Description for MATSIRO6

平成 33 年 1 月 27 日

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1 11 Lake

Up to and including the calculation of the surface flux (11.1-11.2), the method is derived from the land surface model MATSIRO, while the calculation below the lake ice (11.3-11.4) is derived from the ocean model COCO. (The description in this section is also based on Emori (2000) for the first half and Hasumi (2015) for the second half.) For practical use, note, for example, that the unit of temperature is K until section 11.2, while it is $^\circ$ C after section 11.3. It is also noted that because the second half part is based on the old version of COCO, hence it is slightly different from the MIROC6-AOGCM and Hasumi (2015).

Dimensions of the lake scheme is defined in `include/zkg21c.F`. `KLMAX` is the number of vertical layers set to 5 in MIROC6/MATSIRO6. `NLTDIM` is the number of tracers, 1:temperature 2:salt. Since the vertical layers are actually from `KLSTR=2` to `KLEND=KLMAX+1`, `NLZDIM = KLMAX+KLSTR` exists as a parameter for management.

Maximum and minimum thresholds for the lake scheme are given in `matdrv.F`.

1.1 11.1 Lake surface conditions [`LAKEBC`] [100% Written in Jan, 2021]

- Outputs

Meaning	Presentation	Variable	dimension	unit
surface albedo	α	GRALB	IJLSDM, NRDIR, NRBND	-
surface roughness	r_0	GRZ0	IJLSDM, NTYZ0	–
heat flux	G	FOGFLX	IJLSDM	–
heat diffusion coefficient	$\frac{\partial G}{\partial T}$	DGFDS	IJLSDM	–

- Inputs

Meaning	Presentation	Variable	dimension	unit
skin temperature	T_s	GRTS	IJLSDM	K
ice base temperature	T_b	GRTB	IJLSDM	
lake ice amount	Ic	GRICE	IJLSDM	
snow amount	Sn	GRSNW	IJLSDM	
lake ice concentration	R_{ice}	GRICR	IJLSDM	
u surface wind	U_0	GDUA	IJLSDM	
v surface wind	V_0	GDVA	IJLSDM	
cos(solar zenith)	$\cos(\theta)$	RCOSZ	IJLSDM	

- Internal work variables

Meaning	Presentation	Variable	dimension	unit
snow fraction	R_{snow}	GRSNR	IJLSDM	-
	$T_m^{max} - T_m^{min}$	TALSNX	IJLSDM	–
	$\alpha_{snow(2,b)} - \alpha_{snow(1,b)}$	ALBSNX	IJLSDM	–
	α_0	ALB0	IJLSDM	–
	tmp	DALB	IJLSDM	–
	$F(T_s)$	TFACT	IJLSDM	–
	α''	ALBX	IJLSDM	–
	r'	Z00	IJLSDM	–
		DZ0	IJLSDM	–
heat diffusion coefficient		DFGT	IJLSDM	–
		DFGX	IJLSDM	–

In this module, surface albedo and roughness are calculated. They are calculated supposing ice-free conditions, then modified.

First, let us consider the lake albedo. The lake level $\alpha_{(d,b)}$, $b = 1, 2, 3$ represent the visible, near-infrared, and infrared wavelength bands, respectively. Also, $d = 1, 2$ represents direct and scattered light, respectively. The albedo for the visible bands are calculated in `MODULE [LAKEALB]`, supposing ice-free conditions. The albedo for near-infrared is set to same as the visible one. The albedo for infrared is uniformly set to a constant value.

When lake ice is present, the albedo is modified to take into account the ice concentration R_{ice}.

$$\alpha' = \alpha + (\alpha_{ice} - \alpha) R_{ice} \tag{1}$$

where α_{ice} is the lake ice albedo. In addition, we want to consider the albedo change due to snow cover. Assuming that the snow albedo depends on the surface temperature (T_s), we can calculate the function $F(T_s)$

$$F(T_s) = \frac{T_s - T_m^{min}}{T_m^{max} - T_m^{min}} \tag{2}$$

but $0 \leq F(T_s) \leq 1$. The snow cover albedo can be expressed using

$$\alpha'' = \alpha(1, b) + (\alpha_{snow(2,b)} - \alpha_{snow(1,b)}) F(T_s) \tag{3}$$

Therefore, taking into account the snow coverage R_{snow}, we can express it as

$$\alpha = \alpha' + (\alpha'' - \alpha')R_{snow} \tag{4}$$

Second, let us consider the lake surface roughness. The roughnesses of for momentum, heat and vapor are calculated in `[LAKEZOF]`, supposing the ice-free conditions.

When lake ice is present, each roughness is modified to take into account the ice concentration R_{ice}.

$$r_0' = r_0 + (r_{ice} - r_0)R_{ice} \tag{5}$$

Then, taking into account the snow coverage R_{snow}, we can express it as

$$r_0 = r_0' + (r_{snow} - r_0')R_{snow} \tag{6}$$

If the lake ice exists, the heat diffusion coefficient of lake ice D_{ice}

$$\left(\frac{\partial G}{\partial T}\right)_{ice} = \frac{D_{ice}}{R_{ICE}} \tag{7}$$

If the snow exists, the heat diffusion coefficient of snow covered area is

$$\left(\frac{\partial G}{\partial T}\right)_{snow} = \frac{D_{ice}D_{snow}}{D_{ice}R_{snow} + D_{snow}R_{ice}} \tag{8}$$

Therefore, the net heat diffusion coefficient is finally

$$\frac{\partial G}{\partial T} = \left(\frac{\partial G}{\partial T}\right)_{ice}(1 - R_{snow}) + \left(\frac{\partial G}{\partial T}\right)_{snow}R_{snow} \tag{9}$$

The temperature differences between the snow surface (T_S) and the ice bottom (T_B) is saved as heat flux, because the difference should be zero in the ice-free conditions.

$$G = \frac{\partial G}{\partial T}(T_B - T_S) \tag{10}$$

1.1.1 11.1.1. lake surface albedo `[LAKEALB]`

- Inputs

Meaning	Presentation	Variable	dimension	unit
cos(solar zenith)	$cos(\theta)$	COSZ	IJLSDM	[-]

- Outputs

Meaning	Presentation	Variable	dimension	unit
lake surface albedo (direct, diffuse)	$\alpha_{L(d)}$	GALB	IJLSDM ,2	[-]

For lake surface level albedo $\alpha_{L(d)}$, $d = 1, 2$ represents direct and scattered light, respectively. Using the solar zenith angle at latitude θ, the albedo for direct light is presented by

$$\alpha_{L(1)} = e^{(C_3 A^* + C_2) A^* + C_1} \tag{11}$$

where $A = \min(\max(\cos(\theta), 0.03459), 0.961)$
On the other hand, the albedo for scattered light is uniformly set to a constant parameter.

$$\alpha_{L(2)} = 0.06 \tag{12}$$

1.1.2 11.1.2 Lake surface roughness [LAKEZ0F]

- Outputs

Meaning	Presentation	Variable	dimension	unit
surface roughness for momentum	$r_{0,M}$	GRZ0M	IJLSDM	–
surface roughness for heat	$r_{0,H}$	GRZ0H	IJLSDM	–
surface roughness for vapor	$r_{0,E}$	GRZ0E	IJLSDIM	–

- Inputs

Meaning	Presentation	Variable	dimension	unit
u surface wind	U_0	GDUA	IJLSDM	–
v surface wind	V_0	GDVA	IJLSDM	–

surface flux section of MIROC-DOC will be transplanted after modification.
The roughness variation of the lake surface is determined by the friction velocity $u^\star$

$$u^\star = \sqrt{C_{M_0}({U_0}^2 + {V_0}^2)} \tag{13}$$

The bulk coefficient for $u^\star$ (C_{M_0}) is given as a parameter.
$r_{0,M}, r_{0,H}$ and $r_{0,E}$ are surface roughness for momentum, heat, and vapor are presented by

$$r_{0,M} = z_{0,M_0} + z_{0,M_R} + \frac{z_{0,M_R} u^{\star 2}}{g} + \frac{z_{0,M_S} \nu}{u^\star} \tag{14}$$

$$r_{0,H} = z_{0,H_0} + z_{0,H_R} + \frac{z_{0,H_R} u^{\star 2}}{g} + \frac{z_{0,H_S} \nu}{u^\star} \tag{15}$$

$$r_{0,E} = z_{0,E_0} + z_{0,E_R} + \frac{z_{0,E_R} u^{\star 2}}{g} + \frac{z_{0,E_S} \nu}{u^\star} \tag{16}$$

Here, $\nu = 1.5 \times 10^{-5}$[m^2/s] is the kinetic viscosity of the atmosphere. z_{0,M_0}, z_{0,H_0} and z_{0,E_0} are base, and rough factor (z_{0,M_R}, z_{0,M_R} and z_{0,E_R}) and smooth factor (z_{0,M_S}, z_{0,M_S} and z_{0,E_S}), respectively.

1.2 11.2 Lake surface heat balance [LAKEHB] [100% Written in Jan, 2021]

The comments for some variables say "soil", but this is because the program was adapted from a land surface scheme, and has no particular meaning.

- Outputs

Meaning	Presentation	Variable	dimension	unit
surface water flux	$W_{free/ice}$	WFLUXS	IJLSDM,2	–
upward long wave	$LW^{\uparrow}$	RFLXLU	IJLSDM	–
flux balance	F	SFLXBL	IJLSDM	–

- Inputs variables

Meaning	Presentation	Variable
sensible heat flux coefficent	$\frac{\partial H}{\partial T_s}$	DTFDS
latent heat flux coefficient	$\frac{\partial E}{\partial T_s}$	DQFDS
surface heat flux coefficient	$\frac{\partial G}{\partial T_s}$	DGFDS
downward SW radiation	$SW^{\downarrow}$	RFLXSD
upward SW radiation	$SW^{\uparrow}$	RFLXLU

Meaning	Presentation	Variable
downward LW radiation	$LW^{\downarrow}$	RFLXLD
lake surface albedo	α	GRALBL
lake ice concentration	R_{ice}	GRICR

Reference: Hasumi, 2015, Appendices A

Downward radiative fluxes are not directly dependent on the condition of the lake surface, and their observed values are simply specified to drive the model. Shotwave emission from the lake surface is negligible, so the upward part of the shortwave radiative flux is accounted for solely by reflection of the incoming downward flux. Let α_S be the lake surface albedo for shortwave radiation. The upward shortwave radiative flux is represented by

$$SW^{\uparrow} = -\alpha_S SW^{\downarrow} \tag{17}$$

On the other hand, the upward longwave radiative flux has both reflection of the incoming flux and emission from the lake surface. Let α be the lake surface albedo for longwave radiation and ϵ be emissivity of the lake surface relative to the black body radiation. The upward shortwave radiative flux is represented by

$$LW^{\uparrow} = -\alpha LW^{\downarrow} + \epsilon\sigma T_s^4 \tag{18}$$

where σ is the Stefan-Boltzmann constant and T_s is skin temperature. If lake ice exists, snow or lake ice temperature is considered by fractions. When radiative equilibrium is assumed, emissivity becomes identical to co-albedo:

$$\epsilon = 1 - \alpha \tag{19}$$

The net surface flux is presented by

$$F^* = H + (1-\alpha)\sigma T_s^4 + \alpha LW^{\uparrow} - LW^{\downarrow} + SW^{\uparrow} - SW^{\downarrow} \tag{20}$$

The heat flux into the lake surface is presented, with the surface heat flux calculated in `PSFCM`

$$G^* = G - F^* \tag{21}$$

Note that G^* is downward positive.

The temperature derivative term is

$$\frac{\partial G^*}{\partial T_s} = \frac{\partial G}{\partial T_s} + \frac{\partial H}{\partial T_s} + \frac{\partial R}{\partial T_s} \tag{22}$$

When the lake ice exists, the sublimation flux is considered

$$G_{ice} = G^* - l_s E \tag{23}$$

The temperature derivative term is

$$\frac{\partial G_{ice}}{\partial T_s} = \frac{\partial G^*}{\partial T_s} + l_s \frac{\partial E}{\partial T_s} \tag{24}$$

Finally, we can update the surface temperature with the lake ice concentration with $\Delta T_s = G_{ice}(\frac{\partial G_{ice}}{\partial T_s})^{-1}$

$$T_s = T_s + R_{ice}\Delta T_s \tag{25}$$

Then, the sensible and latent heat flux on the lake ice is updated.

$$E_{ice} = E + \frac{\partial E}{\partial T_s}\Delta T_s \tag{26}$$

$$H_{ice} = H + \frac{\partial H}{\partial T_s}\Delta T_s \tag{27}$$

When the lake ice does not existed, otherwise, the evaporation flux is added to the net flux.

$$G_{free} = F^* + l_c E \tag{28}$$

Finally each flux is updated.
For the sensible heat flux, the temperature change on the lake ice is considered.

$$H = H + R_{ice}H_{ice} \tag{29}$$

Then, the heat used for the temperature change is saved.

$$F = R_{ice}H_{ice} \tag{30}$$

For the upward longwave radiative flux, the temperature change on the lake ice is considered.

$$LW^{\uparrow} = LW^{\uparrow} + 4\frac{\sigma}{T_s}R_{ice}\Delta T_s \tag{31}$$

For the surface heat flux, the lake ice concentration is considered.

$$G = (1 - R_{ice})G_{free} + R_{ice}G_{ice} \tag{32}$$

For the latent heat flux, the lake ice concentration is considered.

$$E = (1 - R_{ice})E + R_{ice}E_{ice} \tag{33}$$

Each term above are saved as freshwater flux.

$$W_{free} = (1 - R_{ice})E \tag{34}$$

$$W_{ice} = R_{ice}E_{ice} \tag{35}$$

1.3 11.3 Lake ice submodel `[LAKEIC]` [100% Written in Jan, 2021]

The following is an addition based on Hasumi, 2015, Appendix B1.

A simple lake ice model is based on zero-layer thermodynamics [Semtner, 1976].

There are five prognostic variables in the lake ice model described herein: lake ice concentration A_I, which is area fraction of a grid covered by lake ice and takes a value between zero and unity; mean lake ice thickness h_I over ice-covered part of a grid; mean snow depth h_S over lake ice. The model calculates temperature at snow top (lake ice top when there is no snow cover) T_I, which is a diagnostic variable. Density of lake ice (ρ_I) and snow (ρ_S) are assumed to be constant Lake ice is assumed to have nonzero salinity, and its value S_I is assumed to be a constant parameter.

1.3.1 11.3.1 Heat Flux and Growth Rate `MODULE[FIHEATL]`

The following is an addition based on Hasumi, 2015, Appendix B1.1.

This section is actually the same with

Let us consider here a case that the model is integrated from the n-th time level to the (n+1)-th time level. A_I, h_I and h_S are incrementally modified in the following order.

Temperature at lake ice base is taken to be the lake model's top level temperature $T(k = 2)$. In this model, lake ice exists only when and where $T(k = 2)$ is at the freezing point T_f, which is a decreasing function of salinity ($T_f = -0.0543S$[℃] is used here, where temperature and salinity are measured by C and psu, respectively). In heat budget calculation for snow and lake ice, only latent heat of fusion and sublimation is taken into account, and heat content associated with temperature is neglected. Therefore, temperature inside lake ice and snow are not calculated, and T_I is estimated from surface heat balance.

Nonzero minimum values are prescribed for A_I and h_I , which are denoted by A_I^{min} and h_I^{min}, respectively. These parameters define a minimum possible volume of lake ice in a grid. If a predicted volume $A_I h_I$ is less than that minimum, A_I is reset to zero, and T_1 is lowered to compensate the corresponding latent heat. In this case, the lake model's top level is kept at a supercooled state. Such a state continues until the lake is further cooled and the temperature becomes low enough to produce more lake ice than that minimum by releasing the latent heat corresponding to the supercooling.

Surface heat flux is separately calculated for each of air-lake and air-ice interfaces in one grid. The surface temperature of lake ice T_I is determined such That

$$Q_{AI} = Q_{IO} \tag{36}$$

is satisified, where Q_{IO} is corresponding to $G + SW^{\downarrow}$ and Q_{AI} is corresponding to $G_{ice} - W_{ice}$. However, When the estimated T_I exceeds the melting point of lake ice T_m (which is set to 0 C for convenience), T_I is reset to T_m and Q_{AI} and Q_{IO} are re-estimated by using it. The heat inbalance between Q_{AI} and Q_{IO} is consumed to melt snow (lake ice when there is no snow cover). Snow growth rate due to this heat imbalance is estimated by

$$W_{AS} = \frac{Q_{AI} - Q_{IO}}{\rho L_f} \tag{37}$$

where ρ_O is density of lakewater and L_f is the latent heat of fusion (the same value is applied to snow and lake ice). This growth rate is expressed as a change of equivalent liquid water depth per time. It is zero when $T_I < T_m$ and negative when $T_I = T_m$. Note that W_{AS} is weighted by lake ice concentration.

Although it is assumed that $T(2) = T_f$ when lake ice exists, T_1 could deviated from T_f due to a change of salinity or other factors. Such deviation should be adjusted by forming or melting lake ice. Under a temperature deviation of the top layer of lake,

$$\Delta T = T(k=2) - T_f S(k=2) \tag{38}$$

lake ice growth rate necessary to compensate it in the single time step is given by

$$W_{FZ} = -\frac{C_{po} \Delta T \Delta z_1}{L_f \Delta t} \tag{39}$$

where C_{po} is the heat capacity of lake water and $\Delta z_1 = 100$cm is the thickenss of the lake model's top level (uniformly set to constant in case of the current lake model.) This growth rate is estimated at all grids, irrespective of lake ice existence, for a technical reason. As described below, this growth rate first estimates negative ice volume for ice-free grids, but the same heat flux calculation procedure as for ice-covered grids finally results in the correct heat flux to force the lake. Basal growth rate of lake ice is given by

$$W_{IO} = A_I W_{FZ} + \frac{Q_{IO}}{\rho_O L_f} \tag{40}$$

where, again, W_{IO} is weighted by lake ice concentration.

Lake ice formation could also occur in the ice-free area. Let us define Q_{AO} by

$$Q_{AO} = (1 - A_I)[Q - (1 - \alpha_s) SW^{\downarrow}] \tag{41}$$

i.e., air-lake heat flux except for shortwave, multiplied by the factor of the fraction of ice-free area. Here, Q is air-ice heat flux. Shortwave radiation absorbed at ice-free lake surface, with the factor of ice-free area multiplied, is represented by

$$SW^A = (1 - A_I)(1 - \alpha_S)SW^{\downarrow} \tag{42}$$

Lake ice growth rate in ice-free area is calculated by

$$W_{AO} = (1 - A_I)W_{FZ} + \frac{Q_{AO} + I(k=2)SW^A}{\rho_O L_f} \tag{43}$$

where $I(k=2)$ denotes the fraction of SW^A absorbed by the lake model's top level, which is calculate in `[SVTSETL]` in lakepo.F.

Finaly, the heat flux for freshwater is

$$G_{lake} = \Delta z_1 \frac{\Delta T}{\Delta t} \tag{44}$$

1.3.2 11.3.2 Sublimation of lake Ice`MODULE[FWATERL]`

- variables

Meaning	Presentation	Variable	dimension	unit
lake ice fraction	A'_I	AX	IJLDIM	–
lake ice thickenss	h'_I	HIX	IJLDIM	–
Snow depth	h'_S	HSX	IJLDIM	–
latent heat flux of evaporation	F_W^{EV}	WEV	–	–
latent heat flux of sublimation	F_W^{SB}	WSB	IJLDIM	cm/s
–	ΔF_W	WDIF	–	–
time step	Δt	TS	–	–
latent heat flux of evaporation	F_W^{EV}	EVAP	IJLDIM	–
latent heat flux of sublimation	$F_W^{SB\prime\prime}$	SUBI	IJLDIM	–

- Internal variables

Meaning	Presentation	Variable	dimension	unit
Lake ice concentration	A_I^n	AZ	IJLDIM	–

Meaning	Presentation	Variable	dimension	unit
Snow depth	h_S^n	HSZ	IJLDIM	–
lake ice thickness	h_I^n	HIZ	IJLDIM	–

- parameters

Meaning	Presentation	Variable	unit	value
density of snow	ρ_S	rhos	g/cm^3	0.33
density of lake ice	ρ_I	rhoi	g/cm^3	0.9
Ratio of density (ocean/snow)	R_{ρ_S}	rrs	[-]	ρ_O/ρ_s
Ratio of density (ocean/ice)	R_{ρ_I}	rri	[-]	ρ_O/ρ_I
Minimum thickness of ice	h_I^{min}	himin	–	1.0×10^1

The following is an addition based on Hasumi, 2015, Appendix B1.2.

Sublimation (freshwater) flux, which is practically come from the land ice runoff, is calculated or prescribed over lake ice cover. The flux is first consumed to reduce snow thickness in n-th timstep:

$$h'_S = h_S^n - \frac{\rho_O F_W^{SB} \Delta t}{\rho_S A_I^n} \tag{45}$$

If h'_S becomes less than zero, it is reset to zero. Then, the melted snow flux is added to F_W^{SB}. F_W^{SB} is redefined by

$$F_W^{SB\prime} = F_W^{SB} + \frac{\rho_S A_I^n (h'_S - h_S^n)}{\rho_O \Delta t} \tag{46}$$

Where there no remains snow, but $F_W^{SB\prime}$ is not zero, The remain flux is consumed to reduce lake ice thickness:

$$h'_I = h_I^n - \frac{\rho_O F_W^{SB\prime} \Delta t}{\rho_I A_I^n} \tag{47}$$

If h'_I becomes less than h_I^{min}, it is reset to zero. Then, the melted iceflux is added to $F_W^{SB\prime}$. $F_W^{SB\prime}$ is redefined by

$$F_W^{SB\prime\prime} = F_W^{SB\prime} - A_I^n \frac{\rho_S (h_I^n - h'_I)}{\rho_O \Delta t} \tag{48}$$

Finaly, nonzero $F_W^{SB}{''}$ is consumed to reduce lake ice concentration:

$$A_I' = A_I^n - \frac{R_{\rho I} F_W^{SB}{''} \Delta t}{h_I^{min}} \tag{49}$$

if A_I' becomes less then 0, it is reset to zero. Even if A_I' becomes less than A_I^{min}, on the other hand, it is not adjusted here. If A_I' is adjusted to zero, it means that the sublimation flux is not used up by eliminating snow and lake ice.

The remaining part is consumed to reduce lake water, so the evaporation flux F_W^{EV} is modified as

$$F_W^{EV} = F_W^{EV} + F_W^{SB} + \frac{(A_I' - A_I^n) h_I^{min}}{R_{\rho I} \Delta t} \tag{50}$$

The later two terms cancel out if the adjustment does not take place.

If there is no lake ice, evaporation flux is just as

$$F_W^{EV}{'} = F_W^{EV} + F_W^{SB} \tag{51}$$

The adjusted evaporation flux is saved

$$\Delta F_W^{EV} = F_W^{EV}{'} - F_W^{EV} \tag{52}$$

When sublimation flux is consumed to reduce lake ice amount, salt contained in lake ice has to be added to the remaining lake ice or the underlying water. Otherwise, total salt of the ice-lake system is not coserved. Here, it is added to underlying water, and the way of this adjustment is described later. Nothe that lake ice tends to gradually drain high salinity water contained in brine pockets in reality. Thus, such an adjustment is not very unreasonable. When A_I' is adjusted to zero, on the other hand, the remaining sublimation flux is consumed to reduce lake water. In this case, difference between the latent heat of sublimation and evaporation has to be adjusted, which is also described later.

If the ice and/or snow is too thick, they are converted to snow flux. Here, the overflow snowflux S_{off} is added to F_W^{SN}

$$F_W^{SN} = F_W^{SN} + S_{off} \tag{53}$$

S_{off} is actually calculated in `MATDRV` and handed to `LAKEIC`.

1.3.3 11.3.3 Update snow and ice volume [`PCMPCTL`]

The lake ice fraction is updated, using the lake ice growth (retreat) rate in ice-free area W_{AO}:

$$A_I^{n+1} = A_I' + \frac{\rho_O}{\rho_I h_I \phi W_{AO} \Delta t} \tag{54}$$

If A_I^{n+1} becomes greater than 1, it is reset to 1, and if A_I^{n+1} becomes smaller than zero, it is reset to zero.

1.3.4 11.3.4 Growth and Melting `[PTHICKL]`

- variables

Meaning	Presentation	Variable	dimension	unit
lake ice fraction	A_I^{n+1}	AX	IJLDIM	–
lake ice volume	V_I	AXHIX	IJLDIM	–
lake snow volume	V_S, V_S', V_S^{**}	AXHSX	IJLDIM	–
lake ice volume	V_I	AXHIXN	IJLDIM	–
–	–	AXHSXN	IJLDIM	–
lake ice thickenss	h_I'	HIX	IJLDIM	–
Snow depth	h_S'	HSX	IJLDIM	–
Snow depth	h_S^n	HSZ	IJLDIM	–
lake ice thickness	h_I^n	HIZ	IJLDIM	–
snow growth rate due to heat inbalance	W_{AS}	WAS	IJLDIM	–
lake ice growth rate due to heat inbalance	W_{AI}	WAI	IJLDIM	–
Reduced heat flux	W_{res}	WRES	IJLDIM	–
basal growth rate of lake ice	W_{IO}	WIO	IJLDIM	–
snow fall flux	$F_W^{SN}, F_W^{SN\prime}$	SNOW	IJLDIM	–
Lake ice growth rate in ice-free area	W_{AO}	WAO	IJLDIM	–
precipitation flux	F_W^{PR}	PREC	IJLDIM	–
latent heat flux of evaporation	L_e	EVAP	IJLDIM	–
latent heat flux of sublimation	–	SUBI	IJLDIM	–
Lake ice concentration	A_I'	AZ	IJLDIM	–
–	–	ROFF	IJLDIM	–
–	–	ADJLAT	IJLDIM	–
lake heat flux	H_{lake}	FT	IJLDIM, NLTDIM	–
–	–	FS	IJLDIM	–
time step	Δt	TS		–

以下、Hasumi, 2015, Appendix B.1.3 and B.1.4 をもとに加筆修正。

The lake ice volume V_I' and snow volume V_S' before the snow and ice growth are presented by

$$V_I' = A_I' h_I^n \tag{55}$$

$$V_S' = A_I' h_S^n \tag{56}$$

From here, let us consider the contribution of snowfall and freshwater fluxes to the growth.

- Snowfall flux to snow

Changes of snow depth due to snow fall (freshwater) flux F_W^{SN} (expressed by negative values to be consistent with other freshwater flux components) is first taken into account. F_W^{SN} is not weighted by lake ice concentration or ice-free area are fraction, as snowfall take place for both regions.

If the newly predicted (in `PCMCTL`) lake ice concentration A_I^{n+1} is zero, the amount of snow existed before the growth is added to the snowfall flux.

$$F_W^{SN\prime} = F_W^{SN} + \frac{\rho_S V_S'}{\rho_O \Delta t} \tag{57}$$

Snow depth and amount is set to zero:

$$h_S' = 0, \quad V_S^{**} = 0 \tag{58}$$

Otherwise, snowfall accumulates over the ice covered region. Snow depth is modified by

$$h_S^* = \frac{V_S'}{A_I^{n+1}} + \frac{\rho_O F_W^{SN} \Delta t}{\rho_S} \tag{59}$$

And the snow amount is also modifie by

$$V_S^* = A_I^{n+1} h_S^* \tag{60}$$

The snowfall flux is reduced by that amount:

$$F_W^{SN\prime} = (1 - A_I^{n+1}) F_W^{SN} \tag{61}$$

Then, the snowfall flux is put together with the precipitation flux.

$$F_W^{PR} = F_W^{PR} + F_W^{SN\prime} \tag{62}$$

- Freshwater flux for snow growth

When the lake ice is not existed ($A_I^{n+1} = 0$), the snow amount grown above is converted to ice. Growth rate of the lake ice is presented by:

$$W_{AI}^* = \frac{\rho_O V_S^*}{\rho_S \Delta t} \tag{63}$$

When the lake ice existed, if the snow growth rate

$$W_{AS}^* = W_{AS} + \frac{\rho_O V_S^*}{\rho_S \Delta t} \tag{64}$$

is positive, the energy is used for the snow growing. Otherwise, W_{AS}^* is assumed to reduce the lake ice.

$$W_{AI}^* = W_{AS}^* \tag{65}$$

and deficient flux is come from the snow amount changes.

$$W_{AS}^* = -\frac{\rho_O V_S^*}{\rho_S \Delta t} \tag{66}$$

Then, the snow depth is modified with the accumulation.

$$h_S^{**} = \frac{V_S^* + \rho_O W_{AS} \Delta t}{\rho_S A_I^{n+1}} \tag{67}$$

if h_S' is less than 0, it is reset to zero.

- Freshwater flux for ice growth

When the lake ice is not existed, the flux is just handed to the lake surface.

$$W_{IO}^* = W_{IO} + W_{AI}^* \tag{68}$$

When the lake ice exists ($A_I^{n+1} > 0$), if the ice growth rate

$$W_{AI}^* = W_{AI} + \frac{V_I^*}{R_{\rho_I \Delta t}} \tag{69}$$

is negative, the flux is handed to the lake surface.

$$W_{IO}^* = W_{IO} + W_{AI}^* \tag{70}$$

and deficient flux is come from the lake amount changes.

$$W_{AI}^* = -\frac{V_I^*}{R_{\rho_I \Delta t}} \tag{71}$$

The amount of the ice is then updated,

$$V_I^* = V_I' + \frac{\rho_O W_{AI} \Delta t}{\rho_I} \tag{72}$$

- Get the variables in the new timestep (n+1)

For the snow amount,

$$V_S^{n+1} = A_I^{n+1} h_S^{**} \tag{73}$$

For the ice amount

$$V_I^{n+1} = V_I^* + \frac{\rho_O (W_{IO} + W_{AO}) \Delta t}{\rho_I} \tag{74}$$

If V_I^{n+1} is equal or less than 0, lake ice fraction is set to zero (A_I^{n+1}=0)and its thickness is set to h_I^{min}. Otherwise,

$$h_I^{***} = \frac{V_I^{n+1}}{A^*} \tag{75}$$

If h_I^{***} is smaller than h_I^{min}, it is set t0 h_I^{min} and the lake ice concentration is adjusted.

$$A_I^{n+1} = \frac{V_I^{n+1}}{h_I^{min}} \tag{76}$$

If the A_I^{n+1} is less than A_I^{min}, it is set to 0. If the A_I^{n+1} is larger than $A_I^{max} = 1$, it is set to A_I^{max}.

Let us consider the case of the ice is very thick. Here, the remained volume, which is not covered by ice is considered.

$$V_0^{free} = (A_{max} - A_{min}) h_I^{min} \tag{77}$$

$$V^{free} = (A_{max} - A_I^{n+1}) h_I^{***} \tag{78}$$

If $V^{free} > V_0^{free}$, the ice thickness is increased, by adding V^{free}

$$h_I^{***} = V^{free} + \frac{A_I^{n+1} h_I^{m+1}}{A_I^{max}} \tag{79}$$

The deficient water is come from the snow. The snow depth is now updated

$$h_S^{***} = A_I^{n+1} \frac{h_S^{***}}{A_{max} - \frac{V_0^{free}}{h_I^{***}}} \tag{80}$$

Finally, check if the snow is under water.

$$h_S^{n+1} = \min(h_S^{***}, \frac{\rho_O - \rho_I}{\rho_S} h_I^{***}) \tag{81}$$

and the ice thickness is also updated.

$$h_I^{n+1} = h_I^{***} + \frac{\rho_S}{\rho_I}(h_S^{***} - h_S^{n+1}) \tag{82}$$

The growth rate of the lake ice is

$$W_I^n = \frac{\rho_S A_I^{n+1} h_I^{n+1} - V_I'}{\rho_I \Delta t} \tag{83}$$

$$W_I^{n+1} = \frac{\rho_S A_I^{n+1} h_I^{n+1} - V_I^{n+1}}{\rho_I \Delta t} \tag{84}$$

The growth rate of the snow is

$$W_S^n = \frac{\rho_S A_I^{n+1} h_S^{n+1} - V_S'}{\rho_S \Delta t} \tag{85}$$

$$W_S^{n+1} = \frac{\rho_S A_I^{n+1} h_S^{n+1} - V_S^{n+1}}{\rho_S \Delta t} \tag{86}$$

The surface salinity flux F_S is

$$F_S = S_I(W_I - F_W^{SB\prime\prime}) \tag{87}$$

The freshwater flux F_W is

$$F_W(1) = -F_W + \frac{L_f}{C_p}\left(W_I^{n+1} + W_S^{n+1} - Sn + \Delta F_W^{EV}\right) \tag{88}$$

塩分変化に関係する分？

$$F_W(2) = F_W^{EV} - F_W^{PR} - R_{off} + W_S^n + W_I^n \tag{89}$$

1.4 11.4 Physical formulation & processes [LAKEPO] [20% Written in Jan, 2021]

`lakepo.F`

1.4.1 11.4.1 Diffusion tracer flux [VDIFFL]

- Inputs

Though several valuables are written, they are not used here.

The case considered in COCO4 is that(Hasumi, 2015, Section 1.2): 1. Diffusive flux of tracer follows the Fick's law. 2. Diffusuion coefficient tensor is diagonal, and its horizontal component is identical.

Here, the vertical diffusion coefficent K_V is simply set as

$$K_V(k) = K_{V0}k \tag{90}$$

1.4.2 11.4.2 Estimate the advection and diffusion terms of the tracer equations [FLXTRCL]

- Inputs

Meaning	Presentation	Variable	dimension	unit
vertical diffusion coefficient	K_V	AHV	IJLDIM, NLZDIM	–
water temperature	T	TX	IJLDIM, NLZDIM, NLTDIM	–
water depth	h	HX	IJLDIM	–

- Outputs

Meaning	Presentation	Variable	dimension	unit
vertical component of diffusive tracer flux	F_D	ADT	IJLDIM, NLZDIM, NLTDIM	–
	$\frac{F_D}{D(k-\frac{1}{2})}$	DIFFZ	IJLDIM, NLZDIM	–

- Internal variables

Meaning	Presentation	Variable	dimension	unit
–	–	TH	IJLDIM, NLZDIM, NLTDIM	–
–	h'	HZBOT	IJLDIM	–

Supposing that the thickness of the 1st top layer ($k = 2$) is D_1,

$$D(2) = D_1 \tag{91}$$

the thickness of the remaining water column (h') is presented by

$$h' = h - D_1 \tag{92}$$

The thickness of each remaining layer ($k = 3, 4, 5, 6$) is

$$D(k) = S(k)h' \tag{93}$$

Then, the component of the diffusive tracer flux is represented by

$$F_D = K_V \frac{\partial T}{\partial z} \tag{94}$$

practically,

$$F_D(k) = K_V(k) \frac{T(k-1) - T(k)}{\frac{D(k-1)+D(k)}{2}} - K_V(k+1) \frac{T(k) - T(k+1)}{\frac{D(k)+D(k+1)}{2}} \tag{95}$$

1.4.3 11.4.3 [SLVTRCL]

- Inputs

Meaning	Presentation	Variable	dimension	unit
vertical component of diffusive tracer flux	F_D	ADT	IJLDIM, NLZDIM, NLTDIM	–
	$\frac{F_D}{D(k-\frac{1}{2})}$	DIFFZ	IJLDIM, NLZDIM	–
minimum depth of lake	–	HXMIN		cm
heat flux	–	FT		Kcm/s
absorbed shortwave	–	SWABS		erg/cm
	–	FS		–
time step	Δt	TS		–
surface-type fraction (lake)		LKFRAC	IJLDIM	[-]

- Outputs

Meaning	Presentation	Variable	dimension	unit
lake water deficient	–	XHD	IJLDIM	cm

$$A_A(k) = -\frac{K_V(k)}{\frac{D(k-1)+D(k)}{2}}\Delta t \tag{96}$$

$$A_C(k) = -\frac{K_V(k+1)}{\frac{D(k)+D(k+1)}{2}}\Delta t \tag{97}$$

$$A_B(k) = D)k - A_A(k) - A(k) \tag{98}$$

Solve the linear equations expressed by tri-diagonal matrix in `MODULE: THOASL`. (original files ./ocean/utrdg.F)

Then,

$$T(k) = T(k) + F_D(k)\Delta t \tag{99}$$

$$h_D = -F_T\Delta t \tag{100}$$

$$V_D = \max(h_{min} - h - h_D, 0)R_{lake} \tag{101}$$

$$h_D = \max(h_D, h_{min} - h) \tag{102}$$

Then, the deficient water is added.

$$h = h + h_D \tag{103}$$

$$h_{BOT} = h - D_1 \tag{104}$$

ここで、

If $h_D \geq 0$, $D^{top} = h_D S(\mathrm{KLEND})$ and $D^{bottom} = 0$

If $h_D \geq 0$, 下からループ

$$T(k) = T(k) + \frac{T(k-1)D^{top}}{h_{BOT}S(k)} \tag{105}$$

1.4.4 11.4.4 [OVTURNL]

Reference: Hasumi, 2015, Section 4.1

Let us consider finite difference discretization of a flux-form, one-dimensional advection equation for tracer ψ.

$$\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial z}(w\psi) = 0 \tag{106}$$

- Inputs

Meaning	Presentation	Variable	dimension	unit
$depthofwatercolumn$	h	H	–	–
time step	Δt	TS	–	–

- Outputs

Meaning	Presentation	Variable	dimension	unit
–	–	R	–	–

The unit of XHD will be changed to kg/m2 after this module (in `MATDRV`).