1.)

```
void f1(int n)
{
    int i = 2;
    while (i < n) {
        /* O(1) */
        i = i * i;
    }
}
```

$\Theta(1)\left[\sum_{i=2}^{n}\right]$

k	i		
0	2	$\Rightarrow$	2^1
1	4	$\Rightarrow$	2^2
2	$2^2 \cdot 2^2$	$\Rightarrow$	2^4
3	$2^4 \cdot 2^4$	$\Rightarrow$	2^8
4	$2^8 \cdot 2^8$	$\Rightarrow$	2^{16}

$2^1 \cdot 2^2 \cdot 2^4 \cdot 2^8 \cdots \Rightarrow 2^{2^k}$

$\Rightarrow 2^{2^k} < n$

$2^k < \log(n)$

$k < \log(\log(n)) \Rightarrow$ How many

times the loop will run

$\Rightarrow T(n) = O(1 + \log(\log(n)))$

$\Rightarrow T(n) = O(\log(\log(n)))$

2i) `void f2(int n){`

```
for (int i=1; i<=n; i++){
    if ((i % (int)sqrt(n)) == 0){
        for (int k=0; k<pow(i,3); k++){
            O(1)
        }
    }
}
```

$n = 16$

i	k
4	4^3
8	8^3
12	12^3
16	16^3

# of times inner loop execs when $n = 16$:

$4^3 + 8^3 + 12^3 + 16^3$

$\Rightarrow (1 \cdot \sqrt{16})^3 + (2\sqrt{16})^3 + (3\sqrt{16})^3 + (4\sqrt{16})^3$

$i\sqrt{n}$

$$\Rightarrow \sqrt{n^3} \cdot \sum_{k=1}^{\sqrt{n}} \Theta\left(k\sqrt{n}\right)^3$$

$$\Rightarrow \Theta\left(\sqrt{n}\right)^4 \cdot \left(\sqrt{n}\right)^3$$

$$\Rightarrow T(n) = \sqrt{n^7}$$ //

3-)

```
for (int i = 1; i <= n; i++) {
  for (int k = 1; k <= n; k++) {
    if (A[k] == i) {
      for (int m = 1; m <= n; m = m + m) {
        O(1)
      }
    }
  }
}
```

$n = 20$

j	m
1	1
2	2

$\Rightarrow \log(2n)$

3	4
4	8
5	16
6	32

$\Theta \log(n)$

$$T(n) = \sum_{i=1}^{n} \left(\sum_{k=1}^{n} \left(\Theta(1) + O\left(\sum_{m=1}^{n} \frac{1}{m} \right) \right) \right)$$

$$\Rightarrow \sum_{i=1}^{n} \left(\sum_{k=1}^{n} \Theta(1) + \Theta \sum_{k=1}^{n} \log(n) \right)$$

$$\Rightarrow \sum_{i=1}^{n} n + \sum_{i=1}^{n} n \cdot \log(n)$$

$$\Rightarrow \Theta n^2 + n^2 \log(n)$$

$$T(n) \Rightarrow \Theta n^2$$

4:)

```
int f (int n) {

    int *a = new int [10];
    int size = 10;

    for ( int i=0; i<n ; i++) {
        if (i == size ) {
            int newsize = 3*size/2;
            int *b = new int [newsize];
            for (int j=0 ; j< size ; j++ ) b[j]=a[j];
            delete a;
            a= b;
            size = newsize;
        }
        a[i]= i*i;
    }
}
```

n = 20

i -> 0 -> 20

size = 10

new size = 15

$$T(n) = \sum_{i=0}^{n-1} + \sum_{i=0}^{\log_{\frac{3}{2}}\left(\frac{n}{10}\right)} \left(\sum_{j=0}^{\text{size}-1} \Theta(1) \right)$$

from the inst next size, $10 \cdot \left(\frac{3}{2}\right)^k < n$

$$\left(\frac{3}{2}\right)^k < \frac{n}{10}$$

$$k < \log_{\frac{3}{2}}\left(\frac{n}{10}\right)$$

$$k = \log_{\frac{3}{2}}\left(\frac{n}{10}\right)$$

$$\Rightarrow T(n) = \sum_{i=0}^{n-1} + \sum_{i=0}^{k} \left(\sum_{j=0}^{\text{size}-1} \Theta(1) \right)$$

$$\Rightarrow \sum_{j=0}^{k} 10\left(\frac{3}{2}\right)^j \Rightarrow 10 \cdot \sum_{j=0}^{\log_{\frac{3}{2}}\left(\frac{n}{10}\right)} \left(\frac{3}{2}\right)^j$$

$$\Rightarrow 10 \cdot \frac{3}{2} \cdot \log_{\frac{3}{2}}\left(\frac{n}{10}\right)$$

$$\Rightarrow \Theta(n)$$