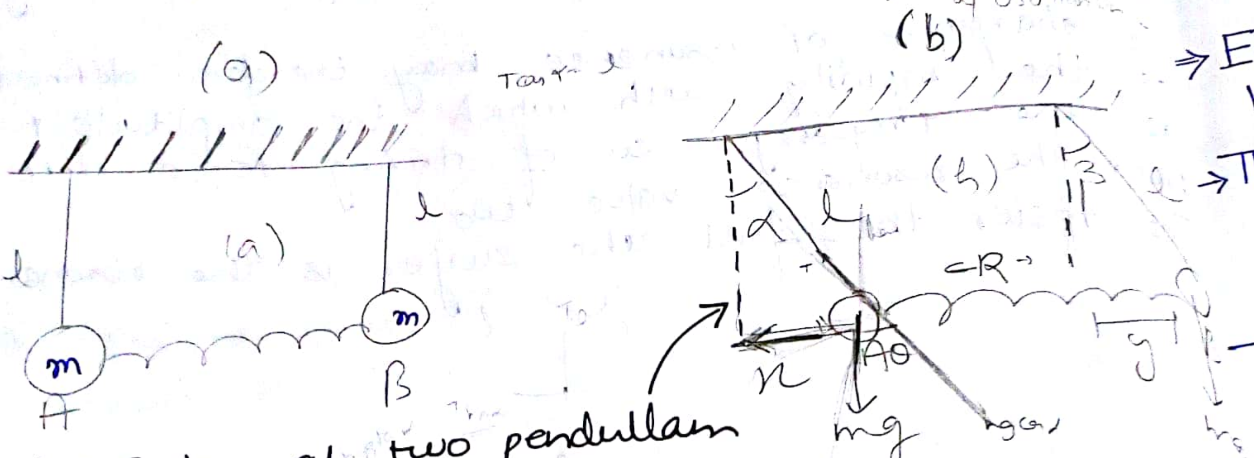


* Coupled Oscillation

→ If two or more oscillator are connected in s[uch] a way that exchange of energy can take place among them the oscillation of th[e] system of oscillator is called coupled oscillation.

Ex- ① Two pendulam connected by a light string
② Prongs of tunning fork.
③ Two oscillatory electrical circuit coupled inductive-ly through mutual inductance.
④ Atoms and ions in molecules and solid.

⑤ Normal modes of oscillation



⇒ Coupled system of two pendullam

→ fig. (a) shows two identical pendulam A & B of mass m suspended by of rigid weightless rod of length 'l' from the rigid support

→ Two bobs are connected by a light spring of force constant k. ~~When~~

⇒ When the Pendullam bobs are sent into oscillation shown in fig. (b). The displacement of the bob A and B are x and y respectively. They are in same direction.

① The restoring force due to spring on A and B are $-k(x-y)$ and $-k(y-x)$ respectively.

② The restoring force on A and B due to the components of gravitational force are $\left[-mg\sin\alpha = -mg\frac{x}{l}\right]$ and $\left[-mg\sin\beta = -mg\frac{y}{l}\right]$

⇒ Equation of
$\frac{md^2x}{dt^2}$
& $\frac{md^2y}{dt^2}$
⇒ Each of th... hence th...
→ This means co-ordinat... Normal
→ The ... eqn of ...
The e... as ...

→ Equation of coupled oscillation

$$m\frac{d^2x}{dt^2} = -mg\frac{x}{l} - k(x-y) \quad \text{---} (1)$$

&

$$m\frac{d^2y}{dt^2} = -mg\frac{y}{l} - k(y-x) \quad \text{---} (2)$$

→ Each of the above eqns involves 'x' and 'y' and hence these are coupled equation.

→ This means each equation involves more than one co-ordinates i.e set of new co-ordinates are called Normal co-ordinates.

→ The second term of the right hand side of eqn (1) and (2) ______ because of the coupling of two pendulums by the spring.

The eqn. of motion (1) and (2) can be written as :-

$$\Rightarrow \frac{d^2x}{dt^2} + \omega_1^2 x + \frac{k}{m}(x-y) = 0 \quad \text{---} (3)$$

$$\Rightarrow \frac{d^2y}{dt^2} + \omega_1^2 y + \frac{k}{m}(y-x) = 0 \quad \text{---} (4)$$

where $\omega_1^2 = g/l$

Equation (3) and (4) represents a pair of coupled equation involving both the variables 'x' & 'y'.

→ To make a pair of de-coupled equation in terms of new pair of variables.

⇒ Add eqn ③ & ④ we have

$$\frac{d^2 (n+y)}{dt^2} + \omega_1^2 (n+y) = 0 \quad \text{—— ⑤}$$

⇒ Subtract eqn ④ from ③ we have

$$\frac{d^2 (n-y)}{dt^2} + \left(\omega_1^2 + \frac{2k}{m}\right)(n-y) = 0 \quad \text{—— ⑥}$$

Suppose $x+y = Q_1$ and $x-y = Q_2$ are two new variable in terms of 'n' and 'y'.

So, Eqn. ⑤ and ⑥ becomes

$$\frac{d^2 Q_1}{dt^2} + \omega_1^2 Q_1 = 0 \quad \text{—— ⑦}$$

& $$\frac{d^2 Q_2}{dt^2} + \omega_2^2 Q_2 = 0 \quad \text{—— ⑧}$$

where, $\omega_2^2 = \omega_1^2 + \frac{2k}{m}$

$\omega_2^2 = \frac{g}{l} + \frac{2k}{m}$

⇒ So, equation ⑦ and ⑧ in term of new co-ordinates 'Q_1' and 'Q_2' are decoupled and represent the oscillation of Simple Harmonic Oscillation.

*Normal co-ordinates

→ The co-ordinates $Q_1 = (n+y)$ and $Q_2 = (n-y)$ are called normal co-ordinates of the coupled system.

→ The normal co-ordinates are linear combination of the orignal Variable x and y.

→ The oscillation of coupled system in terms of each normal co-ordinates is called normal mode of oscillation [Possible : Q_1 - mode & Q_2 mode]

* Normal mode frequency

⇒ from eqn (7) and (8) it is seen that the angular frequency of oscillation for the normal co-ordinates Q_1 and Q_2 are given by :-

$$\omega_1 = \sqrt{g/l} \quad \& \quad \omega_2 = \sqrt{\frac{g}{l} + \frac{2k}{m}}$$

• The corresponding frequency are :=

$$\nu_1 = \frac{\omega_1}{2\pi} = \frac{1}{2\pi}\sqrt{g/l} \quad \& \quad \nu_2 = \frac{\omega_2}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{l} + \frac{2k}{m}}$$

are called normal mode frequency of the coupled oscillation.

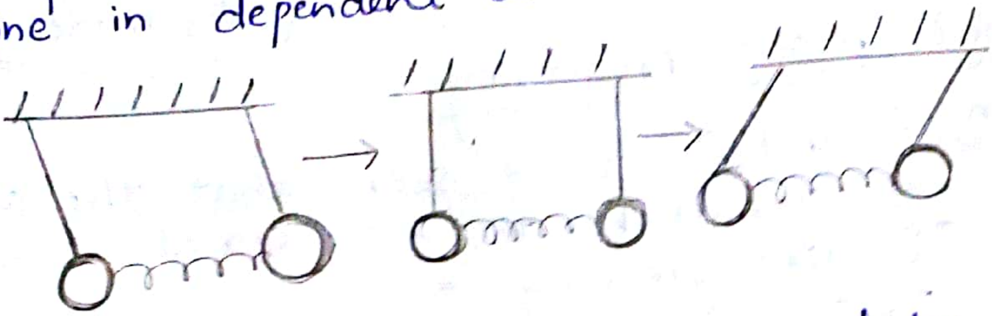
* Normal modes of Oscillation

⇒ The oscillation of coupled system in terms of each normal co-ordinates is called normal modes of oscillation.

⓪⇒ Q_1 mode := If $x=y$, i.e both the pendulum bob's are displaced by same amount in the same direction, $Q_2 = x-y=0$. Thus only Q_1 mode is excited and the eqn's of motion is described by only one eqn.

i.e, $$\frac{d^2Q_1}{dt^2} + \omega_1^2 Q_1 = 0$$

⇒ Since $(x=y)$ ⇒ spring is in normal state i.e. the bobs oscillate with same amplitude, frequency and phase. This is called in-phase mode of oscillation of the oscillation. The frequency of coupled oscillation is same as that of any one independent oscillation.



⇒ **Q_2-mode** :- If $x=-y$; i.e. the bobs are displaced by the same amount in opposite direction.

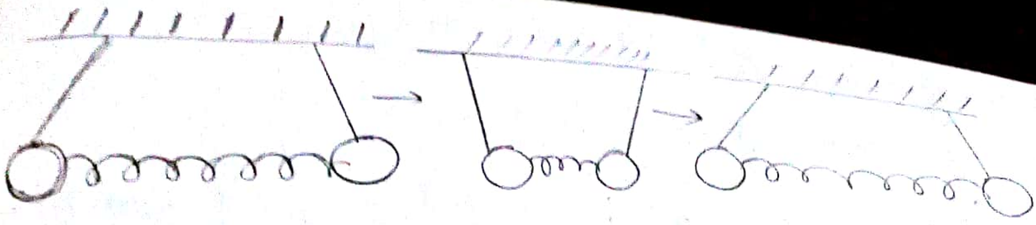
⇒ Q_2 mode is excited and $Q_1=0$, so have only the oscillation is described by the eqn.

$$\frac{d^2Q_2}{dt^2}+\omega_2^2 Q_2=0.$$

⇒ The angular frequency ω_2 is greater than the of the individual oscillation.

⇒ The bobs always vibrate in opposite phase. This mode of oscillation is called out-of-phase.

⇒ Since $\omega_2>\omega_1$, the frequency of out of phase mode is always greater than that of frequency of in-phase mode.



* Equation of a wave moving on a string is $y = 8 \sin \pi (0.02x - 4.00t)$. (x, y) are in sec and t is in second.

Q find Amplitude, frequency and velocity of wave.

Q Two particle at any instant are situated at 20 cm apart calculate phase difference b/w the particle

$\Rightarrow$ $A = 8$, $K = 0.04$, $\frac{2\pi}{T} = 0.04$

$\omega = 8$

$\frac{2\pi}{\lambda} = 8 \qquad \pi = 4\lambda \qquad \boxed{\lambda = \frac{\pi}{4}}$

$\phi = \frac{2\pi}{\lambda} \times x$

$= \frac{2\pi}{\lambda} \times 20$

$= \frac{40\cancel{\pi} \times 4}{\cancel{\pi}} = 160$ cm.

$y = 8 \sin \pi (0.02x - 4t)$

$\because$ $y = a \sin \frac{2\pi}{\lambda} (wt - kx)$

$y = -a \sin 2\pi \left(\frac{kx}{\lambda} - \frac{wt}{\lambda} \right)$

$y = 8 \sin \pi (0.$

$\because$ Amplitude = 8 cm

$\cancel{\frac{k}{\lambda}} = 0.\cancel{0}2$ $\quad$ $k = 0.02\lambda$

$\frac{2\pi}{\lambda} = 0.02\lambda$

$\lambda^2 = \frac{2\pi}{0.02} = \frac{2 \times \pi \times 100}{2} =$

$\because$ $\frac{\omega}{\lambda} = 4$ $\quad$ $0.02 =$

The equation

But the

$\because$ Ampli

Perod

Freq

Wav

w

$\Rightarrow$ If

c

0.01x-2

The equation is $y = 8 \sin \pi (0.02x - 4.00t)$

$$y = 8 \sin 2\pi (0.01x - 2.00t)$$

But the standard eqn of is $y = -a \sin \frac{2\pi}{}\left(\frac{x}{\lambda} - \frac{t}{T}\right)$

Amplitude $= 8$ cm

Periodic time, $T = \frac{1}{2.00} = 0.50$ secn.

Frequency $\nu = \frac{1}{T} = 2.0$ secn.

wavelength $\therefore \lambda = \frac{1}{0.01} = 100$ cm.

wave speed, $v = \nu\lambda$

$= 2 \times 100 = 200$ cm/s.

$\Rightarrow$ If the distance b/w two points is Δx then phase difference $\Delta\phi$ can be

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x.$$

$$= \frac{2\pi}{100} \times 20 = \frac{2\pi}{5} = 72^\circ$$

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

$= a \sin 2\pi \left(\frac{v}{\lambda}t - \frac{x}{\lambda}\right)$ as $v = \nu\lambda$.

$= a \sin 2\pi \left(\nu t - \frac{x}{\lambda}\right)$

Q. phase differ $= 60°$
path differ $= ?$

$$y = -a \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

Q. A mass of 0.5 kg hangs from a spring pulled downward if the mass is let go it execute S.H.M. Calculate time period if the same spring stressed 16 cm by 4 kg mass.

⇒ The instantaneous ... is $A e^{-...}$
let, at ...
after ...
We know

⇒ for a simple mass system in equilibrium

$mg = kx$

$4.0 \times 9.8 = k \times \frac{16}{100}$

$k = \frac{4 \times 9.8 \times 100}{16} = 245$ N/m

Now, 0.5 kg mass is hanged

So, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.5}{245}} = 0.28$ sec

Q. The amplitude of an oscillator of freq. $f = 200/\text{sec}$ falls to $\frac{1}{10}$ of it's initial value (i) after 2000 cycles. Calculate

(i) Relaxation time

(ii) Time in which it's energy falls to $\frac{1}{10}$ of it's initial value. Or

(iii) damping constant.

ring
to the
is
and ⇒ The instantaneous amplitude of damped oscillation is Ae^{-bt}

let, at $t=0$, initial Amplitude $= A_0$

after $t = 10$ sec, it's amplitude $= \frac{A_0}{10}$

we know,

$$A = A_0 e^{-bt}$$

$$\frac{A_0}{10} = A_0 e^{-b \times 10}$$

or,

$$10 = e^{10b}$$

$$\log_e 10 = 10b$$

$$2.3 \log_{10} 10 = 10b$$

$$2.3 = 10b$$

$$b = \frac{2.3}{10} = 0.23$$

(i) Relaxation time $= (\tau) = \frac{1}{2b}$

$$= \frac{1}{2 \times 0.23} = 2.174 \text{ sec.}$$

(ii)

$$E = E_0 e^{-2bt}$$

$$E = E_0 e^{-t/\tau}$$

$$\frac{E_0}{10} = E_0 e^{-t/\tau}$$

$$10 = e^{t/\tau}$$

$$\log 10 = t/\tau$$

$$t = \tau \log_e 10$$

$$= \tau \times 2.3 \log_{10} 10$$

$$= 2.174 \times 2.3 = 5 \text{ sec.}$$

$$2b = \frac{\mu}{m}$$

$$\mu = 2bm$$

$$\mu = 2 \times 0.23 \times m$$

Q. Calculate the average energy stored in 10gm mass attached to a spring and vibrating with an amplitude 2cm in a resonance with periodic force whose frequency is 10Hz.

⇒ During resonance average energy stored

$= \frac{1}{2} m \omega_0^2 A^2$

$= \frac{1}{2} \times (0.01) \times (2\pi \times 10)^2 \times (0.02)^2$

$= 0.0079$ Joule.

Q. Difference b/w $y_1 = A\cos \omega t$
$y_2 = A\cos(\omega t + \pi)$

⇒ Both eqns have same Amplitude and frequency but there is phase difference of π.

Q. A wave is travelling in a gas in a +ve direction with an amplitude of 4cm, velocity 25 m/s and frequency 100 c/s. Calculate the displacement, partial velocity, and acceleration at a distance of 150 cm from the origin after interval of 3s.

⇒ $A = 4cm = \cancel{0.04m}$

$V = 2500\,cm/s$

$\nu = 100\,c/s$

$x = 150cm = \cancel{1.5m}$

$t = 3s$.

∴ $\lambda = \frac{V}{\nu} = \frac{2500}{100} = 25$

∴ $y = 4 \sin \frac{2\pi}{25}(2500 \times 3 - 150)$

$= 4 \sin 2\pi(294)$

$= 4 \sin 580\pi$

$= 0$.

∴ Particle

Particle

Q. Velocity frequen

(a) 51

$\therefore$ Particle velocity = $\frac{dy}{dt} = \frac{2\pi a \nu}{\lambda} \cos \frac{2\pi}{\lambda}(\nu t - x)$

$$\frac{dy}{dt} = \frac{2\pi \nu a}{\lambda} = \frac{2 \times 3.14 \times 25 \times 0.04}{25}$$

$$\frac{dy}{dt} = 25.12.$$

Particle accleration = $\frac{d^2y}{dt^2} = -\frac{4\pi^2 \nu^2}{\lambda^2} a \sin \frac{2\pi}{\lambda}(\nu t - x)$

$$\boxed{\frac{d^2y}{dt^2} = 0}$$

Q. Velocity of radio wave is 4×10^5 Km/s. Calculate the frequency in Mega Hz. the station broadcasting at

(a) 515 (b) 40m

= $V = 4 \times 10^5$ Km/s $\Rightarrow 4 \times 10^8$ m/s

$f = ?$

$\lambda = \frac{515}{\cancel{100}}$ m

$\therefore \nu = \frac{4 \times 10^8}{515} = 0.7767$ MHz.

(b) $V = 4 \times 10^8$ m/s

$\lambda = 40$m

$\nu = \frac{4 \times 10^8}{40} = 10 \times 10^6$

$= 10$ MHz.

Q. Human ear can hear frequencies between 33 Hz to 20 KHz. What wavelength corresponds to these frequencies if the speed of sound is 330 m/s

$V = 330\,m/s$, $\nu_1 = 33\,Hz$, $\nu_2 = 20\times10^3\,Hz$

$$\lambda_1 = \frac{330}{33} = 10\,m$$

$$\lambda_2 = \frac{330}{20\times10^3} = 1.65\,cm$$

Q. A particle executes S.H.M of amplitude 0.1 cm, its velocity passing through the mean position 3 m/s. What is it's frequency.

⇒ $A = 0.1\,cm$

$V = 3\,m/s$

Q. What is lagarithmic decrement

⇒ Measures the rate at which amplitude dies away

or

Natural logarithm of the ratio b/w two successive maximum amplitude which are separated by one period.

$$\boxed{\lambda = \log_e \frac{a_0}{a_1}}$$

Q. Decay modulus ⇒ $\frac{1}{5}$

* Corpuscular

⇒ • Newton, in 1675

• He said corpusc... by a...

• The ... an...

* Ele...

→ In ... an ... call... f...

* W...

⇒ f... m... c...

* Ty...

• S...

• C...

•

Revised Topics

* Corpuscular theory of light

⇒ • Newton, the great among the greatest, proposed in 1675 A.D.

• He said that light consist of tiny particle called corpuscles which are shot out at heigh speed by a luminous object

• The theory could explain the reflection, refraction and rectilinear propagation of light.

* Electromagnetic theory of light

→ In 1983 maxwell suggested that light propagates an Electric and Magnetic field oscillation. These are called Electromagnetic wave which require no medium for their propagtion. Also these waves are transverse in nature.

* Wavefronts

⇒ A continuous locus of all such particle of the medium which are vibrating in the same phase at any instant.

* Types :-

• Spherical wavefront
• Cylindrical wavefront
• Plane wavefront

* Huygen's Principle

- Each point on a wavefront act as a fresh source of new disturbance called secondary wave or wavelets
- The secondary wavelets spread out in all direction with speed of light in a given medium.
- The new wavefront at any later time is given by forward envelope of secondary wavelet at that time.

* Effects of wavelength, frequency and speed during Refraction.

1. frequency remains the same as light travel from one medium to another.
2. frequency is the characterstics of source.
3. wavelength in a medium $\propto$ to phase speed and $\frac{1}{\propto}$ to it's refractive index.
4. $\mu = \frac{c}{v}$

* Condition for obtaining two coherent source of light.

- Obtained from single source.
- Monochromatic light
- Path difference b/w the waves arriving on the screen from two sources must not be large.

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* ~~Interf~~

* Two cohrent source can be obtained from a single source

① Young's double slit Experiment.

② Fresnel biprism method := Obtained from same present source by refraction.

③ Lloyd's mirror method = A source and it's reflected image act as two cohrent source.

* Diffraction of light

⇒ The phenomenon of bending of light around the corners of small obstacles or apertures and its consequent spreading into regions of geometrical shadow.

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Assignment - 1

① What is damped harmonic motion. Set up the differential equation of motion for damped oscillation. Discuss over damped ; critial damped and underdamped motion of the system. ⑧

Q $I = I_0 \frac{\sin^2\alpha}{\alpha^2}$

$\frac{dI}{d\alpha} = ?$

$$\frac{dI}{d\alpha} = I_0 \left[\frac{\frac{d}{d\alpha}\sin^2\alpha \times \alpha^2 - \sin^2\alpha \frac{d\alpha^2}{d\alpha}}{\alpha^4} \right]$$

$$\frac{dI}{d\alpha} = I_0 \left\{ \frac{2\sin\alpha\cos\alpha\,\alpha^2 - 2\sin^2\alpha\,\alpha}{\alpha^4} \right\}$$

$$= 2I_0 \sin\alpha\,\alpha \left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^3} \right]$$

$$= \frac{2\alpha I_0 \sin\alpha\left[\alpha\cos\alpha - \sin\alpha\right]}{\alpha^3}$$

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* following are important cases of superposition of waves :=

(a) Two wave having same frequency moving in same direction [Interferance of waves].

(b) Two waves of slightly different frequencies moving in the same direction. [Beats]

(c) Two waves of same frequency moving in opposite direction [Stationar wayes].

INTERFERENCE 1/08/18

Superposition theorem (observe modification in Amplitude)

⇒ A/c to this theorem the resultant displacement of a particle of the medium acted upon by two or more waves simultaneously is equal to the algebric sum of the displacement of the particle due to individual waves.

- When light from two sources move in a direction then the light wave ~~trains~~ from these sources superimpose upon each other resulting in the modification of distribution of intensity i.e we get position of max. intensity and position of minimum intensity.

- This modification of light energy due to superposition of two or more wave trains is called interferance

Q. What is cohrent → S.F & const P.D

Q. Necessary condition for I.O.L

⇒ Sources should be cohrent.

* Interferance of light

⇒ So the interferance may be defined as the phenomenon in which two or more waves having same frequency and constant phase difference superpose on each other.

⇒ Cohrent sources of light

No.) Two or more sources of light is said to be cohrent if there frequencies are same and they maintain a constant phase relation b/w them. In addition to this there relative Amplitude should be same or nearly same.

* How to obtain coherent

⇒ Method-1 :- Division of wave front.

Method-2 :- Division of Amplitude

* Division of W.F :- Usually Achived by applying mirrors, by prism or lens.

Ex - prism, mirror

Lloyd mirror, young's dable slit Experiment

* Division of Amplitude :- Usually achived by partial reflection or refraction.

Ex - Newton's Ring Experiment.

Micheal's interferometer.

Fabryparat interferometer

* Condition for Interferance - necessary condition for sustained interference pattern

⇒ In addition to coherance (constant phase difference, same frequency). The other conditions,

- The Amplitude of two interfearing waves should nearly same.
- The separation b/w the two sources must be small. d << D
- Each of the sources must be narrow.
- The separation b/w source and screen should be suitable large.

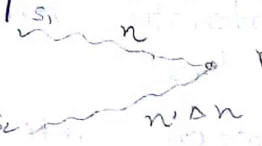
- The sources must be monochromatic
- Background must be dark.

How to obtain

2/08/18

* Analytical Treatment of Interferance

applying → Let two waves having same frequency 'ω' and a constant phase difference superpose on each other. The individual wave is represented by



$$y_1 = a_1 \cos \omega t \quad \text{— (i)}$$

$$y_2 = a_2 \cos(\omega t + \delta) \quad \text{— (ii)}$$

The resultant Amplitude is given by

$$y = y_1 + y_2$$

$$= a_1 \cos \omega t + a_2 \cos \omega t \cos \delta - a_2 \sin \omega t \sin \delta$$

$$\Rightarrow (a_1 + a_2 \cos \delta) \cos \omega t - a_2 \sin \omega t \sin \delta \quad \text{— (3)}$$

Now, $a_1 + a_2 \cos \delta = a \cos \phi$ (3a) , $a_2 \sin \delta = a \sin \phi$ — (3b)

$$\therefore y = a \cos\phi \cos \omega t - a \sin \omega t \sin \phi$$

$$y = a \cos(\omega t + \phi) \quad \text{— (4)}$$

phase difference

with modified Amplitude

Now, squaring and adding eqn (3a) and (3b) we have

$$a^2 = (a_1 + a_2 \cos \delta)^2 + (a_2 \sin \delta)^2$$

$$a^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$$

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} \cos \delta \quad \text{— (5)}$$

Now, dividing eqn (3b) by (3a)

$$\tan\phi = \frac{a_2 \sin\delta}{a_1 + a_2 \cos\delta}$$

$$\phi = \tan^{-1}\left[\frac{a_2 \sin\delta}{a_1 + a_2 \cos\delta}\right]$$

- The intensity remains constant with δ if δ indep. of time.
- for steady interferance pattern phase diff. must be constant. Now,

$* \; I \longrightarrow I_{max}$, when $\delta = +1$; $n = 0, 1, 2 \ldots$
$= 2n\pi$

$$I_{max} = (a_1 + a_2)^2$$

phase difference $= 2n\pi$

path differe $= \frac{\lambda}{2\pi} \times 2n\pi = \boxed{n\lambda}$ or $\boxed{2n\frac{\lambda}{2}}$

when, $\boxed{\text{path difference} = 2n\frac{\lambda}{2}} \rightarrow$ condition for maxima or condition for Constructive Interferance

$* \; I \rightarrow I_{min}$, when $\delta = -1$
$\delta = (2n+1)\pi$

$n = 0, 1, 2, \ldots\ldots$

$$I_{min} = (a_1 - a_2)^2$$

phase ... path ... path ... If ... 1/2M ... (1=) ...

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phase difference = $(2n+1)\pi$

path difference = $\frac{\lambda}{2\pi} \times (2n+1)\pi = (2n+1)\frac{\lambda}{2}$

$\boxed{\text{path difference} = (2n+1)\frac{\lambda}{2}}$ → condition for min Intensity or destructive Interference.

→ Energy is conserved in Interference phenomena show it graphically

Now,

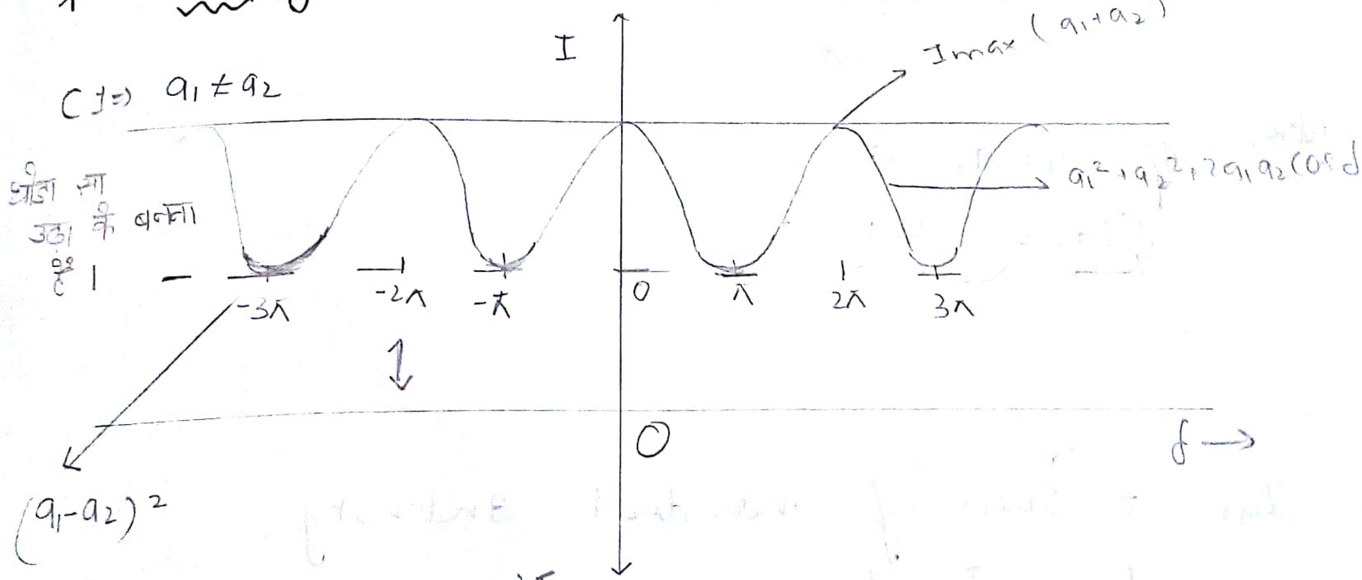
If $a_1 = a_2 = a$

$I_{max} = 4a^2$

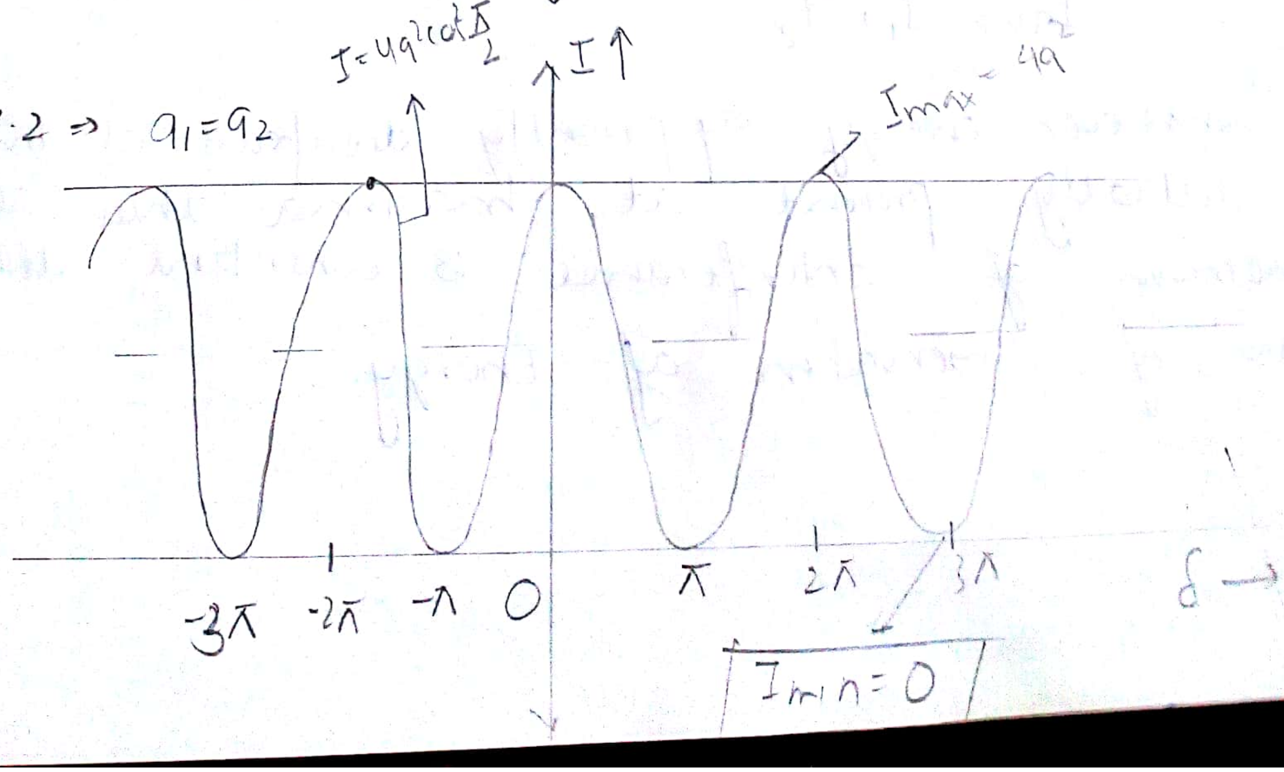
$I_{min} = 0$

1/2M * **Energy Distribution**

C.1 ⇒ $a_1 \neq a_2$



C.2 ⇒ $a_1 = a_2$



* Conservation of Energy (2M)

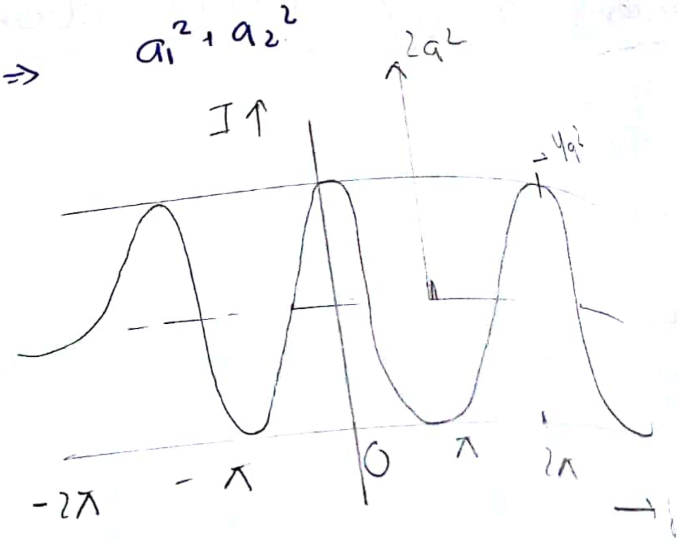
⇒ When we draw a curve b/w resultant intensity I and phase difference δ. It is a periodic curve with it's all heighest and lowest points at $(a_1+a_2)^2$ and $(a_1-a_2)^2$ respectively which are the maximum and minimum intensity b/w $\delta=0$, $\delta=2\pi$ is given

The average intensity by $I_{av} = \dfrac{\int_0^{2\pi} I\, d\delta}{\int_0^{2\pi} d\delta} = \int_0^{2\pi} \dfrac{(a_1^2+a_2^2+2a_1a_2\cos\delta)\, d\delta}{2\pi}$

$$\Rightarrow \frac{\cancel{2\pi}[a_1^2+a_2^2]}{\cancel{2\pi}} \Rightarrow a_1^2+a_2^2$$

Now, If $a_1 = a_2 = a$

$$I_{av} = 2a^2$$



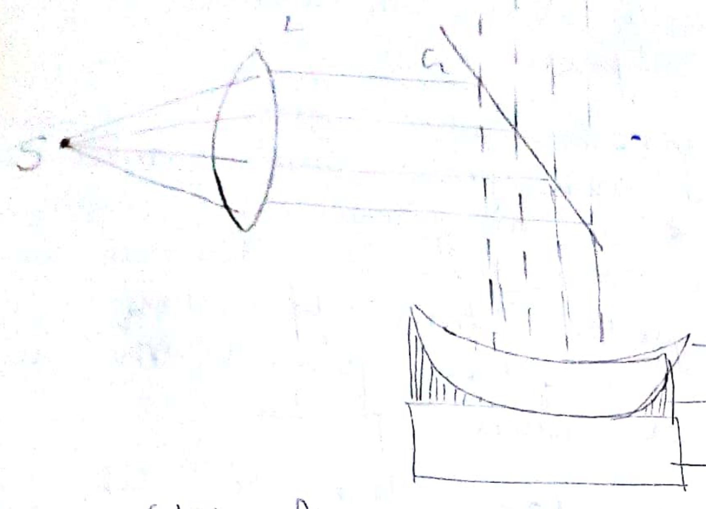
⇒ The I_{av} = sum of individual intensity

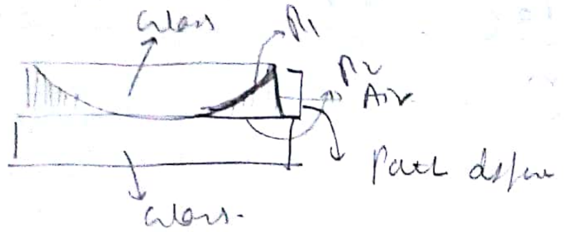
$I_{av} = I_1 + I_2$

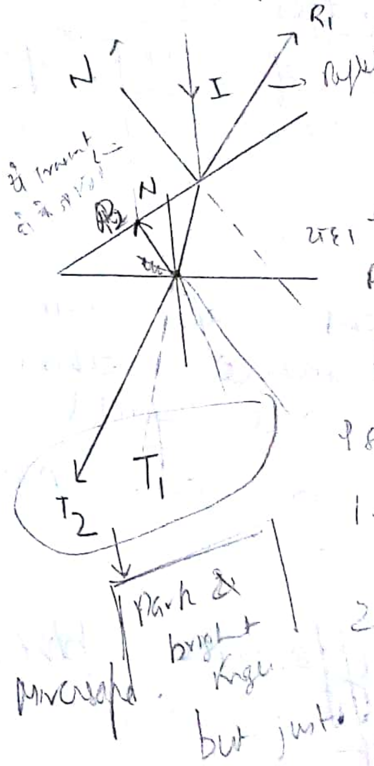
i.e whatever energy apparently dissapears at minima is actually present at maxima. thus the phenomenon of Interferance is consistent with law of conservation of Energy.

intensity reduced lowest ... intensity ... en

* Newton's Ring ✓/IMP → BSM







यहाँ किरण कहाँ है, यहाँ से reflection होगा R_2

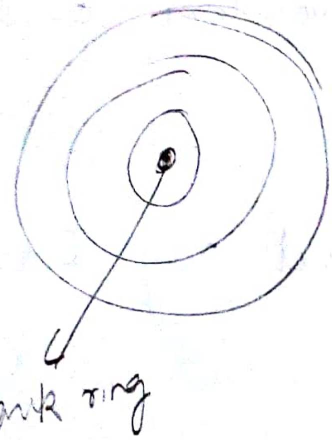
R_1, R_2 = coherent

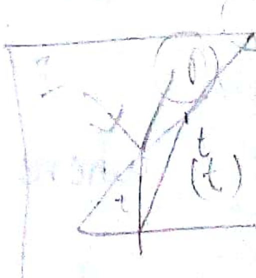
1 single Ray से

1 → upper surface of upper film

2 → ... of lower surface air

but ... observed.





R_1, R_2 → 1 ray is reflected from upper surface of air film

→ 2 ray is reflected from lower surface

became coherent ↓ ...

Path different

effective P.Diff b/w Ray 1 & 2

optical path ⇒ geometrical X μ

geometric path difference :

optical = 2μ...

Add. Path diff = ...

⇒ A plano convex lens is placed on a glass plate in such a way its curved surface lies on a glass plate. An air film of gradually increasing thickness is formed between the two surfaces.

when a beam of monochromatic light is allowed to falls normally on this film and viewed, dark and bright circular fringes are observed. This circular fringes are formed due to interference b/w the rays reflected from upper surface of air film formed b/w lower surface of curved surface and glass plate.

⇒ There fringes are circular due to the symmetry of the air film

formed b/w curved part of plano curved surface and air film

Q. why newton's ring is circular in nature

⇒ The thickness of air film corresponding to fringe is same throughout the circles. These fringes are first observed by Newton and hence known as Newton's ring.

The path difference b/w the reflected ray I & II is given by

$$\Delta = 2\mu t\cos(r+\theta) + \frac{\lambda}{2}$$

where r = is $\angle$ of refraction

θ = is $\angle$ of wedge

Again for normal incidence, $r=0$

$$\Delta = 2\mu t \cos\theta + \frac{\lambda}{2}$$

since radius of curvature of lens is so large then $\theta \cong 0$.

$$\Delta_{eff} = 2\mu t + \frac{\lambda}{2}$$ Effective path difference from I & II.

Now,
At point of contact thickness of air film ($t=0$)

$$\Delta = \frac{\lambda}{2}$$

Path diff $= 2n\frac{\lambda}{2}$ = C.I
Path diff $(2n+1)\frac{\lambda}{2}$ = D.I

Central ring is dark (light)
But this is the condition for minima and hence the central spot is dark for reflected rays.

* Condition for maxima

Path difference $= 2n\frac{\lambda}{2}$

$$2\mu t + \frac{\lambda}{2} = 2n\frac{\lambda}{2}$$

$$2\mu t = (2n-1)\frac{\lambda}{2}$$ — Constructive Interferance

$n = 0, 1, 2, 3 \ldots$

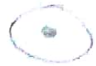
* condition for minima.

path difference $= (2n+1)\frac{\lambda}{2}$

$$2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t = \frac{2n\lambda}{2}$$

$$2\mu t = n\lambda$$ → Destructive Interferance.



In Δ OAB

$R^2 = \cancel{r_n}^2 + (R-1)^2$

$R^2 = \cancel{r_n}^2 + R^2 + t^2 - 2Rt$

$$\boxed{t = \frac{r_n^2}{2R}}$$

- $R \rightarrow$ radius of curvature of lens.
- $O \rightarrow$ centre of curvature

- $BG = t$, thickness of Air fily at Point B.
- $AB = r_n$, radius of n^{th} ringth.

$\rightarrow t^2$ can be neglected due to smallness

$\Rightarrow$ Now, from condition for constructive Interference

$$2\mu t = (2n+1)\frac{\lambda}{2}, \quad n = 0, 1, 2, \ldots\ldots$$

$$2\mu\left(\frac{r_n^2}{2R}\right) = (2n+1)\frac{\lambda}{2}$$

$$r_n^2 = (2n+1)\frac{\lambda R}{2\mu}$$

Air, $\mu = 7$

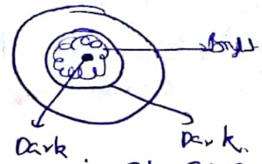
$$r_n = \sqrt{(2n+1)\frac{\lambda R}{2}}$$

$$r_1 = \sqrt{\frac{\lambda R}{2}} \Rightarrow r_1 \propto \sqrt{I}$$

$$r_2 = \sqrt{\frac{3\lambda R}{2}} , \quad r_2 \propto \sqrt{3}$$

$$r_3 = \sqrt{\frac{5\lambda R}{2}} \quad r_3 \propto \sqrt{5}$$

⋮



So this shows that radii of bright rings are proportional to the square root of odd natural no.

* from condition of Destructive Interferance

$$2\mu t = n\lambda$$

$$2\mu \times \left(\frac{r_n^2}{2R}\right) = n\lambda \quad \mu = 1$$

$$r_n^2 = \frac{n\lambda R}{\mu} \Rightarrow \boxed{r_n = \sqrt{n\lambda R}}$$

∴ Now,

S.D.R, $n = 1$

$$r_1 = \sqrt{\lambda R} \quad \Rightarrow \quad r_1 \propto \sqrt{1}$$

$$r_2 = \sqrt{2\lambda R} \quad r_2 \propto \sqrt{2}$$

$$r_3 = \sqrt{3\lambda R} \quad r_3 \propto \sqrt{3}$$

⋮

⇒ So this shows that radius of dark rings are proportional to square root of natural numb

2M

* Application

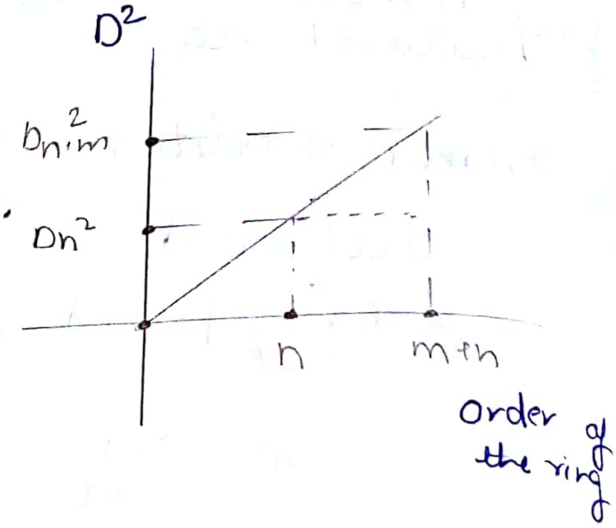
① Determination of wavelength of monochromatic light

* If D_n be the diameter of n^{th} dark ring

$D_n = 2r_n$

$D_n^2 = 4r_n^2$

$$D_n^2 = 4n\lambda R$$



$D_{n+m}^2 = 4(n+m)\lambda R$

$D_{n+m}^2 - D_n^2 = 4m\lambda R.$

$$\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}$$

⇒ from the graph

$$\text{Slope} = \frac{D_{n+m}^2 - D_n^2}{m}$$

$$\lambda = \frac{\text{Slope}}{4R}$$

(2) Determination of refractive index of a liquid.

⇒ A liquid film is formed by putting a few drops of liquid b/w the glass plate and convex lens whose refractive index has to be determined

So, Using monochromatic light of known wavelength the diameter of nth and $(n+m)$th order dark rings are measured.

For destructive,

$$2\mu t = n\lambda$$

$$2\mu \times \frac{r_n^2}{2R} = n\lambda \qquad \left(\because t = \frac{r_n^2}{2R}\right)$$

$$r_n^2 = \frac{n\lambda R}{\mu}$$

$$\therefore D_n^2 = 4 r_n^2 \Rightarrow \frac{4n\lambda R}{\mu}$$

$$\therefore \boxed{D_n^2 = \frac{4n\lambda R}{\mu}}$$

$\therefore$ for air, $\mu = 1$, $\boxed{D_n^2 = 4n\lambda R}$ → 1 diameter, λ, R known

Now,

$$D_{n+m}^2 = \frac{4(n+m)\lambda R}{\mu}$$

$$D_{n+m}^2 - D_n^2 = \frac{4m\lambda R}{\mu}$$

$$\boxed{\mu = \frac{4m\lambda R}{D_{n+m}^2 - D_n^2}}$$

⇒ 2 diameter ring

⇒ λ, R known

* If R is not known then,

D_n' & D_{n+m}' that is diameter of nth and (n+m) th dark ring are to be measured first in air.

$$D_{n+m}'^2 - D_n'^2 = 4m\lambda R \quad - (i) \Leftarrow (\text{Air})$$

$$D_{n+m}^2 - D_n^2 = \frac{4m\lambda R}{\mu} \quad - (ii) \Leftarrow \text{Medium}$$

Dividing eq (i) by (ii)

$$\frac{D_{n+m}'^2 - D_n'^2}{D_{n+m}^2 - D_n^2} = \mu$$

Diameter 5th, 10th in air & medium light of frequency दिया रहेगा then calculate μ

2 diameter - 2 medium use

Hence, ... system

* Newton's Ring by Transmitted Light

⇒ If t is the thickness of air film the path difference b/w the transmitted rays is

$$\Delta = 2\mu t \cos(r+\theta)$$

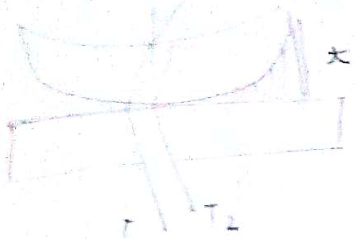
⇒ for normal incidence ($r=0$), large radius of curvature ($\theta \simeq 0$)

$$\therefore \Delta_{eff} \Rightarrow 2\mu t$$

At the point of contact [central point] ($t=0$) i.e. thickness of air film (t)

$\therefore$ $\boxed{\Delta = 0}$, which is condition for maxima

Hence, centre will be bright in transmitted light system.



* Constructive interference

$\Rightarrow$ path difference $= 2n\frac{\lambda}{2} \Rightarrow n\lambda$

$$2\mu t = n\lambda$$

$$\cancel{2}\mu \times \frac{r_n^2}{\cancel{2}R} = n\lambda$$

$$\boxed{r_n^2 = \frac{n\lambda R}{\mu}}$$

for, air, $\mu = 1$

$$\boxed{r_n^2 = n\lambda R}$$

* Destructive interference

path difference $= (2n+1)\frac{\lambda}{2}$

$$2\mu t = (2n+1)\frac{\lambda}{2}$$

$$2\mu \times \frac{r_n^2}{2R} = (2n+1)\frac{\lambda}{2}$$

$$\boxed{r_n^2 = \frac{(2n+1)\lambda R}{2\mu}}$$

for air, $\mu = 1$

$$\boxed{r_n^2 = (2n+1)\frac{\lambda R}{2}}$$

The radius of dark ring due to transmitted ray which is as condition for constructive interference pattern due to reflected light system

They are complementry with each other

Give theory to ...

*. Why ... fringe in N.R is broad source of light is used why?

* (variation of energy with time) in case of damped harmonic oscillator

energy decay

* Diffraction

10/08/18

→ The phenomenon of bending of light waves around obstacles or apertures of size comparable with the wavelength of light resulting there by in their spreading into geometrical shadow of the object is called diffraction.

Interference	Diffraction
• Two separate wavefronts originating from two coherent sources superpose at a point on the screen.	• Secondary wavelets originating from different parts of same wavefront superpose.
• The width of interference fringes may or may not be same.	• The diffraction bands are never of equal width.
• All the bright fringes have same intensity	• The intensity of bright band usually decreases with increase of order number
All the dark fringes have zero Intensity	• The intensity of dark band is not zero.
The intensity distribution of interference pattern is uniform.	• The intensity pattern of diffraction is non-uniform.

* Types

FRESN

The distance or both f aperture / finite.

Wavefront sph

but no

No used

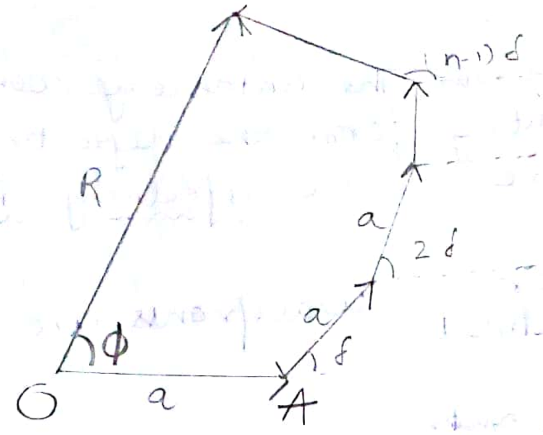
In th the wavele at

* Types

FRESNEL	FRAUNHOFER
The distance of source, screen or both from the defracting aparture/element are finite.	The distance of source, screen from the defracting elements are effectively infinite.
Wavefronts are either spherical or cylindrical but not (plain wave) ↳ J.M.M na equation solution.	wavefronts are plain.
No mirror or lens are used for the study	Converging lens are used to foucus the // rays.
In the plain of appratune the phase of secondary wavelets is not same at all points.	The secondary wavelets are in the same phase at every point in the plain of appratur

* Resultant of 'n- simple harmonic waves

16/08/18



→ Suppose a particle is affected by 'n' simple harmonic vibration of equal amplitude 'a' and having a common phase difference 'δ' b/w two successive vibrations.

~~Resulting~~ Resolving the amplitude in // and ⊥'lr direct to OA we have :

$$\Rightarrow R\cos\phi = a + a\cos\delta + a\cos 2\delta + \dots\dots a\cos(n-1)\delta \quad \text{—(i)}$$

$$\Rightarrow R\sin\phi = 0 + a\sin\delta + a\sin 2\delta + \dots\dots a\sin(n-1)\delta \quad \text{—(ii)}$$

Now, Multiplying eqn (i) by $2\sin(\delta/2)$

$$2R\cos\phi\sin\frac{\delta}{2} = a\left[2\sin\frac{\delta}{2} + 2\cos\delta\sin\frac{\delta}{2} + 2\cos 2\delta\sin\frac{\delta}{2} + \dots \right.$$

$$\left. \dots 2\cos(n-1)\delta\sin\frac{\delta}{2}\right] \text{—(i)}$$

$$2R\cos\phi\sin\frac{\delta}{2} \Rightarrow a\left[2\sin\frac{\delta}{2} + \left(\sin\frac{3\delta}{2} - \sin\frac{\delta}{2}\right) + \left(\sin\frac{5\delta}{2} - \sin\frac{3\delta}{2}\right) + \dots\right.$$

$$\dots\dots \sin\left(n-\frac{1}{2}\right)\delta - \sin\left(n-\frac{3}{2}\right.$$

$$2R\cos\phi \sin\frac{\delta}{2} = a\left[\sin\frac{\delta}{2} + \sin\left(n-\frac{1}{2}\right)\delta\right]$$

$$\cancel{2}R\cos\phi \sin\frac{\delta}{2} = \cancel{2}a\left[\sin\frac{n\delta}{2}\right]\cos\left[\frac{(n-1)\delta}{2}\right]$$

$$R\cos\phi \sin\frac{\delta}{2} = a\sin\frac{n\delta}{2}\cos\frac{(n-1)\delta}{2}$$

$$R\cos\phi = \frac{a\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}} \times \cos\frac{(n-1)\delta}{2} \quad (4)$$

→ Similarly multiplying eqn (2) by $2\sin\left(\frac{\delta}{2}\right)$ and simplifying we have

$$R\sin\phi = \frac{a\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}} \times \sin\frac{(n-1)\delta}{2} \quad (5)$$

→ Dividing eqn (5) by (4)

$$\tan\phi = \tan\left(\frac{(n-1)\delta}{2}\right)$$

$$\phi = \frac{(n-1)\delta}{2} \quad (6)$$

Now,

Squaring and adding eq^n (4) & (5), we have

$$R^2 = \frac{a^2 \sin^2 \frac{n\delta}{2}}{\sin^2 \delta/2}$$

$$R = \frac{a \sin \frac{n\delta}{2}}{\sin \delta/2} \longrightarrow (7)$$

⇒ No. of vibration 'n' is very large and the amplitude and phase are infinetly small

Therefore, na = finite
$n\delta$ = finite

→ Considering $n\delta$ = phase $[2\alpha]$

$$R = \frac{a \sin \frac{2\alpha}{2}}{\sin \frac{2\alpha}{2n}} \Rightarrow \frac{a \sin \alpha}{\sin \frac{\alpha}{n}}$$

$$R = \frac{a \sin \alpha}{\alpha/n} \Rightarrow na \frac{\sin \alpha}{\alpha}$$

$$R = A \frac{\sin \alpha}{\alpha} \longrightarrow (8)$$

where $\boxed{A = na}$

$$\therefore \boxed{\phi = \frac{(n-1)\delta}{2}}$$

- n is large → $10^{20} - 1 = 10^{20}$
δ is small → small → eq

$$\because \boxed{\phi = \frac{n\delta}{2}} \longrightarrow (9)$$

Equation and phase amplitude prog

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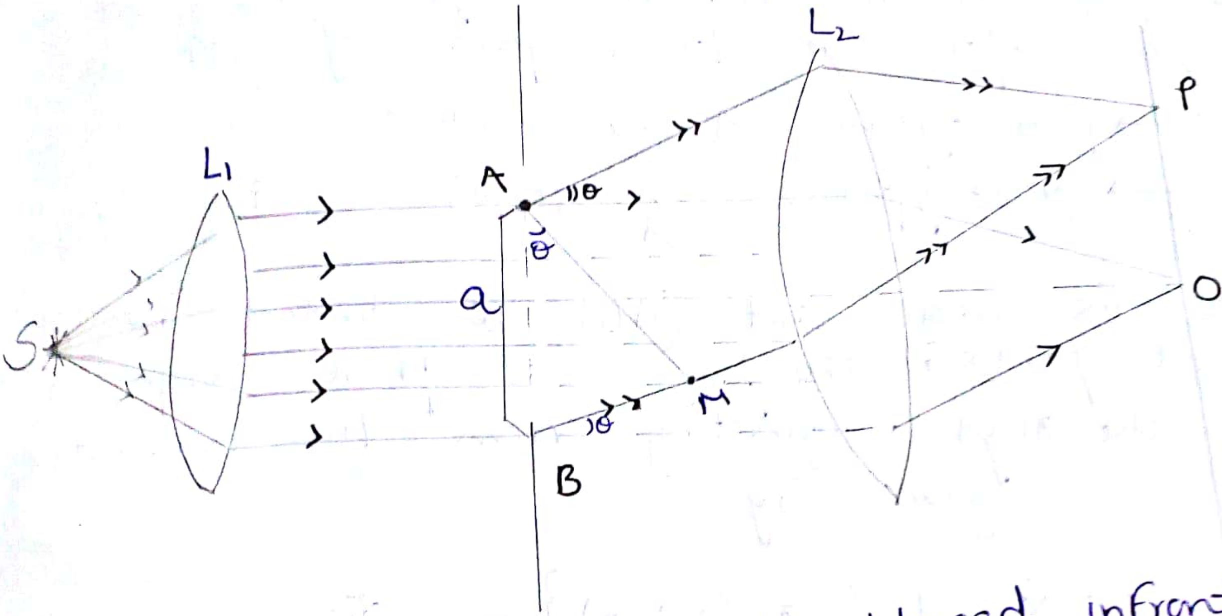
Equation (8) & (9) gives the resultant amplitude and phase of n simple harmonic vibration of equal amplitude and phases increasing in Airthmatic progression.

18/08/18

* Fraunhofen Single Slit diffraction



Say, Slit AB of width 'a' be placed infront of a source 'S'. A beam of monochromatic light of wavelength 'λ' from the source incident on it through lens 'L_1'. The diffracted beam from slit is focous on the screen with the help of a lens 'L_2'.

A diffraction pattern of alternate dark and bright fringe of decreasing intensity is obtained on the screen on both sides of central paint 'O'

Since the path travelled by the beam from upper and lower parts w.r.t 'SO' are same therefore the point 'O' has max. intensity.

So central part due to simple slit has max. intensity

⇒ Suppose some of secondary waves are travelling in a direction making an (θ) from 'SO' and focus at point 'P'.

Now, path difference b/w the rays reaching at P from upper and lower part of slit is

→ path difference = $\Delta = BM = a\sin\theta$ —— (i)

→ phase difference = $\frac{2\pi}{\lambda}(a\sin\theta)$

Suppose the whole slit (AB) is divided into 'n' equal parts therefore the phase difference b/w the rays originating from two consecutive parts is given by

$$\delta = \frac{1}{n}\left[\frac{2\pi}{\lambda}(a\sin\theta)\right] \quad \text{—— (ii)}$$

⇒ But according to the formula the resultant of n- simple harmonic waves of equal Amplitude (a) and phase increasing constant value (δ)

we know,

$$R = \frac{a\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}}$$

$$R = \frac{a \sin(\frac{\pi}{\lambda} a \sin\theta)}{\sin \frac{1}{n}(\frac{\pi}{\lambda} a \sin\theta)}$$

suppose $\frac{\pi}{\lambda} a \sin\theta = \alpha$

$\because$ n is very large

$\therefore R = \frac{a \sin\alpha}{\sin \alpha/n}$

$A = na$

$R = na \frac{\sin\alpha}{\alpha} \Rightarrow R = A \frac{\sin\alpha}{\alpha}$ — (3)

$I = R^2 = A^2 \frac{\sin^2\alpha}{\alpha^2}$

$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$ — (4)

$\because$ Equation -(3) gives Resultant Amplitude of waves & eqn-(4) Resultant Intensity at the screen due to a single slit.

Now, Position of Principle maxima

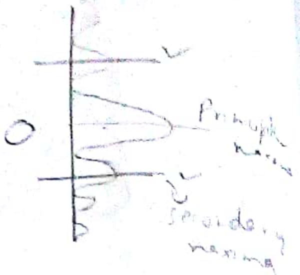
The resultant amplitude is given by :-

$R = A \frac{\sin\alpha}{\alpha}$, $R_{max} \rightarrow$ when,

$R_{max} = \lim \frac{A \sin\alpha}{\alpha} \Rightarrow A$

$$\boxed{I_{max} = A^2}$$

⇒ Intensity will be max at $\alpha \to 0$



But, $\alpha = \frac{\pi}{\lambda} a \sin\theta \to 0$

$$\boxed{\theta = 0^\circ}$$

⇒ The max. will be obtained at the centre

* Position of minima (1)

⇒ The Resultant amplitude is given by

$$R = \frac{A \sin\alpha}{\alpha}$$

The intensity will be minimum when

$R \to R_{min}$, when. $\sin\alpha = 0 = \sin n\pi$

$$\boxed{\alpha = \pm n\pi}$$

$$\frac{\cancel{\pi}}{\lambda} a \sin\theta = \pm n\cancel{\pi}$$

$$a\sin\theta = \pm n\lambda \qquad n = 1, 2, 3 \ldots\ldots$$

20/08/18

* Position of Secondary Maxima

In addition to principle maxima there are less intense secondary maxima on both sides of the principle maxima.

$\Rightarrow$ The position of secondary maxima is given by

$\frac{dI}{d\alpha} = 0$

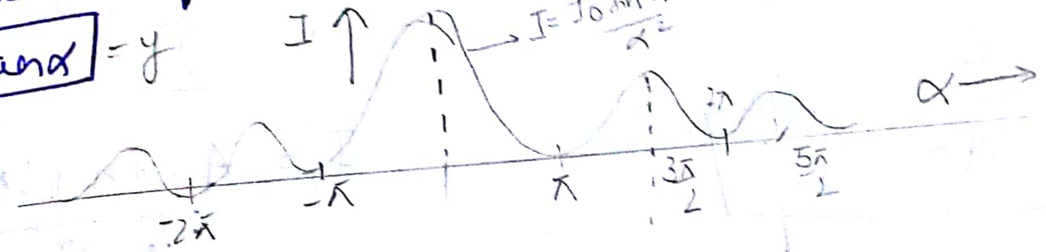
$\frac{d}{d\alpha}\left[A^2 \frac{\sin^2\alpha}{\alpha^2}\right] = 0$

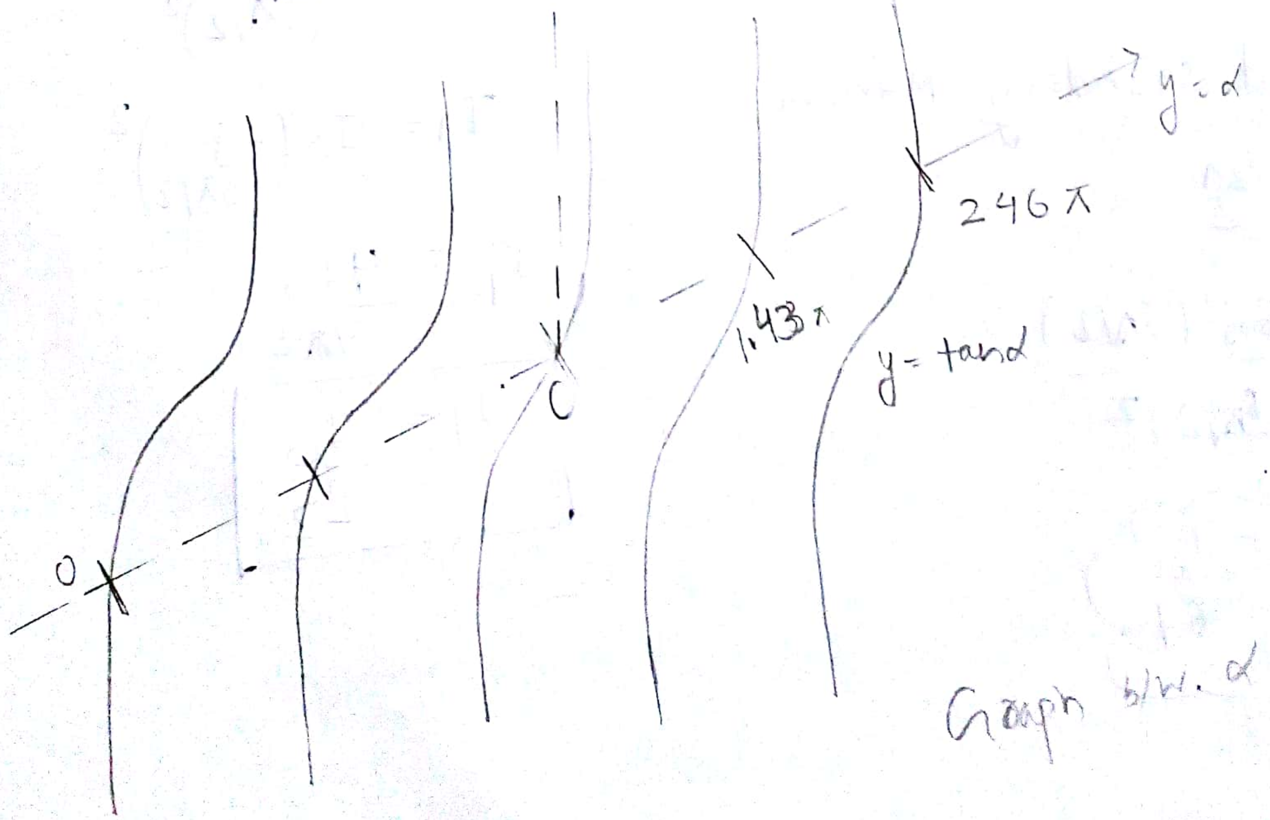
$\Rightarrow A^2 \frac{2\sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$

either, $\boxed{\sin\alpha = 0}$ or $\boxed{\alpha - \tan\alpha = 0}$

$\because \sin\alpha = 0$, Gives the position of primary minima, thus given by the position of secondary maxima is given by

$\boxed{\alpha = \tan\alpha} = y$





Graph b/w. α vs. $\tan\alpha$

Graph b/w α ~ $\tan\alpha$, The point of intersection give you the root of eqn $\boxed{\alpha = \tan\alpha}$ ie The position of secondary maxima.

The intersection occurs at $\boxed{\alpha = 0}$, $\boxed{\alpha = 1.43\pi}$
$\boxed{\alpha = 2.46\pi}$ $\Rightarrow$ Now, taking $\alpha = \overset{I_0}{0}, \overset{I_1}{\frac{3\pi}{2}}, \overset{I_2}{\frac{5\pi}{2}}$

we can find out magnitude of intensity of the principle maximum (I_0), secondary first maxi (I_1), and second maximum (I_2) so on.

* Calculating Intensity

① Principle Maximum

$$I = A^2 \frac{\sin^2\alpha}{\alpha^2}$$

$$\alpha = 0$$

$$\Rightarrow \boxed{I = A^2 = I_0}$$

② first secondary maximum

$$\alpha = \frac{3\pi}{2}$$

$$I_1 = A^2 \frac{\sin^2(3\pi/2)}{(3\pi/2)^2}$$

$$I_1 = I_0 \left(\frac{-1}{3\pi/2}\right)^2$$

$$I_1 = \frac{4I_0}{9\pi^2}$$

$$\boxed{I_1 = \frac{I_0}{22}}$$

③ Second secondary Maxima.

$$\alpha = \frac{5\pi}{2}$$

$$I_2 = A^2 \frac{\sin^2(5\pi/2)}{(5\pi/2)^2}$$

$$I_2 = \frac{4I_0}{25\pi^2} = \frac{I_0}{61}$$

Scanned by CamScanner

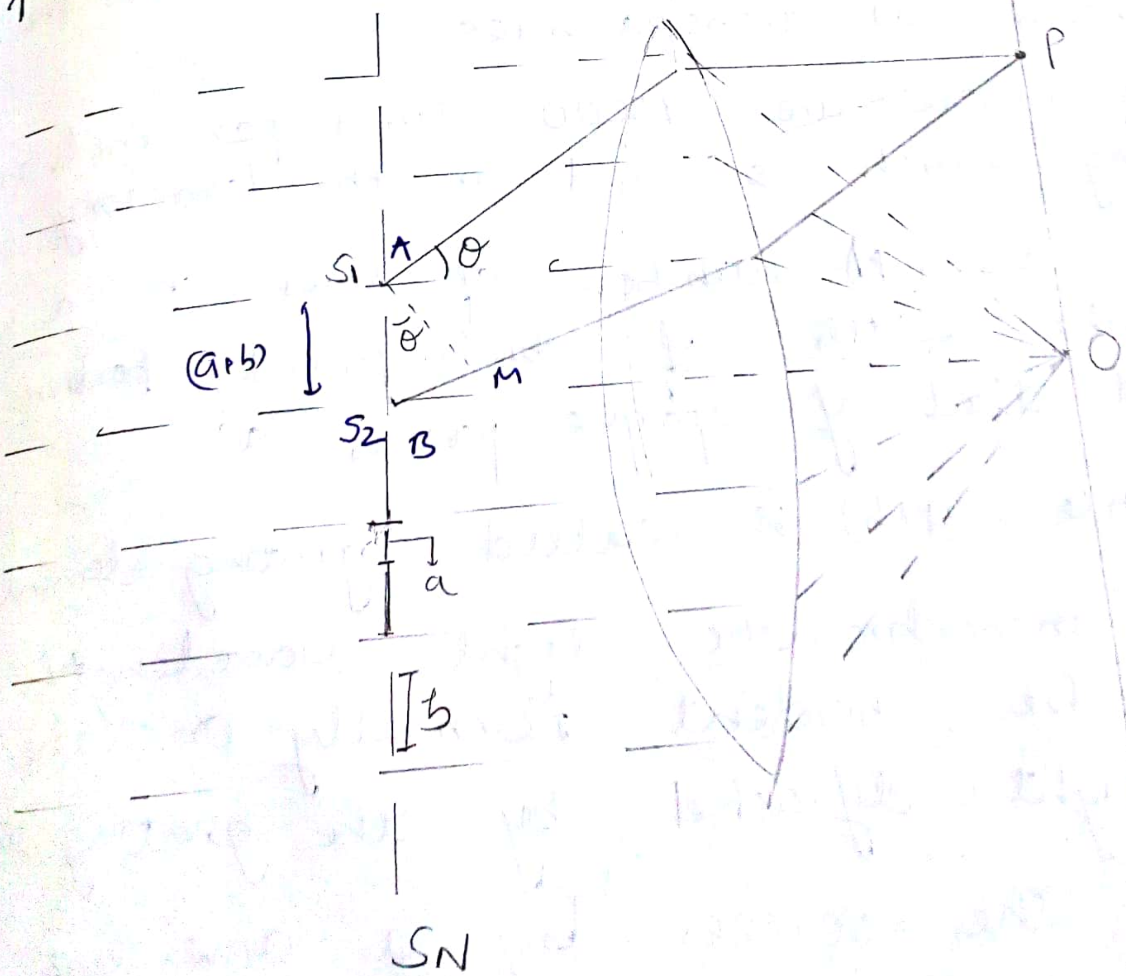
$\Rightarrow I_0 : I_1 : I_2 : I_3 \ldots\ldots$

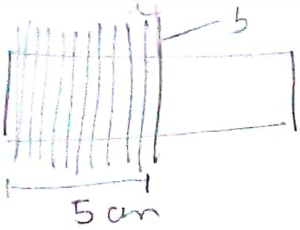
$\Rightarrow 1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} \ldots\ldots$

$\Rightarrow 1 : 0.045 : 0.016 \ldots\ldots$

Conclusion ⇒ This indicates that Intensity of principle maximum is the heighest and the Intensity of secondary maxima decreases on either side of I_0 with the increase of order no. n.

* N-Slit * Fraunhofer * Diffraction





(a+b) grating element

→Q Describe the principle, formation of principle, fringes obtained by N-slits

→Q Missing spectra

15000 → 2.54 cm

1 → $\frac{2.54}{15000}$ = 1.69×10^{-4} cm

⇒The equidistant ruled lines acts as opaque distance and thus do not allow line to pass through them.

These lines are known as opacity. The light passes through the space b/w the lines and are known as transparencies.

Generally there are 15000 lines per inch in a grating which is used in the laboratory.

let there be (N) number of lines in a grating the width of transparent portion (b) and that of opaque portion 'a'.

The distance (a+b) is called grating element.

Consider a monochromatic light wavelength (λ) is to be incident normally on the grating. Light difracted by the grating is focused on the screen by a convex lens.

→ The diffraction pattern obtained on the screen with a large no. of slits consists of extremely sharp interference maxima.

→ And while the intensity of secondary maxima becomes negligibly small so that they are not visible in the diffraction pattern.

Here the resultant amplitude on the screen is due to all waves diffracted from the each slit.

→ Amplitude from of the wave from any slit is $R = A\frac{\sin\alpha}{\alpha}$; where $\alpha = \frac{\pi}{\lambda}(a\sin\theta)$

→ path difference b/w the waves from the slit S_1 and S_2 = $BM = (a+b)\sin\theta$

phase difference b/w two consecutive slits ⇒ $\frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta$ (say)

$$\left\{\beta = \frac{\pi}{\lambda}(a+b)\sin\theta\right\}$$

Resultant Amplitude of n harmonic waves with phase increasing with A.P → is given by ⇒ $R = a\frac{\sin n\delta/2}{\sin\delta/2}$

$a = A\frac{\sin\alpha}{\alpha}$

a = Resultant amplitude due to single slit

$$R = A\frac{\sin\alpha}{\alpha} \times \frac{\sin N\beta}{\sin\beta}$$

$$I = R^2 = A^2\underbrace{\frac{\sin^2\alpha}{\alpha^2}}_{\text{contribution due to diffraction}}\underbrace{\frac{\sin^2 N\beta}{\sin^2\beta}}_{\text{interference}}$$

* Position of Principle maxima
The Resultant Amplitude is given by

$$R = A\frac{\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta}$$

$\Rightarrow R \rightarrow R_{max}$ when,

$$\sin\beta \rightarrow 0$$
$$\beta = \pm n\pi \quad , \quad n = 0, 1, 2, 3 \ldots\ldots$$

$\therefore \boxed{\beta = \pm n\pi} \rightarrow$ position of P. Maxima.

$n = 0 \rightarrow$ zeroth order
$n = 1 \rightarrow$ 1st order
⋮

Now, $\lim\limits_{\beta \to \pm n\pi} \frac{\sin N\beta}{\beta} \Rightarrow N$

$$\because I = A^2\frac{\sin^2\alpha}{\alpha^2} \times \frac{\sin N\beta}{\beta}$$

$$\therefore I = A^2\frac{\sin^2\alpha}{\alpha^2} N^2$$

$\therefore$ position of P. Maxima :=

$$\beta = \pm n\pi$$
$$\therefore \frac{\cancel{\pi}}{\lambda}(a+b)\sin\theta = \pm n\cancel{\pi}$$

$$(a+b)\sin\theta = \pm n\lambda \qquad n = 0, 1, 2, 3, \ldots$$

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Position of minima

$$* \; R = A \frac{\sin\alpha}{\alpha} \times \frac{\sin N\beta}{\sin\beta}$$

$R \to R_{min}$ when, $N\beta = 0 = \pm m\pi$

$$\therefore \; \beta = \frac{\pi}{\lambda}(a+b)\sin\theta$$

$$\Rightarrow \; \boxed{N(a+b)\sin\theta = \pm m\lambda} \qquad \boxed{m = 0, N, 2N \ldots nN}$$

$\Rightarrow m$ can take any values except $(0, N, 2N \cdots nN)$

$\rightarrow$ thus we can say that there are $(N-1)$ minima b/w two consecutive maxima.

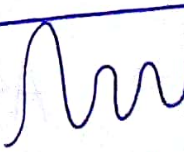
* Position of Secondary Maxima

$\Rightarrow$ Since there are $(N-1)$ no. of minima b/w two consecutive maximum therefore b/w two consecutive maxima. will also $(N-2)$ no. of secondary principle maxima. exist b/w two consecutive

$\Rightarrow$ The position of secondary maxima can be found where,

$$\frac{dI}{d\beta} = 0 \Rightarrow \frac{d}{d\beta}\left[\frac{A^2\sin^2\alpha}{\alpha^2} \times \frac{\sin^2 N\beta}{\sin^2\beta}\right] = 0$$

$$= \frac{A^2\sin^2\alpha}{\alpha^2}\left[\frac{2N\sin N\beta\cos N\beta \times \sin^2\beta - 2\sin^2 N\beta \cos}{\sin^4\beta}\right]$$



b/w two consecutive maxima = $(N-1)$ minima $\;(N-2)$ secondary maxima | 5-Mark

$$\Rightarrow \frac{A^2 \sin^2\alpha}{\alpha^2} \neq 0, \quad [\text{constant, Amplitude}]$$

$$\therefore [\text{Term}] = 0$$

$\therefore$ we get,

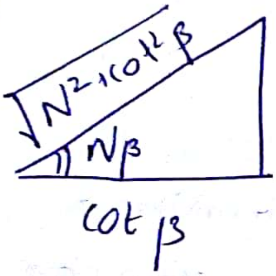
$$\cancel{2N} \sin N\beta \cos N\beta \times \sin^2\beta = \cancel{2} \sin^2 N\beta \sin\beta \cos\beta$$

Q calculate from Missing spectra

Dark band may not have zero intensity

$$\Rightarrow \tan\beta = \frac{\tan N\beta}{N}$$

$$\Rightarrow \boxed{\tan N\beta = N \tan\beta}$$



$$\rightarrow \sin N\beta \Rightarrow \frac{N}{\sqrt{N^2+\cot^2\beta}} \Rightarrow \frac{N\tan\beta}{\sqrt{N^2\tan^2\beta+1}}$$

$$\therefore I' = \frac{A^2\sin^2\alpha}{\alpha^2}\,\frac{\sin^2 N\beta}{\sin^2\beta}$$

$$I' = A^2\frac{\sin^2\alpha}{\alpha^2} \times \frac{1}{\sin^2\beta} \times \frac{N^2\tan^2\beta}{1+N^2\tan^2\beta}$$

$$I' \Rightarrow \frac{A^2\sin^2\alpha}{\alpha^2} \times \frac{1}{\cancel{\sin^2\beta}} \times \frac{N^2}{(1+N^2\tan^2\beta)\cos^2\beta} \times \cancel{\sin^2\beta}$$

$$I' \Rightarrow A^2\frac{\sin^2\alpha}{\alpha^2} \times \frac{N^2}{(\cos^2\beta + N^2\sin^2\beta)} \Rightarrow$$

$$I' = A^2 \frac{\sin^2\alpha}{\alpha^2} \times \frac{N^2}{1+(N-1)\sin^2\beta}$$

→ Intensity of principle maxima

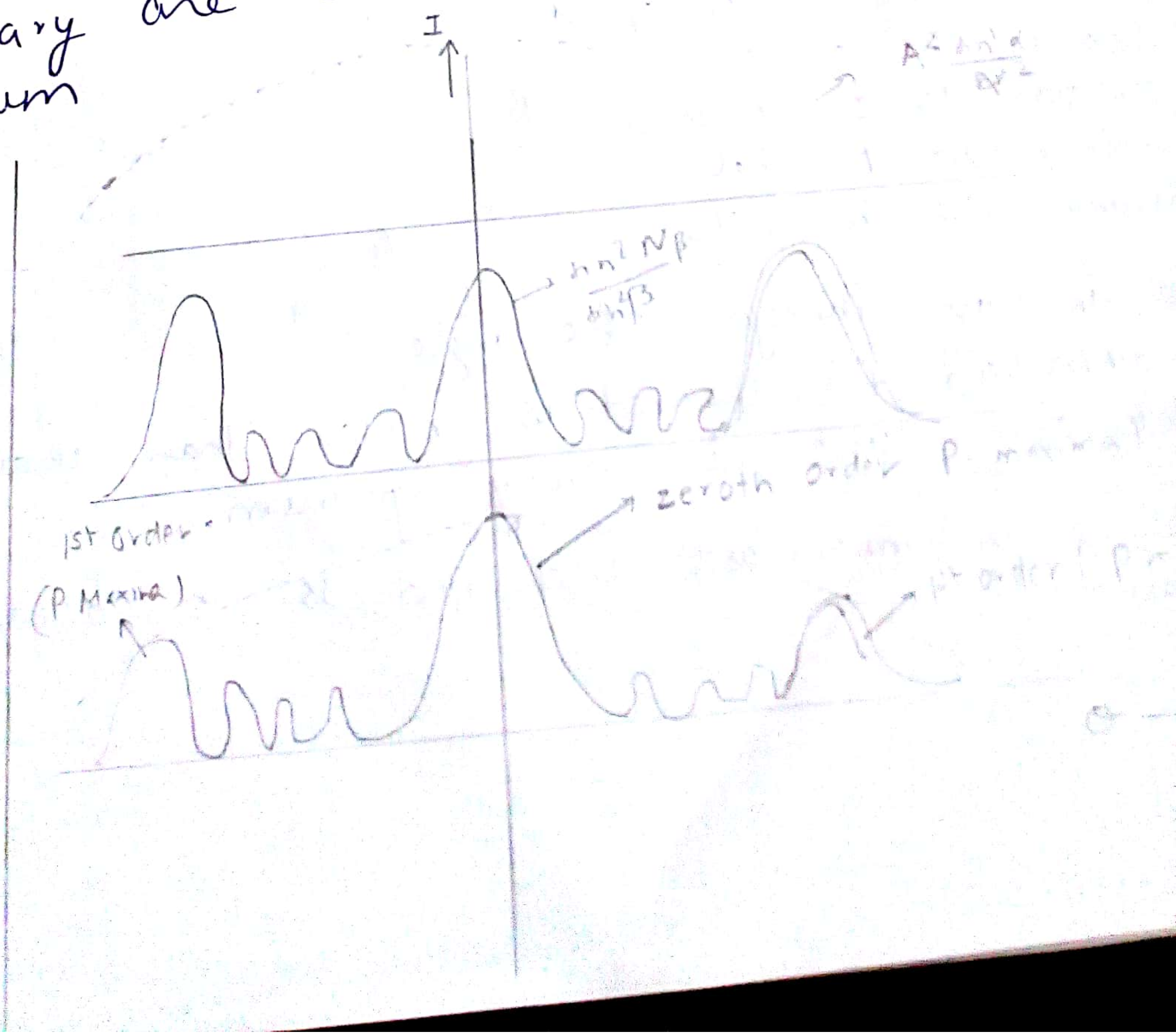
$$\boxed{I = A^2 \frac{\sin^2\alpha}{\alpha^2} N^2}$$

Ratio of $\frac{I'}{I} = \frac{1}{1+(N^2-1)\sin^2\beta}$.

→ This shows that

$$\boxed{I' \propto \frac{1}{N^2}}$$

⇒ Since $I' \propto \frac{1}{N^2}$, and N is very large ~~therefore~~ therfore secondary are not visible in the grating spectrum



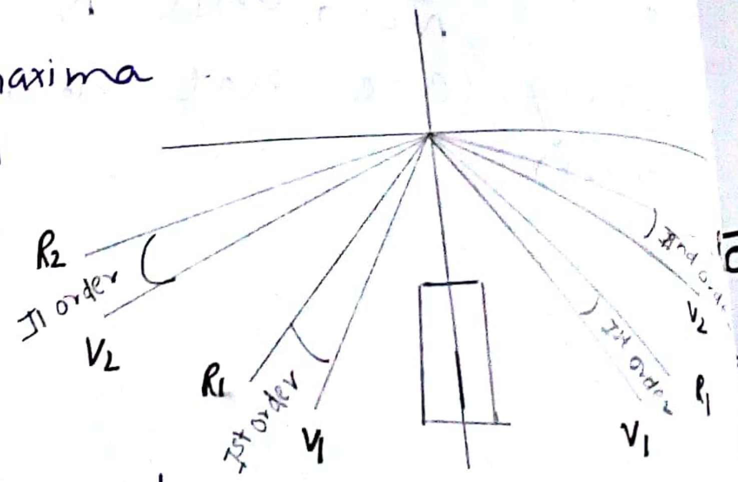
* Formation of Multiple spectra with Grating

→ Suppose the grating is illuminated normally with white light. The direction of principle maxima are $(a+b)\sin\theta = \pm n\lambda$.

→ One can observe :=

- For a particular wavelength, the directions of principle maxima of different order are different.
- For a particular order 'n', the light of different wavelength will be diffracted in different direction.
- For larger wavelength, greater is the angle of diffraction.

* For $n = 0$, $\theta = 0$, for all values of λ



- Thus zero order principle maxima for all the values of λ lies in the same direction. Hence the zero order ($n=0$). Principal maxima will be white.

- Since $\lambda_R > \lambda_V$. Therefor the angle of diffraction for the red is greater than that for violet and hence in every spectrum,

- Violet is in innermost and Red is in outermost position

Absent Spectra with diffraction grating

The condition for the principal maxima for the (n^{th}) order spectrum [also known as grating element],

$$(a+b)\sin\theta = n\lambda$$

Condition for the minima due to single slit is

$$a\sin\theta = m\lambda \qquad m = 1, 2, 3 \ldots\ldots$$

⇒ If both equation are satisfied simultaneously super-position of diffracted rays yield zero. Resultant Intensity and the particular order of the grating spectrum will be absent.

⇒ The above equation suggest that

$$\left(\frac{a+b}{a}\right) = \frac{n}{m}$$

$$\therefore \quad n = \left(\frac{a+b}{a}\right) m$$

CASE-1 :- If $a = b$, then $n = 2m$

⇒ So, 2^{nd}, 4^{th}, 6^{th} …… orders of the spectrum will be missing corresponding to the minima's given by $m = 1, 2, 3, \ldots\ldots$

CASE-2 :- If $b = 2a$, then $n = 3m$

So that 3^{rd}, 6^{th}, 9^{th} … orders of the spectrum will be missing corresponding to the minima given by $m = 1, 2, 3 \ldots\ldots$

~~Maximum number of order~~

* Despersive power of a grating

→ It is defined as the rate of change of angle of diffraction to the change in wavelength is known as dispersive power of grating.

ie $\boxed{\frac{d\theta}{d\lambda}}$

→ The direction of principal maxima are given by $(a+b)\sin\theta = \pm n\lambda$

⇒ Differentiating the above equation

$$(a+b)\cos\theta\, d\theta = n\, d\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} = \frac{n}{(a+b)\sqrt{1-\sin^2\theta}}$$

$$\therefore \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\sqrt{1-\left(\frac{n\lambda}{a+b}\right)^2}} \Rightarrow \frac{1}{\sqrt{\left(\frac{a+b}{n}\right)^2-\lambda^2}}$$

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{\left(\frac{a+b}{n}\right)^2-\lambda^2}}$$

⇒ From the above equation we can see that :-

- Dispersive power is directly proportional to the oder of spectrum.
- Inversely proportional to the grating element
- Inversely proportional to $\cos\theta$.

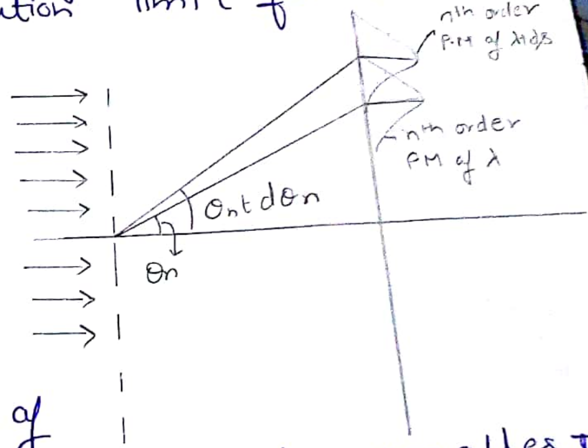
Resolving Power of a Grating

* The capacity of an optical instrument to show two close objects separately is called resolution and the ability of an optical instrument to just resolve images of two close point object is called Resolving power.

angle is known

by

→ The reciprocal of the resolving power is called resolution limit. The resolution limit for a normal human eye is 1.



→ Resolving power of a grating is defined as it's ability to show two neighbouring spectral lines in a spectrum sparatatey.

→ It is given by the ~~notation~~ ratio of wavelength of any spectral line to the smallest wavelength differencebetween neighbouring lines.

i.e $\frac{\lambda}{d\lambda}$

→ For n^{th} order principal maxima of light of wavel-ength (λ) is given by

$$(a+b)\sin\theta = \pm n\lambda$$

or $N(a+b)\sin\theta = \pm Nn\lambda$

→ Let the first minimum adjacent to the n^{th} order maximum be formed in the direction $(\theta_n + d\theta_n)$.

Since the ~~grating~~ direction of minima is given by

$$N(a+b)\sin\theta = \pm m\lambda$$

Therefore $N(a+b)\sin(\theta+d\theta) = \pm(nN+1)\lambda$ — eq (i)

[According [Leave space] : Two spectral lines can be seen separately if the maximum of $\lambda + d\lambda$ falls exactly on the adjacent minimum of wavelength λ]

Q. An harmonic oscillation is represented by the wave function $\psi = (3cm)\sin(0.6x - 2.2t + \pi)$. Determine (i) Amplitude (ii) frequency (iii) wavelength (iv) phase velocity (v) Phase constant

$$\psi(x,t) = A\sin(kx - \omega t \pm \phi)$$

(a) $A = 3cm$

$k = 0.6\ cm^{-1}$, $\omega = 2.2\ rad/sec.$

(b) frequency $= \nu = \frac{\omega}{2\pi} = \frac{2.2}{2\times 3.142} = 0.35\ Hz.$

(c) wavelength $\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\times 3.142}{0.6} \Rightarrow 10.47$

(d) phase velocity, $= v = f\lambda$

$= 0.35 \times 10.47 = 3.66\ cm/s.$

(e) phase constant $(\phi) = \pi$

Q. Two cohrent sources whose intensity ratio is $100:1$ $\Rightarrow$ find the ratio of max. to min intensity. (Q $25:16$ / $16:25$, $9:16$, $16:9$)

Ans $\Rightarrow$ $I_{max} = (a_1+a_2)^2$ & $I_{min} = (a_1-a_2)^2$

$\therefore \frac{I_{max}}{I_{min}} \Rightarrow \frac{(a_1+a_2)^2}{(a_1-a_2)^2} \Rightarrow$

But $\frac{a_1^2}{a_2^2} = \frac{I_1}{I_2} = \frac{100}{1}$

$$\boxed{a_1 = 10a_2}$$

$$\frac{I_{max}}{I_{min}} = \frac{(10a_2 + a_2)^2}{(10a_2 - a_2)^2} = \left(\frac{11}{9}\right)^2 = \frac{121}{81}$$

Q Show that the width of the Newton's rings decreases with order number of rings.

→ The diameters of n^{th} ring dark ring

$$D_n = 2\sqrt{n\lambda R}$$

n →
R →
D_n →

$$\frac{dD_n}{dn} = \frac{2d}{dn}\sqrt{n\lambda R} \Rightarrow \cancel{2}\sqrt{\lambda R} \times \frac{n^{-1/2}}{\cancel{2}}$$

$$\text{"} \Rightarrow 2\sqrt{\lambda R}\,\frac{d\sqrt{n}}{dn}$$

$$\frac{dD_n}{dn} = \sqrt{\frac{\lambda R}{n}}$$

$$\frac{dD_n}{dn} \propto \frac{1}{\sqrt{n}}$$

This shows that, width of the diameter of Newton's Ring decrease with the increase of order no.

Q. A 1 gm particle is subjected to an elastic force of 20 dyne/cm², and a frictional force of 3 dyne/cm². If it is displaced through 2 cm and then relocate. Find whether the resulting motion is oscillater or not. If so find it's period

$$\Rightarrow m\frac{d^2x}{dt^2} + \mu\frac{dx}{dt} + kx = 0$$

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0$$

$$\because 2b = \mu/m$$

$m = 1\,gm$, $k = 20$ unit, $\mu = 3$ unit

$$b = \frac{\mu}{2m} = \frac{3}{2\times1} = 1.5 \text{ unit}$$

$\omega_0 = \sqrt{k/m} = \sqrt{20/4} = 4.47$

$T = \frac{2\pi}{\sqrt{\omega_0^2 - b^2}} \Rightarrow \frac{6.283}{\sqrt{20-2.25}} = 1.495$

Q.> Newtons rings are observed ~~In a~~ normally in the reflected light of ($\lambda = 6000$ Å) Diameter of 10th dark ring is 0.500 m. find the radius of curvature of lens and the thickness of film.

$\Rightarrow$ $D_n^2 = 4n\lambda R$

$(0.5)^2 = 4 \times 10 \times 6000 \times 10^{-8} \times R$

$$\boxed{R = \frac{(0.5)^2}{4 \times 10 \times 6000 \times 10^{-10}}}$$ $\boxed{1.04 \text{ cm}}$

$$t = \frac{r_n^2}{2R} = \frac{(D_n/2)^2}{2R} = \frac{D_n^2}{8R}.$$

$t = 2.99 \times 10^{-4}$ cm

Q. Two cohrent sources of intensity ratio (α) interference. prove that.

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{\alpha}}{1+\alpha}.$$

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Ans. $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \alpha$

$\boxed{a_1 = a_2\sqrt{\alpha}}$

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(a_1+a_2)^2 - (a_1-a_2)^2}{(a_1+a_2)^2 + (a_1-a_2)^2}$$

$$\Rightarrow \frac{2a_1/a_2}{(a_1/a_2)^2+1} = \frac{2\sqrt{\alpha}}{1+\alpha}$$

Q. Same Newton Rings are observed in reflected of $\lambda = 5...$

Q. At time $t=0$, A trains of waves has the form $y = 4\sin 2\pi\left(\frac{x}{100}\right)$. The velocity of wave is 30 cm/sec. Find the equation giving the wave form at a time ($t = 2$ sec)

$\Rightarrow$ we know that, $y = a\sin\frac{2\pi}{\lambda}(vt - x)$

$= a\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$

At $t=0$, $y = a\sin 2\pi\left(\frac{x}{\lambda}\right)$ — (i)

Given eqn, $y = 4\sin 2\pi\left(\frac{x}{100}\right)$ — (2)

$\because$ from (i) & (ii), $\boxed{a=4}$, $\boxed{\lambda = 100}$

$\because$ At, $t = 2$ sec,

$x = vt = 2\times 30 = 60$ cm

Q (a) (b) (c) (d)

Q.

$\therefore y = \ldots \left(\frac{\quad}{100} \ldots \frac{\quad}{100} \right)$

$$\boxed{y = 4\sin 2\pi\left(\frac{x}{100} - 0.6\right)}$$

$\boxed{F = \mu v}$

Due to transmit & bright

Q. What are the sources of damping force.

Loss of energy by radiation of oscillator

(a) Loss of energy by mechanical friction.

(b) Transfer of heat from a layer at ~~of~~ higher temprature to a layer at ~~of~~ lower temprature.

Reflected → Dark

Transmit → Bright

Circular symmetry

(c) Intermolecular exchange of energy

Q. Light containing two wave length λ_1 & λ_2 falls normally on a plano convex lens of Radius of curvature 'R'. If the n^{th} dark ring due to λ_1, concides with $(n+1)^{th}$ dark ring due to λ_2. Show that the radius of n^{th} due to λ_1 is $\sqrt{\frac{\lambda_1\lambda_2 R}{\lambda_1 - \lambda_2}}$

Q. Show with necessary theory. Find out the condition for maxima and minima in N-Ring exp. Also prove that diameter of dark-rings are $\propto$ to square root of natural no.

Q. When seen by transmitted light, why an excessively thin film appears bright in N-