STATISTICA $\underline{X} = (X_1, \dots, X_n)$ $\quad X_i$ IID $\sim f(\cdot\,|\Theta)$ $\quad \underline{X} \sim \prod f(x_k|\Theta)$

MEDIA CAMPIONARIA: $\bar{X} = \frac{1}{n}\sum_k^n X_k$ $\quad$ VARIANZA CAMPIONARIA: $S^2 = \frac{1}{n-1}\sum_k^n (X_K - \bar{X})^2$

LEGGI CHI $\chi^2(1) \sim \Gamma\left(\frac{1}{2}, \frac{1}{2}\right)$

$\chi^2(n) \sim \Gamma\left(\frac{n}{2}, \frac{1}{2}\right)$

$\sum_k^n \Gamma(\alpha_i, \lambda) \sim \Gamma(\sum \alpha_i, \lambda)$

TEOREMA: $X_i \sim N(\mu, \sigma^2)$ IID. Allora: $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{2}\right)$ $\quad \frac{S^2}{\sigma^2} = \frac{1}{n-1}\sum\left(\frac{X_i - \bar{X}}{\sigma}\right)^2$ $\quad \frac{(n-1)S^2}{\sigma^2} = \sum\left(\frac{X_i - \bar{X}}{\sigma}\right)^2 \sim \chi^2(n-1)$ $\quad \bar{X} \sqcup S^2$

TEOREMA: $X_i \sim P_0(\lambda) \Rightarrow \bar{X} \sim N\left(\lambda, \sqrt{\frac{\lambda}{n}}\right)$ $\quad$ **TEOREMA**: $X \sim \exp(\lambda) \Rightarrow \bar{X} = MLE\left(\frac{1}{\lambda}\right)$

t-STUDENT (Se $X_i \sim N(\mu, \sigma^2) \Rightarrow$) $\frac{\bar{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0,1)$, se non conosco σ: $\frac{\bar{X}-\mu}{\sqrt{\frac{S^2}{n}}} \sim t(n-1)$ $\quad$ dove $\quad t(n) \overset{D}{\rightarrow} N(0,1)$ $\quad \frac{S^2(n-1)}{\sigma^2} \sim \chi^2_{n-1}$

STATISTICHE / STIMATORI

Principio sufficienza: $T(\underline{X})$ è statistica sufficiente per Θ se la distribuzione di $\underline{X}$ dato $T(\underline{X})$ non dipende da Θ (se $\mathbb{P}_\Theta(X = \cdots | T = a)$ non dipende da Θ)

TEOREMA: stimatore $T(\underline{X}) \sim q(\underline{x}|\Theta)$, $X \sim f(\underline{x}|\Theta) \Rightarrow T$ è sufficiente se $\forall \underline{x}$ $\quad \Theta \mapsto \frac{f(\underline{x}|\Theta)}{q(\underline{x}|\Theta)}$ è costante

Stimatore corretto: T è corretto di $\psi(\Theta)$ se $\mathbb{E}_\Theta[T] = \psi(\Theta) \quad \forall\Theta$	Stimatore asintoticamente corretto: T_n di $\psi(\Theta)$ se $\lim_{n\to\infty} \mathbb{E}_\Theta[T_n] = \psi(\Theta) \quad \forall\Theta$
Stimatore consistente: T_n di $\psi(\Theta)$ se $T_n \overset{P(\mathbb{P}_\Theta)}{\rightarrow} \psi(\Theta)$	Stimatore asintotico normale: T_n di $\psi(\Theta)$ se $\exists\sigma = \sigma(\Theta)$ tale che: $\frac{T_n - \psi(\Theta)}{\sigma(\Theta)/\sqrt{n}} \overset{D}{\rightarrow} N(0,1) \quad \forall\Theta$

METODO DEI MOMENTI

$X \sim f(x|\Theta)$ $\quad \Theta = (\Theta_1, \dots, \Theta_k)$ $\quad \mathbb{E}_\Theta[X^a] = m_a(\Theta_1, \dots, \Theta_k)$ $\quad a = 1, \dots, k$ $\quad \overline{M_a} = \frac{1}{n}\sum_{j=1}^n X_j^a$ $\quad \overline{m_a} = \frac{1}{n}\sum_{j=1}^n x_j^a$

Se esiste la soluzione di $\begin{cases} \overline{m_1} = m_1(\Theta_1, \dots, \Theta_k) \\ \dots \\ \overline{m_k} = m_k(\Theta_1, \dots, \Theta_k) \end{cases}$ è lo stimatore di $\widehat{\Theta} = (\widehat{\Theta_1}, \dots, \widehat{\Theta_k})$

STIME:

$\mu \sim \bar{X} = \frac{X_1 + \cdots + X_n}{n}$

$\sigma^2 \sim \frac{1}{n}\sum(X_k - \bar{X})^2$

METODO DI MASSIMA VEROSIMIGLIANZA $\quad \mathbb{I}_{[0,\theta]}(x) = \mathbb{I}_{[x,\infty)}(\theta)$, lo tolgo da $L(\theta|x)$ e mi ricordo l'intervallo

(**Non Giro Var**) $L(\Theta|\underline{x}) = \prod f(x_k|\Theta)$ ($= \mathbb{P}_\Theta(X_1 = x_1, \dots, X_n = x_n)$ se discreta, densità se continua) cerco il massimo di L (è in funzione di Θ)

$L(\Theta|x) = \ln(L(\theta|x))) = \sum \ln(f(x_k|\theta))$	$\mu \sim \bar{X}$ corretto, efficiente, consistente, asintoticamente normale $\quad \sigma^2 = \frac{1}{n}\sum(X_k - \bar{X})^2$

TEST DI IPOTESI

$w(X) = \frac{\sup_{\Theta\in\Theta_0} L(\Theta|x)}{L(MLE(\Theta)|x)}$ $\quad \Rightarrow R = \{x: w(x) \le \epsilon\}$ regione di rifiuto Θ_0

POTENZA DEL TEST: $\beta(\Theta) = \mathbb{P}_\Theta(x \in R) = \sup_{\theta\in\Theta_0} \mathbb{P}_\theta(x \in R)$ probabilità ERR TIPO 1. $\quad$ SIGNIFICATIVITA': $\alpha = \sup_{\Theta\in\Theta_0} \beta(\Theta) = \sup_{\Theta\in\Theta_0} \mathbb{P}(w(x) \le \epsilon) = 5\%$

$X \sim N(\mu, \sigma^2)$, σ^2 noto, $H_0: \mu = \mu_0$ $\quad R = \left\{x: |\bar{x} - \mu_0| > \sqrt{\frac{\sigma^2}{n}}\Phi_{1-\frac{\alpha}{2}}\right\}$

$X \sim N(\mu, \sigma^2)$, σ^2 nota, $H_0: \mu \le \mu_0$ $\quad R = \left\{x: \bar{x} - \mu_0 > \sqrt{\frac{\sigma^2}{n}}\Phi_{1-\alpha}\right\}$

$X \sim N(\mu, \sigma^2)$, σ^2 **non** nota, $H_0: \mu = \mu_0$ $\quad R = \left\{x: |\bar{x} - \mu_0| > t_{1-\frac{\alpha}{2}}^{(n-1)}\sqrt{\frac{s^2}{n}}\right\}$ $\quad s^2 = \frac{1}{n-1}\sum(x - \bar{x})^2$

$X \sim p^x(1-p)^{1-x}$, $H_0: p \le p_0$ $\quad R = \left\{casi \ge \sqrt{np_0(1-p_0)}\phi_{1-\alpha} + np_0\right\}$

$X \sim p^x(1-p)^{1-x}$, $H_0: p \ge p_0$ $\quad R = \left\{casi < -\sqrt{np_0(1-p_0)}\phi_{1-\alpha} + np_0\right\}$

$H_0: p = p_0$ $\quad R = \left\{|casi - np_0| > \phi_{1-\frac{\alpha}{2}}\sqrt{np_0(1-p_0)}\right\}$

$X \sim N(\mu, \sigma^2), Y \sim N(\eta, \sigma^2)$ $\quad H_0: \mu = \eta$ $\quad R = \left\{(X,Y): \left|\frac{\bar{X} - \bar{Y}}{\sqrt{(S_{xx} - S_{yy})\left(\frac{1}{n} + \frac{1}{m}\right)}}\right| > t_{1-\frac{\alpha}{2}}^{(n+m-2)}\right\}$

$X \sim N(\mu, \sigma^2), H_0: \sigma^2 \le \sigma_0^2$ $\quad R = \left\{S^2 \ge \frac{\sigma^2}{n-1}\chi^2_{1-\alpha}(n-1)\right\}$

$X \sim N(\mu, \sigma^2), H_0: \sigma^2 = \sigma_0^2$ $\quad R = \left\{\chi^2_{\frac{\alpha}{2}}(n-1) \le \frac{n-1}{\sigma_0^2}s^2 \le \chi^2_{1-\frac{\alpha}{2}}(n-1)\right\}$

$X \sim N(\mu, \sigma_1^2), Y \sim N(\eta, \sigma_2^2)$ $\quad \sigma$ note

$H_0: \mu_1 > (<)\mu_2$ $\quad R = \left\{\frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} > (<-)\phi_\alpha\right\}$ $\quad H_0: \mu_1 = \mu_2$ $\quad R = \left\{\left|\frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}}\right| > \phi_{\frac{\alpha}{2}}\right\}$

$X \sim N(\mu, \sigma_1^2), Y \sim N(\eta, \sigma_2^2)$ $\quad \sigma_1, \sigma_2$ incognite

$H_0: \mu_1 > (<)\mu_2$ $\quad R = \left\{\frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}} > (<-)t_\alpha^{n+m-2}\right\}$ $\quad H_0: \mu_1 = \mu_2$ $\quad R = \left\{\left|\frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}}\right| > t_{\frac{\alpha}{2}}^{n+m-2}\right\}$

P-VALUE Una statistica $p(X)$ è detta p-value se: $0 \le p(x) \le 1$ e $\mathbb{P}_\theta(p(x) \le \alpha) = \alpha \ \forall\theta \in \Theta_0, \forall\alpha \in [0,1]$

$\Rightarrow$Se $R = \{w(x) > \phi_{1-\alpha}\}$ allora il p-value è p tale che: $\phi_{1-p} = w(x)$ $\quad$ dove quindi $\forall\alpha > p \Rightarrow$ rifiuto H_0

DEVIAZIONE STANDARD: $\sqrt{Var(X)}$

REGRESSIONE LINEARE

$\mathbb{E}[Y \mid X = x] = \alpha + \beta x$ dove $Y_i = \alpha + \beta X_i + N_i \Rightarrow \hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$ $\quad \hat{\beta} = \frac{S_{xy}}{S_{xx}}$ $\quad E = S_{yy} - \frac{S_{xy}^2}{S_{xx}}$ $\quad \left[S_{xx} = \frac{1}{n}\sum(x_i - \bar{x})^2 \quad S_{xy} = \frac{1}{n}\sum(x_i - \bar{x})(y_i - \bar{y})\right]$

Stimatore lineare per β: $T_\beta = \frac{1}{S_{xx}}\sum(x_i - \bar{X})Y_i$ $\quad V(T_\beta) = \frac{Var(N)}{S_{xx}}$ $\quad$ Stimatore lineare di α: $T_\alpha = \bar{Y} - T_\beta\bar{X}$

CASO: ERRORE GAUSSIANO $\quad N \sim N(0, \sigma^2 I_n) \Rightarrow Y_i \sim N(\alpha + \beta X_i, \sigma^2)$ $\quad \hat{\sigma}^2 = S^2 = \frac{1}{n-2}\sum(Y_i - \hat{\alpha} - \hat{\beta}X_i)^2 = \frac{1}{n-2}\left(S_{yy} - \frac{S_{xy}^2}{S_{xx}}\right)$

Stimatore per β: $T_\beta = \frac{1}{S_{xx}}\sum(X_i - \bar{X}) \sim N\left(\beta, \frac{\sigma^2}{S_{xx}}\right)$ $\quad$ Stimatore per α: $T_\alpha = \bar{Y} - T_\beta\bar{X} \sim N\left(\alpha, \sigma^2\frac{\bar{X^2}}{S_{xx}}\right)$ $\quad \Rightarrow \frac{T_\beta - \beta}{\sqrt{S^2/S_{xx}}} \sim t(n-2)$ $\quad \frac{T_\alpha - \alpha}{\sqrt{S^2\bar{X^2}/S_{xx}}} \sim t(n-2)$

Segue: $H_0: \beta = 0$ (non c'è relazione lineare), $R = \left\{y: \left|\frac{\hat{\beta}}{\sqrt{S^2/S_{xx}}}\right| > t_{1-\frac{\alpha}{2}}^{(n-2)}\right\}$, dunque se $|\hat{\beta}| > \sqrt{\frac{S^2}{S_{xx}}}t_{1-\frac{\alpha}{2}}^{(n-2)}$ $\Rightarrow$esiste relazione lineare tra X, Y

Segue: $H_0: \beta \ge \beta_0$ $\quad R = \left\{\frac{\hat{\beta} - \beta_0}{\sqrt{s^2/n}} < -t_{1-\alpha}(n-2)\right\}$

INTERVALLO DI CONFIDENZA $\quad$ Confidenza di $I = [\theta_-, \theta_+]$ è $\inf_{\theta\in\Theta} \mathbb{P}_\theta([\theta_-, \theta_+] \ni \theta) = \alpha \sim 95\%$ $\quad \bar{X} = \frac{1}{n}\sum_k^n X_k$ $\quad S^2 = \frac{1}{n-1}\sum_k^n(X_K - \bar{X})^2$

$X \sim N(\mu, \sigma^2)$, σ^2 noto $\Rightarrow [\bar{x} - a, \bar{x} + a]$ $\quad \mathbb{P}_\mu([\bar{x} - a, \bar{x} + a]) = 2\phi\left(a\sqrt{\frac{n}{\sigma^2}}\right) - 1 = \alpha$

$X \sim N(\mu, \sigma^2) \Rightarrow I = [x-1, x+1]$ $\quad \inf_{(\mu,\sigma^2)} \mathbb{P}_\theta([\theta_-, \theta_+] \ni \mu) = \inf_{(\mu,\sigma^2)} 2\phi\left(\sqrt{\frac{n}{\sigma^2}}\right) - 1 = 0$ $\ per \quad \sigma \to \infty$

$X \sim P_0(\lambda) \Rightarrow \bar{X} \sim N\left(\lambda, \sqrt{\frac{\lambda}{n}}\right)$

$I_\lambda = \left(\bar{x} - \phi_{\frac{1+\alpha}{2}}\left(\sqrt{\frac{\bar{X}}{n}}\right), \bar{x} + \phi_{\frac{1+\alpha}{2}}\left(\sqrt{\frac{\bar{X}}{n}}\right)\right)$ $\quad \left(\bar{x} - \phi_\alpha\sqrt{\frac{\bar{x}}{n}}, \infty\right)$ $\quad \left(0, \bar{x} - \phi_{1-\alpha}\sqrt{\frac{\bar{x}}{n}}\right)$

$X \sim N(\mu, \sigma^2)$

$I_\mu = \left(\bar{x} - t_{\frac{1+\alpha}{2}}^{n-1}\frac{s}{\sqrt{n}}, \bar{x} + t_{\frac{1+\alpha}{2}}^{n-1}\frac{s}{\sqrt{n}}\right)$ $\quad \left(\bar{x} - \frac{s}{\sqrt{n}}t_{\frac{1+\alpha}{2}}(n-1), \infty\right)$ $\quad \left(-\infty, \bar{x} + \frac{s}{\sqrt{n}}t_{\frac{1+\alpha}{2}}(n-1)\right)$

$I_{\sigma^2} = \left(\frac{(n-1)s^2}{\chi^2_{\frac{1+\alpha}{2}}(n-1)}, \frac{(n-1)s^2}{\chi^2_{\frac{1-\alpha}{2}}(n-1)}\right)$ $\quad \left(\frac{(n-1)s^2}{\chi^2_\alpha(n-1)}, \infty\right)$ $\quad \left(0, \frac{(n-1)s^2}{\chi^2_{1-\alpha}(n-1)}\right)$ se μ nota$\Rightarrow$in S^2 uso μ invece di $\bar{x}$

$X \sim N(\mu, \sigma^2)$, σ^2 noto

$I_\mu = \left(\bar{x} - \phi_{\frac{1+\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{x} + \phi_{\frac{1+\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right)$ $\quad \left(\bar{x} - \frac{\sigma\phi_\alpha}{\sqrt{n}}, \infty\right)$ $\quad \left(-\infty, \bar{x} + \frac{\sigma\phi_\alpha}{\sqrt{n}}\right)$

$X \sim \exp(\lambda) \Rightarrow I_\lambda = \left(\frac{1}{\bar{X}}\left(1 - \phi_{\frac{1+\alpha}{n}}\left(\frac{1}{\sqrt{n}}\right)\right), \frac{1}{\bar{X}}\left(1 + \phi_{\frac{1+\alpha}{n}}\left(\frac{1}{\sqrt{n}}\right)\right)\right)$ $\quad \left(0, \frac{1}{\bar{x}}\left(1 + \frac{\phi_\alpha}{\sqrt{n}}\right)\right)$ $\quad \left(\frac{1}{\bar{x}}\left(1 + \frac{\phi_{1-\alpha}}{\sqrt{n}}\right), \infty\right)$

$X \sim B(p), \bar{x} = \hat{p} = \frac{\#fav}{\#tot}$ $\quad I_p = \left(\hat{p} \pm \phi_{\frac{1+\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$ $\quad \left(\hat{p} - \phi_\alpha\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, 1\right)$ $\quad \left(0, \hat{p} - \phi_{1-\alpha}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

$\sum X_i \sim \Gamma(n, \lambda)$, c_1, c_2 tali che $\alpha = \mathbb{P}(c_1 < 2\lambda nx < c_2) \Rightarrow I_\lambda = \left(\frac{1}{\bar{x}}\frac{c_1}{2n}, \frac{1}{\bar{x}}\frac{c_2}{2n}\right)$

- $X \sim N(\mu_x, \sigma_1^2), Y \sim N(\mu_Y, \sigma_2^2)$, σ_2^2, σ_2^2 note

$I_{\mu_x - \mu_y} = \left(\bar{x} - \bar{y} - \phi_{\frac{1+\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}, \bar{x} - \bar{y} + \phi_{\frac{1+\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}\right)$

- $X \sim N(\mu_x, \sigma_1^2), Y \sim N(\mu_Y, \sigma_2^2)$, $\sigma_1^2 = \sigma_2^2$ incognite $\quad \sigma^2 = \frac{\sum(x_i - \bar{x})^2 + \sum(y_i - \bar{y})^2}{n+m-2}$

$I_{\mu_x - \mu_y} = \left(\bar{x} - \bar{y} - t_{\frac{1-\alpha}{2}}^{n+m-2}\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}, \bar{x} - \bar{y} + t_{\frac{1-\alpha}{2}}^{n+m-2}\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}\right)$ $\quad \left(\bar{x} - \bar{y} - t_\alpha^{n+m-2}\sqrt{\sigma^2\frac{n+m}{nm}}, \infty\right)$

- $X \sim N(\mu_x, \sigma_1^2), Y \sim N(\mu_Y, \sigma_2^2)$, $\sigma_1^2 = \sigma_2^2$ note $\quad I_{\mu_X - \mu_Y} = \left(\bar{x} - \bar{y} - \phi_\alpha\sigma\sqrt{\frac{n+m}{nm}}, \infty\right)$

CONVERGENZA

QUASI CERTA: $\mathbb{P}(\{w: X_n(w) \to X(w)\}) = 1$ IN PROBABILITÀ: $\forall \epsilon > 0 \;\; \mathbb{P}(|X_n - X| > \epsilon) \to 0$

DISTRIBUZIONE: $F_{X_n}(t) \to F_X(t)$ nei punti di continuità di $F_X \Leftrightarrow \forall f \in C^0(\mathbb{R}) + limitata$ si ha $\lim_{n\to\infty} \mathbb{E}[f(X_n)] = \mathbb{E}[f(X)]$

$X_n \xrightarrow{D} X$ non implica $X_n \xrightarrow{P} X$	$X_n \xrightarrow{QC} X \Rightarrow X_n \xrightarrow{P} X$	$X_n \xrightarrow{P} X$ non implica $X_n \xrightarrow{QC} X$	$X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{D} X$
X_n con media μ_n e varianza σ_n^2, se $\mu_n \to \mu$ e $\sigma_n^2 \to 0 \;\; \Rightarrow X_n \xrightarrow{P} \mu$			$X_n \xrightarrow{D} X \equiv \mu \Rightarrow X_n \xrightarrow{P} X$
$X_n \xrightarrow{D} X \Rightarrow \forall f: \mathbb{R} \to \mathbb{R}$ a supporto compatto, si ha $\mathbb{E}[f(X_n)] \to \mathbb{E}[f(X)]$			$\forall X_n \xrightarrow{D} X \Leftrightarrow \forall x: \mathbb{P}(X_n = x) \to \mathbb{P}(X = x)$

LEGGE DEBOLE (FORTE) DEI GRANDI NUMERI

X_n iid con media μ finita e varianza σ^2 finita. Sia $\overline{X} = \frac{X_1 + \cdots + X_n}{n} \Rightarrow \left(\mathbb{E}[\overline{X}] = \mu, V(\overline{X}) = \frac{\sigma^2}{n}\right) \bar{X} \xrightarrow{p} \mu$ (legge forte: $\bar{X} \xrightarrow{QC} \mu$)

TEOREMA

X_n iid con media μ finita, e fgm $M_X(s)$ definita in un intorno completo dell'origine.

$$\Rightarrow \forall \epsilon > 0 \; \exists p > 1: \mathbb{P}\left(\frac{S_n}{n} - \mu > \epsilon\right) \leq p^n \qquad p = \inf\{e^{-(\mu+\epsilon)s} M_X(s): 0 < s < 1\}$$

ES: $X_n \sim B(p) \Rightarrow p \sim 1 - 2\epsilon^2$

APPROSSIMAZIONE POISSON A BINOMIALE

$$X_n \sim B(n, p_n), p_n \leq 1, np_n \to \lambda \Rightarrow \mathbb{P}(X_n = x) \to e^{-\lambda} \frac{\lambda^x}{x!}$$

POISSON ESERCIZI

$p = \frac{4.9 \to suicidi}{100.000 \to persone}$, su P popolazione$\Rightarrow P_0(\lambda = P \cdot p)$

TEOREMA DEL LIMITE CENTRALE

$$\boxed{B(n,p) = \sum_{i=1} B(p)}$$

$\{X_n\}$ successione di VA IID, $\mu = \mathbb{E}[X_i], \sigma^2 = V(X_i)$

$S_n^* = \frac{S_n - n\mu}{\sqrt{n\sigma^2}} \xrightarrow{D} Z \sim N(0,1)$ $\qquad S_n \sim N(n\mu, n\sigma^2)$ per n grande

$\mathbb{P}(S_n \leq x) \sim \mathbb{P}\left(\frac{-\frac{1}{2} - n\mu}{\sigma\sqrt{n}} < S_n^* \leq \frac{x + \frac{1}{2} - n\mu}{\sigma\sqrt{n}}\right)$ (= se S_n è somma di gauss)

CORREZIONE: $\mathbb{P}(S_n \geq k) = \mathbb{P}(S_n \geq k - 0.5)$

$\mathbb{P}(S_n \leq k) = \mathbb{P}\left(S_n < k + \frac{1}{2}\right) \qquad \mathbb{P}(S_n = 0) = \mathbb{P}(-0.5 < S_n < 0.5)$

VA STANDARDIZZATA X va$\Rightarrow X^* = \frac{X - \mu}{\sqrt{\sigma^2}} \qquad \mathbb{E}[X^*] = 0 \qquad V(X^*) = 1$

VALE: $\left(\frac{S_m}{m}\right)^* = S_m^*$

ENTROPIA

$I(A) = -\log_2(\mathbb{P}(A)) \qquad H(X) = -\sum_k p_k \log_2(p_k)$

$$Z = (X, Y) \sim \{(Z_{ij} = (x_i, y_j), p_{ij})\} \Rightarrow H(Z) = -\sum p_{ij} \log_2(p_{ij})$$

LEMMI

$$X = (X_1, X_2) \quad X_1 \sqcup X_2 \Rightarrow H(X) = H(X_1) + H(X_2)$$
$$H(X) = 0 \Leftrightarrow \exists C: \mathbb{P}(X = C) = 1$$
$$X \sim \{(x_k, p_k): k = 1, \dots, n\} \Rightarrow H(X) \leq \log_2(n)$$
$$X \sim \{(x_k, p_k): k = 1, \dots, n\} \Rightarrow H(X) = \log_2(n) \Leftrightarrow \mathbb{P}(X = x_k) = \frac{1}{n} \forall k$$
$$H(g(x)) \leq H(X), \quad = \Leftrightarrow g \; iniettiva$$

PRINCIPIO DI MASSIMA ENTROPIA

$$L = -\sum p_k \ln(p_k) + (\alpha + 1)\left(\sum p_k - 1\right) + \cdots \Rightarrow \frac{\partial L}{\partial p_i} = 0, \frac{\partial L}{\partial \alpha} = 0, \frac{\partial L}{\partial \beta} = 0, \dots$$

ENTROPIA CONDIZIONATA

$$H_{X=x_i}(Y) = -\sum_j \mathbb{P}_{X=x_i}(Y = y_j) \log_2\left(\mathbb{P}_{X=x_i}(Y = y_j)\right)$$

$$H_X(Y) = H(X,Y) - H(X) = \sum_i \mathbb{P}(X = x_i) H_{X=x_i}(Y) = -\sum_i \sum_j p_{ij} \log_2\left(\frac{p_{ij}}{p_i^x}\right)$$

$$X \sqcup Y \Rightarrow H_{X=x_i}(Y) = H(Y), H_X(Y) = H(Y)$$

LEMMI

$0 \leq H_X(Y) = H(Y) \qquad H_X(Y) = H(Y) \Leftrightarrow X \sqcup Y$

$H_X(Y) = 0 \Leftrightarrow Y = f(X) \qquad H(X,Y) = H(X) + H_X(Y)$

$S = X + Y, X \sqcup Y \Rightarrow \max(H(X), H(Y)) \leq H(S) \leq H(X) + H(Y), H_X(Y) = H_X(S)$

PASSEGGIATA CASUALE

#TRAIETTORIE $\binom{n}{\frac{n+y-x}{2}} = N((0,x) \to (n,y))$ #trai da $x, t = 0$ a $y, t = n$

$$N((0,x) \to (n,y)) = N((m, x+k) \to (n+m, y+k))$$
$$N((0,0) \to (n,y)) = N((0,0) \to (n,-y)) \quad \text{se } p = \frac{1}{2}$$

PRINCIPIO RIFLESSIONE $N_+(A \to B) = N(A \to B) - N(-A \to B)$

TEOREMA VOTAZIONE $N_+((0,0) \to (2n, 2r)) = \frac{r}{n} N((0,0) \to (2n, 2r))$

PROBLEMA: traiettoria da $(0,0)$ a $(p+m, p-m)$ stando sempre sopra lo zero.

#TOTALI: $N((0,0) \to (p+m, p-m))$ #FAV: $N_+((0,0) \to (p+m, p-m))$

(da teo votazione) $\Rightarrow \mathbb{P} = \frac{p-m}{p+m}$

PROBABILITA' $\mathbb{P}_x(S_n = y) = \binom{n}{\frac{n+y-x}{2}} p^{\frac{n+y-x}{2}} q^{\frac{n-y+x}{2}}$

RITORNO ALL'ORIGINE $u_{2n} = \mathbb{P}(S_{2n} = 0 | S_0 = 0) = \binom{2n}{n} \frac{1}{2^{2n}} \sim \frac{1}{\sqrt{\pi n}}$

$$f_{2n} = \mathbb{P}(S_1 \neq 0, \dots, S_{2n} \neq 0 | S_0 = 0) \;\; \text{(anche solo } S_{2k})$$

$$u_{2n} = \sum_{r=1}^{n} f_{2r} u_{2n-2r} \qquad u_{2n} = \mathbb{P}_0(S_1 \neq 0, \dots, S_{2n} \neq 0)$$

$$f_{2n} = u_{2n-2} - u_{2n} = \frac{1}{2n-1} u_{2n}$$

PASSEGGIATE CON BARRIERE

Date due barriere: a, b	T tempo di arrivo a una barriera
$\mathbb{P}_x(S_T = a) + \mathbb{P}_x(S_t = b) = 1$	$\mathbb{E}_x[S_T] = a\mathbb{P}_x(S_T = a) + b\mathbb{P}_x(S_T = b)$
1° identità Wald	$\mathbb{E}_x[S_t] = x + (p - q)\mathbb{E}[T]$
2° identità di Wald	$\mathbb{E}[(S_T - \mu T)^2] = \sigma^2 \mathbb{E}[T]$
CASO SIMMETRICO	**CASO NON SIMMETRICO**
$\mathbb{E}_x[S_T] = x$ $\mathbb{E}_x[T] = (a - x)(x - b)$ $\mathbb{P}_x(S_T = a) = \frac{x-b}{a-b}$ $\mathbb{P}_x(S_T = b) = \frac{a-x}{a-b}$	$\mathbb{P}_x(S_T = b) = \frac{\left(\frac{1-p}{p}\right)^{x-a} - 1}{\left(\frac{1-p}{p}\right)^{b-a} - 1}$ $\mathbb{E}_x[S_T] = (b - a)\mathbb{P}_x(S_T = b) + a$ $\mathbb{E}_x[T] = \frac{\mathbb{E}_x[S_t] - x}{(2p - 1)}$

PASSEGGIATA A 1 BARRIERA $a = 0, b \to \infty$

$$\mathbb{P}_x(S_T = 0) = \frac{\left(\frac{1-p}{p}\right)^x - \left(\frac{1-p}{p}\right)^b}{1 - \left(\frac{1-p}{p}\right)^b} \Rightarrow \begin{cases} p > \frac{1}{2} & per\ b \to \infty\ arrivo\ a\ 0\ con\ \mathbb{P}\ cresc \\ p < \frac{1}{2} & per\ b \to \infty\ arrivo\ a\ 0\ prima\ o\ poi \end{cases}$$

HOW TO:

- **Intervallo di confidenza**

$X \sim N(\mu, \sigma)$ Se posso scrivo: $\mathbb{P}\left(\bar{x} - \phi_{\frac{1+\alpha}{2}} \sqrt{\frac{\sigma^2}{2}} < \mu < \bar{x} + \cdots\right) = \alpha$

Posso approssimare con $\sigma^2 = \bar{x}$. Se le conosco le ficco dentro.

Altrimenti: $\mathbb{P}\left(\bar{x} - t_{\frac{1+\alpha}{2}}^n \sqrt{\frac{s^2}{n}} < \mu < \bar{x} + \cdots\right) = \alpha$

MEMO

$\mathbb{P}(\mu \geq \bar{x} - \cdots) \Rightarrow (\dots, \infty)$	$\mathbb{P}\left(-\phi_{\frac{1+\alpha}{2}} < Z < \phi_{\frac{1+\alpha}{2}}\right) = \alpha$
$\phi_{0.975} = 1.69 \qquad \phi_{0.95} = 1.645 \qquad \phi_{0.05} = -2.575$	

- **TH / IC**

$$R = I^C$$

Noto: TH→$\mu \leq \mu_0 \Rightarrow R = I^C$ con $I = (\dots, \infty)$

Test ipotesi	Intervallo confidenza
$1 - \alpha$	α
$1 - \frac{\alpha}{2}$	$\frac{1 + \alpha}{2}$
$\frac{\alpha}{2}$	$\frac{1 - \alpha}{2}$