



Ramakrishna Mission Vivekananda Educational and Research Institute

PO Belur Math, Howrah, West Bengal 711 202

School of Mathematical Sciences

Department of Computer Science

MSc BDA : Batch 2020-22, Semester II, Mid-Semester Exam

DA310: Multivariate Statistics

Dr. Sudipta Das

Student Name (in block letters):

Date: 03 April 2021

Student Roll No:

Max Marks: 50

Signature:

Time: 1½ hrs

Answers must be properly justified to deserve full credits.

1. (14 points) Let X and Y be r.v.s with the joint density given by

$$f(x, y) = \frac{1}{8\sqrt{3}\pi} \exp\left(-\frac{x^2}{6} - \frac{y^2}{24} + \frac{xy}{12} + \frac{x}{12} + \frac{y}{6} - \frac{7}{24}\right)$$

(a) (8 points) Find the means and variances of X and Y . Also, find the covariance between X and Y .

(b) (6 points) Find the conditional density of Y given $X = x$. Also, calculate $E[Y|x]$.

2. (12 points)

Let X_1 be $N(0, 1)$, and let

$$X_2 = \begin{cases} -X_1 & \text{if } -1 \leq X_1 \leq 1 \\ X_1 & \text{otherwise.} \end{cases}$$

(a) (6 points) Show that $X_2 \sim N(0, 1)$

(b) (6 points) Does $[X_1, X_2]'$ follow a bivariate normal distribution? Explain.

3. (12 points)

The sample mean vector and the sample covariance matrix, as given below, are calculated from pairs of 40 observations.

$$\bar{x} = \begin{bmatrix} 0.64 \\ 0.52 \end{bmatrix}, \text{ and } S = \begin{bmatrix} 0.14 & 0.10 \\ 0.10 & 0.16 \end{bmatrix}.$$

Compare the 95% T^2 and 95% Bonferroni simultaneous confidence intervals.

4. (12 points)

Find the directions of the principal components and the proportion of the total population variance explained by each when the correlation matrix is

$$\rho = \begin{bmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{bmatrix}.$$

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You may need following values:

$$t_{41}(0.025) = 2.020, t_{41}(0.0125) = 2.327, t_{40}(0.025) = 2.021$$

$$t_{40}(0.0125) = 2.323, t_{39}(0.025) = 2.022, t_{39}(0.0125) = 2.331,$$

$$F_{2,38}(0.025) = 4.071, F_{40,2}(0.025) = 39.472, F_{2,38}(0.05) = 3.245, F_{38,2}(0.05) = 19.469$$

$$F_{2,40}(0.025) = 4.051, F_{40,2}(0.025) = 39.473, F_{2,40}(0.05) = 3.232, F_{40,2}(0.05) = 19.471$$

This exam has total 4 questions, for a total of 50 points and 0 bonus points.