$= n^2p^2 + npq$ $\qquad 1-p=q$

$\therefore V(x) = E(x^2) - \{E(x)\}^2$

$= n^2p^2 + npq - n^2p^2$

$= npq$

$S.D.\ (\sigma_x \text{ or } \sigma) = \sqrt{V(x)} = \sqrt{npq}$

cumulative $F(x) = P(X \le x)$ distribution function.

Cumulative binomial distribution (or cumulative distribution function for binomial distribution

For $X \sim B(n;p)$; the cdf is denoted as $B(x;n,p)$ and it is defined as:-

$$B(x;n,p) = P(X \le x) = \sum_{y=0}^{x} b(y;n,p)$$

46/a) $b(3;8,0.35)$

$x=3, n=8, p=0.35, q=0.65$

$= n_{c_3} \times (0.35)^3 \times (0.65)^5 \qquad {}^nc_r = \frac{\lfloor n}{\lfloor r \times \lfloor n-r}$

$= 8c_3 \times (0.35)^3 \times (0.65)^5$

$= \frac{\lfloor 8}{\lfloor 3 \times \lfloor 5} \times 0.004972460546$

$= \frac{8 \times 7 \times \cancel{6}^{3} \times \cancel{\lfloor 5}}{\cancel{2} \times 3\ \cancel{\lfloor 5}} \approx 0.278$

c) $P(3 \le x \le 5),\ n=7, p=0.6, q=0.4$

$= b(3;7,0.6) + b(4;7,0.6) + b(5;7,0.6)$

$= 7c_3 \times (0.6)^3 \times (0.4)^4 + 7c_4 \times (0.6)^4 \times (0.4)^3 + 7c_5 \times (0.6)^5 \times (0.4)^2$

$= (0.6)^3 \times (0.4)^2 \left(7c_3 \times (0.4)^2 + 7c_4 \times 0.6 \times 0.4 + 7c_5 \times (0.6)^2\right)$

≈ 0.745

Ex: 3.4

4(i) b) $b(5; 8, 0.6) =$ ~~8c5~~

$b(x; n, p)$

Here, $x = 5$, $n = 8$, $p = 0.6$, $q = 1 - 0.6 = 0.4$

$\therefore b(5; 8; 0.6) = {}^8C_5 \times (0.6)^5 \times (0.4)^3$

$= \frac{\lfloor 8}{\lfloor 5 \times \lfloor 3} \times (0.6)^5 \times (0.4)^3$

$= \frac{8 \times \cancel{7} \times \cancel{6} \times \cancel{\lfloor 5}}{\cancel{\lfloor 5} \times \cancel{2} \times \cancel{3}} \times (0.6)^5 \times (0.4)^3$

≈ 0.2786

$n = 15$

X: The ...

d) $P(1 \le X), n = 9, p = 0.1$

$P(1 \le X \le 9)$

$P(X \ge 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - B(0; 9; 0.1)$

$= 1 - {}^{9}C_{0} \, p^{0} \times q^{9}$

$= 1 - \dfrac{9!}{0! \times 9!} \times 1 \times (0.9)^{9}$

$= 1 - (0.9)^{9}$

$\simeq 0.612$

$= 0.2206$

4E) X: The no. of auto accidents ~~that involve a single vehicle~~ $n = 15$

a) $P(X \leq 4) = B(4; 15, 0.7)$

It is given that 7 in 10 auto accidents involve a single vehicle

$\therefore p = 0.7$

$q = 1 - 0.7 = 0.3$

$\therefore B(4; 15, 0.7) = 15_{C_4} \times (0.7)^4 \times (0.3)^{15-4}$

$= 0.001$

b) $P(X = 4) = B(4; 15, 0.7) - B(3; 15, 0.7)$

$= 0.001 - 0.000$

$= 0.001$

c) $P(X = 6) = P(6; 15, 0.3) - P(5; 15, 0.3)$

$=$ $-$

d) $P(2 \leq X \leq 4) = B(4; 15, 0.7) - P(1; 15, 0.7)$

$= 0.001 - 0.000$

$= 0.001$

e) $P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1)$

$= 1 - B(1; 15, 0.7)$

$= 1 - 0.000 = 1$

f) Since if exactly 4 involve a single vehicle, the other 11 must be multiple.

$\therefore P(X = 4) = 0.001$

48) $n = 25$, $p = 0.05$, $q = 0.95$ / $B(25, 0.05)$ X: No. of children who have food allergy

a) $P(X \leq 3) = B(0; 25; 0.05)$
$+ B(1; 25, 0.05)$
$+ B(2; 25; 0.05)$
$+ B(3; 25; 0.05)$
$= 0.966$

$P(X < 3) = P(X \leq 2) = B(2; 25; 0.05)$
$= 0.873$

b) $P(X \geq 4) = 1 - P(X < 4) = 1 - P(X \leq 3) = 1 - 0.966 = 0.034$

c) $P(1 \leq X \leq 3) = B(3; 25, 0.05) - B(0; 25; 0.05)$
$= 0.966 - 0.277$
$= 0.689$

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d) $E(x) = np = 25 \times 0.05 = 1.25$

$\sigma_x = \sqrt{npq} = \sqrt{1.25 \times 0.95} = \sqrt{1.1875} \approx 1.089$

e) $n = 50,\ x = 0,\ p = 0.05$

$b(0, 50, 0.05)$

$= {}^{50}C_0 \times (0.05)^0 \times (0.95)^{50}$

$= 0.077$

Ex: 3.3

49) 10% of ~~a~~ the company's goblets have cosmetic flaws

$\therefore p = 0.1$

$q = 0.9$

a) $n = 6$

$\therefore P(X=1) = b(1; 6, 0.1) - \cancel{B(0;6,0.1)}$

$= 6c_1 \times 0.1 \times (0.9)^5 \cancel{- 6c_0 \times 0.1 \times (0.9)^5}$

$= \frac{\lfloor 6}{\lfloor 5} \times 0.1 \times (0.9)^5$

$= 0.354$

b) $n = 6$

$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1) = 1 - 0.354 - b(0; 6, 0.1)$

$= 0.646 - 6c_0 \times 0.1^0 \times (0.9)^6$

≈ 0.114

c) ~~n≥5~~

The probability that the first four goblets are without flaw is $= 4c_0 \times (0.1)^0 \times (0.9)^4 = 0.656$

The probability that one of the first four goblets ~~don't~~ have a flaw but the fifth one doesn't $= 4c_1 \times (0.1)^1 \times (0.9)^3 \times 0.9$

$= 0.262$

$\therefore$ The probability is $= 0.656 + 0.262 = 0.918$

$\lfloor 6 \times \lfloor 19$

~~25×24×23×~~

Continuing classwork

$n = 25,\ p = 0.25,\ q = 0.75$

$\sum_{y=0}^{6} b(y; 25, 0.25)$

50) a) $P(X \le 6) = B(6; 25, 0.25)$ $\quad {}^nC_x\, p^x\, q^{n-x}$

$= {}^{25}C_6 \times (0.25)^6 \times (0.75)^{25-6}$ $\quad = 0.561$

Let, x be the no. of calls that involve fax messages.

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b) $P(X=6) = B(6;25;0.25) - B(5;25;0.25)$

$= 0.561 - 0.378$

$= 0.18$

c) $P(X \geq 6) = 1 - P(X<6)$

$= 1 - P(X \leq 5)$

$= 1 - B(5;25;0.25)$

$= 1 - 0.378$

$= 0.622$

d) $P(X>6) = 1 - P(X \leq 6)$

$= 1 - 0.561$

$= 0.439$

51) a) $E(x) = np = 25 \times 0.25 = 6.25$

b) $\sigma_x = \sqrt{npq} = \sqrt{25 \times 0.25 \times 0.75} = 2.165$

c) $P(X > E(x) + 2\sigma_x) = P(X > 6.25 + 2 \times 2.165)$

$= P(X > 10.58)$

$= 1 - P(X \leq 10.58) = 1 - P(X \leq 10)$ Because not in decimal

$= 1 - B(10;25;0.25)$

$= 1 - 0.97$

$= 0.03$

53) y 0 1 2

52) $n = 25, \quad p = 0.3, \quad q = 0.7$

i) Mean value $= E(x) = np = 25 \times 0.3 = 7.5$

$\sigma = \sqrt{npq} = \sqrt{7.5 \times 0.7} = 2.291$

ii) $P(x > E(x) + 2\sigma_x) = P(x > 7.5 + 2 \times 2.291)$

$= P(x > 12.082)$

$= 1 - P(x \leq 12.082)$

$= 1 - P(x \leq 12)$

$= 1 - B(12; 25, 0.3) = 1 - 0.983$

$= 0.017$

$V(x) = npq = 7.5 \times 0.7 = 5.25$

$P(X > V(x) + 2\sigma_x) = P(X > 5.25 + 2 \times 2.291)$

$= P(X > 9.832)$

$= 1 - P(X \leq 9.832)$

$= 1 - P(X \leq 9)$

$= 1 - B(9; 25, 0.3)$

$= 1 - 0.811$

$= 0.189$

iii) X = The no. of people who want a new copy

$P(10 \leq X \leq 15) = B(15; 25, 0.3) - P(9; 25, 0.3)$

$= 1 - 0.811$

$= 0.189$

iv) $H(x)$ = The revenue when X of the 25 purchasers want new copies $= 100 \times X$

~~G(x) = The~~

$G(x)$ = The revenue when the remaining purchasers want used copies $= (25 - X)70$

$\therefore$ Total revenue $= H(x) + G(x)$

$= 100X + (25 - X)70$

$= \cancel{100X +} 30X + 1750$

$\therefore$ Expected value of the total revenue $= E(30X + 1750)$

$= 30E(x) + 1750$

$= 30 \times 7.5 + 1750$

$= 1975$

54) $n = 10, p = 0.6, q = 0.4$

$= 0.03$

53)

Y	0	1	2	3
$P(Y)$	0.6	0.25	0.1	0.05

Y: The no. of traffic citations for a randomly ~~chosen~~ selected individual.

X → No. of individuals with Y citations.

a) $n = 15,\ p = 0.6,\ q = 0.4$

$P(X \geq 10)$

$1 - P(X < 10)$

$= 1 - P(X \leq 9)$

$= 1 - B(9; 15; 0.6)$

$= 1 - 0.597$

$= 0.403$

b) $P(X < 7.5) = P(X \leq 7)$ $\quad p = 0.25 + 0.1 + 0.05 = 0.4$

$q = 0.6$

$= B(7; 15; 0.4)$

$= 0.787$

c) $P(5 \leq X \leq 10)$, $p = 0.4$, $q = 0.6$

$= B(10; 15; 0.4) - B(4; 15; 0.4)$

$= 0.991 - 0.217$

$= 0.774$

5)

$n=10,\ p=0.6,\ q=0.4$

a) $P(X\geq 6) = 1-P(X<6) = 1-P(X\leq 5)$

$=1-B(5;10,0.6)$

$=1-0.\cancel{800}367$

$=\cancel{0.799}\ 0.633$

54) b) $P(E(x) - \sigma_x \le X \le E(x) + \sigma_x)$

$\cancel{= P(7.8}$

$E(x) = np = 10 \times 0.6 = 6$

$V(x) = npq = 6 \times 0.4 = 2.4$

$\sigma_x = \sqrt{npq} = \sqrt{2.4} = 1.549$

$P(E(x) - \sigma_x \le X \le E(x) + \sigma_x)$

$= P(6 - 1.549 \le X \le 6 + 1.549)$

$= P(4.451 \le X \le 7.549)$

$= P(5 \le X \le 7)$

$= B(7; 10, 0.6) - B(4; 10, 0.6)$

$= 0.833 - 0.166$

$= 0.667$

c) $P(3 \le X \le 7) = B(7; 10, 0.6) - B(2; 10, 0.6)$

$= 0.833 - 0.012$

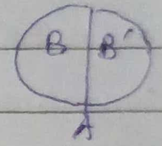
$= 0.821$

55) $n = 10$

~~Let, A be the event~~

$A \rightarrow$ The no. of telephones that are coming under service while under warranty.

$P(A) = 0.2$



$B \rightarrow$ ~~The no. of telephones that can be~~ repaired under warranty.

$B' \rightarrow$ The no. of telephones that can be replaced under warranty.

$P(B|A) = 0.6$

$P(B'|A) = 0.4$

$p = P(B' \cap A) = P(B'|A) \times P(A)$

$= 0.4 \times 0.2$

$= 0.08$

$P(X=2) = b(2; n, p) =$

$= b(2; 10; 0.08)$

$= {}^{10}C_2 \times (0.08)^2 \times (0.92)^8$

$= \dfrac{\lfloor 10}{\lfloor 2 \times \lfloor 8} \times (0.08)^2 \times (0.92)^8$

$= \dfrac{\cancel{10}^{5} \times 9 \times \cancel{\lfloor 8}}{\cancel{2} \times \cancel{\lfloor 8}} \times (0.08)^2 \times (0.92)^8$

$= 0.1478$

56) $n = 25$

$p = 0.02, \; q = 0.98$

a) $P(X=1) = b(1; 25, 0.02) = {}^{25}C_1 \times (0.02)^1 \times (0.98)^{24}$

$= 25 \times 0.02 \times (0.98)^{24}$

$= 0.307$

b) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X \leq 0) = 1 - b(0; 25, 0.02)$

$= 1 - (0.98)^{25}$

$= 0.396$

c) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - P(X \leq 1) = 1 - 0.307 - 0.396 = 0.297$

d) $P(E(x) - 2\sigma_x \leq X \leq E(x) + 2\sigma_x)$

$E(x) = np = 25 \times 0.02 = 0.5$

$V(x) = npq = 0.5 \times 0.98 = 0.49$

$\sigma_x = \sqrt{npq} = \sqrt{0.49} = 0.7$

$= \cancel{P(E(x) - 2x}$

$\therefore P(E(x) - 2\sigma_x \leq X \leq E(x) + 2\sigma_x)$

$= P(0.5 - 2\times 0.7 \leq X \leq 0.5 + 2\times 0.7)$

$= P(-0.9 \leq X \leq 1.9)$

$= P(X \leq 1)$

$= 0.307 + 0.396$

$= 0.703$

9(e) The expected no. of students for students with special accomodation $= 0.5$

The expected no. of students for students with no accomodation $= nq = 25\times 0.98 = 24.5$

The total expected time $= 0.5 \times 4.5 + 24.5 \times 3$

$= 75.75$

$\therefore$ The average time $= \frac{75.75}{25} = 3.03$

57)

X = The no. of flashlights that works.

A : Batteries with accepted voltages = 0.9

~~P(X) = P(A)~~

$p = P(A \cap A) = P(A) \cdot P(A) = 0.9 \times 0.9 = 0.81$

$\therefore q = 0.19$

5E) $P(x \geq 9) = P(x=9) + P(x=10)$

$= b(9;10,0.81) + b(10;10,0.81)$

$= 0.2851 + 0.1215$

$= 0.4066$

$r\left(\frac{1-p}{p}\right)$

Ex: 3·4

59) X = The no. of homes with detectors

$n = 25$

It is given that we'll be rejecting the claim that $p \geq 0.8$ if $x \leq 15$

i) P(The claim is rejected if p is 0.8) $= P(x \leq 15)$

$= B(15; 25, 0.8) = 0.017$

ii) P(The claim is not rejected if $p = 0.7$) $= P(X > 15) = 1 - P(X \leq 15)$

$= 1 - B(15; 25, 0.7)$

$= 1 - 0.189$

$= 0.811$

iii) P(The claim is not rejected if $p = 0.6$) $= 1 - B(15; 25, 0.6)$

$= 1 - 0.575$

$= 0.425$

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c) $B(14; 25, 0.8) = \cancel{{}^{25}C_{14} \times (0.8)^{14} \times (0.2)^{25-14}}$ 0.006

$1 - B(14; 25, 0.7) = 1 - 0.098 = 0.902$

$1 - B(14; 25, 0.6) = 1 - 0.414 = 0.586$

60) X = The no. of passenger cars
$25 - X$ = The no. of other vehicles
$h(x) = 1 \times X$ = Revenue for X
$g(x)$ = The revenue for other cars
$= (25 - X)\, 2.5$
$\therefore$ The total revenue $= X + (25 - X)\,2.5$
$= X + 62.5 - 2.5X$
$= 62.5 - 1.5X$
$\therefore$ The resulting expected toll revenue $= E(62.5 - 1.5x)$
$= 62.5 - 1.5E(x) = 62.5 - 1.5 \times np$
$= 62.5 - 1.5 \times 25 \times 0.6$
$= \$40$

61) $p = 0.9$
$n_A = 2,\ n_B = 4$
$P(X_A \geq 1) = P(X_A = 2) + P(X_A = 1)$
$= b(2; 2, 0.9) + b(1; 2, 0.9)$
$= \cancel{0.19}\ 0.18 + 0.81$
$= 0.99$

$P(X_B \geq 2) = P(X_B = 2) + P(X_B = 3) + P(X_B = 4)$
$= B(2; 4, 0.9) + B(3; 4, 0.9) + B(4; 4, 0.9)$
$= 0.0486 + 0.2916 + 0.6561$
$= 0.9963$

If $p = 0.5$
$P(X_A \geq 1) = P(X_A = 2) + P(X_A = 1)$
$= b(2; 2, 0.5) + b(1; 2, 0.5)$
$= 0.5 + 0.25 = 0.75$

$$P(X_B \geq 2) = P(X_B=2) + P(X_B=3) + P(X_B=4)$$
$$= b(2; 4, 0.5) + b(3; 4, 0.5) + b(4; 4, 0.5)$$
$$= 0.375 + 0.25 + 0.0625$$
$$= 0.6875$$

62) a) We know that, $V(x) = npq$

$$\Rightarrow V(x) = np(1-p)$$

From the above equation, we can see that $V(x)=0$ when $p=0$ or $p=1$

$\therefore V(x)=0$ when $0 \leq p \leq 1$

Hence, proved.

b) $$\frac{d}{dp} V(x) = 0$$

$$\Rightarrow \frac{d}{dp}\{np(1-p)\} = 0$$

$$\Rightarrow n \frac{d}{dp}\{p(1-p)\} = 0$$

$$\Rightarrow n\left(p\times(0-1) + (1-p)\right) = 0$$

$$\Rightarrow n(-p+1-p) = 0$$

$$\Rightarrow n(1-2p) = 0$$

$$\Rightarrow p = 1/2$$

$V(x)$ is maximized when $p = 1/2$.

63) a) $b(x; n, 1-p) = b(n-x; n, p)$

$$b(x; n, 1-p) = {}^nC_x \times (1-p)^x \times (1-(1-p))^{n-x}$$

$$= {}^nC_x \times (1-p)^x \times (p)^{n-x} = \frac{\lfloor n}{\lfloor x \lfloor n-x} \times (1-p)^x \times p^{n-x}$$

$$b(n-x; n, p) = {}^nC_{n-x} \times p^{n-x} \times (1-p)^{n-(n-x)}$$

$$= \frac{\lfloor n}{\lfloor n-x \times \lfloor x} \times (p)^{n-x} \times (1-p)^x$$

$n-(n-x)$
$= x$

$b(x; n, 1-p) = b(n-x; n, p)$

Hence, proved.

b) To prove $B(x; n, 1-p) = 1 - B(n-x-1; n, p)$

$$B(x; n, 1-p) = \sum_{y=0}^{x} b(y; n, 1-p) = P(X \le x)$$

$$P(X > x) = 1 - B(x; n, 1-p)$$
$$= 1 - \sum_{y=0}^{x} b(y; n, 1-p)$$
$$= \sum_{y=x+1}^{n} b(y; n, 1-p)$$

From part (a)

$b(x; n, 1-p) = b(n-x; n, p)$

$$\therefore \sum_{y=x+1}^{n} b(y; n, 1-p) = \sum_{y=x+1}^{n} b(n-y; n, p)$$

$$P(X > x) = \sum_{y=x+1}^{n} b(n-y; n, p) = B(n-x-1; n, p)$$

$$\cancel{\therefore P(X \le x) = 1 - \sum_{y=x+1}^{n} b(n-y; n, p)}$$

$$P(X \le x) = 1 - P(X > x) = 1 - B(n-x-1; n, p)$$
$$\Rightarrow B(x; n, 1-p) = 1 - B(n-x-1; n, p)$$

Hence, proved.

Moment generating function → It gives us ...

For a random variable 'x', then the moment generating function is denoted as $M_x(t)$ and is defined as

$$M_x(t) = E(e^{tx}) = \sum_{x \in D} e^{tx} p(x)$$

kth partial derivative.

Note :- $E(x^k) = \left\{ \frac{\partial^k}{\partial t^k} M_x(t) \right\}_{t=0}$

$\frac{\partial'}{\partial t}$

$$\left\{ \frac{\partial}{\partial t} M_x(t) \right\}_{t=0} = \left\{ \sum_{x \in D} \frac{\partial}{\partial t} (e^{tx} p(x))_{t=0} \right\}$$

$$= \left\{ \sum_{x \in D} x e^{tx} p(x) \right\}_{t=0}$$

$$= \sum_{x \in D} x p(x) = E(x)$$ when $k=1$

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$$\left\{\frac{\partial^2}{\partial t^2} M_x(t)\right\}_{t=0} = \left\{\sum_{x \in D} x^2 e^{tx} p(x)\right\}_{t=0}$$

$$= \sum_{x \in D} x^2 p(x) = E(x^2) \quad k=2$$

3.4 Binomial distribution (It is corresponding

Derivation imp.

Binomial distribution:

Let, 'A' be an event that occurs in x no. of trials where n is the total no. of trials in the experiment.

For $X \sim B(n; p)$, the pmf is given by $p(x) = {}^{n}C_{x} \times p^{x} \times q^{n-x}$

Random variable which follows the binomial distribution

$p \rightarrow$

$= b(x; n, p)$

$x = 0, 1, 2, \ldots, n$

The moment generating function for a random variable X is: $M_X(t) = E(e^{tX}) = \sum_{x=0}^{n} e^{tx}\, b(x; n, p)$ → pmf

$$= \sum_{x=0}^{n} e^{tx} \times {}^{n}C_{x} \times p^{x} \times q^{n-x}$$

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$$= \sum_{x=0}^{n} {}^{n}C_{x} (pe^{t})^{x} \times (q)^{n-x}$$

$$(p+q)^n = p^n + q^n + p^{n-1}q + p^{n-2}q^2 + \dots$$

$$=$$

$$(a+b)^n = a^n + {}^{n}C_{1} a^{n-1} b^1 + {}^{n}C_{2} a^{n-2} b^2 + \dots + {}^{n}C_{r} a^{n-r} b^r + \dots + b^n$$

$$= \sum_{r=0}^{n} {}^{n}C_{r} (a)^r (b)^{n-r} \qquad r = x$$

$$M_x(t) = E(e^{tx}) \quad => (pe^t + q)^n$$

Now, $\left\{\frac{\partial}{\partial t} M_x(t)\right\}_{t=0} = \left\{\frac{\partial}{\partial t}(pe^t+q)^n\right\}_{t=0}$

$$= \left\{ n(pe^t+q)^{n-1} \times pe^t \right\}_{t=0}$$

$$= n(p+q)^{n-1} \times p \quad \text{(Putting } t=0)$$

$$= np \quad (\text{As } p+q=1)$$

$$\therefore E(x) = \left\{\frac{\partial}{\partial x}(M_x(t))\right\}_{t=0} = np$$

~~64) F(x) =~~

65) $n=100, \; p = P(B) = 0.2, \; q = 1-p = 1-0.2 = 0.8$

a) $\text{Mean} = np = 100 \times 0.2 = 20$

$\text{Variance} = npq = 20 \times 0.8 = 16$

b) $n=100, \; p = P(B)+P(A) = 0.7, \; q = 1-p = 0.3$

$\text{Mean} = np = 0.7 \times 100 = 70$

$\text{Variance} = npq = 70 \times 0.3 = 21$

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6?) ~~$P(|x-\mu| \geq k\sigma)$~~

$p = 0.5,\ n = 20,\ q = 0.5$

$\mu = np = 20 \times 0.5 = 10$

$V(x) = npq = 10 \times 0.5 = 5$

$\sigma = \sqrt{npq} = \sqrt{5} = 2.236$

$|X - \mu| \geq k\sigma$ when $k = 2$,

$|x - 10| \geq 2 \times 2.236$

$\Rightarrow |X - 10| \geq 4.472$

$\Rightarrow X - 10 \geq 4.472$ or $(X - 10) \leq -4.472$

$\Rightarrow X \geq 14.472$ or $X \leq 5.528$

$\therefore X \geq 15$ or $X \leq 5$

$\therefore P(X \geq 15) + P(X \leq 5)$

$= 1 - P(X \leq 14) + P(X \leq 5)$

$= 1 - B(14; 20, 0.5) + B(5; 20, 0.5)$

$= 1 - 0.979 + 0.021$

$= 0.042$

$\frac{1}{k^2} = \frac{1}{4} = 0.25$ (so, it is significantly lesser than the upper bound)

when $k = 3$,

$|X - 10| \geq 3 \times 2.236$

$\Rightarrow X - 10 \geq 6.708$ or $X - 10 \leq -6.708$

$\Rightarrow X \geq 16.708$ or $X \leq 3.292$

$X \geq 16$ or $X \leq 3$

$P(X \geq 16) + P(X \leq 3)$

$1 - B(15, 20, 0.5) + B(3; 20, 0.5)$

$= 1 - 0.994 + 0.001$

$= 0.007$

$\frac{1}{k^2} = \frac{1}{9} = 0.111$

(so, it is significantly lesser than the upper bound)

when $p = 0.75$, $q = 0.25$

$\mu = np = 20 \times 0.75 = 15$

$V(x) = npq = 15 \times 0.25 = 3.75$

$\sigma_x = \sqrt{npq} = 1.936$

$|X-\mu| \geq k\sigma$ when $k=2$,

$|X-15| \geq 2 \times 1.936$

$|X-15| \geq 3.872$

$X-15 \geq 3.872$ or $X-15 \leq -3.872$

$\Rightarrow X \geq 18.872$ or $X \leq 11.128$

$X \geq 19$ or $X \leq 11$

$P(X \geq 19) + P(X \leq 11)$

$1 - B(18; 20, 0.25) + B(11; 20, 0.25)$

$= 1 - \cancel{0.909}\ 0.976 + 0.041$

$= \cancel{0.132}\ 0.065$

$\frac{1}{k^2} = \frac{1}{4} = 0.25$

So, it is significantly lesser than the upper bound.

$\cancel{|X-\mu|}\ |X-15| \geq 3 \times 1.936$ when $k=3$

$\Rightarrow |X-15| \geq 5.808$

$\Rightarrow X-15 \geq 5.808$ or $X-15 \leq -5.808$

$\Rightarrow X \geq 20.808$ or $X \leq 9.192$

$\Rightarrow X \geq 21$ or $X \leq 9$

$\therefore P(X \geq 21) + P(X \leq 9)$

$= \cancel{1 - P(X \leq 20)} + P(X \leq 9)$

$= \cancel{1 - B(20; 20, 0.25)} + B(9; 20, 0.25)$

$= \cancel{0.412}\ 0.004$

$\frac{1}{k^2} = \frac{1}{9} = 0.111$

So, it is significantly lesser than the upper bound.