

Finance

Final Exam. / Semester I 2021-22

Time - 3 and 1/2 hours/ Maximum Score - 100

NOTE : THIS IS AN OPEN BOOK AND OPEN NOTE EXAM. HOWEVER, NO INTERNET COMMUNICATIONS AND INTERNET RESOURCES ARE ALLOWED. A standard calculator is allowed as per rule. A normal table may be used, if needed. SHOW ALL YOUR WORK TO GET THE FULL CREDIT. RESULTS USED MUST BE CLEARLY STATED.

1. $[8 + 8 + 8 = 24 \text{ points}]$

Let $\{X_i\}_{i \geq 1}$ be a sequence i.i.d. random variables taking values $\pm b$ (for some $b > 0$) and 0, with $p = P(X_1 = +b)$ and $q = P(X_1 = -b)$ and $r = P(X_1 = 0)$, such that $0 < p, q, r < 1$ and $p + q + r = 1$.

(a) Let $S_n = X_0 + X_1 + \dots + X_n$, with $X_0 = a$, a positive constant. Then show that $W_n = (q/p)^{S_n/b}$ is a martingale with respect to an appropriate filtration $\{\mathcal{F}\}_{n \geq 0}$, and defining the filtration.

(b) Assuming that $\{W_n\}$ is the daily price of an asset. Find the probability that $\{\log(W_n)\}$ will reach a level $(a + 50b)d$ unit before it goes to $(a - 30b)d$, where $d = (\log q - \log p)/b > 0$.

(c) Calculate the expected time to reach either of the boundary.

2. $[6 + 6 + 6 + 6 = 24 \text{ points}]$

Define the following terms with examples

(i) Risk-neutral pricing; (ii) Self-financing strategy.

(iii) No arbitrage market; (iv) Dynamic asset allocation.

3. $[(6 + 6 + 6) + 6 = 24 \text{ points}]$

Let $T = 3$ months, $r = 6\%$ per annum, $S_0 = \text{Rs.}50$, $K = \text{Rs.}52$ and $u = 1.04$ with $d = 1/u$.

(a) Describing the necessary assumption, use 4-step Binomial model to calculate a risk-neutral valuation of (i) European Put option price; (ii) American put option price and (iii) Asian Put option price, where Asian put payoff $= \max(K - S_{\text{aver}}, 0)$ where average may be taken as Geometric mean.

(b) Compare their prices by arranging them in descending order and explain the reasons.

4. $[7 + 7 + 7 + 7 = 28 \text{ points}]$

Write 'yes' or 'no' and justify your answer.

- (i) In the context of (static) portfolio optimization the minimum variance portfolio (mvp) with shortsale restriction and without shortsale restriction are always the same.
- (ii) In a market with one risky asset and a risk-free asset, the Trinomial model is incomplete.
- (iii) If a no arbitrage market is complete then one can price any product with a given (contingent) claim at a maturity time, (say at time $T=1$).
- (iv) If one buys a European call option and a put option on the same underlying asset with the same strike price and maturity time T , then the person would make a profit no matter how the market behaves at time T .

All the best!