Applying Kalman filter on projectile motion of the ball

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***Abstract* — This paper put forward the estimation of location and velocity of the ball's 2D trajectory motion using the best signal processing method, known as the Kalman filter. First, a state-space representation of the 2D moving object is implemented, measuring the system states through stimulated sensor data. By comparing initial parameter modifications, a thorough comparative evaluation has been conducted. The suggested approach can accurately monitor the position and velocity of ball, according to extensive simulation findings. A quick study on Kalman gain was also undertaken.**

Keywords* — *Kalman Filter, Kalman gain, trajectory motion

I Introduction

The majority of contemporary systems are endowed with a wide variety of sensors that, through a sequence of observations, estimate hidden (unknown) parameters. For instance, a GPS receiver may estimate both position and speed, with position and speed acting as hidden variables and measurements coming from the differential time of the satellite signals arriving. The provision of precise and exact estimation of the hidden variables in the presence of uncertainty is one of the main issues facing monitoring and management systems. The calculation of GPS receivers is affected by a variety of extraneous environment, including thermal noise, atmospheric effects, minute variations in satellite locations, receiver clock accuracy, and plenty more. Among the most significant and popular estimate methods is the Kalman Filter. The Kalman Filter generates latent parameter estimates based on unreliable and erroneous data. It further offers a forecast of the future state of the system based on earlier estimates. The filter bears Rudolf E. Kálmán's name (19 May 1930 – 2 July 2016). Kálmán released his well-known work in 1960 outlining a recursive approach to the discrete-data linear filtering issue. The Kalman filter is utilized in many different applications presently, including computer animation, positioning and gps technologies, power systems, and military surveillance (Radar). Here, we are investigating the use of a Kalman filter on the projectile motion of a ball with an initial velocity of "v" falling under gravity.

II Related works

The issue of ball recognition and tracking in a live basketball broadcast is discussed in this article [1]. A methodology for ball recognition and tracing focused on projectile motion is provided here to find and follow the ball in a collection of films of basketball long shot sequences. The ball is identified using a feature-based technique in the first stage of a two-step recognition and monitoring architecture. Using the 2D projectile data of the potential ball possibilities in the screen, the second phase evaluates the ball identification system's classifier. From a collection of possible trajectories, the genuine ball trajectory is selected, and the ball positions along the flight are calculated.

A projectile restoration approach is used to predict the unknown ball locations caused by blockage of the ball and blending of the ball picture with other structures in the background. A thorough assessment of the research on the benefits and drawbacks of applying Kalman filter, optimum signal processing techniques, and moving object tracing is presented in [2]. The suggested approach can accurately monitor the location and speed of moving objects, according to comprehensive experimental findings. The difficulties of following a ball in free-fall across an unconstrained setting are examined in [3].

III Experiments and Evaluation

To properly outputing predicted values, we are creating stimulated measurements also known as noise, to the actual trajectory generated by projectile motion of equations [5].

$$\begin{aligned} x_{t+1} &= x_t + v_x \Delta t \\ y_{t+1} &= y_t + v_y \Delta t - 0.5 g \Delta t^2 \end{aligned} \tag{1}$$

where x_{t+1} and y_{t+1} is the position of ball in horizontal and vertical direction from the imaginary ground position. v_x and v_y are the velocity component in horizontal and vertical direction. Delta t is the time interval between recurrent ball positions.

The probability of the following state, p(xt | ut, xt-1), should be linear in its arguments with additional Gaussian noise.

Algorithm Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$)**:** (2)

$$\begin{aligned} &\bar{\mu}_t = A_t\, \mu_{t-1} + B_t\, u_t \\ &\bar{\Sigma}_t = A_t\, \Sigma_{t-1}\, A_t^T + R_t \\ &K_t = \bar{\Sigma}_t\, C_t^T (C_t\, \bar{\Sigma}_t\, C_t^T + Q_t)^{-1} \\ &\mu_t = \bar{\mu}_t + K_t (z_t - C_t\, \bar{\mu}_t) \\ &\Sigma_t = (I - K_t\, C_t)\, \bar{\Sigma}_t \\ &\text{return } \mu_t, \Sigma_t \end{aligned}$$

Python is used to execute the Kalman filter technique for linear Gaussian state changes and measurements, as stated above [4].

Here, the state vectors are xt and xt-1, and the control vector at time t is ut. Both of these vectors are vertical in our terminology, and ut denotes speed. Matrices At and Bt exist. At is a square matrix with dimensions n * n, where n is the state vector xt's dimension. Bt has a dimension of n*m, where m is the control vector ut's size. The state transition function's inputs are made linear by multiplying the state and control vectors by the matrices At and Bt, correspondingly. Kalman filters therefore presumptively have linear system dynamics. The state transition's unpredictability is modeled by the random variable "t in, which is a Gaussian random vector. It shares the state vector's size. Its covariance shall be referred to as Rt, and its mean is zero. Because it is linear in its arguments with additive Gaussian noise, a state transition probability of the type is referred to as a linear Gaussian. With Ct defined as a matrix of dimension k * n, where k is the size of the measurement vector zt, Zt is the additional Gaussian noise. A covariance matrix is Qt.

Transition model: $\boldsymbol{q}_t = \boldsymbol{A}_t \boldsymbol{q}_{t-1} + \boldsymbol{B}_t \boldsymbol{a}_{t-1} + \boldsymbol{\epsilon}_t$

$$\boldsymbol{q}_t = \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix}_t = \begin{pmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix}_{t-1} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.5\Delta t^2 & 0 \\ 0 & 0 & 0 \\ 0 & \Delta t & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -g \\ 0 \end{pmatrix}_{t-1} + \boldsymbol{\epsilon}_{t-1}$$

Observation model: $\boldsymbol{o}_t = \boldsymbol{C}_t \boldsymbol{q}_t + \boldsymbol{\delta}_t$

$$\boldsymbol{o}_t = \begin{pmatrix} x \\ y \end{pmatrix}_t = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix}_t + \boldsymbol{\delta}_t$$

Fig 1. Transition model and observation model equations

In figure 1 [5] o_t and qt are defined same as z_t and x_t correspondingly. Implemented these equations and concepts using python language on the projectile motion of ball with different parameters were the set up of experiment.

IV Results and discussion

IV.A Results

The acquired results for the examined estimation of position and velocity performance of the trajectory of a ball with the different launch positions (especially the height above an imaginary ground) are shown in Fig.2. It seems like changing the launching position have almost no impact in the prediction of position .

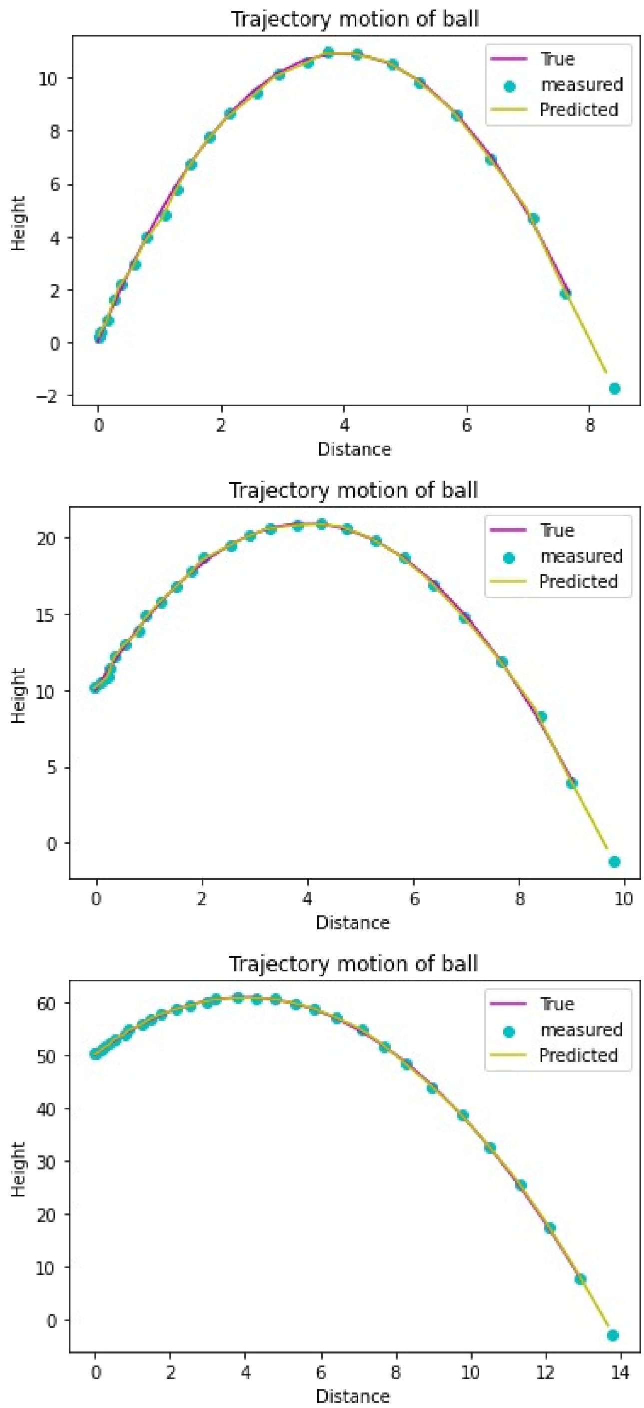

Fig 2. Performance of the prediction of position when the vertical height of the ball from ground = 0, 10, 50 m.

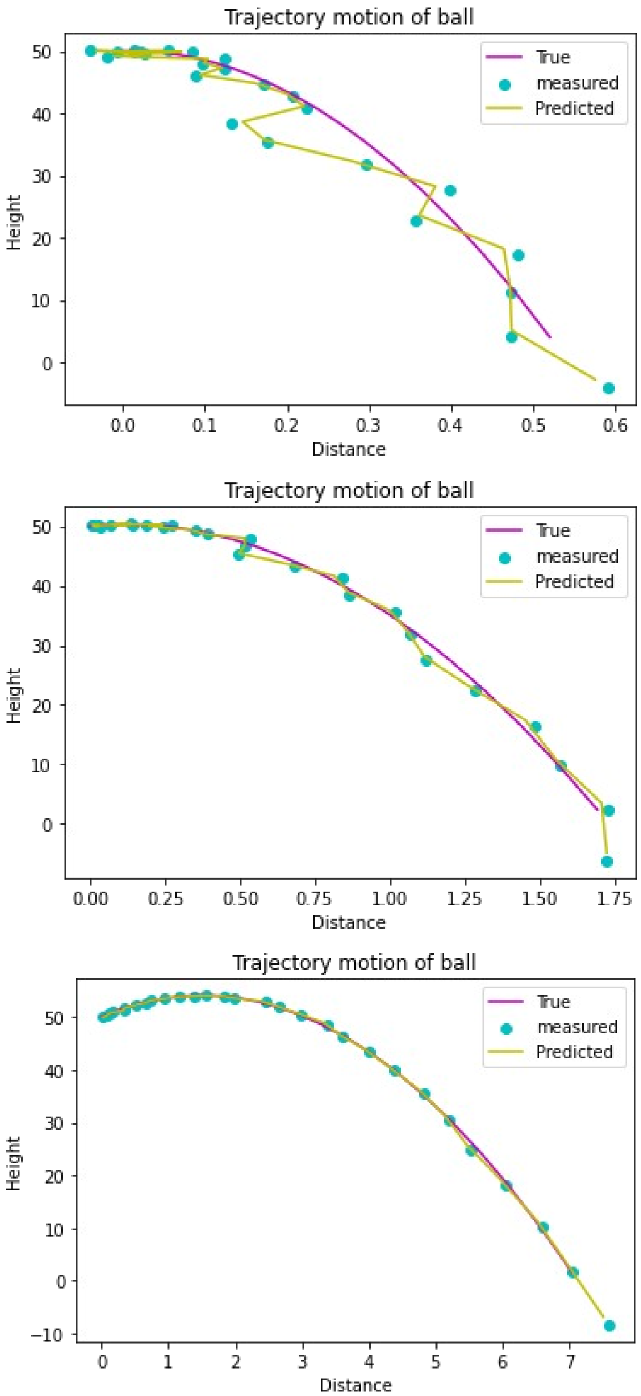

Fig 3. Prediction of position of the ball when changing initial velocity correspond to V = 1, 3, 10 m/s.

In Fig. 3 and Fig. 4, we are experimenting with the parameter of initial velocity. We can see the value of velocity in the initial have a significant impact on the ball position as well as the velocity predictions. As initial velocity increases accuracy of Kalman filter also increases.

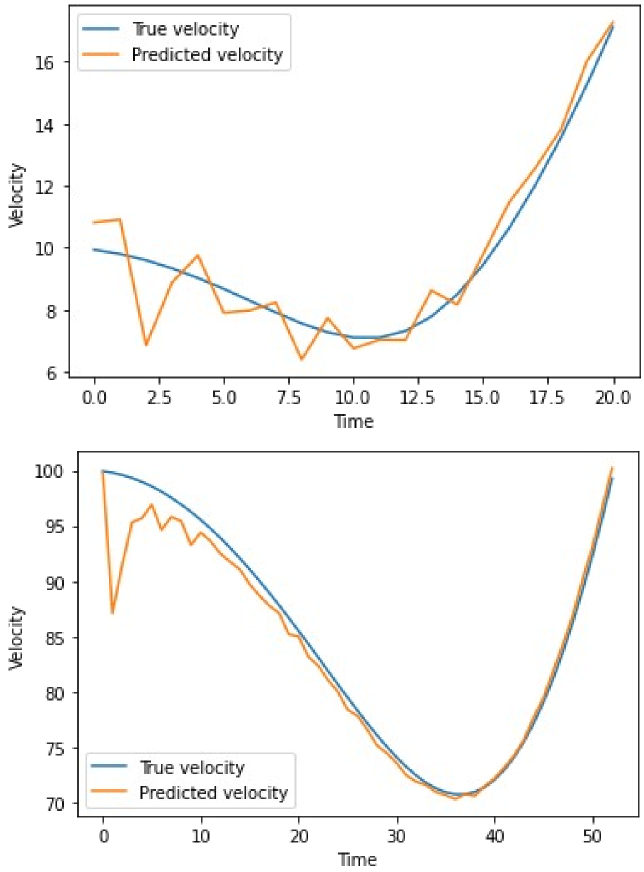

Fig 4. Prediction of velocity of the ball when changing initial velocity correspond to V = 10, 50 m/s.

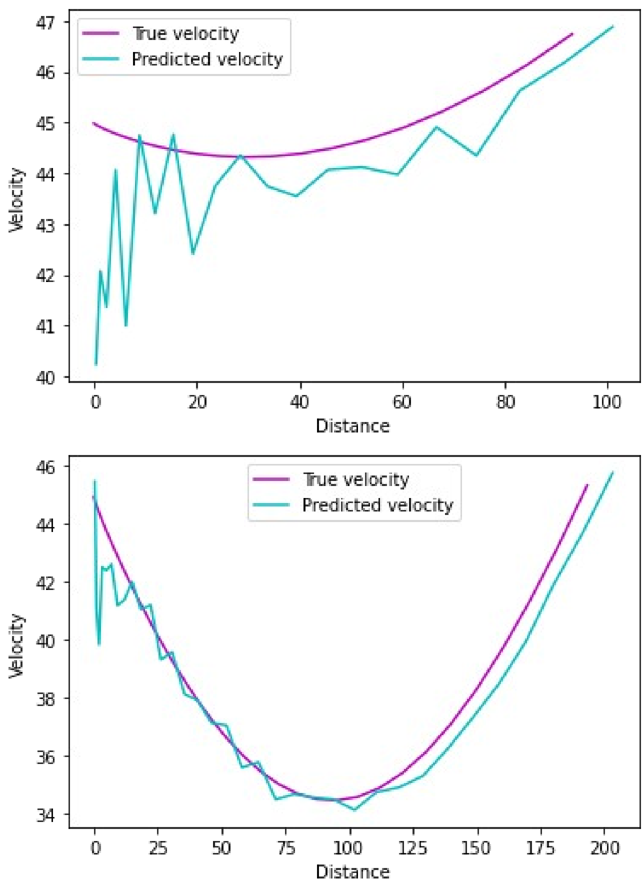

Fig 5. Prediction of velocity of the ball when changing initial inclination correspond to angle = 10, 40 m/s.

In Fig 5, as initial inclination increases, Kalman gain predicts more accurate velocity.

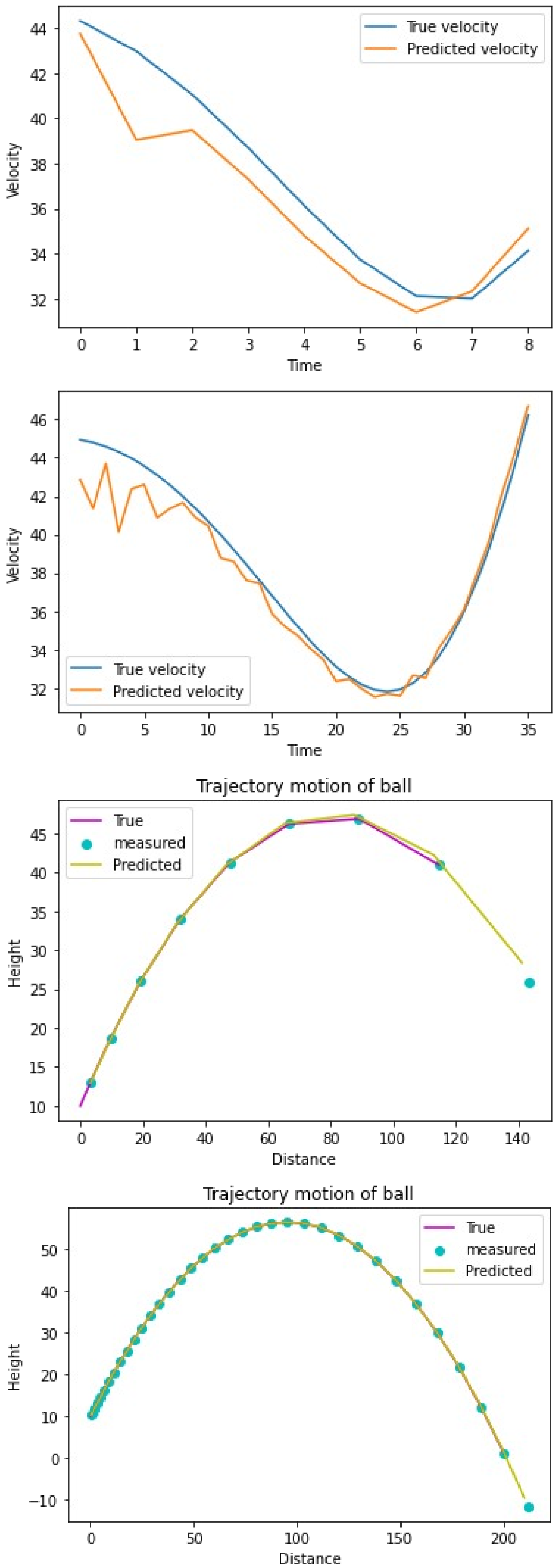

Fig 6. Performance of the prediction of velocity and position when the time interval between two observations correspond to dt = 0.1, 0.01 s.

Fig 6. shows that as time interval between two observations decreases, accuracy in prediction increases in both case of velocity and position.

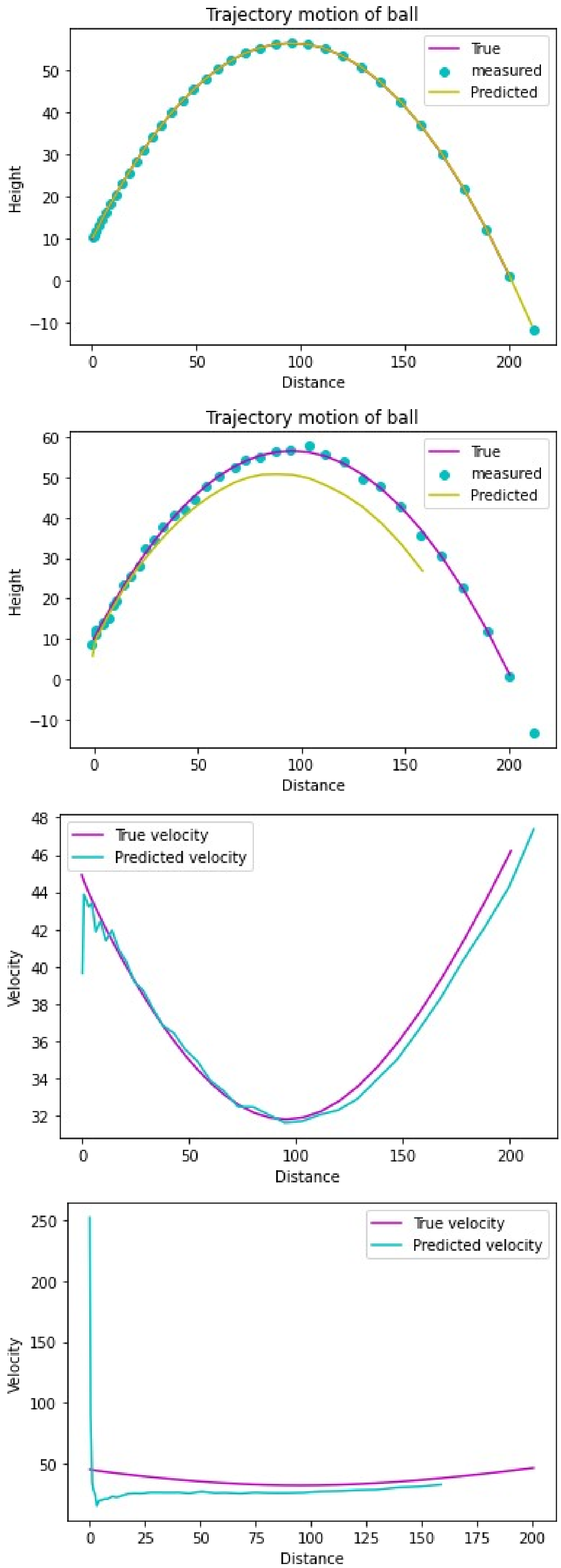

Fig 7. Performance of the Kalman filter when variance value in covariance matrix R changes correspond to 0.001, 0.5.

Fig 7. demonstrates that when variance value in covariance matrix R changes from 0.001 to 0.5 accuracy of Kalman gain decreases. At final stages it also not able to predict the value. But the characteristic of covariance matrix Q behaves just opposite. It shows more accuracy when variance value increases as depicted in Fig 8.

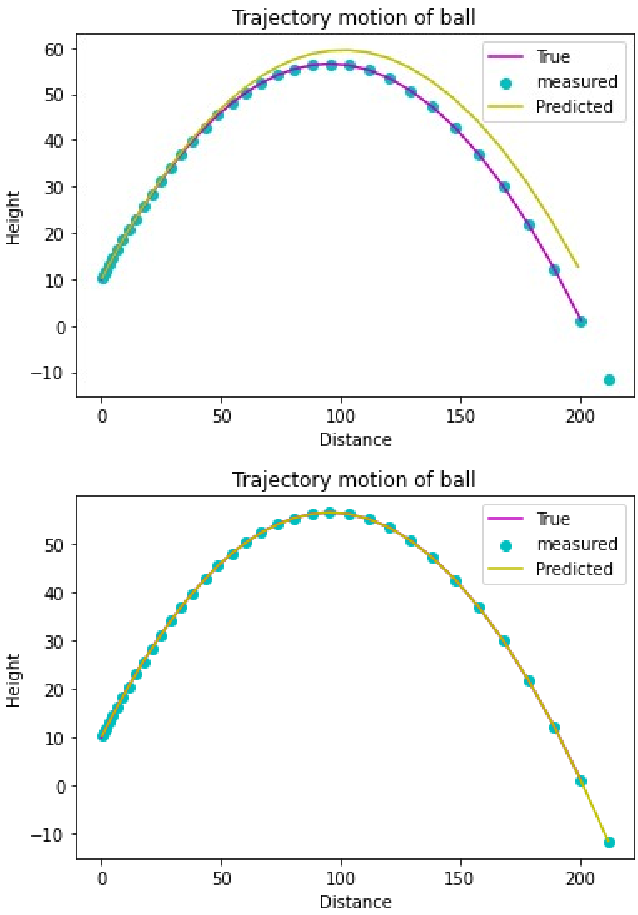

Fig 8. Performance of the Kalman filter when variance value in covariance matrix Q changes correspond to 0.0001, 0.5.

IV.B Discussion

R is based on the sensibility of the detector. If this is a practical experiment, the vendor can provide those. We are utilizing the identity matrix times a scalar smaller than 1 since in this experiment the sensor measurements are stimulated. R describes the precision of the sensors. Q is a metric for gauging a model's accuracy. It is the process noise covariance. Again, if this is a practical experiment, the noise level in the steady-state states of the system may be found. We are presuming Q is a zero-dimensional matrix although it is not, and we are altering the values to find the dependency on Kalman prediction. A non zero Q aids in obtaining favorable convergence properties.

How much we want to adjust an estimate by a measurement is indicated by the Kalman gain. It is a number that falls between 0 and 1.

Kt = (Uncertinity in estimate) / ((Uncertinity in estimate) + (Uncertinity in Measurement)) (3)

((3) represents the Kalman Gain equation's fundamental intuitive explanation. As can be seen in (2), the weight we give to the measurement is the Kalman Gain (Kt), whereas the weight we provide to the prediction is (1-Kt). The Kalman Gain is extremely near to zero when the estimate uncertainty is quite low and the measurement uncertainty is quite high. As a result, we assign the estimate a high weight and the measurement a low weight. From the other side, the Kalman Gain is extremely near to one when the measurement uncertainty is quite low and the estimate uncertainty is quite great. As a result, we assign the measurement a large weight and the estimate a low weight.
The Kalman gain becomes 0.5 if the measurement uncertainty and the estimate uncertainty are same.

The multidimensional equation for Kalman gain expressed as

K_t= Pn,n−1HT(HPn,n−1HT+Rn)−1 (4)

where H is the observation matrix, Rn is the measurement uncertainty (measurement noise covariance matrix), and Pn,n-1 is the preceding estimate uncertainty (covariance) matrix of the present phase (estimated at the prior phase). Hence we look for a Kalman Gain that reduces the estimate variance.

V CONCLUSIONS

In this study, the application of Kalman filter to trace an ball fall with an initial velocity was investigated. Also flexibility is increased by considering different launch positions, launch directions, launch velocities of the ball, different time intervals between observations, different initial parameters and different parameters of the covariance matrices R and Q. Varied launch places, launch orientations, ball launch velocities, different time intervals between observations, variable beginning parameters, and different covariance matrix R and Q parameters are all taken into account in this study to improve flexibility. Additionally, if accurate estimate is to be made, variables like wind and flip that might alter ball flight in the actual world should be taken into account. The tracing model might be customized to the specific circumstance by include prior knowledge of these characteristics, but this would make it harder to follow the general point. Nevertheless, employing better segmentation algorithms and accounting for air drag force with information of the ball's characteristics can boost precision for actual practical world settings

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