Markov Chains

Exam questions. Exam is open-book, all materials are allowed.

1. Conditional mathematical expectation and conditional probability. Basic properties. Regular conditional probability. Probability kernel.
2. Definition of a Markov chain (general state space). Operations on Markov kernels: tensor product, composition. Action of Markov kernels on functions and measures;
3. Total variation distance and Kantorovich-Wasserstein distance. Their properties. Invariant distribution.
4. φ-irreducibility. Aperiodicity. Ergodicity of φ-irreducible and aperiodic chain. Small set and drift conditions. Uniform and V-geometric ergodicity.
5. Invariant distribution (general case). Reversibility. Relations between reversibility and invariance.
6. Asymptotic variance of the Markov chain. Covariance bounds under UGE. Poisson equation. CLT for bounded functions under stationary distribution.
7. Sampling methods: inverse CDF method. Examples. Rejection sampling.
8. Importance sampling and self-normalized importance sampling (SNIS). Bounds for bias and MSE of SNIS.
9. Metropolis-Hasings sampler. Construction, reversibility, basic properties.
10. Gibbs sampler: construction, invariant distribution. Deterministic- and random-update Gibbs samplers.
11. Unadjusted Langevin Algorithm (ULA) and Metropolis-Adjusted Langevin algorithms: construction, invariant distribution. Rate of convergence of the n-th step distribution of the ULA to its invariant distribution under strong convexity.
12. i-SIR (Iterated sequence-importance resampling) algorithm: invariant distribution and geometric ergodicity.
13. Hamiltonian Monte-Carlo (HMC): algorithm, invariant distribution.
14. Brenier theorem. Normalizing flows with maximum likelihood training.
15. Examples of normalizing flows: Real NVP and inverse autoregressive flows. Adaptive i-SIR with normalising flows.
16. Variational autoencoders: basic construction. Energy-based formulation of VAE.
17. GANs: basic construction. Energy-based formulation of GAN and Metropolis-Hastings GAN.