

Finance

Final Exam. / Semester I 2022-23

Time - 3 and 1/2 hours/ Maximum Score - 100

NOTE : THIS IS AN OPEN BOOK AND OPEN NOTE EXAM. HOWEVER, NO INTERNET COMMUNICATIONS AND INTERNET RESOURCES ARE ALLOWED. A standard calculator is allowed as per rule. A normal table may be used, if needed. SHOW ALL YOUR WORK TO GET THE FULL CREDIT. RESULTS USED MUST BE CLEARLY STATED.

1. [12 + 12 = 24 points]

Let $\{S_n\}$ be the daily price of an asset. Assume that, $n \geq 1$, $\log S_n = \log S_0 + Y_1 + \cdots + Y_n$ with $Y_i = \log S_i - \log S_{i-1}$, for $i \geq 1$, are i.i.d. random variables, taking values +0.05, -0.05 and 0, with probabilities p , q and r , respectively, where $p, q, r > 0$, and $p + q + r = 1$. Here log is taken with respect to the base 2.

(i) Assuming the initial price of the asset is Rs.256, find the probability that S_n will reach a level Rs.1024 before it reaches Rs.128.

(ii) Assuming the initial price as above, find the expected time to reach either Rs.1024 or Rs.128.

2. [6 + 6 + 6 + 6 = 24 points]

Define the following terms with examples

(i) Arbitrage; (ii) Discounted martingale;

(iii) Incomplete market; (iv) Mean-variance efficient portfolio;.

3. [(6 + 6 + 6) + 6 = 24 points]

Let $T = 3$ months, $r = 6\%$ per annum, $S_0 = \text{Rs.}60$, $K = \text{Rs.}63$ and $u = 1.06$ with $d = 1/u$.

(a) Describing the necessary assumption, use 3-step Binomial model to calculate a risk-neutral valuation of (i) European Put option price; (ii) American put option price and (iii) Asian Put option price, where Asian put payoff = $\max(K - S_{\text{aver}}, 0)$ where average may be taken as Geometric mean.

(b) Compare their prices by arranging them in descending order and explain the reasons.

4. [7 + 7 + 7 + 7 = 28 points]

Write 'yes' or 'no' and justify your answer.

- (i) In a market with two risky asset and a risk-free asset, the Trinomial model is complete.
- (ii) In a no-arbitrage market with constant risk-free rate $r = 6\%$ per annum, suppose the initial price (i.e., the price at time $t = 0$ month) of a American call option on an asset whose initial price is Rs.60, the strike price is Rs.62 and the maturity time 3 months, is Rs.3. Then the price of the American put option, at time $t = 0$ month, on the same asset with same strike price and the maturity time is Rs.4.
- (iii) If one buys a European call option and a put option on the same underlying asset with the same strike price and maturity time T , then the person would make a profit no matter how the market behaves at time T .
- (iv) Under the Black-Scholes-Merton option pricing model the price of an European call option would increase if the risk-free interest (i.e., r) increases, keeping all other parameters fixed. [Hint: It is given that the price of the European call option under the B-S-M model is given by $S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2)$, where $d_1 = \frac{\log(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$ and Φ is the standard Normal distribution function, while the other notations are as used in the class.]

All the best!