1. Objectives

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Linear Classifiers and Perceptron Algorithm

At the end of this lecture, you will be able to

- understand the concepts of Feature vectors and labels, Training set and Test set, Classifier, Training error, Test error, and the Set of classifiers
- derive the mathematical presentation of linear classifiers
- understand the intuitive and formal definition of linear separation
- use the perceptron algorithm with and without offset

Concept Review Problem: car accident prediction 1

1/1 point (graded)

In this problem, we will put ourselves in the shoes of a car insurance company. Our goal is to find out whether customers were involved in an accident on July 4th, 1998.

For 8 customers, we know the following information:

1. number of accidents the customer made in the past.
2. number of miles the customer has driven.
3. the customer's age

Also, for 5 of the customers, we know whether each of them was involved in an accident on July 4th, 1998.

If we want to learn a model in a supervised way, what is n, the number of training examples?

$n =$ 5 ✔ **Answer:** 5

Solution:

We have 5 data points with known labels.

Show answer

The insurance company recorded relevant information for all 8 customers, as illustrated in the table below.

	number of past accidents	miles customer drove so far	customer's age
customer 1	0	2710.9	21
customer 2	2	13209.2	40
customer 3	1	89001.4	32
customer 4	3	12381.1	18
customer 5	0	1893.5	24
customer 6	2	32493.5	24
customer 7	1	5443.5	30
customer 8	0	4493.5	28

What is the dimension of each feature vector?

$d =$ 3 ✔ **Answer:** 3

Solution:

Each feature vector has length 3 (columns in the table), and thus its dimension is 3.

Concept Review Problem: car accident prediction 3

1/1 point (graded)

How many feature vectors are there in the above table?

$\text{Number of Feature vectors}$ 8 ✔ **Answer:** 8

Solution:

There are 8 rows in the table.

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Concept Review Problem: Classifier and Training Error 1

1/1 point (graded)

Assume we have training data and a classifier like the following: (where $h\left(x\right)$ denotes the value outputted by the classifier with the data point as input)

	$h(x)$	y
data 1	1	1
data 2	-1	1
data 3	1	1
data 4	1	-1
data 5	-1	-1

What is the training error?

$\varepsilon_n(h) =$ 0.4 ✔ **Answer:** 0.4

Solution:

We have 5 data points total, two of which $h(x)$ does not match y (data2 and data4). Thus $\varepsilon_n(h) = \frac{1}{5}\sum_{i=1}^{5}[[h(x_i) \neq y]] = \frac{2}{5}$

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Question 3: What movies are in the **test set**? Select all those apply.

- [ ] movie 1
- [ ] movie 2
- [ ] movie 3
- [ ] movie 4
- [x] movie 5
- [x] movie 6
- [x] movie 7

✔

Solution:

Movies whose labels are not yet available are in the test set. Thus movies $5, 6, 7$ are in the test set. Remember that it is our end goal to predict these movies' labels.

What is the inner product of $[0,1,1]$ and $[1,1,1]$?

2

✔ **Answer:** 2

Solution:

$$0 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 = 2$$

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Linear Classifier Practice

1/1 point (graded)

We saw in the lecture above that for a linear classifier h, $h(x;\theta) = sign(\theta \cdot x)$, i.e. the sign of the dot product of θ and x.
Now consider θ which is given by

$$\theta = (1, -1) \tag{4.1}$$

- [x] $+1$
- [ ] -1

As an aside, classifiers need not be linear. They can be of any shape!

Solution:

$(10, 10)$ belongs to the region where $h(x) = +1$.

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Training Error

1/1 point (graded)

Which of the following points would be classified as positive by θ? Please choose all correct answers.

- [x] $(1,-1)$
- [x] $(1,0)$
- [ ] $(0,1)$
- [ ] $(0,0)$

Solution:

$X$is positively classified by the classifier if and only if $x\cdot\theta>0$. The dot product of $(1,-1)$ with θ is positive. Also, $(1,0)\cdot\theta$ is positive. On the other hand, $(0,1)\cdot\theta$ and $(0,0)\cdot\theta$ are nonpositive. Thus the first and second points are positively classified by θ.

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1/1 point (graded)

Again, we have a linear classifier with θ given by

$$\theta = (1, -1) \tag{4.2}$$

and the offset, θ_0 given by $\theta_0 = -1$ Now which of the following points would be classified as positive by θ? Please choose all correct answers.

- [x] $(1, -1)$
- [ ] $(1, 0)$
- [ ] $(0, 1)$
- [ ] $(0, 0)$

✔

Solution:

$X$is positively classified by the classifier if and only if $x \cdot \theta + \theta_0 > 0$. The dot product of $(1, -1)$ with θ is 2, and adding -1 makes it still positive. However,$x \cdot \theta + \theta_0 \leq 0$ for other data points.

Given θ and θ_0, a **linear classifier** $h : X \to \{-1, 0, +1\}$ is a function that outputs $+1$ if $\theta \cdot x + \theta_0$ is positive, 0 if it is zero, and -1 if it is negative. In other words, $h(x) = \text{sign}(\theta \cdot x + \theta_0)$.

Basics 1

1/1 point (graded)

As described in the lecture above, h is a linear classifier which is defined by the boundary $\theta \cdot x = 0$ (where theta is a vector perpendicular to the plane.) The ith training data is $(x^{(i)}, y^{(i)})$, where $x^{(i)}$ is a vector and $y^{(i)}$ is a scalar quantity. If θ is a vector of the same dimension as $x^{(i)}$, what are $y^{(i)}$ and sign($\theta \cdot x^{(i)}$) respectively?

- ( ) output of the classifier h, label
- ( ) label, dimension of the feature vector
- ( ) label, distance of the point from the linear classifier
- (•) label, output of the classifier h

✔

Solution:

By definition, $y^{(i)}$ is the label of $x^{(i)}$. Also, by the definition of a linear classifier $h(x)$ =sign($\theta \cdot x^{(i)}$), the output of h is given by sign($\theta \cdot x^{(i)}$).

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Basics 2

1/1 point (graded)

For the ith training data (x^i, y^i), what values can $y^{(i)}$ take, **conventionally** (in the context of linear classifiers)? Choose all those apply.

- [x] -1
- [x] $+1$
- [ ] 0
- [ ] $+10$

Solution:

By the convention of linear classification, because $y^{(i)}$ is a label, it can take -1 or $+1$. Note that 0 is not a possible value.

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Answers are displayed within the problem

Basics 3

1/1 point (graded)

For the ith training data (x^{i}, y^{i}), what values can $sign\left(\theta \cdot x^{(i)}\right)$ take? Choose all those apply.

- [x] -1
- [x] $+1$
- [x] 0
- [ ] $+10$

By definition the $sign\left(\theta \cdot x^{(i)}\right)$ function can only take one of $0, -1, +1$ as its value. Remember that a linear classifier outputs one of $-1, 0, 1$.

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Answers are displayed within the problem

When the Product is Positive

1/1 point (graded)

When does $y^{(i)}\left(\theta \cdot x^{(i)}\right) > 0$ happen? Choose all those apply.

- [x] $y^{(i)} > 0$ and $\theta \cdot x^{(i)} > 0$
- [ ] $y^{(i)} < 0$ and $\theta \cdot x^{(i)} > 0$
- [ ] $y^{(i)} > 0$ and $\theta \cdot x^{(i)} < 0$
- [x] $y^{(i)} < 0$ and $\theta \cdot x^{(i)} < 0$

$y^{(i)}\left(\theta \cdot x^{(i)}\right) > 0$ is true if and only if $y^{(i)}$ and $\left(\theta \cdot x^{(i)}\right)$ are both positive both negative. In other words, they have the same sign.

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Intuitive Meanings of Positive Product

1/1 point (graded)

What is the intuitive meaning of $y^{(i)}\left(\theta \cdot x^{(i)}\right) > 0$?

- (selected) x^{i} label and classified result match
- x^{i} label and classified result do not match
- x^{i} is on the boundary of the classifier
- training error is positive

$y^{(i)}\left(\theta\cdot x^{(i)}\right)>0$ is true if and only if $y^{(i)}$ and $\left(\theta\cdot x^{(i)}\right)$ are both positive both negative. In other words, they have the same sign.

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Answers are displayed within the problem

Intuitive Meanings of Negative Product

1/1 point (graded)

What is the intuitive meaning of $y^{(i)}\left(\theta\cdot x^{(i)}\right)<0$?

- ○ x^i label and classified result match
- ● x^i label and classified result do not match
- ○ x^i is on the boundary of the classifier
- ○ training error is negative

$y^{(i)}\,(\theta \cdot x^{(i)}) < 0$ is true if and only if $y^{(i)}$ and $(\theta \cdot x^{(i)})$ have different signs.

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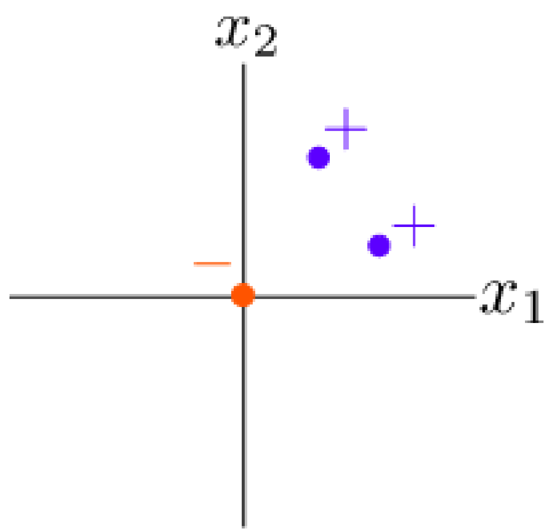
Submit You have used 1 of 1 attempt

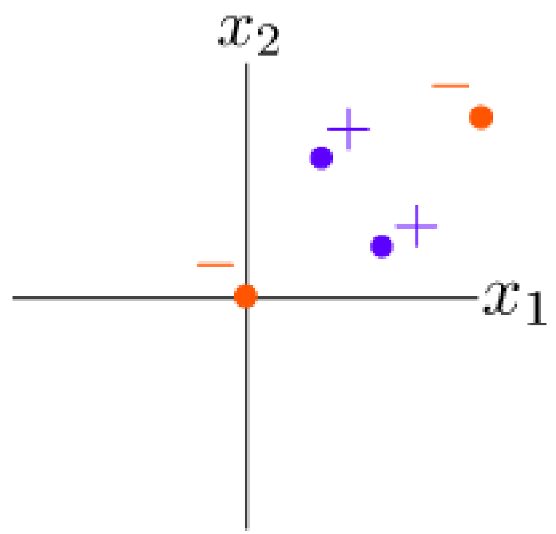
Answers are displayed within the problem

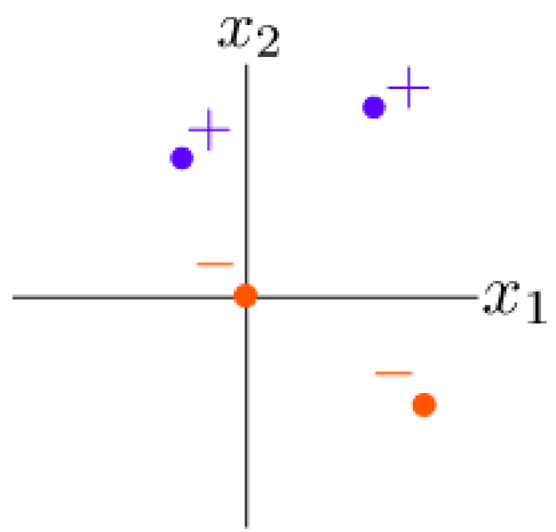
Linear Separation 1

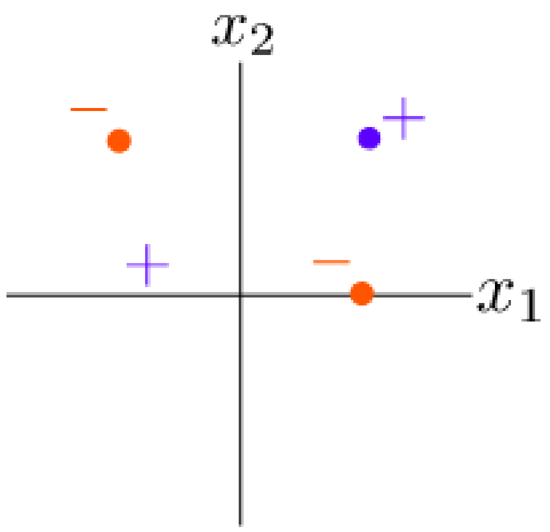
1/1 point (graded)

Of the following, which is linearly separable? Choose all those apply.

- [x] 


x_2
x_1
+
+
−
−


x_2
x_1
+
+
−
−



Solution:

Linearly separable data can be separated with $+$ labels on one side of the line and $-$ labels on the other side, by some line on the plane.

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Answers are displayed within the problem

A set of Training examples is illustrated in the table below, with the classified result by some linear classifier h and the label y^i. Is it linearly separable?

	$h(x^i)$	y^i
example 1	-1	-1
example 2	1	1
example 3	1	1
example 4	-1	-1
example 5	-1	-1

- (•) yes
- ( ) no

✔

Solution:

For linearly separable data, a linear classifier can perfectly separate the data. The provided classifier $h(x)$ classifies all the given points correctly.

Perceptron Concept Questions 1

1/1 point (graded)

Remember that the Perceptron Algorithm (without offset) is stated as the following:

Perceptron $\left(\{(x^{(i)}, y^{(i)}), i = 1, \ldots, n\}, T\right)$:

initialize $\theta = 0$(vector);

for $t = 1, \ldots, T$ do

for $i = 1, \ldots, n$ do

if $y^{(i)}\left(\theta \cdot x^{(i)}\right) \leq 0$ then

update $\theta = \theta + y^{(i)}x^{(i)}$

What does the Perceptron algorithm take as inputs among the following? Choose all those apply.

- [x] Training set
- [x] T - the number of times the algorithm iterates through the whole training set
- [ ] Test set
- [ ] θ
- [ ] θ_0

Solution:

The perceptron algorithm takes T and the training set as input, and aims to learn the optimal "θ","θ_0"

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Answers are displayed within the problem

Perceptron Update 1

1/1 point (graded)

Now consider the Perceptron algorithm with Offset. Whenever there is a "mistake" (or equivalently, whenever $y^{(i)}\left(\theta \cdot x^{(i)} + \theta_0\right) \leq 0$ i.e. when the label y^i and $h\left(x\right)$ do not match), perceptron updates

$$\theta \text{ with } \theta + y^{(i)} x^{(i)}$$

and

$$\theta_0 \text{ with } \theta_0 + y^{(i)}.$$

More formally, the Perceptron Algorithm with Offset is defined as follows:

Perceptron $\left(\left\{\left(x^{(i)}, y^{(i)}\right), i=1, \ldots, n\right\}, T\right)$:

initialize $\theta=0$(vector); $\theta_0=0$(scalar)

for $t=1, \ldots, T$ do

for $i=1, \ldots, n$ do

if $y^{(i)}\left(\theta \cdot x^{(i)}+\theta_0\right) \leq 0$ then

update $\theta=\theta+y^{(i)} x^{(i)}$

update $\theta_0=\theta_0+y^{(i)}$

In the next set of problems, we will try to understand why such an update is a reasonable one.

When a mistake is spotted, do the updated values of θ and θ_0 provide a better prediction? In other words, is

$$y^{(i)}\left(\left(\theta+y^{(i)} x^{(i)}\right) \cdot x^{(i)}+\theta_0+y^{(i)}\right)$$

always greater than or equal to

$$y^{(i)}\left(\theta \cdot x^{(i)}+\theta_0\right)$$

- ( ) Yes, because $\theta + y^{(i)} x^{(i)}$ is always larger than θ
- (•) Yes, because $\left(y^{(i)}\right)^2 \|x^{(i)}\|^2 + \left(y^{(i)}\right)^2 \geq 0$
- ( ) No, because $\left(y^{(i)}\right)^2 \|x^{(i)}\|^2 - \left(y^{i}\right)^2 \leq 0$
- ( ) No, because $\theta + y^{(i)} x^{(i)}$ is always larger than θ

✔

Solution:

Comparing the two terms,

$$y^{(i)}\left(\left(\theta + y^{(i)} x^{(i)}\right) \cdot x^{(i)} + \theta_0 + y^{(i)}\right) - y^{(i)}\left(\theta \cdot x^{(i)} + \theta_0\right) = \left(y^{(i)}\right)^2 \|x^{(i)}\|^2 + \left(y^{(i)}\right)^2 = \left(y^{(i)}\right)^2 \left(\|x^{(i)}\|^2 + 1\right)) > 0$$

the first is always greater than the latter. Considering that our goal is to minimize the training error, the update always makes the training error decrease, which is desirable.

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Perceptron Update 2

0 points possible (ungraded)

For a given example i, we defined the training error as 1 if $y^{(i)}\left(\theta \cdot x^{(i)}+\theta_0\right) \leq 0$, and 0 otherwise:

$$\varepsilon_i\left(\theta, \theta_0\right)=\left[\left[y^{(i)}\left(\theta \cdot x^{(i)}+\theta_0\right) \leq 0\right]\right]$$

Say we have a linear classifier given by θ, θ_0. After the perceptron update using example i, the training error $\varepsilon_i\left(\theta, \theta_0\right)$ for that example can (select all those apply):

- [ ] Increase
- [x] Stay the same
- [x] Decrease

Solution:

From the previous problem, we saw that $y^i\left(\theta \cdot x+\theta_0\right)$ increases after the perceptron update. Thus $\left[\left[y^i\left(\theta \cdot x+\theta_0\right) \leq 0\right]\right]$ becomes zero or stays 1.