15

A.C. Fundamentals

INTRODUCTION

We have dealt so far with cases in which the currents are steady and in one direction; this is called direct current (d.c.). The use of direct currents is limited to a few applications *e.g.* charging of batteries, electroplating, electric traction *etc*. For large-scale power distribution there are, however, many advantages in using alternating current (a.c.). In an a.c. system, the voltage acting in the circuit changes polarity at regular intervals of time and the resulting current (called alternating current) changes direction accordingly. The a.c. system has offered so many advantages that at present, electrical energy is universally generated, transmitted and used in the form of alternating current. Even when d.c. energy is necessary, it is a common practice to convert a.c. into d.c. by means of rotary converters or rectifiers.

Three principal advantages are claimed for a.c. system over the d.c. system. First, alternating voltages can be stepped up or stepped down efficiently by means of a transformer. This permits the transmission of electric power at high voltages to achieve economy and distribute the power at utilization voltages. Secondly, a.c. motors (induction motors) are cheaper and simpler in construction than d.c. motors. Thirdly, the switchgear (*e.g.* switches, circuit breakers *etc.*) for a.c. system is simpler than the d.c. system. In this chapter, we shall confine our attention to the fundamental alternating currents.

15.1. SINUSOIDAL ALTERNATING VOLTAGE AND CURRENT

Commercial alternators produce sinusoidal alternating voltage *i.e.* alternating voltage is a sine wave. A sinusoidal alternating voltage can be produced (For proof, refer to Art. 15.3) by rotating a coil with a constant angular velocity (say ω rad/sec) in a uniform magnetic field. The sinusoidal alternating voltage can be expressed by the equation:

$$e = E_m \sin \omega t$$

where e = Instantaneous value of alternating voltage

E_m = Maximum value of alternating voltage

ω = Angular velocity of the coil

Sinusoidal voltages always produce sinusoidal currents, unless the circuit is non-linear. Therefore, a sinusoidal current can be expressed in the same way as voltage *i.e.*, $i = I_m \sin \omega t$. Fig. 15.1 (*i*) shows the waveform of sinusoidal voltage whereas Fig. 15.1 (*ii*) shows the waveform of sinusoidal current. Note that sinusoidal voltage or current not only changes direction at regular intervals but the magnitude is also changing continuously.

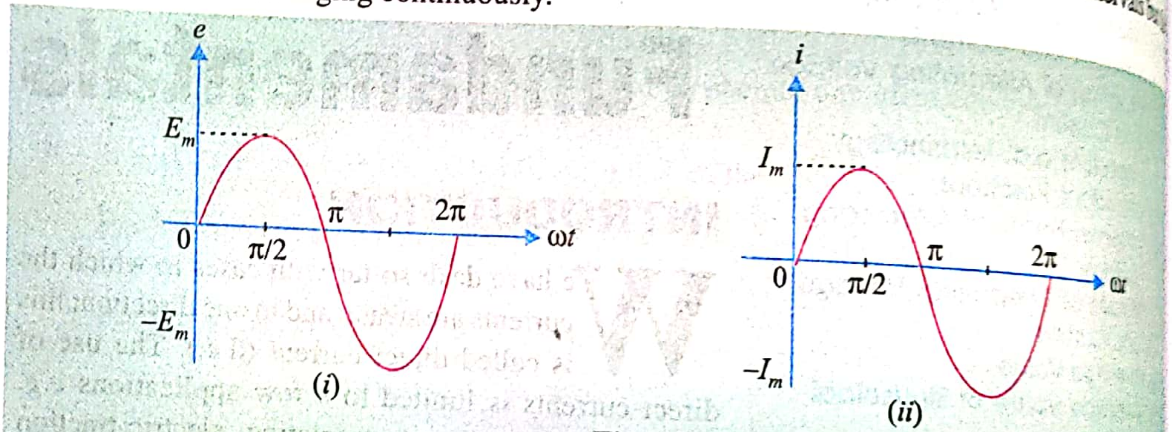


Fig. 15.1

Why Sine Wave form ? Why do we choose sinusoidal wave rather than a simple curve such as a square or triangular wave ? The following are the reasons for doing so :

(*i*) In a.c. machines and transformers, sinusoidal voltages and currents respectively produce the least iron and copper losses for a given output. The efficiency is, therefore, better.

(*ii*) Sinusoidal voltages and currents produce less interference (noise) on telephone lines.

(*iii*) The sine waveform produces the least disturbance in the electrical circuit and is smoothest and efficient waveform.

Due to above advantages, electric supply companies all over the world generate sinusoidal alternating voltages and currents. *It may be noted that alternating voltage and current mean sinusoidal voltage and current unless stated otherwise.*

15.2. GENERATION OF ALTERNATING VOLTAGES AND CURRENTS

An alternating voltage may be generated

(*i*) by rotating a coil at constant angular velocity in a uniform magnetic field as shown in Fig. 15.2.

Or

(*ii*) by rotating a magnetic field at a constant angular velocity within a stationary coil as shown in Fig. 15.3.

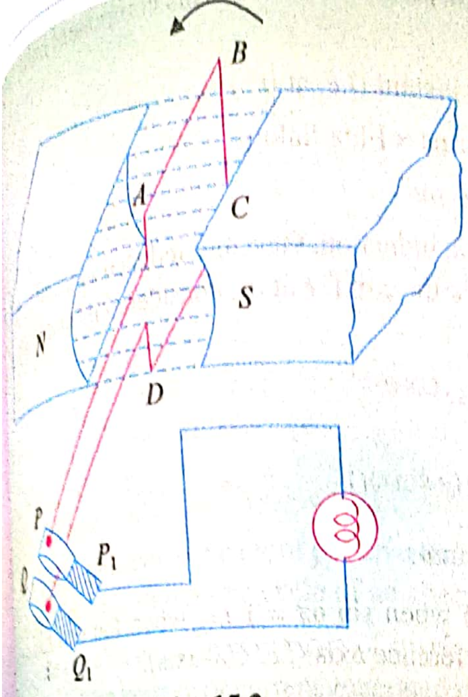


Fig. 15.2

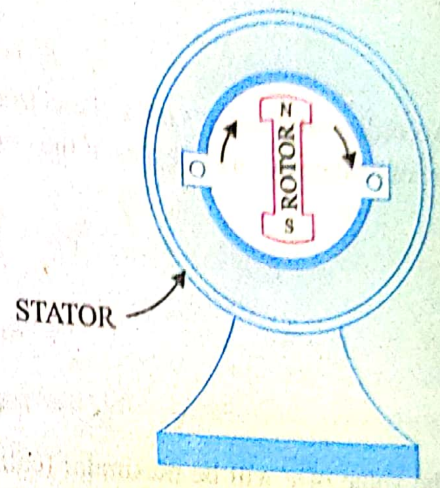


Fig. 15.3

In either case, the generated voltage will be of sinusoidal waveform. The magnitude of generated voltage will depend upon the number of turns of coil, the strength of magnetic field and the speed of rotation. The first method is used for small a.c. generators while the second method is employed for large a.c. generators.

15.3 EQUATION OF ALTERNATING VOLTAGE AND CURRENT

Consider a rectangular coil of n turns rotating in anticlockwise direction with an angular velocity ω rad/sec in a uniform magnetic field as shown in Fig. 15.4. The e.m.f. induced in the coil will be sinusoidal. This can be readily established.

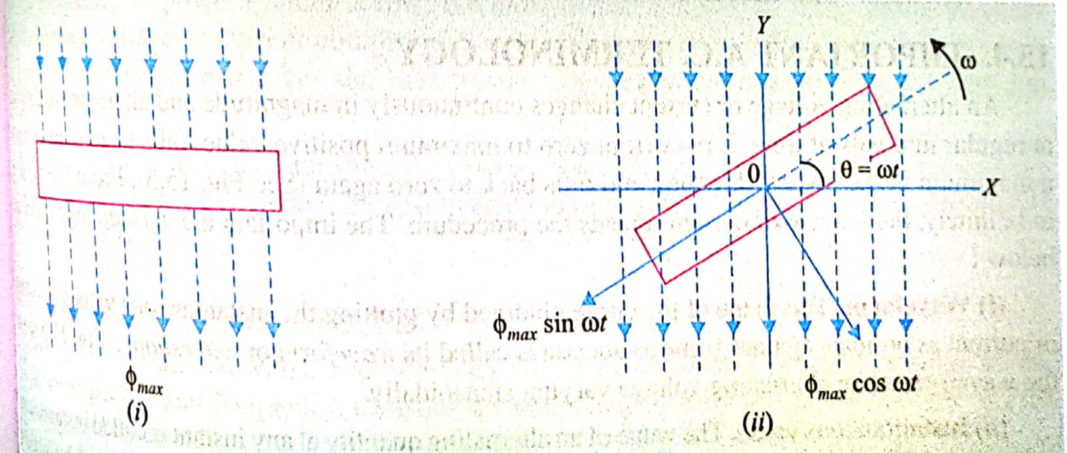


Fig. 15.4

Let the time be measured from the instant the plane of the coil coincides with OX-axis. In this position of the coil [See Fig. 15.4 (*i*)], the flux linking with the coil has its maximum value ϕ_{max}. Let the coil turn through an angle θ (= ωt) in anticlockwise direction in t seconds and assumes the position shown in Fig. 15.4 (*ii*). In this position, the maximum flux ϕ_{max} acting vertically downward can be resolved into two perpendicular components *viz.*

(*i*) component ϕ_{max} sin ωt parallel to the plane of the coil. This component induces *no e.m.f. in the coil.

* A coil moving parallel to the flux has no change of flux linkage with it or there is no "cutting" of flux.

(ii) component $\phi_{max} \cos \omega t$ perpendicular to the plane of the coil. This component induces e.m.f. in the coil.

∴ Flux linkages of the coil at the considered instant (*i.e.* at 0°)

$$= \text{No. of turns} \times \text{Flux linking}$$

$$= n\,\phi_{max} \cos \omega t$$

According to Faraday's laws of electromagnetic induction, the e.m.f. induced in a coil is equal to the rate of change of flux linkages of the coil. Hence the e.m.f. e at the considered instant is given by

$$e = -\frac{d}{dt}(n\phi_{max}\cos\omega t)$$

$$= -n\phi_{max}\omega(-\sin\omega t)$$

$$= n\,\phi_{max}\omega \sin\omega t$$

The value of e will be maximum (call it E_m) when $\sin \omega t = 1$ *i.e.* when the coil has turned through 90° in anticlockwise direction from the reference axis (*i.e.* OX-axis).

$$\therefore \quad E_m = n\,\phi_{max}\omega$$

$$\text{or} \quad e = E_m \sin \omega t = E_m \sin \theta$$

$$\text{where} \quad E_m = n\,\phi_{max}\omega$$

It is clear that e.m.f. induced in the coil is sinusoidal *i.e.* instantaneous value of e.m.f. varies as the sine function of time angle (θ or ωt). If this alternating voltage ($e = E_m \sin \omega t$) is applied across a load, alternating current flows through the circuit which would also vary sinusoidally *i.e.*, following a sine law. The equation of the alternating current is given by :

$i = I_m \sin \omega t$ provided the load is *resistive.

15.4. IMPORTANT A.C. TERMINOLOGY

An alternating voltage or current changes continuously in magnitude and alternates in direction at regular intervals of time. It rises from zero to maximum positive value, falls to zero, increases to a maximum in the reverse direction and falls back to zero again (See Fig. 15.5). From this point on indefinitely, the voltage or current repeats the procedure. The important a.c. terminology is defined below :

(i) Waveform. The shape of the curve obtained by plotting the instantaneous values of voltage or current as ordinate against †time as abcissa is called its *waveform* or *waveshape*. Fig. 15.5 shows the waveform of an alternating voltage varying sinusoidally.

(ii) Instantaneous value. The value of an alternating quantity at any instant is called instantaneous value. The instantaneous values of alternating voltage and current are represented by e and i respectively. As an example, the instantaneous values of voltage (See Fig. 15.5) at 0°, 90° and 270° are 0, $+E_m$, $-E_m$ respectively.

* It will be shown later that if the load is inductive or capacitive, the current equation is changed in time angle.

† We know $\theta = \omega t$. Since ω is constant, $\theta \propto t$. Hence we may take time t or θ or ωt along X-axis.

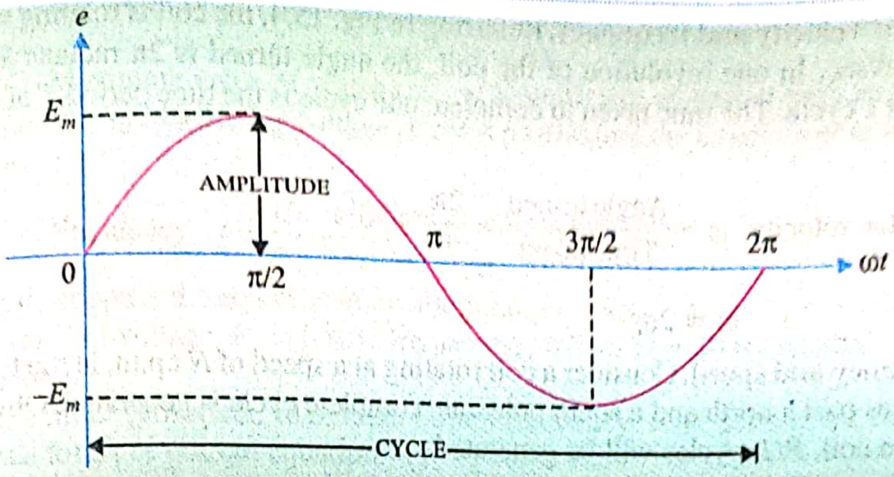


Fig. 15.5

(iii) **Cycle.** One complete set of positive and negative values of an alternating quantity is known as a cycle. Fig. 15.5 shows one cycle of an alternating voltage.

A cycle can also be defined in terms of angular measure. One cycle corresponds to 360° *electrical or 2π radians. The voltage or current generated in a conductor will span 360° electrical (or complete one cycle) when the conductor moves past a north and a south pole.

(iv) **Alternation.** One-half cycle of an alternating quantity is called an alternation. An alternation spans 180° electrical. Thus in Fig. 15.5, the positive or negative half of alternating voltage is the alternation.

(v) **Time period.** The time taken in seconds to complete one cycle of an alternating quantity is called its time period. It is generally represented by T.

(vi) **Frequency.** The number of cycles that occur in one second is called the frequency (f) of the alternating quantity. It is measured in cycles/sec (*c/s*) or hertz (*Hz*). One hertz is equal to 1 c/s.

The frequency of power system is low; the most common being 50 *c/s* or 50Hz. It means that alternating voltage or current completes 50 cycles in one second. The 50 Hz frequency is the most popular because it gives the best results when used for operating both lights and machinery.

(vii) **Amplitude.** The maximum value (positive or negative) attained by an alternating quantity is called its amplitude or peak value. The amplitude of an alternating voltage or current is designated by E_m (or V_m) or I_m.

15.5. IMPORTANT RELATIONS

Having become familiar with a.c. terminology, we shall now establish some important relations.

(i) **Time period and frequency.** Consider an alternating quantity having a frequency of f *c/s* and time period T second.

Time taken to complete f cycles = 1 second (By definition)

Time taken to complete 1 cycle = $1/f$ second

But time taken to complete one cycle is the time period T (by definition).

$$\therefore \quad T = \frac{1}{f} \quad or \quad f = \frac{1}{T}$$

* The time required to generate one cycle is divided into 360 divisions called electrical degrees. They differ from mechanical degrees. In one revolution, the coil will always traverse 360°mechanical. However, the electrical degrees spanned will depend upon the number of poles. For a 2-pole generator, electrical degrees spanned in one revolution of the coil are 360° *i.e.*, one cycle is generated. For a 4-pole generator, 2 cycles (*i.e.*720°) will be generated for one revolution of the coil.

(ii) Angular velocity and frequency. Referring to Fig. 15.4, the coil is rotating with an angular velocity of ω rad/sec. In one revolution of the coil, the angle turned is 2π radians and the voltage wave completes 1 cycle. The time taken to complete one cycle is the time period T of the alternating voltage.

$$\therefore \quad \text{Angular velocity, } \omega = \frac{\text{Angle turned}}{\text{Time taken}} = \frac{2\pi}{T}$$

$$\text{or} \qquad \omega = 2\pi f \qquad (\because f = 1/T)$$

(iii) Frequency and speed. Consider a coil rotating at a speed of N r.p.m. in the field of P poles. As the coil moves past a north and a south pole, one complete cycle is generated. Obviously, in one revolution of the coil, $P/2$ cycles will be generated.

Now, Frequency, f = No. of cycles/sec

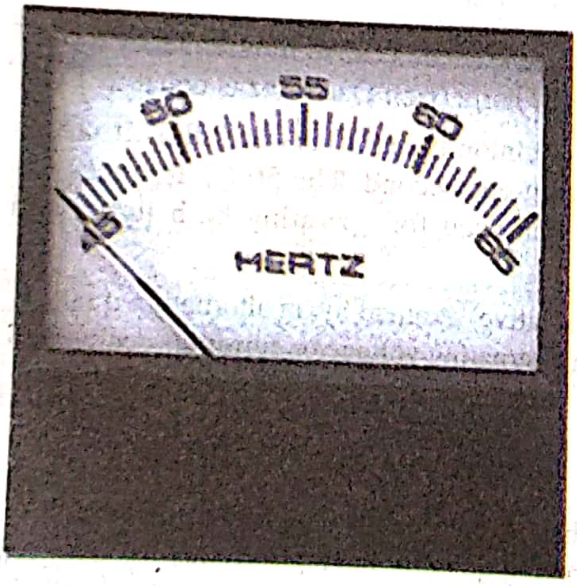
= (No. of cycles/revolution) × (No. of revolutions/sec)

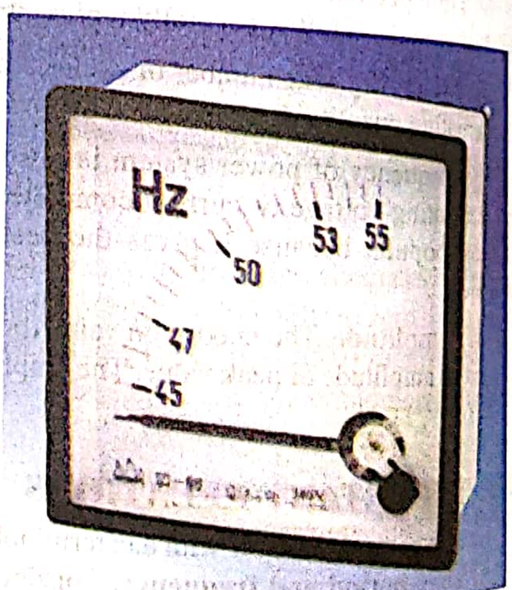
$$= \left(\frac{P}{2}\right) \times \left(\frac{N}{60}\right) = \frac{PN}{120}$$

$$\therefore \qquad f = \frac{PN}{120}$$

For example, an a.c. generator having 10 poles and running at 600 r.p.m. will generate alternating voltage and current whose frequency is :

$$f = \frac{PN}{120} = \frac{10 \times 600}{120} = 50\,\text{Hz}$$





Frequency Meters

15.6. DIFFERENT FORMS OF ALTERNATING VOLTAGE

The standard form of an alternating voltage is given by :

$$e = E_m \sin\theta = E_m \sin\omega t$$

$$= E_m \sin 2\pi f t = E_m \sin\frac{2\pi}{T}t$$

Which of the above form of equations is to be used will depend upon the given data. The following points may be noted carefully:

(i) *The maximum value of alternating voltage is given by the co-efficient of sine of the time angle i.e.*

Maximum value of voltage, E_m = Co-efficient of sine of time angle

(ii) *The frequency f of alternating voltage is given by dividing the co-efficient of time in the angle by 2π i.e.*

$$\text{Frequency, } f = \frac{\text{Co-efficient of time in the angle}}{2\pi}$$

For example, suppose the equation of an alternating voltage is given by $e = 100 \sin 314t$. Then the maximum value of voltage, $E_m = 100$ V; frequency, $f = 314/2\pi = 50$ Hz and time period $T = 1/f = 1/50 = 0.02$ second.

Following similar procedure, maximum value, frequency, time period *etc.* can be found out from the various forms of current equation.

Example 15.1. *A rectangular coil measuring 30 cm by 20 cm and having 40 turns is rotated about an axis coinciding with one of its longer sides at a speed of 1500 r.p.m. in a uniform magnetic field of flux density 0.075 Wb/m². Calculate (i) the frequency of the e.m.f. (ii) maximum value of e.m.f. and (iii) the value of e.m.f. when the coil plane makes an angle of 45° with the field direction.*

Solution. The statement of the problem suggests that the number of poles is 2.

(i) Frequency, $f = \dfrac{NP}{120} = \dfrac{1500 \times 2}{120} = \mathbf{25\ Hz}$

(ii) As proved in Art. 15.3,

Maximum e.m.f., $E_m = n\,\phi_{max}\,\omega$

Here $\phi_{max} = B \times A = 0.075 \times (30 \times 20 \times 10^{-4}) = 0.0045\text{Wb}$

$\omega = 2\pi f = 2\pi \times 25 = 157 \text{ rad./sec}$

$\therefore$ $E_m = 40 \times (0.0045) \times 157 = \mathbf{28.26\ V}$

(iii) When the coil plane makes an angle of 45° with the direction of field,

$$e = E_m \sin\theta = 28.26 \sin 45° = \mathbf{19.98V}$$

Example 15.2. *An alternating current i is given by :*

$$i = 141.4 \sin 314t$$

Find (i) the maximum value (ii) frequency (iii) time period and (iv) the instantaneous value when t is 3ms.

Solution. Comparing the given equation of alternating current with the standard form $i = I_m \sin \omega t$, we have,

(i) Maximum value, $I_m = \mathbf{141.4A}$

(ii) Frequency, $f = \omega/2\pi = 314/2\pi = \mathbf{50\ Hz}$

(iii) Time period, $T = 1/f = 1/50 = \mathbf{0.02s}$

(iv) $i = 141.4 \sin 314t$

When $t = 3 \text{ ms} = 3 \times 10^{-3}\text{s}$

$i = 141.4 \sin 314 \times 3 \times 10^{-3} = \mathbf{114.35\ V}$

Example 15.3. *An alternating current of frequency 60Hz has a maximum value of 120 A.*

(i) *Write down the equation for the instantaneous value.*

(ii) *Reckoning time from the instant the current is zero and becoming positive, find the instantaneous value after 1/360 second.*

(iii) *Time taken to reach 96A for the first time.*

Solution.

(i) The instantaneous value of current is given by ;

$$i = I_m \sin \omega t$$

$= I_m \sin 2\pi ft = 120 \sin 2\pi \times 60 \times t$

$\therefore$ $i = 120 \sin 120\,\pi t$

(*ii*) Fig. 15.6 shows the wave form of the given alternating current. Since point *O* has been taken as the reference, the current equation is :

$i = 120 \sin 120\,\pi t$

When $t = 1/360$ second, then,

$i = 120 \sin 120\pi \times 1/360$

$= 120 \sin (\pi/3) = 103.92\text{A}$

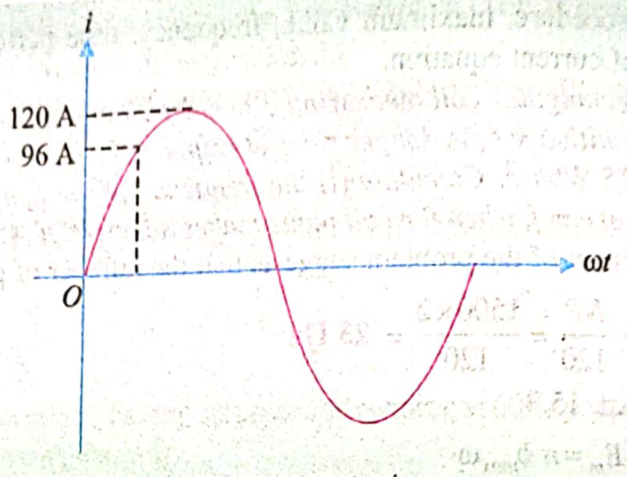


Fig. 15.6

(*iii*) Suppose the current becomes 96A for the first time after *t* second as shown in Fig. 15.6

Then, $96 = 120 \sin 120\pi t$

or $\sin 120\,\pi t = 96/120 = 0.8$

or $120\pi t = \sin^{-1} 0.8 = 0.927$ rad.

$\therefore$ $t = \dfrac{0.927}{120\times\pi} = 0.00246$ s

Example 15.4. *A sinusoidal voltage of 50Hz has a maximum value of $200\sqrt{2}$ volts. At what time measured from a positive maximum value will the instantaneous voltage be 141.4 volts?*

Solution. Fig. 15.7 shows the voltage wave-form. The equation of this wave with point *O* as the reference is given by :

$e = E_m \sin \omega t = E_m \sin 2\pi ft = 200\sqrt{2} \sin 2\pi \times 50 \times t$

$\therefore$ $e = 282.8 \sin 314\,t$

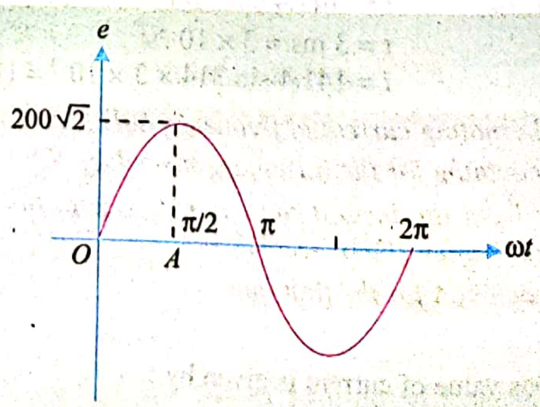


Fig. 15.7

This equation is valid when time is measured from the instant the voltage is zero *i.e.* point *O*. Since the time is measured from the positive maximum value (point *A* in Fig. 15.7), the above equation is modified to :

$$e = 282.8 \sin (314t + \pi/2) = 282.8 \cos 314t$$

Let the value of voltage become 141.4 volts *t* second after passing through the maximum positive value. Then,

$$141.4 = 282.8 \cos 314t$$

or $\cos 314t = 141.4/282.8 = 0.5$

or $314t = \cos^{-1} 0.5 = 1.047 \text{ rad}$

∴ $t = 1.047/314 = 3.33 \times 10^{-3} \text{ s} = \mathbf{3.33 \text{ ms}}$

Example 15.5. *An alternating current of frequency 50Hz has a maximum value of 100A. Calculate (i) its value 1/600 second after the instant the current is zero and its value is decreasing there afterwards (ii) how many seconds after the current is zero and then increasing will attain the value of 86.6A ?*

Solution. Fig. 15.8 shows the current waveform. With *O* as the origin, the equation of the current is :

$$i = I_m \sin \omega t = 100 \sin 2\pi \times 50 \times t$$

∴ $i = 100 \sin 100\pi t$

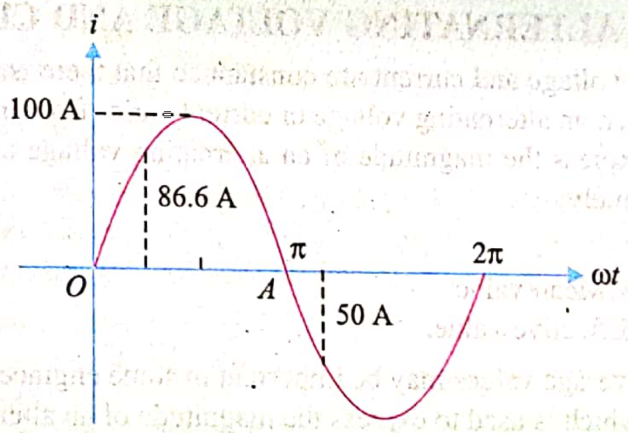


Fig. 15.8

(*i*) Since current is measured from the instant the current is zero and is decreasing there afterwards (point *A* in Fig. 15.8), the equation of the current with point *A* as the origin becomes:

$$i = 100 \sin (100\pi t + \pi) = -100 \sin 100\pi t$$

When $t = 1/600$ second, then,

$$i = -100 \sin 100 \times 180° \times \frac{1}{600} = \mathbf{-50A}$$

(*ii*) When current is measured from the instant the current is zero and is increasing thereafter (*i.e.* point *O* in Fig. 15.8), the current equation is given by :

$$i = 100 \sin 100\pi\, t$$

or $86.6 = 100 \sin 100\pi\, t$

or $\sin 100\pi\, t = 86.6/100 = 0.866$

or $100\pi\, t = \sin^{-1} 0.866 = 1.047 \text{ rad}$

∴ $t = 1.047/100\pi = 3.33 \times 10^{-3}\text{s} = \mathbf{3.33 \text{ ms}}$

TUTORIAL PROBLEMS

1. An alternating voltage is represented by :

$$e = 141.4 \sin 377\, t$$

Find (*i*) the maximum value (*ii*) frequency (*iii*) time period and (*iv*) the instantaneous value of voltage when t is 3 ms. [(*i*) 141.4 V (*ii*) 60 Hz (*iii*) 16.67 ms (*iv*) 127

2. An alternating current of frequency 50 Hz has a maximum value of $200\sqrt{2}$ A. Reckoning time from the instant the current is zero and becoming positive, find the time taken by the current to reach a value of 141.4 A for a first and second time. [1.67 ms; 21.67

3. An alternating current takes 3.375 ms to reach 15 A for the first time after becoming instantaneously zero. The frequency of current is 40 Hz. Find the maximum value of alternating current. [2

4. A 50 Hz sinusoidal voltage has a maximum value of 56.56V. Find the value of voltage 0.0 second after passing through maximum positive value. At what time measured from a positive maximum value will instantaneous voltage be 14.14V ? [40V; 4.2

5. An alternating current of frequency 50Hz has a maximum value of 100A. Calculate its value 1/300 second after the instant the current is zero and its value is decreasing thereafter. [− 86.6

15.7. VALUES OF ALTERNATING VOLTAGE AND CURRENT

In a d.c. system, the voltage and current are constant so that there is no problem of specifying their magnitudes. However, an alternating voltage or current varies from instant to instant. A natural question arises how to express the magnitude of an alternating voltage or current. There are three ways of expressing it, namely;

(*i*) Peak value

(*ii*) Average value or Mean value

(*iii*) R.M.S value or Effective value.

Although peak and average values may be important in some engineering applications, it is the r.m.s. or effective value which is used to express the magnitude of an alternating voltage or current.

15.8. AVERAGE VALUE

The arithmetical average of all the values of an alternating quantity over one cycle is called its average value *i.e.*

$$\text{Average value} = \frac{\text{Area under the curve}}{\text{Base}}$$

(*i*) In case of *symmetrical waves (*e.g.* sinusoidal voltage or current), the average value over one cycle is zero. It is because positive half is exactly equal to the negative half cycle so that net area is zero. However, the average value of positive or negative half is not zero. Hence in case of symmetrical waves, average value means the average value of half-cycle or one alternation.

$$\text{Average value of a symmetrical wave} = \frac{\text{Area of one alternation}}{\text{Base length of one alternation}}$$

$$= \frac{\text{Sum of **mid-ordinates over one alternation}}{\text{No. of mid-ordinates}}$$

* A symmetrical wave is one which has positive half-cycle exactly equal to the negative half-cycle.

** Suppose positive half-cycle is divided into n equal parts. The middle value of each part is the mid-ordinate. The average value will be the sum of mid-ordinates divided by the number of mid-ordinates (*i.e.* n in this case).

(ii) In case of unsymmetrical waves (e.g. half-wave rectified voltage etc.), the average value is taken over the full cycle.

$$\text{Average value of an unsymmetrical wave} = \frac{\text{Area over one cycle}}{\text{Base length of 1 cycle}}$$

15.9. AVERAGE VALUE OF SINUSOIDAL CURRENT

The equation of an alternating current varying sinusoidally is given by :

$$i = I_m \sin\theta$$

Consider an elementary strip of width $d\theta$ in the first half-cycle of current wave as shown in Fig. 15.9. Let i be the mid-ordinate of this strip.

Area of strip $= i d\theta$

Area of half-cycle

$$= \int_0^\pi i\,d\theta = \int_0^\pi I_m \sin\theta\,d\theta = I_m[-\cos\theta]_0^\pi = 2I_m.$$

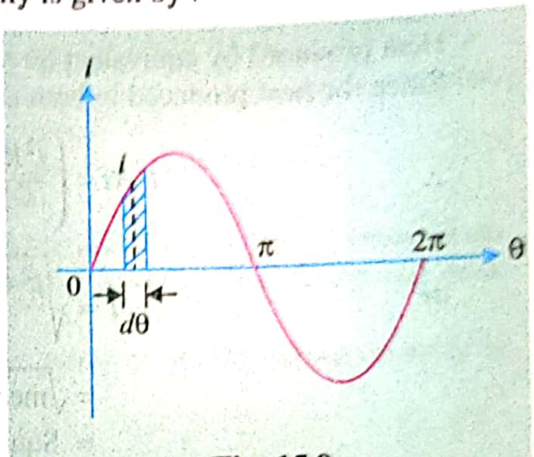


Fig. 15.9

∴ Average value,

$$I_{av} = \frac{\text{Area of half-cycle}}{\text{Base length of half-cycle}} = \frac{2I_m}{\pi}$$

or $\quad$ $\mathbf{I_{av} = 0.637\ I_m}$

Similarly, it can be proved that for alternating voltage varying sinusoidally, $E_{av} = 0.637\ E_m$.

15.10. R.M.S. OR EFFECTIVE VALUE

The **effective** *or* **r.m.s. value** *of an alternating current is that steady current (d.c.) which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current when flowing through the same resistance for the same time.*

For example, when we say that the r.m.s. or effective value of an alternating current is 5A, it means that the alternating current will do work (or produce heat) at the same rate as 5A direct current under similar conditions.

Illustration. The r.m.s. or effective value of an alternating current (or voltage) can be determined as follows. Consider the half-cycle of a non-sinusoidal alternating current i [See Fig. 15.10 (i)] flowing through a resistance $R\Omega$ for t seconds. Divide the time t in n equal intervals of time, each of duration t/n second. Let the mid-ordinates be $i_1, i_2, i_3, \ldots\ldots, i_n$. Each current $i_1, i_2, i_3, \ldots\ldots, i_n$ will produce heating effect when passed through the resistance R as shown in Fig. 15.10 (ii). Suppose the heating effect produced by current i in R is the same as produced by some direct current I flowing through the resistance R for the same time t seconds. Then direct current I is the r.m.s. or effective value of alternating current i.

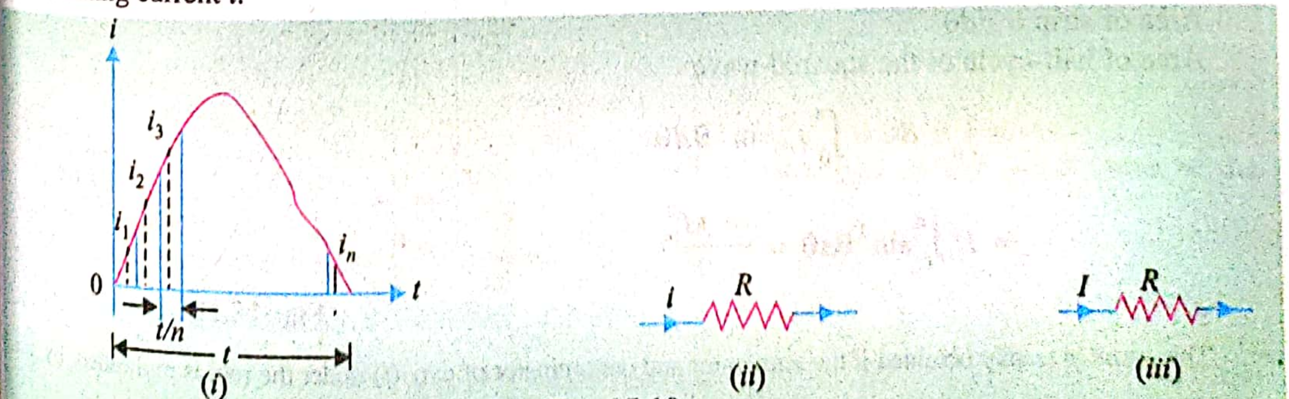


Fig. 15.10

The heating effect of various components of alternating current will be $i_1^2\,Rt/n$, $i_2^2\,Rt/n$,... joules. Since the alternating current is varying, the heating effect will also vary.

Total heat produced by alternating current i is

$$= \left(i_1^2 R + i_2^2 R + i_3^2\, R + ... + i_n^2\, R\right) t/n$$

$$= \left(\frac{i_1^2 R + i_2^2 R + i_3^2\, R + ... + i_n^2\, R}{n}\right) t \text{ joules}$$

Heat produced by equivalent direct current $I = I^2 Rt$ joules

Since the heat produced in both cases is the same,

$$\therefore \qquad I^2 Rt = \left(\frac{i_1^2 R + i_2^2 R + i_3^2\, R + ... + i_n^2\, R}{n}\right) t$$

or

$$I = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + ... + i_n^2}{n}}$$

$$= \sqrt{\text{mean value of } i^2}$$

= Square **root** of the **mean** of the **squares** of the current

= root-mean-square (r.m.s.) value

The r.m.s. or effective value of alternating voltage can similarly be expressed as :

$$E = \sqrt{\frac{e_1^2 + e_2^2 + e_3^2 + ... + e_n^2}{n}}$$

(*i*) For symmetrical waves, the r.m.s. or effective value can be found by considering half-cycle or full-cycle. However, for unsymmetrical waves, full-cycle should be considered.

(*ii*) The r.m.s. value of a wave can also be expressed as :

$$^{*}\text{R.M.S. value} = \sqrt{\frac{\text{Area of half - cycle wave squared}}{\text{Half - cycle base}}}$$

15.11. R.M.S. VALUE OF SINUSOIDAL CURRENT

The equation of the alternating current varying sinusoidally is given by :

$$i = I_m \sin\theta$$

Consider an elementary strip of width $d\theta$ in the first half-cycle of the squared current wave (shown dotted in Fig. 15.11). Let i^2 be the mid-ordinate of this strip.

Area of strip = $i^2 d\theta$.

Area of half-cycle of the squared wave

$$= \int_0^\pi i^2 d\theta = \int_0^\pi I_m^2 \sin^2\theta\, d\theta$$

$$= I_m^2 \int_0^\pi \sin^2\theta d\theta = {}^{**}\frac{\pi I_m^2}{2}$$

$\therefore$

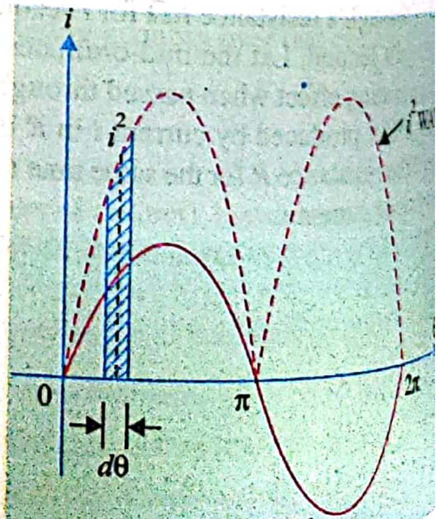


Fig. 15.11

* This result is readily obtained if the numerator and denominator of exp. (*i*) under the root is multiplied by

** $\int_0^\pi \sin^2\theta\, d\theta = \int_0^\pi \frac{1-\cos 2\theta}{2} d\theta = \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi = \frac{\pi}{2}$

$$I_{r.m.s.} = \sqrt{\frac{\text{Area of half - cycle squared wave}}{\text{Half - cycle base}}}$$

$$= \sqrt{\frac{\pi I_m^2/2}{\pi}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

$\therefore$ $\mathbf{I_{r.m.s.} = 0.707\ I_m}$

Similarly, it can be proved that for alternating voltage varying sinusoidally, $E_{r.m.s.} = 0.707 E_m$.

In a.c. circuits, voltages and currents are normally given in r.m.s. values unless stated otherwise. A.C. ammeters and voltmeters are callibrated to record r.m.s. values.

15.12. FORM FACTOR AND PEAK FACTOR

There exists a definite relation among the peak value, average value and r.m.s. value of an alternating quantity. The relationship is expressed by two factors, namely; form factor and peak factor.

(*i*) **Form factor.** The ratio of r.m.s. value to the average value of an alternating quantity is known as form factor *i.e.*

$$\text{Form factor} = \frac{\text{R.M.S. value}}{\text{Average value}}$$

For a sinusoidal voltage or current,

$$\text{Form factor} = \frac{0.707 \times \text{Max. value}}{0.637 \times \text{Max. value}} = 1.11$$

The form factor is useful in rectifier service because it enables us to find the r.m.s. value from average value and *vice-versa.*

(*ii*) **Peak factor.** The ratio of maximum value to the r.m.s. value of an alternating quantity is known as peak factor *i.e.* $\text{Peak factor} = \frac{\text{Max. value}}{\text{R.M.S. value}}$

For a sinusoidal voltage or current : $\text{Peak factor} = \frac{\text{Max. value}}{0.707 \times \text{Max. value}} = 1.414$

The peak factor is of much greater importance because it indicates the maximum voltage being applied to the various parts of the apparatus. For instance, when an alternating voltage is applied across a cable or capacitor, the breakdown of insulation will depend upon the maximum voltage. The insulation must be able to withstand the maximum rather than the r.m.s. value of voltage.

Example 15.6. *An alternating current, when passed through a resistor immersed in water for 5 minutes, just raised the temperature of water to boiling point. When a direct current of 4A was passed through the same resistor under indentical conditions, it took 8 minutes to boil the water. Find the r.m.s. value of the alternating current.*

Solution. Let $I_{r.m.s.}$ amperes be the r.m.s. value of the alternating current and R ohms be the value of the resistor.

Heat produced by the alternating current

$$= I_{r.m.s.}^2 \times R \times t$$

$$= I_{r.m.s.}^2 \times R \times 5 \times 60 \text{ joules} \qquad ...(i)$$

TUTORIAL PROBLEMS

1. An alternating voltage $e = 100 \sin 314t$ is applied to a device which offers an ohmic resistance of 50 Ω in one direction while entirely preventing the flow of current in the opposite direction. Calculate the average and r.m.s. values of current. Also find the form factor. [0.637A ; 1A ; 1.57]

2. A hot-wire ammeter, a moving coil ammeter and a rectifier are connected in series. The combination is connected across a sinusoidal source of 50V r.m.s. The forward resistance of rectifier is 20Ω and the reverse resistance is infinite. Calculate the reading on each ammeter. [1.767A ; 1.124A]

3. The equation of an alternating current is given by $i = 141.4 \sin 314t$. Find (*i*) r.m.s. current (*ii*) frequency and (*iii*) instantaneous value of current when t is 3.6*ms*. [(*i*)100A (*ii*) 50Hz (*iii*) 128A]

4. A current has the following steady values (in amperes) for equal intervals of time, changing instantaneously from one value to the next.
0, 10, 20, 30, 20, 10, 0, –10, –20, –30, –20, –10 etc.
Calculate the r.m.s. value of current and form factor. [17.8A ; 1.18]

15.13. PHASE

Waves of alternating voltage and current are continuous. They do not stop after one cycle is completed but continue to repeat as long as the generator is operating. Consider an alternating voltage wave of time period T second as shown in Fig. 15.15. Note that the time is counted from the instant the voltage is zero and becoming positive. The maximum positive value ($+V_m$) occurs at $T/4$ second or $\pi/2$ radians. We say that *phase* of maximum positive value is $T/4$ second or $\pi/2$ radians. It means that as the fresh cycle starts, $+V_m$ will occur at $T/4$ second or $\pi/2$ radians. Similarly, the phase of negative peak ($-V_m$) is $3T/4$ second or $3\pi/2$ radians.

Hence **phase** *of a particular value of an alternating quantity is the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.*

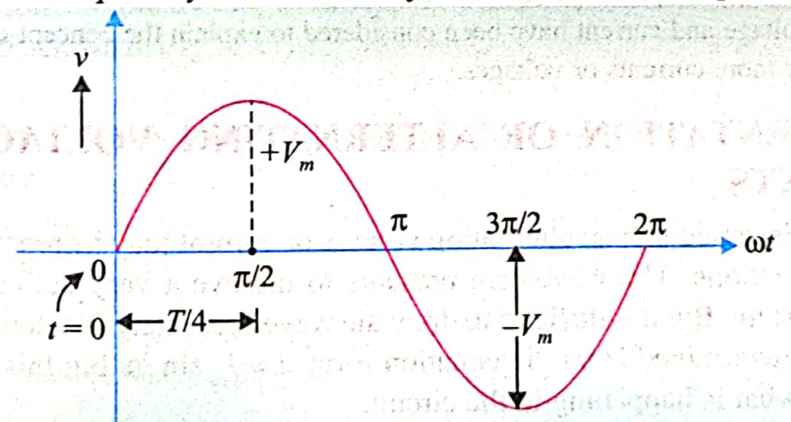


Fig. 15.15

In electrical engineering, we are more concerned with relative phases or phase difference between different alternating quantities rather than with their absolute values.

15.14. PHASE DIFFERENCE

When an alternating voltage is applied to a circuit, an alternating current of the same frequency flows through the circuit. In most of practical circuits, for reasons we will discuss later, voltage and current have different phases. In other words, they do not pass through a particular point, say *zero

* We may select any point but it is more convenient to determine the phase difference by considering zero points or positive maximum values or negative maximum values of the two alternating quantities.

point in the same direction at the same instant. Thus voltage may be passing through its zero point while the current has passed or it is yet to pass through its zero point in the same direction. We say voltage and current have a phase difference.

Hence when two alternating quantities of the same frequency have different zero points, they are said to have a **phase difference.**

The angle between zero points is the angle of phase difference ϕ. It is generally measured in degrees or radians. The quantity which passes through its zero point earlier is said to be leading w.r.t. the other is said to be lagging. It should be noted that those zero points of alternating quantities are to be considered where they pass in the same direction. Thus if voltage has passed through its zero point and is rising in the positive direction, then zero point considered for the current should have similar situation. Since both alternating quantities have the same frequency, the phase difference between them remains the same.

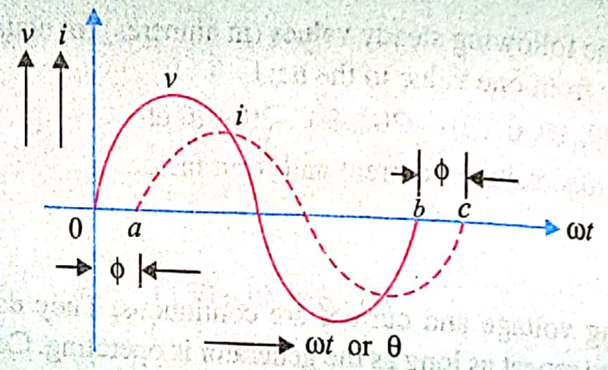


Fig. 15.16

Consider an a.c. circuit in which current i lags behind the voltage v by $\phi°$. This phase relationship is shown by waves in Fig. 15.16. The equations of voltage and current are :

$$v = V_m \sin \omega t$$

$$i = I_m \sin (\omega t - \phi)$$

It is a usual practice to express ωt in radians and ϕ in degrees.

Note. Although voltage and current have been considered to explain the concept of phase difference, it is equally valid for two or more currents or voltages.

15.15. REPRESENTATION OF ALTERNATING VOLTAGES AND CURRENTS

So far we have discussed that an alternating voltage or current may be represented in the form of (*i*) waves and (*ii*) equations. The waveform presents to the eye a very definite picture of what is happening at every instant. But it is difficult to draw the wave accurately. No doubt the current flowing at any instant can be determined from the equation form $i = I_m \sin \omega t$ but this equation presents no picture to the eye of what is happening in the circuit.

The above difficulty has been overcome by representing sinusoidal alternating voltage or current by a line of definite length rotating in *anticlockwise direction at a constant angular velocity (ω). Such a rotating line is called a **phasor**. The length of the phasor is taken equal to the maximum value (on suitable scale) of the alternating quantity and angular velocity equal to the angular velocity of the alternating quantity. As we shall see presently, this phasor (*i.e.*, rotating line) will generate a sine wave.

15.16. PHASOR REPRESENTATION OF SINUSOIDAL QUANTITIES

Consider an alternating current represented by the equation $i = I_m \sin \omega t$. Take a line OP to

* It is a standard convention that the phasor is rotated anticlockwise—a convention that is in harmony with the general use of polar co-ordinates.