Section 5: RNNs & LSTMs for Sequential Data

Advanced Time Series Forecasting for Energy Systems

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Section Overview

RNN Fundamentals

LSTM Architecture

Advanced RNN Architectures

Energy Load Forecasting Applications

Advanced Techniques and Optimization

RNN Fundamentals

Recurrent Neural Networks: Core Concepts

Recurrence Equation:

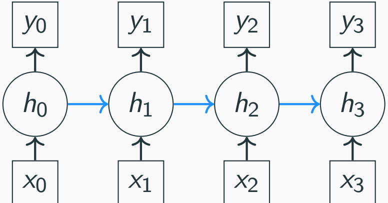
$$\mathbf{h}_t = f(\mathbf{W}_{hh}\mathbf{h}_{t-1} + \mathbf{W}_{xh}\mathbf{x}_t + \mathbf{b}_h)$$

$$\mathbf{y}_t = g(\mathbf{W}_{hy}\mathbf{h}_t + \mathbf{b}_y)$$

Key Properties:

- **Temporal Memory**: Previous states influence curr.
- **Parameter Sharing**: Same weights across time

Unrolled Through Time:



Applications in Energy:

- Load forecasting
- Price prediction
- Demand response
- Fault detection sequences
- Generation scheduling

Challenges:

- Vanishing gradients
- Exploding gradients
- Long-term dependencies
- Computational cost

Backpropagation Through Time (BPTT)

Gradient Flow:

$$\frac{\partial L}{\partial \mathbf{W}} = \sum_{t=1}^{T} \frac{\partial L_t}{\partial \mathbf{W}}$$

Chain Rule Application:

$$\frac{\partial L}{\partial \mathbf{h}_t} = \frac{\partial L}{\partial \mathbf{h}_{t+1}} \cdot \frac{\partial \mathbf{h}_{t+1}}{\partial \mathbf{h}_t} + \frac{\partial L_t}{\partial \mathbf{h}_t}$$

Gradient Problems:

Vanishing: $\prod_i \frac{\partial \mathbf{h}_i}{\partial \mathbf{h}_{i-1}} \to 0$
Exploding: $\prod_i \frac{\partial \mathbf{h}_i}{\partial \mathbf{h}_{i-1}} \to \infty$

Solutions:

- Gradient clipping
- Better initialization
- Gated architectures (LSTM/GRU)
- Skip connections
- Truncated BPTT

Truncated BPTT:

- Limit backprop steps
- Trade-off: memory vs accuracy
- Common: 20-35 steps

LSTM Architecture

Long Short-Term Memory (LSTM)

Gate Equations:

$$\mathbf{f}_t = \sigma(\mathbf{W}_f[\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_f) \tag{1}$$

$$\mathbf{i}_t = \sigma(\mathbf{W}_i[\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_i) \tag{2}$$

$$\tilde{\mathbf{c}}_t = \tanh(\mathbf{W}_c[\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_c) \tag{3}$$

$$\mathbf{c}_t = \mathbf{f}_t \odot \mathbf{c}_{t-1} + \mathbf{i}_t \odot \tilde{\mathbf{c}}_t \tag{4}$$

$$\mathbf{o}_t = \sigma(\mathbf{W}_o[\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_o) \tag{5}$$

$$\mathbf{h}_t = \mathbf{o}_t \odot \tanh(\mathbf{c}_t) \tag{6}$$

Gate Functions:

- **Forget Gate** ($\mathbf{f}_t$): What to discard
- **Input Gate** ($\mathbf{i}_t$): What to store
- **Candidate** ($\tilde{\mathbf{c}}_t$): New information
- **Output Gate** ($\mathbf{o}_t$): What to output

LSTM Deep Dive:

`05_lstm_energy_advanced.ipynb`

Gate visualizations included

GRU: Gated Recurrent Unit

Simplified Architecture:

$$\mathbf{z}_t = \sigma(\mathbf{W}_z[\mathbf{h}_{t-1}, \mathbf{x}_t]) \tag{7}$$
$$\mathbf{r}_t = \sigma(\mathbf{W}_r[\mathbf{h}_{t-1}, \mathbf{x}_t]) \tag{8}$$
$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{W}[\mathbf{r}_t \odot \mathbf{h}_{t-1}, \mathbf{x}_t]) \tag{9}$$
$$\mathbf{h}_t = (1 - \mathbf{z}_t) \odot \mathbf{h}_{t-1} + \mathbf{z}_t \odot \tilde{\mathbf{h}}_t \tag{10}$$

Gates:

- **Update Gate** ($\mathbf{z}_t$): How much to update
- **Reset Gate** ($\mathbf{r}_t$): How much to forget

GRU vs LSTM:

- Fewer parameters (3 vs 4 gates)
- No separate cell state
- Often comparable performance
- Faster training

When to Use:

- **LSTM**: Complex patterns, long sequences
- **GRU**: Limited data, faster training needed

GRU Implementation:

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Advanced RNN Architectures

Bidirectional RNNs

Architecture:

$$\overrightarrow{\mathbf{h}}_t = f(\overrightarrow{\mathbf{W}}_{hh}\overrightarrow{\mathbf{h}}_{t-1} + \mathbf{W}_{xh}\mathbf{x}_t)$$

$$\overleftarrow{\mathbf{h}}_t = f(\overleftarrow{\mathbf{W}}_{hh}\overleftarrow{\mathbf{h}}_{t+1} + \mathbf{W}_{xh}\mathbf{x}_t)$$

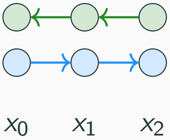
$$\mathbf{h}_t = [\overrightarrow{\mathbf{h}}_t; \overleftarrow{\mathbf{h}}_t]$$

Benefits:

- Context from both directions
- Better feature extraction
- Improved accuracy

Energy Applications:

- Anomaly detection (need full context)
- Data imputation
- Pattern recognition
- NOT for real-time forecasting



Encoder-Decoder Architecture

Sequence-to-Sequence:

- **Encoder**: Compress input sequence
- **Context Vector**: Fixed-size representation
- **Decoder**: Generate output sequence

Mathematical Formulation: Encoder:

$\mathbf{h}_t^{enc} = f_{enc}(\mathbf{x}_t, \mathbf{h}_{t-1}^{enc})$

Context: $\mathbf{c} = g(\mathbf{h}_1^{enc}, ..., \mathbf{h}_T^{enc})$

Decoder: $\mathbf{h}_t^{dec} = f_{dec}(\mathbf{y}_{t-1}, \mathbf{h}_{t-1}^{dec}, \mathbf{c})$

Energy Forecasting:

- Multi-step ahead prediction
- Different input/output lengths
- Weather $\rightarrow$ Load mapping
- Cross-domain translation

Encoder-Decoder:

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Multi-horizon forecasting

Attention Mechanisms

Attention Score:

$$\alpha_{t,s} = \frac{\exp(e_{t,s})}{\sum_{s'=1}^{S} \exp(e_{t,s'})}$$

where $e_{t,s} = a(\mathbf{h}_{t-1}^{dec}, \mathbf{h}_s^{enc})$

Context Vector:

$$\mathbf{c}_t = \sum_{s=1}^{S} \alpha_{t,s} \mathbf{h}_s^{enc}$$

Types:

- **Bahdanau**: Additive attention
- **Luong**: Multiplicative attention
- **Self-Attention**: Query = Key = Value

Benefits for Time Series:

- Focus on relevant time steps
- Handle long sequences
- Interpretability
- Variable importance weighting

c_t

Visualization:

Energy Load Forecasting Applications

Time Series Data Preparation

Feature Engineering:

- **Temporal**: Hour, day, month, season
- **Lag Features**: t-1, t-24, t-168
- **Rolling Statistics**: Mean, std, min, max
- **Calendar**: Holidays, weekends
- **Weather**: Temperature, humidity
- **Economic**: Industrial indices

Normalization:

- StandardScaler: $z = \frac{x-\mu}{\sigma}$
- MinMaxScaler: $x' = \frac{x-x_{min}}{x_{max}-x_{min}}$
- RobustScaler: Using median/IQR

Sequence Creation:

- Input window: 24-168 hours
- Output horizon: 1-24 hours
- Sliding window approach
- Overlap considerations

Data Splits:

- Train: 70%
- Validation: 15%
- Test: 15%
- **Important**: Temporal order

Data Pipeline:

`05 lstm energy advanced.ipynb`

Multi-Step Forecasting Strategies

1. **Recursive (Single-Step):**
   - Train: One step ahead
   - Inference: Use predictions as input
   - Issue: Error accumulation
2. **Direct (Multi-Output):**
   - Train: All horizons simultaneously
   - Separate model per horizon
   - Issue: No correlation between outputs
3. **MIMO (Seq2Seq):**
   - Train: Sequence to sequence
   - Single model for all horizons
   - Best for related outputs

Hybrid Approaches:

- DirRec: Combine direct and recursive
- Multi-stage: Coarse to fine
- Ensemble: Multiple strategies

Evaluation Metrics:

- MAE: $\frac{1}{n}\sum |y - \hat{y}|$
- RMSE: $\sqrt{\frac{1}{n}\sum (y - \hat{y})^2}$
- MAPE: $\frac{100}{n}\sum \frac{|y-\hat{y}|}{y}$
- R^2: Explained variance

Handling Seasonality and Trends

Decomposition:

$$Y_t = T_t + S_t + R_t$$

- T_t: Trend component
- S_t: Seasonal patterns
- R_t: Residual/Random

Multiple Seasonalities:

- Daily: 24-hour cycle
- Weekly: 7-day pattern
- Annual: Seasonal variations

Neural Approaches:

- Seasonal neurons
- Fourier features
- Wavelet decomposition
- STL decomposition

Hybrid Models:

- SARIMA + LSTM
- Prophet + Neural residuals
- Decomposition + Deep learning

Seasonality Handling:

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Advanced Techniques and Optimization

Temporal Fusion Transformer (TFT)

Architecture Components:

- **Variable Selection**: Feature importance
- **Gating**: Suppress unnecessary info
- **Static Enrichment**: Context encoding
- **Temporal Processing**: LSTM encoder
- **Multi-Head Attention**: Self-attention
- **Position-wise FFN**: Final processing

Interpretability:

- Variable importance scores
- Attention weights visualization
- Temporal patterns identification

Advantages:

- Handles multiple time series
- Known/unknown variables
- Multi-horizon forecasting
- Uncertainty quantification
- State-of-the-art performance

TFT Implementation:

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Simplified version included

Training Best Practices

Gradient Management:

- Gradient clipping: $\|\mathbf{g}\| \leq$ threshold
- Gradient normalization
- Adaptive clipping
- Skip connections

Regularization:

- Dropout (standard/variational)
- Weight decay
- Early stopping
- Noise injection
- Zoneout (for RNNs)

Learning Rate Scheduling:

- Warmup phase
- Cosine annealing
- ReduceLROnPlateau
- Cyclical learning rates

Loss Functions:

- MSE: Standard regression
- MAE: Robust to outliers
- Huber: Combination
- Quantile: Uncertainty
- Custom: Peak-aware

Model Optimization and Deployment

Optimization Techniques:

- Teacher forcing ratio decay
- Scheduled sampling
- Curriculum learning
- Transfer learning
- Multi-task learning

Inference Optimization:

- Beam search
- Caching hidden states
- Batch processing
- Model quantization
- ONNX export

Production Deployment:

- Real-time inference pipeline
- Online learning updates
- Model versioning
- A/B testing
- Monitoring & alerts

Performance Metrics:

- Latency (ms/prediction)
- Throughput (predictions/sec)
- Memory usage
- Model size

Uncertainty Quantification

Probabilistic Forecasting:

- Quantile regression
- Prediction intervals
- Monte Carlo dropout
- Deep ensembles
- Bayesian RNNs

Quantile Loss:

$$L_q(y, \hat{y}) = \begin{cases} q(y - \hat{y}) & y \geq \hat{y} \\ (1 - q)(\hat{y} - y) & y < \hat{y} \end{cases}$$

Applications:

- Risk assessment
- Decision making
- Grid stability
- Reserve planning
- Trading strategies

Evaluation:

- Coverage probability
- Interval width
- CRPS (Continuous Ranked Probability Score)
- Pinball loss

Summary: RNNs & LSTMs for Energy Systems

Key Concepts:

- RNN fundamentals
- LSTM/GRU architectures
- Bidirectional processing
- Attention mechanisms
- Encoder-decoder models

Energy Applications:

- Load forecasting
- Multi-step prediction
- Seasonality handling
- Anomaly detection
- Demand response

Advanced Techniques:

- Temporal Fusion Transformer
- Uncertainty quantification
- Online learning
- Production deployment

Complete Implementation:

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All Notebooks Available:

03_deep_learning_advanced.ipynb

04_cnn_solar_advanced.ipynb

05_lstm_energy_advanced.ipynb