Description for MATSIRO6

平成 33 年 1 月 25 日

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1 11 Lake

Up to and including the calculation of the surface flux (11.1-11.2), the method is derived from the land surface model MATSIRO, while the calculation below the lake ice (11.3-11.4) is derived from the ocean model COCO. (The description in this section is also based on Emori (2000) for the first half and Hasumi (2015) for the second half.) For practical use, note, for example, that the unit of temperature is K until section 11.2, while it is $^\circ$ C after section 11.3. It is also noted that because the second half part is based on the old version of COCO, hence it is slightly different from the MIROC6-AOGCM and Hasumi (2015).

Dimensions of the lake scheme is defined in `include/zkg21c.F`. `KLMAX` is the number of vertical layers set to 5 in MIROC6/MATSIRO6. `NLTDIM` is the number of tracers, 1:temperature 2:salt. Since the vertical layers are actually from `KLSTR=2` to `KLEND=KLMAX+1`, `NLZDIM = KLMAX+KLSTR` exists as a parameter for management.

Maximum and minimum thresholds for the lake scheme are given in `matdrv.F`.

Meaning	Presentation	Variable	unit	value
minimum depth of lake	h_{min}	HHMIN	cm	$10^{\times}10^2$
maximum of surface high anomaly	η_{max}	HAMAX	cm	$10^{\times}10^2$

Meaning	Presentation	Variable	unit	value
maximum of lake snow	$h_{snow,max}$	HSMAX	cm	$10^{\times}10^2$
maximum of lake ice thickness	$h_{ice,max}$	HIMAX	cm	$10^{\times}10^2$

1.1 11.1 Lake surface conditions `[LAKEBC]` [100% Written in Jan, 2021]

- Outputs

Meaning	Presentation	Variable	dimension	unit
surface albedo	α	GRALB	IJLSDM, NRDIR, NRBND	-
surface roughness	r_0	GRZ0	IJLSDM, NTYZ0	
heat flux	G	FOGFLX	IJLSDM	
heat diffusion coefficient	$\frac{\partial G}{\partial T}$	DGFDS	IJLSDM	

- Inputs

Meaning	Presentation	Variable	dimension	unit
skin temperature	T_s	GRTS	IJLSDM	K
ice base temperature	T_b	GRTB	IJLSDM	
lake ice amount	Ic	GRICE	IJLSDM	
snow amount	Sn	GRSNW	IJLSDM	
lake ice concentration	R_{ice}	GRICR	IJLSDM	
u surface wind	U_0	GDUA	IJLSDM	
v surface wind	V_0	GDVA	IJLSDM	
cos(solar zenith)	$\cos(\theta)$	RCOSZ	IJLSDM	

- Internal work variables

Meaning	Presentation	Variable	dimension	unit
snow fraction	R_{snow}	GRSNR	IJLSDM	-
	$T_m^{max} - T_m^{min}$	TALSNX	IJLSDM	
	$\alpha_{snow(2,b)} - \alpha_{snow(1,b)}$	ALBSNX	IJLSDM	
	α_0	ALB0	IJLSDM	
	tmp	DALB	IJLSDM	
	$F(T_s)$	TFACT	IJLSDM	

Meaning	Presentation	Variable	dimension	unit
	α''	ALBX	IJLSDM	
	r'	Z00	IJLSDM	
		DZ0	IJLSDM	
heat diffusion coefficient		DFGT	IJLSDM	
		DFGX	IJLSDM	

- Internal parameters

Meaning	Presentation	Variable	unit	Header
diffusion coef. of snow	D_{snow}	DFSNOW		0.4
maximum snow depth		SNWDMX		5.0
minimum snow		EPSSNW		1.0×10^{-8}
ice forming snow		SNWMAX		1000.0
snow albedo	$\alpha_{snow(d,b)}$	ABLSNW(2, NRBND)		$0.75, 0.5, 0.75, 0.5, 0.0, 0.0$
temperature for albedo change	T_m^{min}, T_m^{max}	TALSNW(2)		$258.15, 273.15$
roughness of snow	r_{snow}	Z0SNW(NTYZ0)		$1.0 \times 10^{-2}, 1.0 \times 10^{-3}, 1.0 \times 10^{-3}$
snow amount for fraction=1		SNWCRT		100.0
snow density		SNWDEN		400.0
diffusion coef. of lake ice	D_{ice}	DFICE		2.00
lake ice albedo	$\alpha_{ice(b)}$	ALBICE(NRBND)		$0.5, 0.5, 0.05$
roughness of lake ice	r_{ice}	Z0ICE (NTYZ0)		$2.0 \times 10^{-2}, 2.0 \times 10^{-3}, 2.0 \times 10^{-3}$
ice amount for conc.=1		SICCRT		300.0
sea ice density		SICDEN		1000.0
heat z0/moumentum z0		Z0FCT		0.1
minimum z0		Z0MIN		$.0 \times 10^{-6}$
depth of ML Ocean		DZOCN		50.0
ocean dG/dTs		DFOCN		1.0×10^{10}

Meaning	Presentation	Variable	unit	Header
LW albedo (1-emis)		ALBLO		5.0×10^{-2}

In this module, surface albedo and roughness are calculated. They are calculated supposing ice-free conditions, then modified.

First, let us consider the lake albedo. The lake level $\alpha_{(d,b)}$, $b = 1, 2, 3$ represent the visible, near-infrared, and infrared wavelength bands, respectively. Also, $d = 1, 2$ represents direct and scattered light, respectively. The albedo for the visible bands are calculated in `MODULE [LAKEALB]`, supposing ice-free conditions. The albedo for near-infrared is set to same as the visible one. The albedo for infrared is uniformly set to a constant value.

When lake ice is present, the albedo is modified to take into account the ice concentration R_{ice}.

$$\alpha' = \alpha + (\alpha_{ice} - \alpha)R_{ice} \tag{1}$$

where α_{ice} is the sea ice albedo. In addition, we want to consider the albedo change due to snow cover. Assuming that the snow albedo depends on the surface temperature (T_s), we can calculate the function $F(T_s)$

$$F(T_s) = \frac{T_s - T_m^{min}}{T_m^{max} - T_m^{min}} \tag{2}$$

but $0 \leq F(T_s) \leq 1$. The snow cover albedo can be expressed using

$$\alpha'' = \alpha(1, b) + (\alpha_{snow(2,b)} - \alpha_{snow(1,b)})F(T_s) \tag{3}$$

Therefore, taking into account the snow coverage R_{snow}, we can express it as

$$\alpha = \alpha' + (\alpha'' - \alpha')R_{snow} \tag{4}$$

Second, let us consider the lake surface roughness. The roughnesses of for momentum, heat and vapor are calculated in `[LAKEZOF]`, supposing the ice-free conditions.

When lake ice is present, each roughness is modified to take into account the ice concentration R_{ice}.

$$r_0' = r_0 + (r_{ice} - r_0)R_{ice} \tag{5}$$

Then, taking into account the snow coverage R_{snow}, we can express it as

$$r_0 = r_0' + (r_{snow} - r_0')R_{snow} \tag{6}$$

If the lake ice exists, the heat diffusion coefficient of lake ice D_{ice}

$$\left(\frac{\partial G}{\partial T}\right)_{ice} = \frac{D_{ice}}{R_{ICE}} \tag{7}$$

If the snow exists, the heat diffusion coefficient of snow covered area is

$$\left(\frac{\partial G}{\partial T}\right)_{snow} = \frac{D_{ice}D_{snow}}{D_{ice}R_{snow} + D_{snow}R_{ice}} \tag{8}$$

Therefore, the net heat diffusion coefficient is finally

$$\frac{\partial G}{\partial T} = \left(\frac{\partial G}{\partial T}\right)_{ice}(1 - R_{snow}) + \left(\frac{\partial G}{\partial T}\right)_{snow} R_{snow} \tag{9}$$

The temperature differences between the snow surface (T_S) and the ice bottom (T_B) is saved as heat flux, because the difference should be zero in the ice-free conditions.

$$G = \frac{\partial G}{\partial T}(T_B - T_S) \tag{10}$$

1.1.1 11.1.1. lake surface albedo [LAKEALB]

- Inputs

Meaning	Presentation	Variable	dimension	unit
cos(solar zenith)	$cos(\theta)$	COSZ	IJLSDM	[-]

- Outputs

Meaning	Presentation	Variable	dimension	unit
lake surface albedo (direct, diffuse)	$\alpha_{L(d)}$	GALB	IJLSDM ,2	[-]

- Internal parameters

Meaning	Presentation	Variable	unit	Header
	C_1, C_2, C_3	CC	[-]	$-0.7479, -4.677039, 1.583171$
	$\alpha_{L(2)}$	ALBDIF	[-]	0.06

For lake surface level albedo $\alpha_{L(d)}$, $d = 1, 2$ represents direct and scattered light, respectively.

Using the solar zenith angle at latitude θ, the albedo for direct light is presented by

$$\alpha_{L(1)} = e^{(C_3 A^* + C_2)A^* + C_1} \tag{11}$$

where \$A $= \min(\max(\cos(\theta), 0.03459), 0.961)$\$
On the other hand, the albedo for scattered light is uniformly set to a constant parameter.

$$\alpha_{L(2)} = 0.06 \tag{12}$$

1.1.2 11.1.2 Lake surface roughness [LAKEZ0F]

- Outputs

Meaning	Presentation	Variable	dimension	unit
surface roughness for momentum	$r_{0,M}$	GRZ0M	IJLSDM	
surface roughness for heat	$r_{0,H}$	GRZ0H	IJLSDM	
surface roughness for vapor	$r_{0,E}$	GRZ0E	IJLSDIM	

- Inputs

Meaning	Presentation	Variable	dimension	unit
u surface wind	U_0	GDUA	IJLSDM	
v surface wind	V_0	GDVA	IJLSDM	

surface flux section of MIROC-DOC will be transplanted after modification.
The roughness variation of the lake surface is determined by the friction velocity $u^\star$

$$u^\star = \sqrt{C_{M_0}({U_0}^2 + {V_0}^2)} \tag{13}$$

The bulk coefficient for $u^\star$ (C_{M_0}) is given as a parameter.
$r_{0,M}, r_{0,H}$ and $r_{0,E}$ are surface roughness for momentum, heat, and vapor are presented by

$$r_{0,M} = z_{0,M_0} + z_{0,M_R} + \frac{z_{0,M_R} {u^\star}^2}{g} + \frac{z_{0,M_S}\nu}{u^\star} \tag{14}$$

$$r_{0,H} = z_{0,H_0} + z_{0,H_R} + \frac{z_{0,H_R} {u^\star}^2}{g} + \frac{z_{0,H_S}\nu}{u^\star} \tag{15}$$

$$r_{0,E} = z_{0,E_0} + z_{0,E_R} + \frac{z_{0,E_R} u^{\star 2}}{g} + \frac{z_{0,E_S}\nu}{u^\star} \tag{16}$$

Here, $\nu = 1.5 \times 10^{-5}$[m^2/s] is the kinetic viscosity of the atmosphere. z_{0,M_0}, z_{0,H_0} and z_{0,E_0} are base, and rough factor (z_{0,M_R}, z_{0,M_R} and z_{0,E_R}) and smooth factor (z_{0,M_S}, z_{0,M_S} and z_{0,E_S}), respectively.

1.2 11.2 Lake surface heat balance [LAKEHB] [100% Written in Jan, 2021]

The comments for some variables say "soil", but this is because the program was adapted from a land surface scheme, and has no particular meaning.

- Outputs

Meaning	Presentation	Variable	dimension	unit
surface water flux *1	$W_{free/ice}$	WFLUXS	IJLSDM,2	
upward long wave	$LW^\uparrow$	RFLXLU	IJLSDM	
flux balance	F	SFLXBL	IJLSDM	

- Inputs variables

Meaning	Presentation	Variable
sensible heat flux coefficent	$\frac{\partial H}{\partial T_s}$	DTFDS
latent heat flux coefficient	$\frac{\partial E}{\partial T_s}$	DQFDS
surface heat flux coefficient	$\frac{\partial G}{\partial T_s}$	DGFDS
downward SW radiation	$SW^\downarrow$	RFLXSD
upward SW radiation	$SW^\uparrow$	RFLXLU
downward LW radiation	$LW^\downarrow$	RFLXLD
lake surface albedo	α	GRALBL
lake ice concentration	R_{ice}	GRICR

- Modified in this subroutine

Meaning	Presentation	Variable	dimension	unit
skin temperature	T_s	GDTS	IJLSDM	
surface heat flux from LAKEBC	G	GFLUXS	IJLSDM	
sensible heat flux	H	TFLUXS	IJLSDM	

Meaning	Presentation	Variable	dimension	unit
latent heat flux	E	QFLUXS	IJLDSM	

- Internal work

Meaning	Presentation	Variable	dimension
latent heat for sublimation	l_s	ESUB	
emissivity of the lake surface	ϵ	EMIS	
black body radiation	$\$(1\text{-}\alpha)\sigma T\_s\hat{}4\$$	STG	
dR/dTs	$\frac{\partial R}{\partial T_s}$	DRFDS	
net surface flux	F^*	SFLUX	
net heat flux (downward positive)	G^*	GSFLUX	
The temperature derivative term of G^*	$\frac{dG^*}{dT_s}$	DGSFDS	
surface heat flux for ice-free area	G_{free}	GFLUXF	
sensible heat flux for ice covered area	δH_{ice}	SFLUXBI	
temperature derivative term of G_{ice}	$\frac{\partial G_{ice}}{\partial T_s}$	DSBDSI	
surface temperature change for ice-covored area	ΔT_{ice}	DTI	
latennt heat flux for ice covered area	E_{ice}	EVAPI	
surface heat flux for ice covered area	G_{ice}	GFLUXI	
	$1 - R_{ice}$	FF	
lake ice fraction	R_{ice}	FI	
	$R_{ice}\Delta T_{ice}$	DTX	

- Others (appeared in texts)

Meaning	Presentation	Variable	dimension	unit
lake surface albedo for shortwave radiation (ice-free)	α_S			[-]
the Stefan-Boltzmann constant	σ	STB		

Reference: Hasumi, 2015, Appendices A

Downward radiative fluxes are not directly dependent on the condition of the sea surface, and their observed values are simply specified to drive the model. Shotwave emission from the sea surface is negligible, so the upward part of the shortwave radiative flux is accounted for solely by reflection of the incoming downward flux. Let α_S be the sea surface albedo for shortwave radiation. The upward shortwave radiative flux is represented by

$$SW^{\uparrow} = -\alpha_S SW^{\downarrow} \tag{17}$$

On the other hand, the upward longwave radiative flux has both reflection of the incoming flux and emission from the lake surface. Let α be the lake surface albedo for longwave radiation and ϵ be emissivity of the lake surface relative to the black body radiation. The upward shortwave radiative flux is represented by

$$LW^{\uparrow} = -\alpha LW^{\downarrow} + \epsilon\sigma T_s^4 \tag{18}$$

where σ is the Stefan-Boltzmann constant and T_s is skin temperature. If lake ice exists, snow or lake ice temperature is considered by fractions. When radiative equilibrium is assumed, emissivity becomes identical to co-albedo:

$$\epsilon = 1 - \alpha \tag{19}$$

The net surface flux is presented by

$$F^* = H + (1-\alpha)\sigma T_s^4 + \alpha LW^{\uparrow} - LW^{\downarrow} + SW^{\uparrow} - SW^{\downarrow} \tag{20}$$

The heat flux into the lake surface is presented, with the surface heat flux calculated in `LAKEBC`

$$G^* = G - F^* \tag{21}$$

Note that G^* is downward positive.
The temperature derivative term is

$$\frac{\partial G^*}{\partial T_s} = \frac{\partial G}{\partial T_s} + \frac{\partial H}{\partial T_s} + \frac{\partial R}{\partial T_s} \tag{22}$$

When the lake ice exists, the sublimation flux is considered

$$G_{ice} = G^* - l_s E \tag{23}$$

The temperature derivative term is

$$\frac{\partial G_{ice}}{\partial T_s} = \frac{\partial G^*}{\partial T_s} + l_s \frac{\partial E}{\partial T_s} \tag{24}$$

Finally, we can update the surface temperature with the lake ice concentration with $\Delta T_s = G_{ice}(\frac{\partial G_{ice}}{\partial T_s})^{-1}$

$$T_s = T_s + R_{ice}\Delta T_s \tag{25}$$

Then, the sensible and latent heat flux on the lake ice is updated.

$$E_{ice} = E + \frac{\partial E}{\partial T_s}\Delta T_s \tag{26}$$

$$H_{ice} = H + \frac{\partial H}{\partial T_s}\Delta T_s \tag{27}$$

When the lake ice does not existed, otherwise, the evaporation flux is added to the net flux.

$$G_{free} = F^* + l_c E \tag{28}$$

Finally each flux is updated.
For the sensible heat flux, the temperature change on the lake ice is considered.

$$H = H + R_{ice}H_{ice} \tag{29}$$

Then, the heat used for the temperature change is saved.

$$F = R_{ice}H_{ice} \tag{30}$$

For the upward longwave radiative flux, the temperature change on the lake ice is considered.

$$LW^{\uparrow} = LW^{\uparrow} + 4\frac{\sigma}{T_s}R_{ice}\Delta T_s \tag{31}$$

For the surface heat flux, the lake ice concentration is considered.

$$G = (1 - R_{ice})G_{free} + R_{ice}G_{ice} \tag{32}$$

For the latent heat flux, the lake ice concentration is considered.

$$E = (1 - R_{ice})E + R_{ice}E_{ice} \tag{33}$$

Each term above are saved as freshwater flux.

$$W_{free} = (1 - R_{ice})E \tag{34}$$

$$W_{ice} = R_{ice}E_{ice} \tag{35}$$

1.3 11.3 Lake ice submodel [LAKEIC] [100% Written in Jan, 2021]

The following is an addition based on Hasumi, 2015, Appendix B1.

A relatively simply lake ice model is based on two-category thickness representation, zero-layer thermodynamics [Semtner, 1976] and dynamics with elastic-visous-plastic rheology [Hunke and Dukowicz, 1997].

There are five prognostic variables in the lake ice model described herein: lake ice concentration A_I, which is area fraction of a grid covered by lake ice and takes a value between zero and unity; mean lake ice thickness h_I over ice-covered part of a grid; mean snow depth h_S over lake ice; and x and y direction horizontal velocity components of lake ice motion u_I and v_I. The model calculates temperature at snow top (lake ice top when there is no snow cover) T_I, whic is a diagnostic variable. Density of lake ice (ρ_I) and snow (ρ_S) are assumed to be constant Lake ice is assumed to have nonzero salinity, and its value S_I is assumed to be a constant parameter.

1.3.1 11.3.1 Heat Flux and Growth Rate MODULE[FIHEATL]

- variables

Meaning	Presentation	Variable	dimension	unit
lake ice concentration	A_I	A	IJLDIM	-
Lake ice growth rate in ice-free area	W_{AO}	WAO	IJLDIM	
air-ice heat flux multiplied by the factor of sea ice concentration	Q_{AI}	QAI	IJLDIM	
vertical heat flux through sea ice and snow	Q_{IO}	QIO	IJLDIM	
snow growth rate due to heat inbalance	W_{AS}	WAS	IJLDIM	
basal growth rate of lake ice	W_{IO}	WIO	IJLDIM	
Shortwave radiation absorbed at ice-free lake surface, with the factor of ice-free area multiplied	SW^A	SWABS		
Lake temperature /Salinity	T, S	T	IJLDIM, NLZDIM, NLTDIM	C/psu
time step	Δt	TS		
surfaceheat flux	G	FT	IJLDIM, NLTDIM	

- Internal works

Meaning	Presentation	Variable	dimension	unit
freezing point depression	ΔT	TDEV		
Sea ice growth rate	W_{FZ}	WFRZ		

- parameters

Meaning	Presentation	Variable	unit	value
coeficient for a decreasing function of salinity	$\frac{\partial T}{\partial S}$	dtds		-0.0543
density of sea water	ρ_O	rhoo	g/cm^3	1.0
latent heat cofefficient to melt		emeltl	J/kg	3.4×10^5
latent heat fusion *3	L_f	hfus	erg/g	$E_l \times 1.0 \times 10^4$
	$\frac{1}{\rho_O L_f}$	rrhfus	cm^3/erg	$1.0/\rho_I/L_f$
fraction of SW^A absorbed by the lake model's top level	$I(z=2)$	SWCNVND		
heat capacity of lake water	C_{po}	cpo	erg/g/K	3.990×10^7
thickness of the lake model's top level	D_1	DZ1	cm	1.0×10^2

*3 same value is applied to snow and sea ice.

The following is an addition based on Hasumi, 2015, Appendix B1.1.

This section is actually the same with

Let us consider here a case that the model is integrated from the n-th time level to the (n+1)-th time level. A_I, h_I and h_S are incrementally modified in the following order.

Temperature at lake ice base is taken to be the lake model's top level temperature $T(k=2)$. In this model, lake ice exists only when and where $T(k=2)$ is at the freezing point T_f, which is a decreasing function of salinity ($T_f = -0.0543S$[℃] is used here, where temperature and salinity are measured by C and psu, respectively). In heat budget calculation for snow and lake ice, only latent heat of fusion and sublimation is taken into account, and heat content associated with temperature is neglected. Therefore, temperature inside sea ice and snow are not calculated, and T_I is estimated from surface heat balance.

Nonzero minimum values are prescribed for A_I and h_I , which are denoted by A_I^{min} and h_I^{min}, respectively. These parameters define a minimum possible volume of sea ice in a grid. If a predicted volume $A_I h_I$ is less than that minimum, A_I is reset to zero, and T_1 is lowered to compensate the corresponding latent heat. In this case, the lake model's top level is kept at a supercooled state. Such a state continues until the lake is further cooled and the temperature becomes low enough to produce more lake ice than that minimum by releasing the latent heat corresponding to the supercooling.

Surface heat flux is separately calculated for each of air-sea and air-ice interfaces in one grid.

The surface temperature of lake ice T_I is determined such That

$$Q_{AI} = Q_{IO} \tag{36}$$

is satisified, where Q_{IO} is corresponding to $G + SW^{\downarrow}$ and Q_{AI} is corresponding to $G_{ice} - W_{ice}$. However, When the estimated T_I exceeds the melting point of lake ice T_m (which is set to 0 C for convenience), T_I is reset to T_m and Q_{AI} and Q_{IO} are re-estimated by using it. The heat inbalance between Q_{AI} and Q_{IO} is consumed to melt snow (lake ice when there is no snow cover). Snow growth rate due to this heat imbalance is estimated by

$$W_{AS} = \frac{Q_{AI} - Q_{IO}}{\rho L_f} \tag{37}$$

where ρ_O is density of seawater and L_f is the latent heat of fusion (the same value is applied to snow and lake ice). This growth rate is expressed as a change of equivalent liquid water depth per time. It is zero when $T_I < T_m$ and negative when $T_I = T_m$. Note that W_{AS} is weighted by lake ice concentration.

Although it is assumed that $T(2) = T_f$ when lake ice exists, T_1 could deviated from T_f due to a change of salinity or other factors. Such deviation should be adjusted by forming or melting lake ice. Under a temperature deviation of the top layer of lake,

$$\Delta T = T(k=2) - T_f S(k=2) \tag{38}$$

lake ice growth rate necessary to compensate it in the single time step is given by

$$W_{FZ} = -\frac{C_{po} \Delta T \Delta z_1}{L_f \Delta t} \tag{39}$$

where C_{po} is the heat capacity of lake water and $\Delta z_1 = 100\mathrm{cm}$ is the thickenss of the lake model's top level (uniformly set to constant in case of the current lake model.) This growth rate is estimated at all grids, irrespective of lake ice existence, for a technical reason. As described below, this growth rate first estimates negative ice volume for ice-free grids, but the same heat flux calculation procedure as for ice-covered grids finally results in the correct heat flux to force the lake. Basal growth rate of lake ice is given by

$$W_{IO} = A_I W_{FZ} + \frac{Q_{IO}}{\rho_O L_f} \tag{40}$$

where, again, W_{IO} is weighted by lake ice concentration.

Lake ice formation could also occur in the ice-free area. Let us define Q_{AO} by

$$Q_{AO} = (1 - A_I)[Q - (1 - \alpha_s) SW^{\downarrow}] \tag{41}$$

i.e., air-lake heat flux except for shortwave, multiplied by the factor of the fraction of ice-free area. Here, Q is air-ice heat flux. Shortwave radiation absorbed at ice-free lake surface, with the factor of ice-free area multiplied, is represented by

$$SW^A = (1 - A_I)(1 - \alpha_S) SW^{\downarrow} \tag{42}$$

Lake ice growth rate in ice-free area is calculated by

$$W_{AO} = (1 - A_I)W_{FZ} + \frac{Q_{AO} + I(k=2)SW^A}{\rho_O L_f} \tag{43}$$

where $I(k=2)$ denotes the fraction of SW^A absorbed by the lake model's top level, which is calculate in [SVTSETL] in lakepo.F.

Finaly, the heat flux for freshwater is

$$G_{lake} = \Delta z_1 \frac{\Delta T}{\Delta t} \tag{44}$$

1.3.2 11.3.2 Sublimation of Sea IceMODULE[FWATERL]

- variables

Meaning	Presentation	Variable	dimension	unit
lake ice fraction	A'_I	AX	IJLDIM	
lake ice thickenss	h'_I	HIX	IJLDIM	
Snow depth	h'_S	HSX	IJLDIM	
latent heat flux of evaporation	F_W^{EV}	WEV		
latent heat flux of sublimation	F_W^{SB}	WSB	IJLDIM	cm/s
	ΔF_W	WDIF		
time step	Δt	TS		
latent heat flux of evaporation	F_W^{EV}	EVAP	IJLDIM	
latent heat flux of sublimation	$F_W^{SB\prime\prime}$	SUBI	IJLDIM	

- Internal variables

Meaning	Presentation	Variable	dimension	unit
Lake ice concentration	A_I^n	AZ	IJLDIM	
Snow depth	h_S^n	HSZ	IJLDIM	
lake ice thickness	h_I^n	HIZ	IJLDIM	

- parameters

Meaning	Presentation	Variable	unit	value
density of snow	ρ_S	rhos	g/cm^3	0.33
density of lake ice	ρ_I	rhoi	g/cm^3	0.9
Ratio of density (ocean/snow)	R_{ρ_S}	rrs	[-]	ρ_O/ρ_s

Meaning	Presentation	Variable	unit	value
Ratio of density (ocean/ice)	R_{ρ_I}	rri	[-]	ρ_O/ρ_I
Minimum thickness of ice	h_I^{min}	himin		1.0×10ˆ1

The following is an addition based on Hasumi, 2015, Appendix B1.2.

Sublimation (freshwater) flux, which is practically come from the land ice runoff, is calculated or prescribed over lake ice cover. The flux is first consumed to reduce snow thickness in n-th timstep:

$$h'_S = h^n_S - \frac{\rho_O F^{SB}_W \Delta t}{\rho_S A^n_I} \tag{45}$$

If h'_S becomes less than zero, it is reset to zero. Then, the melted snow flux is added to F^{SB}_W. F^{SB}_W is redefined by

$$F^{SB\prime}_W = F^{SB}_W + \frac{\rho_S A^n_I (h'_S - h^n_S)}{\rho_O \Delta t} \tag{46}$$

Where there no remains snow, but $F^{SB\prime}_W$ is not zero, The remain flux is consumed to reduce sea ice thickness:

$$h'_I = h^n_I - \frac{\rho_O F^{SB\prime}_W \Delta t}{\rho_I A^n_I} \tag{47}$$

If h'_I becomes less than h^{min}_I, it is reset to zero. Then, the melted iceflux is added to $F^{SB\prime}_W$. $F^{SB\prime}_W$ is redefined by

$$F^{SB\prime\prime}_W = F^{SB\prime}_W - A^n_I \frac{\rho_S (h^n_I - h'_I)}{\rho_O \Delta t} \tag{48}$$

Finaly, nonzero $F^{SB\prime\prime}_W$ is consumed to reduce lake ice concentration:

$$A'_I = A^n_I - \frac{R_{\rho_I} F^{SB\prime\prime}_W \Delta t}{h^{min}_I} \tag{49}$$

if A'_I becomes less then 0, it is reset to zero. Even if A'_I becomes less than A^{min}_I, on the other hand, it is not adjusted here. If A'_I is adjusted to zero, it means that the sublimation flux is not used up by eliminating snow and lake ice.

The remaining part is consumed to reduce lake water, so the evaporation flux F^{EV}_W is modified as

$$F^{EV}_W = F^{EV}_W + F^{SB}_W + \frac{(A'_I - A^n_I) h^{min}_I}{R_{\rho_I} \Delta t} \tag{50}$$

The later two terms cancel out if the adjustment does not take place.
If there is no lake ice, evaporation flux is just as

$$F_W^{EV\prime} = F_W^{EV} + F_W^{SB} \tag{51}$$

The adjusted evaporation flux is saved

$$\Delta F_W^{EV} = F_W^{EV\prime} - F_W^{EV} \tag{52}$$

When sublimation flux is consumed to reduce lake ice amount, salt contained in lake ice has to be added to the remaining lake ice or the underlying water. Otherwise, total salt of the ice-lake system is not coserved. Here, it is added to underlying water, and the way of this adjustment is described later. Nothe that lake ice tends to gradually drain high salinity water contained in brine pockets in reality. Thus, such an adjustment is not very unreasonable. When A_I' is adjusted to zero, on the other hand, the remaining sublimation flux is consumed to reduce lake water. In this case, difference between the latent heat of sublimation and evaporation has to be adjusted, which is also described later.

If the ice and/or snow is too thick, they are converted to snow flux. Here, the overflow snowflux S_{off} is added to F_W^{SN}

$$F_W^{SN} = F_W^{SN} + S_{off} \tag{53}$$

S_{off} is actually calculated in `MATDRV` and handed to `LAKEIC`.

1.3.3 11.3.3 Update snow and ice volume `[PCMPCTL]`

The lake ice fraction is updated, using the lake ice growth (retreat) rate in ice-free area W_{AO}:

$$A_I^{n+1} = A_I' + \frac{\rho_O}{\rho_I h_I \phi W_{AO} \Delta t} \tag{54}$$

If A_I^{n+1} becomes greater than 1, it is reset to 1, and if A_I^{n+1} becomes smaller than zero, it is reset to zero.

1.3.4 11.3.4 Growth and Melting `[PTHICKL]`

- variables

Meaning	Presentation	Variable	dimension	unit
lake ice fraction	A_I^{n+1}	AX	IJLDIM	
lake ice volume	V_I	AXHIX	IJLDIM	

Meaning	Presentation	Variable	dimension	unit
lake snow volume	V_S, V_S', V_S^{**}	AXHSX	IJLDIM	
lake ice volume	V_I	AXHIXN	IJLDIM	
		AXHSXN	IJLDIM	
lake ice thickenss	h_I'	HIX	IJLDIM	
Snow depth	h_S'	HSX	IJLDIM	
Snow depth	h_S^n	HSZ	IJLDIM	
lake ice thickness	h_I^n	HIZ	IJLDIM	
snow growth rate due to heat inbalance	W_{AS}	WAS	IJLDIM	
lake ice growth rate due to heat inbalance	W_{AI}	WAI	IJLDIM	
Reduced heat flux	W_{res}	WRES	IJLDIM	
basal growth rate of lake ice	W_{IO}	WIO	IJLDIM	
snow fall flux	$F_W^{SN}, F_W^{SN\prime}$	SNOW	IJLDIM	
Lake ice growth rate in ice-free area	W_{AO}	WAO	IJLDIM	
precipitation flux	F_W^{PR}	PREC	IJLDIM	
latent heat flux of evaporation	L_e	EVAP	IJLDIM	
latent heat flux of sublimation	$$	SUBI	IJLDIM	
Lake ice concentration	A_I'	AZ	IJLDIM	
		ROFF	IJLDIM	
		ADJLAT	IJLDIM	
lake heat flux	H_{lake}	FT	IJLDIM, NLTDIM	
		FS	IJLDIM	
time step	Δt	TS		

- parameters

Meaning	Presentation	Variable	unit	value
density of snow	ρ_S	rhos	g/cm^3	0.33
density of lake ice	ρ_I	rhoi	g/cm^3	0.9
Ratio of density (ocean/snow)	R_{rho_I}	rrs	[-]	ρ_O/ρ_s
Ratio of density (ocean/ice)	R_{rho_I}	rri	[-]	ρ_O/ρ_I
Minimum thickness of ice	h_I^{min}	himin		1.0×10^1
		AMIN		1.0×10^{-6}
		AMAX		1.0
		SI		0.0
		SREF		3.5×10^1
		AIH		2.0×10^8
		AIHB		2.0×10^2

以下、Hasumi, 2015, Appendix B.1.3 and B.1.4 をもとに加筆修正。

The lake ice volume V_I' and snow volume V_S' before the snow and ice growth are presented by

$$V_I' = A_I' h_I^n \tag{55}$$

$$V_S' = A_I' h_S^n \tag{56}$$

From here, let us consider the contribution of snowfall and freshwater fluxes to the growth.

- Snowfall flux to snow

Changes of snow depth due to snow fall (freshwater) flux F_W^{SN} (expressed by negative values to be consistent with other freshwater flux components) is first taken into account. F_W^{SN} is not weighted by lake ice concentration or ice-free area are fraction, as snowfall take place for both regions.

If the newly predicted (in `PCMCTL`) lake ice concentration A_I^{n+1} is zero, the amount of snow existed before the growth is added to the snowfall flux.

$$F_W^{SN\prime} = F_W^{SN} + \frac{\rho_S V_S'}{\rho_O \Delta t} \tag{57}$$

Snow depth and amount is set to zero:

$$h_S' = 0, \quad V_S^{**} = 0 \tag{58}$$

Otherwise, snowfall accumulates over the ice covered region. Snow depth is modified by

$$h_S^* = \frac{V_S'}{A_I^{n+1}} + \frac{\rho_O F_W^{SN} \Delta t}{\rho_S} \tag{59}$$

And the snow amount is also modifie by

$$V_S^* = A_I^{n+1} h_S^* \tag{60}$$

The snowfall flux is reduced by that amount:

$$F_W^{SN\prime} = (1 - A_I^{n+1}) F_W^{SN} \tag{61}$$

Then, the snowfall flux is put together with the precipitation flux.

$$F_W^{PR} = F_W^{PR} + F_W^{SN\prime} \tag{62}$$

- Freshwater flux for snow growth

When the lake ice is not existed ($A_I^{n+1}=0$), the snow amount grown above is converted to ice. Growth rate of the lake ice is presented by:

$$W_{AI}^* = \frac{\rho_O V_S^*}{\rho_S \Delta t} \tag{63}$$

When the lake ice existed, if the snow growth rate

$$W_{AS}^* = W_{AS} + \frac{\rho_O V_S^*}{\rho_S \Delta t} \tag{64}$$

is positive, the energy is used for the snow growing. Otherwise, W_{AS}^* is assumed to reduce the lake ice.

$$W_{AI}^* = W_{AS}^* \tag{65}$$

and deficient flux is come from the snow amount changes.

$$W_{AS}^* = -\frac{\rho_O V_S^*}{\rho_S \Delta t} \tag{66}$$

Then, the snow depth is modified with the accumulation.

$$h_S^{**} = \frac{V_S^* + \rho_O W_{AS} \Delta t}{\rho_S A_I^{n+1}} \tag{67}$$

if h_S' is less than 0, it is reset to zero.

- Freshwater flux for ice growth

When the lake ice is not existed, the flux is just handed to the lake surface.

$$W_{IO}^* = W_{IO} + W_{AI}^* \tag{68}$$

When the lake ice exists ($A_I^{n+1}>0), if the ice growth rate

$$W_{AI}^* = W_{AI} + \frac{V_I^*}{R_{\rho_I \Delta t}} \tag{69}$$

is negative, the flux is handed to the lake surface.

$$W_{IO}^* = W_{IO} + W_{AI}^* \tag{70}$$

and deficient flux is come from the lake amount changes.

$$W_{AI}^* = -\frac{V_I^*}{R_{\rho_I \Delta t}} \tag{71}$$

The amount of the ice is then updated,

$$V_I^* = V_I' + \frac{\rho_O W_{AI} \Delta t}{\rho_I} \tag{72}$$

- Get the variables in the new timestep (n+1)

For the snow amount,

$$V_S^{n+1} = A_I^{n+1} h_S^{**} \tag{73}$$

For the ice amount

$$V_I^{n+1} = V_I^* + \frac{\rho_O (W_{IO} + W_{AO}) \Delta t}{\rho_I} \tag{74}$$

If V_I^{n+1} is equal or less than 0, lake ice fraction is set to zero (A_I^{n+1}=0)and its thickness is set to h_I^{min}. Otherwise,

$$h_I^{***} = \frac{V_I^{n+1}}{A^*} \tag{75}$$

If h_I^{***} is smaller than h_I^{min}, it is set t0 h_I^{min} and the lake ice concentration is adjusted.

$$A_I^{n+1} = \frac{V_I^{n+1}}{h_I^{min}} \tag{76}$$

If the A_I^{n+1} is less than A_I^{min}, it is set to 0. If the A_I^{n+1} is larger than $A_I^{max} = 1$, it is set to A_I^{max}.

Let us consider the case of the ice is very thick. Here, the remained volume, which is not covered by ice is considered.

$$V_0^{free} = (A_{max} - A_{min}) h_I^{min} \tag{77}$$

$$V^{free} = (A_{max} - A_I^{n+1}) h_I^{***} \tag{78}$$

If $V^{free} > V_0^{free}$, the ice thickness is increased, by adding V^{free}

$$h_I^{***} = V^{free} + \frac{A_I^{n+1} h_I^{m+1}}{A_I^{max}} \tag{79}$$

The deficient water is come from the snow. The snow depth is now updated

$$h_S^{***} = A_I^{n+1} \frac{h_S^{***}}{A_{max} - \frac{V_0^{free}}{h_I^{***}}} \tag{80}$$

Finally, check if the snow is under water.

$$h_S^{n+1} = \min(h_S^{***}, \frac{\rho_O - \rho_I}{\rho_S} h_I^{***}) \tag{81}$$

and the ice thickness is also updated.

$$h_I^{n+1} = h_I^{***} + \frac{\rho_S}{\rho_I}(h_S^{***} - h_S^{n+1}) \tag{82}$$

The growth rate of the lake ice is

$$W_I^n = \frac{\rho_S A_I^{n+1} h_I^{n+1} - V_I'}{\rho_I \Delta t} \tag{83}$$

$$W_I^{n+1} = \frac{\rho_S A_I^{n+1} h_I^{n+1} - V_I^{n+1}}{\rho_I \Delta t} \tag{84}$$

The growth rate of the snow is

$$W_S^n = \frac{\rho_S A_I^{n+1} h_S^{n+1} - V_S'}{\rho_S \Delta t} \tag{85}$$

$$W_S^{n+1} = \frac{\rho_S A_I^{n+1} h_S^{n+1} - V_S^{n+1}}{\rho_S \Delta t} \tag{86}$$

The surface salinity flux F_S is

$$F_S = S_I(W_I - F_W^{SB\prime\prime}) \tag{87}$$

The freshwater flux F_W is

$$F_W(1) = -F_W + \frac{L_f}{C_p}\Big(W_I^{n+1} + W_S^{n+1} - Sn + \Delta F_W^{EV}\Big) \tag{88}$$

塩分変化に関係する分？

$$F_W(2) = F_W^{EV} - F_W^{PR} - R_{off} + W_S^n + W_I^n \tag{89}$$

1.4 11.4 Physical formulation & processes [LAKEPO] [20% Written in Jan, 2021]

lakepo.F

1.4.1 Set parameters

Diffusion tracer flux [VDIFFL]

- Inputs

Though several valuables are written, they are not used here.

- Outputs

Meaning	Presentation	Variable	dimension	unit
vertical diffusion coefficient	K_V	AHV	IJLDIM, NLZDIM	

- Parameters

Meaning	Presentation	Variable	unit	value
vertical diffusion coefficient at the surface layer	K_{V0}	AHVL0(NZ=KLMAX)		$k \times 1.0$

The case considered in COCO4 is that(Hasumi, 2015, Section 1.2): 1. Diffusive flux of tracer follows the Fick's law. 2. Diffusuion coefficient tensor is diagonal, and its horizontal component is identical.

Here, the vertical diffusion coefficent K_V is simply set as

$$K_V(k) = K_{V0}k \tag{90}$$

11.3.2 Estimate the advection and diffusion terms of the tracer equations [FLXTRCL]

- Inputs

Meaning	Presentation	Variable	dimension	unit
vertical diffusion coefficient	K_V	AHV	IJLDIM, NLZDIM	
water temperature	T	TX	IJLDIM, NLZDIM, NLTDIM	
water depth	h	HX	IJLDIM	

- Outputs

Meaning	Presentation	Variable	dimension	unit
vertical component of diffusive tracer flux	F_D	ADT	IJLDIM, NLZDIM, NLTDIM	
	$\frac{F_D}{D(k-\frac{1}{2})}$	DIFFZ	IJLDIM, NLZDIM	

- Internal variables

Meaning	Presentation	Variable	dimension	unit
		TH	IJLDIM, NLZDIM, NLTDIM	
	h'	HZBOT	IJLDIM	

-Parameters

Meaning	Presentation	Variable	unit	value
thickness of each lake level	D	DZ(NLZDIM)	cm	$0, S_0(1), S_0(2), S_0(3), S_0(4)$
	S	DS(NLZDIM)	ND	
thickness of the lake model's top level	D_1	DZ1	cm	1.0×10^2
Ratio of the thickness of the each layer in sigma coordinate	S_0	DS0(NLZDIM-1)	ND	$0.1, 0.1, 0.2, 0.6$

Supposing that the thickness of the 1st top layer ($k = 2$) is D_1,

$$D(2) = D_1 \tag{91}$$

the thickness of the remaining water column (h') is presented by

$$h' = h - D_1 \tag{92}$$

The thickness of each remaining layer ($k = 3, 4, 5, 6$) is

$$D(k) = S(k)h' \tag{93}$$

Then, the component of the diffusive tracer flux is represented by

$$F_D = K_V \frac{\partial T}{\partial z} \tag{94}$$

practically,

$$F_D(k) = K_V(k)\frac{T(k-1) - T(k)}{\frac{D(k-1)+D(k)}{2}} - K_V(k+1)\frac{T(k) - T(k+1)}{\frac{D(k)+D(k+1)}{2}} \tag{95}$$

SLVTRCL

- Inputs

Meaning	Presentation	Variable	dimension	unit
vertical component of diffusive tracer flux	F_D	ADT	IJLDIM, NLZDIM, NLTDIM	
	$\frac{F_D}{D(k-\frac{1}{2})}$	DIFFZ	IJLDIM, NLZDIM	
minimum depth of lake		HXMIN		cm
heat flux		FT		Kcm/s
absorbed shortwave		SWABS		erg/cm
		FS		
time step	Δt	TS		
surface-type fraction (lake)	$$	LKFRAC	IJLDIM	[-]

- Outputs

Meaning	Presentation	Variable	dimension	unit
lake water deficient		XHD	IJLDIM	cm

$$A_A(k) = -\frac{K_V(k)}{\frac{D(k-1)+D(k)}{2}}\Delta t \tag{96}$$

$$A_C(k) = -\frac{K_V(k+1)}{\frac{D(k)+D(k+1)}{2}}\Delta t \tag{97}$$

$$A_B(k) = D)k - A_A(k) - A(k) \tag{98}$$

- THOMASL Solve the linear equations expressed by tri-diagonal matrix. (original files ./ocean/utrdg.F)

Then,

$$T(k) = T(k) + F_D(k)\Delta t \tag{99}$$

$$h_D = -F_T \Delta t \tag{100}$$

$$V_D = \max(h_{min} - h - h_D, 0) R_{lake} \tag{101}$$

$$h_D = \max(h_D, h_{min} - h) \tag{102}$$

Then, the deficient water is added.

$$h = h + h_D \tag{103}$$

$$h_{BOT} = h - D_1 \tag{104}$$

ここで、
If $h_D \geq 0$, $D^{top} = h_D S(\text{KLEND})$ and $D^{bottom} = 0$
If $h_D \geq 0$, 下からループ

$$T(k) = T(k) + \frac{T(k-1)D^{top}}{h_{BOT} S(k)} \tag{105}$$

$$D \tag{106}$$

OVTURNL Reference: Hasumi, 2015, Section 4.1

Let us consider finite difference discretization of a flux-form, one-dimensional advection equation for tracer ψ.

$$\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial z}(w\psi) = 0 \tag{107}$$

- Inputs

Meaning	Presentation	Variable	dimension	unit
$depthofwatercolumn$	h		H	
time step	Δt		TS	

- Outputs

Meaning	Presentation	Variable	dimension	unit
		R		

The unit of XHD will be changed to kg/m2 after this module (in `MATDRV`).

1.4.2 11.3.1 Setting vertical diffusion and viscosity coefficients `[VDIFFL]`

Reference: Hasumi, 2015, Section 1

- Variables

Meaning	Presentation	Variable	dimension	unit
the fraction of shortwave radiation absorbed by the lake's z-level		SWCONV	IJLDIM, NLZDIM	ND
		AA	IJLDIM, NLZDIM	
		AB	IJLDIM, NLZDIM	
		AC	IJLDIM, NLZDIM	
		DH	IJLDIM	
thickness of the layers from the 2nd to the bottom	h_B	HZBOT	IJLDIM	cm
		HXBOT	IJLDIM	
		DZB	IJLDIM	
		DZT	IJLDIM	
	$I^{-\frac{1}{2}}(z)$	RADUP		ND
shortwave radiation flux at an arbitrary depth	$I^{+\frac{1}{2}}(z)$	RADDN		ND
Depth of each level	D_H	DEPTH		cm
the fraction of shortwave radiation absorbed by the lake's z-level	$\Delta I(z)$	SWCNV1	NLZDIM	ND
total of abrsorbed shortwave radiation		TSWCNV	IJLDIM	ND

- parameters

Meaning	Presentation	Variable	unit	value
	E_l	emeltl	J/kg	3.4×10^5
density of sea water	ρ_O	rhoo	g/cm^3	1.0

Meaning	Presentation	Variable	unit	value
density of sea ice	ρ_I	rhoi	g/cm^3	0.9
density of snow	ρ_S	rhos	g/cm^3	0.33
latent heat fusion	L_f	hfus	erg/g	$E_l \times 1.0 \times 10^4$
	$\frac{1}{\rho_O L_f}$	rrhfus	cm^3/erg	$1.0/\rho_I/L_f$
heat capacity of lake water	C_{po}	cpo	erg/g/K	3.990×10^7
heat capacity of lake ice	C_{pi}	cpi	erg/g/K	2.093×10^7
coeficient for a decreasing function of salinity	$\frac{\partial T}{\partial S}$	dtds		-0.0543
		dtdz		-7.59×10^{-6}
fraction of the fast-attenuating portion	R	RRR		5.0×10^{-1}
length scale for fast-attenuating portion	ζ_1	ZETA1		3.5×10^1
length scale for deeply penetrating spectral portion	ζ_2	ZETA2		2.3×10^3
thickness of the lake	$D(z)$	DZ(NZ=KLMAX)	cm	`DZ1`
	$S(z)$	DS(NZ=KLMAX)	ND	`DS0(K-KLSTR)`
thickness of the lake model's top level	$D(1) $	DZ1	cm	1.0×10^2
	$S_0(z)$	DS0(NZ-1)	ND	$0.1, 0.1, 0.2, 0.6$

- erg $= 10^{-7}$J

Reference: Hasumi, 2015, Section 3.2.4
Then, the depth of each level is presented by

$$H(k) = \sum_{k=2}^{k-1} H + D(k) \tag{108}$$

While incoming downward longwave radiation is completely absorbed within a very thin surface layer, shortwave radiation can penetrate significantly into depths. In order to take account of its effect, shortwave radiation flux at an arbitary depth in the ocean is parameterized by

$$I^{+\frac{1}{2}}(z) = Re^{-\frac{D(z)}{\zeta_1}} + (1 - R)e^{-\frac{D(z)}{\zeta_2}} \tag{109}$$

$$\Delta I(z) = I^{-\frac{1}{2}}(z) - I^{+\frac{1}{2}}(z) \tag{110}$$

where $I(0) = 1.0$. Here, shortwave radiation is split into two portions: one is a fast-attenuating spectral portion and the other is a deeply penetrating spectral portion, and these two portions attenuate with length scales of ζ_1 and ζ_2, respectively. $\Delta I(z = 2)$ is handed to the lake ice scheme (hence $\Delta I(z = 2)$ is set to zero).

The residual is absorbed by the bottom layer.

$$\Delta I(z_{max} + 1) = 1 - \Sigma_{z=2}^{z_{max}} \tag{111}$$

For the preparation of the next step, the unit of $\Delta I(z)$ is modified. The new variable is represented as $\Delta I_{tmp}(z)$ here.

$$\Delta I_{tmp}(z) = \frac{\Delta I(z)}{\rho_O C_{p_o}} \tag{112}$$

1.4.3 THOMASL

1.4.4 11.3.4 Convective adjustment for the unstable water column [OVTSETL]

Reference: Hasumi, 2015, Section 4.4