1. Dapatkan luas daerah yang dibatasi oleh kurva $x = y^3 - y$ dan $x = 0$

Pertama, cari titik potong

$x_1 = x_2$

$y^3 - y = 0$

$y(y^2 - 1) = 0$

$y(y-1)(y+1) = 0$

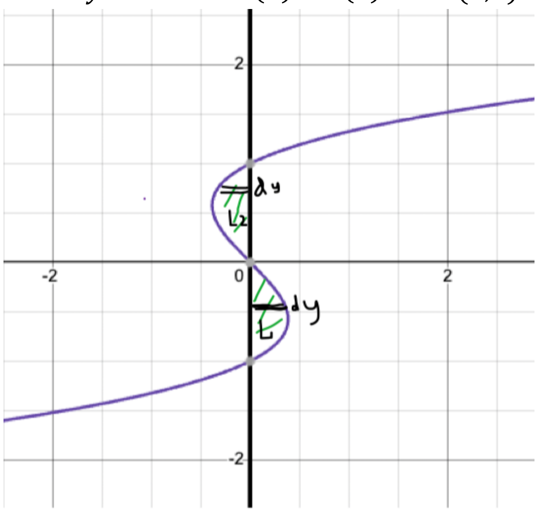
$y = 0, y = -1, y = 1$

Kedua, gambar grafik

Untuk $y = 0 \rightarrow x = 0 \quad (0,0)$

Untuk $y = -1 \rightarrow x = (-1)^3 - (-1) = 0 \quad (0,-1)$

Untuk $y = 1 \rightarrow x = (1)^3 - (1) = 0 \quad (0,1)$



Ketiga, cari luas

$dL = dL_2 + dL_1$

$dL = (y^3 - y - 0)dy + \big(0 - (y^3 - y)\big)dy$

$$L = \int_{-1}^{0} (y^3 - y - 0)\ dy + \int_{0}^{1} (0 - (y^3 - y))\ dy$$

$$= \int_{-1}^{0} (y^3 - y)\ dy + \int_{0}^{1} y - y^3\ dy$$

$$= \left[\frac{1}{4}y^4 - \frac{1}{2}y^2\right]\Big|_{-1}^{0} + \left[\frac{1}{2}y^2 - \frac{1}{4}y^4\right]\Big|_{0}^{1}$$

$$= \left[\frac{1}{4}0^4 - \frac{1}{2}0^2\right] - \left[\frac{1}{4}(-1)^4 - \frac{1}{2}(-1)^2\right] + \left[\frac{1}{2}(1)^2 - \frac{1}{4}(1)^4\right] - \left[\frac{1}{2}0^2 - \frac{1}{4}0^4\right]$$

$$= -\left[\frac{1}{4} - \frac{1}{2}\right] + \left[\frac{1}{2} - \frac{1}{4}\right]$$

$$= -\left[-\frac{1}{4}\right] + \left[\frac{1}{4}\right]$$

$$= \frac{1}{2}\ satuan\ luas$$

2. Gambarkan daerah di kuadran 1 yang dibatasi oleh kurva $y = x^2, y = 8 - 2x, dan\ sumbu\ y$. Dapatkan volume jika diputar pada sumbu x

Pertama, cari titik potong

$y_1 = y_2$

$x^2 = 8 - 2x$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = -4\ (tidak\ memenuhi\ karena\ yang\ diambil\ di\ KW\ 1)$

$x = 2$

Kedua, gambar grafik

Untuk $y = x^2$

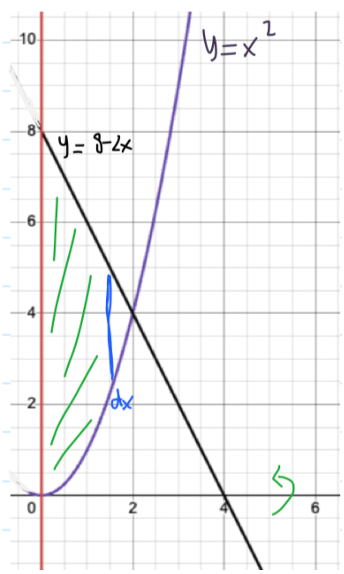
$x = 0 \rightarrow y = 0$

$x = 2 \rightarrow y = 4$

Untuk $y = 8 - 2x$

$x = 0 \rightarrow y = 8$

$x = 2 \rightarrow y = 4$



Ketiga, cari volume (cakram)

$dV = \pi((8 - 2x)^2 - (x^2)^2)dx$

$$V = \int_0^2 \pi((8 - 2x)^2 - (x^2)^2)\ dx$$

$$= \pi \int_0^2 64 - 32x + 4x^2 - x^4 \; dx$$

$$= \pi \left[64x - 16x^2 + \frac{4}{3}x^3 - \frac{1}{5}x^5 \right] \Big|_0^2$$

$$= \pi \left[128 - 64 + \frac{32}{3} - \frac{32}{5} \right]$$

$$= \frac{1024}{15} \; satuan\ luas$$

3. Hitunglah Panjang busur kurva $\boldsymbol{x = a(t - sin\ t), y = a(1 - cos\ t), 0 \leq t \leq 2\pi}$

$x = a(t - \sin t) \rightarrow \frac{dx}{dt} = a(1 - \cos t)$

$y = a(1 - \cos t) \rightarrow \frac{dy}{dt} = \mathrm{a}\sin t$

Sehingga

$$dS = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \; dt$$

$$dS = \sqrt{\left(a(1 - \cos t)\right)^2 + (\mathrm{a}\sin t)^2} \, dt$$

$$S = \int_0^{2\pi} \sqrt{\left(a(1 - \cos t)\right)^2 + (\mathrm{a}\sin t)^2} \; dt$$

$$= \int_0^{2\pi} \sqrt{a^2 - 2a^2 \cos t + a^2 \cos^2 t + a^2 \sin^2 t} \; dt$$

$$= \int_0^{2\pi} a\sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t} \; dt$$

$$= \int_0^{2\pi} a\sqrt{1 - 2\cos t + 1} \; dt$$

$$= \int_0^{2\pi} a\sqrt{2 - 2\cos t} \; dt$$

$$= \int_0^{2\pi} a\sqrt{2(1 - \cos t)} \; dt$$

$$= \int_0^{2\pi} a \sqrt{4 \sin^2 \frac{t}{2}}\, dt$$

$$= \int_0^{2\pi} 2a \sin \frac{t}{2}\, dt$$

$$= -4a \cos \frac{t}{2} \Big|_0^{2\pi} = 8a \; satuan \; panjang$$

4. Dapatkan luas daerah yang diperoleh dari irisan kurva $\boldsymbol{r = 3cos\, \theta}$ dan $\boldsymbol{r = 1 + cos\, \theta}$

Pertama, cari titik potong

Tipot

$r_1 = r_2$

$3 \cos \theta = 1 + cos\theta$

$2\, cos\theta = 1$

$cos\theta = \frac{1}{2}$

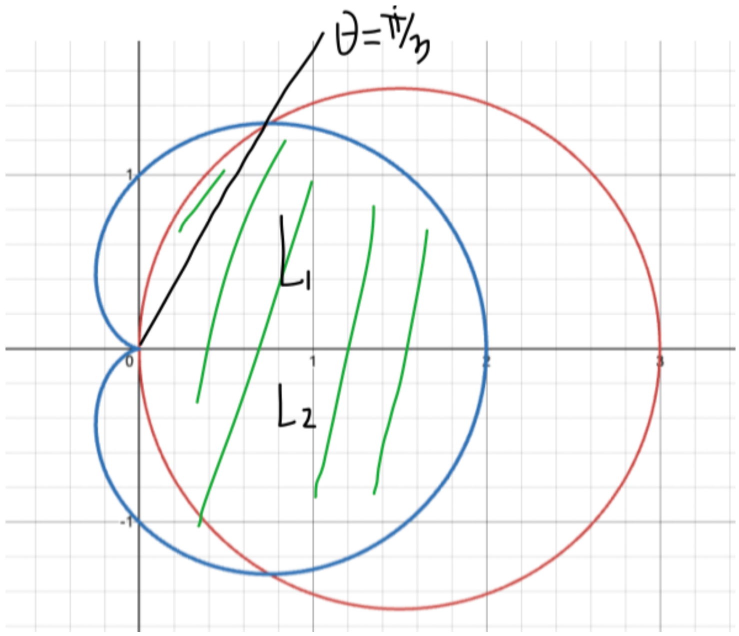
$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

Kedua, gambar grafik

Untuk $r = 3 \cos \theta = 2.\frac{3}{2} \cos \theta$ (lingkaran dengan pusat $\left(\frac{3}{2}, 0\right)$ dan jari jari $\frac{3}{2}$

Untuk $r = 1 + \cos \theta$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r	2	1	0	1



Ketiga, cari luas

$$L = L_1 + L_1$$

$$L = 2L_1$$

$$L = 2\left[\int_0^{\frac{\pi}{3}} \frac{1}{2}(1+\cos\theta)^2 \; d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2}(3\,cos\theta)^2 \; d\theta\right]$$

$$= \int_0^{\frac{\pi}{3}} 1 + 2\,cos\theta + \cos^2\theta \; d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9\cos^2\theta \; d\theta$$

$$= \int_0^{\frac{\pi}{3}} 1 + 2\,cos\theta + \left[\frac{1}{2} + \frac{1}{2}\cos 2\theta\right] \; d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9\left[\frac{1}{2} + \frac{1}{2}\cos 2\theta\right] \; d\theta$$

$$= \left[\theta + 2sin\theta + \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]\Bigg|_0^{\frac{\pi}{3}} + \left[\frac{9}{2}\theta + \frac{9}{4}\sin 2\theta\right]\Bigg|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \left[\frac{3}{2}\theta + 2sin\theta + \frac{1}{4}\sin 2\theta\right]\Bigg|_0^{\frac{\pi}{3}} + \left[\frac{9}{2}\theta + \frac{9}{4}\sin 2\theta\right]\Bigg|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi}{2} + \sqrt{3} + \frac{1}{8}\sqrt{3}\right] + \left[\left(\frac{9\pi}{4}\right) - \left(\frac{3\pi}{2} + \frac{9}{8}\sqrt{3}\right)\right]$$

$$= \frac{1}{8}\left(9\sqrt{3} + 4\pi\right) + \frac{3}{8}\left(2\pi - 3\sqrt{3}\right)$$
$$= \frac{\pi}{2} + \frac{6\pi}{8} \, satuan\ luas$$

5. Dapatkan polynomial taylor untuk fungsi $f(x) = x \cos x$ di sekitar $x = \pi$ hingga sukuk e empat

<u>Polinomial Taylor di $x = a$</u>

$$p_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) = x \cos x \rightarrow f(\pi) = \pi \cos \pi = -\pi$$

$$f'(x) = \frac{d}{dx}[x \cos x]$$

Misal

$$u = x \rightarrow u' = 1$$

$$v = \cos x \rightarrow v' = -\sin x$$

$$f'(x) = u'v + uv'$$

$$= \cos x - x \sin x$$

$$f'(\pi) = \cos \pi - \pi \sin \pi = -1$$

$$f''(x) = \frac{d}{dx}[\cos x - x \sin x]$$

Misal

$$u = x \rightarrow u' = 1$$

$$v = \sin x \rightarrow v' = \cos x$$

$$f''(x) = -\sin x - [u'v + uv']$$

$$= -\sin x - [\sin x + x \cos x]$$

$$= -2 \sin x - x \cos x$$

$$f''(\pi) = -2 \sin \pi - \pi \cos \pi = \pi$$

$$f'''(x) = \frac{d}{dx}[-2 \sin x - x \cos x]$$

$$= -2\cos x - [\cos x - x\sin x]$$

$$= -3\cos x + xsin\ x$$

$$f'''(\pi) = -3\cos\pi + \pi\sin\pi = 3$$

$$f^{(4)}(x) = \tfrac{d}{dx}[-3\cos x + xsin\ x]$$

$$= 3\sin x + \sin x + x\cos x$$

$$= 4\sin x + x\cos x$$

$$f^{(4)}(\pi) = 4\sin\pi + \pi\cos\pi = -\pi$$

Sehingga,

$$p_4(x) = -\pi - (x-\pi) + \frac{\pi}{2!}(x-\pi)^2 + \frac{3}{3!}(x-\pi)^3 - \frac{\pi}{4!}(x-\pi)^4$$