Linear Combinations: Lossy compression

Say you need to store or transmit many 2-megapixel images:

How do we represent the image compactly?

- *Obvious method:* 2 million pixels $\Longrightarrow$ 2 million numbers
- *Strategy 1:* Use sparsity! Find the "nearest" k-sparse vector. Later we'll see this consists of suppressing all but the largest k entries.
- *More sophisticated strategy?*

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Strategy 2: Represent image vector by its coordinate representation:

- Before compressing any images, select vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$.
- Replace each image vector with its coordinate representation in terms of $\mathbf{v}_1, \ldots, \mathbf{v}_n$.

For this strategy to work, we need to ensure that *every* image vector can be represented as a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_n$.

> Given some D-vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ over $\mathbb{F}$, how can we tell whether *every* vector in $\mathbb{F}^D$ can be written as a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_n$?

We also need the number of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ to be much smaller than the number of pixels.

> Given D, what is minimum number of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ such that every vector in $\mathbb{F}^D$ can be written as a linear combination?

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Strategy 3: A hybrid approach

Step 1: Select vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$.

Step 2: For each image to compress, find its coordinate representation $\mathbf{u}$ in terms of $\mathbf{v}_1, \ldots, \mathbf{v}_n$

Step 3: Replace $\mathbf{u}$ with the closest k-sparse vector $\tilde{\mathbf{u}}$, and store $\tilde{\mathbf{u}}$.

Step 4: To recover an image from $\tilde{\mathbf{u}}$, calculate the corresponding linear combination of $\mathbf{v}_1, \ldots \mathbf{v}_n$.

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