Hash Table

Data Structures & Algorithms

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- Introduction
- Static hashing
- Dynamic hashing

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- Introduction
- Static hashing
- Dynamic hashing

- **A *table* has several *fields* (types of information)**
  - A telephone book may have fields **name**, **address**, **phone number**
  - A user account table may have fields **user id**, **password**, **home folder**

⇨ **To find an *entry* in the table, you only need know the contents of one of the fields (not all of them).**

- **The *key* field**
  - In a telephone book, the key is usually **name**
  - In a user account table, the key is usually **user id**

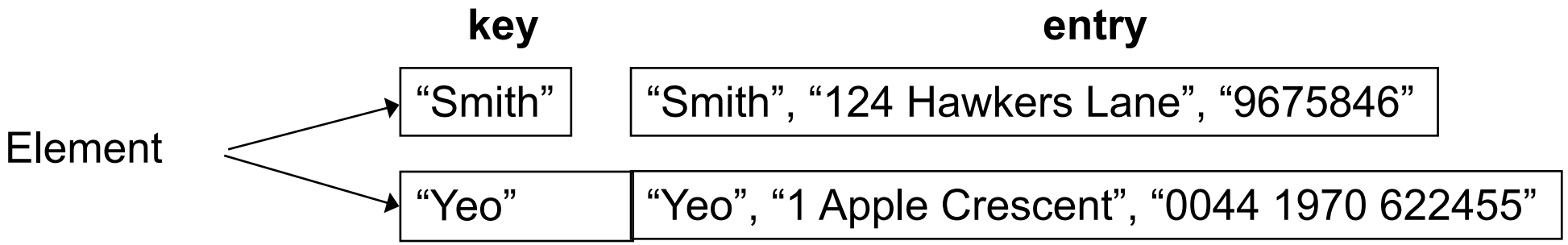
⇨ **Key *uniquely identifies* an entry**

  - If the key is **name** and no two entries in the telephone book have the same name, the key uniquely identifies the entries

- **How should we implement a table?**
  - How often are entries inserted and removed?
  - How many of the possible key values are likely to be used?
  - What is the likely pattern of searching for keys?
    - e.g. Will most of the accesses be to just one or two key values?
  - Is the table small enough to fit into memory?
  - How long will the table exist?

- **Element: a key and its entry**

For searching purposes, it is best to store the key and the entry separately (even though the key's value may be inside the entry)



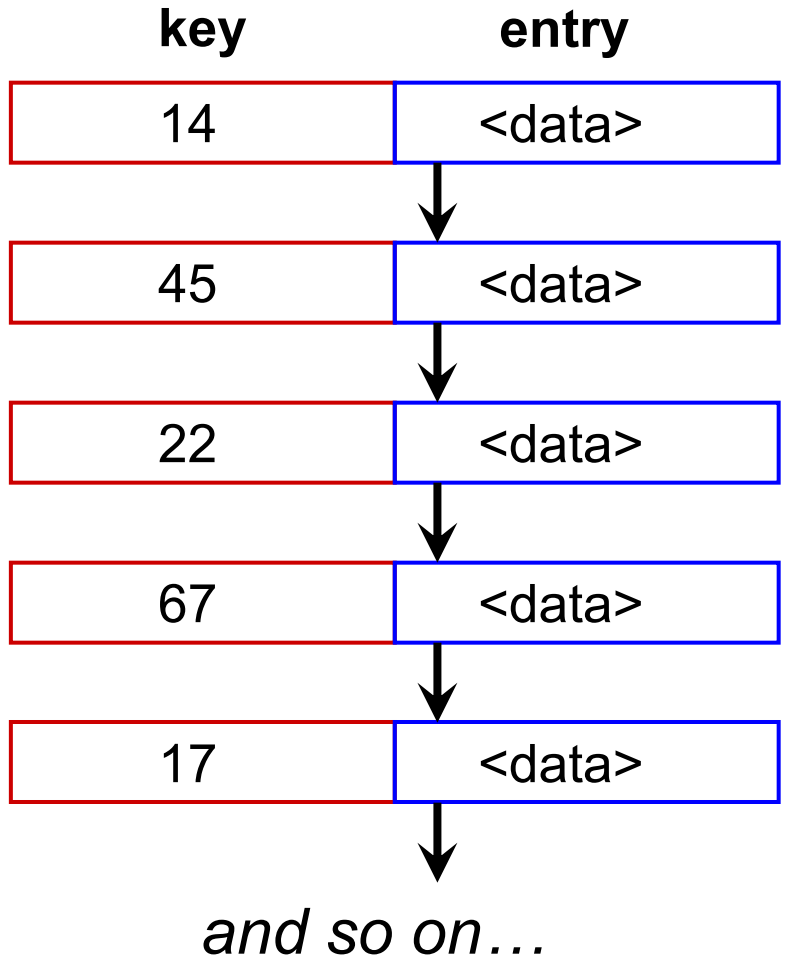
- **Implementation 1- Unsorted Sequential Array**
  - An array in which elements are stored consecutively in *any* order
  - **insert**: add to back of array; O(1)
  - **find**: search through the keys one at a time, potentially all of the keys; O(*n*)
  - **remove**: find + replace removed node with last node; O(*n*)

	key	entry
0	14	<data>
1	45	<data>
2	22	<data>
3	67	<data>
4	17	<data>
⋮	*and so on…*	

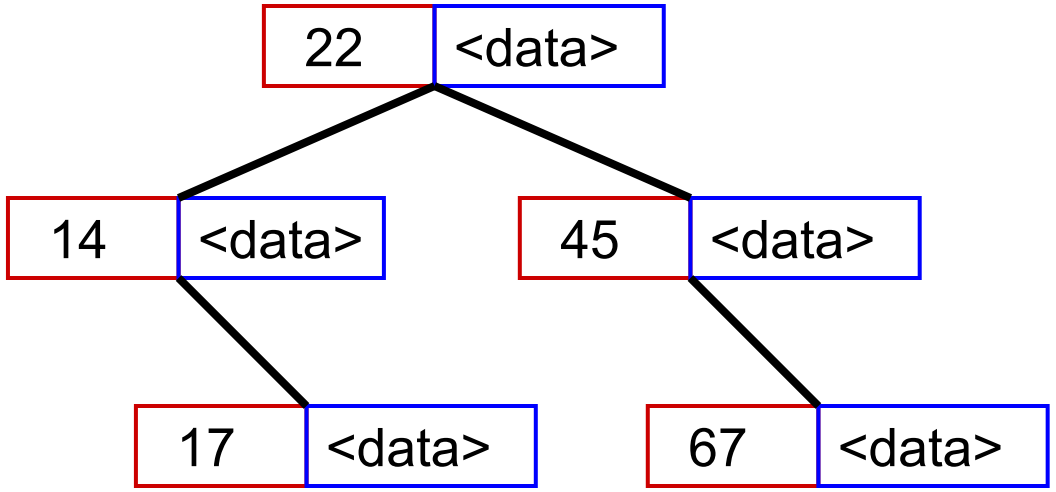
- **Implementation 2 - Sorted Sequential Array**
  - An array in which elements are stored consecutively, *sorted* by key
  - **insert**: add in sorted order; O(*n*)
  - **find**: binary search; O(log *n*)
  - **remove**: find, remove element; O(log *n*)

	key	entry
0	15	<data>
1	17	<data>
2	22	<data>
3	45	<data>
4	67	<data>
⋮	*and so on…*	

- **Implementation 3 - Linked List (Unsorted or Sorted)**
  - Elements are again stored consecutively
  - **insert**: add to front; O(1)
    or *O(n)* for a sorted list
  - **find**: search through potentially all the keys, one at a time; O(*n*) still *O(n)* for a sorted list
  - **remove**: find, remove using pointer alterations; O(*n*)

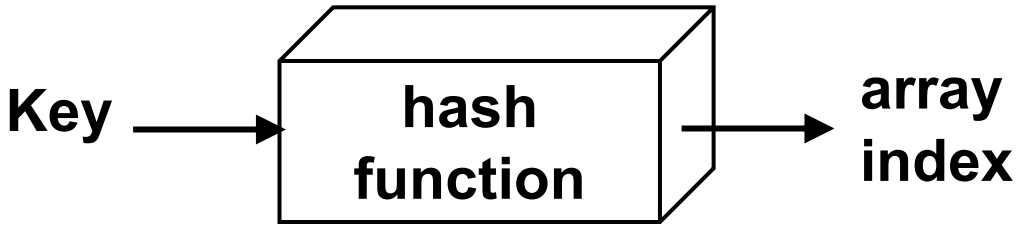


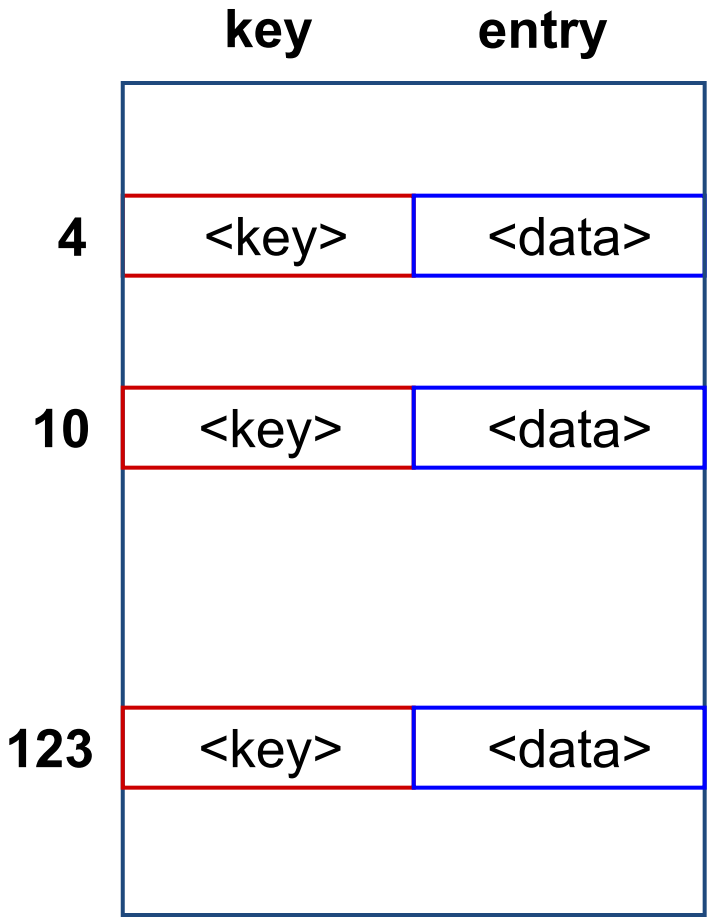
- **Implementation 4 - BST**
  - A BST, ordered by key
  - **insert**: a standard insert; O(log *n*)
  - **find**: a standard find (without removing, of course); O(log *n*)
  - **remove**: a standard remove; O(log *n*)



and so on…

- **Implementation 5 - Hash Table**
  - An array in which elements are ***not*** stored consecutively - their place of storage is calculated using the key and a *hash function*

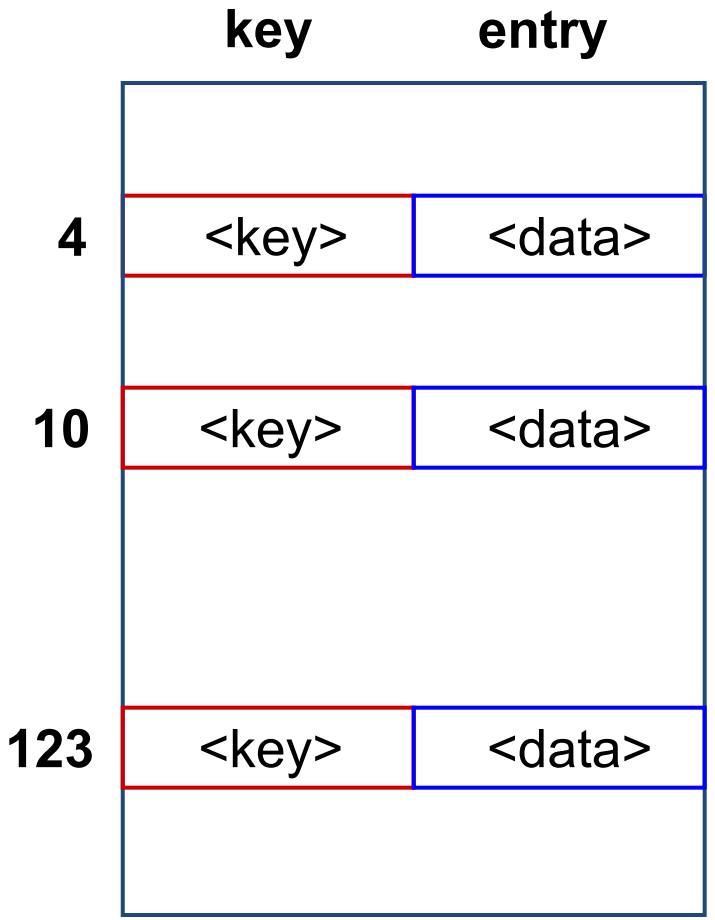




Hash values → mappings of the keys as the indexes in the hash table

- **Implementation 5 - Hash Table**
  - **Hashed key**: result of applying a hash function to a key
  - Keys and entries are scattered throughout the array
  - Array elements are not stored consecutively, their place of storage is calculated using the key & a hash function
  - **insert**: calculate place of storage, insert TableNode; O(1)
  - **find**: calculate place of storage, retrieve entry; O(1)
  - **remove**: calculate place of storage, set it to null; O(1)

All are O(1) !



- **Applications of Hashing**
  - Compilers use hash tables to keep track of declared variables
  - A hash table can be used for on-line spelling checkers — if misspelling detection (rather than correction) is important, an entire dictionary can be hashed and words checked in constant time
  - Game playing programs use hash tables to store seen positions, thereby saving computation time if the position is encountered again
  - Hash functions can be used to quickly check for inequality — if two elements hash to different values they must be different
  - Storing sparse data

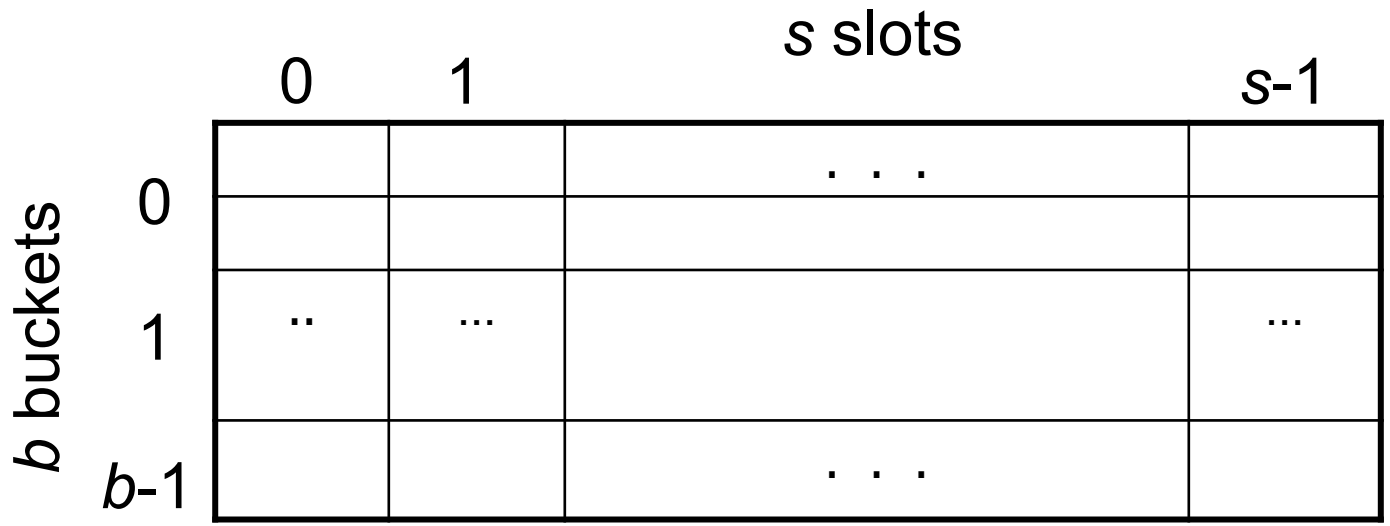
- **When are other representations more suitable than hashing?**
  - Hash tables are very good if there is a need for many searches in a reasonably stable table
  - Hash tables are not so good if there are many insertions and deletions, or if table traversals are needed

- **Types of hashing**
  - Static hashing
    - Tables with a fixed size
  - Dynamic hashing
    - Table sizes may vary

CONTENT

- Introduction
- **Static hashing**
  - **Hash table**
  - **Hash methods**
  - **Collision resolution**
- Dynamic hashing

- **Key-value pairs are stored in a fixed size table called a *hash table***
  - A hash table is partitioned into many ***buckets***
  - Each bucket has many ***slots***
  - Each slot holds one record
  - A hash function $f(x)$ transforms the identifier (i.e. key) into an address in the hash table



- **Uses an array hash_table[0..b-1].**
  - Each position of this array is a **bucket**
  - A bucket can normally hold only one dictionary pair
- **Uses a hash function *f***
  - that converts each key *k* into an index in the range [0, *b*-1].

⇨**Every dictionary pair (key, element) is stored in its home bucket hash_table[f(key)]**

- **Data Structure for Hash Table**

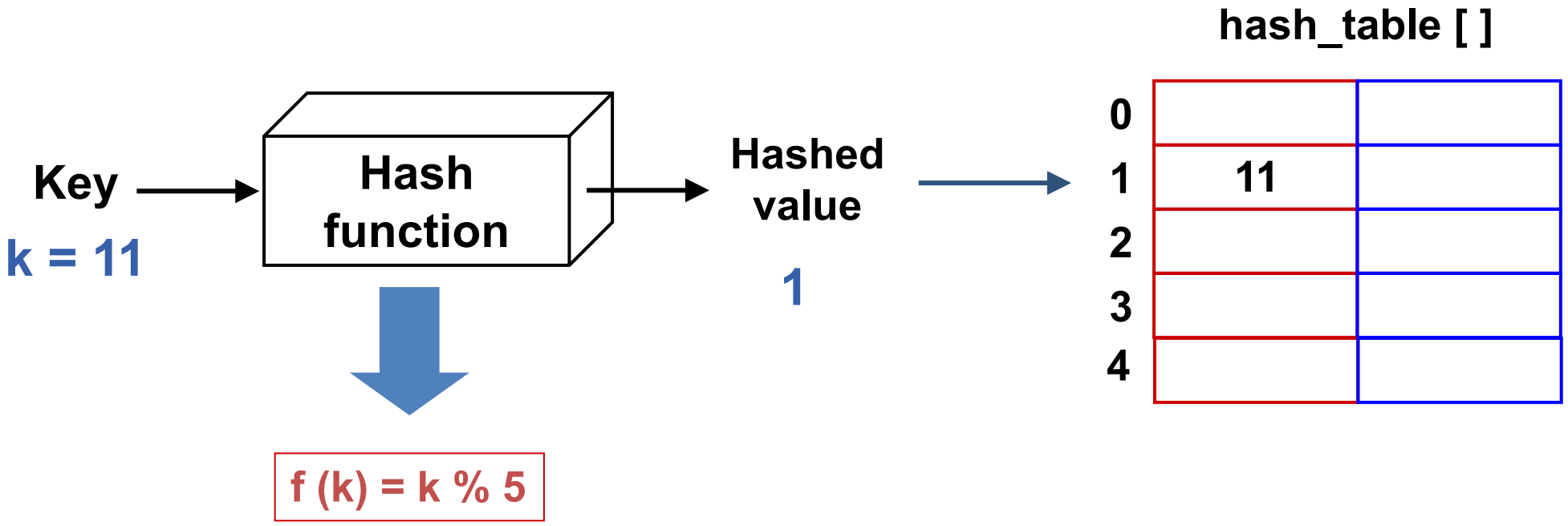
```
#define MAX_CHAR  10
#define TABLE_SIZE  13
typedef struct {
    char key[MAX_CHAR];
    /* other fields */
} element;
element hash_table[TABLE_SIZE];
```

- **Hash Function - Example**

```
void init_table(element ht[]){
        int i;
        for (i=0; i<TABLE_SIZE; i++)
        ht[i].key[0]=NULL;
}

int hash( char *key, int TABLE_SIZE ) {
        unsigned int hash_val = 0;
        while( *key != '\0' )
            hash_val += *key++;
        return( hash_val % H_SIZE );
}
```

- Example


hash_table [ ]
Key
k = 11
Hash function
Hashed value
1
f (k) = k % 5
0
1
2
3
4
11

- **Collision**
  - In a hash table with a single array table (single slot bucket), two different keys may be hashed to **the same hash value**
    - Two different k1 and k2
    - Hash (k1) = Hash (k2)
    - Keys that have the same home bucket are synonyms
  - This is called **collision.** Example: k1 = 11, k2 = 21
    Hash (11) = 11 % 5 = 1
    Hash (21) = 21 % 5 = 1
- **Choice of hash method**
  - To avoid collision (two different pairs are in the same the same bucket)
  - Size (number of buckets) of hash table
- **Overflow handling method**
  - Overflow: there is no space in the bucket for the new pair

- **Overflow Example**

synonyms:
char, ceil,
clock, ctime

overflow

	Slot 0	Slot 1	
0	acos	atan	synonyms
1			
2	char	ceil	synonyms
3	define		
4	exp		
5	float	floor	
6			
…			
25			

- **Choice of Hash Method**
  - Requirements
    - easy to compute
    - minimal number of collisions
  - If a hashing function groups key values together, this is called **clustering** of the keys
    - The larger the cluster, the longer the search
  - A good hashing function **distributes the key values uniformly** throughout the range
    - For a random variable X, $P(X = i) = 1/b$ (b is the number of bucket)

- **Choice of Hash Method**
  - The **worst hash function** maps all keys to the same bucket
  - The **best hash function** maps all keys to distinct addresses
  - **Ideally, distribution of keys to addresses is uniform and random**

- **Many hashing methods**
  - Truncation
  - Division
  - Mid-square
  - Folding
  - Digit analysis
  - and so on

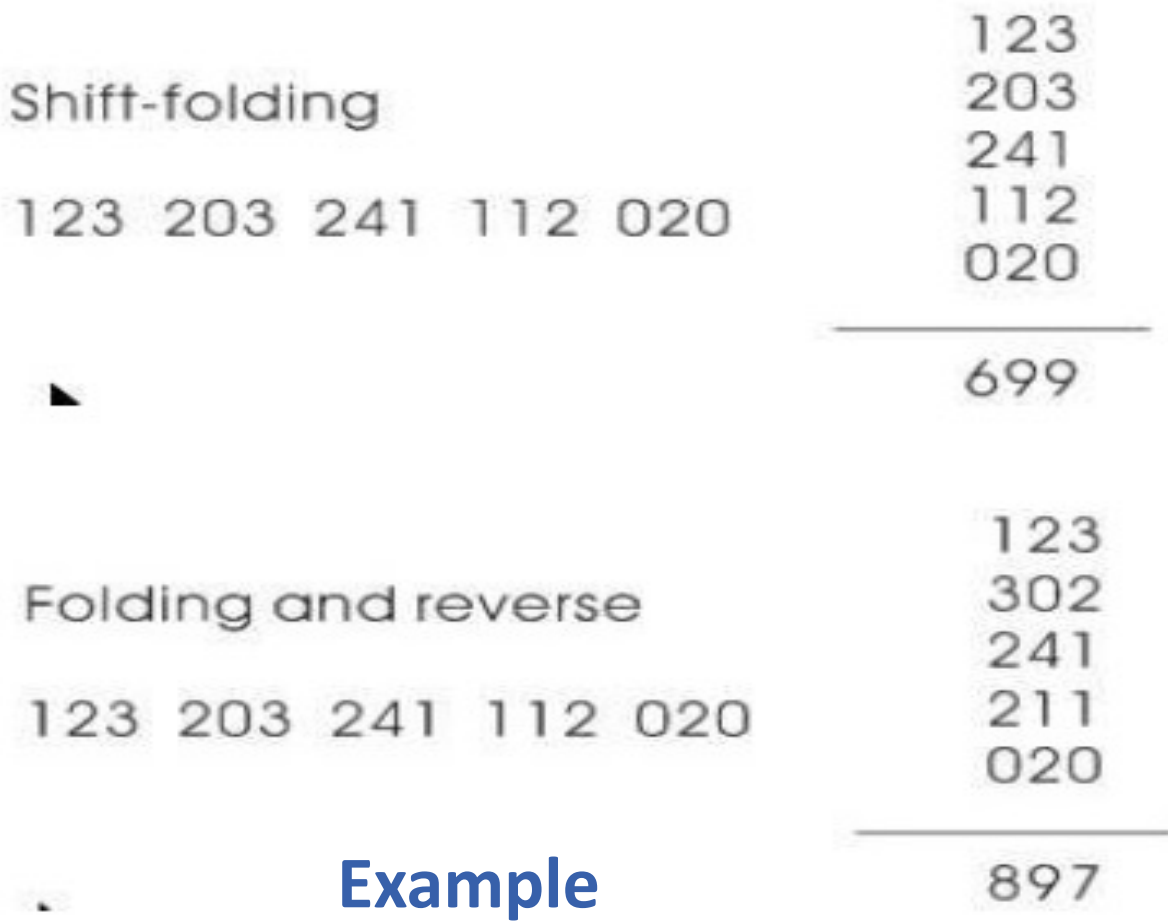
- **Truncation method**
  - Ignore part of the key and use the rest as the array index (converting non-numeric parts)
  - Example
    - If students have an 9-digit identification number, take the last **3** digits as the table position
    - e.g. 925371**622** becomes 622

- **Division method**
  - Hash function $f(k) = k \% b$
    - Requires only a single division operation (quite fast)
  - Certain values of b should be avoided
    - if $b=2^p$, then f(k) is just the p lowest-order bits of k; the hash function does not depend on all the bits
  - It's a good practice to set the table size b to be a **prime number**

- **Mid-square method**
  - Middle of square method
  - This method squares the key value, and then takes out the number of bits from the middle of the square
  - The number of bits to be used to obtain the bucket address depends on the table size
    - If *r* bits are used, the range of values is 0 to 2^r-1
  - This works well because most or all bits of the key value contribute to the result

- **Mid-square method**
  - Example
    - consider records whose keys are 4-digit numbers in base 10
    - The goal is to hash these key values to a table of size 100
    - This range is equivalent to two digits in base 10, that is, r = 2
    - If the input is the number 4567, squaring yields an 8-digit number, 20857489
    - The middle two digits of this result are 57

- **Folding method**
  - Partition the key into several parts of the same length except for the last
  - These parts are then added together to obtain the hash address for the key
  - Two ways of carrying out this addition
    - shift folding
    - folding and reverse



Example

- **Digit analysis method**
  - All the identifiers/keys in the table are known in advance
  - The index is formed by extracting, and then manipulating specific digits from the key
  - For example, the key is 925371622, we select the digits from 5 through 7 resulting 537
  - The manipulation can then take many forms
    - Reversing the digits (735)
    - Performing a circular shift to the right (753)
    - Performing a circular shift to the left (375)
    - Swapping each pair of digits (357)

- **Hash Function Implementations**
  - A generic hashing function does not exist
  - However, there are several forms of a hash function
  - Let's discuss some specific hash function implementations

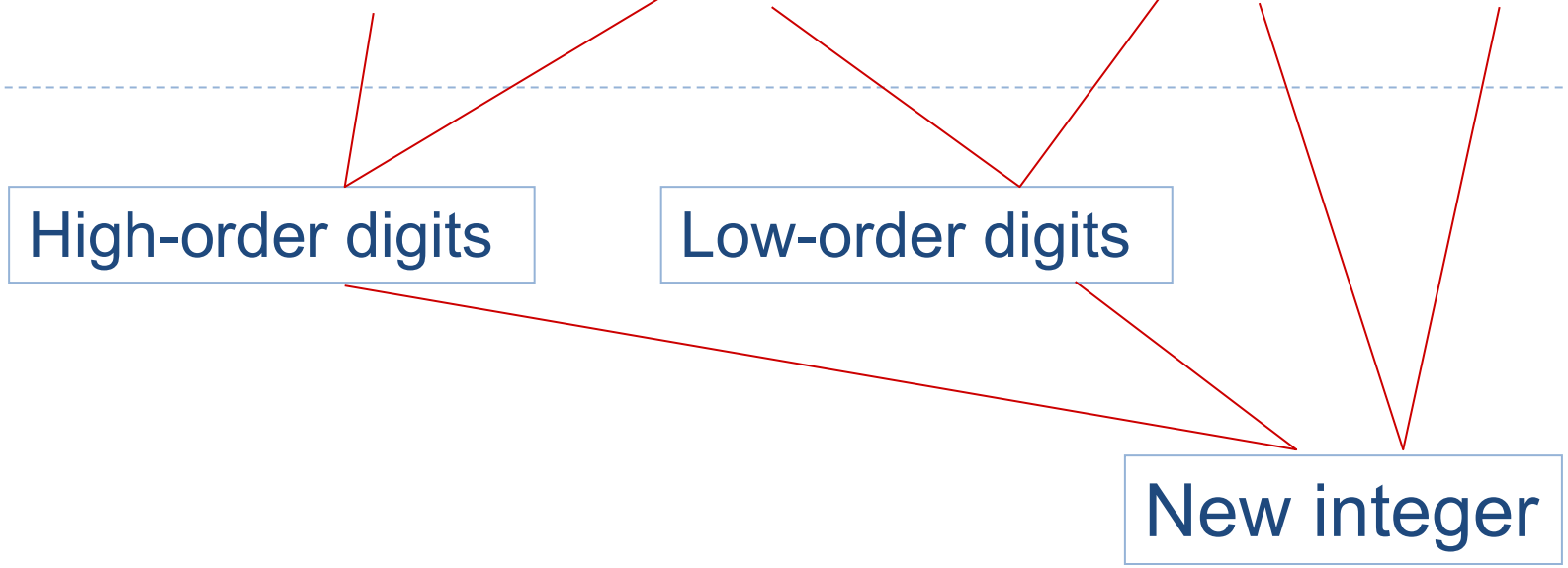
```
typedef unsigned          HASH_VALUE
typedef unsigned short    KEY_TYPE

HASH_VALUE  ModulusHashFunc (KEY_TYPE key){
        return (key % HT_SIZE);
}
```

- **Folding hash function for integers**

```
typedef unsigned long int  KEY_TYPE

HASH_VALUE Fold_Integer_Hash (KEY_TYPE key){
        return ( (key / 10000 + key % 10000) % HT_SIZE);
}
```



Source code above is for a 8-digit integer key

- **Folding hash function for integers**

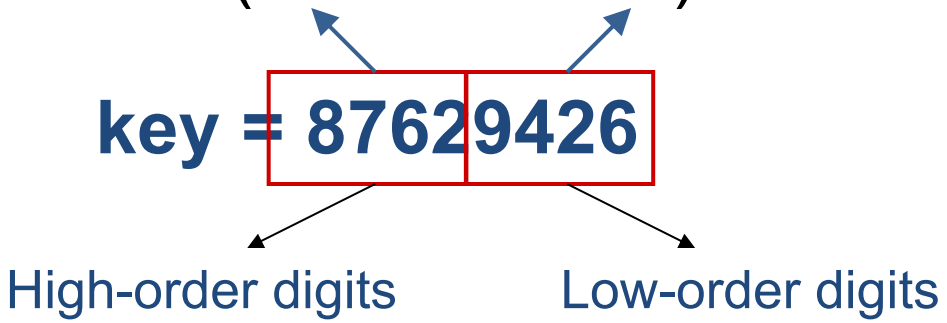
```
typedef unsigned long int  KEY_TYPE

HASH_VALUE Fold_Integer_Hash (KEY_TYPE key){
        return ( (key / 10000 + key % 10000) % HT_SIZE);
}
```

Example: key = 87629426, HT_SIZE = 251

hash value = (87629426 / 10000 + 87629426 % 10000) % 251

hash value = (8762 + 9426) % 251 = 116



- **Folding hash function for pointer-based character strings**

  Useful for applications involving symbols and names.

  Folding hash → adds ASCII values of each character and takes the modulus with respect to the hash table size.

```
typedef char  *KEY_TYPE

HASH_VALUE Fold_String_Hash (KEY_TYPE key){
        unsigned sum_ascii_value = 0;

        while (*key != '\0')
                sum_ascii_values += *key++;

        return (sum_ascii_values % HT_SIZE);
}
```

- **Folding hash function for pointer-based character strings**

Example:

key = “PRATIVA”, HT_SIZE = 31

hash value = (P + R + A + T + I + V + A) % 31

hash value = (80 + 82 + 65 + 84 + 73 + 86 + 65) % 31

hash value = 8

- **Digit Analysis-Based Folding**

```
static unsigned DigitFoldStringHash (char *key){
        unsigned long hash;

        hash = ( (key[0] ^ key[3]) ^ (key[1] ^ key[2]) ) % HT_SIZE;

        return (hash);
}
```

- **Collision Resolution/Overflow Handling**
  - An overflow occurs when the home bucket for a new pair **(key, element)** is full
  - Methods of solving collisions/overflows
    - **Open Addressing**
      - Insert the element into the next free position in the table
    - **Separate Chaining**
      - Each table position is a linked list

- **Open addressing**
  - relocate the key *k* to be inserted if it collides with an existing key. That is, we store *k* at an entry different from *hash_table*[*f*(*k*)].
- **Two issues arise**
  - what is the relocation scheme?
  - how to search for k later?
- **Common methods for resolving a collision in open addressing**
  - Linear probing
  - Quadratic probing
  - Double hashing
  - Rehashing

- **Open Addressing**
  - To insert a key k, compute $f_0(k)$. If $hash\_table[f_0(k)]$ is empty, insert it there.
  - If collision occurs, probe alternative cell $f_1(k)$, $f_2(k)$, ... until an empty cell is found
    - $f_i(k) = (f(k) + g(i)) \% b$, with $g(0) = 0$
    - g: **collision resolution strategy**

- **Linear Probing**
  - **g(i) =i**
    - cells are probed **sequentially** (with wraparound)
    - $f_i(k) = (f(k) + i) \% b$
  - **Insertion**
    - Let k be the new key to be inserted. We compute f(k)
    - For i = 0 to b-1
      - compute L = (f(k) + i) % b
      - hash_table[L] is empty, then we put k there and stop
    - If we cannot find an empty entry to put *k*, it means that the table is full and we should report an error

- **Linear Probing – Insert**

  - divisor = b (number of buckets) = 17
  - Home bucket = key % 17

0				4				8				12				16
34	**0**	**45**				**6**	**23**	**7**			**28**	**12**	**29**	**11**	**30**	**33**

- Insert pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45

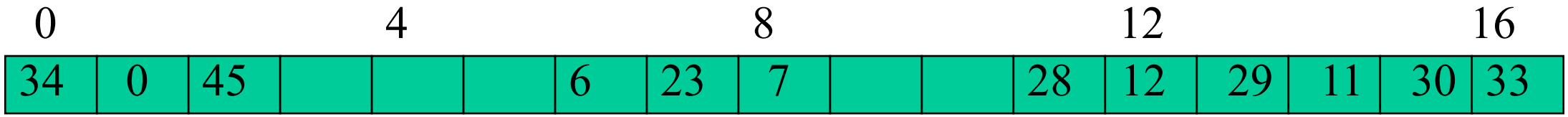
- **Linear Probing – Insert**

```
void linear_insert(element item, element ht[]){
    int i, hash_value;
    i = hash_value = hash(item.key);
    while(strlen(ht[i].key)) {
        if (!strcmp(ht[i].key, item.key)) {
            fprintf(stderr, "Duplicate entry\n");
            exit(1);
        }
        i = (i+1)%TABLE_SIZE;
        if (i == hash_value)
            fprintf(stderr, "The table is full\n"); exit(1);
    }
    ht[i] = item;
}
```

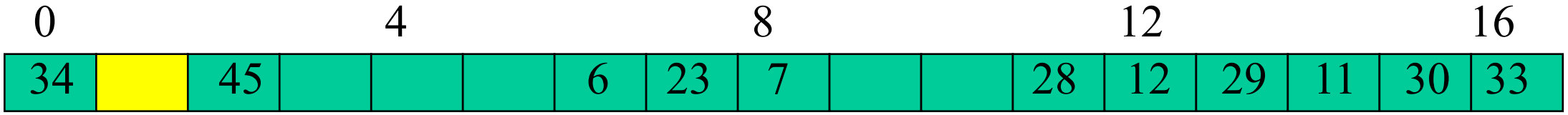
- **Data Structure for Hash Table**

```
#define MAX_CHAR  10
#define TABLE_SIZE  13
typedef struct {
    char key[MAX_CHAR];
    /* other fields */
} element;
element hash_table[TABLE_SIZE];
```

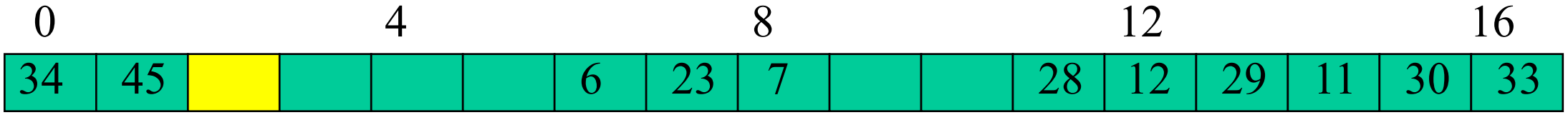
- **Linear Probing – Delete**



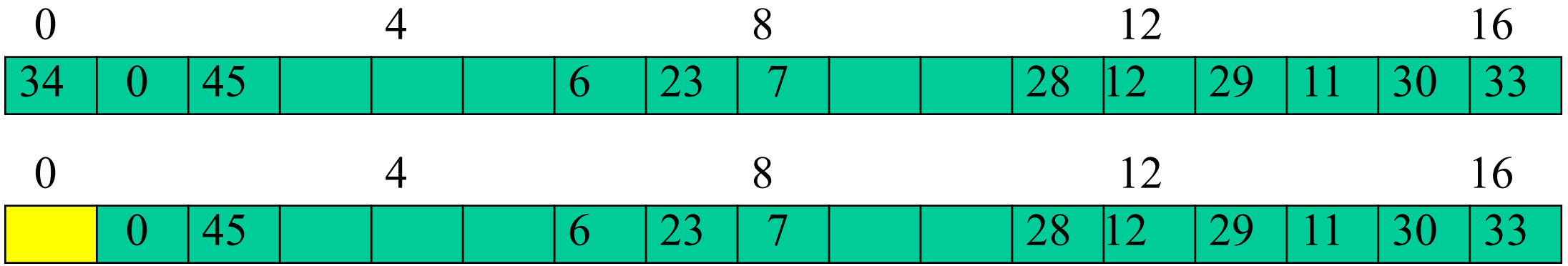
- **Delete(0)**



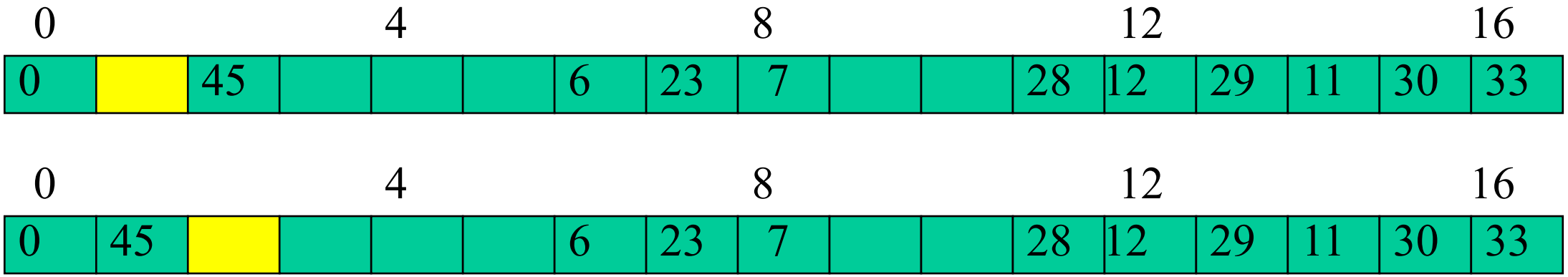
- Search cluster for pair (if any) to fill vacated bucket.



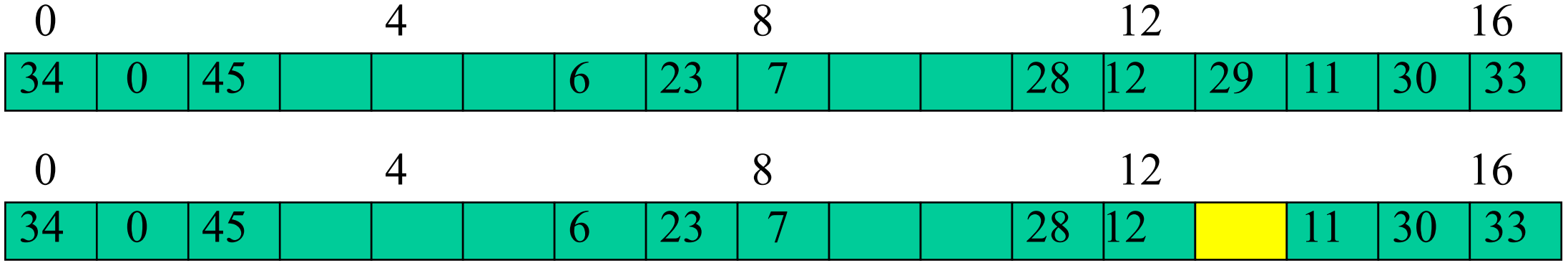
- **Linear Probing – Delete**
  - **Delete(34)**



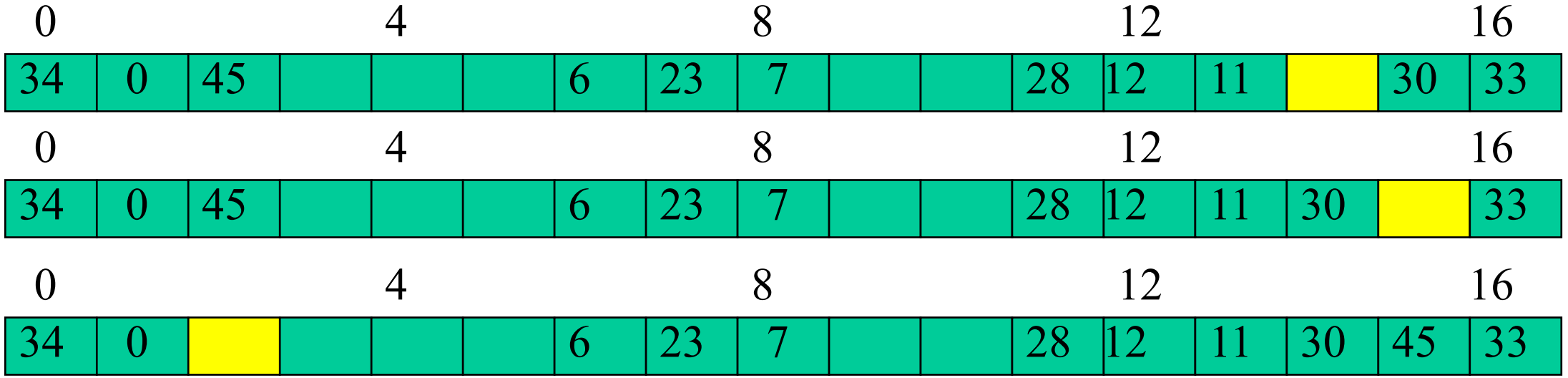
- Search cluster for pair (if any) to fill vacated bucket.



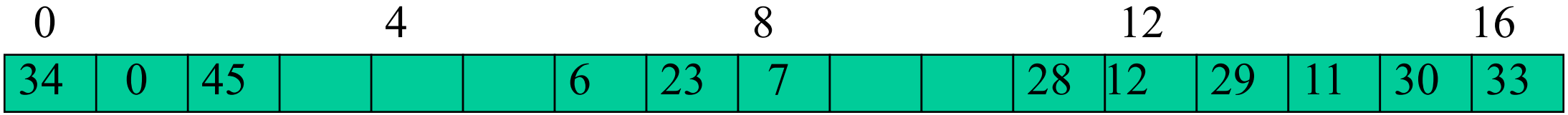
- **Linear Probing – Delete**
  - **Delete(29)**



  - Search cluster for pair (if any) to fill vacated bucket



- **Performance Of Linear Probing**



- Worst-case find/insert/erase time is **Θ(n)**, where n is the number of pairs in the table
- This happens when all pairs are in the same cluster (the same index/bucket)

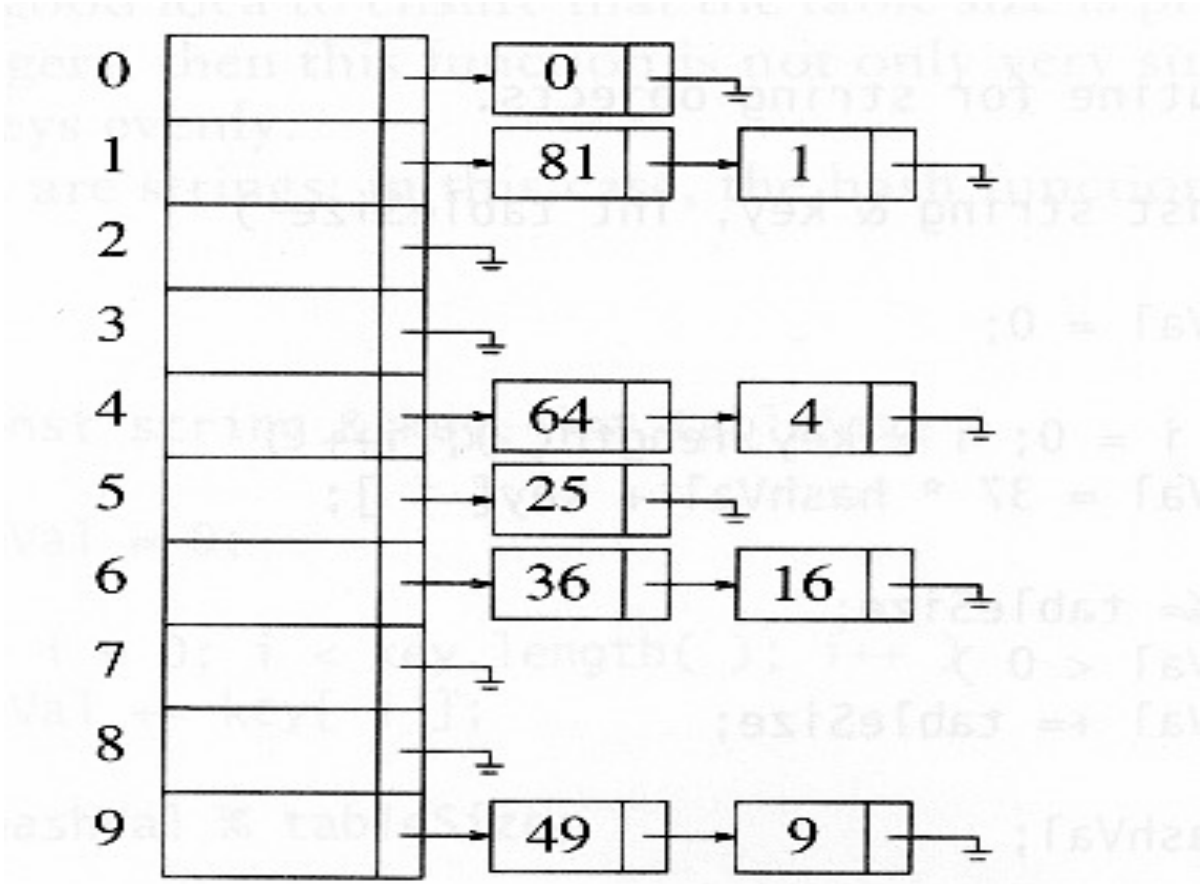
- **Quadratic Probing**
  - Linear probing searches buckets (f(x)+i)%b
  - Quadratic probing uses a quadratic function of *i* as the increment
  - Examine buckets f(x), $(f(x)+i^2)\%b$, $(f(x)-i^2)\%b$, for $1<=i<=(b-1)/2$
  - b is a prime number of the form 4j+3, j is an integer

- **Random Probing**
  - Random Probing works incorporating with random numbers
    - f(x) = (f'(x) + S[i]) % b
      - S[i] is a table with size b-1
      - S[i] is a random permuation of integers [1,b-1]

- **Double hashing**
  - Double hashing is one of the best method for dealing with collisions
  - If the slot is full, then a second hash function (which is different from the first one) is calculated and combined with the first hash function
    - $f(k, i) = (f_1(k) + i\ f_2(k)\ )\ \%\ b$

- **Rehashing**
  - Enlarging the Table
  - To rehashing
    - Create a new table of double the size (adjusting until it is again prime)
    - Transfer the entries in the old table to the new table, by recomputing their positions (using the hash function)
  - Rehashing when the table is completely full

- **Separate Chaining**
  - Instead of a hash table, we use a table of linked list
  - keep a linked list of keys that hash to the same value

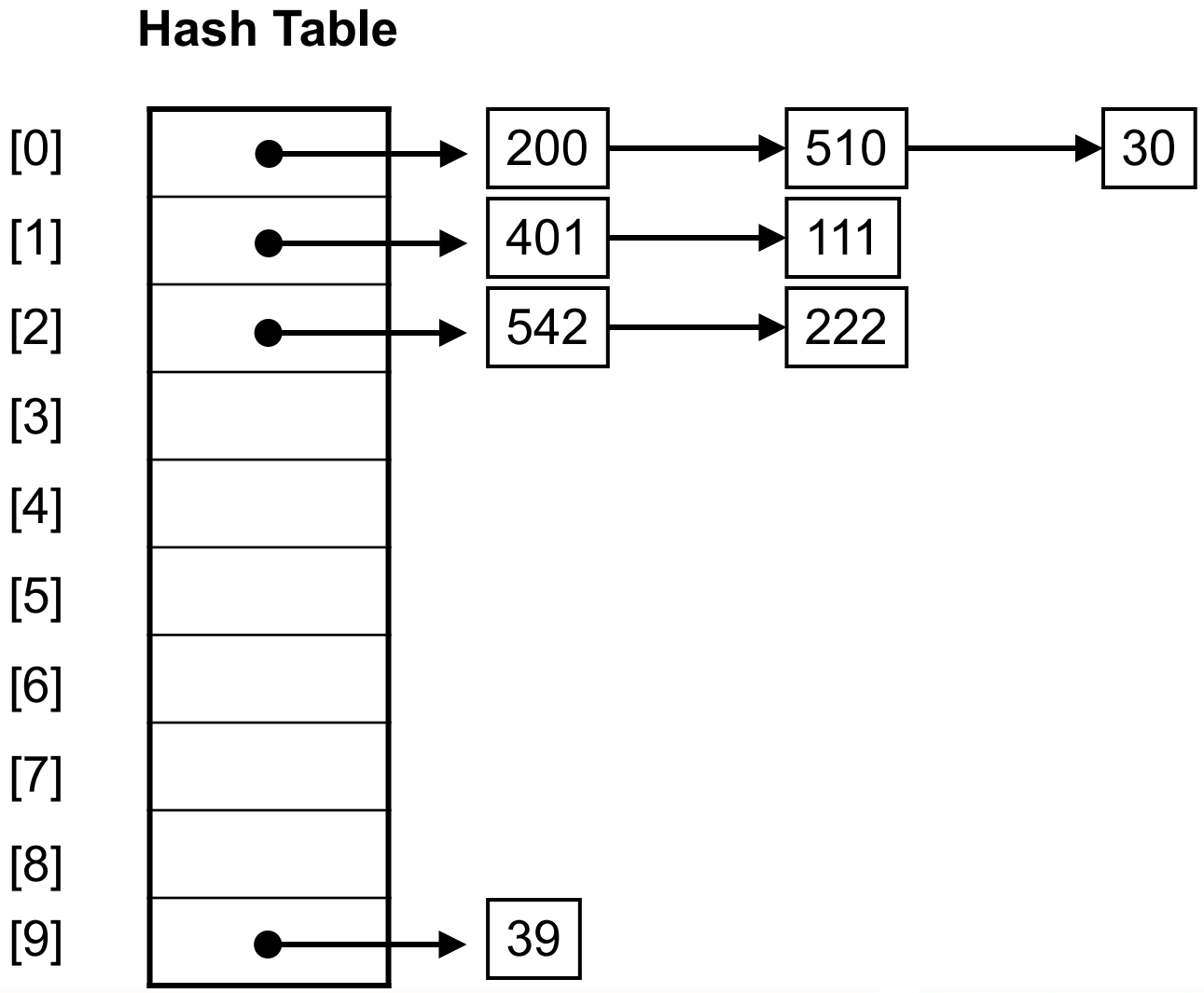


f(k) = k mod 10

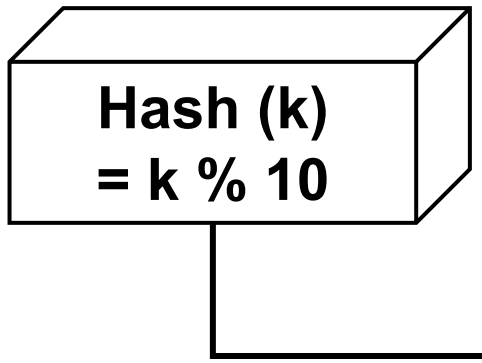
- **Separate Chaining**
  - To insert a key k
    - Compute f(k) to determine which list to traverse
    - If hash_table[f(k)] contains a null pointer, initiatize this entry to point to a linked list that contains k alone
    - If hash_table[f(k)] is a non-empty list, we add k at the beginning of this list
  - To delete a key k
    - compute f(k), then search for k within the list at hash_table[f(k)]. Delete k if it is found.

- **Separate Chaining**
  - If the hash function works well, the number of keys in each linked list will be a small constant
  - Therefore, we expect that each search, insertion, and deletion can be done in constant time
  - Disadvantage
    - Memory allocation in linked list manipulation will slow down the program
  - Advantage
    - Deletion is easy
    - Array size is not a limitation

- **Example**

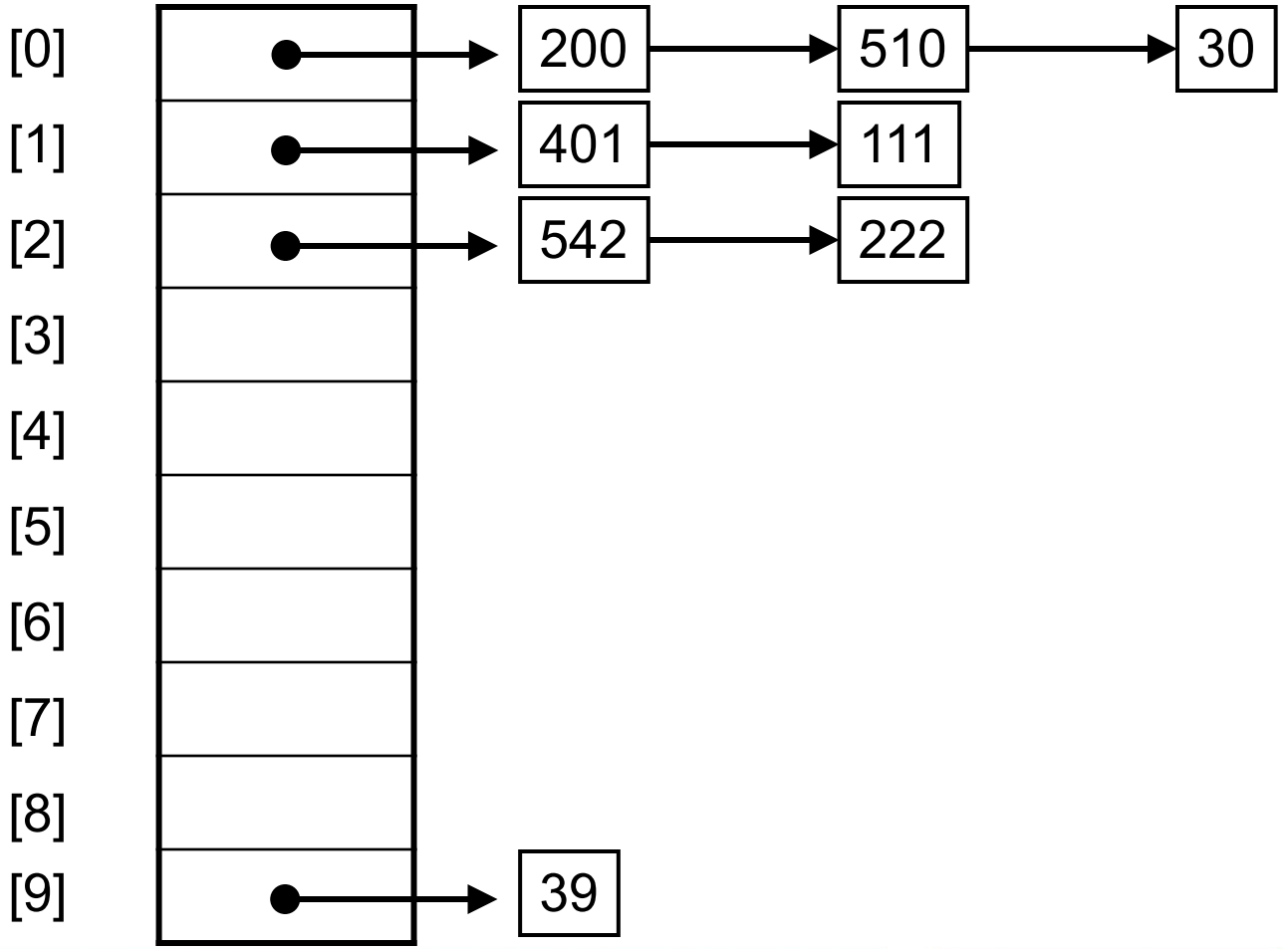


Insert a record with
key = 33

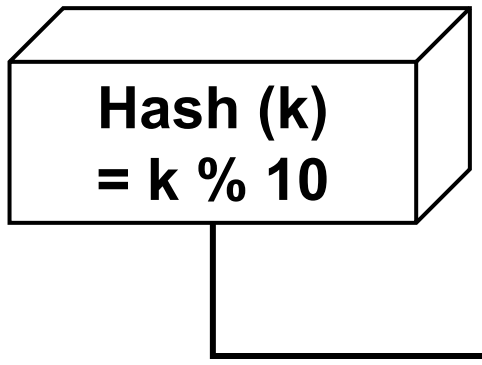


Insert in chain 3

Hash Table

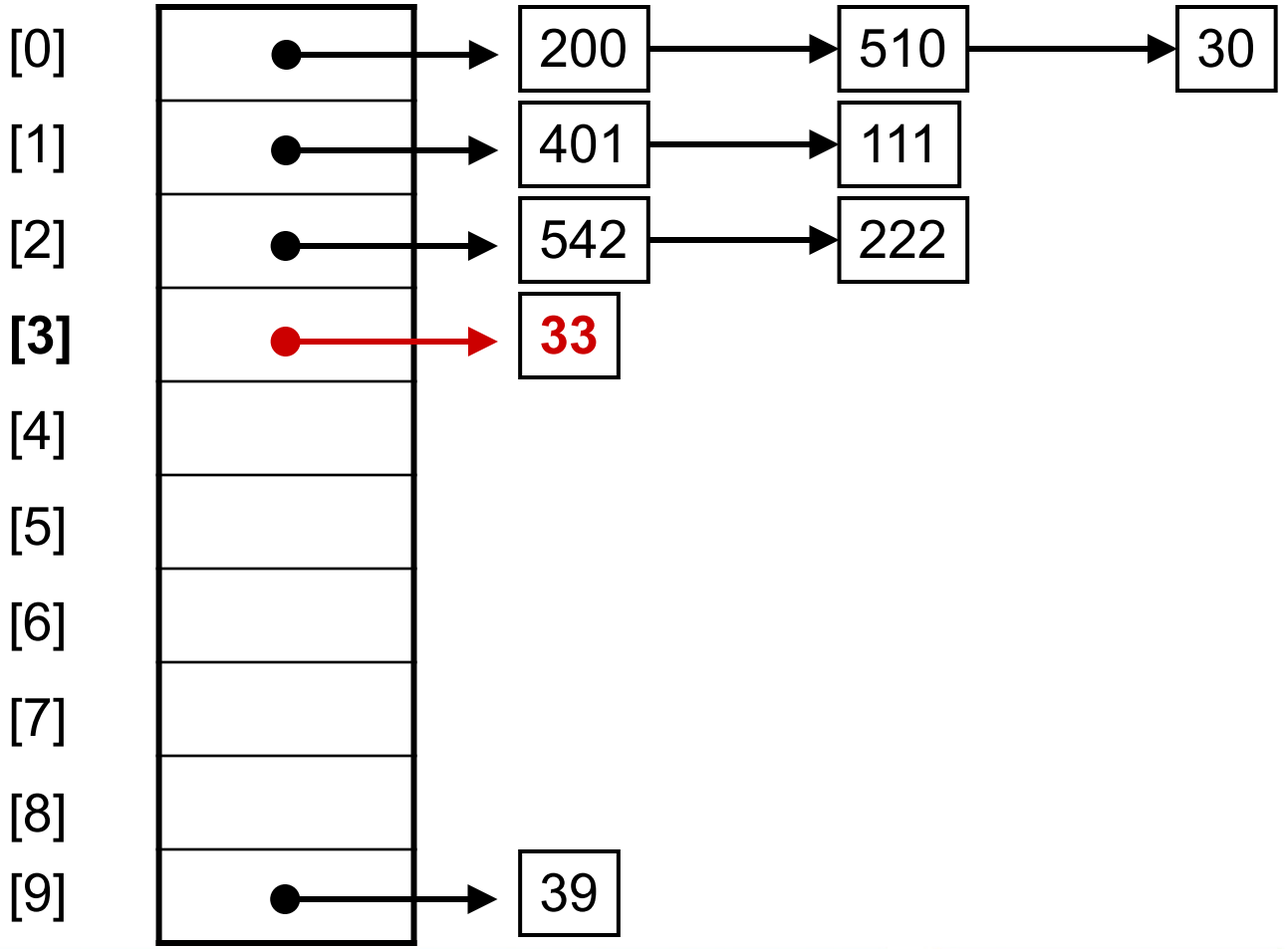


Insert a record with
key = 33

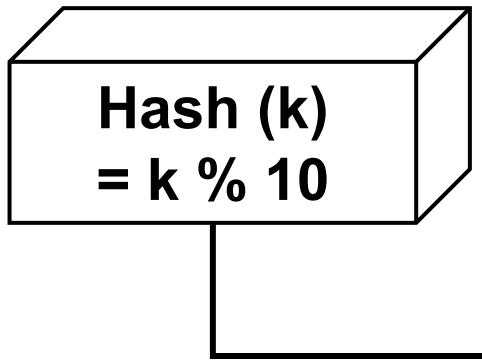


Insert in chain 3

Hash Table



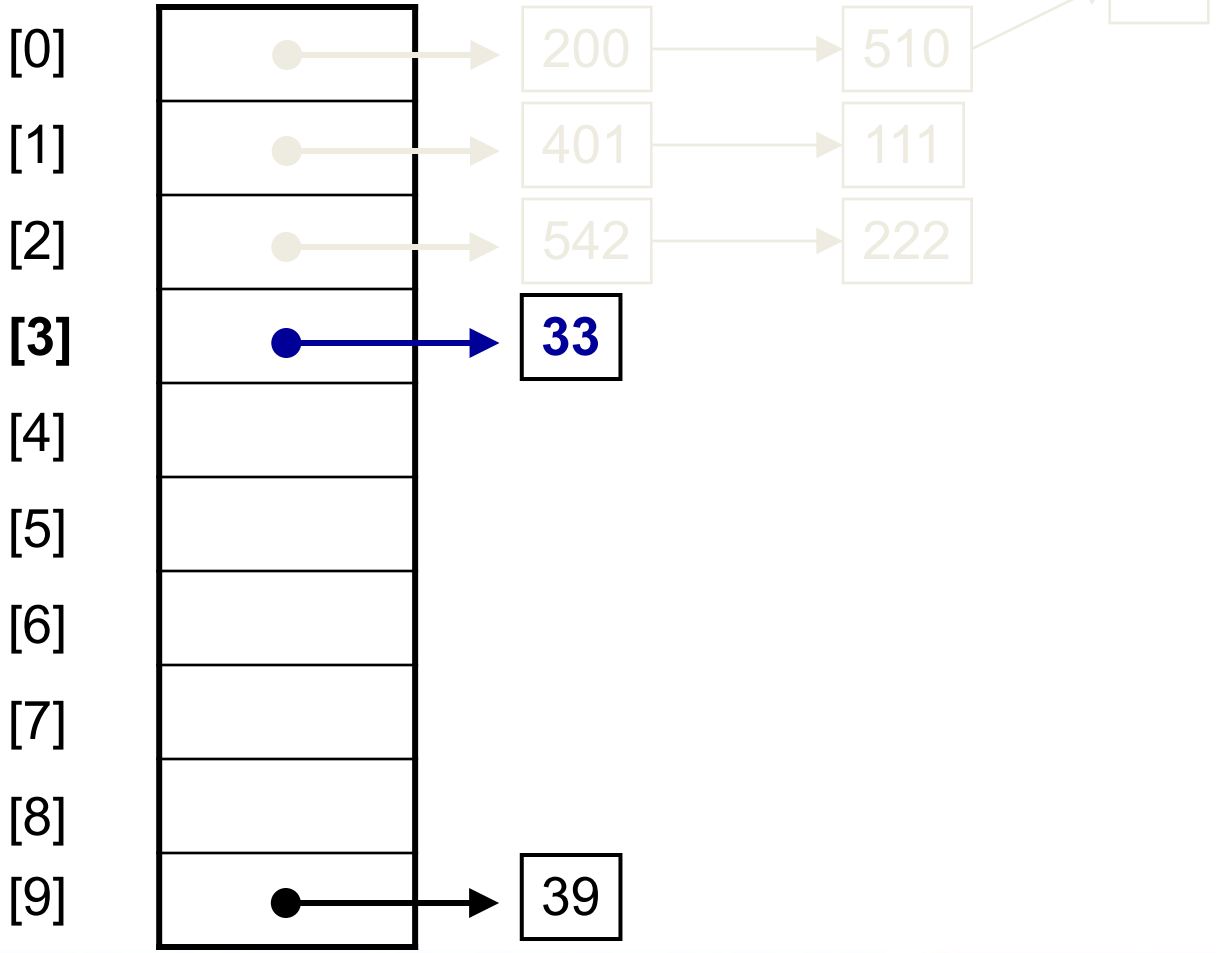
Insert a record with
key = 33



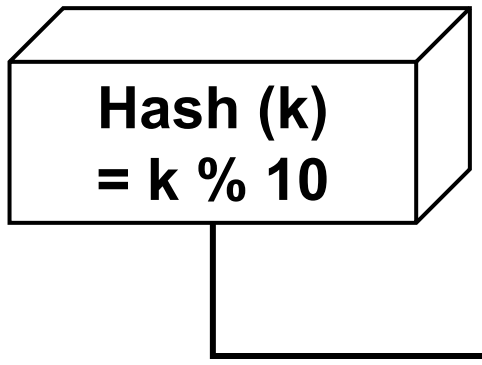
Insert in chain 3

ERROR!

Hash Table

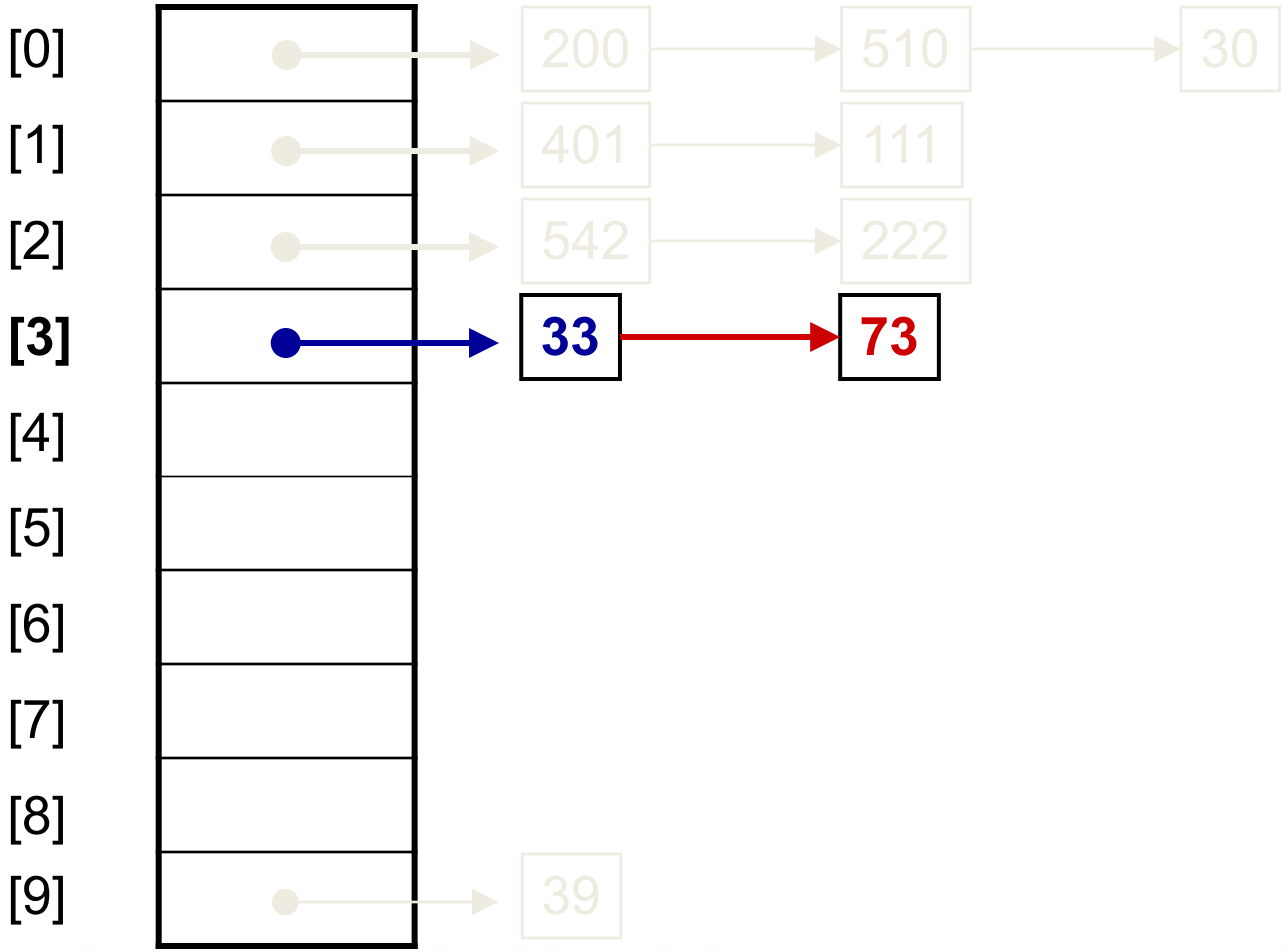


Insert a record with
key = 73

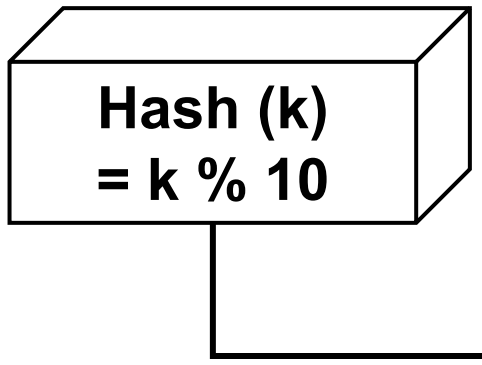


Insert in chain 3

Hash Table

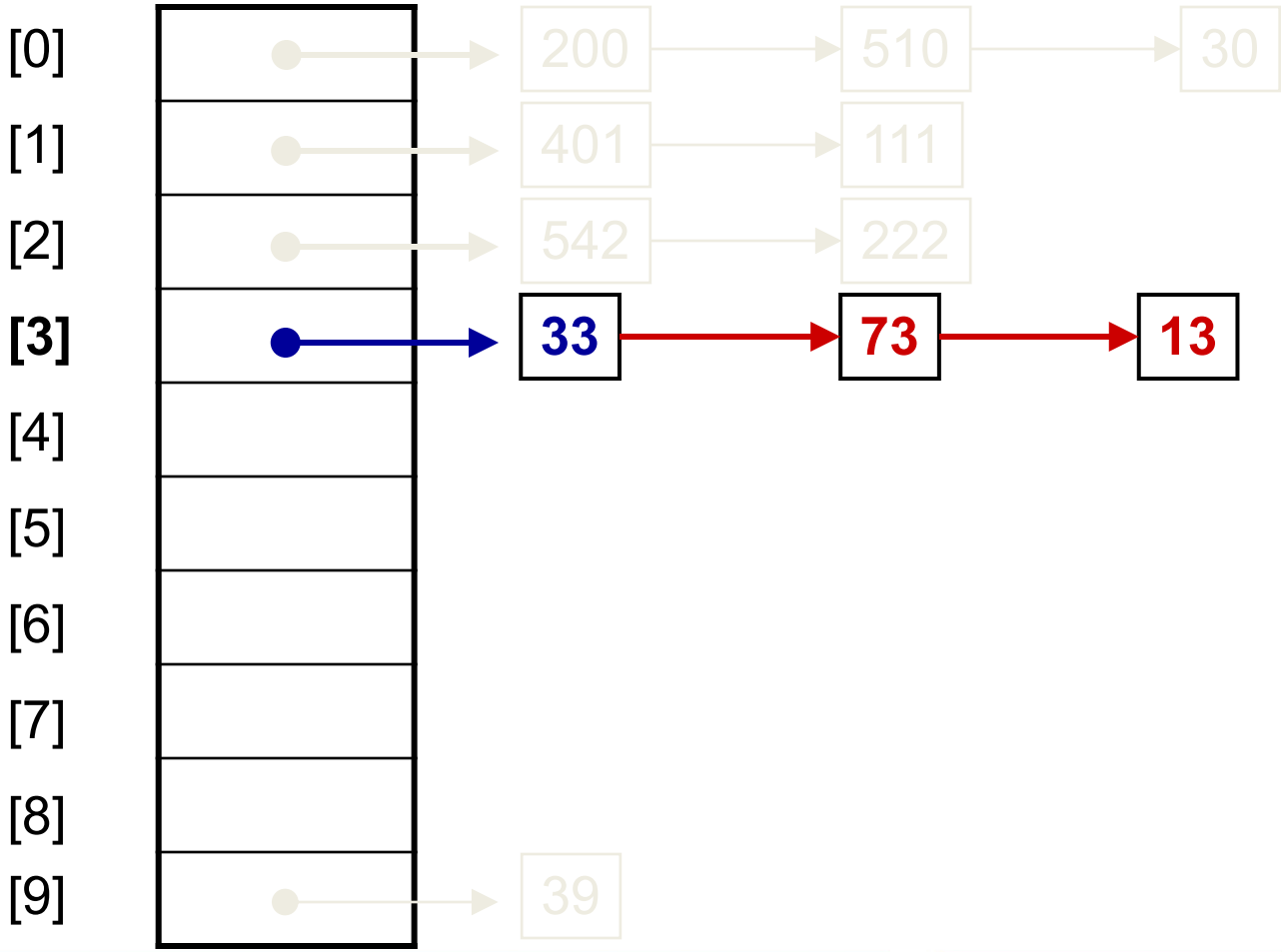


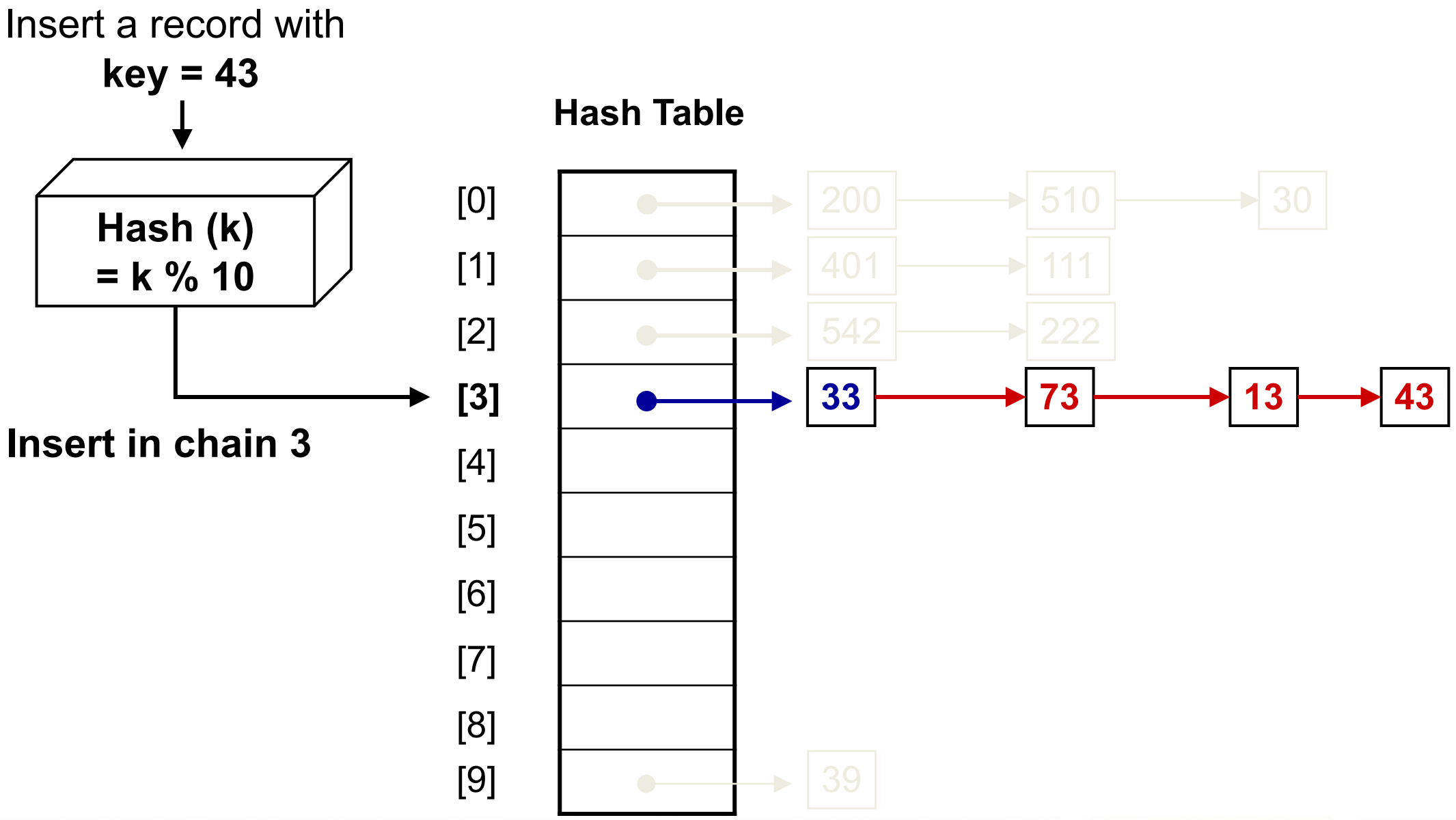
Insert a record with
key = 13



Insert in chain 3

Hash Table




Insert a record with
key = 43
Hash (k)
= k % 10
Insert in chain 3
Hash Table
[0]
[1]
[2]
[3]
[4]
[5]
[6]
[7]
[8]
[9]
200
510
30
401
111
542
222
33
73
13
43
39

- **Data Structure for Chaining**

```
#define MAX_CHAR 10
#define TABLE_SIZE 13
#define IS_FULL(ptr) (!(ptr))
typedef struct {
        char key[MAX_CHAR];
        /* other fields */
} element;
typedef struct list *list_pointer;
typedef struct list {
        element item;
        list_pointer link;
};
list_pointer hash_table[TABLE_SIZE];
```

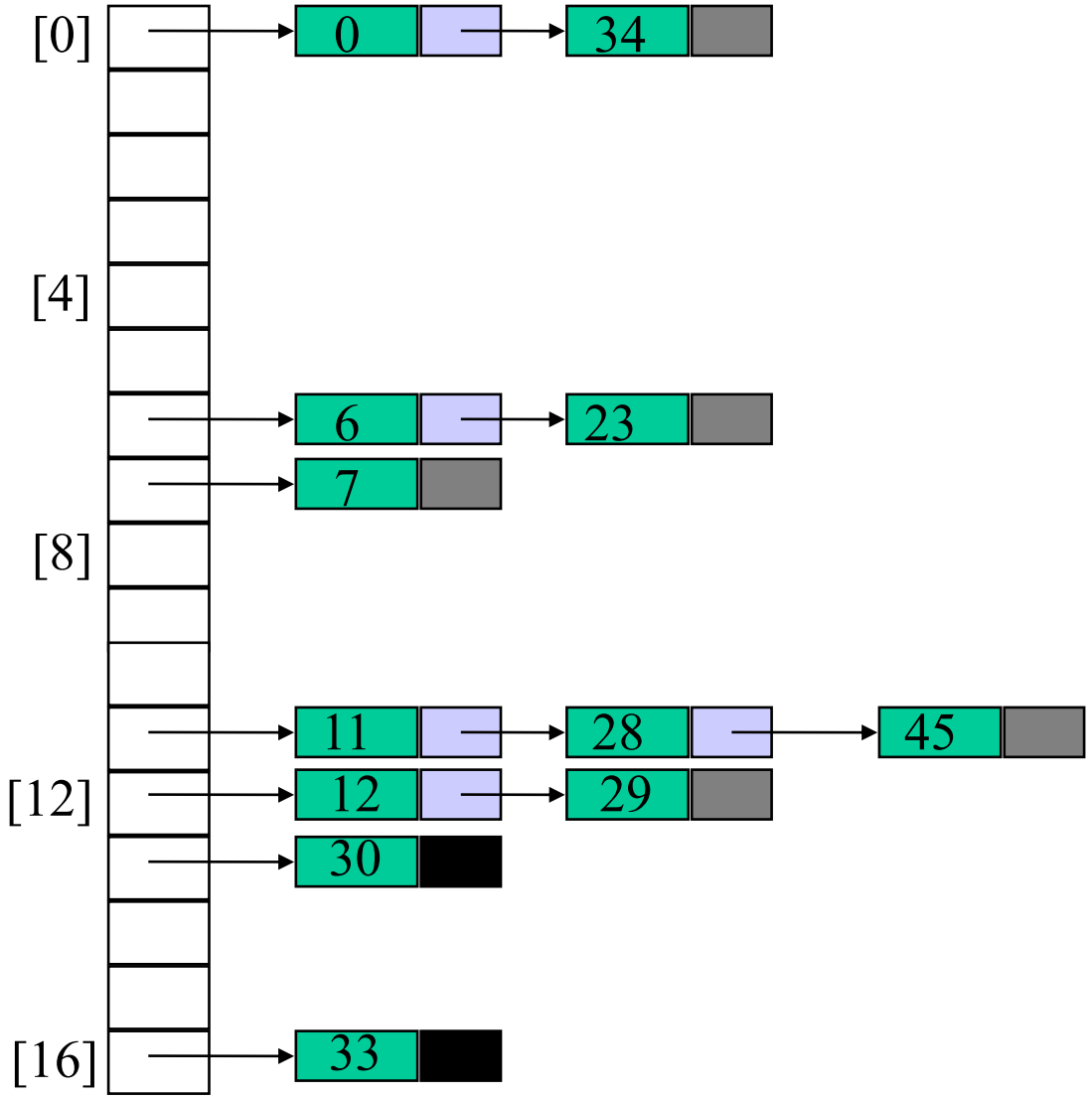
- **Separate Chaining – Insert**

```
void insert( element_type key, HASH_TABLE H ){
    position pos, new_cell; LIST L;
    pos = find( key, H );
    if( pos == NULL ) {
        new_cell = (position) malloc(sizeof(struct list_node));
        if( new_cell == NULL )   fatal_error("Out of space!!!");
        else {
                L = H->the_lists[ hash( key, H->table size ) ];
                new_cell->next = L->next;
                new_cell->element = key; /* Probably need strcpy!! */
                L->next = new_cell;
        }
    }
}
```

- **Other Implementation**

Sorted chains

- Put in pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45
- Bucket = key % 17



CONTENT

- Introduction
- Static hashing
- **Dynamic hashing**

- **Dynamic hashing**
  - The number of identifiers in a hash table may vary
  - Use a small table initially; when a lot of identifiers are inserted into the table, we may increase the table size
  - When a lot of identifiers are deleted from the table, we may reduce the table size
  - This is called **dynamic hashing** or **extendible hashing**
  - Dynamic hashing usually involves databases and buckets may also be called **pages**

- We assume each page contains *p* records
- Each record is identified by a key (i.e., the identifiers in static hashing)
- Space utilization = *n*/*mp*
  - where *n* is the number of actual records
  - *m* is the number of pages reserved
  - *p* is the number of records per page

$$\frac{\text{NumberOfRecord}}{\text{NumberOfPages} * \text{PageCapacity}}$$

- **Objective: Find an extendible hashing function such that**
  - it minimizes the number of pages accessed
  - space utilization is as high as possible

- **There are two approaches**
  - Dynamic Hashing with Using Directories
  - Dynamic Hashing without Using Directories

SUMMARY

- Introduction
- Static hashing
- Dynamic hashing

Enjoy the Course…!