ĐẠI HỌC ĐÀ NẴNG
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Searching & Sorting

Data Structures & Algorithms

CONTENT

- **Searching**
  - Sequential Search
  - Binary Search
- **Sorting**
  - Insertion Sort
  - Selection Sort
  - Bubble Sort
  - Quick Sort
  - Merge Sort

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- How do you find your name on a class list?
- How do you search a book in a library?
- How do you find a word in a dictionary?

⇨ **Search is very often operation**

- Sequential Search
- Binary Search

- Suppose we have an unsorted list of *n* numbers

  ⇨ Find the position of the number 531

```
# define MAX_SIZE 100
typedef struct {
   int  key;
   /* other fileds */
} element;
element  list[MAX_SIZE];
```

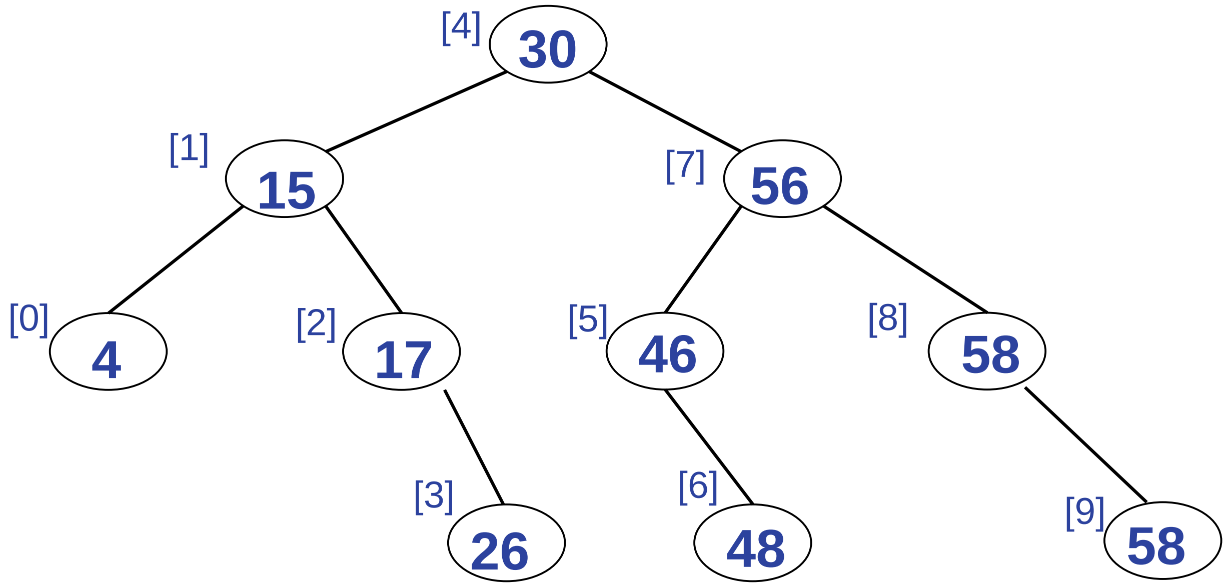
```
int   seqSearch(int list[], int searchNum, int n){
        /* Search an array, list, that has n numbers.
        Return i, if list[i] = searchNum.  Return -1, if searchNum is not in the list */
        list[n].key = searchNum;          // Sentinel that signals the end of the list
        int  i=0;
        while (list[i].key != searchNum)  i++;
        return ((i < n) ? i : -1);
}
```

- **Analysis**
  - Example: 44, 55, 12, 42, 94, 18, 06, 67
  - unsuccessful search: n
  - The average number of comparisons for a successful search is

$$\sum_{i=0}^{n-1}(i+1)/n = \frac{n+1}{2}$$

- **Binary Search**
  - Input: **sorted** list
    (i.e. list[0].key ≤ list[1].key ≤ ... ≤ list[n-1].key)
  - Compare **searchNum** and **list[middle].key**, where middle = (n-1)/2, there are three possible outcomes:
    - list[middle].key < searchNum
      ⇨ Search: list[middle+1] … list[n-1]
    - list[middle].key = searchNum
      ⇨ Search terminal successfully: return **middle**
    - list[middle].key > searchNum
      ⇨ Search: list[0] … list[middle-1]

- **Example**
  - Input : 4, 15, 17, 26, 30, 46, 48, 56, 58, 82



Decision tree for binary search

⇨ Binary search makes no more than **O(log n)** comparisons (i.e. height of tree)

```
int  binSearch(element list[], int searchNum, int n){
        int left = 0, right = n-1, middle;
        while (left <= right){
                middle = (left + right) / 2;
                switch (COMPARE(list[middle].key, searchNum)){
                        case -1 : left = middle + 1;
                                  break;
                        case 0 : return middle;
                        case 1 : right = middle - 1;
                }
        }
        return -1;
}
```

- **Example**
  - Input : 4, 15, 17, 26, 30, 46, 48, 56, 58, 82

searchNum = 46

left right

0	1	2	3	4	5	6	7	8	9
4	15	17	26	30	46	48	56	58	82

left = 0; right = n-1 = 9;
// left <= right
middle = (left + right)/ 2 = 4;
list[middle].key = 30 < searchNum
left = middle + 1 = 5; // right = 9

left right

					5	6	7	8	9
					46	48	56	58	82

// left <= right
middle = (left + right)/ 2 = 7;
list[middle].key = 56 > searchNum
right = middle - 1 = 6; // left = 5

					5				
					46				

// left <= right
middle = (left + right)/ 2 = 5;
list[middle].key = 46 = searchNum
⇨ binSearch()=5

CONTENT

- **Searching**
  - Sequential Search
  - Binary Search
- **Sorting**
  - Insertion Sort
  - Selection Sort
  - Bubble Sort
  - Quick Sort
  - Merge Sort

- **List Verification**
  - Determine if all elements on one list appear on another list
    - Assume the 2 lists have *m* and *n* elements respectively, and there are no repetitions
  - 1st approach (O(*mn*) time)
    1. Assume both lists are not sorted
    2. For each element in the 1st list do
    3. Search the element in the 2nd list
    4. End For
  - 2nd approach (O(mlogm + nlogn + m + n) time)
    1. Sort the 1st list in O(*mlogm*) time
    2. Sort the 2nd list in O(*nlogn*) time
    3. Compare the two sorted lists in O(*m* + *n*) time

- **Sorting Problem**
  - We are given a list of records $(R_0, R_1, \ldots, R_{n-1})$.
    each record R_i, has a key value K_i
  - Find a permutation *p* such that
    - **Sorted:** $K_{p(i-1)} \le K_{p(i)}$, for $0 < i \le n-1$
    - **Stable:** If $i<j$ and $K_i=K_j$ in the list, then R_i precedes R_j in the sorted list.
  - Sorting algorithm is **internal** if it makes use of only memory
  - Sorting algorithm is **external** if it stores intermediate results on hard disks

- **Insertion sort**
  - *n* records are stored in an array *A*
  - *n*-1 steps in the insertion sorting algorithm
  - During step *i*, *i*-1 elements *A*[0],*A*[1], …, *A*[*i*-1] are in sorted order
  - We need to insert *A*[*i*] into *A*[0],*A*[1], …, *A*[*i*-1] such that these *i* elements are sorted
    - the element A[i] is compared with *A*[0],*A*[1], …,*A*[*i*-1], one by one, until a place, say *k*, is decided for *A*[*i*]
    - the elements A[k],A[k+1], …, A[i-1] are moved to the right one position and *A*[*i*] is copied to *A*[*k*]

- **Example**

26	5	77	1	61	11	59	15	48	19
5	26	77	1	61	11	59	15	48	19
5	26	77	1	61	11	59	15	48	19
1	5	26	77	61	11	59	15	48	19
1	5	26	61	77	11	59	15	48	19
1	5	11	26	61	77	59	15	48	19
1	5	11	26	59	61	77	15	48	19
1	5	11	15	26	59	61	77	48	19
1	5	11	15	26	48	59	61	77	19
1	5	11	15	19	26	48	59	61	77

```
void  insertionSort(element list[], int n){
        int i, j;
        element  next;
        for (i=1; i<n; i++) {
                next = list[i];
                for (j=i-1; j>=0 && next.key<list[j].key ; j--)
                        list[j+1] = list[j];
                list[j+1] = next;
        }
}
```

- **Analysis**
  - During step *i*, we need at most *i*+1 comparisons and movements
  - In the **worst case**, we need $(1+2+\ldots+n-1) = n(n-1)/2 = O(n^2)$ comparisons and movements
  - a record $A[i]$ is said ***left-out-of-order*** (LOO) if $A[i] < max\{A[0];A[1];A[2];\ldots;A[i-1]\}$
    - Each LOO record induces at most *n* comparisons
    - Each non-LOO record induces 1 comparison
  - Suppose there are *k* LOO records and $n - k$ non-LOO records
  - We need at most $kn + (n - k)$ comparisons
  - Since $k <= n$, time is $O(n^2)$

⇨This algorithm is **stable**

- **Variation**
  - Binary insertion sort
    - sequential search --> binary search
    - reduce # of comparisons
    - # of moves unchanged
  - List insertion sort
    - array --> linked list
    - sequential search
    - no movement, adjust pointers only

- **Selection Sort**
  - *n* records are stored in an array *A*
  - loop *n*-1 steps in the selection sorting algorithm
  - During step *i*, *A*[0],*A*[1], …, *A*[*i*-1] are the smallest *i* elements, arranged in sorted order
  - Then the smallest among *A*[*i*],*A*[*i*+1], ... ,*A*[*n*-1] is selected and is exchanged with *A*[*i*]

- **Example**

26	5	77	**1**	61	11	59	15	48	19
1	26	77	**5**	61	11	59	15	48	19
1	26	77	**5**	61	11	59	15	48	19
1	**5**	77	**26**	61	11	59	15	48	19
1	5	77	26	61	**11**	59	15	48	19
1	5	**11**	26	61	**77**	59	15	48	19
1	5	11	26	61	77	59	**15**	48	19
1	5	11	**15**	61	77	59	**26**	48	19
1	5	11	15	61	77	59	26	48	**19**
1	5	11	15	**19**	77	59	26	48	**61**
1	5	11	15	19	77	59	**26**	48	61
1	5	11	15	19	**26**	59	**77**	48	61
1	5	11	15	19	26	59	77	**48**	61
1	5	11	15	19	26	**48**	77	**59**	61
1	5	11	15	19	26	48	77	**59**	61
1	5	11	15	19	26	48	**59**	**77**	61
1	5	11	15	19	26	48	59	77	**61**
1	5	11	15	19	26	48	59	61	77

```
void  selectionSort(element list[], int n){
        int i, j, min;
        element tmp;
        for (i=0; i<=n-2; i++)
                for (j=i+1; j<=n-1; j++)
                        if (list[i].key > list[j].key) {
                         /* exchange two elements -> should be improved */
                                tmp = list[i];
                                list[i] = list[j];
                                list[j] = tmp;
                        }
}
```

```
void selectionSort'(element list[], int n){
        int i, j, min;
        element tmp;
        for (i=0; i<=n-2; i++) {
                min = i;                        /* search the smallest element */
                for (j=i+1; j<=n-1; j++)
                        if (list[min].key > list[j].key) min = j;
                If (i != min) {                 /* exchange two elements */
                        tmp = list[i];
                        list[i] = list[min];
                        list[min] = tmp;
                }
        }
}
```

- **Analysis**
  - During step i, there are at most n - i comparisons
  - In the worst case, there are

$$(n - 1 + n - 2 + \ldots + 1) = n(n - 1)/2 = O(n^2) \quad \text{comparisons}$$

- **Bubble Sort**
  - *n* records are stored in an array *A*
  - *n* steps in the bubble sorting algorithm
  - During step *i*, *A*[0],*A*[1], …, *A*[*i*-1] are the smallest *i* elements, arranged in sorted order
  - Then among *A*[*i*],*A*[*i*+1], ... ,*A*[*n*-1], each pair ((*A*[*n*-2], *A*[*n*-1]) , …) will be exchanged if they are out of order, so *A*[*i*] will be the smallest
    - At step *i*, the smallest element bubbles

Example

5	5	5	5	5	**0**	0	0	0	0	0	0	0	0	0	0
1	1	1	1	**0**	**5**	5	5	5	**1**	1	1	1	1	1	1
7	7	7	**0**	**1**	1	1	1	**1**	**5**	5	5	**4**	4	4	4
4	4	**0**	**7**	7	7	7	**4**	**4**	4	4	**4**	**5**	5	**5**	5
0	**0**	**4**	4	4	4	**4**	**7**	7	7	**6**	**6**	6	**6**	**6**	**6**
6	**6**	6	6	6	6	**6**	6	6	6	**7**	7	7	**7**	7	**7**

```
void bubbleSort(element list[], int n){
    int i, j, min;
    element tmp;
    for (i = 0; i < n-1; i++)
        for (j = n-1; j > i; j--)
            if (list[j-1].key > list[j].key) {
                /* exchange two elements */
                tmp = list[j-1];
                list[j-1] = list[j];
                list[j] = tmp;
            }
    }
}
```

- **Quick Sort**
  - n records are stored in an array A
  - Choose a pivot $A[i]$
  - Re-arrange the list so that $A[0], A[1], \ldots, A[i-1]$ are smaller than $A[i]$ while $A[i+1], A[i+2], \ldots, A[n-1]$ are all greater than $A[i]$
  - Then, recursively apply quick sort to the first half $A[0], A[1], \ldots, A[i-1]$ and the second half $A[i+1], A[i+2], \ldots, A[n-1]$, respectively

- **Divide and Conquer algorithm**

Two phases:

- Partition phase: **divide** the list into half

≤ pivot	**pivot**	**> pivot**

- Sort phase:
  - **Conquers** the halves: apply the same algorithm to each half

≤ pivot **> pivot**

≤ p’	**p’**	**> p’**	**pivot**	**≤ p”**	**p”**	**> p”**

- **Example**

R0	R1	R2	R3	R4	R5	R6	R7	R8	R9	left	right
{ 26	5	37	1	61	11	59	15	48	19 }	0	9
{ 11	5	19	1	15 }	26	{ 59	61	48	37 }	0	4
{ 1	5 }	11	{ 19	15 }	26	{ 59	61	48	37 }	0	1
1	5	11	15	19	26	{ 59	61	48	37 }	3	4
1	5	11	15	19	26	{ 48	37 }	59	{ 61 }	6	9
1	5	11	15	19	26	37	48	59	{ 61 }	6	7
1	5	11	15	19	26	37	48	59	61	9	9
1	5	11	15	19	26	37	48	59	61		

```
#define SWAP(a,b,t)   {int  t;   t=a;   a=b;   b=t; }
void quickSort(element list[], int left, int right){
        int pivot, i, j;             element temp;
        if (left < right) {                         /* divide */
                i = left;   j = right+1;
                pivot = list[left].key;
                do {
                do i++;  while (list[i].key < pivot);
                do j--;    while (list[j].key > pivot);
                if (i < j)   SWAP(list[i], list[j], temp);
                   } while (i < j);
           SWAP(list[left], list[j], temp);        /* put pivot a good position*/
           quickSort(list, left, j-1);             /* conquer */
           quickSort(list, j+1, right);
        }
}
```

- **Analysis**
  - Time complexity
    - Worst case: $O(n^2)$
    - Best case: O(nlogn)
    - Average case: O(nlogn)
  - Space complexity
    - Worst case: O(n)
    - Best case: O(logn)
    - Average case: O(logn)
- **Unstable**

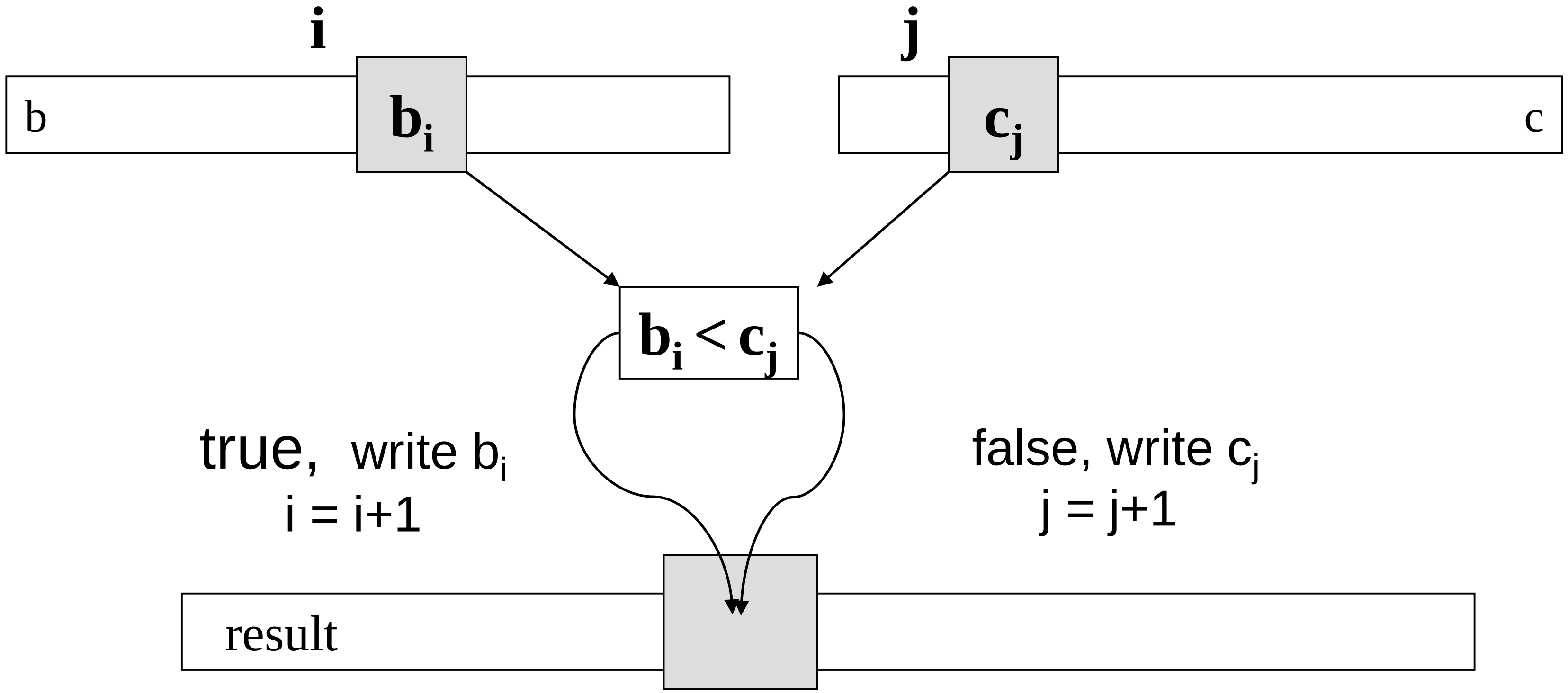
- **Merge Sort**

Divide and Conquer algorithm

1. Cut the list in 2 halves
2. Sort each half, respectively, probably by merge sort recursively
3. Merge the 2 sorted halves

⇨ **How to merge two sorted lists** (list[i], ... , list[m] and list[m+1], ... , list[n]) **into single sorted list,** (sorted[i], ... , sorted[n])**?**

- Merge 2 sorted lists

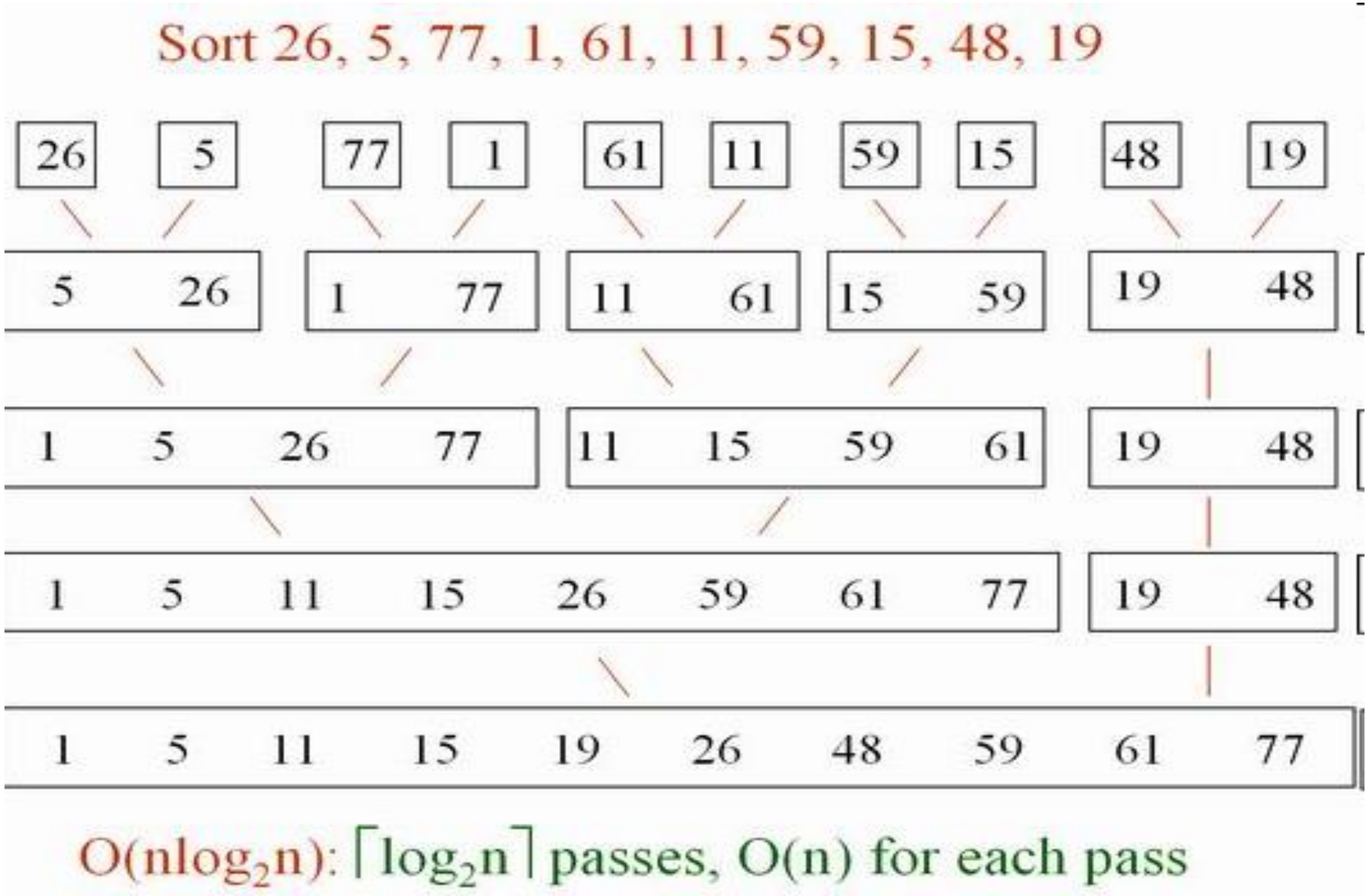


```
void merge(element list[], element sorted[], int i, int m, int n){
        int j, k, t;
        j = m+1;
        k = i;
        while (i<=m && j<=n) {
                if (list[i].key<=list[j].key) sorted[k++]= list[i++];
                else sorted[k++]= list[j++];
        }
        if (i>m)
                for (t=j; t<=n; t++)  sorted[k+t-j]= list[t];
        else
                for (t=i; t<=m; t++) sorted[k+t-i] = list[t];
}
```

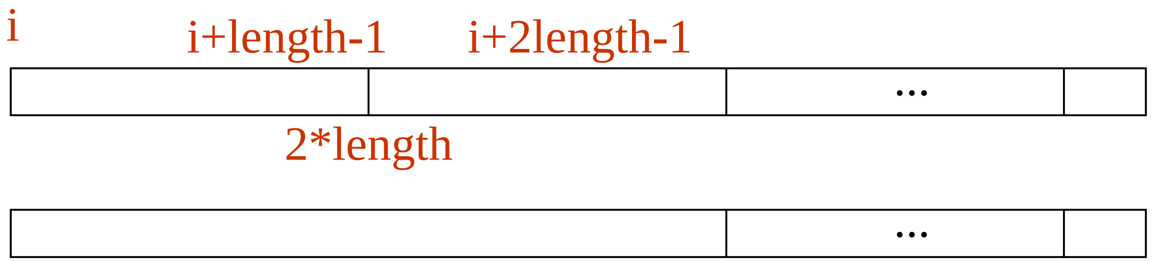
Time complexity: O(n)
Space complexity: O(n)

- **Iterative Merge Sort**
  - We assume that the input sequence has n sorted lists each of length 1
  - We merge these lists pair-wise to obtain n/2 list of size 2
  - We then merge the n/2 lists pair-wise, and so on, until a single list remains

- **Iterative Merge Sort Example**

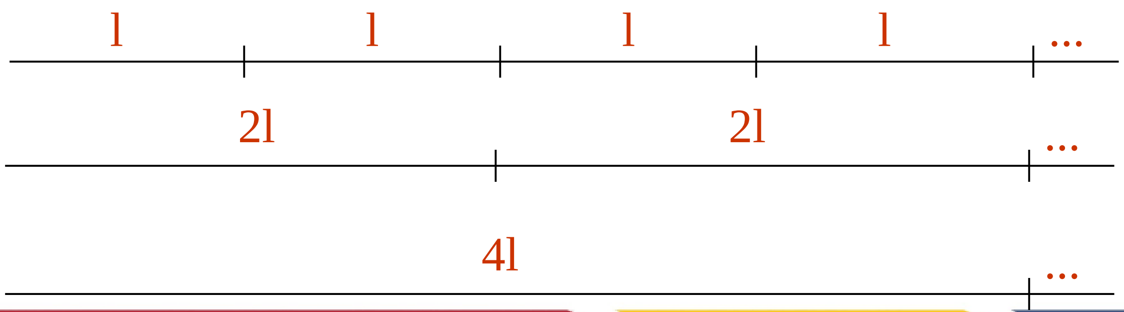


```
void mergePass(element list[], element sorted[],int n, int length){
        int i, j;
        for (i=0; i<n-2*length; i+=2*length)
                merge(list,sorted,i,i+length-1,i+2*length-1);
        if (i+length<n)          //One complement segment and one partial segment
                merge(list, sorted, i, i+length-1, n-1);
        else                     //Only one segment
                for (j=i; j<n; j++)
                        sorted[j]= list[j];
}
```



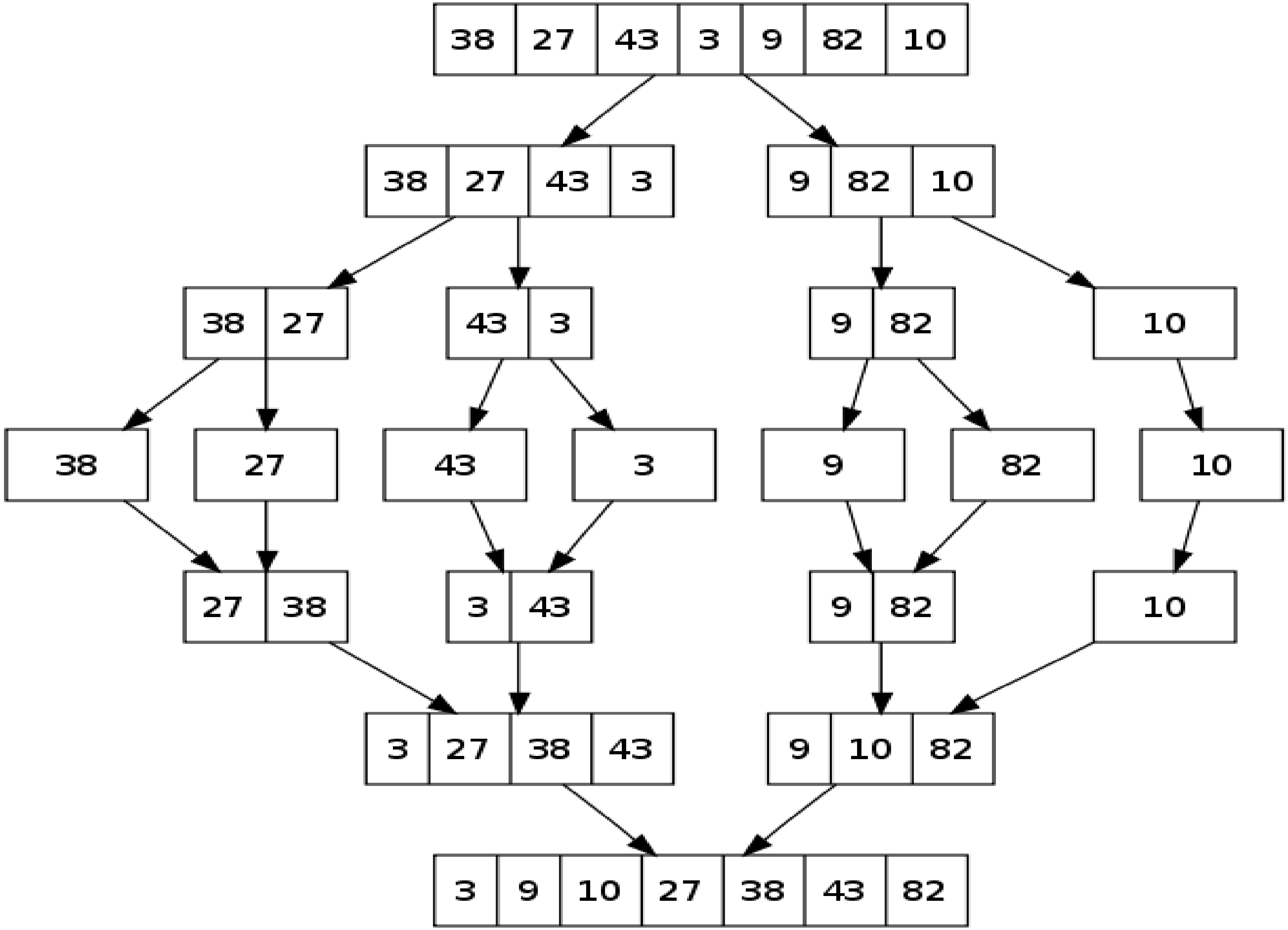
- **Iterative Merge Sort**

```
void mergeSort(element list[], int n){
        int length=1;
        element extra[MAX_SIZE];
        while (length<n) {
                mergePass(list, extra, n, length);
                length *= 2;
                mergePass(extra, list, n, length);
                length *= 2;
         }
}
```



- **Recursive Merge Sort**
  - Given *n* elements (in an array) to be sorted
  - Cut them into two halves, each with *n*/2 elements
  - Sort each half by the same merge sort algorithm recursive
  - Finally merge the two sorted halves into a single array

- **Example**


38 27 43 3 9 82 10
38 27 43 3
9 82 10
38 27
43 3
9 82
10
38
27
43
3
9
82
10
27 38
3 43
9 82
10
3 27 38 43
9 10 82
3 9 10 27 38 43 82

- **Recursive Merge Sort**

```
void mSort(int list[], int left, int right) {
        int mid;
        if (right > left){
                mid = (right + left) / 2;
                mSort(list, left, mid);
                mSort(list, mid+1, right);
                merge(list, left, mid+1, right);
        }
}

void mergeSort(element list[], int array_size) {
        mSort(list, 0, array_size - 1);
}
```

- **Recursive Merge Sort**

```
void merge(element list[], int left, int mid, int right) {
  int i, left_end, num_elements, tmp_pos;
  element temp[right - left + 1];
  left_end = mid - 1;
  tmp_pos = left;
  num_elements = right - left + 1;
  while ((left <= left_end) && (mid <= right)) {
    if (list[left].key <= list[mid]).key {
      temp[tmp_pos] = list[left];
      tmp_pos = tmp_pos + 1;
      left = left +1;
    } else {
      temp[tmp_pos] = list[mid];
      tmp_pos = tmp_pos + 1;
      mid = mid + 1;
    }
  }
while (left <= left_end) {
    temp[tmp_pos] = list[left];
    left = left + 1;
    tmp_pos = tmp_pos + 1;
  }
  while (mid <= right) {
    temp[tmp_pos] = list[mid];
    mid = mid + 1;
    tmp_pos = tmp_pos + 1;
  }
  for (i=0; i < num_elements; i++) {
    list[right] = temp[right];
    right = right - 1;
  }
}
```

- **Analysis**
  - **Time complexity**
    - $T(n) = 2T(n/2) + n-1$
    - $O(n \log n)$ in time
    - $O(n)$ in space

    ⇨Can be improved by an approach with O(1) in space
  - **Stable**

SUMMARY

- **Searching**
    - Sequential Search
    - Binary Search
- **Sorting**
    - Insertion Sort
    - Selection Sort
    - Bubble Sort
    - Quick Sort
    - Merge Sort

Enjoy the Course…!