1. Tentukan luas yang dibatasi oleh kurva $y = -x^2 + 2x + 3$ dan $y + 2x = 3$

$y = -x^2 + 2x + 3$

$y + 2x = 3 \rightarrow y = 3 - 2x$

Pertama, cari titik potong

$y_1 = y_2$

$-x^2 + 2x + 3 = 3 - 2x$

$0 = x^2 - 4x$

$0 = x(x - 4)$

$x = 0, x = 4$

Kedua, gambar grafik

Untuk $y = -x^2 + 2x + 3$

$x = 0 \rightarrow y = 3$

$x = 1 \rightarrow y = 4$

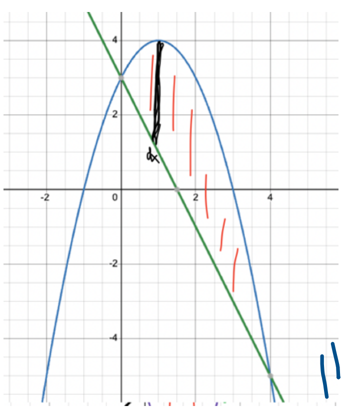
$x = 4 \rightarrow y = -5$

Untuk $y = 3 - 2x$

$x = 0 \rightarrow y = 3$

$x = 1 \rightarrow y = 1$

$x = 4 \rightarrow y = -5$



Ketiga, cari luas

$dL = (y_1 - y_2)dx$

$dL = (-x^2 + 2x + 3 - (3 - 2x))dx$

$$L = \int_0^4 (-x^2 + 2x + 3 - (3 - 2x))\ dx$$

$$= \int_0^4 -x^2 + 4x\ dx$$

$$= \left[-\frac{1}{3}x^3 + 2x^2\right]\Big|_0^4$$

$$= -\frac{64}{3} + 32$$

$$= \frac{32}{3}\ satuan\ luas$$

2. **Gambarkan daerah yang dibatasi kurva $y = 2x - x^2\ dan\ y = x^2 - 2x$ dengan menggunakan dalil guldin 1, dapatkan volume yang diputar terhadap y=2**

Pertama, cari titik potong

$y_1 = y_2$

$2x - x^2 = x^2 - 2x$

$0 = 2x^2 - 4x$

$0 = 2x(x - 2)$

$x = 0, x = 2$

Kedua, gambar grafik

Untuk $y = 2x - x^2$

$x = 2 \rightarrow y = 0$

$x = 1\ \rightarrow y = 1$

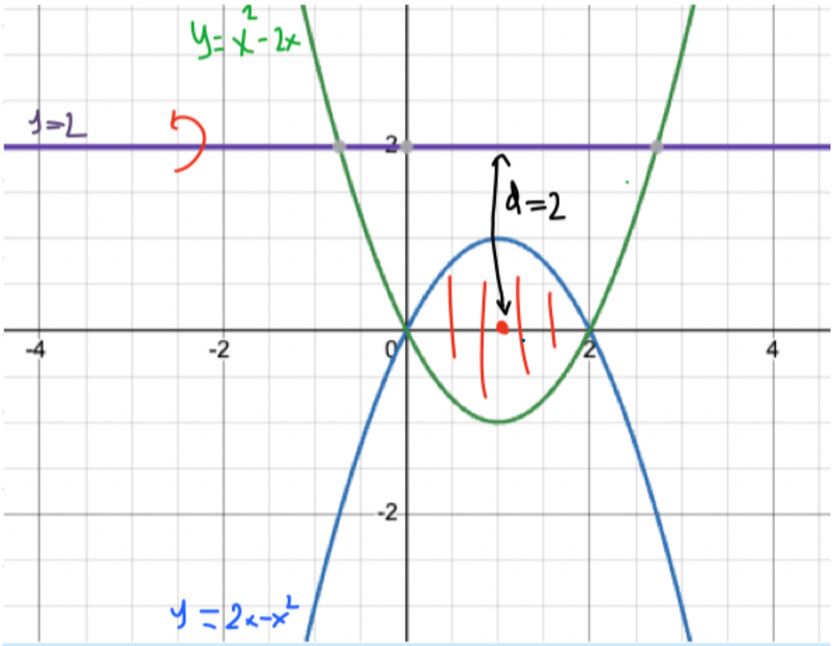
$x = 2 \rightarrow y = 0$

Untuk $y = x^2 - 2x$

$x = 2 \rightarrow y = 0$

$x = 1\ \rightarrow y = -1$

$x = 2 \rightarrow y = 0$



Ketiga, cari jarak titik berat ke sumbu putar

Titik berat di (1,0) karena simetri terhadap garis x=1 dan simetri terhadap sumbu x

Sehingga, jarak titik berat ke sumbu putar adalah d = 2

Keempat, mencari luas

$$dL = (y_1 - y_2)dx$$

$$dL = (2x - x^2 - (x^2 - 2x))dx$$

$$L = \int_0^2 2x - x^2 - (x^2 - 2x)\ dx$$

$$= \int_0^2 -2x^2 + 4x\ dx$$

$$= \left[-\frac{2}{3}x^3 + 2x^2\right]\Big|_0^2$$

$$= -\frac{16}{3} + 8$$

$$= \frac{8}{3}\ satuan\ luas$$

Kelima, cari volume (cakram)

$$dV = \pi(2^2 - \left(\frac{1}{x}\right)^2)dx$$

$$V = 2\pi . d . L = 2\pi . 2 . \frac{8}{3} = \frac{32}{3}\pi \; satuan \; volume$$

3. **Diberikan persamaan parametrik $\boldsymbol{x = sin\ t, y = 1 + 2sin\ t, 0 \leq t \leq \frac{\pi}{2}}$**
   a. Panjang kurva
   b. Sketsa kurva

Pertama, cari Panjang kurva

$$x = \sin t \rightarrow \frac{dx}{dt} = \cos t$$

$$y = 1 + 2\sin t \rightarrow \frac{dy}{dt} = 2\cos t$$

$$dS = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt = \sqrt{(\cos t)^2 + (2\cos t)^2}\ dt$$

$$S = \int_0^{\frac{\pi}{2}} \sqrt{(\cos t)^2 + (2\cos t)^2}\ dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\cos^2 t + 4\cos^2 t}\ \ dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{5\cos^2 t}\ \ dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{5}\cos t\ \ dt$$

$$= \left[\sqrt{5}\sin t\right]\Big|_0^{\frac{\pi}{2}}$$

$$= \left[\sqrt{5}\right] - [0] = \sqrt{5}\ satuan\ panjang$$

Kedua, gambar kurva

$x = \sin t \ldots (1)$

$y = 1 + 2\sin t \ldots (2)$

Substitusi persamaan 1 ke 2

$y = 1 + 2x$

Batas

$t = 0 \rightarrow x = \sin 0 = 0$

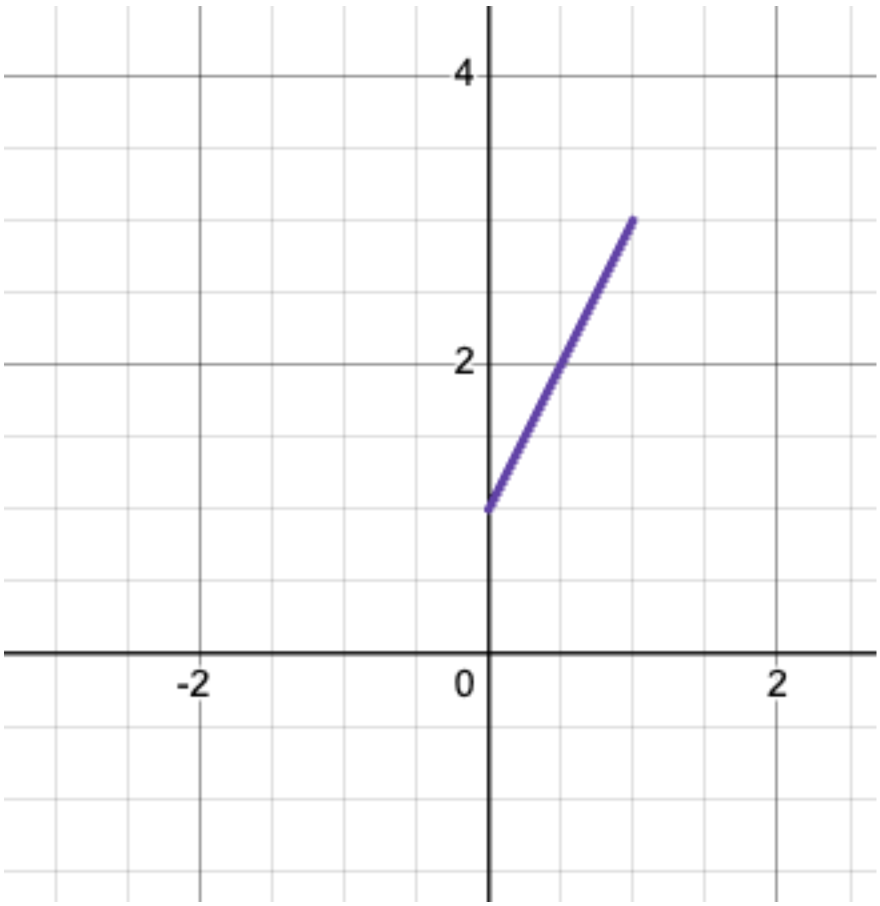
$t = \frac{\pi}{2} \rightarrow x = \sin \frac{\pi}{2} = 1$

sehingga

$y = 1 + 2x, \qquad 0 \leq x \leq 1$

Untuk $x = 0 \rightarrow y = 1$

Untuk $x = 1 \rightarrow y = 3$



4. Dapatkan luas daerah yang berada di dalam lingkaran $r = 4\ sin\ \theta$ dan diluar $r = 4cos\ \theta$

Pertama, cari tiitik potong

Titik potong

$r_1 = r_2$

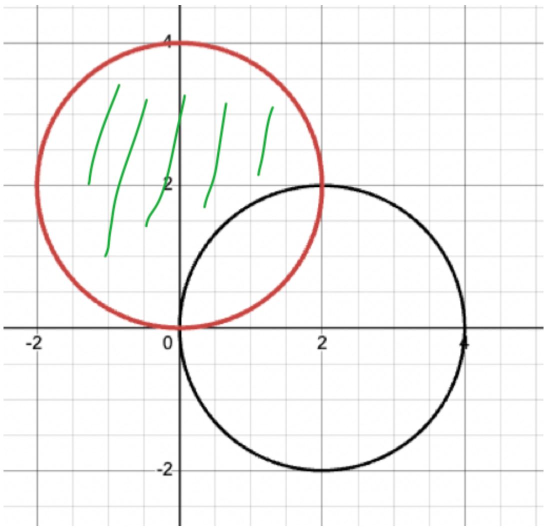
$4 \sin \theta = 4 \cos \theta$

$\sin \theta = \cos \theta$

$\theta = \frac{\pi}{4}$

Untuk $r = 4\sin\theta = 2.2\sin\theta \quad (lingkaran\ dengan\ pusat\ (0{,}2)\ dengan\ r = 2)$

Untuk $r = 4\cos\theta = 2.2\cos\theta \quad (lingkaran\ dengan\ pusat\ (2{,}0)\ dengan\ r = 2)$



Kedua, cari luas

$$L = luas\ lingkaran\ merah - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}(4\cos\theta)^2 d\theta$$

$$= \pi r^2 - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 8\cos^2\theta\, d\theta$$

$$= \pi(4)^2 - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 8\cos^2\theta\, d\theta$$

$$= 16\pi - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 8(\frac{1}{2} + \frac{1}{2}\cos 2\theta) d\theta$$

$$= 16\pi - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 4 + 4\cos 2\theta\, d\theta$$

$$= 16\pi - [4\theta + 2\sin 2\theta]\Bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= 16\pi - (\pi - 2)\ satuan\ luas$$

5. Dapatkan deret taylor untuk fungsi $f(x) = \frac{1}{5-4x}$ di x=1

<u>Deret Taylor dan Notasi sigma</u>

$$\sum_{k=0}^{+\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(k)}(a)}{k!}(x-a)^k + \cdots$$

$$f(x) = \frac{1}{5-4x} = (5-4x)^{-1} \rightarrow f(1) = 1$$

$$f'(x) = 4(5-4x)^{-2} \rightarrow f'(1) = 4$$

$$f''(x) = 32(5-4x)^{-3} \rightarrow f''(1) = 32$$

$$f'''(x) = 384(5-4x)^{-4} \rightarrow f'''(1) = 384$$

<u>Deret Taylor</u>

$$\frac{1}{5-4x} = 1 + 4(x-1) + \frac{32}{2!}(x-1)^2 + \frac{384}{4!}(x-1)^4 + \cdots$$

$$\frac{1}{5-4x} = 1 + 4(x-1) + 16(x-1)^2 + 64(x-1)^4 + \cdots$$

$$\frac{1}{5-4x} = 1 + 4^1(x-1) + 4^2(x-1)^2 + 4^3(x-1)^4 + \cdots + 4^k(x-1)^k + \cdots$$