(1) A random sample of size 10, drawn from a normal population, constitute the observations 62, 63, 64, 65, 67, 67, 68, 69, 70, 72. Test the that the population standard deviation is 2 when

i) the population mean is known to be 66

ii) the population mean is unknown.

ans) To test $H_0 : \sigma = 2$ (or $\sigma^2 = 4$)

against $H_{11} : \sigma > 2$

$H_{12} : \sigma < 2$

$H_{13} : \sigma \neq 2$

population mean unknown then sample mean

$$\bar{x} = \frac{1}{10}(62+63+64+65+67+67+68+69+70+72)$$

$$= 60$$

$$\chi_s^2 = \sum\left(\frac{x_i - \mu}{\sigma_0}\right)^2$$

$$= \frac{1}{\sigma_0^2}\sum(x_i-\mu)^2$$

$$= \frac{1}{2^2}\left[(4)^2+(3)^2+(2)^2+(1)^2+(1)^2+(2)^2+(3)^2+(4)^2+(6)^2\right]$$

$$= \frac{1}{4}(96) = 24$$

$\alpha = 0.05$

$\chi^2_{\alpha, n}$ $\quad \chi^2_{0.05, 10} = 18.307$

$24 > 18.307$

reject the null hypothesis against H_{11}

$\chi^2_{1-0.05, 10} = 3.940$

$24 \not< 3.940$

can not reject the null hypothesis against H_{12}

(ii) $\chi_0^2 = \sum\left(\frac{x_i - \bar{x}}{\sigma_0}\right)^2$

$= \frac{1}{4}\left[(2)^2+(3)^2+(4)^2+(5)^2+(7)^2+(8)^2+(9)^2+(10)^2+(12)^2\right]$

$= 123$

$\chi^2_{0.05,9} = 16.919$

$123 > 16.919$

$\chi^2_{0.95,9} = 3.25$

$123 > 3.25$

so we can not reject the null hypothesis against H_1.

(2)

$\mu = 800$

$\sigma = 40$

$n = 40$

$\bar{x} = 825$

$S_0 = 48.5$

$\alpha = 0.05$

taking unbiased estimation

$\frac{(n-1)s^2}{\sigma^2}$

$= 39 \times \frac{(48.5)^2}{(40)^2}$

$= 57.33$

$\chi^2_{0.05,39} =$

if we take bias estimation

$\frac{nS_0^2}{\sigma^2} = \frac{40 \times (48.5)^2}{(40)^2}$

$= 58.80$

(1) ZCQ scores as a function of dosage.

Z_0	57	27	32	31	34	8	71	33	34	53	36	42	26	52	36	55
Z_{60}	62	49	30	34	38	56	77	51	45	42	43	57	36	58	35	60
diff	5	22	-1	3	4	-2	6	18	11	11	7	15	10	6	1	

36	54	34	29	33	33
33	59	35	37	45	29
-3	5	1	8	12	-4

(a) $H_0 : \mu = 0$;

$H_{11} : \mu > 0$;

$H_{12} : \mu < 0$;

α	0.05	0.025	0.01	0.005
Z_α	1.645	1.96	2.32	2.576

→ the test statistic $ZS = \left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right)$

$$\frac{\sqrt{n}(\bar{x} - \mu)}{\sigma} \qquad \alpha = 0.05$$

$$= \frac{\sqrt{22}(5.54)}{8}$$

$$= 3.064 > Z_\alpha = 1.96$$

$-Z_\alpha < ZS$ reject the null hypothesis against H_{11}

$-1.96 < 3.064$. Can not reject the null hypothesis against H_{12}

(2) When σ is unknown

$ZS = \sqrt{n}\left(\frac{\bar{x} - \mu}{s}\right)$ $\qquad s = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$

$= \frac{1}{21} [(0.54)^2 +$

]

(2)

frequency	45	48	49	49	51	52	53	55	57

$H_0: \mu = 50$

$H_1: \mu \neq 50$

$H_{12}: \mu < 50$

$H_{13}: \mu > 50$

Let, $\alpha = 0.05$

(i) $t_s = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s}$

$\bar{x} = \frac{45+48+49+49+51+52+53+55+57}{9}$

$= 51$

$t_s = \frac{\sqrt{9}(51-50)}{13.75}$

$s^2 = \frac{1}{8}\left\{6^2+3^2+2^2+2^2+1^2+2^2+4^2+6^2\right\}$

$t_s = \cancel{}\ 1.2360$

$= \frac{110}{8}$

$s^2 = 13.75$

$t \sim t_{\alpha, n-1}$

$t_s = \cancel{}\ 1.2360 \ngtr 1.860$

So, we fails to reject the null hypothesis against H_1

(ii)

$\mu < 50$

$t_s = \cancel{}\ 1.2360$

$t_{-0.05}$

here $\cancel{}\ 1.2360 \nless -1.860 = -T_{0.05}$

$-1.860 < 0.2181$

We fails to reject the null hypothesis against the

$\boxed{\text{Or, } > 2 \\ T.S > \text{tablevalue} \\ \downarrow \\ \text{reject the nullhypo}}$

(iii) $\mu \neq 50$

$t_s = 1.2360$

$T_{\alpha/2, n-1}, \quad -T_{\alpha/2, n-1}$

Check $t_s > T_{\alpha/2, n-1}$

$T_{\alpha/2, n-1} = 2.306$

$1.2360 \ngtr 2.306$ / or $t_s < -T_{(\alpha/2, n-1)}$

$1.2360 \nless -2.306$

We fails to reject the null hypothesis

9) Many doctors recommend having a total cholesterol level below 200 mg/dl. Test whether the 1952 population from which the Ethan and Emery sample were gathered is statistically different, on average, from this recommended level.

i) When the population s.d is known to be 63
ii) When the population s.d. is unknown.

209, 200, 173, 185, 239, 270, 274, 254, 223, 209, 219, 281, 257, 228, 222, 269, 209, 238, 285, 325

i) When the population s.d. known to be 63

$H_0 : \mu = 200 mg/dl = \mu_0$

$H_{11} : \mu > \mu_0$

$H_{12} : \mu < \mu_0$

$H_{13} : \mu \neq \mu_0$

$z_0 = \frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma}$ $\bar{x} = 241.35$

$z_0 = \frac{\sqrt{20}\,(241.35 - 200)}{63}$

$z_0 = 2.93$

We choose 0.05 confidence level

$z_0 > z_{0.05}$ $z_{0.05} = 1.96$

We reject the H_0 against H_{11}

(ii) $z_0 \not< -1.96$ $-z_{0.05} = -1.96$

We can not reject the null hypothesis

(1) A psychologist was interested in exploring whether or not male and female college students have different driving behaviours. She opted to focus on the fastest speed ever driven by an individual.

Therefore, the particular statistical question she framed was as follows.

Is the mean fastest speed driven by male college students different than the mean fastest speed driven by female college students?

She conducted a survey of a random $n = 34$ male college students ... Here is the descriptive summary of the result of her survey.

Males (x)	Females (y)
$n = 34$	$m = 29$
$\bar{x} = 105.5$	$\bar{y} = 90.9$
$s_x = 20.1$	$s_y = 12.2$

Carry out the test i) when population S.D.s $\sigma_x = 19$ and $\sigma_y = 12$

ii) When population S.D.s are unknown.

(Ans) (i) We are going to test $H_0: \mu_1 - \mu_2 = 0$

versus

$H_1: \mu_1 - \mu_2 > 0$

$$Z = \frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{(105.5 - 90.9)}{\sqrt{\frac{19^2}{34} + \frac{12^2}{29}}}$$

$$= 3.60$$

Let, μ_1 be the mean fastest speed driven for male c.g. student and μ_2 be the true mean fastest speed driven by female std.

The rejection region for $\alpha = 0.05$ is $\{Z > 1.64\}$

Because $Z = 3.60 > 1.64$

We reject the null hypothesis at $\alpha = 0.05$ we conclude that the speed of male student at this university is higher than female student.

(ii)

$$S = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

$$S = \sqrt{\frac{33(20.1)^2 + 28(12.2)^2}{61}}$$

$$\frac{\overset{3}{7}\overset{4}{2}\overset{4}{7}}{6 \quad 1}$$

$$S = \sqrt{17449}$$

$$\doteq \cancel{132.08}\ 16.93$$

$$t_S = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{16.93\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t_S = \frac{105.5 - 90.9}{16.9\ \cancel{132.08}\sqrt{\frac{1}{34} + \frac{1}{29}}}$$

$t_{0.05, 61}$

$$t_S = \cancel{0.778}\ 3.41$$

for $\alpha = 0.05$

Hence the rejection region is $t > t_{0.05, 61}$

$t_S > 1.67$ $\qquad$ $t > 1.67$

so we reject the null hypothesis again H_1 on $\alpha = 0.05$

(c) An experiment is conducted to determine whether intensive tutoring (covering a great deal of material in a fixed amount of time) is more effective than paced tutoring (covering less material in the same amount of time). Two randomly chosen groups are tutored separately and then administered proficiency test. Their mean marks and s.d. are given in the following table.
Test whether intensive tutoring is better than paced tutoring.

method	sample size	sample mean	sample s.d
intensive	12	46.31	6.44
paced	10	42.79	7.52

Ans

$H_0: (\mu_1 - \mu_2) = 0$

$H_1: \mu_1 > \mu_2$

$$t_s = \frac{(\bar{x}_1 - \bar{x}_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$S_p = \sqrt{\frac{11 \times (6.44)^2 + 9 \times (7.52)^2}{20}}$$

$S_p = \sqrt{}$

$$t_s = \frac{(46.31 - 42.79)}{6.94\sqrt{\left(\frac{1}{12} + \frac{1}{10}\right)}}$$

$S_p = 6.94$

$\alpha = 0.05$

$t_s = 1.18$

$t_{0.05, 20}$

1.725

$t_s < 1.725$

So, we cannot reject the null hypothesis against H_1

① The following dataset contain 480 ceramic strength measurements for two batches of material. The community statistic for each batch are shown below.

batch-1 Number of observations = 240
mean = 638.9987
S.d = 65.54909

batch-2 number of observations = 240
mean = 611.1559
S.d = 61.85425

Test whether the variance of the batches are equal when
i) population means are known to be $\mu_1 = 685$, $\mu_2 = 615$
ii) population means are unknown.

(ans) To test H_0: $\frac{\sigma_1}{\sigma_2} = 1$ vs $\frac{\sigma_1}{\sigma_2} \neq 1$

when population mean are known

Test statistic

$$F_S = \frac{S_{10}^2}{S_{20}^2} = \frac{(65.54009)^2}{(60.854)^2} = 1.12$$

from the F table $F_{\frac{0.05}{2}}(240, 240) = F_{1-\frac{0.05}{2}}(240, 240)$

$$= \frac{1}{F_{\frac{0.05}{2}}(240, 240)}$$

②

Next

SS

$$F_S = \frac{\frac{1}{n_1}\sum_{j=1}^{n_1}(x_{1j}-\mu_1)^2}{\frac{1}{n_2}\sum_{j=1}^{n_2}(x_{2j}-\mu_2)^2}$$

$$= \frac{33/10}{164/12}$$

$= 0.672$

from the F table

$F_{\frac{0.05}{2}}(10, 12) = F_{1-\frac{0.05}{2}}(10, 12) = \frac{1}{F_{\frac{0.05}{2}}(12, 10)}$

Hence the rejection region is

(7) A random sample of size 10 is drawn from a normal population whose population mean is 66. The sample observations 62, 63, 64, 65, 67, 67, 66, 69, 70 and 72. Another random sample of size 12 drawn from a normal distribution 60, 62, 63, 64, 66, 66, 67, 69, 70, 71, 71 and 72. Test whether the two variance can be taken to be equal.

(Ans)

$n_1 = 10, \mu_1 = 66$

$n_2 = 12, \mu_2 = 66$

let σ_1 be the variance of the first population and σ_2 be the variance of second population.

To test whether $H_0: \frac{\sigma_1}{\sigma_2} = 1$ vs $H_A: \frac{\sigma_1}{\sigma_2} \neq 1$

let us define our test statistic

$$A_S = \frac{S_{10}^2}{S_{20}^2}$$

$$= \frac{\frac{1}{n_1}\Sigma(x_i - \mu_1)^2}{\frac{1}{n_2}\Sigma(x_i - \mu_2)^2}$$

$$= \frac{\frac{1}{10}\left(4^2+3^2+2^2+1^2+1^2+1^2+0^2+3^2+4^2+6^2\right)}{\frac{1}{12}(166)}$$

$$= 0.672$$

from the F table $F_{\frac{0.05}{2}}(10,12) = F_{1-\frac{0.05}{2}}(10,12)$

$$= \frac{1}{F_{\frac{0.05}{2}}(12,10)}$$

(4) Consider two independent random ~~variable~~ samples $x_1, \dots, x_{n_1}$ from an $N(\mu_1, \sigma_1^2)$ distribution and $y_1, y_2, \dots, y_{n_2}$ from an $N(\mu_2, \sigma_2^2)$ distribution.

Test $H_0: \sigma_1^2 = \sigma_2^2$ versus $\sigma_1^2 \neq \sigma_2^2$ for the following basic statistic.

$n_1 = 25$, $\bar{x}_1 = 410$, $s_1^2 = 95$, and $n_2 = 16$, $\bar{x}_2 = 300$, $s_2^2 = 300$

(Soln):

$H_0: \sigma_1^2 = \sigma_2^2$ versus $H_1: \sigma_1^2 \neq \sigma_2^2$. This is a two tailed test. Here the degree of freedom are $n_1 = 24$ and $n_2 = 15$, the test statistic is $F_S = \frac{s_1^2}{s_2^2} = \frac{95}{300} = 0.317$

from the F table $F_{0.10}(24, 15) = 1.90$

$F_{0.90}(24, 15) = \frac{1}{F(0.10(15, 24))} = 0.50$

Hence the rejected region is $F_S > 1.90$ or $F_S < 0.50$, because the observed value of the test statistic 0.317 is less than 0.56, we reject the null hypothesis. There is evidence that the population variance are not equal.

Interval Estimation

(1) A random sample of size 10, drawn from a normal population, constitute the observations 62, 63, 64, 65, 67, 67, 68, 69, 70, 72. Construct the confidence interval of σ^2

i) When the population mean is known to be 66

ii) the population mean is unknown

62, 63, 64, 65, 67, 67, 68, 69, 70, 72 $\quad \bar{x} = \frac{\sum x_i}{10}$

i) When population mean is known $\quad s^2 =$

$$P\left\{ \left| \frac{ns^2}{\sigma^2} \right| < \chi^2_{\alpha/2} \right\} = 1-\alpha \quad ; \alpha \text{ be the level of significance}$$

$$\Rightarrow P\left\{ \chi^2_{1-\alpha/2} < \frac{ns^2}{\sigma^2} < \chi^2_{\alpha/2} \right\} = 1-\alpha$$

$$\Rightarrow P\left\{ 1/\chi^2_{\alpha/2, n} < \frac{\sigma^2}{ns^2} < 1/\chi^2_{1-\alpha/2, n} \right\} = 1-\alpha$$

$$\Rightarrow P\left\{ \frac{ns^2}{\chi^2_{\alpha/2, n}} < \sigma^2 < \frac{ns^2}{\chi^2_{1-\alpha/2, n}} \right\} = 1-\alpha \qquad s^2 = \frac{1}{10}\left[\sum(x_i - 66)^2\right] = 9.6$$

$$\Rightarrow P\left\{ \frac{10 \times 9.6}{\chi^2_{0.025, 10}} < \sigma^2 < \frac{10 \times 9.6}{\chi^2_{0.975, 10}} \right\} = 1-\alpha$$

$1.000 - 0.025 = .975$

Let, $\alpha = 0.05$

from χ^2 table –

So we are 95% confident that $\frac{10 \times 9.6}{\chi^2_{0.025, 10}} < \sigma^2 < \frac{10 \times 9.6}{\chi^2_{0.975, 10}}$

the confidence interval is $\left[\frac{10 \times 9.6}{\chi^2_{0.025, 10}}, \frac{10 \times 9.6}{\chi^2_{0.975, 10}} \right]$

the population mean unknown

$s^2 = \frac{1}{n}\sum(x_i-\bar{x})^2$

$= \frac{1}{\cancel{9}_3}\left[\frac{41}{123}\times 4\right]$

$= \frac{164}{3}$

$= \frac{164}{3}$

$P\left\{\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}\right\} = 1-\alpha$

$P\left\{\frac{\cancel{9}^3\times 164}{\cancel{3}\chi^2_{0.025,9}} < \sigma^2 < \frac{9\times 164}{3\cdot\chi^2_{0.975,9}}\right\} = \cdot 95$

so 95% confident that σ^2 lies between

the interval $\frac{492}{\chi^2_{0.025,9}}$ and $\frac{492}{\chi^2_{0.975,9}}$.

(2) Diff : 5 22 -2 3 4 -2 6 18 11 -11 7 15
10 6 -1 5 -3 5 1 8 12 -4

i) the population s.d. is known $\sigma = 8$ $\bar{x} =$

$\frac{\sqrt{n}(\bar{x}-\mu)}{\sigma} \sim N(0,1)$

$P\left[-\tau_{\alpha/2} < \frac{\sqrt{n}(\bar{x}-\mu)}{\sigma} < \tau_{\alpha/2}\right] = 1-\alpha$ let, $\alpha = 0.05$

$P\left[-\tau_{0.025}\frac{\sigma}{\sqrt{n}} < \bar{x}-\mu < \frac{\sigma}{\sqrt{n}}\tau_{0.025}\right] = 1-\alpha$

$\Rightarrow \bar{x} - \frac{\sigma}{\sqrt{n}}\tau_{0.025} < \mu < \bar{x} + \frac{\sigma}{\sqrt{n}}\tau_{0.025}$

$\bar{x} = 5.54$

$\Rightarrow 5.54 - \frac{8}{\sqrt{20}}\times 1.96 < \mu < 5.54 + \frac{8}{\sqrt{20}}\times 1.96$

So, 95% confident that mean lies between

$\left(5.54 - \frac{8}{\sqrt{20}}\times 1.96,\ 5.54 + 8/\sqrt{20}\cdot 1.96\right)$

(ii) σ is unknown

$$P\left[-t_{\alpha/2,n-1} < \frac{\sqrt{n}(\bar{x}-\mu)}{s} < t_{\alpha/2,n-1}\right] = 1-\alpha$$

$$s^2 = \frac{1}{19}\sum(x_i - 5.54)^2$$

$$\Rightarrow P\left[-t_{0.025,19} < \frac{\sqrt{20}}{s}(\bar{x}-\mu) < t_{0.025,19}\right] = 1-\alpha$$

$$\Rightarrow P\left[\bar{x} - t_{0.025,19}\cdot\frac{s}{\sqrt{20}} < \mu < \bar{x} - t_{0.025,19}\frac{s}{\sqrt{20}}\right] = 0.95$$

(3)

Male (x)	Female (y)
$n = 34$	$m = 29$
$\bar{x} = 105.5$	$\bar{y} = 90.9$
$s_x = 20.1$	$s_y = 12.2$

i) We know that

$$\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\sigma_2^2/n_2}} \sim N(0,1)$$

$$\therefore P_\theta\left\{\left|\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\sigma_2^2/n_2}}\right| < \tau_{\alpha/2}\right\} = 1-\alpha$$

; $\tau_{\alpha/2}$ being the upper $\alpha/2$ point of a $N(0,1)$ dist and α is the desired level of sig.

$$\Rightarrow P_\theta\left[-\tau_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} < (\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2) < \tau_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\sigma_2^2/n_2}\right] = 1-0.05$$

$$\Rightarrow P_\theta\left[-\tau_{\frac{0.05}{2}}\sqrt{\frac{19^2}{34}+\frac{13^2}{29}} < (\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2) < \tau_{\frac{0.05}{2}}\sqrt{\frac{19^2}{34}+\frac{13^2}{29}}\right] = 0.95$$

$$\Rightarrow P_\theta\left[(\bar{x}_1-\bar{x}_2)-\tau_{0.025}\sqrt{\frac{19^2}{34}+\frac{13^2}{29}} < \mu_1-\mu_2 < (\bar{x}_1 \ldots\right]$$

$= 0.$

(iii)

then $\left\{(\bar{x}_1-\bar{x}_2) - \tau_{0.025}\sqrt{\frac{19^2}{34}+\frac{13^2}{29}}\right\}$ and $\left\{(\bar{x}_1-\bar{x}_2)+\tau_{0.025}\sqrt{\frac{19^2}{34}+\frac{13^2}{29}}\right\}$ are respectively the lower and the upper confidence limits to $(\mu_1-\mu_2)$ with confidence co-efficient $(1-\alpha)$

(iii)

σ_x and σ_y are common

We know that
$$\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{s\sqrt{1/n_1+1/n_2}} \sim t_{n_1+n_2-2}$$

where
$$s=\sqrt{\frac{(n_1-1)s_x^2+(n_2-1)s_y^2}{n_1+n_2-2}}$$
$$=\sqrt{\frac{33(20.1)^2 + 28(14.2)^2}{61}}$$

$$P\left\{-t_{\alpha/2,n_1+n_2-2} \le \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{s\sqrt{1/n_1+1/n_2}} \le t_{\alpha/2,n_1+n_2-2}\right\} = 1-\alpha$$

$$\Rightarrow P\left\{(\bar{x}_1-\bar{x}_2)-s\sqrt{\frac{1}{n_1}+1/n_2}\, t_{\alpha/2,n_1+n_2-2} \le \mu_1-\mu_2 \le (\bar{x}_1-\bar{x}_2)+s\sqrt{\frac{1}{n_1}+1/n_2}\right\}$$

-0-

The following dataset contains 480 ceramic strength measurement for two batches of material. The summary statistic for each batch are shown below-

Batch-1	Batch-2
number of observation = 240	number of observations = 240
mean = 688.9987	mean = 611
standard deviation = 65.54909	std = 61.85425

Construct the confidence interval of the ratio of variances, when

i) population means are known to be $\mu_1 = 685$, $\mu_2 =$

ii) population means are unknown

)

$$\frac{\sigma_0^2}{\sigma_1^2} \Big/ \frac{s_1^2}{\sigma_2^2} = \frac{\frac{1}{n_1} \sum \left(\frac{x_{ij} - \mu_1}{\sigma_1}\right)^2}{\frac{1}{n_2} \sum \left(\frac{x_{ij} - \mu_2}{\sigma_2}\right)^2}$$

$$F_{n_1, n_2} = \frac{\frac{1}{n_1} \left[\frac{1}{\sigma_1^2} \sum (x_{ij} - \bar{x})^2 + \left(\frac{x_{ij} - \mu_1}{\sigma_1/\sqrt{n}}\right)^2\right]}{\frac{1}{n_2}\left[\frac{1}{\sigma_2^2} \sum (x_{ij} - \bar{x})^2 + \left(\frac{\bar{x} - \mu_2}{\sigma/\sqrt{n}}\right)^2\right]}$$

$$= \frac{\frac{n_1 - 1}{n_1}\left[\frac{1}{\sigma_1^2} \cdot \frac{1}{(n_1 - 1)} \sum (x_{ij} - \bar{x})^2 + \frac{n}{\sigma_1^2}(\bar{x} - \mu_0)^2\right]}{\frac{n_2 - 1}{n_2}\left[\frac{1}{\sigma_2^2} \cdot \frac{1}{n_2 - 1} \sum (x_{ij} - \bar{x})^2 + \frac{n}{\sigma^2}(\bar{x} - \mu_0)^2\right]}$$

$$= \frac{\frac{239}{240} \cdot \frac{1}{\sigma_1^2}\left[(65.54)^2 + 240 \cdot (688 - 685)^2\right]}{\frac{239}{240} \cdot \frac{1}{\sigma_2^2}\left[(61.85)^2 + 240(611 - 615)^2\right]} \quad \frac{239}{240}\left[\frac{1}{(65)^2} \times\right.$$

$$= \frac{\sigma_2^2}{\sigma_1^2} \frac{\left[(65.54)^2 + 240 (688 - 685)^2\right]}{\left[(61.85)^2 + 240 (611 - 615)^2\right]}$$

$$P\left\{ \frac{\sigma_2^2}{\sigma_1^2}\left[\frac{(65.54)^2 + 240(688.685)^2}{1 \cdot (61.85)^2 + 240(611.057)^2} \right] < F_{\alpha/2, n_1, n_2} \right\} = 1-\alpha$$

$\Rightarrow$

(iii)

(c)

$$\frac{\dfrac{s_1^2}{\sigma_1^2}}{\dfrac{s_2^2}{\sigma_2^2}} \sim F_{\alpha/2,\, n_1-1,\, n_2-1}$$

$$P\left\{ \left| \frac{\dfrac{(65.64)^2}{\sigma_1^2}}{\dfrac{(61.85)^2}{\sigma_2^2}} < F_{\alpha/2,\, n_1-1,\, n_2-1} \right| \right\} = 1-\alpha$$

$$\left\{ \frac{1}{F_{\alpha/2,\, n_2-1,\, n_1-1}} < \frac{\sigma_2^2}{\sigma_1^2} \cdot \frac{(65.64)^2}{(61.85)^2} < F_{\alpha/2,\, n_1-1,\, n_2-1} \right\} = 1-0.05$$

$$\Rightarrow \frac{1}{(F_{\alpha/2, 239, 239})}$$

$$\boxed{\begin{array}{c} H_0 = 0 \\ \neq 0 \end{array}}$$

the value of r
distri

(8) Test of parameters under Bivariate normal distribution

① Twelve cars were equipped with radial tires and driven over a test course. Then the same 12 cars (with the same drivers) were equipped with regular belted tires and driven over the same course. After each run, the cars' gas economy (in km/l) was measured. Is there evidence that radial tires produce better fuel economy? (use $\alpha = 0.05$)

If there is no enough evidence belite

	Car											
Gas economy	1	2	3	4	5	6	7	8	9	10	11	12
μ_1 x_1 (radial)	4.2	4.7	6.6	7.0	6.7	4.5	5.7	6.0	7.4	4.9	6.1	5.2
μ_2 x_2 (Belted)	4.1	4.9	6.2	6.9	6.8	4.4	5.7	5.8	6.9	4.7	6.0	4.9
	0.1	-0.2	0.4	0.1	-0.1	0.1	0	0.2	0.5	0.2	0.1	0.3

(ans) $u_2 = (\mu_1 - \mu_2)^2$

$$\bar{x}_2 = \frac{0.1 - 0.2 + 0.4 + 0.1 - 0.1 + 0.1 + 0.2 + 0.5 + 0.2 + 0.1 + 0.3}{12}$$

$$\bar{x}_2 = \frac{1.7}{12}$$

$(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)$

$$\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{n}}$$

$$\sqrt{\frac{s_1^2 + s_2^2}{n}}$$

$s_p^2 = \dfrac{(n_1 - 1)(s_1^2 + s_2^2)}{2(n-1)}$

$s_2 =$

$\sum x_i^2 = 5(0.1)^2 + 3(0.2)^2 + (0.3)^2 + (0.4)^2 + (0.5)^2$

$= 0.09 + 0.16 + 0.25 + 0.12 + 0.05 = 0.67$

$0.04 \times 3 = 0.12$

$$s_2^2 = \frac{1}{11}\left[\left(0.1 - \frac{1.7}{12}\right)^2 + \cancel{\left(0.2-\frac{1.7}{12}\right)^2 + \left(0.4 - \frac{1.7}{12}\right)^2 + \left(0.1-\frac{1.7}{12}\right)^2 + \left(-0.1 - \frac{1.7}{12}\right)^2} + \left(0.1 - \frac{1.7}{12}\right)^2 + \left(\frac{1.7}{12}\right)^2\right.$$

$$= \frac{1}{11}\left[\sum z_i^2 - n(\bar{z})^2\right]$$

$$= \frac{1}{11}\left[(\ ) \quad 12\left(\frac{1.7}{12}\right)^2\right]$$

$$= \frac{1}{11}\sum z_i^2 - \frac{1}{11}\cdot\frac{(1.7)^2}{12}$$

$$= \frac{1}{11}\left[0.67 - \frac{2.89}{11\times 12}\right]$$

$$= \frac{0.67 - 0.021}{11}$$

$$= 0.059$$

$$= s_2 = 0.2428$$

$$\frac{\sqrt{n}\left(\bar{x}_2 - \mu_2 = (\mu_1 - \mu_2)\right)}{s_2}$$

$$\cancel{t_{\alpha/2, n-1}\ P\left(\frac{\sqrt{12}(0.14166 - \mu_2)}{0.2428}\right) \leq t_{\alpha/2, n-1}}$$

$$P\left[-t_{\alpha/2, n-1} \leq \frac{\sqrt{12}(0.14166 - \mu_2)}{0.2428} \leq t_{\alpha/2, n-1}\right] = 1-\alpha$$

$$\Rightarrow P\left[-0.2428\, t_{\alpha/2, n-1} \leq \sqrt{12}(0.14166 - \mu_2) \leq 0.2428\, t_{\alpha/2, n-1}\right]$$

$$\Rightarrow P\left[\frac{-0.2428\, t_{\alpha/2, n-1}}{\sqrt{12}} \leq 0.1416 - \mu_2 \leq \frac{0.2428}{\sqrt{12}} t_{\alpha/2, n-1}\right] = 1-\alpha$$

$$\Rightarrow \left(0.14166 - \frac{0.2428}{\sqrt{12}} t_{0.025, 11},\ 0.14166 + \frac{0.2428}{\sqrt{12}} t_{0.025, 11}\right)$$

is an interval of 95

$H_0 = (\mu_1 - \mu_2) = 0$

$H_1 = \mu_1 \neq \mu_2 \neq 0$

$t_0 = \dfrac{\sqrt{n}(\bar{x}_d - \mu_d)}{s_d} \sim t_{\alpha, n-1}$

$t_0 = \dfrac{\sqrt{12}\left(\frac{1.7}{12}\right)}{0.428}$ $\qquad$ $t_{0.05, 11}$

$0.923 = t_0 < t_{0.05, 11} = 1.796$ $\qquad$ $t_0 < t_{\alpha, n-1}$

fail to

so, we reject the null hypothesis

So, there is no enough evidence that tires in better fuel efficiency.

(2) A study is design to check the relationship between smoking and longevity. A sample of 15 men 50 years and older were taken and the average number of cigarettes smoken per day and the age at death was recorded, as summarized in the following table. Can we conclude from the sample that longevity is independent of smoking.

Cigarettes	5	23	25	48	17	8	4	26	11	19	14	35	29	4	[illegible]
Longevity	80	78	60	53	85	84	73	79	81	75	68	72	58	92	65

(3) For the above data, can two variance be considered to be equal? Justify your answer.

$\sqrt{n-2}$ $\quad$ $\sqrt{1-r^2}$

$\rho = 0$, independent $\quad$ $\rho \neq 0$

$P(\text{long}/\text{Ci}) = P(\text{long})$

$\rho \neq 0$