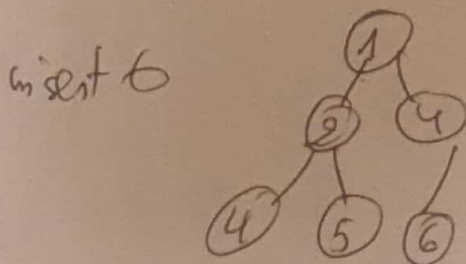
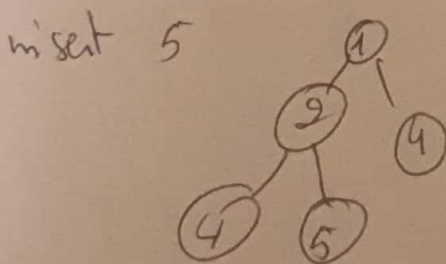
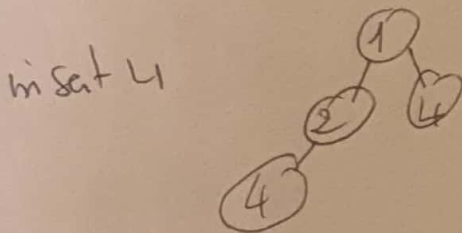
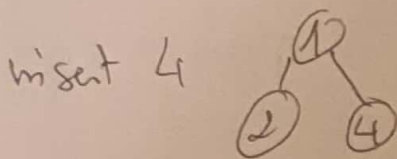
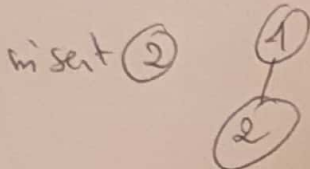
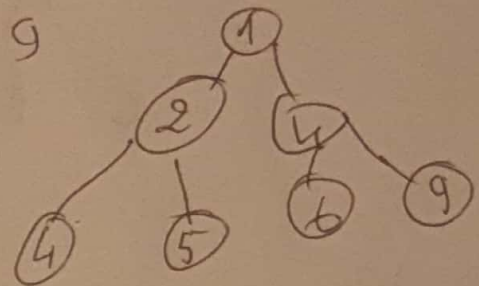


Problem 1: 1, 2, 4, 4, 5, 6, 9, 11, 12, 12, 17

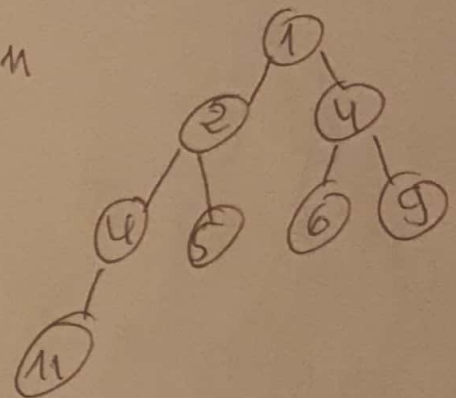
insert 1 ①



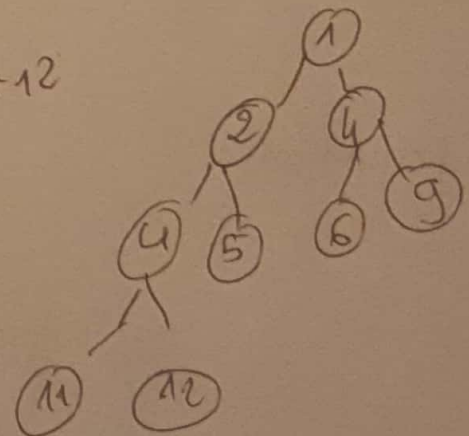
insert 9



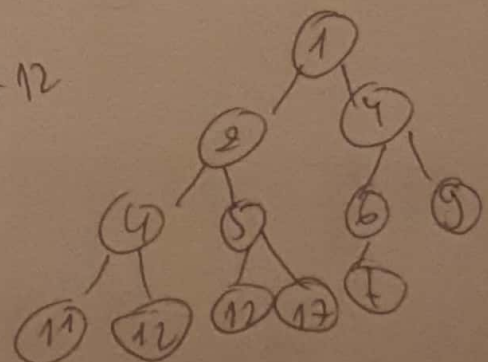
insert 11



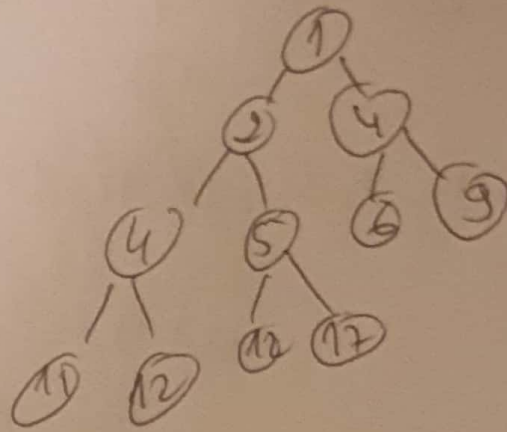
insert 12



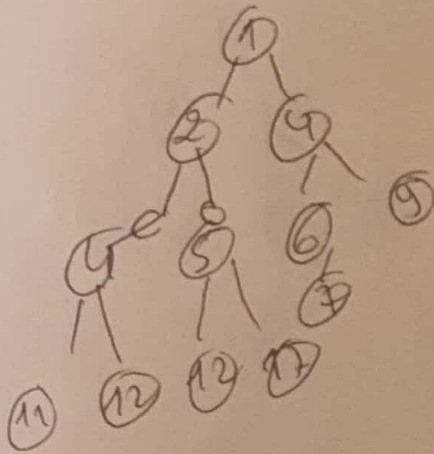
insert 12



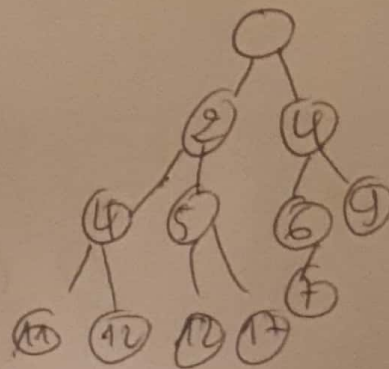
insert 12



b) insert 7

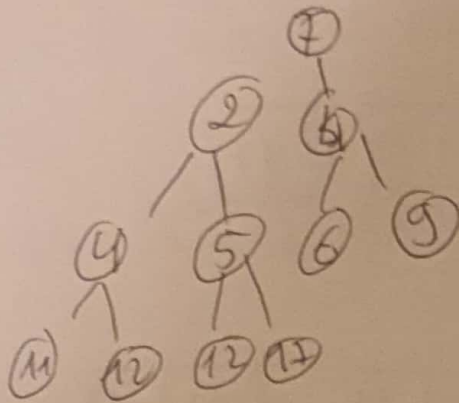


c) remove min

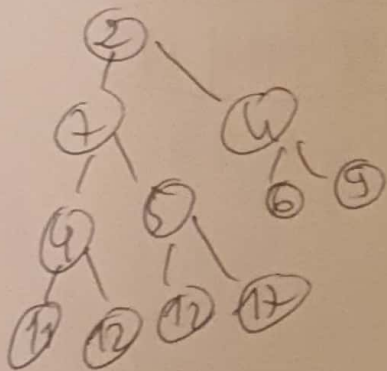


bring the last element to pos min

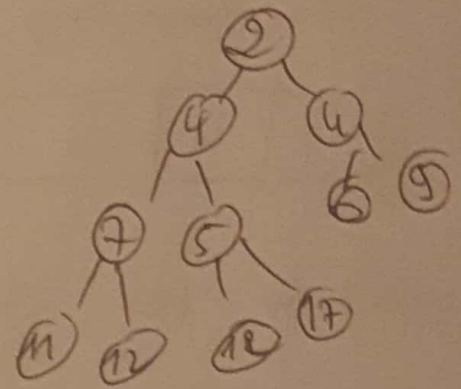
(3)



- do down heap



→ down heap



D - present in an array

2	4	4	7	5	6	9	11	12	12	17
0	1	2	3	4	5	6	7	8	9	10

E - an

left child  $\rightarrow 2i+1$

right child  $\rightarrow 2i+2$

parent node  $\left(\frac{i-1}{2}\right)$

2	4	4	7	5	6	9	11	12	12	17	17
0	1	2	3	4	5	6	7	8	9	10	11

$$\text{parent} = \frac{11-1}{2} = 5$$

2	4	4	7	5	6	9	11	12	12	17	14
---	---	---	---	---	---	---	----	----	----	----	----

memory 2

14	4	4	2	5	6	9	11	12	12	17
----	---	---	---	---	---	---	----	----	----	----

then down heap

4	4	14	7	5	6	9	11	12	12	17
---	---	----	---	---	---	---	----	----	----	----

then down heap again

4	4	6	7	5	14	9	11	12	12	17
---	---	---	---	---	----	---	----	----	----	----

problem 2 : sort [1, 4, 3, 9, 12, 2, 4]

phase 1: heap fixation

1) 1 | 4, 3, 9, 12, 2, 4

2)  $\begin{matrix} \swarrow & \searrow \\ 1 & 4 \end{matrix}$  | 3 9 12 2 4  $\xrightarrow{\text{upheap}}$  4 1 | 3 9 12 2 4

3) 4 1 3 | 9 12 2 4

4)  $\begin{matrix} & & \text{upheadex} \\ 4 & 1 & 3 & 9 \end{matrix}$  | 12 2 4  $\rightarrow$  9 4 3 1 | 12 2 4

5) 9 4 3 1 12 12 | 4  $\xrightarrow{\text{no heap}}$

6) 12 9 3 1 4 2  $\rightarrow$  upheap 12 9 4 2 3



+ remove Max

<sup>12</sup>  
□ 9 4 14 23 → 3 9 14 21 \_

down heap

9 3 4 14 2 → <sup>down heap</sup> 9 4 4 13 2 | 12

9 □ 4 4 1 3 2 | 12 → 4 3 4 12 | 9 12

4 □ 3 4 12 | 9 12 → 2 3 4 1 | 9 12

down heap

4 3 2 1 \_ | 9 12 → 4 3 2 1 | 4 9 12

4 □ 3 2 1 | 4 9 12 → 1 3 2 \_ | 4 9 12

down heap

3 1 2 \_ | 4 9 12 → 3 1 2 | 4 4 9 12

↓

1 □ 1 2 3 4 4 9 12 → no element to swap

1 2 3 4 4 9 12 → sorted

problem 3:

11, 5, 2, 3, 17, 24, 1

$K=11$

$A_1 = [5, 2, 3]$      $A_2 = [17, 24, 11]$

BUT  $(A_1, [5, 2, 3])$

$K=5$

$A_1 = [2]$ ,  $A_2 = [3]$

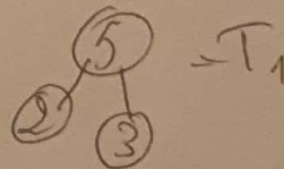
BUT  $(A_1, [2])$

~~$K=2$~~   $A_1 = \emptyset$  (2)

BUT  $A_1 = [3]$

$K=3$   $A_2 = \emptyset$  (3)

Take these nodes and the key



Now BUT  $(A_2, [17, 24, 11])$

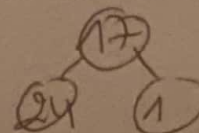
$K=17$

$A_1 = [24]$      $A_2 = 1$

BUT  $(A_1, [24])$   $A_1 = \emptyset$  (24)

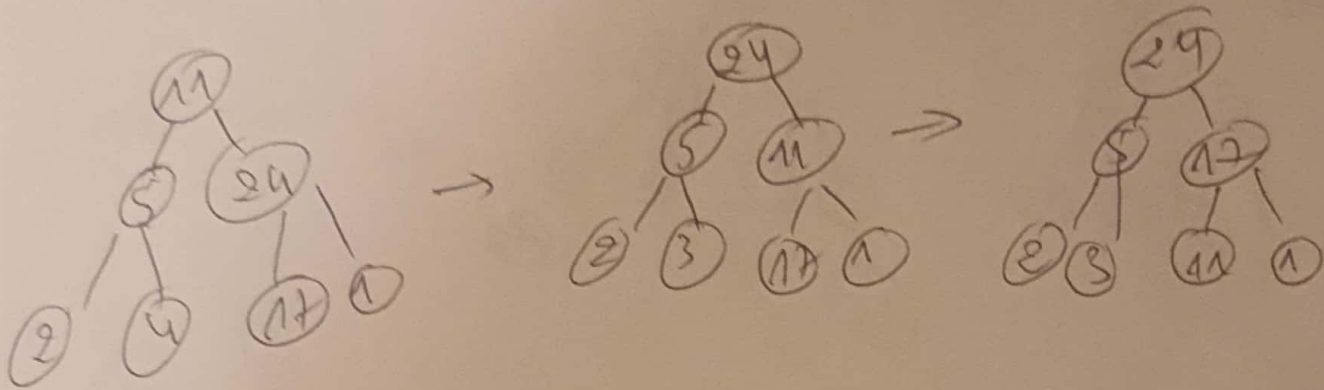
BUT  $A_2$   $A_2 = [1]$  (1)

Take 2 nodes and the key



take 2 trees and the key

②



4) draw Max heap using keys [1, 21]

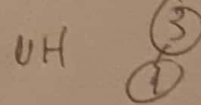
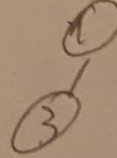
Our keys will be

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21

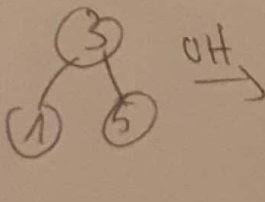
insert 1



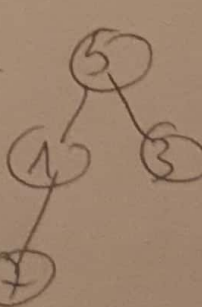
insert 3



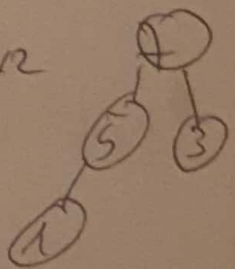
insert 5



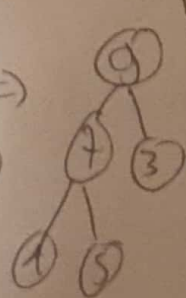
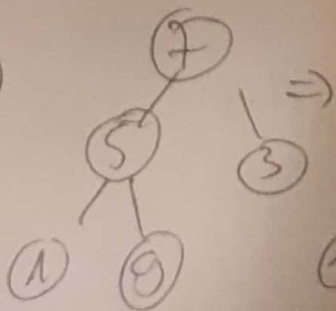
insert 7



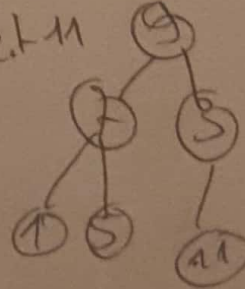
UPH x2



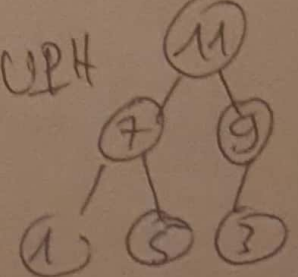
insert 9

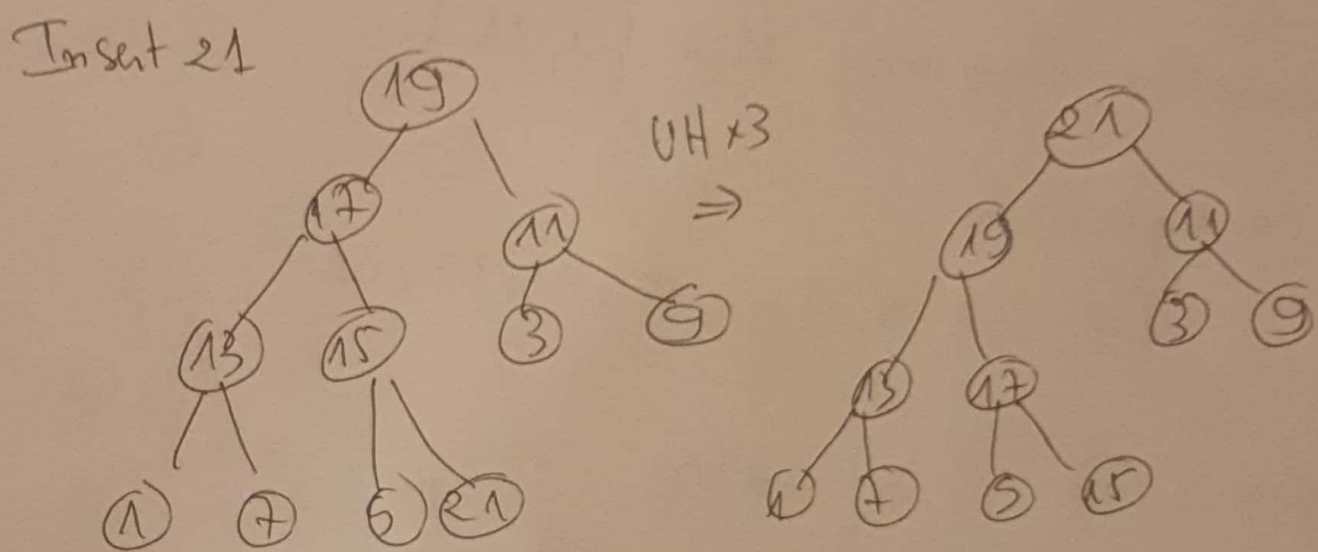
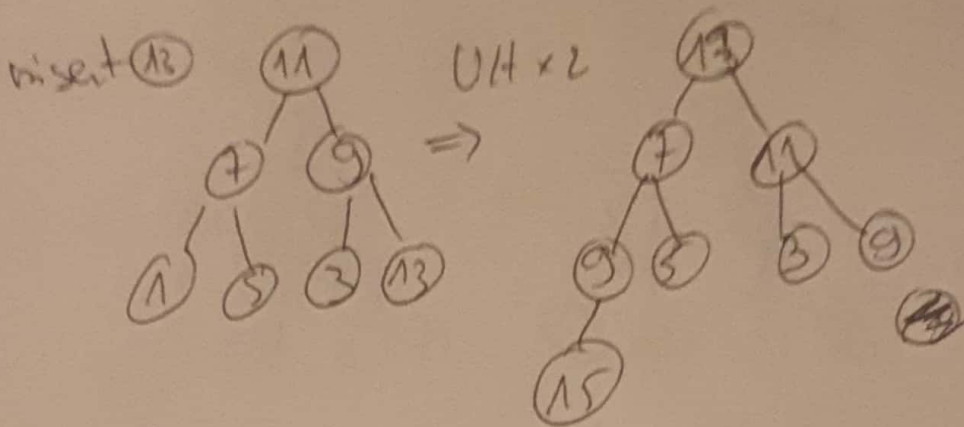


insert 11



UPH





Problem 5:

10, 3, 7, 6, 4, 12, 5, 2, 12, 10, 1

$n = 11$

Step 1: make  $\frac{n+1}{2}$  single item heap tree

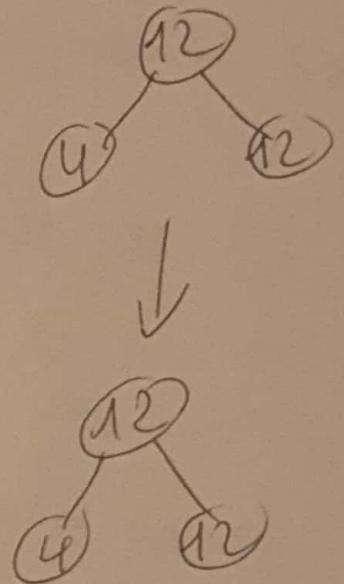
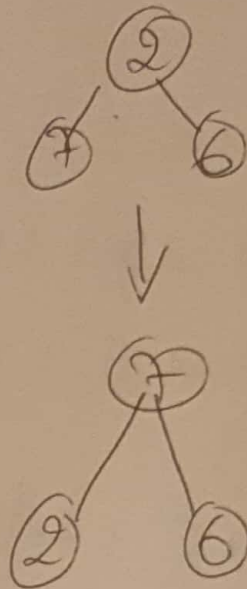
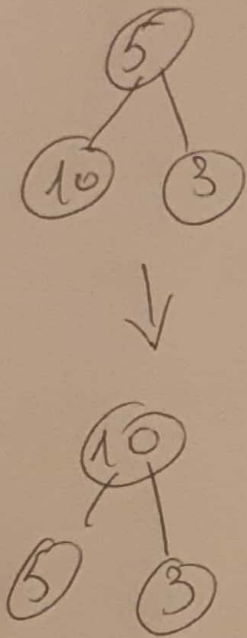
$$\frac{12}{2} = 6$$

10, 3, 7, 6, 4, 12

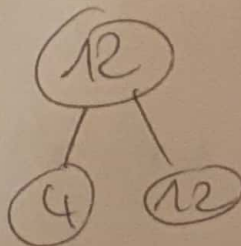
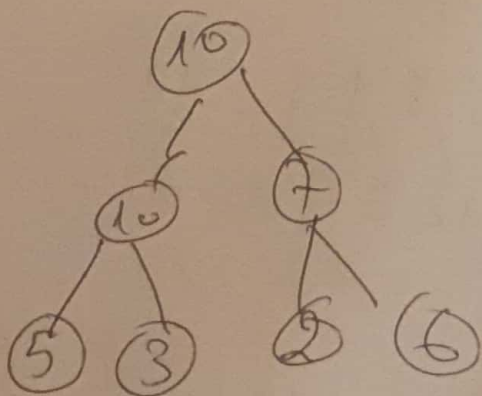


10 3 7 6 4 12

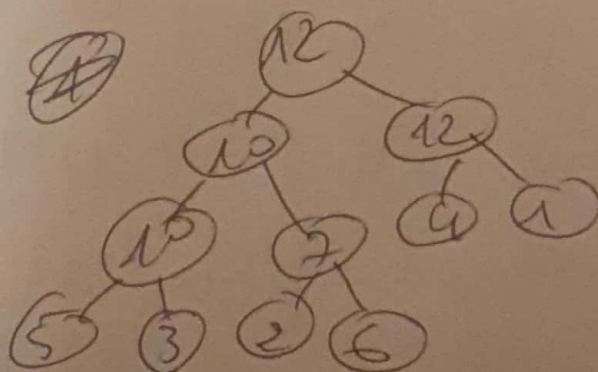
pull next  $\frac{n+1}{4} = 3$



pull next  $\frac{n+1}{8} = \frac{11+1}{8} = 1$

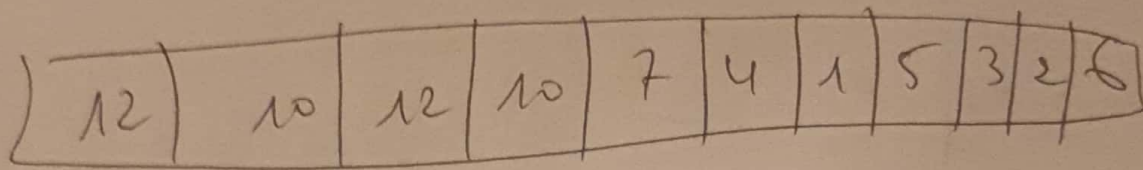


Finally



The array obtained is

(10)



Phase 2: remove max

<sup>12</sup>   10 12 10 7 4 1 5 3 2 6  
 6 10 12 10 7 4 1 5 3 2 | -  
 ↗

12 10 6 10 7 4 1 5 3 2 | ~~6~~

12 10 7 10 6 4 1 5 3 ~~2~~ | 12

<sup>12</sup>   10 7 10 6 4 1 5 3 2 | 12

downheap x3 2 ~~10~~ 7 10 6 4 1 5 3 1 | - 12

10 10 7 5 6 4 1 2 3 | 12 12

<sup>10</sup>   6 7 5 3 4 12 | 10 12 12

2 6 7 5 3 4 | - 10 12 12

7 6 4 5 3 2 1 | 10 12 12

7  $\square$  64 5321 | 10 10 12 12  
 1 645 32 | - 10 10 12 12  
 654 132 | 7 10 10 12 12

6  $\square$  54 132 | 7 10 10 12 12  
 2 54 13 | - 7 10 10 12 12  
 534 12 | 6 7 10 10 12 12

5  $\square$  34 12 | 67 10 10 12 12  
 2 34 1 | - 6 7 10 10 12 12  
 43 44 1 | 5 6 7 10 10 12 12

4  $\square$  321 | 5 67 10 10 12 12  
 13 21 | - 5 6 7 10 10 12 12  
 321 | 4 5 6 7 10 10 12 12

3  $\square$  12 | 45 678 10 10 12 12  
 21 | - 45 678 10 10 12 12  
 1234 5678 10 10 12 12

(2)

□ 1 3 4 5 6 7 10 10 12 12

11 - 3 4 5 6 7 10 12 12

1 2 3 4 5 6 7 10 10 12 12

1

□ 1 2 3 4 5 6 7 10 10 12 12

finally 1 2 3 4 5 6 7 10 10 12 12

(12)