


LATIHAN SOAL 5.1

11.  $r = \text{jari}^2$
 $V = \text{Volume bola}$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt}$$

$$= 4 \pi (15)^2$$

$$\frac{dV}{dt} = -9860 \pi \text{ cm/menit.}$$

20. Diket: $r = 10 = \frac{5}{12} h$
 $h = 24$

$$V = \frac{1}{3} \pi r^2 h$$

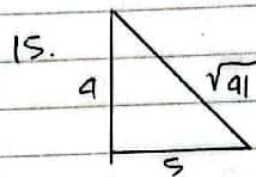
$$= \frac{1}{3} \pi \left(\frac{5}{12} h \right)^2 h = \frac{25 \pi h^3}{432}$$

$$\frac{dV}{dt} = \frac{25 \pi h^2}{432} \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{25 \pi h^2}{144} \cdot \frac{dh}{dt}$$

$$20 = \frac{25 \pi (16)^2}{144} \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2880}{6400 \pi} = \frac{9}{20 \pi} \text{ m/menit}$$



$$s^2 = x^2 + y^2$$

$$2s \frac{ds}{dt} = 0 + 2y \frac{dy}{dt}$$

$$25 \frac{ds}{dt} = 0 + 2y \frac{dy}{dt} \text{ (x tetap)}$$

$$2 \sqrt{41} \cdot 2000 = 2 \cdot 4 \cdot \frac{dy}{dt}$$

$$500 \sqrt{41} = \frac{dy}{dt}$$

Jadi, kecepatan roket $500 \sqrt{41}$ km/jam

LATIHAN SOAL 5.2

12. $F(x) = \frac{x}{x^2+2}$

$$F'(x) = \frac{1 \cdot (x^2+2) - x \cdot 2x}{(x^2+2)^2} = \frac{-x^2+2}{(x^2+2)^2}$$

$$\Delta -x^2+2=0$$

$$x^2=2 \Rightarrow x = \pm \sqrt{2}$$

$$\begin{array}{c} - & + & - \\ -\sqrt{2} & 0 & \sqrt{2} \end{array}$$

$$x=0, \frac{-0^2+2}{(0^2+2)^2} = +$$

a. $[-\sqrt{2}, \sqrt{2}]$

b. $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty)$

$$x=-2, \frac{-(-2)^2+2}{(-2^2+2)^2} = -$$

$$x=2, \frac{-2^2+2}{(-2^2+2)^2} = -$$

$$x=2, \frac{-2^2+2}{(-2^2+2)^2} = -$$

$$F''(x) = \frac{-2x(x^2+2)^2 - (-x^2+2)2(x^2+2)2x}{(x^2+2)^4}$$

$$= \frac{2x^3-12x}{(x^2+2)^3}$$

$$2x^3-12x=0$$

$$2x(x^2-6)=0$$

$$2x=0 \quad x^2-6=0$$

$$x=0 \quad x=\sqrt{6} \quad x=-\sqrt{6}$$

$$\begin{array}{c} - & + & - & + \\ -\sqrt{6} & 0 & \sqrt{6} \end{array}$$

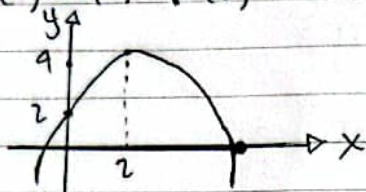
c. $(-\infty, -\sqrt{6}) \cup [0, \sqrt{6}]$

x=-3, - x=1, - d. $[-\sqrt{6}, 0] \cup [\sqrt{6}, \infty)$

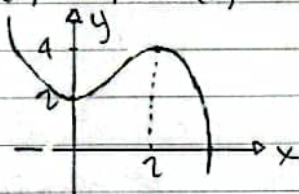
x=-1, + x=3, + e. x=-\sqrt{6}, x=0, x=\sqrt{6}

22. $S = F(x)$

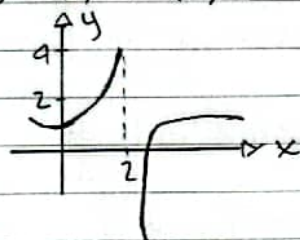
a). $F(2) = 4$, $F'(2) = 0$, $F''(x) < 0$ untuk semua x



b). $F(2) = 4$, $F'(2) = 0$, $F''(x) > 0$ untuk $x < 2$, $F''(x) < 0$ untuk $x > 2$



c). $F(2) = 4$, $F''(x) > 0$ untuk $x \neq 2$ dan $\lim_{x \rightarrow 2^+} F'(x) = -\infty$, $\lim_{x \rightarrow 2^-} F'(x) = +\infty$



17. $F(x) = x + \sin x$

$F'(x) = 1 + \cos x$

Pada interval $[0, 360]$

$F(x)$ naik pada $[0, 180]$

$F(x)$ naik pada $[180, 360]$

$\Rightarrow F$ naik pada semua titik

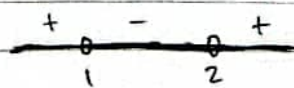
LATIHAN SOAL 5.3

22. $F(x) = 2x^3 - 9x^2 + 12x$

a). $F'(x) = 6x^2 - 10x + 12$

$= (6x - 6)(x - 2)$

$x = 1$ $x = 2$



Jadi, terdapat ekstrim relatif

di $x = 1$ (maksimum) dan $x = 2$ (minimum)

b). $F''(x) = 12x - 10$

$x = 1$, $12(1) - 10 = < 0$ maka $x = 1$ maksimum relatif

$x = 2$, $12(2) - 10 = > 0$ maka $x = 2$ minimum relatif

$$32. F(x) = \frac{x}{x+2}$$

$$F'(x) = \frac{1 \cdot (x+2) - x \cdot 1}{(x+2)^2} = \frac{2}{(x+2)^2}$$

Karena turunan Pertama tidak bisa menjadi 0 maka tdk ada ekstrim relatif

$$37. F(x) = \tan(x^2+1)$$

$$F'(x) = 2x \sec^2(x^2+1)$$

Karena turunan Pertama tidak bisa menjadi 0,

Maka tdk ada ekstrim relatif.

LATIHAN SOAL 5.4

$$14. y = \frac{4x+1}{3x+2}$$

Asimtot tegak:

$$\lim_{x \rightarrow -\frac{2}{3}^-} \frac{4x+1}{3x+2} = +\infty$$

$$\lim_{x \rightarrow -\frac{2}{3}^+} \frac{4x+1}{3x+2} = -\infty$$

Asimtot tegak $= x = -\frac{2}{3}$

Asimtot datar:

$$\lim_{x \rightarrow -\infty} \frac{4x+1}{3x+2} = \frac{4}{3}$$

$$\lim_{x \rightarrow +\infty} \frac{4x+1}{3x+2} = \frac{4}{3}$$

Asimtot datar $y = \frac{4}{3}$

$$35. y = \frac{x-1}{x^2-4}$$

$$y' = \frac{-x^2+2x-4}{(x^2-4)^2}$$

$$x=0, y = \frac{1}{4}$$

$$y=0, x=1$$

$$\text{Asimtot tegak} = \lim_{x \rightarrow -2^-} \frac{x-1}{x^2-4} = -\infty$$

$$\lim_{x \rightarrow -2^+} y = +\infty$$

$$\lim_{x \rightarrow 2^-} y = -\infty$$

$$\lim_{x \rightarrow 2^+} y = +\infty$$

$$\text{Asimtot Datar} = \lim_{x \rightarrow \pm\infty} y = 0$$

$$47. F(x) = (x-2)^{\frac{1}{3}}$$

a). Karena Fungsi tdk memiliki ~~asimtot~~ penyebut, maka tdk memiliki asimtot tegak, karena $\lim_{x \rightarrow \pm\infty} F(x)$ tdk terdefinisi maka tdk memiliki asimtot datar

b). $F'(x) = \frac{1}{3}(x-2)^{-\frac{2}{3}}$, karena $F'(x)$ tdk bisa diurung dari 0, maka $F(x)$ naik di $(-\infty, +\infty)$

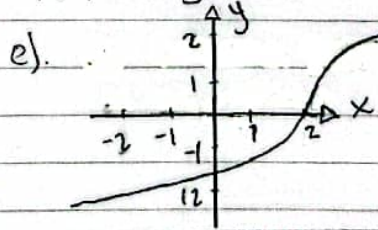
c). $(x-2)^{\frac{1}{3}} = 0$, titik belok di titik $(2,0)$
 $x=2$

$$d). F''(x) = -\frac{2}{9(x-2)^{\frac{5}{3}}}, \dots -\frac{2}{9(x-2)^{\frac{5}{3}}} > 0 \text{ saat } x < 2$$

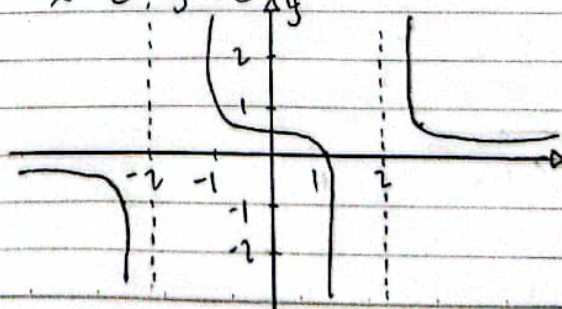
$$-\frac{2}{9(x-2)^{\frac{5}{3}}} < 0 \text{ saat } x > 2$$

$F(x)$ cekung keatas saat $(-\infty, 2)$

$F(x)$ cekung kebawah saat $(2, +\infty)$



uji turunan Pertama
 $x=0, y' < 0$



LATIHAN SOAL 5.5

19. $f(x) = x^4 + 4x : (-\infty, +\infty)$

$$f'(x) = 4x^3 + 4$$

$$4x^3 + 4 = 0$$

$$x = -1$$

$$\lim_{x \rightarrow -\infty} x^4 + 4x = +\infty$$

$$\lim_{x \rightarrow +\infty} x^4 + 4x = +\infty$$

$f(x)$ mempunyai sebuah minimum tetapi tidak maksimum pada $(-\infty, +\infty)$

$$y = (-1)^4 + 4 \cdot (-1) = -3$$

f minimum pada $x = -1$ $(-1, -3)$

f tidak memiliki maksimum

18. $f(x) = \frac{x+3}{x-3}$

Karena diskontinu di $x = 3$. maka

f tidak memiliki nilai maksimum & minimum

20. $f(x) = 3 \cos \frac{x}{3} + 2 \cos \frac{x}{2}$

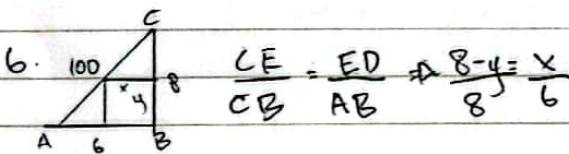
$$f'(x) = -\sin \frac{x}{3} - \sin \frac{x}{2}$$

$$-\sin \frac{x}{3} = \sin \frac{x}{2} = 0$$

$$x = 0$$

Maksimumnya di $x = 0$

LATIHAN SOAL 5.6



$$48 - 6y = 8x$$

$$6y = 48 - 8x$$

$$y = 8 - \frac{8}{6}x$$

$$L = x \cdot y = x \left(8 - \frac{8}{6}x \right) = 8x - \frac{8}{6}x^2$$

$$L' = 8 - 2 \cdot \frac{8}{6}x$$

$$8 - \frac{8}{3}x = 0 \Rightarrow 8 = \frac{8}{3}x$$

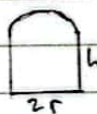
$$x = 3$$

$$y = 8 - \frac{8}{6} \cdot 3$$

$$= 8 - \frac{4}{6} \cdot 3$$

$$= 4$$

$$L = x \cdot y = 3 \cdot 4 = 12 \text{ cm}^2$$

15.  $\frac{1}{2}(2\pi r) + 2r + 2h = P$
 $2h = P - \pi r - 2r$
 $h = \frac{P - \pi r - 2r}{2}$

$$L = \frac{1}{2}\pi r^2 + 2rh = \frac{1}{2}\pi r^2 + 2r \left(\frac{P - \pi r - 2r}{2} \right) = \frac{1}{2}\pi r^2 + r(P - \pi r - 2r)$$

$$L' = \pi r + P - 2\pi r - 4r = 0$$

$$= r(\pi - 2\pi - 4) = -P$$

$$r = \frac{-P}{-\pi - 4} = \frac{P}{\pi + 4}$$

21. $P^2 = \text{Bola}$, $V^2 = \text{tabung}$

$$P^2 = V^2 + \left(\frac{h}{2}\right)^2 = V^2 + \frac{h^2}{4}$$

$$V^2 = P^2 - \frac{h^2}{4}$$

$$V^2 = P^2 - \frac{h^2}{4}$$

$$V^2 = P^2 - \frac{1}{4} \left(\frac{2}{3}R \right)^2$$

$$V = \pi V^2 h = \pi \left(P^2 - \frac{h^2}{4} \right) h$$

$$V^2 = P^2 - \frac{1}{3}P^2$$

$$= \pi \left(P^2 h - \frac{h^3}{4} \right)$$

$$V = \sqrt{\frac{2}{3}}P = R\sqrt{\frac{2}{3}}$$

$$V' = \pi \left(P^2 - \frac{3h^2}{4} \right) = 0$$

Tinggi $\frac{2P}{3}$ dan

$$\bullet P^2 - \frac{3h^2}{4} = 0$$

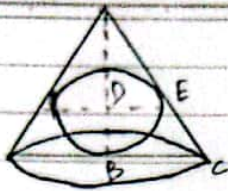
Jari-jari $= \sqrt{\frac{2}{3}}P$

$$h^2 = \frac{4}{3}P^2$$

$$h = P\sqrt{\frac{2}{3}}$$



39.



$$AE = \sqrt{AD^2 - DE^2}$$

$$= \sqrt{x^2 - R^2}$$

$$\frac{AE}{DE} = \frac{AB}{BC}$$

$$\frac{\sqrt{x^2 - R^2}}{R} = \frac{x + R}{r}$$

$$V = \frac{Rx + R^2}{\sqrt{x^2 - R^2}}$$

$$V = \frac{1}{3} \pi r^2 t$$

$$= \frac{1}{3} \pi \left(\frac{Rx + R^2}{\sqrt{x^2 - R^2}} \right)^2 (x + R)$$

$$= \frac{1}{3} \pi \left(\frac{R^2 (x + R)^2 (x + R)}{(x - R)(x + R)} \right)$$

$$= \frac{R^2}{3} \pi \left(\frac{(x + R)^2}{(x - R)} \right)$$

$$V^2 = \frac{R^2}{3} \pi \left(\frac{2(x + R)(x - R) - (x + R)^2}{(x - R)^2} \right)$$

$$= \frac{R^3}{3} \pi \left(\frac{x^2 - 2Rx - 3R^2}{(x - R)^2} \right)$$

$$x^2 - 2Rx - 3R^2 = 0$$

$$(x - 3R)(x + R)$$

$$x = 3R \quad x = -R \text{ (Tidak Memenuhi)}$$

$$t \cdot x + R = 3R + R = 4R$$

$$= \frac{Rx + R^2}{\sqrt{x^2 - R^2}}$$

$$V = \frac{3R^2 + R^2}{\sqrt{9R^2 - R^2}} = \frac{4R^2}{\sqrt{8R^2}} = \frac{4R^2}{2R\sqrt{2}} = \frac{2R}{\sqrt{2}} = R\sqrt{2}$$

Tinggi kerucut = $4R$

Jari-jari kerucut = $\sqrt{2}R$