Diskusi EAS Kalkulus 2 2024

Tuesday, June 25, 2024 7:23 PM

1) Luas antara dua kurva (4.1)
2) Menghitung volume benda putar (4.2)
   Luas permukaan benda putar (4.4)
   Titik berat (4.5)
3) Persamaan parametrik (5.1)
4) Luas dalam koordinat kutub (5.4)
5) Deret Taylor dan Deret Maclaurin (6.4)

titik potong (titik dilewati 2 kurva)

kurva 1 (x_1, y_1) ✓ kurva 2 (x_2, y_2)

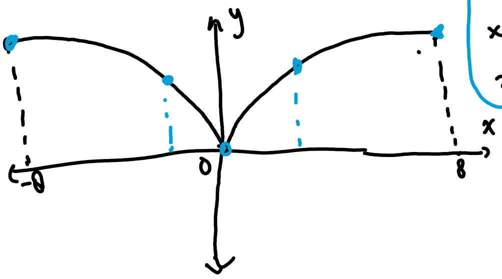
✓ $x_1 = x_2$ ✓ → ✓ y? ✓..

$y_1 = y_2$ ✓ → x?

1. ~~Diketahui kurva fungsi dengan persamaan $y = x^{\frac{2}{3}}$~~
   a. Gambarkan kurva yang dari persamaan tersebut di atas untuk $-8 \le x \le 8$
   b. Tentukan panjang busur untuk $0 \le x \le 8$, Jelaskan mengapa rumus panjang busur $\int_a^b \sqrt{1 + (f'(x))^2}\,dx$ tidak dapat digunakan secara langsung.

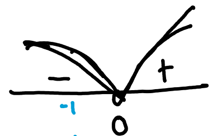
$\boxed{y = x^{2/3}}$ ✓

$x = \pm 1 \to y = 1$
$x = 0 \to y = 0$
$x = \pm 8 \to y = 4$



✓ $\frac{dy}{dx} = \frac{2}{3} x^{-1/3} = \frac{2}{3x^{1/3}}$

$x^{1/3} = 0 \to x = 0$

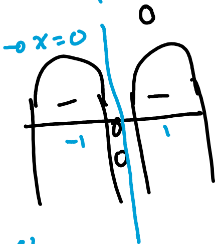


naik turun

✓ $\frac{d^2y}{dx^2} = -\frac{2}{9} x^{-4/3}$

$= -\frac{2}{9x^{4/3}} \to x^{4/3} = 0$

$x = 0$



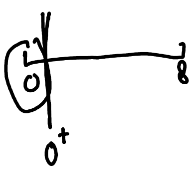
kecekungan

b) $y = x^{2/3} \to \frac{dy}{dx} = \frac{2}{3} x^{-1/3} = \frac{2}{3x^{1/3}}$ ✓

$0 \le x \le 8$ ✓

$f'(x)$ tidak terdefinisi untuk $x = 0$

$$S = \int_{\boxed{x=0}}^{8} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$



$$= \lim_{a \to 0^+} \int_a^8 \sqrt{1 + \left(\frac{4}{9} x^{-2/3}\right)}\,dx$$

$$= \lim_{a \to 0^+} \int_a^8 \sqrt{\frac{9x^{2/3}}{9x^{2/3}} + \frac{4}{9x^{2/3}}}\,dx$$

$$= \lim_{a \to 0^+} \int_a^8 \frac{1}{3x^{1/3}} \sqrt{9x^{2/3}+4} \; dx$$

$$= \lim_{a \to 0^+} \int_a^8 \frac{1}{3} x^{-1/3} \sqrt{9x^{2/3}+4} \; dx$$

Hitung $\int \frac{1}{3} x^{-1/3} \sqrt{9x^{2/3}+4} \; dx = \int \frac{1}{3} \cdot \frac{1}{6} u^{1/2} du$

$$= \frac{1}{18} \cdot \frac{2}{3} u^{3/2}$$

$$= \frac{1}{27} (9x^{2/3} + 4)$$

misal $u = 9x^{2/3} + 4$

$$\frac{du}{dx} = 9 \cdot \frac{2}{3} x^{-1/3}$$

$$\frac{1}{6} du = x^{-1/3} \; dx$$

$$\lim_{a \to 0^+} \frac{1}{27} (9x^{2/3} + 4) \Big|_a^8$$

$$= \lim_{a \to 0^+} \frac{1}{27} \left[\left(9 \cdot (2^3)^{2/3} + 4\right) - \left(9a^{2/3} + 4\right) \right]$$

$$= \lim_{a \to 0^+} \frac{1}{27} \left[40 - 9a^{2/3} - 4 \right]$$

$$= \lim_{a \to 0^+} \boxed{\frac{36}{27}} - \underbrace{\boxed{\frac{9}{27} a^{2/3}}}_{\to 0^+}$$

$$S = \frac{4}{3} \; //$$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

4. Diberikan fungsi $f(x) = sinhx$. [Petunjuk: $sinhx = \frac{1}{2}(e^x - e^{-x}), x \in (-\infty, \infty)$]
   a. Dapatkan polinomial Maclaurin derajat 3 dari fungsi tersebut. $\to a = 0$
   b. Dapatkan deret Maclaurin fungsi tersebut dan nyatakan dalam notasi sigma.

$$\sum_{n=0}^{\infty} \frac{\boxed{f^{(n)}(a)}}{n!} \cdot (x-a)^n \quad ✓$$

$f(x) = \sinh x$ ✓

a) $n=0 \to f^{(0)}(0) = \sinh 0 = \frac{1}{2}(e^0 - e^{-0}) = \frac{1}{2}\left(1 - \frac{1}{1}\right) = 0$ ✓

$n=1 \to f^{(1)}(x) = \cosh x \to f^{(1)}(0) = \cosh 0 = \frac{1}{2}(e^0 + e^{-0}) = \frac{1}{2}\left(1 + \frac{1}{1}\right) = 1$

$n=2 \to f^{(2)}(x) = \sinh x \to f^{(2)}(0) = \sinh 0 = 0$

$n=3 \to f^{(3)}(x) = \cosh x \to f^{(3)}(0) = \cosh 0 = 1$.

$$\sum_{n=0}^{3} \frac{f^{(n)}(0)}{n!}(x)^n = \frac{0}{0!}x^0 + \frac{1}{1!}x^1 + \frac{0}{2!}x^2 + \frac{1}{3!}x^3$$

$$= \frac{1}{1!}x^1 + \frac{1}{3!}x^3$$

b.) $$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n = \frac{1}{1!}x^1 + \frac{1}{3!}x^3 + \dots$$

$$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} \cdot x^{2n+1} \quad //$$

$n=0 \to 1 : \boxed{2}\cdot 0 + 1$

$n=1 \to 3 : \boxed{2}\cdot 1 + 1$

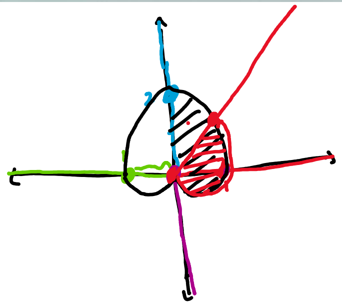
$n=2 \to 5 : \boxed{2}\cdot 2 + 1$

$n=3 \to 7 : \boxed{2}\cdot 3 + 1$

4. Dapatkan luas daerah di kwadran I dan IV di dalam kardioda $r = 1 + \sin\theta$ dan $\theta = \frac{\pi}{4}$.

θ	r
0	1
$\pi/2$	2
π	1
$3\pi/2$	0
2π	1

$2\pi - \frac{\pi}{2}$



θ	2θ	r
0	0	
$\pi/4$	$\pi/2$	
$\pi/2$	π	
$3\pi/4$	$3\pi/2$	
π	2π	
$5\pi/4$	$5\pi/2$	
$\vdots$		
2π	4π	

$$A = \frac{1}{2}\int_{-\pi/2}^{\pi/4} (1+\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/4} 1 + 2\sin\theta + \sin^2\theta \, d\theta$$

$$\boxed{\begin{aligned} \sin^2\theta &= \tfrac{1}{2}(1-\cos 2\theta) \\ \cos^2\theta &= \tfrac{1}{2}(1+\cos 2\theta) \end{aligned}}$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/4} 1 + 2\sin\theta + \frac{1}{2} - \frac{1}{2}\cos\boxed{2\theta} \, d\theta$$

$$\frac{d}{d\theta}(\sin 2\theta) = 2 \cdot \cos 2\theta$$

$$= \frac{1}{2}\left(\frac{3}{2}\theta + 2(-\cos\theta) - \frac{1}{2}\cdot\boxed{\frac{1}{2}\sin 2\theta}\right)\Bigg|_{-\pi/2}^{\pi/4}$$

$$\sin 2\theta = \int 2\boxed{\cos 2\theta}\,d\theta$$

$$= \frac{1}{2}\left(\frac{3}{2}\left(\frac{\pi}{4} - \left(-\frac{\pi}{2}\right)\right) - 2\left(\cos\frac{\pi}{4} - \cos\left(-\frac{\pi}{2}\right)\right) - \frac{1}{4}\left(\sin\frac{\pi}{2} - \sin(-\pi)\right)\right)$$

$+2\pi$, $3\pi/2$; $+2\pi$, π

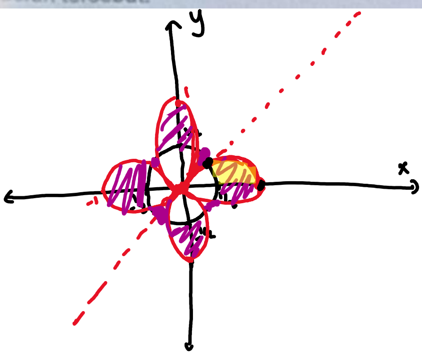
$$= \frac{1}{2}\left(\frac{3}{2}\cdot\frac{3\pi}{4} - 2\left(\frac{1}{2}\sqrt{2} - 0\right) - \frac{1}{4}(1-0)\right)$$

$$= \frac{1}{2}\left(\frac{9\pi}{8} - \sqrt{2} - \frac{1}{4}\right)$$

$$A = \frac{9\pi}{16} - \frac{1}{2}\sqrt{2} - \frac{1}{8} \quad //$$

$$A = \frac{9\pi}{16} - \frac{1}{2}\sqrt{2} - \frac{1}{8} \;//$$

3. Dapatkan luas daerah di luar lingkaran $r_2 = \frac{1}{2}$ dan di dalam rose $r_1 = \cos 2\theta$. Sketsa daerah tersebut.



θ	2θ	r
0	0	1
$\pi/4$	$\pi/2$	0 ✓
$\pi/2$	π	-1
$3\pi/4$	$3\pi/2$	0
π	2π	1
$5\pi/4$	$5\pi/2$	0
$3\pi/2$	3π	-1
$7\pi/4$	$\frac{7\pi}{2}$	0
π	2π	1

$r = r$

$\frac{1}{2} = \cos 2\theta$

$\frac{\pi}{3} = 2\theta$

$\frac{\pi}{6} = \theta$

$$A_t = \frac{1}{2}\int_0^{\pi/6} (\cos 2\theta)^2 - \frac{1}{2}^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/6} \cos^2 2\theta - \frac{1}{4} \, d\theta$$

$$= \frac{1}{2}\left[\int_0^{\pi/6} \frac{1}{2}(1 + \cos 4\theta) - \frac{1}{4} \, d\theta\right] = \frac{1}{2}\left[\int_0^{\pi/6} \frac{1}{4} + \frac{1}{2}\cos 4\theta \, d\theta\right]$$

$$= \frac{1}{2}\left[\frac{1}{4}\theta + \frac{1}{2}\cdot\frac{1}{4}\cdot\sin 4\theta \Big|_0^{\pi/6}\right]$$

$$= \frac{1}{2}\left[\frac{1}{4}\left(\frac{\pi}{6} - 0\right) + \frac{1}{8}\left(\sin\frac{4\pi}{6} - \sin 0\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{24} + \frac{1}{8}\,\frac{1}{2}\sqrt{3}\right] = \frac{\pi}{48} + \frac{1}{32}\sqrt{3}$$

$$A_t = 8\left(\frac{\pi}{48} + \frac{1}{32}\sqrt{3}\right) = \frac{\pi}{6} + \frac{1}{4}\sqrt{3} \;//$$

1. Dapatkan volume benda putar jika daerah yang dibatasi oleh $y = \frac{1}{x}$, $y = 2$ dan garis $x = 2$ diputar terhadap garis $y = 2$ serta sketsa daerahnya.

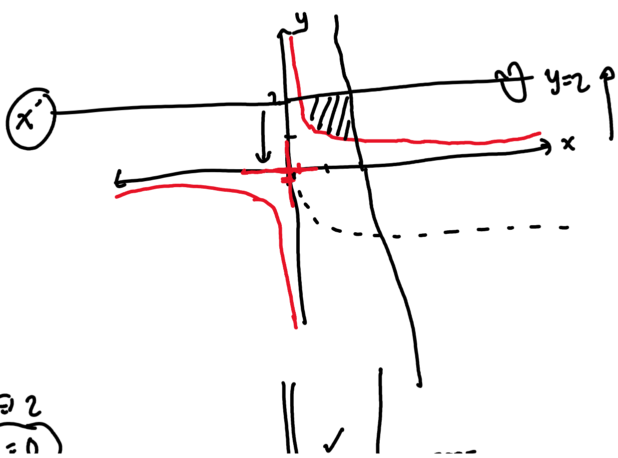
$y = \frac{1}{x}$

$x \to 0^+ \Rightarrow y \to +\infty$

$x \to 0^- \Rightarrow y \to -\infty$

$x \to +\infty \Rightarrow y \to 0^+$

$x \to -\infty \Rightarrow y \to 0^-$



$y = 2 \longrightarrow y - 2 = 2$

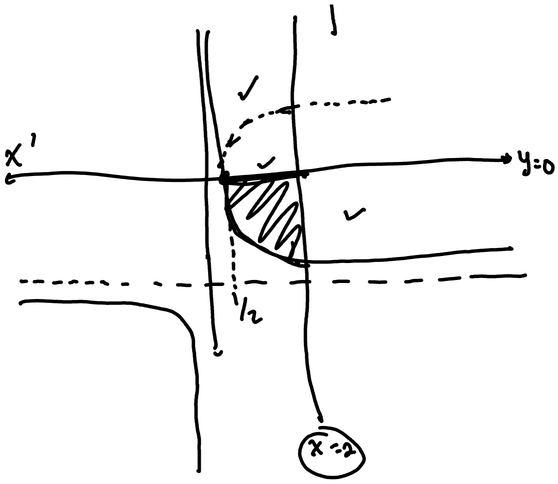
$y = 0$

$y = 2 \longrightarrow y + 2 = 2$

$y_1 = 0$

$y = \frac{1}{x} \rightarrow y + 2 = \frac{1}{x}$

$y_2 = \frac{1}{x} - 2$



titik potong $y=0$ dan $y = \frac{1}{x} - 2$

$0 = \frac{1}{x} - 2$

$2 = \frac{1}{x}$

$x = \frac{1}{2}$

$$V = \pi \int_{x=1/2}^{2} (0)^2 - \left(\frac{1}{x} - 2\right)^2 \, dx$$

$$= \pi \int_{x=1/2}^{2} + \left(\frac{1}{x^2} + 4 - \frac{4}{x}\right) dx$$

$$= \pi \int_{x=1/2}^{2} + x^{-2} + 4 - 4x^{-1} \, dx$$

$$= \pi \left[-(-1)x^{-1} - 4x + \ln|x| \right]_{1/2}^{2}$$

$$= -\pi \left[\left(\frac{1}{2} - \frac{1}{1/2}\right) - 4\left(2 - \frac{1}{2}\right) + \ln|2| - \ln\left|\frac{1}{2}\right| \right]$$

$$= -\pi \left[-\frac{3}{2} - 8 + 2 + \ln|2| - \ln\left|\frac{1}{2}\right| \right]$$

$$= -\pi \left[-\frac{15}{2} + \ln|2| - \ln\left|\frac{1}{2}\right| \right]$$

$$= \frac{15}{2}\pi - \pi \ln|2| + \pi \ln\left|\frac{1}{2}\right| .$$

$\ln|2^{-1}| = -\ln 2$

$$= \frac{15}{2}\pi - 2\pi \ln 2 \quad //$$