Computer Vision and Machine Learning

(Camera calibration and correction)

Bhabatosh Chanda
chanda@isical.ac.in

Position and orientation of camera

- Camera may be placed anywhere in a scene.
- Same object may appear differently while captured by same camera at different position and orientation.
- It would be easier to handle if the relation between their geometry is known.

Coordinate transformation

- World coordinate system
  - World (object) centric scene coordinates
- Camera coordinate system
  - Camera centric scene coordinates
- Transforming world (scene) coordinate to camera coordinate
  - Rotation, translation and (optional) scaling
  - Linear transformation is advantageous.

Projection matrix for image formation

In practice: lots of coordinate transformations…

$$\begin{pmatrix}\text{2D}\\ \text{point}\\ \text{(3x1)}\end{pmatrix} = \begin{pmatrix}\text{Camera to}\\ \text{pixel coord.}\\ \text{trans. matrix}\\ \text{(3x3)}\end{pmatrix}\begin{pmatrix}\text{Perspective}\\ \text{projection matrix}\\ \text{(3x4)}\end{pmatrix}\begin{pmatrix}\text{World to}\\ \text{camera coord.}\\ \text{trans. matrix}\\ \text{(4x4)}\end{pmatrix}\begin{pmatrix}\text{3D}\\ \text{point}\\ \text{(4x1)}\end{pmatrix}$$

$$\begin{bmatrix} u^c\varphi_x + w^c d_x/f \\ v^c\varphi_y + w^c d_y/f \\ w^c/f \end{bmatrix} = \begin{bmatrix} \varphi_x & 0 & d_x \\ 0 & \varphi_y & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_u \\ r_{32} & r_{22} & r_{23} & t_v \\ r_{31} & r_{32} & r_{33} & t_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u^c\varphi_x + w^c d_x/f \\ v^c\varphi_y + w^c d_y/f \\ w^c/f \end{bmatrix} = \begin{bmatrix} \varphi_x & 0 & d_x/f & 0 \\ 0 & \varphi_y & d_y/f & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_u \\ r_{32} & r_{22} & r_{23} & t_v \\ r_{31} & r_{32} & r_{33} & t_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}$$

Projection matrix for image formation

- Combining the matrices of

$$\begin{bmatrix} u^c\varphi_x + w^c d_x/f \\ v^c\varphi_y + w^c d_y/f \\ w^c/f \end{bmatrix} = \begin{bmatrix} \varphi_x & 0 & d_x/f \\ 0 & \varphi_y & d_y/f \\ 0 & 0 & 1/f \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_u \\ r_{32} & r_{22} & r_{23} & t_v \\ r_{31} & r_{32} & r_{33} & t_w \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}$$

- We obtain $\begin{bmatrix} u^c\varphi_x + w^c d_x/f \\ v^c\varphi_y + w^c d_y/f \\ w^c/f \end{bmatrix} = K[R \quad t] \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix} = A \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}$

- Finally, $\begin{bmatrix} x' \\ y' \\ w^c/f \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{32} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}$

Camera calibration

Finally, $\begin{bmatrix} x' \\ y' \\ w_c/f \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}$

$$\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}$$

where $x = \frac{x'}{\lambda}$; $y = \frac{y'}{\lambda}$; i.e., (x, y) is image coordinate.

Camera calibration

- For i-th world point $(u_i, v_i, w_i)$and corresponding image point (x_i, y_i) we have

$$\begin{bmatrix} \lambda_i x_i \\ \lambda_i y_i \\ \lambda_i \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ 1 \end{bmatrix}$$

- Determining the elements $a_{ij}$of matrix A is called ***camera calibration.***
- Note that ***matrix A*** contains ***both intrinsic and extrinsic parameters***.

Camera calibration

- Matrix A has 12 elements: 3 rows each with 4 elements.

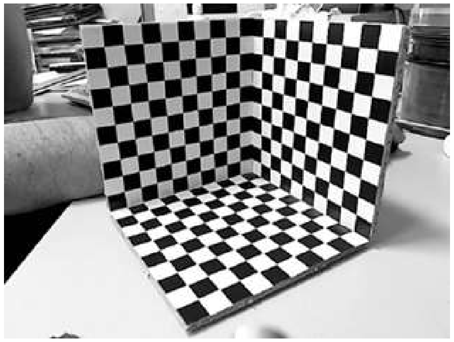
$$p^I = \begin{bmatrix} \lambda_i x_i \\ \lambda_i y_i \\ \lambda_i \end{bmatrix} = Ap = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ 1 \end{bmatrix}$$

- Each image point provides two linear equations.

$$x_i = \frac{\lambda_i x_i}{\lambda_i} = \frac{r_1 \cdot p}{r_3 \cdot p} \quad \text{and} \quad y_i = \frac{\lambda_i y_i}{\lambda_i} = \frac{r_2 \cdot p}{r_3 \cdot p}$$

- 6 world points and corresponding image points give solution.

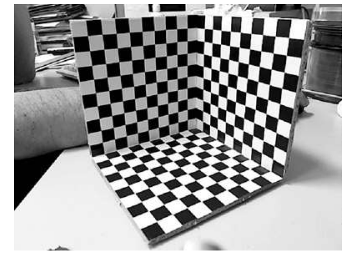
Camera calibration grid



Origin may be assumed at the junction of three faces.

Camera calibration

- World points are located on *calibration grid* with known *geometric structure*.
- Much more than six points are taken and solution is obtained by least square estimation.
- Solutions are correct up to a scale.

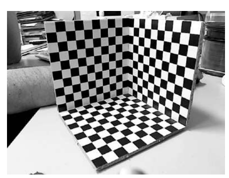


Camera calibration

- For i-th world point $(u_i, v_i, w_i)$and corresponding image point (x_i, y_i) we have

$$\begin{bmatrix} \lambda_i x_i \\ \lambda_i y_i \\ \lambda_i \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ 1 \end{bmatrix}$$

- There are 12 unknowns a_{ij}.
- So to solve we need at least 12 linear equations relating image points and world points.
- Solution is correct up to a scale.



Camera calibration

- Once we determine the camera projection matrix A

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = K[R \quad t]$$

- Thus we can write

$$\hat{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = KR \text{ and } \begin{bmatrix} a_{14} \\ a_{24} \\ a_{34} \end{bmatrix} = Kt$$

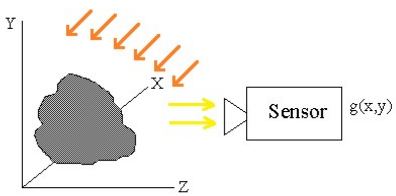
($\hat{A} = KR$ can be solved by RQ-decomposition, where R is an upper-triangular matrix and Q is an orthonormal matrix.)

Stereo: Anaglyph

Numerical problems

- Suppose the tip of a lamppost forms an image on the image plane of a digital camera having focal length 1.5 cm and image plane of size 4 cm x 3 cm. The tip of the lamppost is located at (2 m., 5 m. 10 m.) with respect to the centre of the lens of the camera. Determine the image coordinate in terms of pixels, if the photoreceptor density is 1200/cm in both horizontal and vertical direction.
- A digital camera has the intrinsic parameters as: focal length = 0.8 cm, photoreceptor density 1000 per cm in both directions, image plane height = 2 cm and width = 3 cm. When imaged a particular object point appears at (1580, 1320) in pixels. The camera is shifted by 5m along horizontal axis. Now the image of the same object point appears at (1380, 1320) in pixels. Calculate the location of the object point with respect to the initial position of the camera.

Image formation

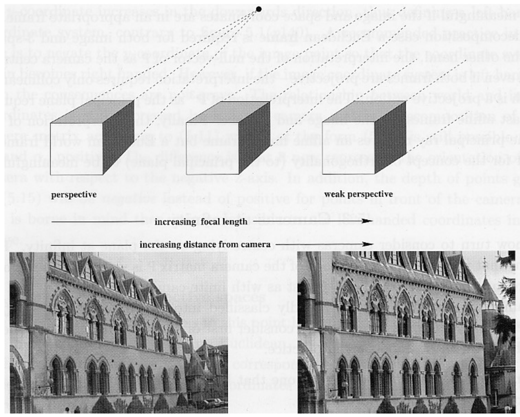


- Reflected light is intercepted by the sensor (camera) and an image is formed,
- Within camera two types of transformations takes place
  - Geometric transformation
  - Photometric transformation

Distortions and limitations

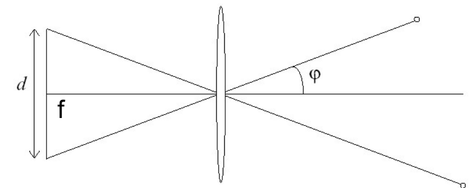
- There are various types of distortions in the mage formed due to camera parameters.
  - Some of these may be considered as the limitations of the camera model
  - Most of these results in photometric distortions / degradations.
  - Others cause geometric distortion in the image.
- Example: low photoreceptor density φ_x and φ_y
  - Resulting in low resolution image
  - Loosing finer details

Approximating an affine camera



Source: Hartley & Zisserman

Field of View



FOV depends on focal length and size of the camera retina

Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Larger Focal Length → Smaller FoV

Slide by A. Efros

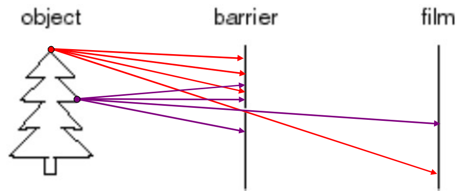
Field of View

- Individually, our eyes have
  - a horizontal FoV of about 135 degrees and
  - a vertical FoV of just over 180 degrees.
- Our brains stitches two monocular FoVs together to form one binocular FoV.
  - Binocular horizontal FoV is 114 degrees and
  - Peripheral vision FoV about 60 – 70 degrees.

Field of View

- Different camera has different FoV (depending on its manufacturer and model)
  - A typical point-and-shoot camera has FOV ranging from about 50 degrees.
  - Wide angle lens has larger FoV.

Pinhole camera



Add a barrier with a small hole to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**

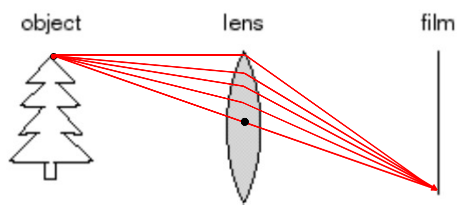
Slide inspired by by Steve Seitz

Problem with pinhole aperture

- Not enough light enters into and reaches the retina (image plane) of the camera.
- Increase the size of the aperture.
- New problem: blurring

- Solution: increase aperture size and add lens.

Adding a lens

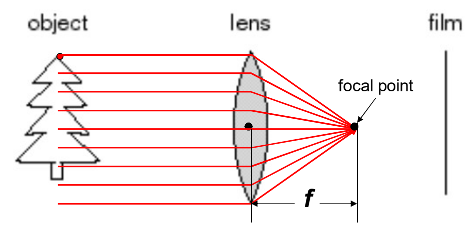


A lens focuses light onto the film

- Rays passing through the center are not deviated

Slide by Steve Seitz

Adding a lens

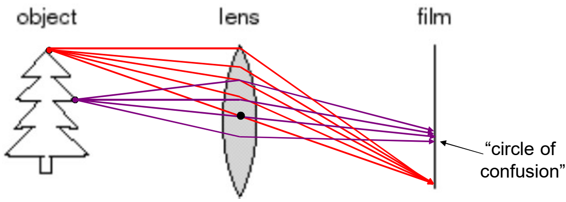


A lens focuses light onto the film

- Rays passing through the center are not deviated
- All parallel rays converge to one point on a plane located at the *focal length f*

Slide by Steve Seitz

Adding a lens

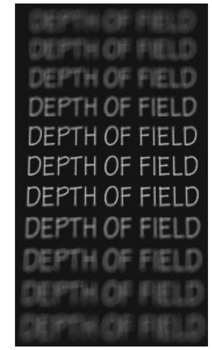


A lens focuses light onto the film

- There is a specific distance at which objects are "in focus".
- other points project to a "circle of confusion" in the image.

Slide by Steve Seitz

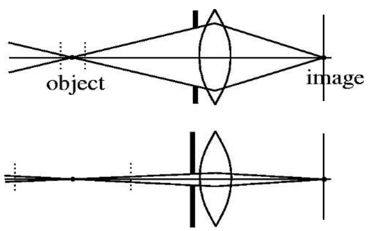
Depth of Field



http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide by A. Efros

How can we control the depth of field?



Changing the aperture size affects depth of field

- A smaller aperture increases the range in which the object is approximately in focus
- But small aperture reduces amount of light – need to increase exposure

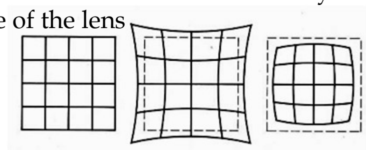
Slide by A. Efros

Depth of Field

f/2.8 f/5.6 f/8.0

Radial Distortion

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens



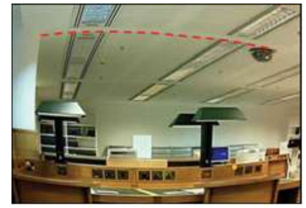
No distortion Pincushion Barrel

Pincushion

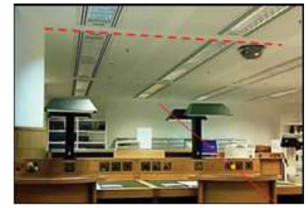
Barrel

Fisheye

Radial distortion



Radial distortion



Radial distortion Correction

$$\tilde{x} = x\,(1 + \beta_1 r^2 + \beta_2 r^4)$$
$$\tilde{y} = y\,(1 + \beta_1 r^2 + \beta_2 r^4)$$

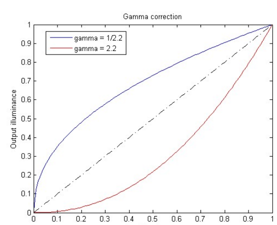
Gamma correction

- Response of photosensor to input is non-linear (red curve).

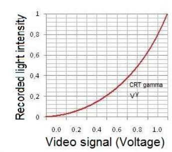
$$B = V^{\gamma}$$

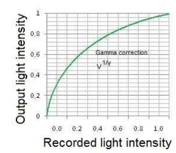
- That we may compensate by applying inverse operation (blue curve).
- $I = B^{1/\gamma}$

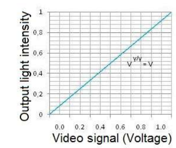
  $= (V^{\gamma})^{1/\gamma} = \mathrm{V} \rightarrow$ **linear**



Gamma correction







→ **Gamma Correction** →

White balance

- When looking at a picture on screen or print, we adapt to the illuminant of the room, not to that of the scene in the picture
- When the white balance is not correct, the picture has an unnatural color "cast".

incorrect white balance | correct white balance

http://www.cambridgeincolour.com/tutorials/white-balance.htm

White balance

- We need to "guess" which pixels correspond to white objects (highest average brightness)!
- Gray world assumption
  - The image average r_{ave}, g_{ave}, b_{ave} is gray
  - Use weights $1/r_{ave}$, $1/g_{ave}$, $1/b_{ave}$
- Brightest pixel assumption
  - Highlights usually have the color of the light source
  - Use weights inversely proportional to the values of the brightest pixels
- Use image statistics, learning techniques

Color balance

Raw image

Color balanced image (brightest pixels)

Thank you!

Any question?