Lab 9

(1)

1) show that G contains no cycle, then $m < n$

we have already proven this ~~for~~ for the connected graph

suppose G is not connected with component $G_1, G_2 \dots G_k$

with $K > 1$, since contains no cycle none of the components

contains a cycle either, therefore for each i we have

that G_i is a tree $\to m_i$ number of ~~the~~ edges

n_i number of vertices then

$$m = m_1 + m_2 \dots m_k$$

$$= (n_1 - 1) + \dots (n_k - 1)$$

$$= (n_1 + n_2 + \dots n_k) - k = n - k \leq n - 1 < n$$

2) ~~Suppose~~ U is connected: if u, v are vertices in U, the only

difficult case is the one in which u is in V_S and v is in V_T.

To find a path in that case, get a path p from u to x,

another path q from y to v and then combined $p \cup \{\{x, y\}\}$ is

the needed path from u to v in U

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u contains no cycle, we need to show that U has the right number of edges let $m_S = |V_S|$, $n_S = |E_S|$ and $M_T = |N_T|$ $n_T = |E_T|$ then u has $m_S + m_T$ vertices, we show u has $m_S + m_T - 1$ edges let m be the number of edges in U, then

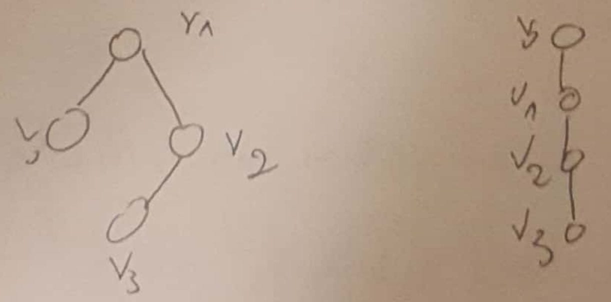
$$m = m_S + m_T + 1 = n_S - 1 + (n_T - 1) + 1 = m_S + m_T - 1 = n - 1$$

as required.

Problem 5

* proof by contradiction

let T b a non trival tree, with atleast 2 vertices



let p be the path in T of the longest possible length, it is alright if the tree have more than one path of equal length u and v represent the begining and the end vertices of p so an P is u, v_1, v_2, v

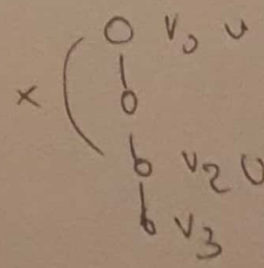
⇒ suppose, that there are not two vertices of degree one, this bring us to why we care about P, the longest path in any tree contains start and end vertices of degree one

Both ends of P must be vertices with degree one, satisfying the "at least 2 vertices of degree one"

part of the original proposition, but we are supposing that there are not two vertices of degree might

that means either u or v need to be adjecent to one more vertex in T, let's pick u, the start vertex and we will call the adcejent vertex U

which vertex is U? u is already adjecent to a vertex on the path P(v,n), if u' was another vertex on the path P then we will get a cycle.



we can't have a cycle in the tree

since the tree is undirected acyclic graph → <u>contradiction</u>

or we can show it like this

Case one : There are no vertex of degree one, since $n-1$ edges in tree total degree of any tree $2(n-1)$, but for this case, since no vertex has degree 1 then every vertex have atleast a degree of 2 and since there are n vertices, the total degree is $\geq 2n$ which contradiction.

Case 2 : There is only one vertex of degree one, similarly the total degree of any tree has to be $2(n-1)$, then there are $(n-1)$ vertices with which have degree of ≥ 2 while only one vertex which ~~have degree~~ only one vertex with degree of one, thus summing up to find the total of the vertices we have total degree of vertices is $2(n-1)+1 = 2n-1$

$\downarrow$

contradiction