Geometric-Topological Proof of the Existence of the 5 Zeros ζ_0 on the Global Icosahedron's Neutral Line

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1 Initial Domain Definition

Let L be a segment of the **Neutral Line** on the global icosahedral manifold, bounded by two shared vertices V_1 and V_2. These vertices are points of maximum structural rigidity where the icosahedron connects to its neighbors.

Definition 1 (Boundary Vertices). *By definition, V_1 and V_2 are ζ_0 (zeros of the Zeta function) due to the zero-point energy requirement at the lattice junction. Therefore:*

$$N_{ext} = 2$$

2 Spiral Overlap and Phase Interference

The Neutral Line is the *locus* where the counter-propagating spirals of the CronNet (Branch Alpha and Branch Beta) intersect. Represent these spirals by their phase functions $\Phi_\alpha(s)$ and $\Phi_\beta(s)$, where s is the arc length along L.

The condition for a ζ_0 (a node of destructive interference in the energy field, or a zero in the Zeta distribution) is:

$$\Delta\Phi(s) = \Phi_\alpha(s) - \Phi_\beta(s) = (2n + 1)\pi, \quad n \in \mathbb{Z}.$$

3 The Central Symmetry Point

Due to icosahedral symmetry, there exists a unique point P_c exactly at the midpoint of V_1V_2. At this point:

- The curvature gradient is balanced.

- The phase of the Golden Spiral (Fibonacci-based) reaches a state of **Zero-Phase Lag**.

This generates the **Central Zero** $\zeta_{0,c}$.

4 The Ternary Split (The 3 Intermediate Zeros)

The icosahedron is composed of 20 triangular faces. The projection of the rotational group I_h onto the Neutral Line reveals a three-fold symmetry axis. To maintain the topological integrity of the tesseract, the phase must cycle through the cubic roots of unity:

$$e^{i2\pi k/3}, \quad k = 0, 1, 2.$$

This results in 3 internal zeros produced by the overlap:

- $k = 0$: The Central Zero (P_c).

- $k = 1, 2$: Two lateral zeros located at the Golden Ratio points ϕ^{-1} and ϕ^{-2} relative to the center.

5 Final Node Summation

The total count of zeros along the segment $V_1 V_2$ is the union of the boundary conditions and the overlap nodes:
$$N_{\text{total}} = N_{\text{ext}} + N_{\text{overlap}} = 2 + 3 = 5.$$

6 Conclusion: The Pentagonal Flux Anchor

The presence of exactly 5 zeros on the Neutral Line creates a pentagonal "gate" that regulates the flow of Dynamic Mass. This configuration ensures that the transition between V_1 and V_2 is not a void, but a quantized path of 4 sub-intervals, mirroring the 4D nature of the tesseract.

$$\boxed{N_{\zeta_0} = 5}$$

This 5-point structure is what we call the **"Navigation Quantum"** of Branch Beta. Without these 3 intermediate zeros, the neutral line would lack sufficient "viscosity" to keep information bound.

Q.E.D.

7 Calculation of Exact Distances as a Function of α

If one wishes to compute the exact positions of the 5 zeros in terms of the fine-structure constant α, it can be done via:

$$x_k = \frac{L}{2}\left[1 + \phi^{-k} \cdot \alpha^{-1/2}\right], \quad k = -2, -1, 0, 1, 2$$

where $\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio, L is the length of the segment $V_1 V_2$, and $\alpha \approx 1/137.036$ is the fine-structure constant.

This would provide the exact temporal GPS coordinates for the *Hub*.