problem 1: no it doesn't need to be connected to be dense.

proof: let G is a graph with |V| vertices and |E| edges and it is disconnected

G has components G_1 G_2 G_3 each has V, E

since it is possible that all those components be complete graph

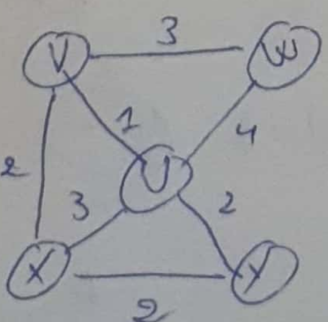
$$m_G = m_1 + m_2 + \dots m_k$$

$$= \frac{m_1 - 1}{2} + \frac{m_2 - 1}{2} + \dots \frac{m_k - 1}{2}$$

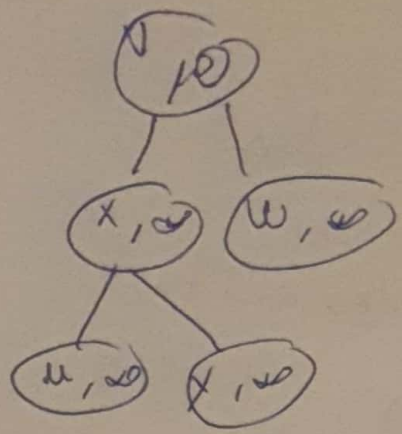
$$= \sum_{i=1}^{k} m_i \left(\frac{m_i - 1}{2}\right) = O(n^2)$$

So the graph G has $\Theta(n^2)$, number of edges which is equal with the amount expected by dense graphs.

problem 2



	A
A[V]	0
A[X]	∞
A[U]	∞
A[U]	∞
A[W	∞

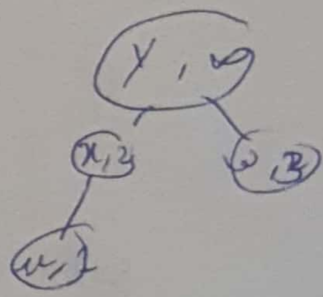


$X = \{\}$

$H = \{V, x, w, u, y)$

Step 1: remove min $(V, 0)$

Vertices adject to V : x, w, u



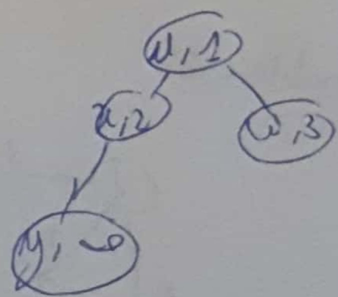
process x: greedylen $A[V] + wt(V, x) = 0 + 2 = 2$

greedylen $< A[0]$ -> yes

old = $A[x]$ $A[x] = 2$

~~old = A[x] A[x] = 2~~

update $(x, old, (x, 2))$



process w : greedylen $A[V] + wt(V, w) = 0 + 3$

update $(w, old, w, 3)$

process u : greedylen $A[V] + wt(V, u) = 1$

greedylen $< A[u]$? yes

old = $A[u]$, $A[u] = 1$

update $(u, old, u, 1)$

A	
V	0
x	2
w	3
u	1
Y	∞

step remove min (u)

M. remove (u)

vertices adjecent to u, x, w, y

proces x

greedy len = A[u] + wt(u, x) = 4

proces w

greedylen = A[u] + wt (u, w) = 5

proces y

greedylen = A[u] + wt(u, y) = 3

old = A[y] A[y] = 3

update ((y, old), (y, 3))

A	
A[v]	0
A[x]	2
A[w]	3
A[u]	1
A[y]	3

X = {v, u}

M {x, y, w}

A	
A[v]	0
A[x]	2
A[w]	3
A[u]	1
A[y]	3

(u, 3)

(y, 3)

step 4: removemin (w)

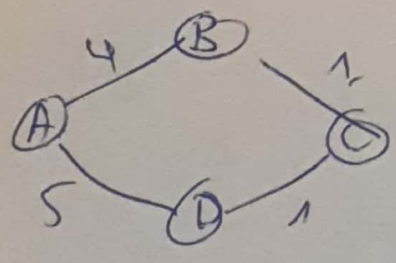
X = {V, u, x, w, u}

	A
A[V]	0
A[x]	2
A[w]	3
A[u]	1
A[y]	3

problem 3: shortest path

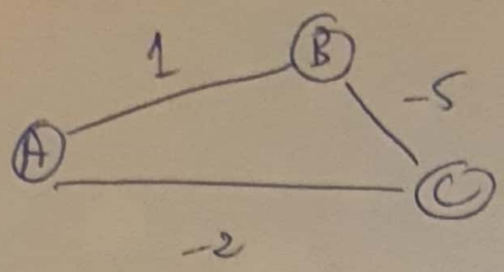
A) There are no algorithm that truly computes the smallest path on undirected negative weight edge graphs

Let's add 3 to the edge.



the shortest path from A to c is A-B-c which is 5-4 = -1

however by following the path A-B-C-B-C which is -7 we can get small paths.



1- what goes wrong by applying dijkistra algorithm
it will compute $A[C] = -2$ because of the greedy length choice.

2) $D[A) = 0$

$D[B] = \min\{D[x] + wt(x, v) \mid (x, v) \in E\}$

$= D[A] + wt(A, B) = 0 + 1 = 1$

$$D[C] = \min \begin{cases} D[A] + wt(A,C) = -2 \\ D[B] + wt(B,C) = -4 \end{cases}$$

$\underline{D[C] = -4}$.