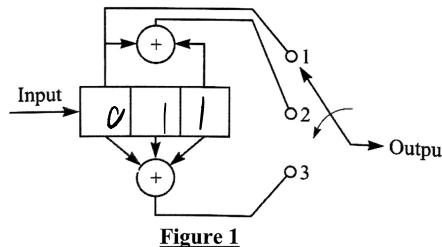
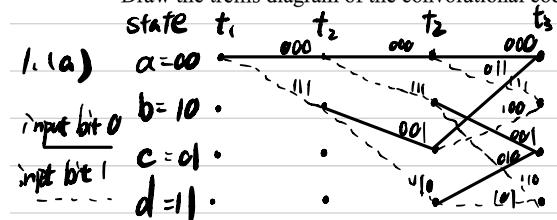


EE6111 24S2

1. (a) Consider the convolutional encoder as shown in Figure 1.



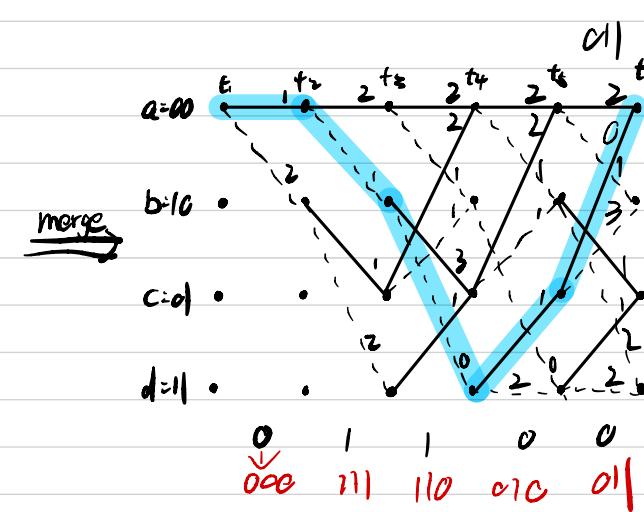
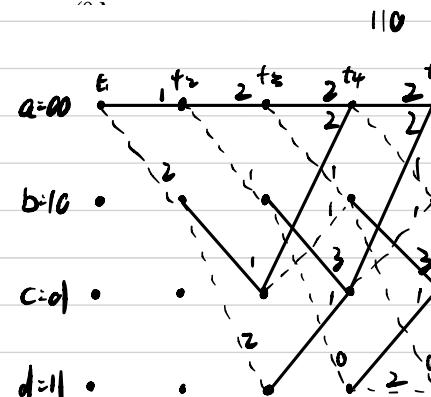
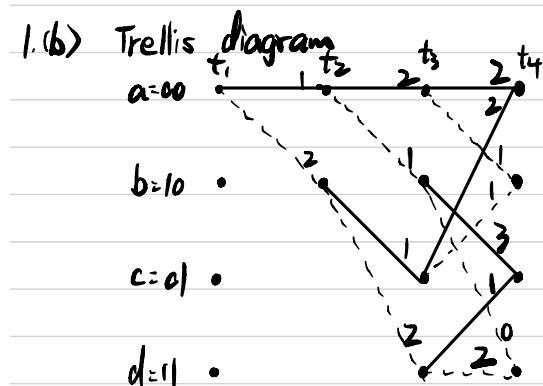
Draw the trellis diagram of the convolutional code.



- (b) Based on the encoder in (a). What is the encoded message if you receive the followings bits:

(first to receive) 001 101 110 110 011 (last to receive)?

Assuming the registers of the encoder are of zero value at the initial stage.



- (c) Designing a $k=2, n=8$ code. Just need to show the final codeword. Given the following properties:

- All-zero is one of the codeword.
- Systematic.
- Minimum distance $d_{\min} = 5$.
- The sum of any two codewords must yield a valid codeword in the space.

1.(c)

$$\begin{matrix} 00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 01 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 10 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 11 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{matrix}$$

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} ?$$

2. (a) Consider a message as follows:

a b b a b | a a a b c | a a a b c | d e f a a | a e f c d | d e f a a | c e f d b | b a b a a

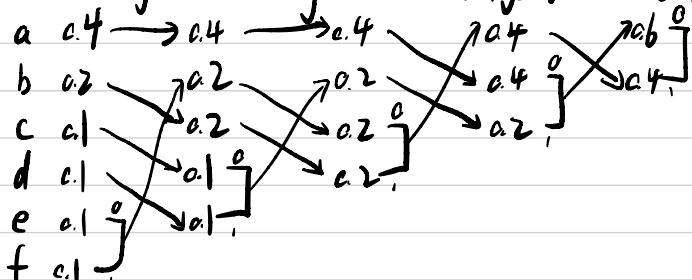
Determine the entropy of the above message, and compress the above message using Huffman code.

$$2.(a) N_a = 16 \quad N_b = 8 \quad N_c = 4 \quad N_d = 4 \quad N_e = 4 \quad N_f = 4 \quad N = 40$$

$$P_a = \frac{N_a}{N} = 0.4 \quad P_b = N_b/N = 0.2 \quad P_c = P_d = P_e = P_f = 0.1$$

$$H = -\sum_i P_i \log_2 P_i \quad (i = a, b, c, d, e, f)$$

$$= -0.4 \log_2 0.4 - 0.2 \log_2 0.2 - 4 \times 0.1 \times \log_2 0.1 \approx 2.32 \text{ bits}$$



message a b c d e f

Huffman code 00 11 010 011 100 101

- (b) Calculate the efficiency of the constructed codeword in (a), and propose a way to improve the efficiency.

$$2.(b) I = 2 \times 0.4 + 2 \times 0.2 + 3 \times 0.1 \times 4 = 2.4 \text{ bits}$$

$$L_{\min} = H \approx 2.32 \text{ bits}$$

$$\eta = \frac{L_{\min}}{I} = 2.32 / 2.4 \approx 96.67\%$$

- (d) Orthogonal frequency division multiplexing (OFDM) system is an effective multicarrier modulation with overlapping subchannels. It has the following two key elements:

- Guard Interval
- Cyclic Prefix

Please state the respective functions of the two elements, and how it helps OFDM to achieve multicarrier modulation with overlapping subchannels.

- (c) Compress the following message with Lempel-Ziv code:

0 1 0 0 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 1 1 1 0 1 0 1 0 0 1 0 0 0 1 0 0 1 1

Assuming the first two entry of the dictionary are:

Location 1 0001 Content: 0 Codeword: 00000

Location 2 0010 Content: 1 Codeword: 00001

1.(c) location content codeword

0001	0	00000
0010	1	00001
0011	01	00011
0100	00	00010
0101	011	00111
0110	11	00101
0111	000	01000
1000	001	01001
1001	111	01011
1010	0000	01110
1011	0111	01011
1100	10	00100
1101	101	11001
1110	0001	01111
1111	00001	10101
0011	1000	10001

Guard Interval (GI)

Function: Acts as a "time buffer" between consecutive OFDM symbols. It separates adjacent symbols to prevent Inter-Symbol Interference (ISI) — a problem where delayed signals (from multipath propagation) of the previous symbol overlap with the current symbol.

Role in overlapping subchannels: By isolating symbols in time, GI ensures that even if subchannels overlap in frequency, the time-domain separation avoids cross-talk between symbols, allowing subchannels to share spectrum efficiently.

Cyclic Prefix (CP)

Function: A segment copied from the end of an OFDM symbol and prepended to its start. It serves two key purposes: Extends the symbol duration to exceed the channel's multipath delay spread, eliminating ISI (building on GI's effect).

Maintains subchannel orthogonality (the core of OFDM's overlapping subchannels). By making the channel's impulse response "appear cyclic" in the symbol duration, CP ensures that overlapping subchannels (which are orthogonal in frequency) remain independent during demodulation (via FFT).

Role in overlapping subchannels: Orthogonality (preserved by CP) allows subchannels to overlap in frequency without interfering with each other. This maximizes spectrum efficiency — OFDM can use all available bandwidth, unlike non-overlapping schemes that waste spectrum between channels.