Markov Chains

Midterm questions. Midterm is open-book, all materials are allowed.

1. Conditional mathematical expectation and conditional probability. Basic properties;
2. Regular conditional probability. Probability kernel. Existence of the probability kernel.
3. Definition of a Markov chain (general state space). Operations on Markov kernels: tensor product, composition. Action of Markov kernels on functions and measures;
4. Discrete state-space Markov chains. Irreducible chains. Recurrent and non-recurrent states.
5. Total variation distance and Kantorovich-Wasserstein distance. Their properties. Invariant distribution. Exponential convergence in total variation for ergodic transition matrices.
6. Invariant measures for countable state-space Markov Chain. Random walk on $\mathbb{Z}$ and its properties. Detailed balance condition and its relation with invariant measure.
7. Invariant distribution (general case). Reversibility. Relations between reversibility and invariance. Example: Metropolis-Hastings kernel.
8. φ-irreducibility. Aperiodicity. Ergodicity of φ-irreducible and aperiodic chain. Small set and drift conditions. Uniform and V-geometric ergodicity.
9. Asymptotic variance of the Markov chain. Covariance bounds under UGE.
10. Poisson equation. CLT for bounded functions under stationary distribution.
11. CLT under arbitrary initial distribution for UGE chains.