7. Frequency Chi-square Tests

Suppose a population consists of k mutually exclusive classes with proportions $p_1, p_2, \ldots p_n$ $\left(\sum_{i=1}^{k} p_i = 1\right)$. This classification may be with respect to either an attribute or a variable.

Now, a random sample of size n (large) is taken from the population, each sample observation being independent of each other. Let the frequencies in the classes be $n_1, n_2, \ldots, n_k$ with $\sum_{i=1}^{k} n_i = n =$ total frequency, n_i being the frequency of the ith class, $i = 1, 2, \ldots, k$. The frequency distribution for the sample as well as the population distribution is shown below (Table 17.2)

Table 17.2
Population proportion and sample frequencies

Class	Probability (Population proportion)	Sample frequency
1	p_1	n_1
2	p_2	n_2
3	p_3	n_3
4	p_4	n_4
$\vdots$	$\vdots$	$\vdots$
k	p_k	n_k
Total	$\sum_i^k p_i = 1$	$n = \sum_i^k n_i$

Now, the sample frequencies $n_1, n_2, ..., n_k$ are random variables, the expected value of n_i is np_i, *i.e.*, $E(x_i) = np_i$. Now, it can be seen that $\sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$ approximately follows a χ^2 distribution with $(k-1)$ degrees of freedom.

In many cases we see that the results obtained in samples do not agree exactly with theoretical results expected according to rules of probability. For example, if we toss a fair coin 50 times, theoretically we may expect 25 heads and 25 tails, but it is rare to obtain these results in practice.

If, in a particular sample, a set of possible events $A_1, A_2, ..., A_k$ are observed to occur with frequencies. $O_1, O_2, ..., O_k$ and according to probability rules $A_1, A_2, ..., A_k$ are expected to occur with frequencies. $E_1, E_2, ..., E_k$ (see table), then $O_1, O_2, ..., O_k$ are called **observed frequencies** and $E_1, E_2, ... E_k$ are called **expected frequencies.**

Table

Event	Observed frequency	Expected frequency
A_1	O_1	E_1
A_2	O_2	E_2
A_3	O_3	E_3
.	.	.
.	.	.
A_k	O_k	E_k

If now the O_i and E_i *for* $i = 1, ..., k$ are available then we can write $\chi^2_{k-1} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$

where O_i = observed frequency of the *i*th class and E_i = Expected frequency of the *i*th class. Such a χ^2 is called *Pearsonian* χ^2 or *Frequency* χ^2.

17.28. Test of Goodness of Fit

Suppose we like to test whether the population distribution is of a particular kind. Let us draw a random sample of size n, (each sample observation being independent of each other) from the population and let the frequencies (observed) in k mutually exclusive classes be $n_1, n_2, ..., n_k$ with $\sum_{i=1}^{k} n_i = n$ = total frequency.

We now like to test the null hypothesis $H_0 : p_1 = p_1^0, p_2 = p_2^0, p_3 = p_3^0, \ldots, p_k = p_k^0 \left(\sum_{i=1}^{k} p_i = 1 \right)$ where p_i represents the proportion of population members in the ith class and p_i^0 is a specified value on the basis of the specified distribution. In that case the test criterion is $\chi_{k-1}^2 = \sum_{i=1}^{k} \frac{(n_i - np_i^0)^2}{np_i^0} = \sum_{i=1}^{k} \frac{n_i^2}{np_i^0} - n$ which follows asymptotically a χ^2 distribution with $df = k-1$ provided $np_i^0 \geq 5$ for every i. It should be noted that in computing the value of the χ^2 for a given sample, we use the simpler form

$$\chi_{k-1}^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}.$$

Clearly, more we move away from the null hypothesis, *i.e.*, greater the divergence between the observed (O_i) and expected (E_i) frequencies the higher will be the value of χ^2.

Now, on the basis of the given samples H_0 will be rejected at $100\alpha\%$ level of significance if observed

$$\chi_{k-1}^2 \left(= \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \right)$$ exceeds $\chi_{\alpha,k-1}^2$ (table value). Otherwise H_0 is accepted.

Sometimes we see that the null hypothesis specifies the form of the population but the proportions cannot be specified because they depend on some unknown parameters of the population. For example. it may state that the population distribution is of the binomial form without specifying the parameter p or may state that it is of the Poisson form without specifying the parameter λ. In that case we may estimate these parameters by some suitable method. Let the corresponding estimates of the proportions of population members in the ith class be $\hat{p}_i$. In that case the test criterion becomes,

$$\chi_{k-r-1}^2 = \sum_{i=1}^{k} \frac{(n_i - n\hat{p}_i)^2}{n\hat{p}} = \sum_{i=1}^{k} \frac{(O_i - \hat{E}_i)^2}{\hat{E}_i}$$

which follows a χ^2 distribution with $df = k-r-1$ provided $n\,\hat{p}_i \geq 5$ for every i (*i.e.*, for large samples). Here r is the number of independent unknown parameters. Both for Binomial and Poisson distributions $r = 1$. Hence, the degrees of freedom of χ^2 would be $k-r-1 = k-2$. For normal distribution there are two parameters (μ, σ) and hence $r = 2$ and hence $df = k-r-1 = k-3$.

Example : 17.37. A die was thrown 90 times with the following results :

Face :	1	2	3	4	5	6	Total
Frequency :	10	12	16	14	18	20	90

Are these data consistent with the hypothesis that the die is unbiased ?

Solution : Here we have to test the null hypothesis H_0 : the die is unbiased against the alternative, H_1 : the die is biased. The sample size $n = 90$ (large) and the observed frequencies (O_i) are given. The expected frequencies in each case would be $E_i = np_i = 90 \times 1/6 = 15$.

[When an unbiased die is thrown, out of 6 faces any one face may occur with equal probability $p_i = 1/6$]

Now, to test H_0 : the die is unbiased against the alternative H_1 : the die is biased the appropriate test statistic would be, $\chi^2 = \sum_i^k \frac{(O_i - E_i)^2}{E_i}$ with $df = k-1$. Here $k = 6$, as there are 6 faces.

Now, for the given sample H_0 will be rejected at $100\alpha\%$ level of significance if χ^2_{k-1} (observed) > $\chi^2_{\alpha,k-1}$ (table value) and will be accepted otherwise. Here observed $\chi^2_{6-1} = \chi^2_5 = \sum_{i=1}^{6} \frac{(O_i - E_i)^2}{E_i} = \frac{70}{15} = 4.67$. and $\chi^2_{\alpha,k-1} = \chi^2_{\alpha,5}$.

Calculations for χ^2

Event (A_i)	Observed frequency (O_i)	Expected frequency (E_i)	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
$A_1 = 1$	10	15	25	25/15
$A_2 = 2$	12	15	9	9/15
$A_3 = 3$	16	15	1	1/15
$A_4 = 4$	14	15	1	1/15
$A_5 = 5$	18	15	9	9/15
$A_6 = 6$	20	15	25	25/15
Total				$70/15 = \Sigma(O_i - E_i)^2/E_i$

When $= 0.01$, $\chi^2_{0.01,5} = 15.086$ and when $\alpha == 0.05$, $\chi^2_{0.05,5} = 11.070$.

Thus, we see that χ^2 (observed) < tabulated value of χ^2 both at 1% and 5% levels of significance and hence we accept H_0 and conclude that the die is unbiased.

Example : 17.38. The following mistakes per page were observed in a book :

No. of mistakes per page (x) :	0	1	2	3	4	Total
No. of pages (f) :	211	90	19	5	0	$325 = n$

Fit a Poisson distribution and test the goodness of fit. [Given $e^{-0.44} = 0.644$]

Solution : Let x be a random variable denoting the number of mistakes per page. According to the Poisson model, $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ where λ is the parameter of the distribution. The proper estimate of λ, the population mean, is the sample mean $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{\Sigma fx}{n}$

$$= \frac{0\times211 + 1\times90 + 2\times19 + 3\times5 + 4\times0}{325}$$

$$= \frac{0+90+38+15+0}{325} = \frac{143}{325} = 0.44.$$

Now, the expected frequency of x mistakes per page is given by Poisson law :

$$f(x) = n.e^{-\lambda}\frac{\lambda^x}{x!} = \frac{325\times e^{-0.44}\times(0.44)^x}{x!} \text{ where } x = 0, 1, 2, 3, 4.$$

When $x = 0$, $f(0) = \frac{325\times0.644\times1}{0!} = 209.3$

$$x = 1, f(1) = \frac{325 \times 0.644 \times (0.44)}{1!} = 92.1$$

$$x = 2, f(2) = \frac{325 \times 0.644 \times (0.44)^2}{2!} = 20.3$$

When $x = 3, f(3) = \dfrac{325 \times 0.644 \times (0.44)^3}{3!} = 3.0$

$$x = 4, f(4) = 325 \times \frac{0.644 \times (0.44)^4}{4!} = 0.30.$$

Calculation of χ^2

Observed frequency (O_i)	Expected frequency (E_i)	$(O_i - E_i)^2$	$(O_i - E_i)^2/E_i$
211	209.3	2.89	0.0138
90	92.1	–2.1	0.0479
$\left.\begin{matrix}19\\5\\0\end{matrix}\right] = 24$	$\left.\begin{matrix}20.3\\3.0\\0.3\end{matrix}\right] = 23.6$	0.16	0.0068
Total 325	—	—	$\sum_i (O_i - E_i)^2/E_i = 0.0685.$

$$\therefore \chi^2_{k-2} = \chi^2_{3-2} = \chi^2_1 = \sum_i \frac{(O_i - E_i)^2}{E_i} = 0.0685.$$

[Here the last 3 classes have been combined into a single class to ensure that the expected frequency is not smaller than 5.] Hence, we have three classes *i.e.*, $k = 3$. Now, on the basis of the observed data the distribution is said to be a good fit if χ^2_{k-2} (observed) $< \chi^2_{\alpha, k-2}$ *i.e.*, when the null hypothesis H_0 : Frequencies correspond to a Poisson distribution against the alternative, H_1 : Frequencies do not correspond to a Poisson distribution is accepted. Here observed $\chi^2_{k-2} = \chi^2_1 = 0.0685$ and when $\alpha = 0.01$, $\chi^2_{0.01} = 6.635$. When $\alpha = 0.05$, $\chi^2_{0.05,1} =$ 3.841. Thus, we see that H_0 is accepted both at 1% and 5% levels of significance. Hence, fitting by a Poisson distribution seems to be quite satisfactory.

...ing by a Poisson distribution seems to be quite satisfactory.

Example : 17.39. A survey of 320 families with 5 children each revealed the following distribution.

No. of boys :	5	4	3	2	1	0
No. of girls :	0	1	2	3	4	5
No. of families :	14	56	110	88	40	12

Is the result consistent with the hypothesis that male and female births are equally probable ? [Delhi. U.M. Com 1975, B.U. M. Com. 1981]

[or Fit a binomial distribution and test the goodness of fit.]

Solution : We have to fit a binomial distribution and also we have to check the goodness of fit. If the distribution is a good fit on the basis of observed data then we can conclude that male and female births are equally probable. We have to test the null hypothesis H_0 : male and female births are equally probable *i.e.*, $p = q = 1/2$ against the alternative, H_1 : male and female births are not equally probable.

The expected frequency x, of female birth is given by,

$f(x) = n. {}^mc_x p^x q^{m-x}$ where $x = = 0, 1, 2, 3, 4, 5$ and $n = 320$ [where ${}^mc_x p^x q^{m-x}$ is the *p.m.f.* of binomial distribution]. Thus, expected frequencies are

$$f(0) = 320 \times {}^5C_0 \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^{5-0} = 320 \times 1 \times 1 \times \frac{1}{32} = 10$$

$$f(1) = 320 \times {}^5c_1 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^{5-1} = 320 \times 5 \times \frac{1}{32} = 50$$

$$f(2) = 320 \times {}^5c_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{5-2} = 320 \times 10 \times \frac{1}{32} = 100$$

$$f(3) = 320 \times {}^5C_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^{5-3} = 100$$

$$f(4) = 320 \times {}^5c_4 \times \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^{5-4} = 320 \times 5 \times \frac{1}{32} = 50$$

$$f(5) = 320 \times {}^5C_5 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^{5-5} = 320 \times 1 \times \frac{1}{32} = 10$$

Now, the expected frequencies of female births are :

x :	0	1	2	3	4	5
$f(x)$:	10	50	100	100	50	10

Let us now apply χ^2 test to examine the goodness of fit of the given data to the above binomial distribution. Now,

$\chi^2_{k-2} = \sum_i (O_i - E_i)^2/E_i$, where $O_i =$ observed frequency of the ith class, E_i = expected frequency of the ith class and k = number of classes. Here $k = 6$.

Calculations for χ^2

Observed frequency (O_i)	Expected frequency (E_i)	$(O_i - E_i)^2$	$(O_i - E_i)^2/E_i$
14	10	16	1.60
56	50	36	0.72
110	100	100	1.00
88	100	144	1.44
40	50	100	2.00
12	10	4	0.40
Total	—	—	$\sum_i (O_i - E_i)^2/E_i = 7.16$

$$\therefore \chi^2_{k-2} = \chi^2_{6-2} = \chi^2_4 = \frac{\sum_i (O_i - E_i)^2}{E_i} = 7.16$$

If now $\alpha = 0.01$, $\chi^2_{\alpha, k-2} = \chi^2_{0.05, 4} = 13.277$

and if $\alpha = 0.05$, $\chi^2_{\alpha, k-2} = \chi^2_{0.01, 4} = 14.860$.

Now, H_0 is accepted both at 1% and 5% levels of significance as χ^2 (observed) < $\chi^2_{0.01, 4}$ and $\chi^2_{0.05, 4}$. Thus, we may conclude that both at 1% and 5% level of significance we accept the null hypothesis and hence the male and female births are equally probable.

.Test for Independence

3. We now consider a 2×2 table (*i.e.*, when $k = l = 2$) with observed cell frequencies as follows (Table 17.8).

Table 17.8

Row \ Column	1	2	Total
1	a	b	$a + b$
2	c	d	$c + d$
Total	$a + c$	$b + d$	$n = a + b + c + d$

$$\text{Now, } \chi^2 = \frac{\left(a - \dfrac{a+b)(a+c)}{a+b+c+d}\right)^2}{\dfrac{(a+b)(a+c)}{a+b+c+d}} + \frac{\left(b - \dfrac{(a+b)(b+d)}{a+b+c+d}\right)^2}{\dfrac{(a+b)(b+d)}{a+b+c+d}}$$

$$+ \frac{\left(c - \dfrac{(c+d)(a+c)}{a+b+c+d}\right)^2}{\dfrac{(c+d)(a+c)}{a+b+c+d}} + \frac{\left(d - \dfrac{(c+d)(b+d)}{a+b+c+d}\right)^2}{\dfrac{(c+d)(b+d)}{a+b+c+d}}$$

$$= \frac{(ad - bc)^2}{a+b+c+d}\left(\frac{1}{(a+b)(a+c)} + \frac{1}{(a+b)(b+d)} + \frac{1}{(c+d)(a+c)} + \frac{1}{(c+d)(b+d)}\right)$$

$$= \frac{(ad-bc)^2 (a+b+c+d)}{(a+b)(c+d)(a+c)(b+d)} = \frac{n(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

This is distributed as a χ^2 with $df = 1$.

17.32. Yates' Continuity Correction

Since we know that for the validity of χ^2 approximation it is necessary that the expected frequency in each class should be sufficiently large (at least 5). When some expected frequency is too small (*i.e.*, < 5), it is needed to coalesce that class with some neighbouring class or classes to satisfy the condition. Clearly this is not possible in the case of (2 × 2) table, if a cell frequency is less than 5.

Yates' has suggested a method in this case when a cell frequency is < 5. Let us take a 2 x 2 table (Table 17.8).

If $ad > bc$. a and d are replaced by $a - 1/2$ and $d - 1/2$ and b and c are replaced by $b + 1/2$ and $c + 1/2$. But if $ad < bc$, then a, d are increased by 1/2 and b, c are decreased by 1/2.

After Yates' correction, the formula for χ^2 becomes,

$$\chi^2 = \frac{n[(a \pm 1/2)(d \pm 1/2) - (b \pm 1/2)(c \pm 1/2)]^2}{(a+b)(c+d)(a+c)(b+d)}.$$

Simplifying we get,

$$\chi^2 = \frac{n[|ad-bc| - n/2]^2}{(a+b)(c+d)(a+c)(b+d)}$$

This χ^2 has $df = 1$.

A Simple Method to Test for Independence of Two Attributes

when the observations are classified according to two attributes and the observed frequencies O_i in the different categories are shown in a two-way table, called *contingency table*. Now we want to test on the basis of the cell frequencies whether the two attributes are independent or not.

Under the null hypothesis H_0 that the two attributes are independent, the expected frequencies (E_i) of any cell

$$= \frac{\text{Row total} \times \text{Column total}}{\text{Grand total}}$$

The test statistic $\chi^2 = \sum_i (O_i - E_i)^2/E_i$ approximately follows a χ^2 distribution with df = (No. of rows – 1) × (No. of columns – 1) = $(k-1)(l-1)$.

If now χ^2 (observed) > $\chi^2_{\alpha,(k-1)(l-1)}$ then we reject the null hypothesis at $100\alpha\%$ level of significance and we accept H_0 otherwise.

Example : 17.40. The number of defectives in random samples drawn from lots supplied by four producers A,B,C,D are given in the following table.

Producer :	I	II	III	IV
Sample size :	100	200	150	250
Number of defectives :	20	35	37	43

Do you think that there is no difference among the producers as regards quality of the articles ?

Solution : Let p_1, p_2, p_3, p_4 represent the proportions of defectives in the lots supplied by the producers I, II, III and IV. Now, we like to test the null hypothesis $H_0 : p_1 = p_2 = p_3 = p_4$ against the alternative, $H_1 : p_1, p_2, p_3, p_4$ are not all equal. This is equivalent to testing the homogeneity of lots supplied by I, II, III, IV from the point of classification into defectives against non homogeneity.

Now, to test homogeneity the appropriate test statistic would be,

$$\chi^2_{(k-1)} = \frac{n^2}{T_a T_b}\left(\sum_{i=1}^{k}\frac{a_i^2}{T_i} - \frac{T_a^2}{n}\right), \text{(from simplified formula of } \chi^2 \text{)}.$$

In the case of $(k \times 2)$ way table. T_a = Total of first column, T_b = Total of second column, n = total frequency. Calculations are shown in the following table.

Name of items / Producer	No of defectives (1)	No. of non defectives (2)	Total
I	$a_1 = 20$	$b_1 = 80$	$T_1 = 100$
II	$a_2 = 35$	$b_2 = 165$	$T_2 = 200$
III	$a_3 = 37$	$b_3 = 113$	$T_3 = 150$
IV	$a_4 = 43$	$b_4 = 207$	$T_4 = 250$
Total	$T_a = 135$	$T_b = 565$	$n = 700$

Now, under H_0

$$\chi^2 = \frac{n^2}{T_a \times T_b}\left[\frac{\sum_{i=1}^{k} a_i^2}{T_i} - \frac{T_a^2}{n}\right] = \frac{(700)^2}{135 \times 565}\left[\frac{20^2}{100} + \frac{35^2}{200} + \frac{37^2}{150} + \frac{43^2}{250}\right]$$

$= 3.931$ with $df = k - 1 = 4 - 1 = 3$ (Here number of rows $= k = 4$).

Now, for the given sample H_0 will be rejected at $100\alpha\%$ level of significance if χ^2 (observed) $> \chi^2_{\alpha, k-1}$.

When $\alpha = 0.05$, then $\chi^2_{\alpha, k-1} = \chi^2_{0.05, 3} = 7.815$. Thus, we see that χ^2 (observed) $<$ $\chi^2_{\alpha, k-1}$ at 5% level of significance and hence at 5% level of significance H_0 is accepted. Hence, there does not seem to exist any real difference in quality of articles supplied by the four producers.

Example : 17.41 D...

four producers

Example : 1 . During a small pox epidemic the following data were collected on the basis of a survey of 222 persons vaccinated against the disease. Do you think that the standard vaccination affects the power to resist the disease?

	Attacked with small pox	Not attacked	Total
Well vaccinated	33	120	153
Badly vaccinated	18	51	69
Total	51	171	222

Solution : We have to test the null hypothesis H_0 : Attack of the disease is independent of vaccination against the alternative,

H_1 : Attack of the disease is dependent on vaccination.

Now, under H_0 the appropriate test statistic is,

$$\chi^2 = \frac{(ad-bc)^2 \times n}{(a+b)(c+d)(a+c)(b+d)} \text{ with } df = 1.$$

We have the following 2 × 2 table.

	Attacked with small pox	Not attacked	Total
Well vaccinated	$33\ (=a)$	$120\ (=b)$	$153\ (=a+b)$
Badly vaccinated	$18\ (=c)$	$51\ (=d)$	$69 = (c+d)$
Total	$51 = (a+c)$	$171 = (b+d)$	$222 = n$

Now, $\chi^2 = \dfrac{n(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$

$$= \frac{222\times(33\times51-18\times120)^2}{153\times51\times171\times69} = \frac{50511438}{92067597} = 0.5486.$$

Now, H_0 will be rejected for the given sample at $100\alpha\%$ level of significance if χ^2 (observed) $> \chi_{\alpha.1}$.

When $\alpha = 0.05$, $\chi^2_{\alpha.1} = \chi^2_{0.05,1} = 3.841$ and when $\alpha = 0.01$, $\chi^2_{\alpha.1} = \chi^2_{0.01,1} = 6.635$.

Here we see that both at 1% and 5% levels of significance H_0 is accepted. Thus, we may conclude that the attack of the disease is independent of vaccination.

Example : 17.42. 1,072 school boys were classified according to intelligence, and at the same time their economic conditions were recorded. The results are shown in the following table. Judge whether there is any association between intelligence and economic conditions.

Economic Conditions	Intelligence				Total
	Excellent	Good	Mediocre	Dull	
Good	48	199	181	82	510
Not good	81	185	190	106	562
Total	129	384	371	188	1072

Solution : Here the two attributes are A = Economic conditions and B = Intelligence. There are 2 classes of A and 4 classes of B. On the basis of the given cell frequencies and total frequency we have to test whether the two attributes are related or not (independent). We have to test the null hypothesis H_0 : A and B are independent against the alternative, H_1 : A and B are associated.

Under H_0, the appropriate test statistic would be,

$$\chi^2 = n\left[\sum_i\sum_j \frac{(f_{ij})^2}{f_{i0}\times f_{0j}} - 1\right]$$

with $df = (k-1)(l-1)$. Here $k = 2, l = 4$. For the given samples, f_{ij} and the products $f_{io} \times f_{oj}$ are shown in the table below, together with $(f_{ij})^2 / f_{io} \times f_{oj}$

(1) Observed frequency (f_{ij})	(2) Row total × Col. total ($f_{io} \times f_{oj}$)	(3) = (1)²/(2) $\left[\frac{(f_{ij})^2}{f_{io} \times f_{oj}}\right]$
48	65790	0.0350205
199	195840	0.2022109
181	189210	0.1731462
82	95880	0.0701293
81	72498	0.0904990
185	215808	0.1585900
190	208502	0.1731398
106	105656	0.1063451
Total 1072 = $\sum_i \sum_j f_{ij} = n$		1.0090808 $= \sum_i \sum_j (f_{ij})^2 / f_{io} \times f_{oj}$

$$\therefore \chi^2 = n\left[\sum_i \sum_i \frac{(f_{ij})^2}{f_{io} \times f_{oj}} - 1\right]$$

$= 1072\,[1.0090808 - 1] = 9.73439 = 9.735$ (approx) with $df = (2-1) \times (4-1) = 3$.

Now, on the basis of the sample H_0 will be rejected at $100 \times \alpha\,\%$ level of significance if χ^2 (observed) $> \chi^2_{\alpha,(k-1)(l-1)}$ and will be accepted otherwise. When $\alpha = 0.01$, $\chi^2_{\alpha,(k-1)(l-1)} = \chi^2_{0.01,3} = 11.345$.

When $\alpha = 0.05$, $\chi^2_{\alpha,(k-1)(l-1)} = \chi^2_{0.01,3} = 7.815$. Thus, we see that H_0 is accepted at 1% but rejected at 5% level of significance and hence we may conclude that the two attributes in the population are not associated significantly at 1% level but they are said to be associated at 5% level.