

LATIHAN SOAL 9.2

13. Diket $F(3) = -1$ dan $F'(3) = 3$. Dapatkan Persamaan garis Singgung Pada Kurva $y = F(x)$ di $x = 3$

$$F(3) = -1 \text{ dan } F'(3) = 3$$

Gradien garis singgung di $x = 3$

$$\rightarrow F'(3) = 3$$

Persamaan garis singgung $= y - y_1 = m(x - x_1)$

$$y_1 = F(3) = -1 \quad x_1 = 3$$

$$y - (-1) = 3(x - 3)$$

$$y + 1 = 3x - 9$$

$$y = 3x - 10$$

17. Tunjukkan bahwa $F(x) = \begin{cases} 2x+1, & x \leq 1 \\ x+2, & x > 1 \end{cases}$

adalah Fungsi kontinu

tetapi tdk dapat diturunkan di $x = 1$

Kontinu karena $\lim_{x \rightarrow 1^-} = \lim_{x \rightarrow 1^+}$

$$2x+1 = x+2$$

$$2(1)+1 = (1)+2$$

$$3 = 3 \text{ (kontinu)}$$

Tidak dapat diturunkan di $x = 1$ karena $\lim_{h \rightarrow 0^-} \neq \lim_{h \rightarrow 0^+}$

$$\Rightarrow \lim_{h \rightarrow 0^-} \frac{F(x+h) - F(x)}{h}$$

$$\lim_{h \rightarrow 0^-} \frac{2(1+h)+1-3}{h}$$

$$\lim_{h \rightarrow 0^-} \frac{2+2h+1-3}{h}$$

$$\lim_{h \rightarrow 0^-} \frac{2h}{h} = 2$$

$$\Rightarrow \lim_{h \rightarrow 0^+} \frac{f(x+h) - F(x)}{h}$$

$$\lim_{h \rightarrow 0^+} \frac{(1+h)+2-3}{h}$$

$$\lim_{h \rightarrow 0^+} \frac{h}{h} = 1$$

Maka Fungsi $F(x)$ tidak dapat diturunkan di $x = 1$
dan kontinu di $x = 1$

LATIHAN SOAL 4.3

12. Dapatkan $F'(x)$

a). $F(x) = x^5 + 3x^2 + 5x$

b). $F(x) = 2x - 3x^2 - 2x^3$

Jawab:

a). $F(x) = x^5 + 3x^2 + 5x$

$$F'(x) = 5x^4 + 6x + 5$$

b). $F(x) = 2x - 3x^2 - 2x^3$

$$F'(x) = 2 - 6x - 6x^2$$

16. Diberikan fungsi $F(x) = \frac{x+3}{x+2}$

a. Dapatkan $F'(x)$

$$\Rightarrow F(x) = \frac{x+3}{x+2} \rightarrow u, u' = 1$$

$$x+2 \rightarrow v, v' = 1$$

$$F'(x) = \frac{u'v - uv'}{v^2}$$

$$= \frac{1(x+2) - (x+3) \cdot 1}{(x+2)^2}$$

$$= \frac{x+2 - x-3}{(x+2)^2} \Rightarrow \frac{-1}{(x+2)^2}$$

b. M garis $y = x \rightarrow 1$

Garis singgung tegak lurus maka $M \cdot 1 = -1 \rightarrow M = -1$

$$M = \frac{-1}{(x+2)^2}$$

$$-1 = \frac{-1}{(x+2)^2}$$

$$-(x+2)^2 = -1$$

$$(x+2)^2 = 1$$

$$x^2 + 4x + 4 = 1$$

$$x^2 + 4x + 3 = 0$$

$$(x+1)(x+3)$$

$$x_1 = -1 \vee x_2 = -3$$

$$y_1 = \frac{-1+3}{-1+2} = 2, (x_1, y_1) = (-1, 2)$$

$$y_2 = \frac{-3+3}{-3+2} = 0, (x_2, y_2) = (-3, 0)$$

Persamaan garis singgung

$$\Rightarrow y - 2 = -1(x - (-1)) \text{ dan } y - 0 = -1(x - (-3))$$

$$y - 2 = -x - 1$$

$$y = -x + 2$$

$$y = -x - 3$$

LATIHAN SOAL 4.4

16. $\frac{d}{dx} [\sqrt{x^3 + \csc x}]$

$$\Rightarrow \frac{d}{dx} [(x^3 + \csc x)^{\frac{1}{2}}]$$

$$= \frac{1}{2} (x^3 + \csc x)^{-\frac{1}{2}} (3x^2 - \csc x \cdot \cot x)$$

$$= \frac{3x^2 - \csc x \cdot \cot x}{2\sqrt{x^3 + \csc x}}$$

19. $3x^2 + y^2 - 2xy + 5y - x = 10$

$$\frac{d}{dx} [3x^2 + y^2 - 2xy + 5y - x] = \frac{d}{dx} (10)$$

$$6x + 2y \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} + 5 \frac{dy}{dx} - 1 = 0$$

$$2y \frac{dy}{dx} - 2x \frac{dy}{dx} + 5 \frac{dy}{dx} = 1 - 6x + 2y$$

$$\frac{dy}{dx} (2y - 2x + 5) = 2y - 6x + 1$$

$$\frac{dy}{dx} = \frac{2y - 6x + 1}{2y - 2x + 5}$$

$$\frac{dy}{dx} \cdot \begin{matrix} x=0 \\ y=-3 \end{matrix} \Rightarrow \frac{2(-3) - 6(0) + 1}{2(-3) - 2(0) + 5} = \frac{-6 + 1}{-6 + 5} = \frac{-5}{-1} = 5$$

22. $x^2y + 5x - 2y = 4$

$$\frac{d}{dx} [x^2y + 5x - 2y] = \frac{d}{dx} (4)$$

$$2xy + x^2 \frac{dy}{dx} + 5 - 2 \frac{dy}{dx} = 0$$

$$x^2 \frac{dy}{dx} - 2 \frac{dy}{dx} = -2xy - 5$$

$$\frac{dy}{dx} (x^2 - 2) = -2xy - 5$$

$$\frac{dy}{dx} = \frac{-2xy - 5}{x^2 - 2}$$

$$M = \frac{-2(1)(1) - 5}{1^2 - 2} = \frac{-7}{-1} = 7$$

Persamaan garis singgung (1,1)

$$y - 1 = 7(x - 1)$$

$$y - 1 = 7x - 7$$

$$y = 7x - 6$$