Chapter 7

FLOW ROUTING

I. INTRODUCTION

- Flow routing is a procedure to estimate downstream hydrograph from a known upstream hydrograph
- Flow routing is often called flood routing
- Downstream hydrograph is delayed by the time lag (translation) and is attenuated
- Two types of flow routing : River flow routing and reservoir flow routing

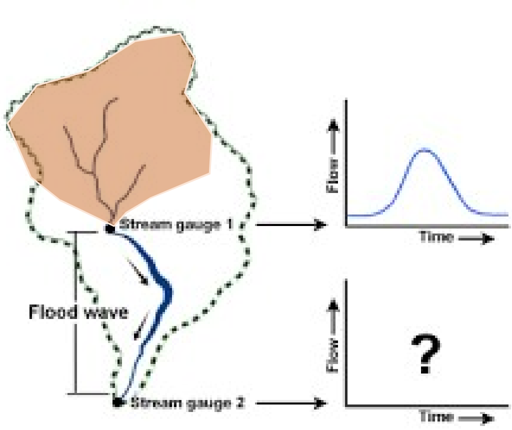


Fig.1: Flow routing

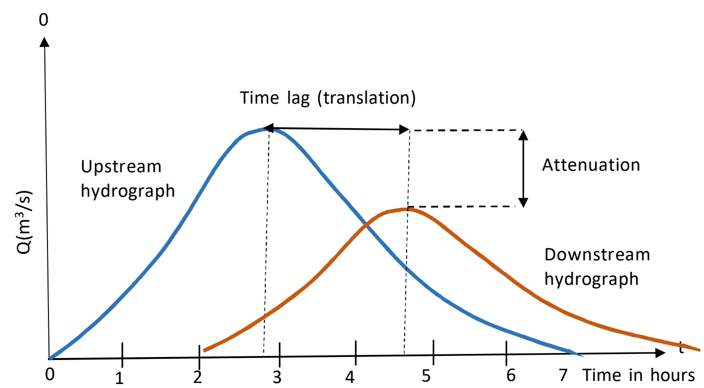


Fig.2: Upstream and downstream hydrograph

II. BASIC EQUATIONS

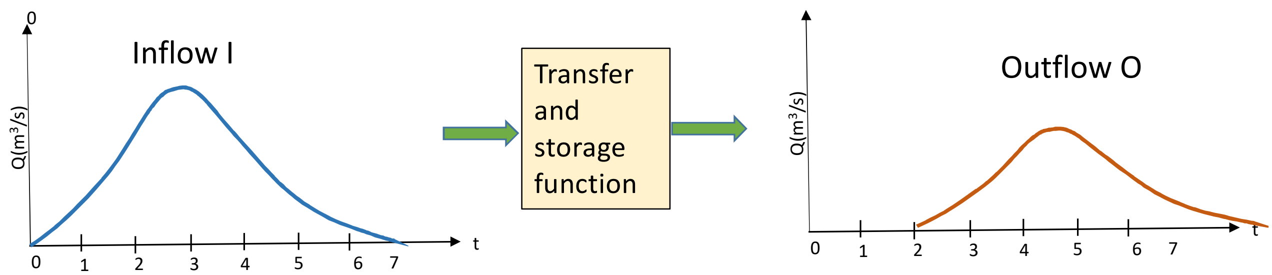


Fig.3 Flow routing system

The water balance for the system can be expressed

$$I - O = \frac{dS}{dt}$$

Where: I : upstream inflow , O: downstream outflow, S : storage in reservoir or a river reach

In practical calculation, the equation of water balance is rewritten in finite difference form for a Δt duration, I and O are replaced by mean values.

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t}$$

Where: I_1 , I_2 : upstream inflow at time t_1 and t_2 , O_1 , O_2 : downstream inflow at time t_1 and t_2 , S : storage in reservoir or a river reach and $\Delta t = t_2 - t_1$

If storage S is known, the downstream outflow O can be estimated.

III. RIVER FLOW ROUTING (MUSKINGUM method)

Assumptions:

- Area of cross section is proportional to the discharge and K is the proportional constant

Volume in prism : KQ

Volume in wedge : KX(I-Q)

X is the weighing factor

Volume stores in the river reach

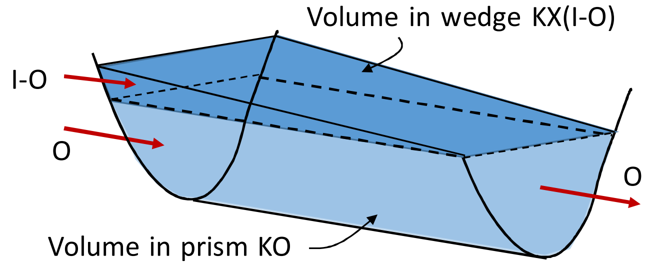


Fig.3 Flow routing system

$$S = KO + KX(I - O)$$

or

$$S = K[XI + (1 - X)O]$$

Replace S into the water balance equation

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t}$$

Obtain

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{K[XI_2 + (1 - X)O_2] - K[XI_1 + (1 - X)O_1]}{\Delta t}$$

Get O_2

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

Where:

$$C_0 = (0{,}5\Delta t - KX)/D$$

$$C_1 = (KX + 0{,}5\Delta t)/D$$

$$C_2 = (K - KX - 0{,}5\Delta t)/D$$

$$D = K - KX + 0{,}5\Delta t$$

It's important to check if : $C_0 + C_1 + C_2 = 1$, if not, some parameters are needed to adjustment and the largest value of C is selected to change first.

Since upstream inflow I_1, I_2 are known at time step t_1, t_2 and downstream outflow O_1 at time step t_1 is known, the downstream outflow can be obtained.

If the initial condition is not given, the downstream outflow O_1 is assumed the same as I_1 at the beginning

Practice 1 (Concise Hydrology – Dawei Han)

Estimate the downstream hydrograph using the Muskingum method with K=3hr and X=0.3. The time interval is 3 hours. The upstream hydrograph is as follows

Time (hr)	0	3	6	9	12	15	18
I (m^3/s)	1	3	9	15	13	10	6

Solution

At the beginning (time step t_1) assuming upstream inflow I_1 and downstream outflow O_1 are the same

$$O_1 = I_1$$

Downstream outflow at time step t_2 $\quad O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$

Calculate the basic parameters:

$$D = K - KX + 0{,}5\Delta t \;= 3 - 3.0{,}3 + 0{,}5.3 = 3{,}6$$

$$C_0 = (0{,}5\Delta t - KX)/D \;= (0{,}5.3\text{-}3.0{,}3)/3{,}6 = 0{,}17$$

$$C_1 = (KX + 0{,}5\Delta t)/D \;= (3.0{,}3\text{+}0{,}5.\ 3)/3{,}6 = 0{,}67$$

$$C_2 = (K - KX - 0{,}5\Delta t)/D \;= (3\text{-}3.0{,}3\text{-}0{,}5.\ 3)/3{,}6 = 0{,}17$$

Check if $C_0 + C_1 + C_2 = 1$, $\longrightarrow$ 0,17+ 0,67 + 0,17 = 1,01 $\longrightarrow$ Change the largest weight , let $C_1 = 0{,}66$

Replace basic parameters into the equation

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1 = 0{,}17 I_2 + 0{,}66 I_1 + 0{,}17 O_1$$

From the given upstream inflow

Time (hr)	0	3	6	9	12	15	18
I (m^3/s)	1	3	9	15	13	10	6

At the beginning (time step t_1)

$O_1 = I_1$ $\qquad$ $O_1 = 1$

Downstream outflow at time step t_2

$$O_2 = 0{,}17.3 + 0{,}66.1 + 0{,}17.\ 1 = 1{,}3 \text{ m}^3/\text{s}$$

Similarly at time step t_3, t_4, t_5 ......

Time	1	3	6	9	12	15	18
I(m3/s)	1	3	9	15	13	10	6
O(m3/s)	1	1.3	3.7	9.1	13.7	12.6	9.8

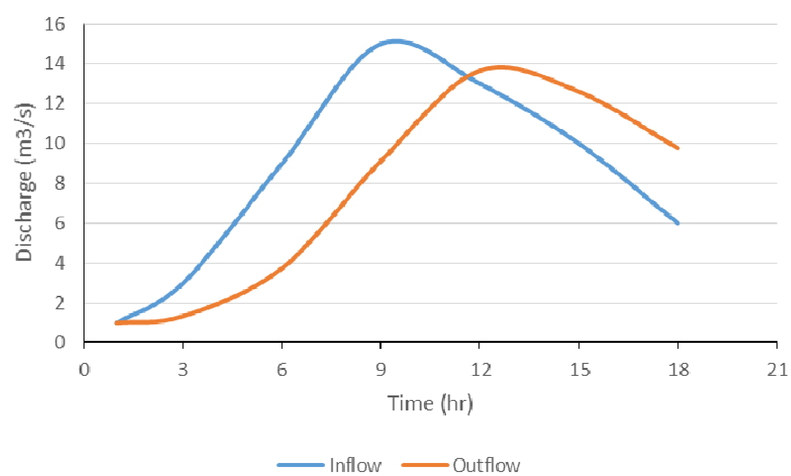


Fig.4 Inflow and outflow

IV. ESTIMATION OF K AND X

In case there are the measured outflow records, parameter K and X can be estimated by following procedure.

Water volume stores in the river reach:

$$S = K[XI + (1 - X)O]$$

So storage S and [XI+(1-X)O] have a linear relationship and its slope is K.

The storage S can be worked out by accumulating ($I_{mean} - O_{mean}$) from each step

The term [XI+(1-X)O] can be calculated by using the instantaneous value of O

Plot storage S and the term [XI+(1-X)O] in a graph to estimate K from the slope of graph

Practice 2 (Concise Hydrology – Dawei Han)

The record data of upstream inflow and the down stream outflow in a river are given in the table below. Estimate the parameter proportional constant K and weighing factor X

Time (hr)	I(m3/s)	O(m3/s)
0	31	31
6	50	27
12	86	25
18	123	30
24	145	44
30	150	63
36	144	82
42	128	97
48	113	106
54	95	111
60	79	111
66	65	108
72	55	101
78	46	94
84	40	85
90	35	77
96	31	70
102	27	63
108	25	56
114	24	50
120	23	45
126	22	41

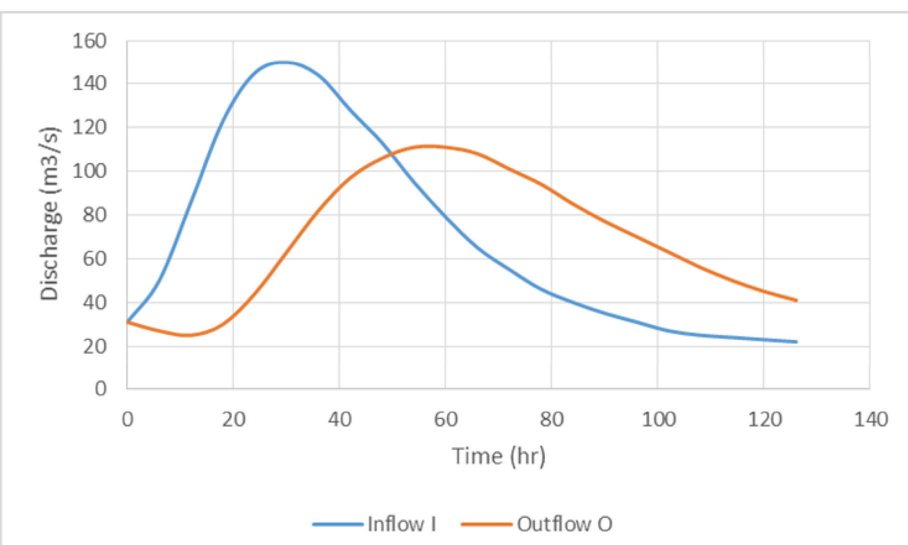


Fig.5 Inflow and outflow

Solution

- Calculate S by accumulating ($I_{mean} – O_{mean}$) from each step

At time step t_1 = 0, S_1 = 0

At time step t_2 = 6 hr

$$S_2 = \left(\frac{I_1+I_2}{2}\right)-\left(\frac{O_1+O_2}{2}\right)=\left(\frac{31+50}{2}\right)-\left(\frac{31+27}{2}\right)= 11{,}5$$

Similarly for the time step t_3 , t_4 , t_5 ….. and obtain S_3 , S_4, S_5 …..

- Calculate the term [XI+(1-X)O] by using the instantaneous value of O

Assume X = 0,2

At time step t_1 : [XI+(1-X)O] = [0,2x31 +(1-0,2)x31 = 31

Similarly for the time step t_3 , t_4 , t_5 …..

Assume X = 0,3 , X = 0,25 and repeat the procedure above to obtain the value of [XI+(1-X)O] for each step

Computed results are shown in table 1

Table 1

Time step	Time (hr)	I(m3/s)	O(m3/s)	S	X= 0.2	X= 0.3	X= 0.25
t1	0	31	31	0	31	31	31.0
t2	6	50	27	11.5	31.6	33.9	32.8
t3	12	86	25	53.5	37.2	43.3	40.3
t4	18	123	30	130.5	48.6	57.9	53.3
t5	24	145	44	227.5	64.2	74.3	69.3
t6	30	150	63	321.5	80.4	89.1	84.8
t7	36	144	82	396	94.4	100.6	97.5
t8	42	128	97	442.5	103.2	106.3	104.8
t9	48	113	106	461.5	107.4	108.1	107.8
t10	54	95	111	457	107.8	106.2	107.0
t11	60	79	111	433	104.6	101.4	103.0
t12	66	65	108	395.5	99.4	95.1	97.3
t13	72	55	101	351	91.8	87.2	89.5
t14	78	46	94	304	84.4	79.6	82.0
t15	84	40	85	257.5	76	71.5	73.8
t16	90	35	77	214	68.6	64.4	66.5
t17	96	31	70	173.5	62.2	58.3	60.3
t18	102	27	63	136	55.8	52.2	54.0
t19	108	25	56	102.5	49.8	46.7	48.3
t20	114	24	50	74	44.8	42.2	43.5
t21	120	23	45	50	40.6	38.4	39.5
t22	126	22	41	29.5	37.2	35.3	36.3

- Plot storage S and the term [XO-(1-X)O] in a graph

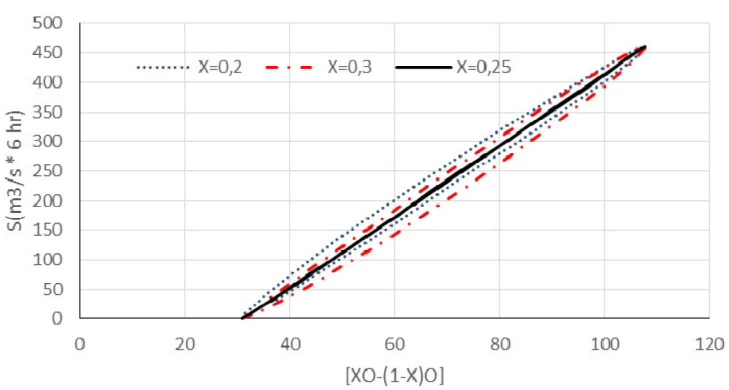


Fig.6 Storage loop diagram with X =0,2, X=0,3 and X=0,2!

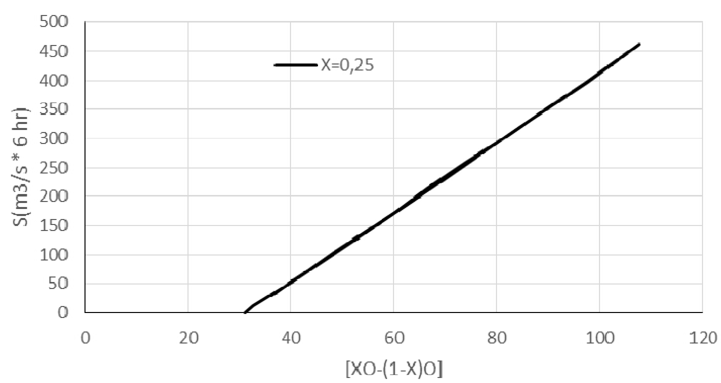


Fig.7 Storage loop diagram with X=0,25

It’s found that the X =0,25 curve has the narrowest loop, hence X=0,25 curve gives a linear relationship between S and [XI+(1-X)O]. The X=0,25 curve is chosen and the slope of this line is K.

From the graph, the slope of the line is 6 , but the time interval is 6 hr, so K = 6x 6 hr = 36 hr

V. RESERVOIR FLOW ROUTING

Assumptions:

- The water level in reservoir is horizontal
- The storage S is a function of h

S = f(h)

h : the water depth on the crest of spillway

The discharge over spillway crest is a function of h and determine by formula

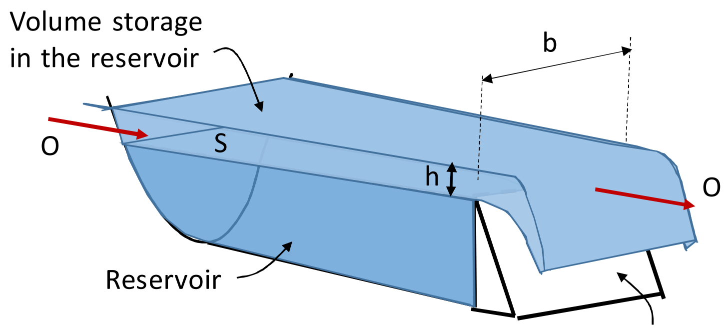


Fig.3 Flow routing in reservoir

$$O = Cbh^{1,5}$$

C is discharge coefficient and b is the width of the spillway

The water balance equation

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t}$$

or

$$(I_1 + I_2) - (O_1 + O_2) = \frac{2S_2 - 2S_1}{\Delta t}$$

rewrite

$$I_1 + I_2 - O_1 + \frac{2S_1}{\Delta t} = \frac{2S_2}{\Delta t} + O_2$$

$$I_1 + I_2 - O_1 + \frac{2S_1}{\Delta t} = \frac{2S_2}{\Delta t} + O_2$$

$$I_1 + I_2 - O_1 + \frac{2\,f(h_1)}{\Delta t} = \frac{2\,f(h_2)}{\Delta t} + Cbh_2^{1,5}$$

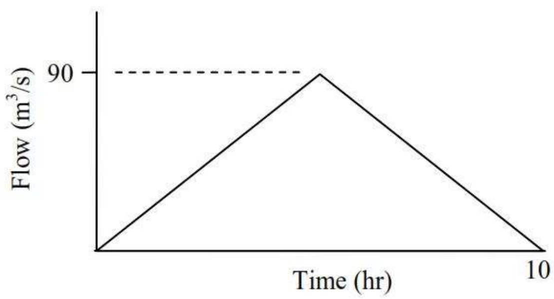
The knowns are on the right side (RS) and the unknowns (S_2, O_2 or h_2) are on the left side. The equation is non linear , h_2 is solved then O_2 can be obtained.

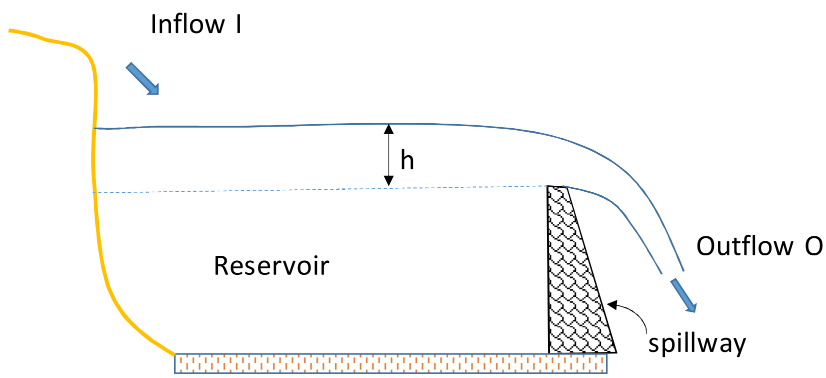
The computer time interval is often taken as

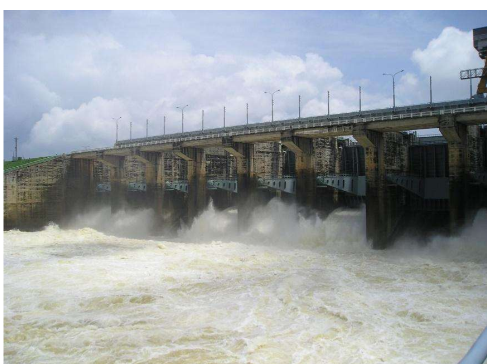
$$\Delta t = \frac{Duration\ of\ the\ inflow\ rising\ limb}{5}$$

Practice 3 (Concise Hydrology – Dawei Han)

A triangular-shaped inflow hydrograph is routed through a reservoir assuming it is completely full at the beginning of the storm. The spillway crest is 20 m wide and has a coefficient of 2.7. The reservoir area is 0.5 km² and has vertical sides. What are the maximum outflow and the maximum height of water in the reservoir during this flow event?







Trị An spillway
Q_{max} = 18450 m³/s

Tri An reservoir