[CS2102]

Skeletonization by Thinning

Approaches:

- *Zhang-Suen*
- *Guo-Hall*

Diego Enciso

Juan Galvez

Luis Jáuregui

Macarena Oyague

Miguel Yurivilca

01. Skeletonization by Thinning

Explanation of the concept

02. Applications

Medical and Biology fields

03. Zhang Suen Approach

Pseudocode and theoretical Analysis

04. Guo Hall Approach

Pseudocode and theoretical Analysis

05. Analysis

Of both approaches

06. Experiments and Results

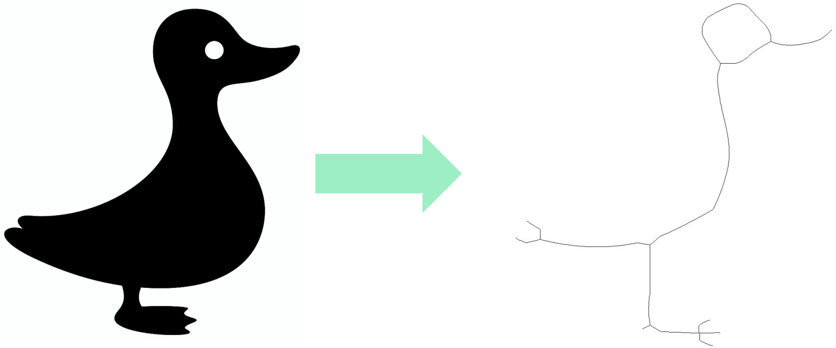
Of both approaches

Skeletonization by Thinning

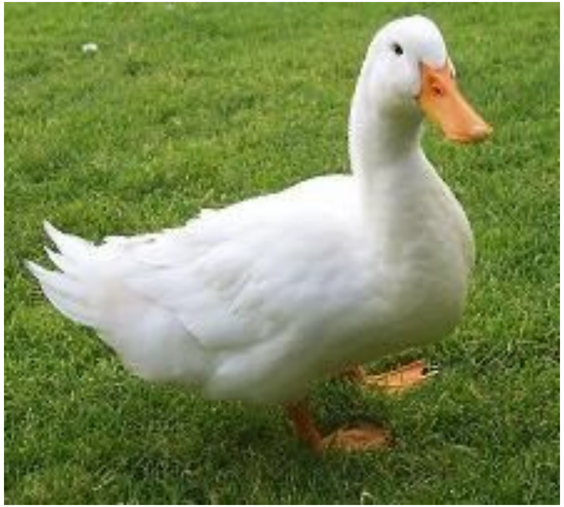
01

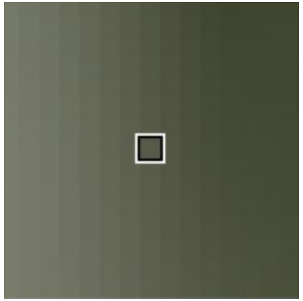
Explanation of the concept

Skeletonization is a procedure to transform the width of an image into just one single pixel.



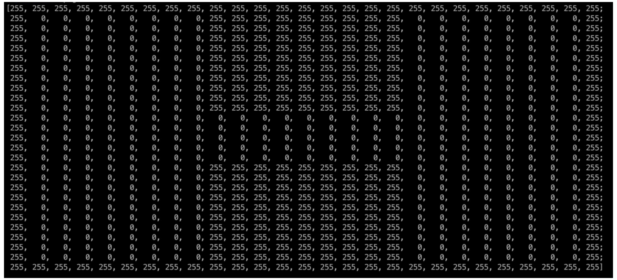
An image is composed by pixels





An image contains a pattern, and that is the reason why an algorithm use that pattern to make an image transformation and change the value of the numbers.

Skeletonization



Receive: a binary pattern/image

Return: a skeleton

Skeletonization by thinning

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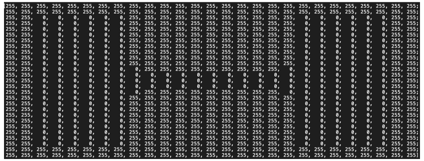
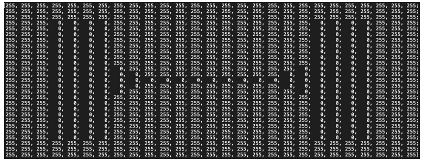
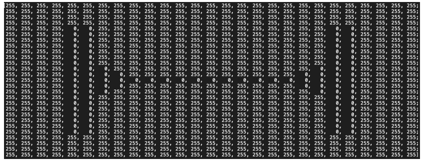
Skeletonization Algorithm guarantee:

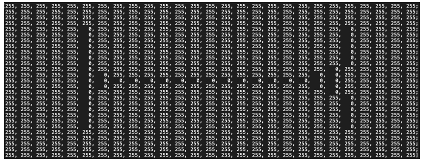
Preservation the topological and geometric properties of the object in the image

Skeletonization by thinning









The border points of a binary object that satisfy certain topological and geometric constraints are deleted in iteration steps.

The entire process is then repeated until only the skeleton is left .

02

Applications

Medical and Biology fields

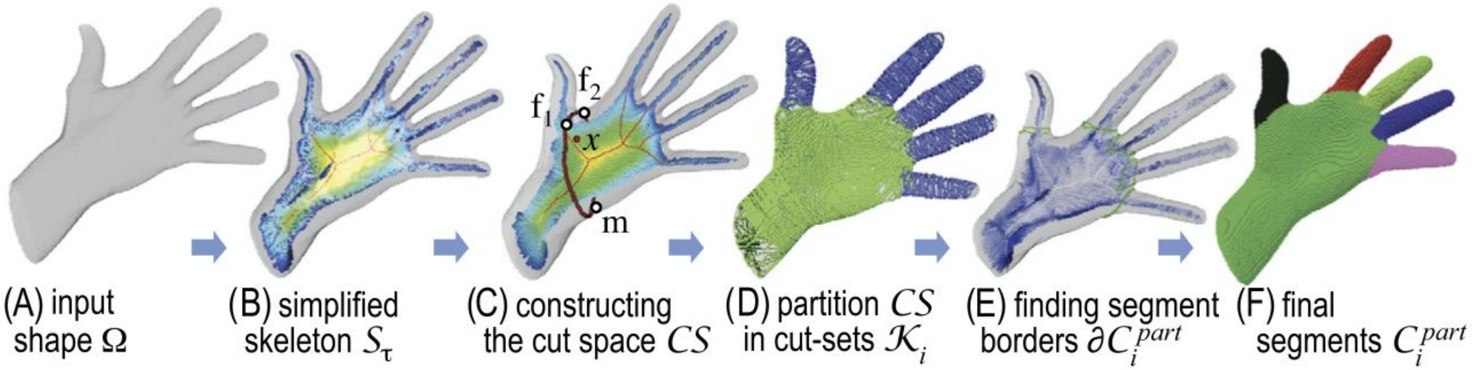
Medicine

Analysis of blood vessels

Skeletonization process

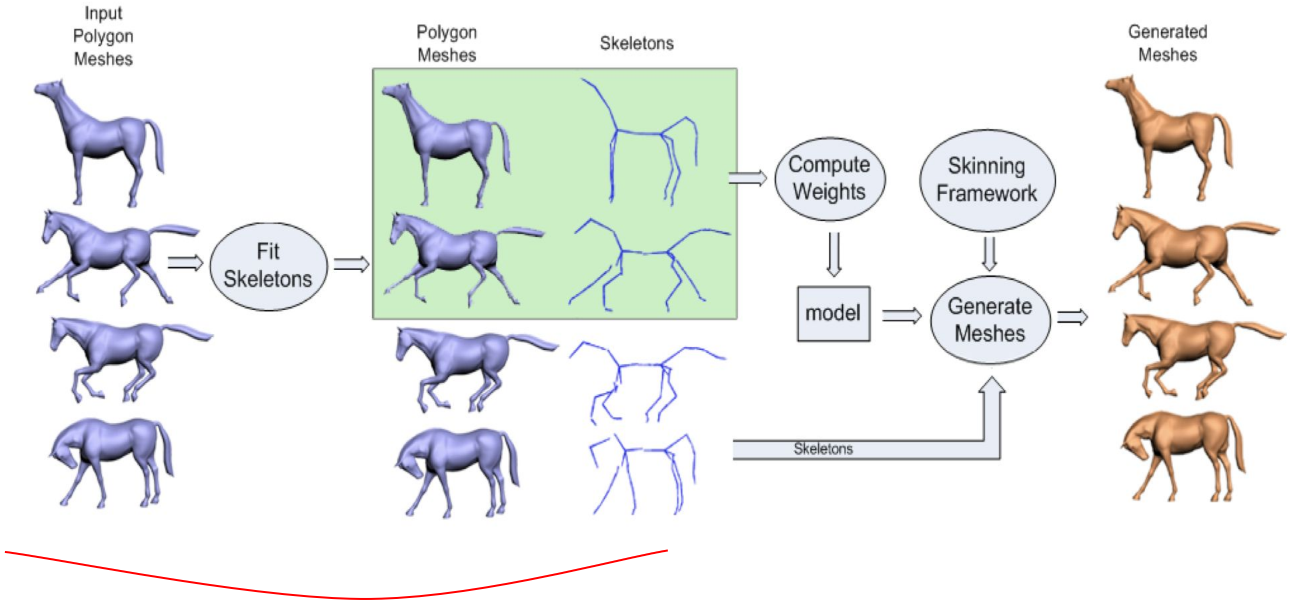
Biology

Voxel cut-space segmentation VCS



Skeletonization process

Generation of mesh approximations



Skeletonization process

Zhang-Suen Approach

03

Pseudocode and theoretical analysis

Algorithm

Verify conditions (A)

While points are deleted do

For all pixels p(i,j) = P1 do

if 2 <= B(P1) <= 6 and A(P1) = 1

P2 x P4 x P6 = 0

P4 x P6 x P8 = 0

then

delete pixel p(i,j)

For all pixels p(i,j) = P1 do

if 2 <= B(Pi) <= 6 and A(p1) = 1

P2 x P4 x P8 = 0

P2 x P6 x P8 = 0

then

delete pixel p(i,j)

$$
\begin{array}{ll}
(a) & 2 \leq B(P_1) \leq 6 \\
(b) & A(P_1) = 1 \\
(c) & P_2 * P_4 * P_6 = 0 \\
(d) & P_4 * P_6 * P_8 = 0
\end{array}
$$

where $A(P_1)$ is the number of 01 patterns in the sequence $P_2, P_3, \cdots, P_8, P_9, P_2$ that are the neighbours of P_1 and $B(P_1)$ is the number of nonzero neighbours of P_1.

Guo-Hall Approach

04

Pseudocode and theoretical analysis

Algorithm

```
1.  While points are deleted do
2.      For all pixels p(i,j) do
3.          if  (a) C(p1) = 1
                (b) 2 <= N(p1) <= 3
                (c) Apply one of the following:
        (p2 | p3 | !p5) & p4 = 0 in odd iterations
        (p6 | p7 | !p9) & p8 = 0 in even iterations
            then
4.              Delete pixel p(i,j)
5.          end if
6.      end for
7.  end while
```

P9	P2	P3
P8	P1	P4
P7	P6	P5

```
Where:
C(p1)  = !p2 & (p3 | p4) + !p4 & (p5 | p6) + !p6 & (p7 | p8) + !p8 & (p9 | p2)
N1(p1) = (p9 | p2) + (p3 | p4) + (p5 | p6) + (p7 | p8)
N2(p1) = (p2 | p3) + (p4 | p5) + (p6 | p7) + (p8 | p9)
N(p1)  = MIN[N1, N2]
```

05

Analysis

Of both approaches

Analysis

Theoretical:

We considered the worst case to be when all pixels are distributed in such way that only one pixel is deleted in each iteration, which results in approximately (rows-1*cols-1) iterations. In each one, we traverse all pixels, thus, we get a bound of $O(n^4)$. The best case is when 0 or 1 pixel is deleted, resulting in 1 iteration and a bound of $\Omega(n^2)$.

Empirical:

Guo Hall approach was presented as an improvement of the former Zhang Suen, hence, it runs faster as we will see in results chart below, even though, it does way more iterations than the other. Both are based in a set of 2 sub-iterations, but consider different criteria to determine if a pixel is deleted or not.

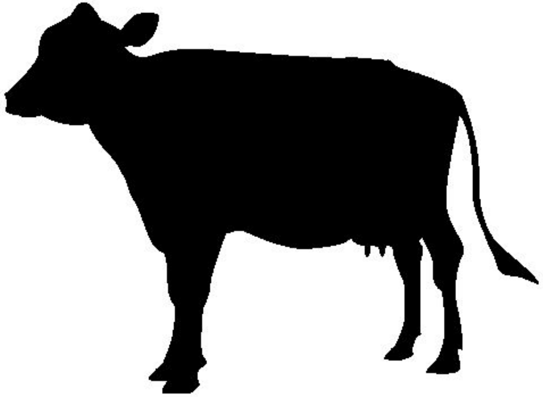
CPU 1	Cow	Duck
Zhang-Suen	5.05005 sec	24.4098 sec
Gou-Hall	2.40053 sec	16.4013 sec

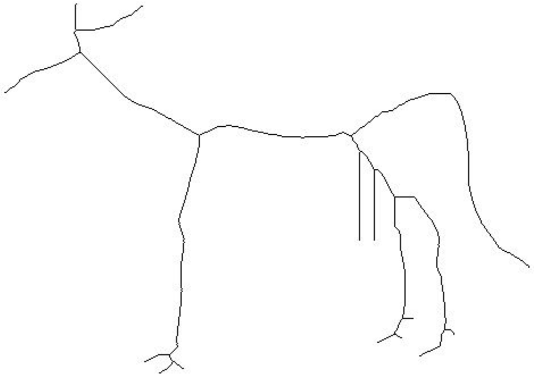
CPU 2	Cow	Duck
Zhang-Suen	12.3745 sec	54.2858 sec
Gou-Hall	2.5839 sec	15.8636 sec

Experiments and results

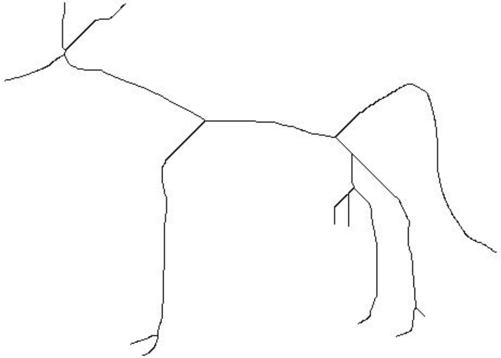
06

Pseudocode and theoretical approaches

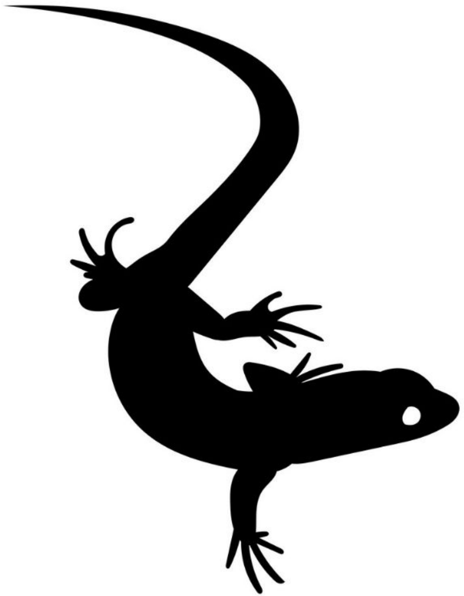


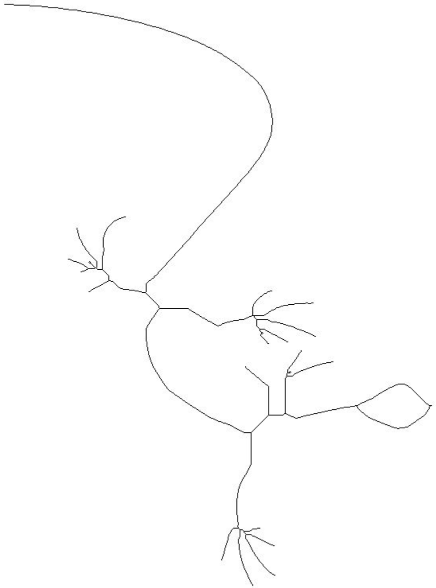


Guo Hall

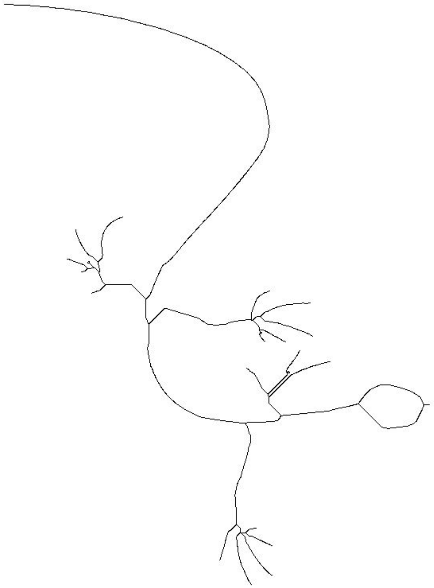


Zhang Suen

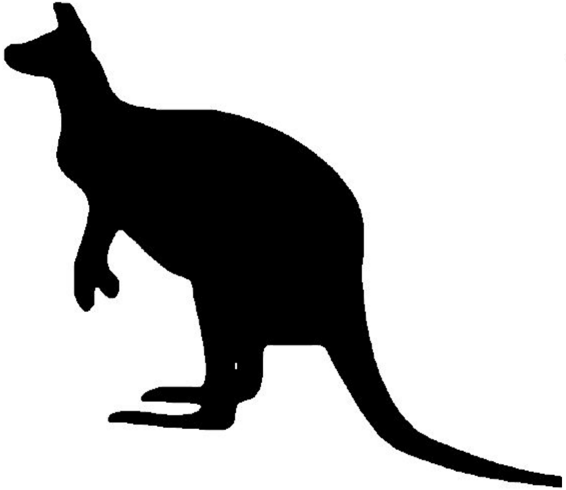


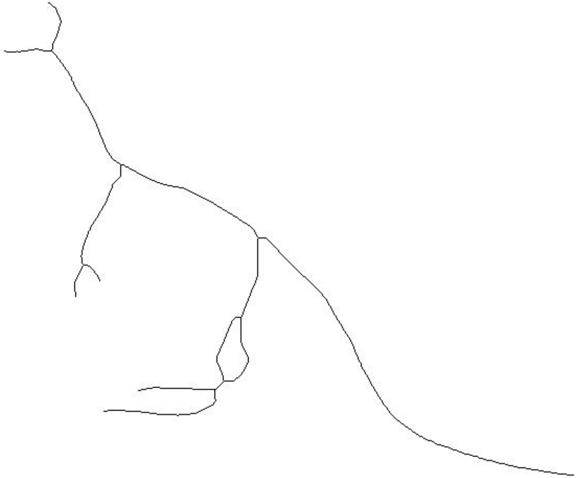


Guo Hall

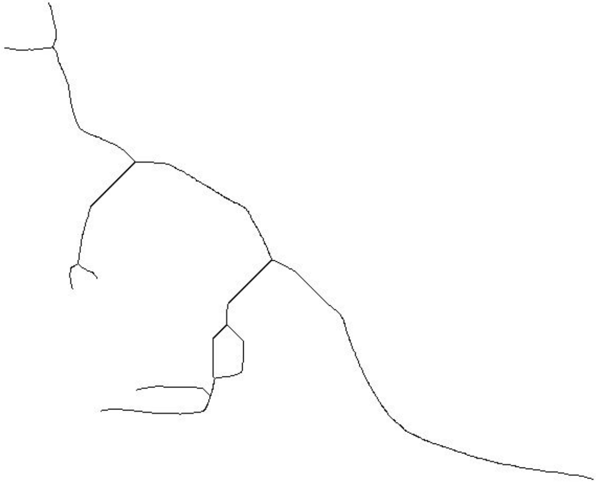


Zhang Suen





Guo Hall



Zhang Suen