1. Tentukan luas yang dibatasi oleh kurva $x = y^2$ dan $2y + x = 3$

$x = y^2$

$2y + x = 3 \rightarrow x = 3 - 2y$

Pertama, cari titik potong

$x_1 = x_2$

$y^2 = 3 - 2y$

$y^2 + 2y - 3 = 0$

$(y + 3)(y - 1) = 0$

$y = -3, y = 1$

Kedua, gambar grafik

Untuk $x = y^2$

$y = -3 \rightarrow x = 9$

$y = 0 \rightarrow x = 0$

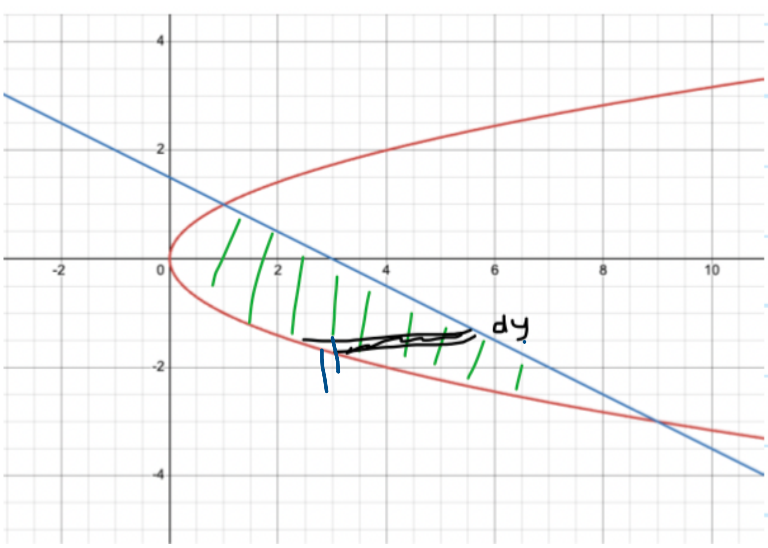
$y = 1 \rightarrow x = 1$

Untuk $x = 3 - 2y$

$y = -3 \rightarrow x = 9$

$y = 0 \rightarrow x = 3$

$y = 1 \rightarrow x = 1$



Ketiga, cari luas

$dL = (x_1 - x_2)dy$

$dL = (3 - 2y - y^2)dy$

$$L = \int_{-3}^{1} 3 - 2y - y^2 \; dy$$

$$= \left[3y - y^2 - \frac{1}{3}y^3\right] \Big|_{-3}^{1}$$

$$= \left[3(1) - 1^2 - \frac{1}{3}(1)^3\right] - [3(-3) - (-3)^2 - \frac{1}{3}(-3)^3]$$

$$= \frac{32}{3} \; satuan \; luas$$

2. **Sketsa grafik yang dibatasi oleh $y = \sqrt{x}, y = 2, x = 0$ dan tentukan volume yang diputar terhadap $x = -2$**

Pertama, cari titik potong

$y_1 = y_2$

$\sqrt{x} = 2$

$x = 4$

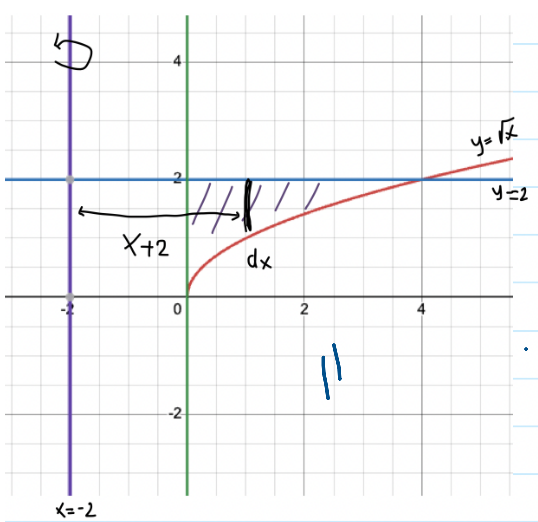
Kedua, gambar grafik

Untuk $y = \sqrt{x}$

$x = 0 \rightarrow y = 0$

$x = 2 \rightarrow y = \sqrt{2}$

$x = 4 \rightarrow y = 2$



Ketiga, cari volume (cincin)

$dV = 2\pi(2+x)(2-\sqrt{x})dx$

$$V = \int_0^4 2\pi(2+x)(2-\sqrt{x})\,dx$$

$$= 2\pi \int_0^4 (2+x)(2-\sqrt{x})\,dx$$

$$= 2\pi \int_0^4 4 - 2\sqrt{x} + 2x - x\sqrt{x}\;dx$$

$$= 2\pi \int_0^4 4 - 2x^{\frac{1}{2}} + 2x - x^{\frac{3}{2}}\;dx$$

$$= 2\pi \left[4x - \frac{4}{3}x^{\frac{3}{2}} + x^2 - \frac{2}{5}x^{\frac{5}{2}}\right]\Big|_0^4$$

$$= 2\pi \left[4x - \frac{4}{3}x\sqrt{x} + x^2 - \frac{2}{5}x^2\sqrt{x}\right]\Big|_0^4$$

$$= 2\pi \left[4(4) - \frac{4}{3}.4\sqrt{4} + 4^2 - \frac{2}{5}.4^2\sqrt{4}\right] - 0$$

$$= \frac{256\pi}{15}\ satuan\ luas$$

3. **Diberikan persamaan parametrik $\boldsymbol{x = t^2 + 1, y = t, 0 \leq t \leq 5}$**
   a. Gambar kurva dengan mengeliminasi parameter t
   b. Tentukan persamaan garis singgung Ketika $t = \frac{1}{2}$

Pertama, gambar kurva

$x = t^2 + 1 \ldots (1)$

$y = t. \ldots (2)$

Substitusi persamaan 2 ke 1

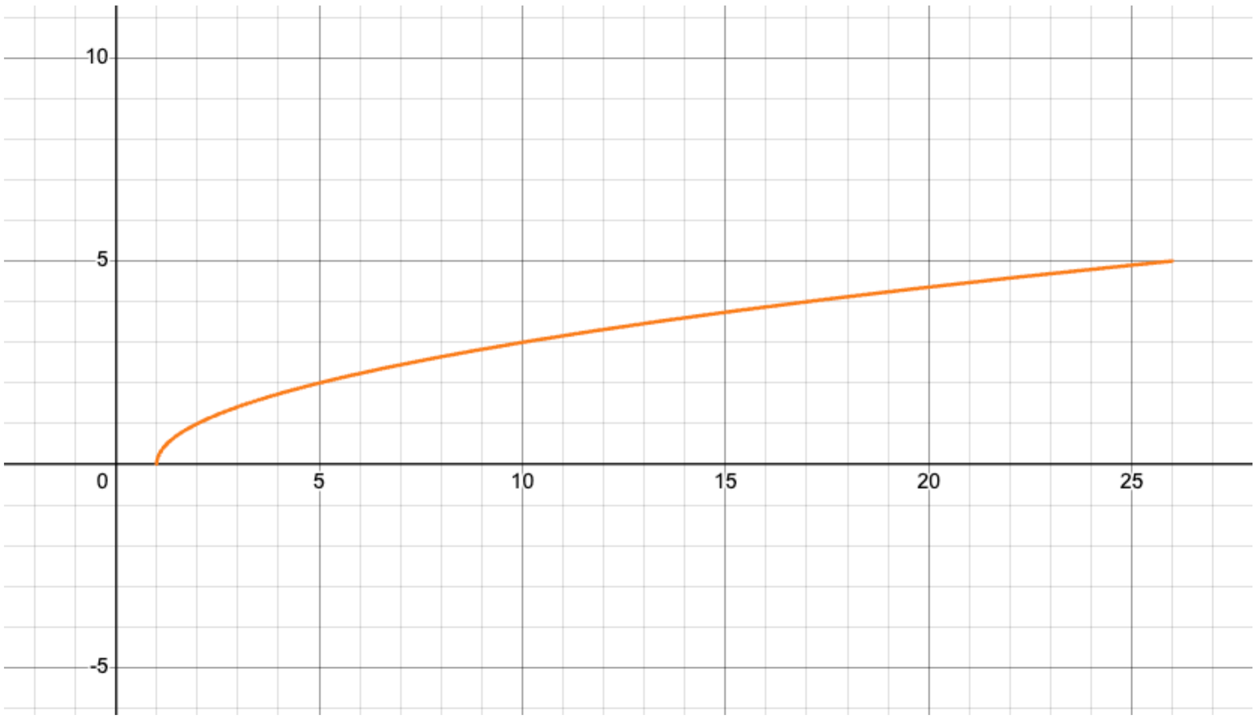
$x = y^2 + 1,\;\; 0 \leq y \leq 5$

Sehingga,

$y = 0 \rightarrow x = 1$

$y = 1 \rightarrow x = 2$

$y = 5 \rightarrow y = 26$



Kedua, cari persamaan garis singgung Ketika $t = \frac{1}{2}$

$$x_1 = x\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 1 = \frac{1}{4} + 1 = \frac{5}{4}$$

$$y_1 = y\left(\frac{1}{2}\right) = \frac{1}{2}$$

Lalu, cari gradien

$$x = t^2 + 1 \to \frac{dx}{dt} = 2t$$

$$y = t \to \frac{dy}{dt} = 1$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1}{2t}$$

$$m = \frac{dy}{dx}\Big|_{t=\frac{1}{2}} = \frac{1}{2\left(\frac{1}{2}\right)} = 1$$

Jadi, persamaan garis singgungnya adala

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{2} = 1\left(x - \frac{5}{4}\right)$$

4. Tentukan luas dari irisan kardioida $r = 2 - 2\cos\theta$ dan $r = 2 + 2\cos\theta$

Pertama, cari tiitik potong

Titik potong

$r_1 = r_2$

$2 - 2\cos\theta = 2 + 2\cos\theta$

$0 = 4\cos\theta$

$0 = \cos\theta$

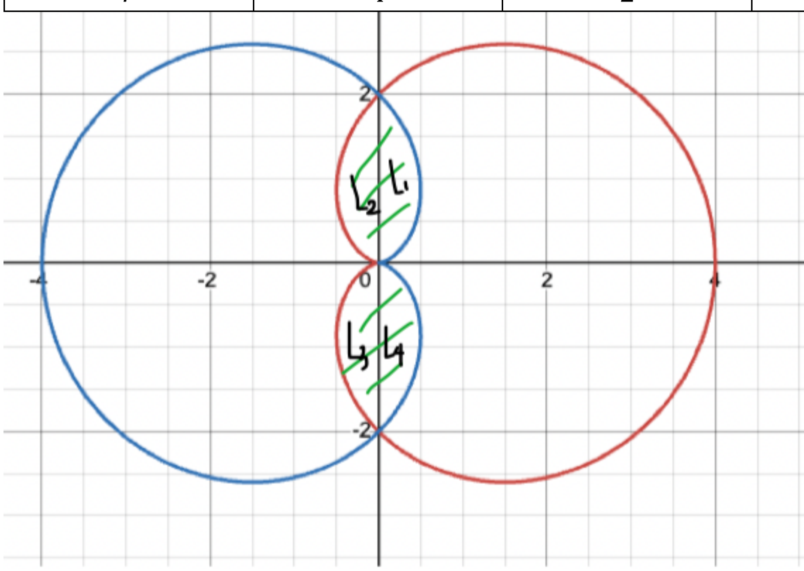
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

Untuk $r = 2 - 2\cos\theta$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r	0	2	4	2

Untuk $r = 2 + 2\cos\theta$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r	4	2	0	2



Perhatikan bahwa $L_1 = L_2 = L_3 = L_4$

Kedua, cari luas

$L = L_1 + L_2 + L_3 + L_4$

$L = 4L_1$

$$L = 4\int_0^{\frac{\pi}{2}} \frac{1}{2}(2-2\cos\theta)^2 d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} 4 - 8\cos\theta + 4\cos^2\theta \, d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} 4 - 8\cos\theta + 4\left[\frac{1}{2} + \frac{1}{2}\cos 2\theta\right] d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} 4 - 8\cos\theta + 2 + 2\cos 2\theta \, d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} 6 - 8\cos\theta + 2\cos 2\theta \, d\theta$$

$$= 2[6\theta - 8\sin\theta + \sin 2\theta]\Big|_0^{\frac{\pi}{2}}$$

$$= 2\left[3\pi - 8\sin\frac{\pi}{2} + \sin\pi\right] - 0 = 6\pi - 16 \; satuan\ luas$$

5. Dapatkan lima suku pertama polynomial maclaurine untuk fungsi $\boldsymbol{f(x) = e^{-x^2}}$

Polinomial maclaurine

$$p_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

$$f(x) = e^{-x^2} \rightarrow f(0) = 1$$

$$f'(x) = -2xe^{-x^2} \rightarrow f'(0) = 0$$

$$f''(x) = e^{-x^2}(4x^2 - 2) \rightarrow f''(0) = e^0(0-2) = -2$$

$$f'''(x) = -4e^{-x^2}(2x^3 - 3x) \rightarrow f'''(0) = -4(0) = 0$$

$$f^{(4)}(x) = 4e^{-x^2}(4x^4 - 12x^2 + 3) \rightarrow f^{(4)}(0) = 4e^0(0 - 0 + 3) = 4(3) = 12$$

$$f^{(5)}(x) = -8e^{-x^2}(4x^5 - 20x^3 + 15x) \rightarrow f^{(5)}(0) = 0$$

Sehingga,

$$p_5(x) = 1 + 0x - \frac{2}{2!}x^2 + \frac{0x^3}{3!} + \frac{12x^4}{4!} + \frac{0x^5}{5!}$$

$$p_5(x) = 1 - x^2 + \frac{x^4}{2}$$

$$p_5(x) = 1 - x^2 + \frac{x^4}{2!}$$

Perhatian !

Untuk memperoleh turunannya contohnya seperti ini

$f(x) = e^{-x^2}$

Misal

$$u = -x^2 \rightarrow \frac{du}{dx} = -2x$$

$$f(x) = e^u \rightarrow f'(x) = e^u . \frac{du}{dx} = e^{-x^2}(-2x) = -2xe^{-x^2}$$