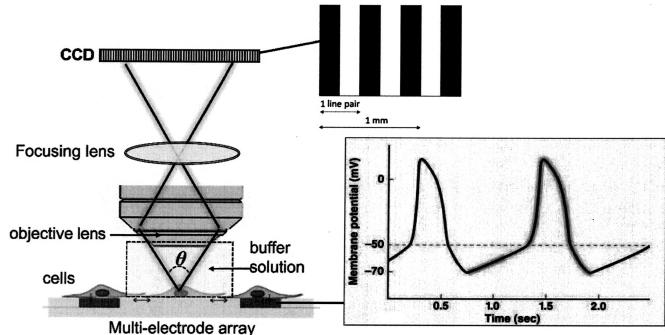


**EE6301 24S2**

1. A group of students are trying to study the spiking behavior and action potentials of neurons. Figure 1 on page 2 shows the system of a two-photon fluorescence microscope imaging system, where a biological cell is excited by light passing through a set of filters and then into the 10X magnifying objective lens with a full convergence angle  $\theta = 30^\circ$ . The objective is formed by a focusing lens with refractive index of  $n_{\text{lens}} = 1.5$  and the sample is immersed in a buffer solution with refractive index of  $n_{\text{solution}} = 1.35$ . The cell is labeled with a two-photon absorption fluorescent dye which emits a green fluorescence at 550 nm.

- (a) Calculate the diffraction limited spatial resolution and the axial resolution of the microscope when observing a two-photon fluorescence microscopic image. The smallest distance that can be resolved by the detector is 1.



$$1. (a) \text{NA} = n_{\text{solution}} \cdot \sin(\frac{\theta}{2}) = 1.35 \times \sin(\frac{30^\circ}{2}) = \frac{27}{80} (\sqrt{6} - \sqrt{2}) \approx 0.3494$$

$$\lambda_{\text{excited}} \approx 2\lambda_{\text{emit}} = 2 \times 550 \text{ nm} = 1100 \text{ nm}$$

$$\Delta d = \frac{0.61 \lambda_{\text{excited}}}{\text{NA}} = 0.61 \times 1100 \div 0.3494 \approx 1920.4036 \text{ nm} \approx 1.92 \mu\text{m}$$

- (b) To obtain the above-mentioned two photon fluorescence, what would the excited photon energy for each photon of light to be? (units in eV).

$$1. (b) V_{\text{excited}} = \frac{c}{\lambda_{\text{excited}}} = 3 \times 10^8 \text{ m/s} \div 1100 \text{ nm} \approx 2.7273 \times 10^{14} \text{ Hz}$$

$$E_{\text{excited}} = h\nu_{\text{excited}} = 6.63 \times 10^{-34} \text{ J s} \times 2.7273 \times 10^{14} \text{ Hz} = 1.8082 \times 10^{-19} \text{ J}$$

$$E_{\text{excited}} = 1.8082 \times 10^{-19} \text{ J} \approx 1.1287 \text{ eV}$$

$$(E_{\text{excited}} \text{ (eV)}) \approx \frac{1.239.8}{1100} = \frac{1.239.8}{1100} \approx 1.127 \text{ eV}$$

- (c) The action potentials were measured at the same time by connecting the neurons to a multielectrode array below the cells. The results are shown in the right side inset of Figure 1. Determine the threshold potential and the temporal resolution required for recording.

$$1. (c) \text{threshold potential} = -50 \text{ mV} \text{ (spike)} \\ \text{temporal resolution} = 2.5 / 2 = 1.25 \text{ s}$$

- (d) The charge-coupling device (CCD) camera is formed by a chip of dimensions 5 mm x 5 mm and 2048 x 2048 sensing elements which is focused on a square, flat area, located 100 mm away. If the camera is equipped with a focusing lens (focal length  $f=20$  mm), how many line pairs per mm will this camera be able to resolve? (line pair/millimeter indicates the camera's ability to resolve distinct lines or patterns within a given area).

(5 Marks)

$$1. (d) \text{像素间距 } P = \frac{5 \text{ mm}}{2048} \approx 0.002441 \text{ mm}$$

$$P_{\text{CCD}} = f_{\text{max}} = \frac{1}{2P} = \frac{1}{2 \times 5 \text{ mm}} = 204.81 \text{ lp/mm}$$

$$\text{imaging distance } S_2 = 100 \text{ mm}$$

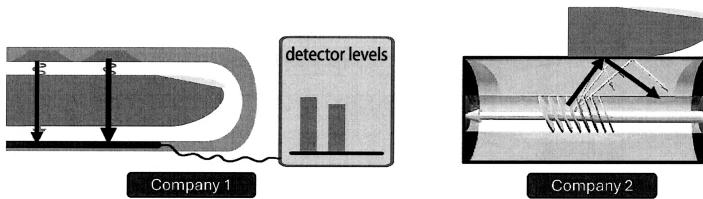
$$\text{object distance } S_1 = \left( \frac{1}{f} - \frac{1}{S_2} \right)^{-1} = \left( \frac{1}{20} \text{ mm} - \frac{1}{100} \text{ mm} \right)^{-1} = 25 \text{ mm}$$

$$\text{magnification } M = \frac{S_2}{S_1} = 100 \text{ mm} \div 25 \text{ mm} = 4$$

$$\text{camera} = P_{\text{CCD}} / M = 2048 \text{ lp/mm} \div 4 = 51.2 \text{ lp/mm}$$

2. A pulse oximeter is a device that measures how much light is absorbed by your blood. Two companies are competing for their new product to measure blood oxygen saturation level ( $SaO_2$ ), as shown in Figure 2 on page 3.

- (a) The first company detects  $SaO_2$  based on absorption ratio of light at  $\lambda_1=660\text{ nm}$  and  $\lambda_2=895\text{ nm}$ . According to literature, Hb has extinction coefficients at respective wavelengths  $\epsilon_{Hb}(\lambda_1)=3000\text{ L mol}^{-1}\text{ cm}^{-1}$  and  $\epsilon_{Hb}(\lambda_2)=820\text{ L mol}^{-1}\text{ cm}^{-1}$ ;  $HbO_2$  has extinction  $\epsilon$  coefficients of  $\epsilon_{HbO_2}(\lambda_1)=400\text{ L mol}^{-1}\text{ cm}^{-1}$  and  $\epsilon_{HbO_2}(\lambda_2)=1150\text{ L mol}^{-1}\text{ cm}^{-1}$ . Suppose a patient's oximeter measures the ratio of absorbances to be  $A(\lambda_2)/A(\lambda_1)=1.5$ . What is the patient's oxygen saturation level, defined as  $SaO_2=[HbO_2]/([Hb]+[HbO_2])$ ?



$$2.(a) A(\lambda) = \epsilon(\lambda) c L$$

$$A(\lambda_2) = \epsilon_{Hb}(\lambda_2) [Hb] + \epsilon_{HbO_2}(\lambda_2) [HbO_2] \cdot L$$

$$A(\lambda_1) = \epsilon_{Hb}(\lambda_1) [Hb] + \epsilon_{HbO_2}(\lambda_1) [HbO_2] \cdot L$$

$$\therefore A(\lambda_2)/A(\lambda_1) = \frac{820 \cdot [Hb] + 1150 \cdot [HbO_2]}{3000 \cdot [Hb] + 400 \cdot [HbO_2]} = 1.5$$

$$\therefore [HbO_2] / [Hb] = 368 / 55$$

$$\therefore SaO_2 = [HbO_2] / ([Hb] + [HbO_2]) = 368 / (368 + 55) \approx 0.8700$$

- (b) The second company directly measures optical refractive index change of blood at one single wavelength at  $\lambda_1=660\text{ nm}$  to determine the  $SaO_2$  level using a photonic fiber coated with gold film with a thickness,  $d$ . The corresponding resonance wavelength change can be calculated by  $\lambda(\text{resonance})=\lambda_1 \cdot n^2(\text{blood}) + 2d$ . Suppose a patient's oximeter measures the refractive index  $n$  of blood which varies from 1.37 to 1.39 during low oxygen to high oxygen level. Gold film thickness  $d=10\text{ nm}$ . Calculate the sensitivity in terms of (nm/RIU).

$$2.(b) \left( S = \frac{d\lambda_{\text{resonance}}}{dn} = \frac{d\lambda_1 \cdot n^2 + 2d}{dn} = 2\lambda_1 n = 2 \times 660\text{ nm} \times \frac{1.37 + 1.39}{2} = 1821.6\text{ nm/RIU} \right)$$

$$\lambda_{\text{resonance1}} = \lambda_1 \cdot n_{\text{low}}^2 + 2d = 660\text{ nm} \cdot 1.37^2 + 2 \times 10\text{ nm} = 1258.754\text{ nm}$$

$$\lambda_{\text{resonance2}} = \lambda_1 \cdot n_{\text{high}}^2 + 2d = 660\text{ nm} \cdot 1.39^2 + 2 \times 10\text{ nm} = 1285.186\text{ nm}$$

$$S = \frac{\Delta\lambda_{\text{resonance}}}{dn} = (1285.186 - 1258.754) / (1.39 - 1.37) = 1821.6\text{ nm/RIU}$$

- (c) A hyperspectral camera was used to capture the signals. A customer requested a minimum spectral resolution  $\Delta\lambda=0.01\text{ nm}$  at  $\lambda_1=660\text{ nm}$ . Calculate the minimum groove density ( $N/\text{mm}$ ) of grating to support the first-order diffraction pattern. Beam diameter is 10 mm.

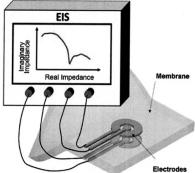
$$2.(c) R = \frac{\lambda}{\Delta\lambda} = \frac{660\text{ nm}}{0.01\text{ nm}} = 66000$$

$$N = \frac{R}{m} = 66000 \div 1 = 66000$$

$$\text{the minimum groove density of grating} = \frac{N}{\text{diameter}} = \frac{66000}{10\text{ mm}} = 6600 \text{ N/mm}$$

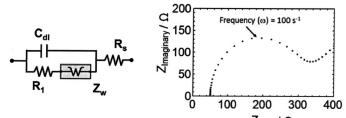
3. An electrochemical membrane biosensor is designed for detecting DNA biomarkers. The sensor was analyzed by using the EIS (electrochemical impedance spectroscopy). The corresponding Randles circuit (bottom, left) and Nyquist plot (bottom, right) are provided in Figure 3 on page 4. Answer the following questions.

- (a) What is the value of charge transfer resistance  $R_t$  and the value of series resistance  $R_s$  (unit: Ohms)?



$$3.(a) R_s = 50\Omega$$

$$(R_s + R_t + R_s)/2 = 200\Omega \Rightarrow R_t = 300\Omega$$



- (b) What is the double layer capacitance ( $C_{dl}$ )? What is the electrode area assuming a specific capacitance of  $30\mu\text{F}/\text{cm}^2$ ?

$$3.(b) W_{max} = 2\pi f_{max} = 2\pi \cdot 100 = 200\pi \text{ rad/s}$$

$$C_{dl} = \frac{1}{W_{max} \cdot R_t} = \frac{1}{200\pi \cdot 300} \approx 5.3052 \times 10^{-6} \text{ F} = 5.3052 \mu\text{F}$$

$$A = \frac{C_{dl}}{\text{specific}} = 5.3052 \mu\text{F} \div 30 \mu\text{F}/\text{cm}^2 \approx 0.17684 \text{ cm}^2$$

- (c) Assuming  $R_t = 50$  Ohm,  $R_s = 100$  Ohm, sketch the Bode plot (log of impedance magnitude versus log of frequency) of the electrode corresponding to this Randles cell model using the frequency range of 0.01 Hz–1MHz. When the sensor is exposed to an increment of other cells, the charge transfer resistance increases by 20%. How does the Nyquist plot shift (To the right or left)?

3.(c) Nyquist plot shift:

$$R' = R_t (1+20\%) = 60\Omega$$

The (first) intersection point of the Nyquist plot with the real axis remains unchanged, while the radius of the semicircle increase by  $5\Omega$  and the center shifts  $5\Omega$  to the right.

Bode plot:  $0.01/\text{Hz} - 1/\text{MHz} \Rightarrow -2 \sim 6$

$$Z = R_s + \left( \frac{1}{C_{dl}} + \frac{1}{R_t + Z_w} \right)^{-1} = R_s + \frac{1}{j2\pi f(C_{dl} + R_t + \frac{1}{j2\pi f Z_w})}$$

$$\textcircled{1} f \rightarrow \infty$$

$$Z_w \rightarrow 0 \quad Z_{dl} \rightarrow 0$$

$$Z \approx R_s$$

$$\therefore f = 100 \text{ MHz} : |g|/|Z| \approx |g|/R_s = 2$$

$$\textcircled{2} f \rightarrow 0$$

$$Z_{dl} \rightarrow \infty \quad Z_w \rightarrow \infty$$

$$Z \approx R_s + R_t$$

$$\therefore f = 1 \text{ Hz} : |g|/|Z| \approx |g|/(R_s + R_t) \approx 2.1761$$

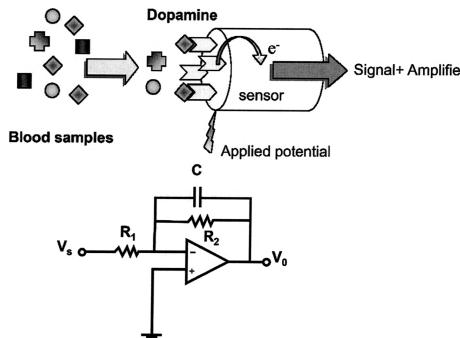
$$\textcircled{3} f_{max} = \frac{W_{max}}{2\pi} = \frac{R_s C_{dl} \cdot 2\pi}{R_s C_{dl} + R_t + 2\pi}$$

$$2.1761 = R_s + \frac{R_t}{1 + \frac{R_t}{R_s}}$$

$$= R_s + \frac{R_t}{1 + \frac{R_t}{R_s}} = (R_s + \frac{1}{2} R_t) - \frac{1}{2} R_t$$

$$|Z| = \sqrt{(R_s + \frac{1}{2} R_t)^2 + (\frac{1}{2} R_t)^2} = \sqrt{(R_s^2 + R_s R_t + \frac{1}{4} R_t^2)} \approx 2.1054$$

1. When human brains are stimulated by external environment, they release critical chemicals called Dopamine (molecular weight= 150.00) into our blood, which is a crucial biomarker for Alzheimer's diseases. Figure 4 on page 5 shows a biosensor by measuring the ultralow current contributed from dopamine in blood samples.



- (d) List two advantages for electrical-based biosensor and two advantages for optical-based biosensors.

*(1.(d)) electrical-based : High sensitivity . miniaturization . low cost . ease of operation.  
optical-based : High specificity . real time performance . label free detection*

- (a) Determine the Coulombs of charge that can be detected in 0.1 mL of blood? Assume concentration of dopamine in a patient is 15.0 g/L. (Reaction: 1 Dopamine + 6H<sub>2</sub>O → 1 product + 10 e<sup>-</sup>).

$$\begin{aligned} 4.(a) \quad & 0.1 \text{ mL} \times 15 \text{ g/L} = 1.5 \times 10^{-3} \text{ g} \\ & 1.5 \times 10^{-3} \div 150 \text{ g/mol} = 1 \times 10^{-5} \text{ mol} \\ & c: 10^{-5} \times 10 = 10^{-4} \text{ mol} \\ & Q = 10^{-4} \text{ mol} \times 96485 \text{ C/mol} = 9.6485 \text{ C} \end{aligned}$$

- (b) To improve the signal readout and remove the noise from the sensor, a biopotential amplifier was integrated to the above sensor. The added filter is shown in the inset of Figure 4. Determine the value of the output voltage  $V_o$ , for a signal  $V_s = 5 \cos(2000t + \pi/4)$  mV that feeds into the filter. Assume that the time,  $t$  is in seconds;  $R_1 = 1\text{k}$ ;  $R_2 = 2\text{k}$ ;  $C = 1 \mu\text{F}$ .

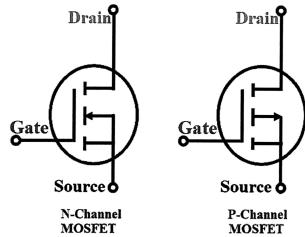
$$\begin{aligned} 4.(b) \quad & Z_i = R_1, \quad Z_f = (j\omega C + R_2)^{-1} = -\frac{R_2}{1+j\omega CR_2} \\ & A(j\omega) = -\frac{Z_f}{Z_i} = -\frac{R_2}{R_1(1+j\omega CR_2)} \\ & w = 2000 \text{ rad/s} \\ & A(j2000) = -\frac{2k}{1k} \cdot (1+j2000 \cdot 10^{-6} \cdot 2000)^{-1} = -\frac{2}{1+j4} = \frac{2}{\sqrt{17}} e^{j104.0362^\circ} \\ & V_o = -A(j2000) \cdot V_s = \frac{V_s}{\sqrt{17}} \cos(2000t + \frac{\pi}{4} + 104.0362^\circ) \approx 2.43 \cos(2000t + 0.83\pi) \end{aligned}$$

- (c) Following question part (b), calculate the cut-off frequency of the filter (amplifier).

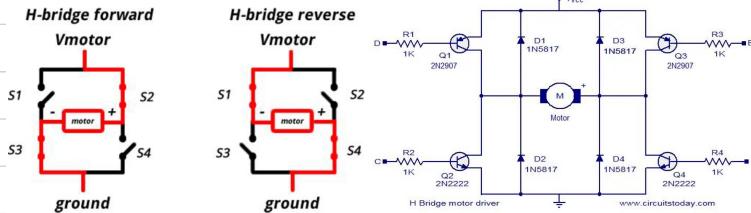
$$4.(c) \quad f_c = \frac{1}{2\pi R_2 C} = \frac{1}{2\pi \cdot 2k \cdot 10^{-6}} \approx 79.5775 \text{ Hz}$$

5. Imagine that you are designing a robot driven by direct current (DC) brushed motors.

- (a) Draw a driver circuit that can control the direction and speed of a DC brushed motor using the provided N channel metal-oxide-semiconductor field-effect transistor (MOSFET) and P channel MOSFET, as shown in Figure 5.



5. (a)



P-MOSFET 代  $\text{NPN BJT } (V_{\text{ce}} - \text{Source})$  (Gate - ABCD)

N-MOSFET 代  $\text{NPN BJT } (V_{\text{GND}} - \text{Source})$

- (b) An optical rotatory encoder is chosen to monitor the motor status. If the encoder's sampling frequency is 125kHz and the maximum shaft speed is 1,000 revolutions per minute (RPM), what are the maximum pulses per revolution that the encoder can capture? If you require measuring down to 0.01 degrees, what is the minimum analog-to-digital (ADC) converter resolution?

$$f_{\text{Nyquist}} = f_s/2 = 62.5 \text{ kHz}$$

$$\text{RPS} = 1000 \text{ RPM}/60s = 16.67 \text{ RPS}$$

$$\text{PPR}_{\text{max}} = f_{\text{Nyquist}} / \text{RPS} \approx 3750 \text{ pulse/second}$$

$$R = \frac{\text{PPR}}{0.01^\circ} = \frac{3750}{0.01^\circ} = 360000$$

$$2^{15} < 360000 < 2^{16}$$

$$\therefore \text{minimum ADC resolution} = 2^6$$

- (c) If a low dropout regulator (LDO, TPS74201) with a soft starter is given to drive the motor as shown in Figure 6 on page 7, how do we generate output voltages of 1.8 V, 2.5 V, and 3.3 V via the LDO?

$$5.(c) V_{\text{out}} = V_{\text{ref}} (1 + \frac{R_1}{R_2})$$

$$V_{\text{out}} = 1.8 : R_1/R_2 = 1.8/0.8 - 1 = 1.25$$

$$V_{\text{out}} = 2.5 : R_1/R_2 = 2.125$$

$$V_{\text{out}} = 3.3 : R_1/R_2 = 3.125$$

- (d) A current response plot of a DC brushed motor with various driving voltages is given in Figure 7 on page 7. A strong inrush currents was observed when immediately turn on the motor and the inrush currents are proportional to the driving voltages. TPS74201 is equipped with a soft start designed for mitigating the inrush current by slowly ramping up the driving voltage. The soft-start ramp time depends on the soft-start charging current ( $I_{SS}$ ),  $C_{SS}$ , and the internal reference voltage ( $V_{REF}$ ), and can be calculated using  $t_{SS} = \frac{V_{REF} \times C_{SS}}{I_{SS}}$  where  $I_{SS}$  is soft start current and the  $V_{REF}$  is the reference voltage. What  $C_{SS}$  should be used if the inrush current of 3.3V driving voltage is completely removed?

$$1.(d) t_{SS} \approx 100 \text{ ms} \quad I_{SS} \approx 220 \text{ mA}$$

$$C_{SS} = \frac{t_{SS} I_{SS}}{V_{REF}} = \frac{100 \text{ ms} \times 220 \text{ mA}}{0.8 \text{ V}} = 0.275 \text{ F} = 27.5 \text{ mF}$$