

1.)

Total ways to ask 8 questions to 15 ppl.: 15^8

of ways to select 8 students from 15 in order: $15P8$

probability no student will have to answer more than 1 question: $15P8 / 15^8$

$\Rightarrow 0.107 //$

2:) 0 - 100 all 2 digit

So not possible to fake even with 3 odd.

100 - 1000

No. of required integers: $5 \cdot 4 \cdot 5 = 100$

1000 - 10000

No. of required integers: $5 \cdot 4 \cdot 7 \cdot 5 = 700$

10000 - 99999

No. of required integers: $5 \cdot 4 \cdot 7 \cdot 6 \cdot 5 = 4200$

Total # of required int. = 5000

Total # of int from 0 - 99999 = 10^5

probability of getting a req. integer = $5000/10^5 = 0.05$

Generate 8 numbers

$$n = 8 \quad p = 0.05$$

$$P(X=5) = {}^8C_5 (0.05)^5 (1-0.05)^3$$
$$P(X=5) = 1.5004 \cdot 10^{-5}$$

3) probability of Value $\geq 4 = 5/6$

here, $P(A) = P(\geq 2 \text{ dice show } \geq 4)$

$$\Rightarrow P(X=2) + P(X=3) = {}^3C_2 \cdot \left(\frac{1}{5}\right)^2 \cdot \left(\frac{1}{5}\right) + {}^3C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{1}{5}\right)$$
$$= \frac{1}{2}$$

$P(B) = P(\text{all 3 shows same number})$

$$\Rightarrow \frac{1}{6^3} = 1/216$$

$P(A \cap B) = P(\text{all 3 same and } \geq 4)$

$$\Rightarrow \frac{1}{6^3} + \frac{1}{6^3} + \frac{1}{6^3} = 1/72$$

Since $P(A) \cdot P(B) = 1/72 = P(A \cap B)$

A, B are independent

4.)

Probability of flush on 5-card hand

$$\Rightarrow p = \frac{4 \cdot 13C5}{52C5} = 0.00198$$

$$E[X] = \frac{1}{p} = 504.8486$$

$$5.) P(\text{win} \mid \text{Superstar}) = 0.7$$

$$P(\text{win} \mid \text{No Super}) = 0.5$$

$$P(\text{Super plays}) = 0.75 \quad \text{for next 5 games}$$

therefore,

$$P(\text{win } 4/5 \mid \text{Super plays}) = 5C4 \cdot (0.7)^4 \cdot 0.3 = 0.36015$$

$$P(\text{win } 4/5 \mid \text{Super no plays}) = 5C4 \cdot (0.5)^5 = 0.15625$$

$$P(\text{win } 4/5) = 0.15625 \cdot 0.25 + 0.36015 \cdot 0.75 = 0.309775$$

$$\text{therefore, } P(\text{super plays} \mid \text{win } 4/5) = 0.36015 \cdot 0.75 / 0.309775$$

$$\Rightarrow 0.87377$$