Department of Computer Science

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Optimization for Machine Learning, BDA 2022 batch

Problem set on Constrained Optimization

A. Problems on KKT Optimality conditions

Chapter 4 , *Nonlinear Programming* by Sherali and Shetty,

Problems: 4.1,4.4 (a,b, d), 4.5, 4.7, 4.10, 4.37 (a,b), 4.38, 4.39

Additional problem: Consider the optimization problem

$$Min\, f(x_1, x_2)\text{ , s.t., , } 2x_1 + x_2 \geq 1,\ x_1 + 3x_2 \geq 1,\ x_1 \geq 0, x_2 \geq 0$$

Make a sketch of the feasible set. For each of the following objective functions, find (graphically) the optimal solution and optimal objective value

a) $f(x_1, x_2) = x_1 + x_2$
b) $f(x_1, x_2) = -x_1 - x_2$
c) $f(x_1, x_2) = x_1$
d) $f(x_1, x_2) = \{x_1, x_2\}$
e) $f(x_1, x_2) = x_1^2 + 9x_2^2$

B. Problems on Lagrangian Duality

Chapter 6, *Nonlinear Programming* by Bazaraa, Sherali and Shetty

Problem 6.10 (a,b), 6.11 (a,b), 6.16, 6.17, 6.26

***Additional problems*:**

1. Consider the problem

$$\text{minimize} f(x) = x_1^2 + x_2^2$$

$$\text{subject to: } (x_1 - 1)^2 + (x_2 - 1)^2 \leq 1, (x_1 - 1)^2 + (x_2 + 1)^2 \leq 1,\ x \in \mathbb{R}^2$$

   (a) Sketch the feasible set and level sets of the objective function. Find optimal point $x^\star$ and optimal value $f(x^*)$.
   (b) Give the KKT conditions. Do there exist Lagrange multipliers λ_1 and λ_2 that prove that $x^\star$ is optimal?
   (c) Derive and solve the Lagrange dual problem. Does strong duality hold?

2. Consider the optimization problem

$$\text{minimize} f(x) = x^2 + 1$$

$$\text{subject to: } (x - 2)(x - 4) \leq 0,\ x \in \mathbb{R}$$

   (a) Plot the objective f(x) versus x. On the same plot, show the feasible set, optimal point and value, and plot the Lagrangian L(x, λ) versus x for a few positive values of λ. Verify the Weak Duality theorem. Derive and sketch the Lagrange dual function
   (b) State the Lagrangian dual problem, and verify that it is a concave maximization problem. Find the dual optimal value and dual optimal solution $\lambda^\star$ Does strong duality hold?