Linear Algebra and Matrix Computation (LAMC) notes

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Contents

Lecture 1

Motivation, Prerequisites and Resources

1.1 Motivation

Please go through the class slides.

1.2 Prerequisites:

Student should have some knowledge in

- Basic concept on algebra, vectors and co-ordinate geometry
- Computer programming: Any one from C/C++/Python(recommended for the class assignments/project)/MATLAB/Octave

1.3 Tentative syllabus:

- Introduction to vectors
  - Vectors and its geometry
  - Operation on vectors: addition, multiplication by a scalar, dot production, length
- Vector space

  - Vector space
  - Subspace
  - Basis and dimension
  - Change of basis
- Linear transformations
  - Rank-Nullity theorem
  - Matrix of a linear transformation
  - Linear operators and isomorphism
  - Linear functionals
- Matrix algebra
  - Matrix addition and multiplication, transpose, inversion
  - Special matrices
  - Row rank and column rank of a matrix
  - Determinant of a matrix and its geometric interpretation
  - Cramer's rule to solve system of linear equations
  - Various matrix decompositions
- Eigenvalues and Eigenvectors
  - Introduction to eigenvalues and eigenvectors of matrices
  - Characteristic polynomial
  - Cayley-Hamilton theorem
  - Algebraic and geometric multiplicities of eigenvalues
  - Matrix diagonalization
  - Positive (semi-) definite matrices
  - Solving linear recurrences
- Normed linear spaces
  - Normed spaces

  - Cauchy-Schwarz inequality and triangle inequality
  - Projection
  - Gram-Schmidt orthogonalization
  - Hermitian operators
  - The Spectral theorem

- Matrix Computations
  - Floating point numbers and operations, Error Analysis
  - Solving system of linear equations:
    * Direct methods: Gaussian elimination, LU factorization
    * Iterative methods: Jacobi method, Gauss-Seidel method
  - Solving least square problems:
    * QR decomposition
    * Gram-Schmidt orthogonalisation
    * Singular value-decomposition (SVD)
  - Solving Eigenvalue problems:
    * Tridiagonal QR iteration, Jacobi method
  - Some practical applications (if time permit)

1.4 Related books

We will follow these books:

[1] Sheldon Axler. *Linear Algebra Done Right*. Springer, 3rd edition, 2015. [sample chapter online]

[2] Kenneth Hoffman and Ray Kunze. *Linear Algebra*. Prentice Hall of India, 2nd edition, 1971. [library] or [online]

[3] Gilbert Strang. *Introduction to linear algebra*. Wellesley-Cambridge Press, 5th edition, 2016. [sample chapter online] and [online]

[4] Gene H. Golub and Charles F. Van Loan. *Matrix Computations*. Hindustan Book Agency, 4th edition, 2015. [library] and [online (3rd ed.)]

[5] Holger Wendland. *Numerical linear algebra : an introduction*. Cambridge University Press, Cambridge texts in applied mathematics, 2018. [library]

1.5 Computational tools

Here are some popular LAMC tools:

- Numpy (python) - `https://numpy.org/`
- Octave - `https://octave.org/`
- CLAPAC (C) - `https://netlib.org/clapack/`
- Matlab - `https://in.mathworks.com/`
- PyTorch (Python and GPU support) - `https://pytorch.org/`
- ...

1.6 Datasets repository

You can find some datasets to evaluate yous programming assignment in UCI Machine Learning Repository (`https://archive.ics.uci.edu/ml/datasets.php`)

1.7 For recent updates on LAMC you can follow the arXiv

You can go to Mathematics section in arXiv and under that you can find Numerical Analysis

- NA - `https://arxiv.org/list/math.NA/recent`

Lecture 2

Vector space and Subspace

2.1 Suggested reading

Please read *Chapter-1* of Axler's book [1] or *Chapter-2* of Hoffman and Kunze's [2] book.

Lecture 3

Span, Linearly dependent and independent, Basis

3.1 Suggested reading

Please read *Chapter-2* of Axler's book [1] or *Chapter-2* of Hoffman and Kunze's [2] book.

Lecture 4

Dimension of a vector space and related problems

4.1 Suggested reading

Please read *Chapter-2* of Axler's book [1] or *Chapter-2* of Hoffman and Kunze's [2] book.

Lecture 5

Linear maps/transformation

5.1 Suggested reading

Please read *Chapter-3* of Axler's book [1] or *Chapter-3* of Hoffman and Kunze's [2] book.

Lecture 6

Null space, range of linear maps and rank-nullity theorem

6.1 Suggested reading

Please read *Chapter-3* of Axler's book [1] or *Chapter-3* of Hoffman and Kunze's [2] book.

Lecture 7

Matrix, Linear map represent by a matrix, and introduction to system of linear equations and relation with linear map

7.1 Suggested reading

For representation of *Linear map* by a matrix in *Chapter-3, Section 3.C* of Axler's book [1] or *Chapter-3, Section 3.4* of Hoffman and Kunze's [2] book. Also, read surrounding sections in each book's.

Lecture 8

Invertibility and Isomorphic Vector Spaces

8.1 Suggested reading

Please read *Chapter-3* of Axler's book [1] or *Chapter-3* of Hoffman and Kunze's [2] book.

Bibliography

[1] Sheldon Axler. *Linear Algebra Done Right*. Springer, 3rd edition, 2015.

[2] Kenneth Hoffman and Ray Kunze. *Linear Algebra*. Prentice Hall of India, 2nd edition, 1971.