# Wykład 3 – zadania domowe

1) **Wykonaj działania:**

   a) $(1-3i)+(4-5i)=5-8i$

   b) $(1+\sqrt{2}i)-(\sqrt{3}-6i)i=-5+(\sqrt{2}-\sqrt{3})i$

   c) $(\sqrt{7}-\sqrt{3}i)\cdot(\sqrt{7}+\sqrt{3}i)=(\sqrt{7})^2-(\sqrt{3}i)^2=7-3i^2=10$

   d) $\dfrac{2+3i}{1+i}=\dfrac{(2+3i)\cdot(1-i)}{(1+i)\cdot(1-i)}=\dfrac{2+3i-2i-3i^2}{(1-i^2)}=\dfrac{5+i}{2}=\dfrac{5}{2}+\dfrac{1}{2}i$

2) **Oblicz wartości podanych wyrażeń (wyniki podać w postaci algebraicznej)**

   **a)** $(1-i)^{12}$

$$|z|=\sqrt{1^2+(-1)^2}=\sqrt{2}$$

$$\left.\begin{array}{l}\cos\varphi=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}\\[2mm]\sin\varphi=-\dfrac{1}{\sqrt{2}}=-\dfrac{\sqrt{2}}{2}\\[2mm]ze\ wzorow\ redukcyjnych:\\[1mm]\sin(-\alpha)=-\sin\alpha\\[1mm]\sin\left(-\dfrac{\pi}{4}\right)=-\sin\left(\dfrac{\pi}{4}\right)\\[1mm]\cos(-\alpha)=\cos\alpha\\[1mm]\cos\left(-\dfrac{\pi}{4}\right)=\cos\left(\dfrac{\pi}{4}\right)\end{array}\right\}\Rightarrow\varphi=-\dfrac{\pi}{4}+2k\pi$$

$$(1-i)^{12}=(\sqrt{2})^{12}\left(\cos\left(12\left(-\dfrac{\pi}{4}\right)\right)+i\sin\left(12\left(-\dfrac{\pi}{4}\right)\right)\right)=$$

$$=2^6(\cos(-3\pi)+i\sin(-3\pi))=2^6(-1+0)=-64$$

**b)** $\left(1+\sqrt{3}i\right)^8$

$$|z| = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$$

$$\left.\begin{array}{l} \cos\varphi = \dfrac{1}{2} \\[2mm] \sin\varphi = \dfrac{\sqrt{3}}{2} \end{array}\right\} \Rightarrow \varphi = \dfrac{\pi}{3} + 2k\pi$$

$$\left(1+\sqrt{3}i\right)^8 = 2^8\left(\cos\left(\dfrac{8\pi}{3}\right) + i\sin\left(\dfrac{8\pi}{3}\right)\right) = 256\left(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right) = 128\left(-1+\sqrt{3}i\right)$$

$$\cos\left(\dfrac{8\pi}{3}\right) = \cos\left(2\pi + \dfrac{2}{3}\pi\right) = \cos\left(\dfrac{2}{3}\pi\right) = \cos\left(\pi - \dfrac{\pi}{3}\right) = -\cos\left(\dfrac{\pi}{3}\right) = -\dfrac{1}{2}$$

$$\sin\left(\dfrac{8\pi}{3}\right) = \sin\left(2\pi + \dfrac{2}{3}\pi\right) = \sin\left(\dfrac{2}{3}\pi\right) = \sin\left(\pi - \dfrac{\pi}{3}\right) = \sin\left(\dfrac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$$

**3)  Podane liczby zespolone zapisać w postaci trygonometrycznej**

**a)**  $7 + 7i$

$$|z| = \sqrt{7^2 + 7^2} = \sqrt{98} = 7\sqrt{2}$$

$$\left.\begin{array}{l} \cos\varphi = \dfrac{7}{7\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \\[3mm] \sin\varphi = \dfrac{7}{7\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \end{array}\right\} \Rightarrow \varphi = \dfrac{\pi}{4}$$

$$7 + 7i = 7\sqrt{2}\left(\cos\left(\dfrac{\pi}{4}\right) + i\sin\left(\dfrac{\pi}{4}\right)\right)$$

**b)**  $\sqrt{3} - i$

$$|z| = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = 2$$

$$\cos\varphi = \frac{\sqrt{3}}{2}$$

$$\sin\varphi = -\frac{1}{2}$$

$ze\ wzorow\ redukcyjnych:$

$$\sin(-\alpha) = -\sin\alpha$$

$$\sin\left(-\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right)$$

$$\cos(-\alpha) = \cos\alpha$$

$$\cos\left(-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right)$$

$$\Rightarrow \quad \varphi = -\frac{\pi}{6} + 2k\pi$$

$$\sqrt{3} - i = 2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$$

**4) Rozwiąż równanie**

a) $4x^2 - x + 1 = 0$

$$\Delta = 1 - 16 = -15 = \sqrt{15i^2} = \sqrt{15}i$$

$$x_1 = \frac{1 - \sqrt{15}i}{8}$$

$$x_2 = \frac{1 + \sqrt{15}i}{8}$$