Exc: 4.3

28) a) $P(0 \leq Z \leq 2.17) = \phi(2.17) - \phi(0)$

(All are calculated from the distribution table)

$= 0.985 - 0.5$

$= 0.485$

f) $P(Z \geq -1.75) = 1 - P(Z < -1.75)$

$= 1 - 0.0401$

$= 0.9599$

i) $P(Z \geq 1.5) = 1 - P(Z < 1.5)$

$= 1 - 0.9332$

$= 0.0668$

j) $P(|Z| \leq 2.50) = P(-2.5 \leq Z \leq 2.5)$

$= \phi(2.5) - \phi(-2.5)$

$= 0.9938 - 0.0062 = 0.9876$

If we use $\phi(-z) = 1 - \phi(z)$

$$2\phi(2.5) - 1$$

$$= 2\{0.9938\} - 1$$

$$= 0.9876$$

Exc:- 4.3 We'll use distribution table.

28) b) $P(0 \leq Z \leq 1) = \phi(1) - \phi(0)$

$= 0.8413 - 0.5$

$= 0.3413$

c) $P(-2.5 \leq Z \leq 0) = \phi(0) - \phi(-2.5)$

$= 0.5 - 0.0062$

$= 0.4938$

d) $P(-2.5 \leq Z \leq 2.5) = \phi(2.5) - \phi(-2.5)$

$= 0.9938 - 0.0062$

$= 0.9876$

e) $P(Z \leq 1.37) = \phi(1.37) = 0.9147$

g) $P(-1.5 \leq Z \leq 2) = \phi(2) - \phi(-1.5) = 0.9772 - 0.0668$

$= 0.9104$

h) $P(1.37 \leq Z \leq 2.5) = \phi(2.5) - \phi(1.37) = 0.9938 - 0.9147 = 0.0791$

29) a) $\phi(c) = 0.9838$

Find the value of c

$c = 2.14$

c) $P(C \leq Z) = P(Z \geq c) = 1 - P(Z < C)$

$\Rightarrow 1 - \phi(c) = 0.121$

$\Rightarrow \phi(c) = 1 - 0.121$

$\Rightarrow \phi(c) = 0.879$

$\Rightarrow c = 1.17$

e) $P(C \leq |Z|) = P(|Z| \geq c)$

$= 1 - P(|Z| < c)$

$\Rightarrow 1 - P(-c < Z < c) = 0.16$

$\Rightarrow 1 - \{\phi(c) - \phi(-c)\} = 0.16$

$\Rightarrow \phi(c) - \phi(-c) = 1 - 0.16$

$\Rightarrow$

$\Rightarrow 2\phi(c) - 1 = 0.984$

$\Rightarrow \phi(c) = \frac{1.984}{2}$

$\Rightarrow \phi(c) = 0.992$

$\Rightarrow c = 2.41$

29) b) $$P(0 \le Z \le c) = 0.291$$

$$\Rightarrow \phi(c) - \phi(0) = 0.291$$

$$\Rightarrow \phi(c) = 0.5 + 0.291$$

$$\Rightarrow \phi(c) = 0.791$$

$$\Rightarrow c = 0.81$$

d) $$P(-c \le Z \le c) = 0.668$$

$$\Rightarrow \phi(c) - \phi(-c) = 0.668$$

$$\Rightarrow 2\phi(c) - 1 = 0.668 \quad [\because \phi(-x) = 1 - \phi(x)]$$

$$\Rightarrow \phi(c) = \frac{1.668}{2}$$

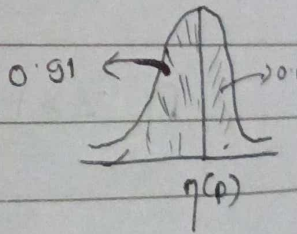
$$\Rightarrow \phi(c) = 0.834$$

$$\Rightarrow c = 0.97$$

30)

We know, for crv

$$p = F(\eta(p)) = \int_{-\infty}^{\eta(p)} f(y)\,dy$$



We need to find

a) $\phi(\eta(p)) = \phi(\eta) = p = 0.91$

$\Rightarrow \eta = 1.32$ (left side se dekho)

b) $\phi(\eta) = p = 0.09$

$\Rightarrow \eta = -1.3\,2$

0.24

c) $\phi(\eta(p)) = 0.75$

$\Rightarrow \eta = 0.67$

d) $\phi(\eta(p) = 0.25$

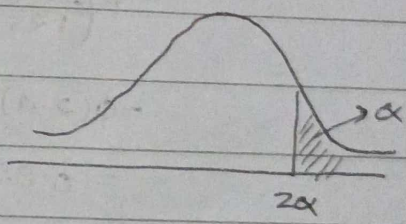
$\Rightarrow \eta = -0.67$

30) e) ~~$\phi(p) = 0.06$~~

$\phi(\eta) = p = 0.06$

$\Rightarrow \eta = -1.55$

31) a) $\alpha = 0.0055$



$P(Z \geq z_\alpha) = \alpha$

$\Rightarrow 1 - P(Z < z_\alpha) = \alpha$

$\Rightarrow 1 - \phi(z_\alpha) = \alpha$

$\Rightarrow \phi(z_\alpha) = 1 - \alpha$

$\Rightarrow \phi(z_\alpha) = 1 - 0.0055$

$\Rightarrow \phi(z_\alpha) = 0.9945$

$\Rightarrow z_\alpha = 2.54$

c) $\alpha = 0.663$

$\phi(z_\alpha) = 1 - 0.663$

$\Rightarrow \phi(z_\alpha) = 0.337$

$\Rightarrow z_\alpha = -0.42$

31) b) $\alpha = 0.09$

$$P(Z \geq Z_\alpha) = \alpha$$

$$\Rightarrow 1 - P(Z < Z_\alpha) = \alpha$$

$$\Rightarrow 1 - \phi(Z_\alpha) = \alpha$$

$$\Rightarrow \phi(Z_\alpha) = 1 - 0.09$$

$$\Rightarrow \phi(Z_\alpha) = 0.91$$

$$\Rightarrow Z_\alpha = 1.32$$

32) $\mu = 15, \sigma = 1.25$

a) $P(X \leq 15) = P\left(\frac{X-\mu}{\sigma} \leq \frac{15-15}{1.25}\right) = P(Z \leq 0)$

$= \phi(0) = 0.5$

c) $P(X \geq 10) = 1 - P(X < 10) = 1 - P\left(Z < \frac{10-15}{1.25}\right)$

$= 1 - P(Z < -4)$

$= 1 - \phi(-4)$

d) $P(14 \leq X \leq 18) = \phi\left(\frac{18-15}{1.25}\right) - \phi\left(\frac{14-15}{1.25}\right)$

$= \phi(2.4) - \phi(-0.8)$

$= 0.9918 - 0.2119$

$= 0.7799$

32) $\mu = 15, \quad \sigma = 1.25$

→ convert this to normal random variable

b) $\cancel{P(14 \le X \le}$ $P(X \le 17.5)$

$$P\left(\frac{x-\mu}{\sigma} \le \frac{17.5-\mu}{\sigma}\right)$$

$$= P\left(Z \le \frac{17.5-15}{1.25}\right)$$

$$= P(Z \le 2)$$

$$= \Phi(2)$$

$$= 0.9772$$

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Q) $P(|X-15| \le 3)$

$P(-3 \le X-15 \le 3)$

$\cancel{\Rightarrow P(12 \le X \le 18)}$

$\cancel{\Rightarrow P\left(\frac{12-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{18-\mu}{\sigma}\right)}$

$\Rightarrow \cancel{P} = P(-3 \le x-\mu \le 3)$

$= P\left(\frac{-3}{1.25} \le z \le \frac{3}{1.25}\right)$

$= \phi\left(\frac{3}{1.25}\right) - \phi\left(\frac{-3}{1.25}\right)$

$= \phi(2.4) - \phi(-2.4)$

$= 0.9918 - 0.0082$

$= 0.9836$

34) $\mu = 0.3, \quad \sigma = 0.06$

a) $P(X > 0.5) = P\left(\frac{X-\mu}{\sigma} > \frac{0.5-0.3}{0.06}\right)$

$= P(Z > 3.33)$

$= 1 - P(Z \leq 3.33) = 1 - \phi(3.33) = 1 - 0.9996$

$= 0.0004$

b) $P(X \leq 0.2)$

$= P\left(Z \leq \frac{0.2 - 0.3}{0.06}\right)$

$= P(Z \leq -1.67)$

$= \phi(-1.67)$

$= 0.0475$

c) we want 95 percentile, c of this normal distribution so that 5% of the values are higher.

$\phi(c) = 0.95$

$\Rightarrow c = 1.65$

$\frac{x - \mu}{\sigma} = 1.65$

$\Rightarrow \frac{x - 0.3}{0.06} = 1.65$

$\Rightarrow x = 1.65 \times 0.06 + 0.3$

$\Rightarrow x = 0.399$

35) $\mu = 8.46$ $\qquad$ x = truck haul time.

$\sigma = 0.913$

a) $P(X \geq 10) = 1 - P(X < 10)$

$$= 1 - P\left(\frac{X-\mu}{\sigma} < \frac{10 - 8.46}{0.913}\right)$$

$$= 1 - P(Z < 1.69)$$

$$= 1 - 0.9545$$

$$= 0.0455$$

$P(X > 10) = 0.0455$

b) $P(X > 15) = 1 - P(X \leq 15)$

$$= 1 - P\left(Z < \frac{15 - 8.46}{0.913}\right)$$

$$= 1 - P(Z < 7.16)$$

$$= 1 - \phi(7.16)$$

$$= 1 - 1$$

$$= 0$$

c) $P(8 \le X < 10)$

$$= \phi\left(\frac{10-8.46}{0.913}\right) - \phi\left(\frac{8-8.46}{0.913}\right)$$

$$= \phi(1.69) - \phi(-0.50)$$

$$= 0.9545 - 0.3085$$

$$= 0.646$$

d) $P(8.46 - c < X < 8.46 + c) = 0.98$

$$\Rightarrow P\left(\frac{8.46 - c - 8.46}{0.913} < Z < \frac{8.46 + c - 8.46}{0.913}\right) = 0.98$$

$$\Rightarrow P\left(\frac{-c}{0.913} < Z < \frac{c}{0.913}\right) = 0.98$$

$$\Rightarrow \phi\left(\frac{c}{0.913}\right) - \phi\left(\frac{-c}{0.913}\right) = 0.98$$

$$\Rightarrow 2\phi\left(\frac{c}{0.913}\right) - 1 = 0.98$$

$$\Rightarrow \phi\left(\frac{c}{0.913}\right) = \frac{1.98}{2}$$

$$\Rightarrow \phi\left(\frac{c}{0.913}\right) = 0.99$$

$$\Rightarrow \frac{c}{0.913} = 0.8389$$

$$\Rightarrow c = 0.766$$

e) ~~$P(X>10)$ $P($~~ A_1, A_2, A_3, A_4

$P(X>10) \cdot P(X>10)$

P(Hall time exceeds 10 mins) $= P(X>10)$
$= 0.0455$

P(Hall time is less than 10 mins)
$= P(X<10) = 1 - P(X \ge 10) = 1 - 0.0455 = 0.9545$

P(All the hall times are less than 10 mins)
$= (0.9545)^4 = 0.83$

P(~~all~~ atleast one of the hall times exceeds 10 mins)
P(All the hall times are less than 10 mins)
$= 1 - P$(all the hall times are less than 10 mins)
$= 1 - 0.83 = 0.17$

41) $\mu = 200, \sigma = 30$

The probability of equipment damage can be represented by

$$P(X < 100)$$

$$= P\left(\frac{X-\mu}{\sigma} < \frac{100-200}{30}\right)$$

$$= P(Z < -3.33)$$

$$= \Phi(-3.33)$$

$$= 0.0004$$

P(parachute doesn't suffer damage) $= 1 - 0.0004 = 0.9996$

P(atleast one of the five independently dropped parachutes suffer damage) $= 1 -$ P(none of the five independently dropped parachutes suffer damage)

$$= 1 - (0.9996)^5 = 0.002$$

51) $P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$

$P(|X-\mu| \geq k\sigma)$

Dividing by σ

$P\left(\frac{|X-\mu|}{\sigma} \geq k\right)$

$\Rightarrow P\left(\frac{X-\mu}{\sigma} \geq k\right)$ or $P\left(\frac{X-\mu}{\sigma} \leq -k\right)$

$\Rightarrow P(Z \geq k)$ or $P(Z \leq -k)$

$\Rightarrow 1 - P(Z < k)$ $\quad = \phi(-k)$

$\Rightarrow 1 - \phi(k)$

~~when k=1,~~

$= 1 - P\left(-k < \frac{x-\mu}{\sigma} < k\right)$

$\cancel{1-\phi(1)}$

$= 1 - P(-k < Z < k)$

$= 1 - \{\phi(k) - \phi(-k)\}$

When $k=1$,

$= 1 - \{\phi(1) - \phi(-1)\}$

$= 1 - \{0.8413 - 0.1587\}$

$= 0.3174$

upper bound $\frac{1}{1^2} = 1$

When $k=2$,

$1 - \{\phi(2) - \phi(-2)\}$

$= 1 - \{0.9772 - 0.0228\}$

$= 0.0456$

upper bound $\frac{1}{2^2} = 0.25$

When $k=3$,

$1 - \{\phi(3) - \phi(-3)\}$

$= 1 - \{0.9987 - 0.0013\}$

$= 0.0026$

upper bound $\frac{1}{3^2} = 0.11$

$\therefore$ It is observed that all the calculated values are less than the upper bound.

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53) $n = 25$, p

when $n=25, p=0.5, 0.6, 0.8$

$np = 12.5 \geq 10$, $15 \geq 10$, $20 \geq 10$ respectively

So, we can apply normal distribution for every given values of p.

when $n=25, p=0.5, q=0.5$

$$P(15 \leq X \leq 20) = \phi\left(\frac{20+0.5-np}{\sigma}\right) - \phi\left(\frac{14+0.5-np}{\sigma}\right)$$

$$\sigma = \sqrt{npq} = \sqrt{25 \times 0.5 \times 0.5} = 2.5$$

$$= \phi\left(\frac{20+0.5-12.5}{2.5}\right) - \phi\left(\frac{14+0.5-12.5}{2.5}\right)$$

$$= \phi(3.2) - \phi(0.8)$$

$$= 0.9993 - 0.7881$$

$$= 0.2112$$

when $n=25, p=0.6, q=0.4$

$$\sigma = \sqrt{25 \times 0.6 \times 0.4} = 2.45$$

$$P(15 \leq X \leq 20) = \phi\left(\frac{20+0.5-15}{2.45}\right) - \phi\left(\frac{14+0.5-15}{2.45}\right)$$

$$= \phi(2.24) - \phi(-0.2)$$

$$= 0.9875 - 0.4207$$

$$= 0.5668$$

when $n=25, p=0.8, q=0.2$

$$\sigma = \sqrt{25 \times 0.8 \times 0.2} = 2$$

$$P(15 \leq X \leq 20) = \phi\left(\frac{20+0.5-20}{2}\right) - \phi\left(\frac{14+0.5-20}{2}\right)$$

$$= \phi(0.25) - \phi(-2.75)$$

$$= 0.5987 - 0.003$$

$$= 0.5957$$

b) $P(X \leq 15)$ when $p=0.5$,

$= \phi\left(\frac{15+0.5-12.5}{2.5}\right)$

$= \phi(1.2)$

$= 0.8849$

when $p=0.6$

$P(X \leq 15)$

$= \phi\left(\frac{15+0.5-15}{2.45}\right)$

$= \phi(0.2)$

$= 0.5793$

when $p=0.8$

$P(X \leq 15)$

$= \phi\left(\frac{15+0.5-20}{2}\right)$

$= \phi(-2.25)$

$= 0.0122$

c) $P(X \geq 20)$

$= 1 - P(X < 20)$

when $p=0.5$ $\quad = 1 - \phi(19)$

$1 - \phi\left(\frac{19+0.5-12.5}{2.5}\right)$

$= 1 - \phi(2.8)$

$= 1 - 0.9974$

$= 0.0026$

when $p=0.6$

$1 - \phi\left(\frac{19+0.5-15}{2.45}\right)$

$= 1 - \phi(1.84)$

$= 1 - 0.9671$

$= 0.0329$

when $p = 0.8$,

$1 - \phi\left(\frac{19+0.5-20}{2}\right)$

$= 1 - \phi(-0.25)$

$= 1 - 0.4013$

$= 0.5987$

54) a) X: The no. among all steel shafts that are non conforming and can be reworked.

$p = 0.1, \; n = 200, \; q = 0.9$

$np = 0.1 \times 200 = 20 \geq 10$

So, we can apply normal distribution.

$P(X \leq 30)$

$= P\left(Z \leq \dfrac{30 + 0.5 - np}{\sqrt{npq}}\right)$

$\sqrt{npq} = \sqrt{200 \times 0.1 \times 0.9} = 4.24$

$\therefore P\left(Z \leq \dfrac{30 + 0.5 - 20}{4.24}\right)$

$= \Phi(2.48)$

$= 0.9934$

b) $P(X < 30)$ ~~[∴ This is~~

$= P\left(Z < \dfrac{29 + 0.5 - 20}{4.24}\right)$

$= P(Z < 2.24)$

$= \Phi(2.24)$

$= 0.9875$

c) $P(15 \leq X \leq 25)$

$= \Phi\left(\dfrac{25 + 0.5 - 20}{4.24}\right) - \Phi\left(\dfrac{14 + 0.5 - 20}{4.24}\right)$

$= \Phi(1.3) - \Phi(-1.3)$

$= 0.9032 - 0.0968$

$= 0.8064$

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55) a) $p = 0.75,\ n = 500,\ np = 375 \geq 10 = \mu$

So, we need to use normal distribution instead of binomial distribution.

$\sigma = \sqrt{npq} = \sqrt{500 \times 0.75 \times 0.25} = 9.68$

$P(360 \leq X \leq 400)$

$= \phi\left(\frac{400 + 0.5 - 375}{9.68}\right) - \phi\left(\frac{359 + 0.5 - 375}{9.68}\right)$

$= \phi(2.63) - \phi(-1.6)$

$= 0.9957 - 0.0548$

$= 0.94$

b) $P(X < 400) = \phi\left(\frac{400 + 0.5 - 375}{9.68}\right)$

$= \phi(2.63)$

$= 0.9957$

$= 0.996$

Do it yourself.

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5. b) $E(C) = 115$

$\sigma_c = 2$

$$E\left(\frac{9}{5}C+32\right) = \frac{9}{5}E(C)+32 = 239$$

$$\sigma_{\frac{9}{5}C+32} = |a|\,\sigma_c = \frac{9}{5}\times 2 = 3.6$$

a) we know that,

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -(i)$$

$$Y = aX+b$$

$$F_Y(y) = P(Y \le y)$$
$$= P(aX+b \le y)$$
$$= P\left(X \le \frac{y-b}{a}\right)$$
$$= F_X\left(\frac{y-b}{a}\right)$$

$$f_Y(y) = F_Y'(y)$$
$$= F_X'\left(\frac{y-b}{a}\right)$$
$$= \frac{1}{a} f_X\left(\frac{y-b}{a}\right) \quad -(ii)$$

From eqn (i) and (ii), we get:-

$$f_Y(y) = \frac{1}{a} \times \frac{1}{\sigma\sqrt{2\pi}} \times e^{-\frac{1}{2}\left(\frac{\frac{y-b}{a}-\mu}{\sigma}\right)^2}$$

From the above equation, we can say that Y is normally distribu

$$E(Y) = E(aX+b) = aE(X)+b$$

~~$\sigma_Y = \sigma_{aX+b} = |a|\sigma_X$~~

$$V(Y) = V(aX+b) = a^2 V(x)$$