

1 Cryptographic primitives

We rely on a public key signing scheme for verification of spending.

Types & functions

```
SKey VKey Sig Ser : Set
isKeyPair : SKey → VKey → Set
isSigned : VKey → Ser → Sig → Set
sign : SKey → Ser → Sig

KeyPair = Σ[ sk ∈ SKey ] Σ[ vk ∈ VKey ] isKeyPair sk vk
```

Property of signatures

```
((sk , vk , __) : KeyPair) (d : Ser) (σ : Sig) → sign sk d ≡ σ → isSigned vk d σ
```

Figure 1: Definitions for the public key signature scheme

2 Addresses

We define credentials and various types of addresses here.

Abstract types

Network
KeyHash
ScriptHash

Derived types

Credential = *KeyHash* \uplus *ScriptHash*

record **BaseAddr** : **Set** **where**
 field **net** : *Network*
 pay : **Credential**
 stake : **Credential**

record **BootstrapAddr** : **Set** **where**
 field **net** : *Network*
 pay : **Credential**
 attrsSize : \mathbb{N}

record **RwdAddr** : **Set** **where**
 field **net** : *Network*
 stake : **Credential**

Addr = **BaseAddr** \uplus **BootstrapAddr**

VKeyBaseAddr = $\Sigma[\text{addr} \in \text{BaseAddr}] \text{isVKey} (\text{BaseAddr.pay} \text{ addr})$

VKeyBootstrapAddr = $\Sigma[\text{addr} \in \text{BootstrapAddr}] \text{isVKey} (\text{BootstrapAddr.pay} \text{ addr})$

ScriptBaseAddr = $\Sigma[\text{addr} \in \text{BaseAddr}] \text{isScript} (\text{BaseAddr.pay} \text{ addr})$

ScriptBootstrapAddr = $\Sigma[\text{addr} \in \text{BootstrapAddr}] \text{isScript} (\text{BootstrapAddr.pay} \text{ addr})$

VKeyAddr = **VKeyBaseAddr** \uplus **VKeyBootstrapAddr**

ScriptAddr = **ScriptBaseAddr** \uplus **ScriptBootstrapAddr**

Helper functions

payCred : **Addr** \rightarrow **Credential**
netId : **Addr** \rightarrow *Network*
isVKeyAddr : **Addr** \rightarrow **Set**

Figure 2: Definitions used in Addresses

3 Scripts

We define Timelock scripts here. They can verify the presence of keys and whether a transaction happens in a certain slot interval. These scripts are executed as part of the regular witnessing.

```

data Timelock : Set where
  RequireAllOf      : List Timelock      → Timelock
  RequireAnyOf      : List Timelock      → Timelock
  RequireMOF        : ℕ → List Timelock → Timelock
  RequireSig        : KeyHash            → Timelock
  RequireTimeStart   : Slot              → Timelock
  RequireTimeExpire  : Slot              → Timelock

data evalTimelock (khs : ℙ KeyHash) (I : Maybe Slot × Maybe Slot) : Timelock → Set where
  evalAll : All (evalTimelock khs I) ss → evalTimelock khs I (RequireAllOf ss)
  evalAny : Any (evalTimelock khs I) ss → evalTimelock khs I (RequireAnyOf ss)
  evalMOF : ss' S.⊆ ss → All (evalTimelock khs I) ss' → evalTimelock khs I (RequireMOF (length ss') ss)
  evalSig : x ∈ khs → evalTimelock khs I (RequireSig x)
  evalTSt : proj1 I ≡ just l → a ≤ l → evalTimelock khs I (RequireTimeStart a)
  evalTEx : proj2 I ≡ just r → r ≤ a → evalTimelock khs I (RequireTimeStart a)

```

Figure 3: Timelock scripts and their evaluation

4 Protocol parameters

```
ProtVer =  $\mathbb{N} \times \mathbb{N}$   
  
record PParams : Set where  
  field a          :  $\mathbb{N}$   
      b          :  $\mathbb{N}$   
      maxBlockSize :  $\mathbb{N}$   
      maxTxSize   :  $\mathbb{N}$   
      maxHeaderSize :  $\mathbb{N}$   
      maxValSize   :  $\mathbb{N}$   
      minUtxOValue : Coin  
      poolDeposit  : Coin  
      Emax         : Epoch  
      pv           : ProtVer
```

Figure 4: Definitions used for protocol parameters

5 Transactions

Transactions are defined in Figure 5. A transaction is made up of three pieces:

- A set of transaction inputs. This derived type identifies an output from a previous transaction. It consists of a transaction id and an index to uniquely identify the output.
- An indexed collection of transaction outputs. The TxOut type is an address paired with a coin value.
- A transaction fee. This value will be added to the fee pot.

Finally, txid computes the transaction id of a given transaction. This function must produce a unique id for each unique transaction body. **We assume that txid is injective.**

$\text{getValue} : \text{TxOut} \rightarrow \text{Value}$

$\text{getValue } (fst, snd) = snd$

$\text{txinsVKey} : \mathbb{P} \text{ TxIn} \rightarrow \text{UTxO} \rightarrow \mathbb{P} \text{ TxIn}$

$\text{txinsVKey } txins \ utxo = txins \cap \text{dom } ((utxo \mid^{\wedge} \text{to-sp } (\text{isVKeyAddr?} \circ \text{proj}_1)) \circ s)$

Abstract types

$\text{Ix TxId AuxiliaryData} : \text{Set}$

Derived types

$\text{TxIn} = \text{TxId} \times \text{Ix}$

$\text{TxOut} = \text{Addr} \times \text{Value}$

$\text{UTxO} = \text{TxIn} \rightarrow \text{TxOut}$

$\text{Wdrl} = \text{RwdAddr} \rightarrow \text{Coin}$

$\text{ProposedPPUpdates} = \text{KeyHash} \rightarrow \text{PParamsUpdate}$

$\text{Update} = \text{ProposedPPUpdates} \times \text{Epoch}$

Transaction types

```
record TxBody : Set where
  field txins      :  $\mathbb{P}$  TxIn
        txouts     :  $\text{Ix} \rightarrow \text{TxOut}$ 
        --txcerts  : List DCert
        mint       : Value
        txfee      : Coin
        txvldt     : Maybe Slot  $\times$  Maybe Slot
        txwdrls    : Wdrl
        txup       : Maybe Update
        txADhash   : Maybe ADHash
        netwrk     : Maybe Network
        txsize     :  $\mathbb{N}$ 
        txid       : TxId
```

```
record TxWitnesses : Set where
  field vkSigs : VKey  $\rightarrow$  Sig
        scripts :  $\mathbb{P}$  Script
```

```
record Tx : Set where
  field body : TxBody
        wits : TxWitnesses
        txAD : Maybe AuxiliaryData
```

Abstract functions

Figure 5: Definitions used in the UTxO transition system

6 UTxO

6.1 Accounting

Figure 6 defines functions needed for the UTxO transition system. Figure 7 defines the types needed for the UTxO transition system. The UTxO transition system is given in Figure 8.

- The function `outs` creates the unspent outputs generated by a transaction. It maps the transaction id and output index to the output.
- The `balance` function calculates sum total of all the coin in a given UTxO.

```
outs : TxBODY → UTxO
outs tx = mapKeys (txid tx ,_) (λ where refl → refl) $ txouts tx

balance : UTxO → Value
balance utxo = Σm[ x ← utxo ] proj2 (proj2 x)

cbalance : UTxO → Coin
cbalance utxo = coin (balance utxo)

minfee : PParams → TxBODY → Coin
minfee pp tx = a * txsize tx + b
  where open PParams pp

-- need to add withdrawals to consumed
consumed : PParams → UTxO → TxBODY → Value
consumed pp utxo txb = balance (utxo | txins txb)
  +⊥ mint txb
  --+ inject (wbalance (txwdrls txb) + keyRefunds pp txb)

-- need to add deposits to produced
produced : PParams → UTxO → TxBODY → Value
produced pp utxo txb = balance (outs txb)
  +⊥ inject (txfee txb)
  --+ totalDeposits pp stpools (txcerts txb))

-- this has to be a type definition for inference to work
data inInterval (slot : Slot) : (Maybe Slot × Maybe Slot) → Set where
  both : ∀ {l r} → l ≤s slot × slot ≤s r → inInterval slot (just l , just r)
  lower : ∀ {l} → l ≤s slot → inInterval slot (just l , nothing)
  upper : ∀ {r} → slot ≤s r → inInterval slot (nothing , just r)
  none : inInterval slot (nothing , nothing)
```

Figure 6: Functions used in UTxO rules

UTxO environment

```
record UTxOEnv : Set where
  field slot : Slot
  pparams : PParams
```

UTxO states

```
record UTxOState : Set where
  constructor [[_,_]]u
  field utxo : UTxO
  fees : Coin
```

UTxO transitions

```
_⊢_→(⟦_,UTXO⟧)_ : UTxOEnv → UTxOState → TxBody → UTxOState → Set
```

Figure 7: UTxO transition-system types

UTXO-inductive :

```

 $\forall \{ \Gamma \} \{ s \} \{ tx \}$ 
 $\rightarrow$  let slot =  $\text{UTxOEnv.slot } \Gamma$ 
      pp =  $\text{UTxOEnv.pparams } \Gamma$ 
      utxo =  $\text{UTxOState.utxo } s$ 
      fees =  $\text{UTxOState.fees } s$ 
    in
  txins tx  $\neq \emptyset$ 
 $\rightarrow$  inInterval slot (txvldt tx)
 $\rightarrow$  txins tx  $\subseteq \text{dom } (utxo^s)$ 
 $\rightarrow$  let f = txfee tx in minfee pp tx  $\leq f$ 
 $\rightarrow$  consumed pp utxo tx  $\equiv$  produced pp utxo tx
 $\rightarrow$  coin (mint tx)  $\equiv 0$ 

{- these break deriveComputational but don't matter at the moment
   $\rightarrow$  txout  $\rightarrow$  txout proj1 (txouts tx)
       $\rightarrow$  (getValue (proj2 txout)) (inject (utxoEntrySize (proj2 txout) * PParams.maxValSize pp)
   $\rightarrow$  txout  $\rightarrow$  txout proj1 (txouts tx)
       $\rightarrow$  (serSize (getValue (proj2 txout)))  $\leq$  PParams.maxValSize pp
-}

-- PPUP
-- these fail with some reduceDec error
--  $\rightarrow$  All ( { (inj2 a , _)  $\rightarrow$  BootstrapAddr.attrsSize a  $\leq$  64 ; _  $\rightarrow$  } ) (range ((txouts tx)
--  $\rightarrow$  All ( a  $\rightarrow$  netId (proj1 a) networkId) (range ((txouts tx) ))
--  $\rightarrow$  All ( a  $\rightarrow$  RwdAddr.net a networkId) (dom ((txwdrls tx) ))
 $\rightarrow$  txsize tx  $\leq$  PParams.maxTxSize pp
-- Add Deposits

```

Γ
 $\vdash s$
 $\rightarrow \langle tx, \text{UTXO} \rangle$
 $\llbracket (utxo \mid \text{txins tx}^c) \cup^{ml} \text{outs tx}, \text{fees} + f \rrbracket^u$

Figure 8: UTXO inference rules

6.2 Witnessing

```

witsVKeyNeeded : UTxO → TxBODY →  $\mathbb{P}$  KeyHash
witsVKeyNeeded utxo txb =
  mapPartial (( $\lambda$  { (inj1 kh) → just kh ; _ → nothing }) • payCred • proj1) ((utxo s)  $\ll \$ \gg$  txins txb)

scriptsNeeded : UTxO → TxBODY →  $\mathbb{P}$  ScriptHash
scriptsNeeded utxo txb =
  mapPartial (( $\lambda$  { (inj2 sh) → just sh ; _ → nothing }) • payCred • proj1) ((utxo s)  $\ll \$ \gg$  txins txb)

scriptsP1 : TxWitnesses →  $\mathbb{P}$  PIScript
scriptsP1 txw = mapPartial ( $\lambda$  { (inj1 s) → just s ; _ → nothing }) (scripts txw)

```

Figure 9: Functions used for witnessing

```

_⊢_→(⟦_,UTXOW⟧_) : UTxOEnv → UTxOState → Tx → UTxOState → Set

```

Figure 10: UTxOW transition-system types

```

UTXOW-inductive :
  ∀ {Γ} {s} {tx} {s'}
  → let utxo = UTxOState.utxo s
      txb = body tx
      txw = wits tx
      witsKeyHashes = map hash (dom (vkSigs txw s))
      witsScriptHashes = map hash (scripts txw)
  in
  All (λ where (vk , σ) → isSigned vk (txidBytes (txid txb)) σ) (vkSigs txw s)
  → All (validP1Script witsKeyHashes (txvldt txb)) (scriptsP1 txw)
  → witsVKeyNeeded utxo txb ⊆ witsKeyHashes
  → scriptsNeeded utxo txb ≡e witsScriptHashes
  -- TODO: check genesis signatures
  → txADhash txb ≡ M.map hash (txAD tx)
  → Γ ⊢ s →(| txb ,UTxO|) s'
  -----
  Γ ⊢ s →(| tx ,UTXOW|) s'

```

Figure 11: UTXOW inference rules

7 Update Proposal Mechanism

GenesisDelegation = KeyHash \rightarrow (KeyHash \times KeyHash)

record PPUUpdateState : Set where
 field pup : ProposedPPUpdates
 fpup : ProposedPPUpdates

record PPUUpdateEnv : Set where
 field slot : Slot
 pparams : PParams
 genDelegs : GenesisDelegation

Figure 12: PPUP types

data pvCanFollow : ProtVer \rightarrow ProtVer \rightarrow Set where
 canFollowMajor : pvCanFollow (m, n) (m $\mathbb{N}.$ + 1, 0)
 canFollowMinor : pvCanFollow (m, n) (m, n $\mathbb{N}.$ + 1)

viablePParams : PParams \rightarrow Set
viablePParams pp = T -- TODO: block size check

isViableUpdate : PParams \rightarrow PParamsUpdate \rightarrow Set
isViableUpdate pp pup with applyUpdate pp pup
... | pp' = pvCanFollow (PParams.pv pp) (PParams.pv pp') \times viablePParams pp'

Figure 13: Definitions for PPUP

$\text{PPUUpdateEmpty} : \Gamma \vdash s \rightarrow \langle \text{nothing}, \text{PPUP} \rangle s$

$\text{PPUUpdateCurrent} : \text{let open PPUUpdateEnv } \Gamma \text{ in}$
 $\text{dom } (pup^s) \subseteq \text{dom } (\text{genDelegs }^s)$
 $\rightarrow \text{All } (\text{isViableUpdate } pparams) (\text{range } (pup^s))$
 $\rightarrow (\text{slot} + (2 * \text{StabilityWindow})) <^s \text{firstSlot } (\text{suc}^e (\text{epoch slot}))$
 $\rightarrow \text{epoch slot} \equiv e$

 $\Gamma \vdash \text{record } \{ \text{pup} = pup ; \text{fpup} = fpup \} \rightarrow \langle \text{just } (pup, e), \text{PPUP} \rangle$
 $\text{record } \{ \text{pup} = pup \cup^{m1} pup ; \text{fpup} = fpup \}$

$\text{PPUUpdateFuture} : \text{let open PPUUpdateEnv } \Gamma \text{ in}$
 $\text{dom } (pup^s) \subseteq \text{dom } (\text{genDelegs }^s)$
 $\rightarrow \text{All } (\text{isViableUpdate } pparams) (\text{range } (pup^s))$
 $\rightarrow (\text{slot} + (2 * \text{StabilityWindow})) \geq^s \text{firstSlot } (\text{suc}^e (\text{epoch slot}))$
 $\rightarrow \text{suc}^e (\text{epoch slot}) \equiv e$

 $\Gamma \vdash \text{record } \{ \text{pup} = pup ; \text{fpup} = fpup \} \rightarrow \langle \text{just } (pup, e), \text{PPUP} \rangle$
 $\text{record } \{ \text{pup} = pup ; \text{fpup} = pup \cup^{m1} fpup \}$

Figure 14: PPUP inference rules

8 Ledger State Transition

```

-- Only include accounting & governance info for now
record LEnv : Set where
  constructor [[_,_,_]]le
  field slot : Slot
         --txix : Ix
         pparams : PParams
         --acct : Acnt
         genDelegs : GenesisDelegation -- part of DPState

record LState : Set where
  constructor [[_,_]]l
  field utxoSt : UTxOState
         ppup : PPUUpdateState
         --dpstate : DPState

```

Figure 15: Types for the LEDGER transition system

```

LEDGER : let open LState s in
  record { LEnv  $\Gamma$  }  $\vdash$  utxoSt  $\rightarrow$  ( tx ,UTXOW ) utxoSt'
   $\rightarrow$  record { LEnv  $\Gamma$  }  $\vdash$  ppup  $\rightarrow$  ( txup (body tx) ,PPUP ) ppup'
  -- DELEGS

```

```

 $\Gamma \vdash s \rightarrow$  ( tx ,LEDGER ) [ [ utxoSt' , ppup' ] ]l

```

Figure 16: LEDGER transition system

LEDGERS-base : $\Gamma \vdash s \rightarrow (\langle \square, \text{LEDGERS} \rangle s$

LEDGERS-ind : $\forall \{txs\}$

$\rightarrow \Gamma \vdash s \rightarrow (\langle tx, \text{LEDGER} \rangle s'$

$\rightarrow \Gamma \vdash s' \rightarrow (\langle txs, \text{LEDGERS} \rangle s''$

$\Gamma \vdash s \rightarrow (\langle tx :: txs, \text{LEDGERS} \rangle s''$

Figure 17: LEDGERS transition system


```

record NewPParamState : Set where
  constructor [[_,_]]np
  field pparams : PParams
      ppup       : PPUUpdateState

updatePPUp : PParams → PPUUpdateState → PPUUpdateState
updatePPUp pparams record { fpup = fpup } with allb (isViableUpdate? pparams) (range (fpup s))
... | false = record { pup =  $\emptyset^m$  ; fpup =  $\emptyset^m$  }
... | true  = record { pup = fpup ; fpup =  $\emptyset^m$  }

votedValue : ProposedPPUpdates → PParams → ℕ → Maybe PParamsUpdate
votedValue pup pparams quorum =
  case any? (λ u → lengths ((pup |^ fromList [ u ]) s) ≥? quorum) (range (pup s)) of λ where
    (no _)      → nothing
    (yes (u , _) ) → just u

```

$$\frac{\text{NEWPP-Accept} : \forall \{ \Gamma \} \rightarrow \text{let open NewPPParamState } s; \text{newpp} = \text{applyUpdate pparams } \text{upd in} \\ \text{viablePPParams } \text{newpp}}{\Gamma \vdash s \rightarrow (\text{just } \text{upd} , \text{NEWPP}) \llbracket \text{newpp} , \text{updatePPUp } \text{newpp} \text{ ppup} \rrbracket^{\text{np}}}$$

$$\text{NEWPP-Reject} : \forall \{ \Gamma \} \rightarrow \text{let open NewPPParamState } s \text{ in} \\ \Gamma \vdash s \rightarrow (\text{nothing} , \text{NEWPP}) \llbracket \text{pparams} , \text{updatePPUp } \text{newpp} \text{ ppup} \rrbracket^{\text{np}}$$

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10 Blockchain layer

```
record Acnt : Set where
  field treasury : Coin
  reserves : Coin

record NewEpochState : Set where
  constructor [[_,_,_,_]] ne
  field lastEpoch : Epoch
  acnt : Acnt
  ls : LState
  prevPP : PParams
  pparams : PParams

record ChainState : Set where
  field newEpochState : NewEpochState
  genDelegs : GenesisDelegation

record Block : Set where
  field ts : List Tx
  slot : Slot
```

Figure 20: Definitions for the NEWEPOCH and CHAIN transition systems

11 Properties

11.1 UTxO

Property 11.1 (Preserve Balance) *For all $env \in \text{UTxOEnv}$, $utxo, utxo' \in \text{UTxO}$, $fee, fee' \in \text{Coin}$ and $tx \in \text{TxBody}$, if $\text{txid } tx \notin \text{map proj}_1 (\text{dom } (utxo^s))$ and $\Gamma \vdash \llbracket utxo, fee \rrbracket^u \rightarrow \langle tx, \text{UTXO} \rangle \llbracket utxo', fee' \rrbracket^u$ then*

$$\text{getCoin } \llbracket utxo, fee \rrbracket^u \equiv \text{getCoin } \llbracket utxo', fee' \rrbracket^u$$

Here, we state the fact that the UTxO relation is computable. This just follows from our automation.

$\text{UTXO-step} : \text{UTxOEnv} \rightarrow \text{UTxOState} \rightarrow \text{TxBody} \rightarrow \text{Maybe UTxOState}$

$\text{UTXO-step} = \text{compute Computational-UTXO}$

$\text{UTXO-step-computes-UTXO} :$

$\text{UTXO-step } \Gamma \text{ utxoState } tx \equiv \text{just } utxoState' \Leftrightarrow \Gamma \vdash utxoState \rightarrow \langle tx, \text{UTXO} \rangle utxoState'$

$\text{UTXO-step-computes-UTXO} = \equiv\text{-just}\Leftrightarrow\text{STS Computational-UTXO}$

Figure 23: Computing the UTxO transition system