Artificial Intelligence Notes

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1 | INTRODUCTION

This course will provide an introduction to AI techniques and approach analyzed nowadays and to understand the current state of art we have to provide an Timeline to see progress and discover done during the time, so in figure 1 we will see all important events related with AI.

The major discover happened on 2020 are the following:

GPT3 (GENERATIVE PRE-TRAINED TRANSFORMER): produced by OpenAI in May 2020, is a larger and richer language model consisting in 175 billion machine learning parameters used for automatic text generation, translation, user interface synthesis

DARPA CHALLENGE (ALPHADOGFIGHTS) with simulated F-16 Air Fighters where on 18-20 August 2020 there was the final Event, where AI system was against each other and the winner was a system by Heron system, that was also able to defeated a human expert top gun fighter $5-0$.

Andrew NG says that AI will transform many industries, but it's not magic and almost all of AI's recent progress is based on one type of AI, in which some input data (A) is used to quickly generate some simple response (B) $[A \to B]$.
Also Andrew Ng says that if a typical person can do a mental task with less than one second of thought, we can probably automate it using AI either now or in the near future.
Choosing A and B creatively has already revolutionized many industries, it is poised to revolutionize many more.

ML systems are not (yet?) able to justify in human terms their results, so for some application it is essential the human knowledge to be able to generate explanations, infact some regulations requires the right to an explanation in decision-making, and seek to prevent discrimination based on race, opinions, health, sex and so on, like GPDR.
ML systems learn what's in the data, without understanding what's true or false, real or imaginary, fair or unfair and so it is possible to develop bad/unfair models.

The goal of building AI systems is far from being solved and is still quite challenging in its own. Building complex AI systems requires the combination of several techniques and approaches, not only ML.
One of the most challenging tasks ahead of us is integration of perception and reasoning in AI systems.

AI fundamentals is mostly about "Slow thinking" or "Reasoning" and AI fundamentals has the role, within the AI curriculum, of teaching you about the foundations of a discipline which is now 60 year old.
We will cover different approaches, also some coming of the "Good Old- Fashioned Artificial Intelligence" (GOFAI) or symbolic AI.

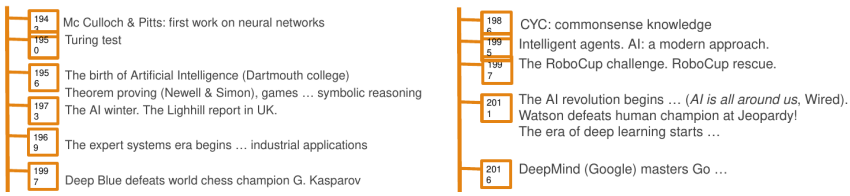


Figura 1: AI Timeline evolution

Def. Symbolic AI is an high-level "symbolic" (human-readable) representations of problems, the general paradigm of searching for a solution, knowledge representation and reasoning, planning.
Symbolic AI was the dominant paradigm of AI research from the mid 1950s until the late 1980s and central to the building of AI systems is the *Physical symbol systems hypothesis*, formulated by Newell and Simon.

The approach is based on the assumption that many aspects of intelligence can be achieved by the manipulation of symbols (the physical symbol system hypothesis):

Def. A physical symbol system has the necessary and sufficient means for general intelligent action

Human thinking is a kind of symbol manipulation system (a symbol system is necessary for intelligence) and machines can be intelligent (a symbol system is sufficient for intelligence).
The hypothesis cannot be proven, we can only collect empirical evidence and observations and experiments on human behavior in tasks requiring intelligence.

We have two different typologies of AI, that was introduced and considered:

STRONG AI: relies on the strong assumption that human intelligence can be reproduced in all its aspects (general A.I.).
It includes adaptivity, learning, consciousness and not only pre-programmed behavior.

WEAK AI: simulation of human-like behavior, without effective thinking/understanding and no claim that it works like human mind; it is the dominant approach today.

A problem of AI is that computer can't have needs, cravings or desires and Abraham Maslow's define a hierarchy of human needs:

1. Biological needs (food, sleep, sex, ...)
2. Safety, protection from environment
3. Love and belonging, friendship
4. Self esteem and respect from others
5. Self-actualization

2 | AGENTS

Artificial intelligence, or AI, is the field that studies the synthesis and analysis of computational agents that act intelligently.
An agent is something that acts in an environment/it does something and we are interested in what an agent does, that is, how it acts, so we judge an agent by its actions.

An agent acts *intelligently* when what it does is appropriate given the circumstances and its goals, it is flexible to changing environments and changing goals, it learns from experience and it makes appropriate choices given its perceptual and computational limitations.

Def. A *computational agent* is an agent whose decisions about its actions can be explained in terms of computation.

We have that the central scientific goal of AI is to understand the principles that make intelligent behavior possible in natural or artificial systems, instead the central engineering goal of AI is the design and synthesis of useful, intelligent artefacts, agents, that are useful in many applications.
This is done by the analysis of natural and artificial agents, formulating and testing hypotheses about what it takes to construct intelligent agents and in the end designing, building, and experimenting with computational systems that perform tasks commonly viewed as requiring intelligence.

Artificial Intelligence is not the opposite of real Intelligence, infact intelligence cannot be fake, so if an artificial agent behaves intelligently, it is intelligent and it is only the external behavior that defines intelligence (weak AI).

Artificial intelligence is real intelligence created artificially and we can use different test to estabilish if AI is intelligent: *Turing test* where only external behavior counts and *Winograd schemas* as a test of intelligence, where we asks "The city councilmen refused the demonstrators a permit because they feared violence. Who feared violence?" and "The city councilmen refused the demonstrators a permit because they advocated violence. Who advocated violence?".
These questions are difficult for a machine because it has not the knowledge of context.

The obvious naturally intelligent agent is the human being and the human intelligence comes from three main sources:

1. biology: Humans have evolved into adaptable animals that can survive in various habitats.

2. culture: Culture provides not only language, but also useful tools, useful concepts, and the wisdom that is passed from parents and teachers to children. Language, which is part of culture, provides distinctions in the world that should be noticed for learning.

3. life-long learning (experience): Humans learn throughout their life and accumulate knowledge and skills.

Another form of intelligence is *social intelligence*, the one exhibited by communities and organizations.

Three aspects of computation that must be distinguished:

1. Design time computation, that goes into the design of the agent

2. Offline computation, that the agent can do before acting in the world

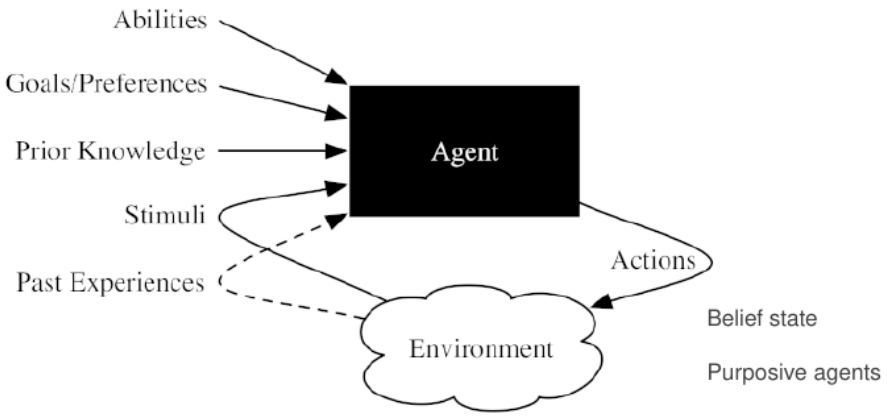


Figura 2: Design Process of AI agents

3. Online computation, the computation that is done by the agent as it is acting.

Designing an intelligent agent that can adapt to complex environments and changing goals is a major challenge, so to reach this ultimate goal, two strategies are possible:

1. simplify environments and build complex reasoning systems for these simple environments.

2. build simple agents for natural/complex environments, simplifying the tasks.

The design process of an agents has the following phases, that can be viewed on figure 2:

1. Define the task: specify what needs to be computed

2. Define what constitutes a *solution* and its quality: optimal solution, satisficing solution, approximately optimal solution, probable solution.

3. Choose a formal representation for the task; this means choosing how to represent knowledge for the task and this includes representations suitable for learning.

4. Compute an output

5. Interpret output as a solution

A model of the world is a symbolic representation of the beliefs of the agents about the world, so it is necessarily an abstraction.
More abstract representations are simpler and human-understandable, but they may be not effective enough and low level descriptions are more detailed and accurate but introduce complexity.

Multiple level of abstractions are possible (hierarchical design), but usually two levels are considered:

1. The *knowledge level*: what the agent knows and its goals

2. The *symbol level*: the internal representation and reasoning algorithms

In agent design we consider these aspects, with a summary foundable in figure 3:

MODULARITY: is the extent to which a system can be decomposed into interacting modules and it is a key factor for reducing complexity.
In the modularity dimension, an agent's structure is one of the following:

- *flat* where there is no organizational structure.

- *modular*, which the system is decomposed into interacting modules that can be understood on their own.
- *hierarchical*, which the system is modular, and the modules themselves are decomposed into simpler modules and the agent reasons at multiple levels of abstraction.

PLANNING HORIZON: is how far ahead in time the agent plans and in this dimension an agent is one of the following:

- *Non-planning agent*, that does not look at the future.
- *Finite horizon* planner, where agent looks for a fixed finite number of stages and it is greedy if only looks one time step ahead.
- *Indefinite horizon* planner is an agent that looks ahead some finite, but not predetermined, number of stages.
- *Infinite horizon* planner is an agent that keeps planning forever

REPRESENTATION: concerns on how the state of the world is described and a state of the world specifies the agent's internal state (its belief state) and the environment state.
We have 3 type of representation, from simple to complex:

- *atomic* states, as in problem solving.
- *feature-based* representation: set of propositions that are true or false of the state, properties with a set of possible values.
- Individuals and relations (often called relational representations) and is the Representations at the expressive level of FOL (or contractions).

COMPUTATIONAL LIMIT: an agent must decide on its best action within time constraints or other constraints in computational resources (memory, precision, ...).
The computational limits dimension determines whether an agent has *perfect rationality*, where an agent is able to reasons about the best action without constraints and *bounded rationality*, where an agent decides on the best action that it can find given its computational limitations.
An *anytime algorithm* is an algorithm where the solution quality improves with time and to take into account bounded rationality, an agent must decide whether it should act or reason for longer.

LEARNING: is necessary when the designer does not have a good model and the learning dimension determines whether knowledge is given in advance or knowledge is learned from data or past experience.
Learning typically means finding the best model that fits the data and produces a good predictive model and in this course only modelling formalisms and approaches are dealt, infact all the issues concerned with learning are dealt in the Machine Learning course.

UNCERTAINTY: is divided into two dimensions:

1. uncertainty from sensing/perception (fully observable, partially observable states).
2. uncertainty about the effects of actions (deterministic, stochastic) and when the effect is stochastic, there is only a probability distribution over the resulting states.

PREFERENCE: considers whether the agent has goals or richer preferences:

- A *goal* is either an achievement goal, which is a proposition to be true in some final state, or a maintenance goal, a proposition that must be true in all visited states.

Dimension	Values
Modularity	flat, modular, hierarchical
Planning horizon	non–planning, finite stage, indefinite stage, infinite stage
Representation	states, features, relations
Computational limits	perfect rationality, bounded rationality
Learning	knowledge is given, knowledge is learned
Sensing uncertainty	fully observable, partially observable
Effect uncertainty	deterministic, stochastic
Preference	goals, complex preferences
Number of agents	single agent, multiple agents
Interaction	offline, online

Figura 3: Summary of Agent Design consideration

- *Complex preferences* involve trade-offs among the desirability of various outcomes, perhaps at different times.
  An *ordinal preference* is where only the ordering of the preferences is important, instead a *cardinal preference* is where the magnitude of the values matters and States are evaluated by utility functions.

NUMBER OF AGENTS: considers whether the agent explicitly considers other agents:

- *Single agent* reasoning means the agent assumes that there are no other agents in the environment or that all other agents are "part of nature", and so are non-purposive.
- *Multiple agent* reasoning means the agent takes the reasoning of other agents into account and this occurs when there are other intelligent agents whose goals or preferences depend, in part, on what the agent does or if the agent must communicate with other agents.

INTERACTION: considers whether the agent does:

- *offline reasoning*, where the agent determines what to do before interacting with the environment.
- *online reasoning*, where the agent must determine what action to do while interacting in the environment, and needs to make timely decisions.

More sophisticated agents reason while acting and this includes long-range strategic reasoning as well as reasoning for reacting in a timely manner to the environment.

Def. An agent is something that interacts with an environment, receiving information through its sensors and acts in the world through actuators (effectors).
A robot is an artificial purposive embodied agent and also a computer program is a software agent

An agent is made up of a body and a controller, where the controller receives percepts from the body and sends commands to the body.
A body includes sensors that convert stimuli into percepts and actuators that convert

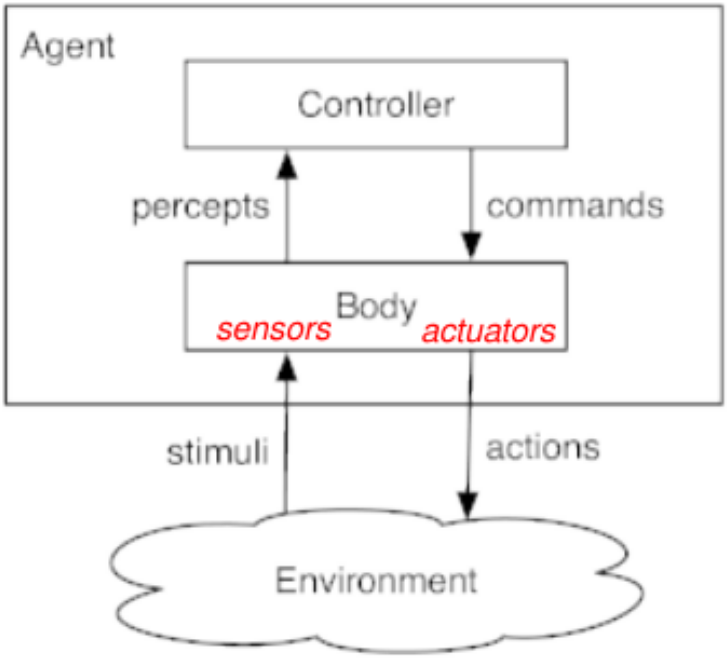


Figura 4: Structure of an generic agent system

commands into actions. Both sensor and actuators can be uncertain, the controller is the brain of an agent and in the end an agent system includes an agent and the environment in which it acts.
In figure 4 is possible to see the structure of an generic agent system.

Agents act in time, we have T that is the set of time points, and we assume that T has a start (0) totally ordered, discrete, and each t has a next time $t + 1$.
We have an agent history that at time t has percepts up to t and commands up to $t - 1$ and we have also *causal transduction* (usually implements by a controller), a function from history to commands, called causal because only previous and current percepts and previous commands can be considered.

However, complete history is usually not available and we have only the memory of it: the memory or belief state of an agent at time t is all the information the agent has remembered from the previous times and the behavior of an agent is described by two functions:

- A *belief state* transition function $S \times P \rightarrow S$, where S is the set of belief states and P is the set of percepts.
- A *command* function $S \times P \rightarrow C$, where C is the set of commands.

The controller implements a command function (an approximation of a causal transduction) and with a single controller it is difficult to reconcile the slow reasoning about complex high-level goals with the fast reaction that an agent needs for lower-level tasks such as avoiding obstacles.
In figure 5 is possible to note the agent functions that should be implemented at each layer of our agent.

high-level reasoning, is often discrete and qualitative • low-level reasoning is often continuous and quantitative A controller that reasons in terms of both discrete and continuous values is called a hybrid system.

The notion of belief state is quite general, most agents need to keep a model of the word and update it while acting and there are 2 extremes:

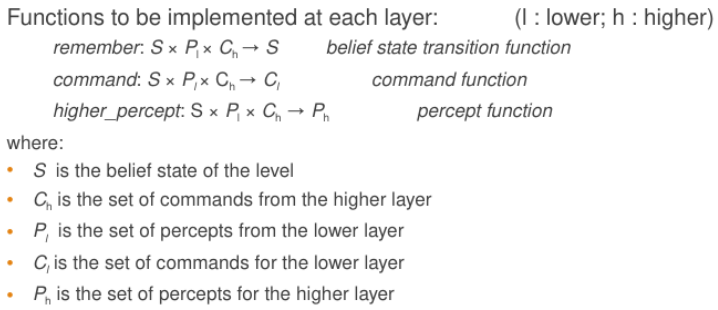


Figura 5: Agent Functions to implement at each layer

1. the agent possess a very good predictive model, so it does not need to use perceptions to update the model
2. purely reactive systems do not have a model, and decide only on the basis of perceptions

In the general case the agent uses a combination of prediction and sensing:

- In Bayesian reasoning (under uncertain information) the estimation of the next belief state is called *filtering*.
- In alternative, more complex models of the world can be kept and updated, for example through vision and image processing.

Knowledge of a specific domain may also be represented explicitly and used to decide the action and the *knowledge base* contains general rules and specific/contingent facts in declarative form.
The KB is built offline, built by designers or learned from data and a domain ontology gives meaning to symbols used to represent knowledge; knowledge may be then updated and used to decide actions during operation.

3 CSP (CONSTRAINT SATISFACTION PROBLEMS

It is often better to describe states in terms of features and then to reason in terms of these features and we have called this a *factored representation*.

This representation may be more natural and efficient than explicitly enumerating the states, so with 10 binary features we can describe $2^{10} = 1024$ states, often these features are not independent and there are constraints that specify legal combinations of assignments of values to them.

A CSP is a problem composed of a finite set of variables, each variable is associated with a finite domain and a set of constraints that restrict the values of the variables can simultaneously take, so the task is to assign a value (from the associated domain) to each variable satisfying all the constraints; this problem is in NP hard in the worst cases but general heuristics exist, and structure can be exploited for efficiency.

A Constraint Satisfaction Problem consists of three components, X, D, and C, so $CSP = (X, D, C)$ where X is a set of variables $\{x_1, \ldots, x_n\}$, D is a set of domains $\{D_1, \ldots, D_n\}$, one for each variable, and C is a set of contraints that specify allowable combinations of values.

A [partial] assignment of values to a set of variables (also called compound label) is a set of pairs $A = \{(x_i, v_i), (x_i, v_j), \ldots\}$ where values are taken from the variable domain and we have that a complete assignment is an assignment to all the variables of the problem (a possible world).
A complete assignment can be projected to a smaller partial assignment by restricting the variables to a subset and we will use the projection operatorfrom relational algebra as notation.

A constraint on a set of variables is a set of possible assignments for those variables and each constraint C can be represented as a pair (scope, rel),where scope is a tuple of variables participating in the constraint $(x_1, x_2, \ldots, x_k)$ and rel is a relation that defines the allowable combinations of values for those variables, taken from their respective domains.

To solve a CSP problem (X, D, C), seen as a search problem, we need to define a state space and the notion of a solution: a state in a CSP is an assignment of values to some or all of the variables and we have a partial assignment when we assigns values to only some of the variables, a complete assignment when every variable is assigned and in the end a consistent assignment is the one that satisfies all the constraints.
A solution to a CSP (a goal state) is a consistent, complete assignment.

The simplest kind of CSP involves variables that have discrete, finite domains and values can be numbers, strings, Booleans (True, False); when variables are numbers, and the constraints are inequalities we can deal with variables with infinite domains or continuous domains with linear or integer programming (techniques used in Operations research).

According to the number of variables involved constraints can be unary, binary or higher-order constraints and there can be some preferences constraints and that case we can solve using some optimization methods called *Constraint Optimization Problems*.

Problem reduction/Constraint propagation are techniques for transforming a CSP into an equivalent one which is easier to solve or recognizable as insoluble (removing values from domains and tightening constraints) and we can also exploits the structure of the problem as we will deal later.

We will majority consider CSP with unary and binary constraints only and a binary CSP may be represented as an undirected graph (V, E), where V correspond to variables and edges correspond to binary constraints among V.

We define that graph as *Constraint graph* and also a node x is adjacent to node y if and only if $(x, y) \in E$.

We can consider only binary CSP, since all problems can be transformed into binary constraint problems (not always worthwhile) using dual graph transformation where constraints became variables and if two constraints share variables they are connected by an arc, correspondig to the constraint that the shared variables receive the same value.

In general, every CSP is associated with a *constraint hypergraph*, which are a generalization of graphs, so in a hypergraph a hypernode may connect more than two nodes.

The constraint hypergraph of a CSP(X, D, C) is a hypergraph in which each node represents a variable in X and each hyper-node represents a higher order constraint in C.

Reducing a problem means removing from the constraints those assignments which appear in no solution tuples and we have that two CSP problems are equivalent if they have identical sets of variables and solutions.

A CSP problem P_1 is reduced to a problem P_2 when P_1 is equivalent to P_2, domains of variables in P_2 are subsets of those in P_1 and the constraints in P_2 are at least as restrictive than in P_1.

These conditions guarantee that a solution to P_2 is also a solution to P_1 and only redundant values and assignments are removed.

Problem reduction involves two possible tasks:

1. removing redundant values from the domains of the variables.
2. tightening the constraints so that fewer compound labels satisfy them.

Constraints are sets, so if the domain of any variable or any constraint is reduced to an empty set, the n one can conclude that the problem is unsolvable.

Problem reduction is also called *consistency checking* since it relies on estabilishing local consistency properties, that are the following:

NODE CONSISTENCY: a node is consistent if all the values in its domain satisfy unary constraints on the associated variable.

Node consistency is done using the algorithm called NC-1 that has time complexity $O(dn)$.

ARC CONSISTENCY: a variable is arc-consistent if every value in its domain satisfies the binary constraints of this variable with other variables.
x_i is arc-consistent with respect to another variable x_j if for every value in its domain D_i there is some value in the domain D_j that satisfies the binary constraints on the arc (x_i, x_j).

The most popular algorithm for arc consistency is called AC-3, with pseudocode visible in figure 6, and assuming a CSP with n variables, each with domain size at most d, and with c binary constraints, each arc (x_k, x_i) can be inserted in the queue only d times since x_i has at most d values to delete and checking consistency of an arc can be done in $O(d^2)$ so we get $O(cd^3)$ as total worst-case time.

An improvement is provided by AC-4, based on the notion of support, that doesn't need to consider all the incoming arcs and yields to $O(cd^2)$.

FORWARD CHECKING: a very weak, local and quick form of consistency which is triggered during the search process, so when we assign a value v to a variable x in the process of searching for a consistent assignment, we check the neighbor's variables and exclude values that are not compatible with v from their domains.

```
function AC-3(csp) returns false if an inconsistency is found and true otherwise
  inputs: csp, a binary CSP with components (X, D, C)
  local variables: queue, a queue of arcs, initially all the arcs in csp

  while queue is not empty do
    (X_i, X_j) ← REMOVE-FIRST(queue)
    if REVISE(csp, X_i, X_j) then
      if size of D_i = 0 then return false
      for each X_k in X_i.NEIGHBORS - {X_j} do
        add (X_k, X_i) to queue
  return true
```

```
function REVISE(csp, X_i, X_j) returns true iff we revise the domain of X_i
  revised ← false
  for each x in D_i do
    if no value y in D_j allows (x,y) to satisfy the constraint between X_i and X_j then
      delete x from D_i
      revised ← true
  return revised
```

Figura 6: Pseudocode of AC-3

GENERALIZED ARC CONSISTENCY: an extension of the notion of arc consistency to handle n-ary constraints and we says that a variable x_i is generalized arc consistent with respect to a n-ary constraint if for every value v in the domain of x_i there exists a tuple of values that is a member of the constraint and has its x_i component equal to v.

The GAC algorithm is a generalization of AC-3 and it uses hypergraphs.

PATH CONSISTENCY: it tightens the binary constraints by using implicit constraints that are inferred by looking at triples of variables and a path of length 2 between variables X_i, X_j with respect to a third intermediate variable x_m if, for every consistent assignment there is an assignment to x_m that satisfies the constrains on (x_i, x_m) and (x_m, x_j); the algorithm used is called PC-2 and Montanari says that if all path of length 2 are made consistent, then all path of any length are consistent.

k-CONSISTENCY: a generalization of the other properties, so a CSP is k-consistent if for any set of $k-1$ variables and for any constraint assignment to those variables, a consistent value can always be assigned to any k variable.

DOMAIN SPLITTING: split a problem into a number of disjoint cases and solve each case separately and the set of solutions to the initial problem is the union of the solutions to each case.

VARIABLE ELIMINATION: simplifies the network by removing variables and we can eliminate x, having taken into account constraints of x with other variables and obtain a simpler network.

Problem reduction techniques are used in combination with search and in figure 7 is possible to note the cost of problem reduction and the cost of search, so we discover that more effort one spends on problem reduction the less effort one needs in searching.

Most problems cannot be solved by problem reduction alone, in this case we must search for solutions or combine problem reduction with search.

In this context we talk about *constraint propagation* and we can exploit *commutativity* since the order of variable assignment does not change the result so we have only d^n leaves instead of $n!d^n$.

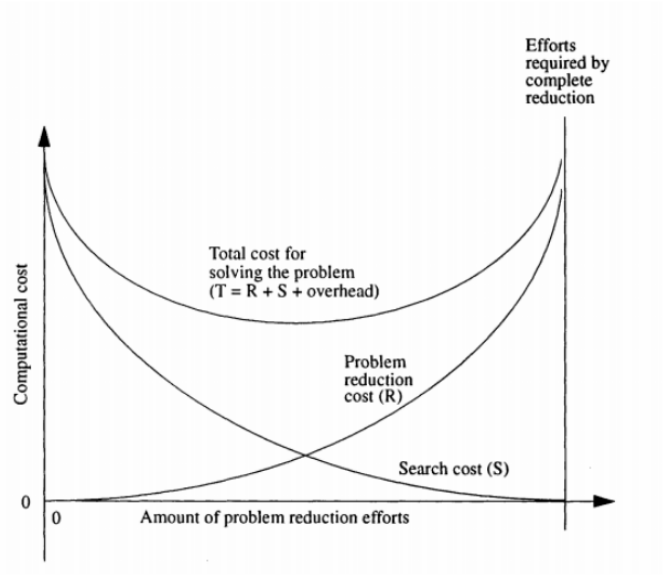


Figura 7: Comparison of Search and Forward Checking cost in CSP problems

To solve CSP we use the *backtracking* search algorithm, which pseudocode is visible in figure 8, and we use some heuristics and search strategies that can be the following:

1. To choose the next variable to assign we can use *minimum-remaining-values* heuristic, which choose the variable with the fewest "legal" remaining values, and *degree heuristic*, which select the variable that is involved in the largest number of constraints on other unassigned variables.

2. To choose the value to assign we can use the *Least-constraining-value*, which prefers the value that rules out the fewest choices for the neighboring variables in the constraint graph.

3. To choose the inferences to perform at each step in the search we can perform forward checking, which ensure arc consistency of last assigned variable, but we can also perform *Mac* (Maintaining Arc Consistency) where calls AC-3 but start with only the arcs (X_j, X_i) for all X_j that are unassigned variables that are neighbors of X_i.

4. To choose which variable to backtrack we can use *chronological* backtracking, where we backtrack to previous variables that sometimes is not useful to solve the problem, so we can use an "intelligent" way that is the *Conflict backjumping*, that consist to find the variable responsible for the violation of constraints, using the *conflict set*.

   The rule for computing the conflict set is:

   Def. If every possible value for X_j fails, backjump to the most recent variable $X_i \in conf(X_j)$ and update its conflict set

   $$conf(X_i) = conf(X_i) \cup conf(X_j) - \{X_i\}$$

   Constraint learning is the idea of finding a minimum set of variables from the conflict set that causes the problem and this set of variables is called a *no-good*.

```
function BACKTRACKING-SEARCH(csp) returns a solution, or failure
  return BACKTRACK({ }, csp)

function BACKTRACK(assignment, csp) returns a solution, or failure
  if assignment is complete then return assignment
  var ← SELECT-UNASSIGNED-VARIABLE(csp)
  for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
    if value is consistent with assignment then
      add {var = value} to assignment
      inferences ← INFERENCE(csp, var, value)
      if inferences ≠ failure then
        add inferences to assignment
        result ← BACKTRACK(assignment, csp)
        if result ≠ failure then
          return result
    remove {var = value} and inferences from assignment
  return failure
```

Figura 8: Pseudocode of Backtracking algorithm

```
function Local_search(V, Dom, C) returns a complete & consistent assignment
  Inputs: V: a set of variables
          Dom: a function such that Dom(x) is the domain of variable x
          C: set of constraints to be satisfied
  Local:  A (complete assignement)  an array of values indexed by variables in V

repeat until termination
    for each variable x in V do          # random initialization or random restart
      A[x] := a random value in Dom(x)
    while not stop_walk( ) & A is not a satisfying assignment do    # local search
      Select a variable y and a value w ∈ Dom(y), w ≠ A[y]          # a successors
      A[y] := w
      if A is a satisfying assignment then return A                 # solution found
```

Figura 9: Pseudocode for a generic Local Search algorithm

3.1 REVIEW OF LOCAL SEARCH METHODS

Local search methods require a complete state formulation of the problem:all the elements of the solution in the current state, so for CSP a complete assignment.

They keep in memory only the current state and try to improve it, iteratively and do not guarantee that a solution is found even if it exists, so cannot be used to prove that a solution does not exist.

To be used when the search space is too large for a systematic search and we need to be very efficient in time and space, we need to provide a solution but it is not important to produce the set of actions leading to it (the solution path) and we know in advance that solutions exist.

In figure 9 is possible to note a generic algorithm for local search in CSP and we have some extreme cases:

RANDOM SAMPLING: no walking is done to improve solution, so stop_walk is always true, just generating random assignments and testing them.

RANDOM WALK: no restarting is done (stop_walk is always false).

We can inject heuristics in the selection of the variable and the value by means of an evaluation function and in CSP we use f =number of violated constraints or conflicts (perhaps with weights), which we stop when $f = 0$.

```
function MIN-CONFLICTS(csp, max_steps) returns a solution or failure
  inputs: csp, a constraint satisfaction problem
          max_steps, the number of steps allowed before giving up

  current ← an initial complete assignment for csp
  for i = 1 to max_steps do
    if current is a solution for csp then return current
    var ← a randomly chosen conflicted variable from csp.VARIABLES
    value ← the value v for var that minimizes CONFLICTS(var, v, current, csp)
    set var = value in current
  return failure
```

Figura 10: Pseudocode of Min-conflicts

Iterative best improvement consist to choose the successor that most improves the current state according to an evaluation function f (greedy ascent/descent) and if more than one we choose at random.

Hill-climbing stops when no improvement is possible and it can be stuck in local maxima instead iterative best improvement algorithms move to the best successor, even if worse than current state and they may be stuck in a loop, so is not complete.

Stochastic local search combine iterative best improvement with randomness, which can be used to escape local minima, and we can use 2 different type of random:

RANDOM RESTART: is a global random move, where the search start from a completely different part of the search state.

RANDOM WALK: is a local random move, where random steps are taken interleaved with the optimizing steps.

All the local search techniques are candidates for application to CSPs, but *Min-conflict heuristics* is widely used that consist to select a variable at random among conflicting variables and then we select the value that results in the minimum number of conflicts with other variables.

The pseudocode of Min-conflict algorithm is visible in figure 10 and we have that the run time of min-conflict is roughly independent of problem size.

The landscape of a CSP under the min-conflict heuristic usually has a series of plateaux and possible improvements are:

TABU SEARCH: local search has no memory, so the idea is keeping a small list of the last t steps and forbidding the algorithm to change the value of a variable whose value was changed recently.

CONSTRAINT WEIGHTING: can help concentrate the search on the important constraints and we assign a numeric weight to each constraint, which is incremented each time the constraint is violated.

SIMULATED ANNEALING: is a technique for allowing downhill moves at the beginning of the algorithm and slowly freezing this possibility as the algorithm progresses.

POPULATION BASED METHODS: inspired by biological evolution like genetic algorithms, *local beam search*, which proceed with the best k successors according to the evaluation function and also *stochastic local beam search* selects k of the individuals at random with a probability that depends on the evaluation function.

Another advantage of local search methods is that theyy can be used in an online setting when the problem changes dynamically, but randomized algorithms are difficult to evaluate since they give a different result and a different execution time each time they are run, so we have to take *average/median runtime* but this definition is ill defined since in case of algorithms that never ends we can't evaluate it.

function TREE-CSP-SOLVER(*csp*) **returns** a solution, or failure
  inputs: *csp*, a CSP with components X, D, C

  $n \leftarrow$ number of variables in X
  assignment $\leftarrow$ an empty assignment
  root $\leftarrow$ any variable in X
  $X \leftarrow$ TOPOLOGICALSORT(X, *root*)
  for $j = n$ **down to** 2 **do**
    MAKE-ARC-CONSISTENT(PARENT(X_j), X_j)
    if it cannot be made consistent **then return** *failure*
  for $i = 1$ **to** n **do**
    assignment[X_i] $\leftarrow$ any consistent value from D_i
    if there is no consistent value **then return** *failure*
  return *assignment*

Figura 11: Pseudocode for Tree CSP Solver

3.2 THE STRUCTURE OF PROBLEMS

The structure of the problem reflects in properties of the constraint graph and when problems have a specific structure, we have strategies for improving the process of finding a solution.

A first obvious case is that of *independent subproblems*, so each connected components of the constraint graph corresponds to a subproblem CSP_i, so if assignment S_i is a solution of CSP_i, then $\cup_i S_i$ is a solution of $\cup_i CSP_i$.

This can save dramaticly computation time so we can solve CSP in $O(d^c n/c)$ linear on the number of variables n rather than $O(d^n)$ which is exponential.

If we can estabilish a total ordering of the variables we can compute a weaker form of consistency called Directional Arc Consistency, and this property is less expensive to compute, but in addition we need to solve a CSP problem when the shape of its constraint graph is a tree.
A CSP is Directional Arc Consistent (DAC) under an ordering of the variables if and only if for every label (x, a), which satisfies the constraints on x, there exists a compatible label (y, b) for every variable y, which comes after x according to the ordering.
In the algorithm for estabilishing DAC (DAC-1), each arc is examined exactly once by proceedings from the last in the ordering, so the complexity is $O(cd^2)$.

In a tree-structured constraint graph two nodes are connected by only one path and er can choose any variable as the root of tree and chosen a variable as the root, the tree induces a topological sort on the variables.

A CSP is defined to be directed arc-consistent under an ordering of variables $X_1, X_2, \ldots, X_n$ if and only if every X_i is arc-consistent with each X_j for $j > i$.

We can make a tree-like graph directed arc-consistent in one pass over the n variables and each step must compare up to d possible domain values for two variables d^2 for a total time of $O(nd^2)$.

To solve a tree-constraint graph for CSP we use the pseudocode in figure 11, and to make a graph in a Tree for CSP we have the following two heuristics:

CUTSET CONDITIONING: in general we must apply a domain splitting strategy, and we choose a subset S on the CSP's variables such that the constraint graph becomes a tree after removal of S (S is called a *cycle cutset*.

For each possible consistent assignment to the variables in S, we remove the domains of the remaining variables any values that are inconsistent with the assignment for S and if the remaining CSP has a solution we return it together with the assignment for S.

With this approach we have $O(d^c(n-c)d^2)$ time complexity, where c is the size of the cycle cutset and d is the size of the domain.

TREE DECOMPISITION: this appraoch consists in a tree decomposition of the constraint graph into a set of connected sub-problems and each sub-problem is solved independently, and the resulting solutions are then combined.

A tree decomposition must satisfy the following three requirements:

1. Every variable in the original problem appears in at least of one of the sub-problems.
2. If two variables are connected by a constraint in the original problem, they must appear together in at least one of the sub-problems.
3. If a variable appears in two sub-problems in the tree, it must appear in every subproblem along the path connecting those those sub-problems.

Simmetry is an important factor for reducing the complexity of CSP problems, and we have that *Value simmetry*, which the value does not really matters, instead *Symmetry-breaking constraints* we might impose an arbitrary ordering constraint, that requires the three values to be in alphabetical order and in practice breaking value symmetry has proved to be important and effective on a wide range of problems.

4 KNOWLEDGE REPRESENTATION & REASONING

We introduce and discuss about Knowledge representation and Reasoning (KR & R), that is the field of Artificial Intelligence dedicated to representing information about the world in a form that a computer system can utilize to solve complex tasks.

The class of systems that derive from this approach are *knowledge based agents* and a KB agent maintains a knowledge base of facts expressed in a declarative language.

A representation is a surrogate for reasoning about things that exists externally, it is necessarily imperfect, it is also a set of ontological commitments and a fragmentary theory of intelligent reasoning.

Knowledge representation is about the use of formal symbolic structures to represent a collection of propositions believed by some agent, instead reasoning is the formal manipulation of the symbols representing a collection of beliefs to produce representations of new ones, so logical deduction is a well know example of reasoning.

Reasoning is not only done by logical entailment/deduction but we have also default reasoning, probabilistic reasoning and so on, that we will introduce later in this chapter.

The Knowledge Representation hypothesis formulated by Brian C. Smith in 1985 estabilish that any mechanically embodied intelligent process will be comprised of structural ingredients that:

- we as external observers naturally take to represent a propositional account of the knowledge that the overall process exhibits.
- independent of such external semantic attribution, play a formal but causal and essential role in engendering the behavior that manifests that knowledge.

In simpler words, we want to construct A.I. systems that contain symbolic representations that we can understand these symbolic structures as propositions and these symbolic structures determine the behavior of the system; we have that Knowledge based systems have these properties.

There are two competing approaches for KB systems:

PROCEDURAL APPROACH: knowledge is embedded in programs.

CONNECTIONIST APPROACH: avoids symbolic representation and reasoning, and instead sees computing with networks of weighted links between "neurons".

The KB approach has the following advantages:

1. Separation of knowledge and "inference engine".
2. Extensibility: we can extend the existing behavior by simply adding new propositions and the knowledge is modular, so the reasoning mechanism does not change.
3. Understandability: the system can be understood at the knowledge level, so we debug faulty behavior by changing erroneous beliefs, and we can explain and justify current behaviour in terms of the beliefs.

There is a fundamental tradeoff in knowledg representation and reasoning who estabilish that the more expressive is the representation language, the more complex is reasoning.

So we need to find a best compremise between this two aspects, so for example databases use only positive and concrete facts and also FOL avoid to represent some default aspects.

Much of AI involves building systems that are knowledge-based: their ability derives [in part] from reasoning over explicitly represented knowledge and typical KB systems are expert systems, language understanding, machine reading (common sense knowledge is required)and planning; KR&R today has many applications outside AI, like Bio-medicine, Engineering, Business and commerce, Databases, Software engineering.

We will consider two "modern" applications:

- Cognitive assistants/digital personal assistants (SIRI/Alexa/Google home/Jibo), spoken dialog in a natural language in open domain.
- Computational Knowledge Engine (Wolfram Alpha): scientific and medical thinking.

4.1 PROPOSIZIONAL AND FIRST ORDER LOGIC

We can understand KB systems at two different levels:

KNOWLEDGE LEVEL: representation language and its semantics, expressive adequacy (what can be expressed), and what can be inferred.

SYMBOL LEVEL: computational aspects, efficiency of encoding, data structures and efficiency of reasoning procedures, including their complexity.

The tools of symbolic logic seem especially suited for the knowledge level.

We have that a sentence is true (or false) with respect to an interpretation, where an interpretation (possible world) assigns truth values to atomic formulas.

Truth values of compound formulas follow as a consequence and an interpretation that makes a set of formulas true, is called a model.
A formula is satisfiable if there is at least one interpretation that makes it true and unsatisfiable if is false in all interpretations.
A formula is valid (a tautology in PROP) if it is true in all interpretations and note that the negation of a satisfiable formula can be satisfiable or unsatisfiable and the negation of a valid formula is unsatisfiable, and vice versa.

A set of sentences KB [logically] entails a sentence α iff any model of KB is also a model of α and we also say that α is a logical consequence of KB.

We write $KB \models \alpha$ and we have an alternative definition that says

$$KB \models \alpha \iff M(KB) \subseteq M(\alpha)$$

where M stands for the set of models; it is also possible define the logical equivalence as $\alpha \equiv \beta$ iff $\alpha \models \beta$ and $\beta \models \alpha$.

A deductive system is defined by a set of axioms and inference rules and axioms can be logical or part of the agent's KB.
A proof is a sequence of formulas, starting from the axioms, where each formula can be obtained from previous ones by application of inference rules and we write $KB \vdash \alpha$ when there is a proof of α from KB.

We have connection between deduction and entailment with these two definition:

SOUNDNESS: if $KB \vdash \alpha$ then $KB \models \alpha$.

COMPLETENESS: if $KB \models \alpha$ then $KB \vdash \alpha$.

Proof by refutation is based on the following meta-theorem $KB \models \alpha$ iff $KB \cup \{\neg\alpha\}$ is unsatisfiable, and this means that entailment can be formulated as a satisfiability

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*
  inputs: s, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of s
  symbols ← a list of the proposition symbols in s
  return DPLL(*clauses*, *symbols*, { })

---

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

  if every clause in *clauses* is true in *model* **then return** *true*
  if some clause in *clauses* is false in *model* **then return** *false*
  P, *value* ← FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)
  if P is non-null **then return** DPLL(*clauses*, *symbols* − P, *model* ∪ {P=*value*})
  P, *value* ← FIND-UNIT-CLAUSE(*clauses*, *model*)
  if P is non-null **then return** DPLL(*clauses*, *symbols* − P, *model* ∪ {P=*value*})
  P ← FIRST(*symbols*); *rest* ← REST(*symbols*)
  return DPLL(*clauses*, *rest*, *model* ∪ {P=*true*}) **or**
      DPLL(*clauses*, *rest*, *model* ∪ {P=*false*}))

Figura 12: Pseudocode of DPLL

problem (SAT): we can use SAT algorithms for checking entailment and if $KB \cup \{\not\alpha\}$ derives a contradiction, and the proof system is sound, then $KB \models \alpha$.

It is possible to convert any finite domain CSP into a propositional satisfiability problem, so if Y has domain $\{v_1, \ldots, v_k\}$ introduce k boolean variables $Y_1, \ldots, Y_k$, with Y_i iff $Y = v_i$.
Additional constraints are that if at least one of $Y_1, \ldots, Y_k$ is true and if $i \neq j$ then Y_i and Y_j not both true and we define a clause for each disallowed combination of values $(v_1, \ldots, v_k) : \{\neg v_1 \vee \cdots \vee \neg v_k\}$.

Satisfiability (SAT) algorithms for PROP can be made more efficient than general CSP solvers and strategies for computing entailment in PROP:

1. Model checking (reduce to SAT problem) that consist in several approach: check satisfiability using different heuristics (DPLL) or use a local and incomplete search method (Walk-SAT).

2. Deduction (uses a proof system and deduction strategies): the resolution method uses only one inference rule (resolution rule) and resolution strategies for searching are more efficiently.

Clausal form (PROP) is a Conjunctive normal form (a conjunction of disjunctions of atomic formulas) and any PROP formula can be converted in an equivalent set of clauses: each conjunct is a clause, a disjunction of literals (positive o negative atoms).

DPLL (Davis, Putman, Lovemann, Loveland) requires a formula in clausal form and it enumerates, with a depth first strategy, all interpretations, looking for a model. It uses three strategies:

1. Anticipated control, so if one clause is false backtrack or if one literal is true the clause is satisfied.

2. Pure symbols heuristics, where assign first pure symbols (which appear everywhere with the same sign).

3. Unit clauses heuristics, which assign first unit clauses (only one literal).

In figure 12 is possible to note the pseudocode of DPLL.

Walk-SAT is one (of many) local search methods and is to be used for SAT problems where solutions exist and are evenly distributed; it is not complete and is very effective for large problems.

```
function WALKSAT(clauses, p, max_flips) returns a satisfying model or failure
  inputs: clauses, a set of clauses in propositional logic
          p, the probability of choosing to do a "random walk" move, typically around 0.5
          max_flips, number of flips allowed before giving up

  model ← a random assignment of true/false to the symbols in clauses
  for i = 1 to max_flips do
      if model satisfies clauses then return model
      clause ← a randomly selected clause from clauses that is false in model
      with probability p flip the value in model of a randomly selected symbol from clause
      else flip whichever symbol in clause maximizes the number of satisfied clauses
  return failure
```

Figura 13: Pseudocode of WalkSat

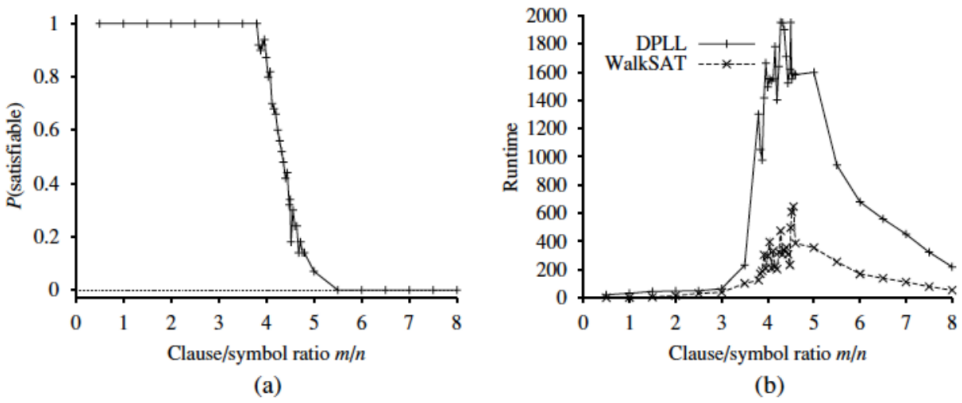


Figura 14: Comparison between DPLL and WalkSat

In figure 13 is possible to note the pseudocode to compute Walk-Sat and in figure 14 is possible to see a comparison between DPLL and WalkSat.

Also in the case of FOL you can obtain a clausal form in an effective way, and the transformation involves the elimination of existential quantifiers by Skolemization, so

$$\exists x\, father(x, G) \text{ becomes } father(k, G)$$

This transformation preserves satisfiability, but not equivalence.

Given the fundamental problem $KB \models \alpha$ an equivalent problem is $KB \cup \neg\alpha$ is unsatisfiable and this can be solved by a deductive system showing that $KB \cup \neg\alpha \vdash \{\}$, where $\{\}$ is the empty clause meaning False.

Resolution by refutation is a method which is correct and complete:

1. transform the KB in clausal form (a conjunction of disjunction of literals)
2. add to KB the negation of the goal in clausal form
3. use the resolution rule as unique inference rule.

This strategy works for PROP and FOL with different complexity results: it is decidable and NP-complete for PROP, instead is semi-decidable for FOL.

Clauses are set of literals, so the resolution rule for PROP is the following

$$\frac{c_1 \cup \{p\} \quad c_2 \cup \{\neg p\}}{c_1 \cup c_2}$$

In a resolution refutation we aim to deduce the empty clause and the resolution rule for FOL is the following:

$$\frac{c_1 \cup \{p\} \quad c_2 \cup \{\neg q\}}{[c_1 \cup c_2]\gamma}$$

and $\gamma = MGU(p, q)$ and γ is not fail, so a fundamental operation is *unification*, who is a process to determine whether two expressions can be made identical by a substitution of terms to variables, so the result is the unifying substitution, the unifier, or FAIL, if the expressions are not unifiable.

Given two expressions there may be different substitutions that make them identical and we are interested in computing the most general unifier (MGU), the one that does only the essential instantiations.

We present now the Unification algorithm, invented in 1982 by Martelli and Montanari, which consist in the following passes:

1. Computes the MGU by means of a rule-based equation-rewriting system
2. Initially the working memory (WM) contains the equality of the two expressions to be unified.
3. The rules modify equations in the WM
4. The algorithm terminates with failure or when there are no applicable rules (success)
5. At the end, if there is no failure, the WM contains the MGU.

The rules used to compute the MGU are the following:

1. $f(s_1, \ldots, s_n) = f(t_1, \ldots, t_n) \rightarrow s_1 = t_1, \ldots, s_n = t_n$
2. $f(s_1, \ldots, s_n) = g(t_1, \ldots, t_m) \rightarrow fail$ when $f \neq g$ or $n \neq m$.
3. $x = x$ we can remove the equation
4. $t = x \rightarrow x = t$
5. $x = t, x$ does not occur in t the nwe apply $\{x/t\}$ to other equations.
6. $x = t, t$ is not x, x occur in t then we have fail.

Note that when we compare two different constants, rule 2 applies, as a special case where $n = m = 0$, and we fail.

4.2 KNOWLEDGE & ONTOLOGICAL ENGINEERING

Knowledge engineering is the activity to formalize a specific problem or task domain and it involves decisions about what are the relevant fact, objects and so on, but also which is the right level of abstaction.
Ontology engineering seeks to build general-purpose ontologies which should be applicable in any special-purpose domain (with the addition of domain-specific axioms).

Before implementing, need to understand clearly, like in software engineering what is to be computed, what kind of knowledge, why and where inference is necessary.

Sometimes useful to reduce n-ary predicates to 1-place predicates and 1-place functions, that involves creating new individuals and new functions for properties/-roles, this is typical of description logics/frame languages, which we will talking later.

The use of KR languages and logic in A.I. is representing "common sense" knowledge about the world, rather than mathematics or properties of programs.
Common sense knowledge is difficult since it comes in different varieties and it requires formalisms able to represent actions, events, time, physical objects, beliefs, categories that occur in many different domains.

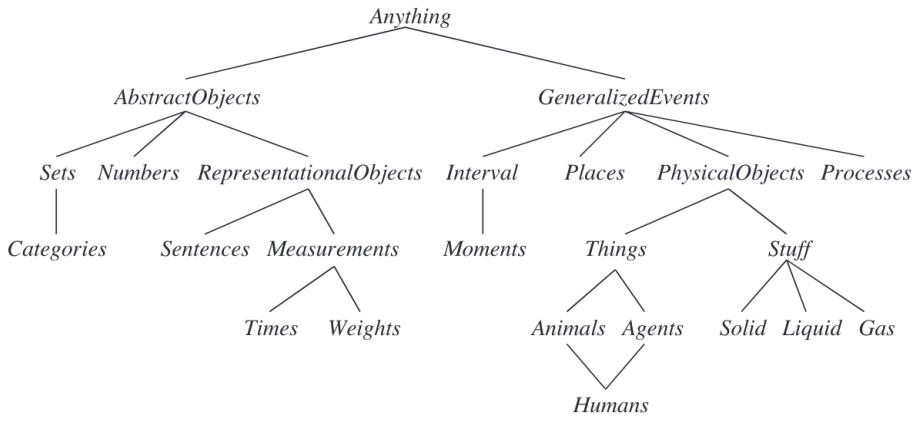


Figura 15: General Ontology specification

In figure 15 is possible to note that a general ontology organizes everything in the world into a hierarchy of categories and should be applicable in any special-purpose domain (with the addition of domain-specific axioms).
In any non trivial domain, different areas of knowledge must be combined, because reasoning and problem solving could involve several areas simultaneously and it is difficult to construct one best ontology, infact "Every ontology is a treaty—a social agreement—among people with some common interest in sharing."

Much reasoning takes place at the level of categories, so we can infer category membership from the perceived properties of an object, and then uses category information to derive specific properties of the object.
There are two choices for representing categories in first-order logic:

1. Predicates, categories are unary predicates, that we assert of individuals:

2. Objects: categories are objects that we talk about (*reification*), so for example the predicate $WinterSport(Ski)$ become $Ski \in WinterSports$

In this way we can organize categories in taxonomies (like in natural sciences), define disjoint categories, partitions and use specialized inference mechanisms, such as inheritance.

We use the general PartOf relation to say that one thing is part of another, composite objects can be seen as part-of hierarchies, similar to the Subset hierarchy and these are called *mereological hierarchies*.

Composite objects is a structural relations among parts so for example, a biped has two legs attached to a body will be represented as a relation among biped and leg.

Bunch is a composite objects with definite parts but no particular structure, so "a bag of three apples" will be represented by BunchOf($\{Apple_1, Apple_2, Apple_3\}$) that should not be confused with the set of 3 apples and unlike sets, bunches have weight; also we have that each element of category s is part of BunchOf(s) and also BunchOf(s) is the smallest object satisfying this condition (logical minimization).

Physical objects have height, weight, mass, cost, and so on and the values that we assign for these properties are called measures.
A solution is to represent measures with units functions that take a number as argument so for example we have Length(L_1) = Inches(1.5) = Centimeters(3.81).
The most important aspect of measures is not the particular numerical values/scale, but the fact that measures can be ordered, and to perform some sort of qualitative inference, often it is enough to be able to order values and to compare quantities (qualitative physics).

There are countable objects, things such as apples, holes, and theorems, and mass objects, such as butter, water, and energy, these are called Stuff.
The properties of stuff are the following:

1. Any part of butter is still butter, so we have in symbols
$$b \in Butter \cap PartOf(p, b) \Rightarrow p \in Butter$$
2. Stuff has a number of intrinsic properties (color, high-fat content, density ...), shared by all its subparts, but no extrinsic properties, so it is a substance.

4.3 SITUATION CALCULUS

The situation calculus is a specific ontology in FOL dealing with actions and change, with the following concepts:

SITUATIONS: snapshots of the world at a given instant of time, the result of an action.

FLUENTS: time dependent properties.

ACTIONS: performed by an agent, but also events.

CHANGE: how the world changes as a result of actions.

We define Result as the effect of a sequence of actions, defined as a function
$$Result : [A*] \times S \rightarrow S$$
with Result([], s) = s and Result([a | seq], s) = Result(seq, Result(a, s)).

The frame problem is one the most classical A.I. problems and the name comes from an analogy with the animation world, where the problem is to distinguish background (the fixed part) from the foreground (things that change) from one frame to the other. Let's try to fix the problem writing frame axioms, such that an action remains true unless someone makes false, but with frame axioms we have too many axioms (representational frame problem).

We can combine preconditions, effect and frame axioms to obtain a more compact representation for each fluent f and the schema is as follows: f true after true preconditions and some action made f true or f was true before and no action made it false.

The representational frame problem is considered to be (more or less) solved with this new definition and we have another problem, called *Qualification problem*, since in real situations it is almost impossible to list all the necessary and relevant preconditions.
Ramification problem is the problem of among derived propertied which ones persist and which ones change?

What we would need is the ability to formalize a notion of persistence, that estabilish that "in the absence of information to the contrary (by default) things remain as they were".
Unfortunately this leads out of classical logic and the closure assumption we used is already an ad hoc form of completion and we will see more of this strategy in nonmonotonic reasoning.
In planning we end up using other languages that make stronger assumptions and are more limited in their expressivity.

Situation calculus is limited in its applicability: Single agent, Actions are discrete and instantaneous (no duration in time), Actions happen one at a time: no concurrency, no simultaneous actions and only primitive actions, so there is no way to combine actions (conditionals, iterations and so on).
To handle such cases we introduce an alternative formalism/ontology known as *event calculus*, which is based on events, points in time, intervals rather than situations.

4.4 EVENT CALCULUS

Event calculus reifies fluents and events and the fluent is an object (represented by a function), so to assert that a fluent is true at some point in time t we use the predicate T, so for example T(At(Shankar, Berkeley), t).

Events are described as instances of *event categories*, so for example the event E_1 of Shankar flying from San Francisco to Washington, D.C. is described as

$$E_1 \in Flyings \cap Flyer(E_1, Shankar) \cap Origin(E_1, SF) \cap Destination(E_1, DC)$$

By reifying events we make it possible to add any amount of arbitrary information about them, such participants in the event or properties.

Time intervals are a pair of times $(start, end)$, so $i = (t_1, t_2)$ is the time interval that starts at t_1 and ends at t_2.
We have the event Happens(E_1, i) to say that the event E_1 took place over the time interval i.
The complete set of predicates for one version of the event calculus is the following:

$t(f, t)$ fluent f is true at time t.

HAPPENS(E, I) event e happens over the time interval i.

INITIATES(E, F, T) event e causes fluent f to start at time t.

TERMINATES(E, F, T) event e causes fluent f to cease at time t.

CLIPPED(F, I) fluent f ceases to be true at some point during time interval i.

RESTORED(F, I) fluent f becomes true sometime during time interval i.

A fluent holds at a point in time if the fluent was initiated by an event at some time in the past and was not made false (clipped) by an intervening event, so formally

$$Happens(e, (t_1, t_2)) \cap Initiates(e, f, t_1) \cap \not Clipped(f, (t_1, t)) \cap t_1 < t \Rightarrow T(f, t)$$

A fluent does not hold at a point in time if the fluent was terminated by an event at some time in the past and was not restored by an event occurring at a later time, so formally we have

$$Happens(e, (t_1, t_2)) \cap Terminates(e, f, t_1) \cap \not Restored(f, (t_1, t)) \cap t_1 < t \Rightarrow \not T(f, t)$$

where Clipped and Restored are defined by

$$Clipped(f, (t_1, t_2)) \iff \exists e, t, t_3 Happens(e, (t, t_3)) \cap t_1 \leq t < t_2 \cap Terminates(e, f, t)$$
$$Restored(f, (t_1, t_2)) \iff \exists e, t, t_3 Happens(e, (t, t_3)) \cap t_1 \leq t < t_2 \cap Initiates(e, f, t)$$

We can extend the predicate T to hold over intervals, so we says that a fluent holds over an interval if it is true at every point within the interval

$$T(f, (t_1, t_2)) \iff [\forall t \, (t_1 \leq t < t_2) \Rightarrow T(f, t)]$$

Actions are modeled as events, and we have that fluents and actions are related with domain-specific axioms that are similar to successor-state axioms.
We can extend event calculus to make it possible to represent simultaneous events, continuous events and so on.

Processes or liquid events are events with the property that if they happen over an interval also happen over any subinterval

$$(e \in Processes) \cap Happens(e, (t_1, t_4)) \cap (t_1 < t_2 < t_3 < t_4) \Rightarrow Happens(e, (t_2, t_3))$$

The distinction between liquid and nonliquid events is analogous to the difference between substances, or stuff, and individual objects, or things.

We can model in detail the behavior of intervals, providing more vocabulary:

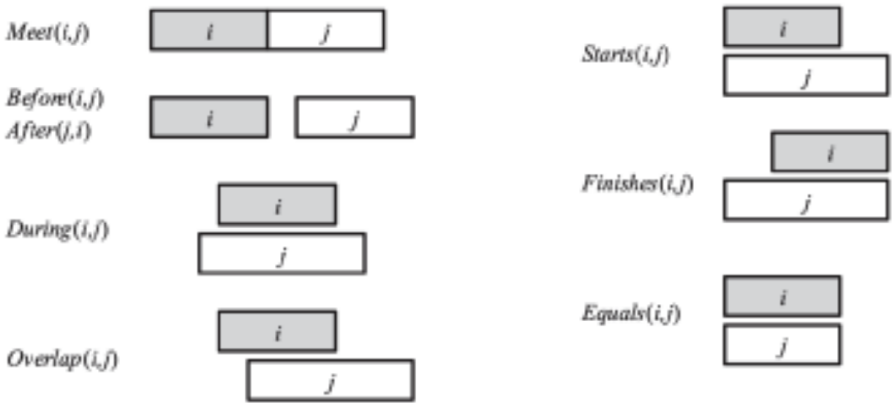


Figura 16: Visual representation of interval relations

TIME(X) points in a time scale, giving us absolute times in seconds.

BEGIN(I) earliest moment in an interval.

END(I) latest moment in an interval.

DURATION(I) the duration of an interval.

The Complete set of interval relations, proposed by Allen (1983) and visible in figure 16, are the following:

- $Meet(i,j) \iff End(i) = Begin(j)$
- $Before(i,j) \iff End(i) < Begin(j)$
- $After(j,i) \iff Before(i,j)$
- $During(i,j) \iff Begin(j) < Begin(i) < End(i) < End(j)$
- $Overlap(i,j) \iff Begin(i) < Begin(j) < End(i) < End(j)$
- $Begins(i,j) \iff Begin(i) = Begin(j)$
- $Finishes(i,j) \iff End(i) = End(j)$
- $Equals(i,j) \iff Begin(i) = Begin(j) \cap End(i) = End(j)$

4.5 NONMONOTONIC REASONING

Failures of monotonicity are widespread in commonsense reasoning, infact it seems that humans often "jump to conclusions", when they think it is safe to do so (lacking information to the contrary).
These conclusions are only "reasonable", given what you know, rather than classically entailed and also most of the inference we do is defeasible: additional information, may lead to to retract those tentative conclusions.
Any time the set of beliefs does not grow monotonically as new evidence arrives, the monotonicity property is violated and including defeasible reasoning leads us to consider nonsound inferences.

Some common instances of nonmonotonic reasoning are the following:

DEFAULT REASONING: reasonable assumptions unless evidence of the contrary

PERSISTENCE: things stay the same, according to a principle of inertia, unless we know they change.

ECONOMY OF REPRESENTATION: only true facts are stored, false facts are only assumed.

REASONING ABOUT KNOWLEDGE: if you have $\neg Know(p)$ and you learn p we have $Know(p)$.

ABDUCTIVE REASONING: most likely explanations to known facts.

Universal rules, like $\forall x(P(x) \Rightarrow Q(x))$, express properties that apply to all instances, but most of what we learn about the world is in terms of generics rather than universals.
Listing exceptions to generic properties is not a viable solution, so the goal is to be able to say a P is a Q in general, normally, but not necessarily.
In this way it is reasonable to conclude $Q(a)$, given $P(a)$, unless there is a good reason not to and this is what we call a *default* and *default reasoning* the tentative conclusion.

There are three ways to approach the problem, that we will analyze:

MODEL THEORETIC FORMALIZATIONS (CWA, CIRCUMSCRIPTION): consist in a restriction to the possible interpretations, redefining the notion of entailment and we can still have systems sound and complete wrt the new semantics.

PROOF THEORETIC FORMALIZATIONS (DEFAULT LOGIC, AUTOEPISTEMIC LOGIC): A proof system with non-monotonic inference rules and autoepistemic logic (under the heading "logics for knowledge and beliefs").

SYSTEMS SUPPORTING BELIEF REVISION TMS, ATMS

Under Closed World Assumption (CWA) only positive facts are stored, any other basic fact is assumed false and CWA assumption is used in [deductive] databases and in logic programming with negation as failure.

CWA corresponds to a new version of entailment ($\models_c$) defined as

Def (CWA). $KB \models_c a$ if and only if $CWA(KB) \models a$ where

$$CWA(KB) = KB \cup \{\neg p : \text{ p ground atom and } KB \not\models p\}$$

$CWA(KB)$ is the completion under CWA of KB and note that the CWA is nonmonotonic.

KB with consistent knowledge (satisfiable) happens when for no α $KB \models \alpha$ and $KB \models \neg\alpha$, but normally a KB has incomplete knowledge.
CWA can be seen as an assumption about complete knowledge, or a way to make a theory complete, using the following theorem:

Thm 4.1. For every α (without quantifiers), $KB \models_c \alpha$ or $KB \models_c \neg\alpha$.

$CWA(KB)$ is not always consistent when KB is consistent, infact there are problem with disjunctions, so a solution is to restrict CWA to atoms that are "uncontroversial"; CWA limited in such a way is called Generalized CWA (GCWA) and is a weaker form of completion than unrestricted CWA (the assumed beliefs are less).

Thm 4.2 (Consistency of CWA). $CWA(KB)$ is consistent iff whenever $KB \models (q_1 \vee \cdots \vee q_n)$ then $KB \models q_i$ for some q_i.
GCWA comes if $KB \models \{p \vee q_1, \cdots \vee q_n\}$ and $KB \not\models p$ add $\neg p$ only if at least one ground literal q_i is entailed.

Since it may be difficult the condition of the theorem, the following corollary, which restricts the application, is also of practical importance:

Corol 4.1. If the clause form of KB is Horn and consistent, then $CWA(KB)$ is consistent.

The application of the theorem of consistency of CWA depends on the terms that we allow as part of the language, and the Domain Closure Assumption (DCA) may be used to restrict the constants to those explicitly mentioned in the KB:

$$\forall x.[x = c_1 \vee \cdots \vee x = c_n]$$

where c_i are all the finite constants appearing in KB.
Under this restriction quantifiers can be replaced by finite conjunctions and disjunctions and we also introduce the Unique Names assumption (UNA), that can be used to deal with terms equality $c_i \neq c_j$ for $i \neq j$ but with functions things get more complicated.

With CWA we can reduce queries (without quantifiers) to atomic queries, by repeated applications of the following properties:

1. $KB \models_c (a \wedge b)$ iff $KB \models_c a$ and $KB \models_c b$
2. $KB \models_c \neg\neg a$ iff $KB \models_c a$
3. $KB \models_c \neg(a \vee b)$ iff $KB \models_c \neg a$ and $KB \models_c \neg b$
4. $KB \models_c (a \vee b)$ iff $KB \models_c a$ or $KB \models_c b$
5. $KB \models_c \neg(a \wedge b)$ iff $KB \models_c \neg a$ or $KB \models_c \neg b$

If $CWA(KB)$ is consistent, any query reduces to a set of atomic queries and we further assume that $CWA(KB)$ is consistent we get

$$KB \models_c \not\alpha \iff KB \not\models_c a$$

Much more efficient than ordinary logic reasoning (e.g. no reasoning by cases), in fact we have restricted reasoning to a unique interpretation and instead of checking validity/entailment we check truth in that interpretation.

In a *vivid KB* we store this unique interpretation (a consistent and complete set of literals) and answer questions retrieving from it, so a vivid KB has the CWA builtin. If atoms are stored as a table, deciding if $KB \models_c \alpha$ is like DB-retrieval, so instead of reasoning with sentences we reason about an analogical representation of the world, or model.

The CWA is too strong for many applications, so we do not want to assume that any ground atom not provable from the KB is false, because certain predicates are considered complete, others are not.
CWA wrt to a predicate P [set of predicates P]: the set of assumed beliefs is only for ground atoms in P [predicates in P] and a similar theory is predicate completion which consists in adding a set of completion axioms, so if only $P(A)$ and $P(B)$ are in KB we have that the completion axiom is defined as

$$\forall(x = a) \vee (x = b) \iff P(x)$$

The theory accounting for if and when this leads to consistent augmentation is quite complex and a generalization of CWA and predicate completion is circumscription.

Circumscription can be seen as a more powerful and precise version of the CWA, working also for FOL, which was problematic and required further assumptions.
The idea is to specify special abnormality predicates for dealing with exceptions and the definition of Circumscription is defined as

Def (Circumscription)**.** Given the unary predicate Ab, consider only interpretations where $I[AB_f]$ is as small as possible, relative to KB.

Note that Circumscription is a semantic notion based on minimal models (a kind of model preference logics) due to MacCarthy 1980.

Let P be a set of unary abnormality predicates and let I_1 and I_2 two interpretations that agree on the values of constants and functions so we define the ordering and minimal entailment as

Def (Minimal Entailment)**.** $I_1 < I_2$ iff same domain and $\forall p \in P I_1[p] \subset I_2[p]$ holds and the minimal entailment estabilish that $KB \models_{\leq} \alpha$ iff for every interpretation I, if $I[KB] = T$ and such that there is no other interpretation $I' < I$ such that $I'[KB] = T$ then α is true in I.

In simpler words, α need not be true in all interpretations satisfying KB but only in all those that minimize abnormalities (all the interpretations that are most "normal") and also that Circumscription need not produce a unique minimal interpretation.

Although the default assumptions made by circumscription are usually weaker than those of the CWA, there are cases where they appear too strong, so suppose for example, that we have the following KB:

$$\begin{aligned} \forall x[Bird(x) \wedge \not Ab(x) \Rightarrow Flies(x)] \\ Bird(tweety) \\ \forall x[Penguin(x) \Rightarrow (Bird(x) \wedge \not Flies(x))] \end{aligned}$$

From this follows that

$$\forall x[Penguin(x) \Rightarrow Ab(x)]$$

Minimizing abnormalities leads to $KB \models_{\leq} \not\exists x Ab(x)$ but also $KB \models_{\leq} \not\exists x Penguin(x)$.

Partial fix is done using McCarthy's definition related to predicate completion, so distinguish between P (variable predicates) and Q (fixed predicates).
Ordering on interpretations is different for the two sets, so only predicates in P are allowed to be minimized and $\forall Q \in \mathbf{Q} I_1[Q] = I_2[Q]$.

Default logic uses beliefs as deductive theory and uses $KB =< F, D >$ where F is a set of sentences (facts) and D is a set of default rules of type

$$\frac{\alpha : \beta}{\gamma}$$

Default rules where $\beta = \gamma$ are called *normal defaults*.

Problem is now how to characterize theorems/entailments, because we cannot write a derivation, since do not know when to apply default rules and there is no guarantee of unique set of theorems, so we define *extensions* as sets of sentences that are "reasonable" beliefs, given explicit facts and default rules.

E is an extension of $< F, D >$ iff for every sentence π, E satisfies the following $\pi \in E \iff F \cup \Delta \models \pi$, where

$$\Delta = \{\gamma : \frac{\alpha : \beta}{\gamma} \in D, \alpha \in E, \not\beta \notin E\}$$

So, an extension E is the set of entailments of $F \cup \Delta$, where the Δ are a "suitable" set of assumptions given D.
Note that α has to be in E , not in F and this has the effect of allowing the prerequisite to be believed as the result of other default assumptions.

If E is inconsistent we can conclude anything we want, so we have this theorem

Thm 4.3. An extension of a default theory is inconsistent iff the original F is inconsistent.

In this case the extension is unique, but in general a default theory can have multiple extensions.

The properties of Default logics are the following:

1. If a default theory has distinct extensions, they are mutually inconsistent.
2. There are default theories with no extensions.
3. Any normal default theory has an extension.
4. Adding new normal default rules does not require the withdrawal of beliefs, even if adding new beliefs might, some normal default theories are semi-nonmonotonic.

We have a problem that leads to a more complex definition of extension, so suppose $F = \{\}$ and $D = \{: p/p\}$ then $E =$ entailments of $\{p\}$ is an extension since $p \in E$ and $\not p \notin E$.
However, we have no good reason to believe p, since only support for p is the default rule, which requires p itself as a prerequisite, so the default should have no effect; for this reason we define a revision of definition of extension with

Def (Grounded extension)**.** For any set S, let $\Gamma(S)$ be the least set containing F, closed under entailment, and satisfying

$$\alpha : \beta/\gamma \in D, \alpha \in \Gamma(S), \not\beta \notin S \Rightarrow \gamma \in \Gamma(S)$$

A set E is an extension of $<F, D>$ iff $E = \Gamma(E)$, so E is a fixed point of the Γ operator.

4.6 KNOWLEDGE AND BELIEFS

Human intelligence is intrinsically social, since humans need to negotiate and coordinate with other agents, so in multi-agent scenarios, we need methods for one agent to model mental states of other agents: high level representations of other agent's belief, intentions and goals may be relevant for acting; by mental states we mean the relation of an agent to a proposition and propositional attitudes that an agent can have include Believes, Knows, Wants, Intends, Desires, Informs, so called because the argument is a proposition, and also propositional attitudes do not behave as regular predicates.

A property important is called *referential transparency*, which means that what matters is the object that the term names, not the form of the term, but propositional attitudes like Believes and knows, require referential opacity, the terms used do matter, because an agent may not be aware of which terms are co-referential.

To implement knowledge and beliefs there are three approaches:

REITIFICATION: we remain within FOL, as we did for the situation calculus, using terms to represent propositions, so for example $Bel(a, On(b, c))$, but suffer for Referential transparency problem.

META-LINGUISTIC REPRESENTATION: we remain within FOL and represent propositions as strings, so for example Bel(a, "On(b, c)").

MODAL LOGICS: propositional attitudes are represented as modal operators in specialized modal logics, with alternative semantics and modal operators are an extension of classical logical operators, so for example we have B(a, On(b, c)) or B_A(On(b, c)) to say that agent a believes(B)/ knows(K) that block B is on C.

Classical logic has only one modality (the modality of truth), so P is the same as saying "P is true".

Strictly speaking modal logic is about necessity and possibility, however, the term is used more broadly to cover logics with different modelling goals. $\Box A$ means that

it is necessary that A instead $\diamond A$ means that it is possible that A, and these two operators are related, as $\exists$ and $\forall$ in First Order logic.

The simplest logic is called K (after Saul Kripke), that results from adding the following to the principles of propositional logic:

NECESSITATION RULE: if A is a theorem of K, then so is $\Box A$.

DISTRIBUTION AXIOM: $\Box(A \Rightarrow B) \Rightarrow (\Box A \Rightarrow \Box B)$

$\Box$ is some sort of universal quantification over interpretations and $\diamond$ is some sort of existential quantification over interpretations.

Logic T adds axiom $\Box A \Rightarrow A$, that may be relevant for some modal operators and not for others, so for example Knows $A \Rightarrow A$ seems plausible instead Bel $A \Rightarrow A$ is not.
Logic $S4$ adds $\Box A \Rightarrow \Box\Box A$ and logic $S5$ adds $\diamond A \Rightarrow \Box \diamond A$.

Semantics for modal logics is defined by introducing a set W of possible worlds and an accessibility relation R between worlds, so the interpretation of a formula is now with respect to a possible world w and we write $I(A, w)$.
The interpretation formula for operators from classic logic are the same instead the interpretation formula for $\Box$ and $\diamond$ are the following

$$I(\Box A, w) = T \text{ iff for every world } w' \in W \text{ such that } wRw' I(A, w') = T$$
$$I(\diamond A, w) = T \text{ iff for some world} w' \in W \text{ such that} wRw' I(A, w') T$$

Different modal logics are defined according to the properties of the accessibility relation R (and corresponding axioms).

Modal logics address the problem of referential transparency, since the truth of a complex formula does not depend on the truth of the components in the same world/interpretation.
Modal operators are not compositional, so the truth of Knows(A, P) cannot simply be determined by the components of Knows , the denotation of the agent and the truth value of P.
Modal logics for knowledge are easier than those of beliefs, so we start with these.

The syntax of modal logic for knowledge is the following:

1. All the wff of ordinary FOL are also wff of the modal language.
2. If Φ is a closed wff of the modal language and a is an agent, then $K(a, \Phi)$ is a formula of the modal language.
3. If Φ and Ψ are wff so are the formulas that can be constructed from them with the usual logic connectives.

Examples of Formulas are the following:

$$K(A_1, K(A_2, On(B, C)))$$

$$K(A_1, On(B, C)) \vee K(A_1, \not On(B, C))$$

$$\not K(A_1, On(B, C))$$

Properties of knowledge which are desirable to have are:

- One agent can hold false beliefs but cannot hold false knowledge and if an agent knows something than this must be true, so knowledge is justified by true belief.
- An agent does not know all the truths: something may be true without the agent knowing it.

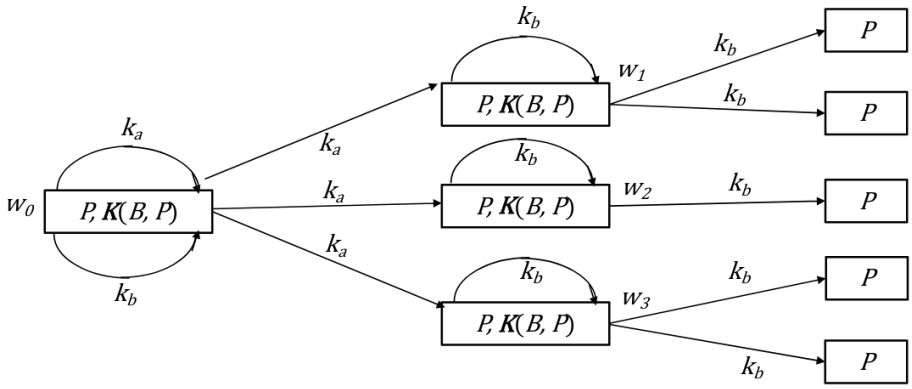


Figura 17: Example of Nested knowledge statements

- If two formulas Φ and Ψ are equivalent not necessarily $K(A, \Psi)$ implies $K(A, \Psi)$

The semantics of modal logic is given in terms of possible worlds and specific accessibility relations among them, one for each agent.
An agent knows a proposition just when that proposition is true in all the worlds accessible from the agent's world (those that the agent considers possible).

Possible worlds roughly correspond to interpretations and an accessibility relation (k for knowledge) is defined between agents and possible worlds:

Def (Accessibility Relation)**.** If $k(a, w_i, w_j)$ is satisfied, then world w_j is accessible from world w_i for agent a.

The semantic of Modal logic for knowledge is defined with the following rules:

1. Regular wffs (with no modal operators) are not simply true or false but they are true or false wrt a possible world, so $I(w_1, \Phi)$ may be different from $I(w_2, \Psi)$.

2. A modal formula $K(a, \Psi)$ is true in w iff Ψ is true in all the worlds accessible from w for agent a.

3. The semantics of complex formulas is determined by regular truth recursive rules.

$K(A, \Psi)$ means that agent A knows the proposition denoted by Ψ and "Not knowing Ψ" in w_0 (a specific world) is modelled by allowing worlds, accessible from w_0, in which Ψ is true and some worlds in which Ψ is false.

The accessibility relation also accounts for nested knowledge statements and involving different agents, so as we can see in figure 17, $K(A, K(B, P))$ holds in w_0 since $K(B, P)$ holds in w_0, w_1, w_2 and w_3.

Many of the properties that we desire for knowledge (and belief) can be achieved by imposing constraints to the accessibility relation, so we define:

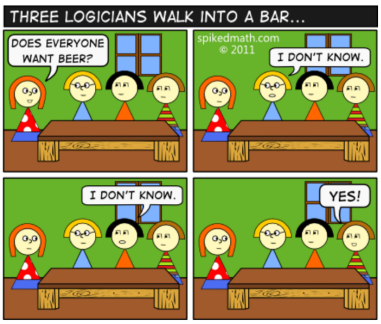
1. Agents should be able to reason with the knowledge they have

$$K(a, \alpha \Rightarrow \beta) \Rightarrow (K(a, \alpha) \Rightarrow K(a, \beta))$$

   This is implicit in possible world semantics and is called *Distribution axiom*.

2. Agents cannot have false knowledge (different for beliefs) $K(a, \alpha) \Rightarrow \alpha$ and this is called *Knowledge axiom*.

   The knowledge axiom is satisfied if the accessibility relation is reflexive and this implies that there is at least a world accessible from w.



Three wise man who are told by their king that at least one of them has a white spot in his forehead; actually all three have white spots.

Each wise-man can see the other's foreheads but not his own.

The first wise man says "I don't know whether I have a white spot".

The second wise man says "I don't know whether I have a white spot".

The third wise man can then conclude that he has a white spot.

Figura 18: Description of Wise-men puzzle

3. It is also reasonable to assume that if an agent knows something, than it knows that it knows, that is formally

$$K(a,\alpha) \Rightarrow K(a,K(a,\alpha))$$

The accessibility relation must be transitive and this property is called *positive introspection*.

4. In some axiomatization we also assume that if an agent doesn't know something, then it knows that it doesn't know it, that is formally

$$\not{K}(a,\alpha) \Rightarrow K(a,\not{K}(a,\alpha))$$

The accessibility relation must be euclidean and this property is called *Negative introspection*

5. It is also intrinsic in possible world semantics that an agent knows all the logical theorems, including the ones characterizing knowledge.
From $\vdash \alpha$ infer $K(a,\alpha)$ and this is *Epistemic necessitation rule*.

6. From 1 and 5, in the propositional case we also get rules

$$\alpha \vdash \beta \wedge K(a,\alpha) \text{ we infer } K(a,\beta)$$

$$\vdash \alpha \Rightarrow \beta \text{ we infer} K(a,\alpha) \Rightarrow K(a,\beta)$$

This is called *Logical omniscience*, that is considered problematic, since we are assuming unbounded reasoning capabilities.
As a corollary of logical omniscence we can infer K distribution over and

$$K(a,\alpha \wedge \beta) \equiv K(a,\alpha) \wedge K(a,\beta)$$

Modal epistemic logics are obtained with various combinations of axioms 1-4 plus inference rule 5, so we have System K has only axiom 1, system T has axioms 1-2, logic $S4$ has axioms 1-3 and logic $S5$ has axioms 1-4.
Not any combination is possible since the properties of accessibility relations are interdependent, so for example reflexive implies serial and if a relation is reflexive and Euclidian it is also transitive (axiom 2 and 4 imply 3).

We have also consider the wise-men puzzle visible in figure 18, where we can also see an explanation of what consist.

Since an agent can hold wrong beliefs the knowledge axiom is not appropriate, so we include as axiom the following instead

$$\not{B}(a,False)$$

The distribution axiom and the necessitation rule are controversial, since an agent cannot realistically believe all the logical consequences of its beliefs but only those that he is able to derive (limited/bounded rationality), so

$$B(a,\alpha) \Rightarrow B(a,B(a,\alpha))$$

$$B(a,\alpha) \Rightarrow K(a,B(a,\alpha))$$

Negative introspection is problematicm but with the following special case of the knowledge axioms is safe $B(a,B(a,\alpha)) \Rightarrow B(a,\alpha)$

One disadvantage of default logic is that default rules are not sentences and cannot be combined or reasoned about, so for example $\alpha : \beta/\gamma$ does not derive $\alpha : \beta/(\gamma \vee \delta)$.
A different approach is to reason about defaults within a logic with a belief operator B, where $B\alpha$ stands for "α is believed to be true", and this is *autoepistemic logic*.
The B operator could be used to represent defaults, for example, as follows

$$\forall x Bird(x) \wedge \not B \not Flies(x) \Rightarrow Flies(x)$$

Note that $B \not Flies(x)$ is different from $\not Flies(x)$

Given a KB that contains sentences using the B "auto-epistemic" operator, the minimal properties for a set of beliefs E to be considered stable are the reasonable set of beliefs to hold:

CLOSURE UNDER ENTAILMENT: if $E \models \alpha$, then $\alpha \in E$.

POSITIVE INTROSPECTION: if $\alpha \in E$, then $B\alpha \in E$.

NEGATIVE INTROSPECTION: if $\alpha \notin E$, then $\not B\alpha \in E$

This leads to the following definition of stable expansion of a KB (a minimal set satisfying the properties yet described)

Def (Stable expansion)**.** A set E is a stable expansion of KB if and only if for every sentence ϕ, it is the case that

$$\phi \in E \iff KB \cup \{B\alpha : \alpha \in E\} \cup \{\not B\alpha : \alpha \notin E\} \models \phi$$

The implicit beliefs E are those sentences that are entailed by KB plus the assumptions deriving from positive and negative introspection.

The KB consisting of the sentence $(\not Bp \Rightarrow p)$ has no stable expansion, since if Bp is false, then the expansion entails p and if Bp is true, then the expansion does not include p.
The KB consisting of the sentences $(\not Bp \Rightarrow q)$ and $(\not Bq \Rightarrow p)$ has exactly two stable expansions, so if Bp is true and Bq false the KB entails p and does not entail q, another stable expansion is when Bp is false and Bq is true.

4.7 REASON MAINTENANCE SYSTEMS

Reason Maintenance Systems are the monotonic view of reasoning, so truth never changes, the only allowed reasoning is to discover new truths.
If the KB becomes inconsistent, there is not much you can do and common sense reasoning requires belief revision mechanisms.
We want to be able to make assumptions (defeasible reasoning) and if the world changes, RETRACT is also an option (belief update).

RMS are practical systems designed to support beliefs revision, since they record the reasons for believing something and do the job of maintaning consistency in the belief set on behalf of the problem solver.

Figura 19: General Architecture of RMS

Many of the inferences drawn by a knowledge representation system will have only default status or tentative nature and inevitably, some of these inferred facts will have to be retracted; this process is called *belief revision* and can be problematic. One simple approach to belief revision is to number sentences according to the order of assertion $P_1, P_2, \ldots P_n$, so when the call $RETRACT(KB, P_i)$ is made, the system revert to the state just before P_i was added, removing both P_i and any inferences derived from P_i.
Sentences P_{i+1} through P_n can be added again if it is the case.

Reason Maintenance Systems (JTMS, ATMS), are reasoning mechanisms designed to handle these problems efficiently and general architecture is visible in figure 19, where the problem solver communicates facts, rules, assumptions along with their justifications to the RMS and the PS may retract assertions, or may request to handle inconsistencies.
The RMS maintains beliefs, handles contradictions, restores consistency, performs beliefs revision, generates explanations and provides the PS with an up-to-date set of beliefs.

Rational thought is the process of finding reasons for attitudes, so has justified belief or reasoned argument, rather than truth and the RMS handles a network of nodes as propositional variables, representing propositions, rules and justifications.

Nodes are of different types: premises, assumptions, contradictions and have different support according to the type of RMS and we will look at two of them:

- JTMS (Justification-based Truth Maintenance Systems).
- ATMS (Assumption-based Truth Maintenance Systems)

In JTMS Each assertion in the knowledge base is represented as a node in the TMS with an associated set of SL-justifications.
Each SL-justification has two components:

INLIST: the set of sentences from which it was inferred.

OUTLIST: a nonmonotonic justification the set of propositions that should be false for the beliefs to be supported.

Examples:

$$R(SL\{\}\{\})$$

$$Q(SL\{P, P \Rightarrow Q\}, \{\})$$

$$P(SL\{\}\{\neg P\})$$

A node can have more than one SL justification (a justification set), since there can be different reasons.

With JTMS, $RETRACT(KB, P)$ will delete exactly those sentences for which P is a member of every justification, so we have the following cases:

- If a sentence Q had the single justification $\{P, P \Rightarrow Q\}$, then Q would be removed.

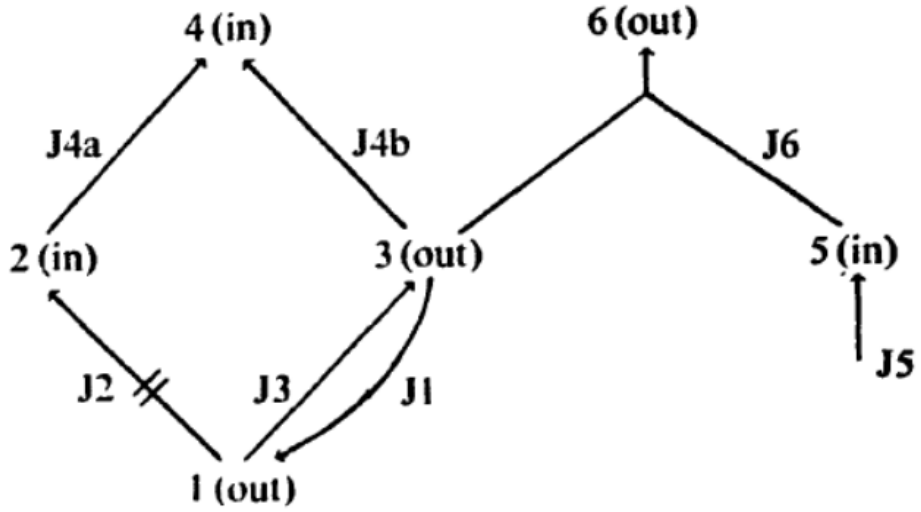


Figura 20: Example of SlList

	Propositions	Justifications	Context In	Context Out
	A: Temperature >= 25	SL({ }, {B}) assumption	A	B
	B: Temperature < 25			
	C: Not raining	SL({ }, {D}) assumption	A, C	B, D
	D: Raining			
	E: Day	SL({ }, {F}) assumption	A, C, E	B, D, F
	F: Night			
PS ⇒	G: Nice weather	SL({A, C}, { })	A, C, E, G	B, D, F
PS ⇒	H: Swim	SL({E, G}, { })	A, C, E, G, H	B, D, F
PS ⇒	I: Contradiction	SL({C}, { }) dep. dir. backtracking		
	X: NG I	(CP (I, {D}, { })		
	D: Raining	({X}, { })	A, D, E	B, C, F, H, I
PS ⇒	J: Read	({D, E}, { })	A, D, E, J	B, C, F, H, I

Figura 21: Complete example of Sllist in JTMS system

- If it had the additional justification $\{P, P \vee R \Rightarrow Q\}$, then Q would still be removed.
- If it also had the justification $\{R, P \vee R \Rightarrow Q\}$, then Q would be maintained.

The JTMS, rather than deleting a sentence from the knowledge base entirely when it loses all justifications, marks the sentence as being "out".
If a subsequent assertion restores one of the justifications, then the sentence is marked as "in" again, so we no need to re-compute inferences.

The node for proposition P can be in one of two support states:

IN: P has at least one currently valid reason/justification, and thus is a member of the current beliefs.

OUT: P has no currently acceptable reason/justifications and thus is not a member of the current beliefs.

A justification (SL inlist outlist) is valid iff each node in its inlist is IN, and each node in its outlist is OUT and this creates a network of dependencies.
Beliefs revision algorithms operate on these networks propagating effects and an example of that is visible in figure 20 and 21.

sentence	consequent	justifications	label
node(p,	[r],	[(p :- a, b), (p :- b, c, d)],	{{a, b}, {b, c, d}})
node(q,	[r],	[(q :- a, c), (q :- d, e)],	{{a, c}, {d, e}})
node(r,	[],	[(r :- p, q)],	{{a, b, c}, {b, c, d, e}})

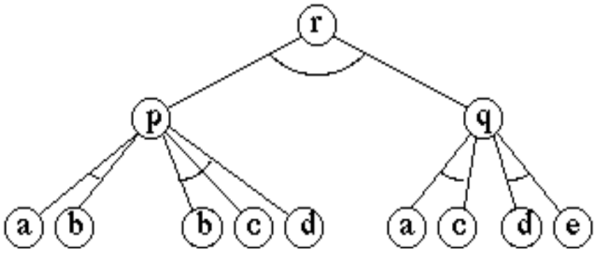


Figura 22: Example of ATMS Dependency graph

JTMSs can be used to speed up the analysis of multiple hypothetical situations, so in a JTMS, the maintenance of justifications allows you to move quickly from one state to another by making a few retractions and assertions, but at any time only one contexts is represented.
Taking the idea one step forward we could let multiple contexts to co-exist rather than a global state of INs and OUT's.

An Assumption based Truth Maintenance System (ATMS) represents all the contexts been considered at the same time, so alternative contexts are explicitly stored.
An ATMS-node is characterized by a label and justifications:

- The label consists in a number of assumption sets (environments) and the node holds when there is at least one supporting environment (with true assumptions).
- ATMS justifications are Horn formulas of the form $C : -L_1, L_2, \ldots, L_n$, where $L_1, L_2, \ldots, L_n$ are the antecendents and C is the consequent of justistification.

Thus a node is a triple $< n, E, J >$, where n stands for an external sentence, the description and in figure 22 is possible to note an example of dependency graph.

We have four types of nodes, according to the form of E and of J:

PREMISE NODES: nodes always true, so the proposition can be believed without having to assume anything ($< n, \{\{\}\}, \{()\} >$.

ASSUMPTION NODES: these nodes are justified by themselves $< n, \{(n)\}, \{(n)\} >$

DERIVED NODES: Maintain the set of assumptions made and the justifications.

CONTRADICTIONS: an environment can be inconsistent when its assumptions are contradictory, therefore must be removed, and these environments are called *nogoods*.

The fundamental operation is updating environments and deciding whether a proposition holds in a given environment, so a node n holds in a given environment E, iff it can be derived from E given the set of justifications $J : E, J \vdash n$.

ATMS allow for quick generation of explanations, so an explanation of a sentence P is a set of sentences E such that $E \models P$, usually a minimal one is preferred and explanations can be assumptions.
ATMS can generate explanations by making tentative assumptions and looking if the label for the sentence "car won't start" contains the assumptions that would justify the sentence.

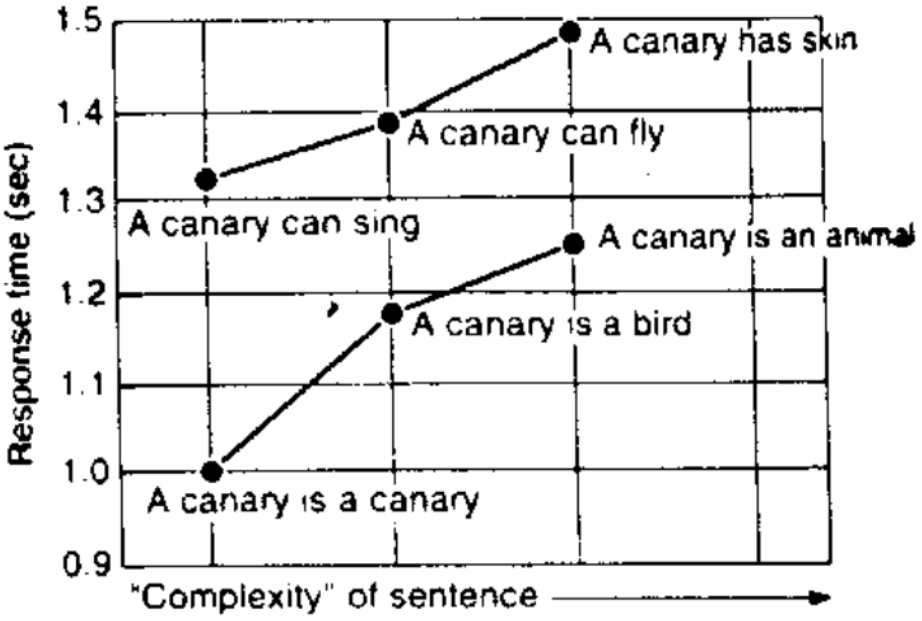


Figura 23: Example of a cognitive psychology definition

4.8 SEMANTIC NETWORKS AND FRAMES

We have that the logical approach is suitable to model rational reasoning, instead the cognitive–linguistic approach is more concerned with understanding the mechanisms for knowledge acquisition, representation and use of knowledge in human minds and has synergies with other fields, such as cognitive psychology and linguistics.

In logic systems, symbolic expressions are modular and are syntactically transformed without paying attention to the symbols used or to their "meaning", infact the symbol themselves are arbitrary and it is all a matter of writing axioms to restrict interpretations.
Associationist theories are instead concerned with the connections among symbols and the meaning that emerges from these connections.
The idea is that meaning of a word emerges as a result of the connections to other words and connections are shaped by experience, as for example from reading texts.

The question is how the meaning of words is acquired, represented and used, and we have that the memory itself is distinguished in:

EPISODIC MEMORY: specific facts and events.

SEMANTIC MEMORY: abstract and general knowledge.

Semantic networks is a graphical model proposed for semantic memory and we represent two kinds of knowledge:

CONCEPTS: the semantic counterpart of words, represented as nodes.

PROPOSITIONS: relations among concepts, represented as labelled arcs.

It is not accounting for dynamic aspects of memory and learning and there are other models like *Distributional models*, like word embeddings, or connectionist models for learning.

Cognitive psychology is an experimental discipline and we have an example in figure 23.
Evidence are for hierarchical organization, properties are attached to the most general

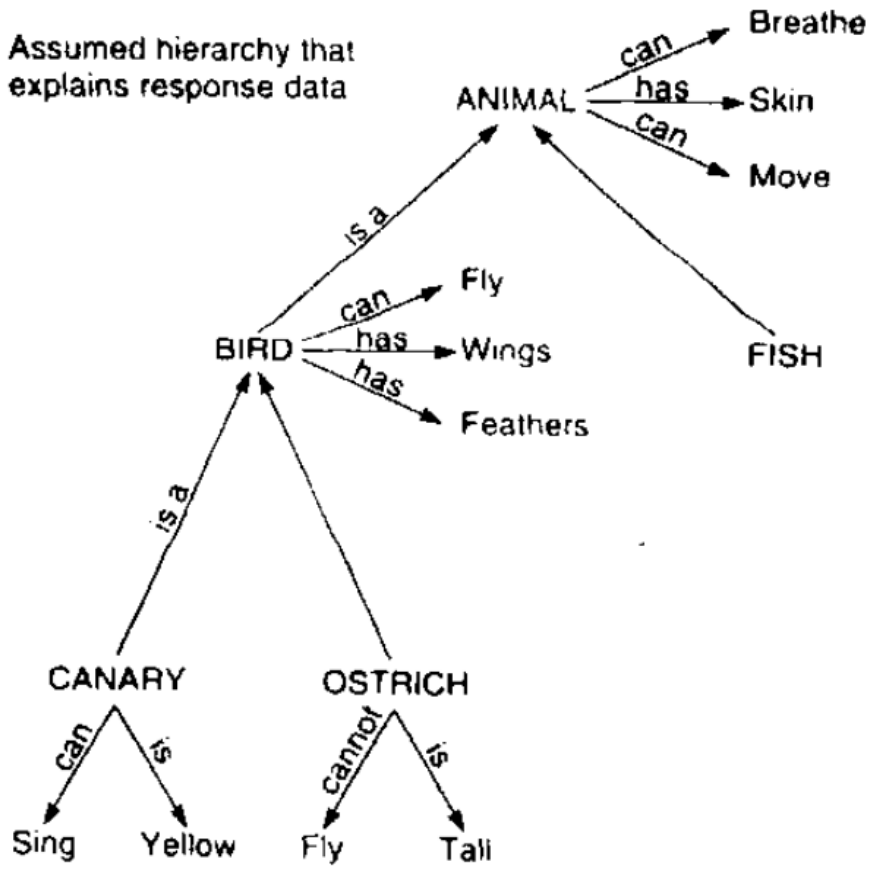


Figura 24: Example of Hierarchical organization

concept to which they apply and exceptions are attached directly to the objects, like as possible to note in figure 24.
Success of hierarchical organization of concepts in computer science and influence on OOP languages in SW engineering.

Semantic networks are a large family of graphical representation schemes and a semantic network is a graph where nodes, with a label, correspond to concepts and arcs, labelled and directed, correspond to binary relations between concepts, often called roles.
Nodes come in two flavors:

GENERIC CONCEPTS corresponding to categories/classes

INDIVIDUAL CONCEPTS , corresponding to individuals

Two special relations are always present, with different names:

IS-A: holds between two generic concepts (subclass).

INST-OF: holds between an individual concept and a class (member of).

Inheritance is very conveniently implemented as link traversal, as we can see in figure 25, and Multiple inheritance is also allowed, but is banned in OOP.

The presence of exceptions does not create any problem, so we just take the most specific information: the first that is found going up the hierarchy.
Even if they can express n-ary predicates, semantic networks do not have the same expressive power of FOL: § Existentials, $\vee$ and $\Rightarrow$ are not expressible or are expressible only in special cases and this is not necessarily a bad thing since it suggests a subset of FOL with interesting computational properties, explored in description logics.

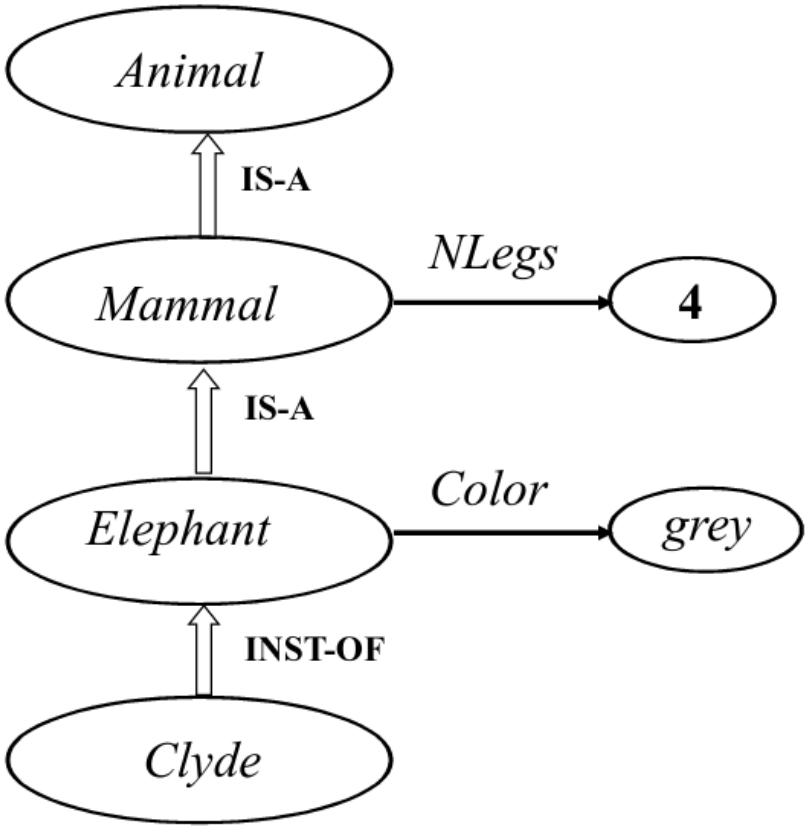


Figura 25: Example of Inheritance in Semantic Network

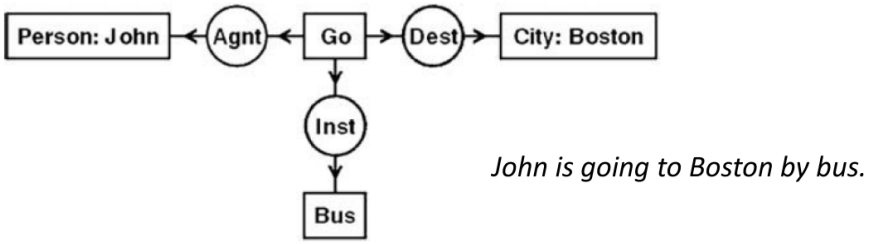


Figura 26: Example of Conceptual graph

A well -known example in AI are Sowa's conceptual graphs, inspired by Pierce's existential graphs and they candidate as an intermediate schema for representing natural language.
Conceptual graphs, visible in figure 26, rectangles are concepts (possibly typed as in Person: John), circles are called conceptual relations: the label is the name of the relation.

A more complex proposition, including implication, uses context boxes, so for example "If a farmer owns a donkey, then he beats it" is represented in figure 27.

Woods and others point out the lack of "semantics" of semantic nets, so as a result in the 80's KL-One introduces important ideas:

- Concepts and roles (they are also nodes with a different status).
- Value restrictions (v/r)
- Numerical restrictions (1, NIL)
- A formal semantics

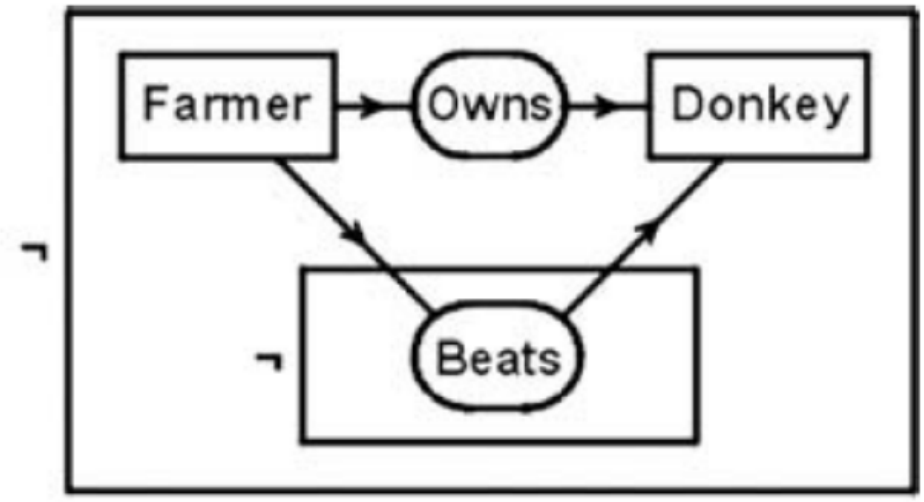


Figura 27: Complex example of Conceptual graph

The double arrow is a IS-A relation and an example in figure 28.

We may look at semantic networks as a convenient implementation for a part of FOL and a representation and mechanism at the symbol level rather than at the knowledge level, with concepts defined in figure 29.

A more essential notation for inheritance networks are negated arcs, so more specific information should win and there is also multiple inheritance, as we can see in figure 30.

Not so easy and there are two strategies:

- Shortest path heuristics, based on minimal path length.
- Inferential distance, based on the topology (not path length).

The shortest path strategy fails in presence of redundant links such as q (shortcuts), as we can see in figure 31 and the inferential distance strategy: A closer to B than C iff there is a path from a A to C through B.
With shortest path it would conclude that Clyde is grey, instead inferential distance it would conclude that Clyde is not grey.

WordNet, a big lexical resource organized as a Semantic network (122.000 terms), names, verbs, adjectives, adverbs are organized in sets of synonims (synsets) which are a representation of a concepts (117.000 synset); a syntactic word is associated with a set of synsets: the word senses and uses of WordNet in computational linguistics are:

- Query expansion with synonyms or hyperonyms.
- Semantics distance among words.
- Word sense disambiguation.
- Semantic category of a word (or supersense).

The structure of WordNet is visible in figure 32.

Knowledge graphs are large-scale knowledge bases have been built including:

OPEN ACCESS: Dbpedia, WikiData, Freebase, YAGO.

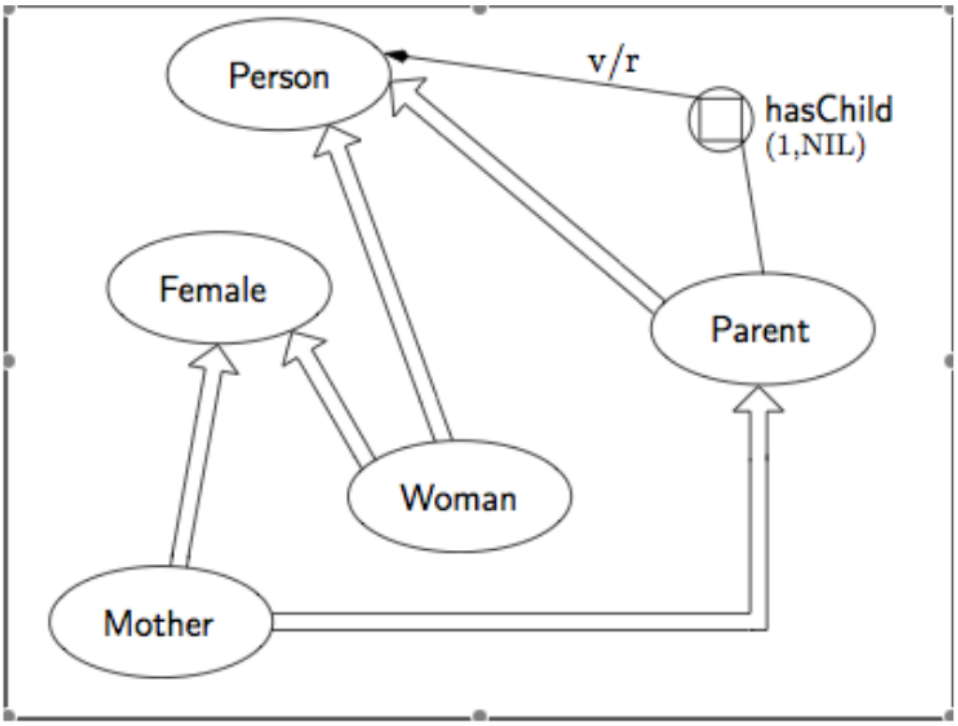


Figura 28: Example of IS-A relation in KL One

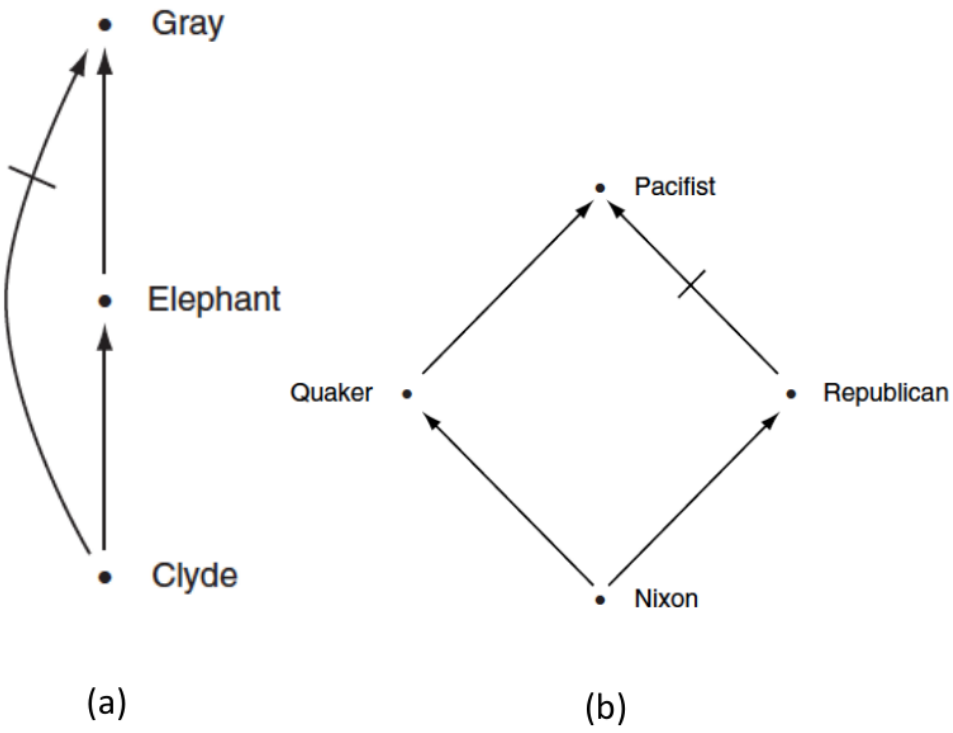
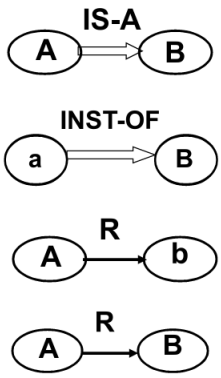


Figura 29: Definition of representation done at the symbol level



$\forall x\, A(x) \Rightarrow B(x)$
All members of class A are also member of class B.

$B(a)$
a belongs to class B

$\forall x\ x \in A \Rightarrow R(x, b)$
All members of class A are in relation R with b

$\forall x\ x \in A \Rightarrow \exists y\ y \in B \wedge R(x, y)$
For all members A, there is a R-related element in B

Figura 30: Example of Inheritance Network

PROPRIETARY: Microsoft's Satori, Google's Knowledge Graph, Google's Knowledge vault.

From 2012 Google uses the knowledge graph with its search engine, structured data coming from many sources, including the CIA World Factbook, Wikidata, Wikipedia, Freebase and has more than 70 billion facts in 2016.
Since 2014, Google's Knowledge Vault contains facts derived automatically from the web with machine learning techniques and facts have an associated confidence value.

It is very natural to think of knowledge not as a flat collection of sentences, but rather as structured and organized in terms of the objects the knowledge is about.
Complex objects have attributes, parts constrained in various ways, objects might have a behavior that is better expressed as procedures and this is very much as in Object Oriented Programming.
Marvin Minsky in 1975 suggested the idea of using a structured representation of objects, called frames, to recognize and deal with new situations.

Knowledge is organized in complex mental structures called *frames*, and the essence of the theory is "When one encounters a new situation (or makes a substantial change in one's view of the present problem) one selects from memory a structure called a frame.
This is a remembered framework to be adapted to fit reality by changing details as necessary.".
A frame is a data-structure for representing a stereotypical object or situation.

There are two types of frames:

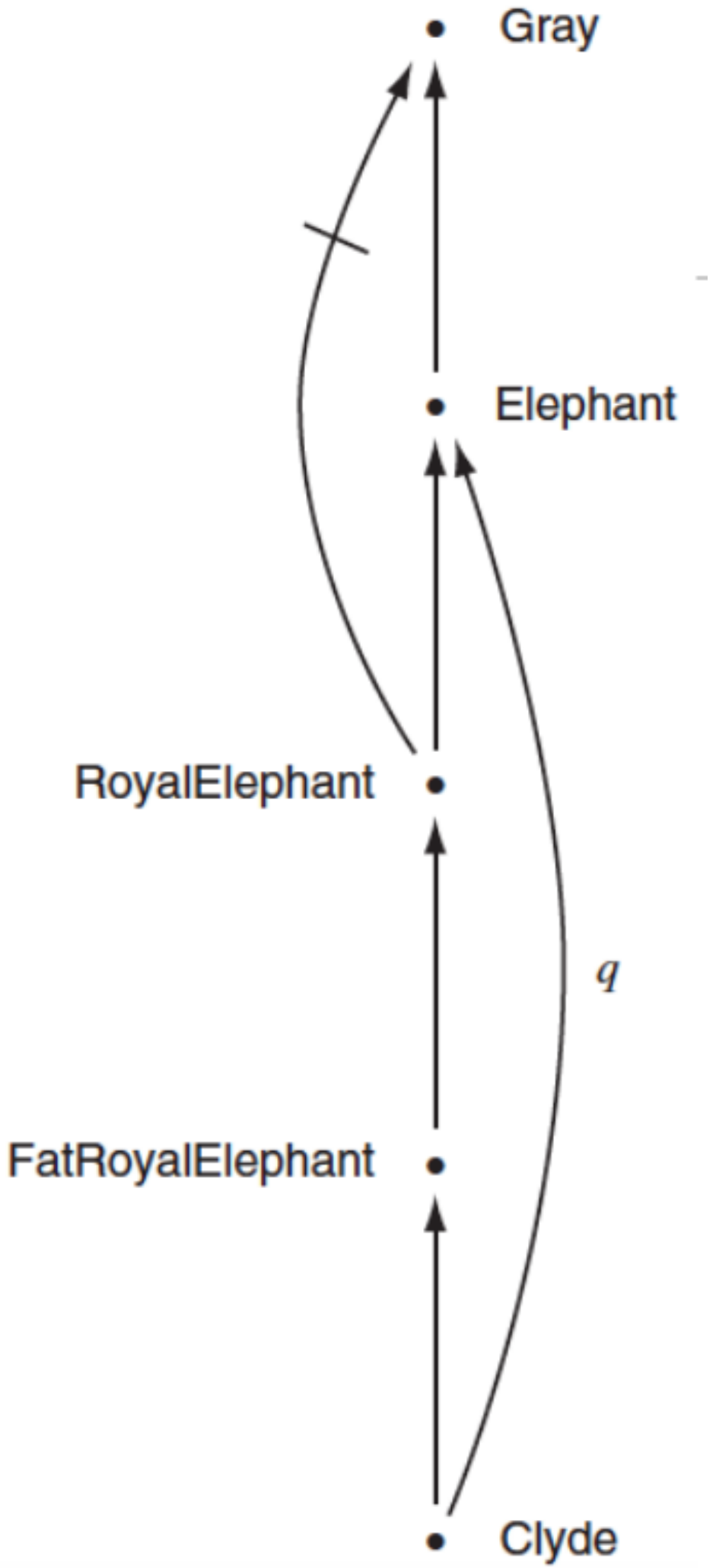
INDIVIDUAL FRAMES used to represent single objects.

GENERIC FRAMES used to represent categories or classes of objects.

An individual frame is a collection of slot-fillers pairs, and fillers can be:

- values, usually default values.
- constraints on values.
- the names of other individual frames.
- the special slot INSTANCE-OF

Generic frames are similar and fillers can be the special slot IS-A or procedures like IF-ADDED, activated when the slot receives a value and IF-NEEDED, activated when the value is requested.
These procedures are called procedural attachments or demons, so the :INSTANCE-OF and :IS-A slots organize frames in frame systems.
They have the special role of activating inheritance of properties and procedures.


Gray
Elephant
RoyalElephant
FatRoyalElephant
Clyde
q

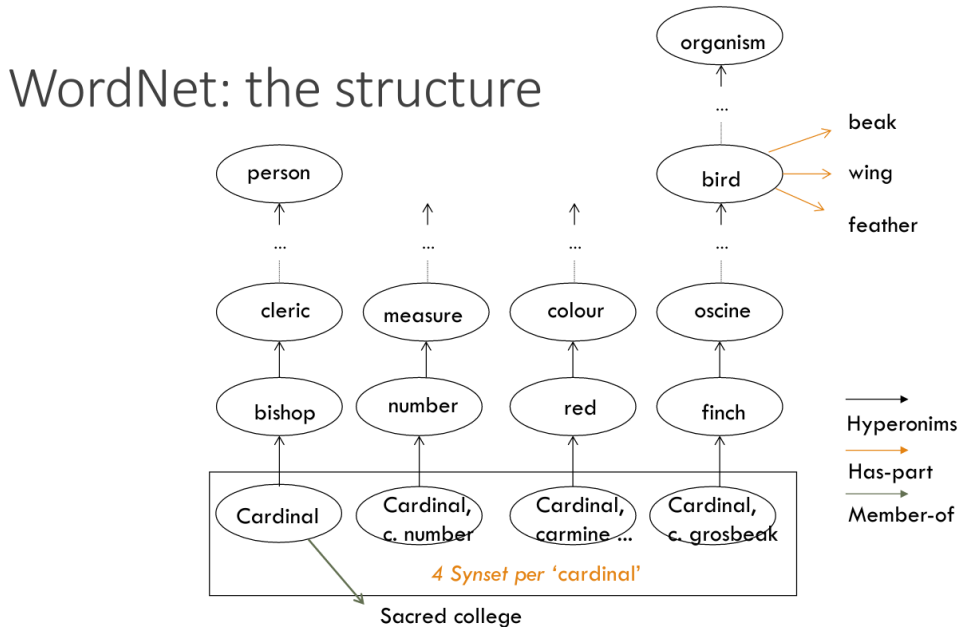


Figura 32: Structure of WordNet

In frames, all values are understood as default values, which can be overridden and example of frames are visible in figure 33.
Attached procedures provide a flexible, organized framework for computation and reasoning has a procedural flavor.
A basic reasoning loop in a frame system has three steps:

RECOGNITION: a new object or situation is recognized as instance of a generic frame.

INHERITANCE: any slot fillers that are not provided explicitly but can be inherited by the new frame instance are inherited.

DEMONS: for each slot with a filler, any inherited IF-ADDED procedure is run, possibly causing new slots to be filled, or new frames to be instantiated, until the system stabilizes, then the cycle repeats.

When the filler of a slot is requested:

1. if there is a value stored in the slot, the value is returned.
2. otherwise, any inherited IF-NEEDED procedure is run to compute the filler for the slot, and this may cause other slots to be filled, or new frames to be instantiated.

Frame-based representation languages and OOP systems were developed at the same time, and they look similar and certainly one could implement a frame system with OOP.
Important difference is that frame systems tend to work in a cycle:

- Instantiate a frame and declare some slot fillers
- inherit values from more general frames
- trigger appropriate forward-chaining procedures
- when the system is quiescent, stop and wait for the next input

```
(CanadianCity
   <:IS-A City>
   <:Province CanadianProvince>
   <:Country canada> )

(Lecture
  <:DayOfWeek WeekDay>
  <:Date  [IF-ADDED ComputeDayOfWeek ]>
  … )

(Table
  <:Clearance  [IF-NEEDED
  ComputeClearanceFromLegs ]> … )
```

Figura 33: Example of Frames

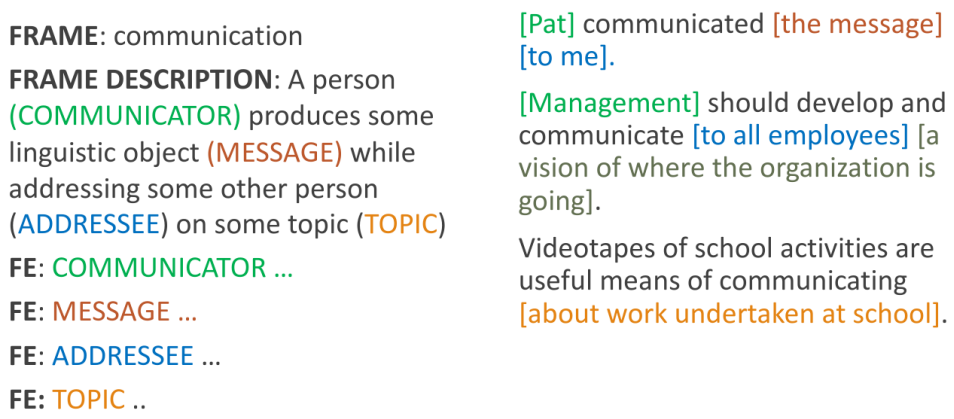


Figura 34: Example of FrameNet

The designer can control the amount of "forward" reasoning that should be done (in a data-directed fashion) or "backward" (in a goal-directed fashion).

Applications of frames are story understanding, Scene recognition in vision, tools for building expert systems (possibly together with rules) and the idea of frames was also used in building "FrameNet": a large NL resource based on the theory of "frame-based" semantics.

FrameNet is a resource consisting of collections of NL sentences syntactically and semantically annotated, organized in frames.
Frame semantics, the meaning of words emerges from the role they have in the conceptual structure of sentences and knowledge is structures in 16 general domains: time, space, communications, cognition, health and also has 6000 Lexical elements, and 130.000 annotated sentences; an example in FrameNet is visible in figure 34.

4.9 DESCRIPTION LOGICS

Most of the reasoning takes place at the level of categories rather than on individuals, so if we organize knowledge in categories and subcategories (in a hierarchy) it is enough to classify an object, according to its perceived properties, in order to infer properties of the categories to which it belongs.
Inheritance is a common form of inference, which exploits structure and also ontologies will play a crucial role, providing a source of shared and precisely defined terms that can be used in meta-data of digital objects and real world objects.

In the 80's we assist to a formalization of the ideas coming from semantic networks and frames resulting in specialized logics.
These logics, called terminological logics and later description logics find an important application in describing "domain ontologies" and represent the theoretical foundations for adding reasoning capabilities to the Semantic web.
Ontology is a formal model of an application domain (a conceptualization) and subclass relations are important in defining the terminology and serve to organize knowledge in hierarchical taxonomies (shared ontologies are the basis for the semantic web).

The *Semantic Web* is the vision of Tim Berners-Lee (1998) to gradually develop alongside the "syntactic web" (or web of documents), for communication among people, a "semantic web" (or web of data) for communication among machines.
The semantic web is a huge distributed network of linked data which can be used by programs as well, provided their semantics is shared and made clear (this is the role of formal ontologies).

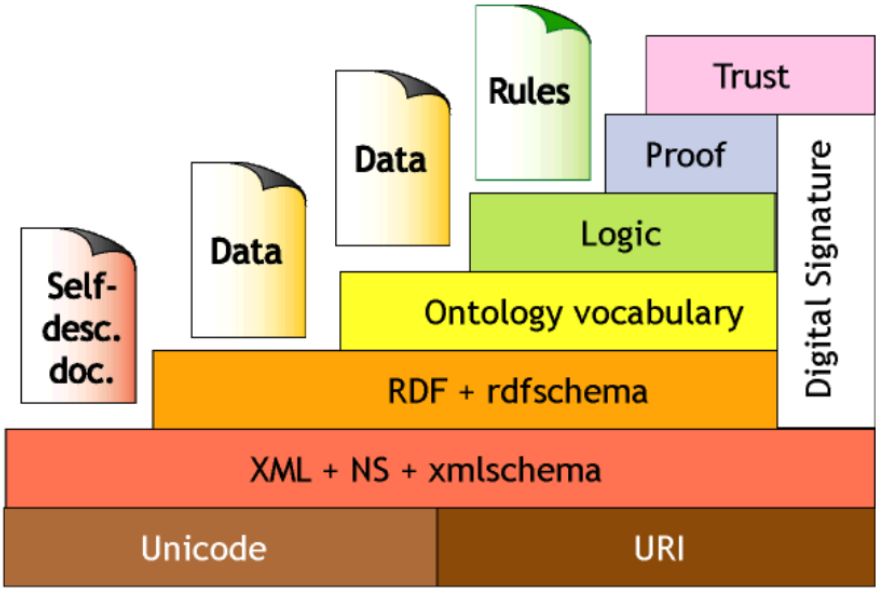


Figura 35: Technologies used in Semantic Web

These data comply with standard web technologies: Unicode encoding, XML, URI, HTTP web protocol.

The technologies used in Semantic Web, visible in 35, are the following:

- Unicode and URI (Universal Resource Identifier)
- XML for syntactic interoperability
- RDF (Resource Description Framework) is a language to describe binary relations between resources (subject, predicate, object).
- RDF schema (RDFS) used to define classes, relations between classes, to constrain domains and co-domains of relations and this is basic language for ontologies.
- OWL is the web ontology language, one among many description logics elected as standard by the W3C.

Description logics can be be seen as:

1. Logical counterparts of network knowledge representation schema, frames and semantic networks; in this formalization effort, defaults and exceptions are abandoned and the ideas and terminology (concept, roles, inheritance hierarchies) are very similar (to KLOne in particular).
2. Contractions of first order logic (FOL), investigated to obtain better computational properties and attention to computational complexity/decidability of the inference mechanisms.

Each DL is characterized by operators for the construction of terms, that are of 3 types:

CONCEPTS: corresponding to unary relations, with operators for the construction of complex concepts: and($\cap$), or($\cup$), not($\neg$), all($\forall$), some ($\exists$), atleast ($\geq n$), almost ($\leq n$) and so on.

ROLES: corresponding to binary relations, possibly together with operators for construction of complex roles.

INDIVIDUALS: only used in assertions.

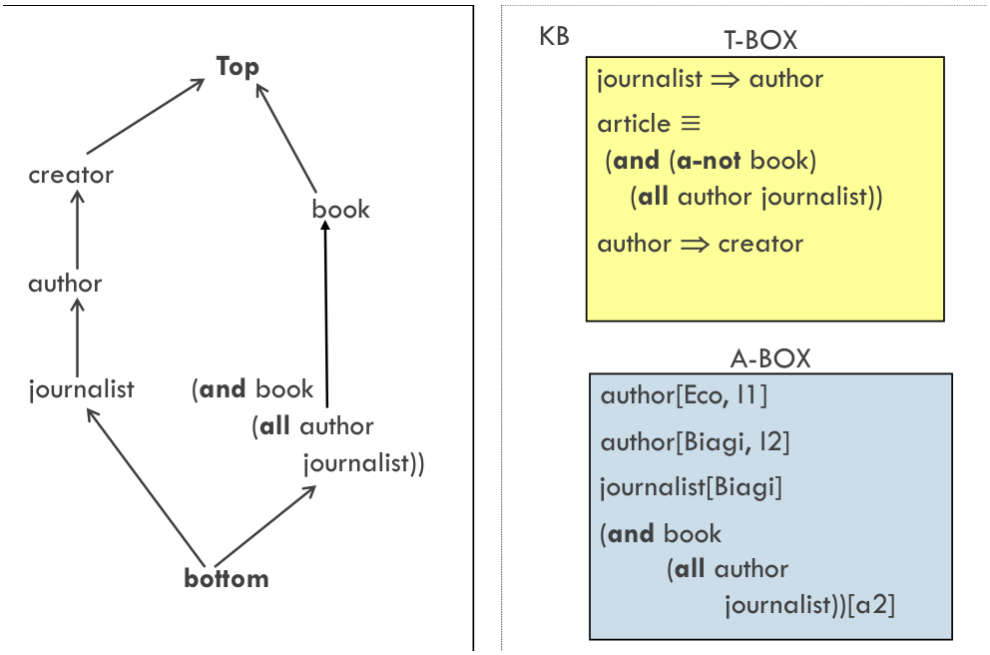


Figura 36: Example of KB Description

<concept> → A

| $\top$	(**top**, *universal concept*)
| $\bot$	(**bottom**)
| $\neg A$	(*atomic negation*)
| $C \sqcap D$	(*intersection*)
| $\forall R.C$	(*value restriction*)
| $\exists R.\top$	($\top$ for top, *weak existential*)

<role> → R

A, B *primitive concepts*

R *primitive role*

C, D *concepts*

Examples:
Person ⊓ Female
Person ⊓ ¬Female
Person ⊓ ∃ hasChild. ⊤
Person ⊓ ∀ hasChild . Female
Person ⊓ ∀ hasChild . ⊥

Figura 37: Definition of syntax of Logic AL

Assertions are kept separate and can be only of two types:

- $i : C$, where i an individual and C is a concept.
- $(i, j) : R$, where i and j are individual and R is a role.

In figure 36 is possible to note an example of KB based on description logic and we define the logic AL as the syntax of terms in figure 37.

The semantics of AL consist of Δ^I, interpretation domain (a set of individuals) and I an interpretation function which assign for atomic concepts $A : A^I \subseteq \Delta^I$, for atomic roles $R : R^I \subseteq \Delta^I \times \Delta^I$ and for individual constants $a : a^I \in \Delta^I$.
We have some rules visible in figure 38 and we have also 4 more expressive logics, defined as:

u **UNION:** $(C \cup D)^I = (C^I \cup D^I)$

e **FULL EXISTENTIAL:** $(\exists R.C)^I = \{a \in \Delta^I : \exists b.(a, b) \in R^I \cap b \in C^I\}$

n **NUMERICAL RESTRICTIONS:**

$$(\geq nR)^I = \{a \in \Delta^I : |\{b : (a, b) \in R^I\}| \geq n\}$$

$$\top^I = \Delta^I \qquad \textit{the interpretation domain}$$
$$\bot^I = \varnothing \qquad \textit{the empty set}$$
$$(\neg A)^I = \Delta^I \setminus A^I \qquad \textit{the complement of } A^I \textit{ to } \Delta$$
$$(C \sqcap D)^I = C^I \cap D^I \qquad \textit{the intersection of the denoted sets}$$
$$(\forall\, R\,.\,C)^I = \{a \in \Delta^I \mid \forall b\,.\,(a,b) \in R^I \rightarrow b \in C^I\}$$
$$(\exists\, R\,.\,\top)^I = \{a \in \Delta^I \mid \exists b\,.\,(a,b) \in R^I\}$$

Figura 38: Some rules for logic AL

$$(\leq nR)^I = \{a \in \Delta^I : |\{b : (a,b) \in R^I\}| \leq n\}$$

c **FULL COMPLEMENT:** $(\not C)^I = \Delta^I - C^I$

Different description logics are obtained by adding other constructors to AL and we obtain the lattice visible in figure 39, where not all of them are distinct so for example $ALUE = ALC$.

We have the terminological axioms T defined as:

$$C \subseteq D \text{ inclusion of concepts } C^I \subseteq D^I$$
$$R \subseteq S \text{ inclusion of roles } R^I \subseteq S^I$$
$$C \equiv D \text{ equality of concepts } C^I \equiv D^I$$
$$R \equiv S \text{ equality of roles } R^I \equiv S^I$$

We have that equalities introduce a symbol on the left and we have a *terminology*, when symbols appear on the left not more than once; we have a *primitive symbols* in case appear only on the right and a *defined symbols* if appear also on the left.

If a terminology is acyclic, an example is visible in figure 40, it can be expanded by substituting to defined symbols their definitions.
In the case of acyclic terminologies the process converges and the expansion T^e is unique.

Properties of T^e are the following:

- In T^e each equality has the form $C \equiv D^e$ where D^e contains only primitive symbols.
- T^e contains the same primitive and defined symbols of T
- T^e is equivalent to T.

An example of expansion is visible in figure 41 and inclusion axioms are called *specializations*, like for example $Woman \subseteq Person$.
A *generalized terminolohy*, with inclusion axioms, if acyclic, can be transformed in an equivalent terminology with just equivalence axioms

$$A \subseteq C \rightarrow A \equiv A' \cap C$$

where A' is a new primitive symbol.
This also means that specializations do not add expressive power to the language, at least in the case of acyclic terminologies.

An A-BOX is a set of assertions of the following type $a : C$ assertion over concepts (meaning $a^I \in C^I$) and $(b,c) : R$ assertions over roles (meaning $(b^I, c^I) \in R^I$, where $a, b, c, d, \ldots$ are individuals.
In description logic we make an assumption that different indivudal constants refer to different individuals, and that is called *Unique Name Assumption* (UNA).

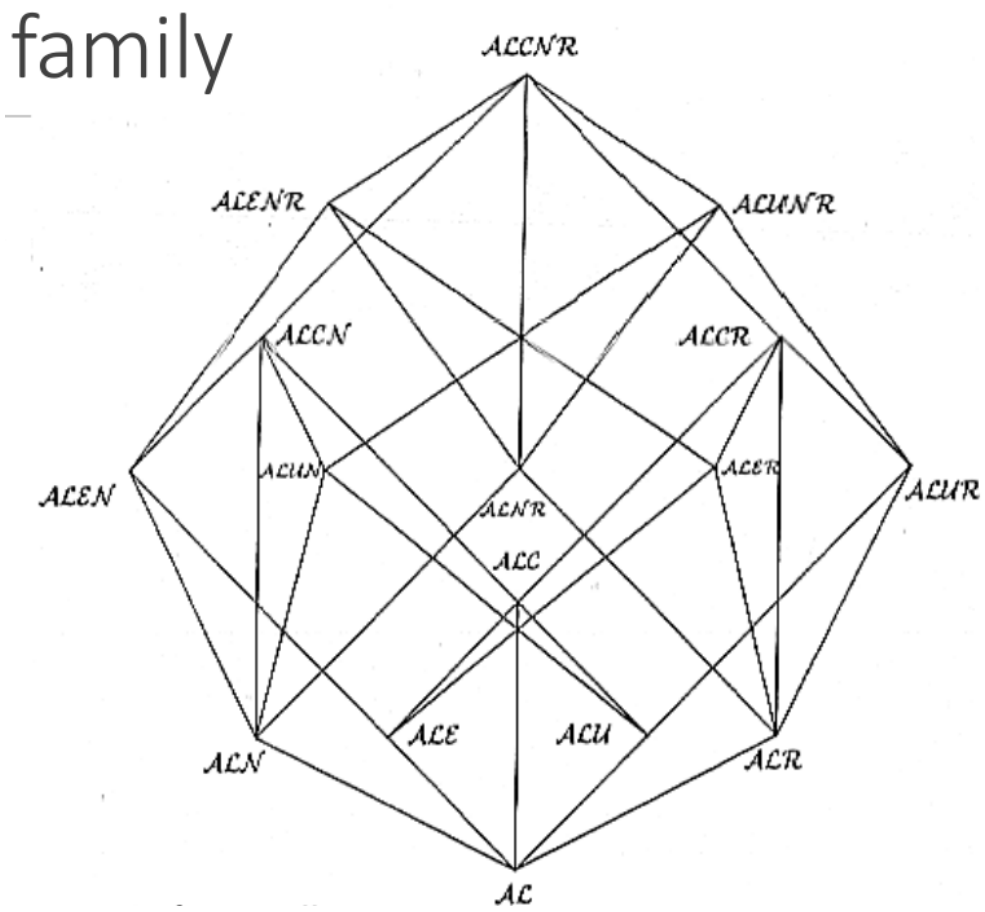


Figura 39: Lattice of AL logics

$$
\begin{aligned}
\text{Woman} &\equiv \text{Person} \sqcap \text{Female} \\
\text{Man} &\equiv \text{Person} \sqcap \neg(\text{Person} \sqcap \text{Female}) \\
\text{Mother} &\equiv (\text{Person} \sqcap \text{Female}) \sqcap \exists \text{hasChild.Person} \\
\text{Father} &\equiv (\text{Person} \sqcap \neg(\text{Person} \sqcap \text{Female})) \sqcap \exists \text{hasChild.Person} \\
\text{Parent} &\equiv ((\text{Person} \sqcap \neg(\text{Person} \sqcap \text{Female})) \sqcap \exists \text{hasChild.Person}) \\
&\quad \sqcup ((\text{Person} \sqcap \text{Female}) \sqcap \exists \text{hasChild.Person}) \\
\text{Grandmother} &\equiv ((\text{Person} \sqcap \text{Female}) \sqcap \exists \text{hasChild.Person}) \\
&\quad \sqcap \exists \text{hasChild.}(((\text{Person} \sqcap \neg(\text{Person} \sqcap \text{Female})) \\
&\qquad \sqcap \exists \text{hasChild.Person}) \\
&\qquad \sqcup ((\text{Person} \sqcap \text{Female}) \\
&\qquad\quad \sqcap \exists \text{hasChild.Person})) \\
\text{MotherWithManyChildren} &\equiv ((\text{Person} \sqcap \text{Female}) \sqcap \exists \text{hasChild.Person}) \sqcap \geqslant 3\, \text{hasChild} \\
\text{MotherWithoutDaughter} &\equiv ((\text{Person} \sqcap \text{Female}) \sqcap \exists \text{hasChild.Person}) \\
&\quad \sqcap \forall \text{hasChild.}(\neg(\text{Person} \sqcap \text{Female})) \\
\text{Wife} &\equiv (\text{Person} \sqcap \text{Female}) \\
&\quad \sqcap \exists \text{hasHusband.}(\text{Person} \sqcap \neg(\text{Person} \sqcap \text{Female}))
\end{aligned}
$$

Figura 40: Example of Acyclic description logics

$$
\begin{aligned}
\text{Woman} &\equiv \text{Person} \sqcap \text{Female} \\
\text{Man} &\equiv \text{Person} \sqcap \neg\, \text{Woman} \\
\text{Mother} &\equiv \text{Woman} \sqcap \exists \text{hasChild.Person} \\
\text{Father} &\equiv \text{Man} \sqcap \exists \text{hasChild.Person} \\
\text{Parent} &\equiv \text{Father} \sqcup \text{Mother} \\
\text{Grandmother} &\equiv \text{Mother} \sqcap \exists \text{hasChild.}\ \text{Parent} \\
\text{MotherWithManyChildren} &\equiv \text{Mother} \sqcap \geqslant 3\ \text{hasChild} \\
\text{MotherWithoutDaughter} &\equiv \text{Mother} \sqcap \forall \text{hasChild.} \neg\ \text{Woman} \\
\text{Wife} &\equiv \text{Woman} \sqcap \exists \text{hasHusband.}\ \text{Man}
\end{aligned}
$$

Figura 41: Example of an expansion of terminology

Translation rules for assertions:

$$
\begin{aligned}
t(C \sqsubseteq D) &\mapsto \forall x.\, t(C, x) \Rightarrow t(D, x) \\
t(a : C) &\mapsto C(a) \\
t((a, b) : R) &\mapsto R(a, b)
\end{aligned}
$$

Figura 42: Translation rules for assertions

It is always possible to translate DL assertions into FOL, so we define a translation function $t(C, x)$ which returns a FOL formula with x free

$$t(C, x) \rightarrow C(x)$$

In figure 42 we have translation rules for assertions and in figure 43 we have translation rules for terms.

We have that the knowledge base in description logics is defined by $K = (T, A)$, where T is the T-BOX terminological component and A is the A-BOX the assertional component.

An interpretation I satisfies A and T (therefore K) iff it satisfies any assertion in A and definition in T (I is a model of K).

Classical decision problems in DL are the following:

SATISFIABILITY OF A KB: $KBS(K)$ answer if there is a model for $K = (T, A)$.

LOGICAL CONSEQUENCE OF A KB: $K \models a : C$ also called instance checking.

CONCEPT SATISFIABILITY $cs(c)$**:** estabilish if is there an interpretation different from the empty set.

SUBSUMPTION: $K \models C \subseteq D$ if for every model I di T, $C^I \subseteq D^I$.

CONCEPT EQUIVALENCE: $K \models C \equiv D$.

Translation rules for terms:

$$
\begin{aligned}
t(\top, x) &\mapsto \mathit{true} \\
t(\bot, x) &\mapsto \mathit{false} \\
t(\mathrm{A}, x) &\mapsto \mathrm{A}(x) \\
t(\mathrm{C} \sqcap \mathrm{D}, x) &\mapsto t(\mathrm{C}, x) \wedge t(\mathrm{D}, x) \\
t(\mathrm{C} \sqcup \mathrm{D}, x) &\mapsto t(\mathrm{C}, x) \vee t(\mathrm{D}, x) \\
t(\neg \mathrm{C}, x) &\mapsto \neg t(\mathrm{C}, x) \\
t(\exists \mathrm{R}.\mathrm{C}, x) &\mapsto \exists y.\, R(x, y) \wedge t(\mathrm{C}, y) \\
t(\forall \mathrm{R}.\mathrm{C}, x) &\mapsto \forall y.\, R(x, y) \Rightarrow t(\mathrm{C}, y)
\end{aligned}
$$

Figura 43: Translation rules for terms

DISJOINTNESS: answer if $C^I \cap D^I = \emptyset$ for any model I of T.

RETRIEVAL: find all individuals which are instances of C.

MOST SPECIFIC CONCEPT (MSC): given a set of individuals, find the most specific concept of which they are instances and this is used for classification.

LEAST COMMON SUBSUMER (LCS): given a set of concepts, find the most specific concept which subsumes all of them and is also used for classification.

Decision problems are not independent, infact all problems can be reduced to KB satisfiability, so we have for esample that concept consistency is satisfied iff $K \cup \{a : C\}$ is satisfiable, instance checking $K \models a : C$ iff $K \cup \{a : \neg C\}$ is unsatisfiable; this is for some aspect reconduble to classic logic deduction.

The most used method is a technique for verifying satisfiability of a KB (KBS), and it is a variant of a method for natural deduction, called *semantic tableaux* (It applies constraint propagation).
Basic idea is that each formula in KB is a constraint on possible interpretations and complex constraints are decomposed in simpler constraints by means of propagation rules until we obtain, in a finite number of steps, atomicvconstraints, which cannot further decomposed.
If the set of atomic constraints contains an evident contradiction then the KB is not satisfiable, otherwise a model has been found and this technique is simple, modular, useful for evaluating complexity of decision algorithm.

Preliminary steps before KBS are the following:

TERMINOLOGY EXPANSION: a preliminary step consisting in resolving specializations, getting rid of the terminology by substituting defined concepts in A with their definitions.

NORMALIZATION: assertions are transformed in negation normal form, by applying the following rules until every occurrence of negation is in front of a primitive concept.

Rule		Description
($\sqcap$)	if 1.	$C_1 \sqcap C_2 \in \mathcal{L}(x)$ and
	2.	$\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$ *not both in* $\mathcal{L}(x)$
	then	$\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C_1, C_2\}$
($\sqcup$)	if 1.	$C_1 \sqcup C_2 \in \mathcal{L}(x)$ and
	2.	$\{C_1, C_2\} \cap \mathcal{L}(x) = \emptyset$ *neither one is in* $\mathcal{L}(x)$
	then	$\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C\}$ for some $C \in \{C_1, C_2\}$
($\exists$)	if 1.	$\exists R.C \in \mathcal{L}(x)$ and
	2.	x has no R-successor y with $C \in \mathcal{L}(y)$
	then	create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{R\}$ and $\mathcal{L}(y) = \{C\}$
($\forall$)	if 1.	$\forall R.C \in \mathcal{L}(x)$ and
	2.	x has an R-successor y with $C \notin \mathcal{L}(y)$
	then	$\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{C\}$

Figura 44: Rules for constraint propagation

These transformed assertions constitute the initial set of constraints for the *KBS* algorithm.

A constraint is an assertion of the form $a : C$ or $(b, c) : R$ where a, b and c are constants (distinct individuals) or variables $(x, y, \dots)$ referring to individuals but not necessarily distinct ones.
A constraint set A is satisfiable iff there exists an interpretation satisfying all the constraints in A and each step of the algorithm decomposes a constraint into a simpler one until we get a set of elementary constraints, or a contradiction (clash) is found.
For ALC a clash is one of the following types $\{a : C, a : \neg C\}$ or $\{a : \bot\}$.

Completion forest are data structures for supporting the execution of the algorithm, where for each individual a appearing in assertions in A, a labelled tree is initialized.

The label is a set of the constraints that apply to a, so if A contains $a : C$, we add the constraint C to the label of a, or if A contains $(a, b) : R$, we create a successor node of a for b to represent the R relation between them.

In figure 44 we have the rules for constraint propagation and also most rules are deterministic, but the rule for disjunction is non deterministic: its application results in alternative constraints sets, so we have a fork in the proof.

A is satisfiable iff at least one of the resulting constraint set is satisfiable, and A is unsatisfiable iff all the alternatives end up with a clash.

The result is invariant with respect to the order of application of the rules, and we have the following result for correctness and completeness:

CORRECTNESS: if the algorithm terminates with at least one primitive constraint set and no clashes, then A is satisfiable and from the constraints we can derive a model.

COMPLETENESS: if a knoweldge base A is satisfiable, then the algorithm terminates producing at least a finite model without clashes.

We have *KBS* decidable for *ALC* and also for *ALCN*.

In figure 45 is possible to note possible additional constructors to description logics, ans we have that *OWL-DL* is equivalent to *SHOIN*, instead *OWL-Lite* is equivalent to *SHIF*; these two language are the ontology language for Semantic Web.

The syntax for OWL is defined by figure 46, axioms by figure 47, the XML syntax used in visible in figure 48 and in the end in figure 49 is possible to see complexity and decitability results for DL's.

$\mathcal{H}$: inclusion between roles

$R \sqsubseteq S$ *iff* $R^I \subseteq S^I$

$\mathcal{Q}$: qualified numerical restrictions

$(\geq n\, R.C)^I = \{a \in \Delta^I \mid |\{b \mid (a, b) \in R^I \wedge b \in C^I\}| \geq n\}$

$(\leq n\, R.C)^I = \{a \in \Delta^I \mid |\{b \mid (a, b) \in R^I \wedge b \in C^I\}| \leq n\}$

O : nominals (singletons) $\{a\}^I = \{a^I\}$

$\mathcal{I}$: inverse roles, $(R^-)^I = \{(a, b) \mid (b, a) \in R^I\}$

$\mathcal{F}$: functional roles

$fun(F)$ *iff* $\forall x, y, z\, (x, y) \in F^I \wedge (x, z) \in F^I \Rightarrow y = z$

$\mathcal{R}^+$: transitive role

$(R^+)^I = \{(a, b) \mid \exists c \text{ such that } (a, c) \in R^I \wedge (c, b) \in R^I\}$

S: $\mathcal{ALC} + \mathcal{R}^+$

Figura 45: Additional constructors to description logics

Constructor	DL Syntax	Example
A (URI)	A	Conference
thing	$\top$	
nothing	$\bot$	
intersectionOf	$C_1 \sqcap \ldots \sqcap C_n$	Reference ∏ Journal
unionOf	$C_1 \sqcup \ldots \sqcup C_n$	Organization ⊔ Institution
complementOf	$\neg C$	¬MasterThesis
oneOf	$\{x_1\} \sqcup \ldots \sqcup \{x_n\}$	{WISE, ISWC, …}
allValuesFrom	$\forall P.C$	∀date.Date
someValuesFrom	$\exists P.C$	∃date.{2005}
maxCardinality	$\leqslant nP$	(≤ 1 location)
minCardinality	$\geqslant nP$	(≥ 1 publisher)

Figura 46: Syntax for OWL

Axiom	DL Syntax	Example
subClassOf	$C_1 \sqsubseteq C_2$	Human $\sqsubseteq$ Animal $\sqcap$ Biped
equivalentClass	$C_1 \equiv C_2$	Man $\equiv$ Human $\sqcap$ Male
disjointWith	$C_1 \sqsubseteq \neg C_2$	Male $\sqsubseteq \neg$Female
sameIndividualAs	$\{x_1\} \equiv \{x_2\}$	{President_Bush} $\equiv$ {G_W_Bush}
differentFrom	$\{x_1\} \sqsubseteq \neg\{x_2\}$	{john} $\sqsubseteq \neg$ {peter}
subPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter $\sqsubseteq$ hasChild
equivalentProperty	$P_1 \equiv P_2$	cost $\equiv$ price
inverseOf	$P_1 \equiv P_2^-$	hasChild $\equiv$ hasParent$^-$
transitiveProperty	$P^+ \sqsubseteq P$	ancestor$^+ \sqsubseteq$ ancestor
functionalProperty	$\top \sqsubseteq \leqslant 1P$	$\top \sqsubseteq \leqslant$ 1hasMother
inverseFunctionalProperty	$\top \sqsubseteq \leqslant 1P^-$	$\top \sqsubseteq \leqslant$ 1hasSSN$^-$

Figura 47: Axioms defined for OWL

E.g., Person $\sqcap \forall$hasChild.(Doctor $\sqcup \exists$hasChild.Doctor)

```
<owl:Class>
  <owl:intersectionOf rdf:parseType=" collection">
    <owl:Class rdf:about="#Person"/>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#hasChild"/>
      <owl:toClass>
        <owl:unionOf rdf:parseType=" collection">
          <owl:Class rdf:about="#Doctor"/>
          <owl:Restriction>
            <owl:onProperty rdf:resource="#hasChild"/>
            <owl:hasClass rdf:resource="#Doctor"/>
          </owl:Restriction>
        </owl:unionOf>
      </owl:toClass>
    </owl:Restriction>
  </owl:intersectionOf>
</owl:Class>
```

Figura 48: XML syntax used for OWL

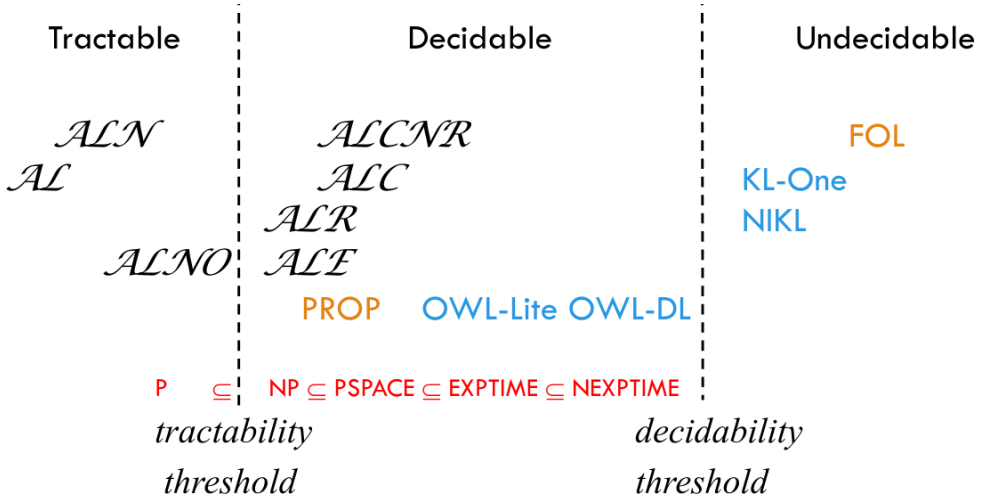


Figura 49: Complexity and decidability results for DL

5 BAYESIAN NETWORK

In this chapter we will talk about Bayesian Network, but before we introduce and analyze it we will refresh our knowledge of probability.

5.1 UNCERTAIN REASONING

Agents are inevitably forced to reason and make decisions based on incomplete information and they need a way to handle uncertainty deriving from:

1. partial observability (uncertainty in sensors).
2. nondeterministic actions (uncertainty in actions).

A partial answer would be to consider, instead of a single world, a set of possible worlds (those that the agent considers possible, so a belief set) but planning by anticipating all the possible contingencies can be really complex.
Moreover, if no plan guarantees the achievement of the goal, but still the agents needs to act, we have that probability theory offers a clean way to quantify uncertainty (common sense reduced to calculus).

Suppose the goal for a taxi-driver agent is "delivering a passenger to the airport on time for the flight" and consider action A_t = leave for airport t minutes before flight.
How can we be sure that A_{90} will succeed, because there are many sources of uncertainty:

1. partial observability or noisy sensors: road state, other drivers plans, police control, inaccurate traffic reports and so on.
2. uncertainty in action outcomes (flat tire, car problems, bad weather and so on).

With a logic approach it is difficult to anticipate everything that can go wrong (qualification problem), A_{90} may be the most rational action, given that the airport is 5 miles away and you want to avoid long waits at the airport and still catch the flight.

The rational decision depends on both the relative importance of various goals and the likelihood that, and degree to which, they will be achieved.
When there are conflicting goals the agent may express preferences among them by means of a utility function and utilities are combined with probabilities in the general theory of rational decisions called *decision theory*.
An agent is rational if and only if it chooses the action that yields the maximum expected utility, averaged over all the possible outcomes of the action.
This is called the principle of *Maximum Expected Utility* (MEU).

Logic theory and probability theory both talk about a world make of propositions which are true or false and they share the ontological commitment. What is different is the *epistemological* commitment: a logical agent believes each sentence to be true or false or has no opinion, whereas a probabilistic agent may have a numerical degree of belief between 0 (for sentences that are certainly false) and 1 (certainly true).
One example can be to define the patient who has a toothache has a cavity with 0.8 probability.
The uncertainty is not in the world, but in the beliefs of the agent (state of knowledge), so if the knowledge about the world changes (we learn more information about the patient) the probability changes, but there is no contradiction.

We assume now a knowledge of Probability, like conditional probabilities, Bayes formula, definition of probability and so on.
We now refresh now how to compute the probability of a variable from the full joint distribution, so given a joint distribution $P(X, Y)$ over variables X and Y, the distribution of the single variable X is given by

$$P(X) = \sum_{y \in dom(Y)} P(X, y) = \sum_{y} P(X, y)$$

This operation is also called *marginalization*. A variant of the marginalization rule, called *conditioning*, involves conditional probabilities instead of joint probabilities. It can be obtained from marginalization using the product rule and it is

$$P(Y) = \sum_{z \in Z} P(Y|z)P(z)$$

If the query involves a single variable X, e is the list of the observed values and Y the rest of unobserved variables, we have that

$$P(X|e) = \alpha P(X, e) = \alpha \sum_{y} P(X, e, y)$$

However the complexity of the joint distribution table is intractable, so if n is the number of boolean variables, it requires an input table of size $O(2^n)$ and requires $O(2^n)$ time to process.
Better reasoning involves using Bayes theorem or leveraging on the notion of independence, that can reduce the size of representation.
Bayes rule tells us how to update the agent's belief in hypothesis h as new evidence e arrives, given the background knowledge k, as follows

$$P(h|e, k) = \frac{P(e|h, k)P(h|k)}{P(e|k)}$$

We define a refinement of the independence property, called *conditional independece*, which is defined that X and Y are conditionally independent given Z, when

$$P(X, Y|Z) = P(X|Z)P(Y|Z)$$

Using product rule and conditional independence is possible to obtain this alternative formulation which estabilish that X and Y are conditionally independent given Z when

$$P(X|Y, Z) = P(X|Z)$$
$$P(Y|X, Z) = P(Y|Z)$$

We also cite *Naive Bayes model*, used in Naive Bayes classifiers, where the full joint distribution can be computed as

$$P(Cause, Effect_1, Effect_2, \ldots, Effect_n) = P(Cause) \prod_{i} P(Effect_i|Cause)$$

This distribution can be also be obtained from observations, anyway better explanation will be provided in other courses, like Intelligent Systems for Pattern Recognition.

5.2 BAYESIAN NETWORK

Bayesian networks (also called belief networks) are graphs for representing dependencies among variables and the network makes explicit conditional dependencies: the intuitive meaning of an arc from X to Y is typically that X has a direct influence on Y.
Bayesian networks are directed acyclic graphs (DAG) so defined:

1. Each node corresponds to a random variable, which may be discrete or continuous.

2. Directed links or arcs connect pairs of nodes and if there is an arc from node X to node Y , X is said to be a parent of Y.
   Parents (Y) is the set of variables directly influencing Y.

3. Each node X has an associated conditional probability distribution table $P(X|Parents(X))$ that quantifies the effect of the parents on the node.

Easier for domain experts to decide what direct influences exist in the domain than actually specifying the probabilities themselves and the network naturally represents independence and conditional independence, infact the lack of an arc connection between two variables is interpreted as *independence*.

We define now two alternative and equivalent ways to look at the semantics of Bayesian networks:

1. Distributed representation of the full joint probability distribution, which suggests a methodology for constructing networks.

2. An encoding of a collection of conditional independence statements, which helps in designing inference procedures.

A Bayesian network can be seen as a representation of the full joint distribution, since a joint probability distribution can be expressed in terms of conditional probabilities

$$P(x_1, \ldots, x_n) = P(x_n|x_{n-1}, \ldots, x_1)P(x_{n-1}, \ldots, x_1)$$

By iterating the process we get the so-called chain rule

$$\begin{aligned} P(x_1, \ldots, x_n) &= P(x_n|x_{n-1}, \ldots, x_1)P(x_{n-1}|x_{n-2} \ldots x_1) \ldots P(x_2|x_1)P(x_1) \\ &= \prod_{i=1}^{n} P(x_i|x_{i-1}, dots, x_1) \end{aligned}$$

This amounts to assuming an ordering of the variables, computing the posterior probabilities of each variable, given all previous variables and in the end taking the product of these posteriors.

Assuming that each variable appears after its parents in the ordering, we simplify the computation by conditioning the computation only to values of the parent variables (assuming the others are independent) and the numbers in the CPT's are actually conditional probabilities.
Each entry in the joint distribution can be computed as the product of the appropriate entries in the conditional probability tables (CPTs) in the Bayesian network.

We get the simplified rule for computing joint distributions

$$P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i|parents(X_i))$$

where $parents(X_i)$ denotes the set of values $x_1, \ldots, x_n$ for $parents(X_i)$.
A Bayesian network is a distributed representation of the full joint distribution.

We present now a procedure for building a Bayesian network which is a good representation of a domain:

NODES: determine the set of variables required to model the domain and we order them in $\{X_1, \ldots, X_n\}$.
Ordering influences the result: the resulting network will be more compact if the variables are ordered such that causes precede effects.

LINKS: For $i = 1$ to n choose from $X_1, \ldots, X_{i-1}$ a minimal set of parents for X_i, such that equation

$$P(X_i|X_{i-1}, \ldots, X_1) = P(X_i|Parents(X_i)$$

is satisfied.
For each parent insert an arc from the parent to X_i and write down the conditional propability table $P(X_i|Parents(X_i))$.

Note that the parents of node X_i should contain all those nodes in $X_1, \ldots, X_{i-1}$ that directly influence X_i and also note that the network is a DAG by construction and contains no redundant probability values.

If we use a causal model (with links from causes to effect) we obtain a better network, with less connections (more compact), fewer probabilities to specify and the numbers will often be easier to obtain.
In locally structured systems, each subcomponent interacts directly with only a bounded number of other components and this is a huge savings wrt full joint distribution tables.

Locally structured domains are usually associated with linear rather than exponential growth in complexity, and in the case of Bayesian networks, it is reasonable to suppose that in most domains each random variable is directly influenced by at most k others, for some constant k.
If we assume n Boolean variables, then each conditional probability table will have at most 2^k numbers, and the complete network can be specified by $n2^k$ numbers, in contrast, the joint distribution contains 2^n numbers.

We can also extract the independence assumptions encoded in the graph structure to do inference and the topological semantics specifies that each variable is conditionally independent of its non-descendants, given its parents.
A node X is conditionally independent of all other nodes in the network given its parents, children, and children's parents and this is its Markov blanket.

Often the relationship between a node and its parent follows a canonical distribution and the construction of the CPT's can be simplified, so we present two examples:

DETERMINISTIC NODES: nodes whose value is specified exactly by the values of their parents.

NOISY-OR RELATIONS: a generalization of the logical OR, so for example we can define the following proposition

$$Cold \vee Flu \vee Malaria \iff Fever$$

We can specify inhibiting factors for Cold, Flu, Malaria and we have two assumpions with Noisy-OR:

- all the possible causes are listed; if not, we can add a leak condition/node.
- the inhibiting factor of each parent is independent of any other parent.

One possible way to handle continuous variables (such as temperature) is to avoid them by using discrete intervals, but the most common solution is to define standard families of probability density functions that are specified by a finite number of parameters, like for example a normal distribution $N(\mu, \sigma^2)$.

A network with both discrete and continuous variables is called a *hybrid Bayesian network*.
Hybrid Bayesian networks combine discrete and continuous variables and there are two new kinds of distributions:

1. the conditional distribution for a continuous variable given discrete and continuous parents.

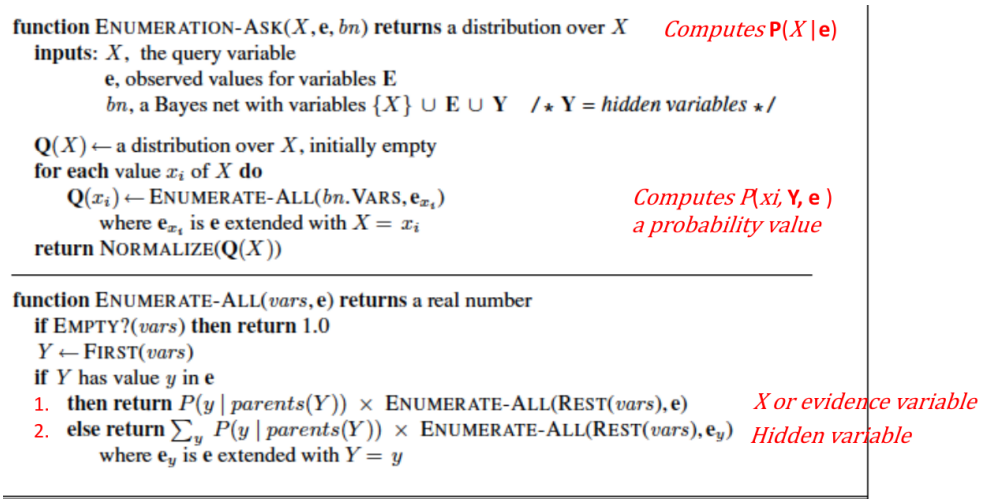


Figura 50: Pseudocode for Enumeration Ask

2. the conditional distribution for a discrete variable given continuous parents.

Given X, the query variable (we assume one), E the set of evidence variables $\{E_1, \ldots, E_m\}$, and Y the set of unknown variables, a typical query asks for $P(X|E)$.

We defined a procedure for the task by enumeration as

$$P(X|e) = \alpha P(X, e) = \alpha \sum_t P(X, e, y)$$

where α is a normalization factor.
The query can be answered using a Bayesian network by computing sums of products of conditional probabilities from the network.

In figure 50 there is the pseudocode for the inference procedure using enumeration, that can be improved substantially, by storing and reusing the results of repeated computations (a kind of dynamic programming).
The variable elimination algorithm, proceeds right-to-left (bottom-up) and conditional probabilities are represented as factors, matrices resulting from conditional probabilities and the names make explicit the argument.
For example if we have $P(a|b, e)$ we define the factor $f_i(a, b, e)$ and two operations on factors: pointwise-product (x) and summing out variables.

Given a variable and some evidence, a factor can be built by looking at the variable part of CPTS's in the network, using MAKE-FACTOR(VAR, E) and the *pointwise product* of two factors f_1 and f_2 yields a new factor f_3 such that variables are the union of the variables in f_1 and f_2 and elements are given by the product of the corresponding values in the two factors, so for example computation of the poitnwise product $f_1(A, B) \times f_2(B, C)$ gives $f_3(A, B, C)$ and the result is not necessarily a probability distribution.

Computational saving comes from the realization that any factor that does not depend on the variable to be summed out can be moved outside the summation and this operation of "distributing out" common factors is part of the operation, which in the general case returns a set of factors.
In figure 51 is possible note this improved version of variable elimination, where MAKE-FACTOR creates a factor for each variable, given the evidence and SUM-OUT sums out over the possible values of a hidden variable, producing a set of factors; it takes care of distributing out factors which do not depend on the variable.

Every choice of ordering yields a valid algorithm, but different orderings of variables can produce differences in efficiency, but determining the optimal ordering is intractable, but several good heuristics are available, for example eliminate whichever variable minimizes the size of the next factor to be constructed.

function ELIMINATION-ASK(X, **e**, bn) **returns** a distribution over X — *Computes* $\mathbf{P}(X \mid \mathbf{e})$
  inputs: X, the query variable
    e, observed values for variables **E**
    bn, a Bayesian network specifying joint distribution $\mathbf{P}(X_1, \ldots, X_n)$

  $factors \leftarrow []$
  for each var **in** ORDER(bn.VARS) **do** — *Elimination ordering is important*
    $factors \leftarrow [\text{MAKE-FACTOR}(var, \mathbf{e}) | factors]$ — *Add new factor to factors*
    if var is a hidden variable **then** $factors \leftarrow \text{SUM-OUT}(var, factors)$
  return NORMALIZE(POINTWISE-PRODUCT($factors$))

Figura 51: Pseudocode for Variable elimination algorithm

We can remove any leaf node that is not a query variable or an evidence variable, and continuing this process, we can remove any variable that is not an ancestor of a query variable or evidence variable.

The complexity of exact inference in Bayesian networks depends strongly on the structure of the network, singly connected networks or polytrees are such that there is at most one undirected path between any two nodes.
The time and space complexity of exact inference in polytrees is linear in the size of the network (the number of CPT entries).
For multiply connected networks variable elimination can have exponential time and space complexity in the worst case, infact inference in Bayesian networks includes as a special case propositional inference, which is NP-complete.

5.3 PROBABILISTIC REASONING OVER TIME

So far, we have been doing uncertain reasoning in a static world and for reasoning in an evolving world an agent needs:

BELIEF STATE: the states of the world that are possible.

TRANSITION MODEL: to predict how the world will evolve.

SENSOR MODEL: to update the belief state from perceptions.

A changing world is modeled using a variable for each aspect of the world state at each point in time (fluents) and we use probabilities to quantify how likely a world is.
The transition and sensor model themselves may be uncertain:

- the *transition model* gives the probability distribution of the variables at time t, given the state of the world at past times.
- the *sensor model* describes the probability of each percept at time t, given the current state of the world.

We view the world as a series of snapshots, or time slices, each of which contains a set of random variables, some observable and some not.
X_t will denote the set of state variables, unobservable (hidden) at time t and $E_t = e_t$ will denote the set of observations (evidence variables) at time t, with e_t their values.
We will assume that the state sequence starts at $t = 0$ and the distance between time slices is fixed and we will use the notation $a : b$ to denote the sequence of integers from a to b (inclusive), and the notation $X_{a:b}$ to denote the set of variables from X_a to X_b.

The transition model specifies how the world evolves, the probability distribution over the latest state variables, given the previous values starting from time 0

$$P(X_t | X_{0:t-1})$$

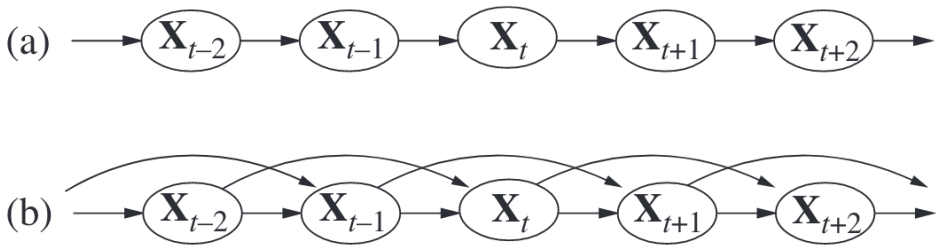


Figura 52: Bayesian Network of Markov chain

The sequence of states can become very large, unbounded as t increases and we define the *Markov assumptions*: the transition model specifies the probability distribution over the latest state variables, given a finite fixed number of previous states

$$P(X_t|X_{0:t-1}) = P(X_t|X_{t-1}) \quad \text{first order Markov chain}$$

$$P(X_t|X_{0:t-1}) = P(X_t|X_{t-1}, X_{t-2}) \quad \text{second order Markov chain}$$

Additionally, we assume a stationary process, so the conditional probability distribution is the same for all t, since sometimes change is governed by laws that do not themselves change over time.

In figure 52 is possible note the transition model using Markov chain in Bayes networks and the sensor/observation model, under Markov sensor assumption, postulates that evidence only depends on the current state

$$P(E_t|X_{0:t}, E_{0:t-1}) = P(E_t|X_t)$$

This is a reasonable assumption to make, given the availability of sensors.
Assuming for example that "Rain" only depends on rain the previous day may be a too strong assumption, so there are two ways to improve the accuracy of the approximation:

1. Increasing the order of the Markov process model, for example using a second order assumption.

2. Increasing the set of state variables: we could add Season, Temperature, Humidity, Pressure and so on as state variables.
   This may imply more computation for predicting state variables or adding new sensors.

We also need the prior probability distribution at time $0, P(X_0)$, so putting all together, the complete joint distribution over all the variables, for any t, computed from the network is

$$P(X_{0:t}, E_{1:t}) = P(X_0)\prod_{i=1}^{t} P(X_i|X_{i-1})P(E_i|X_i)$$

Basic inference tasks based on the temporal model, consist in the following operations:

FILTERING: computing the belief state (posterior probability distribution of state variables) given evidence from previous and current states.
A subtask is the likelihood of the evidence sequence $P(X_t|e_{1:t})$.

PREDICTION: computing the posterior distribution over a future state, given all evidence to date, so we compute $P(X_{t+k}|e_{1:t})$ with $k > 1$.

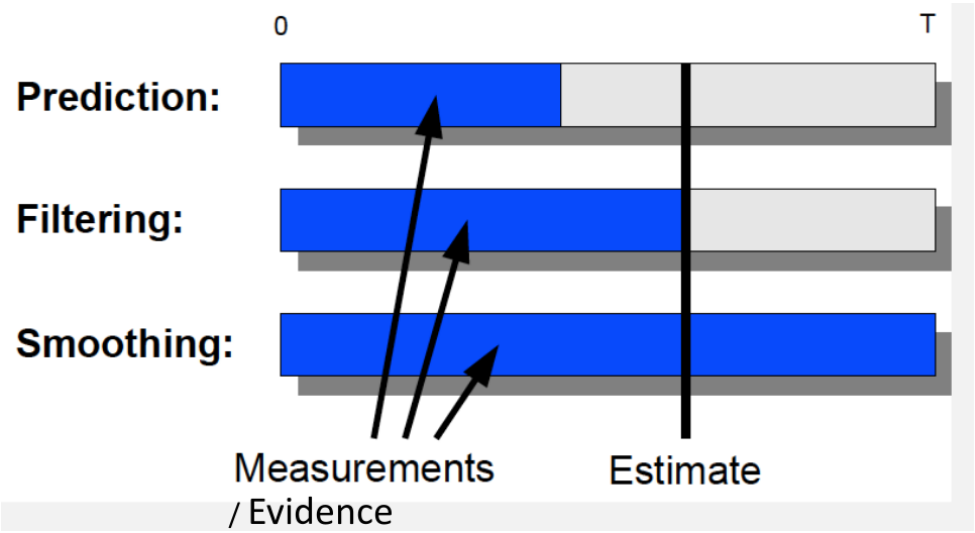


Figura 53: Interaction between time and inference operations

SMOOTHING: computing the posterior distribution over a past state, given all evidence up to the present.
Looking back with the knowledge of today provides a more accurate estimate, so we have $P(X_k|e_{1:t})$ with $0 \leq k < t$.

MOST LIKELY EXPLANATION/SEQUENCE: given a sequence of observations, we might wish to find the sequence of states that is most likely to have generated those observations.

LEARNING: the transition and sensor models, can be learned from observations.

In figure 53 is possible to note how some inference operations interface with past, current and future state and a good filtering algorithm maintains a current state estimate and updates it, rather than going back over the entire history of percepts for each time.
The filtering function f takes into account the state estimation computed up to the present and the new evidence.

$$P(X_{t+1}|e_{1:t+1}) = f(e_{t+1}, P(X_t|e_{1:t})$$

This process is called *recursive estimation* and is made of two parts:

PREDICTION: the current state distribution is projected forward from t to $t+1$:

$$P(X_{t+1}|e_{1:t})$$

UPDATE: it is updated using the new evidence $e_{t+1}P(e_{t+1}|X_{t+1})$.

Filtering can be computed as follows:

$$\begin{aligned}
P(X_{t+1}|e_{1:t+1}) &= P(X_{t+1}|e_{1:t}, e_{t+1}) \\
&= \alpha P(e_{t+1}|X_{t+1}, e_{1:t}) P(X_{t+1}|e_{1:t}) \\
&= \alpha P(e_{t+1}|X_{t+1}) P(X_{t+1}|e_{1:t}) \\
&= \alpha P(e_{t+1}|X_{t+1}) \sum_{x_t} P(X_{t+1}|x_t, e_{1:t}) P(x_t|e_{1:t}) \\
&= \alpha P(e_{t+1}|X_{t+1}) \sum_{x_t} P(X_{t+1}|x_t) P(x_t|e_{1:t})
\end{aligned}$$

We can think of $P(X_t|e_{1:t})$ as a message $f_{1:t}$ that is propagated forward in the sequence and this makes evident the recursive structure:

$$f_0 = P(X_0)$$

$$f_{1:t+1} = \alpha Forward(f_{1:t}, e_{t+1})$$

where *Forward* implements the filtering update in constant time.

The task of prediction can be seen simply as filtering without the contribution of new evidence and also the filtering process already incorporates a one-step prediction.

In general, looking ahead k steps, at time $t+k+1$, given evidence up to t is done by

$$P(X_{t+k+1}|e_{1:t}) = \sum_{x_{t+k}} P(X_{t+k+1}|x_{t+k})P(x_{t+k}|e_{1:t})$$

This computation involves only the transition model and not the sensor model and we can show that the predicted distribution for Rain converges to a fixed point $(0.5, 0.5)$, after which it remains constant for all time (the stationary distribution of the Markov process).

The *mixing time* is the time to reach the fixed point and we have that the more uncertainty there is in the transition model, the shorter will be the mixing time and the more the future is obscure.

We can use a forward recursion also to compute the likelihood of an evidence sequence P e 1:t , useful if we want to compare two models producing the same evidence sequence.

We can derive a recursive equation similar to filtering and a similar likelihood message

$$l_{1:t}(X_t) = P(X_t, e_{1:t})$$

Once we have $l_{1:t}(X_t)$ we can compute the likelihood of the evidence sequence by summing out on the values of X_t as follows

$$L_{1:t} = P(e_{1:t}) = \sum_{x_t} l_{1:t}(x_t)$$

Note that the likelihood becomes smaller and smaller as t increases.

Smoothing is the process of computing the posterior distribution of the state at some past time k given a complete sequence of observations up to the present t

$$P(X_k|e_{1:t}) \quad \text{for } 0 \leq k < t$$

The additional evidence is expected to provide more information and more accurate predictions on the past.

To compute the smoothing we have the following phases

$$\begin{aligned} P(X_k|e_{1:t}) &= P(X_k|e_{1:k}, e_{k+1:t}) \\ &= \alpha P(X_k|e_{1:k})P(e_{k+1:t}|X_k, e_{1:k}) \\ &= \alpha P(X_k|e_{1:k})P(e_{k+1:t}|X_k) \\ &= \alpha f_{1:k} \times b_{k+1:t} \end{aligned}$$

$P(X_k|e_{1:k})$ corresponds to the forward message $f_{1:k}$, computed up to t, as before in filtering and we define another message $b_{k+1:t} = P(e_{k+1:t}|X_k)$ that can be computed by a recursive process that runs backward from t.

The terms in $f_{1:k}$ and $b_{k+1:t}$ can be implemented by two recursive calls, one running forward from 1 to k and using the filtering equation and the other running backward from t to $k+1$ for computing $P(e_{k+1:t}|X_k)$.

function FORWARD-BACKWARD(**ev**, *prior*) **returns** a vector of probability distributions
  inputs: **ev**, a vector of evidence values for steps $1, \ldots, t$ — *Smoothing all the sequence*
    prior, the prior distribution on the initial state, $\mathbf{P}(\mathbf{X}_0)$
  local variables: **fv**, a vector of forward messages for steps $0, \ldots, t$
    b, a representation of the backward message, initially all 1s
    sv, a vector of smoothed estimates for steps $1, \ldots, t$

  fv$[0] \leftarrow$ *prior*
  for $i = 1$ **to** t **do**
    fv$[i] \leftarrow$ FORWARD(**fv**$[i-1]$, **ev**$[i]$) — *Forward filtering phase*
  for $i = t$ **downto** 1 **do**
    sv$[i] \leftarrow$ NORMALIZE(**fv**$[i] \times$ **b**) — *Backward phase reusing result from the filtering*
    b $\leftarrow$ BACKWARD(**b**, **ev**$[i]$)
  return sv

Figura 54: Pseudocode for ForwardBackward algorithm

To compute smoothing going backwards we have

$$\begin{aligned} P(e_{k+1:t}|X_k) &= \sum_{x_{k+1}} P(e_{k+1:t}|X_k, x_k+1)P(x_{k+1}|X_k) \\ &= \sum_{x_{k+1}} P(e_{k+1:t}|x_{k+1})P(x_{k+1}|X_k) \\ &= \sum_{x_{k+1}} P(e_{k+1}, e_{k+2:t}|x_{k+1})P(x_{k+1}|X_k) \\ &= \sum_{x_{k+1}} P(e_{k+1}|x_{k+1})P(e_{k+2:t}|x_{k+1})P(x_{k+1}|X_k) \end{aligned}$$

$b_{k+1:t} = P(e_{k+2:t}|x_{k+1})$ is defined by the previous equation computing backward and the backward phases start with $b_{t+1:t} = 1$ and then we compute

$$b_{k+1:t} = Backward(b_{k+2:t}, e_{k+1})$$

where *Backward* implements the update defined above.

To improve efficincy we use an algorithm, called *forward–backward*, visible in figure 54, which uses dynamic programming to reduce the complexity of running the algorithm over the whole sequence.
The trick is to record the results computed during the forward filtering phase, over the whole sequence, and reuse them during the backward phase, so we obtain that the complexity is $O(t)$ and is consistent with the fact that the Bayesian network structure is a polytree.
The forward–backward algorithm is the basis for many applications that deal with sequences of noisy observations and improvements are required to deal with space complexity for long sequences and online computations, where new observations continuously arrive (fixed-lag smoothing).

Suppose we observe [true, true, false, true, true] for the Umbrella variable, so what is the most likely sequence for Rain given these oservations?
There are 2^5 possible Rain sequences, and each sequence is a path through a graph whose nodes are the possible states at each time step.
We want to discover which is the one maximizing the likelihood, in linear time and the likelihood of a path is the product of the transition probabilities along the path and the probabilities of the given observations at each state.

There is a recursive relationship between the most likely path to each state x_{t+1} and most likely paths to each previous state x_t.
We can write a recursive equation, similar to the one for filtering as

$$\max_{x_1,\ldots,x_t} P(x_1, \ldots, x_t, X_{t+1}|e_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1}) \max_{x_t}(P(X_{t+1}|X_t) \max_{x_1,\ldots,x_{t-1}} P(x_1, \ldots, x_{t-1}, x_t|e_{1:t}))$$

The forward message in this case is $\max P(x_1, \ldots, x_{t-1}, X_t|e_{1:t})$, the probabilities of the best path to each state x_t; from those we can compute the probabilities of the extended paths at time $t+1$ and take the max.
The most likely sequence overall can be computed in one pass and for each state, the best state that leads to it is recorded (marked as black arrows in the example) so that the optimal sequence is identified by following black arrows backwards from the best final state.
This algorithm is the famous *Viterbi algorithm*, named after A. Viterbi [1967]

The original application of Viterbi was in telecommunications: Viterbi decoders are used for decoding a bitstream encoded using a technique called convolutional code or trellis code, but is also used in NLP as different kinds of "sequence tagger" or also Speech recognition (speech-to-text), speech synthesis, speech diarization.

We now present an overview of other approaches: Probability theory and Bayesian networks are the dominant approaches today, despite the disadvantage of having to specify many probability values (lots of numbers), so other approaches are used to help humans to understand reasoning behind:

1. Rule-based methods for uncertain reasoning in expert systems.
2. Representing ignorance: Dempster–Shafer theory.
3. Representing vagueness: fuzzy sets and fuzzy logic.

Three good properties of classical logic-based rules:

LOCALITY: In logical systems, from A and $A \rightarrow B$, we can conclude B, without worrying about any other rules, instead in probabilistic systems, we need to consider all the evidence.

DETACHMENT: once B is proved, it can be used regardless of how it was derived and it can be detached from its justification.

TRUTH-FUNCTIONALITY: the truth of complex sentences can be computed from the truth of the components, instead probability combination does not work this way.

In Rule-based methods the idea is to attach degree of belief to facts and rules and to combine and propagate them, and the most famous example is the certainty factors model, which was developed for the MYCIN medical diagnosis program.
The system was carefully engineered to avoid mixing different kind of rules (diagnostic vs causal), trying to control non plausible results.

The theory of *fuzzy sets* is a way to specify how well an object satisfies a vague description (non categorical) property, like "being tall".
Fuzziness is not uncertainty in the world, but is the uncertainty in the use of qualifiers/properties.
Fuzzy logic is way of reasoning about membership in fuzzy sets and a fuzzy predicate implicitly defines a fuzzy set.
Fuzzy logic is a method for reasoning with logical expressions describing membership in fuzzy sets and standard rules used are

$$T(A \wedge B) = \min(T(A), T(B))$$

$$T(A \vee B) = \max(T(A), T(B))$$

$$T(\neg A) = 1 - T(A)$$

Fuzzy control is a methodology for constructing control systems in which the mapping between real-valued input and output parameters is represented by fuzzy rules.
It consist on the following operations:

FUZZIFICATION: continous variables are mapped into a set of linguistic variables with fuzzy values; temperature can be mapped in fuzzy variables such as low, medium, high and their membership functions.

REASONING: with rules expressed in terms of these fuzzy variables, reaching fuzzy conclusions.

DEFUZZIFICATION: map the results into numerical valueo far, we have been doing uncertain reasoning in a static world.

6 RULE-BASED SYSTEMS

We introduce now rule-based systems, which are a contractions of PROP and FOL, since by limiting expressivity to a specific interesting subset of first-order logic, resolution procedures become much more manageable.
We limit the degree of uncertainty we can express by considering clauses that have at most one positive literal, *Horn clauses*.
We have two cases:

1. The KB is a set of definite Horn clauses (exactly one positive literal), written as $a_1 \wedge a_2 \wedge \cdots \wedge a_m \Rightarrow h$, where rules happens when $m > 0$ and facts are when $m = 0$.

2. Goals or queries include only negative literals (negative clauses).

We have limited expressivity, so we cannot represent disjunctive conclusions and also Propositional Horn clauses KB's have linear-time deduction algorithms.

Once we have a KB of fact and rules the inference procedure can work in two directions:

FORWARD REASONING: use of rules from antecedent to consequent to compute all the logical consequences of the initial facts (also called bottom-up) and it is used in deductive databases, like Datalog, and in Production systems.

BACKWARD REASONING: use of the rules from the consequent to the antecedent until goals match initial facts (also called top-down) and the most effective strategy is called SLD and is used in Logic programming languages and PROLOG.

We start considering the forward reasoning, so for the first-order case inference is performed on the basis of the following rule, which makes use of unification:

$$\frac{p'_1, p'_2, \ldots, p'_n \; (p_1 \wedge p_2 \wedge \cdots \wedge p_n \Rightarrow q)}{q\gamma}$$

where γ is an MGU such that $p'_i\gamma = p_i\gamma$ for each i. At each iteration new sentences are added to the KB and terminates when no new sentences are generated (fixed point) or goal is found.
This inferential process is sound, the rule is correct and complete for Definite Horn clauses, for Datalog databases (without function symbols, only constants) convergence is also guaranteed since there are a finite number of consequences.

In figure 55 is possible to note the pseudocode for Forward chaining and the interpreter is required to perform many unification operations, so if we have r rules to try (in OR), n preconditions in each rules to be satisfied (in AND) and w facts in the KB (in OR) and everything repeated for c cycles so we have $r \times n \times w \times c$ unification operations.

Finding all the facts that unify with a given pattern can be made more efficient with appropriate predicate indexing, so a hash table on the head of the predicate, in the simplest case, but in complex case we can use a hash table with a combined key head + first argument and/or a discrimination network on the antecedents of the rules.
Another improvents is that each rule does not need to be checked again at each iteration, so only the ones which are affected by recent additions. This two strategies

FOL-FC-Ask

```
function FOL-FC-ASK(KB, α) returns a substitution or false
  inputs: KB, the knowledge base, a set of first-order definite clauses
          α, the query, an atomic sentence
  local variables: new, the new sentences inferred on each iteration

  repeat until new is empty                    Fixed point is reached
    new ← { }
    for each rule in KB do
      (p1 ∧ ... ∧ pn ⇒ q) ← STANDARDIZE-VARIABLES(rule)
      for each θ such that SUBST(θ, p1 ∧ ... ∧ pn) = SUBST(θ, p'1 ∧ ... ∧ p'n)
                  for some p'1, ..., p'n in KB
        q' ← SUBST(θ, q)
        if q' does not unify with some sentence already in KB or new then
          add q' to new
          φ ← UNIFY(q', α)
          if φ is not fail then return φ       Goal found?
    add new to KB
  return false
```

Figura 55: Pseudocode for Forward chaining

are among the strategies implemented by the RETE algorithm included in Production Systems and rule-based programming languages such as CLIPS.

Another improvement is the *Conjunct ordering*, which the order in which you list antecedents of rules is important for efficiency.
Each antecedent is a constraint and each rule a CSP, so we can apply heuristics from CSP for ordering and we can try to write rules which correspond to CSP with a tree-structure.
The last improvement is that proceeding forward may lead to derive many irrelevant facts, so we restrict the forward chaining to the relevant rules for the goal by proceeding backward and marking them (in deductive databases are called *Magic sets*).

Rule-based systems are one of the first paradigms for representing knowledge in A.I. and most expert systems are rule-based.
Rules are used forward to produce new facts, hence the names productions and production systems and they are a general computational model based guided by patterns: the rule to be applied next is determined by the operation of pattern matching, a simplified form of unification.

CLIPS (C Language Integrated Production System") is a successor of OPS-5 (Forgy), developed by NASA and now freely available at .

A typical rule-based system has four basic components:

1. A list of facts (no variables), the temporary *Working Memory* (WM), defined as

```
(predicate arg_1 arg_2 \dots arg_k) %ordered facts
(predicate (slot_1 val_1) \dots (slot_k val_k)) %structured facts}
```

2. A list of rules or rule base, a specific type of knowledge base, represented as

```
(defrule rule\_name ["comment"]
  (pattern_1) (pattern_2) \dots (pattern_k) \Rightarrow (action_1) (a
  \dots (action_m)
```

3. An interpreter, called *inference engine*, which infers new facts or takes action based on the interaction of the WM and the rule base.

4. A conflict resolution strategy.

The interpreter executes a match-resolve-act cycle, which consist to

MATCH: the Left-Hand-Sides (LHS) of all rules are matched against the contents of the working memory and this produces a conflict set: instantiations of all the rules that are satisfied/applicable.

CONFLICT-RESOLUTION: one of the rules instantiations in the conflict set is chosen for execution and if no applicable rule, the interpreter halts.

ACT: the actions in the Right-Hand-Side (RHS) of the rule selected in the conflict-resolution phase are executed and these actions may change the contents of working memory and this cycle repeats.

Matching rules (activations) are put in an agenda from where one will be selected and newly activated rules are added to the agenda and the agenda is reordered according to the salience of the rules.
Among the rules of equal salience, the current conflict resolution strategy is used to determine the rule to be fired (analogy with neurons) and the depth strategy is the standard default strategy of CLIPS (new rules on top).
Different predefined conflict resolution strategies under user control:

BREADTH: newly activated rules are placed below all rules of the same salience.

SIMPLICITY: among rules of the same salience, less specific rules are preferred.

COMPLEXITY: among rules of the same salience, most specific rules are preferred.

RANDOM: among rules of the same salience, choose at random and this is useful for debugging.

The advantages of production systems are that writing rules is very natural for experts or end-users, justification of conclusions is possible and it is a general programming paradigm, which can be extensible with user defined functions and OOP for object definition.
Disadvantages are that it may be difficult to control the firing of rules and expressing knowledge as rules may be a bottleneck.

We consider now backward chaining and we start from SLD resolution, where a SLD derivation of a clause c from KB is a sequence S of clauses $c_1, c_2, \ldots, c_n$ such that $c_1 \in KB, c_n = c$ and each c_{i+1} is obtained by the resolution rule applied to c_i and some clause in KB.
In backward reasoning systems the SLD strategy is used by refutation, so we starting from the negation of the goal and trying to derive the empty clause.
The SLD strategy is complete for Horn clauses, so $KB \models c \Rightarrow S \vdash_{SLD} c$ and propositional Horn clauses KB's have linear-time deduction algorithms.

A logic program is a set of definite Horn clauses (facts and rules), so for example

$$A.$$

$$A :- B_1, B_2, \ldots, B_n.$$

The declarative interpretation of this example is that A is true and $B_1, B_2, \ldots, B_n \Rightarrow A$.
The goal is a negative clause, written as $? - G_1, G_2, \ldots, G_k$ (a conjunction of subgoals).

The procedural interpretation is that the head of a rule can be seen as a function call and the body as functions to be called in sequence, so when they all return the main procedure returns.

Given a logic program and a goal $G_1, G_2, \ldots, G_k$ the SLD goal tree is constructed as follows: each node of the tree corresponds to a conjunctive goal to be solved.
The root node is $? - G_1, G_2, \ldots, G_k$, let $? - G_1, G_2, \ldots, G_k$ a node in the tree and the node successors are obtained by considering the facts and rules in the program whose head unifies with G_1.
If A is a fact and $\gamma = MGU(A, G_1)$, a descendent is the new goal $? - (G_2, \ldots, G_k)\gamma$,

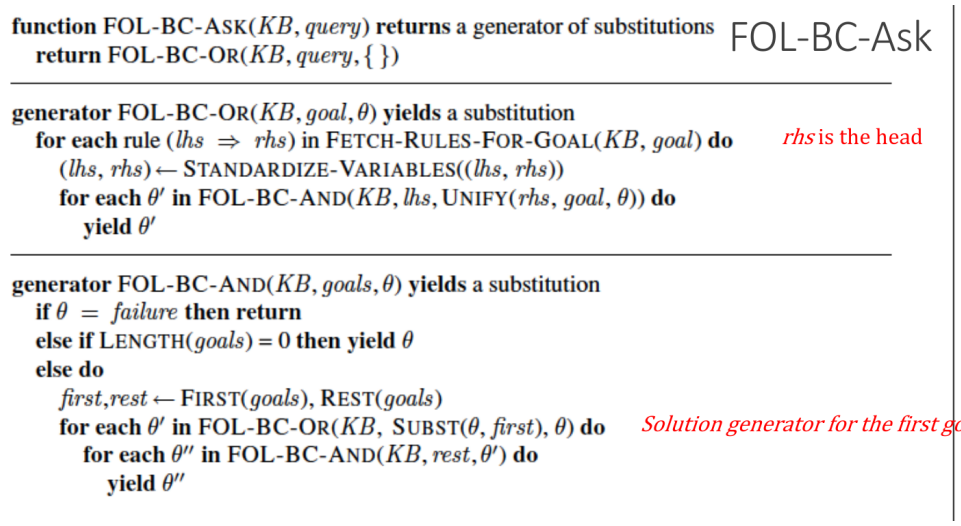


Figura 56: Pseudocode for SLD resolution strategy

instead if $A : -B_1, \ldots, B_m$ is a rule and $\gamma = MGU(A, G_1)$, a descendent is the new goal $? - (B_1, \ldots, B_m, G_2, \ldots, G_k)\gamma$. Note that variables in rules are renamed before using them, nodes that correspond to empty clauses are successes and nodes without successors are failures.

The SLD resolution strategy is complete for definite Horn clauses, with pseudo-code visible in figure 56, so this means that if $P \cup \{\neg G\}$ is unsatisfiable, then at least one of the leaves of the SLD tree produces the empty clause (success).
Moreover trying to satisfy the subgoals in the order they appear it is not restrictive, since in the end all of them must be satisfied and when there are variables in the goal, the substitution that we obtain is the computed answer.
Completeness and efficiency are however influenced by the order of expansion of the nodes (the visit strategy of the SLD tree), the order in which we consider successor nodes at each level and the order of literals in the body.

Observations arrive online from the user or from sensors and we cannot expect users or sensors to provide all relevant information.
The solution is to introduce an "ask-the-user" mechanism into the backward procedure, so define askable atoms: those atom for which the user is expected to provide an answer when unknown.
In the top-down procedure if the system is not able to prove an atom, and the atom is askable, it asks the user.

Explanations are required and a strong pro of rule-based decision support systems wrt other AI techniques, so support can be provided for three different kinds of requests:

1. How questions: how a fact was proved.
2. Why questions: why the system is asking about this?
3. Whynot questions: why a fact was not proved?

Given that the inference engine is correct, misbehaviors can only be caused by faulty knowledge, and debugging can be done by the domain expert, provided he/she is aware of the meaning of symbols.
Four types of errors can be supported in debugging:

1. Incorrect answers
2. Missing answers
3. Infinite loops

4. Irrelevant questions

We need an extension of the language to deal with integrity constraints of this form

$$F = a_1 \wedge \ldots a_k \equiv \not a_1 \vee \cdots \not a_k$$

We have that a Definite Horn database is always satisfiable and a Horn database (including integrity constraints) can be unsatisfiable.
Discovering a contradiction in the KB can be important in many applications and for these tasks you must declare a set of assumables, atoms that can be assumed in a proof by contradiction.
The system can then collect all the assumable that are used in proving false and that is

$$KB \cup \{c_1, \ldots, c_r\} \models F$$

where $C = \{c_1, \ldots, c_r\}$ is a conflict of KB and a possible answer is

$$KB \models \not c_1 \vee \cdots \vee \not c_r$$

A *minimal conflict* is a conflict such that no strict subset is also a conflict.

The aim of consistency-based diagnosis is to determine the possible faults based on a model of the system and observations of the system.
Background knowledge provides the model of the system, we make the absence of faults assumable, conflicts with observations can be used to derive what is wrong with the system and in the end from set of conflicts we can derive a minimal diagnosis.
A consistency-based diagnosis is a set of assumables that has at least one element in each conflict and a minimal diagnosis is such that no subset is a diagnosis.

We have $a \iff b_1 \vee \cdots \vee b_n$, that is the *Clark's completion*, and under Clark's completion if there are no rules for a then a is false.
With this, the system is able to derive negations, so we extend the language of definite Horn clauses with not and note that this negation is not the same as logical $\not$ so we use a different notation.
In backward reasoning systems this not can be implemented by negation as failure.

The top-down procedure for negation as failure is defined as follows: not p can be added to the set of consequences whenever proving p fails.
The failure must occur in finite time, there is no conclusion if the proof procedure does not halt and *Negation as failure* is the same as a proof of a logical negation from the KB under Clark's completion KB_c, so formally is

$$KB_c \models \not a \text{ iff } KB \not\vdash a$$

Negation as failure is a form of non-monotonic reasoning, so we can express defaults.

Abduction is a form of reasoning which consists in providing explanations for observations and the term abduction was coined by Peirce (1839–1914) to differentiate this type of reasoning from deduction, which involves determining what logically follows from a set of axioms, and induction, which involves inferring general relationships from examples.

Given a knowledge base KB, which is a set of of Horn clauses, a set A of assumable atoms, the building blocks of hypotheses.
An explanation of g is a set H such that $H \subseteq A$ such that $KB \cup H \not\models F$ and $KB \cup H \models g$ and we usually want a minimal explanation, since there can be more than one explanation.

Abductive diagnosis, provides explanation of symptoms in terms of diseases or malfunctions, so both the normal and the faulty behavior need to be modelled in order to infer observations and observations are not added to the KB.
Consistency based diagnosis needs only to represent normal behavior and observations are added to the KB to obtain inconsistencies and abductive diagnosis requires more detailed models, allowing to derive faulty behavior as well.

6.1 PROLOG

Prolog is the most widely used logic programming language and we have already introduced the syntax and the execution model (SLD resolution).
The declarative semantics is given by Horn clause knowledge bases and the procedural semantics is given by a specific strategy for exploring SLD trees, successors are generated in the order they appear in the logic program and the SLD tree is generated left-to-right depth first, also in the end the occur check is omitted from PROLOG's unification algorithms, so as a consequence Prolog is not complete and also is not correct.

Prolog uses the database semantics rather than first-order semantics, so it has unique name and closed word assumption (a form of nonmonotonic reasoning like negation as failure).
Prolog is a full-fledged programming language, since has an efficient implementation of the interpreter, there are built-in functions for arithmetic and list manipulation.

A Prolog interpreter is similar to Forward Chaining ask algorithm but is more efficient, since rely on a global data structure, a stacj of choice points.
Logic variables in a path remember their bindings in a trail and when failure occurs, the interpreter backtracks to a previous choice point and undoes the bindings of the variables.

Prolog compiles into an intermediate abstract machine (the Warren abstract Machine) and parallelism cna be exploited by OR-parallelism or AND-parallelism.

In forward chaining we do not have the problem of repeated computation, and we can obtain a similar saving in backward systems with a technique called *memoization*, which consists in caching solutions to subgoals and reusing them.
Tabled logic programming systems use efficient storage and retrieval mechanisms to perform memoization.

Basic data types in Prolog are numbers, atoms (identifiers with initial lowecase) and Variables (identifiers with initial uppercase and anonymous variables use _).
Structured objects are functions with arguments (functor applied to arguments), like $date(1, may, 2001)$ and lists, like for example $[ann, john, tom, alice]$.

We have as basic arithmetic operations $+, -, *, /, //, **, mod$ and so on, where // is the integer division.
The *is* infix operator forces the evaluation of the expressions and we use as comparison operators

```
<, >, <=, >=, =:=, =\=
```

where the last two are the equal and not equal operators which force evaluation.

To improve performance of Prolog programs it is better that programmers choose the order of clauses and subgoals that require less nodes to consider.
Prolog will automatically backtrack if this is necessary to satisfy a goal and uncontrolled backtracking however may cause inefficiency in a Prolog program so we introduce cut !, so $G : -T, !, R.$ and $G : -S.$ will execute S if T is not true otherwise we will execute R.

The negation as failure is implemented in Prolog using the built-in procedure not, that fails if P succeeds otherwise fails and failure must occur in a finite number of steps.

Using cut has advantages and drawbacks, with cut we can often improve the efficiency of the program and the idea is to explicitly tell Prolog to do not try other alternatives because they are bound to fail.
Using cut we can specify mutually exclusive rules, so we can add expressivity to the language.

The main disadvantage is that we can lose the correspondence between the declarative and procedural menaning of programs and two kinds of cut can be distinguished:

GREEN CUTS: that do not change the meaning (safe).

```
convert(Euro, USD) :-
    USD is Euro * 1.11.

?- convert(250, USD).
USD = 277.5

?- convert(Euro, 150).
Arguments are not sufficiently instantiated
150 is _x*1.11
```

```
convert(Euro, USD) :-
    {USD = Euro * 1.11}.   Special syntax for constraints

?- convert(250, USD).
USD = 277.5

?- convert(Euro, 150).
Euro = 135.13513513513513
?- convert(Euro, USD).
{USD=1.11*Euro}
```

Figura 57: Comparison between Prolog and CLP program

RED CUTS: that change the meaning, we have to be careful to the actual meaning.

Constraint logic programming (CLP) combines the constraint satisfaction approach with logic programming, creating a new language where a logic program works along a specialized constraint solver.
The basic Prolog can be seen as a very specific constraint satisfaction language where the constraints are of a limited form, that is equality of terms (unifications constraints or bindings).
Prolog is extended introducing other types of constraints and $CLP(X)$ differ in the domain and type of constraints they can handle, so for example $CLP(R)$ handles constraints on real numbers, $CLP(Z)$ for integers, $CLP(Q)$ is for rational numbers, $CLP(B)$ for boolean values and $CLP(FD)$ are for finite domains.

A CLP solution is the most specific set of constraintson the variables that can be derived from the knowledge base and is a specific solution if the constraints are tight enough.
In figure 57 is possible to compare the behavior a classical Prolog program with a CLP program.

A meta-interpreter for a language is an interpreter that is written in the language itself and Prolog has a powerful features for writing meta programs because Prolog treats programs and data both as terms.
A program can be input to another program and also one can write meta interpreters for various applications, extending the implementation of Prolog in different directions.
Applications is exploring different execution strategies for the interpreter, like on breadth first, limited depth search, combination of depth-first and breadth-first searches and so on.
It is also possible generating proof trees and implementing new languages and/or an OOP implementation in Prolog.
To build the meta-interpreter we can rely on the built-in predicate $clause(Goal, Body)$ and given a program and a goal, it retrieves a clause from the consulted program that matches Goal and Body then can be executed and in figure 58 is visible an interpreter implementation done using Prolog.

6.2 ANSWER SET PROGRAMMING

Answer Set programming/Prolog is a language for KR&R based on the stable model semantics of logic programs (an alternative, more declarative, semantics) and it includes, in a coherent framework, ideas from declarative programming, expressive KR language, deductive databases (forward reasoning), but also disjunctive databases; it includes also syntax and semantics of standard Prolog as a special case (but also disjunction, "classical" and "strong" negation, constraints) and from nonmonotonic logic (defaults, preference models, autoepistemic logic).

```
member1(X, [X |_]).      %  example program
member1(X, [_| Tail]) :-
    member1(X, Tail).
%?- member1(3, [1, 2, 3]).
%-------------------------------------------------------
% Vanilla meta-interpreter
prove(true).
prove(Goal) :-
    clause(Goal, Body),
    prove(Body).
prove((Goal1, Goal2)) :-
    prove(Goal1), prove(Goal2).
%?- prove(member1(3, [1,2, 3])).
```

Figura 58: Meta interpreter implementation with Prolog

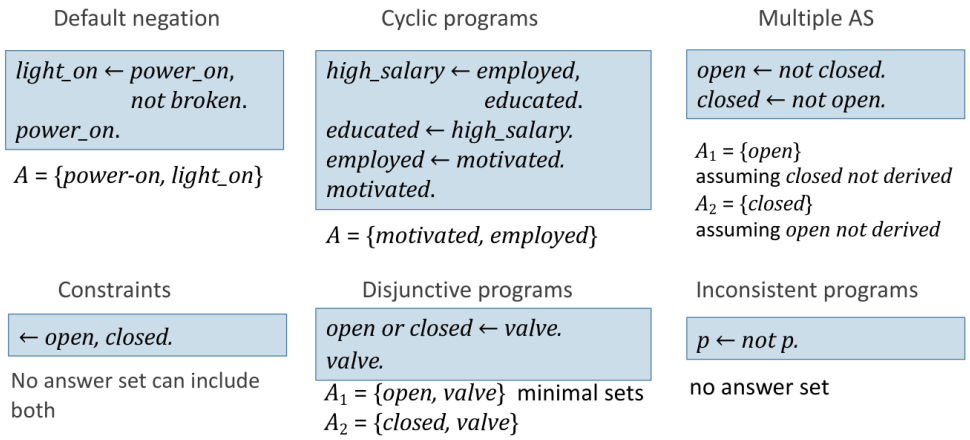


Figura 59: Example of Answer Set

Useful real world applications are team building, bio-informatics, linguistics, but are not exhaustive since there are several real world applications.

Answer sets are sets of beliefs (AS) that can be justified on the basis of the program, as we can see in figure 59, and we assume sorted signatures from classic logic, including natural numbers and arithmetic operators.
Literals are $p(t)$ and $\neg p(t)$ where t are sequences of terms and a rule is an expression of the form

$$l_0 \vee \cdots \vee l_k = l_{k+1}, \ldots, l_m, \neg l_{m+1}, \ldots, \neg l_n.$$

where the head is all terms before $=$, the body are all terms after $=$.

Programs without $\neg$(the strong negation) and without or ($k = 0$) are called *normal logic programs* (nlp).
When $head(r) = \{\}$ the rule is called a *constraint* and when $body(r) = \{\}$ the rule is called a *fact*.

A program of Answer Set Prolog/Programming is a pair $\{\sigma, Pi\}$ where σ is a signature and Π is a collection of logic programming rules over σ.
The signature σ is defined by the following rules:

- Sorts/constants $\tau_1 = \{a, b\}$ and $\tau_2 = N$ where $N = \{0, 1, \ldots\}$.
- Predicates $p(\tau_1), q(\tau_1, \tau_2), r(\tau_1)$ and the standard relation $<$ on N.

Note that the signature may be derived from the program, $\sigma(\Pi)$, rather than being given explicitly and it consist of all the constants which appear in the program.
Terms, literals, and rules are called ground if they contain no variables and no symbols for arithmetic functions and we define the following concepts:

HERBRAND UNIVERSE: the set of all ground istances of terms.

HERBRAND BASE: the set of all ground atomic sentences.

A program $gr(\Pi)$ consisting of all ground instances of all rules of Π is called the *ground instantiation* of Π.
In figure 60 is possible to note an example of Answer program and the relative ground instances.

A (partial) interpretation is a consistent subset of the Herbrand base and a partial interpretation S is a consistent set of ground literals over σ. Given two contrary literals l and l^-, l is true if $l \in S$ and l is false if $l^- \in S$, otherwise l is unknown in S.
An extended literal $notl$ is true in S if $l \notin S$ otherwise $notl$ is false in S; a set of extended literals U, represented as conjuction is true if all of them are true and is false if at least one is false otherwise is unknown.
A disjunction of literals $\{l_0 or \ldots or l_k\}$ is true if at least one is true, false if all of them

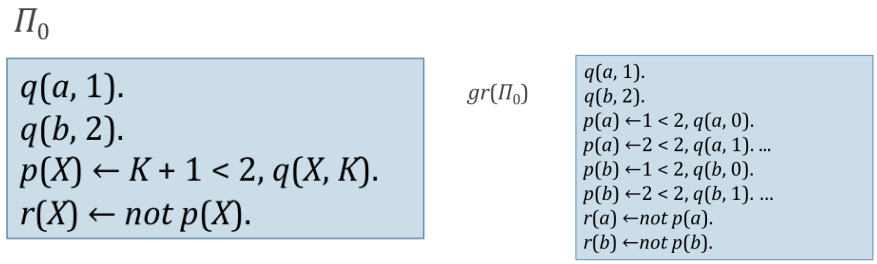


Figura 60: Example of Ground program

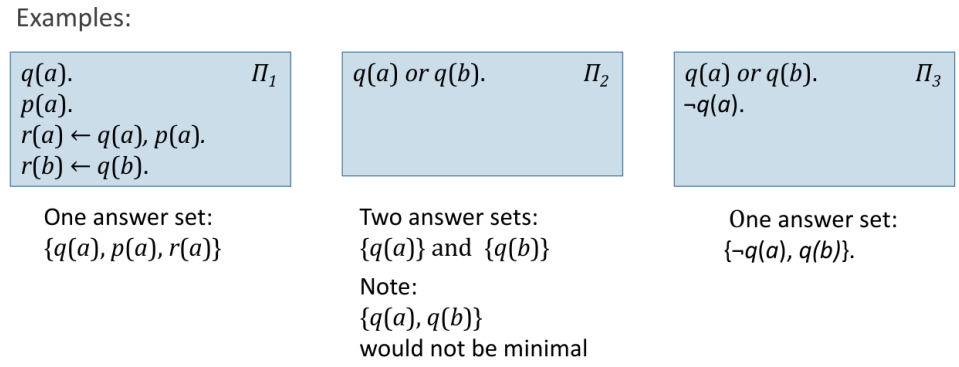


Figura 61: Example of Answer Set without default negative

are false in S otherwise is unknown.
We have that S satisfies a rule r if S satisfies r's head or does not satisfy its body.

The answer set semantics of a logic program Π assigns to Π a collection of answer sets S and an answer set is a partial interpretations over $\sigma(\Pi)$ corresponding to possible sets of beliefs which can be built by a rational reasoner on the basis of rules of Π.
Each answer set S obeys the following principles:

CONSISTENCY: S must satisfy the rules of Π.

MINIMALITY: a rationality principle, "do not believe anything you are not forced to believe".

A partial interpretation S of $\sigma(\Pi)$ is an answer set for Π if S is minimal (in the sense of set-theoretic inclusion) among the partial interpretations satisfying the rules of Π.
Epistemic or is different from logical disjunction and also the operator $=$ is different

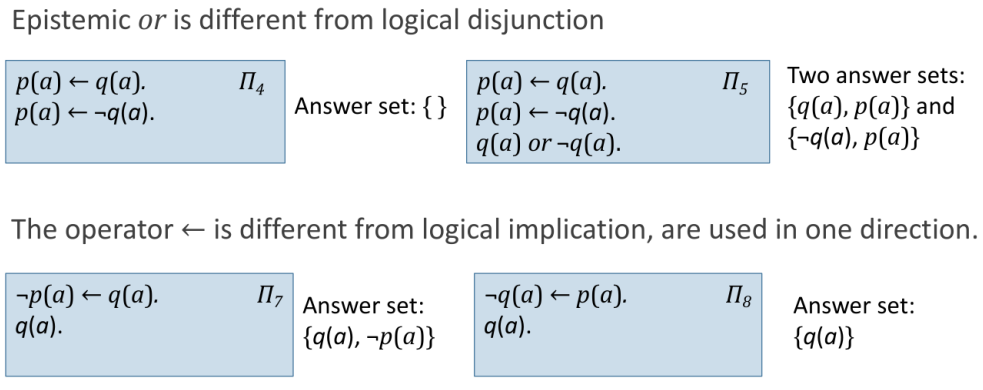


Figura 62: Another Example of Answer Set without negative

from logical implication, since are used in one direction.
In figure **??** and 62 are possible to note some example about answer set in program without default negative.
To consider also default negative the approach consist to generate answer sets, simplify rules and check.
Reduct Π^S of Π wrt a partial interpretation S is the set of rules obtained from Π consist to dropping every rule $A = B_1, \ldots, B_m, notC_1, \ldots, notC_n$ such that at least one of the negated atoms C_i in its body belongs to S (rule is not applicable) and also dropping $notC_j$ from the body of the rules when C_j does not belong to S (condition is satisfied).
A partial interpretation S of $\sigma(\Pi)$ is an answer set for Π if S is an answer set for Π^S , the reduct of Π, as defined before.

A program Π entails a ground literal l ($\Pi \models l$) if l is satisfied by every answer set of Π and we say that the program Π's answer to a query q is yes if $\Pi \models q$, no if $\Pi \models \not q$ and unknown otherwise.

A logic program is called consistent if it has an answer set and inconsistencies may be due to improper use of logical negation.
We can transform a ASP program Π to one without negation Π^+ (the positive form) and the transformation works as follows:

1. for each predicate symbol p we introduce a symbol p' with the same arity.
2. we replace each $l = \not p(t)$ with $p'(t)$, its positive form, l^+ and if l is positive we set $l = l^+$.

A logic program Π can be transformed in its positive form Π^+ by replacing each rule with

$$\{l_0^+, \ldots, l_k^+\} = l_{k+1}^+, \ldots l_m^+, notl_{m+1}^+, \ldots, notl_n^+$$

and adding the constraints $= p(t), p'(t)$ for each p' added.
A property is that a set S of literals of $\sigma(\Pi)$ is an answer set of Π iff S^+ is an answer set of Π^+.

A logic program can be inconsistent for the use of the default negation and in general the problem of checking consistency is undecidable or decidable and very complex for finite domains.
The theory helps us in making simplifying assumptions to guarantee consistency, so we define

Def (Level Mapping)**.** A function $||.||$ which maps ground atoms in P (the Herbrand base of P) to natural numbers and if D is a disjunction or conjunction of literals, $||D||$ is defined as the minimum level of atoms occurring in literals from D' (the positive form).
This also implies that $||\not l|| = ||l||$.

Def (Locally stratified program)**.** A logic program Π is locally stratified if it does not contain $\not$ and there is a level mapping of the grounded version of $\Pi, gr(\Pi)$ such that for every rule:

$$\forall l \in pos(r)\ ||l|| \leq ||head(r)||$$
$$\forall l \in neg(r)\ ||l|| < ||head(r)||$$

Def. If a program is locally stratified and, in addition, for any predicate symbol p, $||p(t_1)|| = ||p(t_2)||$ for any t_1 and t_2, the program is called stratified.

A stratified program is also locally stratified and a program without not is stratified.

Properties are the following:

1. A locally stratified program is consistent.
2. A locally stratified program without disjunction has exactly one answer set.

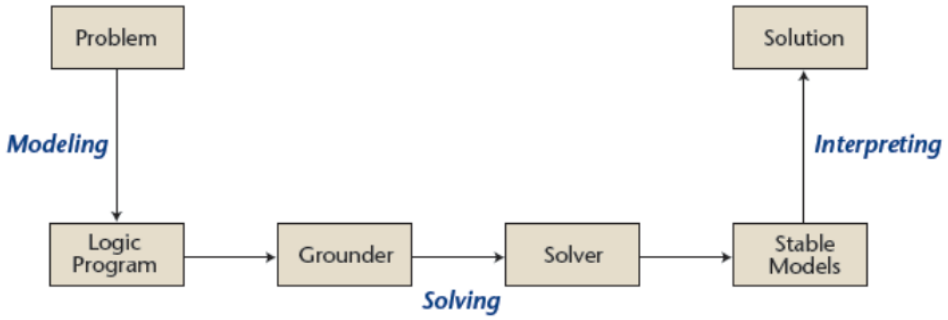


Figura 63: Workflow of Answer Set programming

3. The above conditions hold by adding to a locally stratified programs a collection of closed world assumptions of the form $\not p(X) = notp(X)$.

There are weaker syntactic conditions that still guarantee consistency, for example order-consistent programs.

The choice of the algorithms used depend on the structure of problems and the type of queries, so we consider for example Normal logic programming acyclic programs and Tight programs.

Def (Acyclic programs)**.** A normal logic program Π is called acyclic if there is a level mapping of Π such that for every rule r of $gr(\Pi)$ and every literal l which occurs in $pos(r)$ or $neg(r)$, $||l|| < ||head(r)||$

If Π is acyclic then the unique answer set of Π is the unique Herbrand/classical model of Clark's completion of Π, $Comp(\Pi)$.
SLDNF resolution-based interpreter of Prolog will always terminate on atomic queries and produce the intended answers.

Def (Tight programs)**.** A normal logic program Π is called tight if there is a level mapping of Π such that for every rule r of $gr(\Pi)$ and every literal $l \in pos(r), head(r) > l$.

Acyclic programs are tight but no vice versa and if program Π is tight then answer sets of Π are the models of the Clark's completion of Π.
The problem of computing answer sets can be reduced to SAT for propositional formulas and a SAT solver can be used and also in the case of not tight programs there are theoretical results that allow translating into SAT problems, even if very large.

These theoretical results have led to the development of answer set solvers such as ASET, CMODELS, and so on, which are based on (possibly multiple) calls to propositional solvers.

Traditional answer set (AS) solvers typically have a two level architecture:

GROUNDING STEP: compute $gr(P)$ and grounders can be integrated (e.g. DLV) or provided as separate modules.
The size of the grounding (even if partial) is a major bottleneck and smart and lazy grounding techniques have been developed.

MODEL SEARCH: the answer sets of the grounded (propositional) program are computed.

The workflow of Answer Set programming is possible to note in figure 63 and applications are in repairing large scale biological networks, planning, decision making but also Industrial integration and Music composition system.

7 PLANNING

Planning combines two major areas of AI, search and logic, and representing actions and change in logic we can cast the problem of planning as a SAT problem in the propositional case and as theorem planning in the case of FOL.
In classical planning we will introduce a restricted language suitable for describing planning problems and this allows to circumvent representation and computational problems by resorting to a restricted FOL representation.

A Planning agent has an explicit representation of the goal state, of actions and their effects, it is able to inspect the goal and decompose it and make abstractions; it can also work freely on the plan construction, manipulating goals and plans and some general heuristics and algorithms for planning become possible, leveraging on this representation.

In figure 64 there is the pseudocode to planning as satisfiability in PROP, where the planning problem is translated into a CNF sentence for increasing values of t until a solution is found or the upper limit to the plan length is reached.
Planning is the generation of a sequence of actions, a plan p, to reach the goal G, so this amounts to proving that $\exists p G(Result(p, s_0))$.
The Green planner used a theorem prover based on resolution and the task is made complex by different sources of non-determinism: the length of the sequence of actions is not known in advance, frame actions may infer many things that are irrelevant, we need to resort to ad hoc strategies, we do not have any guarantee of the efficiency of the generated plan and a general theorem prover is inefficient and semi-decidable, so completeness is not guaranteed.

In classical planning we assume fully observable, deterministic, static environments with single agents and we assume a factored representation, so a state of the world is represented by a collection of variables.
We use PDDL (Planning Domain Definition Language) is a specialized language for describing planning problems, in particular states, initial, goal states, applicable actions and transition model through action schemas.

In PDDL states are conjuctions/set of fluents, ground positive atoms, no variables and also no function; database semantics is used, since we assume closed world assumption and unique name assumption.
Actions are defined by a set of action schemas that implicitly define the $Actions(s)$ and their result $Result(s, a)$ and actions are defined in a way that avoids the frame problem since they require that we carefully specify all the charges and all the rest it assumed to persist.

```
function SATPLAN(init, transition, goal, T_max) returns solution or failure
  inputs: init, transition, goal, constitute a description of the problem
          T_max, an upper limit for plan length

  for t = 0 to T_max do
    cnf ← TRANSLATE-TO-SAT(init, transition, goal, t)
    model ← SAT-SOLVER(cnf)
    if model is not null then
      return EXTRACT-SOLUTION(model)
  return failure
```

Figura 64: Pseudocode of SatPlan approach

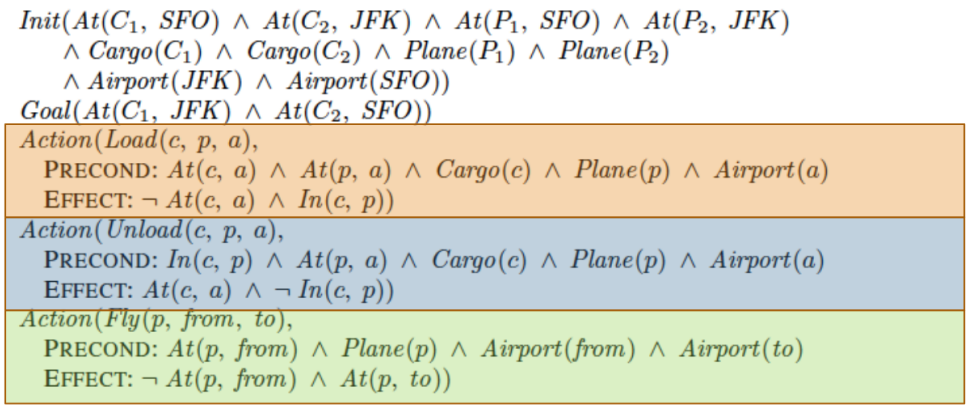


Figura 65: Example of PDDL actions

Action schemas correspond to parametric actions or operators and action are represented by the following two components:

precond: a list of preconditions to be satisfied in s for the action to be applicable

$$(a \in ACTIONS(s)) \iff s \models PRECOND(a)$$

$effect$: the successor state s' is obtained from s as result of the action by

$$RESULT(s, a) = (s - DEL(a)) \cup ADD(a)$$

In figure 65 is possible to note an example of PDDL actions.

PlanSAT answer if does a plan exist, instead Bounded PlanSAT solve the problem to answer if there is a solution of length k or less.
Both problems are decidable for classical planning, with PlanSAT decidable only without functions instead Bounded PlanSAT is always decidable, also with functions.
Theoretical complexity for both PlanSAT and Bounded PlanSAT is very high and if we disallow negative effects both problems are still NP-Hard and if we also disallow negative preconditions PlanSAT reduces to the class P.

If we view Planning as state-space search, where nodes in the search space are states and arcs are the actions, we have two different type of planning:

PROGRESSION PLANNING forward search from the initial state to the goal state and it is believed to be inefficient.
Prone to exploring irrelevant actions and often have large state spaces.

REGRESSION PLANNING backward search from the goal state to the initial state.

We start with the goal , a conjuction of literals, describing a set of worlds and the PDDL representation allows to regress action

$$g' = (g - ADD(a)) \cup Precond(a)$$

Actions that are useful to reach a goal are defined *relevant* and relevant actions contribute to the goal but must be the last step to the solution and given a goal g containing a literal g_i, an action a is relevant action towards g if action schema A has an effect literal e such that $Unify(g_i, e) = \Theta$, $a = SUBST(\Theta, A)$ and also there is no effect in a that is the negation of a literal in g.
One of the earlier planning system was a linear regression planner called *STRIPS*.

Given the factored representation we can devise good general heuristics for planning, and *problem relaxation* is a common technique for finding admissible heuristics, which consists in looking at the problem with less constraints and computing the cost of the solution in the relaxed problem.
This cost can be used as an admissible heuristics for the relaxed problem and there are two strategies to relax the problem:

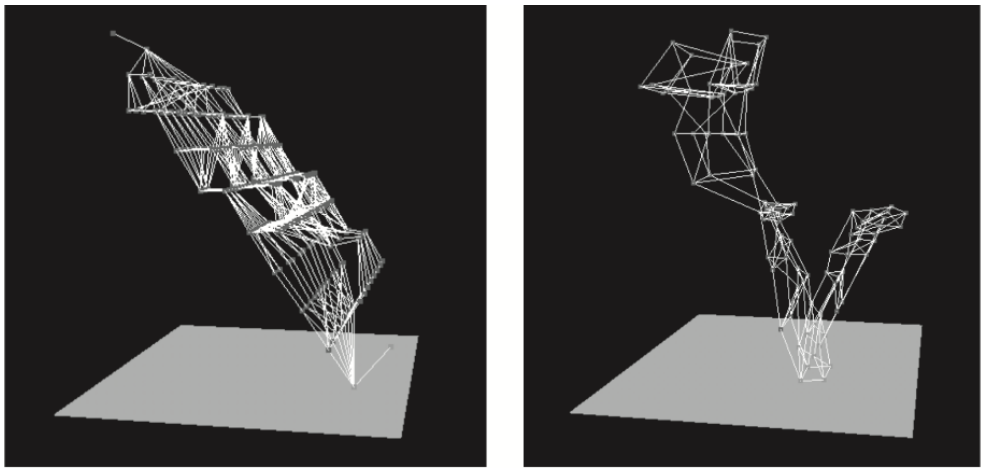


Figura 66: Comparison between two state space without and with ignore delete lists heuristics

1. Add more arcs to the graph, so it is easier to find a path to the goal, and *Ignore preconditions heuristics* and *Ignore delete list heuristics* are of this type.

2. Group multiple nodes together, forming an abstraction of the state space (with fewer states, it is easier to search).

The *ignore preconditions* heuristic drops all preconditions from actions and every action is applicable in any state, any single goal literal can be satisfied in one step or there is no solution.
The number of steps to solve a goal is approximated by the number of unsatisfied subgoals, but one action may achieve more than one subgoal (non admissible estimate) and one action may undo the effect of another one (admissible).
An accurate heuristics consist to remove all preconditions and all effects except those that are literals in the goal and count the minimum number of actions required such that the union of those actions' effects satisfies the goal (NP-Complete but greedy approximations exist).

As an alternative we could ignore only some selected preconditions from the actions.

Ignore delete list heuristic consist to assume that all goals and preconditions contain only positive literals and remove the delete lists from all actions (removing all negative literals from effects) and no action will ever undo the effect of actions, so there is a monotonic progress towards the goal.
We use the length of the solution as admissible heuristics and is still NP-Hard to find the optimal solution of the relaxed problem to compute the heuristic function but this can be approximated in polynomial time, with hill-climbing.

In figure 66 is possible to see two state spaces from planning problems with the ignore delete lists heuristic.

In order to reduce the number of states, we need other forms of relaxations, so state abstractions reduces the number of states.
A state abstraction is a many-to-one mapping from states in the ground/original representation of the problem to a more abstract representation, instead *Decomposition* divide a problem into parts, solve each part independently and then combine the subplans.
The subgoals *independence assumption* is that the cost of solving a conjuction of subgoals is approximated by the sum of the costs of solving each subgoal independently and this assumption can be *optimistic* (admissible), when there are negative interactions, and *pessimistic* (inadmissible), when subplans contain redundant actions.
Two techniques used to find useful independent sub-problems are *pattern database* (solve subproblems and memorize their cost) and *hierarchical decomposition*.

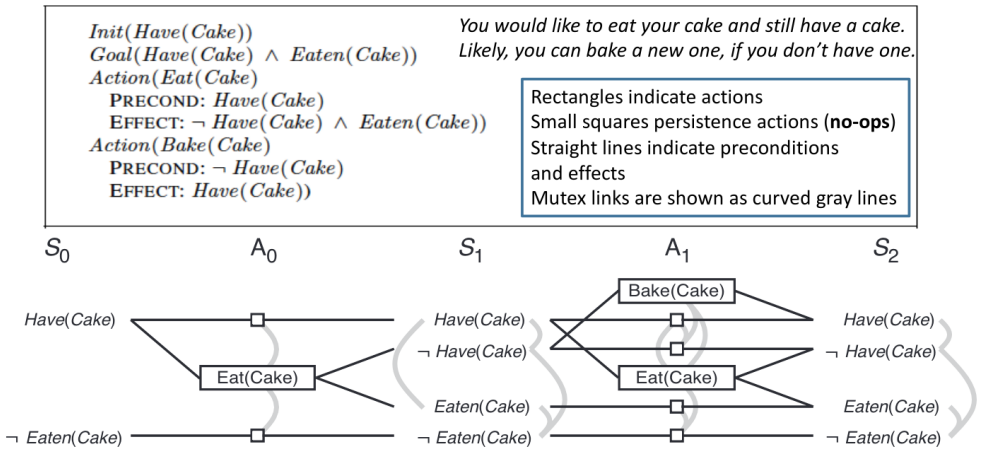


Figura 67: Example of Planning Graph

A data structure commonly used in Planning is *Planning Graph*, that can be used to give better heuristics estimates to employ in conjuction with search algorithms and it is the search space of an algorithm called *GraphPlan*.

A search tree is exponential in size and a planning graph is a polynomial size approximation of the search tree and can be constructed quickly.
The planning graph can't answer definitively whether G is reachable from S_0, but it may discover that the goal is not reachable and can estimate how many steps it takes, in the most optimistic case, to reach G, so it can be used to derive an admissible heuristic.

Planning graphs work only for propositional planning problems, with no variables and is a directed graph which is built forward, organized into levels: a level S_0 for the initial state, representing each fluent that holds in S_0, a level A_0 consisting of nodes for each ground action applicable in S_0 and alternating levels S_i followed by A_i are built until we reach a termination condition.

S_i contains all the literals that could hold at time i (even contrary literals P and $\not P$) and A_i contains all the actions that could have their preconditions satisfied at time i.

Mutual exclusion links (mutex) connect incompatible pairs of literals and actions:

1. Mutex between literals mean that two literals cannot appear in the same belief state.

2. Mutex between actions mean that two actions cannot occur at the same time.

In figure 67 is possible to note an example of planning graph.

Each level S_i represents a set of possible belief states and two literals connected by a mutex belong to different belief states.
The levels, alternating S's an A's, are computed until we reach a point where two consecutive levels are identical.
The level j at which a literal first appears is never greater than the level at which it can be achieved and we call this the *level cost* of a literal/goal.
A planning graph is polynomial in the size of the planning problem: an entire graph with n levels, a actions, l literals, has size $O(n(a+l)^2)$ also the time complexity is the same.

Planning graphs can provide useful informations, so we have that if any goal literal fails to appear in the final level of the graph, then the problem is unsolvable.
We can estimate the cost of achieving a goal literal g_i as by its level cost and a better estimate can be obtained by serial planning graphs: by enforcing only one action at each level (adding mutex).
Estimating the heuristic cost of a conjuction of goals can be done with 3 heuristics:

```
function GRAPHPLAN(problem) returns solution or failure
  graph ← INITIAL-PLANNING-GRAPH(problem)
  goals ← CONJUNCTS(problem.GOAL)
  nogoods ← an empty hash table
  for tl = 0 to ∞ do
    if goals all non-mutex in S_t of graph then
      solution ← EXTRACT-SOLUTION(graph, goals, NUMLEVELS(graph), nogoods)
      if solution ≠ failure then return solution
    if graph and nogoods have both leveled off then return failure
    graph ← EXPAND-GRAPH(graph, problem)
```

Figura 68: Pseudocode of GRAPHPLAN algorithm

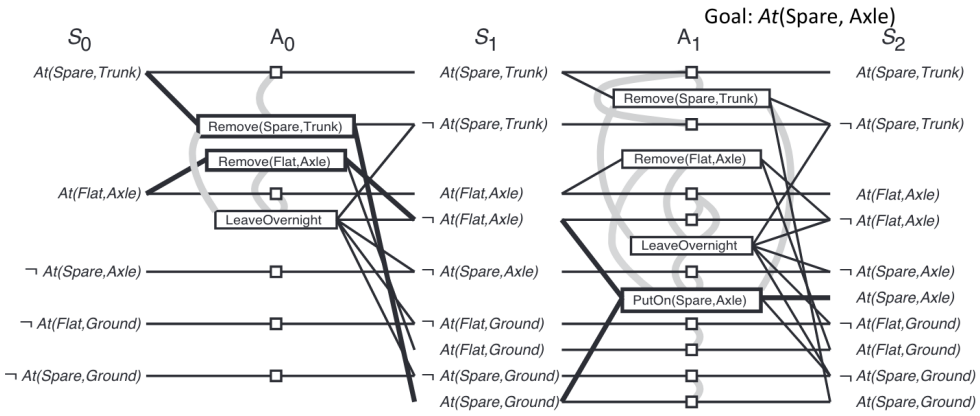


Figura 69: Example of Graph Plan on the spare-tire problem

MAX-LEVEL: the maximum level cost of any of the sub-goals and this is admissible.

LEVEL SUM: the sum of the level costs of the goals and this can be inadmssible when goals are not independent, but it may work well in practice.

SET LEVEL: finds the level at which all the literals in the goal appear together in the planning graph, without any mutex between pairs of them.
It is admissible, accurate but not perfect.

The planning graph can be seen as a relaxed problem with the following characteristics, so when g appears at level S_i we can prove that if there exists a plan with i action levels that achieves g then g will appear at level i, and if g does not appear there is no plan, but not viceversa, so the fact that g appears does not mean that there is a plan.

If g does appear at level i, the plan possibly exists but to be sure we need to check the mutex relations, pairs of conflicting actions and pairs of conflicting literals.
The *GraphPlan* algorithm, with pseudocode in figure 68, is a strategy for extracting a plan from the planning graph, that can be computed incrementally by the function EXPAND-GRAPH.
Once a level is reached where all the literals in the goal show up as non-mutex, an attempt to extract a plan is made with EXTRACT-SOLUTION.
If EXTRACT-SOLUTION fails, the failure is recorded as a no-good, another level is expanded and the process repeats until a termination condition is met.
In figure 69 is possible to see the graphplan on the spare-tire example.

To extract solution there are two approaches:

1. Solve as a boolean CSP: the variables are the actions at each level, the values for each variable are in or out of the plan, and the constraints are the mutexes and the need to satisfy each goal and precondition.

2. Solve as a backward search problem, we start with S_n and the goal, and for each level S_i select a number of non-conflicting actions in A_{i-1}, whose effects cover the goals in S_i.
   The resulting state is S_{i-1} with goals the preconditions of the selected actions and the process is repeated until level S_0 hoping all the goals are satisfied.

If EXTRACT-SOLUTION fails to find a solution for a set of goals at a given level, we record the (level, goals) pair as no-good, so that we can avoid to repeat the computation.

Constructing the planning graph takes polynomial time and solution extraction is intractable in the worst case, but fortunately an heuristics exist.
It is a greedy algorithm based on the level cost of the literals, which consist to pick first the literal with the highest level cost and to achieve that literal, prefer actions with easier preconditions, that is choose an action such that the sum (or maximum) of the level costs of its preconditions is smallest.

We can prove that GRAPHPLAN will in fact terminate and return failure when there is no solution, but we may need to expand the graph even after it levels off.

Thm 7.1. If the graph and the no-goods have both leveled off, and no solution is found we can safely terminate with failure

The sketch of the proof consist that literals and actions increase monotonically and are finite, so we need to reach a level where they stabilize.
Mutex and no-goods decrease monotonically and cannot become less that zero, so they too must level off.
When we reach this stable state if one of the goals is missing or is mutex with another goal it will remain so we may as well stop computation.

We consider now other approaches for planning, starting from translating to a boolean SAT problem, where we execute the following steps:

1. Propositionalize the actions: create ground instances of the actions.

2. Define the initial state F^0 for every fluent F in the initial state, $\neg F^0$ for every fluent F not in the initial state.

3. Proposizionalize the goal, by instantiating variables to a disjunction of constants.

4. Add successor-state axioms, so for each fluent we have

$$F^{t+1} \iff ActionCausesF^T \vee (F^t \wedge \neg ActionCausesNotF^t)$$

   where $ActionCausesF^t$ is a disjunction of all the ground actions that have F in their add list and $ActionCausesNotF^t$ is a disjunction of all the ground actions that have F in their delete list.

5. Add precondition axioms, so for every ground action $A^t \Rightarrow PRE(A)^t$.

6. Add action exlusion axioms, so every action is distinct from every other action.

The resulting translation is in the form that can be given in input to SATPLAN to find a solution.

Partial Order Planning is an interesting approach, very popular in the nineties, since it addresses the issue of independent subgoals can be performed in parallel.
For some specific tasks, such as operations scheduling is the technology of choice and is interesting, because it represents a change of paradigm: planning as search in

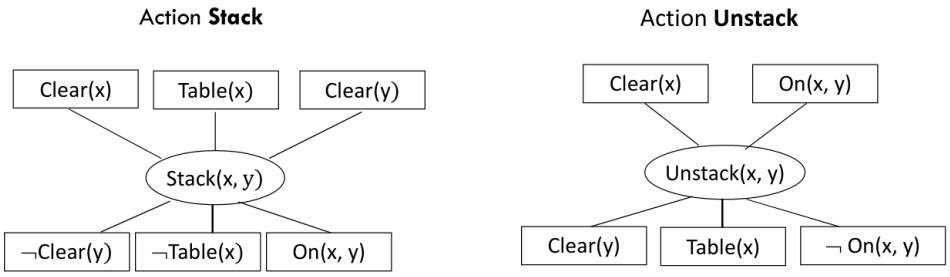


Figura 70: Representation of POP actions

the state of partial plans rather than in space of states and this refinement is also more explainable: it makes easier for humans to understand what the planning algorithms are doing and verify that they are correct.
The driving principle of POP is least commitment, and partially ordered plans do not order steps in the plan unless necessary to do so, in a partial-order plan steps are partially ordered, and also there is *Plan linearization*, to impose a total order to a partially ordered plan.

Partially instantiated plans leave variables uninstantiated until is necessary to instantiate them and a plan without variables is said to be totally instantiated.

Instead of searching in space of states as in the classical formulation, in POP we search in the space of partial plans, so we start with an empty plan, at each step we use operators for plan construction and refinement (we can add actions, instantiate variables and add ordering constraints between steps) and in the end we stop when we obtain a consistent and complete plan where all the preconditions of all the steps are satisfied and ordering constranits do not create cycles.

Partial plan are represented as a set of actions, among them Start and Finish, a set of open preconditions and constraints among actions of two different types (Ordering relations $S_1 < S_2$ and causal links $S_1 \rightarrow_{cond} S_2$).
In figure 70 and 71 is possible to note how actions are represented in POP approach and also how empty plan are represented.

In POP algorithm we start with the empty plan, with Start and Finish, and at each step we choose a step B and one of its open preconditions p and we generate a successor plan for each action A having p among the effects.
After choosing an action A consistency is re-estabilished by add to the plan the constraints $A < B$ and $A \rightarrow_p B$, and possible actions C having $\neg p$ as effect, are potential conflicts (or threats).
They need to be anticipated or delayed by adding the constraints $C < A$ or $B < C$, and we stop when the set of open pre-conditions is empty.

In figure 72 is possible to see how we can remove threats and in figure 73 is possible to see POP in action.

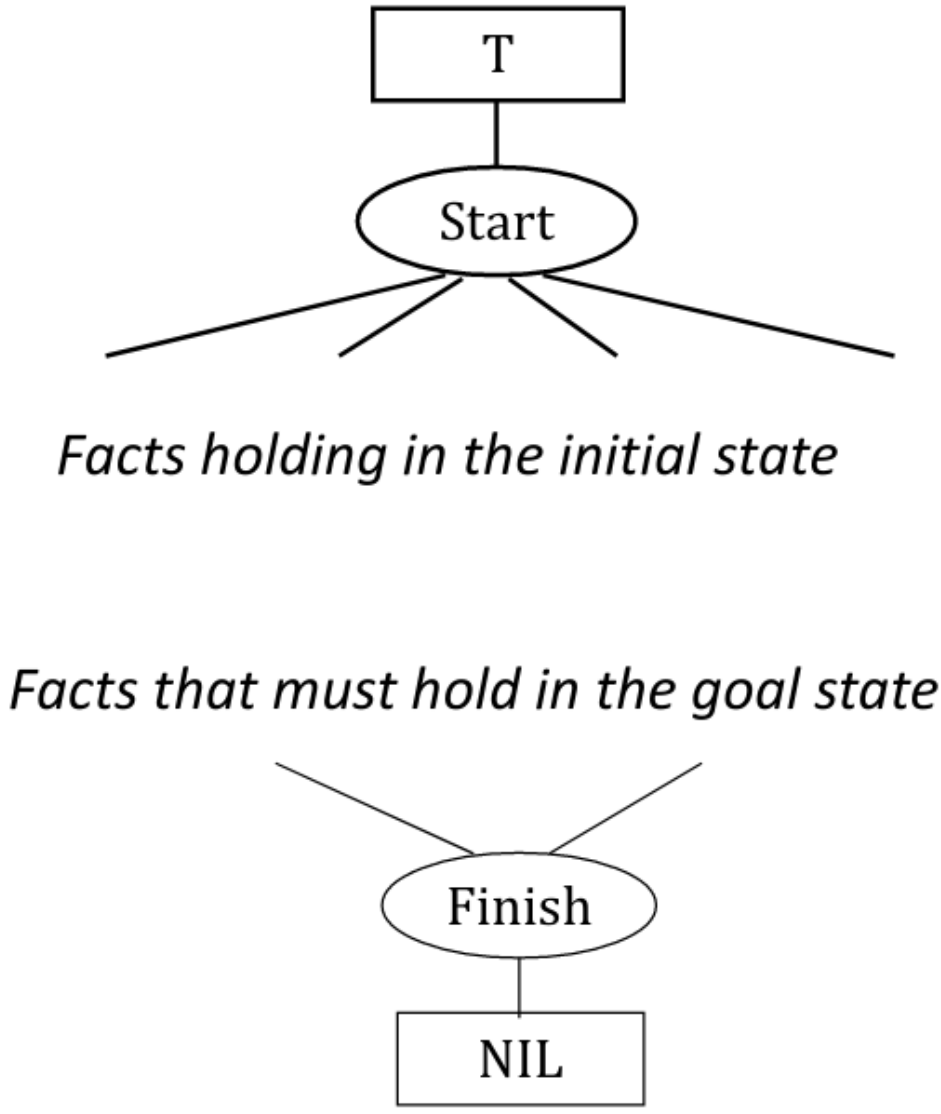
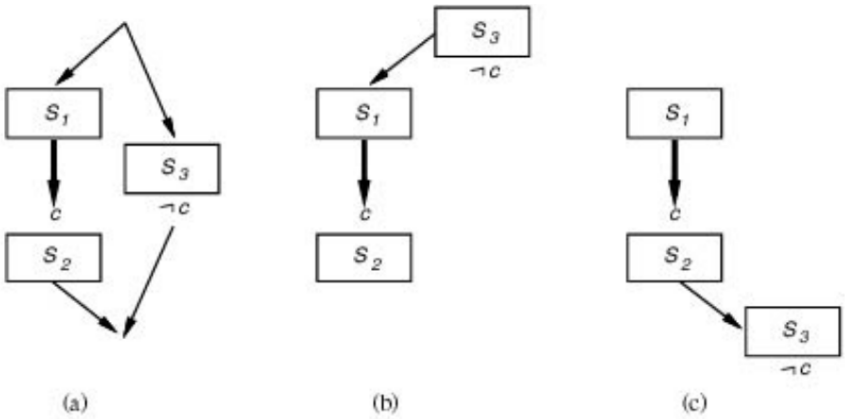


Figura 71: Empty Plan on POP approach



(a) S_3 is a threat for pre-condition c di S_2, achieved by S_1
(b) The *threat* is resolved by *demotion*
(c) The *threat* is resolved by *promotion*

Figura 72: Example how to remove threats in POP

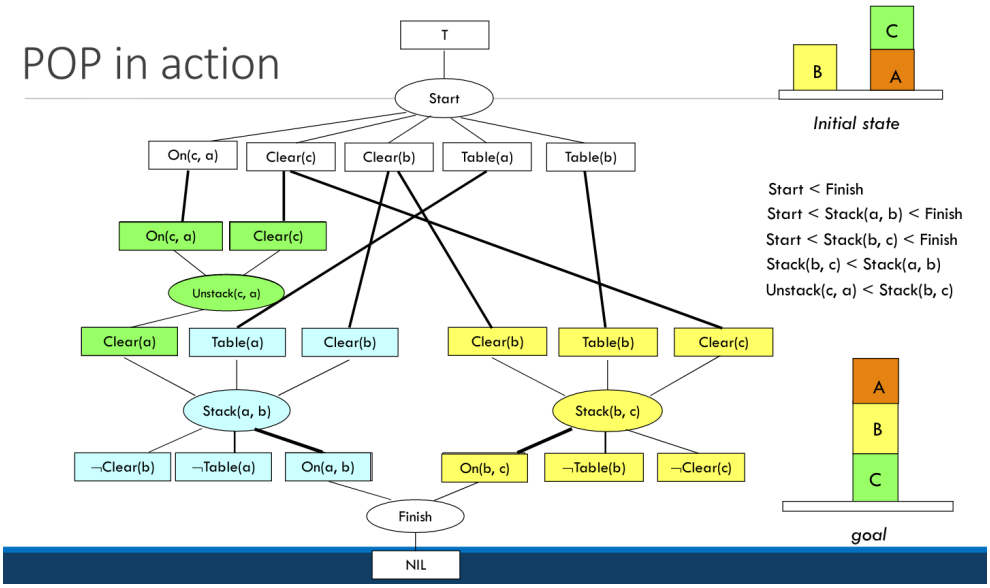


Figura 73: Example of POP in action