Solution: Doubly Reinforced Beam (Practice Question 3)

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Problem Statement

A doubly reinforced concrete beam has a cross-section of 300 mm (width) $\times$ 550 mm (overall depth).

- The tension reinforcement consists of 4 bars of 25 mm diameter.
- The compression reinforcement consists of 2 bars of 16 mm diameter.

The clear cover to all reinforcement is 30 mm. Use M25 grade concrete ($f_{ck} = 25$ N/mm^2) and Fe415 grade steel ($f_y = 415$ N/mm^2). Find the ultimate Moment of Resistance of the section.

Solution

Step 1: Given Data and Properties

- Width, $b = 300$ mm
- Overall Depth, $D = 550$ mm
- Concrete, M25: $f_{ck} = 25$ N/mm^2
- Steel, Fe415: $f_y = 415$ N/mm^2
- Area of Tension Steel (A_{st}):

$$A_{st} = 4 \times \frac{\pi}{4} \times (25)^2 = 1963.5 \text{ mm}^2$$

- Area of Compression Steel (A_{sc}):

$$A_{sc} = 2 \times \frac{\pi}{4} \times (16)^2 = 402.1 \text{ mm}^2$$

- Effective Depth (Tension, d):

$$d = D - \text{clear\_cover} - \frac{\text{dia}_{\text{tension}}}{2} = 550 - 30 - \frac{25}{2} = 507.5 \text{ mm}$$

- Effective Cover (Compression, d'):

$$d' = \text{clear\_cover} + \frac{\text{dia}_{\text{comp}}}{2} = 30 + \frac{16}{2} = 38 \text{ mm}$$

Step 2: Find Limiting Neutral Axis ($x_{u,max}$)

For Fe415 grade steel, the maximum allowed neutral axis depth is:

$$x_{u,max} = 0.48 \times d$$

$$x_{u,max} = 0.48 \times 507.5 = \mathbf{243.6} \text{ mm}$$

Step 3: Diagnose the Beam (Find Actual x_u)

We find the actual neutral axis, x_u, by equating the total compressive force (C_u) and total tensile force (T_u).

$$C_u = T_u$$

- **Total Tension (T_u):** (Assuming tension steel yields)

$$T_u = 0.87 f_y A_{st} = 0.87 \times 415 \times 1963.5 = \mathbf{708,575} \text{ N}$$

- **Total Compression (C_u):** (By trial and error)

$$C_u = C_{concrete} + C_{steel} = (0.36 f_{ck} b x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc}$$

Let's try $x_u = 211.5$ mm:

First, find the strain in compression steel (ϵ_{sc}) at this x_u:

$$\epsilon_{sc} = \frac{0.0035(x_u - d')}{x_u} = \frac{0.0035(211.5 - 38)}{211.5} = 0.00288$$

From the IS 456 stress-strain curve for Fe415, the yield strain is ≈ 0.00276. Since $\epsilon_{sc} > 0.00276$, the steel yields. For $\epsilon_{sc} = 0.00288$, $f_{sc} \approx \mathbf{353}$ $\text{N/mm}^{\mathbf{2}}$.

Now, calculate C_u:

$$\begin{aligned}
C_u &= (0.36 \times 25 \times 300 \times 211.5) + (353 - 0.45 \times 25) \times 402.1 \\
C_u &= 571,050 + (353 - 11.25) \times 402.1 \\
C_u &= 571,050 + (341.75) \times 402.1 \\
C_u &= 571,050 + 137,420 = \mathbf{708,470} \text{ N}
\end{aligned}$$

Diagnosis: Since $C_u(708,470 \text{ N}) \approx T_u(708,575 \text{ N})$, our trial $x_u = 211.5$ mm is correct. We compare x_u to $x_{u,max}$:

$$x_u(211.5 \text{ mm}) < x_{u,max}(243.6 \text{ mm})$$

The section is **Under-Reinforced**, which is a valid ductile design.

Step 4: Calculate Ultimate Moment of Resistance (M_u)

Since the section is under-reinforced, we use $x_u = 211.5$ mm. We find M_u by taking moments of the compression forces about the tension steel.

$$M_u = M_{u,c} + M_{u,s}$$

- **Moment from Concrete ($M_{u,c}$):**

$$\begin{aligned}
M_{u,c} &= 0.36 f_{ck} b x_u (d - 0.42 x_u) \\
&= (0.36 \times 25 \times 300 \times 211.5) \times (507.5 - 0.42 \times 211.5) \\
&= 571,050 \times (507.5 - 88.83) \\
&= 571,050 \times 418.67 \\
&= 239,076,818 \text{ N-mm} \\
M_{u,c} &= \mathbf{239.08} \text{ kNm}
\end{aligned}$$

- **Moment from Compression Steel ($M_{u,s}$):**

$$\begin{aligned}
M_{u,s} &= (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d') \\
&= (353 - 0.45 \times 25) \times 402.1 \times (507.5 - 38) \\
&= (341.75) \times 402.1 \times 469.5 \\
&= 64,484,720 \text{ N-mm} \\
M_{u,s} &= \mathbf{64.48} \text{ kNm}
\end{aligned}$$

- **Total Moment (M_u):**

$$\begin{aligned}
M_u &= M_{u,c} + M_{u,s} \\
M_u &= 239.08 \text{ kNm} + 64.48 \text{ kNm} \\
M_u &= \mathbf{303.56} \text{ kNm}
\end{aligned}$$

Final Answer

The ultimate Moment of Resistance (M_u) of the section is **303.56 kNm**.