

[SKPB - ITS]

EVALUASI AKHIR SEMESTER BERSAMA GASAL 2023/2024

Mata Kuliah/SKSK : Kalkulus 1 (SM224101) / 3 SKS
Hari, Tanggal : Selasa, 12 Desember 2023
Waktu : 15.30-17.10 WIB (100 menit)
Sifat : Tertutup
Kelas : 51-56, 61

Diberikan 5 soal, dengan bobot nilai masing-masing soal sama dan boleh dikerjakan tidak berurutan.

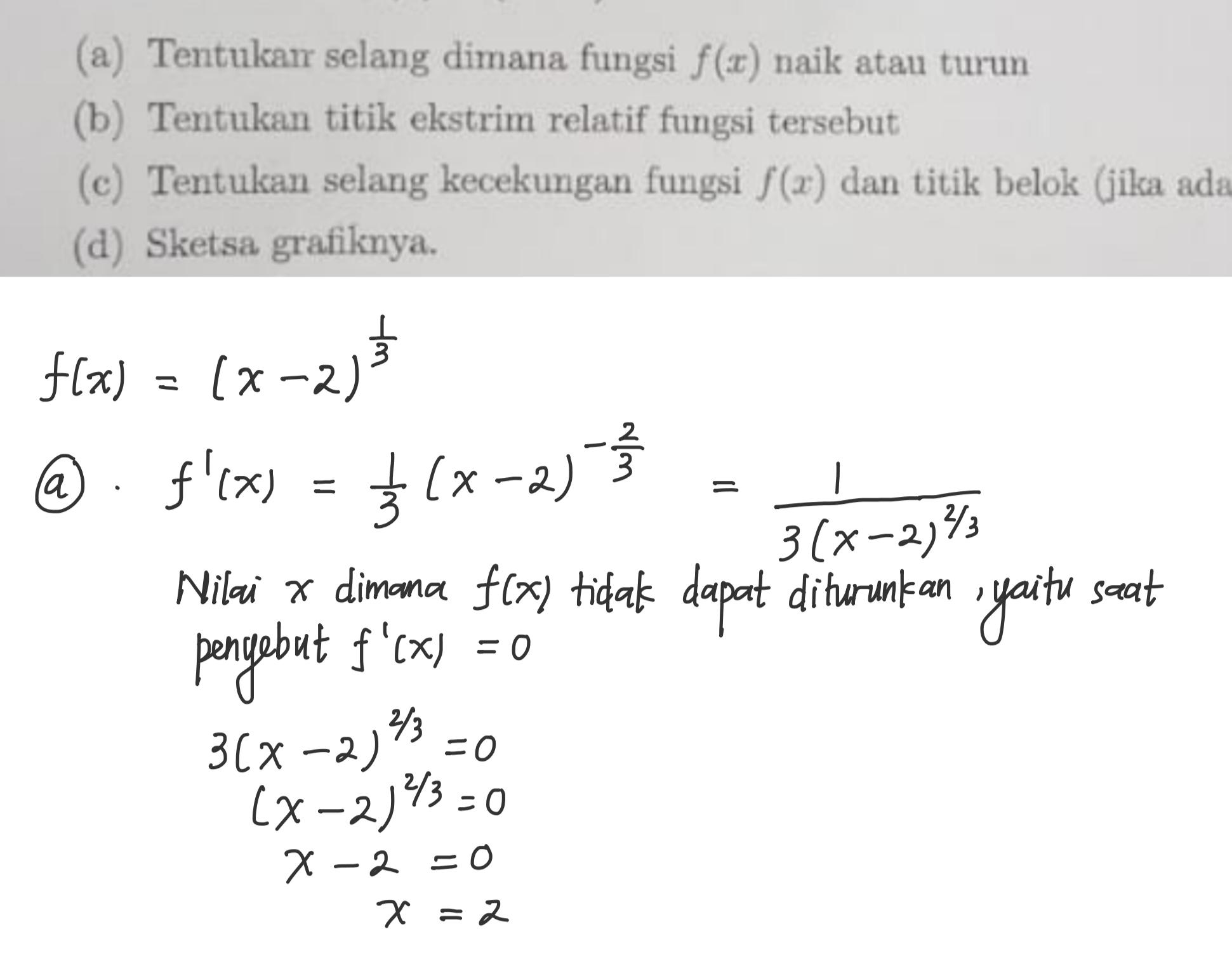
Tuliskan Nama, NRP, dan Nomor Kelas pada lembar jawaban Anda.

DILARANG MEMBAWA/MENGUNAKAN KALKULATOR DAN ALAT KOMUNIKASI

DILARANG MEMBERIKAN/MENERIMI JAWABAN SELAMA UJIAN

"Setiap tindak kecurangan akan mendapat sanksi akademik."

1. Roket naik vertikal dan dipantau oleh stasiun radar yang terletak 10 km dari landasan peluncuran. Berapa cepat roket naik jika tingginya 6 km dan jaraknya dari stasiun radar bertambah dengan laju 3000 km/jam?



Perhatikan segitiga siku-siku.

Note: saat $y = 6$, maka $x = ?$

$$y^2 + 10^2 = x^2$$

$$y^2 + 100 = x^2$$

$$y^2 = x^2 - 100$$

$$6^2 = x^2 - 100$$

$$\frac{d}{dt} [y^2 + 100] = \frac{d}{dt} [x^2]$$

$$36 + 100 = x^2$$

$$2y \cdot \frac{dy}{dt} = 2x \cdot \frac{dx}{dt}$$

$$136 = x^2$$

$$\frac{dy}{dt} = \frac{2x}{2y} \cdot \frac{dx}{dt}$$

$$x = \sqrt{136}$$

$$= \frac{2\sqrt{34}}{6} \cdot \frac{3000}{500}$$

$$x = 2\sqrt{34}$$

$$= 1000\sqrt{34} \text{ km/jam}$$

2. Diberikan fungsi $f(x) = (x-2)^{\frac{1}{3}}$.

(a) Tentukan selang dimana fungsi $f(x)$ naik atau turun

(b) Tentukan titik ekstrim relatif fungsi tersebut

(c) Tentukan selang kecekungan fungsi $f(x)$ dan titik belok (jika ada)

(d) Sketsa grafiknya.

$$f(x) = (x-2)^{\frac{1}{3}}$$

$$\textcircled{a} \quad f'(x) = \frac{1}{3}(x-2)^{-\frac{2}{3}} = \frac{1}{3(x-2)^{\frac{2}{3}}}$$

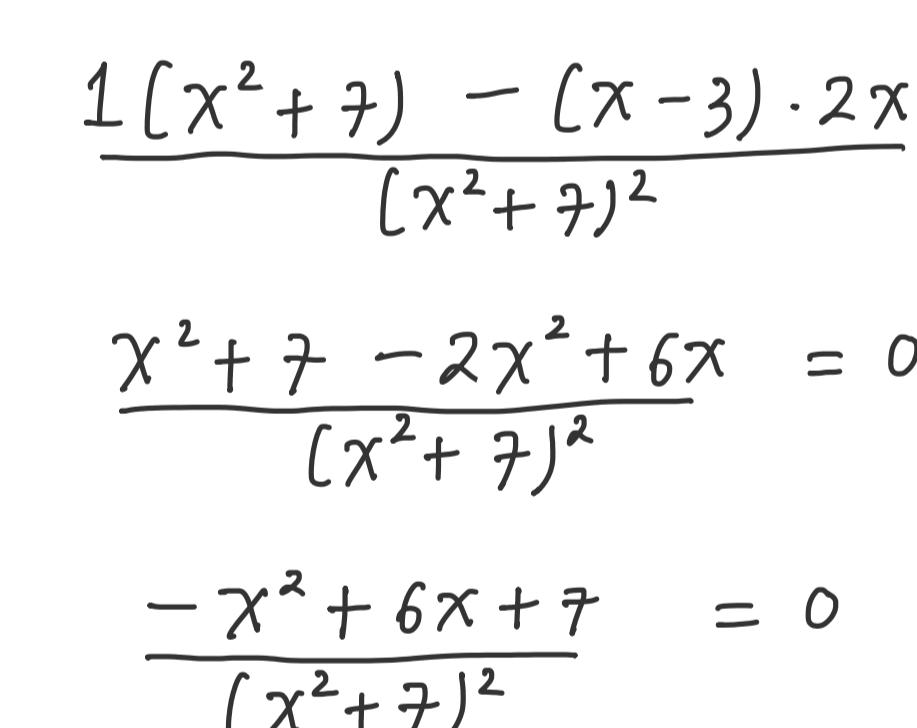
Nilai x dimana $f(x)$ tidak dapat di turunkan, yaitu saat
penyebut $f'(x) = 0$

$$3(x-2)^{\frac{2}{3}} = 0$$

$$(x-2)^{\frac{2}{3}} = 0$$

$$x-2 = 0$$

$$x = 2$$



$$\text{Uji titik ke } f'(x) = \frac{x-3}{3(x-2)^{\frac{2}{3}}}$$

cekung keatas : $(-\infty, 2)$

cekung kebawah : $(2, +\infty)$

Titik belok terjadi di $x = 2$, maka $y = f(2) = 0$

$$(2, 0) \checkmark$$

\textcircled{d} Sketsa Grafik

$$f(x) = (x-2)^{\frac{1}{3}}$$

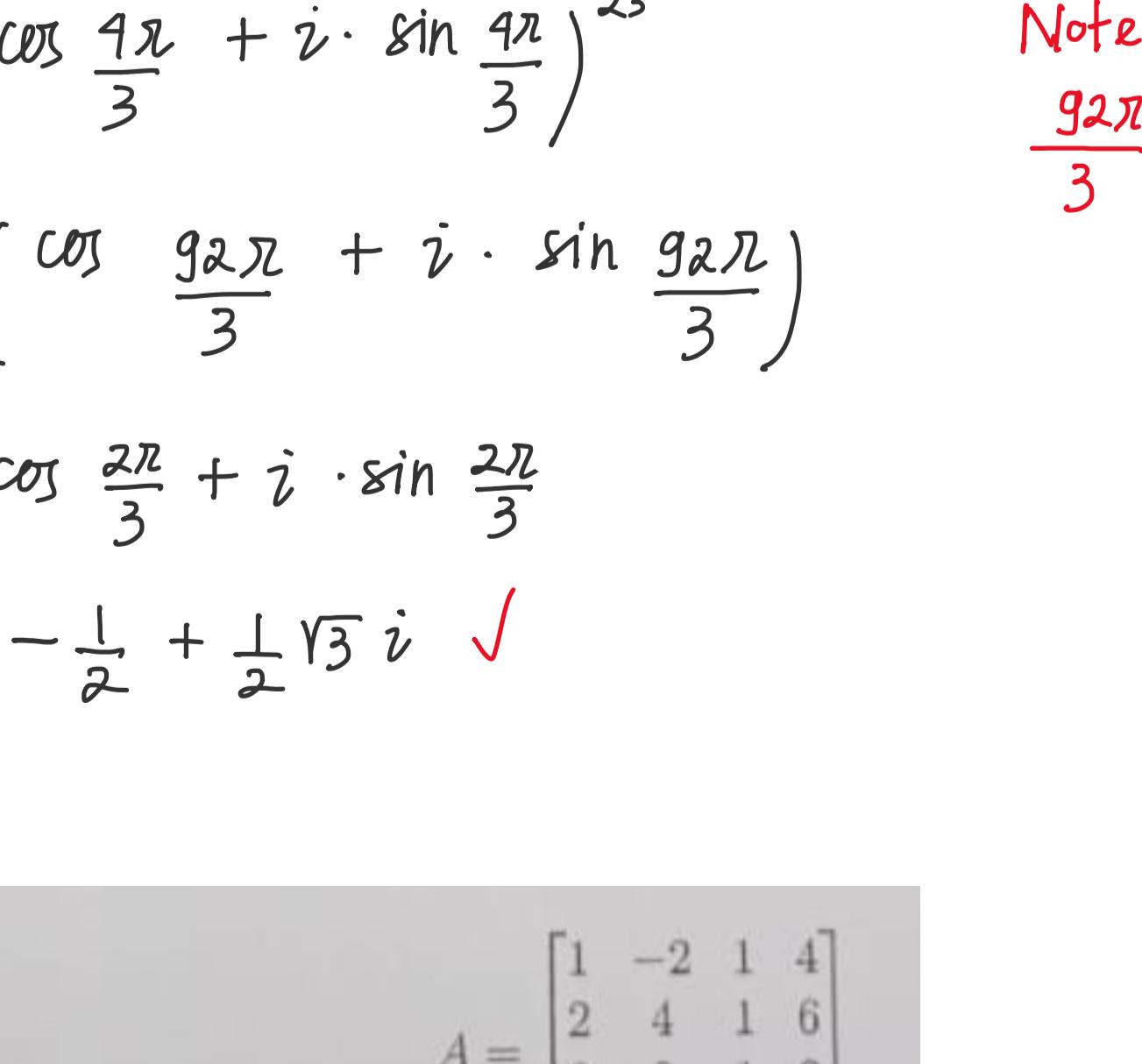
\rightarrow Titik potong thd sb. x , maka $y = 0$

$$(x-2)^{\frac{1}{3}} = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} (2, 0)$$

\textcircled{d} Titik potong thd sb. y , maka $x = 0$

$$y = f(0) = (0-2)^{\frac{1}{3}} = (-2)^{\frac{1}{3}} = \sqrt[3]{-2}$$

$$(0, \sqrt[3]{-2})$$



3. Diberikan $F(x) = \int_0^x \frac{t-3}{t^2+7} dt$, $-\infty < x < \infty$.

(a) Dapatkan selang dimana fungsi $F(x)$ naik atau turun

(b) Dapatkan selang kecekungan fungsi $F(x)$.

$$\textcircled{a} \quad F'(x) = \frac{d}{dx} [F(x)]$$

$$= \frac{d}{dx} \left[\int_0^x \frac{t-3}{t^2+7} dt \right]$$

$$= \frac{x-3}{x^2+7}$$

Pada Teorema Fundamental Kalkulus II

$$\frac{x-3}{x^2+7} = 0$$

$$x-3 = 0$$

$$x = 3$$

$$\text{Uji titik ke } f'(x) = \frac{x-3}{x^2+7}$$

f-naik : $(3, +\infty) \checkmark$

f-turun : $(-\infty, 3) \checkmark$

$$\textcircled{b} \quad F'(x) = \frac{x-3}{x^2+7} \rightarrow \frac{u}{v} = \frac{u'v - v'u}{v^2}$$

$$F''(x) = 0$$

$$\frac{1(x^2+7) - (x-3) \cdot 2x}{(x^2+7)^2} = 0$$

$$\frac{x^2+7 - 2x^2+6x}{(x^2+7)^2} = 0$$

$$\frac{-x^2+6x+7}{(x^2+7)^2} = 0$$

$$\frac{(-x-1)(x-7)}{(x^2+7)^2} = 0$$

$$-x-1 = 0 \quad ; \quad x-7 = 0$$

$$x = -1 \quad ; \quad x = 7$$

$$\text{Uji titik ke } f''(x) = \frac{(-x-1)(x-7)}{(x^2+7)^2}$$

cekung keatas : $(-1, 7) \checkmark$

cekung kebawah : $(-\infty, -1) \cup (7, +\infty)$

Titik belok terjadi di $x = -1$ dan $x = 7$, maka $y = f(-1) = f(7) = 0$

$$(-1, 0) \checkmark$$

$$(7, 0) \checkmark$$

4. Hitung

$$\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}} \right)^{23}$$

$$\text{Misal } z = \frac{1-i\sqrt{3}}{1+i\sqrt{3}} \quad ; \quad z^{23} = ?$$

$$\Rightarrow z = \frac{1-i\sqrt{3}}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$z = \frac{1-2\sqrt{3}i}{1+3}$$

$$z = -\frac{1}{2} + \frac{1}{2}\sqrt{3}i$$

$$z = -\frac{1}{2} - \frac{1}{2}\sqrt{3}i$$

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$$z = -\frac{1}{2}$$