Models with long memory

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Some slides are from Zabolotnyi Artem

Benchmarking for Efficient Transformers for transactions data

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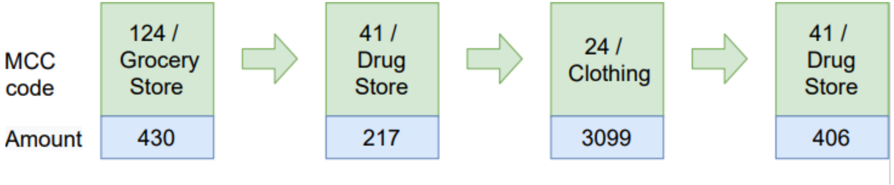
Dataset description
Transaction dataset

Task
Predict the probability of user default

Features
~10 categorical features (MCC code, type etc)
~10 numerical (amount, time etc)

Train test split
Train – 1 million users
Validation – 250k users
Out-of-time – 500k users

Length
700 - mean transaction count
(*Typical transformer length 512)*
Metric
Gini coefficient



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Hyperparameters search for Longformer

Quality

Longformer model shows better quality than baseline LSTM

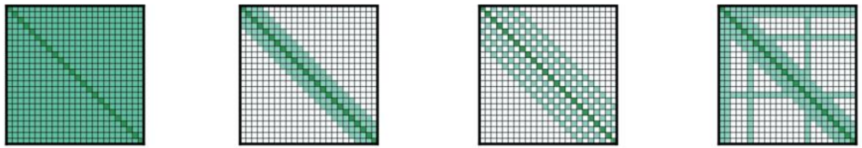
Hyperparameters

- Very deep model required a huge amount of memory for training
- Sliding window size W does not change quality significant

Baseline metric

LSTM - 60.12 ± 0.17

№	Heads	Embedding size	W	Layers	Hidden dropout	OOT Gini	RAM (Mb)
1	8	512	128	1	0.1	59.59	**7647**
2	8	256	128	1	0.1	58.84	4929
3	8	64	128	1	0.1	60.11	4581
4	8	64	128	2	0.1	61.02	6039
5	8	64	256	2	0.1	61.03	10131
6	8	64	64	2	0.1	60.58	4500
7	4	64	128	8	0.3	62.21	13971
8	4	32	128	8	0.3	62.58	13505
9	4	64	128	8	0.5	62.39	13971
10	4	64	128	12	0.3	62.39	20255
11	2	64	128	8	0.5	62.47	12891
12	4	64	128	12	0.6	**62.83**	20255



(a) Full n^2 attention (b) Sliding window attention (c) Dilated sliding window (d) Global+sliding window

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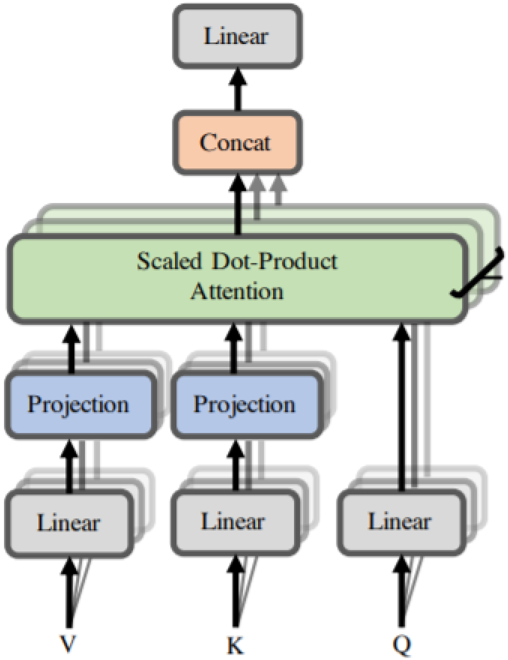
Results of the experiment
Matrix projection Linformer

Quality
Linformer with projection matrix shows poor results in comparing with recurrent neural network models

Shift invariance of a projection matrix
The projection matrix not being shifting invariant

Shift invariance test
We train the model to prove this property, choose a subsample, and calculate the metric on it. Then we shift all transactions by one and calculate the metric again.



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Results of the experiment
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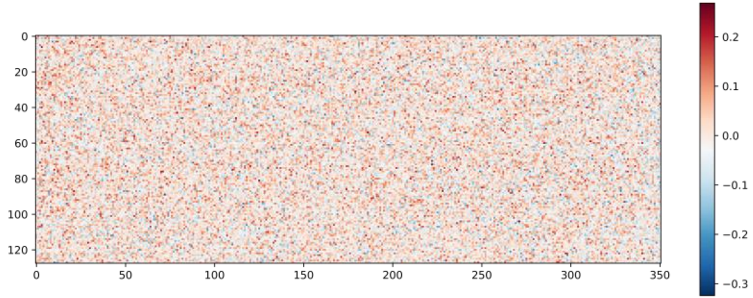

Linformer projection matrix

Dataset	Gini
Original dataset	53.47
Shift by one transaction	47.31

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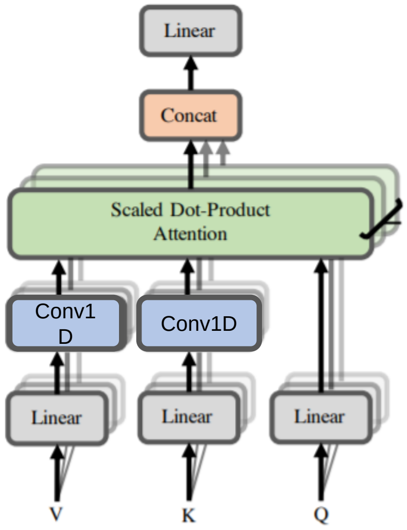
Results of the experiment

Convolution Linformer

Quality
Convolution Linformer shows a better result using less memory and shorter training time.

Convolution modification
To make projection shift-invariant, we replace a projection matrix with a convolution layer.

Cycle-shift augmentation
The result of convolution with kernel and stride equals size could change after adding a new object in sequence. (+1 Gini)



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Wang, Sinong, et al. "Linformer: Self-attention with linear complexity." *arXiv preprint arXiv:2006.04768* (2020).

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Results of the experiment

Convolution Linformer

Metric
LSTM - 60.12±0.17
Longformer - 62.80

Quality
Convolution Linformer shows a better result using less memory and shorter training time.

Convolution modification
To make projection shift-invariant, we replace a projection matrix with a convolution layer.

Cycle-shift augmentation
The result of convolution with kernel and stride equals size could change after adding a new object in sequence. (+1 Gini)

№	Projection size	Heads	Embedding size	Layers	Hidden dropout	dim ff	OOT gini	RAM (Mb)
1	128	8	64	4	0.2	2048	63.5	8757
2	32	8	64	4	0.2	2048	62.02	5124
3	64	8	64	4	0.2	2048	62.40	6789
4	128	4	64	4	0.2	2048	62.73	7159
5	128	8	64	2	0.2	2048	64.04	5333
6	128	8	64	4	0.2	2048	62.49	8715
7	128	8	32	4	0.2	2048	64.17	8179
8	128	8	64	4	0.2	2048	62.98	3635
9	128	8	64	3	0.2	2048	63.53	7051
10	128	8	64	2	0.2	512	**64.3 ± 0.19**	5333
11	128	8	64	2	0.2	256	63.5	**3395**
12	128	8	128	2	0.2	512	63.26	3833

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Results of the experiment

Convolution Linformer

Curriculum learning: Pre-trained weights
Training first on short sequences and the fine-tune on long show better quality and reduce training time

Non linear complexity
We need to increase the projection size: increasing length of sequence without increasing projection size leads to quality degradation.

№	Length	K	Training type	OOT gini	RAM (Mb)	Train time (hours)
1	350	128	From scratch	64.37	9787	3
2	1500	128	From scratch	63.75	12451	4.5
3	1500	256	From scratch	64.81	16047	11.61
4	1500	256	Pre-trained weights	**65.2**	16081	9.15
5	1500	400	From scratch	65.39	23481	17.3
6	1500	400	Pre-trained/train only conv layer	64.05	2299	7.1
7	1500	400	Pre-trained weights	**66.00 ± 0.21**	23489	8.5

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Results of the experiment
Comparison LSTM and Transformer

Transaction count	LSTM	Longformer	Linformer	Convolution Linformer
350	60.12 ± 0.17	62.80	53.47	64.37 ± 0.19
1500	63.01 ± 0.14	None	None	66.00 ± 0.21

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Performer: making transformers faster

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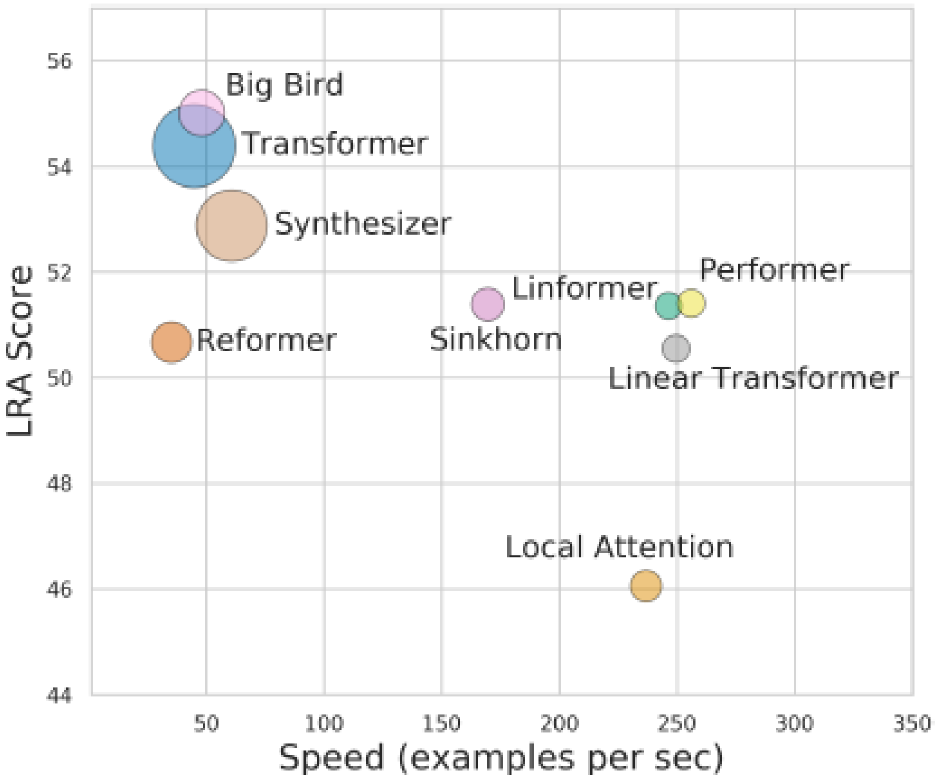
Tay, Yi, et al. "Long range arena: A benchmark for efficient transformers." *arXiv preprint arXiv:2011.04006* (2020).

Overall results

Speed (x axis), Performance (y axis) and Memory (size of the circles) for different models

BigBird is better than the vanilla transformer

Performer is more efficient



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Performer

$$\exp(q_i^T k_j) = \phi(q_i)^T \phi(k_j) = \mathrm{E}(\phi(q_i)^T \phi(k_j))$$

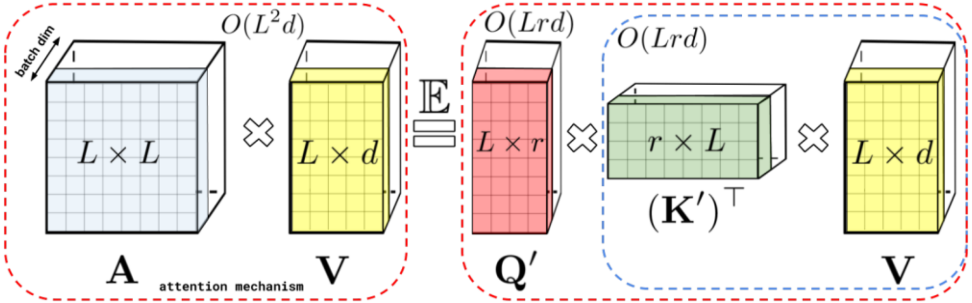
Vanilla attention formula:

$$\mathrm{Att}_{\leftrightarrow}(\mathbf{Q},\mathbf{K},\mathbf{V}) = \mathbf{D}^{-1}\mathbf{A}\mathbf{V}, \quad \mathbf{A} = \exp(\mathbf{Q}\mathbf{K}^{\top}/\sqrt{d}), \quad \mathbf{D} = \mathrm{diag}(\mathbf{A}\mathbf{1}_L).$$

Efficient bidirectional attention:

$$\widehat{\mathrm{Att}_{\leftrightarrow}}(\mathbf{Q},\mathbf{K},\mathbf{V}) = \widehat{\mathbf{D}}^{-1}(\mathbf{Q}'((\mathbf{K}')^{\top}\mathbf{V})), \qquad \widehat{\mathbf{D}} = \mathrm{diag}(\mathbf{Q}'((\mathbf{K}')^{\top}\mathbf{1}_L)).$$

It uses low-rank approximation to query and key matrices: $Q' \in \mathrm{R}^{T\times r}$, $K' \in \mathrm{R}^{r\times T}$



Choromanski, Krzysztof Marcin, et al. *Rethinking Attention with Performers.* ICLR. 2020.

Exponential kernel decomposition

Kernel representation

$$\exp\left(q_i^T k_j\right) = \phi(q_i)^T \phi\left(k_j\right) = \mathrm{E}\left(\phi(q_i)^T \phi\left(k_j\right)\right)$$

Feature vector

$$\phi(\mathbf{x}) = \frac{h(\mathbf{x})}{\sqrt{m}}\left(f_1(\omega_1^\top \mathbf{x}), ..., f_1(\omega_m^\top \mathbf{x}), ..., f_l(\omega_1^\top \mathbf{x}), ..., f_l(\omega_m^\top \mathbf{x})\right)$$

$$\mathbf{x}, \mathbf{y} \in \mathbb{R}^d,\ \mathbf{z} = \mathbf{x} + \mathbf{y}$$

$$\Lambda = \exp\left(-\frac{\|\mathbf{x}\|^2 + \|\mathbf{y}\|^2}{2}\right)$$

Our kernel

$$\mathrm{SM}(\mathbf{x}, \mathbf{y}) = \exp\left(\frac{\|\mathbf{x}\|^2}{2}\right) \mathrm{K}_{\mathrm{gauss}}(\mathbf{x}, \mathbf{y}) \exp\left(\frac{\|\mathbf{y}\|^2}{2}\right)$$

$$\mathrm{SM}(\mathbf{x}, \mathbf{y}) = \mathbb{E}_{\omega \sim \mathcal{N}(0, \mathbf{I}_d)}\left[\exp\left(\omega^\top \mathbf{x} - \frac{\|\mathbf{x}\|^2}{2}\right) \exp\left(\omega^\top \mathbf{y} - \frac{\|\mathbf{y}\|^2}{2}\right)\right] = \Lambda \mathbb{E}_{\omega \sim \mathcal{N}(0, \mathbf{I}_d)} \cosh(\omega^\top \mathbf{z})$$

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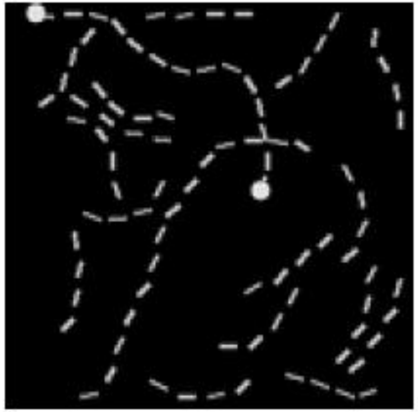
Choromanski, K.M., et al. *Rethinking Attention with Performers.* ICLR. 2020.

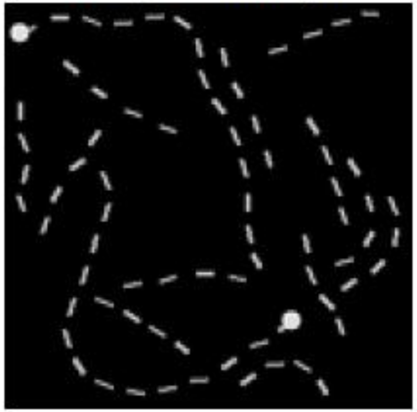
Pathfinder problem

For an image reshaped to the image find out, if two points are connected by a dashed line or not.

- Pathfinder: image 32x32 -> sequence 1024
- Pathfinder-X: images128x128 -> sequence 16K, extreme length (c.t. 1K limit for GPT-3)



Positive example



Negative example

Tay, Yi, et al. "Long range arena: A benchmark for efficient transformers." *arXiv preprint arXiv:2011.04006* (2020).

All transformers fail for Path-X problem

Model	ListOps	Text	Retrieval	Image	Pathfinder	Path-X	Avg
Transformer	36.37	64.27	57.46	42.44	71.40	FAIL	54.39
Local Attention	15.82	52.98	53.39	41.46	66.63	FAIL	46.06
Sparse Trans.	17.07	63.58	**59.59**	**44.24**	71.71	FAIL	51.24
Longformer	35.63	62.85	56.89	42.22	69.71	FAIL	53.46
Linformer	35.70	53.94	52.27	38.56	76.34	FAIL	51.36
Reformer	**37.27**	56.10	53.40	38.07	68.50	FAIL	50.67
Sinkhorn Trans.	33.67	61.20	53.83	41.23	67.45	FAIL	51.39
Synthesizer	36.99	61.68	54.67	41.61	69.45	FAIL	52.88
BigBird	36.05	64.02	59.29	40.83	74.87	FAIL	**55.01**
Linear Trans.	16.13	**65.90**	53.09	42.34	75.30	FAIL	50.55
Performer	18.01	65.40	53.82	42.77	**77.05**	FAIL	51.41
Task Avg (Std)	29 (9.7)	61 (4.6)	55 (2.6)	41 (1.8)	72 (3.7)	FAIL	52 (2.4)

Experimental results for Long-Range Arena Benchmark

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Tay, Yi, et al. "Long range arena: A benchmark for efficient transformers." *arXiv preprint arXiv:2011.04006* (2020).

Efficiently Modeling Long Sequences with Structured State Spaces

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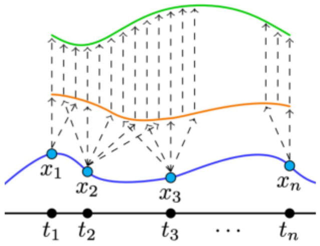
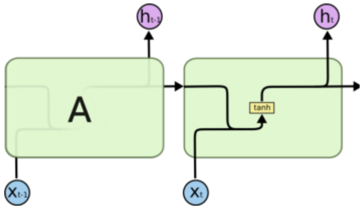
Gu, Albert, Karan Goel, and Christopher Re. Efficiently Modeling Long Sequences with Structured State Spaces. *ICLR*. 2021.

Paradigms for sequence modelling

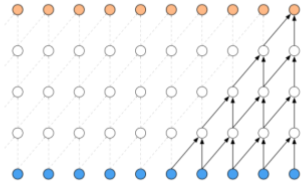
Continuous-time Model



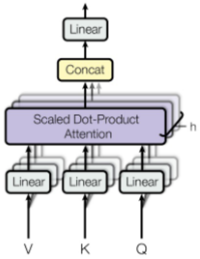
Recurrent Neural Net.



Convolutional Neural Net.



Transformer



Deep Sequence Model
Sequence-to-sequence map

(batch, length, dim)
↑
Sequence Model Layer
|
(batch, length, dim)

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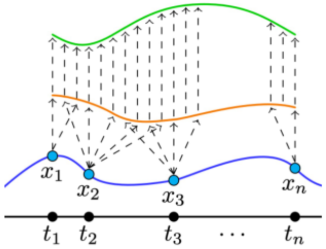
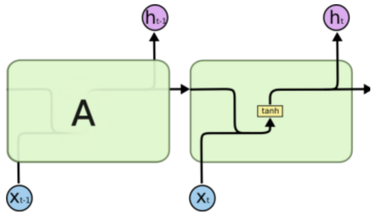
Pros and cons for each paradigm



✓ Continuous data
Irregular sampling

✗ Complex, very inefficient
Vanishing gradients

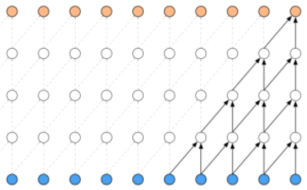
Continuous-time (CTM)



✓ Unbounded context
Stateful inference

✗ Inefficient training
Vanishing gradients

Recurrent (RNN)



✓ Easy optimization
Parallelizable training

✗ Inefficient inference
Bounded context

Convolutional (CNN)

Existing model families have clear tradeoffs
All struggle with long-range dependencies (LRD)

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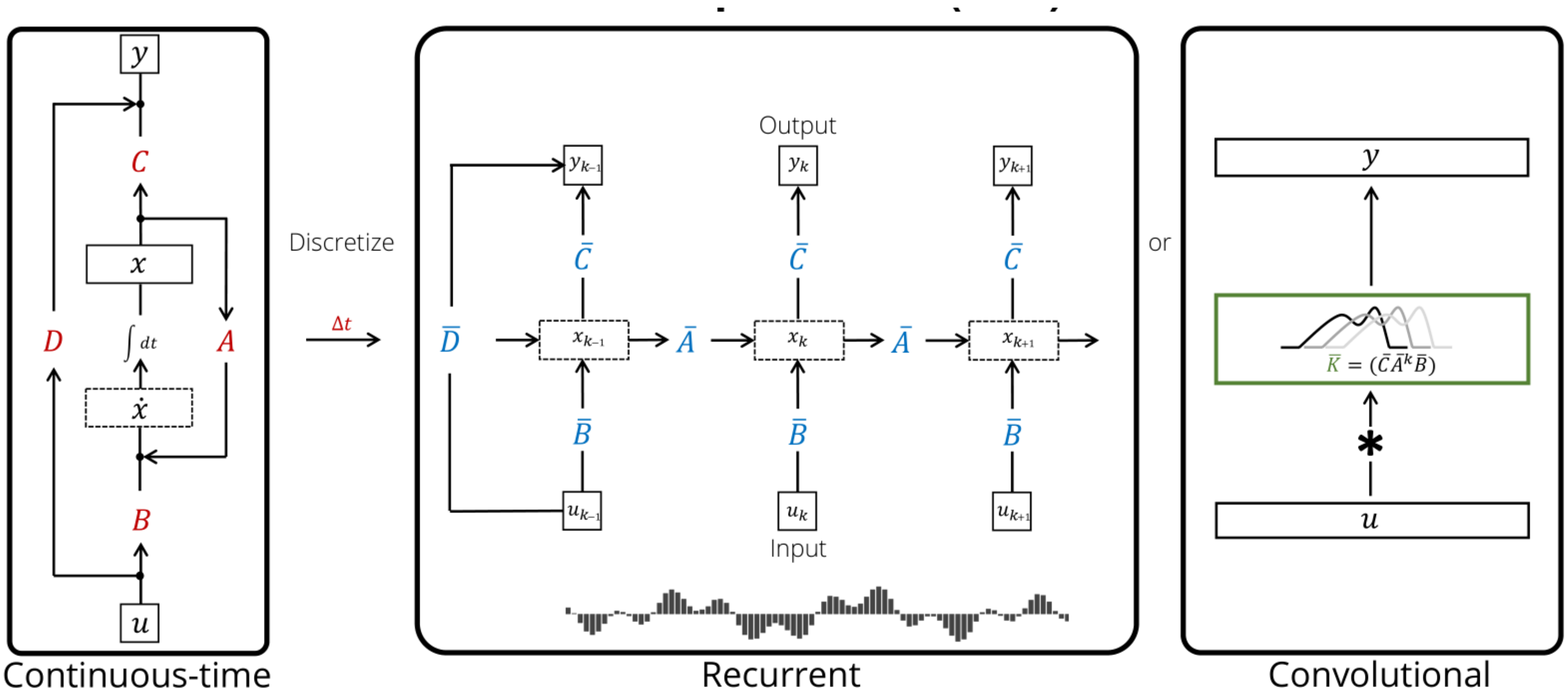
Structured state-space models



They are all similar! And you will soon know how…

Gu, Albert, Karan Goel, and Christopher Re. Efficiently Modeling Long Sequences with Structured State Spaces. *ICLR*. 2021.

Long-range arena benchmark

Benchmark spanning text, images, symbolic reasoning (length 1K-16K)

Model	ListOps	Text	Retrieval	Image	Pathfinder	Path-X	Avg
Random	10.00	50.00	50.00	10.00	50.00	50.00	36.67
Transformer	36.37	64.27	57.46	42.44	71.40	✗	53.66
Local Attention	15.82	52.98	53.39	41.46	66.63	✗	46.71
Sparse Trans.	17.07	63.58	59.59	44.24	71.71	✗	51.03
Longformer	35.63	62.85	56.89	42.22	69.71	✗	52.88
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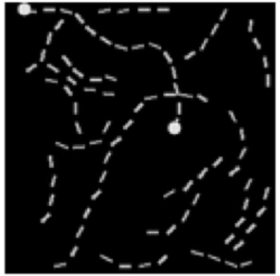
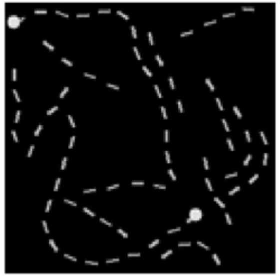
Reformer	37.27	56.10	53.40	38.07	68.50	✗	50.56
Sinkhorn Trans.	33.67	61.20	53.83	41.23	67.45	✗	51.23
Synthesizer	36.99	61.68	54.67	41.61	69.45	✗	52.40
BigBird	36.05	64.02	59.29	40.83	74.87	✗	54.17
Linear Trans.	16.13	65.90	53.09	42.34	75.30	✗	50.46
Performer	18.01	65.40	53.82	42.77	77.05	✗	51.18
FNet	35.33	65.11	59.61	38.67	77.80	✗	54.42
Nyströmformer	37.15	65.52	79.56	41.58	70.94	✗	57.46
Luna-256	37.25	64.57	79.29	47.38	77.72	✗	59.37
S4	**58.35**	**76.02**	**87.09**	**87.26**	**86.05**	**88.10**	**80.48**

Path-X



(a) A positive example.



(b) A negative example.

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State Space Models (SSM)

Input → State

$$\boxed{x'(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)}$$

$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

Parameters

$\boldsymbol{A} \in \mathbb{R}^{N\times N}$

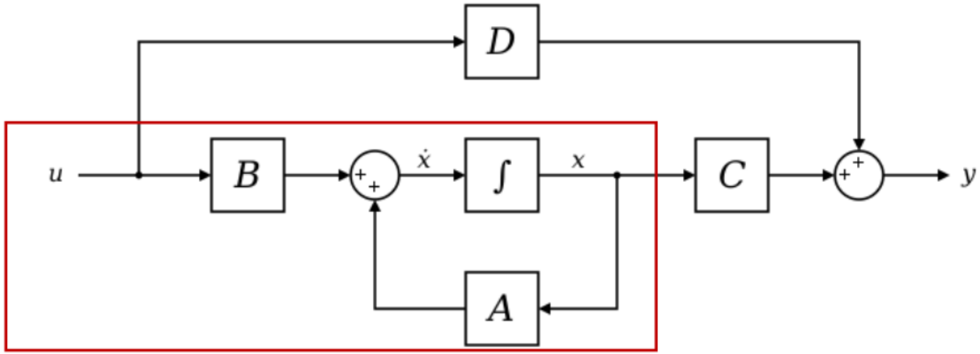
$\boldsymbol{B} \in \mathbb{R}^{N\times 1}$

$\boldsymbol{C} \in \mathbb{R}^{1\times N}$

$\boldsymbol{D} \in \mathbb{R}^{1\times 1}$

Function-to-function map $u(t) \mapsto y(t)$

1D input
u(t)



1D output
y(t)

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State Space Models (SSM)

$$x'(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$

State → Output $$\boxed{y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)}$$

Parameters

$\boldsymbol{A} \in \mathbb{R}^{N \times N}$

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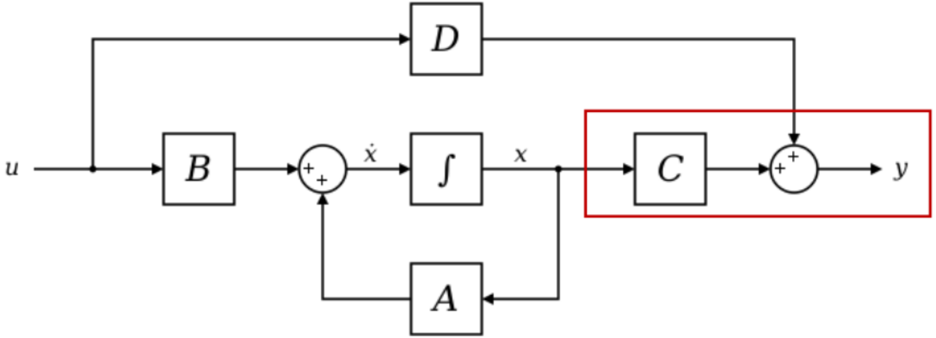
$\boldsymbol{C} \in \mathbb{R}^{1 \times N}$

$\boldsymbol{D} \in \mathbb{R}^{1 \times 1}$

Function-to-function map $$u(t) \mapsto y(t)$$

1D input
u(t)



1D output
y(t)

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Computing with SSMs: Recurrent View

$$x'(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$

$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

Parameters

$\boldsymbol{A} \in \mathbb{R}^{N \times N}$

$\boldsymbol{B} \in \mathbb{R}^{N \times 1}$

$\boldsymbol{C} \in \mathbb{R}^{1 \times N}$

$\boldsymbol{D} \in \mathbb{R}^{1 \times 1}$

$\Delta \in \mathbb{R}$

1. Discretize

$$\overline{\boldsymbol{A}} = \boldsymbol{I} + \Delta \boldsymbol{A}$$

2. Recurrent "hidden state"

$$x_k = \overline{\boldsymbol{A}} x_{k-1} + \overline{\boldsymbol{B}} u_k$$

3. Out projection

$$y_k = \overline{\boldsymbol{C}} x_k + \overline{\boldsymbol{D}} u_k$$

Can be computed with **linear recurrence**, similar to RNNs

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Computing with SSMs: Convolution View

$$x_k = \overline{\boldsymbol{A}} x_{k-1} + \overline{\boldsymbol{B}} u_k$$

$$\boxed{y_k = \overline{\boldsymbol{C}} x_k}$$

Can explicitly unroll the linear recurrence in closed form

$$x_0 = \overline{\boldsymbol{B}} u_0 \qquad x_1 = \overline{\boldsymbol{A}\boldsymbol{B}} u_0 + \overline{\boldsymbol{B}} u_1 \qquad x_2 = \overline{\boldsymbol{A}}^2 \overline{\boldsymbol{B}} u_0 + \overline{\boldsymbol{A}\boldsymbol{B}} u_1 + \overline{\boldsymbol{B}} u_2 \qquad \dots$$

$$y_0 = \overline{\boldsymbol{C}\boldsymbol{B}} u_0 \quad y_1 = \overline{\boldsymbol{C}\boldsymbol{A}\boldsymbol{B}} u_0 + \overline{\boldsymbol{C}\boldsymbol{B}} u_1 \quad y_2 = \overline{\boldsymbol{C}\boldsymbol{A}}^2 \overline{\boldsymbol{B}} u_0 + \overline{\boldsymbol{C}\boldsymbol{A}\boldsymbol{B}} u_1 + \overline{\boldsymbol{C}\boldsymbol{B}} u_2 \quad \dots$$

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Computing with SSMs: Convolution View

$$y_k = \overline{\boldsymbol{C}}\,\overline{\boldsymbol{A}}^k\overline{\boldsymbol{B}}u_0 + \overline{\boldsymbol{C}}\,\overline{\boldsymbol{A}}^{k-1}\overline{\boldsymbol{B}}u_1 + \cdots + \overline{\boldsymbol{C}\boldsymbol{A}\boldsymbol{B}}u_{k-1} + \overline{\boldsymbol{C}\boldsymbol{B}}u_k$$

$$\overline{\boldsymbol{K}} \in \mathbb{R}^L := (\overline{\boldsymbol{C}\boldsymbol{B}}, \overline{\boldsymbol{C}\boldsymbol{A}\boldsymbol{B}}, \dots, \overline{\boldsymbol{C}}\,\overline{\boldsymbol{A}}^{L-1}\overline{\boldsymbol{B}})$$

$$y = \overline{\boldsymbol{K}} * u$$

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Can be computed with **convolutions**, similar to CNNs

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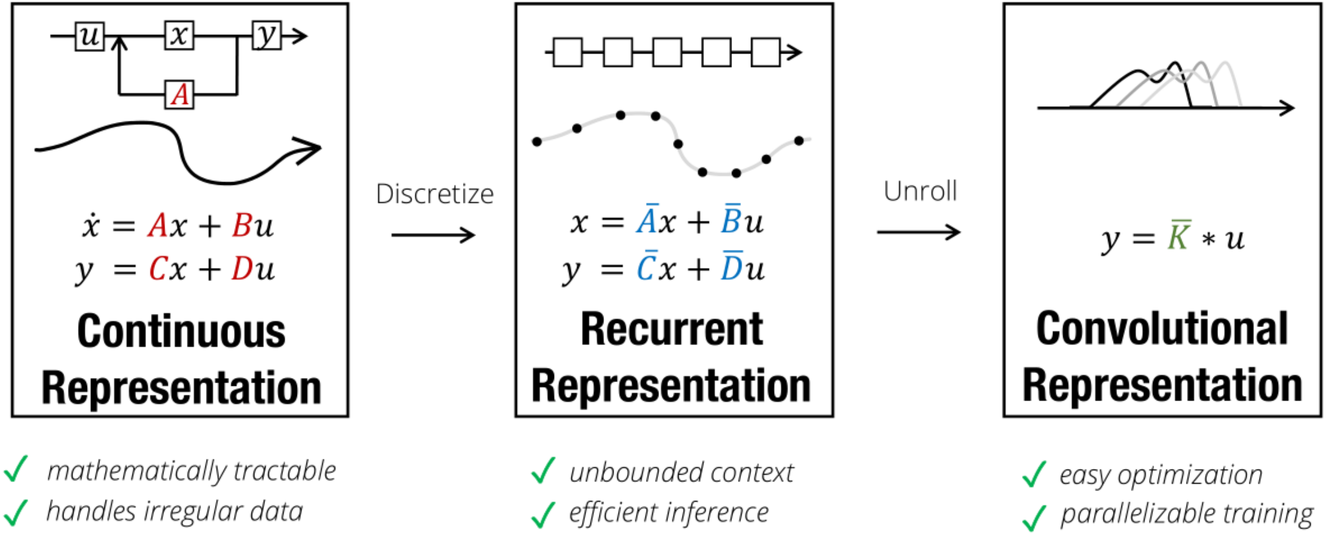
Summary: Properties of SSMs



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Conclusions

- We can improve Transformers memory and computational requirements with some heuristics and different types of attention

- The main ideas are:
  - combine different sparse attention mechanisms
  - bucket processing

- But there are some limitations in terms of reducing number of parameters and memory as we have large window sizes and large number of global attention nodes