

陸面過程パラメタリゼーション  
“Minimal Advanced Treatments of  
Surface Interaction and RunOff  
(MATSIRO)”  
の記述

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# 1 Introduction.

Minimal Advanced Treatments of Surface Interaction and RunOff (MATSIRO) is a land-surface process parameterization that has been developed to introduce the global climate model CCSR/NIES AGCM and other models. MATSIRO is mainly designed to be used for long-term climate calculations for integration from one month to several hundred years, coupled with atmospheric models with grid scales of several 10 km or more. In the development of the system, we paid attention to the fact that the processes to be taken into account in the water-energy cycle of the atmospheric land system are represented as adequately as possible and are modeled as concisely as possible so that the results can be easily interpreted. The development of MATSIRO was based on the CCSR/NIES AGCM5.4g land surface sub-model, combined with the vegetation canopy parameterization by Watanabe (1994), and at the same time improved the processes of snow accumulation and runoff. Subsequently, the AGCM was modified to correspond to the current AGCM5.6 through the modification of the AGCM structure, including the use of a flux coupler and parallelization. As for the physiological processes of the vegetation, the Farquhar-type photosynthesis scheme, which was initially used as a Jarvis-type function for stomatal resistance, has been transferred from the SiB2 code to the Farquhar-type photosynthesis scheme, which is now used as a quasi-standard scheme due to recent advances in climate-ecosystem interaction research.

## 2 Composition of MATSIRO

### 2.1 MATSIRO Overall

#### 2.1.1 Overall Structure.

MATSIRO consists of ,

- Flux Calculation Unit (LNDFLX)
- Land Surface Integrator (LNDSTP)

This is similar to the standard land process routine of AGCM5.6. This is the same as the standard land process routine of AGCM5.6.

The flux calculator is called in the time step of the atmospheric model as part of the physical process of the atmospheric model. In contrast, the land surface integrator is called from the coupled model main routine as the land surface model equivalent to the atmospheric and oceanic models. The time step is set independently from the atmospheric model.

The data exchange between the fluxing section and the land surface integrating section is carried out through the flux coupler. The flux calculation section treats snow-covered and snow-free surfaces separately and obtains surface fluxes for each of them. The fluxes are weighted and averaged according to the ratio of the snow-covered and snow-free surfaces, and the fluxes are passed through the flux coupler to the land surface integrating unit. At the same time, the temporal averaging operation is also carried out.

Programmatically, the driver program (matdrv.F) contains an entry for the flux calculation part, LNDFLX, and an entry for the land surface integration part, LNDSTP, from which the subroutines of the necessary processes are called. LNDFLX and LNDSTP share the internal variables of MATSIRO.

### 2.1.2 Internal variables of MATSIRO

MATSIRO has the following internal variables.

	Header0	Header1	Header2	Header3	$T_{s(l)}$
$(l = 1, 2)$	surface	.L.A.[K.R.I.E.D]	$T_{c(l)}$	$(l = 1, 2)$	
	temperature				
Canopy	.L.A.[K.R.I.E.D.]				
temperature					
	$T_{g(k)}$	$(k = 1, \dots, K_g)$	Soil	.L.A.[K.R.I.E.D]	$w_{\{(k)\}}$
			temperature		
$(k = 1, \dots, K_g)$	soil moisture	$[m^3/m^3]$	$w_{i(k)}$	$(k = 1, \dots, K_g)$	
	content				

Frozen soil  
moisture  
content

$[m^3/m^3]$

$w_c$

amount of  
snowfall

$[kg/m^2]$

snow

$\alpha_{sn(b)}$

temperature  
state of affairs  
in the state of  
affairs of others

$[-]$

Water content  
in the canopy

$T_{Sn(k)}$  ( $k =$   
 $1, \dots, K_{Sn}$ )

( $b = 1, 2, 3$ ) snow albedo

where  $l = 1, 2$  are snow-free and snow-covered areas, respectively,  $k$  is the vertical layer number of soil and snow cover (top layer is 1 and increases downward),  $K_g$  is the number of soil layers,  $K_{Sn}$  is the number of snow-covered layers,  $b = 1, 2, 3$  are It represents the visible, near-infrared and infrared wavelengths.

In standard practice, there are five layers of soil, with the thicknesses of each layer being 5cm, 20cm, 75cm, 1m, and 2m from the top. The definition points of soil temperature, soil moisture and frozen soil moisture are the same.

The number of layers of the snowpack is variable, and the number of layers increases as the snowpack increases. The standard number of layers is three at most.

The surface temperature and the canopy temperature are so-called skin temperature, which have no heat capacity, but are formally predictive variables (the current calculation method uses the stability evaluated in the previous step, and therefore depends on the value of the previous step. If we evaluate the stability and so on with the updated value and repeat the calculation until the convergence is achieved, then we have a complete diagnostic variable that does not depend on the value of the previous step.) The other variables are all forecast variables that require the values of the previous step.

The surface temperature and the canopy temperature are updated in

the flux calculation section. The other variables (the original forecast variables) are all updated in the land surface integration part.

## 2.2 Flux calculation section

### 2.2.1 Structure of the flux calculation section

The flux calculation section proceeds as follows.

1. receive input variables from the coupler.
2. diagnose the snow area ratio of the subgrid.

For the snow-free area ( $l = 1$ ) and the snow-covered area ( $l = 2$ ), sub-routines are called to calculate the fluxes and to update the surface and canopy temperatures. Concretely, the following subroutines are called in sequence.

MATLAI vegetation shape parameter (LAI, vegetation height)

2. calculation of MATRAD radiation parameters (albedo, vegetation permeability)

MATBLK calculation of turbulent parameters (bulk coefficient) (momentum and he

4. MATRST calculation of stomatal resistance, bare ground evaporation resistan

5. MATBLQ calculation of turbulent parameters (bulk coefficient) (water vapor)

6. calculation of MATFLX surface flux

7. calculation of MATGHC ground heat transfer

8. MATSHB Deciphering the energy balance of the ground surface and canopy

4. pass output variables to the coupler.

Register and add history output variables. 5.



(a) to (e) are classified into boundary value submodels and the programs are summarized in matbnd.

(f)-(h) are classified as surface submodels, and the programs are summarized in matsfc.

The photosynthesis scheme MATPHT is called from (d). The photosynthesis scheme is summarized in matpht.

### 2.2.2 The input variables in the flux calculation section.

The following variables are entered into the flux calculation section.

[]@llll@		Header0	Header1	Header2	Header3	$u_a$
		Atmospheric	"m/score	$v_a$		
		1st layer				
		east-west wind				
First layer of	"m/score	$T_a$			Atmospheric	
atmospheric					1st layer	
north-south					temperature	
wind						
.L.A.[K.R.I.E.D]	$q_a$			Specific	[kg/kg\]	$P_a$
				humidity in the		
				first layer of the		
				atmosphere		
		Atmospheric	state of affairs	$P_s$		
		1st layer	of the city of			
		pressure	Los			
			Angeles[Pahren]			
surface pressure	state of affairs					
	of the city of					
	Los					
	Angeles[Pahren]					
	$R_{(d,b)}^\downarrow$	$(d = 1, 2; b =$	Surface	/m <sup>2</sup> ]		$\cos \zeta$
		$1, 2, 3)$	downward			
			radiation flux			

cosine of the	state of affairs
solar zenith	of the company
angle	in the first
	place is in the
	form of the
	following.

where  $d = 1, 2$  represent the direct line and scattering, respectively, and  $b = 1, 2, 3$  represent the visible, near-infrared, and infrared spectral bands, respectively.

### 2.2.3 Output variables in the flux calculation section

The following variables are output from the flux calculation section.

$\tau_x$  surface-to-west wind stress  $\tau_y$  surface-to-south wind stress  $H$  surface sensible heat flux  $E$  Surface Water Vapor Flux  $R_S^\uparrow$  Upward Bound Shortwave Radiation Flux  $R_L^\uparrow$  Upward Bound Longwave Radiation Flux  $\alpha_{s(b)}$  ( $b = 1, 2, 3$ ) surface albedo state of affairs of the world  $T_{sR}$  surface radiation temperature .L.A.[K.R.I.E.D.]  $F_{g(1/2)}$  Surface Heat Transfer Flux  $F_{Sn(1/2)}$  Heat Transfer Flux for Snow Surface  $Et_{(i,j)}$  ( $i = 1, 2; j = 1, 2, 3$ ) Evapotranspiration components  $\Delta F_{conv}$  surface energy convergence  $F_{root(k)}$  ( $k = 1, \dots, K_g$ ) Root sucking flux  $LAI$  leaf area index  $A_{Snc}$  Canopy freezing area ratio shit... [-]

Here,  $i = 1, 2$  in evapotranspiration represent liquid and solid, respectively, and  $j = 1, 2, 3$  represent bare ground (forest floor) evaporation, evaporation and evaporation of water on the canopy, respectively. The other subscripts are the same as those in the previous section.

## 2.3 Land surface integral part

### 2.3.1 Composition of the Land Surface Integrator

The land surface integration section proceeds as follows.

1. receive input variables from the coupler.

The subroutines of each process are called and the land surface forecast variables are updated. Specifically, the following subroutines are called in sequence.

1. calculation of the water balance of the MATCNW canopy

Calculation of MATSNW snowpack, snow temperature and snow albedo

3. calculation of MATROF runoff

MATGND Calculation of soil temperature, soil moisture, and frozen ground

3. pass output variables to the coupler.

Register and add history output variables. 4.

Each subroutine called from the main part of the land surface integrator consists of a submodel. The programs of each sub-model are organized in a single file. In concrete terms, each subroutine is described as follows

- MATCNW (matchnw.F) Canopy water balance sub-model
- MATSNW (matsnw.F) snow sub-model
- MATROF (matrof.F) Spillover sub-model
- MATGND (matgnd.F) Soil sub-model

The submodels are basically executed in this order, but if necessary, subroutines may be called among the submodels in order to refer to the values of the parameters managed by other submodels. Subroutines within the above submodels may also be called by the fluxing part for the same purpose.

### 2.3.2 Input variables in the land surface integration section

The following variables are entered into the land surface integrator.

Header0	Header1	Header2	Header3	
Convective	[kg/m <sup>2</sup> /s]	$Pr_l$		$Pr_c$
rainfall flux				

Layered Rainfall Flux	[kg/m <sup>2</sup> /s]	$P_{Snc}$		Convective snowfall flux
	[kg/m <sup>2</sup> /s]	$P_{Snl}$	Layered Snowfall Flux	[kg/m <sup>2</sup> /s] $F_{g(1/2)}$
		Surface Heat Transfer Flux	"W/m <sup>2</sup> \	$F_{Sn(1/2)}$
Heat Transfer Flux for Snow Surface	/m <sup>2</sup> /s]	$Et_{(i,j)}$	( $i = 1, 2; j =$ $1, 2, 3)$	Evapotranspiration components
	[kg/m <sup>2</sup> /s]	$\Delta F_{conv}$	surface energy convergence	$F_{root(k)}$
( $k = 1, \dots, K_g$ )	Root sucking flux	[kg/m <sup>2</sup> /s]	$LAI$	
leaf area index	state of affairs of the state [m <sup>2</sup> /m <sup>2</sup> ]	$A_{Snc}$		Canopy freezing area ratio
state of affairs in the state of affairs of the company				
[\$-0101]				

### 2.3.3 Output variables of the land surface integration section

The land surface integrator outputs the following variables.

Ro runoff [kg/m<sup>2</sup>/s]

The runoff is used as an input variable for the river network model.

## 2.4 External Parameters.

There are two types of external parameters for executing MATSIRO: one is the parameter values of each grid by the horizontal distribution (map), and the other is the parameter values for each land cover type or soil type by the table. Land cover type and soil type are one of the

parameters given by the map, and each parameter given by the table is assigned to each grid. Namely,

$$\phi(i, j) \quad (1)$$

\.066} or .066.066.

where  $(i, j)$  are indices of the horizontal position of the grid,  $I_L$  is the land use type, and  $I_S$  is the soil type. where  $(i, j)$  are the indexes of the horizontal position of the grid,  $I_L$  is the land use type, and  $I_S$  is the soil type.

#### 2.4.1 External parameters given by the map

The types of external parameters given by the map are as follows.

[]@lllll@						
	Header0	Header1	Header2	Header3	Header4	$I_L$
	Land cover type	constant	\.com	$I_S$		
Soil Type	constant	\.com	$LAI_0$		Leaf Area Index (LAI)	
every month	$[m^2/m^2]$	$\alpha_{0(b)}$	$(b = 1, 2, 3)$	Ground surface (forest floor) albedo	constant	
\.com	$\tan \beta_s$		Tangent of the mean surface slope	constant	\.com	$\sigma_z$
	elevation standard deviation	constant	\0.25\0.25\0.25\0.25\0.25\0.25\0.25\0.00}.			

### 2.4.2 External parameters given by the table for each land cover type

The external parameters given by the table for each land cover type are as follows

	Header0	Header1	Header2	Header3	$h_0$
	vegetation	$\backslash 0.25 \backslash 0.25 \backslash 0.25 \backslash 0.25 \backslash 0.25 \backslash 0.25 \backslash 0.00 \}$ .			
	height				
Height of the bottom of the canopy	$\backslash 0.25 \backslash 0.25 \backslash 0.25 \backslash 0.25 \backslash 0.25 \backslash 0.25 \backslash 0.00 \}$ .			Reflectivity of individual leaves	
$\backslash .com$	$t_{f(b)}$	$(b=1,2)$	Transmittance of individual leaves	$\backslash .com$	$f_{root(k)}$
$(k = 1, \dots, K_g)$	Percentage of root presence	$\backslash .com$	$c_d$		
Momentum exchange coefficient between the individual leaves and the atmosphere	$\backslash .com$	$c_h$		Heat Exchange Coefficient between individual leaves and the atmosphere	
$\backslash .com$	$f_V$		vegetation coverage	$\backslash .com$	$V_{\max}$
	Rubisco Reaction Capacity	"m/score	$m$		
$A_n-g_s$ Slope of the relationship	$\backslash .com$	$b$		$A_n-g_s$ relationship intercepts	
"m/score	$\epsilon_3, \epsilon_4$		Photosynthetic efficiency per photon		



### 3 Boundary Value Submodel MATBND

#### 3.1 Set of vegetation shape parameters.

The leaf area index (LAI) and vegetation height are set as vegetation shape parameters.

The LAI reads the seasonally varying horizontal distributions, and the top and bottom canopy heights are determined by land use type as external parameters. If there is snowfall, the LAI considers only the vegetation above the snow depth and corrects the geometry parameters.

$$h = \max(h_0 - D_{Sn}, 0) \quad (2)$$

$$h_B = \max(h_{B0} - D_{Sn}, 0) \quad (3)$$

$$LAI = LAI_0 \frac{h - h_B}{h_0 - h_{B0}} \quad (4)$$

Here,  $h$  is the height at the top of the canopy (vegetation height),  $h_B$  is the height at the bottom of the canopy (dead height),  $LAI$  is the leaf area index, and  $h_0$ ,  $h_{B0}$ , and  $LAI_0$  are the values without snow, respectively.  $D_{Sn}$  is the snow cover depth. It is assumed that the LAI is uniformly distributed vertically between the top and bottom edges of the canopy.

Afterwards, the average of snow-free and snow-covered surfaces weighted by the snow area ratio ( $A_{Sn}$ ) is calculated, but since the snow-free surface and the snow-covered surface are calculated separately,  $A_{Sn}$  requires either 0 (snow-free surface) or 1 (snow-covered surface). Note that it contains either of the values of the (surface), and mixing of values does not occur here (there are several similar locations later).

#### 3.2 Calculating radiant parameters.

Calculation of radiative parameters (albedo, vegetation permeability, etc.).

##### 3.2.1 Calculation of surface (forest floor) albedo

The horizontal distribution of the ground (forest floor) albedo  $\alpha_{0(b)}$  ( $b = 1, 2$ ) is read as an external parameter.  $b = 1, 2$  represent the visible and



near-infrared wavelengths, respectively. The  $\alpha_{0(3)}$  value of the infrared surface albedo ( $\alpha_{0(3)}$ ) is set to a fixed value (horizontal distribution may be prepared).

The dependence of the incident angle of albedo on ice and snow cover is considered as a function of the angle of incidence as follows

$$\alpha_{0(d,b)} = \hat{\alpha}_{0(b)} + (1 - \hat{\alpha}_{0(b)}) \cdot 0.4(1 - \cos \phi_{in(d)})^5 \quad (5)$$

where  $b = 1, 2$  are wavelengths,  $d = 1, 2$  are direct and scattered, respectively, and  $\hat{\alpha}_{0(b)}$  is an albedo value for a direct angle of incidence of 0 (from directly above).  $\cos \psi_{in(d)}$  is the cosine of the incident angle of incidence for direct and scattered light, respectively,

$$\cos \psi_{in(1)} = \cos \zeta, \quad \cos \psi_{in(2)} = \cos 50^\circ \quad (6)$$

We give the  $\zeta$  is the solar zenith angle.

Except for the ice and snow cover surfaces, the albedo of the ground (forest floor) gives the same values for direct and diffuse light, without considering the dependence on the zenith angle. In other words, the results are as follows.

$$\alpha_{0(d,b)} = \alpha_{0(b)} \quad (d = 1, 2; b = 1, 2) \quad (7)$$

For infrared wavelengths, we only need to consider the scattered light. The infrared albedo gives a value that is independent of the zenith angle for all surfaces.

$$\alpha_{0(2,3)} = \alpha_{0(3)} \quad (8)$$

### 3.2.2 Canopy albedo and transmittance calculations

The calculation of albedo and transmittance of the canopy is based on the radiation calculation in the canopy layer by Watanabe and Otani (1995).

Considering a vertically uniform canopy and making some simplifying assumptions, the transfer equation and boundary conditions for the insolation within the canopy are expressed as follows

$$\frac{dS_d^\downarrow}{dL} = -F \sec \zeta S_d^\downarrow \quad (9)$$

$$\frac{dS_r^\downarrow}{dL} = -F(1 - t_{f(b)})d_f S_r^\downarrow + F t_{f(b)} \sec \zeta S_d^\downarrow + F r_{f(b)} d_f S_r^\uparrow \quad (10)$$

$$\frac{dS_r^\uparrow}{dL} = F(1 - t_{f(b)})d_f S_r^\uparrow - F r_{f(b)}(d_f S_r^\downarrow + \sec \zeta S_d^\downarrow) \quad (11)$$

$$S_d^\downarrow(0) = S_d^{top} \quad (12)$$

$$S_r^\downarrow(0) = S_r^{top} \quad (13)$$

$$S_r^\uparrow(LAI) = \alpha_{0(1,b)} S_d^\downarrow(LAI) + \alpha_{0(2,b)} S_r^\downarrow(LAI) \quad (14)$$

where  $S_d^\downarrow$  is direct downward light,  $S_r^\uparrow$  and  $S_r^\downarrow$  are upward and downward scattered light, respectively,  $L$  is the leaf area accumulated from the top of the canopy downward,  $d_f$  is the scatter factor ( $= \sec 53^\circ$ ),  $r_{f(b)}$  and  $t_{f(b)}$  is the reflection coefficient and transmission coefficient of the leaf surface (the same values are used for scattered light and direct light), respectively, and  $F$  is a factor indicating the orientation of the leaf relative to radiation. For simplicity, we assume that the distribution of leaf orientation is random ( $F = 0.5$ ).

These can be solved analytically and the solution is as follows.

$$S_d^\downarrow(L) = S_d^{top} \exp(-F \cdot L \cdot \sec \zeta) \quad (15)$$

$$S_r^\downarrow(L) = C_1 e^{aL} + C_2 e^{-aL} + C_3 S_d^\downarrow(L) \quad (16)$$

$$S_r^\uparrow(L) = A_1 C_1 e^{aL} + A_2 C_2 e^{-aL} + C_4 S_d^\downarrow(L) \quad (17)$$

Here,

$$a = Fd_f[(1 - t_{f(b)})^2 - r_{f(b)}^2]^{1/2} \quad (18)$$

$$A_1 = \{1 - t_{f(b)} + [(1 - t_{f(b)})^2 - r_{f(b)}^2]^{1/2}\} / r_{f(b)} \quad (19)$$

$$A_2 = \{1 - t_{f(b)} - [(1 - t_{f(b)})^2 - r_{f(b)}^2]^{1/2}\} / r_{f(b)} \quad (20)$$

$$A_3 = (A_1 - \alpha_{0(2,b)})e^{aLAI} - (A_2 - \alpha_{0(2,b)})e^{-aLAI} \quad (21)$$

$$C_1 = \{-(A_2 - \alpha_{0(2,b)})e^{-aLAI}(S_r^{top} - C_3S_d^{top}) + [C_3\alpha_{0(2,b)} + \alpha_{0(1,b)} - C_4]S_d^\downarrow(LAI)\} / A_3 \quad (22)$$

$$C_2 = \{(A_1 - \alpha_{0(2,b)})e^{aLAI}(S_r^{top} - C_3S_d^{top}) - [C_3\alpha_{0(2,b)} + \alpha_{0(1,b)} - C_4]S_d^\downarrow(LAI)\} / A_3 \quad (23)$$

$$C_3 = \frac{\sec \zeta [t_{f(b)} \sec \zeta + d_f t_{f(b)} (1 - t_{f(b)}) + d_f r_{f(b)}^2]}{d_f^2 [(1 - t_{f(b)})^2 - r_{f(b)}^2] - \sec^2 \zeta} \quad (24)$$

$$C_4 = \frac{r_{f(b)}(d_f - \sec \zeta) \sec \zeta}{d_f^2 [(1 - t_{f(b)})^2 - r_{f(b)}^2] - \sec^2 \zeta} \quad (25)$$

It is.

The Albedo  $\alpha_s$  seen at the top of the canopy,

$$S_r^\uparrow(0) = \alpha_{s(1,b)}S_d^\downarrow(0) + \alpha_{s(2,b)}S_r^\downarrow(0) \quad (26)$$

So,

$$\alpha_{s(2,b)} = \{A_2(A_1 - \alpha_{0(2,b)})e^{aLAI} - A_1(A_2 - \alpha_{0(2,b)})e^{-aLAI}\} / A_3 \quad (27)$$

$$\alpha_{s(1,b)} = -C_3\alpha_{s(2,b)} + C_4 + (A_1 - A_2)(C_3\alpha_{0(2,b)} + \alpha_{0(1,b)} - C_4)e^{-F \cdot LAI \cdot \sec \zeta} / A_3 \quad (28)$$

Get.

The transmission coefficient of the canopy ( $\mathcal{T}_c$ ), or more precisely, the percentage of the incident light absorbed by the forest floor at the top of the canopy, is

$$S_d^\downarrow(LAI) + S_r^\downarrow(LAI) - S_r^\uparrow(LAI) = \mathcal{T}_{c(1,b)}S_d^\downarrow(0) + \mathcal{T}_{c(2,b)}S_r^\downarrow(0) \quad (29)$$

Defined by ,

$$\mathcal{T}_{c(2,b)} = \{(1 - A_2)(A_1 - \alpha_{0(2,b)}) - (1 - A_1)(A_2 - \alpha_{0(2,b)})\} / A_3 \quad (30)$$

$$\mathcal{T}_{c(1,b)} = -C_3\mathcal{T}_{c(2,b)} \quad (31)$$

$$+ \{(C_3\alpha_{0(2,b)} + \alpha_{0(1,b)} - C_4)((1 - A_1)e^{aLAI} - (1 - A_2)e^{-aLAI}) / A_3 + C_3 - C_4 + 1\} e^{-F \cdot LAI \cdot \sec \zeta} \quad (32)$$

$$(33)$$

The above is performed for  $b = 1, 2$  (visible and near-infrared), respectively. The above procedure is performed for  $b = 1, 2$  (visible and near-infrared), respectively.

The reflectance  $r_f$  and transmission  $t_f$  are read as external parameters for each land cover type, but before they are used in the above calculations, the following two modifications are made.

1. the effect of snow (ice) on the leaves When the canopy temperature is less than  $0^\circ \text{C}$ , the water above the canopy is regarded as snow (ice). In this case, the snow albedo ( $\alpha_{Sn(b)}$ ) and the water content in the canopy ( $w_c$ ) are used to determine the snow (ice),

$$r_{f(b)} = (1 - f_{cwet})r_{f(b)} + f_{cwet}\alpha_{Sn(b)} \quad (34)$$

$$f_{cwet} = w_c/w_{c,cap} \quad (35)$$

The following table shows the volume of water on the canopy.  $w_{c,cap}$  is the water content in the canopy. The transmittance is given in the following formula for convenience to avoid negative absorption ( $1 - r_f - t_f$ ), i.e.

$$t_{f(b)} = (1 - f_{cwet})t_{f(b)} + f_{cwet}t_{Sn(b)}, \quad t_{Sn(b)} = \min(0.5(1 - \alpha_{Sn(b)}), t_{f(b)}) \quad (36)$$

When the water on the canopy is liquid, we should ignore the change in the radiative parameters of the leaf surface. When the liquid water on the canopy is liquid, changes in the radiative parameters of the leaf surface due to the liquid water on the canopy are ignored. In the case of snowfall trapping by the canopy (snow), and in the case of freezing of the liquid water on the canopy (ice), the same albedo as that of the snow cover on the forest floor is used here, although the radiative characteristics of each case may be different.

2. the effect of considering the direction of reflection and transmission In the solution of the above equation, it is assumed that all the reflected light returns to the direction of the incident light, but considering that some of it is scattered in the same direction as the incident light, we can replace the radial parameters of the leaf surface with the following (Watanabe, in press).

$$r_{f(b)} = 0.75r_{f(b)} + 0.25t_{f(b)} \quad (37)$$

$$t_{f(b)} = 0.75t_{f(b)} + 0.25r_{f(b)} \quad (38)$$

The above is done for  $b = 1, 2$  (visible and near-infrared), respectively.

We also take into account cases where vegetation is unevenly distributed in parts of the grid (e.g., savannahs), prior to the calculation of albedo, etc,

$$LAI = LAI/f_V \quad (39)$$

The LAI of the vegetation cover (the original LAI is considered to be the grid average) is calculated as the LAI of the vegetation cover, which is used in the calculation of the albedo described above.  $f_V$  is the vegetation coverage of the grid. After the albedo and other data are obtained, the LAI of the grid is calculated by

$$\alpha_{s(d,b)} = f_V\alpha_{s(d,b)} + (1 - f_V)\alpha_{0(d,b)} \quad (40)$$

$$\mathcal{T}_{c(d,b)} = f_V\mathcal{T}_{c(d,b)} + (1 - f_V)(1 - \alpha_{0(d,b)}) \quad (41)$$

We take the area-weighted average of the vegetation-covered and non-vegetation-covered portions, as

### 3.2.3 Calculations such as surface radiation flux

Using the downward radiation flux ( $R_{(d,b)}^\downarrow$ ) and the albedo obtained above, the following radiation fluxes are obtained.

$$R_S^\downarrow = \sum_{b=1}^2 \sum_{d=1}^2 R_{(d,b)}^\downarrow \quad (42)$$

$$R_S^\uparrow = \sum_{b=1}^2 \sum_{d=1}^2 \alpha_{s(d,b)} R_{(d,b)}^\downarrow \quad (43)$$

$$R_L^\downarrow = R_{(2,3)}^\downarrow \quad (44)$$

$$R_S^{gnd} = \sum_{b=1}^2 \sum_{d=1}^2 \mathcal{T}_{s(d,b)} R_{(d,b)}^\downarrow \quad (45)$$

$$PAR = \sum_{d=1}^2 R_{(d,1)}^\downarrow \quad (46)$$

$R_S^\downarrow$  and  $R_S^\uparrow$  represent the downward and upward shortwave radiation fluxes,  $R_L^\downarrow$  represents the downward longwave radiation flux,  $R_S^{gnd}$  represents the shortwave radiation flux absorbed by the forest floor, and  $PAR$  represents the downward Photosynthetically Active Radiation (PAR) flux.

The canopy transmittance of the shortwave and longwave canopies and the longwave emission coefficient are obtained as follows.

$$\mathcal{T}_{cS} = R_S^{gnd} / (R_S^\downarrow - R_S^\uparrow) \quad (47)$$

$$\mathcal{T}_{cL} = \exp(-F \cdot LAI \cdot d_f) \quad (48)$$

$$\epsilon = 1 - \alpha_{s(2,3)} \quad (49)$$

### 3.3 Calculation of turbulent parameters (bulk coefficients).

Calculate the turbulence parameters (bulk coefficient).

#### 3.3.1 roughness calculations for momentum and heat.

The calculation of roughness is based on Watanabe (1994). Using the results of Kondo and Watanabe's (1992) multi-layered canopy model,

Watanabe (1994) proposed the following roughness functions for the bulk model that best fit the results.

$$\left(\ln \frac{h-d}{z_0}\right)^{-1} = \left[1 - \exp(-A^+) + \left(-\ln \frac{z_{0s}}{h}\right)^{-1/0.45} \exp(-2A^+)\right]^{0.45} \quad (50)$$

$$\left(\ln \frac{h-d}{z_T^\dagger}\right)^{-1} = \frac{1}{-\ln(z_{Ts}/h)} \left[\frac{P_1}{P_1 + A^+ \exp(A^+)}\right]^{P_2} \quad (51)$$

$$\left(\ln \frac{h-d}{z_0}\right)^{-1} \left(\ln \frac{h-d}{z_T}\right)^{-1} = C_T^\infty \left[1 - \exp(-P_3 A^+) + \left(\frac{C_T^0}{C_T^\infty}\right)^{1/0.9} \exp(-P_4 A^+)\right]^{0.9} \quad (52)$$

$$h-d = h[1 - \exp(-A^+)]/A^+ \quad (53)$$

$$A^+ = \frac{c_d LAI}{2k^2} \quad (54)$$

$$\frac{1}{C_T^0} = \ln \frac{h-d}{z_0} \ln \frac{h-d}{z_T^\dagger} \quad (55)$$

$$C_T^\infty = \frac{-1 + (1 + 8F_T)^{1/2}}{2} \quad (56)$$

$$P_1 = 0.0115 \left(\frac{z_{Ts}}{h}\right)^{0.1} \exp \left[5 \left(\frac{z_{Ts}}{h}\right)^{0.22}\right] \quad (57)$$

$$P_2 = 0.55 \exp \left[-0.58 \left(\frac{z_{Ts}}{h}\right)^{0.35}\right] \quad (58)$$

$$P_3 = [F_T + 0.084 \exp(-15F_T)]^{0.15} \quad (59)$$

$$P_4 = 2F_T^{1.1} \quad (60)$$

$$F_T = c_h/c_d \quad (61)$$

where  $z_0$  and  $z_T$  are the roughness of the entire canopy with respect to momentum and heat, respectively,  $z_{0s}$  and  $z_{Ts}$  are the roughness of the ground surface (forest floor) with respect to momentum and heat, respectively,  $c_d$  and  $c_h$  are the roughness of the ground surface with respect to momentum and heat, respectively  $h$  is the vegetation height,  $d$  is the zero-plane displacement, and  $LAI$  is the LAI for the heat transfer coefficient between the individual leaves and the atmosphere for the  $z_T^\dagger$  is the roughness of the heat flux at the leaf surface under the assumption of no heat flux, and is used to determine the heat transport coefficient from the forest floor.

$z_{0s}$  and  $z_{Ts}$  are given as external data for each land cover type, but

the standard values ( $z_{0s} = 0.05$  m,  $z_{Ts} = 0.005$  m) are fixed regardless of land cover type. However, for snow cover, the following modifications are made.

$$z_{0s} = \max(f_{Sn}z_{0s}, z_{0Sn}) \quad (62)$$

$$z_{Ts} = \max(f_{Sn}z_{0s}, z_{TSn}) \quad (63)$$

$$f_{Sn} = 1 - D_{Sn}/z_{0s} \quad (64)$$

where  $D_{Sn}$ ,  $z_{0Sn}$  and  $z_{TSn}$  are the roughness of the snow surface with respect to momentum and heat, respectively.

$c_d$  and  $c_h$  are parameters determined by the shape of the leaves and are given as external data for each land cover type.

### 3.3.2 Calculating Bulk Coefficients for Momentum and Heat

The bulk coefficients are also derived following Watanabe (1994), using Monin-Obukhov's law of similarity, as follows

$$C_M = k^2 \left[ \ln \frac{z_a - d}{z_0} + \Psi_m(\zeta) \right]^{-2} \quad (65)$$

$$C_H = k^2 \left[ \ln \frac{z_a - d}{z_0} + \Psi_m(\zeta) \right]^{-1} \left[ \ln \frac{z_a - d}{z_T} + \Psi_h(\zeta) \right]^{-1} \quad (66)$$

$$C_{Hs} = k^2 \left[ \ln \frac{z_a - d}{z_0} + \Psi_m(\zeta_g) \right]^{-1} \left[ \ln \frac{z_a - d}{z_T^\dagger} + \Psi_h(\zeta_g) \right]^{-1} \quad (67)$$

$$C_{Hc} = C_H - C_{Hs} \quad (68)$$

where  $C_M$  and  $C_H$  are the bulk coefficient of total canopy (foliage + forest floor) for momentum and heat, respectively,  $C_{Hs}$  is the bulk coefficient of surface (forest floor) flux for heat, and  $C_{Hc}$  is the bulk coefficient of canopy (leaf surface) flux for heat. Bulk coefficients,  $\Psi_m$  and  $\Psi_h$  are Monin-Obukhov shear functions for momentum and heat, respectively, and  $z_a$  is the reference altitude of the atmosphere (altitude of the first layer of the atmosphere).  $\zeta$  and  $\zeta_g$  are calculated using the Monin-Obukhov length  $L$  and  $L_g$  for the whole canopy and ground surface (forest floor), respectively,



$$\zeta = \frac{z_a - d}{L} \quad (69)$$

$$\zeta_g = \frac{z_a - d}{L_g} \quad (70)$$

where the length is denoted by The Monin-Obukhov length is expressed in

$$L = \frac{\Theta_0 C_M^{3/2} |V_a|^2}{kg(C_{Hs}(T_s - T_a) + C_{Hc}(T_c - T_a))} \quad (71)$$

$$L_s = \frac{\Theta_0 C_M^{3/2} |V_a|^2}{kgC_{Hs}(T_s - T_a)} \quad (72)$$

is expressed in where  $\Theta_0 = 300\text{K}$ ,  $|V_a|$  is the absolute surface wind speed,  $k$  is the Kalman constant,  $g$  is the acceleration due to gravity,  $T_a$ ,  $T_c$ , and  $T_s$  are the first atmospheric layer, the canopy (leaf surface) and the ground (forest floor), respectively ) temperature.

Since the bulk coefficients are required for the calculation of the Monin-Obukhov length and the Monin-Obukhov length is required for the calculation of the bulk coefficients, the neutral bulk coefficients are used as the initial values and are repeated (twice in the standard case).

Prior to this calculation, the snow depth is added to the zero-plane displacements, but the zero-plane displacements should be limited to a maximum value so that they are not too large compared to the  $z_a$ .

$$d = \min(d + D_{Sn}, f_{\max} \cdot z_a) \quad (73)$$

The standard is taking  $f_{\max}$  to 0.5.

### 3.3.3 Calculating Bulk Coefficients for Water Vapor

This calculation is performed after the calculation of the stomatal resistance, which is described below.

Once the stomatal resistance ( $r_{st}$ ) and surface evaporation resistance ( $r_{soil}$ ) are obtained, the bulk coefficient for water vapor is calculated as follows

$$C_{Ec}|V_a| = [(C_{Hc}|V_a|)^{-1} + r_{st}/LAI]^{-1} \quad (74)$$

$$C_{Es}|V_a| = [(C_{Hs}|V_a|)^{-1} + r_{soil}]^{-1} \quad (75)$$

(Previously, the pore resistance was calculated via roughness by converting the pore resistance into a reduction in the exchange coefficient, but this method was changed to a simpler method because it seemed to be problematic.)

Note that the bulk coefficient of water vapor is the same as the bulk coefficient of heat when no stomatal resistance is applied (e.g., evaporation from a wet surface).

### 3.4 calculation of stomatal resistance.

The calculation of the stomatal resistance is based on the photosynthesis-stomatal model based on Farquhar et al. (1980), Ball (1988), Collatz et al. The code of SiB2 (Sellers et al., 1996) is used almost verbatim, except for the method to determine the resistance of the entire canopy. It is also possible to use Jarvis-type empirical equations instead, but the explanation is omitted here.

#### 3.4.1 Calculating Soil Moisture Stress Factors.

Determine the soil water stress on transpiration. The soil water stress factor for each soil layer is determined and the overall soil stress factor is calculated by weighting the stress factor by the distribution of roots in each layer.

Soil water stress in each layer is assessed using SiB2 (Sellers et al., 1996) as a guide, using the following equation

$$f_{w(k)} = [1 + \exp(0.02(\psi_{cr} - \psi_k))]^{-1} \quad (k = 1, \dots, K_g) \quad (76)$$

Overall soil stressors are ,

$$f_w = \sum_{k=1}^{K_g} f_{w(k)} f_{root(k)} \quad (77)$$

Here,  $f_{root(k)}$  is the ratio of the presence of roots in each layer and is an external parameter for each land cover type. This is  $\sum_{k=1}^{K_g} f_{root(k)} = 1$ .

In addition, the weights that distribute the transpiration to the siphoning flux in each layer are calculated as follows.

$$f_{rootup(k)} = f_{w(k)} f_{root(k)} / f_w \quad (k = 1, \dots, K_g) \quad (78)$$

Note that it is  $\sum_{k=1}^{K_g} f_{rootup(k)} = 1$ .

### 3.4.2 Calculating Photosynthesis

Following SiB2 (Sellers et al., 1996), we calculate the amount of photosynthesis.

We believe that the amount of photosynthesis is determined by three upper limits.

$$A \leq \min(w_c, w_e, w_s) \quad (79)$$

$w_c$  is the upper limit of photosynthetic enzyme (Rubisco) efficiency, and  $w_e$  is the upper limit of photosynthetically available radiation.  $w_s$  is the upper limit of the sink of photosynthetic products in the case of plants, and  $C_4$  is the upper limit of the concentration of  $\text{CO}_2$  in the case of plants (Collatz et al., 1991, 1992).

The size of each is estimated as follows.

$$w_c = \begin{cases} V_m \left[ \frac{c_i - \Gamma^*}{c_i + K_c(1 + O_2/K_O)} \right] & (C_3 \text{ 植物の場合}) \\ V_m & (C_4 \text{ 植物の場合}) \end{cases}$$

$$\begin{aligned}
& ) \\
w_e &= \begin{cases} PAR \cdot \epsilon_3 \left[ \frac{c_i - \Gamma^*}{c_i + 2\Gamma^*} \right] (C_3 \text{ 植物の場合} \\ )PAR \cdot \epsilon_4 (C_4 \text{ 植物の場合} \end{cases} \\
w_s &= \begin{cases} V_m/2 (C_3 \text{ 植物の場合} \\ )V_m c_i/5 (C_4 \text{ 植物の場合} \end{cases} \quad ) \quad (80)
\end{aligned}$$

where  $V_m$  is the Rubisco reaction capacity,  $c_i$  is the partial pressure of  $\text{CO}_2$  in the stomatal chamber,  $O_2$  is the partial pressure of oxygen in the stomatal chamber, and  $PAR$  is the photosynthetically available radiation (PAR).  $\Gamma^*$  is the compensation point of  $\text{CO}_2$  and is represented by  $\Gamma^* = 0.5O_2/S$ .  $K_c$ ,  $K_O$ , and  $S$  are functions of temperature and will be shown in functional form later.  $\epsilon_3$  and  $\epsilon_4$  are constants determined by vegetation type.

(76) is actually solved as follows to represent a smooth transition between the different upper bounds.

$$\beta_{ce} w_p^2 - w_p(w_c + w_e) + w_c w_e = 0 \quad (81)$$

$$\beta_{ps} A^2 - A(w_p + w_s) + w_p w_s = 0 \quad (82)$$

Solving the two equations in sequence, choosing the smaller of the two solutions for each equation, yields  $A_n$ .  $\beta_{ce}, \beta_{ps}$  are constants determined by vegetation type. Note that when  $\beta = 1$ , it coincides with a simple minimum value operation.

Once the amount of photosynthesis is determined, the net photosynthesis amount ( $A_n$ ) is determined as follows.

$$A_n = A - R_d \quad (83)$$

$R_d$  is the respiratory rate and is expressed as

$$R_d = f_d V_m \quad (84)$$

$f_d$  is a constant determined by vegetation type.

$V_m$ , for example, depends on temperature and soil moisture as follows ( $V_m$  depends differently on temperature depending on the term that appears, but is represented by the same  $V_m$ ).

$$V_m = V_{\max} f_T(T_c) f_w \quad (85)$$

$$K_c = 30 \times 2.1^{Q_T} \quad (86)$$

$$K_O = 30000 \times 1.2^{Q_T} \quad (87)$$

$$S = 2600 \times 0.57^{Q_T} \quad (88)$$

$$f_T(T_c) = \begin{cases} 2.1^{Q_T} / \{1 + \exp[s_1(T_c - s_2)]\} & (C_3 \text{ の } w_c, w_e \text{ のとき}) \\ 1.8^{Q_T} / \{1 + \exp[s_3(s_4 - T_c)]\} & (C_3 \text{ の } w_s \text{ のとき}) \end{cases} \quad (89)$$

$$2.1^{Q_T} / \{1 + \exp[s_1(T_c - s_2)]\} / \{1 + \exp[s_3(s_4 - T_c)]\} \quad (C_4 \text{ の } w_c, w_e \text{ のとき}) \quad (90)$$

$$1.8^{Q_T} \quad (C_4 \text{ の } w_s \text{ のとき}) \quad (91)$$

$$2^{Q_T} / \{1 + \exp[s_5(T_c - s_6)]\} \quad (R_d \text{ のとき})$$

$$Q^T = (T_c - 298) / 10 \quad (92)$$

Here,  $V_{\max}$ ,  $s_1, \dots, s_6$  are constants determined by the vegetation type.

Given the above values for  $V_{\max}$ ,  $PAR$ ,  $c_i$ ,  $T_c$  and  $f_w$ , the amount of photosynthesis in each individual leaf can be calculated. In reality, these values can be considered to be distributed evenly within the same canopy, but we assumed that  $c_i$ ,  $T_c$ , and  $f_w$  are the same for all leaves and that the vertical distributions of  $V_{\max}$  and  $PAR$  are considered. The vertical distribution of the  $PAR$  is large at the top of the canopy and diminishes as it moves downward, and the distribution of the  $V_{\max}$  is considered to be similar to that of the  $PAR$  following this distribution of the  $PAR$ .

The average vertical distribution of the  $PAR$  (and therefore the vertical distribution of the  $V_{\max}$ ) is shown in the following table.

$$PAR(L) = PAR^{top} \exp(-f_{atn} a L) \quad (93)$$

Here,  $L$  is the leaf area accumulated from the top of the canopy,  $PAR^{top}$  is the  $PAR$  at the top of the canopy,  $a$  is the attenuation factor defined

in (17), and  $f_{atn}$  is a constant for adjustment. Using this value, the factor ( $f_{avr}$ ) which represents the average value of  $PAR$  is defined as follows.

$$f_{avr} = \int_0^{LAI} PAR(L)dL / (LAI \cdot PAR^{top}) = \frac{1 - \exp(-f_{atn}aL)}{f_{atn}a} \quad (94)$$

Since each term in  $A_n$  ( $w_c, w_s, w_e, R_d$ ) is proportional to  $V_{\max}$  or  $PAR$ , on the assumption that the vertical distributions of  $V_{\max}$  and  $PAR$  are proportional to those of  $V_{\max}$  at the top end of the canopy. By multiplying the  $A_n$  calculated using the  $PAR$  value by  $f_{avr}$ , the average leaf photosynthetic rate ( $\overline{A_n}$ ) can be obtained.

$$\overline{A_n} = f_{avr}A_n \quad (95)$$

Hereinafter, we will refer to it again as  $A_n$ .

### 3.4.3 Stomatal Resistance Calculations.

Net photosynthesis ( $A_n$ ) and stomatal conductance ( $g_s$ ) are related by the quasi-empirical formula of Ball (1988) as follows

$$g_s = m \frac{A_n}{c_s} h_s + b f_w \quad (96)$$

where  $c_s$  is the  $CO_2$  molar fraction at the leaf surface (the number of mol of  $CO_2$  per 1mol of air),  $f_w$  is the soil moisture stress factor, and  $m$  and  $b$  are constants determined by vegetation type.

$h_s$  is the relative humidity at the leaf surface, defined as

$$h_s = e_s / e_i \quad (97)$$

$e_s$  is the mole fraction of water vapor at the leaf surface,  $e_i$  is the mole fraction of water vapor in the stomata, and  $e_i = e^*(T_c)$  is the mole fraction of water vapor in the stomata.  $e^*$  is the mole fraction of saturated water vapor.

Assuming that the water vapor flux from the stomatal surface to the leaf surface is equal to the water vapor flux from the leaf surface to the atmosphere (i.e., there is no convergent water vapor divergence at the leaf surface),

$$g_s(e_i - e_s) = g_l(e_s - e_a) \quad (98)$$

than ,

$$e_s = (g_l e_a + g_s e_i) / (g_l + g_s) \quad (99)$$

is obtained. where  $e_a$  is the atmospheric water vapor mole fraction and  $g_l$  is the conductance between the leaf surface and the atmosphere.  $g_l$  is represented by  $g_l = C_{Hc}|V_a|/LAI$  using the bulk coefficient.

Similarly, given the lack of convergent divergence of  $CO_2$  on the leaf surface,

$$A_n = g_l(c_a - c_s)/1.4 = g_s(c_s - c_i)/1.6 \quad (100)$$

than ,

$$c_s = c_a - 1.4A_n/g_l \quad (101)$$

$$c_i = c_s - 1.6A_n/g_s \quad (102)$$

where  $c_a$  and  $c_i$  are obtained from the atmosphere and pores, respectively. where  $c_a$  and  $c_i$  are the  $CO_2$  mole fractions in the atmosphere and in the pores, respectively. 1.4 and 1.6 are constants that appear due to the difference in diffusion coefficients of water vapor and  $CO_2$ .

Substituting (94) and (96) into (93), we obtain the following equation for  $g_s$ .

$$Hg_s^2 + (Hg_l - e_i - Hbf_w)g_s - g_l(Hbf_w + e_a) = 0 \quad (103)$$

However,

$$H = (e_i c_s) / (m A_n) \quad (104)$$

and use (99) for  $c_s$ .

Of the two solutions of (100), the larger of the two solutions makes more sense. From the above, assuming that  $A_n$  is known, we can solve for  $g_s$ , but we use  $c_i$  to solve for  $A_n$ .  $c_i$  can be obtained by (99) if  $g_s$  is obtained. In other words, finding  $g_s$  requires  $A_n$  and finding  $A_n$  requires  $c_i$  and thus  $g_s$ , so the calculation must be repeated.

The algorithm for iterative computation is ported from SiB2. Six iterations are performed and the next solution is estimated in the order of increasing errors to accelerate the convergence.

Finally, using the stomatal conductance, the stomatal resistance is expressed as

$$r_{st} = 1/g_{st} \quad (105)$$

#### 3.4.4 Calculation of Surface Evaporation Resistance

Calculate the surface evaporation resistance ( $r_{soil}$ ) and the relative humidity ( $h_{soil}$ ) of the first layer of soil as follows.

$$r_{soil} = a_1(1 - W_{(1)}) / (a_2 + W_{(1)}) \quad (106)$$

$$h_{soil} = \exp\left(\frac{\psi_{(1)}g}{R_{air}T_{g(1)}}\right) \quad (107)$$

where  $W_{(1)} = w_{(1)}/w_{sat(1)}$  is the saturation degree of the first soil layer,  $\psi_1$  is the moisture potential of the first soil layer,  $g$  is the gravitational acceleration,  $R_{air}$  is the gas constant of air, and  $T_{g(1)}$  is the temperature of the first soil layer.  $a_1$  and  $a_2$  are constants and the standard uses  $a_1 = 800$ ,  $a_2=0.2$ .



## 4 Earth Surface Submodel MATSFC

### 4.1 Calculation of surface turbulence flux.

The bulk method is used to obtain the turbulent flux at the surface as follows. When the surface energy balance is solved and the surface temperature ( $T_s$ ) and the canopy temperature ( $T_c$ ) are updated, the surface flux is also updated with respect to these values. The value obtained here is a provisional value until then. In order to linearize the energy balance for  $T_s$  and  $T_c$ , the derivatives of each flux for  $T_s$  and  $T_c$  have been calculated.

- momentum flux

$$\tau_x = -\rho C_M |V_a| u_a \quad (108)$$

$$\tau_y = -\rho C_M |V_a| v_a \quad (109)$$

Here,  $\tau_x$  and  $\tau_y$  are the momentum fluxes (surface stresses) in the east-west and north-south directions, respectively.

- Sensible Heat Flux

$$H_s = c_p \rho C_{Hs} |V_a| (T_s - (P_s/P_a)^\kappa T_a) \quad (110)$$

$$H_c = c_p \rho C_{Hc} |V_a| (T_c - (P_s/P_a)^\kappa T_a) \quad (111)$$

$$\partial H_s / \partial T_s = c_p \rho C_{Hs} |V_a| \quad (112)$$

$$\partial H_c / \partial T_c = c_p \rho C_{Hc} |V_a| \quad (113)$$

where  $H_s$  and  $H_c$  are sensible heat fluxes from the ground surface (forest floor) and canopy (leaf surface), respectively,  $\kappa = R_{air}/c_p$  and  $R_{air}$  are the gas constant of air, and  $c_p$  is the specific heat of air.

- Bare ground (forest floor) evaporation flux

$$Et_{(1,1)} = (1 - A_{Sn})(1 - f_{ice}) \cdot \rho \widetilde{C_{Es}} |V_a| (h_{soil} q^*(T_s) - q_a) \quad (114)$$

$$Et_{(2,1)} = (1 - A_{Sn}) f_{ice} \cdot \rho \widetilde{C_{Es}} |V_a| (h_{soil} q^*(T_s) - q_a) \quad (115)$$

$$\partial Et_{(1,1)} / \partial T_s = (1 - A_{Sn})(1 - f_{ice}) \cdot \rho \widetilde{C_{Es}} |V_a| h_{soil} \cdot dq^* / dT|_{T_s} \quad (116)$$

$$\partial Et_{(2,1)} / \partial T_s = (1 - A_{Sn}) f_{ice} \cdot \rho \widetilde{C_{Es}} |V_a| h_{soil} \cdot dq^* / dT|_{T_s} \quad (117)$$

where  $Et_{(1,1)}$  and  $Et_{(2,1)}$  are water evaporation and ice sublimation fluxes on bare ground, respectively,  $q^*(T_s)$  is the specific humidity at the saturated surface temperature,  $h_{soil}$  is the relative humidity at the soil surface,  $A_{Sn}$  is the snow cover area fraction, and  $f_{ice}$  is the percentage of ice in the first soil layer

$$f_{ice} = w_{i(1)} / w_{(1)} \quad (118)$$

in  $A_{Sn}$ . Note that the value of  $A_{Sn}$  should be either 0 (snow-free surface) or 1 (snow-covered surface) because snow-free and snow-covered surfaces are calculated separately. In the case of downward-facing (condensation) fluxes, no soil moisture resistance is applied, so the bulk coefficient should be calculated as follows

$$\widetilde{C_{Es}} = \begin{cases} C_{Es}(h_{soil} q^*(T_s) - q_a > 0 \text{ のとき}) \\ C_{Hs}(h_{soil} q^*(T_s) - q_a \leq 0 \text{ のとき}) \end{cases} \quad (119)$$

- Transpiration Flux

$$Et_{(1,2)} = (1 - f_{cwt}) \cdot \rho \widetilde{C_{Ec}} |V_a| (q^*(T_c) - q_a) \quad (120)$$

$$Et_{(2,2)} = 0 \quad (121)$$

$$\partial Et_{(1,2)} / \partial T_c = (1 - f_{cwt}) \cdot \rho \widetilde{C_{Ec}} |V_a| \cdot dq^* / dT|_{T_c} \quad (122)$$

$$\partial Et_{(2,2)} / \partial T_c = 0 \quad (123)$$

Here,  $Et_{(1,2)}$  and  $Et_{(2,2)}$  are water and ice transpiration, while  $Et_{(2,2)}$  is always zero.  $f_{cwt} = w_c / w_{c,cap}$  is the wetting area ratio of the canopy.

In the case of downward-facing fluxes, the bulk factor is considered to be condensation on dry parts of the leaves and is calculated as follows

$$\widetilde{C}_{Ec} = \begin{cases} C_{Ec}(q^*(T_c) - q_a > 0 \text{ のとき}) \\ C_{Hc}(q^*(T_c) - q_a \leq 0 \text{ のとき}) \end{cases} \quad (124)$$

- Evaporated flux on the canopy  $T_c \geq 0^\circ \text{C}$

$$Et_{(1,3)} = f_{cwet} \cdot \rho C_{Hc} |V_a| (q^*(T_c) - q_a) \quad (125)$$

$$Et_{(2,3)} = 0 \quad (126)$$

$$\partial Et_{(1,3)} / \partial T_c = f_{cwet} \cdot \rho C_{Hc} |V_a| \cdot dq^*/dT|_{T_c} \quad (127)$$

$$\partial Et_{(2,3)} / \partial T_c = 0 \quad (128)$$

$T_c \geq 0^\circ \text{C}$  In case of  $\text{C}$

$$Et_{(1,3)} = 0 \quad (129)$$

$$Et_{(2,3)} = f_{cwet} \cdot \rho C_{Hc} |V_a| (q^*(T_c) - q_a) \quad (130)$$

$$\partial Et_{(1,3)} / \partial T_c = 0 \quad (131)$$

$$\partial Et_{(2,3)} / \partial T_c = f_{cwet} \cdot \rho C_{Hc} |V_a| \cdot dq^*/dT|_{T_c} \quad (132)$$

Here,  $Et_{(1,3)}$  and  $Et_{(2,3)}$  are the evaporation of water and ice sublimation on the canopy.

- Snow Sublimation Flux

$$E_{Sn} = A_{Sn} \cdot \rho C_{Hs} |V_a| (q^*(T_s) - q_a) \quad (133)$$

$$\partial E_{Sn} / \partial T_s = A_{Sn} \cdot \rho C_{Hs} |V_a| \cdot dq^*/dT|_{T_s} \quad (134)$$

$E_{Sn}$  is a snow sublimation flux. Note that since snow-free and snow-covered

## 4.2 Calculating heat transfer flux.

Calculating the heat conduction fluxes on snow-free and snow-covered surfaces. As well as the turbulent fluxes, the heat conduction fluxes are also updated when the surface temperature is updated after the energy balance is solved.

Note that since snow-free and snow-covered surfaces are calculated separately, the snow coverage area ratio ( $A_{Sn}$ ) should be set to either 0 (snow-free surface) or 1 (snow-covered surface), as shown below.

- Heat Transfer Flux on Snow-Free Surfaces

$$F_{g(1/2)} = (1 - A_{Sn}) \cdot k_{g(1/2)} / \Delta z_{g(1/2)} (T_{g(1)} - T_s) \quad (135)$$

$$\partial F_{g(1/2)} / \partial T_s = -(1 - A_{Sn}) \cdot k_{g(1/2)} / \Delta z_{g(1/2)} \quad (136)$$

where  $F_{g(1/2)}$  is the heat transfer flux,  $k_{g(1/2)}$  is the thermal conductivity of the soil,  $\Delta z_{g(1/2)}$  is the thickness of the first layer of soil from the temperature definition point to the ground surface, and  $T_{g(1)}$  is the temperature of the first layer of soil.

- Heat Transfer Flux of Snow Surface

$$F_{Sn(1/2)} = A_{Sn} \cdot k_{Sn(1/2)} / \Delta z_{Sn(1/2)} (T_{Sn(1)} - T_s) \quad (137)$$

$$\partial F_{Sn(1/2)} / \partial T_s = A_{Sn} \cdot k_{Sn(1/2)} / \Delta z_{Sn(1/2)} \quad (138)$$

where  $F_{Sn(1/2)}$  is the heat transfer flux,  $k_{Sn(1/2)}$  is the thermal conductivity of the snowpack,  $\Delta z_{Sn(1/2)}$  is the thickness of the first layer of snow from the temperature definition point to the ground surface, and  $T_{Sn(1)}$  is the temperature of the first layer of snowpack.

## 4.3 Surface, Solving the Canopy Energy Balance

The energy balance is solved for two cases: 1: the case of no melting and 2: the case of melting at the surface. In Case 2, we fix the surface

temperature ( $T_s$ ) to 0° C, and diagnose the energy available for melting based on the energy balance. Since the snow melting on vegetation will be processed by correction later, we do not solve this case separately here. The case in which the snow melts completely within a time step is also processed by later corrections.

#### 4.3.1 Surface, Canopy Energy Balance

The amount of energy dissipation at the ground surface (forest floor) is ,

$$\Delta F_s = H_s + R_s^{net} + lEt_{(1,1)} + l_s(Et_{(2,1)} + E_{Sn}) - F_{g(1/2)} - F_{Sn(1/2)} \quad (139)$$

However,  $l$  and  $l_s$  are the latent heat of evaporation and sublimation, respectively, and  $R_s^{net}$  is the net radiative divergence at the ground surface,

$$R_s^{net} = -(R_S^\downarrow - R_S^\uparrow)\mathcal{T}_{cS} - \epsilon R_L^\downarrow \mathcal{T}_{cL} + \epsilon \sigma T_s^4 - \epsilon \sigma T_c^4(1 - \mathcal{T}_{cL}) \quad (140)$$

$\sigma$  is the Stefan-Boltzman constant.

The amount of energy dissipation in the canopy (leaf surface) is ,

$$\Delta F_c = H_c + R_c^{net} + l(Et_{(1,2)} + Et_{(1,3)}) + l_s(Et_{(2,2)} + Et_{(2,3)}) \quad (141)$$

However,  $R_c^{net}$  is the net radiative divergence in the canopy,

$$R_c^{net} = -(R_S^\downarrow - R_S^\uparrow)(1 - \mathcal{T}_{cS}) - \epsilon R_L^\downarrow(1 - \mathcal{T}_{cL}) + (2\epsilon \sigma T_c^4 - \epsilon \sigma T_s^4)(1 - \mathcal{T}_{cL}) \quad (142)$$

#### 4.3.2 Case 1: In the absence of surface melting

In the absence of melting of the ground surface, as  $\Delta F_s = \Delta F_c = 0$ , we solve for  $T_s$  and  $T_c$  so that the energy balance between the ground surface and the canopy is maintained.

The linearized energy balance equation for each term for  $T_s$  and  $T_c$  is,

$$\begin{pmatrix} \Delta F_s \\ \Delta F_c \end{pmatrix}^{current} = \begin{pmatrix} \Delta F_s \\ \Delta F_c \end{pmatrix}^{past} + \begin{pmatrix} \partial \Delta F_s / \partial T_s & \partial \Delta F_s / \partial T_c \\ \partial \Delta F_c / \partial T_s & \partial \Delta F_c / \partial T_c \end{pmatrix} \begin{pmatrix} \Delta T_s \\ \Delta T_c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (143)$$

I could write.

The part marked with *past* on the right-hand side is the flux calculated from (107) to (134) using the values of  $T_s$  and  $T_c$  in the previous step and substituted into (136) to (139).

The differential term is ,

$$\frac{\partial \Delta F_s}{\partial T_s} = \frac{\partial H_s}{\partial T_s} + \frac{\partial R_s^{net}}{\partial T_s} + l \frac{\partial Et_{(1,1)}}{\partial T_s} + l_s \left( \frac{\partial Et_{(2,1)}}{\partial T_s} + \frac{\partial E_{Sn}}{\partial T_s} \right) - \frac{\partial F_{g(1/2)}}{\partial T_s} - \frac{\partial F_{Sn(1/2)}}{\partial T_s} \quad (144)$$

$$\frac{\partial \Delta F_s}{\partial T_c} = \frac{\partial R_s^{net}}{\partial T_c} \quad (145)$$

$$\frac{\partial \Delta F_c}{\partial T_s} = \frac{\partial R_c^{net}}{\partial T_s} \quad (146)$$

$$\frac{\partial \Delta F_c}{\partial T_c} = \frac{\partial H_c}{\partial T_c} + \frac{\partial R_c^{net}}{\partial T_c} + l \left( \frac{\partial Et_{(1,2)}}{\partial T_c} + \frac{\partial Et_{(1,3)}}{\partial T_c} \right) + l_s \left( \frac{\partial Et_{(2,2)}}{\partial T_c} + \frac{\partial Et_{(2,3)}}{\partial T_c} \right) \quad (147)$$

However,

$$\frac{\partial R_s^{net}}{\partial T_s} = \epsilon 4 \sigma T_s^3 \quad (148)$$

$$\frac{\partial R_s^{net}}{\partial T_c} = -(1 - \mathcal{T}_{cL}) \epsilon 4 \sigma T_c^3 \quad (149)$$

$$\frac{\partial R_c^{net}}{\partial T_s} = -(1 - \mathcal{T}_{cL}) \epsilon 4 \sigma T_s^3 \quad (150)$$

$$\frac{\partial R_c^{net}}{\partial T_c} = 2(1 - \mathcal{T}_{cL}) \epsilon 4 \sigma T_c^3 \quad (151)$$

Using the above, solve (140) for  $T_s$  and  $T_c$ .

#### 4.3.3 Case 2: When there is ground surface melting

Melting of the ground surface occurs when there is snow or ice cover on the ground surface and the temperature of the ground surface ( $T_s^{current} = T_s^{past} + \Delta T_s$ ) is higher than 0 ° C after the solution in Case 1. If there

is ground surface melting, the surface temperature is fixed at 0 ° C. In other words, the surface temperature is fixed at 0 ° C,

$$\Delta T_s = \Delta T_s^{melt} = T_{melt} - T_s^{past} \quad (152)$$

is the melting point of ice (0 ° C).  $T_{melt}$  is the melting point of ice (0 ° C).

Assuming that  $T_c$  is known,  $\Delta T_s$  is obtained by the following equation as well as (140).

$$\Delta T_c = \left( -\Delta F_c^{past} - \frac{\partial \Delta F_c}{\partial T_s} \Delta T_s^{melt} \right) / \frac{\partial \Delta F_c}{\partial T_c} \quad (153)$$

If the  $\Delta T_s$  and  $\Delta T_c$  are thus known, the energy convergence at the surface used for melting will be

$$\Delta F_{conv} = -\Delta F_s^{current} = -\Delta F_s^{past} - \frac{\partial \Delta F_s}{\partial T_s} \Delta T_s^{melt} - \frac{\partial \Delta F_s}{\partial T_c} \Delta T_c \quad (154)$$

It is obtained by.

#### 4.3.4 Constraints on solutions.

We set some constraints on the solution of the surface energy balance. If the condition is violated, the energy balance is re-solved by fixing the violated flux at the limit of the condition to be met.

1. don't take too much water vapor from the first layer of the atmosphere A large downward latent heat may be generated due to temporary computational instability. Even in such a case, the condition is set so that all the water vapor from the surface to the first layer of the atmosphere is not lost.

$$Et_{(i,j)}^{current} > -q_a(P_s - P_a)/(g\Delta t) \quad (i = 1, 2; j = 1, 2, 3) \quad (155)$$

$$E_{Sn}^{current} > -q_a(P_s - P_a)/(g\Delta t) \quad (156)$$

Here,  $g$  and  $\Delta t$  are the acceleration due to gravity and the time step of the atmospheric model. For the values such as  $Et$  used in the judgment, an updated flux value (*current*) is used with respect to the value of  $T_s$  and  $T_c$ , which have been updated to satisfy the energy balance. This is the same for all other cases below. Updating the flux value will be described later.

2. soil moisture is not negative. Prevent soil moisture from becoming negative through transpiration.

$$Et_{(1,2)}^{current} < \sum_{k \in \text{rootzone}} \rho_w w_k \Delta z_{g(k)} / \Delta t_L \quad (157)$$

where  $\rho_w$  is the density of water and  $\Delta t_L$  is the time step of the land surface model.

3. no negative water content on the canopy Ensure that water on the canopy does not become negative by evaporation.

$$Et_{(i,3)}^{current} < \rho_w w_c / \Delta t_L \quad (i = 1, 2) \quad (158)$$

4. the snowpack is not negative Ensure that the snowpack does not become negative due to sublimation of the snowpack.

$$E_{Sn}^{current} < Sn / \Delta t_L \quad (159)$$

#### 4.3.5 Ground Surface, Canopy Temperature Update

Update surface and canopy temperatures.

$$T_s^{current} = T_s^{past} + \Delta T_s \quad (160)$$

$$T_c^{current} = T_c^{past} + \Delta T_c \quad (161)$$



Based on the updated canopy temperature, it is necessary to diagnose whether the water in the canopy is liquid or solid. This information will be used in the future when dealing with the freeze and thaw of the water on the canopy.

$$A_{Snc} = \begin{cases} 0 & (T_c \geq T_{melt}) \\ 1 & (T_c < T_{melt}) \end{cases} \quad (162)$$

The  $A_{Snc}$  is the freezing area fraction of water on the canopy.

#### 4.3.6 Update the value of the flux

Update the flux value with respect to the updated  $T_s$  and  $T_c$  values. If you set the  $F$  to an arbitrary flux, updating the value is done as follows

$$F^{current} = F^{past} + \frac{\partial F}{\partial T_s} \Delta T_s + \frac{\partial F}{\partial T_c} \Delta T_c \quad (163)$$

The updated flux value is used to calculate the flux output to the atmosphere.

$$H = H_s + H_c \quad (164)$$

$$E = \sum_{j=1}^3 \sum_{i=1}^2 Et_{(i,j)} + E_{Sn} \quad (165)$$

$$R_L^\uparrow = \mathcal{T}_{cL} \epsilon \sigma T_s^4 + (1 - \mathcal{T}_{cL}) \epsilon \sigma T_c^4 + (1 - \epsilon) R_L^\downarrow \quad (166)$$

$$T_{sR} = (R_L^\uparrow / \sigma)^{1/4} \quad (167)$$

$T_{sR}$  is the radiant temperature of the ground surface.

The root uptake flux of each soil layer is calculated.

$$F_{root(k)} = f_{rootup(k)} Et_{(1,2)} \quad (k = 1, \dots, K_g) \quad (168)$$

$F_{root(k)}$  is the weight of the uptake flux of the roots, and  $f_{rootup(k)}$  is the weight that distributes the transpiration rate to the uptake flux of the roots in each layer.

## 5 Canopy Water Balance Submodel MATCNW

Calculating the water balance of the upper canopy water component .

### 5.1 Diagnosis of the phases of the canopy upper water portion.

Moisture in the canopy is considered separately as liquid (interrupted precipitation, condensation, or solid moisture that has melted) and solid (interrupted snow, ice, or liquid moisture that has frozen) and allows for a mixture of them. The predictor is only the amount of moisture ( $w_c$ ) of the liquid and solid combined, and depending on whether the canopy temperature ( $T_c$ ) is higher or lower than  $T_{melt} = 0^\circ \text{ C}$ , the case is diagnosed as either liquid or solid, respectively. The reason why liquid and solid can coexist is that the  $T_c$  of snow covered areas and snow-free areas are calculated separately. That is, the freezing area fraction ( $A_{Snc}$ ) of water on the canopy is defined as follows (in reality, the averaged result by the coupler is as follows),

$$A_{Snc} = \begin{cases} 0(T_{c(1)} \geq T_{melt}, T_{c(2)} \geq T_{melt} \text{のとき}) \\ (1 - A_{Sn})(T_{c(1)} < T_{melt}, T_{c(2)} \geq T_{melt} \text{のとき}) \\ A_{Sn}(T_{c(1)} \geq T_{melt}, T_{c(2)} < T_{melt} \text{のとき}) \\ 1(T_{c(1)} < T_{melt}, T_{c(2)} < T_{melt} \text{のとき}) \end{cases} \quad (169)$$

$$w_{cl} = w_c(1 - A_{Snc}) \quad (170)$$

$$w_{ci} = w_c A_{Snc} \quad (171)$$

The following table shows the values for each of the two types of moisture in the canopy.  $w_{cl}$  and  $w_{ci}$  are the liquid and solid moisture on the canopy respectively.

$A_{Snc}$  is given by the coupler as the updated value  $A_{Snc}^{\tau+1}$  in the flux calculation section, but the value of the previous step,  $A_{Snc}^\tau$ , should be stored in the MATCNW.  $\tau$  represents a time step. This will not become a new predictor since it is obtained at the start of the calculation from the initial values of  $T_c$  and  $Sn$ .

## 5.2 Forecast for water content in the canopy

The predictive equations for water content in the canopy are given below for the liquid and the solid, respectively.

$$\rho_w \frac{w_{cl}^{\tau+1} - w_{cl}^{\tau}}{\Delta t_L} = P_{Il} - E_l - D_l + M_c \quad (172)$$

$$\rho_w \frac{w_{ci}^{\tau+1} - w_{ci}^{\tau}}{\Delta t_L} = P_{Ii} - E_i - D_i - M_c \quad (173)$$

$P_{Il}$  and  $P_{Ii}$  are the precipitation cutoff values for a liquid and a solid, respectively,  $E_l$  and  $E_i$  are the evaporation (sublimation) value,  $D_l$ ,  $D_i$  is the drop value, and  $M_c$  is the melting value. Note that the former values of  $w_{cl}^{\tau}$  and  $w_{ci}^{\tau}$  are defined by the following using the former  $A_{Snc}^{\tau}$ .

$$w_{cl}^{\tau} = w_c^{\tau}(1 - A_{Snc}^{\tau}) \quad (174)$$

$$w_{ci}^{\tau} = w_c^{\tau} A_{Snc}^{\tau} \quad (175)$$

### 5.2.1 Evaporation (sublimation) of water on the canopy.

First, the evaporation (sublimation) rate is subtracted and the water content in the canopy is partially updated. The amount of evaporation (sublimation) is determined in the flux calculation section.

$$w_{cl}^* = w_{cl}^{\tau} - E_l \Delta t_L / \rho_w \quad (176)$$

$$w_{ci}^* = w_{ci}^{\tau} - E_i \Delta t_L / \rho_w \quad (177)$$

$$E_l = Et_{(1,3)} \quad (178)$$

$$E_i = Et_{(2,3)} \quad (179)$$

If one of the  $w_{cl}$  or  $w_{ci}$  becomes negative at this time, the other is compensated until the value returns to zero, and the amount of thawing that occurs in this case (or a negative value in the case of freezing) is set in  $M_c$ .

### 5.2.2 Interruption of precipitation by canopy

Precipitation interception and dripping are considered separately for convective and non-convective precipitation areas. The area fraction of convective precipitation area ( $A_c$ ) is assumed to be uniform (with the standard value of 0.1). It is assumed that stratified precipitation is uniform.

$$P_{Il}^c = f_{int}(Pr_c/A_c + Pr_l) \quad (180)$$

$$P_{Il}^{nc} = f_{int}Pr_l \quad (181)$$

$$P_{Ii}^c = f_{int}(P_{Snc}/A_c + P_{Snl}) \quad (182)$$

$$P_{Ii}^{nc} = f_{int}P_{Snl} \quad (183)$$

$P_{Il}^c$  and  $P_{Ii}^c$  are cutoffs for convective precipitation areas and  $P_{Il}^{nc}$  and  $P_{Ii}^{nc}$  are cutoffs for non-convective areas.  $f_{int}$  is the cutoff efficiency, easy

$$f_{int} = \begin{cases} LAI(LAI < 1 \text{ のとき}) \\ 1(LAI \geq 1 \text{ のとき}) \end{cases} \quad (184)$$

This is given by

The water content on the canopy is further partially updated by adding the intercepted precipitation rate.

$$w_{cl}^{c*} = w_{cl}^* + P_{Il}^c \Delta t_L / \rho_w \quad (185)$$

$$w_{cl}^{nc*} = w_{cl}^* + P_{Il}^{nc} \Delta t_L / \rho_w \quad (186)$$

$$w_{ci}^{c*} = w_{ci}^* + P_{Ii}^c \Delta t_L / \rho_w \quad (187)$$

$$w_{ci}^{nc*} = w_{ci}^* + P_{Ii}^{nc} \Delta t_L / \rho_w \quad (188)$$

### 5.2.3 Dropping water into the canopy.

The drop rate is based on both natural drops due to gravity and water overflow in the canopy.

$$D_l^c = \max(w_{cl}^{c*} - w_{c,cap}, 0) + D_g(w_{cl}^{c*}) \quad (189)$$

$$D_l^{nc} = \max(w_{cl}^{nc*} - w_{c,cap}, 0) + D_g(w_{cl}^{nc*}) \quad (190)$$

$$D_i^c = \max(w_{ci}^{c*} - w_{c,cap}, 0) + D_g(w_{ci}^{c*}) \quad (191)$$

$$D_i^{nc} = \max(w_{ci}^{nc*} - w_{c,cap}, 0) + D_g(w_{ci}^{nc*}) \quad (192)$$

Moisture volume in the canopy ( $w_{c,cap}$ ) is derived from the moisture volume per unit leaf area ( $w_{c,max}$ ) and the LAI,

$$W_{c,cap} = W_{c,max} LAI \quad (193)$$

The standard value of  $W_{c,max}$  is 0.2 mm and the same for liquids and solids. The standard value of  $W_{c,max}$  is 0.2mm and the same value is used for liquids and solids.

Gravity-induced spontaneous dropping  $D_g$  follows Rutter et al,

$$D_g(w_c) = D_1 \exp(D_2 w_c) \quad (194)$$

The following values are assumed to be the same for both liquids and solids. The standard value is  $D_1 = 1.14 \times 10^{-11}$ ,  $D_2 = 3.7 \times 10^{-3}$  and the same value is used for liquid and solid.

The values are updated by subtracting the drop volume.

$$w_{cl}^{c**} = w_{cl}^{c*} - D_{Il}^c \Delta t_L / \rho_w \quad (195)$$

$$w_{cl}^{nc**} = w_{cl}^{nc*} - D_{Il}^{nc} \Delta t_L / \rho_w \quad (196)$$

$$w_{ci}^{c**} = w_{ci}^{c*} - D_{Ii}^c \Delta t_L / \rho_w \quad (197)$$

$$w_{ci}^{nc**} = w_{ci}^{nc*} - D_{Ii}^{nc} \Delta t_L / \rho_w \quad (198)$$

#### 5.2.4 Updating and melting the water content in the canopy.

Furthermore, averaging the convective and non-convective precipitation areas gives the updated water content in the canopy.

$$w_{cl}^{**} = A_c w_{cl}^{c**} + (1 - A_c) w_{cl}^{nc**} \quad (199)$$

$$w_{ci}^{**} = A_c w_{ci}^{c**} + (1 - A_c) w_{ci}^{nc**} \quad (200)$$

$$w_c^{\tau+1} = w_{cl}^{**} + w_{ci}^{**} \quad (201)$$

However, if we consider the updated freezing area fraction ( $A_{Snc}$ ), we can conclude that

$$w_{cl}^{\tau+1} = w_c^{\tau+1} (1 - A_{Snc}^{\tau+1}) \quad (202)$$

$$w_{ci}^{\tau+1} = w_c^{\tau+1} A_{Snc}^{\tau+1} \quad (203)$$

Therefore, the melting value  $M_c$  is diagnosed as follows.

$$M_c = -\rho_w (w_{ci}^{\tau+1} - w_{ci}^{**}) / \Delta t_L \quad (204)$$

However, the amount resulting from the evaporation process, if any, is added.

Here, the temperature of the canopy should be changed by the latent heat of melting, but this is not possible because the heat capacity of the canopy is ignored. The temperature of the surrounding atmosphere should be changed, but this is not possible if we wish to close the system with the land surface integrator. To conserve energy in the system, the latent heat of melting is provided as a heat flux to the soil (or to the snowpack).

### 5.3 Fluxes given to the soil, snowpack, and runoff processes

The water flux  $F_w$ , which is fed to the snowpack or runoff process via canopy interception, is applied to convective and non-convective precipitation areas, and to liquids and solids, respectively,

$$F_{wl}^c = (1 - f_{int})(Pr_c/A_c + Pr_l) + D_l^c \quad (205)$$

$$F_{wl}^{nc} = (1 - f_{int})Pr_l + D_l^{nc} \quad (206)$$

$$F_{wi}^c = (1 - f_{int})(P_{Snc}/A_c + P_{Snl}) + D_i^c \quad (207)$$

$$F_{wi}^{nc} = (1 - f_{int})P_{Snl} + D_i^{nc} \quad (208)$$

Convective and stratified precipitation are given separately for use in runoff calculations; for snowfall, they are given together as they are not necessary.

$$Pr_c^* = Ac(F_{wl}^c - F_{wl}^{nc}) \quad (209)$$

$$Pr_l^* = F_{wl}^{nc} \quad (210)$$

$$P_{Sn}^* = AcF_{wl}^c + (1 - Ac)F_{wl}^{nc} \quad (211)$$

$Pr_c^*$ ,  $Pr_l^*$ , and  $P_{Sn}^*$  are the amount of convective precipitation, stratified precipitation, and snowfall after interception by a canopy, respectively.

The energy flux corrections to the soil or snow cover are the same as those in the above cases,

$$\Delta F_{c,conv} = -l_m M_c \quad (212)$$

is the latent heat of melting.  $l_m$  is the latent heat of melting.

## 6 Snow Submodel MATSNW

Calculate the snowpack, snow temperature and snow albedo.

### 6.1 Diagnosis of snow area ratio

In the case of small snow cover, the sub-grid snow cover is taken into account. The area fraction of snow cover,  $A_{Sn}$ , is a unique function of the snow cover,  $Sn$ ,

$$A_{Sn} = \min(Sn/Sn_c, 1)^{1/2} \quad (213)$$

given in . The standard position is  $Sn_c=100[\text{kg/m}^2]$ .

In practice, various factors, such as topography and differences in snow-fall and snowmelt, are expected to affect the snow coverage ratio. For this purpose, the introduction of a sub-grid snow coverage model (SSNOWD) based on Liston (private communication) is under consideration.

$A_{Sn}$  is referred to at the beginning of the flux calculation section and is used to compute an area-weighted average of the various fluxes computed there as follows

$$\overline{F} = (1 - A_{Sn})F_{(1)} + A_{Sn}F_{(2)} \quad (214)$$

Here,  $F_{(1)}$  and  $F_{(2)}$  represent fluxes on a snow-free surface and a snow-covered surface, respectively. Actually, this operation is carried out through the flux coupler.

### 6.2 Vertical division of the snowpack layer

In order to represent the vertical distribution of the snow temperature, the snow is divided into multiple layers and the temperature is defined for each layer when the snow cover is large. The number of layers is variable, and the number of layers is increased as the amount of snowfall increases. The number of layers is variable, and the number of layers increases as the amount of snowfall increases.



The number of layers and the mass of each layer are uniquely determined by the amount of snow cover. In this way, the mass of each layer is not a new forecast variable.

The mass of each layer ( $\Delta\widetilde{S}n_{(k)}$  ( $k = 1, 2, 3$ )) is determined as follows ( $k = 1$  is the top layer).

$$\Delta\widetilde{S}n_{(1)} = \begin{cases} \widetilde{S}n(\widetilde{S}n < 20) \\ 0.5\widetilde{S}n(20 \leq \widetilde{S}n < 40) \\ 20(\widetilde{S}n \geq 40) \end{cases} \quad (215)$$

$$\Delta\widetilde{S}n_{(2)} = \begin{cases} 0(\widetilde{S}n < 20) \\ \widetilde{S}n - \Delta\widetilde{S}n_{(1)}(20 \leq \widetilde{S}n < 60) \\ 0.5(\widetilde{S}n - 20)(60 \leq \widetilde{S}n < 100) \\ 40(\widetilde{S}n \geq 100) \end{cases} \quad (216)$$

$$\Delta\widetilde{S}n_{(3)} = \begin{cases} 0(\widetilde{S}n < 60) \\ \widetilde{S}n - (\Delta\widetilde{S}n_{(1)} + \Delta\widetilde{S}n_{(2)})(\widetilde{S}n \geq 60) \end{cases} \quad (217)$$

Here,

$$\widetilde{S}n = Sn/A_{Sn} \quad (218)$$

and while  $Sn$  represents the grid-averaged snow cover,  $\widetilde{S}n$  represents the snow cover in the snow region. Note that the mass of each layer ( $\Delta\widetilde{S}n_{(k)}$ ) is also a value for the snow cover and not a grid average. The unit for each layer is  $\text{kg/m}^2$ .

It is clear from the above that the standard number of snow layers ( $K_{Sn}$ ) is as follows.

$$K_{Sn} = \begin{cases} 0(\widetilde{S}n = 0) \\ 1(0 < \widetilde{S}n < 20) \\ 2(20 \leq \widetilde{S}n < 60) \\ 3(\widetilde{S}n \geq 60) \end{cases} \quad (219)$$

### 6.3 Calculating the snowpack.

The forecasting equation for snow cover is given by

$$\frac{S_{n^{\tau+1}} - S_{n^{\tau}}}{\Delta t_L} = P_{S_n}^* - E_{S_n} - M_{S_n} + Fr_{S_n} \quad (220)$$

$P_{S_n}^*$  is snowfall flux after interception by canopy,  $E_{S_n}$  is sublimation flux,  $M_{S_n}$  is snowmelt, and  $Fr_{S_n}$  is refreezing of snowmelt water or freezing of rainfall.

### 6.3.1 Sublimation of the snowpack

First, we subtract the amount of sublimation and partially update the snowpack.

$$S_{n^*} = S_{n^{\tau}} - E_{S_n} \Delta t_L \quad (221)$$

$$\Delta \widetilde{S}_{n(1)}^* = \Delta \widetilde{S}_{n(1)}^{\tau} - E_{S_n} / A_{S_n} \Delta t_L \quad (222)$$

If, by chance, the sublimation is greater than the snow accumulation in the first layer, the remaining amount is subtracted from the lower layer. The same is true if the second layer is insufficient.

### 6.3.2 Melting Snowpack.

Next, the heat conduction during the snow cover is calculated and the amount of snowmelt is determined. The method for calculating the heat conduction in the snowpack is described later. The temperature of the snow cover updated by heat conduction is set to  $T_{S_{n(k)}}^*$ . If the temperature of the first snow layer is higher than  $T_{melt} = 0^\circ \text{ C}$ , the temperature of the first snow layer is fixed at  $T_{melt}$  and the calculation is redone. In this case, the energy convergence  $\Delta \widetilde{F}_{conv}$  is calculated for the first layer. This is not a grid-averaged value, but a value for the snow cover area. The amount of snowmelt in the first layer is the same as that in the first layer,

$$\widetilde{M}_{S_{n(1)}} = \min(\Delta \widetilde{F}_{conv} / l_m, \Delta \widetilde{S}_{n(1)}^* / \Delta t_L) \quad (223)$$

It is.

If the temperature in the second and lower levels is higher than  $T_{melt}$ , the temperature is returned to  $T_{melt}$  and the internal energy of the temperature change is used to melt the snow. In other words,

$$T_{Sn(k)}^{**} = T_{melt} \quad (224)$$

and added  $\Delta\tilde{F}_{conv}$  as a new

$$\Delta\tilde{F}_{conv} = (T_{Sn(k)}^* - T_{melt})c_{pi}\Delta\tilde{S}n_{(k)}^*/\Delta t_L \quad (225)$$

The amount of snowmelt is defined in (11) and is obtained in the same way as in (11).

Update the mass of each layer by subtracting the amount of snowmelt.

$$\Delta\tilde{S}n_{(k)}^{**} = \Delta\tilde{S}n_{(k)}^* - \tilde{M}_{Sn(k)} \quad (226)$$

In the middle of these calculations, if all the layers are melted, the remainder of  $\Delta\tilde{F}_{conv}$  is given to the layer below to raise the temperature of the layer below. That is, the temperature of the lower layer is increased,

$$\Delta\tilde{F}_{conv}^* = \Delta\tilde{F}_{conv} - l_m\tilde{M}_{Sn(k)} \quad (227)$$

$$T_{Sn(k+1)}^{**} = T_{Sn(k+1)}^* + \Delta\tilde{F}_{conv}^*/(c_{pi}\Delta\tilde{S}n_{(k+1)}^*)\Delta t_L \quad (228)$$

Here,  $c_{pi}$  is the specific heat of snow (ice). When all the snow cover is melted,  $\Delta\tilde{F}_{conv}^*$  is given to the soil.

The total snowmelt of the entire snowpack is the sum of the snowmelt in each layer (note that this is a grid average).

$$M_{Sn} = \sum_{k=1}^{K_{Sn}} \tilde{M}_{Sn(k)} A_{Sn} \quad (229)$$

Subtracting the amount of snowmelt, the snowpack is partially updated.

$$Sn^{**} = Sn^* - M_{Sn}\Delta t_L \quad (230)$$

### 6.3.3 Freezing snowmelt water and rainfall in snowpack

The freezing of snowmelt and rainfall in the snowpack is calculated. For the snowmelt, the effect of the liquid water generated by the melting of the upper layers of snow on the refreezing of the lower layers of snow is taken into account. The amount of liquid water retained in the snowpack is not taken into account; it is assumed that all the liquid water either freezes in the snowpack or runs down the snowpack.

Liquid water fluxes at the top of the snow cover in snow regions,

$$\widetilde{F}_{wSn(1)} = Pr_c^* + Pr_l^* + M_{Sn}/A_{Sn} \quad (231)$$

Here, the snow melt below the second layer of the snowpack is also flushed from the top of the snowpack. (As a practical matter, melting below the second layer is unlikely to occur in this case.)

The temperature of snowmelt water is considered to be 0 ° C, but the temperature of rainfall on the snowpack is assumed to be 0 ° C in terms of the toilet bowl. The latent heat of freezing water causes the temperature of the snow cover to rise, but if the temperature of a layer of snow cover rises to 0 ° C, water cannot freeze any higher and flows to the next layer below. In addition, there is an upper limit to the amount of water that can be frozen at a certain ratio to the mass of the snow cover at that layer. In other words, the freezing amount in a layer,  $\widetilde{Fr}_{Sn(k)}$ , is

$$\widetilde{Fr}_{Sn(k)} = \min \left( \widetilde{F}_{wSn(k)}, \frac{c_{pi}(T_{melt} - T_{Sn(k)}^{**})}{l_m} \frac{\Delta \widetilde{Sn}_{(k)}^{**}}{\Delta t_L}, f_{Fmax} \frac{\Delta \widetilde{Sn}_{(k)}^{**}}{\Delta t_L} \right) \quad (232)$$

It is determined by the following formula. The  $\widetilde{F}_{wSn(k)}$  is the liquid water flux flowing from the top of the  $k$ th layer of snow cover. The standard value of the  $f_{Fmax}$  is 0.1.

The temperature change in the snowpack is ,

$$T_{Sn(k)}^{***} = \frac{l_m \widetilde{Fr}_{Sn(k)} \Delta t_L + c_{pi}(T_{Sn(k)}^{**} \Delta \widetilde{Sn}_{(k)}^{**} + T_{melt} \widetilde{Fr}_{Sn(k)} \Delta t_L)}{c_{pi}(\Delta \widetilde{Sn}_{(k)}^{**} + \widetilde{Fr}_{Sn(k)} \Delta t_L)} \quad (233)$$

and update the mass to

$$\Delta \widetilde{S}n_{(k)}^{***} = \Delta \widetilde{S}n_{(k)}^{**} + \widetilde{Fr}_{Sn(k)} \Delta t_L \quad (234)$$

And update.

The total amount of freezing in the entire snowpack is the sum of the amount of freezing in each layer (but it is a grid average).

$$Fr_{Sn} = \sum_{k=1}^{K_{Sn}} \widetilde{Fr}_{Sn(k)} A_{Sn} \quad (235)$$

Add the amount of freeze and partially update the snowpack.

$$Sn^{***} = Sn^{**} + Fr_{Sn} \Delta t_L \quad (236)$$

The liquid water that passes through the snowpack down to the bottom is given to the soil.

#### 6.3.4 Snowfall.

Finally, the amount of snowfall that has been intercepted by the canopy is added to obtain the final updated snowpack.

$$Sn^{\tau+1} = Sn^{***} + P_{Sn}^* \Delta t_L \quad (237)$$

However, when the temperature of the first layer of soil is higher than 0° C, the snowfall is assumed to melt on the ground. In this case, the energy of the latent heat of melting is absorbed by the soil.

When snow accumulation occurs due to a snowfall in a grid where there has been no snowfall, the snow coverage ratio ( $A_{Sn}$ ) is newly diagnosed based on (1), and snow temperature ( $T_{Sn(1)}$ ) is assumed to be equal to the temperature of the first layer of soil.

We also add the amount of snowfall to the mass of the first layer.

$$\Delta \widetilde{S}n_{(k)}^{\tau+1} = \Delta \widetilde{S}n_{(k)}^{***} + P_{Sn}^* \Delta t_L / A_{Sn} \quad (238)$$

### 6.3.5 Redividing the snow layer and re-diagnosing the temperature

When the snow cover is updated, the area fraction is re-diagnosed by (1) and the mass of each layer is reconstructed by (3) ~ (5). The temperature of each layer is re-diagnosed so that the energy is conserved.

$$T_{Sn(k)}^{new} = \left( \sum_{l=1}^{K_{Sn}^{old}} f_{(l^{old} \in k^{new})} T_{Sn(l)}^{old} \Delta \widetilde{S}n_{(l)}^{old} A_{Sn}^{old} \right) / (\Delta \widetilde{S}n_{(k)}^{new} A_{Sn}^{new}) \quad (239)$$

Note that the variables are those subscripted with *old* and *new* are those before and after the reconstruction.  $f_{(l^{old} \in k^{new})}$  is the percentage of the mass of the  $l$  layer before the reconstruction that is contained in the  $k$  layer after the reconstruction.

## 6.4 Calculating heat transfer during snowfall.

### 6.4.1 The Heat Transfer Equation in Snowpack

The prediction equation for snowfall temperature due to heat conduction during snow cover is as follows.

$$c_{pi} \Delta \widetilde{S}n_{(k)} \frac{T_{Sn(k)}^* - T_{Sn(k)}^\tau}{\Delta t_L} = \widetilde{F}_{Sn(k+1/2)} - \widetilde{F}_{Sn(k-1/2)} \quad (k = 1, \dots, K_{Sn}) \quad (240)$$

where the heat transfer flux  $\widetilde{F}_{Sn}$  is given by

$$\widetilde{F}_{Sn(k+1/2)} = \begin{cases} (F_{Sn(1/2)} - \Delta F_{conv}) / A_{Sn} - \Delta F_{c,conv} (k = 0) \\ k_{Sn(k+1/2)} \frac{T_{Sn(k+1)} - T_{Sn(k)}}{\Delta z_{Sn(k+1/2)}} (k = 1, \dots, K_{Sn} - 1) \\ k_{Sn(k+1/2)} \frac{T_{Sn(B)} - T_{Sn(k)}}{\Delta z_{Sn(k+1/2)}} (k = K_{Sn}) \end{cases} \quad (241)$$

$k_{Sn(k+1/2)}$  is the thermal conductivity of the snow cover, which gives a constant value of 0.3 W/m/K in standard practice.  $\Delta z_{Sn(k+1/2)}$  is the thickness of each snow layer,

$$\Delta z_{Sn(k+1/2)} = \begin{cases} 0.5\Delta\widetilde{Sn}_{(1)}/\rho_{Sn}(k=1) \\ 0.5(\Delta\widetilde{Sn}_{(k)} + \Delta\widetilde{Sn}_{(k+1)})/\rho_{Sn}(k=2, \dots, K_{Sn}-1) \\ 0.5\Delta\widetilde{Sn}_{(K_{Sn})}/\rho_{Sn}(k=K_{Sn}) \end{cases} \quad (242)$$

The density of snow cover is defined by  $\rho_{Sn}$  as 300 kg/m<sup>3</sup>.  $\rho_{Sn}$  is the density of the snow cover, and a constant value of 300 kg/m<sup>3</sup> is given in the standard definition. It is assumed that the density and thermal conductivity of the snowpack will change with the passage of time (aging) due to consolidation and alteration of the snowpack, but these effects are not taken into account here.

In (29), the flux  $\widetilde{F}_{-}\{Sn(1/2)\}$  at the top of the snowpack is the heat transfer flux  $F_{Sn(1/2)}$  determined at the surface energy balance, the surface energy convergence  $\Delta F_{conv}$  generated by melting the surface temperature under snow melting conditions, and the phase of water content in the canopy. In the case of a change, the energy is given using the energy correction  $\Delta F_{c,conv}$ . We assume that the energy is given only for the snow covered surface ( $\Delta F_{conv}$ ) and that the energy is given uniformly for the grid ( $\Delta F_{c,conv}$ ). Since the sign of the fluxes is positively oriented upward, the convergence is negatively signified.

In the equation of snow flux ( $\widetilde{F}_{Sn(K_{Sn}+1/2)}$ ),  $T_{Sn(B)}$  is the temperature at the bottom of the snowpack (i.e., the boundary between the snowpack and the soil). However, the flux from the first layer of soil to the bottom of the snowpack is

$$\widetilde{F}_{g(1/2)} = k_{g(1/2)} \frac{T_{g(1)} - T_{Sn(B)}}{\Delta z_{g(1/2)}} \quad (243)$$

Therefore, we assume that there is no flux convergence at the lower end of the snowpack,

$$\widetilde{F}_{Sn(K_{Sn}+1/2)} = \widetilde{F}_{g(1/2)} \quad (244)$$

Substituting this into (33), we obtain the following

$$\widetilde{F}_{Sn(K_{Sn}+1/2)} = \left[ \frac{\Delta z_{g(1/2)}}{k_{g(1/2)}} + \frac{\Delta z_{Sn(K_{Sn}+1/2)}}{k_{Sn(K_{Sn}+1/2)}} \right]^{-1} (T_{g(1)} - T_{Sn(K_{Sn})}) \quad (245)$$

#### 6.4.2 Case 1: If snowmelt does not occur in the first layer

The implicit method is used for the temperature of the first to lowest layer of snow cover. That is, the implicit method is used to determine the temperature from the first to the lowest layer of snow cover,

$$\tilde{F}_{Sn(k+1/2)}^* = \tilde{F}_{Sn(k+1/2)}^\tau + \frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k)}} \Delta T_{Sn(k)} + \frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k+1)}} \Delta T_{Sn(k+1)} \quad (246)$$

$$\tilde{F}_{Sn(k+1/2)}^\tau = \begin{cases} (F_{Sn(1/2)} - \Delta F_{conv})/A_{Sn} - \Delta F_{c,conv} (k=0) \\ \frac{k_{Sn(k+1/2)}}{\Delta z_{Sn(k+1/2)}} (T_{Sn(k+1)}^\tau - T_{Sn(k)}^\tau) (k=1, \dots, K_{Sn}-1) \\ \left[ \frac{\Delta z_{g(1/2)}}{k_{g(1/2)}} + \frac{\Delta z_{Sn(K_{Sn}+1/2)}}{k_{Sn(K_{Sn}+1/2)}} \right]^{-1} (T_{g(1)} - T_{Sn(K_{Sn})}^\tau) (k=K_{Sn}) \end{cases} \quad (247)$$

$$\frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k)}} = \begin{cases} -\frac{k_{Sn(k+1/2)}}{\Delta z_{Sn(k+1/2)}} (k=1, \dots, K_{Sn}-1) \\ -\left[ \frac{\Delta z_{g(1/2)}}{k_{g(1/2)}} + \frac{\Delta z_{Sn(K_{Sn}+1/2)}}{k_{Sn(K_{Sn}+1/2)}} \right]^{-1} (k=K_{Sn}) \end{cases} \quad (248)$$

$$\frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k+1)}} = \begin{cases} 0 & (k=0) \\ \frac{k_{Sn(k+1/2)}}{\Delta z_{Sn(k+1/2)}} & (k=1, \dots, K_{Sn}-1) \end{cases} \quad (249)$$

and put (28)

$$c_{pi} \Delta \tilde{S}_{n(k)} \frac{\Delta T_{Sn(k)}}{\Delta t_L} = \tilde{F}_{Sn(k+1/2)}^* - \tilde{F}_{Sn(k-1/2)}^* \quad (250)$$

$$= \tilde{F}_{Sn(k+1/2)}^\tau + \frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k)}} \Delta T_{Sn(k)} + \frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k+1)}} \Delta T_{Sn(k+1)} \quad (251)$$

$$- \tilde{F}_{Sn(k-1/2)}^\tau - \frac{\partial \tilde{F}_{Sn(k-1/2)}}{\partial T_{Sn(k-1)}} \Delta T_{Sn(k-1)} - \frac{\partial \tilde{F}_{Sn(k-1/2)}}{\partial T_{Sn(k)}} \Delta T_{Sn(k)} \quad (252)$$

The fluxes are treated as in the following equations, and are solved by the LU decomposition method as a series of coupled equations of



$\Delta T_{Sn(k)}$  ( $k = 1, \dots, K_{Sn}$ ) by the  $K_{Sn}$  method. Note that the fluxes at the top of the snowpack are fixed as a boundary condition, and that the boundary condition at the bottom of the snowpack is the temperature of the first layer of soil, and that the fluxes at the bottom of the snowpack are explicitly treated as the temperature of the first layer of soil.

$$T_{Sn(k)}^* = T_{Sn(k)}^\tau + \Delta T_{Sn(k)} \quad (253)$$

Partially update the snowpack temperature by

#### 6.4.3 Case 2: When snowmelt occurs in the first layer

If the temperature of the first layer of snow cover melted in Case 1 is higher than the temperature of the first layer of snow cover melted in Case 1, snow melts in the first layer of snow cover. In this case, the temperature of the first snow layer is fixed at 0° C. In other words, the temperature of the first snow layer is fixed from the second snow layer to the first snow layer. In other words, the flux from the second snow layer to the first snow layer is

$$\tilde{F}_{3/2}^* = \frac{k_{Sn(3/2)}}{\Delta z_{Sn(3/2)}} (T_{Sn(2)}^\tau - T_{melt}) + \frac{\partial \tilde{F}_{Sn(3/2)}}{\partial T_{Sn(2)}} \Delta T_{Sn(2)} \quad (254)$$

and solve as in case 1 (when there is only one layer of snow, the snow temperature is fixed in the flux from soil to snow in the same way).

The energy convergence used to melt the first layer of snow is given by

$$\Delta \tilde{F}_{conv} = (\tilde{F}_{3/2}^* - \tilde{F}_{1/2}) - c_{pi} \Delta \tilde{S}_{n(1)} \frac{T_{melt} - T_{Sn(1)}^*}{\Delta t_L} \quad (255)$$

Even if the temperature below the second layer of snow accumulation is higher than the  $T_{melt}$ , the snow melting is not performed again, and the snow melting is processed as a correction.

## 6.5 Generating Glaciers.

A maximum value is set for the snowpack, and the excess snowfall is considered to be glacial runoff.

$$Ro_{gl} = \max(Sn - Sn_{\max})/\Delta t_L \quad (256)$$

$$Sn = Sn - Ro_{gl}\Delta t_L \quad (257)$$

$$\Delta \widetilde{Sn}_{(K_{Sn})} = \Delta \widetilde{Sn}_{(K_{Sn})} - Ro_{gl}/A_{Sn}\Delta t_L \quad (258)$$

$Ro_{gl}$  is the amount of glacial runoff. This mass is subtracted from the bottom of the snowpack.  $Sn_{\max}$  is given uniformly as 1000 kg/m<sup>2</sup> by default.

## 6.6 Fluxes fed to the soil or runoff process

The heat flux to the soil through the snow accumulation process is as follows.

$$\Delta F_{conv}^* = A_{Sn}(\Delta \widetilde{F}_{conv}^* - \widetilde{F}_{Sn_{K_{Sn}}}) - l_m P_{Sn,melt}^* \quad (259)$$

$\Delta \widetilde{F}_{conv}^*$  is the energy convergence of the remaining snowpack,  $\widetilde{F}_{Sn_{K_{Sn}}}$  is the heat conduction flux in the lowest layer of the snowpack, and  $P_{Sn,melt}^*$  is the amount of snowfall that melted immediately after reaching the ground.

The energy correction term for the phase change of water on the canopy is given directly to the soil as follows.

$$\Delta F_{c,conv}^* = (1 - A_{Sn})\Delta F_{c,conv} \quad (260)$$

The water fluxes fed to the runoff process through the snow accumulation process are as follows.

$$Pr_c^{**} = (1 - A_{Sn})Pr_c^* \quad (261)$$

$$Pr_l^{**} = (1 - A_{Sn})Pr_l^* + A_{Sn}\tilde{F}_{wSn}^* + P_{Sn,melt}^* \quad (262)$$

$\tilde{F}_{wSn}^*$  is a flux of rainfall or snowmelt water that has passed through the lowest layer of snow cover.

## 6.7 Snow albedo calculations.

The snow albedo is large in fresh snow, but decreases with time due to consolidation, deterioration and accumulation of dirt. In order to take this effect into account, snow albedo is treated as a predictor variable.

The time evolution of the “age” (age) of the snow cover follows Wiscombe and Warren (1980) and follows the following equation.

$$\frac{A_g^{\tau+1} - A_g^\tau}{\Delta t_L} = \left\{ \exp \left[ f_{ageT} \left( \frac{1}{T_{melt}} - \frac{1}{T_{Sn(1)}} \right) \right] + r_{dirt} \right\} / \tau_{age} \quad (263)$$

$f_{ageT} = 5000$ ,  $\tau_{age} = 1 \times 10^6$ .  $r_{dirt}$  is a parameter for dirt adhesion, giving a value of 0.01 on the ice sheet and 0.3 at other locations.

With this, the albedo of the snowpack is ,

$$\alpha_{Sn(b)}^{\tau+1} = \alpha_{Sn(b)}^{new} + \frac{A_g^{\tau+1}}{1 + A_g^{\tau+1}} (\alpha_{Sn(b)}^{old} - \alpha_{Sn(b)}^{new}) \quad (b = 1, 2, 3) \quad (264)$$

This can be calculated by Here,  $A_g^\tau$  is calculated backward from the predictor,  $\alpha_{Sn(1)}^\tau$ , using the same formula as above.

When there is a snowfall, the albedo is updated to the new snow value according to the amount of snowfall.

$$\alpha_{Sn(b)}^{\tau+1} = \alpha_{Sn(b)}^{\tau+1} + \min \left( \frac{P_{Sn}^* \Delta t_L}{\Delta S n_c}, 1 \right) (\alpha_{Sn(b)}^{new} - \alpha_{Sn(b)}^{\tau+1}) \quad (b = 1, 2, 3) \quad (265)$$

$\Delta S n_c$  is the amount of snow cover required for the albedo to return to a completely fresh snow value.

## 7 Spillover sub-model MATROF

Surface runoff and groundwater runoff are calculated using a simplified version of TOPMODEL (Beven and Kirkby, 1979).

### 7.1 Overview of TOPMODEL

In the TOPMODEL, we consider the horizontal distribution of groundwater level along the slope in the basin. It is assumed that the groundwater flow down a slope is balanced with the total recharge rate of the groundwater recharge rate at the slope above the point (quasi-stationary assumption). Then, the lower the slope is, the larger the groundwater flow should be. Based on another assumption, which will be discussed later, it is assumed that the shallow groundwater surface is necessary for the groundwater flow to be high. Thus, the distribution of shallow groundwater level is derived for the lower slopes. When the average groundwater level is shallower than a certain level, the groundwater level rises to the ground below a certain point of the slope and forms a saturated area. Thus, TOPMODEL is characterized by the fact that the concepts of mean groundwater level, saturated area, and velocity of groundwater flow, which are important for the estimation of runoff, are physically consistent with each other.

In the TOPMODEL, three main assumptions are made as follows

The saturated hydraulic conductivity of soil decays exponentially with depth.

The slope of the groundwater surface is nearly equal to the slope locally.

The groundwater flow down a slope corresponds to the accumulated groundwater recharge rate above the point of slope.

In the following, the use of symbols follows the conventions of TOPMODEL (Sivapalan et al., 1987 ; Stieglitz et al., 1997).

The assumption 1 can be written as

$$K_s(z) = K_0 \exp(-fz) \quad (266)$$

$K_s(z)$  is the saturated hydraulic conductivity of soil at the depth  $z$ ,  $K_0$  is the saturated hydraulic conductivity at the ground surface, and  $f$  is the attenuation coefficient.

If the depth of the groundwater surface at a certain point ( $i$ ) is defined as  $z_i$ , then the groundwater flux down slope at that point ( $q_i$ ) is expressed as

$$q_i = \int_{z_i}^Z K_s(z) dz \cdot \tan \beta = \frac{K_0}{f} \tan \beta [\exp(-f z_i) - \exp(-f Z)] \quad (267)$$

$\beta$  is the slope slope slope, using assumption 2. Although  $Z$  is the depth of the impermeable surface, the term  $Z$  is usually assumed to be deeper than that of  $1/f$ , and the term  $\exp(-f Z)$  is omitted. The slope directional soil water flux in the unsaturated zone above the groundwater surface is neglected because it is small.

Assuming that the groundwater recharge rate is uniformly set to the  $R$ , Assumption 3 can be expressed as follows

$$aR = \frac{K_0}{f} \tan \beta \exp(-f z_i) \quad (268)$$

Here,  $a$  is the total upstream area (per unit contour length at the point  $i$ ) relative to the point  $i$ .

Solving this for  $z_i$  yields the following.

$$z_i = -\frac{1}{f} \ln \left( \frac{f a R}{K_0 \tan \beta} \right) \quad (269)$$

The average groundwater depth in the region ( $A$ ) is the depth of the groundwater table ( $\bar{z}$ ),

$$\bar{z} = \frac{1}{A} \int_A z_i dA = -\Lambda - \frac{1}{f} \ln R \quad (270)$$

$$\Lambda \equiv \frac{1}{A} \int_A \ln \left( \frac{f a}{K_0 \tan \beta} \right) dA \quad (271)$$

Thus, the recharge rate ( $R$ ) as a function of mean groundwater depth ( $\bar{z}$ ) is expressed as

$$R = \exp(-f\bar{z} - \Lambda) \quad (272)$$

According to Assumption 3, this is the amount of groundwater discharged from the area ( $A$ ).

Substituting  $R$  into (4) gives the following relationship between  $z_i$  and  $\bar{z}$ .

$$z_i = \bar{z} - \frac{1}{f} \left[ \ln \left( \frac{fa}{K_0 \tan \beta} \right) - \Lambda \right] \quad (273)$$

The region satisfying the  $z_i \leq 0$  is the surface saturation region.

## 7.2 Application of TOPMODEL under the assumption of simplified terrain

When TOPMODEL is used, detailed topographical data of the target area are usually required, but here we estimate roughly the average shape of the slopes in the grid based on the data of average slope and standard deviation of the elevation of the grid (this method is currently provisional and requires further study).

The topography of the grid is represented by the slope of  $\beta_s$  with a uniform slope of  $\beta_s$  and the distance from the ridge to the valley of  $L_s$ .

$L_s$  is estimated using the standard deviation of elevation ( $\sigma_z$ ) as follows.

$$L_s = 2\sqrt{3}\sigma_z / \tan \beta_s \quad (274)$$

$2\sqrt{3}\sigma_z$  is the difference between the elevation of the ridge and the valley in the serrated terrain with the standard deviation of the elevation of the  $\sigma_z$ .

Taking the  $x$  axis from the ridge to the valley on the horizontal plane. Since the total upstream area of the curve at the  $x$  is  $x$ , (4) becomes the following.

$$z(x) = -\frac{1}{f} \ln \left( \frac{fxR}{K_0 \tan \beta_s} \right) \quad (275)$$

Based on this, the mean groundwater surface is calculated from (5) as

$$\bar{z} = \frac{1}{L_s} \int_0^{L_s} z(x) dx = -\frac{1}{f} \left[ \ln \left( \frac{fL_s R}{K_0 \tan \beta_s} \right) - 1 \right] \quad (276)$$

Groundwater recharge rate is from (7)

$$R = \frac{K_0 \tan \beta_s}{fL_s} \exp(1 - f\bar{z}) \quad (277)$$

The relationship between groundwater level and mean groundwater level in  $x$  is from (8)

$$z(x) = \bar{z} - \frac{1}{f} \left( \ln \frac{x}{L_s} + 1 \right) \quad (278)$$

The result is Solving for  $z(x) \leq 0$  with respect to  $x$  yields the following.

$$x \geq x_0 \quad (279)$$

$$x_0 = L_s \exp(f\bar{z} - 1) \quad (280)$$

Therefore, the area factor of the saturation region is

$$A_{sat} = (L_s - x_0)/L_s = 1 - \exp(f\bar{z} - 1) \quad (281)$$

In the case of  $A_{sat} \geq 0$  and  $\bar{z} > 1/f$ , the saturation region does not exist. However, for the  $A_{sat} \geq 0$  and  $\bar{z} > 1/f$ , there is no saturation region.

### 7.3 Calculation of flow rate

Four discharge mechanisms are considered and the total amount of discharge by each mechanism is defined as the total amount of discharge from the grid.

$$Ro = Ro_s + Ro_i + Ro_o + Ro_b \quad (282)$$

$Ro_s$  is a saturation excess runoff (Dunne runoff),  $Ro_i$  is an infiltration excess runoff (Horton runoff),  $Ro_o$  is an overflow of the first layer of soil, and the above is the total amount of runoff from the grid by each mechanism. Classified.  $Ro_b$  is a groundwater runoff .

#### 7.3.1 Estimates of Mean Groundwater Depth.

Assuming that soil moisture content starts from the lowest layer of soil and that the layer that becomes unsaturated for the first time is the  $k_{WT}$ th layer, the average depth of the groundwater table ( $\bar{z}$ ) is estimated as follows

$$\bar{z} = z_{g(k_{WT}-1/2)} - \psi_{k_{WT}} \quad (283)$$

This corresponds to the assumption that the moisture potential at the top of the unsaturated layer is set to  $\psi_{k_{WT}}$ , under which the distribution of soil moisture is considered to be in equilibrium (i.e., the equilibrium condition between gravity and capillary force).

If the  $\bar{z} > z_{g(k_{WT}+1/2)}$  is the lowest level, no groundwater surface is assumed to exist in the  $k_{WT}$  region. If the  $k_{WT}$  is not the bottom layer, the uppermost layer of the saturated groundwater layer (the uppermost layer) is assumed to be the  $k_{WT}$  and the above equation is applied to the lower layer.

In the case where the frozen ground surface exists in the middle of the soil, the depth of the groundwater surface is estimated above the frozen ground surface.



### 7.3.2 Calculation of Groundwater Runoff

Since the groundwater discharge is equal to the recharge rate of (12) under quasi-steady assumptions, we can assume that the groundwater recharge rate is the same as that of (12),

$$Ro_b = \frac{K_0 \tan \beta_s}{f L_s} \exp(1 - f \bar{z}) \quad (284)$$

(1). However, in the case of a frozen surface under the ground surface, see the case where the term  $\exp(-fZ)$  in (2) is not omitted,

$$Ro_b = \frac{K_0 \tan \beta_s}{f L_s} [\exp(1 - f \bar{z}) - \exp(1 - f z_f)] \quad (285)$$

The depth of the frozen ground surface is defined as  $z_f$  is the depth of the frozen ground. In this case, the other formulas of the TOPMODEL system should be different, but we do not change them for the sake of simplicity.

If there is an antifreeze layer beneath the frozen ground surface and a groundwater surface exists there, the runoff of groundwater is calculated and added in the same way.

The groundwater runoff is later removed from the  $k_{WT}$  layer.

$$Ro_{(k_{WT})} = Ro_b \quad (286)$$

$Ro_{(k)}$  represents the runoff flux from the soil in the  $k$ th layer.

### 7.3.3 Calculation of Surface Runoff.

All precipitation that falls in the saturated area of the earth's surface will runoff intact (saturation excess runoff).

$$Ro_s = (Pr_c^{**} + Pr_l^{**}) A_{sat} \quad (287)$$

The area fraction of saturated area  $A_{sat}$  is given by (16). Here, the correlation between precipitation distribution on the subgrid and the topography is ignored.

The precipitation that falls in the unsaturated area is infiltrated excess runoff (infiltration excess runoff). The soil infiltration capacity is given in terms of the saturated hydraulic conductivity of the first layer of soil for simplicity. Convective precipitation is assumed to be localized, and the area fraction of the precipitation area ( $A_c$ ) is assumed to be uniform (the standard value is 0.1). Stratified precipitation is assumed to be uniform.

$$Ro_i^c = \max(Pr_c^{**}/A_c + Pr_l^{**} - K_{s(1)}, 0)(1 - A_{sat}) \quad (288)$$

$$Ro_i^{nc} = \max(Pr_l^{**} - K_{s(1)}, 0)(1 - A_{sat}) \quad (289)$$

$$Ro_i = A_c Ro_i^c + (1 - A_c) Ro_i^{nc} \quad (290)$$

$Ro_i^c$  and  $Ro_i^{nc}$  are  $Ro_i$  for convective and non-convective precipitation areas, respectively, and  $K_{s(1)}$  is the saturated hydraulic conductivity of the first soil layer.

The overflow of the first layer of soil allows for a small amount of waterlogging  $w_{str}$  (1 mm by default),

$$Ro_o = \max(w_{(1)} - w_{sat(1)} - w_{str}, 0)\rho_w \Delta z_{g(1)} / \Delta t_L \quad (291)$$

This amount will be subtracted from the first layer of soil later. This amount will later be subtracted from the first layer of soil, and is therefore included in the runoff from the first layer.

$$Ro_{(1)} = Ro_{(1)} + Ro_o \quad (292)$$

## 7.4 Water flux to the soil

The water fluxes fed to the soil through the runoff process are as follows.

$$Pr^{***} = Pr_c^{**} + Pr_l^{**} - Ro_s - Ro_i \quad (293)$$

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## 8 Soil Submodel MATGND

Calculating soil temperature, soil moisture and frozen ground.

### 8.1 Calculating heat transfer in soil.

#### 8.1.1 The Heat Transfer Equation in Soil

The prediction equation for soil temperature due to heat transfer in the soil is as follows.

$$C_{g(k)} \frac{T_{g(k)}^* - T_{g(k)}^\tau}{\Delta t_L} = F_{g(k+1/2)} - F_{g(k-1/2)} \quad (k = 1, \dots, K_g) \quad (294)$$

$C_{g(k)}$  is the heat capacity of the soil and is defined by

$$C_{g(k)} = (c_{g(k)} + \rho_w c_{pw} w_{(k)}) \Delta z_{g(k)} \quad (295)$$

$c_{g(k)}$  is the specific heat of the soil and is given as a parameter for each soil type.  $c_{pw}$  is the specific heat of water and  $w_{(k)}$  is the soil water content (volume moisture content).  $\Delta z_{g(k)}$  is the thickness of the  $k$  layer of soil. Thus, when the heat capacity of the soil is included in the heat capacity of the soil, the energy is not conserved unless the heat transport due to soil moisture transfer is taken into account. Since the heat transport associated with soil moisture transfer is not considered in MATGND, we are now discussing its introduction. However, it should be noted that the energy conservation is somehow broken unless the heat capacity of water vapor and precipitation in the atmosphere is taken into account.

The heat transfer flux  $F_g$  is given by

$$F_{g(k+1/2)} = \begin{cases} F_{g(1/2)} - \Delta F_{conv}^* - \Delta F_{c,conv}^* & (k = 0) \\ k_{g(k+1/2)} \frac{T_{g(k+1)} - T_{g(k)}}{\Delta z_{g(k+1/2)}} & (k = 1, \dots, K_g - 1) \\ 0 & (k = K_g) \end{cases} \quad (296)$$

where  $k_{g(k+1/2)}$  is the thermal conductivity of the soil and is given as follows

$$k_{g(k+1/2)} = k_{g0(k+1/2)}[1 + f_{kg} \tanh(w_{(k)}/w_{kg})] \quad (297)$$

$k_{g0(k+1/2)}$  is the thermal conductivity of the soil when the soil moisture content is 0, and  $f_{kg} = 6$  and  $w_{kg} = 0.25$  are constants.

$\Delta z_{g(k+1/2)}$  is the thickness between the temperature definition point of the first  $k$  layer and the soil temperature definition point of the  $k + 1$  layer (for  $k = 0$ , it is the thickness between the temperature definition point of the first layer and the top of the soil, and for  $k = K_g$ , it is the thickness between the temperature definition point of the lowest layer and the bottom of the soil).

In (3), the boundary condition ( $F_{g(1/2)}$ ) is obtained by adding the energy convergence at the bottom edge of the snowpack (including the heat flux at the bottom edge of the snowpack) and the assignment of the energy correction term to the snow-free surface due to the phase change of water content in the canopy. The fluxes are given. The fluxes are positive upward and are negative when adding the convergence amount. The boundary condition  $F_{g(K_g+1/2)}$  at the lower edge of the soil is assumed to be zero flux.

### 8.1.2 Solving the heat transfer equation.

These equations are solved in terms of soil temperature from the first to the lowest layer using the implicit method. In other words, for  $k = 1, \dots, K_g - 1$ , the heat transfer fluxes are estimated by

$$F_{g(k+1/2)}^* = F_{g(k+1/2)}^\tau + \frac{\partial F_{g(k+1/2)}}{\partial T_{g(k)}} \Delta T_{g(k)} + \frac{\partial F_{g(k+1/2)}}{\partial T_{g(k+1)}} \Delta T_{g(k+1)} \quad (298)$$

$$F_{g(k+1/2)}^\tau = \frac{k_{g(k+1/2)}}{\Delta z_{g(k+1/2)}} (T_{g(k+1)}^\tau - T_{g(k)}^\tau) \quad (299)$$

$$\frac{\partial F_{g(k+1/2)}}{\partial T_{g(k)}} = -\frac{k_{g(k+1/2)}}{\Delta z_{g(k+1/2)}} \quad (300)$$

$$\frac{\partial F_{g(k+1/2)}}{\partial T_{g(k+1)}} = \frac{k_{g(k+1/2)}}{\Delta z_{g(k+1/2)}} \quad (301)$$

and then add (1) to

$$C_{g(k)} \frac{\Delta T_{g(k)}}{\Delta t_L} = F_{g(k+1/2)}^* - F_{g(k-1/2)}^* \quad (302)$$

$$= F_{g(k+1/2)}^\tau + \frac{\partial F_{g(k+1/2)}}{\partial T_{g(k)}} \Delta T_{g(k)} + \frac{\partial F_{g(k+1/2)}}{\partial T_{g(k+1)}} \Delta T_{g(k+1)} \quad (303)$$

$$- F_{g(k-1/2)}^\tau - \frac{\partial F_{g(k-1/2)}}{\partial T_{g(k-1)}} \Delta T_{g(k-1)} - \frac{\partial F_{g(k-1/2)}}{\partial T_{g(k)}} \Delta T_{g(k)} \quad (304)$$

and solved by the LU decomposition method as a series of  $K_g$  equations for  $\Delta T_{g(k)}$  ( $k = 1, \dots, K_g$ ). Note that the fluxes at the top and bottom of the soil are fixed as boundary conditions.

$$T_{g(k)}^* = T_{g(k)}^\tau + \Delta T_{g(k)} \quad (305)$$

After correction for the phase change of soil moisture content, which will be described later, the soil temperature is completely updated.

## 8.2 Calculation of soil moisture transfer

### 8.2.1 Equation for Soil Moisture Transfer

The equation for soil moisture transfer (Richards' equation) is given by

$$\rho_w \frac{w_{(k)}^{\tau+1} - w_{(k)}^\tau}{\Delta t_L} = \frac{F_{w(k+1/2)} - F_{w(k-1/2)}}{\Delta z_{g(k)}} + S_{w(k)} \quad (k = 1, \dots, K_g) \quad (306)$$

Soil moisture flux  $F_w$  is given by

$$F_{w(k+1/2)} = \begin{cases} Pr^{***} - Et_{(1,1)}(k=0) \\ K_{(k+1/2)} \left( \frac{\psi_{(k+1)} - \psi_{(k)}}{\Delta z_{g(k+1/2)}} - 1 \right) (k=1, \dots, K_g-1) \\ 0 (k=K_g) \end{cases} \quad (307)$$

where  $K_{(k+1/2)}$  is the soil permeability coefficient based on Clapp and Hornberger (1978) and is given as follows

$$K_{(k+1/2)} = K_{s(k+1/2)} (\max(W_{(k)}, W_{(k+1)}))^{2b_{(k)}+3} f_i \quad (308)$$

$K_{s(k+1/2)}$  is the saturated hydraulic conductivity and  $b_{(k)}$  is the index of moisture potential curve as an external parameter for each soil type.  $W_{(k)}$  is the saturation degree excluding freezing soil moisture and is given by

$$W_{(k)} = \frac{w_{(k)} - w_{i(k)}}{w_{sat(k)} - w_{i(k)}} \quad (309)$$

$w_{sat(k)}$  is the porosity of soil, which is also given as a parameter for each soil type.  $f_i$  is the parameter that indicates the suppression of soil moisture migration due to the presence of frozen soil, and is currently given as follows.

$$f_i = (1 - W_{i(k)}) (1 - W_{i(k+1)}) \quad (310)$$

This parameter is  $W_{i(k)} = w_{i(k)} / (w_{sat(k)} - w_{i(k)})$ .

$\psi$  is the soil moisture potential given by Clapp and Hornberger as follows

$$\psi_{(k)} = \psi_{s(k)} W_{(k)}^{-b_{(k)}} \quad (311)$$

$\psi_{s(k)}$  is given as an external parameter for each soil type.

In (11),  $S_{w(k)}$  is the source term and, considering the absorption and runoff by roots, is given by

$$S_{w(k)} = -F_{root(k)} - Ro_{(k)} \quad (312)$$

In (12), the boundary condition  $F_{w(1/2)}$  is the difference between the water flux ( $P^{***}$ ) and the evaporation flux ( $Et_{(1,1)}$ ) at the top of the soil through the runoff process. Apart from this, the sublimation flux

is subtracted from the frozen soil moisture in the first layer prior to the calculation of soil moisture transfer.

$$w_{i(k)}^\tau = w_{i(k)}^\tau - Et_{(2,1)}\Delta t_L / (\rho\Delta z_{g(1)}) \quad (313)$$

$$w_{(k)}^\tau = w_{(k)}^\tau - Et_{(2,1)}\Delta t_L / (\rho\Delta z_{g(1)}) \quad (314)$$

### 8.2.2 Solving the soil moisture transfer equation

These equations are solved from the first to the lowest layer using the implicit method. In other words, for  $k = 1, \dots, K_g - 1$ , the soil moisture fluxes are estimated by

$$F_{w(k+1/2)}^{\tau+1} = F_{w(k+1/2)}^\tau + \frac{\partial F_{w(k+1/2)}}{\partial w_{(k)}} \Delta w_{(k)} + \frac{\partial F_{w(k+1/2)}}{\partial w_{(k+1)}} \Delta w_{(k+1)} \quad (315)$$

$$F_{w(k+1/2)}^\tau = K_{(k+1/2)} \left( \frac{\psi_{(k+1)}^\tau - \psi_{(k)}^\tau}{\Delta z_{g(k+1/2)}} - 1 \right) \quad (316)$$

$$\frac{\partial F_{w(k+1/2)}}{\partial w_{(k)}} = -\frac{K_{(k+1/2)}}{\Delta z_{g(k+1/2)}} \left[ -b_{(k)} \frac{\psi_{s(k)}}{w_{sat(k)} - w_{i(k)}} W_{(k)}^{-b_{(k)}-1} \right] \quad (317)$$

$$\frac{\partial F_{w(k+1/2)}}{\partial w_{(k+1)}} = \frac{K_{(k+1/2)}}{\Delta z_{g(k+1/2)}} \left[ -b_{(k)} \frac{\psi_{s(k+1)}}{w_{sat(k+1)} - w_{i(k+1)}} W_{(k+1)}^{-b_{(k)}-1} \right] \quad (318)$$

and (11) as

$$\rho_w \Delta z_{g(k)} \frac{\Delta w_{(k)}}{\Delta t_L} = F_{w(k+1/2)}^{\tau+1} - F_{w(k-1/2)}^{\tau+1} + S_{w(k)} \Delta z_{g(k)} \quad (319)$$

$$= F_{w(k+1/2)}^\tau + \frac{\partial F_{w(k+1/2)}}{\partial w_{(k)}} \Delta w_{(k)} + \frac{\partial F_{w(k+1/2)}}{\partial w_{(k+1)}} \Delta w_{(k+1)} \quad (320)$$

$$-F_{w(k-1/2)}^\tau - \frac{\partial F_{w(k-1/2)}}{\partial w_{(k-1)}} \Delta w_{(k-1)} - \frac{\partial F_{w(k-1/2)}}{\partial w_{(k)}} \Delta w_{(k)} + S_{w(k)} \Delta z_{g(k)} \quad (321)$$

The equations are treated as follows for  $\Delta T_{g(k)}$  ( $k = 1, \dots, K_g$ ), and are solved by the LU decomposition method as a series of  $K_g$  equations

for  $\Delta T_{g(k)}$  ( $k = 1, \dots, K_g$ ). Note that the fluxes at the top and bottom of the soil are fixed as boundary conditions.

$$w_{(k)}^{\tau+1} = w_{(k)}^{\tau} + \Delta w_{(k)} \quad (322)$$

The soil moisture content is updated using the LU decomposition method.

If this calculation results in supersaturation of soil moisture, the supersaturation is removed by vertical adjustment. The supersaturation is not considered as runoff because this supersaturation is artificial and is caused by the solution of the vertical soil moisture transfer without information about the saturation. First, supersaturated soil moisture is applied from the second soil layer downward. Then, from the lowermost layer of soil to the uppermost layer, the supersaturated soil moisture is added to the uppermost layer. This operation results in the formation of a saturated layer near the bottom of the soil when soil moisture is sufficiently high to define the groundwater level (with a certain amount of water content of the ground surface in the vicinity of the lowest layer of soil (with a certain amount of water content of the ground surface in the vicinity of the lowest level of soil)).

### 8.3 Phase changes in soil moisture.

As a result of the heat conduction in the soil, the phase change of soil moisture content is calculated when the temperature of the layer with liquid moisture is below  $T_{melt} = 0^\circ \text{ C}$  or when the temperature of the layer with solid moisture is above  $T_{melt}$ . Assuming that the freezing rate of soil moisture in the  $k$ st layer is set to  $\Delta w_{i(k)}$ ,

In case of  $T_{g(k)}^* < T_{melt}$  and  $w_{(k)}^{\tau+1} - w_{i(k)}^{\tau} > 0$  (frozen)

$$\Delta w_{i(k)} = \min \left( \frac{C_{g(k)}(T_{melt} - T_{g(k)}^*)}{l_m \rho_w \Delta z_{g(k)}}, w_{(k)}^{\tau+1} - w_{i(k)}^{\tau} \right) \quad (323)$$

In case of  $T_{g(k)}^* > T_{melt}$  and  $w_{i(k)}^{\tau} > 0$  (melting)

$$\Delta w_{i(k)} = \max \left( \frac{C_{g(k)}(T_{melt} - T_{g(k)}^*)}{l_m \rho_w \Delta z_{g(k)}}, -w_{i(k)}^{\tau} \right) \quad (324)$$



Update soil freezing moisture and soil temperature as follows.

$$w_{i(k)}^{\tau+1} = w_{i(k)}^{\tau} + \Delta w_{i(k)} \quad (325)$$

$$T_{g(k)}^{\tau+1} = T_{g(k)}^* + l_m \rho_w \Delta z_{g(k)} \Delta w_{i(k)} / C_{g(k)} \quad (326)$$

### 8.3.1 Ice sheet process.

If land cover type is ice-cover, return to  $T_{melt}$  when soil temperature exceeds  $T_{melt}$ .

$$T_{g(k)}^{\tau+1} = \min(T_{g(k)}^*, T_{melt}) \quad (327)$$

The rate of change in the amount of ice cover,  $F_{ice}$ , is diagnosed as

$$F_{ice} = -Et_{(2,1)} - \frac{C_{g(k)} \max(T_{g(k)}^* - T_{melt}, 0)}{l_m \Delta t_L} \quad (328)$$

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