Lab 12

Problem 1: No, Not every dense graph has a hamiltonian cycle.

if we have a disconnected graph G that each components have a complete subgraph of G then it is dense but we can't form a cycle to reach every edge.

assuming the graph is connected

$G = (V, E)$ it is dense if $m = C(n, 2) = \frac{n(n-1)}{2}$

if we take the degree of vertice to be average

$$\deg(v) = \frac{m}{n} \Rightarrow \frac{n-1}{2}$$

by Ore's theorem

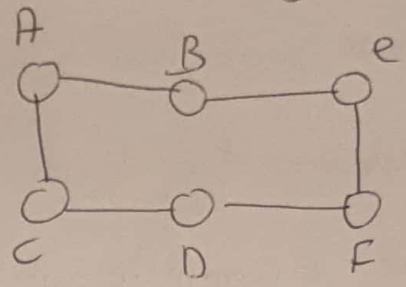
$$\deg(u) + \deg(v) \geq n$$

$$\frac{n-1}{2} + \frac{n-1}{2} \geq n$$

$n - 1 \geq n \rightarrow$ contradiction.

So being dense does not show existance of hamiltonian cycle.

Problem 2: graph G having $n=6$ vertices



A) yes, because there exists a simple cycle that connects every vertices in G

B) Ore's theorem $deg(u) + deg(v) \geq n$ when u ~~x~~ and v non adjecent then the graph a hamiltonian cycle.

$n=6$ $\quad deg(u) + deg(v) = deg(A) + deg(F) = 4$

$4 < 6$

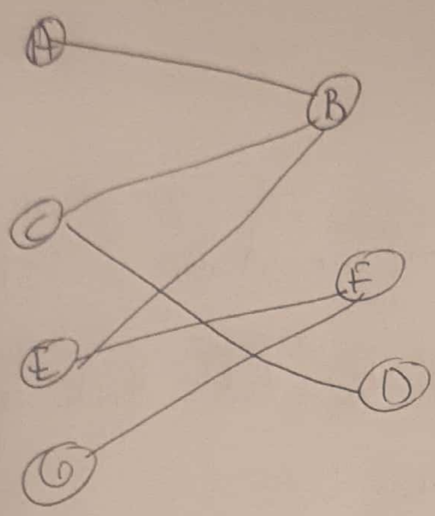
C) no, because Ore's theorem suggests that if deg(u) and degree(v) assuming that u and v are not adjecent vatices if their sum is greater or equal to ~~n~~, than it has a hamiltonian cycle.

it doesn't suggest that there is no other criteria that a graph can be shown to have hamiltonian cycle other than that

- suppose C is hamiltonian Cycle in G, G is a solution. ⑤

③

problem 3) a)



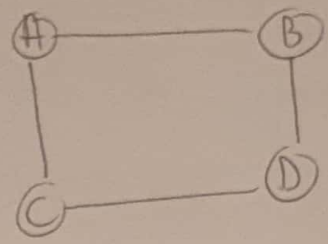
b) maximum matching is AB, CD, FG , size = 3

c) MVC - $\mu = \{B, G, D\}$, size = 3

So König's theorem that if there is maximum matching of size n then the minimum vertex cover will also be of that same size

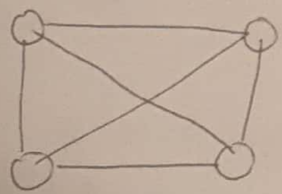
8

Problem 4)



given $G = (V, E)$ with n vertices we obtain a TSP problem

→ we need a complete graph $H = K_n$, let's think of getting H by adding the missing vertices to G



→ G is a subgraph of H

$H = K_4$

$H = (V_H, E_H)$

let's define k and function c

let $k = 0$

$c : E_H \rightarrow$

$$c(e) = \begin{cases} 0 \text{ if } e \in E \\ 1 \text{ if } e \notin E \end{cases}$$

it takes $O(n^2)$ time to do this

Now let's verify that a solution to HC problem have a solution to TSP problem

- suppose C is hamiltonian cycle in G, G is a solution.
- check C is HC in H (H → new graph for TSP)
- Since $V = V_H$ is C spaming ? yes
- if C still simple → yes

then lets verify that the solution to the TSP problem yield a soluti[on] to the HC problem, suppose C is a solution to the TSP problem

Problem 5 :

assume hamiltonian cycle is NP problem and

TSP: given a graph G with cost function $C \to N$ and positive integer k. is there a hamiltonian cycle that the sum of costs of the edges in C is at most k ?

on problem 4 we proved that hamiltonian cycle is polynomial reducable to TSP

To show that a given problem is NP complete you can show that it is reducable to another NP complete problem, Since HC is NP complete because V_C is NP Complete

TSP is also NP Complete.

Problem 6: G has a smallest vertex cover of size s (6)

b) Vertex Cover Approx outputs size $2 \times s$ as it is approximation to optimal size

Solution: Consider the following disconnected with two edges and for vertices, the smallest vertex cover has size 2 but the vertex cover approximation outputs a vertex cover of size 4