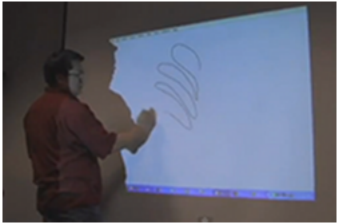
Wiimote whiteboard

For location of infrared point, wiimote provides coordinate representation in terms of its camera basis).



Johnny Chung Lee, wiimote whiteboard

To use as a mouse, need to find corresponding location on screen (coordinate representation in tems of screen basis)

How to transform from one coordinate representation to the other?

Can do this using a matrix H.

The challenge is to calculate the matrix H.

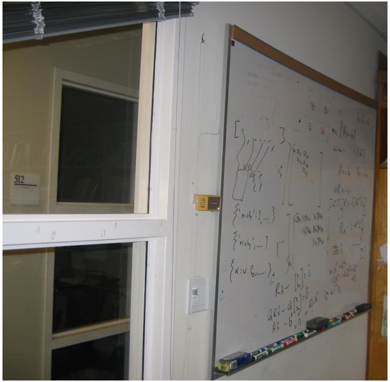
Can do this if you know the camera coordinate representation of four points whose screen coordinate representations are known.

You'll do exactly the same computation but for a slightly different problem....

Removing perspective

Given an image of a whiteboard, taken from an angle...

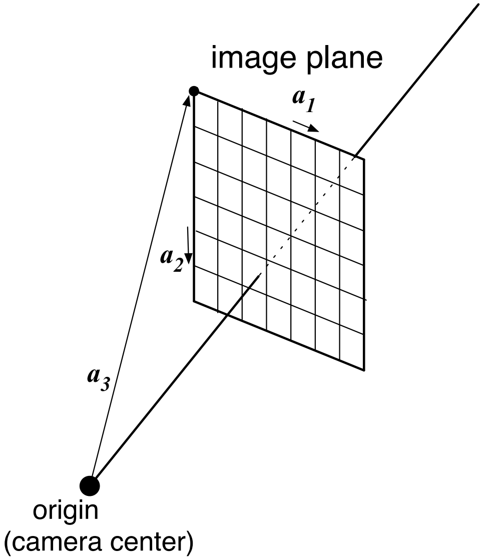
synthesize an image from straight ahead with no perspective



Camera coordinate system

We use same camera-oriented basis $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$:

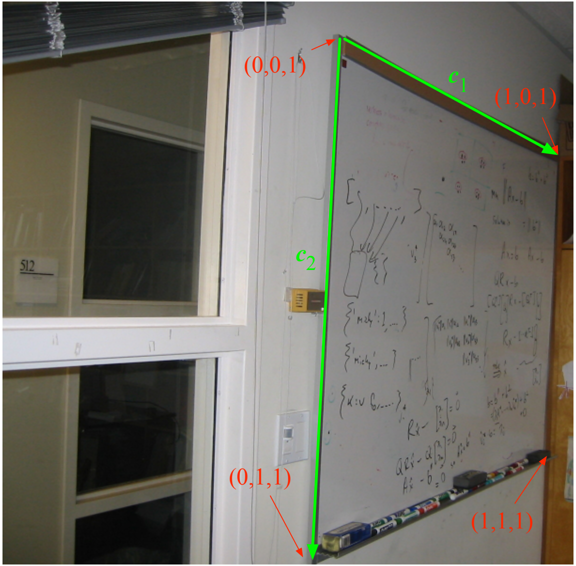
- The origin is the camera center.
- The first vector $\mathbf{a}_1$ goes horizontally from the top-left corner of the sensor element (0,0) to the top-right corner.
- The second vector $\mathbf{a}_2$ goes vertically from the top-left corner of the sensor element (0,0) to the bottom-left corner.
- The third vector $\mathbf{a}_3$ goes from the origin (the camera center) to the top-left corner of sensor element (0,0).



Converting from one basis to another

In addition, we define a *whiteboard basis* $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3$

- The origin is the camera center.
- The first vector $\mathbf{c}_1$ goes horizontally from the top-left corner of whiteboard to top-right corner.
- The second vector $\mathbf{c}_2$ goes vertically from the top-left corner of whiteboard to the bottom-left corner.
- The third vector $\mathbf{c}_3$ goes from the origin (the camera center) to the top-right corner of whiteboard.



Converting between different basis representations

Start with a point $\mathbf{p}$ written in terms of in camera coordinates

$$\mathbf{p} = \left[\begin{array}{c|c|c} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

We write the same point $\mathbf{p}$ in the whiteboard coordinate system as

$$\mathbf{p} = \left[\begin{array}{c|c|c} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{array} \right] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Combining the two equations, we obtain

$$\left[\begin{array}{c|c|c} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \left[\begin{array}{c|c|c} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{array} \right] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Converting...

$$\left[\begin{array}{c|c|c} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{array}\right]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \left[\begin{array}{c|c|c} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{array}\right]\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Let A and C be the two matrices. As before, C has an inverse C^{-1}.
Multiplying equation on the left by C^{-1}, we obtain

$$\begin{bmatrix} \quad C^{-1} \quad \end{bmatrix}\begin{bmatrix} \quad A \quad \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \quad C^{-1} \quad \end{bmatrix}\begin{bmatrix} \quad C \quad \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Since C^{-1} and C cancel out, we obtain

$$\begin{bmatrix} \quad C^{-1} \quad \end{bmatrix}\begin{bmatrix} \quad A \quad \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

We have shown that there is a matrix H (namely $H = C^{-1}A$) such that

$$\begin{bmatrix} \quad H \quad \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

How to almost compute H

Write $H = \begin{bmatrix} h_{y_1,x_1} & h_{y_1,x_2} & h_{y_1,x_3} \\ h_{y_2,x_1} & h_{y_2,x_2} & h_{y_2,x_3} \\ h_{y_3,x_1} & h_{y_3,x_2} & h_{y_3,x_3} \end{bmatrix}$

The h_{ij}'s are the unknowns.

To derive equations, let $\mathbf{p}$ be some point on the whiteboard, and let $\mathbf{q}$ be the corresponding point on the image plane. Let $(x_1, x_2, 1)$ be the camera coordinates of $\mathbf{q}$, and let (y_1, y_2, y_3) be the whiteboard coordinates of $\mathbf{q}$. We have

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_{y_1,x_1} & h_{y_1,x_2} & h_{y_1,x_3} \\ h_{y_2,x_1} & h_{y_2,x_2} & h_{y_2,x_3} \\ h_{y_3,x_1} & h_{y_3,x_2} & h_{y_3,x_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

Multiplying out, we obtain

$$\begin{aligned} y_1 &= h_{y_1,x_1} x_1 + h_{y_1,x_2} x_2 + h_{y_1,x_3} \\ y_2 &= h_{y_2,x_1} x_1 + h_{y_2,x_2} x_2 + h_{y_2,x_3} \\ y_3 &= h_{y_3,x_1} x_1 + h_{y_3,x_2} x_2 + h_{y_3,x_3} \end{aligned}$$

Almost computing H

$$\begin{aligned} y_1 &= h_{y_1,x_1}x_1 + h_{y_1,x_2}x_2 + h_{y_1,x_3} \\ y_2 &= h_{y_2,x_1}x_1 + h_{y_2,x_2}x_2 + h_{y_2,x_3} \\ y_3 &= h_{y_3,x_1}x_1 + h_{y_3,x_2}x_2 + h_{y_3,x_3} \end{aligned}$$

Whiteboard coordinates of the original point **p** are $(y_1/y_3, y_2/y_3, 1)$. Define

$$\begin{aligned} w_1 &= y_1/y_3 \\ w_2 &= y_2/y_3 \end{aligned}$$

so the whiteboard coordinates of **p** are $(w_1, w_2, 1)$.
Multiplying through by y_3, we obtain

$$\begin{aligned} w_1 y_3 &= y_1 \\ w_2 y_3 &= y_2 \end{aligned}$$

Substituting our expressions for y_1, y_2, y_3, we obtain

$$w_1(h_{y_3,x_1}x_1 + h_{y_3,x_2}x_2 + h_{y_3,x_3}) = h_{y_1,x_1}x_1 + h_{y_1,x_2}x_2 + h_{y_1,x_3}$$
$$w_2(h_{y_3,x_1}x_1 + h_{y_3,x_2}x_2 + h_{y_3,x_3}) = h_{y_2,x_1}x_1 + h_{y_2,x_2}x_2 + h_{y_2,x_3}$$

$$w_1(h_{y_3,x_1}x_1 + h_{y_3,x_2}x_2 + h_{y_3,x_3}) = h_{y_1,x_1}x_1 + h_{y_1,x_2}x_2 + h_{y_1,x_3}$$
$$w_2(h_{y_3,x_1}x_1 + h_{y_3,x_2}x_2 + h_{y_3,x_3}) = h_{y_2,x_1}x_1 + h_{y_2,x_2}x_2 + h_{y_2,x_3}$$

Multiplying through and moving everything to the same side, we obtain

$$(w_1x_1)h_{y_3,x_1} + (w_1x_2)h_{y_3,x_2} + w_1h_{y_3,x_3} - x_1h_{y_1,x_1} - x_2h_{y_1,x_2} - 1h_{y_1,x_3} = 0$$
$$(w_2x_1)h_{y_3,x_1} + (w_2x_2)h_{y_3,x_2} + w_2h_{y_3,x_3} - x_1h_{y_2,x_1} - x_2h_{y_2,x_2} - 1h_{y_2,x_3} = 0$$

Thus we get two linear equations in the unknowns. The coeffients are expressed in terms of x_1, x_2, w_1, w_2.

For four points, get eight equations. Need one more...

One more equation

We can't pin down H precisely.

This corresponds to the fact that we cannot recover the scale of the picture (a tiny building that is nearby looks just like a huge building that is far away).

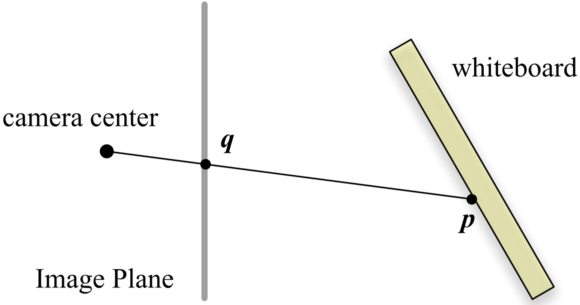
Fortunately, we don't need the true H.

As long as the H we compute is a scalar multiple of the true H, things will work out.

To arbitrarily select a scale, we add the equation $h_{y_1,x_1} = 1$.

Once you know H

1. For each point $\mathbf{q}$ in the representation of the image, we have the camera coordinates $(x_1, x_2, 1)$ of $\mathbf{q}$. We multiply by H to obtain the whiteboard coordinates (y_1, y_2, y_3) of the same point $\mathbf{q}$.
2. Recall the situation as viewed from above:



   The whiteboard coordinates of the corresponding point $\mathbf{p}$ on the whiteboard are $(y_1/y_3, y_2/y_3, 1)$. Use this formula to compute these coordinates.
3. Display the updated points with the same color matrix