Imperial College London
Department of Mathematics
MSc in Mathematics and Finance
Academic year 2018–2019, Autumn term

M5MF10 Quantitative Risk Management

Coursework — Assignment 1 (weight: 20%), 12 November 2018 (updated: 28 Nov)

General rules:

- ⋆ You may work in groups (at most 4 persons per group).
- ⋆ Present the results in a written report (at most 15 pages).
- ⋆ You may use any *publicly available*[1] software and packages. Please indicate in your report, which software (and packages) you have used.
- ⋆ Hand in your report to the lecturer as a PDF via email — one message per group sent to `m.pakkanen@imperial.ac.uk` suffices. **However, please make sure to include the email addresses of all group members in the message body.**
- ⋆ **Deadline: Friday, 30 November 2018, 11:00am.**

Suggested software and packages:

- ⋆ R with rugarch or fGarch packages
- ⋆ Python with arch 4.6.0[2] package

Part A: Stylised facts and GARCH modelling

This part is about statistical analysis and modelling of the price of *West Texas Intermediate* (WTI) crude oil.[3] The daily spot prices per barrel of WTI (delivered at Cushing, Oklahoma, USA) from 5 November 2008 to 5 November 2018 are provided in the file `QRM-2018-cw1-data-a.csv`. Using these rates, compute first daily log returns.

[1] free or commercial
[2] https://pypi.python.org/pypi/arch
[3] https://en.wikipedia.org/wiki/West_Texas_Intermediate

(i) Compute the mean, standard deviation, skewness, and kurtosis of the log returns. Draw also a histogram including a kernel density estimate and a normal distribution (density) fitted to the log returns. What can you conclude?

(ii) Draw the empirical ACFs of the log returns, their absolute values, and their squares. What can you conclude?

(iii) Fit a standard GARCH$(1,1)$ model, with constant conditional mean and standard normal innovations, to the log return data. Report the estimated parameter values and their standard errors. Plot the absolute returns and fitted volatilities.

(iv) Assess the goodness of fit of the model estimated in (iii). Do the standardised residuals look like an iid sample from the standard normal distribution?

(v) Fit an ARMA$(1,1)$–GARCH$(1,1)$ model with Student t-distributed innovations. Does the new specification improve the fit, compared to the model in (iii)?

Part B: Risk measures

Pietro[4] wants to invest in US equities, but eager to take more risk than what the S&P 500 index offers, he is now interested in a *leveraged* exchange-traded fund (ETF) based on S&P 500, the *ProShares Ultra S&P 500* (symbol: SSO). This ETF aims to deliver twice the daily return on S&P 500. Pietro wants to monitor his daily linearised loss, relative to the value of the investment, given by

$$\bar{L}^{\Delta}_{t+1} = \frac{L^{\Delta}_{t+1}}{V_t} = -r_{t+1}, \tag{1}$$

where r_{t+1} is the log return on SSO. Daily closing prices of SSO from 9 November 2006 to 9 November 2018 are provided in the file `QRM-2018-cw1-data-b.csv`.

As Pietro is currently busy with other things, help him by producing *one-day-ahead VaR and ES forecasts* for the loss (1), at 95% and 99% confidence levels, starting from 10 November 2008 until 9 November 2018. Compute such forecasts using each of the following three methods:

[4] a fictional character

(i) *Historical simulation* (HS) — use a rolling window scheme, whereby for each forecast you use the data over the past 500 trading days to compute the HS forecasts of VaR and ES (see Part 2, slide 45).

(ii) *Filtered historical simulation* (FHS) with *exponentially-weighted moving average* (EWMA) — generate first volatility forecasts for the entire time series by applying the EWMA scheme with $\alpha = 0.06$ and $\hat{\mu}_t = 0$ for all t (see Part 3, slide 98). As the starting value, use for example $\hat{\sigma}_0 = 1$ percent. Use the formulae in Part 3, slide 102 to compute the VaR and ES forecasts. Determine the distribution F_Z by applying historical simulation to the standardised EWMA residuals, within a rolling window over the past 500 trading days.

(iii) FHS with *GARCH* — use a rolling window scheme, whereby you estimate a GARCH(1,1) model, with constant conditional mean and standard normal innovations, every day using the data over the past 500 trading days. Determine the distribution F_Z by applying historical simulation to the standardised GARCH residuals and use then the formulae in Part 3, slide 102 and the one-day-ahead GARCH volatility forecast to compute the VaR and ES forecasts.

Present the results, for each risk measure and confidence level, by drawing a figure where the actual losses and forecasts, using (i), (ii), and (iii), are plotted.

Finally, *backtest* all forecasts; in the case of VaR, use both unconditional coverage and joint coverage–independence tests. Interpret your results. Which method would you recommend?

SOME MISCELLANEOUS TIPS:

- ⋆ When reporting numerical results, use a sensible number of digits — typically no more than five significant figures.
- ⋆ Use either author–year (e.g., "Gauss and Jacquier (1840)") or numbered (e.g., "Brigo and Leibniz [21]") references (but do not mix them).
- ⋆ Typically, all axes in a graph should be labelled.
- ⋆ Importing graphs to your document in PDF or Postscript form is recommended.
- ⋆ The function `kurtosis()` in R returns the *excess* kurtosis by default.
