Tutor ETS Ganjil IT 2023

Sunday, October 15, 2023 1:31 PM

EVALUASI TENGAH SEMESTER BERSAMA GASAL 2022/2023

Mata kuliah/SKS	:	Matematika 1 (KM18 4 101) / 3 SKS
Hari, Tanggal	:	Senin, 17 Oktober 2022
Waktu	:	07.00-08.40 WIB (100 menit)
Sifat	:	Tertutup
Kelas	:	1-6

1. Tentukan persamaan garis l yang melalui titik $(2, -1)$ jika diketahui bahwa garis l tegak lurus dengan $\underbrace{3x - ky = 1}_{I}$ dan sejajar dengan $\underbrace{k^2x + 3(y-1) = 0}_{II}$.

Bab 1

1.4

$l \xrightarrow{I} (2,-1)$ & m_l

garis l tegak lurus garis I $\rightarrow$ $\boxed{m_l \times m_I = -1}$

$3x - ky = 1 \rightarrow ax + by + c = 0$

$3x - ky - 1 = 0 \rightarrow m_I = -\frac{a}{b} = \frac{-3}{-k} = \frac{3}{k}$

atau $\rightarrow 3x - 1 = ky$

$\frac{3x}{k} - \frac{1}{k} = y \rightarrow y = mx + c$

$m_I = 3/k$ ✓

$m_l \times m_I = -1$

$m_l \times 3/k = -1$

$m_l = -\frac{k}{3} \quad \ldots (1)$

- garis l sejajar dengan garis II

$k^2x + 3(y-1) = 0$

$k^2x + 3y - 3 = 0 \rightarrow m_{II} = -\frac{a}{b} = \frac{-k^2}{3}$

$m_l = m_{II}$

$m_l = -\frac{k^2}{3} \ldots (2)$

kombinasikan persamaan (1) dan (2)

$\frac{-k}{3} = \frac{-k^2}{3}$

$$\frac{k^2}{3} - \frac{k}{3} = 0$$

$$\frac{k}{3}(k-1) = 0$$

$$\frac{k}{3} = 0 \quad \vee \quad k - 1 = 0$$

$$k = 0 \qquad\qquad k = 1 \checkmark$$

TM

karena $m_I = \frac{3}{k}$

dan $k \neq 0$

$$m_l = \frac{-k}{3} = -\frac{k^2}{3} = -\frac{1}{3}$$

garis $l \rightarrow (2, -1)$

(x_1, y_1)

$$y - y_1 = m_l(x - x_1)$$
$$y - (-1) = -\tfrac{1}{3}(x - 2)$$
$$y + 1 = -\frac{1}{3}x + \frac{2}{3}$$
$$y = -\frac{1}{3}x + \frac{2}{3} - 1$$
$$y = -\tfrac{1}{3}x - \tfrac{1}{3} \quad //$$

4. **Diberikan fungsi $f(x) = \sqrt{x+5}$ dan $g(x) = \sqrt{x-4}$. Dapatkan**

   (a) Domain f dan g

   (b) $(g \circ f)(x)$ beserta domainnya

(a) Domain $\rightarrow x$ yg menyebabkan fungsinya terdefinisi di bilangan real

$f \rightarrow$ syarat akar (tak negatif

$x + 5 \geq 0$ $\quad D(f) = D_f = [-5, +\infty)$

$x \geq -5$

$g \rightarrow$ syarat akar

$x - 4 \geq 0$ $\quad [4, +\infty)$

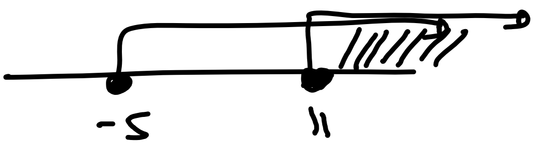
$y \to$ syarat akar

$$x - 4 \geq 0$$
$$x \geq 4 \qquad D(g) = D_g = [4, +\infty)$$

(b) $(g \circ f)(x)$ dan $D_{g \circ f}$?

$$D_{g \circ f} = \{ x \in D_f \mid f(x) \in D_g \}$$

$$x \in [-5, +\infty) \mid \overset{y}{\sqrt{x+5}} \in [4, +\infty) \quad ✓$$



$$\sqrt{x+5} \geq 4$$
$$x + 5 \geq 16$$

irisan $\quad x \geq 11$

$$= [11, +\infty)$$

$$g(f(x)) = g\left(\sqrt{x+5}\right) = \sqrt{\left(\sqrt{x+5}\right) - 4} \quad //$$

$\cancel{D?}$

5. Diberikan fungsi $f(x) = (x-1)^3 + 1$.
   (a) Dapatkan $f^{-1}(x)$ beserta domainnya
   (b) Sketsa grafik $f(x)$ dan $f^{-1}(x)$ pada satu bidang koordinat.

(a) $f(x) = (x-1)^3 + 1 \quad f(x) = y \to f^{-1}(y) = x$

$$y = (x-1)^3 + 1$$
$$y - 1 = (x-1)^3$$
$$\sqrt[3]{y-1} = x - 1$$
$$\sqrt[3]{y-1} + 1 = x$$

? $\sqrt[3]{y-1} + 1 = f^{-1}(y)$

✓ $\boxed{\sqrt[3]{x-1} + 1 = f^{-1}(x)}$

$\cancel{Go?}$

$D_{f^{-1}} = R_f \qquad f(x) = (x-1)^3 + 1. \qquad R_f = (-\infty, +\infty)$

$R_{f^{-1}} = D_f$

$R_{f^{-1}} = D_f$

$f(x) = (x-1)^3+1$ → semua $x \in R$

$D_f = (-\infty, +\infty) = R_{f^{-1}}$.

$R_f = (-\infty, +\infty) = D_{f^{-1}}$

$f(x) = (x-1)^3+1$

$y = (x-1)^3+1$

$y-1 = (x-1)^3$ → $y \to y-1$, $x \to x-1$

geser atas(+) 1 satuan

geser kanan(+) 1 satuan

$y = x^3$

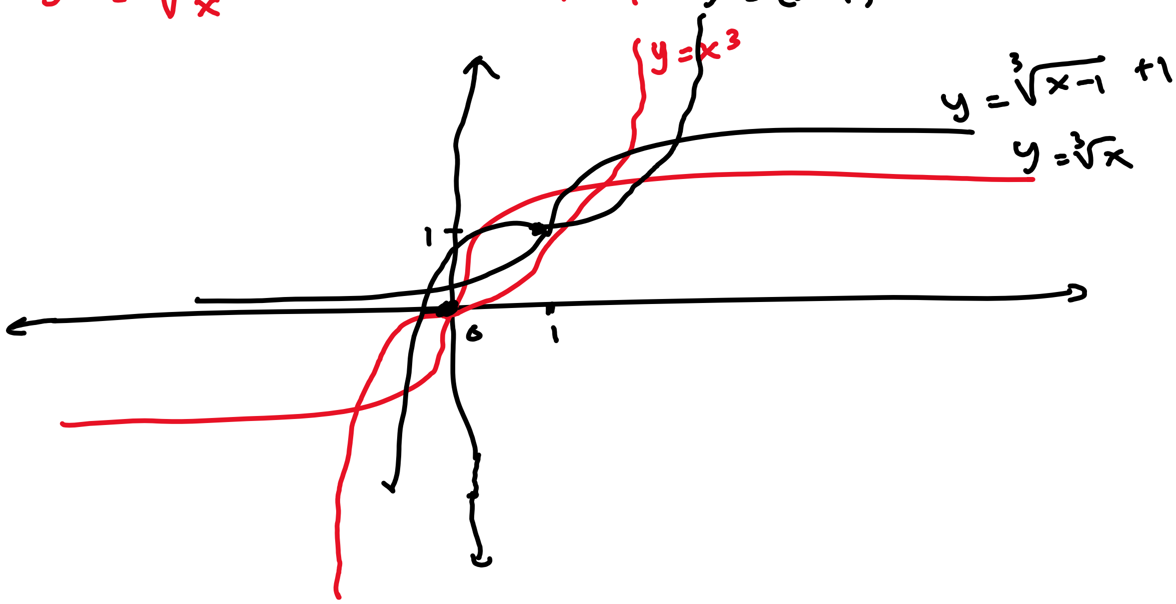
$f(x) = \sqrt[3]{(x-1)} + 1$

$y-1 = \sqrt[3]{x-1}$

$y = \sqrt[3]{x}$

$y \to y-1$

$x \to x-1$



① Hitung limit

$$\lim_{y \to -\infty} \frac{2-y}{\sqrt{7+6y^2}}$$

$|y| = \sqrt{y^2}$ ✓

Bab 3.

$$|y| = \begin{cases} y, & \text{jika } y \geq 0 \\ -y, & \text{jika } y < 0 \end{cases}$$

$\lim \quad 2-y$

$|y| = \sqrt{y^2}$ ... $-y$, jika $y<0$

$$\lim_{y\to-\infty} \frac{\frac{2-y}{|y|}}{\frac{\sqrt{7+6y^2}}{|y|}} = \lim_{y\to-\infty} \frac{\frac{2-y}{-y}}{\frac{\sqrt{7+6y^2}}{\sqrt{y^2}}} = \lim_{y\to-\infty} \frac{-\frac{2}{y}+1}{\sqrt{\frac{7}{y^2}+6}}$$

$$= \frac{0+1}{\sqrt{0+6}} = \frac{1}{\sqrt{6}} = \frac{1}{6}\sqrt{6} \quad //$$

(2) Dapatkan persamaan garis singgung dari fungsi $\frac{xy^3}{1+y^2} = 2$. di titik $(4,1)$

$$m_s = \left.\frac{dy}{dx}\right|_{(x_1, y_1)}.$$

$$\frac{xy^3}{1+y^2} = 2$$

$$\frac{d}{dx}\left(\frac{xy^3}{1+y^2}\right) = \frac{d}{dx}(2)$$

$$\frac{\frac{d}{dx}(xy^3)(1+y^2) - (xy^3)\frac{d}{dx}(1+y^2)}{(1+y^2)^2} = 0$$

$$\frac{\left[\frac{d}{dx}(x)\cdot y^3 + x\cdot\frac{d}{dx}(y^3)\right](1+y^2) - (xy^3)\left(0+2y\cdot\frac{dy}{dx}\right)}{(1+y^2)^2} = 0$$

$$\left(1\cdot y^3 + x\cdot 3y^2\frac{dy}{dx}\right)(1+y^2) - 2xy^4\frac{dy}{dx} = 0$$

$y^3 + y^5 + 3xy^2 \frac{dy}{dx} + 3xy^4\frac{dy}{dx} - 2xy^4\frac{dy}{dx} = 0$

$$y^3 + y^5 + 3xy^2 \frac{dy}{dx} + 3xy^4 \frac{dy}{dx} - 2xy^4 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(3xy^2 + 3xy^4 - 2xy^4\right) = -y^3 - y^5$$

$$\frac{dy}{dx} = \frac{-y^3 - y^6}{3xy^2 + xy^4}$$

$$= \frac{\cancel{y^2}(-y - y^4)}{\cancel{y^2}(3x + xy^2)}$$

$$\frac{dy}{dx} = \frac{-y - y^4}{3x + xy^2}$$

$$m_s = \frac{dy}{dx}\Big|_{(4,1)}$$

$$= \frac{-1-(1^4)}{3(4) + 4(1)^2} = \frac{-2}{12+4} = \frac{-2}{16} = -\frac{1}{8}$$

$$y - y_1 = m_s(x - x_1)$$

$$y - 1 = -\frac{1}{8}(x - 4)$$

$$y - 1 = -\frac{1}{8}x + \frac{1}{2} \Rightarrow y = -\frac{1}{8}x + \frac{3}{2}$$