Week 4 Assignment

1. Medical Diagnosis Using Bayes' Theorem

Example1:

Child	Disease	Develops Rashes
1	Flu (F)	No
2	Flu (F)	Yes
3	Measles (M)	Yes
4	Flu (F)	No
5	Flu (F)	No
6	Flu (F)	No
7	Measles (M)	Yes
8	Flu (F)	Yes
9	Flu (F)	No
10	Flu (F)	Yes

Analysis and Application of Bayes' Theorem

Given probabilities:

- Probability of flu (F) = 0.90 (90%)
- Probability of measles (M) = 0.10 (10%)
- Probability of observing rashes given measles $P(R \mid M)=0.95$
- Probability of observing rashes given flu $P(R \mid F)=0.08$

Step-by-Step Calculation

1. **Calculate P(R)** - the total probability of developing rashes:

   $P(R) = P(R|F)*P(F) + P(R|M)*P(M)$

   $P(R) = (0.08*0.90) + (0.95*0.10)$

   $P(R) = 0.072+0.095$

   $P(R)=0.167$

2. **Calculate P(F|R)** using Bayes' theorem:

   $P(F|R) = P(R|F)*P(F) / P(R)$

   $P(F|R) = 0.08*0.90 / 0.167$

   $P(F|R) = 0.072 / 0.167$

   $P(F|R) \approx 0.431$

Based on the analysis using Bayes' theorem, the probability that a child has the flu given that they have developed rashes is approximately 0.431, or 43.1%. This result reflects the likelihood of having flu based on the observed symptom (rashes) and the prevalence of flu and measles in the simulated dataset.

Example 2:

In this example, Bayes' Theorem is used to find the probability of having a disease given a positive test result. Given the following probabilities:

$P(D) = 0.01$ (Probability of having the disease)
$P(\neg D) = 0.99$ (Probability of not having the disease)
$P(T|D) = 0.95$ (Probability of testing positive given the disease)
$P(T|\neg D) = 0.05$ (Probability of testing positive given no disease)

First, calculating the total probability of testing positive, P(T):
$P(T) = P(T|D)P(D) + P(T|\neg D)P(\neg D)$
$P(T) = (0.95 * 0.01) + (0.05 * 0.99) = 0.059$

Next, using Bayes' Theorem to find P(D|T):
$P(D|T) = (P(T|D) * P(D)) / P(T)$
$P(D|T) = (0.95 * 0.01) / 0.059$
$P(D|T) = 0.0095 / 0.059 \approx 0.161$

Therefore, the probability of having the disease given a positive test result is approximately 0.161.

2. Eigenvalues and Eigenvectors of a Matrix

Given matrix A:
$A = [[4, 1], [2, 3]]$

Step-by-Step Calculation of Eigenvalues:
The characteristic equation is:
$\det(A - \lambda I) = 0$

Where I is the identity matrix:
$\lambda I = [[\lambda, 0], [0, \lambda]]$

So, $A - \lambda I$ is:
$A - \lambda I = [[4 - \lambda, 1], [2, 3 - \lambda]]$

The determinant is calculated as:
$\det(A - \lambda I) = (4 - \lambda)(3 - \lambda) - 2 * 1$
$= \lambda^2 - 7\lambda + 10 = 0$

Solving the quadratic equation for λ:
$\lambda = (7 \pm \sqrt{(49 - 40)}) / 2$
$\lambda = (7 \pm 3) / 2$
Thus, the eigenvalues are:
$\lambda_1 = 5, \lambda_2 = 2$

Step-by-Step Calculation of Eigenvectors:
For $\lambda_1 = 5$:
$A - 5I = [[-1, 1], [2, -2]]$

Solving $(A - 5I)v = 0$:
$[[-1, 1], [2, -2]] [[x], [y]] = [[0], [0]]$

This simplifies to $x = y$, so one eigenvector is:
$v_1 = [[1], [1]]$

For $\lambda_2 = 2$:
$A - 2I = [[2, 1], [2, 1]]$

Solving $(A - 2I)v = 0$:
$[[2, 1], [2, 1]] [[x], [y]] = [[0], [0]]$

This simplifies to 2x + y = 0 or y = -2x, so one eigenvector is:
v_2 = [[1], [-2]]

3. Determinant and Inverse of a 3x3 Matrix

Given matrix B:
B = [[1, 2, 3], [0, 1, 4], [5, 6, 0]]

Step-by-Step Calculation of Determinant:
The determinant is calculated as:
det(B) = 1*(1*0 - 4*6) - 2*(0*0 - 4*5) + 3*(0*6 - 1*5)
det(B) = 1*(-24) - 2*(-20) + 3*(-5)
det(B) = -24 + 40 - 15 = 1

Step-by-Step Calculation of Inverse:
Since the determinant is non-zero, the matrix is invertible. The inverse is:
B^{-1} = 1/det(B) * adj(B)

Where adj(B) is the adjugate matrix.Inverse of B is:
B^{-1} = [[-24, 18, 5], [20, -15, -4], [-5, 4, 1]]

4. Properties and Applications of a Normal Distribution

The normal distribution is a continuous probability distribution characterized by a bell-shaped curve.

Properties:
- Symmetry about the mean (μ)
- Mean, median, and mode are equal
- Defined by two parameters: mean (μ) and standard deviation (σ)
- The total area under the curve is 1

Applications:
- Modeling natural phenomena (e.g., heights, test scores)
- Central limit theorem applications
- Quality control and standardization

5. Calculating Probabilities Using Normal Distribution

Given a standard normal distribution with mean (μ) = 0 and standard deviation (σ) = 1, to find the probability $P(Z < 1.96)$.

The Z-score represents the number of standard deviations a data point is from the mean. In a standard normal distribution:

- $Z = (X - \mu) / \sigma$
- For Z = 1.96, looking at the area to the left of Z in the standard normal distribution.

Using standard normal distribution tables, find the cumulative probability up to Z = 1.96.

The cumulative distribution function (CDF) value for Z = 1.96 is approximately 0.975. This means that the probability that a value from this distribution is less than 1.96 is 0.975.

In other words, $P(Z < 1.96) \approx 0.975$.

Steps to calculate using a Z-table:
1. Locate the row for the first two digits of the Z-score (1.9).
2. Locate the column for the second decimal place (0.06).
3. The intersection gives the cumulative probability.

Therefore, the probability that a value from this standard normal distribution is less than 1.96 is approximately 0.975.