

SKPB - ITS

EVALUASI AKHIR SEMESTER BERSAMA GASAL 2023/2024

Mata Kuliah/SKSK : Kalkulus 1 (SM234101) / 3 SKS
 Hari, Tanggal : Selasa, 12 Desember 2023
 Waktu : 07.00-08.40 WIB (100 menit)
 Sifat : Ter tutup
 Kelas : 17-23

Diberikan 5 soal, dengan bobot nilai masing-masing soal sama dan boleh dikerjakan tidak berurutan.
 Tuliskan: Nama, NRP, dan Nomor Kelas pada lembar jawaban Anda.

DILARANG MEMBACA/MENGUNAKAN KALKULATOR DAN ALAT KOMUNIKASI
 DILARANG MEMBERIKAN/MENERIMA JAWABAN SELAMA UJIAN
 "Setiap tindak kecurangan akan mendapat sanksi akademik."

1. Kaleng silinder dengan bagian atas terbuka berisi 1000 cm^3 cairan. Tentukan tinggi dan jari-jari yang meminimumkan jumlah material yang diperlukan untuk membuat kaleng tersebut.



Luas $\rightarrow L$

$$L = \text{luas alas} + \text{luas selimut}$$

$$L = \pi r^2 + 2\pi r t$$

Diketahui $V = 1000$
 $\pi r^2 t = 1000$

$$t = \frac{1000}{\pi r^2}$$

$$L(r) = \pi r^2 + \frac{2000}{r}$$

> Menentukan titik stationer

$$L'(r) = \pi r^2 + 2000 \cdot r^{-1}$$

$$2\pi r - 2000 r^{-2} = 0$$

$$2\pi r - \frac{2000}{r^2} = 0$$

$$\frac{2\pi r^3 - 2000}{r^2} = 0$$

$$2\pi r^3 - 2000 = 0$$

$$2(\pi r^3 - 1000) = 0$$

$$\pi r^3 - 1000 = 0$$

$$r^3 = \frac{1000}{\pi}$$

$$r = \frac{10}{\sqrt[3]{\pi}}$$

Diperoleh jari-jari fabung $\rightarrow r = \frac{10}{\sqrt[3]{\pi}}$ ✓

$$\text{tinggi fabung} \rightarrow t = \frac{1000}{\pi r^2}$$

$$= \frac{1000}{\pi \left(\frac{10}{\sqrt[3]{\pi}} \right)^2}$$

$$= \frac{1000}{\pi \cdot \frac{100}{\sqrt[3]{\pi^2}}}$$

$$= \frac{10}{\sqrt[3]{\pi^3}}$$

$$= \frac{10}{\sqrt[3]{\pi}} \quad \checkmark$$

2. Diberikan fungsi $f(x) = 1 + \frac{1}{x}$.

- (a) Tentukan asimtot datar dan tegaknya (jika ada)
 (b) Tentukan selang dimana fungsi $f(x)$ naik atau turun
 (c) Tentukan titik ekstrim relatif fungsi tersebut
 (d) Tentukan selang kecekungan fungsi $f(x)$ dan titik belok (jika ada)
 (e) Sketsa grafiknya.

$$f(x) = 1 + \frac{1}{x} = \frac{x+1}{x}$$

a). \rightarrow Asimtot Datar

$$\lim_{x \rightarrow +\infty} \frac{x+1}{x} = 1 \quad \lim_{x \rightarrow -\infty} \frac{x+1}{x} = 1 \quad \left. \begin{array}{l} y = 1 \\ \end{array} \right\} \quad \checkmark$$

> Asimtot Tegak

$$\text{Penyebut } 0 = 0$$

$$x = 0$$

b). $f(x) = \frac{x+1}{x} \rightarrow u = \frac{u^1 u - u^{1+1}}{v^2}$

$$f'(x) = \frac{1 \cdot x - (x+1) \cdot 1}{x^2} = -\frac{1}{x^2}$$

$$\frac{-}{0} \frac{-}{-}$$

$$\text{Uji titik ke } f'(x) = -\frac{1}{x^2}$$

$$f \cdot \text{turun} : (-\infty, 0) \cup (0, +\infty)$$

c). $x = 0$ adalah titik dimana $f(x)$ tidak dapat diturunkan & merupakan asimtot tegak,

$$x = 0 \rightarrow y = 1 + \frac{1}{0} = \text{tidak terdefinisi}.$$

Jadi f tidak mempunyai titik ekstrim relatif

d). $f'(x) = -\frac{1}{x^2} = -x^{-2}$

$$f''(x) = 2x^{-3} = \frac{2}{x^3}$$

$$\frac{-}{0} \frac{+}{+}$$

$$\text{Uji titik ke } f''(x) = \frac{2}{x^3}$$

$$\text{cukup kertas} : (0, +\infty)$$

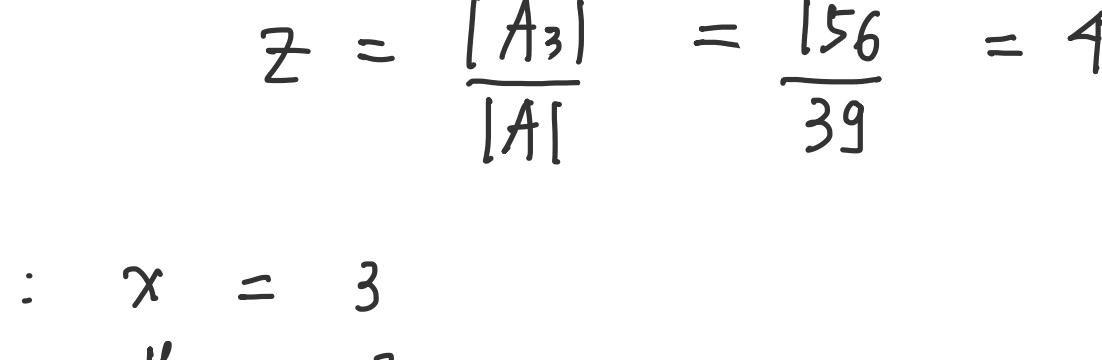
$$\text{cukup kebaikan} : (-\infty, 0)$$

e). Sketsa Grafik

$$f(x) = 1 + \frac{1}{x}$$

> Titik potong thd sb. x , maka $y = 0$

$$\left. \begin{array}{l} 1 + \frac{1}{x} = 0 \\ x = -1 \end{array} \right\} (-1, 0)$$



3. Hitung integral

$$\int_0^1 \frac{y^2}{\sqrt{4-3y}} dy$$

Substitusi : $u = 4 - 3y \rightarrow y = \frac{4-u}{3}$

$$du = -3 \cdot dy \rightarrow -\frac{1}{3} du = dy$$

$$-\frac{1}{3} du = dy$$

$$3y = 4 - u \rightarrow y = \frac{4-u}{3}$$

$$y = \frac{4-u}{3}$$

Batas integral :

$$y = 0 \rightarrow u = 4 - 3 \cdot 0 = 4$$

$$y = 1 \rightarrow u = 4 - 3 \cdot 1 = 1$$

$$\int_0^1 \frac{y^2}{\sqrt{4-3y}} dy = \int_4^1 \frac{\left(\frac{4-u}{3}\right)^2}{\sqrt{u}} \cdot -\frac{1}{3} du$$

$$= -\int_4^1 \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{(4-u)^2}{u^{1/2}} du$$

$$= \int_1^4 \frac{1}{27} \left(\frac{16-8u+u^2}{u^{1/2}} \right) du$$

$$= \frac{1}{27} \int_1^4 \left(16u^{-1/2} - 8u^{1/2} + u^{3/2} \right) du$$

$$= \frac{1}{27} \left[32u^{1/2} - \frac{16}{3}u^{3/2} + \frac{2}{5}u^{5/2} \right]_1^4$$

$$= \frac{1}{27} \left[32 \cdot 4^{1/2} - \frac{16}{3} \cdot 4^{3/2} + \frac{2}{5} \cdot 4^{5/2} \right] - \left[32 \cdot 1^{1/2} - \frac{16}{3} \cdot 1^{3/2} + \frac{2}{5} \cdot 1^{5/2} \right]$$

$$= \frac{1}{27} \left(64 - \frac{128}{3} + \frac{64}{5} \right) - \left(32 - \frac{16}{3} + \frac{2}{5} \right)$$

$$= \frac{1}{27} \left(64 - 32 - \frac{128}{3} + \frac{16}{3} + \frac{64}{5} - \frac{2}{5} \right)$$

$$= \frac{1}{27} \left(32 - \frac{112}{3} + \frac{62}{5} \right)$$

$$= \frac{1}{27} \left(\frac{480 - 560 + 186}{15} \right)$$

$$= \frac{1}{27} \cdot \frac{106}{15}$$

$$= \frac{106}{405} \quad \checkmark$$

4. Carilah semua bilangan kompleks w yang memenuhi persamaan $w^3 = -1 + i$.

$$w^3 = -1 + i$$

$$\rightarrow r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$w^3 = r[\cos \theta + i \sin \theta]$$

$$\rightarrow \theta = ?$$

$$\tan \theta = \frac{1}{-1}$$

$\tan \theta = -1 \rightarrow$ Kuadran ?

$$\theta = 180^\circ - 45^\circ \rightarrow \theta = 135^\circ$$

$$\theta = \frac{3\pi}{4}$$

$$\text{II}$$

$$\text{Im}(z)$$

$$\text{Rz}$$

$$\text{Iz}$$

$$\text{Rz}$$