8
Magnetic Circuits

Introduction

We have seen that magnetic lines of force form closed loops around and through the magnetic material. The closed path followed by magnetic flux is called a magnetic circuit just as the closed path followed by current is called an electric circuit. Many electrical devices (*e.g.* generator, motor, transformer etc.) depend upon magnetism for their operation. Therefore, such devices have magnetic circuits *i.e.* closed flux paths. In order that these devices function efficiently, their magnetic circuits must be properly designed to obtain the required magnetic conditions. In this chapter, we shall focus our attention on the basic principles of magnetic circuits and methods to obtain their solution.

8.1. Magnetic Circuit

The closed path followed by magnetic flux is called a **magnetic circuit.**

In a magnetic circuit, the magnetic flux leaves the *N*-pole, passes through the entire circuit, and returns to the starting point. A magnetic circuit usually consists of materials having high permeability *e.g.* iron, soft steel *etc*. It is because these materials offer very small opposition to the 'flow' of magnetic flux. The most usual way of producing magnetic flux is by passing electric current through a wire of number of turns wound over a magnetic material. This helps in exercising excellent control over the magnitude, density and direction of magnetic flux.

Consider a coil of N turns wound on an iron core as shown in Fig. 8.1. When current I is passed through the coil, magnetic flux ϕ is set up in the core. The flux follows the closed path *ABCDA* and hence *ABCDA* is the magnetic circuit. The following points may be noted carefully :

(*i*) The amount of magnetic flux set up in the core depends upon current (I) and number of turns (N). If we increase the current or number of turns, the amount of magnetic flux also increases and *vice-versa*. The product *$N\,I$ is called the **magnetomotive force (m.m.f.)** and determines the amount of flux set up in the magnetic circuit.

$$\text{m.m.f.} = N\,I \text{ ampere-turns}$$

It can just be compared to electromotive force (e.m.f.) which sends current in an electric circuit.

(*ii*) The opposition that the magnetic circuit offers to the magnetic flux is called **reluctance.** It depends upon length of magnetic circuit (*i.e.* length *ABCDA* in this case), area of *X*-section of the circuit and the nature of material that makes up the magnetic circuit.

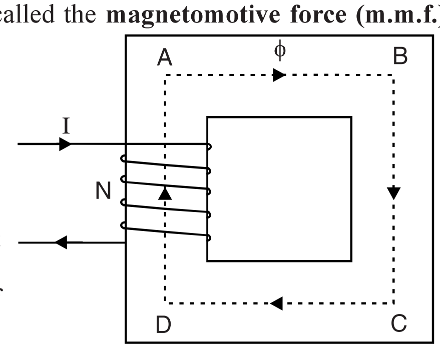

Fig. 8.1

8.2. Analysis of Magnetic Circuit

Consider the magnetic circuit shown in Fig. 8.1. Suppose the mean length of the magnetic circuit (*i.e.* length *ABCDA*) is l metres, cross-sectional area of the **core is 'a' m^2 and relative

* Coiling a conductor into two or more turns has the effect of using the same current for more than once. For example, 5-turn coil carrying a current of 10A produces the same magnetic flux in a given magnetic circuit as a 1-turn coil carrying a current of 50A. Hence m.m.f. is equal to the product of N and I.

** The arrangement of magnetic materials to form a magnetic circuit is generally called a *core*.

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permeability of core material is μ_r. When current I is passed through the coil, it will set up flux ϕ in the material.

Flux density in the material, $B = \dfrac{\phi}{a}\ \text{Wb/m}^2$

Magnetising force in the material, $H = \dfrac{B}{\mu_0\mu_r} = \dfrac{\phi}{a\mu_0\mu_r}\ \text{AT/m}$

According to work law, the work done in moving a unit magnetic pole once around the magnetic circuit (*i.e.* path *ABCDA* in this case) is equal to the ampere-turns enclosed by the magnetic circuit.

$$\therefore \qquad {}^*H \times l = N I$$

$$\text{or} \qquad \frac{\phi}{a\mu_0\mu_r} \times l = N I$$

$$\text{or} \qquad \phi = \frac{NI}{(l/a\mu_0\mu_r)}$$

The quantity NI which produces the magnetic flux is called the magnetomotive force (m.m.f.) and is measured in ampere-turns. The quantity $l/a\ \mu_0\ \mu_r$ is called the reluctance of the magnetic circuit. Reluctance is the opposition that the magnetic circuit offers to magnetic flux.

$$\therefore \qquad \text{Flux, } \phi = \frac{\text{m.m.f.}}{\text{reluctance}} \qquad \ldots(i)$$

Note that the relationship expressed in eq. (*i*) has a strong resemblance to Ohm's law of electric circuit ($I = E/R$). The m.m.f. is analogous to e.m.f. in the electric circuit, reluctance is analogous to resistance and flux is analogous to current. Because of this similarity, eq. (*i*) is sometimes referred to as **Ohm's law of magnetic circuit.**

8.3. Important Terms

In the study of magnetic circuits, we generally come across the following terms :

(*i*) Magnetomotive force (m.m.f.). It is a magnetic pressure which sets up or tends to set up flux in a magnetic circuit and may be defined as under :

The work done in moving a unit magnetic pole once around the magnetic circuit is called the magnetomotive force (m.m.f.). It is equal to the product of current and number of turns of the coil *i.e.*

$$\text{m.m.f.} = N I \text{ ampere-turns (or AT)}$$

Magnetomotive force in a magnetic circuit corresponds to e.m.f. in an electric circuit. The only change in the definition is the substitution of unit magnetic pole in place of unit charge.

(*ii*) Reluctance. *The opposition that the magnetic circuit offers to magnetic flux is called reluctance.* The reluctance of a magnetic circuit depends upon its length, area of *X*-section and permeability of the material that makes up the magnetic circuit. Its unit is †AT/Wb.

$$\text{Reluctance, } S = \frac{l}{a\,\mu_0\,\mu_r}$$

Reluctance in a magnetic circuit corresponds to resistance ($R = \rho\ l/a$) in an electric circuit. Both of them vary as length ÷ area and are dependent upon the nature of material of the circuit. Magnetic materials (*e.g.* iron, steel *etc.*) have a low reluctance because the value of μ_r is very large in their case. On the other hand, non-magnetic materials (*e.g.* air, wood, copper, brass *etc.*) have a high reluctance because they possess least value of μ_r ; being 1 in case of all non-magnetic materials.

* You may recall that H means force acting on a unit magnetic pole. If the unit pole is moved once around the magnetic circuit (*i.e.* distance covered is l), then work done $= H \times l$.

† Reluctance $= \dfrac{\text{m.m.f.}}{\text{flux}} = \dfrac{\text{AT}}{\text{Wb}} = \text{AT/Wb}$

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The reciprocal of permeability $\mu(= \mu_0\mu_r)$ corresponds to resistivity ρ of the electrical circuit and is called *reluctivity. It may be noted that magnetic permeability (μ) is the analog of electrical conductivity.*

(iii) **Permeance.** *It is the reciprocal of reluctance* and is a measure of the ease with which flux can pass through the material. Its unit is Wb/AT.

$$\text{Permeance} = \frac{1}{\text{Reluctance}} = \frac{a\mu_0\mu_r}{l}$$

Permeance of a magnetic circuit corresponds to conductance (reciprocal of resistance) in an electric circuit.

8.4. Comparison Between Magnetic and Electric Circuits

There are many points of similarity between magnetic and electric circuits. However, the two circuits are not anologous in all respects. A comparison of the two circuits is given below in the tabular form.

Magnetic Circuit

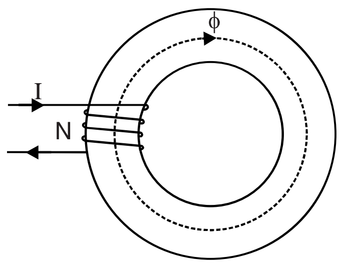

Fig. 8.2

Electric Circuit

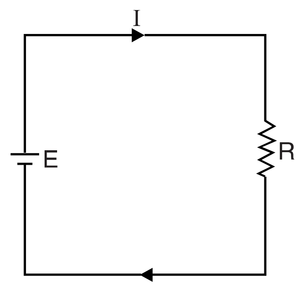

Fig. 8.3

Similarities

1. The closed path for magnetic flux is called a magnetic circuit.	**1.** The closed path for electric current is called an electric circuit.
2. Flux, $\phi = \dfrac{\text{m.m.f.}}{\text{reluctance}}$	**2.** Current, $I = \dfrac{\text{e.m.f.}}{\text{resistance}}$
3. m.m.f. (ampere-turns)	**3.** e.m.f. (volts)
4. Reluctance, $S = \dfrac{l}{a\mu_0\mu_r}$	**4.** Resistance, $R = \rho\dfrac{l}{a}$
5. Flux density, $B = \dfrac{\phi}{a}\,\text{Wb/m}^2$	**5.** Current density, $J = \dfrac{I}{a}\,\text{A/m}^2$
6. m.m.f. drop $= \phi\, S$	**6.** Voltage drop $= I\, R$
7. Magnetic intensity, $H = N\,I/l$	**7.** Electric intensity, $E = V/d$
8. Permeance	**8.** Conductance.
9. Permeability	**9.** Conductivity

Dissimilarities

1. Truly speaking, magnetic flux does not flow.	**1.** The electric current acutally flows in an electric circuit.
2. There is no magnetic insulator. For example, flux can be set up even in air (the best known magnetic insulator) with reasonable m.m.f.	**2.** There are a number of electric insulators. For instance, air is a very good insulator and current cannot pass through it.

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3. The value of μ_r is not constant for a given magnetic material. It varies considerably with flux density (B) in the material. This implies that reluctance of a magnetic circuit is not constant rather it depends upon B.	**3.** The value of resistivity (ρ) varies very slightly with temperature. Therefore, the resistance of an electric circuit is practically constant. This salient feature calls for different approach to the solution of magnetic and electric circuits.
4. No energy is expended in a magnetic circuit. In other words, energy is required in creating the flux, and not in maintaining it.	**4.** When current flows through an electric circuit, energy is expended so long as the current flows. The expended energy is dissipated in the form of heat.

8.5. Calculation of Ampere-Turns

In any magnetic circuit, flux produced is given by ;

$$\text{Flux, } \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{AT}{(l/a\mu_0\mu_r)}$$

$$\therefore \quad AT \text{ required} = \phi \times \frac{l}{a\mu_0\mu_r} = \frac{\phi}{a} \times \frac{l}{\mu_0\mu_r}$$

$$= \frac{B}{\mu_0\mu_r} \times l \qquad \left(\because \; B = \frac{\phi}{a}\right)$$

$$= H \times l \qquad (\because \; H = B/\mu_0\,\mu_r)$$

i.e. **AT required for any part = Field strength H in that part × length of that part of magnetic circuit**

8.6. Series Magnetic Circuits

In a series magnetic circuit, the same flux ϕ flows through each part of the circuit. It can just be compared to a series electric circuit which carries the same current throughout.

Consider a ***composite series magnetic circuit** consisting of three different magnetic materials of different relative permeabilities along with an air gap as shown in Fig. 8.4. Each part of this series magnetic circuit will offer reluctance to the magnetic flux ϕ. The reluctance offered by each part will depend upon dimensions and μ_r of that part. Since the different parts of the circuit are in series, the total reluctance is equal to the sum of reluctances of individual parts, *i.e.*

$$\text{Total reluctance} = \frac{l_1}{a_1\,\mu_0\,\mu_{r1}} + \frac{l_2}{a_2\,\mu_0\,\mu_{r2}} + \frac{l_3}{a_3\,\mu_0\,\mu_{r3}} + \frac{l_g}{a_g\,\mu_0}^{**}$$

$$\text{Total m.m.f.} = \text{Flux} \times \text{Total reluctance}$$

$$= \phi\left[\frac{l_1}{a_1\,\mu_0\,\mu_{r1}} + \frac{l_2}{a_2\,\mu_0\,\mu_{r2}} + \frac{l_3}{a_3\,\mu_0\,\mu_{r3}} + \frac{l_g}{a_g\,\mu_0}\right]$$

$$= \frac{\phi}{a_1\,\mu_0\,\mu_{r1}} \times l_1 + \frac{\phi}{a_2\,\mu_0\,\mu_{r2}} \times l_2 + \frac{\phi}{a_3\,\mu_0\,\mu_{r3}} \times l_3 + \frac{\phi}{a_g\,\mu_0} \times l_g$$

$$= \frac{B_1}{\mu_0\,\mu_{r1}} \times l_1 + \frac{B_2}{\mu_0\,\mu_{r2}} \times l_2 + \frac{B_3}{\mu_0\,\mu_{r3}} \times l_3 + \frac{B_g}{\mu_0} \times l_g$$

$$= H_1\,l_1 + H_2\,l_2 + H_3\,l_3 + H_g\,l_g \qquad (\because \; H = B/\mu_0\,\mu_r)$$

* A series magnetic circuit that has parts of different dimensions and materials is called a composite series circuit.

** For air, $\mu_r = 1$.

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Hence the total ampere-turns required for a series magnetic circuit can be found as under :

(*i*) Find H for each part of the series magnetic circuit. For air, $H = B/\mu_0$ whereas for magnetic material, $H = B/\mu_0\mu_r$.

(*ii*) Find the mean length (l) of magnetic path for each part of the circuit.

(*iii*) Find AT required for each part of the magnetic circuit using the relation, $AT = H \times l$.

(*iv*) The total AT required for the entire series circuit is equal to the sum of AT for various parts.

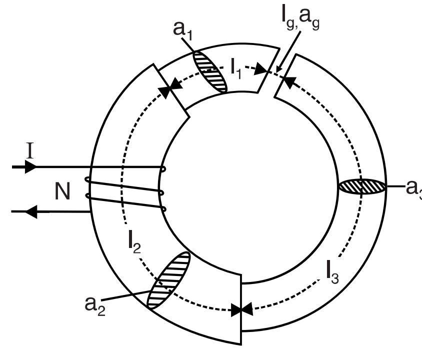

Fig. 8.4

8.7. Air Gaps in Magnetic Circuits

In many practical magnetic circuits, air gap is indispensable. For example, in electromechanical conversion devices like electric motors and generators, the magnetic flux must pass through stator as well as rotor. This necessitates to have a small air gap between the stator and rotor to permit mechanical clearance.

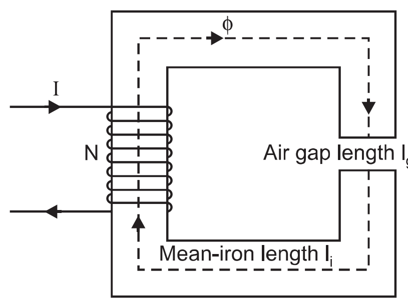

Fig. 8.5

The magnitude of AT required for air gap is much greater than that required for iron part of the magnetic circuit. It is because reluctance of air is very large compared to that offered by iron. Consider a magnetic circuit of uniform cross-sectional area a with an air gap as shown in Fig. 8.5. The length of the air gap is l_g and the mean length of iron part is l_i. The flux density $B(= \phi/a)$ is constant in the magnetic circuit.

$$\therefore \qquad \text{Reluctance of air gap} = \frac{l_g}{a\mu_0}$$

$$\text{Reluctance of iron part} = \frac{l_i}{a\mu_0\mu_r}$$

Now relative permeability μ_r of iron is very high (> 6000) so that reluctance of iron part is very small as compared to that of air gap inspite of the fact that $l_i > l_g$. In fact, most of ampere-turns (AT) are required in a magnetic circuit to force the flux through the air gap than through the iron part. In some magnetic circuits, we neglect reluctance of iron part compared to the air gap/gaps. This assumption leads to reasonable accuracy.

8.8. Parallel Magnetic Circuits

A magnetic circuit which has more than one path for flux is called a parallel magnetic circuit. It can just be compared to a parallel electric circuit which has more than one path for electric current.

The concept of parallel magnetic circuit is illustrated in Fig. 8.6. Here a coil of N turns wounded on limb AF carries a current of I amperes. The flux ϕ_1 set up by the coil divides at B into two paths, namely ;

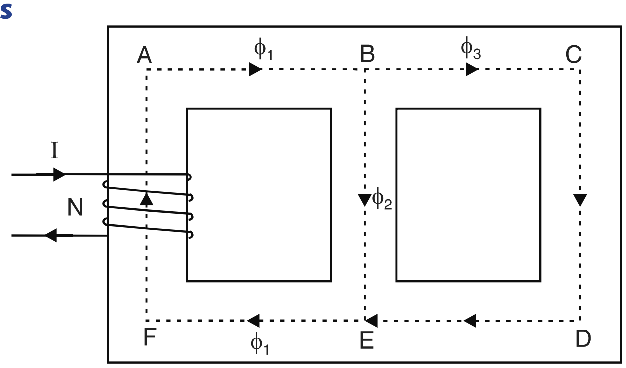

Fig. 8.6

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(*i*) flux ϕ_2 passes along the path *BE*

(*ii*) flux ϕ_3 follows the path *BCDE*

$$\text{Clearly, } \phi_1 = \phi_2 + \phi_3$$

The magnetic paths *BE* and *BCDE* are in parallel and form a parallel magnetic circuit. The *AT* required for this parallel circuit is equal to *AT* required for any *one of the paths.

Let S_1 = reluctance of path *EFAB*

S_2 = reluctance of path *BE*

S_3 = reluctance of path *BCDE*

$\therefore$ Total m.m.f. required = m.m.f. for path *EFAB* + m.m.f. for path *BE* or path *BCDE*

or
$$NI = \phi_1 S_1 + \phi_2 S_2$$
$$= \phi_1 S_1 + \phi_3 S_3$$

The reluctances S_1, S_2 and S_3 must be determined from a calculation of $l/a\mu_0\mu_r$ for those paths of the magnetic circuit in which ϕ_1, ϕ_2 and ϕ_3 exist respectively.

8.9. Magnetic Leakage and Fringing

The flux that does not follow the desired path in a magnetic circuit is called a **leakage flux.**

In most of practical magnetic circuits, a large part of flux path is through a magnetic material and the remainder part of flux path is through air. The flux in the air gap is known as *useful flux* because it can be utilised for various useful purposes. Fig. 8.7 shows an iron ring wound with a coil and having a narrow air gap. The total flux produced by the coil does not pass through the air gap as some of it **leaks through the air (path at '*a*') surrounding the iron. These flux lines as at '*a*' are called leakage flux.

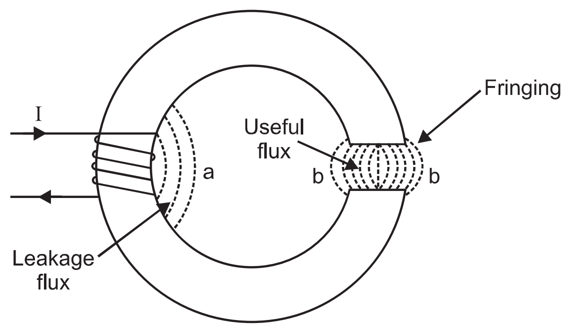

Fig. 8.7

Let ϕ_i = total flux produced *i.e.,* flux in the ***iron ring

ϕ_g = useful flux across the air gap

$\therefore$ Leakage flux, $\phi_{leak} = \phi_i - \phi_g$

$$\text{Leakage coefficient, } \lambda = \frac{\text{Total flux}}{\text{Useful flux}} = \frac{\phi_i}{\phi_g}$$

The value of leakage coefficients for electrical machines is usually about 1.15 to 1.25.

Magnetic leakage is undesirable in electrical machines because it increases the weight as well as cost of the machine. Magnetic leakage can be greatly reduced by placing source of m.m.f. close to the air gap.

Fringing. When crossing an air gap, magnetic lines of force tend to bulge out such as lines of force at *bb* in Fig. 8.7. It is because lines of force repel each other when passing through non-

* This means that we may consider either path, say path *BE*, and calculate *AT* required for it. The same *AT* will also send the flux (ϕ_3 in this case) through the other parallel path *BCDE.* The situation is similar to that of two resistors R_1 and R_2 in parallel in an electric circuit. The voltage *V* required to send currents (say I_1 and I_2) in the resistors is equal to that appearing across either resistor *i.e.* $V = I_1 R_1 = I_2 R_2$.

** Air is not a good magnetic insulator. Therefore, leakage of flux from iron to air takes place easily.

*** The flux ϕ_i is not constant all around the ring. However, for reasonable accuracy, it is assumed that the iron carries the whole of the flux produced by the coil.

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magnetic material such as air. This effect is known as *fringing*. The result of bulging or fringing is to increase the effective area of air gap and thus decrease the flux density in the gap. The longer the air gap, the greater is the fringing and *vice-versa*.

Note. In a short air gap with large cross-sectional area, the fringing may be insignificant. In other situations, 10% is added to the air gap's cross-sectional area to allow for fringing.

8.10. Solenoid

A long coil of wire consisting of closely packed loops is called a **solenoid.**

The word solenoid comes from Greek word meaning 'tube-like'. By a long solenoid we mean that length of the solenoid is very large as compared to its diameter. When current is passed through the coil of air-cored solenoid, magnetic field is set up as shown in Fig. 8.8. The path of the magnetic flux is made up of two components :

(*i*) length l_1 of the path within the coil

(*ii*) length l_2 of the path outside the coil.

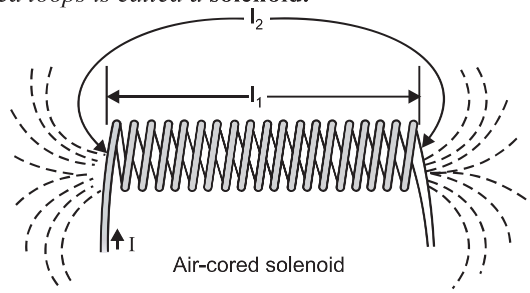

Fig. 8.8

The total m.m.f. required for the solenoid is the sum of m.m.f.s required for these two paths *i.e.*

Total m.m.f. = m.m.f. for path l_1 + m.m.f. for path l_2

But m.m.f. for path l_1 *>> m.m.f. for path l_2

$\therefore$ Total m.m.f. = m.m.f. for path l_1

Hence, for a solenoid (air-cored or iron-cored), the length of the magnetic circuit is the coil length l_1. We can use right-hand rule to determine the direction of magnetic field in the core of the solenoid.

Example 8.1. *A cast steel electromagnet has an air gap length of 3 mm and an iron path of length 40 cm. Find the number of ampere-turns necessary to produce a flux density of 0.7 Wb/m² in the gap. Neglect leakage and fringing. Assume ampere-turns required for air gap to be 70% of the total ampere-turns.*

Solution. Air-gap length, $l_g = 3 \text{ mm} = 3 \times 10^{-3}$ m

Flux density in air gap, $B_g = 0.7 \text{ Wb/m}^2$

$$\therefore \quad \text{Magnetising force, } H_g = \frac{B_g}{\mu_0 \mu_r} = \frac{0.7}{4\pi \times 10^{-7} \times 1} = 5.57 \times 10^5 \text{ AT/m}$$

$$AT \text{ required for air gap, } AT_g = H_g \times l_g = 5.57 \times 10^5 \times 3 \times 10^{-3} = 1671 \text{ AT}$$

It is given that : AT_g = 70% of total AT

$$\therefore \quad \text{Total } AT = \frac{AT_g}{0.7} = \frac{1671}{0.7} = \mathbf{2387 \ AT}$$

Example 8.2. *An iron ring has a cross-sectional area of 400 mm² and a mean diameter of 25 cm. It is wound with 500 turns. If the value of relative permeability is 250, find the total flux set up in the ring. The coil resistance is 474 Ω and the supply voltage is 240 V.*

* The lengths l_2 and l_1 do not differ very much. However, the cross-sectional area of path l_2 is very large as compared to that of path l_1. Therefore, reluctance of path l_2 is very small as compared to that of path l_1.

Now, m.m.f. = flux × reluctance

Since reluctance of path l_2 is very small, the m.m.f. required for this path is negligible compared to that for path l_1.

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Solution. The conditions of the problem are represented in Fig. 8.9.

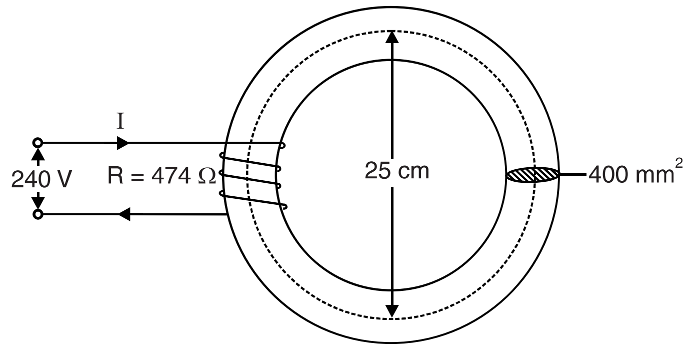

Fig. 8.9

Current through the coil, $I = V/R = 240/474 = 0{\cdot}506$ A

Mean length of magnetic circuit is given by ;

$$l = \pi \times (25 \times 10^{-2}) = 0{\cdot}7854 \text{ m}$$

$$\text{Magnetising force, } H = \frac{Nl}{l} = \frac{500 \times 0.506}{0.7854} = 322.13 \text{ AT/m}$$

$$\text{Flux density, } B = \mu_0 \mu_r H = (4\pi \times 10^{-7}) \times 250 \times 322{\cdot}13 = 0{\cdot}1012 \text{ Wb/m}^2$$

$$\therefore \quad \text{Flux in the ring, } \phi = B \times a = 0{\cdot}1012 \times (400 \times 10^{-6}) = \mathbf{40{\cdot}48 \times 10^{-6}\ Wb}$$

Example 8.3. *An iron ring of cross-sectional area 6 cm^2 is wound with a wire of 100 turns and has a saw cut of 2 mm. Calculate the magnetising current required to produce a flux of 0·1 mWb if mean length of magnetic path is 30 cm and relative permeability of iron is 470.*

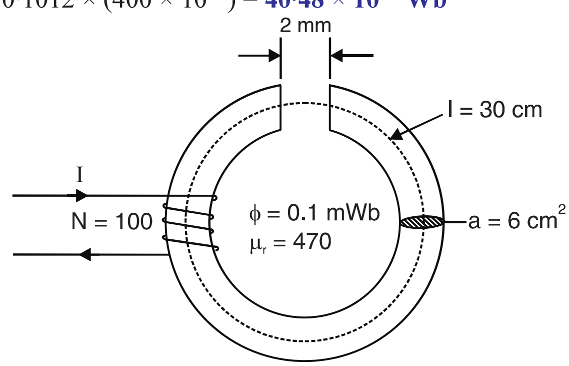

Fig. 8.10

Solution. The conditions of the problem are represented in Fig. 8.10. It will be assumed that flux density in the air gap is equal to the flux density in the core *i.e.* fringing is neglected. This assumption is quite reasonable in this case.

$$\text{Flux density, } B = \frac{\phi}{a} = \frac{0.1 \times 10^{-3}}{6 \times 10^{-4}} = 0.167 \text{ Wb/m}^2$$

Ampere-turns required for iron will be :

$$AT_i = H_i \times l_i$$

$$= \frac{B}{\mu_0 \mu_r} \times l_i = \frac{0.167}{4\pi \times 10^{-7} \times 470} \times 0.3 = 84.83 \text{ AT}$$

Ampere-turns required for air will be :

$$AT_g = \frac{B}{\mu_0} \times l_g = \frac{0 \cdot 167}{4\pi \times 10^{-7}} \times (2 \times 10^{-3}) = 265 \cdot 8 \text{ AT}$$

$$\therefore \quad \text{Total } AT = 265{\cdot}8 + 84{\cdot}83 = 350{\cdot}63 \text{ AT}$$

$$\therefore \quad \text{Magnetising current, } I = 350{\cdot}63/N = 350{\cdot}63/100 = \mathbf{3{\cdot}51\ A}$$

It may be seen that many more ampere-turns are required to produce the magnetic flux through 2 mm of air gap than through the iron part. This is expected because reluctance of air is much more than that of iron.

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Example 8.4. *A circular iron ring has a mean circumference of 1·5 m and a cross-sectional area of 0·01 m^2. A saw-cut of 4 mm wide is made in the ring. Calculate the magnetising current required to produce a flux of 0·8 mWb in the air gap if the ring is wound with a coil of 175 turns. Assume relative permeability of iron as 400 and leakage factor 1·25.*

Solution. $\phi_g = 0{\cdot}8 \times 10^{-3}$ Wb ; $a = 0{\cdot}01$ m^2 ; $l_i = 1{\cdot}5$m ; $l_g = 4 \times 10^{-3}$ m

AT for air gap

$$B_g = \frac{\phi_g}{a} = \frac{0.8\times10^{-3}}{0.01} = 0.08 \text{ Wb/m}^2$$

$$H_g = \frac{B_g}{\mu_0} = \frac{0\cdot08}{4\pi\times10^{-7}} = 63662 \text{ AT/m}$$

$$\therefore \quad AT_g = H_g \times l_g = 63662 \times (4 \times 10^{-3}) = 254{\cdot}6 \text{ AT}$$

AT for iron path

$$\phi_i = \phi_g \times \lambda = 0{\cdot}8 \times 10^{-3} \times 1{\cdot}25 = 10^{-3} \text{ Wb}$$

$$B_i = \phi_i/a = 10^{-3}/0{\cdot}01 = 0{\cdot}1 \text{ Wb/m}^2$$

$$H_i = \frac{B_i}{\mu_0\mu_r} = \frac{0\cdot1}{4\pi\times10^{-7}\times400} = 199 \text{ AT/m}$$

$$\therefore \quad AT_i = H_i \times l_i = 199 \times 1{\cdot}5 = 298{\cdot}5 \text{ AT}$$

$$\therefore \quad \text{Total } AT = 254{\cdot}6 + 298{\cdot}5 = 553{\cdot}1 \text{ AT}$$

$$\therefore \quad \text{Magnetising current, } I = 553{\cdot}1/N = 553{\cdot}1/175 = \mathbf{3{\cdot}16\ A}$$

Example 8.5. *A shunt field coil is required to develop 1500 AT with an applied voltage of 60 V. The rectangular coil is having a mean length of 50 cm. Calculate the wire size. Resistivity of copper may be assumed to be 2 × 10^{-6} Ω-cm at the operating temperature of the coil. Estimate also the number of turns if the coil is to be worked at a current density of 3 A/mm^2.*

Solution. Suppose the number of turns of coil is N.

Then the total length of the coil, $l = 50 \times N$ cm

Current in coil, $I = V/R = 60/R$

$$\text{Resistance of coil, } R = \rho\frac{l}{A} = 2\times10^{-6}\times\frac{50\times N}{A} = \frac{N\times10^{-4}}{A} \quad ...(i)$$

Also $NI = 1500$ or $N \times (60/R) = 1500$ $\therefore$ $R = N/25$ ...(*ii*)

From eqs. (*i*) and (*ii*), $\dfrac{N}{25} = \dfrac{N\times10^{-4}}{A}$ or $A = 25 \times 10^{-4}$ cm^2 = 0·25 mm^2

If D is the diameter of the wire, then,

$$\frac{\pi}{4}D^2 = 0{\cdot}25 \quad \text{or} \quad D = \mathbf{0{\cdot}568\ mm}$$

In order to operate the coil at a current density of 3 A/mm^2, the current in the coil is

$$I' = A \times \text{current density} = 0{\cdot}25 \times 3 = 0{\cdot}75 \text{ A}$$

$$\therefore \quad N'I' = 1500 \quad \text{or} \quad N' = 1500/I' = 1500/0{\cdot}75 = \mathbf{2000}$$

Example 8.6. *An iron ring has a mean diameter of 15 cm, a cross-section of 20 cm^2 and a radial gap of 0·5 mm cut in it. It is uniformly wound with 1500 turns of insulated wire and a magnetising current of 1 A produces a flux of 1 mWb. Neglecting the effect of magnetic leakage and fringing, calculate (i) reluctance of the magnetic circuit, (ii) relative permeability of iron and (iii) inductance of the winding.*

Solution. ***(ii)*** $a = 20 \times 10^{-4}$ m^2 ; $l_i = \pi \times 0{\cdot}15 = 0{\cdot}471$ m ; $l_g = 0{\cdot}5 \times 10^{-3}$ m

$$\text{Flux density in air gap, } B = \frac{\phi}{a} = \frac{1\times10^{-3}}{20\times10^{-4}} = 0{\cdot}5 \text{ Wb/m}^2$$

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Magnetising force in air gap, $H_g = B/\mu_0 = 0{\cdot}5/4\pi \times 10^{-7} = 398 \times 10^3$ AT/m

Ampere-turns for air gap, $AT_g = H_g \times l_g = (398 \times 10^3) \times 0{\cdot}5 \times 10^{-3} = 199$ AT

Total AT provided $= NI = 1500 \times 1 = 1500$ AT

$\therefore$ AT available for iron part, $AT_i = 1500 - 199 = 1301$ AT

Magnetising force in iron, $H_i = \dfrac{AT_i}{l_i} = \dfrac{1301}{0.471} = 2762$ AT/m

Now, $B = \mu_0\mu_r H_i$

$$\therefore \quad \mu_r = \frac{B}{\mu_0 H_i} = \frac{0.5}{4\pi \times 10^{-7} \times 2762} = \mathbf{144}$$

(*i*) Reluctance of air gap $= \dfrac{l_g}{a\mu_0} = \dfrac{0\cdot 5 \times 10^{-3}}{(20 \times 10^{-4}) \times 4\pi \times 10^{-7}} = 1{\cdot}99 \times 10^5$ AT/Wb

Reluctance of iron part $= \dfrac{l_i}{a\mu_0\mu_r} = \dfrac{0.471}{(20 \times 10^{-4}) \times 4\pi \times 10^{-7} \times 144} = 13{\cdot}01 \times 10^5$ AT/Wb

$\therefore$ Total circuit reluctance $= 10^5\,(1.99 + 13.01) =$ **15×10^5 AT/Wb**

(*iii*) Inductance of winding $= \dfrac{N\phi}{I} = \dfrac{(1500) \times (1 \times 10^{-3})}{1} =$ **1.5 H**

Example 8.7. *A magnetic circuit is constructed as shown in Fig. 8.11. Both sections A and B are of 20 mm by 20 mm square cross-section and the mean dimensions are 100 mm by 80 mm. The relative permeability of section A is 250 and of section B is 500. Find the reluctance of each section and the total circuit reluctance.*

If the joints between sections A and B have an air gap of 0·5 mm at each joint, find the total reluctance of the circuit.

Solution. The conditions of the problem are represented in Fig. 8.11. The area of X-section of the core, $a = 20 \times 20 = 400\ \text{mm}^2 = 4 \times 10^{-4}\ \text{m}^2$.

Section A

Length of magnetic path, $l_A = 80 + 10 + 10 = 100$ mm $= 0{\cdot}1$ m

$\therefore$ Reluctance of section $A = \dfrac{l_A}{a\mu_0\mu_r} = \dfrac{0\cdot 1}{(4 \times 10^{-4}) \times 4\pi \times 10^{-7} \times 250} =$ **$0{\cdot}796 \times 10^6$ AT/Wb**

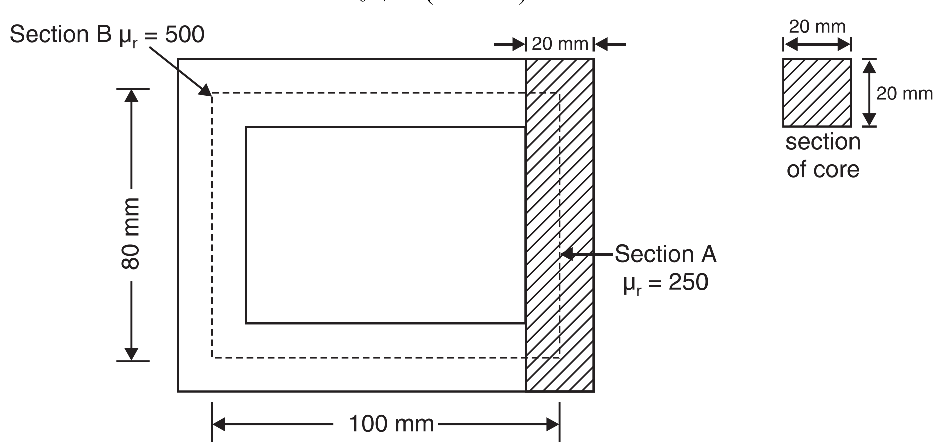

Fig. 8.11

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permeability 400. Find the current needed to produce a flux of 0·4 Wb in the core if it is wound with 1000 turns of insulated wire. Ignore leakage and fringing effects. **[636·8 A]**

4. An iron ring has a cross-sectional area of 400 mm^2 and a mean diameter of 250 mm. An air gap of 1 mm has been made by a saw-cut across the section of the ring. If a magnetic flux of 0·3 mWb is required in the air gap, find the current necessary to produce this flux when a coil of 400 turns is wound on the ring. The iron has a relative permeability of 500. **[3·84 A]**
5. An iron ring has a mean circumferential length of 60 cm and a uniform winding of 300 turns. An air gap has been made by a saw-cut across the section of the ring. When a current of 1 A flows through the coil, the flux density in the air gap is found to be 0·126 Wb/m^2. How long is the air gap ? Assume iron has a relative permeability of 300. **[1 mm]**
6. An iron magnetic circuit has a uniform cross-sectional area of 5 cm^2 and a length of 25 cm. A coil of 120 turns is wound uniformly over the magnetic circuit. When the current in the coil is 1·5 A, the total flux is 0·3 Wb. Find the relative permeability of iron. **[663]**
7. The uneven ring-shaped core shown in Fig. 8.20 has μ_r = 1000 and the flux density in the thicker section is to be 0.75 T. If the current through a coil wound on the core is to be 500 mA, determine number of coil turns required. **[567]**

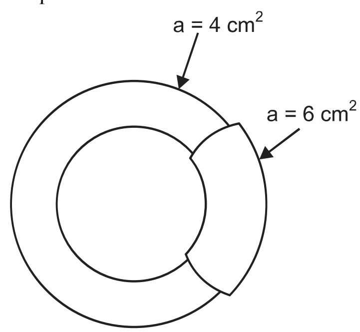

Fig. 8.20

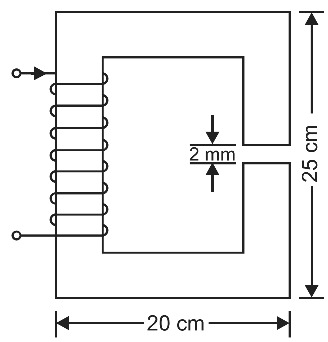

Fig. 8.21

8. A rectangular magnetic core shown in Fig. 8.21 has square cross section of area 16 cm^2. An air gap of 2 mm is cut across one of its limbs. Find the exciting current needed in the coil having 1,000 turns wound on the core to create an air-gap flux of 4 mWb. The relative permeability of the core is 2000. **[4.713 A]**
9. The magnetic circuit of Fig. 8.22 is energised by a current of 3A. If the coil has 1500 turns, find the flux produced in the air gap. The relative permeability of the core material is 3000. **[65.25 × 10^{-4} Wb]**

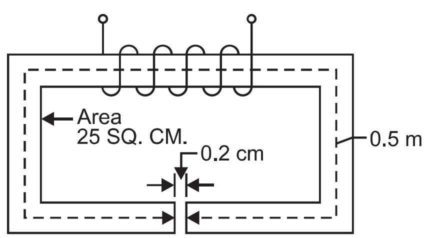

Fig. 8.22

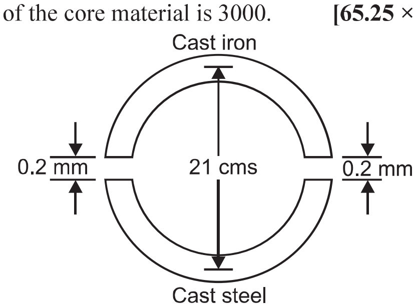

Fig. 8.23

10. A ring [See Fig. 8.23] has a diameter of 21 cm and a cross-sectional area of 10 cm^2. The ring is made up of semicircular sections of cast iron and cast steel with each joint having a reluctance equal to an air gap of 0.2 mm. Find the ampere turns required to produce a flux of 8 × 10^{-4} Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively. Neglect leakage and fringing effects. **[1783 AT]**

8.11. B-H Curve

The *B-H* curve (or magnetisation curve) indicates the manner in which the flux density (*B*) varies with the magnetising force (*H*).

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(*i*) For non-magnetic materials. For non-magnetic materials (*e.g.* air, copper, rubber, wood *etc.*), the relation between B and H is given by ;

$$B = \mu_0 H$$

Since μ_0 (= $4\pi \times 10^{-7}$ H/m) is constant,

$$\therefore \qquad B \propto H$$

Hence, the B-H curve of a non-magnetic material is a straight line passing through the origin as shown in Fig. 8.24. Two things are worth noting. First, the curve never saturates no matter how great the flux density may be. Secondly, a large m.m.f. is required to produce a given flux in the non-magnetic material *e.g.* air.

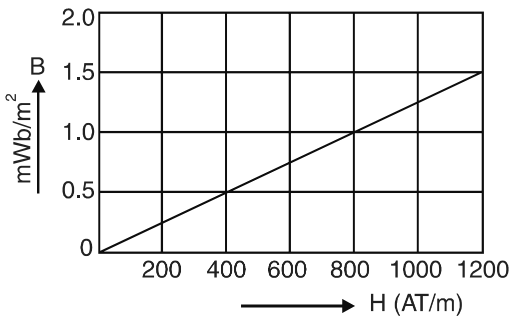
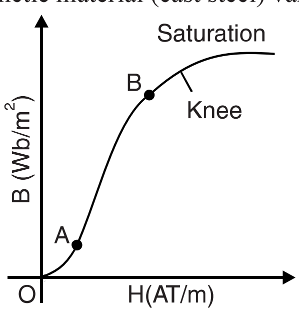


Fig. 8.24

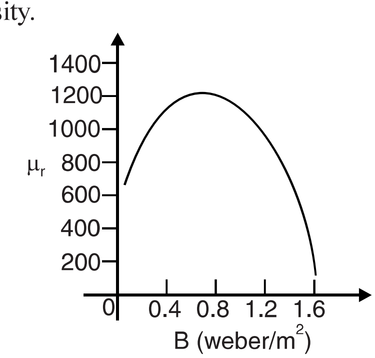
(*ii*) For magnetic materials. For magnetic materials (*e.g.* iron, steel *etc.*), the relation between B and H is given by ;

$$B = \mu_0 \mu_r H$$

Unfortunately, μ_r is not constant but varies with the flux density. Consequently, the B-H curve of a magnetic material is not linear. Fig. 8.25 (*i*) shows the general *shape of B-H curve of a magnetic material. The non-linearity of the curve indicates that relative permeability μ_r (= $B/\mu_0 H$) of a material is not constant but depends upon the flux density. Fig. 8.25 (*ii*) shows how relative permeability μ_r of a magnetic material (cast steel) varies with flux density.



(*i*)



(*ii*)

Fig. 8.25

While carrying out magnetic calculations, it should be ensured that the values of μ_r and H are taken at the working flux density. For this purpose, the B-H curve of the material in question may be very helpful. In fact, the use of B-H curves permits the calculations of magnetic circuits with a fair degree of ease.

8.12. Magnetic Calculations From B-H Curves

The solution of magnetic circuits can be easily obtained by the use of B-H curves. The procedure is as under :

(*i*) Corresponding to the flux density B in the material, find the magnetising force H from the B-H curve of the material.

* Note the shape of the curve. It is slightly concave up for 'low' flux densities (portion OA) and exhibits a straight line character (portion AB) for 'medium' flux densities. In the portion AB of the curve, the μ_r of the material is almost constant. For higher flux 'densities', the curve concaves down (called the *knee* of the curve). After knee of the curve, any further increase in H does not increase B. From now onwards, the curve is almost flat and the material is said to be *saturated.* In terms of molecular theory, saturation can be explained as the point at which all the molecular magnets are oriented in the direction of applied H.

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$$\therefore \qquad \mu_r = \frac{B}{\mu_0 H} = \frac{1 \cdot 28}{4\pi \times 10^{-7} \times 720} = \mathbf{1400}$$

Tutorial Problems

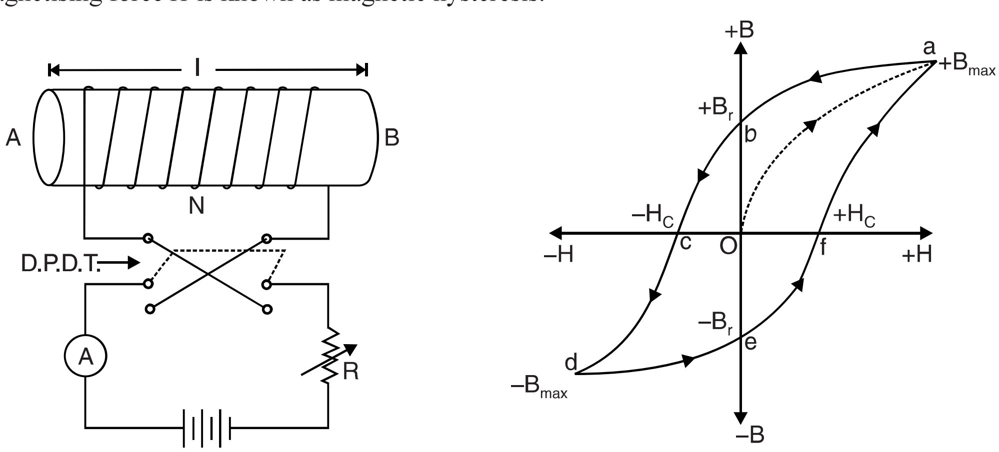
1. A moving coil ballistic galvanometer of 150Ω resistance gives a throw of 75 divisions when the flux through a search coil to which it is connected is reversed. Find the flux density given that the galvanometer constant is 110 μC per scale division and the search coil has 1400 turns, a mean area of 50 cm^2 and a resistance of 20Ω. **[0.1T]**
2. A fluxmeter is connected to a search coil having 500 turns and mean area of 5 cm^2. The search coil is placed at the centre of a solenoid one metre long wound with 800 turns. When a current of 5A is reversed, there is a deflection of 25 scale divisions on the fluxmeter. Calculate the fluxmeter constant. **[10^{-4} Wb-turn/division]**
3. A ballistic galvanometer connected to a search coil for measuring flux density in a core gives a throw of 100 scale divisions on reversal of flux. The galvanometer coil has a resistance of 180Ω. The galvanometer constant is 100μC per scale division. The search coil has an area of 50 cm^2 wound with 1000 turns having a resistance of 20Ω . Calculate the flux density in the core. **[0.2 T]**

8.16. Magnetic Hysteresis

When a magnetic material is subjected to a cycle of magnetisation (*i.e.* it is magnetised first in one direction and then in the other), it is found that flux density B in the material lags behind the applied magnetising force H. This phenomenon is known as hysteresis.

The phenomenon of lagging of flux density (B) behind the magnetising force (H) in a magnetic material subjected to cycles of magnetisation is known as **magnetic hysteresis.**

The term 'hysteresis' is derived from the Greek word *hysterein* meaning to lag behind. If a piece of magnetic material is subjected to *one cycle of magnetisation, the resultant B-H curve is a closed loop *abcdefa* called *hysteresis loop* [See Fig. 8.36 (*ii*)]. Note that B always lags behind H. Thus at point '*b*', H is zero but flux density B has a positive finite value *ob*. Similarly at point '*e*', H is zero, but flux density has a finite negative value *oe*. This tendency of flux density B to lag behind magnetising force H is known as magnetic hysteresis.



(*i*) (*ii*)

Fig. 8.36

* If we start with unmagnetised iron piece, then magnetise it in one direction and then in the other direction and finally demagnetise it (*i.e.* obtain the original condition we started with), the piece is said to go through one cycle of magnetisation. Compare it with one cycle of alternating current or voltage.

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Hysteresis Loop. Consider an unmagnetised iron bar *AB* wound with *N* turns as shown in Fig. 8.36 (*i*). The magnetising force H (= NI/l) produced by this solenoid can be changed by varying the current through the coil. The double-pole, double-throw switch (DPDT) is used to reverse the direction of current through the coil. We shall see that *when the iron piece is subjected to a cycle of magnetisation, the resultant B-H curve traces a loop abcdefa called hysteresis loop.*

(*i*) We start with unmagnetised solenoid *AB*. When the current in the solenoid is zero, $H = 0$ and hence B in the iron piece is 0. As H is increased (by increasing solenoid current), the flux density (+ B) also increases until the point of maximum flux density (+ B_{max}) is reached. The material is saturated and beyond this point, the flux density will not increase regardless of any increase in current or magnetising force. Note that *B-H* curve of the iron follows the path *oa*.

(*ii*) If now H is gradually reduced (by reducing solenoid current), it is found that the flux density B does not decrease along the same line by which it had increased but follows the path *ab*. At point *b*, the magnetising force H is zero but flux density in the material has a finite value + B_r (= *ob*) called **residual flux density.** It means that after the removal of H, the iron piece still retains some magnetism (*i.e.* + B_r). In other words, B lags behind H. The greater the lag, the greater is the residual magnetism (*i.e.* ordinate *ob*) retained by the iron piece. The power of retaining residual magnetism is called **retentivity** of the material.

The hysteresis effect (*i.e.* lagging of B behind H) in a magnetic material is due to the opposition offered by the magnetic domains (or molecular magnets) to the turning effect of magnetising force. Once arranged in an orderly position by the magnetising force, the magnetic domains do not return exactly to the original positions. In other words, the material retains some magnetism even after the removal of magnetising force. This results in the lagging of B behind H.

(*iii*) To demagnetise the iron piece (*i.e.* to remove the residual magnetism *ob*), the magnetising force H is reversed by reversing the current through the coil. When H is graudally increased in the reverse direction, the *B-H* curve follows the path *bc* so that when $H = oc$, the residual magnetism is zero. The value of H (= *oc*) required to wipe out residual magnetism is known as **coercive force** (H_c).

(*iv*) If H is further increased in the reverse direction, the flux density increases in the reverse direction (− B). This process continues (curve *cd*) till the material is saturated in the reverse direction ($-B_{max}$ point) and can hold no more flux.

(*v*) If H is now gradually decreased to zero, the flux density also decreases and the curve follows the path *de*. At point *e*, the magnetising force is zero but flux density has a finite value $-B_r$ (= *oe*) — the residual magnetism.

(*vi*) In order to neutralise the residual magnetism *oe*, magnetising force is applied in the positive direction (*i.e.* original direction) so that when $H = of$ (coercive force H_c), the flux density in the iron piece is zero. Note that the curve follows the path *ef*. If H is further increased in the positive direction, the curve follows the path *fa* to complete the loop *abcdefa.*

Thus when a magnetic material is subjected to one cycle of magnetisation, B always lags behind H so that the resultant B-H curve forms a closed loop, called **hysteresis loop.**

For the second cycle of magnetisation, a *similar loop *abcdefa* is formed. If a magnetic material is located within a coil through which alternating current (50 Hz frequency) flows, 50 loops will be formed every second. This hysteresis effect is present in all those electrical machines where the iron parts are subjected to cycles of magnetisation *e.g.* armature of a d.c. machine rotating in a stationary magnetic field, transformer core subjected to alternating flux etc.

* Owing to the nature of magnetic material, a second or even third cycle of H would not exactly lie on the tops of the first one. After a relatively few cycles, the successive loops would follow a fixed path.

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