# Interesting Application of Euler's Fomula

Souta

$$\sin x = 2$$

This equation seems to be unsolvable because $\sin x$ only takes values from $-1$ to 1 if x is real. However, when $x$ takes complex values, a solution to this equation does indeed exist.

From Euler's identity, $e^{ix} = \cos x + i \sin x$. Here, simplifying for $\sin x$, we obtain $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$.

$$\therefore \sin x = 2 \iff \frac{e^{ix} - e^{-ix}}{2i} = 2 \iff e^{ix} - e^{-ix} - 4i = 0$$

Multiple by $e^{ix}$, we can get $\left(e^{ix}\right)^2 - 1 - 4ie^{ix} = 0 \left(\because e^{ix} \cdot e^{-ix} = 1\right)$. This is an quadratic equation for $e^{ix}$. From quadratic formula:

$$e^{ix} = \left(2 \pm \sqrt{3}\right) i = \left(2 + \sqrt{3}\right) \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right).$$

Now from $x \in \mathbb{C}$, let $x = a + bi (a, b \in \mathbb{R})$.

$$\begin{aligned} e^{ix} &= e^{i(a+bi)} \\ &= e^{-b+ai} \\ &= e^{-b} \cdot e^{ai} \\ &= e^{-b} \cdot (\cos a + i \sin a) \end{aligned}$$

$$\therefore a = \frac{\pi}{2} + 2n\pi (n \in \mathbb{Z}), b = \pm \log 2 + \sqrt{3}$$

Substituting $a$ and $b$ for $x$:

$$x = \left(\frac{4n+1}{2}\pi \pm \log\left(2\sqrt{3}\right) i\right) (n \in \mathbb{Z}).$$