Probability and Stochastic Processes

1. Basic Probability
    - Introduction to sample spaces, classical definition of probability
    - Countable probability space
    - Counting methods using combinatorial tools
    - Conditional probability, Independence of events, Bayes theorem
2. Discrete Random Variables
    - Probability mass function
    - Expectation
    - Examples including Binomial, Poisson, Geometric etc.
3. Continuous random variables
    - Probability density function
    - Cumulative distribution function
    - Expectation
    - Examples including Uniform, Exponential, Normal etc.
4. Joint distribution
    - Joint PMF for discrete random variables
    - Joint density and CDF
    - Independence of random variables
    - Conditional distribution and expectation
    - Covariance and correlation
5. Limit theorems
    - Characteristic function
    - Lévy continuity theorem (at least the statement)
    - Central limit theorem
    - Weak law of large numbers for finite variance
    - Borel-Cantelli lemma
    - Strong law of large numbers for finite fourth moment

6. Random walks
    - Reflection principle
    - Distribution of the maximum
    - Ballot theorem
    - Arc sine law
7. Markov chains
    - Definition and examples
    - Decomposition of states into communicating classes
    - Recurrence and transience as class properties
    - Invariant distributions for finite irreducible chains
    - Expected return time
    - Convergence of finite irreducible aperiodic chains to equilibrium
8. Poisson processes
    - Definition and basic properties
    - Conditional distribution of arrival times given the number of arrivals
    - Inhomogeneous Poisson processes