Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
E		2209563

Trees Fade, Carbon Retains

"Things are not always what they seem." — Phaedrus

As environmental issues become more of a concern, efficient ways of carbon sequestration are increasingly in demand. Forest ecosystems are rich in biodiversity and vast in size, and it is not only the surviving vegetation that plays a role in carbon sequestration; carbon is also sequestered in wood products. Appropriate deforestation helps to allow more carbon to be sequestered, while too little or too much deforestation can harm the forest to some extent and is detrimental to carbon sequestration. Therefore, it is necessary to come up with a well-thought-out, wise forest management strategy.

Firstly, we built the **Wave-up Carbon Sequestration Model (WCSM)**. The total forestry carbon sequestration can be divided into two parts, namely forest carbon sequestration and wood product carbon sequestration. Forests mainly sequester carbon in vegetation and soil. Carbon in soil is derived from vegetation, so the carbon sequestration in forests is strongly correlated with their **biomass**. To clarify this relationship, we introduced the **Logistic Model** to study the forest biomass. The relationship between soil carbon sequestration and biomass carbon sequestration is then calculated based on the **decomposition rate** of litterfall. We can then obtain an expression for the total forest carbon sequestration. The amount of carbon sequestered in wood products varies with time. We used the **Exponential Decay Model** to calculate the carbon sequestration of short-lifespan wood products; for long-lifespan wood products, we modeled their decay rates with **chi-square distribution**. We derived an expression for total carbon sequestration in forestry, from which we solved the numerical results for the optimal **rotation period** and optimal **initial biomass** using a computer.

Then we built the **Felling-Allocation Decision Model (FADM)**. The first part of this model is designed to find the best felling solution, but in reality there are differences between forests. The optimal initial biomass at the time of harvesting varies from forest to forest, so the solution given by the WCSM cannot be implemented immediately. We devided the forests into **three categories** based on the ratio of current biomass to maximum biomass, and for each category we designed transition plans that would make them agree with the WCSM solutions. Wood product lines can be divided into four main categories. We used the **Entropy Weighting Method (EWM)** to assign weights to each wood product industry. From this we get a timber allocation influenced by six indicators: **economic benefits**, **demand**, **distribution of different tree species** in a forest, **emission reduction**, **biodiversity**, **local policies**.

We also did a case study on the forests of Greater Khingan Range in China. The model predictions show that the maximum value of total forestry carbon sequestration in 100 years will be $5.31 \times 10^9 Mg\ ha^{-1}$. The best forest felling strategy is to harvest $21.98 Mg\ ha^{-1}$ trees every five years from the beginning; the best industry allocation strategy is to assign 43.9% of the lumber to the paper and pulp industry.

Finally we performed a comprehensive test or evaluation for our model. Though the allocation and transition scheme could be further improved, the model is intuitive, specific and adaptable.

Keywords: Carbon sequestration; Wood product; Logistic; Entropy Weighting Method

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1 Introduction

1.1 Background

Carbon sequestration is defined as the transfer of carbon dioxide into carbon pools such as long-standing oceanic, pedologic, biotic and geological strata, and thus the net rate of increase in atmospheric greenhouse gas reduces (Lal, Rattan, 2008) [1]. Dead wood, harvested wood products, living biomass, litter and soil can all be sequestered for carbon, which demonstrates the hindering effect of forestry on global warming (Pan, Yude, et al., 2011) [2]. Fire and decomposition activities release carbon dioxide back into the atmosphere; bioenergy from forest biomass can substitute for fossil fuel energy, where carbon dioxide is also returned to the air [3](Figure 1). Many governments have shown commitment in the face of climate change, such as China, which has set goals for carbon neutrality and peak carbon dioxide emissions (Xi J.P., 2020) [4].

Most forest managers place great emphasis on afforestation and some even advocate replacing wood products with bamboo products where possible, ignoring the carbon storage role of trees even after they have lost their lives. Forests serve as "carbon scrubbers" or "carbon removers", and selective harvesting and storage of some wood can be effective for carbon sequestration (Zeng, Ning, et al., 2013) [5]. If new construction utilized 90% wood products during the next 30 years, then 700 million tons of carbon would be sequestered, which is the equivalent of approximately 7 days worth of global emissions in 2019 [6], implying the importance of the wise forest management. According to different climate, population and interests, forest management is site-specific as well as long-term. It is challenging but well worth the efforts to design a forest management plan that satisfies all forest managers and users, while balancing multiple factors both inside and outside the forest.

Figure 1: Forest Carbon Sequastration [7]

1.2 Our Work

To analyze the changes in carbon sequestration, we have built the **Wave-up Carbon Sequestration Model (WCSM)**. In this model, we studied **forest carbon sequestration** and **wood product carbon sequestration** separately. The amount of carbon sequestered by forestry was further divided into the carbon sequestration by the **vegetation** and the **soil**, and we used the **Logistic Model** and **SOC Model** respectively, both of which are based on **differential equations**. We divided wood products into two categories, **long-term products** and **short-term products**, described by **chi-squared decay model** and **exponential decay model** respectively. The combined benefits of carbon sequestration are obtained by **cyclically** harvesting and observing the carbon sequestration of wood products and forests. Finally, the optimal solution is obtained by analysing the **wave-like rise** in the amount of carbon sequestered.

In order to identify reasonable forest management options, **Felling Allocation Decision Model (FADM)** was developed.

1) We first divide the forest into three categories according to their growth and make appropriate measures for these three categories. Once the forest reaches the **transition point**, it is harvested according to the plan given by **WCSM**.

2) The factors influencing the production of different wood products are then determined and the different industries are assigned weights using the **Entropy Weighting Method (EWM)** to obtain an allocation scheme.

The scenarios given by **WCSM** guide the harvest scheme in **FADM**, which then integrate social, economic, cultural and ecological aspects to give allocation scheme, resulting in a complete forest management scenario.

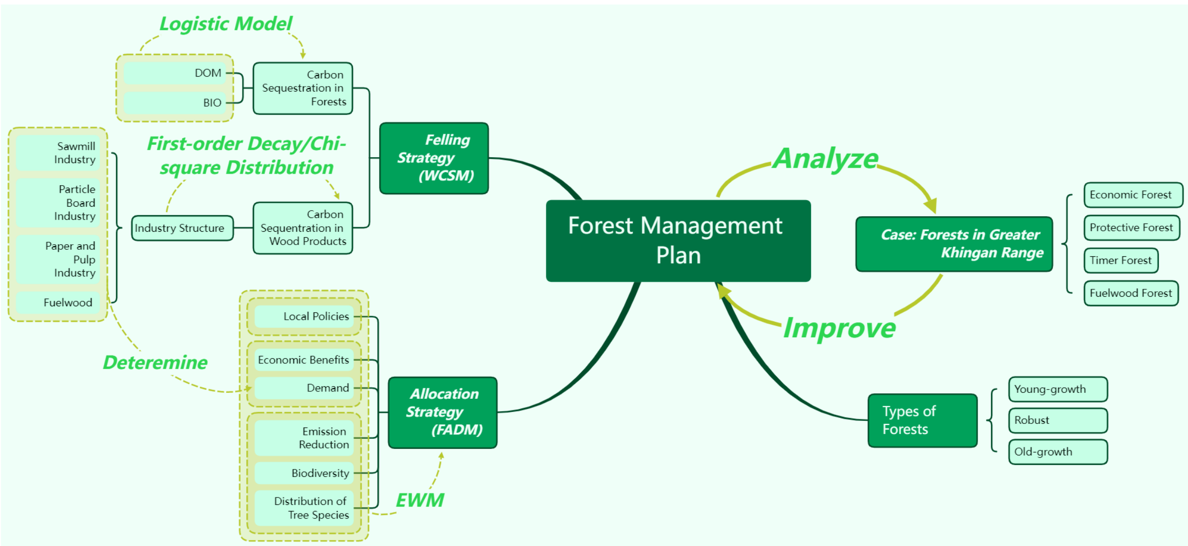

Figure 2: Model Overview

2 Assumptions and Justification

To simplify our model and eliminate the complexity, we make the following main assumptions in this literature.

- **Assumption 1**. Assume that different felling practices and the part of the tree felled do not affect the forest growth rate.
- **Assumption 2**. Assumed that the time consumed by a single felling is negligible compared to the time taken by the organisms to grow.
- **Assumption 3**. Assume that the proportion of various vegetation in the forest will not change in a short period of time.

- **Assumption 4**. Assumed that the forest will not be damaged by land use planning, vandalism, forest fires, earthquakes, etc.
- **Assumption 5**. Assume that all operations have a time span in years.

3 Notations

Symbols	Description	Unit
B_t	the forest vegetation biomass	Mg$\cdot ha^{-1}$
$B_{\max}$	the maximum vegetation biomass in mature forests	Mg$\cdot ha^{-1}$
α	the average carbon sequestration ratio of the forest	1
A_c	the elemental carbon withering coefficient	a^{-1}
LP_t	the annual production of litter	Mg$\cdot ha^{-1}$
k_2	the decomposition coefficient of litter	a^{-1}
C_{ECO}	the carbon sequestration of the forest ecosystem	Mg$\cdot ha^{-1}$
k_{short}	he decay coefficient of short-life products	1
r	the proportion of remaining carbon elements in the products	1
T	the harvesting cycle	a
T_h	the interval between the harvest of the same woodland	a
c_{inc}	the increase in carbon sequestration	Mg$\cdot ha^{-1}$
G_c	growth function of carbon sequestration after felling	Mg$\cdot ha^{-1}$
$< C >_{\mathrm{n}}$	the average amount of carbon stored during the n$^{th} rotation$	Mg$\cdot ha^{-1}$
$< C >_{\infty}$	the upper bound of carbon sequestration	Mg$\cdot ha^{-1}$
$V_{\max}$	the maximum of rate of biomass growth	Mg$\cdot ha^{-1}a^{-1}$
$C_{\mathrm{cut}}, G_c(T)$	the forest vegetation biomass cut down every time	Mg$\cdot ha^{-1}$

where we define the main parameters while specific value of those parameters will be given later.

4 Model WCSM: Wave-up Carbon Sequestration Model

4.1 Model I Overview

The general idea of our model can be shown by Fig.3, which divides the carbon sequestration into three parts: soil, vegetation and wood products. Our model is established on the carbon flow among the three parts.

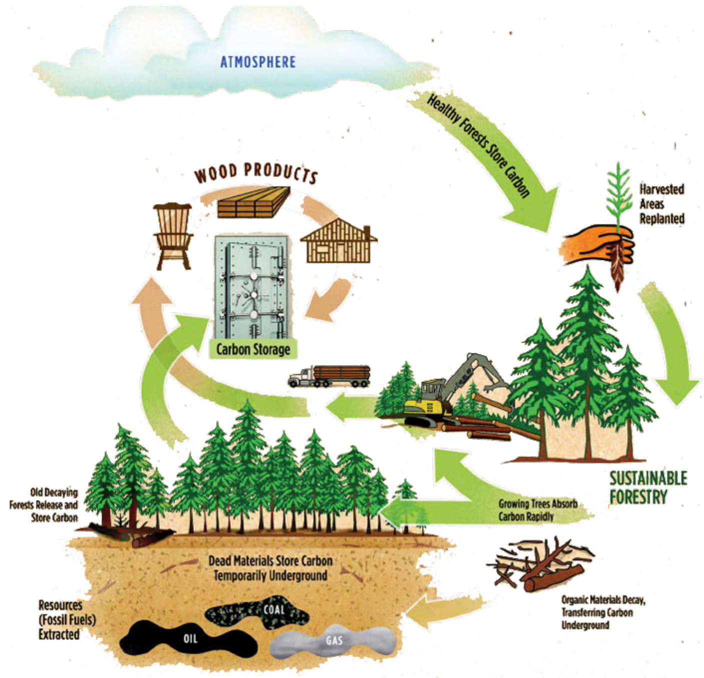

Figure 3: The Carbon Cycle in Forests

4.2 Calculation of Carbon Content in Forests

The amount of carbon sequestered by forestry mainly consists of two parts, namely carbon sequestered by forests and carbon in wood products. The amount of carbon sequestered by forests can be divided into carbon sequestered by living things (especially plants) and carbon sequestered by soils (mainly dead wood, coarse woody debris and soil organic matter). As a result, there are two carbon pool systems, i.e. the biomass(BIO) carbon pool and the dissolved organic matter(DOM) carbon pool. Therefore, the total amount of carbon stored in the forest is

$$\triangle \text{ECO} = \sum \triangle \text{DOM} + \sum \triangle \text{BIO} \tag{1}$$

4.2.1 Calculation of Forest Vegetation Carbon Sequestration

According to biological analysis, due to the limitation of natural resources and environmental conditions, the forests cannot grow indefinitely. And the growth rate increases first and decreases with the increase of year.So we base on Logistic model to stimulate the change in forest vegetation biomass over time:

$$\frac{\mathrm{d}B_t}{\mathrm{d}t} = v_0 \left(1 - \frac{B_t}{B_{\max}}\right) B_t \tag{2}$$

where

- t represents the forest age (year);
- B_t represents the forest vegetation biomass (Mg ha^{-1});
- v_0 represents intrinsic growth rate, i.e. the maximum growth rate of a plant without the limit of several environmental factors;
- $B_{\max}$ represents the maximum vegetation biomass in mature forests (Mg ha^{-1});

- $(1 - \frac{B_t}{B_{\max}})$ represents the fractional deficiency of the current biomass to its saturation level.

Equation 2 can be expressed as Equation (3), as follows:

$$B_t = \frac{B_{\max}}{1 + e^{-v_0 \cdot (t-t_0)} \cdot \left(\frac{B_{\max}}{B_{t_0}} - 1\right)} \tag{3}$$

where B_{t_0} is the initial forest vegetation biomass.

The carbon content in plants is generally within a stable range. Letting α be the average carbon sequestration ratio of the forest, we have

$$C_{\text{BIO}}(t) = B_t \cdot \alpha \tag{4}$$

where C_{BIO} is the carbon sequestration by the forest vegetation.

4.2.2 Calculation of Soil Carbon Sequestration

When the forest is relatively stable, the mineralization process of humification will tend to balance. A decomposition factor is required because a part of the litterfall and the organic matter in the soil will be decomposed by microorganisms to carbon dioxide. The higher the content of litterfall in the soil, the slower the rate of change in carbon content, which leads to the following differential equation:

$$\frac{\mathrm{d}C_{DOM}}{\mathrm{d}t} = h \cdot LP_t - k_2 C_{DOM} \tag{5}$$

where

- C_{DOM} represents the carbon sequestration in the DOM pool;
- h represents the decay coefficient [8];
- LP_t represents the annual production of litter (Mg $\text{ha}^{-1} a^{-1}$);
- k_2 represents the decomposition coefficient of litter.

The amount of litterfall accumulates over a period of time and reaches a stable value, which is positively correlated with forest biomass. Assuming that trees allocate the biomass to three parts, which are leaves, branches and roots, with T_1, T_2, and T_3 respectively denoting their withering cycles and a_1, a_2, and a_3 respectively denoting their allocation ratios, we have

$$LP_t = B_t \cdot A_c \tag{6}$$

where A_c is the elemental carbon withering coefficient, and

$$A_c = \sum_{i=1}^{3} \frac{a_i}{T_i} \tag{7}$$

Solving Equation (5) yields

$$C_{DOM} = \frac{h\, LP_t}{k_2} - \left(\frac{h\, LP_t}{k_2} - C_{DOM_0}\right) \cdot e^{-k_2(t-t_0)} \tag{8}$$

Substituting Equation (6) into Equation gives the equation to calculate the amount of carbon sequestered by the soil, as follows:

$$C_{DOM} = \frac{h\ A_c}{k_2} B_t - \left(\frac{h\ A_c}{k_2} \cdot B_t - C_{DOM_0} \right) e^{-k_2(t-t_0)} \tag{9}$$

For the simplicity of subsequent calculations and to be closer to reality, we default to $C_{DOM_0} = 0$, i.e., the amount of soil carbon sequestration at the moment t = t_0 is 0.

4.2.3 Calculation of Total Carbon Sequestration in Forests

With a defined B_t, the value of the C_{DOM} tends to a stable value. In other words the main determinant of C_{ECO} is B_t.

$$C_{\text{ECO}} = C_{\text{BIO}} + C_{\text{DOM}} \approx \alpha B_t + \beta B_t = B_t\,(\alpha + \beta) \tag{10}$$

where B_t is defined by equation (3), and $\beta = \frac{hA_c}{k_2}$.

4.3 Calculation of Carbon Sequestration in Wood Products

4.3.1 Carbon Sequestration of Short-lifespan Products

For short-lifespan, easily consumed wood products such as paper and fuel, a first-order decay model as follows can be used to describe the degree of decay of their carbon elements over time:

$$\frac{dr}{dt} = -k_{\text{short}} r \tag{11}$$

where

- r represents the proportion of remaining carbon elements in the products;
- k_{short} represents the decay coefficient of short-life products and varies with product types.

From Equation (11) we get

$$r = e^{-kt}\ (t > 0) \tag{12}$$

As yellow curve shown in Fig.4.

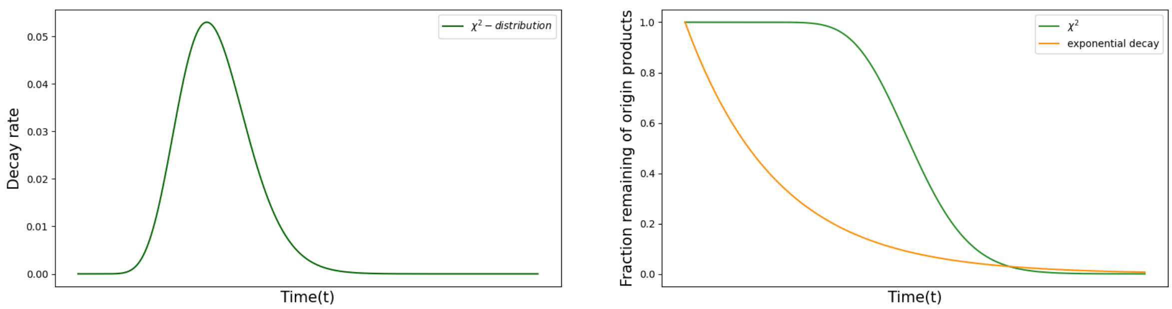

Figure 4: Related Curves of Carbon Sequestration in Wood Products

4.3.2 Carbon Sequestration of Long-lifespan Products

The decay rate of long-lifespan wood products such as furniture and buildings will not peak at the beginning, thus not conforming to the first-order decay model. Instead, it reaches its maximum when the duration of use reaches the average product life. This can be well fitted when the decay rate, defined as the degree of product loss or decay per unit of time, satisfies the chi-square distribution, namely

$$\text{decay rate} \sim \chi^2 \left(T_{\mathrm{Av}}\right), \; T_{\mathrm{Av}} \in Z^+ \tag{13}$$

where $\chi^2 \left(T_{\mathrm{Av}}\right)$ satisfies the chi-square distribution of $n = T_{\mathrm{Av}}$. Then we get

$$r = 1 - \int_0^1 \frac{1}{2^{\frac{T_{\mathrm{Av}}}{2}}\Gamma\left(\frac{T_{\mathrm{Av}}}{2}\right)} x^{\frac{T_{\mathrm{Av}}-1}{2}} e^{-\frac{x}{2}} \mathrm{d}x \tag{14}$$

where $D \equiv \int_0^\infty R_c\left(t\right) \mathrm{d}t$

- r is the proportion of remaining carbon elements in the products;
- $\Gamma\left(\frac{T_{\mathrm{Av}}}{2}\right)$ is Gamma Function.

4.4 Comprehensive Proceeds from Cyclical Felling and Wood Products

4.4.1 Derivation of Average Growth Curve of Carbon Sequestration

In general, the harvesting volume should not exceed the growth volume according to the regulations, and the felling is cyclical. If the harvest is less than the growth, the biomass of the forest gradually increases after each harvest. However, according to Equation (3), the growth rate gradually slows down as the biomass increases. As a result, the amount of growth in a cycle will be equal to the amount harvested after a certain period of time, at which point a dynamic equilibrium is reached. Therefore, under the assumption that the forest growth rate is known, we may again assume that the harvest is equal to the growth in a cycle.

Let the amount of carbon sequestered in a forest after harvesting be $C_{\mathrm{ECO}}\left(t_0\right)$. After T years, the total amount of carbon sequestered in the forest reaches $C_{\mathrm{ECO}}\left(t_0 + T\right)$. Then the amount of harvesting is $C_{\mathrm{ECO}}\left(t_0 + T\right) - C_{\mathrm{ECO}}\left(t_0\right)$.

We now define the growth function of carbon sequestration after felling

$$G_c\left(t\right) = C_{\mathrm{ECO}}\left(t + t_o\right) - C_{\mathrm{ECO}}\left(t_o\right) \tag{15}$$

to represent the growth of carbon sequestration in one cycle, and the harvesting amount is $G_c\left(T\right)$.

$C_{\mathrm{inc}}\left(t\right)$ represents the increase in carbon sequestration between the present moment and moment $t\left(t \geqslant 0\right)$. It is calculated from the sum of the current forest growth curve and the contribution of the retention curves of wood products that have not decayed or been reconverted to CO_2 in all previous cycles or rotations. The result during successive rotations is:

First rotation: $0 \leqslant t \leqslant T$

$$C_{\mathrm{inc}}\left(t\right) = G_c\left(t\right) \tag{16}$$

Second rotation: $T \leqslant t \leqslant 2T$

$$C_{\mathrm{inc}}\left(t\right) = G_c\left(t - T\right) + \sum_{i=1}^{p} u_i G_c\left(T\right) r_c^{(i)}\left(t - T\right) \tag{17}$$

Third rotation: $2T \leqslant t \leqslant 3T$

$$C_{\text{inc}}(t) = G_c(t-2T) + \sum_{i=1}^{p} u_i G_c(T)\left[r_c^{(i)}(t-T) + r_c^{(i)}(t-2T)\right] \tag{18}$$

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n$^{\text{th}}$ rotation: (n-1)$T \leqslant t \leqslant$ nT

$$C_{\text{inc}}(t) = G_c(t-(\text{n-1})T) + G_c(T)\sum_{m=1}^{\text{n-1}} R_c(t-mT) \tag{19}$$

where

- $R_c(t) \equiv \sum_{i=1}^{p} u_i r_c^{(i)}(t)$ is the weighted-average of the carbon retention curves for all wood products;
- u_i is the fraction of harvested wood used for product i.

The first term to the right of the equal sign of Equation (20) represents the periodic contribution of surviving trees to C(t) in successive cycles or rotations. The second term represents contribution of carbon in wood products. For a better grasp of the overall trend, the periodic fluctuations can average out. From Equation (20), the following equation can be calculated for the average amount of carbon stored during the n$^{\text{th}}$ rotation:

$$< C >_{\text{n}} \equiv \frac{1}{T}\int_{(\text{n-1})T}^{\text{n}T} C_{\text{inc}}(t)\,\mathrm{d}t = \frac{1}{T}\int_0^T G_c(t)\,\mathrm{d}t + \frac{G_c(T)}{T}\int_0^{(\text{n-1})T} R_c(t)\,\mathrm{d}t \tag{20}$$

As this cycle continues, more and more timber products will decay. The amount of carbon sequestration will reach an upper bound when the decay rate of timber products and the growth rate of vegetation are in balance, i.e.

$$< C >_{\infty} = \frac{1}{T}\int_0^T G_c(t)\,\mathrm{d}t + \frac{G_c(T)}{T}D \tag{21}$$

where

$$D \equiv \int_0^{\infty} R_c(t)\,\mathrm{d}t \tag{22}$$

depends only on wood products and is a measure of their average decay time of properties or half-life. From Equation (21) it follows the average increase in carbon sequestration:

$$C_{\text{Av}}(t) = \frac{1}{T}\int_0^T G_c(x)\,\mathrm{d}x + \frac{G_c(T)}{T}\int_0^t R_c(x)\,\mathrm{d}x \tag{23}$$

As shown in Fig.5, the yellow curve is $G_c(t-kT_0)$, the green curve is actual amount of carbon sequestered, the blue curve is $C_{Av}(t)$. From the diagram we can also roughly predict the presence of $< C >_{\infty}$.

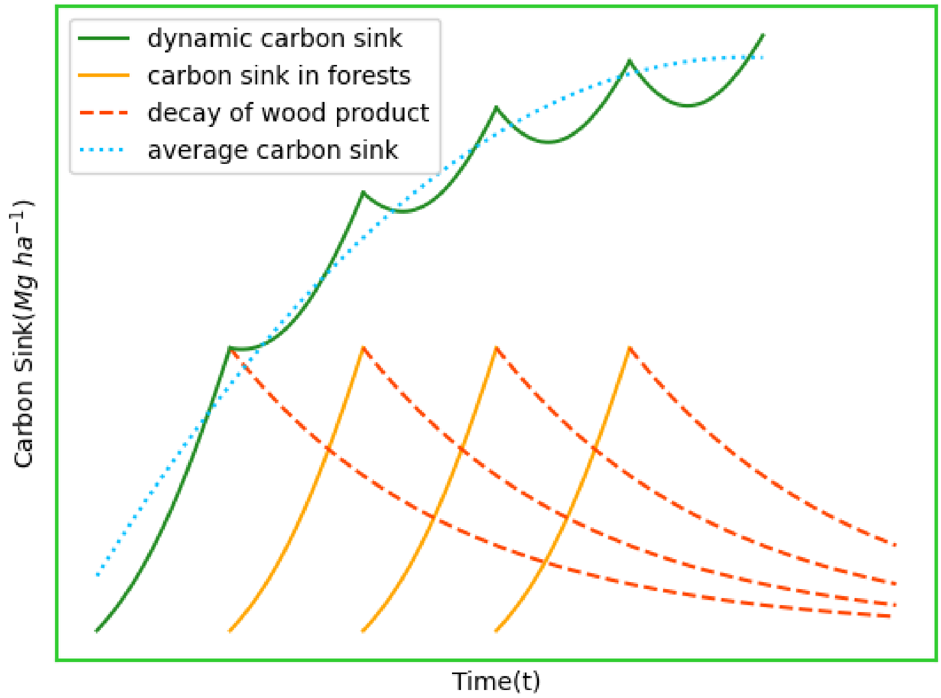


Figure 5: The Wave up of Carbon Sequestration

4.4.2 Calculation of Optimal Cycle and Initial Biomass

The model seeks an optimal solution for the following two main objectives:

1) Reaching the upper bound of carbon sequestration $< C >_\infty$ as soon as possible.

2) Make the upper bound of carbon sequestration $< C >_\infty$ as large as possible.

To achieve the first objective, derive the function $C_{\mathrm{Av}}(t)$

$$C'_{\mathrm{Av}}(t) = \frac{G_c(T)}{T} R_c(t) \tag{24}$$

When the value of u_i, r and $r_c^{(i)}$ is determined, $R_c(t)$ is a constant. So the rate of growth of the $C_{\mathrm{Av}}(t)$ is determined by $\frac{G_c(T)}{T}$, which represents the average rate of carbon sequestration over a cycle T.

Under ideal conditions, when the initial forest vegetation biomass $B_{t_0} = \frac{1}{2} B_{\max}$, forest vegetation biomass B_t grows at the fastest rate, so as C_{all}. Therefore, when $T \to 0$ and $B_{t_0} \to \frac{1}{2} B_{\max}$, $C_{\mathrm{Av}}(t)$ takes the maximum value of the growth rate. But this holds only in the ideal case; in reality T and B_{t_0} will be subject to other conditions.

To achieve the second objective, i.e. to find the maximum value of $< C >_\infty$, calculate the zeros of its derivative.

$$T\left(G_c(T) + DG'_c(t)\right) - \left(\int_0^T G_c(x)\,\mathrm{d}x + DG_c(T)\right) = 0 \tag{25}$$

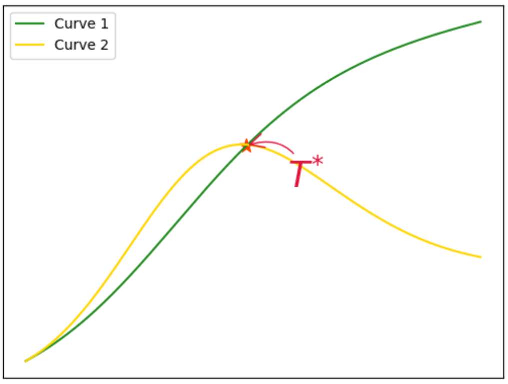
where the expression for $G_c(t)$ can be obtained from the equation (15), (10), (3).

The equation (25) is a transcendental equation, which needs computer simulation to solve.

As shown in Fig.6, the $Curve1$ is $\left(\int_0^T G_c(x)\,\mathrm{d}x + DG_c(T)\right)$ and the $Curve2$ is $T\left(G_c(T) + DG_c'(t)\right)$. T^* is the optimal cycle.

1) When the function $G_c(t)$ is known, the solution to this equation provides an optimal cycle T^* .

2) When the value of T is fixed, the solution to this equation provides an optimal $G_c^*(t)$, i.e. an optimal initial forest vegetation biomass $B_{t_0}^*$.



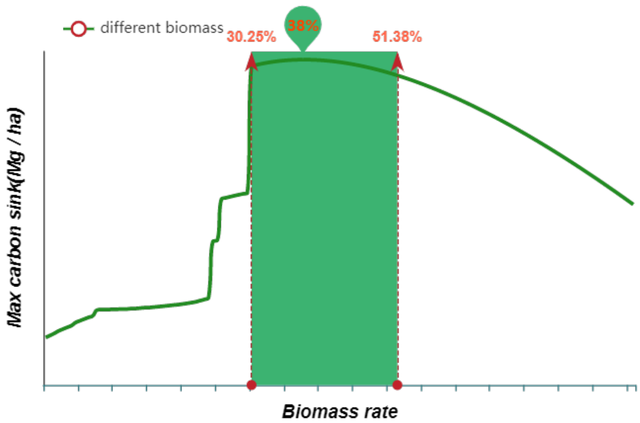


Figure 6: The Solution of Equation (25)

Figure 7: Biomass Rate-Max Carbon Sink Curves

Shown by the results of calculation as Fig.7, the theoretical maximum of carbon sequestration firstly rise with the initial forest vegetation and then fall. The interval with larger value is between 30.25% *and* 51.38% of the max forest vegetation biomass ,which is almost the interval with largest growth rate.

5 Model FADM : Felling-Allocation Decision Model

5.1 Harvesting Scheme Adapted to Local Conditions

Forests grow differently, so we need to classify forests at different stages of growth.We call the point in the biomass growth curve with the fastest growth rate the FGR point. To find a reasonable interval where the biomass growth rate is close to that at the FGR point, we judge according to the following formula

$$\frac{dB_t}{dt} \geq 0.9 \cdot V_{max} \tag{26}$$

The interval obtained is $[0.342B_{max}, 0.658B_{max}]$.So far, based on this interval we can classify forests into three categories according to the ratio of current growth to maximum growth: Old-growth forests, Robust forests and Young-growth forests.

Table 1: A comparison of three types of forests

Type name	**Young-growth forests**	**Robust forests**	**Old-growth forests**
Ratio($\frac{B_t}{B_{\max}}$)	$Ratio \leq 34.2\%$	$34.2\% \leq Ratio \leq 65.8\%$	$Ratio \geq 65.8\%$
transition point	$0.648B_{max}$	$0.5 \sim 0.648\text{B}_{max}$	$0.648B_{max}$

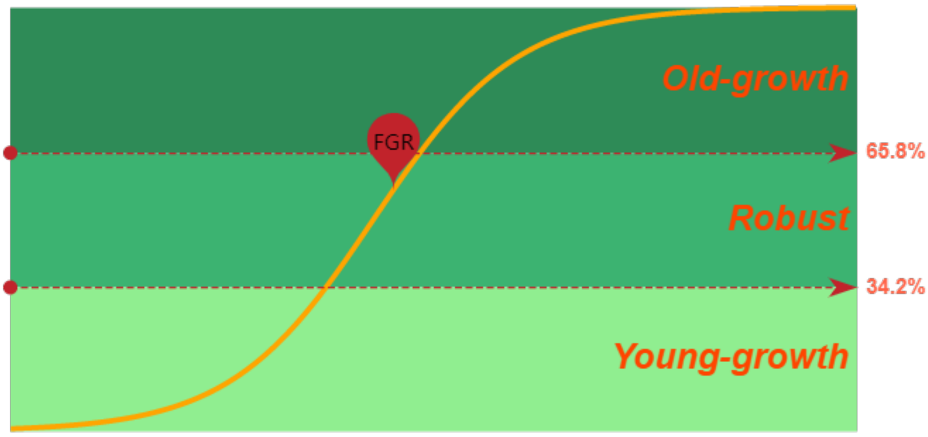


Figure 8: Classification of Forests

- For young forests, the ecosystem itself is not stable and cannot be harvested any further. Instead, certain measures should be taken to protect the forest and accelerate the growth of its biomass. When the biomass of the forest is close to $65.8\%B_{max}$, it means that the forest has evolved into an old-growth forest and can then be harvested according to the scheme given in WCSM.Tree planting is an investment that allows for the economic benefits of forests to be used efficiently over a long period of time.
- For robust forests, their own biomass already allows them to be harvested. It is straightforward to follow the strategy given in WCSM for harvesting.
- In old-growth forests, high biomass has a dampening effect on the rate of carbon sequestration. In order to keep forests robust, harvesting in the first few cycles should ensure that harvesting is greater than growth. When the biomass of the forest is close to $65.8\%B_{max}$, the transition point is reached, and the harvesting can be carried out according to the scheme given in WCSM.

5.2 Allocation Scheme Based on Entropy Weighting Method (EWM)

Wooden production lines are made up of four main categories: PL1 = sawmill industry, PL2 = particle board/plywood industry, PL3 = pulp and paper industry, and PL4 = fuelwood. These four production lines represent four categories of wood products. Whereas the proportion of these products allocated is determined by a combination of multiple factors, we intend to use the **Entropy Weighting Method(EWM)** to calculate the weighting of the allocation between the various industries.

5.2.1 Determination of Indicators

- **Economic Indicators**

  1) **Economic benefits.**The profitability of different wood product industries is different. People often prefer to choose the more profitable wooden product to produce.
  2) **Demand.** People in different regions have different levels of demand for different wooden products. Production needs to be tailored to the needs of the market.
  3) **Distribution of forest tree species.** There are many different kinds of plants in the forest, each of which provides timber with its own characteristics. Each wood product industry has a unique and most suitable timber. The proportion of numbers in the forest will therefore also affect the final wood products industry.

- **Ecological Indicators**

  1) **Emission reduction.** Wood products are themselves a form of carbon. In addition to this, the replacement of steel products by wood products contributes significantly to the reduction of CO2 emissions.
  2) **Biodiversity.** Note that biodiversity contributes to the long-term stability of forests. If fewer trees are harvested there is a risk that the ecosystem will become less resilient and eventually lead to the loss of the forest.

- **Political Indicators**

  1) **Local policies.**Government policies often inadvertently restructure and upgrade industries. For some wood products industries, the government can restrict or guide them by changing tax rates, for example.

5.2.2 Calculation of Entropy Weights of the Four Industries

1) **Data standardization**

The units of data obtained from each of the six dimensions above vary. The measurement dimensions for obtaining the data also differ. So in order for these data to be compared with each other, we should normalise the raw data obtained.Here we use the extreme difference method for normalisation.

When we analyse all the above indicators, we will see that there are two types of indicators.Some of these indicators are positive, such as emission reductions and demand. The larger these figures are, the higher the corresponding output for that industry should be. For these indicators, the equation should be:

$$r_{ij} = \frac{x_{ij} - x_{\min}}{x_{\max} - x_{\min}} \tag{27}$$

There are also some indicators that are negative, such as tax rates. The higher the tax rate, the less wood products are produced.For these indicators, the equation should be:

$$r_{ij} = \frac{x_{\max} - x_{ij}}{x_{\max} - x_{\min}} \tag{28}$$

where

- r_{ij} and x_{ij} represents the standardized value and original value of wood products industry j at the ith indicator.
- $x_{\max}$ and $x_{\min}$ represents the minimum and maximum value of wood products industry j among all indicators.

2) **Weight determination**

In order to finalize the weights for each wood product industry, we need to find the entropy weights for each industry.Suppose there are k industries and n indicators.

First, we calculate the weight of the jth indicator in the ith industry.

$$f_{ij} = \frac{r_{ij}}{\sum_{i=0}^{n} r_{ij}} \tag{29}$$

Then calculate the entropy value e_j of the ith wood products industry according to the definition of information entropy.

$$e_j = -ln(n)^{-1} \sum_{i=0}^{n} f_{ij} ln(f_{ij}) \tag{30}$$

Then find the entropy weight based on the information entropy.

$$w_j = \frac{1 - e_j}{k - \sum_{i=0}^{k} e_i}, j = 1, 2, 3, 4 \tag{31}$$

This gives the weights assigned to timber in the four wood product industries. The final allocation can then be obtained based on the previously determined harvesting volumes.

6 Case Study: Forests in Greater Khingan Range

6.1 Acquisition of Key Parameters

To verify the applicability of the model, we chose forests in Greater Khingan Range in China for our case study. Based on the model, we collected and processed the raw data to obtain key parameters.

As a result, the current forest vegetation biomass(B_{t0}) is 89.4 $Mg\ ha^{-1}$, the intrinsic growth rate(v_0) is 0.118 a^{-1}, and the maximum vegetation biomass(B_{max}) is 160.28 $Mg\ ha^{-1}$. [8]

According to the Yasso Model, the decomposition coefficient for the leaves, branches, and roots(k_1) is 0.462, and the decomposition rate of humus (k_2) is 0.00057.

As can be seen from the volume of production transactions of wood products, paper and paperboard have the largest share, accounting for 97% of all production transactions [9]. The average lifespan of paper is about 32 years, so k_{short} is approximately 0.005.

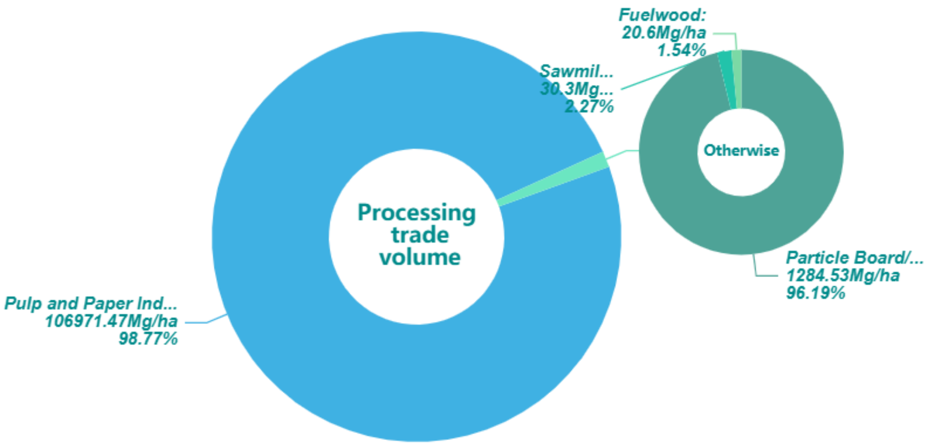


Figure 9: Industry Structure

6.2 Calculation of Carbon Sequestration in 100 Years

After inputting parameters into the model, we could firstly find the optimal solution in 100 years, which is different from the solution before, mainly focusing on larger time range. We chose traversal method to solve the problems more easily. Inputting every possible T into the model yields the maximum carbon sequestration. The change of carbon sequestration is as shown in Fig.10.

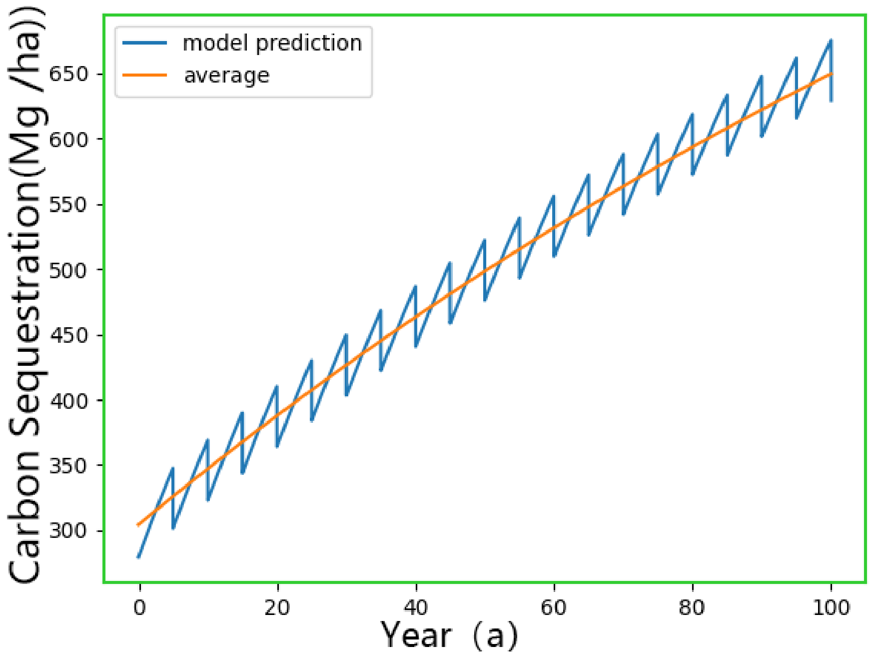


Figure 10: Carbon Sequestration of Greater Khingan Range in 100 years

The maximum carbon sequestration per unit of area is 649.74 $Mg\ ha^{-1}$. The area of forests in Greater Khingan Range is $8.17 \times 10^6\ ha$, so the whole carbon sequestration amount of the forests is

$$C_{\text{all}} = C_t \times S_{\text{forest}} \tag{32}$$

where

- C_{all} represents the whole carbon sequestration of the forests;
- S_{forest} represents the area of forests;
- C_t represents carbon sequestration per unit of area.

Then we get the maximum carbon sequestration amount is $5.31 \times 10^9 Mg\ ha^{-1}$.

6.3 Felling Strategy in the Case

By comparing the current forest vegetation biomass(B_{t0}) and the maximum vegetation biomass(B_{max}), we find the forests in Greater Khingan Range is "Robust Forests", which are in prime of life. According to the model, the best forest felling strategy is to harvest $21.98 Mg\ ha^{-1}$ trees every five years from the beginning.

In this way, we can make the carbon sequestration maximum, obtain the maximum amount of wood, and keep forests in a sustainable and healthy state.

6.4 Allocation Strategy in the Case

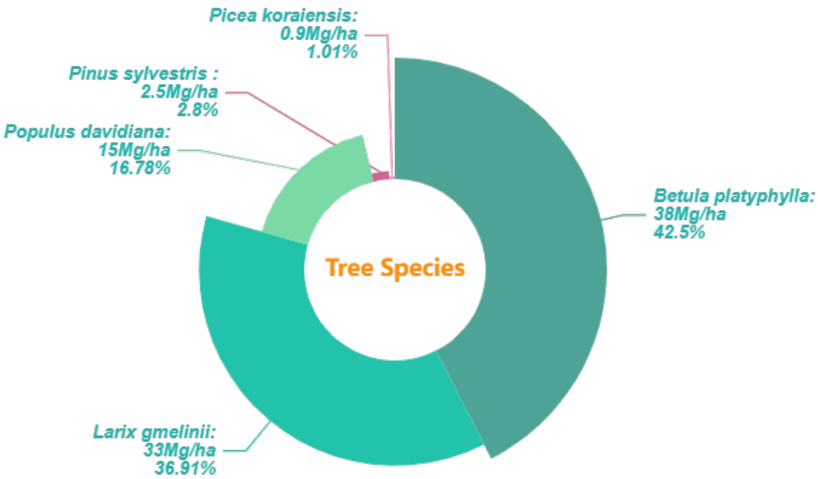


Figure 11: Rose Chart of Tree Species

Tree species in Greater Khingan Range, which is as shown in Fig.11 can be mainly divided into five categories. Larix gmelinli takes the largest proportion, an excellent material for Sawmill Industry. Pinus sylvestris is a kind of protective tree for local ecology.

To balance the biodiversity, we prefer to cut down the tree species with larger numbers. As a result, we prefer to choose a more suitable industry for these tree species. In order to protect the local ecology, we could not cut down those protective tree species and have to make them grow better. With the collected raw data and EWM model, 6 indicators are as shown in Table 2.

Table 2: 6 Indicators, Entropy, and Weight

Industry	Economic benefits	Demand	Distribution of tree species	Emission reduction	Biodiversity	Local policies	Entropy	Weight
Sawmill Industry	0.024	0	0.488	0	0	0.488	0.441	0.362
Particle Board Industry	0.289	0.133	0	0.289	0.289	0	0.751	0.161
Paper and Pulp Industry	0	0.077	0.300	0.073	0.231	0.319	0.322	0.439
Fuelwood	0.124	0.303	0.215	0.121	0.155	0.064	0.943	0.036

The best industry allocation scheme is to assign more lumber to the paper and pulp industry, which accounts for about 43.9%, followed by the sawmill industry with about 36.2%. The rank of four industries is as shown in Fig.12.

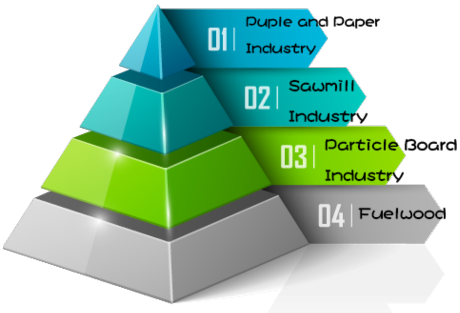


Figure 12: Proper Industry Structure

6.5 Transition Strategy in the Case

Although we cut down trees every five years, under the assumption the interval between two harvests has to be much longer. The interval between the harvest of the same woodland can be defined as follows:

$$T_h = \frac{B_0}{C_{cut}} \times T \tag{33}$$

where

- B_0 represents current forest vegetation biomass;
- C_{cut} represents forest vegetation biomass cut down every time;
- T represents interval of cutting down forest

In the best management strategy, T_h is equal to 20 years. Under the assumption, the whole forest will be harvested once every 10 years. Let T converge to 0:

$$\lim_{T \to 0} T_h = 19 \tag{34}$$

It means the harvest overloads the recovery capability of the forests, and the amount of harvest is larger than that of growth of forests. Excessive logging can reduce soil nutrients , which may make the rate of growth v_0 lower, and make the forests reduce to "young-growth forest".

Therefore, our priority has to be to increase the interval between harvest and to reduce the felling volume. Considering the profit of economy, the felling volume ought to decline year by year in order to satisfy the local need of wood. Meanwhile, the authority should try to improve the industrial structure and reduce the demand for wood, while profiting more.

Besides, it is wise to upgrade land nutrients through small-scale mountain fires or fertilizer. More trees should be planted to help forests return to healthy and sustainable condition.

7 Model Evaluation

7.1 Strengths and Weaknesses

7.1.1 Strengths

1) A specific expression for the amount of carbon sequestered is derived, which visually demonstrates the process of increasing carbon sequestration.

2) By adopting an analytical approach it is possible to study quite generally the relation between carbon sequestration, vegetative growth, rotation length, and the carbon retention properties of products. In this way, the effectiveness of different management strategies for fixing carbon dioxide can be assessed and understood in a general context.

3) The model is able to combine various factors to give a detailed allocation scheme.It provides greater guidance on the allocation of local wood products.

4) A very detailed classification of forest carbon sequestration and wood product types is presented, and various submodels are proposed to bring the models closer to reality and make the results more reliable.

7.1.2 Weaknesses

1) No consideration of details such as the age of the different trees, felling methods, etc.

2) No analytical solution is available, computer simulation based on experimental data is required.

3) Lack of specificity in the management scheme for the transition phase.

7.2 Scalability and Adaptability Analysis

The model is suitable for large scale forest ecosystems where the main plant species are distributed in relatively close proportions. It can be very effective in giving a statistically significant solution to forest management.

However, for small-scale forests or forests with an uneven distribution of plant species, the felling scheme of the model is not always feasible. This is because felling in proportion to the number of plant species may destroy the competitive relationships between the original forest populations and eventually lead to changes in the forest community and a reduction in ecological stability. Even if the felling scheme is not ideal, the model's allocation is still very informative and effective in the short term.

Based on this, there is still room for improvement in the model, such as taking into account the distribution of forest species, the age of the trees, and quantifying the opportunity cost of carbon sequestration, so that a more rational and detailed felling scheme can be given.

7.3 Sensitivity Analysis

To the robust of our model, we change the value of growth rate(v_0) from its 90% to 110% and plot the relationship between relative error and the change of growth rate under the condition of 100 years and 1000 years as Fig.13 and Fig.14 show:

The relative error is very low, showing that out model have an excellent robust.

8 Conclusion

Model I: WCSM can be used to acquire the optimal T, and meanwhile B_{t_0} can be changed. This means a simulation system can be completed by the computer, where T and B_{t_0} can be changed arbitrarily. Changes in carbon sequestration over a long period of time can be observed during this process. The data analysis is as shown in Fig.7

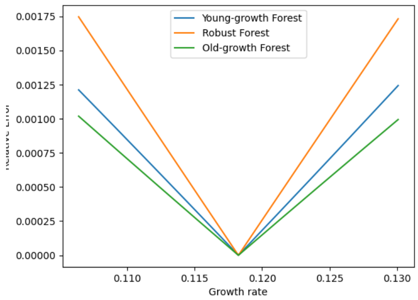


Figure 13: Relative Error over 100 Years

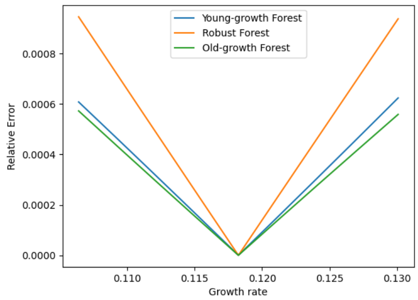


Figure 14: Relative Error over 1000 Years

After fully considering social, economic, cultural and ecological factors, we built a decision model: FADM. This model is based on the Entropy Weight Method, so the more indicators we consider, the higher the accuracy of the results will be. Although the Entropy Weight Method gives the weight from the perspective of information theory, it often ignores the importance of indicators. In order to further improve the model, we consider adding weights to various indicators, so that the allocation scheme will be more reasonable for decision makers.

One of the advantages of this model is that it does not require specific information such as the age of the tree. But to put it in another way, this is also where the model could be improved in the future. The introduction of more detailed information about the forest would make the felling scheme given by the whole model more efficient and effective.

"Things are not always what they seem."
— Phaedrus

Forest Times

Greater Khingan Range

VOL.I. . . No.1 *FEBRUARY 22, 2022* ZERO YUAN

Fell Forests Fairly for Fortune and Future

By TEAM#2209563

Forests are meritorious for sequestering the carbon, conserving the water and fixing the sand, and forest products have great economic benefits. However, a case study example of Greater Khingan Range in China shows that natural and economic values of forests do not necessarily contradict each other. Contrary to popular belief, the research indicates that well-planned deforestation not only raises local incomes, but also contributes to controlling carbon emissions and reducing global warming.

Forests with a variety of resources and a vast area yield high economic benefits. In 2020, the forestry output value of Da Hinggan Ling Prefecture (a prefecture-level administrative region on the northeast slope of Greater Khingan Range) was 4.54 billion yuan, up 4.4% from 2019 and accounting for 32% of the regional GDP. In addition to traditional currencies, forestry can be integrated into the carbon currey and carbon trading market system. A sound forest management strategy is conducive to a bright future of the carbon market. It is in line with the expectation that China will be carbon neutral as well as the world's largest forestry carbon market by mid-century, entering the virtuous development cycle of "Forestry Investment — Forestry Growth — Carbon Sink Increase — Carbon Sink Trading — Forestry Investment".

The carbon actually remains sequestered in timber products which may even live longer than the surviving vegetation, and thus appropriate logging can mitigate climate change while ensuring economic returns. The northeast China forest region, where Greater Khingan Range is located, is the main production area of disposable chopsticks. The raw materials for disposable chopsticks are mainly wood trimmings and fast-growing forests with poor quality and few uses. Contrary to wasting the lumber, disposable chopsticks make use of the trimmings and fast-growing forests instead, also reducing the time and money cost of sterilizing tableware and improving economic efficiency while being environmentally friendly.

Speaking of environmental protection, the first to occur to some people may be reforestation or forest conservation. But according to the latest findings of an anonymous research team on the forests of Greater Khingan Range, a wise forest management plan should include harvesting without excessive logging. It is said that the best tree felling scheme is to cut 21.98 $Mg{\cdot}ha^{-1}$ of trees every five years from the beginning, according to the charateristics of Robuster Forests, which are in prime of life. If a forest is left untouched for a long time, the average tree ages will increase and the growth of carbon sequestration will actually be significantly reduced. The best industry allocation scheme is to assign more lumber to the paper and pulp industry, which accounts for about 43.9%, followed by the sawmill industry with about 36.2%. The research team recommends to increase harvest intervals and reduce logging volumes year by year while meeting the local demand for timber and economic benefits. One of the researchers, who wished to remain unidentified, concluded, "The local industry structure should be improved to reduce the demand. It is wise to improve land nutrients through small-scale hill fires or fertilization. And the authorities can consider planting more trees to help restore the forest to healthy condition for better long-term development."

Living in a modern society, we should think rationally and calmly about what the environment actually needs and what the relationship between humans and nature should be, instead of simply being brainwashed by environmentalism. 21st-century environmental concept should be practical and sensible, not to mention proper logging have both ecological and economic benefits. Managers of Greater Khingan Range and other forest areas with similar environments should be justified in considering felling forests fairly for fortune as well as future.

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Appendices

Input python source:

```
import math
import numpy as np
import matplotlib.pyplot as plt
from scipy import integrate


class carbon_model:
    def __init__(self, MAT, MAP, AET, B_0, T, k3):
        self.MAT = MAT
        self.MAP = MAP
        self.AET = AET
```

```
        self.B_0 = B_0
        self.T = T
        self.k3 = k3
        self.v_0 = self.v0_definition()
        self.k1 = self.k1_definition()
        self.k2 = self.k2_definition()
        self.B_max = self.B_max_definition()
        self.cut = self.growth_single_T()
        self.TT = self.B_0 / self.cut * self.T

    def v0_definition(self):
        return 0.081 * math.exp(0.038 * self.MAT) + \
               0.065 * np.log(self.MAP) - 0.38

    def k1_definition(self):
        return 0.7 * pow(10, -1.4553 + 0.0014175 * self.AET)

    def k2_definition(self):
        beta = 0.105
        gamma = 0.00274
        D_0 = -32
        s_1 = 0.00815 * self.MAT + 0.731719
        return 0.012 * (1 + s_1 * beta * (self.MAT - 3.3) +
                        gamma * (self.MAP - self.AET - D_0))

    def B_max_definition(self):
        return 3442.194 * (0.05 + math.exp(0.000158 * self.MAP)) / \
               (1 + math.exp(-0.037 * (self.MAT - 95.606))) + 33.128

    def bio(self, t, t_start):
        return self.B_max / (1 + (self.B_max / self.B_0 - 1) *
                             math.exp(-self.v_0 * (t - t_start)))

    def soil(self, B_t):
        I_t = B_t * 0.05 * 0.03
        return I_t / self.k2

    def carbon_sink(self, t, t_start):
        B_t = self.bio(t, t_start)
        return 0.5 * B_t + self.soil(B_t)

    def growth_show(self, year, data_num):
        x = np.linspace(0, year, data_num).tolist()
        y = [self.bio(i, 0) for i in x]
        print(x)
        print(y)
        plt.plot(x, y)
        plt.show()

    def carbon_sink_show(self, year, data_num):
        x = np.linspace(0, year, data_num).tolist()
        y = [self.carbon_sink(i, 0) for i in x]
        plt.plot(x, y)
        plt.show()

    def decay_of_WP(self, t, t_start):
        return self.cut * math.exp(-self.k3 * (t - t_start))
```

```
    def growth_single_T(self):
        return self.bio(self.T, 0) - self.B_0

    def growth_T(self, t):
        cut_num = int(t / self.T)
        bio_num = self.carbon_sink(t, cut_num * self.T)
        ciw = 0
        for i in range(cut_num):
            ciw += self.decay_of_WP(t, (i + 1) * self.T)
        return bio_num + ciw

    def carbon_all_show(self, year, data_num):
        x = np.linspace(0, year, data_num).tolist()
        y = [self.growth_T(i) for i in x]
        y2 = [self.average(i) for i in x]
        TK = plt.gca()

    def function1(self, T_):
        cut = self.bio(T_, 0) - self.B_0
        alpha = (self.B_max / self.B_0 - 1)
        beta = self.B_max
        return T_ * cut + T_ * (1 / self.k3) * \
               beta * alpha * self.v_0 * math.exp(- self.v_0 * T_) / (
                        (1 + alpha * math.exp(-self.v_0 * T_)) ** 2)

    def function2(self, T_):
        cut = self.bio(T_, 0) - self.B_0

        def f(x):
            return self.B_max / (1 + (self.B_max / self.B_0 - 1) *
                                 math.exp(-self.v_0 * x))

        v, err = integrate.quad(f, 0, T_)
        return v + 1 / self.k3 * cut

    def solution(self):
        x = np.linspace(1, 50, 10000).tolist()
        y1 = [self.function2(i) for i in x]
        y2 = [self.function1(i) for i in x]
        pos = 0
        for i in range(2, len(y1)):
            if abs(y1[i] - y2[i]) < 100:
                pos = i
        print(pos / (10000 / 50))
        answer = round(pos / (10000 / 50))
        # print(answer)
        if answer == 0:
            answer = 1
        self.T = answer
        return answer

    def average(self, t):

        def f(x):
            return self.B_max / (1 + (self.B_max / self.B_0 - 1) *
                                 math.exp(-self.v_0 * x))

        v, err = integrate.quad(f, 0, self.T)
```

```
        return 0.5 * self.B_0 + 158.3 + 1 / self.T * v + \
               self.cut / self.T * (-1 / self.k3 *
                                    math.exp(-self.k3 * t) + 1 / self.k3)

    def max_carbon(self, year):
        return self.average(year)
```