3.2) Ex-3

a) Type-I censoring time - 60 years (right)
Left truncated at 30.

b) It is interval censoring and the time is $(40+12, 40+15)$
Left tr. at $= 40$
$= (52, 55)$

c) Random censoring (year of ~~th~~ censoring is choosen at random)
Left trunc at $= 50$

d) right censoring time 55
Left truncated at 42

e) ~~tex~~

$$L \propto \frac{S(60)}{S(30)} \times \frac{S(52)-S(55)}{S(40)} \times \frac{S(61)}{S(42)} \times \frac{S(55)}{S(50)}$$

c d

3.3) a) Left censored at 42 day

b) Right (Type-I) censoring at 140

c) Interval censoring at (84-91) days

d) Right (random) censoring at 37 days

e) $L \propto (1-S(42))(S(140))\left[S(84)-S(91)\right] S(37)$

3.8) f has log-logistic distribution.

$$f(x:\beta,\lambda) = \frac{\beta\lambda(\lambda x)^{\beta-1}}{[(1+(\lambda x)^{\beta}]^2}$$

$$S(x) = \frac{1}{1+(\lambda x)^{\beta}}$$

we know,

$$L \propto \prod f_i(x_i) \prod S_i(c_r) \prod S_i(c_i) \prod (1-S_i(c_r)) \prod (1-S_i(c_i))$$

$$\therefore L \propto \frac{\beta\lambda(\lambda \cdot 0.5)^{\beta-1}}{([1+(\lambda\cdot 0.5)^{\beta}]^2} \cdot \frac{\beta\lambda(\lambda)^{\beta-1}}{[1+\lambda^{\beta}]^2}$$

$$\frac{1}{(1+(0.75\lambda)^{\beta}} \cdot \frac{1}{1+(1.5\lambda)^{\beta}}$$

$$[\quad]$$

3.07)

a) $(55,56] \rightarrow$ right censored (interval)

$(58,59] \rightarrow$ " "

$(52,53] \rightarrow$ " "

$(59,60] \rightarrow$ " "

$60 \geq \longrightarrow$ Type-I (right)

b) $L = \left[e^{-(55\lambda)^{\beta}} - e^{-(56\lambda)^{\beta}}\right]\left[e^{-(58\lambda)^{\beta}} - e^{-(59\lambda)^{\beta}}\right]$

$\left[e^{-(59\lambda)^{\beta}} - e^{-(60)^{\beta}}\right]\left[e^{-(\lambda 60)^{\beta}}\right]^{2}$

3.8) (a) $P(\delta=1) = P(X \leq C)$

$= \int_0^{\infty} P(X \leq C \mid C=t)\, \theta e^{-\theta t} dt$

$= \int_0^{\infty} P(X \leq t \mid C=t)\, \theta e^{-\theta t} dt$

$= \int_0^{\infty} (1-e^{-\lambda t})\, \theta e^{-\theta t} dt$

$= \int_0^{\infty} \theta e^{-\theta t} - \theta \int e^{-(\lambda+\theta)t} dt$

$= \frac{\theta}{\theta}\left(\frac{e^{-\theta\lambda}}{-1}\right)_0^{\infty} - \frac{\theta}{-(\lambda+\theta)}\left(e^{-(\lambda+\theta)\lambda}\right)_0^{\infty}$

$= 1 - \frac{\theta}{\lambda+\theta}$

$= \frac{\lambda}{\lambda+\theta}$

b) $P(T > t) = S(t)$

$= P(\min(X, C) > t)$

$= P(X > t, C > t)$

$= P(X > t)\,P(C > t)$

$= (1 - P(X < t))(1 - P(C < t))$

$= [1 - \{1 - e^{-\lambda t}\}][1 - \{1 - e^{-\theta t}\}]$

$= e^{-(\lambda + \theta)t}$

We need to show,

c) $P(\delta = 1, T > t) = P(\delta = 1)P(T > t)$

$P(\delta = 0, T > t) = P(\delta = 0)P(T > t)$

now, $P(\delta = 1, T > t)$

$= P(X \le C, \cancel{X > t, T > t})$ $\min(X, C) > t$

$= P(X \le C, X > t, C > t)$

$= \int_t^\infty P(X \le C, X > t \mid C = s)\,\theta e^{-\theta s}\,ds$

$= \int_t^\infty [e^{-\lambda t} - e^{-\lambda s}]\,\theta e^{-\theta s}\,ds$

$= e^{-\lambda t} \cdot e^{-\theta t} - \frac{\theta}{\theta + \lambda} e^{-(\theta + \lambda)t}$

$= e^{-(\theta + t)t}\left[1 - \frac{\theta}{\theta + \lambda}\right]$

$= e^{--}\left[\frac{\lambda}{\theta + \lambda}\right] = P(T > t)P(\delta = 1)$

now,

$$P(\delta = 0, T \geq t) = P(X \geq C, \min(X, C) \geq t)$$

$$= \int_t^{\infty} P(X > s, X > t \mid C = s)\, \theta e^{-\theta s}\, ds$$

$$= \int_t^{\infty} e^{-\lambda s} \cdot \theta \cdot e^{-\theta \cdot s}\, ds$$

$$= \theta \int_t^{\infty} e^{-s(\lambda+\theta)}\, ds$$

$$= \frac{\theta}{\lambda + \theta} \left[\frac{e^{-s(\lambda+\theta)}}{-1} \right]_t^{\infty}$$

$$= \frac{\theta}{\lambda+\theta} \left[e^{-t(\lambda+\theta)} - 0 \right]$$

$$= e^{-(\theta+\lambda)t} \cdot \frac{\theta}{\theta+\lambda}$$

$$= \cancel{P(\delta = 0)}$$

$$= P(T \geq t)\, P(\delta = 0)$$

$$\left[\because P(\delta = 0) = 1 - P(\delta = 1) = 1 - \frac{\lambda}{\theta + \lambda} = \frac{\theta}{\lambda + \theta} \right]$$

d)

$$L = \prod_{i=1}^{n} f_X(T_i)^{\delta_i} [S_C(T_i)]^{\delta_i} [f_C(T_i)]^{1-\delta_i} [S_X(T_i)]^{1-\delta_i}$$

$$= \prod_{i=1}^{n} \left(\lambda e^{-\lambda T_i} e^{-\theta T_i}\right)^{\delta_i} \left(\theta e^{-\theta T_i} e^{-\lambda T_i}\right)^{1-\delta_i}$$

3.6)

$f(x) = \lambda e^{-\lambda x}$

$S(x) = e^{-\lambda x}$

(a)

$$L = \underbrace{\lambda^6 \cdot e^{-\lambda(98)}}_{\text{survive}} \cdot \underbrace{e^{-\lambda(82)}}_{\text{censored}}$$

$$(\log L) = 6 \log \lambda - 180\lambda$$

$$\frac{\partial(\log L)}{\partial \lambda} = \frac{6}{\lambda} - 180$$

$$\Rightarrow \lambda = \frac{6}{180} = \frac{1}{30}$$

(b) Now for deaths,

$$L = \lambda^4 e^{-\lambda(11+12+15+45)} e^{-\lambda(33+28+16+17+19+30)}$$

$$= \lambda^4 e^{-83\lambda} \cdot e^{-143\lambda}$$

$$\log L = 4 \log \lambda - 83\lambda - 143\lambda$$

$$\frac{\partial(\log L)}{\partial \lambda} = 0$$

$$\Rightarrow \frac{4}{\lambda} - 226 = 0$$

$$\Rightarrow \lambda = \frac{4}{226} = \frac{2}{113}$$ [Ans]

now λ independent terms are redundant

$$\therefore L = \prod_{i=1}^{n} \lambda^{\delta_i} \cdot (e^{-\lambda T_i}) \left[\cdot \text{---} \right]$$
of λ-inde

$$\log L = \sum \delta_i \log \lambda - \lambda \sum T_i$$

$$\frac{\partial (\log L)}{\partial \lambda} = \frac{\sum \delta_i}{\lambda} - \sum T_i = 0$$

$$\Rightarrow \hat{\lambda} = \frac{\sum \delta_i}{\sum T_i}$$

3.4) from 1.2.

$f(x, \lambda) = \lambda e^{-\lambda x}$

$S(x) = e^{-\lambda x}$

now here censored = {32+, 34+, 32+
25+, 11+, 20+, 19+
17+, 35+, 9+, 6+, 10+}

not {10, 7, 23, 22, 6, 16, 6, 6
13.}

$$L = \lambda^{12} \cdot e^{-\lambda(109)} e^{-\lambda(318)}$$

$$\log L = 12 \log \lambda - 109\lambda - 318\lambda$$

$$\frac{\partial (\log L)}{\partial \lambda} = \frac{12}{\lambda} - 427 = 0 \quad \Rightarrow \lambda = \frac{12}{427}$$