1. Tentukan luas yang dibatasi oleh kurva $y = x^2 - 4x + 3$ dan $y = x + 3$

Pertama, cari titik potong

$y_1 = y_2$

$x^2 - 4x + 3 = x + 3$

$x^2 - 5x = 0$

$x(x - 5) = 0$

$x = 0, x = 5$

Kedua, gambar grafik

Untuk $y = x^2 - 4x + 3$

$x = 0 \rightarrow y = 3$

$x = 1 \rightarrow y = 0$

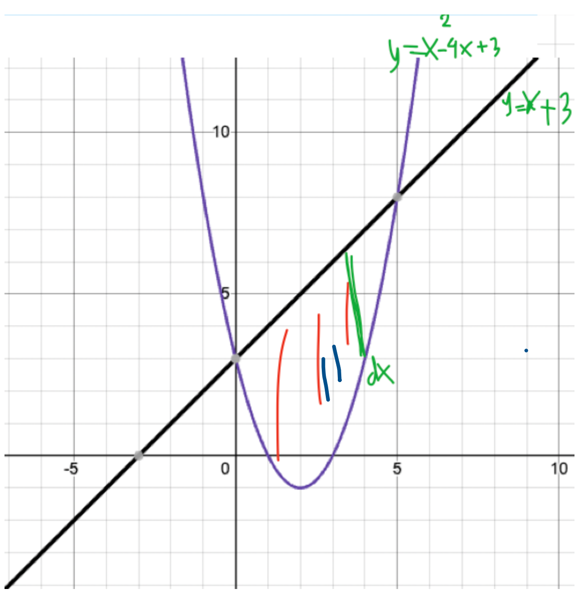
$x = 5 \rightarrow y = 8$

Untuk $x = x + 3$

$x = 0 \rightarrow y = 3$

$x = 1 \rightarrow y = 4$

$x = 5 \rightarrow y = 8$



Ketiga, cari luas

$dL = (y_1 - y_2)dx$

$dL = (x + 3 - (x^2 - 4x + 3))dx$

$$L = \int_{0}^{5} (x + 3 - (x^2 - 4x + 3))\ dx$$

$$= \int_{0}^{5} -x^2 + 5x\ dx$$

$$= \left[-\frac{1}{3}x^3 + \frac{5}{2}x^2\right]\Big|_0^5$$

$$= -\frac{125}{3} + \frac{125}{2}$$

$$= \frac{125}{6}\ satuan\ luas$$

2. **Volume benda putar yang dibatasi $y = \frac{1}{x}, x = 2, y = 2$ yang diputar terhadap sumbu x**

Pertama, cari titik potong

$y_1 = y_2$

$$\frac{1}{x} = 2$$

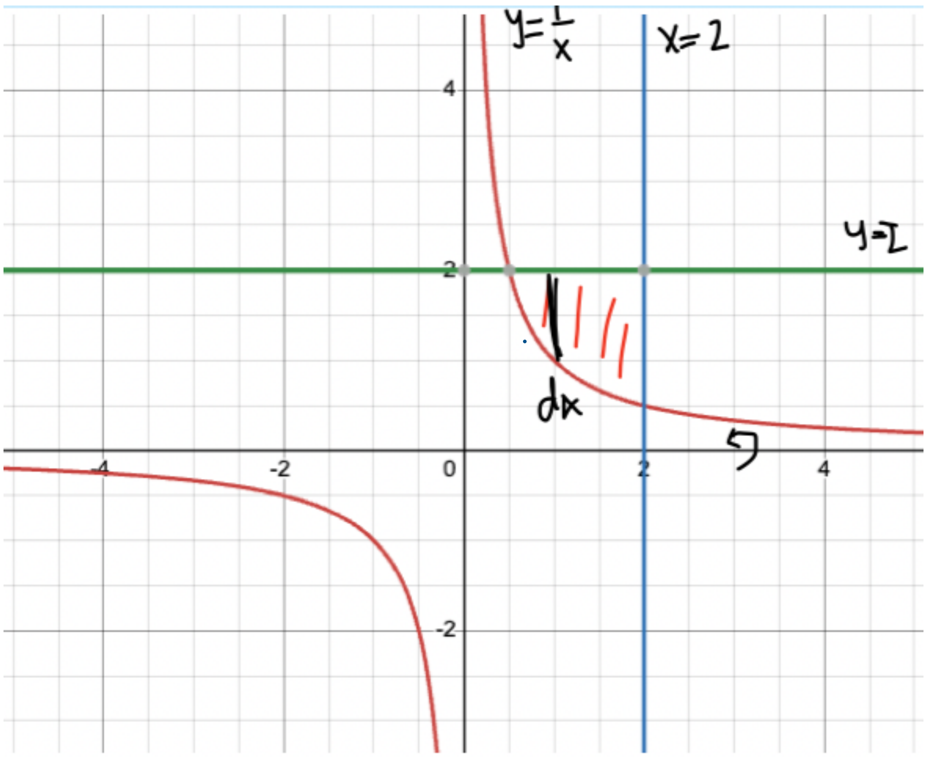
$$x = \frac{1}{2}$$

Kedua, gambar grafik

Untuk $y = \frac{1}{x}$

$$x = \frac{1}{2} \rightarrow y = 2$$

$$x = 2 \rightarrow y = \frac{1}{2}$$



Ketiga, cari volume (cakram)

$$dV = \pi(2^2 - \left(\frac{1}{x}\right)^2)dx$$

$$V = \int_{\frac{1}{2}}^{2} \pi\left(2^2 - \left(\frac{1}{x}\right)^2\right)\, dx$$

$$= \pi \int_{\frac{1}{2}}^{2} 4 - \frac{1}{x^2}\, dx$$

$$= \pi\left[4x + \frac{1}{x}\right]\Big|_{\frac{1}{2}}^{2}$$

$$= \pi\left[8 + \frac{1}{2}\right] - \pi[2 + 2]$$

$$= \frac{9}{2}\ satuan\ luas$$

3. Diberikan persamaan parametrik $x = \cos 2t, y = 3 - 2\cos 2t, 0 \le t \le \frac{\pi}{2}$

a. Panjang kurva

b. Sketsa kurva

Pertama, cari Panjang kurva

$$x = \cos 2t \rightarrow \frac{dx}{dt} = -2\sin 2t$$

$$y = 3 - 2\cos 2t \rightarrow \frac{dy}{dt} = 2\sin 2t$$

$$dS = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt = \sqrt{(-2\sin 2t)^2 + (2\sin 2t)^2}\, dt$$

$$S = \int_0^{\frac{\pi}{2}} \sqrt{(-2\sin 2t)^2 + (2\sin 2t)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{4\sin^2 2t + 4\sin^2 2t}\ dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{8\sin^2 2t}\ dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{8}\sin 2t\ dt$$

$$= \left[-\frac{\sqrt{8}}{2}\cos 2t\right]\Bigg|_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\sqrt{8}}{2}\right] - \left[-\frac{\sqrt{8}}{2}\right] = \sqrt{8}\ satuan\ panjang$$

Kedua, gambar kurva

$x = \cos 2t \ldots (1)$

$y = 3 - 2\cos 2t \ldots (2)$

Substitusi persamaan 1 ke 2

$y = 3 - 2x$

Batas

$t = 0 \rightarrow x = \cos 0 = 1$

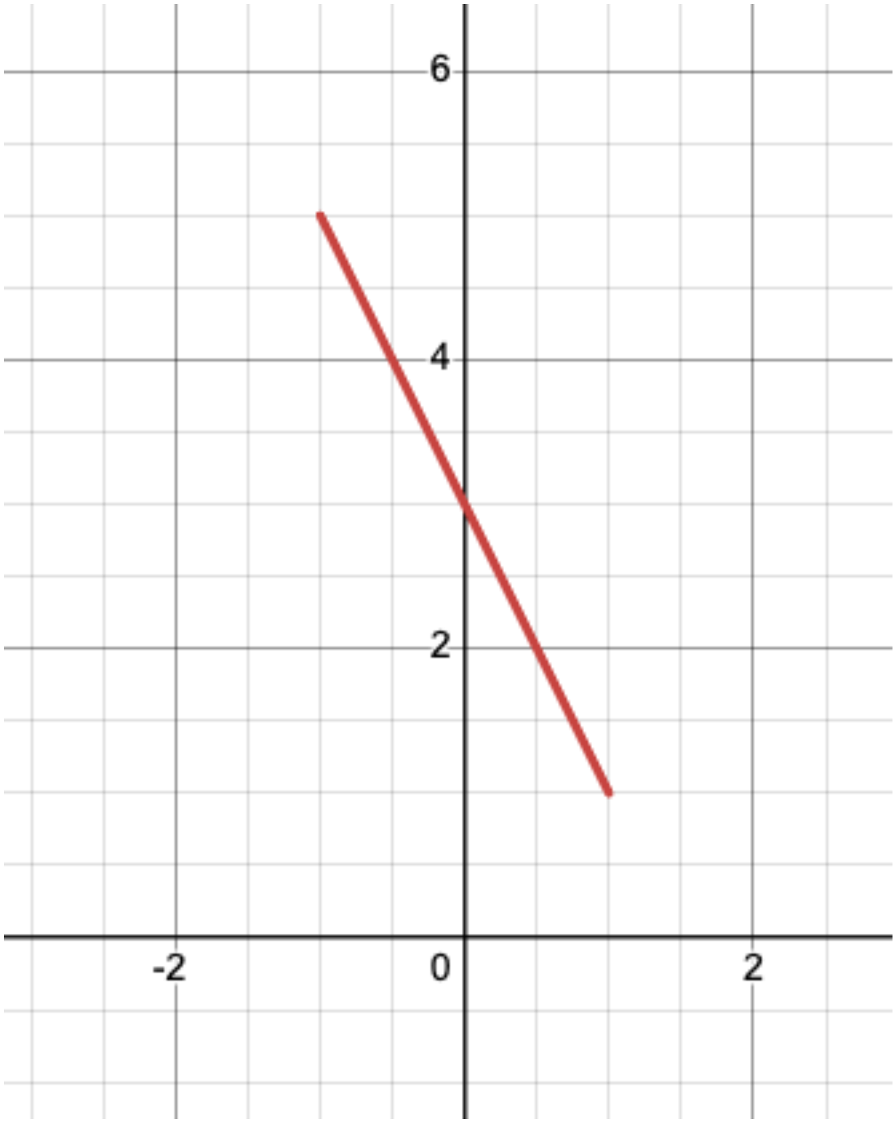
$t = \frac{\pi}{2} \rightarrow x = -1$

sehingga

$y = 3 - 2x, \qquad -1 \leq x \leq 1$

Untuk $x = -1 \rightarrow y = 5$

Untuk $x = 1 \rightarrow y = 1$



4. Dapatkan luas daerah yang beara di dalam kardioida $r = 2 - 2\cos\theta$ dan diluar $r = 2 + 2\cos\theta$

Pertama, cari tiitik potong

Titik potong

$r_1 = r_2$

$2 - 2\cos\theta = 2 + 2\cos\theta$

$0 = 4\cos\theta$

$0 = \cos\theta$

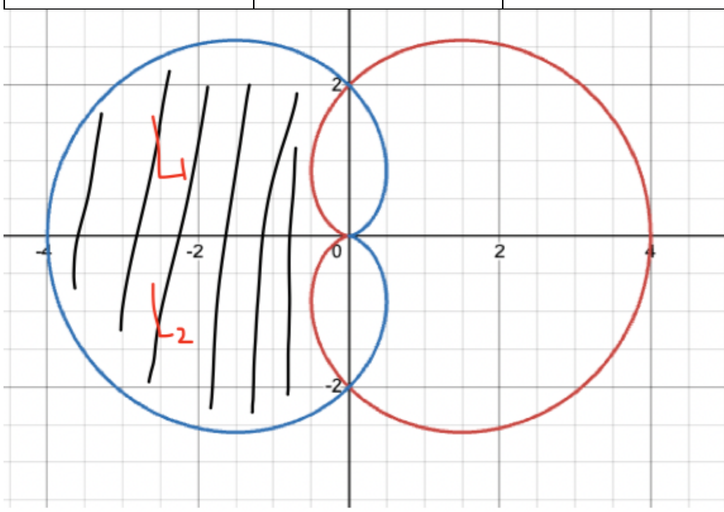
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

Untuk $r = 2 - 2\cos\theta$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r	0	2	4	2

Untuk $r = 2 + 2\cos\theta$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r	4	2	0	2



Perhatikan bahwa $L_1 = L_2$

Kedua, cari luas

$L = L_1 + L_2$

$L = 2L_1$

$$L = 2\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}[(2 - 2\cos\theta)^2 - (2 + 2\cos\theta)^2]d\theta$$

$$= \int_{\frac{\pi}{2}}^{\pi} (2 - 2\cos\theta)^2 - (2 + 2\cos\theta)^2 d\theta$$

$$= \int_{\frac{\pi}{2}}^{\pi} 4 - 8\cos\theta + 4\cos^2\theta - (4 + 8\cos\theta + 4\cos^2\theta)d\theta$$

$$= \int_{\frac{\pi}{2}}^{\pi} -16 \cos\theta \, d\theta$$

$$= [-16 \sin\theta] \Big|_{\frac{\pi}{2}}^{\pi}$$

$$= [0] - [-16] = 16 \; satuan \; luas$$

5. Dapatkan deret maclaurin untuk fungsi $f(x) = \ln(1+x)$

Deret Maclaurin dan Notasi sigma

$$\sum_{k=0}^{+\infty} \frac{f^{(k)}(a)}{k!} x^k = f(0) + f'(0)x + \frac{f''(0)}{2!} x^2 + \cdots + \frac{f^{(k)}(0)}{k!} x^k + \cdots$$

Akan dicari deret maclaurine dan notasi sigma dari fungsi $f(x) = \ln(1+x)$

$$f(x) = \ln(1+x) \rightarrow f(0) = \ln 1 = 0$$

$$f'(x) = \frac{1}{1+x} = (1+x)^{-1} = 0!\,(1+x)^{-1} \rightarrow f'(0) = 0!$$

$$f''(x) = -(1+x)^{-2} = -1!\,(1+x)^{-2} \rightarrow f''(0) = -1!$$

$$f'''(x) = 2(1+x)^{-3} = 2!\,(1+x)^{-3} \rightarrow f'''(0) = 2!$$

$$f''''(x) = -6(1+x)^{-4} = -3!\,(1+x)^{-4} \rightarrow f''''^{(0)} = -3!$$

$\vdots$

$$f^{(k)}(x) = (-1)^{k+1}(k-1)!\,(1+x)^{-k} \rightarrow f^{(k)}(0) = (-1)^{k+1}(k-1)!$$

Deret Maclaurin dan Notasi sigma

$$\ln(1+x) = 0!\,x - \frac{1!}{2!}x^2 + \frac{2!}{3!}x^3 - \frac{3!}{4!}x^4 + \cdots + \frac{(-1)^{k+1}(k-1)!}{k!}x^k + \cdots$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + \frac{(-1)^{k+1}x^k}{k} + \cdots = \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}x^k}{k}$$