

EE6111 24S1

1. Consider convolutional encoder as shown in Figure 1.

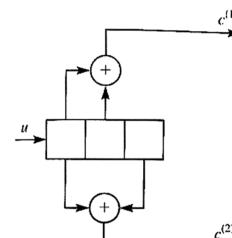
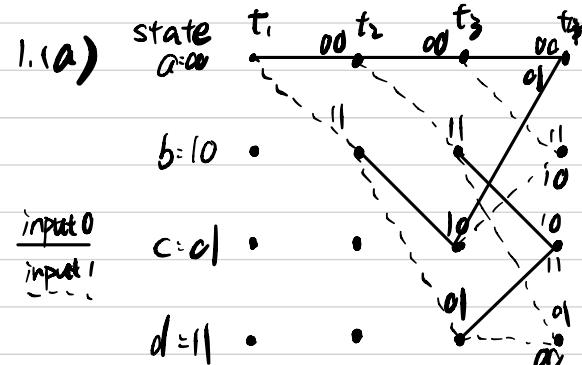


Figure 1

- (a) Draw the trellis diagram of the convolutional code.

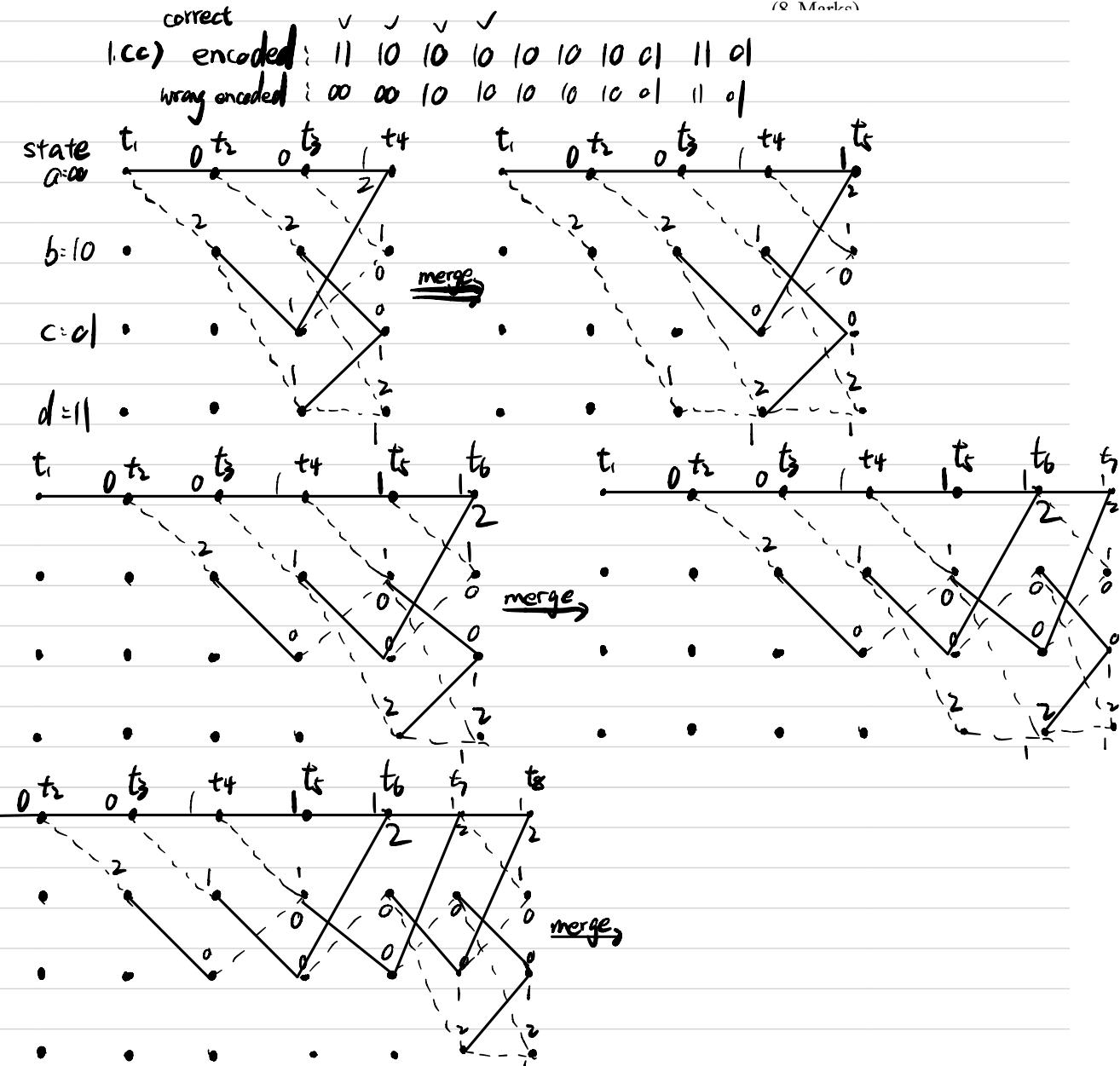


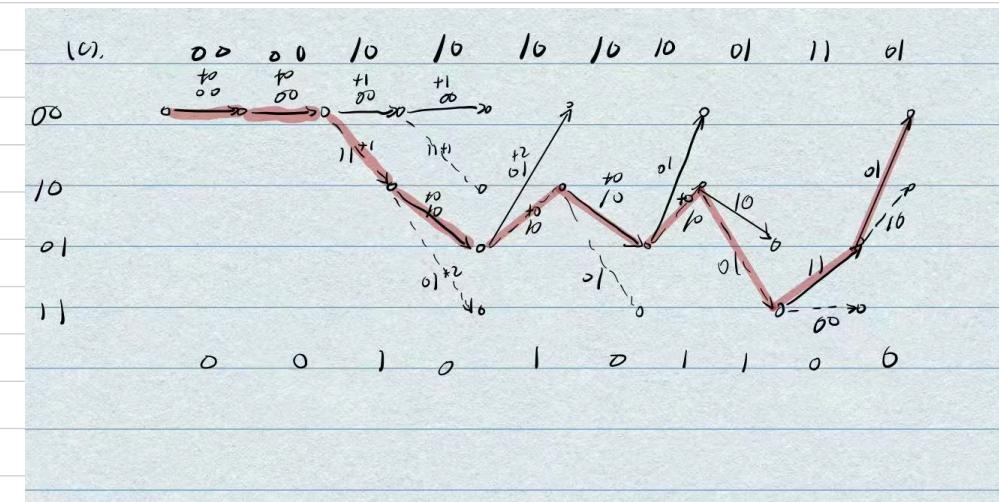
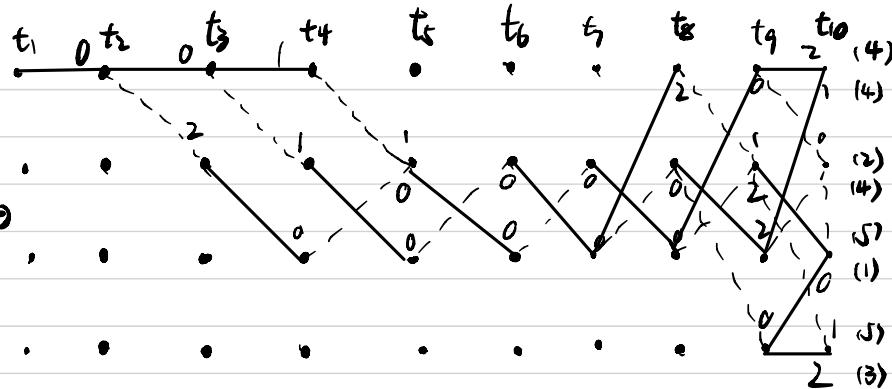
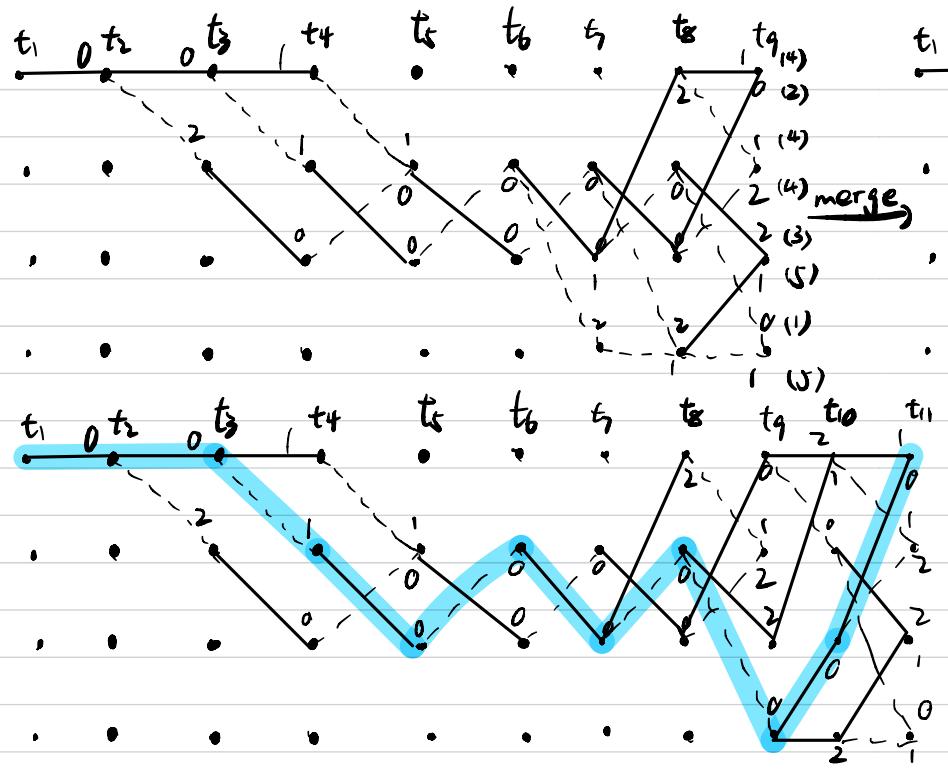
- (b) What is the encoded message if $u =$ (first bit) 10101011 (last bit)?
(assuming the registers of the encoder are of zero value at the initial stage)

1.(b)

register	state at t	state at $t+1$	output
100	00	10	11
010	10	01	10
101	01	10	10
010	10	01	10
101	01	10	10
010	10	01	10
101	01	10	10
110	10	11	01
011	11	01	11
001	01	00	01

- (c) What is the decoded message if there are mistakes in the first three bits when you received the encoded message from (b)?





0 0 1 0 1 0 1 1 0 0

2. (a) Consider a received LZ (Lempel-Ziv) message as follows:

00001 00000 00010 00111 01000 00101 00011 00110
01011 01110 01011 01110 10010 10111 10001 11011

Given the first two entry of the dictionary are:

Location 1	0001	Content: 1	Codeword: 00001
Location 2	0010	Content: 0	Codeword: 00000

Please decode the received LZ message as shown above.

2.(a) location content codeword

1	0001	1	00001
2	0010	0	00000
3	0011	10	00010
4	0100	101	00111
5	0101	1010	01000
6	0110	01	00101
7	0111	11	00011
8	1000	100	00110
9	1001	10101	01011
10	1010	110	01110
11	1011	10100	01010
12	1100	111	01111
13	1101	101010	10010
14	1110	10100	10111
15	1111	1001	10001
16		101010	11011

- (b) A source is generating two messages: S₁ with probability p , and S₂ with probability $1-p$.

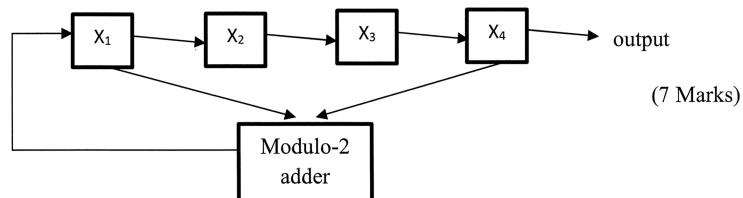
- (i) What is the value of p that requires the highest channel capacity for the source to transmit the information? What is the required channel capacity in this case, assuming that S₁ and S₂ are both single bit, and the source is generating one message per second?

$$\begin{aligned} I.(b)(i) \quad H &= -p \log_2 p - (1-p) \log_2 (1-p) \\ H(p) &= -\log_2 p - p \cdot \frac{1}{p} \cdot \frac{1}{m_2} + \log_2 (1-p) + (1-p) \cdot \frac{1}{1-p} \cdot \frac{1}{m_2} \\ &= \log_2 \frac{1-p}{p} \\ H'(p) = 0 &\Rightarrow \frac{1-p}{p} = 1 \Rightarrow p = 0.5 \\ \therefore \text{when } p = 0.5 \quad H_{\max} &= 1 \text{ bit} \\ C &= 1 \text{ message/second} \times 1 \text{ bit/message} = 1 \text{ bit/second} \end{aligned}$$

- (ii) What is the value of p that requires the lowest channel capacity for the source to transmit the information? What is the required channel capacity in this case assuming that S₁ and S₂ are both single bit, and the source is generating one message per second?

$$\begin{aligned} I.(b).(ii) \quad \text{lowest channel capacity: } p=0 \text{ or } p=1 \\ H_{\min} &= 0 \text{ bit/message} \\ C_{\min} &= H_{\min} \times 1 \text{ message/second} = 0 \text{ bit/second} \end{aligned}$$

- (c) Consider the generator in the circuit in Figure 2, assuming initial state is 1000, what is the 15-bits output sequence? Using the 15-bits output sequence, show the values of the autocorrelation with zero shift, and the values of the autocorrelation with two cyclic shift to the right.



(7 Marks)

1.(c) state adder output

	state	adder	output
1	1000	1	—
2	1100	1	0
3	1110	1	0
4	1111	0	0
5	0111	1	1
6	1011	0	1
7	1010	1	1
8	1101	0	0
9	0110	0	1
10	0011	1	0
11	1001	1	1
12	1100	1	1
13	1110	1	0
14	1111	0	0
15	0111	1	1

$$s[n] = [0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ 0\ 1] \quad s'[n] = [-1\ -1\ -1\ 1\ 1\ -1\ 1\ -1\ 1\ -1\ 1]$$

$$R(0) = \sum_{n=0}^{N-1} s[n] \cdot s[n+k] \bmod N \quad 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$$

$$R(0) = \sum_{n=0}^{N-1} s[n]^2 = 1^2 \times 15 = 15$$

$$R(2) = 1 - 1 - 1 + 1 + 1 - 1 + 1 + 1 + 1 - 1 - 1 - 1 - 1 = -1$$

3. In a 5G NR system, consider the information that is provided in Table 1 below:

Table 1

Subcarrier Spacing (kHz)	Allocated Bandwidth (MHz)	Resource Blocks
30	50	120

Given that the number of subcarriers occupied by 1 resource block is 12,

- (a) Calculate the number of occupied subcarriers of the system.

$$3.(a) \quad 120 \times 12 = 1440$$

- (b) Calculate the theoretical occupied bandwidth of the system.

$$3.(b) \quad 1440 \times 30 \text{ kHz} = 43200 \text{ kHz} = 4.32 \text{ MHz}$$

- (c) Given that possible FFT sizes are 2^n , where n is an integer. What is the most appropriate FFT size for the configuration above?

$$3.(c) \quad 2^10 = 1024 < 1440 < 2048 = 2^{11}$$

most appropriate FFT size is 2048

- (d) Given the FFT size in (c), how many unoccupied/guard subcarriers are there?

$$3.(d) \quad 2^{11} - 1440 = 608$$

- (e) Assuming that the channel quality is uniform throughout the subcarriers and is able support up to 64 QAM in each subcarrier, calculate the maximum physical layer data rate given that the duration of each slot is 500μs. [Hint: 1 slot has 14 Orthogonal Frequency Division Multiplexing (OFDM) symbols; assume all resource elements carry data]

$$3.(e) \quad \log_{2} 64 = 6 \text{ bits/symbol}$$

$$1 \div 500 \mu\text{s} / \text{slot} = 2000 \text{ slots}$$

$$2000 \times 14 = 28000 \text{ symbols/RE}$$

$$28000 \times 1440 = 4.032 \times 10^8 \text{ symbols}$$

$$4.032 \times 10^8 \times 6 = 2.4192 \times 10^8 \text{ bits/s} = 241.92 \text{ Mbps}$$

4. A 5G NR gNodeB has a channel bandwidth of 400 MHz and is operating in a hypothetical environment where the root mean square delay spread is measured to be 350 ns. Table 2 shows the subcarrier spacings and corresponding cyclic prefix durations that are being considered for the system.

Table 2

Subcarrier Spacing (kHz)	Cyclic Prefix Duration (μs)	Slot Duration (ms)
15	4.7	1
30	2.3	0.5
60	1.2	0.25
120	0.59	0.125
240	0.29	0.0625

- (a) Which subcarrier spacing from the above list are most suitable for the channel conditions and can achieve maximum bandwidth occupancy with the least number of subcarriers? [Hint: Maximum number of subcarriers is 3300]

↓
subcarrier spacing 尺量大

$$4.(a) \quad 350 \text{ ns} = 0.35 \mu\text{s}$$

$$0.29 \mu\text{s} < 0.35 \mu\text{s} < 0.59 \mu\text{s}$$

∴ most suitable subcarrier spacing = 120 kHz

$$120 \text{ kHz} \times 3300 = 396 \text{ MHz} < 400 \text{ MHz}$$

- (b) The gNodeB has 4 bandwidth parts it can configure for any User Equipment (UE) and they are shown in Table 3 as follows:

Table 3

Bandwidth Part #	Subcarrier Spacing (kHz)	Number of Resource Blocks
1	15	20
2	30	80
3	60	80
4	120	20

- (i) A UE connects to the gNodeB and reports that it can only support an operating bandwidth of up to 5 MHz. Which bandwidth part(s) can be configured for the UE? [Hint: each resource block has 12 subcarriers]

4.(b). (i) bandwidth:

$$\text{Part 1: } 15 \text{ kHz} \times 20 \times 12 = 3.6 \text{ MHz} \quad \checkmark$$

$$\text{Part 2: } 30 \text{ kHz} \times 80 \times 12 = 28.8 \text{ MHz}$$

$$\text{Part 3: } 60 \text{ kHz} \times 80 \times 12 = 57.6 \text{ MHz}$$

$$\text{Part 4: } 120 \text{ kHz} \times 20 \times 12 = 28.8 \text{ MHz}$$

- (ii) A second UE with higher capability connecting to the gNodeB can support an operating bandwidth of up to 30 MHz. Calculate bandwidth of each bandwidth parts and state if each of the bandwidth part in Table 3 can or cannot be configured for this UE.

4.(b).(ii) Part 1 can
Part 2 can
Part 3 cannot
Part 4 can