SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

AS
FURTHER MATHEMATICS
Paper 1

Exam Date Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A reflection is represented by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

State the equation of the line of invariant points.

Circle your answer.

[1 mark]

$x = 0$ $\qquad$ $y = 0$ $\qquad$ $y = x$ $\qquad$ $y = -x$

2 Find the mean value of $3x^2$ over the interval $1 \leq x \leq 3$

Circle your answer.

[1 mark]

$8\frac{2}{3}$ $\qquad$ 10 $\qquad$ 13 $\qquad$ 26

3 Find the equations of the asymptotes of the curve $x^2 - 3y^2 = 1$

Circle your answer.

[1 mark]

$y = \pm\, 3x$ $\qquad y = \pm \dfrac{1}{3} x$ $\qquad y = \pm \sqrt{3}\, x$ $\qquad y = \pm \dfrac{1}{\sqrt{3}} x$

Turn over for the next question

4 $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 1 & k \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

4 (a) Find the value of k for which matrix **A** is singular.

[1 mark]

4 (b) Describe the transformation represented by matrix **B**.

[1 mark]

4 (c) (i) Given that **A** and **B** are both non-singular, verify that $\mathbf{A}^{-1}\mathbf{B}^{-1} = (\mathbf{BA})^{-1}$.

[4 marks]

4 (c) (ii) Prove the result $\mathbf{M}^{-1}\mathbf{N}^{-1} = (\mathbf{NM})^{-1}$ for all non-singular square matrices **M** and **N** of the same size.

[4 marks]

Turn over for the next question

5 The region bounded by the curve with equation $y = 3 + \sqrt{x}$, the x-axis and the lines $x = 1$ and $x = 4$ is rotated through 2π radians about the x-axis.

Use integration to show that the volume generated is $\frac{125\pi}{2}$

[5 marks]

6 (a) Use the definitions of $\sinh x$ and $\cosh x$ in terms of e^{x} and e^{-x} to show that

$x = \frac{1}{2}\ln\left(\frac{1+t}{1-t}\right)$ where $t = \tanh x$

[4 marks]

Question 6 continues on the next page

6 (b) (i) Prove $\cosh^3 x = \frac{1}{4}\cosh 3x + \frac{3}{4}\cosh x$

[4 marks]

6 (b) (ii) Show that the equation $\cosh 3x = 13\cosh x$ has only one positive solution.

Find this solution in exact logarithmic form.

[4 marks]

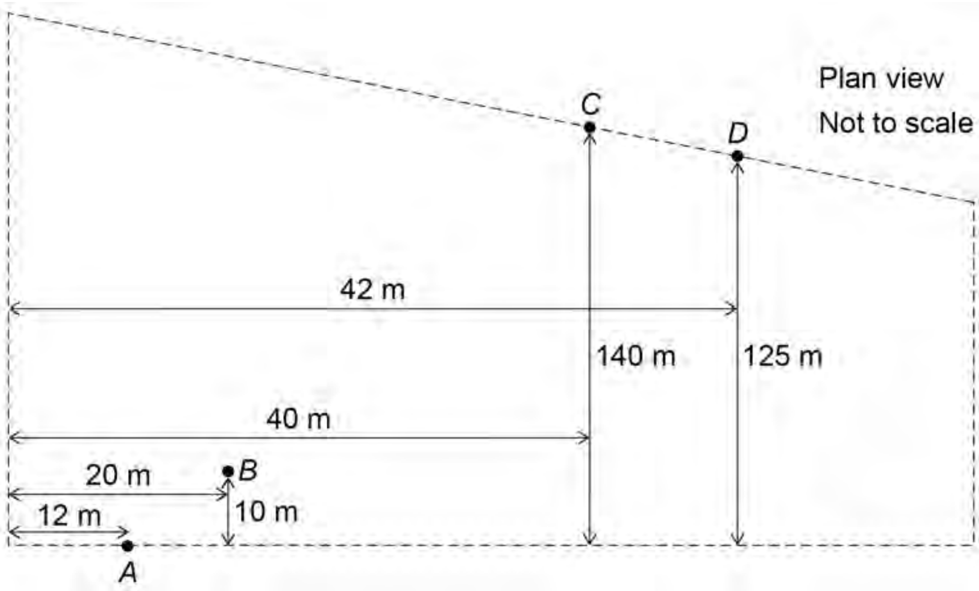
Turn over for the next question

7 A lighting engineer is setting up part of a display inside a large building. The diagram shows a plan view of the area in which he is working.

He has two lights, which project narrow beams of light.

One is set up at a point 3 metres above the point *A* and the beam from this light hits the wall 23 metres above the point *D*.

The other is set up 1 metre above the point *B* and the beam from this light hits the wall 29 metres above the point *C*.



7 (a) By creating a suitable model, show that the beams of light intersect.

[6 marks]

7 (b) Find the angle between the two beams of light.

[3 marks]

7 (c) State one way in which the model you created in part **(a)** could be refined.

[1 mark]

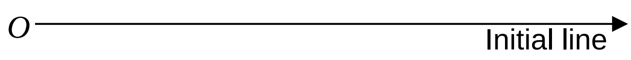
8 A curve has polar equation $r = 3 + 2\cos\theta$, where $0 \leq \theta < 2\pi$

8 (a) (i) State the maximum and minimum values of r.

[2 marks]

8 (a) (ii) Sketch the curve.

[2 marks]



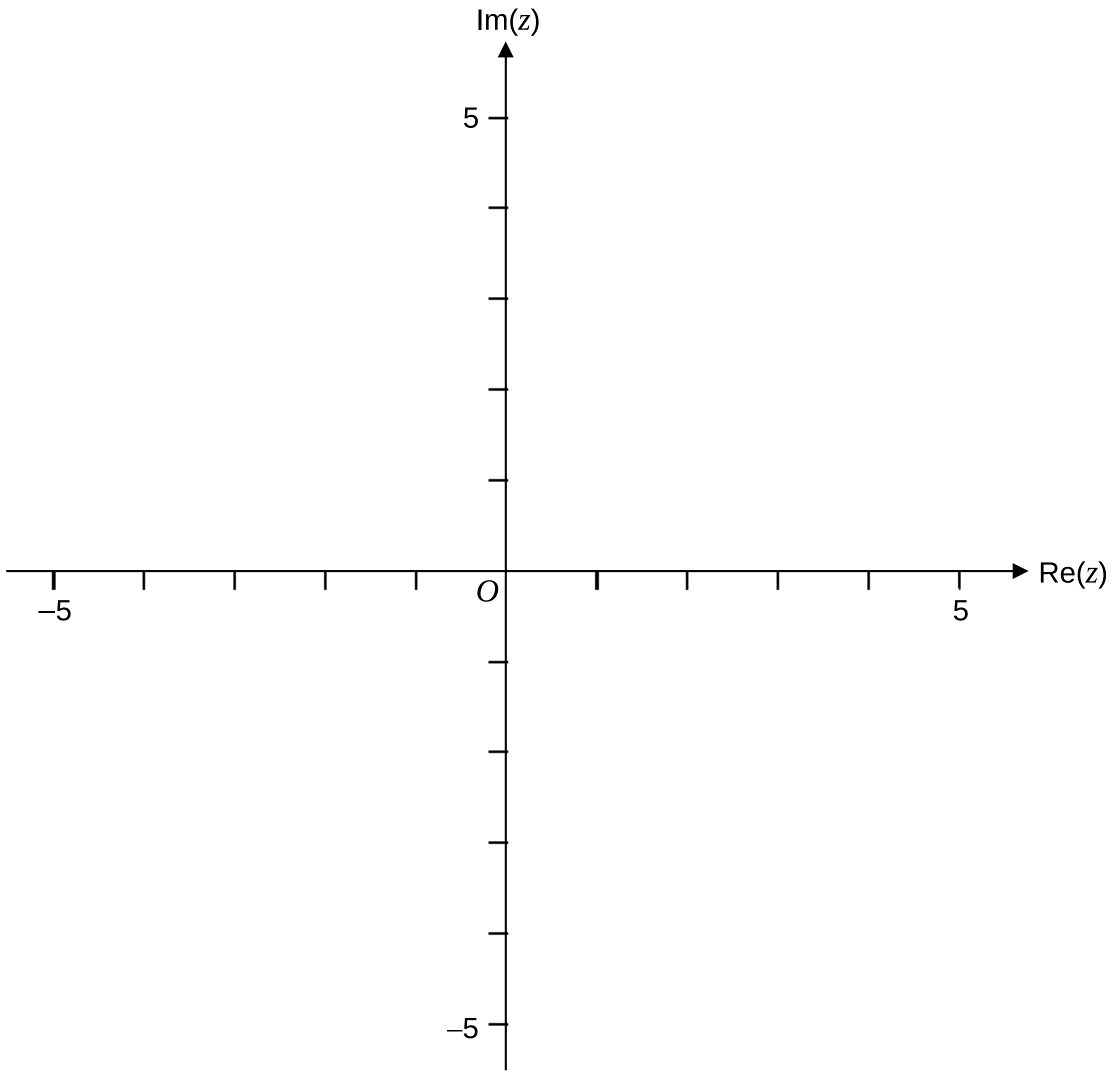
8 (b) The curve $r = 3 + 2\cos\theta$ intersects the curve with polar equation $r = 8\cos^2\theta$, where $0 \le \theta < 2\pi$

Find all of the points of intersection of the two curves in the form $[r, \theta]$.

[6 marks]

9 (a) Sketch on the Argand diagram below, the locus of points satisfying the equation $|z - 2| = 2$

[2 marks]



9 (b) Given that $|z - 2| = 2$ and $\arg(z - 2) = -\dfrac{\pi}{3}$, express z in the form $a + b\text{i}$, where a and b are real numbers.

[3 marks]

Turn over for the next question

10 (a) Prove that

$$6 + 3\sum_{r=1}^{n}(r + 1)(r + 2) = (n + 1)(n + 2)(n + 3)$$

[6 marks]

10 (b) Alex substituted a few values of n into the expression $(n + 1)(n + 2)(n + 3)$ and made the statement:

"For all positive integers n,

$$6 + 3\sum_{r=1}^{n}(r + 1)(r + 2)$$

is divisible by 12."

Disprove Alex's statement.

[2 marks]

Turn over for the next question

11 The equation $x^3 - 8x^2 + cx + d = 0$ where c and d are real numbers, has roots α, β, γ.

When plotted on an Argand diagram, the triangle with vertices at α, β, γ has an area of 8.

Given $\alpha = 2$, find the values of c and d.

Fully justify your solution.

[5 marks]

12 A curve, C_1 has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{5x^2 - 12x + 12}{x^2 + 4x - 4}$

The line $y = k$ intersects the curve, C_1

12 (a) (i) Show that $(k + 3)(k - 1) \geq 0$

[5 marks]

Question 12 continues on the next page

12 (a) (ii) Hence find the coordinates of the stationary point of C_1 that is a maximum point.

[4 marks]

12 (b) Show that the curve C_2 whose equation is $y = \dfrac{1}{\mathrm{f}(x)}$, has no vertical asymptotes.

[2 marks]

12 (c) State the equation of the line that is a tangent to both C_1 and C_2.

[1 mark]

END OF QUESTIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

AS
FURTHER MATHEMATICS
Paper 1

Time allowed: 1 hour 30 minutes

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- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

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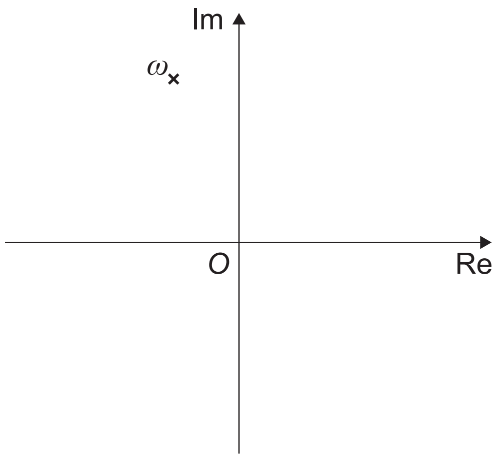
Answer **all** questions in the spaces provided.

1 The complex number ω is shown below on the Argand diagram.



Which of the following complex numbers could be ω?

Tick (✓) **one** box.

[1 mark]

$\cos(-2) + \mathrm{i}\sin(-2)$ ☐

$\cos(-1) + \mathrm{i}\sin(-1)$ ☐

$\cos(1) + \mathrm{i}\sin(1)$ ☐

$\cos(2) + \mathrm{i}\sin(2)$ ☐

2 Given that $\mathrm{f}(x) = 3x - 1$ find the mean value of $\mathrm{f}(x)$ over the interval $4 \leq x \leq 8$

Circle your answer.

[1 mark]

6 11 17 23

3 The matrix **M** represents a rotation about the x-axis.

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & \frac{\sqrt{3}}{2} \\ 0 & b & -\frac{1}{2} \end{bmatrix}$$

Which of the following pairs of values is correct?

Tick (✓) **one** box.

[1 mark]

$a = \frac{1}{2}$ and $b = \frac{\sqrt{3}}{2}$ ☐

$a = \frac{1}{2}$ and $b = -\frac{\sqrt{3}}{2}$ ☐

$a = -\frac{1}{2}$ and $b = \frac{\sqrt{3}}{2}$ ☐

$a = -\frac{1}{2}$ and $b = -\frac{\sqrt{3}}{2}$ ☐

4 The point $(2, -1)$ is invariant under the transformation represented by the matrix **N**

Which of the following matrices could be **N**?

Circle your answer.

[1 mark]

$\begin{bmatrix} 4 & 6 \\ 2 & 5 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 6 & 5 \\ 4 & 2 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$

Turn over for the next question

Do not write outside the box

5 Show that the vectors $\begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix}$ are perpendicular.

[2 marks]

6 Prove the identity

$$\cosh^2 x - \sinh^2 x = 1$$

[2 marks]

Do not write outside the box

7 Show that the Maclaurin series for $\ln(e + 2ex)$ is

$$1 + 2x - 2x^2 + ax^3 - ...$$

where a is to be determined.

[3 marks]

Turn over for the next question

Do not write outside the box

8 Stephen is correctly told that $(1 + \mathrm{i})$ and -1 are two roots of the polynomial equation

$$z^3 - 2\mathrm{i}z^2 + pz + q = 0$$

where p and q are complex numbers.

8 (a) Stephen states that $(1 - \mathrm{i})$ **must** also be a root of the equation because roots of polynomial equations occur in conjugate pairs.

Explain why Stephen's reasoning is wrong.

[1 mark]

8 (b) Find p and q

[5 marks]

Turn over for the next question

Do not write outside the box

9 (a) Use the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} r(r+3) = an(n+1)(n+b)$$

where a and b are constants to be determined.

[4 marks]

Do not write outside the box

9 (b) Hence, or otherwise, find a fully factorised expression for

$$\sum_{r=n+1}^{5n} r(r+3)$$

[3 marks]

10 Matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 3 & \mathrm{i}-1 \\ \mathrm{i} & 2 \end{bmatrix}$$

10 (a) Show that $\det \mathbf{A} = a + \mathrm{i}$ where a is an integer to be determined.

[2 marks]

10 (b) Matrix **B** is given by

$$\mathbf{B} = \begin{bmatrix} 14-2\mathrm{i} & b \\ c & d \end{bmatrix} \quad \text{and} \quad \mathbf{AB} = p\mathbf{I}$$

where b, c, $d \in \mathbb{C}$ and $p \in \mathbb{N}$

Find b, c, d and p

[6 marks]

Turn over for the next question

11 (a) Show that, for all positive integers r,

$$\frac{1}{(r-1)!} - \frac{1}{r!} = \frac{r-1}{r!}$$

[1 mark]

11 (b) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \frac{r-1}{r!} = a + \frac{b}{n!}$$

where a and b are integers to be determined.

[3 marks]

Do not write outside the box

Turn over for the next question

Do not write outside the box

12 The equation $x^3 - 2x^2 - x + 2 = 0$ has three roots. One of the roots is 2

12 (a) Find the other two roots of the equation.

[1 mark]

12 (b) Hence, or otherwise, solve

$$\cosh^3\theta - 2\cosh^2\theta - \cosh\theta + 2 = 0$$

giving your answers in an exact form.

[4 marks]

Do not write outside the box

13 Prove by induction that, for all integers $n \geq 1$

$$\sum_{r=1}^{n} 2^{-r} = 1 - 2^{-n}$$

[4 marks]

Turn over ▶

Do not write outside the box

14 Curve C_1 has equation

$$\frac{x^2}{16}+\frac{y^2}{4}=1$$

14 (a) Curve C_2 is a reflection of C_1 in the line $y=x$

Write down an equation of C_2

[1 mark]

14 (b) Curve C_3 is a circle of radius 4, centred at the origin.

Describe a single transformation which maps C_1 onto C_3

[2 marks]

14 (c) Curve C_4 is a translation of C_1
The positive x-axis and the positive y-axis are tangents to C_4

14 (c) (i) Sketch the graphs of C_1 and C_4 on the axes opposite. Indicate the coordinates of the x and y intercepts on your graphs.

[2 marks]

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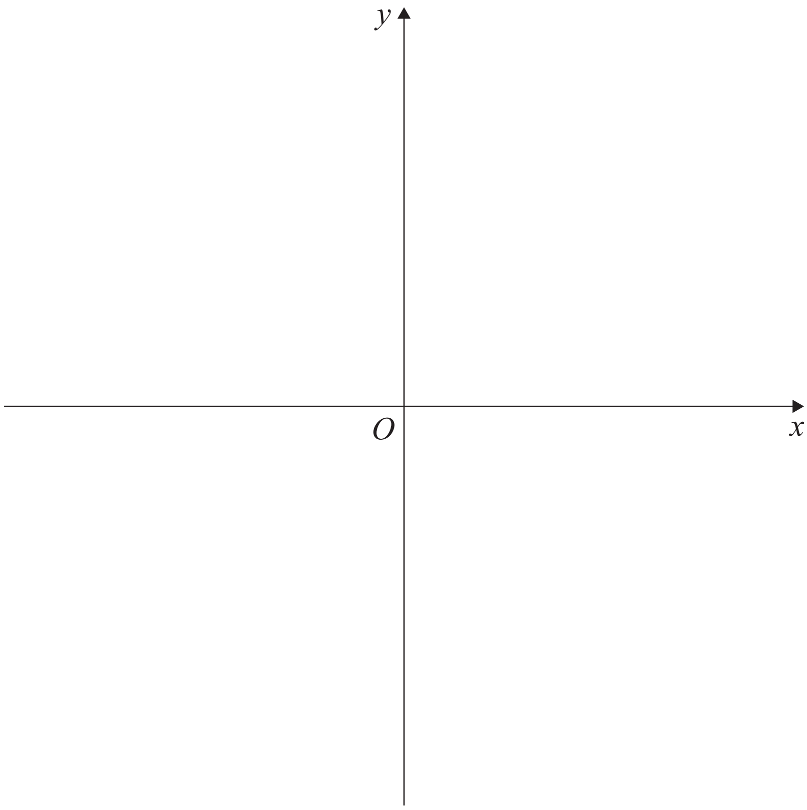


14 (c) (ii) Determine the translation vector.

[2 marks]

14 (c) (iii) The line $y = mx + c$ is a tangent to both C_1 and C_4
Find the value of m

[2 marks]

Turn over ▶

Do not write outside the box

15 Two submarines are travelling on different straight lines.
The two lines are described by the equations

$$\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} \quad \text{and} \quad \frac{x-5}{4} = \frac{y}{2} = 4 - z$$

15 (a) (i) Show that the two lines intersect.

[3 marks]

15 (a) (ii) Find the position vector of the point of intersection.

[1 mark]

Do not write outside the box

15 (b) Tracey says that the submarines will collide because there is a common point on the two lines.

Explain why Tracey is not necessarily correct.

[1 mark]

15 (c) Calculate the acute angle between the lines

$$\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} \quad \text{and} \quad \frac{x-5}{4} = \frac{y}{2} = 4 - z$$

Give your angle to the nearest 0.1°

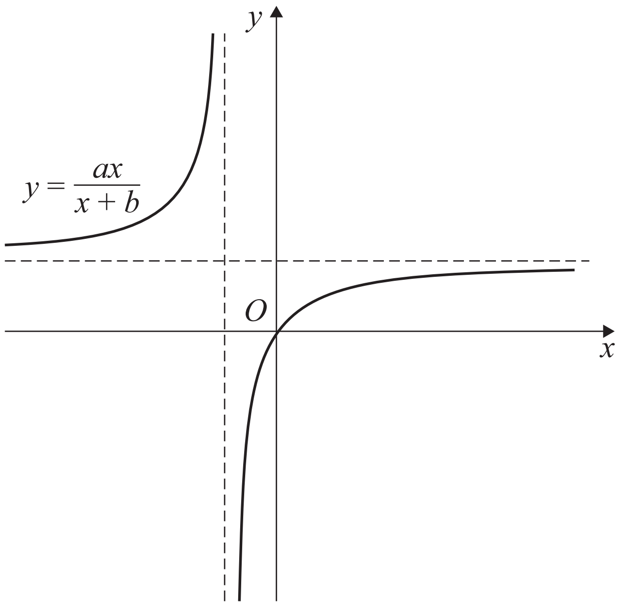
[3 marks]

Turn over ▶

Do not write outside the box

16 Curve C has equation $y = \dfrac{ax}{x+b}$ where a and b are constants.

The equations of the asymptotes to C are $x = -2$ and $y = 3$



16 (a) Write down the value of a and the value of b

[2 marks]

16 (b) The gradient of C at the origin is $\dfrac{3}{2}$

With reference to the graph, explain why there is exactly one root of the equation

$$\frac{ax}{x+b} = \frac{3x}{2}$$

[2 marks]

Do not write outside the box

16 (c) Using the values found in part **(a)**, solve the inequality

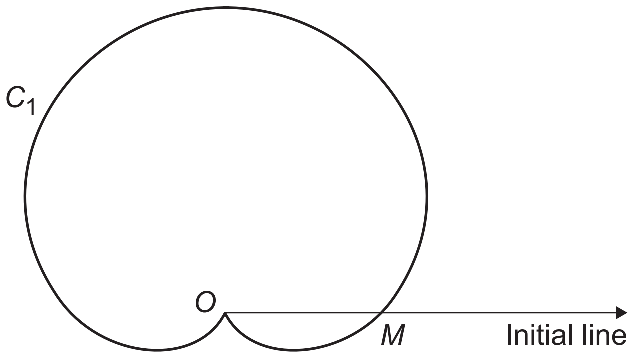
$$\frac{ax}{x+b} \leq 1 - x$$

[4 marks]

Turn over ▶

Do not write outside the box

17 The curve C_1 has polar equation $r = 2a(1 + \sin\theta)$ for $-\pi < \theta \leq \pi$ where a is a positive constant.



The point M lies on C_1 and the initial line.

17 (a) Write down, in terms of a, the polar coordinates of M

[1 mark]

17 (b) N is the point on C_1 that is furthest from the pole O

Find, in terms of a, the polar coordinates of N

[2 marks]

Do not write outside the box

17 (c) The curve C_2 has polar equation $r = 3a$ for $-\pi < \theta \leq \pi$
C_2 intersects C_1 at points P and Q

Show that the area of triangle NPQ can be written in the form

$$m\sqrt{3}a^2$$

where m is a rational number to be determined.

[5 marks]

Question 17 continues on the next page

Do not write outside the box

17 (d) On the initial line below, sketch the graph of $r = 2a(1 + \cos\theta)$ for $-\pi < \theta \leq \pi$

Include the polar coordinates, in terms of a, of any intersection points with the initial line.

[2 marks]

END OF QUESTIONS

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ANSWER IN THE SPACES PROVIDED

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I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 1

Monday 11 May 2020 Afternoon Time allowed: 1 hour 30 minutes

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Answer **all** questions in the spaces provided.

1 Express the complex number $1 - \mathrm{i}\sqrt{3}$ in modulus-argument form.

Tick (✓) **one** box.

[1 mark]

$2\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$ ☐

$2\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right)$ ☐

$2\left(\cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$ ☐

$2\left(\cos\left(-\frac{2\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{2\pi}{3}\right)\right)$ ☐

2 Given that $1 - \mathrm{i}$ is a root of the equation $z^3 - 3z^2 + 4z - 2 = 0$, find the other two roots.

Tick (✓) **one** box.

[1 mark]

$-1 + \mathrm{i}$ and -1 ☐

$1 + \mathrm{i}$ and 1 ☐

$-1 + \mathrm{i}$ and 1 ☐

$1 + \mathrm{i}$ and -1 ☐

Do not write outside the box

3 Given $(x-1)(x-2)(x-a) < 0$ and $a > 2$

Find the set of possible values of x.

Tick (✓) **one** box.

[1 mark]

$\{x : x < 1\} \cup \{x : 2 < x < a\}$ ☐

$\{x : 1 < x < 2\} \cup \{x : x > a\}$ ☐

$\{x : x < -a\} \cup \{x : -2 < x < -1\}$ ☐

$\{x : -a < x < -2\} \cup \{x : x > -1\}$ ☐

Turn over for the next question

Do not write outside the box

4 The matrices **A** and **B** are such that

$$\mathbf{A} = \begin{bmatrix} 2 & a & 3 \\ 0 & -2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -3 \\ -2 & 4a \\ 0 & 5 \end{bmatrix}$$

4 (a) Find the product **AB** in terms of a.

[2 marks]

4 (b) Find the determinant of **AB** in terms of a.

[1 mark]

Do not write outside the box

4 (c) Show that **AB** is singular when $a = -1$

[2 marks]

Turn over for the next question

Do not write outside the box

5 (a) Show that

$$r^2(r+1)^2 - (r-1)^2 r^2 = pr^3$$

where p is an integer to be found.

[1 mark]

Do not write outside the box

5 (b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

[3 marks]

Do not write outside the box

6 Anna has been asked to describe the transformation given by the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

She writes her answer as follows:

The transformation is a rotation about the x-axis through an angle of θ, where

$$\sin\theta = \frac{1}{2} \quad \text{and} \quad -\sin\theta = -\frac{1}{2}$$

$$\theta = 30°$$

Identify and correct the error in Anna's work.

[2 marks]

Do not write outside the box

7 Prove by induction that, for all integers $n \geq 1$, the expression $7^n - 3^n$ is divisible by 4

[4 marks]

Turn over ▶

Do not write outside the box

8 (a) Prove that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

[5 marks]

8 (b) Prove that the graphs of

$$y = \sinh x \quad \text{and} \quad y = \cosh x$$

do **not** intersect.

[3 marks]

Do not write outside the box

9 The quadratic equation $2x^2 + px + 3 = 0$ has two roots, α and β, where $\alpha > \beta$.

9 (a) (i) Write down the value of $\alpha\beta$.

[1 mark]

9 (a) (ii) Express $\alpha + \beta$ in terms of p.

[1 mark]

9 (b) Hence find $(\alpha - \beta)^2$ in terms of p.

[2 marks]

Do not write outside the box

9 (c) **Hence** find, in terms of p, a quadratic equation with roots $\alpha - 1$ and $\beta + 1$

[4 marks]

10 (a) Show that the equation

$$y = \frac{3x - 5}{2x + 4}$$

can be written in the form

$$(x + a)(y + b) = c$$

where a, b and c are integers to be found.

[3 marks]

10 (b) Write down the equations of the asymptotes of the graph of

$$y = \frac{3x - 5}{2x + 4}$$

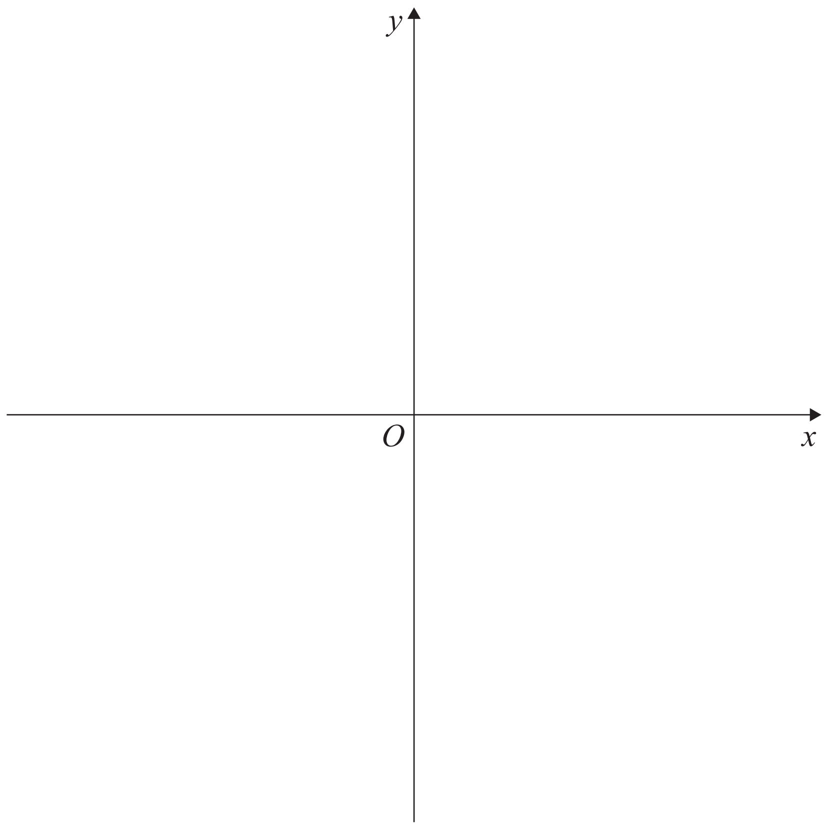
[2 marks]

Do not write outside the box

10 (c) Sketch, on the axes provided, the graph of

$$y = \frac{3x - 5}{2x + 4}$$

[3 marks]



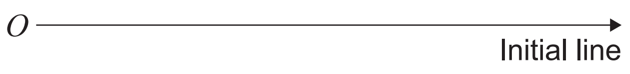
Do not write outside the box

11 Sketch the polar graph of

$$r = \sinh\theta + \cosh\theta$$

for $0 \leq \theta \leq 2\pi$

[3 marks]



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12 The mean value of the function f over the interval $1 \leq x \leq 5$ is m.

The graph of $y = \mathrm{g}(x)$ is a reflection in the x-axis of $y = \mathrm{f}(x)$.

The graph of $y = \mathrm{h}(x)$ is a translation of $y = \mathrm{g}(x)$ by $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Determine, in terms of m, the mean value of the function h over the interval $4 \leq x \leq 8$

[2 marks]

Turn over for the next question

13 Line l_1 has equation

$$\frac{x-2}{3} = \frac{1-2y}{4} = -z$$

and line l_2 has equation

$$\mathbf{r} = \begin{bmatrix} -7 \\ 4 \\ -2 \end{bmatrix} + \mu \begin{bmatrix} 12 \\ a+3 \\ 2b \end{bmatrix}$$

13 (a) In the case when l_1 and l_2 are parallel, show that $a = -11$ and find the value of b.

[4 marks]

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13 (b) In a **different** case, the lines l_1 and l_2 intersect at exactly one point, and the value of b is 3

Find the value of a.

[5 marks]

Turn over ▶

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14 (a) Given

$$\frac{x+7}{x+1} \leq x+1$$

show that

$$\frac{(x+a)(x+b)}{x+c} \geq 0$$

where a, b, and c are integers to be found.

[4 marks]

14 (b) Briefly explain why this statement is incorrect.

$$\frac{(x+p)(x+q)}{x+r} \geq 0 \Leftrightarrow (x+p)(x+q)(x+r) \geq 0$$

[1 mark]

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14 (c) Solve

$$\frac{x+7}{x+1} \leq x+1$$

[2 marks]

Turn over for the next question

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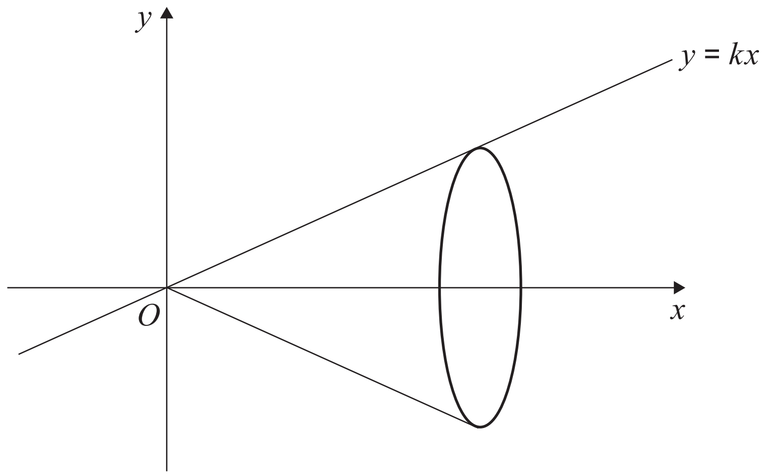
15 A segment of the line $y = kx$ is rotated about the x-axis to generate a cone with vertex O.

The distance of O from the centre of the base of the cone is h.

The radius of the base of the cone is r.



15 (a) Find k in terms of r and h.

[1 mark]

15 (b) Use calculus to prove that the volume of the cone is

$$\frac{1}{3}\pi r^2 h$$

[3 marks]

16 **A** and **B** are non-singular square matrices.

16 (a) Write down the product $\mathbf{AA}^{-1}$ as a single matrix.

[1 mark]

16 (b) **M** is a matrix such that $\mathbf{M} = \mathbf{AB}$.

Prove that $\mathbf{M}^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

[3 marks]

17 The polar equation of the circle C is

$$r = a(\cos\theta + \sin\theta)$$

Find, in terms of a, the radius of C.

Fully justify your answer.

[4 marks]

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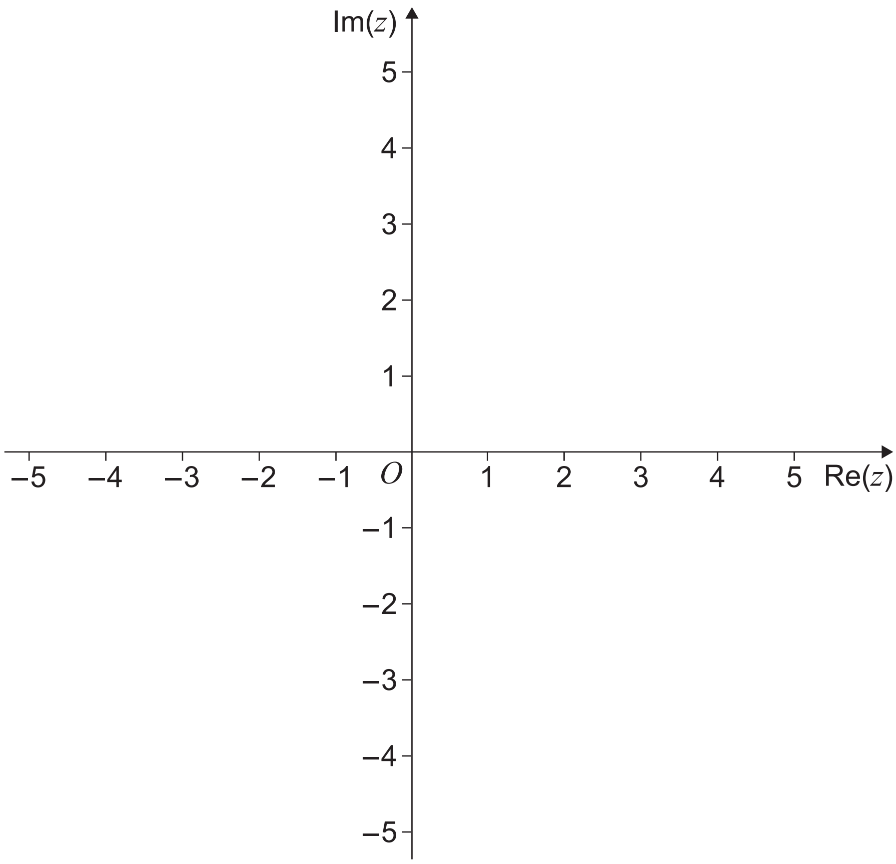
18 The locus of points L_1 satisfies the equation $|z| = 2$

The locus of points L_2 satisfies the equation $\arg(z + 4) = \frac{\pi}{4}$

18 (a) Sketch L_1 on the Argand diagram below.

[1 mark]



18 (b) Sketch L_2 on the Argand diagram above.

[1 mark]

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18 (c) The complex number $a + \mathrm{i}b$, where a and b are real, lies on L_1

The complex number $c + \mathrm{i}d$, where c and d are real, lies on L_2

Calculate the least possible value of the expression

$$(c - a)^2 + (d - b)^2$$

[3 marks]

END OF QUESTIONS

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