LINEAR CIRCUIT THEORY

Matrices in Computer Applications

Jiri Vlach

Apple Academic Press

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TORONTO NEW JERSEY

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Apple Academic Press, Inc
3333 Mistwell Crescent
Oakville, ON L6L 0A2
Canada

Exclusive worldwide distribution by CRC Press an imprint of Taylor & Francis Group, an Informa business

Version Date: 20140128

International Standard Book Number-13: 978-1-4822-1961-6 (eBook - PDF)

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PREFACE

Circuit theory, developed in the twentieth century, supports today a huge industrial section of analog networks. Most designs are done by computers, but the need to thoroughly understand the theory remains.

In about five hundred pages, this book introduces most subjects of the linear circuit theory. Some methods, like the Laplace transformation, sensitivities, active network design and modified nodal formulation, will be taught in later years and the student can keep the book throughout the undergraduate program.

Emphasis of the book is on solving small examples by hand, because only practice leads to understanding and knowledge. Almost all problems in the book lead to only two or three linear equations. A complete solutions manual is available.

The style of explanation is also important and here is what makes this book different. Solutions by matrix algebra are started in the third chapter. First year students, most likely, have not heard about matrices, so we start with an absolute minimum: how to write two or three linear equations in matrix form, how to evaluate their determinants and how to apply the Cramer's rule. This can be taught in a one hour lecture. Additional subjects of the matrix algebra can be taught later or left to mathematics courses. Cramer's rule avoids divisions and fractions of the elimination method and is suitable for up to four linear equations. Matrices have to be used for more complicated problems anyway and the students will be acquainted with their use.

When designing a network for a certain performance, we may not know numerical values of some or all elements and we must keep them as variables (letters). This is called symbolic analysis. To use elimination is almost unthinkable. Cramer's rule makes the process reasonably simple, as demonstrated on many examples. Programs for any kind of analysis, including symbolic, do exist, but are not useful for introductory studies.

Another simplification in the presentation is based on the fact that linear networks can be scaled to unit element values and unit frequency (see the Appendix). To learn circit analysis, there is no need to use kiloohms, picofarads and miliampers. In all problems we use basic units, ohms, farads and henries, and entire numbers for element values.

Network equations can be based on mesh or nodal analysis. In this book, both are given approximately the same amount of attention, but they are not equally important. The mesh analysis has severe limitations and nodal formulation is preferred. All widely available circuit analysis programs are based on its extension, called Modified Nodal

Formulation, MNF (Chapter 17). These programs automatically convert all linear resistors into conductances, to keep the number of equations small. In this book, when dealing with nodal formulation, we adhere to this practice and use conductances, G, to avoid fractions of the type $1/R$.

It is advantageous to use different letters for circuit variables and for independent sources. The book uses V and I for variables and E and J for independent sources. This is a minor point, but it helps the student to avoid confusion of what should be on the left or right side of the equal sign.

Circuits with capacitors and inductors are described by differential equations, but the classical mathematical theory of differential equations is rarely used. We introduce these elements in Chapter 6 by means of the s variable. It is used in the Laplace transformation. If we need frequency domain responses, all we do is replace s by $j\omega$.

The Laplace transformation chapter has a section on multiple poles. Theoretically, it is important, simply because multiple poles can exist. From a practical point of view, networks with multiple poles are always inferior to networks with simple poles. The instructor may decide to skip the section on multiple poles.

Active networks can be analyzed by nodal formulation, as explained in Chapter 12, while Chapter 16 shows the importance of sensitivity analysis. These subjects are for later years of undergraduate studies.

In Chapter 15, the student is exposed for the first time to modeling of actual devices. Only linear models are given for field-effect and bipolar transistors. Nonlinearities are introduced by means of the simplest nonlinear device, the diode. We explain the standard Newton-Raphson iteration and use it to solve a few primitive nonlinear networks. We also added a section on numerical time domain integration, to show that integration with the backward Euler formula leads to the substitution of s by $1/h$, where h is the integration step size.

Modified Nodal Formulation, MNF, a method used in computers, is in Chapter 17. It leads to larger matrices and is not suitable for hand calculations. Most computer programs use it. Some calculators may perform time domain and frequency domain evaluations. The students are, of course, welcome to use them, but we do not require them.

In Chapter 18, we briefly introduce the Fourier series and Fourier transformation. Fourier series is important because it shows that any periodic signal can be decomposed into a sum of harmonic frequencies and in linear networks each frequency can be handled separately.

Fourier integral is presented in a limited version, for signals starting at $t = 0$. This is the case of all practical signals and, with this simplification, Fourrier integral does not differ from the Laplace transformation. Everything we have learned in Chapter 9 can be applied; all we have to do is substitute $j\omega$ for s to obtain spectra of signals.

To summarize, the book prepares the student for later studies by using matrix algebra for all solutions. It explains most subjects of the linear circuit theory and prepares the student for further studies of nonlinear systems.

The author of this book, with a colleague, Kishore Singhal, wrote an advanced book, "Computer Methods for Circuit Analysis and Design," (Van Nostrand Reinhold, Second Edition, 1994). Its title indicates the content. It can be considered as a continuation of this introductory text.

1 BASIC CONCEPTS

INTRODUCTION

This chapter starts with the concept of electric current and voltage, introduces basic terminology and shows how networks are drawn. It gives a brief overview of all linear elements which are used in the network theory, plus a few of the most important practical devices. After this informal explanation we give definitions of the independent current and voltage sources, of resistors, of power delivered or consumed. Three fundamental laws are stated: the Ohm's law and the Kirchhoff's current and voltage laws. Based on this knowledge we study the voltage and current dividers and also explain how to obtain equivalent resistances for combinations of resistors.

1.1. VOLTAGES AND CURRENTS

Network theory deals with voltages, currents, network elements and signals; in this section we will consider in detail the first two, and also introduce some basic concepts and typical signals.

Electric *current* is a flow of electrons and is measured in *amperes*, denoted by the letter *A*. Voltage is a force which causes electrons to flow and is measured in *volts*, denoted by *V*. The concept of current is usually somewhat easier to grasp because we can visualize it as a current of water.

When we talk about the flow of water, we associate it with a certain direction and we know, from daily experience, that the direction is from a higher point to a lower point. We also accept without difficulties that there must be some force which determines the direction of the flow. A similar situation exists in electrical networks and the force that pushes the current through the elements is the voltage. The word *potential* can be used as well.

In the early days of electricity, only chemical batteries were available and the concept of electrons was unknown. One of the connecting points of the battery was arbitrarily marked with a + sign, the other with a − sign and it was assumed that the current flowed into the external network from the + terminal to the − terminal. The current direction is usually marked by an arrow, similarly as in Fig. 1.1.1 where the box denotes some element not yet specified. It is a standard practice to use letters i or I for the current and letters v or V for the voltage.

Since voltage is a force and force must always be referred to some point with respect to which it acts, we must indicate a reference point when talking about voltage. Such point is usually *ground*. In electrical instruments ground is understood to be the metal base of the equipment and not earth, to which the instrument may or may not be connected. We will use the term ground even if the equipment is in a flying aircraft.

The concept of a positive direction of current is fundamental to network theory. If the arrow denoting the current is directed from a point with higher voltage to a point with lower voltage, e.g. from + to –, then we say that the current has *positive* direction. If the arrow points from – to +, a negative sign is associated with the current. Positive direction of current is indicated in Fig. 1.1.1. We will say more about the current direction and its implications later in this chapter.

Each idealized element has its own symbol but for a start we will represent the simplest one by a small box with leads as shown in Fig. 1.1.1. The lines coming out of the box have no special electrical properties; their only purpose is to connect the element to other elements. The small circles at the ends of the lines are called *terminals*. The element has two terminals and is thus called a *two-terminal* element. We will see later that we may have three- and four-terminal elements as well.

Several elements may be connected together to form a network, for instance as shown in Fig. 1.1.2a. The drawing, although showing correctly what we intended to do, is not very pleasing to the eye and is also not simple to draw. In order to avoid difficulties in drawing schematics of networks and also in understanding them, we have a convention that a line which connects elements has no special meaning; it just indicates a connection. Using this rule, we can redraw the same network as shown in Fig. 1.1.2b. This figure is clearly much easier to produce and is also easier to interpret. The full dots indicate that there is a true connection of the lines (we could think that they are soldered together). A point connecting two or more elements is called the *node*. Connection of two lines is indicated in Fig. 1.1.3a.

In more complicated situations we may not be able to draw the lines without some intersections, although there is no electrical connection at the point where they intersect.

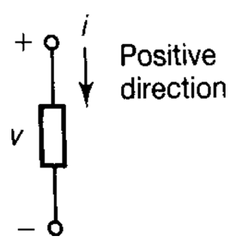

FIGURE 1.1.1 A two-terminal element.

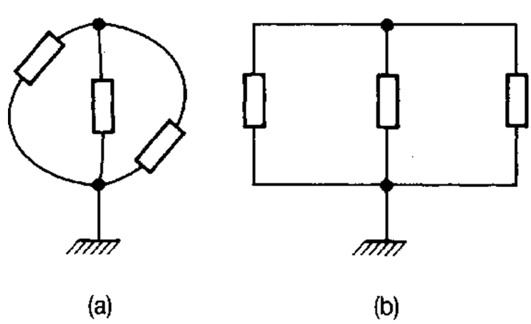

FIGURE 1.1.2 Drawing Networks: (a) Possible way. (b) Accepted method.

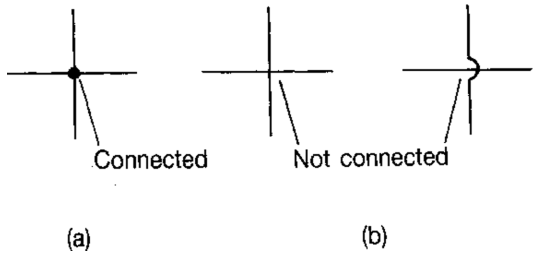

FIGURE 1.1.3 Conventions for connections: (a) Electrically connected. (b) Electrically not connected.

In order to clearly indicate that the lines are *not* connected, there are two conventions: Intersection *without* a full dot means that the lines are not connected and can be thought of as running one above the other. This is shown in Fig. 1.1.3b. Another convention draws a small semicircle, shown in the same figure. True connection of *three or more* wires will *always* be marked by a full dot. We may use any of the above two conventions to indicate that the lines are not connected.

Another common expression says that a certain point is *grounded*. *Ground* is marked by the symbol at the bottom of Fig. 1.1.2. It is a somewhat misleading concept because we do not ground the network in the way the lightning rod is grounded. The symbol only indicates a reference point. Every network must always have one such reference point. In many network drawings, the bottom long line is automatically assumed to represent the reference point.

We still need a few more fundamental concepts. A voltage at a node, measured with respect to ground, is called *nodal voltage*. Two nodes are sketched in Fig. 1.1.4a. It is a common practice to give the node a name, usually a positive integer number, and the voltage may be indicated at the node, as shown in the Fig. 1.1.4a. The two nodes are not connected and we use the expression *open circuit* for such a situation. Rather obviously, no current can flow through an open circuit and arbitrary voltages may exist at the two nodes and thus across the open circuit. The opposite situation would be a *short circuit*, shown in Fig. 1.1.4b. It is represented by a connecting line, the same line as used in Fig. 1.1.2. Two nodes connected by a short circuit have the same voltage (zero voltage difference between the two nodes) and an arbitrary current can flow through a short circuit in any direction. The node numbers may be in circles, as in Fig. 1.1.4b.

The current (or voltage) can have a value which does not change with time, or it can vary with time. We can draw a diagram in which time is measured on the horizontal axis and current is marked on the vertical axis. If the current value does not change with time,

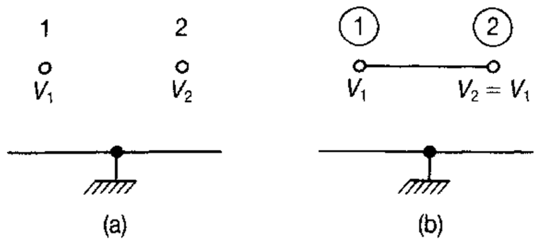

FIGURE 1.1.4 Two nodes: (a) Open circuit, (b) Short circuit.

1.1
1.2
1.3
1.4
1.5
1.6
1.7

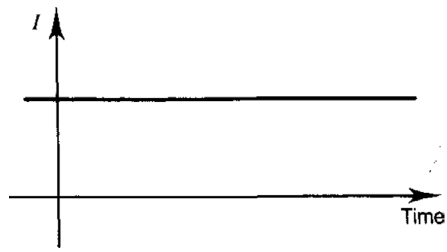

FIGURE 1.1.5 Direct current (dc) as function of time.

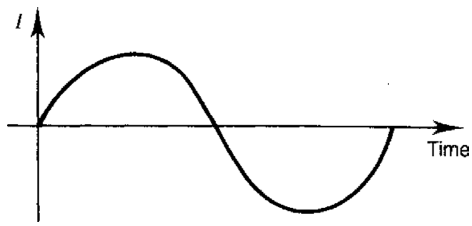

FIGURE 1.1.6 Alternating current (ac) as function of time.

FIGURE 1.1.7 Graph of current in pulses.

such diagram will be a horizontal line, shown in Fig. 1.1.5. We speak about *direct current* and denote it by *dc*. A dc current is supplied, for instance, by a car battery. In other situations we may have currents whose instant values change. Fig. 1.1.6 shows another typical case called *alternating current*, denoted as *ac*. This type of current is supplied by the outlet at your home. It is a periodic function which repeatedly changes its value. An example of still other type of current is in Fig. 1.1.7 where the current flows in short pulses with periods of time where the current does not flow at all. The three examples belong to more typical cases. There are many ways in which current can change with time. Also, whatever we said about currents, will also apply to the voltages.

1.2. NETWORK ELEMENTS

Our first section introduced some of the most important conventions and explained the concepts of currents and voltages. In this section we will give a brief, informal overview of elements which are used in network theory. All the elements will be discussed in detail later.

1.2.1. Independent Voltage and Current Sources

An electric network can function only if a source of a voltage or current is attached to it. Network theory uses two sources: the *independent voltage source*, and the *independent*

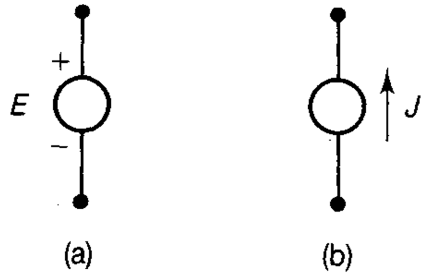

FIGURE 1.2.1 Symbols for independent voltage (a) and current (b) sources.

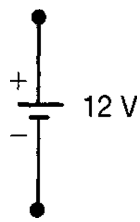

FIGURE 1.2.2 Symbol for battery.

current source. Their symbols are in Fig. 1.2.1. The same symbols are usually used for a *dc*, *ac* or, in fact, any type of source. Sometimes, in order to clearly indicate that there is a special source, another symbol may be used. For instance, it is common to use the symbol in Fig. 1.2.2 for a battery.

1.2.2. Resistor

A resistor is an element produced from some material which "resists" the flow of electrons. The property is called *resistance*. At normal temperatures, all materials exhibit some resistance. Silver and copper have small resistances and copper is commonly used in wires for connections. Some alloys or carbon mixed with other materials have larger resistances. The letter R is used for the resistance; it is measured in Ohms and the symbol is Ω. As an example, if a resistor has the resistance of ten Ohms, we would write next to its symbol $R = 10\ \Omega$, $R = 10$, or just $10\ \Omega$. The symbol for a resistor is shown in Fig. 1.2.3. The voltage across the resistor and the current flowing through the resistor are coupled by the Ohm's law:

$$V = RI,$$

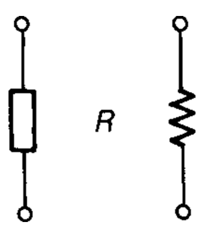

FIGURE 1.2.3 Symbol for a resistor.

1.1 1.2 1.3 1.4 1.5 1.6 1.7

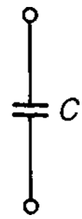

FIGURE 1.2.4 Symbol for a capacitor.

Sometimes we use the inverted value of the resistance,

$$G = \frac{1}{R}.$$

It is called *conductance*, is measured in Siemens and the symbol for the unit is S. We will learn more about resistors later in this chapter.

1.2.3. Capacitor

The symbol for a capacitor is in Fig. 1.2.4 and in schematics is denoted by the letter C. It is an element formed by two layers of conducting material, separated by a material which does not conduct, an insulator. Many materials can serve as insulators and even waxed paper is sometimes used. Electrons can be stored on the capacitor; in such case we say that there is a charge on the capacitor. Charge is usually denoted by the letter Q or q, and is measured in Coulombs, C (same letter as for a capacitor). If the charge is changing with time, a current flows through the capacitor according to

$$i(t) = \frac{dq(t)}{dt}$$

The equation indicates that no current can flow through the capacitor if the charge does not change with time (derivative of a constant is zero). If the capacitor is independent of any external influences, the equation simplifies to

$$i(t) = C\frac{dv(t)}{dt}$$

where C is the symbol for the *capacitance*, the property of the capacitor to store electrons. *Capacitance* is measured in Farads, F. If the capacitor has a capacitance of 2 Farads, we would write next to the symbol either $C = 2F$, or $C = 2$ or just $2F$. The element will be introduced later in this book, with all details postponed until then.

1.2.4. Inductor

The symbol for an inductor is shown in Fig. 1.2.5. The classical form of an inductor is a coil of insulated wire, possibly placed into a core of ferromagnetic material, such as iron.

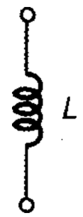

FIGURE 1.2.5 Symbol for an inductor.

A current flowing through the inductor creates a magnetic field, or flux. Flux is usually denoted by the letter Φ and is measured in Webers, *Wb*. There is a relationship between the flux and the voltage across the inductor,

$$v(t) = \frac{d\Phi(t)}{dt}$$

This equation indicates that if the flux is constant (created by a dc current), then there is no voltage across the inductor, because the derivative of a constant is zero. If the inductor is independent of external influences, then the above equation simplifies to

$$v(t) = L\frac{di(t)}{dt}$$

where L is the property of the inductor, called the *inductance*. Inductance is measured in Henries, H. In schematics, the element is denoted by the letter L. For instance, if an inductor has an inductance of 3 Henries we would write next to its symbol $L = 3H$ or $L = 3$ or just $3H$. This element will also be discussed in greater detail later in the book.

1.2.5. Dependent Sources

Unlike the independent sources introduced above, dependent sources deliver voltages or currents whose values depend on a voltage or current somewhere else in the network. This means that the elements have two controlling terminals, where they do the sensing, and two controlled terminals, where they deliver the voltage or current. There are four possible types of dependent sources:

1. *Voltage controlled voltage source*, *VV*, Fig. 1.2.6a. The voltage difference at the terminals on the left, V_1, is multiplied by a coefficient, μ, and delivered by the source on the right:

   $$V_2 = \mu V_1$$

   The coefficient μ is often called the *gain* of the VV.

2. *Voltage controlled current source*, *VC*, Fig. 1.2.6b. The voltage difference at the terminals on the left, V_1, is multiplied by a conversion constant, g, and is delivered as a current at the terminals on the right:

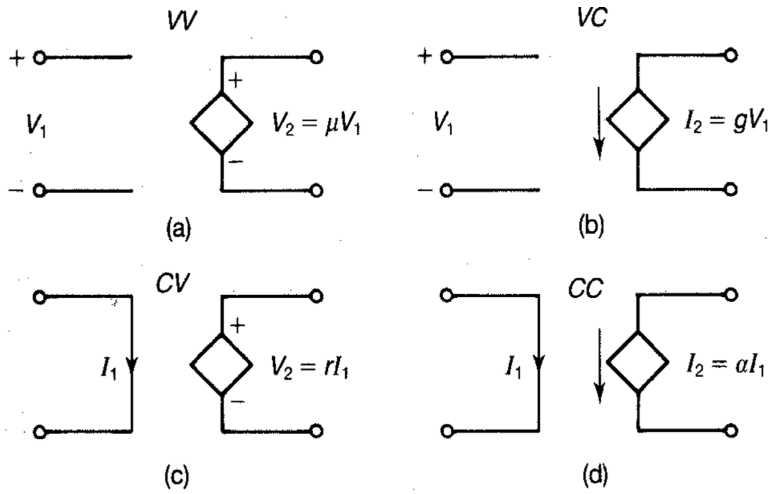

FIGURE 1.2.6 Symbols for dependent sources: (a) Voltage controlled voltage source, VV. (b) Voltage-controlled current source, VC. (c) Current-controlled voltage source, CV. (d) Current-controlled current source, CC.

$$I_2 = gV_1$$

3. *Current controlled voltage source*, *CV*, Fig. 1.2.6c. The current through the short circuit on the left, I_1, is multiplied by a conversion constant, r, and applied as a voltage to the terminals on the right:

$$V_2 = rI_1$$

4. *Current controlled current source*, *CC*, Fig. 1.2.6d. The current flowing through the short circuit on the left, I_1, is multiplied by a constant, α, and applied as a current on the terminals on the right:

$$I_2 = \alpha I_1$$

The elements will be discussed in detail in Chapter 4. They are very important because they are used to model properties of useful devices in real-life situations.

1.2.6. Transformer

If two (or more) coils are near each other, or are placed on one ferromagnetic core, then they form a transformer. There are many forms of transformers, but the most common one has two coils and its symbol is in Fig. 1.2.7. The element will be discussed in detail later.

1.2.7. Operational Amplifier

Operational amplifier, OPAMP, is a complicated electronic device whose simplified properties can be summarized as follows: it acts as a voltage controlled voltage source, with one fundamental difference. In the ideal case, the amplification factor, μ, approaches infinity. The most common symbol for the operational amplifier is in Fig. 1.2.8 and we will use it in our studies of active networks.

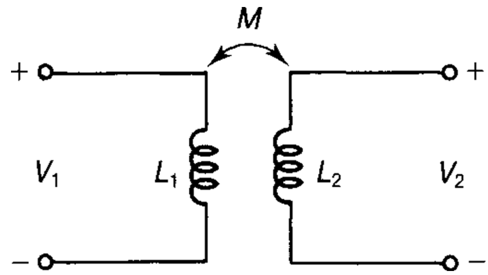

FIGURE 1.2.7 Symbol for a transformer.

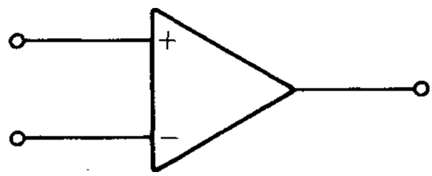

FIGURE 1.2.8 Symbol for an operational amplifier.

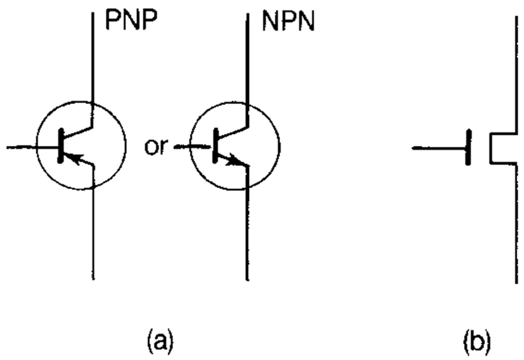

FIGURE 1.2.9 Symbols for transistors: (a) Bipolar transistor, (b) Field-effect transistor, FET.

1.2.8. Other Elements

The above elements are all the elements used in network theory. In various combinations, they are used to describe the properties of actual, physically existing elements. We will not go into details now, but we will mention two more elements, because they are widely used. They are the field effect and bipolar transistors. In Fig. 1.2.9a is the symbol for a bipolar transistor, and in Fig. 1.2.9b is the symbol for a field effect transistor, FET. Their properties are modeled by combinations of dependent sources, resistors, capacitors, and possibly inductors. Transistors are also used in the design of operational amplifiers.

1.2.9. Units and Their Prefixes

To complete this section we summarize the terminology introduced so far. It is in Table 1.2.1. Theoretical units like Ohms for a resistor, or Henries for the inductor, may in some cases be either too large or too small in practical applications. For instance, a unit to measure capacitance is one Farad, $1F$. This happens to be an impractically large unit. To express that a practical value is, for instance, 10^{-12} times smaller, we write the letter *p* before the

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TABLE 1.2.1 Units and symbols.

Name	Notation	Unit Name	Unit Symbol
Resistance	R	Ohm	Ω
Conductance	G	Siemens	S
Capacitance	C	Farad	F
Inductance	L	Henry	H
Current	I,i	Ampere	A
Voltage	V,v	Volt	V
Energy	W,w	Joule	J
Power	P,p	Watt	W
Charge	Q	Coulomb	C
Flux	Φ	Weber	Wb
Time	t	Second	s

TABLE 1.2.2 Prefixes for electrical units.

Prefix	Name	Value
f	femto	10^{-15}
p	Pico	10^{-12}
n	nano	10^{-9}
μ	micro	10^{-6}
m	milli	10^{-3}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}

symbol F and have one picofarad, 1 $pF = 10^{-12}F$. The prefixes for all units are summarized in Table 1.2.2.

1.2.10. Examples of Networks

Since we already know the elements which we will be using in this book, we now give some examples. They will show how we draw the networks and what information we usually write into the schematics.

Figure 1.2.10 shows a network with one independent voltage source, E =5V, two resistors, two capacitors, one inductor and a current controlled current source. If there are several elements of the same type, we either give them different names or use subscripts, like in the figure. Subscripts are not needed here for the inductor, L, and the current controlled current source, CC, but can be given. The controlling current is denoted by I and its direction is indicated by the arrow. It is the current flowing through the resistor R_2. The source part of the CC, connected between ground and node four, has a current gain $\alpha = 3$. We have marked the nodes of the network by numbers in circles; this is not a standard way, but is quite convenient. We have also written nodal voltages to these nodes. The ground node, or reference node, is at the bottom of the picture. Note especially the short circuit

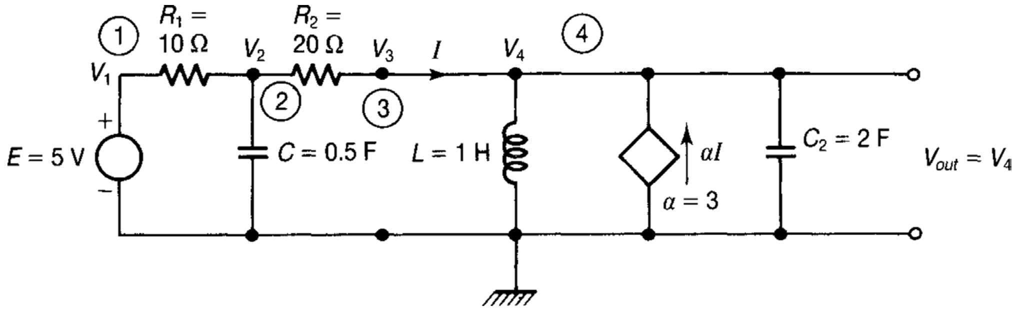

FIGURE 1.2.10 First example of a network.

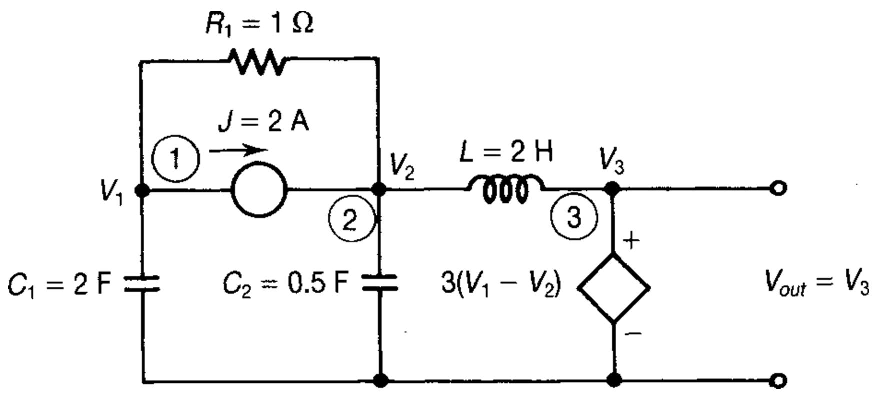

FIGURE 1.2.11 Second example of a network.

between nodes 3 and 4. It is the sensing part of the current controlled current source and is indicated separately. Nodes 3 and 4 are connected by a short circuit.

The voltage source is the *input* of the network. Also marked, on the right side, is the *output*, in this case a voltage, V_{out}. We will use this notation quite often. By output we mean that the variable (voltage or current) will be measured (or calculated) because we need to know it.

Another network is in Fig. 1.2.11. It has a floating current source. The word *floating* refers to elements which are not connected to ground. In this case the source current is $J = 2\ A$. The network has two capacitors, an inductor, a resistor and a voltage controlled voltage source. The controlling voltage is taken between nodes 1 and 2 and the gain of the dependent source is $\mu = 3$. The output voltage is $V_{out} = V_3$. The nodes as well as the nodal voltages are written into the figure. In some schematics the symbol for ground is omitted and the bottom line is understood to be the reference node. We have done so in this example.

In general, the output can be any nodal voltage, a difference of two nodal voltages, for instance $V_{out} = V_2 - V_3$, or the current through any of the elements. A network may have several outputs.

1.3. INDEPENDENT VOLTAGE AND CURRENT SOURCES

The first set of idealized elements which we discuss in detail are the independent voltage and current sources. An ideal *independent voltage source* is an element for which we will use the symbol shown in Fig. 1.3.1a. The word *independent* means that the properties of the source will not change due to any external influence. A voltage source always has the symbol + at one of its terminals and the symbol − at the other terminal. Because of the rule we introduced in section 1.1, positive direction of the current flowing through the voltage source will be from the + terminal to the − terminal. It has to be understood that our definition does not mean that the current will actually flow that way; more about this will be said when we have covered the concept of resistors and power.

We will *always* denote an independent voltage source by the letter E. The use of this letter may differ from other books, but there is a strong advantage to give the independent source a special letter. When we get later to the point that we will write the equations, we will see that the independent source always appears on the other side of the equation than the network variables, voltages and currents. By assigning special letters we eliminate many possible mistakes.

The voltage of an independent voltage source is given, but we can say nothing about the current flowing through it. An ideal voltage source can supply *any* amount of current and the current can flow in *any* direction, depending on the network to which it is connected. Since the voltage delivered by the independent voltage source is fixed and any amount of current can flow through it in any direction, we can draw a graph (sometimes also called the characteristic) describing these properties. If we measure current on the horizontal axis and the voltage on the vertical axis, then the characteristic is a horizontal line, shown in Fig. 1.3.1b. If the voltage available from the source changes, for instance from 2 to 4 volts, the horizontal line will shift upwards. Should we apply a voltage − 2 V without changing the positions of the + and − signs, the horizontal line would shift below the horizontal axis. One very important case is what happens if we have a voltage source which is delivering zero voltage. In such case the horizontal line in Fig. 1.3.1 drops down on the horizontal axis. Such element will have zero voltage across it but a current of arbitrary magnitude can flow through it in any direction. If we recall the definition of the short circuit from our previous discussion, then we see that *a voltage source with zero voltage is equivalent to a short circuit.*

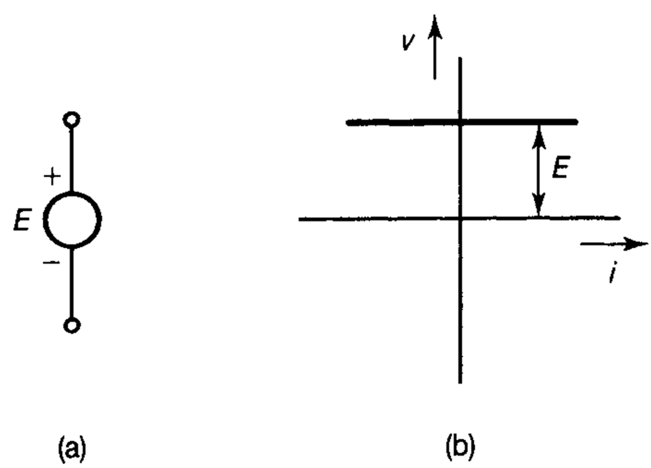

FIGURE 1.3.1 Independent voltage source: (a) Symbol. (b) Its characteristic.

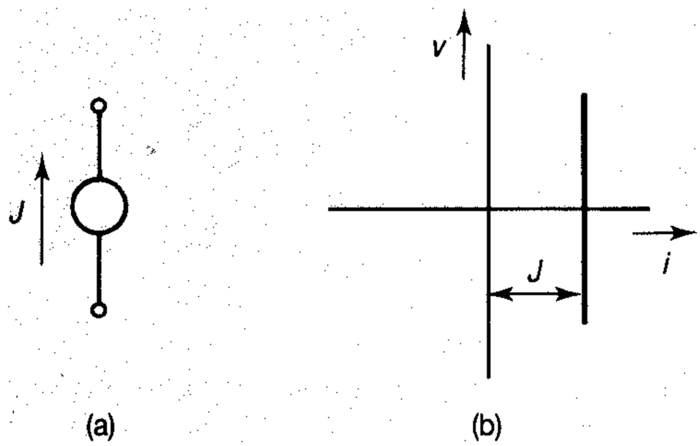

FIGURE 1.3.2 Independent current source: (a) Symbol. (b) Its characteristic.

An ideal *independent current source* is an element for which we will use the symbol shown in Fig. 1.3.2a. Similarly as in the above case, there exists no possibility of changing the amount of the current delivered by some external influence. The direction of the current is indicated by an arrow. We will *always* denote an independent current source by the letter *J*, for similar reasons as discussed above for the independent voltage source. The current of an independent current source is given, but we can say nothing about the voltages at its terminals, because they depend on the network to which the source is connected. Since the current delivered by the independent current source is fixed, but any voltage difference can be measured between its terminals, we can draw a graph (characteristic) describing these properties. It is shown in Fig. 1.3.2b. If the amount of current delivered by the source changes from, say, $2A$ to $4A$, the straight line will shift to the right. Should we apply, say, $-2\ A$ without changing the direction of the arrow in the figure, the vertical line will shift to the left of the vertical axis. If we consider a current source which delivers zero current, then the vertical line will coincide with the vertical axis. In such case no current can flow through the element but arbitrary voltage can appear across its terminals; we conclude that *a current source with zero current is equivalent to an open circuit.*

A battery is an example of a nonideal *dc* source, because the voltage at the terminals depends on the amount of current we draw from it. Nevertheless, in many cases we take it as an ideal source and denote it by the letter *E*. The outlet at your home is another source of voltage, again non-ideal, but it delivers an alternating current, *ac*. This voltage changes with time but we will still use the symbol *E*.

Independent sources cannot be connected together arbitrarily and we now discuss the acceptable and the prohibited cases. If we connect two voltage sources *in series*, as shown in Fig. 1.3.3a, then the voltage between the external terminals will be $E_1 + E_2$. Should one of them have reversed direction, as shown in Fig. 1.3.3b, then the voltage between the external terminals will be $E_1 - E_2$. We can *never* connect two *different* voltage sources *in parallel*, something like in Fig. 1.3.4, because it is not possible to have simultaneously two different voltages between two nodes. There is, of course, the possibility of connecting in parallel two voltage sources with *exactly the same voltage* and this is sometimes used in theoretical considerations, as we will learn later. From a practical point of view, this is a meaningless situation, since an ideal voltage source can deliver any current and can thus do the same job as two or more such sources in parallel.

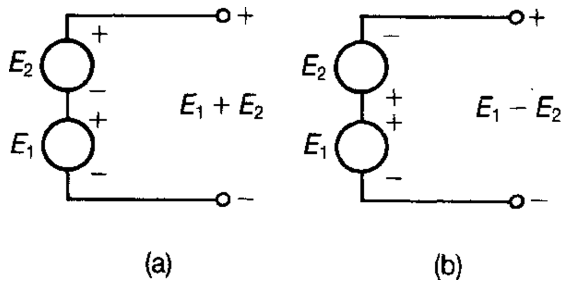

FIGURE 1.3.3 Permitted connections of voltage sources: (a) The voltages add. (b) The voltages subtract.

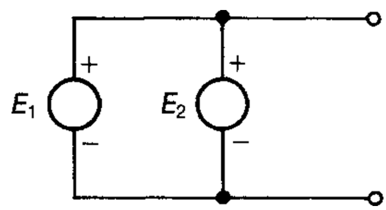

FIGURE 1.3.4 Prohibited connections for $E_1 \neq E_2$.

external terminals will be $E_1 - E_2$. We can *never* connect two *different* voltage sources *in parallel*, something like in Fig. 1.3.4, because it is not possible to have simultaneously two different voltages between two nodes. There is, of course, the possibility of connecting in parallel two voltage sources with *exactly the same voltage* and this is sometimes used in theoretical considerations, as we will learn later. From a practical point of view, this is a meaningless situation, since an ideal voltage source can deliver any current and can thus do the same job as two or more such sources in parallel.

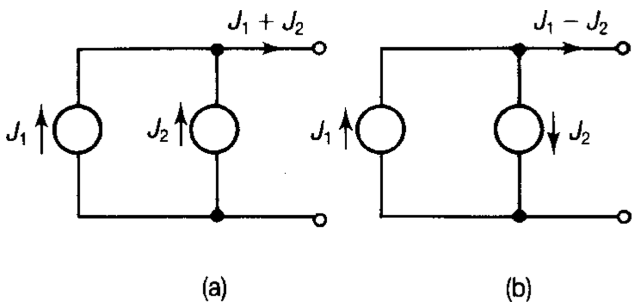

FIGURE 1.3.5 Permitted connections of current sources: (a) The currents add; (b) The currents subtract.

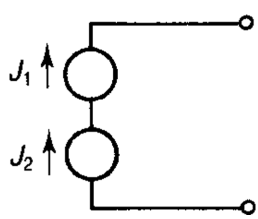

FIGURE 1.3.6 Prohibited connection for $J_1 \neq J_2$.

this is sometimes used in theoretical considerations, but is meaningless practically, because one ideal source can do exactly the same job as two equal sources in series.

Connecting one voltage and one current source either in series or in parallel is not prohibited theoretically, but is not useful practically. Finally, we should mention that in a network schematic a voltage source should never be shortcircuited. By our definition, it should still maintain the same voltage across its terminals, but the current flowing into a short circuit would be infinitely large. Similarly, a current source should never be left without a connection to some circuit which can return the current back to the other terminal of the source.

Example 1.

Consider the combination of voltage sources in Fig. 1.3.7a. What is the voltage we obtain at the terminals of the network? The voltage sources E_1 and E_3 have one orientation, the other sources act in opposite direction. The resulting voltage is

$$E = E_1 - E_2 + E_3 - E_4 - E_5 = -7$$

as shown in Fig. 1.3.7b. The same is achieved by reversing the + and – signs, as shown on the right of Fig. 1.3.7c, and making the voltage positive $7V$.

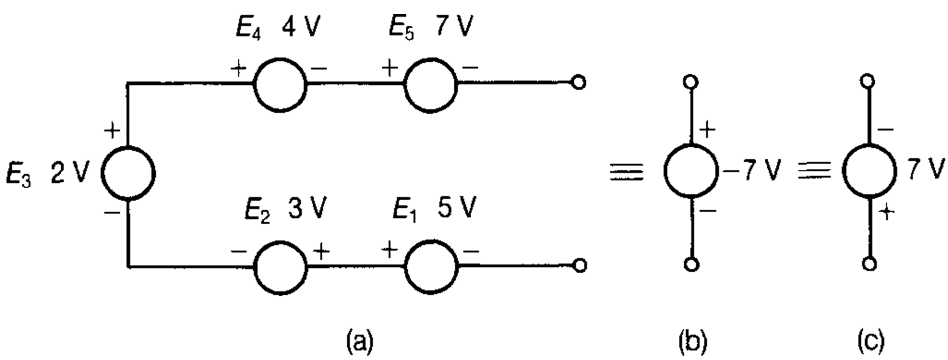

FIGURE 1.3.7 Equivalent voltage of five voltage sources: (a) Original connection. (b) and (c) Two equivalent results.

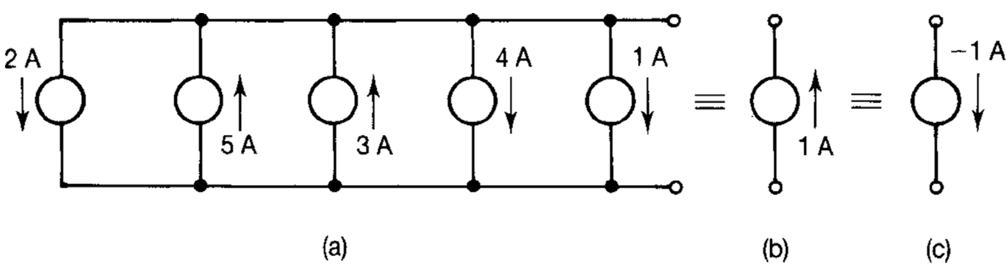

FIGURE 1.3.8 Equivalent current of five current sources: (a) Original connections. (b) and (c) Two equivalent results.

Example 2.

Consider the parallel connection of 5 current sources, Fig. 1.3.8a. The sum of currents flowing upwards is 8, the sum flowing downwards is 7. Thus the equivalent current source will be 1 *A* upwards, as shown n Fig. 1.3.8b. Equivalently, as shown in Fig. 1.3.8c, this could also be drawn as a negative current of 1 *A* pointing downwards.

1.4. RESISTORS AND OHM'S LAW

In this section we consider the simplest of all elements, the resistor, and the way it relates the voltage across it to the current flowing through it.

The electrical property of the resistor, called the *resistance*, can be best understood by considering a small experiment, at least in our mind, if not in reality. Take several materials (silver, copper, iron, wood, china, glass etc.), make thin rods with exactly equal dimensions, connect them to the same source of voltage and measure the current flowing through them. We will discover that a large current will flow through the rod made of silver, less current through copper, still less through iron, very little through wood and next to nothing through china or glass. The properties of the materials are such that they offer different resistances to the flow of current. More detailed measurements would lead us to the conclusion that the voltage, the current, and the properties of the material are related by Ohm's law which states

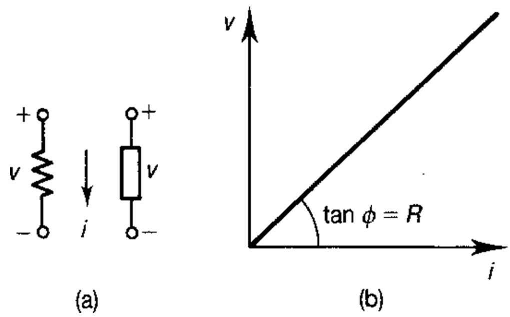

FIGURE 1.4.1 Linear resistor: (a) symbol. (b) Its characteristic.

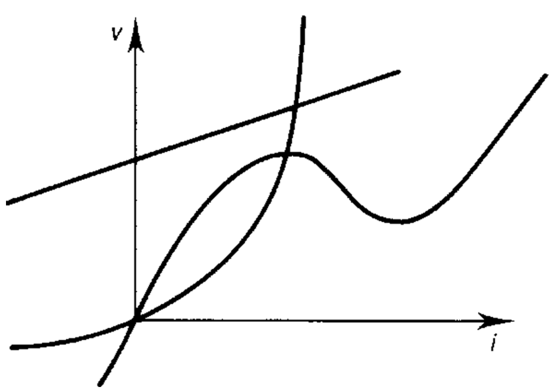

FIGURE 1.4.2 Characteristics of nonlinear resistors.

$$V = IR \tag{1.4.1}$$

Here V is the voltage across the element, measured in *Volts*, V, I is the current flowing through the element, measured in *Amperes*, A, and R is the resistance, measured in *Ohms*, Ω. A resistor will have a resistance of 1 Ω if the voltage across it is 1 V and the current flowing through it is 1 A. For the resistor we will use the symbol shown in Fig. 1.4.1a.

If we draw a graph with perpendicular axes, measure the current on the horizontal axis and the voltage on the vertical axis, then Eq. (1.4.1) represents a straight line passing through the origin. We call it the *characteristic* of the resistor, see the graph in Fig. 1.4.1b. The tangent of the angle, ϕ, is the value of the resistance. The equation describes a *linear* resistor, a resistor whose value is independent of any external influences. In true life, nothing is as simple as that. The resistor will change its value with time or temperature or with the current which flows through it. Fig. 1.4.2 shows several cases of nonlinear resistors. Note that the straight line which *does not* pass through the origin would also represent a nonlinear resistor.

Since the resistance of a linear resistor is a constant and Eq. (1.4.1) represents a straight line, we can invert the equation and write

$$I = \frac{1}{R}V = GV \tag{1.4.2}$$

The inverted value

$$G = \frac{1}{R} \tag{1.4.3}$$

is called the *conductance* and is measured in *Siemens*, S.

Should the resistor change with time, t, we could denote it by the symbol $R(t)$. Its resistance can also depend on the voltage across it. In such case the resistor depends on v and we could express the dependence by writing $R(v)$. Whatever the situation may be, the Ohm's law is always valid. In practical life we talk mostly about resistors and their values in ohms (Ω), kiloohms ($1k\,\Omega = 10^3\Omega$) or megaohms ($1M\Omega = 10^6\Omega$). The notation is standardized and is summarized in Tables 1.2.1 and 1.2.2.

Example 1.

A voltage source delivering 100 V is connected to the resistor having the value $R = 10\ k\Omega$. What is the current through the resistor? The answer is $I = \dfrac{10^2}{10^4} = 10^{-2}\ A = 10 \times 10^{-3}\ A =$ 10 mA.

Example 2.

Let the resistor be described by the relationship $V = I^3$. If the current through through it is 0.5 A, what is the voltage across it? Answer: is $V = 0.5^3 = 0.125\ V$.

1.5. POWER AND ENERGY

Power and energy are the next two fundamental concepts to be covered in this chapter. Energy, usually denoted by $w(t)$ or W, is measured in joules, J; it is the amount of work which a source of energy can deliver. In network theory, energy is delivered by the voltage or current sources and is consumed by resistors. The notation $w(t)$ indicates that the energy is changing with time while W indicates that it is constant.

Closely related to energy is power, denoted by $p(t)$ or P and measured in watts, W. The two definitions are related by

$$\begin{aligned} p(t) &= \frac{dw(t)}{dt} \\ w(t) &= \int_{t_1}^{t_2} p(t) \end{aligned} \tag{1.5.1}$$

The relation of their units is $1W = 1J/s$. In electrical terms, power is expressed by the product of current and voltage,

$$p(t) = v(t)i(t) \tag{1.5.2}$$

and the expression is valid at any instant of time. Substituting from the Ohm's law we can also write

$$p(t) = \frac{v^2(t)}{R} = i^2(t)R \tag{1.5.3}$$

When dealing with direct current, then

$$P = VI = \frac{V^2}{R} = I^2 R \tag{1.5.4}$$

or replacing $R = 1/G$,

$$P = VI = V^2 G = \frac{I^2}{G} \tag{1.5.5}$$

In applications, we must distinguish between power which is delivered by the source and power consumed by the network or element. A resistor *always* consumes power and the current flows through it from + to −. If the network has only one voltage source, then the source delivers power into the network and the current will flow from the + terminal into the network and return through the network to the − terminal. If the network has a single current source, then the terminal at the arrow will be the positive one and the other will be negative.

Example 1.

Consider the network in Fig. 1.5.1 and calculate the currents and powers delivered and consumed in the network.

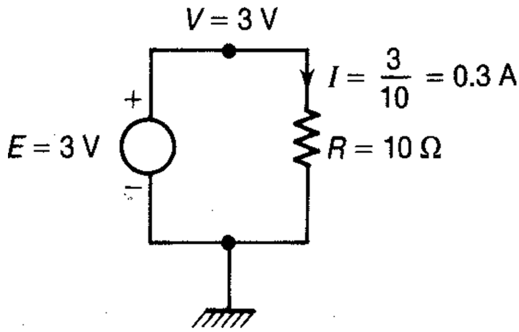

FIGURE 1.5.1 Power from a voltage source.

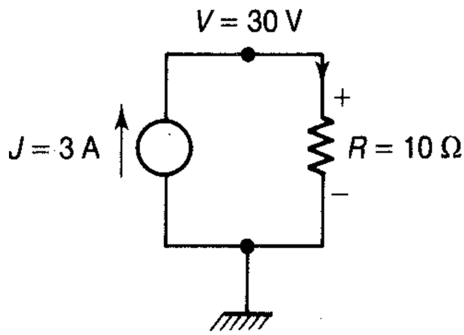

FIGURE 1.5.2 Power from a current source.

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Applying the Ohm's law we get the current $I = \dfrac{3}{10} = 0.3\ A$. It will flow in the indicated direction. The power *consumed* by the resistor will be $P = VI = 0.3 \times 3\ W$ and the same amount of power will be *delivered* by the source.

Example 2.

In the network Fig. 1.5.2 the current source delivers 3 A. Calculate the voltages and powers delivered and consumed.

The voltage across the resistor is $V = 3 \times 10 = 30\ V$. The power consumed by the resistor (and converted into heat) is $P = V \times I = 3 \times 30 = 90\ W$. The same power is delivered by the current source.

The situation becomes more complicated if there are more than one independent source. In a general case, the question how the powers are distributed can be answered only by setting up the appropriate equations and by solving them. We will learn how to do that later. Here we will take a simple network where the answer can be found by the theory covered until now.

Example 3.

Consider the network in Fig. 1.5.3 and calculate the current and powers delivered or consumed by the elements.

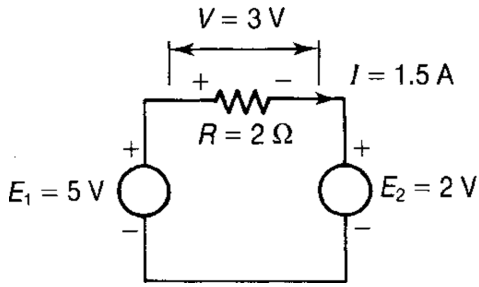

FIGURE 1.5.3 Network with two voltage sources.

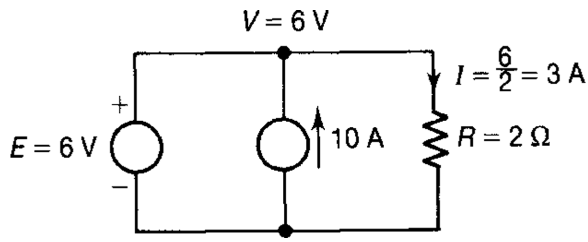

FIGURE 1.5.4 Network with current and voltage source.

The voltage across the resistor is $V = 5 - 2 = 3\ V$ and the current in the network is $I = \frac{3}{2} = 1.5\ A$. The power consumed by the resistor is $P_R = I\ V = 3 \times 1.5 = 4.5\ W$. The source on the left is delivering the power $P = 5 \times 1.5 = 7.5\ W$. On the right, the current flows through the source in the direction from + to − and thus this source *consumes* power. The amount is $P = 2 \times 1.5 = 3\ W$. The power delivered by the left source is exactly equal to the power consumed by the resistor and the other source.

Example 4.

A parallel connection of a voltage and current source is in Fig. 1.5.4. Calculate the powers delivered and consumed by the respective elements.

Due to the voltage source, the voltage across the resistor *must* be 6 V and the current through the resistor must be, due to the Ohm's law, $I_R = \frac{6}{2} = 3\ A$. The current source is supplying 10 A. Three amperes will be forced through the resistor, the rest must flow through the voltage source, from + to −. The powers are distributed as follows: The current source delivers $P = 6 \times 10 = 60\ W$. Of this the resistor consumes $P = 3 \times 6 = 18\ W$ while the voltage source consumes $P = 6 \times 7 = 42\ W$. As always, the power delivered is equal to the power consumed.

1.6. KIRCHHOFF'S LAWS

The laws discovered by Kirchhoff are fundamental to the theory of electricity. They are valid in *any* situation, even if the elements are nonlinear or time varying.

1.6.1. Kirchhoff Current Law

The Kirchhoff current law (*KCL*) states that *the sum of currents flowing away from a node is zero*. The definition remains valid if we replace the words *away from* by the word *to*. A *node* is a point where we connect several elements and one such case is shown in Fig. 1.6.1a. Since the node is only a connection of wires, it is easily understood that it cannot store electrons and thus any amount of electrons flowing into the node through some of the connecting lines must also leave through some other lines. In the statement of the law we used the word *away* because in this book we will consider a current flowing *away* from a node as having *positive* direction.

In Fig. 1.6.1a we have five elements connected to the central node. *At this node*, the currents i_1, i_2 and i_3 have positive directions while currents i_4 and i_5 have negative directions. The Kirchhoff current law can be written as

$$i_1 + i_2 + i_3 - i_4 - i_5 = 0 \tag{1.6.1}$$

We drew the directions of the currents in the figure arbitrarily. There is, however, one situation where we do not have this freedom of choice, namely when there are independent current sources connected to the node. Fig. 1.6.1b shows a node with two independent current sources and three elements which are not sources. We arbitrarily select directions of currents for the resistors. Following the *KCL* it must be true that

$$i_1 - i_2 + i_3 + J_1 - J_2 = 0 \tag{1.6.2}$$

In mathematics, it is common to write unknown variables on the left side and known quantities on the right side of the equation. Because the currents and directions of independent current sources are known, we can follow the usual mathematical strategy and rewrite the above equation in the form

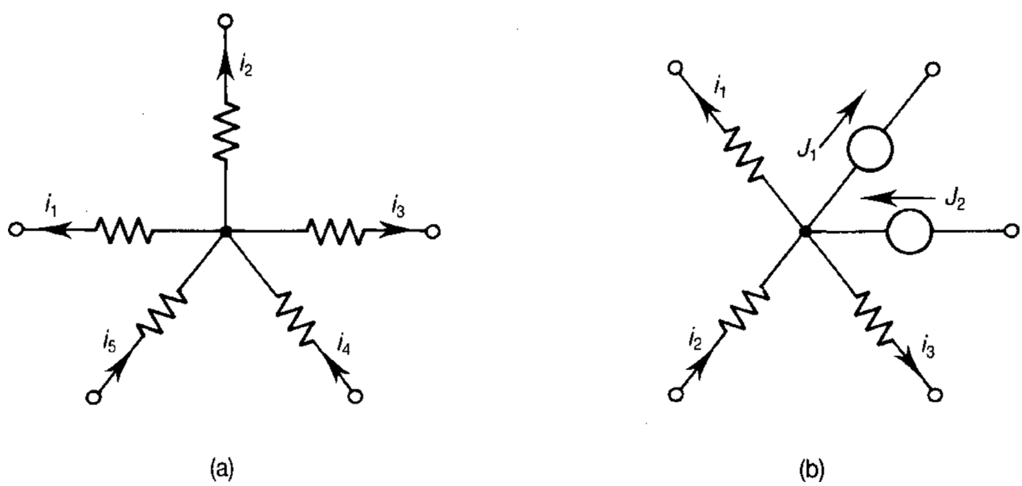

FIGURE 1.6.1 A node with five elements: (a) All current directions are arbitrary. (b) Directions of current sources given, rest arbitrary.

$$i_1 - i_2 + i_3 = -J_1 + J_2 \tag{1.6.3}$$

1.1
1.2
1.3
1.4
1.5
1.6
1.7

We now see the reason why we used different letters to describe properties of independent sources: without much thinking what is dependent and what is independent, we know immediately that the letter J must go to the right side of the equation *with a change in sign*.

1.6.2. Kirchhoff Voltage Law

The Kirchhoff voltage law (*KVL*) states that **the sum of voltages around a loop is zero**. A loop is such a connection of elements that we can walk around it and reach the same point from which we started. An example of a loop with five elements is shown in Fig. 1.6.2a. We know nothing about the voltages and currents in this loop, because they will be influenced by the connections to other parts of the network (indicated in the figure by the openended lines out of the loop). We can assign, completely arbitrarily, the signs + and – to the elements, as was done in the figure. However, once we have done that, there is nothing arbitrary any more, except for the direction of our walk around the loop. Let us choose the one indicated in the figure by the arrow. If this direction goes through the element from + to –, then we add the voltage of this element. If the direction of our walk is such that we go through the element from – to +, we subtract the voltage. For the network in Fig. 1.6.2a this results in the equation

$$-v_1 + v_2 - v_3 + v_4 - v_5 = 0 \tag{1.6.4}$$

We have the freedom to assign the directions of voltages to all elements except the independent voltage sources where the + and – signs are given.

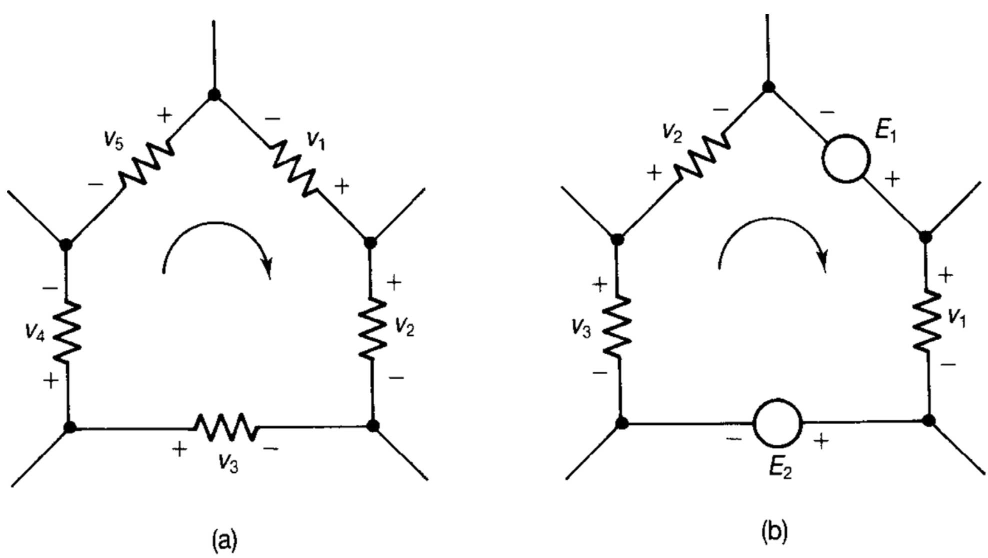

FIGURE 1.6.2 A loop with five elements: (a) Individual voltages arbitrary. (b) Voltages of voltage sources given, rest arbitrary.

The network in Fig. 1.6.2b has two voltage sources with their + and − specified. For the remaining elements we choose arbitrary positions of + and −, for instance as shown. Once we have done this, the only arbitrary thing is the direction of the walk around the loop. If we decide to walk as shown, the equation must be written as

$$-E_1 + v_1 + E_2 - v_3 + v_2 = 0 \tag{1.6.5}$$

Since the independent sources are known, we follow the usual mathematical practice and write their voltages on the right side of the equation. The sum of voltages around the loop can be rewritten as

$$v_1 + v_2 - v_3 = +E_1 - E_2 \tag{1.6.6}$$

Here we take advantage of our rule that independent voltage sources are marked by the letter E. Since we use different letters for the unknowns, v, and for the knowns, E, we do not need much thinking to determine what should go to the right side of the equation *with opposite sign* and what should stay on the left side.

Example 1.

The sketch in Fig. 1.6.3 represents part of some larger network. The nodal voltage at the center is V, the other have subscripts 1 through 4. Current directions are also marked. Applying the KCL we can write for the node in the center

$$-I_1 + I_2 - I_3 + I_4 = 0 \tag{1.6.7}$$

From these directions of the currents we conclude that $V_1 > V$, $V_2 < V$, $V_3 > V$ and $V_4 < V$. Express the currents in terms of the voltages across the conductances:

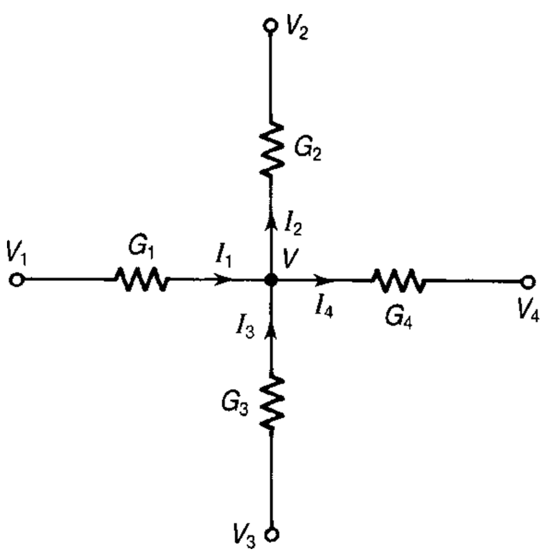

FIGURE 1.6.3 A node with specified current directions.

$$I_1 = (V_1 - V)G_1$$
$$I_2 = (V - V_2)G_2 \qquad (1.6.8)$$
$$I_3 = (V_3 - V)G_3$$
$$I_4 = (V - V_4)G_4$$

These equations can be inserted into (1.6.7) to get

$$-(V_1-V)G_1+(V-V_2)G_2-(V_3-V)G_3(V-V_4)G_4=0$$

We can open the brackets and rewrite in the form

$$V\,(G_1+G_2+G_3+G_4)\;-V_1G_1-V_2G_2-V_3G_3-V_4G_4=0 \qquad (1.6.9)$$

Example 2.

In this example we take the same network, but assume that the voltage V is larger then all the other indicated voltages. Due to this assumptions, the directions of the currents must change. They will be as indicated in Fig. 1.6.4. KCL for the central node provides

$$I_1 + I_2 + I_3 + I_4=0 \qquad (1.6.10)$$

The currents through the branches are

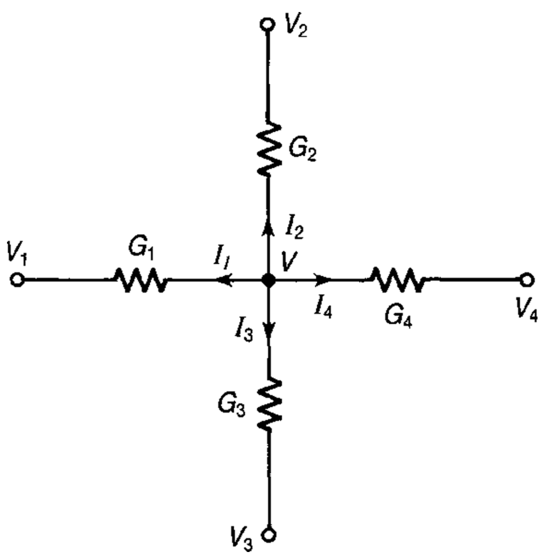

FIGURE 1.6.4 A node with other current directions.

$$I_1 = (V - V_1)G_1$$

$$I_2 = (V - V_2)G_2 \qquad (1.6.11)$$

$$I_3 = (V - V_3)G_3$$

$$I_4 = (V - V_4)G_4$$

Inserting these currents into (1.6.10) results in

$$(V - V_1)G_1 + (V - V_2)G_2 + (V - V_3)G_3 + (V - V_4)G_4 = 0$$

Rearranging terms we get

$$V(G_1 + G_2 + G_3 + G_4) - V_1G_1 - V_2G_2 - V_3G_3 - V_4G_4 = 0 \qquad (1.6.12)$$

The result is interesting, in both cases we get *exactly* the same equation. This is an important result, because it clearly does not matter which voltage we think is higher and which is lower. All we have to do is correctly relate the currents in terms of the voltages and conductances. We will make use of this fact in the next chapter.

1.7. CONNECTIONS OF RESISTORS

The first application of Ohm's and Kirchhoff's laws comes in the study of series and parallel connections of resistors. The simplest case is one loop of resistors, for instance as shown in Fig. 1.7.1. The arrow indicates the direction of our walk around the loop and in this case also indicates the direction of the current, I, which will flow through the combination. Since the same current flows through all resistors, the + and − signs of the voltages across the individual resistors are as shown. Let the source be a *dc* source, $E = 22\ V$. From the Kirchhoff 's voltage law we have

$$V_1 + V_2 + V_3 + V_4 - E = 0,$$

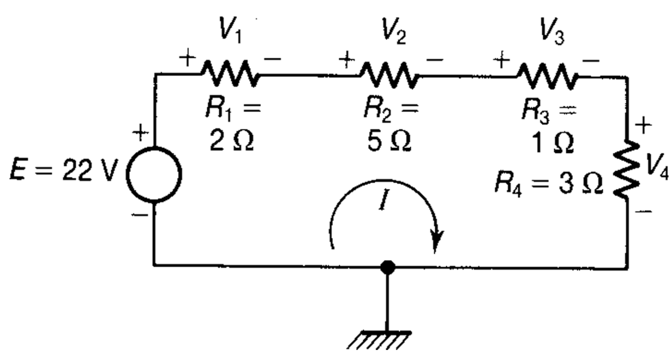

FIGURE 1.7.1 Voltage divider with four resistors.

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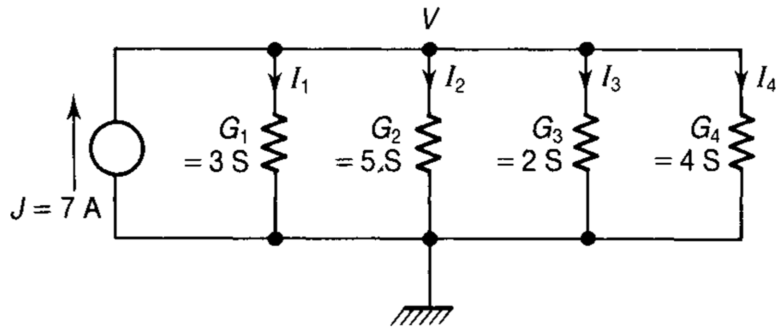


FIGURE 1.7.2 Current divider with four branches.

the minus sign before E coming from the fact that the circulating current enters the voltage source at its minus sign and thus has negative direction according to our rules. The voltage of the source is known and we transfer it on the other side of the equation:

$$V_1 + V_2 + V_3 + V_4 = E \tag{1.7.1}$$

Each of the resistors follows the Ohm's law:

$$\begin{aligned} V_1 &= IR_1 \\ V_2 &= IR_2 \\ V_3 &= IR_3 \\ V_4 &= IR_4 \end{aligned} \tag{1.7.2}$$

Substituting into Eq. (1.7.1)

$$IR_1 + IR_2 + IR_3 + IR_4 = E$$

or

$$I\,(R_1 + R_2 + R_3 + R_4) = E \tag{1.7.3}$$

We can introduce the notation

$$R_{tot} = R_1 + R_2 + R_3 + R_4 \tag{1.7.4}$$

and obtain the overall resistance of the combination, in our case $R_{tot} = 11\Omega$. The voltage of the source is $22V$ and the equation is solved for the current

$$I = \frac{22}{11} = 2A.$$

Returning to (1.7.2), we can now obtain the voltage across each resistor: $V_1 = 4\ V$, $V_2 = 10$, $V_3 = 2\ V$ and $V_4 = 6\ V$. The sum of these individual voltages must be equal to the voltage of the voltage source.

Since the voltage of the source was divided into several partial voltages, the network in Fig. 1.7.1 is also called a *voltage divider*. We say that the resistors are connected in *series*. We will also remember that the overall resistance of the resistors connected in series is equal to the sum of the individual resistances. In other words, we do not have to go through all the steps derived above. All we have to do is add the resistances of resistors connected in series.

A different situation is shown in Fig. 1.7.2 where we have one current source and several resistors connected *in parallel*. We would like to know the voltage common to all these resistors and the currents flowing through them. It is obvious that the same voltage, V, appears across all the resistors and the sum of the currents flowing through them must be equal to the current delivered by the current source. The Kirchhoff current law states that the sum of currents at the upper node must be equal to zero:

$$I_1 + I_2 + I_3 + I_4 - J = 0, \tag{1.7.5}$$

where the negative sign for the current source comes from the fact that its current flows *into* the node. As always, known values are transferred to the right hand side of the equation with a change of sign:

$$I_1 + I_2 + I_3 + I_4 = J \tag{1.7.6}$$

The current through each resistor can be expressed in terms of its conductance:

$$\begin{aligned} I_1 &= VG_1 \\ I_2 &= VG_2 \\ I_3 &= VG_3 \\ I_4 &= VG_4 \end{aligned} \tag{1.7.7}$$

Inserting these values into (1.7.5)

$$VG_1 + VG_2 + VG_3 + VG_4 = J$$

or

$$V(G_1 + G_2 + G_3 + G_4) = J. \tag{1.7.8}$$

The total conductance is

$$G_{tot} = G_1 + G_2 + G_3 + G_4$$

1.1 1.2 1.3 1.4 1.5 1.6 1.7

and the equation simplifies into

$$V = \frac{J}{G_{tot}} = \frac{7}{14} = 0.5$$

Going back to the individual equations (1.7.6), we calculate the currents through all resistors as $I_1 = 1.5\ A$, $I_2 = 2.5\ A$, $I_3 = 1\ A$ and $I_4 = 2\ A$. The sum is equal $7\ A$. The current of the source was divided into individual currents of the resistors and for this reason the network is also called the *current divider*.

The steps explained above can be summarized into two simple rules:

(a) If all resistors are in series, add their resistances to obtain the overall resistance.
(b) If all resistors are in parallel, add their conductances to obtain the overall conductance.

The rules help us to find the overall resistance of more complicated networks without the need to calculate the currents. We give several examples with increasing complexity.

Example 1.

In the network Fig. 1.7.3 find the current and power delivered into the network by the source. On the right, the two resistors are in parallel and thus their conductances are added to get $G = \frac{3}{4}$ or $R = \frac{4}{3}$. This is redrawn in the middle of the figure. Now we have two resistors in series and we can add their resistances to obtain the overall resistance $R_{tot} = 1 + \frac{4}{3} = \frac{7}{3}$. The network is again redrawn on the right. We now have a source $E = 14\ V$, a resistance $R_{tot} = \frac{7}{3}$ and thus the current flowing from the source is $I = E/R = 6\ A$. The power delivered into the combination is $P = E \times I = 14 \times 6 = 84\ W$.

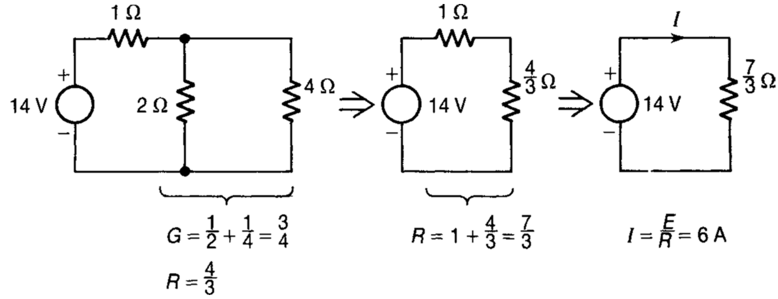


FIGURE 1.7.3 Calculating equivalent resistance.

Example 2.

Find the voltage across the current source in Fig. 1.7.4. Also find the power delivered by the source.

In the solution we first add the resistances of the two resistors on the right, $R = 3 + 9 = 12\Omega$. This is redrawn in the middle. Now we have two resistors in parallel and we must add their conductances, $G = \frac{1}{2} + \frac{1}{12} = \frac{7}{12}$. The network can be redrawn again, shown on the right, where we have a resistance $R = \frac{12}{7}$. The volt age across the source (and also across the equivalent resistor) is $V = R \times J = 12\ V$ and the power delivered into the network is $P = J \times V = 7 \times 12 = 84\ W$.

Example 3.

Find the equivalent resistance of the combination in Fig. 1.7.5.

Starting from the right we first add $R_3 + R_4 = R' = 3\Omega$. We now have two resistors in parallel and we must add their conductances. This is done in the middle of the figure. Their resistance is $R'' = \frac{6}{5}$. Finally, the resistance of R_1 is added to R'' to obtain the total resistance $R_{tot} = \frac{11}{5}\Omega$.

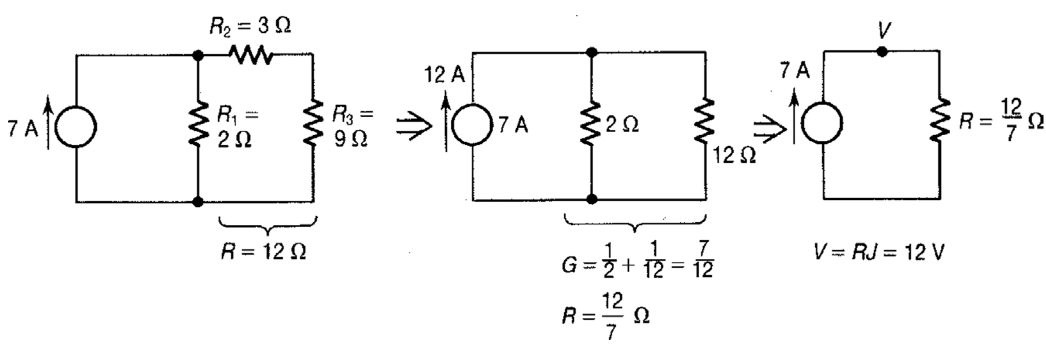


FIGURE 1.7.4 Calculating equivalent resistance.

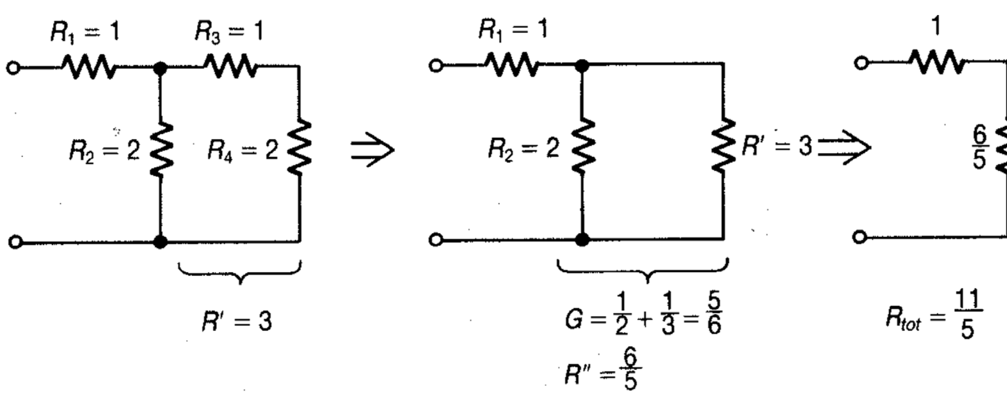


FIGURE 1.7.5 Calculating equivalent resistance.

Example 4.

Find the equivalent resistance of the combination in Fig. 1.7.6.

The steps are all indicated in the figure, with the result $R_{tot} = \dfrac{292}{141}\Omega.$

Example 5.

We will use the voltage divider to introduce the concept of an internal resistance of a *non-ideal* voltage source. In Fig. 1.7.7 is one ideal voltage source and two resistors. This is a simple voltage divider but consider the voltage source with the resistor R_1 as a model of a nonideal source, for instance a battery. The current flowing from the source will be

$$I = \frac{E}{R_1 + R_2}$$

and the voltage across R_2 will be

$$V = IR_2 = \frac{ER_2}{R_1 + R_2}$$

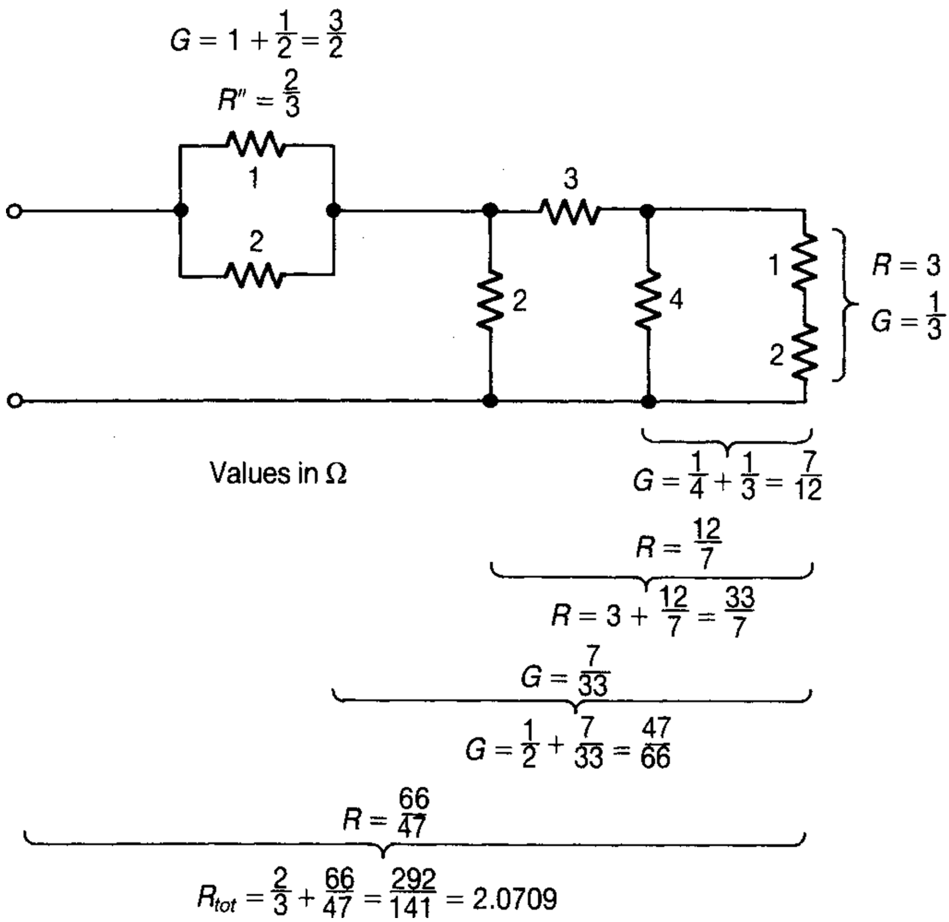


FIGURE 1.7.6 Calculating equivalent resistance.

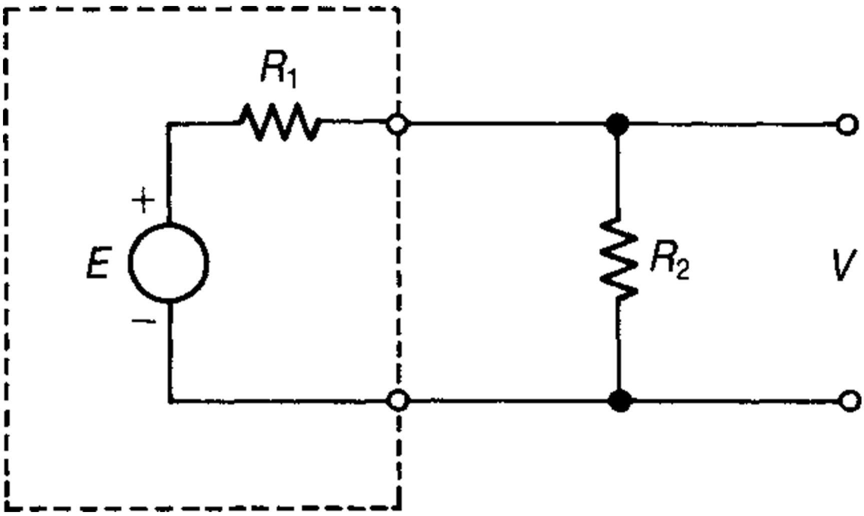


FIGURE 1.7.7 Voltage source with internal resistance.

If R_2 is very large, $R_2 \to \infty$, then there will be no current and the voltage V will be equal to E. If we replace R_2 by a short circuit, then the voltage will be zero. Any finite value of R_2 will make the output voltage $0 < V < E$. We can also calculate the power delivered by the source into the resistor R_2. It will be equal to

$$P = I \times V = \frac{E}{R_1 + R_2} \times \frac{ER_2}{R_1 + R_2} \times E^2 \frac{R_2}{(R_1 + R_2)^2}$$

If we select $R_2 = 0$, then the power consumed in it is zero. If we select $R_2 \to \infty$ (the same as if we disconnect R_2 from the network), then the power consumed by it will also be zero. For all other cases there will be some power consumed in R_2. We could plot such curve for a number of values and discover that the largest power is delivered when $R_2 = R_1$, in which case the voltage $V = \frac{E}{2}$.

Example 6.

A nonideal current source can be modeled as in Fig. 1.7.8. Using the KCL, we have

$$V\,(G_1 + G_2) = J$$

or

$$V = \frac{J}{G_1 + G_2}$$

The current flowing through G_2 is

$$I = VG_2 = \frac{JG_2}{G_1 + G_2}$$

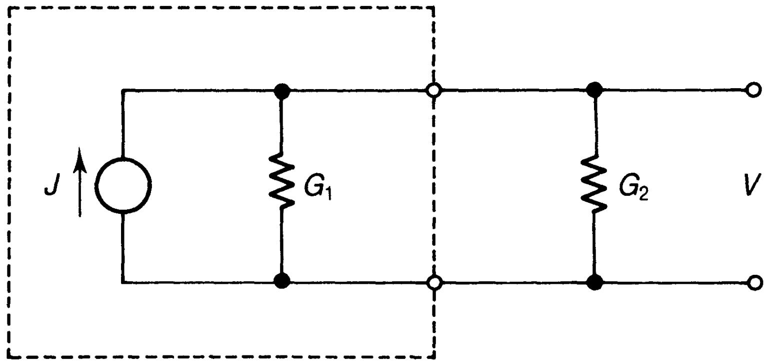


FIGURE 1.7.8 Current source with internal conductance.

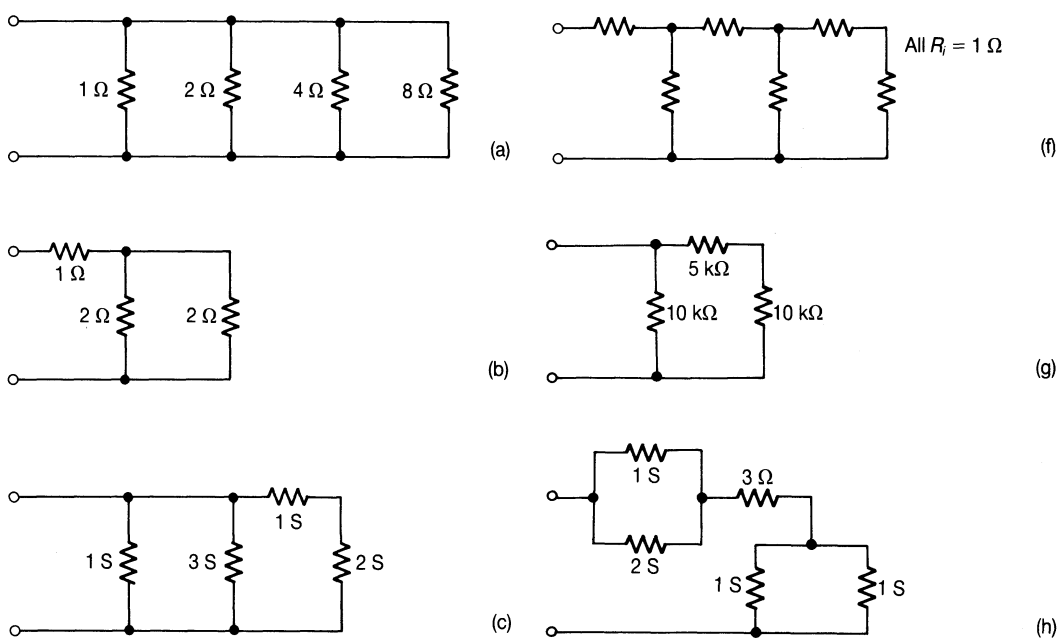
and the power consumed (or as we say dissipated) in G_2 is

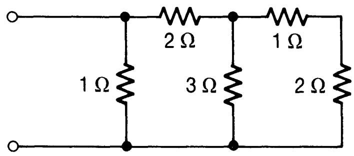
$$P = V \times I = J^2 \frac{G_2}{(G_1 + G_2)^2}$$

For the two extreme values of G_2, namely zero and infinity, there will be no power consumed in G_2. In all other cases the power will be nonzero and maximum transfer of power will occur for If $G_1 = G_2$.

PROBLEMS CHAPTER 1

P.1.1 Find equivalent resistances of the networks.





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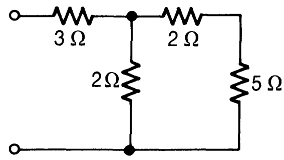
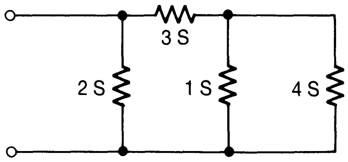
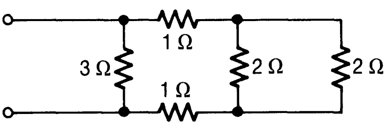


(i)

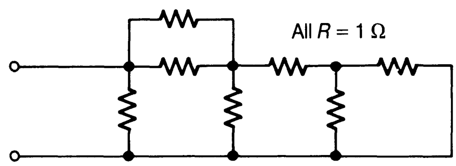


(e)

P.1.2 Find equivalent resistances.



(a)



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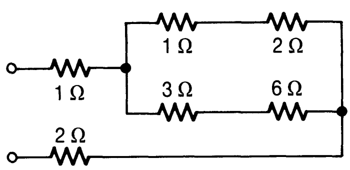

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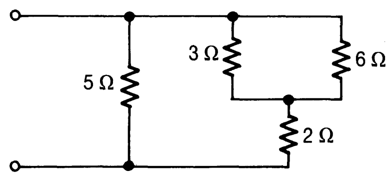

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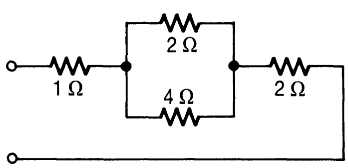
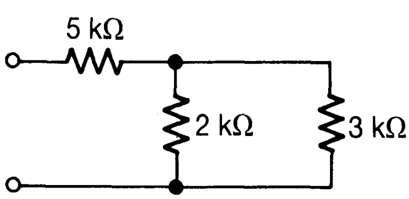
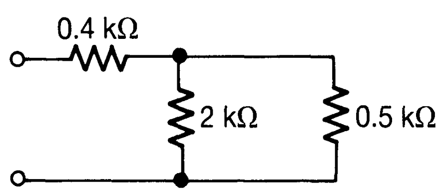


(c)

P.1.3 Find equivalent resistances.



(a)



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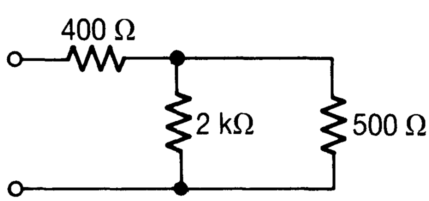
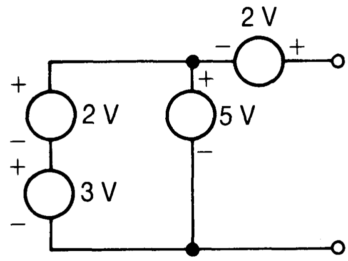


(b)

P.1.4 Find equivalent sources for the given combinations. Are a and b possible?



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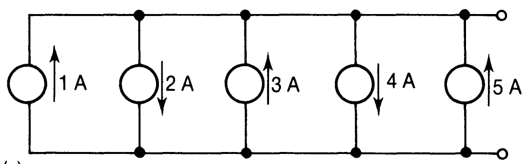

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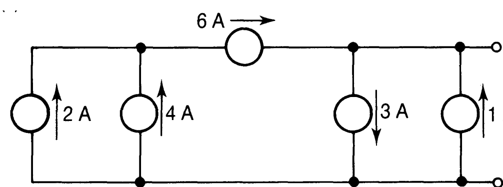

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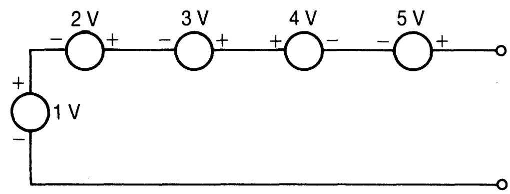

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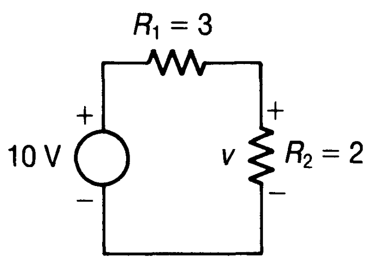
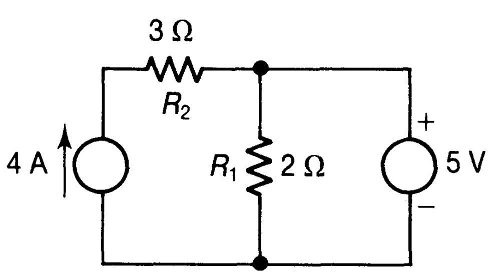
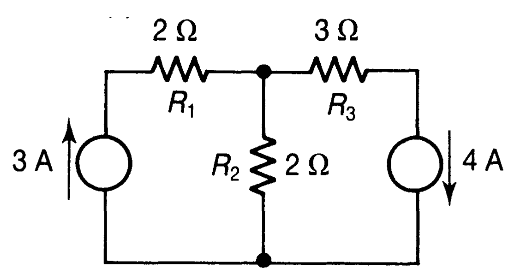
P.1.5 Find the voltage across the resistor R_2.



P.1.6 Find the voltages across the resistors and currents flowing through them.

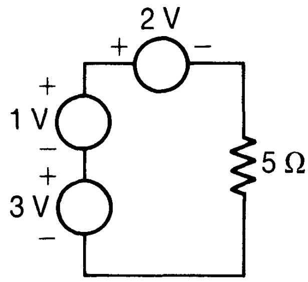


(a)

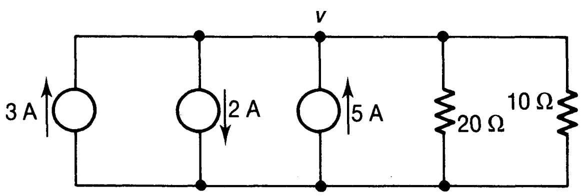


(b)

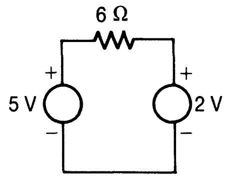
P.1.7 Find the current flowing through the resistor.



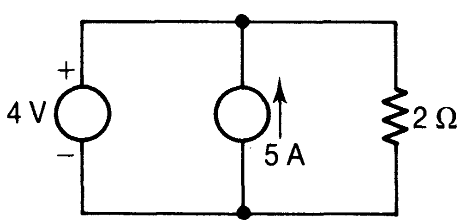
P.1.8 Find the nodal voltage V.



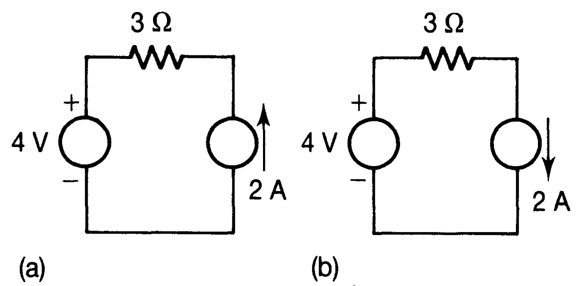
P.1.9 Find the current flowing through the resistor.



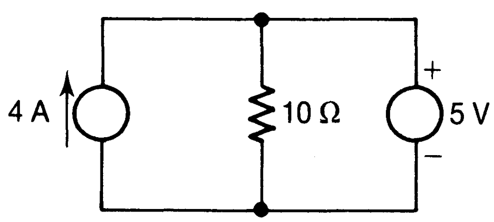
P.1.10 What current is flowing through the resistor? Which source delivers and which consumes power?



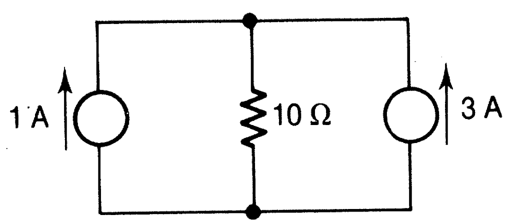
P.1.11 Find the powers delivered and onsumed in the three elements.



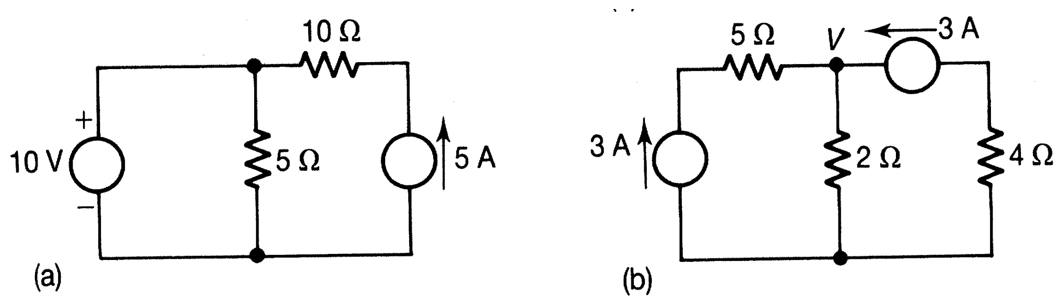
P.1.12 Find the powers delivered and consumed by the three elements.



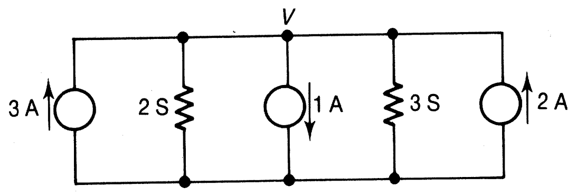
P.1.13 Find the power consumed in the resistor.



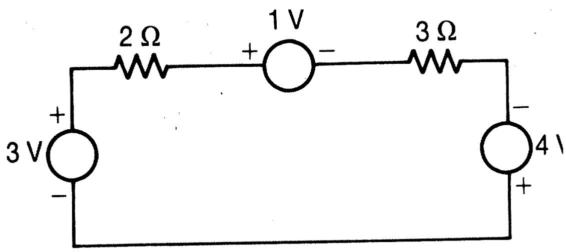
P.1.14 Find the voltages, currents and powers delivered and consumed.



P.1.15 Find the voltage across the combination and the powers delivered or consumed by all elements.



P.1.16 Find the voltages across the elements and powers delivered or consumed by each element.



P.1.17 Find the voltages across the resistors, currents flowing through them and powers consumed in them.