

SAMPLE SOLUTION
Sub: AFL
Sub Code: CS2010

SECTION-A(Answer All Questions. All questions carry 2 Marks)

Time:20 Minutes
(5×2=10 Marks)

<u>Ques tion No</u>	<u>Question Type(M CQ/SAT)</u>	<u>Question</u>	<u>Answer Key(if MCQ)</u>	<u>CO Mapping</u>
<u>Q.No :1(a)</u>	<u>MCQ</u>	Consider the language L given by the regular expression $(a + b)^*b(a + b)$ over the alphabet {a, b}. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is _____. (A) 6 (B) 5 (C) 4 (D) 3	C) 4	
	<u>MCQ</u>	The number of states in a minimal deterministic finite automaton corresponding to the language $L = \{ a^n \mid n \geq 4 \}$ is (A) 6 (B) 5 (C) 4 (D) 3	B) 5	
	<u>MCQ</u>	Number of final and non-final states in the minimal DFA which accepts the language $L = \{ w \text{ has odd length and exactly 1 } b \text{ for } \Sigma = \{a, b\} \}$ are respectively A) 3, 1 B) 1, 3 C) 1, 4 D) 4, 1	C) 1, 4	
	<u>MCQ</u>	The minimal number of states required to construct a DFA for set of all strings of length at most 5 ($ w \leq 5$) will be. A) 5	C) 7	

		B)6 C)7 D)None of the above		
<u>Q.No :1(b)</u>		<p>Consider the following regular expressions.</p> <p>$r_1 = a^*$ $r_2 = \emptyset$ $r_3 = \epsilon$</p> <p>Let $r_4 = (r_1.r_2)^*$ and $r_5 = (r_2+r_3)$ Select the correct option.</p> <p>A) $r_4 = r_5$ B) $L(r_4)$ contains infinite strings while $L(r_5)$ contains finite strings. C) $L(r_5) \subset L(r_4)$ D) $L(r_4) \subset L(r_5)$</p>	$A)r_4 = r_5$	
		<p>Let $R=(1+0)^*1$, $S=11^*01$ and $T=1^*01$ be three regular expressions.</p> <p>Then which one of the following is true?</p> <p>A. $L(T) \subseteq L(S)$ and $L(T) \subseteq L(R)$ B. $L(S) \subseteq L(R)$ and $L(R) \subseteq L(T)$ C. $L(S) \subseteq L(T)$ and $L(T) \subseteq L(R)$ D. $L(T) \subseteq L(S)$ and $L(R) \subseteq L(T)$</p>	C)$L(S) \subseteq L(T)$ and $L(T) \subseteq L(R)$	
		<p>Consider the following regular expressions:</p> <p>i. $(ab)^*a = a(ba)^*$ ii. $(a+b)^*ab(a+b)^* + b^*a^* = (a+b)^*$ iii. $(a+b)a(a+b)^*b(a+b)^* = (a+b)^*ab(a+b)^*$ iv. $(a+b)^* = a^* + b^*$</p> <p>Which of the following is/are true?</p> <p>A. I and II only B. I and III only C. II and III only D. I, II and III only E. All are true</p>	A)I and II only	
		<p>What is the length of the shortest string not in the language over $\Sigma = \{a,b\}$ for regular expression $(ba+a)^*(b+ba)^*$</p> <p>A. 1 B. 2 C. 3 D. 4 E. None of the above</p>	D)4	
<u>Q.No</u>		Using the following Table	A) {1,	

:1(c)

δ	a	b	λ
1	φ	φ	{2}
2	{3}	φ	{5}
3	φ	{4}	φ
4	{4}	φ	{1}
5	φ	{6,7}	{7}
		}	
6	{5}	φ	φ
7	φ	φ	{1}

Select the correct option for $\delta^*(1, bab)$

- A) {1, 2, 4, 5, 6, 7}
- B) {1, 2, 3, 4, 5, 6, 7}
- C){1, 2, 3, 4, 5}
- D){3, 5}
- E) None of the above

**2, 4, 5,
6, 7}**

Using the following Table

δ	a	b	λ
1	φ	φ	{2}
2	{3}	φ	{5}
3	φ	{4}	φ
4	{4}	φ	{1}
5	φ	{6,7}	{3}
		}	
6	{5}	φ	φ
7	φ	{4}	{1}

Select the correct option for $\delta^*(1, bab)$

- A){1, 2, 4, 5, 6, 7}
- B){1, 2, 3, 4, 5, 6, 7}
- C){1, 2, 3, 4, 5}
- D){3, 5}
- E) None of the above

**B){1, 2,
3, 4, 5,
6, 7}**

Using the following Table

δ	a	b	λ
1	φ	φ	{2}
2	{3}	φ	{5}
3	φ	{4}	φ
4	{4}	φ	{1}
5	φ	{6,7}	{3}
		}	
6	{5}	φ	φ
7	φ	{4}	{1}

Select the correct option for $\delta^*(1, aba)$

- A){1, 2, 4, 5, 6, 7}
- B){1, 2, 3, 4, 5, 6, 7}
- C){1, 2, 3, 4, 5}

**C){1, 2,
3, 4, 5}**

		D) $\{3, 5\}$ E) None of the above																																		
		<p>Using the following Table</p> <table border="1"> <tr> <th>δ</th> <th>a</th> <th>b</th> <th>λ</th> </tr> <tr> <td>1</td> <td>φ</td> <td>φ</td> <td>{2}</td> </tr> <tr> <td>2</td> <td>{3}</td> <td>φ</td> <td>{5}</td> </tr> <tr> <td>3</td> <td>φ</td> <td>{6,7 }</td> <td>φ</td> </tr> <tr> <td>4</td> <td>{4}</td> <td>φ</td> <td>{1}</td> </tr> <tr> <td>5</td> <td>φ</td> <td>{4}</td> <td>φ</td> </tr> <tr> <td>6</td> <td>{5}</td> <td>φ</td> <td>φ</td> </tr> <tr> <td>7</td> <td>φ</td> <td>φ</td> <td>{1}</td> </tr> </table> <p>Select the correct option for $\delta^*(1, aba)$</p> <p>A){1, 2, 4, 5, 6, 7} B){1, 2, 3, 4, 5, 6, 7} C){1, 2, 3, 4, 5} D){3, 5} E) None of the above</p>	δ	a	b	λ	1	φ	φ	{2}	2	{3}	φ	{5}	3	φ	{6,7 }	φ	4	{4}	φ	{1}	5	φ	{4}	φ	6	{5}	φ	φ	7	φ	φ	{1}	D) $\{3, 5\}$	
δ	a	b	λ																																	
1	φ	φ	{2}																																	
2	{3}	φ	{5}																																	
3	φ	{6,7 }	φ																																	
4	{4}	φ	{1}																																	
5	φ	{4}	φ																																	
6	{5}	φ	φ																																	
7	φ	φ	{1}																																	
<u>Q.No :1(d)</u>		<p>If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider</p> <p>I)$L_1.L_2 = \{a^n.b^n \mid n \geq 0\}$ II)$L_1.L_2$ is a regular language III)$L_1.L_2$ is not a regular language</p> <p>Which of the following is Correct?</p> <p>A)Both (I) and (III) B) Only (II) C)Both (I) and (II) D)None of the above</p>	B) Only (II)																																	
		<p>Given that language L_1 is regular and that the language $L_1 \cap L_2$ is regular. Which of the following is true about L_2?</p> <p>A)L_2 must be regular B)L_2 can not be regular C)L_2 may not to be regular D)None of the above</p>	C) L_2 may not to be regular																																	
		<p>If L_1 is infinite and L_2 and L_3 are regular and $L_3 = L_1.L_2$ then which of the statement is true?</p> <p>A) L_1 is regular B) L_1 may not to be regular C) L_2 is finite D) None of the above</p>	D) L_1 may not to be regular																																	
		<p>If $L_1.L_2$ is regular and L_1 is regular then which is the correct statement for L_2?</p> <p>A)L_2 is regular B)L_2 may not to be regular</p>	E) L_2 may not to be regular																																	

		C) L_2 is \emptyset D) L_2 is finite		
<u>Q.No :1(e)</u>		<p>Consider the following statements.</p> <p>I.Empty string is regular II.Empty set is regular set Which of the following is true?</p> <p>A)I is true only B)II is true only C)Both I and II are true D)None</p>	C)Both I and II are correct	
		<p>Consider the following statements and identify which option(s) is/ are correct</p> <p>I. λ is a regular expression. II. \emptyset is a Regular expression. III. \emptyset^* is not a Regular Expression</p> <p>A)Only III is correct B)II is not correct but III is correct C)Only I is correct D)Both I and II are correct.</p>	D)Both I and II are correct.	
		<p>Consider the following DFA</p> <p>R1: Regular Expression equivalent to the DFA, assuming q_0 to be the initial state and q_1 to be the final state</p> <p>R2: Regular Expression, expressing the machine assuming q_1 to be the initial state, and q_0 to be the final state</p> <p>R3: Regular Expression, expressing the machine assuming q_0 to be the both initial and final state</p> <p>Consider the following.</p> <p>I. $R_1 = (R_2)$ II. $R_2 = R_3 - R_1$ III. $R_3 = \Sigma^* - R_1$</p> <p>What relationship exists between, R_1, R_2 and R_3</p> <p>A)I and II B) I and III</p>	B) I and III	

		C)I , II and III D)I only		
		Consider the following statements : Regular Expressions represent I.Finite sets II.Countable sets III.Both finite and infinite sets IV.Regular sets V.Uncountable sets A)Only III B)Only IV C)I & II only, nothing else D)None of the above combination are Correct	D)None of the above combination are Correct	

SECTION - B

Q₀₂-2

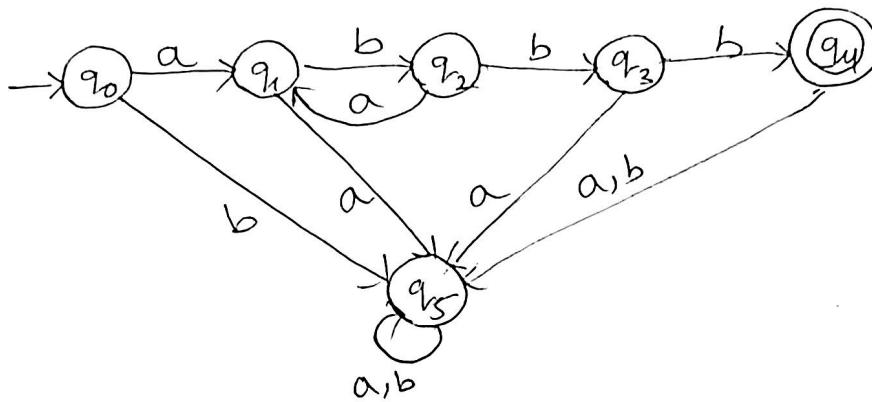
(A) Design a DFA for the following language:

$$L = \{(ab)^i b^{2j} : i \geq 1, j \geq 1\}$$

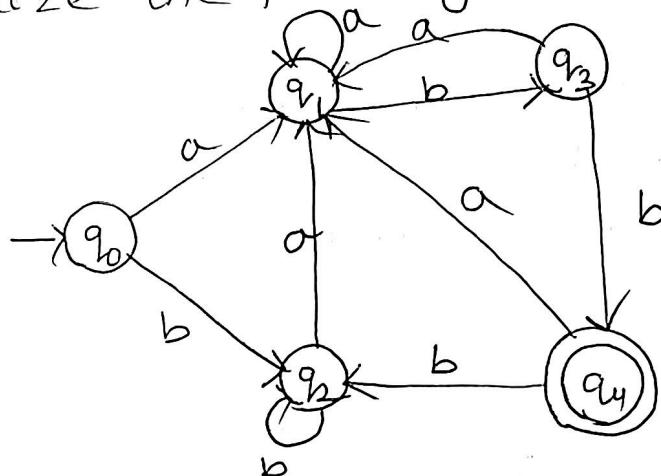
[4 marks]

[4 marks]

Ans:-



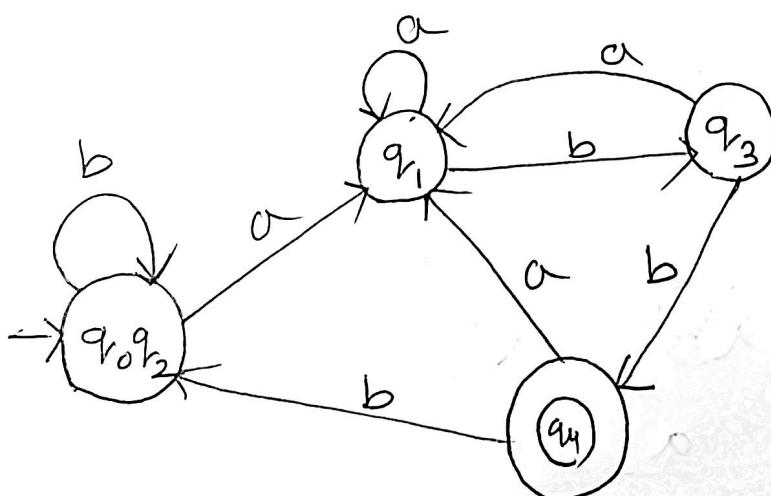
(B) Minimize the following DFA:



[3 marks]

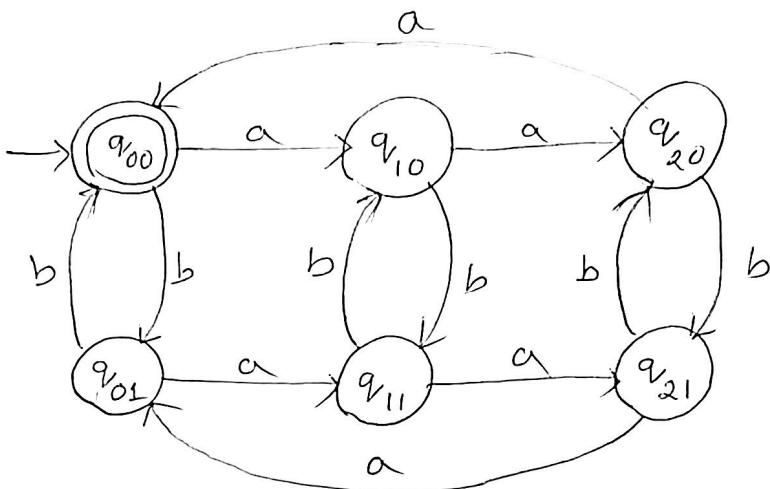
[3 marks]

Ans:-



(C) Design an NFA which will accept the given language L over alphabets {a,b}. L = {set of all strings which the number of a's is divisible by 3 and the number of b's is divisible by 2. [3 marks]}

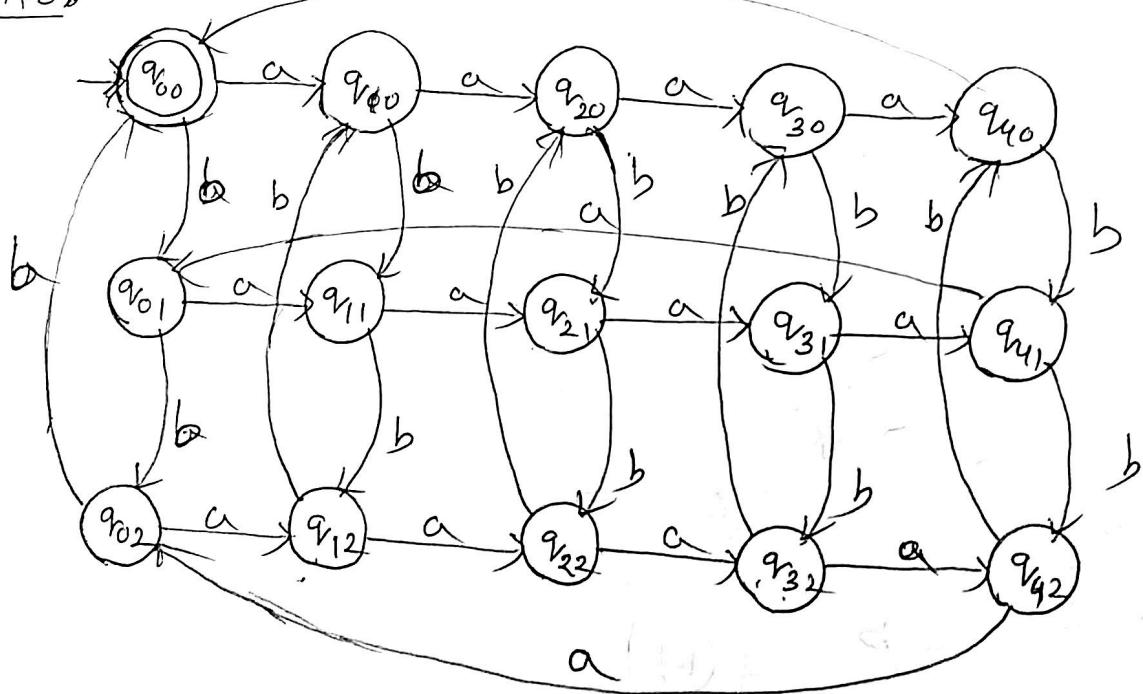
Ans:-



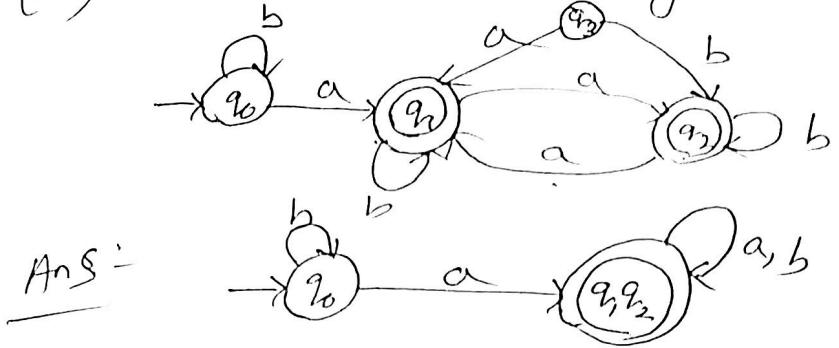
Q₀-3

(A) Design a DFA accepting the set strings over alphabet $\Sigma = \{x, y\}$ such that in each string number of x's is divisible by 5 and the number of y's is divisible by 3.

Ans $\frac{1}{6}$



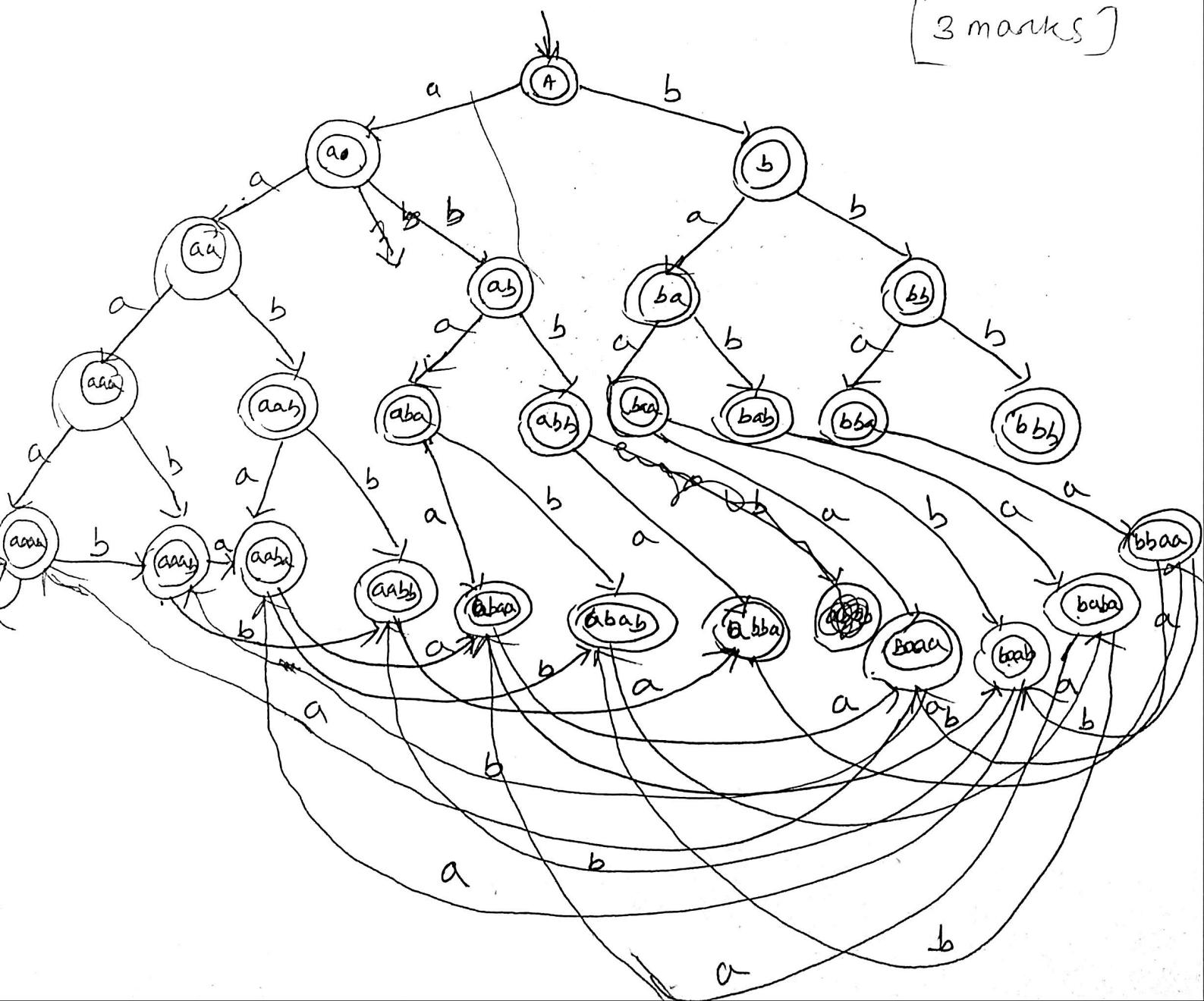
(B) Minimize the following DFA:



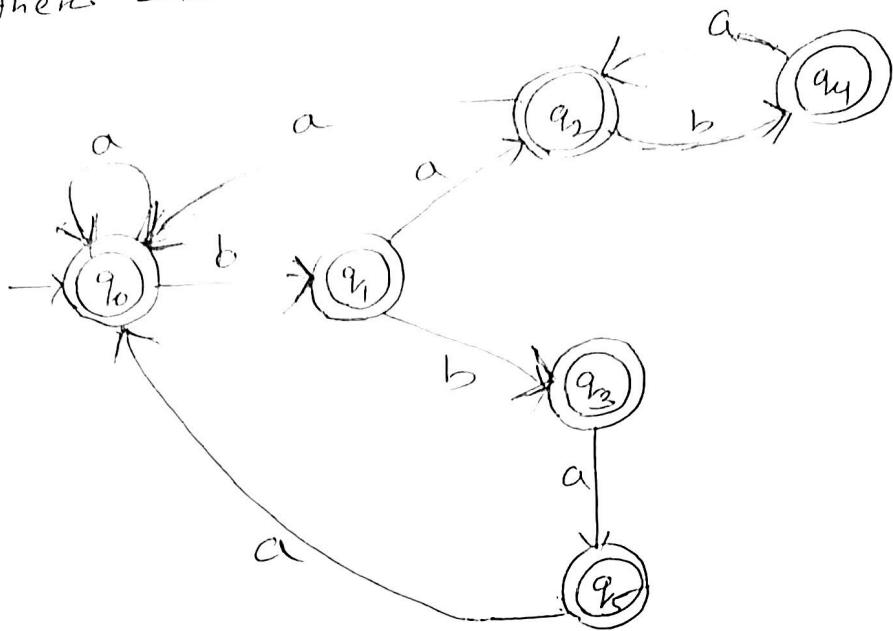
[3 marks]

(C) Design an NFA that accepts the given language L over the alphabets $\{a, b\}$. $L = \{ \text{The set of all the strings in which any four consecutive symbols contain at least two a's} \}$

[3 marks]



Another solution of B(c)



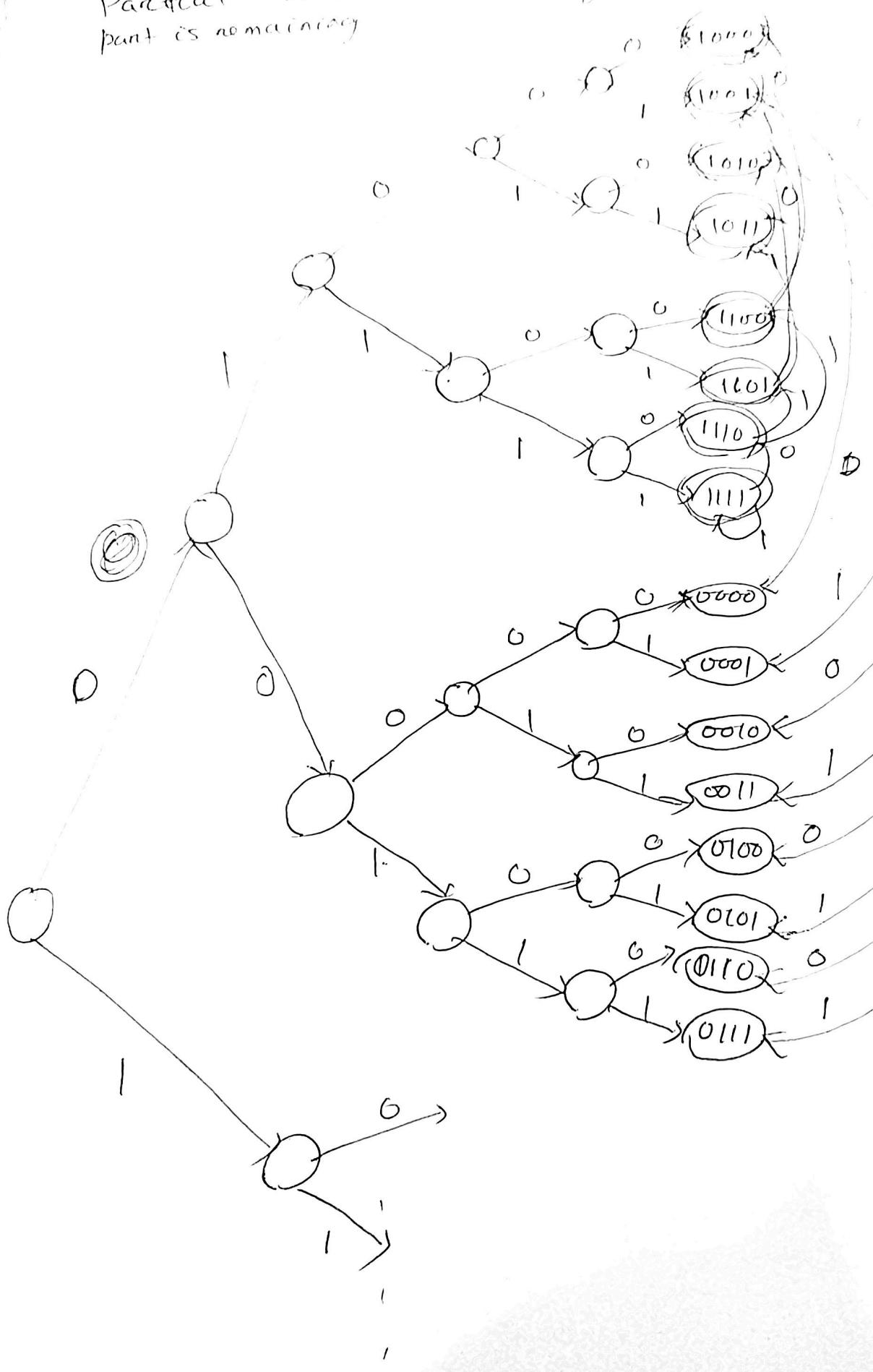
Q₀ → q₄

(A) Consider the set of strings on $\{0, 1\}$. Construct a DFA for all strings of length five or more in which the fourth symbol from the right end is different from the leftmost symbol. [4 marks]

Ans:- Minimum number of states is 33.

When we evaluate this question we have to see the approach of the student towards this question. If student mentioned in the answersheet that this questions require many states to solve and also partially solved this then by seeing his/her approach we will provide 70% to 80% marks.

Partial DFA. I draw only one part other
part is remaining

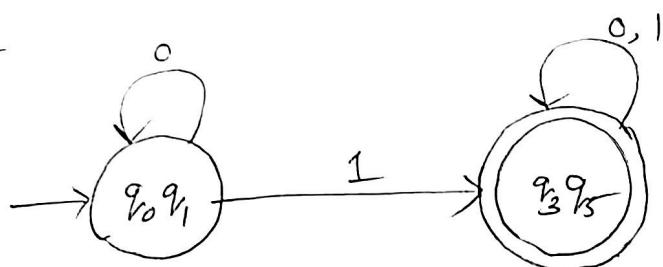


(B) Minimize the following DFA :
 $M = (Q, \Sigma, \delta, q_0, F)$ where $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$
 $\Sigma = \{0, 1\}$, $F = \{q_3, q_5\}$

[3 marks]

δ	0	1.
q_0	q_1	q_3
q_1	q_0	q_2
q_2	q_1	q_4
q_3	q_5	q_5
q_4	q_2	q_3
q_5^*	q_5	q_5

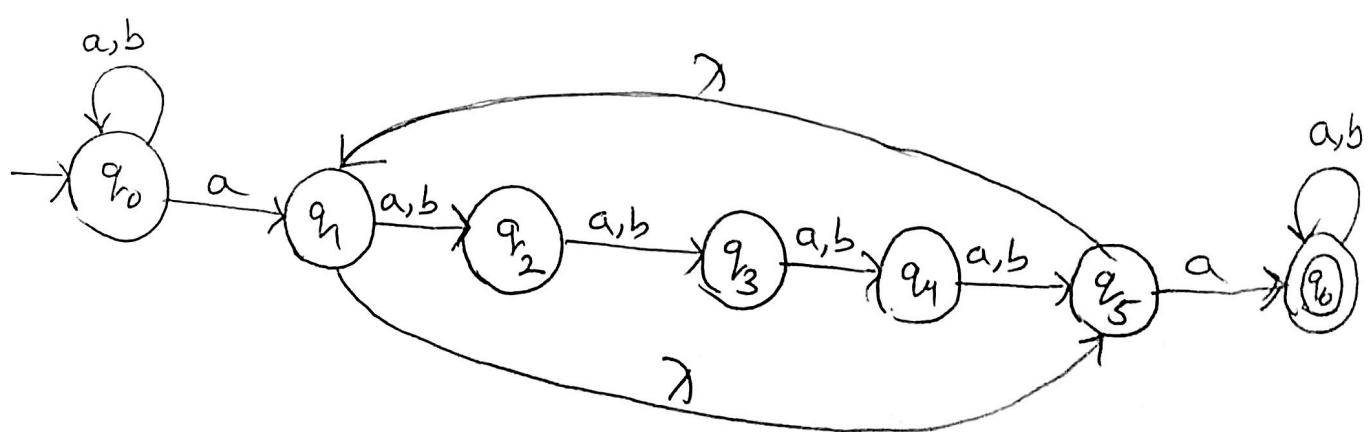
Ans:-



(C) Design an NFA which will accept the given language L over the alphabet $\{a, b\}$. $L = \{ \text{The set of strings such that there are two } a's \text{ separated by a substring of length which is a multiple of 4} \}$.

Ans:-

[3 marks]



Q_o-5

(A) Write the language a regular expression for each of the following languages. [1x5 = 5 marks]

(I) $L = \{ w \mid \text{decimal equivalent of } w \text{ is neither a power of 2 nor an odd number} \}$ over $\{0, 1\}$

Ans:- $[0^* + \overline{0^* 1 0^* 1} (0+1)^* 0]$ or $[0^* + \overline{0^* 1} \overline{0^* 1 1^* 0} (0+1)^* 0]$

(II) $L = \{ w \mid w \text{ is your KIIT roll number} \}$ over $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, @, \dots, #, a, b, - z\}$

Ans:- 2005001 sample ans

(III) $L = \{ w \mid \text{length of } w \text{ is between 20 to 50} \}$ over $\{0, 1\}$

Ans:- $\boxed{(0+1)^{20} (0+1)^{30}}$

(IV) $L = \{ 0^n 1^m \mid m+n \text{ is odd and } m*n \text{ is even} \}$ over $\{0, 1\}$

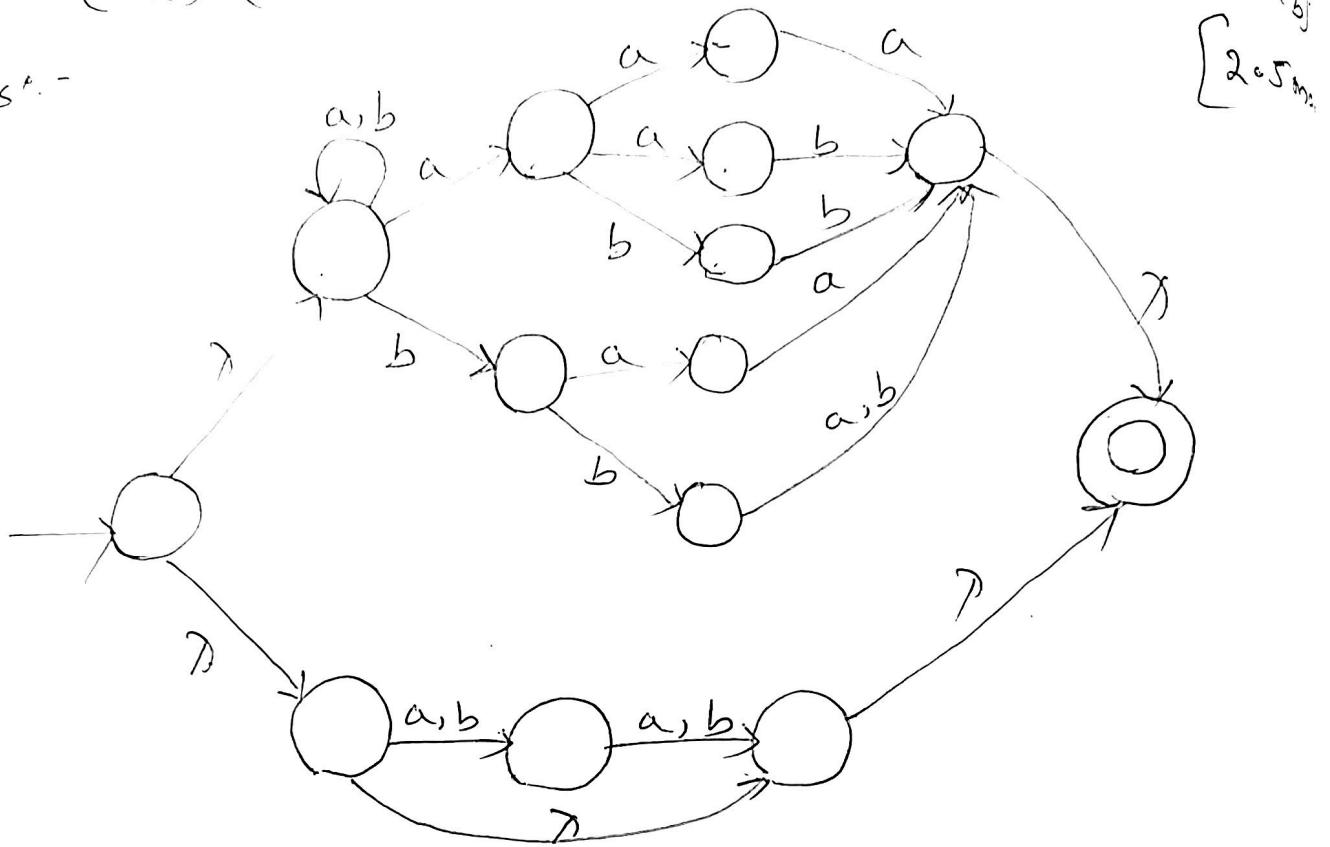
Ans:- $\boxed{0(00)^*(11)^* + (00)^* 1(11)^*}$

(V) $L = \{ w \mid w \text{ doesn't contain } 1101 \}$ over $\{0, 1\}$

Ans:- $\boxed{(0+10+11(11)^* 00)^* (0+1+11(11)^* + 11(11)^* 0)}$

(B) Convert the regular expression to NFA
 $(a+b)^*(aaa+aab+abb+baa+bba+bbb) + (\gamma + a+b)$

Ans :-



(C) Prove that the following language is not regular using Pumping Lemma for regular language:

$$L = \{a^n b^l n^k : k \geq n+l\}$$

[2.5 marks]

Ans :-

Statement - 0.5

Selection of string - 1 mark

Decomposition - 1 mark

(Q: 6A) Write the regular expression for the following languages:

[1x5 = 5 marks]

(I) $L = \{b^m a b^n \mid m > 0, n > 0\}$

Ans:- $b^+ a b^+$ or $bb^* a b b^*$

(II) Strings of length multiple of 5 over $\{0, 1, 2\}$

Ans:- $((0+1+2)^5)^*$

(III) Strings where 1st symbol is 0 and 3rd symbol from right is 1 over $\{0, 1\}$

Ans:- $0(0+1)^* 1(0+1)^2$

(IV) $L = \{\omega : n_a(\omega) \bmod 5 > 0\}$ over $\{a, b\}$

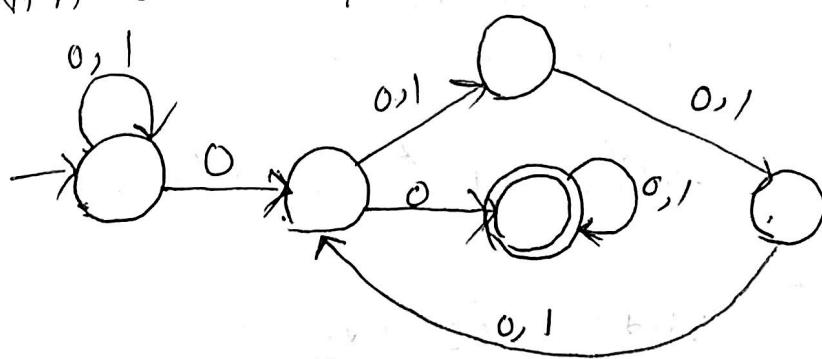
Ans:- $((bab^*ab^*ab^*ab^*)^* (b^*ab^* + b^*ab^*ab^* + b^*ab^*ab^*ab^* + b^*ab^*ab^*ab^*ab^*))$

(V) Strings of length not a multiple of 3 over $\{a, b\}$

Ans:- $((a+b)^3)^* ((a+b) + (a+b)^2)$

(B) Find the regular expression equivalent to the following NFA over alphabet $\{0, 1\}$.

[2x5 marks]



Ans:- $(0+1)^* 0 ((0+1)^3)^* 0 (0+1)^*$

(C) Determine whether or not the following language
 $\Sigma = \{a\}^*$ is regular using Pumping Lemma.
 $L = \{a^n : n \text{ is not a prime number}\}$

[2.5 marks]

Ans:- Statement - 0.5 mark

Selection of string - 1 mark

Decomposition - 1 mark

Q. 7

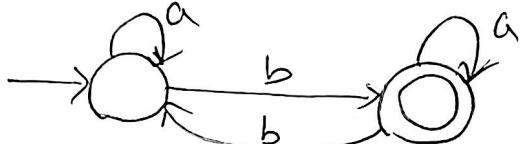
(A) Write regular expressions for the following languages

(I) $L = \{\omega \mid \omega \text{ doesn't contain } 1101\}$ over $\{0, 1\}$

Ans:- $\boxed{(0+10+11(11)^*00)^*(\lambda + 1 + 11(11)^* + 11(11)^*0)}$

(II) $L = \{\omega \mid 2\eta_a(\omega) + 3\eta_b(\omega) \text{ is odd}\}$

Ans:-



$\boxed{a^*b(a+ba^*b)^*}$

(III) $L = \{01^n\omega \mid n \geq 0, \omega \in \{0, 1\}^*\}$

Ans:- $\boxed{01^*(0+1)^*} \cup \boxed{0^*(0+1)^*}$

(IV) $L = \{\omega \mid \omega \text{ contains at least one occurrence of each symbol in alphabet set } \Sigma = \{0, 1\}\}$

Ans:-

$\boxed{(0+1)^*0(0+1)^*1(0+1)^* + (0+1)^*1(0+1)^*0(0+1)^*}$

(V) $L = \{a^n b^m \mid (n+m) \text{ is even}\}$

Ans:-

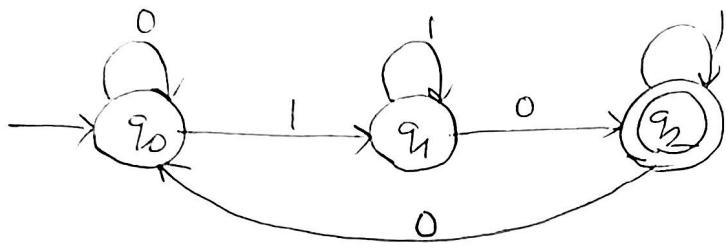
$\boxed{(aa)^*(bb)^* + a(aa)^*b(bb)^*}$

(B) Draw the transition diagram for the following transition table. Find the regular expression for the transition diagram.

[2.5 marks]

$\delta =$	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_2

Ans:-



$$r = 0^* 1^* 0 (1 + 00^* 11^* 0)^*$$

(C) Determine whether or not the following language on $\Sigma = \{a\}$ is regular using Pumping Lemma.

$$L = \{a^n : n \text{ is the product of two prime numbers}\}$$

[2.5 marks]

Ans:- Statement - 0.5 mark

selection of string - 1 mark

Decomposition - 1 mark