Computer Vision
and
Machine Learning
**(Image smoothing)**

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# Outline

- Introduction
  - Signal and noise characteristics
- Noise cleaning or smoothing
  - Mean and Order statistics filters
  - Different kernels
- Sharpening
  - Laplacian
  - Smoothing method



# Types of processing

- Spatial domain processing
  - Directly operates on the pixel values in the spatial domain.
  - Point process
  - Neighbourhood process
    - Most common is convolution operation.
- Frequency domain processing
  - First transforms the image data to frequency domain using an orthogonal transform.
  - Appropriate filtering is applied on transformed data.
  - Inverse transform is applied on filtered data to get back into spatial domain.



# Histogram equalization

- Transfer function: $s = \int_{-\infty}^{r} p_r(\alpha) d\alpha$
- Assuming both *r* and *s* are continuous, and $0 \leq r, s \leq 1$.
- In discrete domain: $s_j = L_{max} \sum_{i=0}^{j} p_{r_i}$

where $p_{r_i} = \frac{n_{r_i}}{N}$ and $n_{r_i}$ is the frequency of occurrence of *i*-th levele $r_i$.

## Colour histogram equalization

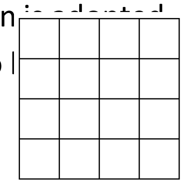
**Enhanced**

**Original**



## Block histogram equalization

- In the cases where even after enhancing global contrast, contrast over a small region is not adequate.
- Block histogram equalization
- Whole image is divided into
- Computationally intensive.
- Blocking effect visible.





## Block histogram equalization

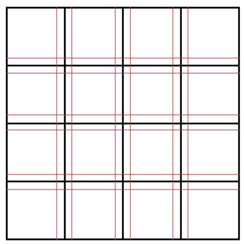
**Original color** **Block histogram equalization**

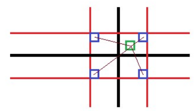


## Adaptive histogram equalization

- Block are non-overlapping.
  - Result is blocking effect.
  - Overlapping blocks may reduce blocking effect.
- Extreme proposition: block is around every pixel.
  - Removes blocking effect completely.
  - Time consuming.
- Non-overlapping and smoothing at their boundaries.

# Adaptive histogram equalization





Weighted sum
of border
pixels



# Adaptive histogram equalization

**Block histogram equalization**

**Adaptive histogram equalization**

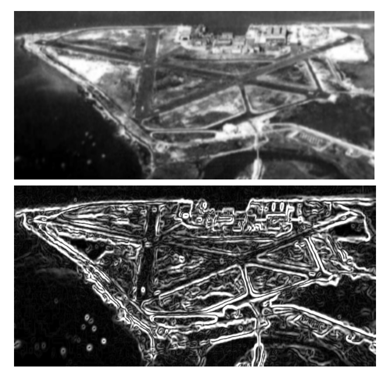


# Effect of noise and smoothing

along with the “correct” edges, contains too many false edges.



# Effect of noise and smoothing



Many false edges are smoothed, unfortunately so are true edges.

# Smoothing

**Assumptions**

(i) regarding noise

Signal independent and additive

Zero-mean and symmetrically distributed

(ii) regarding intensity

May be modeled by smooth surface (e.g. plane)



# Noisy image: Example

- Let us consider a 5x5 block of a noise-free image

| | | | | |
|---|---|---|---|---|
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |

Original image



# Noisy image: Example

- Let us consider a 5x5 block of a noise-free image
- A zero mean symmetrically distributed random noise is added to it.

| | | | | |
|---|---|---|---|---|
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |

Original image

+

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 2 | 0 | 1 |
| -3 | 0 | 0 | 3 | 0 |
| 0 | -1 | 4 | 0 | -2 |
| 3 | 0 | 0 | -1 | 0 |
| 0 | -3 | 1 | 0 | -4 |

Noise terms



# Noisy image: Example

- Let us consider a 5x5 block of a noise-free image.
- A zero mean symmetrically distributed random noise is added to it.
- Average of pixel values of the original image and that of the noisy image is same!

| | | | | |
|---|---|---|---|---|
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |

Original image

+

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| 0 | -1 | 4 | 0 | -2 |
| 3 | 0 | 0 | -1 | 0 |
| 0 | -3 | 1 | 0 | -4 |

Noise terms

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| | | | | |
|---|---|---|---|---|
| 6 | 7 | 9 | 7 | 9 |
| 3 | 7 | 7 | 10 | 8 |
| 6 | 6 | 11 | 7 | 6 |
| 9 | 7 | 7 | 6 | 8 |
| 6 | 4 | 8 | 7 | 4 |

Noisy image

# Degradation model

- Noise is signal independent and additive

$$g(r,c) = f(r,c) + \eta(r,c)$$

- For $n$ no. of noisy version of same image

$$g_i(r,c) = f(r,c) + \eta_i(r,c) \quad \text{for} \quad i = 1,2,3,\ldots,n$$

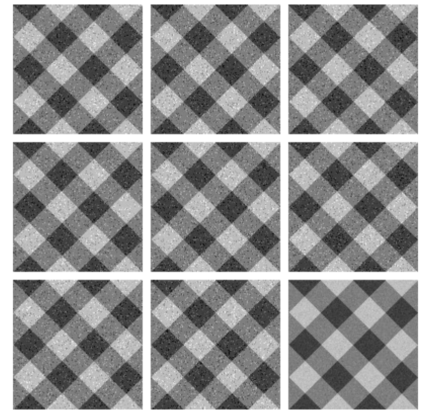
- Let us take pixel-wise average over $n$ image

$$\bar{g}(r,c) = \frac{1}{n}\sum_{i=1}^{n} g_i(r,c) = \frac{1}{n}\sum_{i=1}^{n} f(r,c) + \frac{1}{n}\sum_{i=1}^{n} \eta_i(r,c)$$

$$= f(r,c)$$



# Multiple noisy image





# Neighborhood process: Smoothing

- Noise causes abrupt change in graylevel.
- Noisy pixel is either much brighter or much darker than its neighbouring pixels.
- A pixel and its neighbourhood is considered to compute the value (colour) of the corresponding pixel in the output image.

$$f(x,y) = T_{(u,v)\,\in\, N(x,y)}[g(u,v)]$$



# Mean square estimation

- Image graylevel over a patch is approximated by a plane

$$f(x,y) = A(x - x_0) + B(y - y_0) + C \qquad \text{given } f(x_0, y_0) = C$$

- Noisy graylevel may be modeled as

$$g(x,y) = f(x,y) + \eta(x,y) = A(x - x_0) + B(y - y_0) + C + \eta(x,y)$$

- Least square error is then defined as

$$e = \sum_{(x,y)\in W} [g(x,y) - A(x - x_0) - B(y - y_0) - C]^2 - \sum_{(x,y)\in W} [\eta(x,y)]^2$$

- Estimated noise free graylevel is

$$\bar{g}(x_0, y_0) = C = \frac{1}{|W|}\sum_{(x,y)\in W} g(x,y)$$

## Noisy image: Example

- Let us consider a 5x5 block of a noise-free image

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| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
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Original image



## Noisy image: Example

- Let us consider a 5x5 block of a noise-free image
- A zero mean symmetrically distributed random noise is added to it.

| | | | | |
|---|---|---|---|---|
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| 6 | 7 | 7 | 7 | 8 |
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Original image

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| | | | | |
|---|---|---|---|---|
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Noise terms



## Noisy image: Example

- Let us consider a 5x5 block of a noise-free image.
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Original image

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| | | | | |
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Noise terms

=

| | | | | |
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| 6 | 6 | 11 | 7 | 6 |
| 9 | 7 | 7 | 6 | 8 |
| 6 | 4 | 8 | 7 | 4 |

Noisy image



## Noisy image: Example

- Because of linear variation in graylevel in the original image, centre pixel has the average of the pixel values.
- Hence, if we replace the graylevel of the centre pixel of the noisy image by the average value of the block, we get back original value at that position.

| | | | | |
|---|---|---|---|---|
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |
| 6 | 7 | 7 | 7 | 8 |

Original image

+

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 2 | 0 | 1 |
| -3 | 0 | 0 | 3 | 0 |
| 0 | -1 | 4 | 0 | -2 |
| 3 | 0 | 0 | -1 | 0 |
| 0 | -3 | 1 | 0 | -4 |

Noise terms

=

| | | | | |
|---|---|---|---|---|
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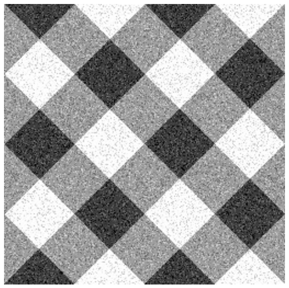
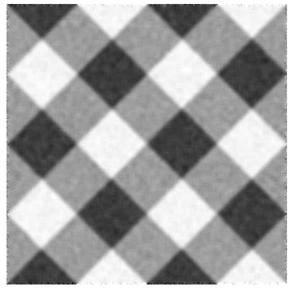
| 6 | 6 | 7 | 7 | 6 |
| 9 | 7 | 7 | 6 | 8 |
| 6 | 4 | 8 | 7 | 4 |

Noisy image

**Noisy image:** **Mean filter:**

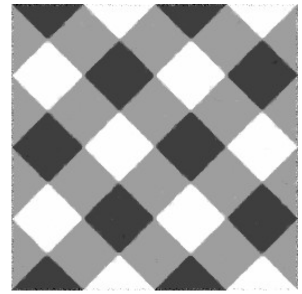


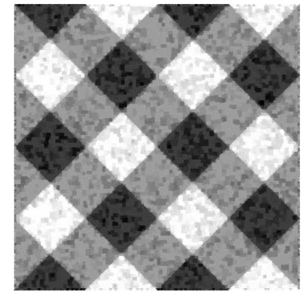


**Advantage:** Low computational cost.
**Disadvantage:** Blurs edge information.



**Median filter:** **Max-min-min-max filter:**





**Advantage:** Preserves edge information.
**Disadvantage:** High computational cost.



# Mean vs. median

- Mean is linear filter, while median is non-linear.
- Mean is affected by the outliers, but median is not.
- Mean is computationally less costly than median.
- Median can preserve edge much better than mean filter.
- Weighted averaging (with suitable set of weights) may lead to edge preserving smoothing by
  - sufficient intra-region smoothing
  - Insignificant inter-region smoothing



# Linear neighborhood operation

Convolution: $g_{smooth}(r,c) = g_{noisy}(r,c) * h_{mask}(r,c)$

**Mask:** $h_{mask}(r,c)$ may be one such shown as follows.

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} \quad \frac{1}{25}\begin{bmatrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{bmatrix} \quad \frac{1}{16}\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} \quad \frac{1}{81}\begin{bmatrix}1&2&3&2&1\\2&4&6&4&2\\3&6&9&6&3\\2&4&6&4&2\\1&2&3&2&1\end{bmatrix}$$

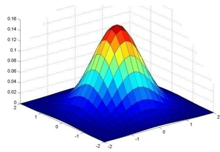
| | | | | |
|---|---|---|---|---|
| 0.003 | 0.013 | 0.022 | 0.013 | 0.003 |
| 0.013 | 0.059 | 0.097 | 0.059 | 0.013 |
| 0.022 | 0.097 | 0.159 | 0.097 | 0.022 |
| 0.013 | 0.059 | 0.097 | 0.059 | 0.013 |
| 0.003 | 0.013 | 0.022 | 0.013 | 0.003 |

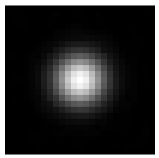
**Non-linear neighborhood operation:** Uses order statistic

**Window:** symmetric neighborhood (domain of the masks).

# Gaussian Kernel

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$





| | | | | |
|---|---|---|---|---|
| 0.003 | 0.013 | 0.022 | 0.013 | 0.003 |
| 0.013 | 0.059 | 0.097 | 0.059 | 0.013 |
| 0.022 | 0.097 | 0.159 | 0.097 | 0.022 |
| 0.013 | 0.059 | 0.097 | 0.059 | 0.013 |
| 0.003 | 0.013 | 0.022 | 0.013 | 0.003 |

5 x 5, $\sigma$ = 1

- Constant factor at front makes volume sum to 1 (can be ignored, as we should re-normalize weights to sum to 1 in any case)
- Replicates *isotropic* diffusion.



Source: C. Rasmussen

# Gaussian smoothing

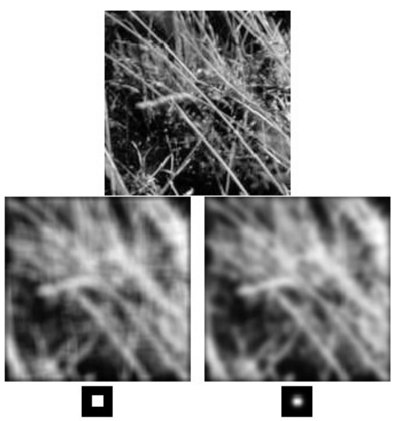
- Based on convolving a Gaussian kernel of size NxN with each and every pixel.
- A pixel's brightness value is determined by its own value as well as the values of its neighbor pixels.
- an appropriate definition of the transformation would be:

$$f_{t+1}(x,y) = f_t * G(x,y)$$

where $G(x,y) = \frac{1}{2\pi\sigma^2} * e^{-\frac{x^2+y^2}{2\sigma^2}}$ and $f_0(x,y) = f(x,y)$



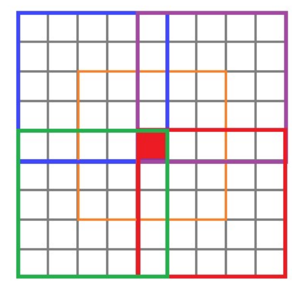
# Mean vs. Gaussian filtering





# Edge preserving smoothing

- An edge divides two regions.
- A window covering single region may be characterized low variance of gray values.
- A window containing pixels from several region should have high variance.
- Neighborhood of a candidate pixel may be partitioned into various windows.



# Thank you !

**Any question?**