

[SKPB - ITS]

EVALUASI AKHIR SEMESTER BERSAMA GASAL 2023/2024

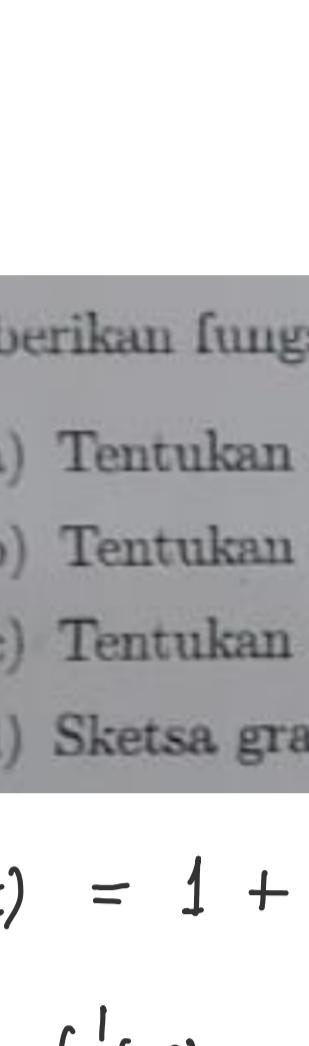
Mata Kuliah/SKS : Kalkulus 1 (SM234101) / 3 SKS
 Hari, Tanggal : Selasa, 12 Desember 2023
 Waktu : 09.00-10.40 WIB (100 menit)
 Sifat : Tertutup
 Kelas : 24-30

Diberikan 5 soal, dengan bobot nilai masing-masing soal sama dan boleh dikerjakan tidak berurutan.

Tuliskan: Nama, NRP, dan Nomor Kelas pada lembar jawaban Anda.

DILARANG MEMBAWA/MENGGUNAKAN KALKULATOR DAN ALAT KOMUNIKASI
 DILARANG MEMBERIKAN/MENERIMA JAWABAN SELAMA UJIAN
 "Setiap tindak kecurangan akan mendapat sanksi akademik."

1. Pasir dituangkan dari suatu luncuran akan membentuk berupa kerucut dengan ketinggian selalu sama dengan diameter. Jika ketinggian bertambah dengan laju 8 m/menit, dengan laju berapa pasir dituangkan dari peluncuran jika ketinggian tumpukan 12 m?



$$\begin{aligned} \text{Diketahui : } & h = d, \text{ maka} \\ & h = 2r \rightarrow r = \frac{h}{2} \\ & \frac{dh}{dt} = 8 \text{ m/menit} \\ & h = 12 \text{ m} \end{aligned}$$

$$\text{Ditanya : } \frac{dV}{dt} = ? \quad (\text{laju volume pasir yang dituangkan})$$

$$\Rightarrow \text{Volume Kerucut} = \frac{1}{3} \times \text{luas alas} \times \text{tinggi}$$

$$V = \frac{1}{3} \cdot \pi r^2 \cdot h$$

$$V = \frac{1}{3} \cdot \pi \left(\frac{h}{2}\right)^2 \cdot h$$

$$V = \frac{\pi}{12} \cdot h^3$$

$$\Rightarrow \frac{d}{dt} [V] = \frac{d}{dt} \left[\frac{\pi}{12} \cdot h^3 \right]$$

$$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{h^2}{4} \cdot \frac{dh}{dt}$$

$$= \frac{12^2}{4} \cdot 8$$

$$= 144 \times 2 \\ = 288 \text{ m}^3/\text{menit} \quad \checkmark$$

2. Diberikan fungsi $f(x) = 1 + (1 - 2x)^3$.

(a) Tentukan selang dimana fungsi $f(x)$ naik atau turun

(b) Tentukan titik ekstrim relatif fungsi tersebut

(c) Tentukan selang kecekungan fungsi $f(x)$ dan titik belok (jika ada)

(d) Sketsa grafiknya.

$$f(x) = 1 + (1 - 2x)^3$$

$$\begin{aligned} \text{(a)} \quad f'(x) &= 0 \\ 3(1-2x)^2 \cdot (-2) &= 0 \\ -6(1-2x)^2 &= 0 \\ (1-2x)^2 &= 0 \\ 1-2x &= 0 \\ x &= \frac{1}{2} \end{aligned}$$

$$\text{Uji titik k : } f'(x) = -6(1-2x)^2$$

f. turun : $(-\infty, +\infty)$

$$\text{(b)} \quad \text{Dari hasil (a) diperoleh titik stasioner } x = \frac{1}{2},$$

$$\text{maka } y = f\left(\frac{1}{2}\right)$$

$$= 1 + (1 - 2 \cdot \frac{1}{2})^3$$

$$= 1$$

$$\left(\frac{1}{2}, 1\right) \quad \checkmark$$

$$\text{(c)} \quad f'(x) = -6(1-2x)^2$$

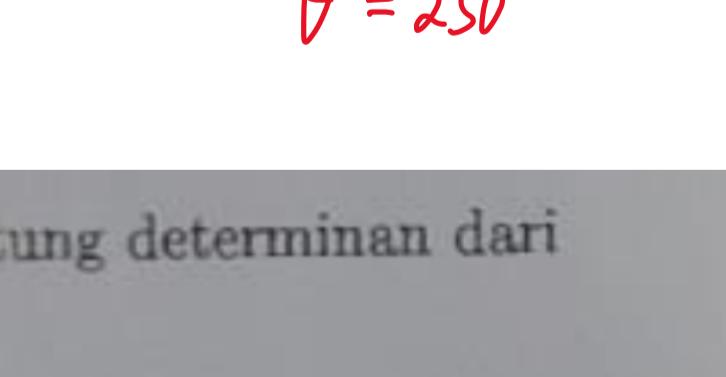
$$f''(x) = 0$$

$$-6 \cdot 2(1-2x) \cdot (-2) = 0$$

$$24(1-2x) = 0$$

$$1-2x = 0$$

$$x = \frac{1}{2}$$



$$\text{Uji titik k : } f''(x) = 24(1-2x)$$

cekung keatas : $(-\infty, \frac{1}{2})$

cekung kebawah : $(\frac{1}{2}, +\infty)$

Titik belok terjadi di $x = \frac{1}{2} \rightarrow y = f\left(\frac{1}{2}\right) = 1$

$$\left(\frac{1}{2}, 1\right) \quad \checkmark$$

d). Sketsa Grafik

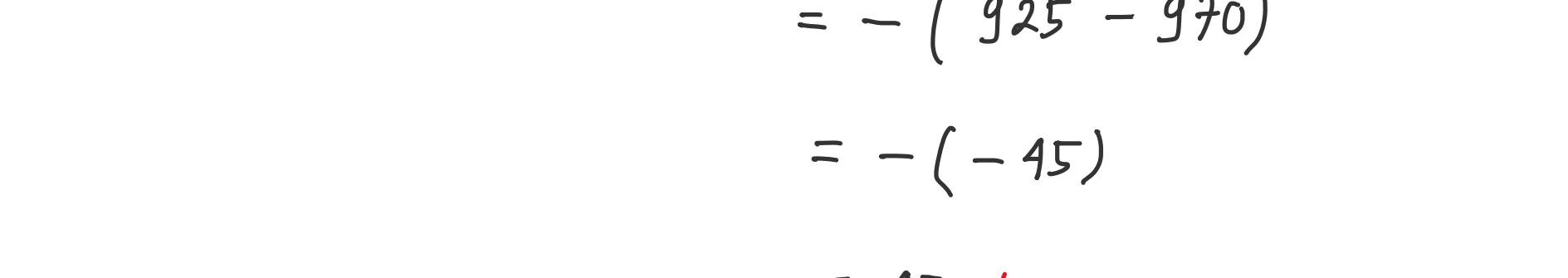
$$f(x) = 1 + (1 - 2x)^3$$

\Rightarrow Titik potong thd sb.y, maka $x = 0$

$$y = f(0)$$

$$= 1 + (1 - 2 \cdot 0)^3$$

$$(0, 2)$$



3. Diberikan $F(x) = \int_4^x \sqrt{t^2 + 9} dt$. Dapatkan $F(4)$, $F'(4)$ dan $F''(4)$.

$$\Rightarrow F(4) = \int_4^4 \sqrt{t^2 + 9} dt = 0$$

$$\Rightarrow F'(4) = ?$$

$$F'(x) = \frac{d}{dx} \left[\int_4^x \sqrt{t^2 + 9} dt \right] = \sqrt{x^2 + 9}$$

$$F'(4) = \sqrt{4^2 + 9} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$\Rightarrow F''(4) = ?$$

$$F'(x) = (x^2 + 9)^{\frac{1}{2}}$$

$$F''(x) = \frac{1}{2} (x^2 + 9)^{-\frac{1}{2}} \cdot 2x = \frac{x}{(x^2 + 9)^{\frac{1}{2}}} = \frac{x}{\sqrt{x^2 + 9}}$$

$$F''(4) = \frac{4}{\sqrt{16+9}} = \frac{4}{\sqrt{25}} = \frac{4}{5}$$

4. Dapatkan semua bilangan kompleks z yang memenuhi persamaan $z^3 = \sqrt{3} + i$, serta sketsa semua bilangan kompleks tersebut.

$$z^3 = \sqrt{3} + i$$

$$\Rightarrow r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$z^3 = r(\cos \theta + i \sin \theta)$$

$$\Rightarrow \theta = ?$$

$$\tan \theta = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan \theta = \frac{1}{3} \sqrt{3} \rightarrow \text{Kuatran?}$$

$$\theta = 30^\circ$$

$$\theta = \frac{\pi}{6}$$

$$z^3 = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$z = 2^{\frac{1}{3}} \left[\cos \left(\frac{\pi}{6} + 2k\pi \right) + i \sin \left(\frac{\pi}{6} + 2k\pi \right) \right]; k = 0, 1, 2$$

$$z_0 = 2^{\frac{1}{3}} \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$\theta = 10^\circ$$

$$z_1 = 2^{\frac{1}{3}} \left(\cos \frac{13\pi}{6} + i \sin \frac{13\pi}{6} \right)$$

$$\theta = 130^\circ$$

$$z_2 = 2^{\frac{1}{3}} \left(\cos \frac{25\pi}{6} + i \sin \frac{25\pi}{6} \right)$$

$$\theta = 250^\circ$$

$$\theta = 250^\circ$$