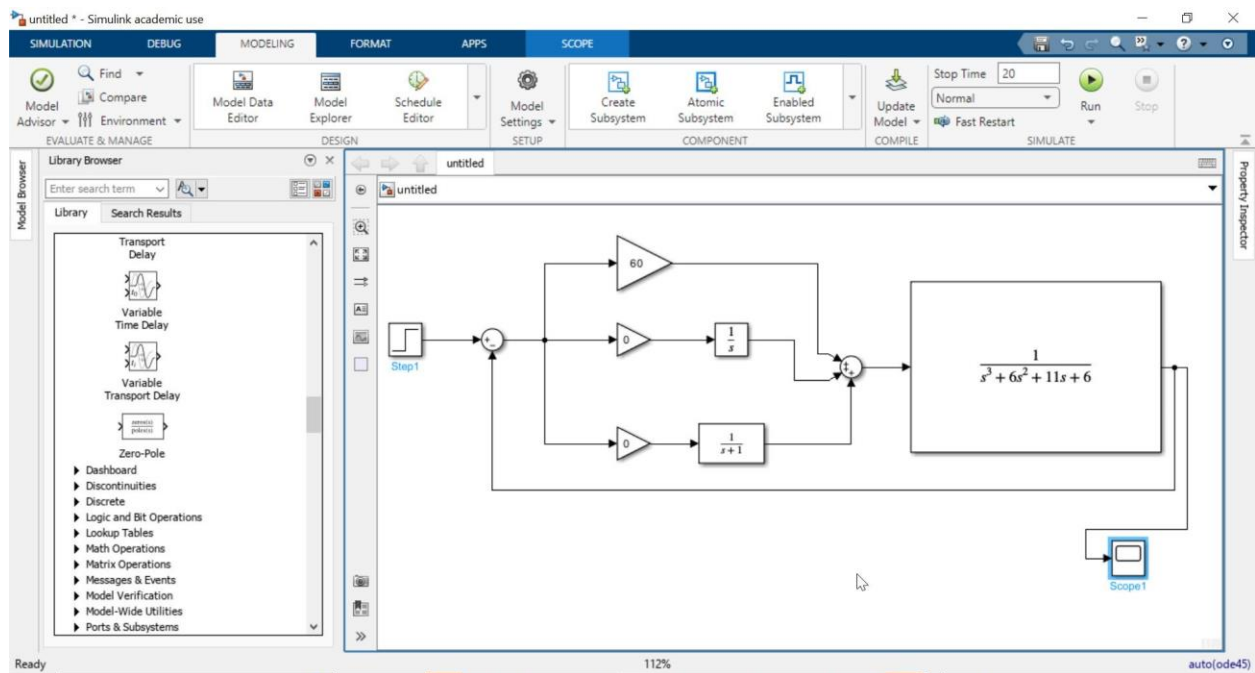


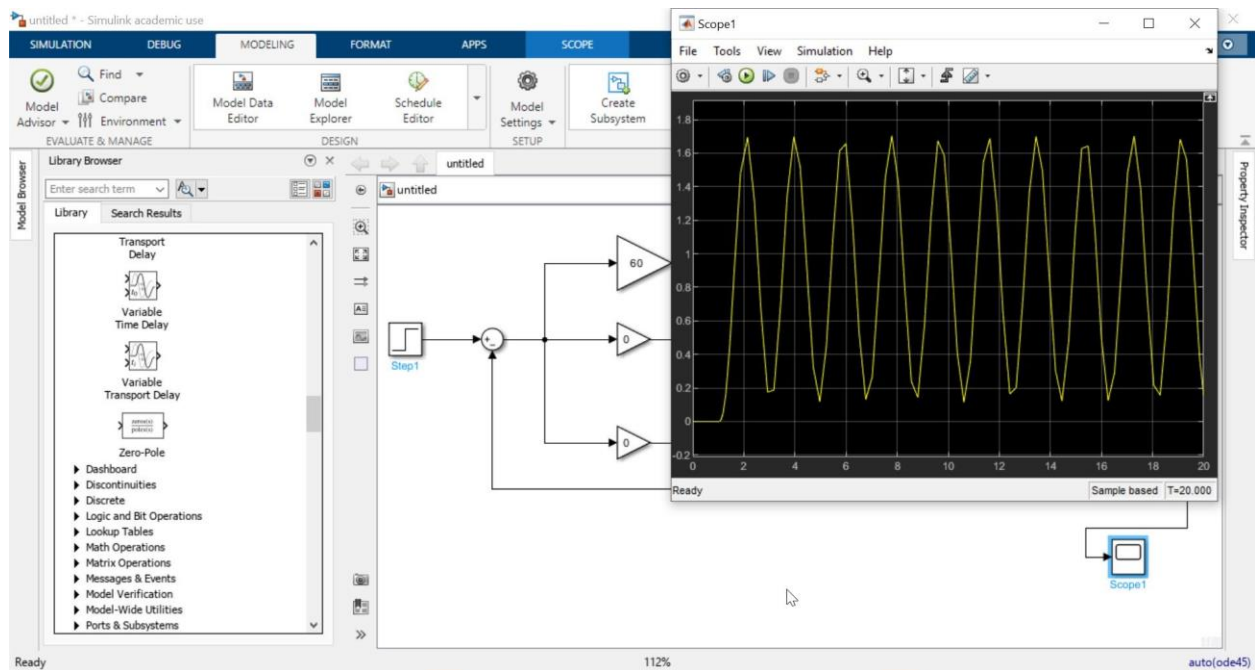
## Ziegler-Nichols PID tuning method

Use the Ziegler-Nichols tuning method to obtain the control gains for the system below:

$$\frac{1}{s^3 + 6s^2 + 11s + 6}$$

- Start with the Kp only (Ki and Kd are zero)
- Start with a small value for Kp (for example Kp=1).
- Increase Kp until the system is on the verge of stability (oscillates with a constant amplitude without settling down)





This proportional control gain is called the ultimate gain  $K_u$ :

$K_u=60$

Find the period of the oscillation corresponding to  $K_u$ .

$T_u=2$  seconds

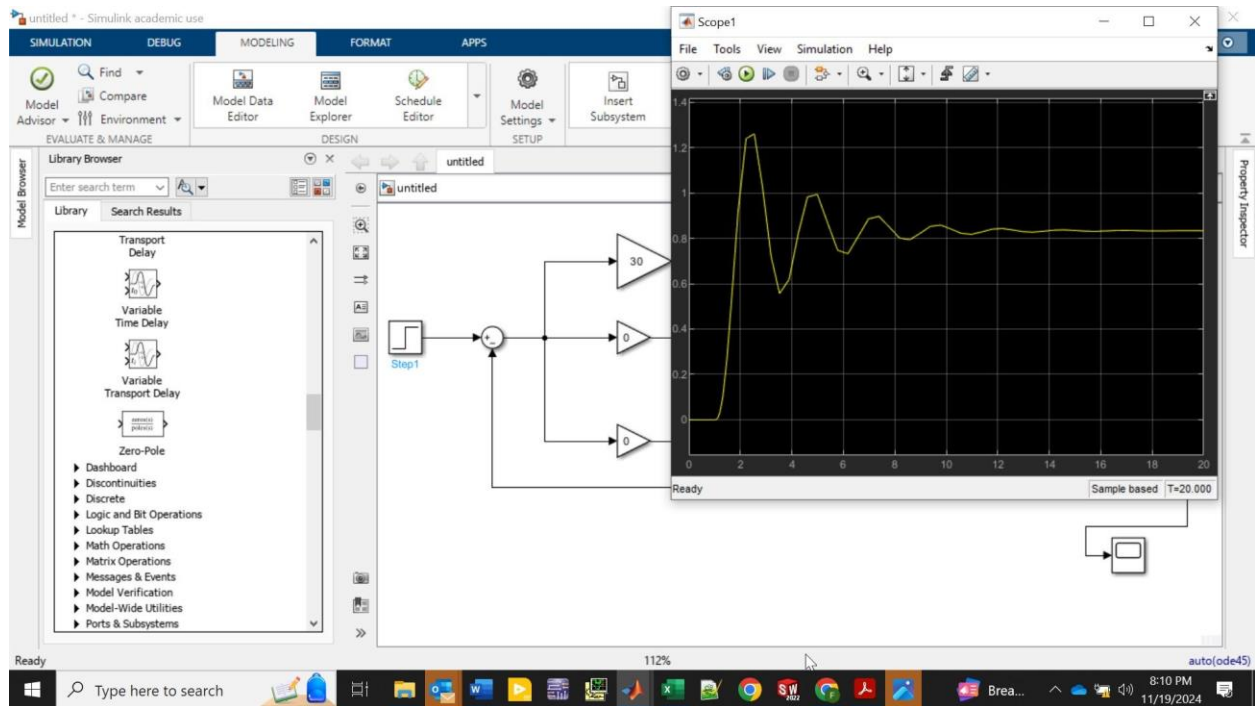
### Ziegler–Nichols method<sup>[1]</sup>

Control Type	$K_p$	$T_i$	$T_d$	$K_i$	$K_d$
<b>P</b>	$0.5K_u$	—	—	—	—
<b>PI</b>	$0.45K_u$	$0.83T_u$	—	$0.54K_u/T_u$	—
<b>PD</b>	$0.8K_u$	—	$0.125T_u$	—	$0.10K_uT_u$
<b>classic PID<sup>[2]</sup></b>	$0.6K_u$	$0.5T_u$	$0.125T_u$	$1.2K_u/T_u$	$0.075K_uT_u$

**Control type: P**

$0.5 K_u = 30$

Update the Kp gain and run the simulation:



Steady-state error exists.

Now use the Integral controller:

### Ziegler–Nichols method<sup>[1]</sup>

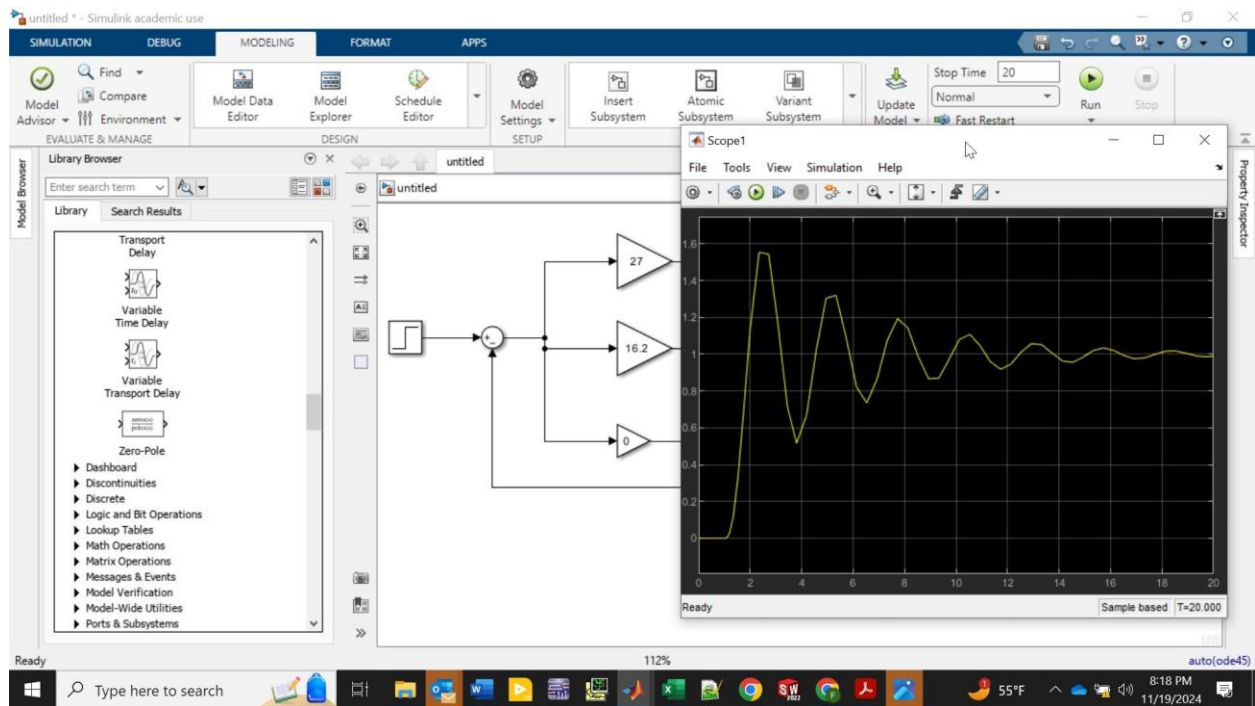
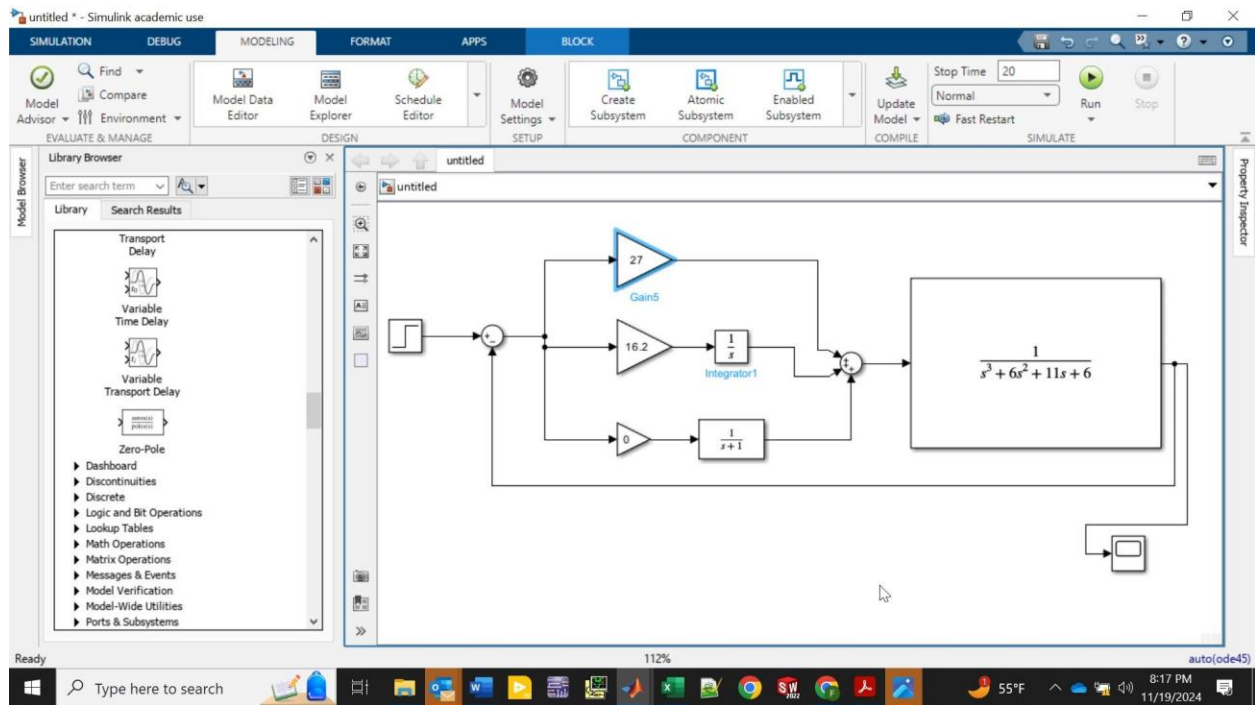
Control Type	$K_p$	$T_i$	$T_d$	$K_i$	$K_d$
P	$0.5K_u$	—	—	—	—
PI	$0.45K_u$	$0.83T_u$	—	$0.54K_u/T_u$	—

**Control type: PI**

$K_p = 0.45 K_u = 27$

$K_i = 0.54 K_u / T_u = 16.2$

Update the Kp and Ki and run the simulation:



The overshoot can be reduced using the derivative controller.

## PID controller:

Note: The derivative controller can not be used on it's own. It requires a filter to remove the high frequency noise (also a derivative term is predicting the future which is not realistic in a practical situation). Therefore a filter has to be added to the derivative controller. The derivative controller with the filter looks like:  $N.S/(S+N)$

In this example we are using a filter with  $N=100$  (to remove the frequency higher than 100 rad/s). Therefore and the derivative controller with the filter looks like:  $100S/(S+100)$ .

### Ziegler–Nichols method<sup>[1]</sup>

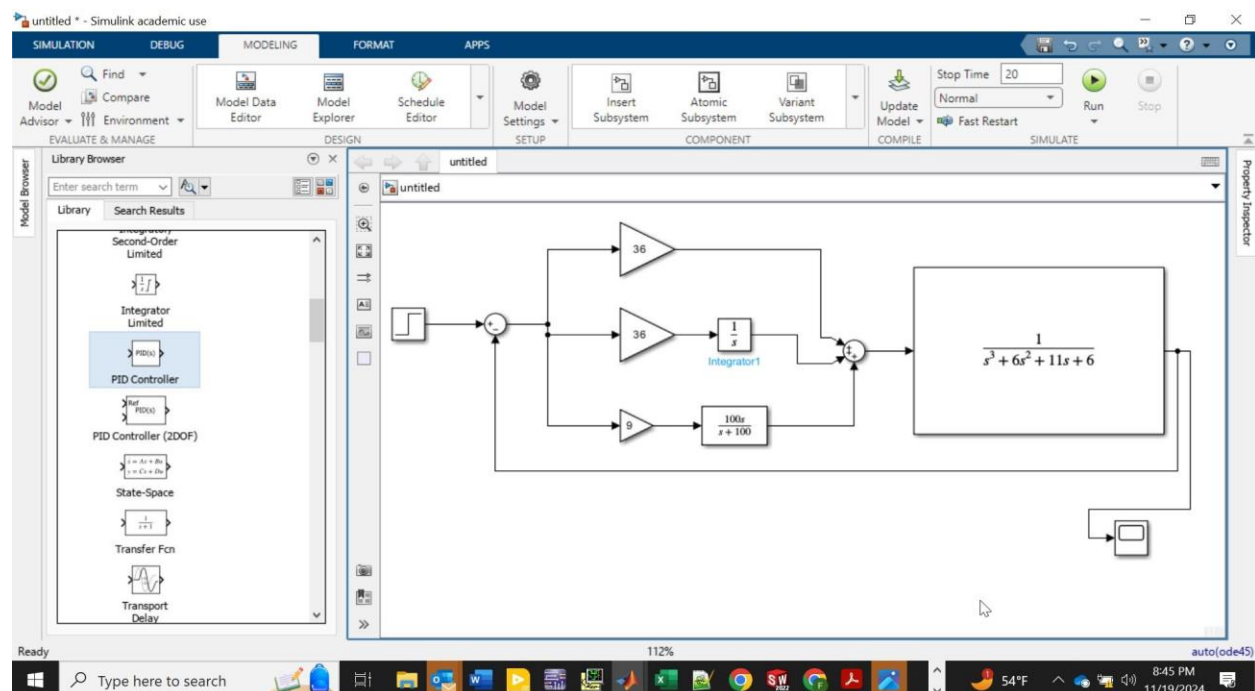
Control Type	$K_p$	$T_i$	$T_d$	$K_i$	$K_d$
P	$0.5K_u$	–	–	–	–
PI	$0.45K_u$	$0.83T_u$	–	$0.54K_u/T_u$	–
PD	$0.8K_u$	–	$0.125T_u$	–	$0.10K_uT_u$
classic PID <sup>[2]</sup>	$0.6K_u$	$0.5T_u$	$0.125T_u$	$1.2K_u/T_u$	$0.075K_uT_u$

### Control type: PID

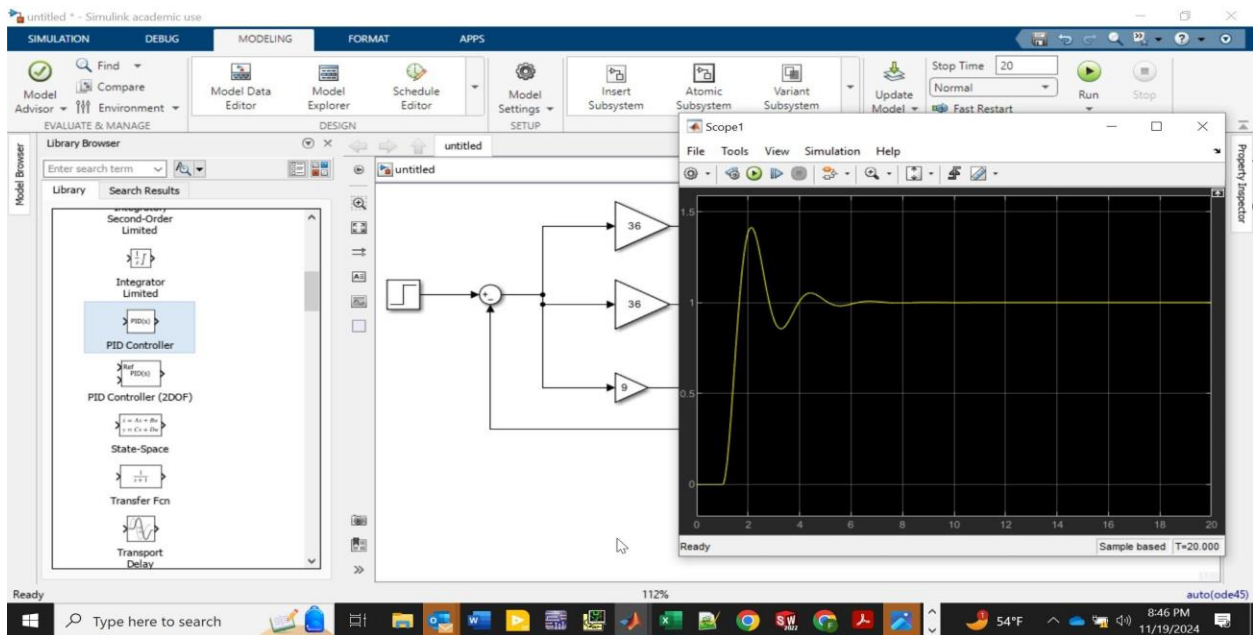
$$K_p = 0.6 K_u = 36$$

$$K_i = 1.2K_u/T_u = 36$$

$$K_d = 0.075 K_u T_u = 9$$



These values



initial of the

controller gains for  $K_p$ ,  $K_i$ , and  $K_d$  are the starting values for the tuning. Further tuning can be performed by trial and error (after this initial tuning) to improve the response and fine tuning the response, if needed.

### Improved Response

$K_p = 42$

$K_i = 52.5$

$K_d = 12.6$

### Control type: For no overshoot

$K_p = 12$

$K_i = 12$

$K_d = 7.92$