A New Efficient Algorithm for Tracking LEO Satellites

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***Abstract*—The methods of tracking Low Earth Orbit (LEO) satellites can be grouped into two different categories: "signal-based" and "program-based" methods. In this paper, we employ a new program-based tracking method to trace satellites, using Two Line orbital Elements (TLE). Using TLE is a simple and well-known method for tracking satellites; however, this paper presents a new algorithm for tracking satellites, based on TLE files. This algorithm, uses an available TLE file to estimate the satellite's position and to update the orbital elements, for any desired time that we are interested in tracing the satellite. The results show that the algorithm is accurate and reliable.**

Keywords—Program-based tracking; LEO satellites, Orbital elements; TLE component.

I. Introduction

Along with satellite technology development, researchers offered various ways for tracking LEO satellites, but many of the presented ways are either expensive or inaccurate. Therefore, there is a need to present accurate and inexpensive ways for tracking satellites. The purpose of this paper is to present a new reliable algorithm for tracking LEO satellites, which employs an available TLE file as the primary data and predicts satellites' orbital elements.

In order to uniquely identify a satellite orbit, we need its orbital elements, including a set of six parameters. These six parameters are [1]:

1. *Eccentricity (e):* This parameter determines how a satellite's orbit deviates from a perfectly circular orbit. Indeed, it determines the shape of orbit.
2. *Semi-major axis (a):* This parameter defines the size of orbit and equals to the sum of apogee and perigee, divided by two.
3. *Inclination (i)*: Inclination determines the vertical tilt of the orbit measured at the ascending node, with respect to the earth's equatorial plane.
4. *Right ascension of ascending node* (Ω): This parameter horizontally orients the ascending node of the orbit, where it passes upward (with respect to the earth's equatorial plane).

   These two elements (inclination and right ascension of ascending node) are used to determine the orientation of the orbital plane of a satellite.
5. Argument of perigee (w): This parameter is an angle measured from the ascending node to perigee, and determines the orientation of the orbit.
6. True anomaly (θ): This parameter is an angle measured from the perigee to the position of the satellite at any given time. True anomaly determines the real geometric angle of the satellite.

These six orbital elements are shown in Fig. 1.

The NORAD website [2] provides some text files that include orbital elements for various satellites. These text files are called Two-Line Elements (TLE) files. Although there are a few algorithms, such as SGP, SGP4, SDP4, SGP8, and SDP8, that use TLE files to track satellites [3], this paper presents a new reliable algorithm for tracking LEO satellites, using TLE file.

II. PROPOSED ALGORITHM

In the suggested algorithm in this paper, an available TLE file is used for extracting the initial values of orbital elements (semi-major axis, inclination, eccentricity, argument of perigee, Right Ascension of Ascending Node (RAAN), and True Anomaly (TA)). Using these initial values, the initial velocity vector and position vector, are obtained. These initial vectors are for the time that the TLE file has been generated. Using the initial velocity and position vectors, and Lagrange coefficients, the velocity and position vectors for any desired time are estimated. Finally, these new vectors are used to calculate the orbital elements and to estimate the satellite's location.

This algorithm includes the following steps:

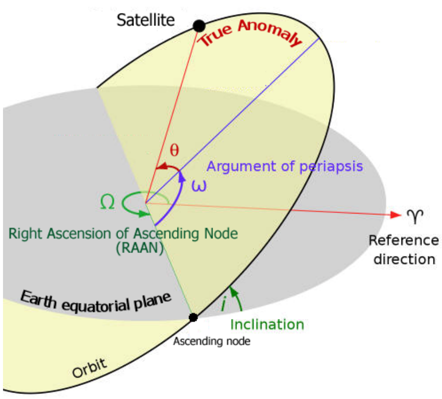

Fig. 1. Orbital elements

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A. Calculating the initial position vector and velocity vector:

Using an available TLE file, the numerical values of orbital elements, which are available in the TLE file, are extracted. Using these orbital elements, the initial position vector and velocity vector will be obtained as follow [4]:

$$\{r_0\} = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta} * \begin{Bmatrix}\cos\theta\\ \sin\theta\\ 0\end{Bmatrix} * \begin{bmatrix}\cos\Omega\cos w-\sin\Omega\sin w\cos i & -\cos\Omega\sin w-\sin\Omega\cos w\cos i & \sin\Omega\sin i\\ \cos w\sin\Omega+\sin w\cos\Omega\cos i & -\sin\Omega\sin w+\cos\Omega\cos w\cos i & -\cos\Omega\sin i\\ \sin w\sin i & \cos w\sin i & \cos i\end{bmatrix} \tag{1}$$

$$\{v_0\} = \frac{\mu}{h} * \begin{Bmatrix}-\sin\theta\\ e+\cos\theta\\ 0\end{Bmatrix} * \begin{bmatrix}\cos\Omega\cos w-\sin\Omega\sin w\cos i & -\cos\Omega\sin w-\sin\Omega\cos w\cos i & \sin\Omega\sin i\\ \cos w\sin\Omega+\sin w\cos\Omega\cos i & -\sin\Omega\sin w+\cos\Omega\cos w\cos i & -\cos\Omega\sin i\\ \sin w\sin i & \cos w\sin i & \cos i\end{bmatrix} \tag{2}$$

where θ is the initial value of true anomaly, e is initial eccentricity, w is the initial value of the argument of perigee, h is the magnitude of angular momentum, Ω is the initial value of RAAN, and μ is the standard gravitational parameter.

B. Calculating new position and velocity vectors of the satellite:

Using the initial position and velocity vectors, the new position and velocity vectors of the satellite will be calculated using (3) and (4) [4]:

$$\{r\} = f\{r_0\} + g\{v_0\} \tag{3}$$

$$\{v\} = \dot{f}\{r_0\} + \dot{g}\{v_0\}\ , \tag{4}$$

where f and g are the Lagrange coefficients. For calculating the Lagrange coefficients, the Kepler's equation is solved using Newton's method. After simplification, we have an iterative equation [4] , [5]:

$$E_{i+1} = E_i - \frac{E_i - e\sin E_i - M_e}{1 - e\cos E_i}, \tag{5}$$

where M_e is called mean anomaly and defines the angular position of the satellite along the orbit, at a specific time, and e is the eccentricity. In the above equation, E is called *eccentric anomaly* that is obtained by drawing an auxiliary circle of the ellipse with center O and focus F, and drawing a line perpendicular to the semi major and intersecting it at A. Fig. 2, shows the eccentric anomaly.

In order to determine the Lagrange coefficients, the Kepler's equation is rewritten in terms of χ , $S(a\chi_i)$, $C(a\chi_i)$:

$$\chi_{i+1} = \chi_i - \frac{\frac{r_0 v_{ro}}{\sqrt{\mu}}\chi_i^2 C(a\chi_i) + (1-\frac{r_0}{a})\chi_i^3 S(a\chi_i) + r_0\chi_i - \sqrt{\mu}\Delta t}{\frac{r_0 v_{ro}}{\sqrt{\mu}}\chi_i[1-\alpha\chi_i^2 S(a\chi_i)] + (1-\alpha r_0)\chi_i^2 C(a\chi_i) + r_0} \tag{6}$$

where 'a' is the semi-major axis, and v_{ro} is the radial velocity component:

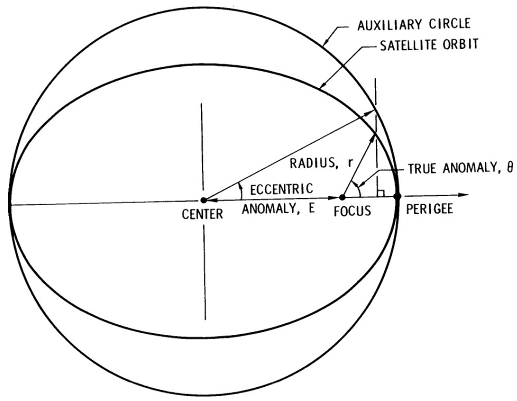

Fig. 2. Definition of eccentric anomaly ([6])

$$v_{ro} = \frac{r_o . v_o}{r_o}\ . \tag{7}$$

Equation (8) describes the relationship between χ_i and E_i [7]:

$$\chi_i = \begin{cases} \frac{h}{\sqrt{\mu}}\tan\frac{\theta}{2} & parabola \\ \sqrt{a}E_i & ellipse \\ \sqrt{-a}E_i & hyperbola \end{cases}, \tag{8}$$

where an initial estimation of χ_i can be calculated as:

$$\chi_0 = \sqrt{\mu_i}\left|\frac{1}{a}\right|\Delta t\ . \tag{9}$$

Furthermore, $S(a\chi_i)$ and $C(a\chi_i)$ are defined as follow [4]:

$$S(a\chi_i) = \begin{cases} \frac{\sqrt{a\chi_i^2} - \sin\sqrt{a\chi_i^2}}{(\sqrt{a\chi_i^2})^3} & a\chi_i^2 > o \\ \frac{\sinh\sqrt{-a\chi_i^2} - \sqrt{-a\chi_i^2}}{(\sqrt{-a\chi_i^2})^3} & a\chi_i^2 < o \\ \frac{1}{6} & a\chi_i^2 = o \end{cases} \tag{10}$$

$$C(a\chi_i) = \begin{cases} \frac{1-\cos\sqrt{a\chi_i^2}}{a\chi_i^2} & a\chi_i^2 > o \\ \frac{\cosh\sqrt{-a\chi_i^2} - 1}{-a\chi_i^2} & a\chi_i^2 < o \\ \frac{1}{2} & a\chi_i^2 = o \end{cases} \tag{11}$$

These parameters are used to calculate the Lagrange coefficients, using the following steps [4]:

1. In each step, with χ_i obtained in the previous step, $S(a\chi_i)$, $C(a\chi_i)$, $f(\chi_i)$, and $f'(\chi_i)$ will be calculated:

$$f(\chi_i) = \left(\frac{r_0 v_{r0}}{\sqrt{\mu}}\right)\chi_i^2 C(a\chi_i^2) + (1-\frac{r_0}{a})\chi_i^3 S(a\chi_i^2) + r_0\chi_i - \sqrt{\mu}\Delta t \quad (12)$$

$$f'(\chi_i) = \left(\frac{r_0 v_{r0}}{\sqrt{\mu}}\right)\chi_i \,[1-\frac{\chi_i^2}{a} S(a\chi_i^2)] + (1-\frac{r_0}{a})\chi_i^2 C(a\chi_i^2) + r \quad (13)$$

If $\left| f(\chi_i) \big/ f'(\chi_i) \right|$ is less than the desired error, this χ_i will be accepted, and will be used for calculating the Lagrange coefficients, using (15) and (16). Otherwise, the new value of χ_i will be calculated using (14), and the algorithm will return to (10) for calculating new values of $S(a\chi_i)$, $C(a\chi_i)$, $f(\chi_i)$, and $f'(\chi_i)$:

$$\chi_{i+1} = \chi_i - \left|\frac{f(\chi_i)}{f'(\chi_i)}\right|. \quad (14)$$

2. Finally, f and g that are the Lagrange coefficients are calculated using (15) and (16):

$$g = \Delta t - \frac{1}{\sqrt{\mu}}\chi^3 S(a\chi^2) \quad (15)$$

$$f = 1 - \frac{\chi^2}{r_0} C(a\chi^2)\,, \quad (16)$$

where a is the semi major axis that will be calculated as follow:

$$\frac{1}{a} = \frac{2}{r_0} - \frac{v_0^2}{\mu} \quad (17)$$

$\dot{f}$ and $\dot{g}$ can be calculated using r_0, χ, a and r:

$$\dot{g} = 1 - \frac{\chi^2}{r} C(\alpha\chi^2) \quad (18)$$

$$\dot{f} = \frac{\sqrt{\mu}}{r r_0}[\alpha\chi^3 S(\alpha\chi^2) - \chi]\,. \quad (19)$$

After calculating f, g, $\dot{f}$, and $\dot{g}$, the new position vector and velocity vector can be calculated using (3) and (4).

C. Calculating new orbital elements

After computing new position vector (r) and velocity vector (v), new orbital elements that are for any desired time will be calculated as follow [8], [9]:

- for calculating orbital elements, the angular momentum is calculated using (20):

$$\mathrm{h} = \mathrm{r} \times \mathrm{v} \quad (20)$$

- The inclination is calculated as the second orbital elements, using the following equation:

$$\mathrm{i} = \cos^{-1}\left(\frac{h_z}{|h|}\right) \quad (21)$$

In the above equation, if $90 < i \le 180$, the direction of the satellite motion is against the direction of the earth motion.

- The Right Ascension of Ascending Node (RAAN), that is the third orbital element, can be calculated using (22):

$$\Omega = \begin{cases} \cos^{-1}(N_X \big/ |N|) & N_Y \ge 0 \\ 360 - \cos^{-1}(N_X \big/ |N|) & N_Y < 0 \end{cases} \quad (22)$$

where $\{\mathrm{N}\}$ is a vector between the ascending node and the descending node, and is calculated using:

$$\{\mathrm{N}\} = \hat{\mathrm{K}} \times \mathrm{h} \quad (23)$$

- In this step, the eccentricity vector and its magnitude will be calculated as the fourth orbital element:

$$e = \frac{1}{\mu}\sqrt{\left(2\mu - rv^2\right) r v_r^2 + \left(\mu - rv^2\right)^2} \quad (24)$$

- Using h and e, the first orbital element that is the semi-major axis is calculated:

$$a = h^2 \big/ \mu(1-e^2) \quad (25)$$

- The argument of perigee can be calculated using the following equation:

$$w = \begin{cases} \cos^{-1}(N.e \big/ |N||e|) & e_z \ge 0 \\ 360 - \cos^{-1}(N.e \big/ |N||e|) & e_z < 0 \end{cases} \quad (26)$$

In the above equation, if $N.e > 0$, the w is located in either the first or fourth quadrant. Using the following equation, the True Anomaly (TA) can be calculated:

$$\theta = \begin{cases} \cos^{-1}(\mathrm{e.r} \big/ |e||r|) & v_r \ge 0 \\ 360 - \cos^{-1}(\mathrm{e.r} \big/ |e||r|) & v_r < 0 \end{cases} \quad (27)$$

III. CHECKING THE ACCURACY OF THE SUGGESTED ALGORITHM

The described algorithm has been written using C++ software. In order to check the accuracy of this program, six

TLE files for the IRS-1B satellite are used. Based on the available information in these TLE files, the last five TLE files that we used are extracted 0.8586, 1.5762, 2.939, 3.9382, and 5.0156 days after the first TLE file, respectively.

First, we executed the program using the first TLE file, that is downloaded from the NORAD website, to calculate the orbital elements of the IRS-1B satellite, after 0.8586, 1.5762, 2.939, 3.9382, 5.0156 days. After calculating the orbital elements, we compared the results of the program with the orbital elements that are available in the corresponding TLE files, that have been downloaded from the NORAD website. In Fig. 3, which is plotted using MATLAB, we compared the numerical values of one of the orbital elements (True Anomaly) that are extracted using the program for the last four days, and the real values of True Anomaly that are available in the corresponding TLE files. As the Fig. 3 shows, this algorithm precisely predicts the geometric angle of the satellite (True Anomaly).

Table I and table II show the results of the simulation for 2.939 and 5.0156 days after extracting the first TLE file, and compare them with the real values. As the tables show, this program is accurate and can precisely predict the orbital elements of a satellite, based on an initial TLE file.

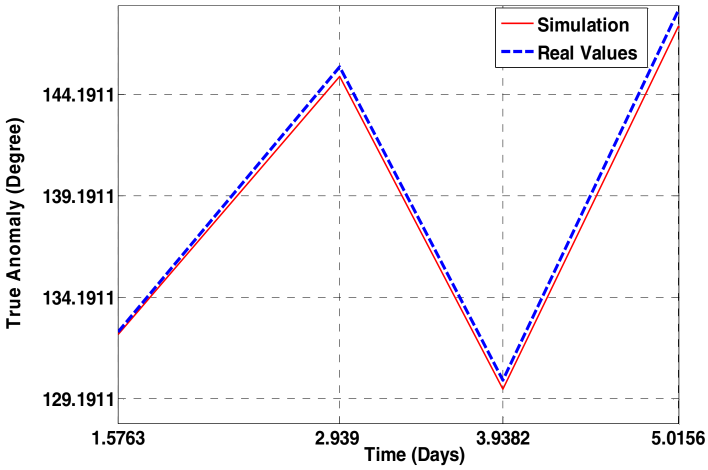

Fig. 3. Comparing the values of True Anomaly that are extracted from the program, with the values that are available in TLE files.

TABLE I. ORBITAL ELEMENTS EXTRACTED FROM AN AVAILABLE TLE FILE AND EXECUTING PROGRAM ON THE PRIMARY TLE (2.9382 DAYS LATER)

Parameters	From TLE	From program	errors
Semi major axis	7282.95	7282.95	0%
Eccentricity	0.0024287	0.0024416	0.53%
RAAN	57.0689	56.2254	1.48%
Inclination	98.8958	98.8958	0%
Argument of Perigee	336.3878	342.131	1.70%
TA	145.5506	145.071	0.33%

TABLE II. ORBITAL ELEMENTS EXTRACTED FROM AN AVAILABLE TLE FILE AND EXECUTING PROGRAM ON THE PRIMARY TLE (5.0156 DAYS LATER)

Parameters	From TLE	From program	errors
Semi major axis	7282.95	7282.95	0%
Eccentricity	0.0024203	0.0024416	0.88%
RAAN	59.0781	58.2254	1.44%
Inclination	98.8958	98.8958	0%
Argument of Perigee	330.3620	337.131	2.04%
TA	148.3167	147.535	0.53%

IV. CONCLUSION

Using TLE files is a well-known method for tracking LEO satellites. However, using an available TLE file for tracking satellites is not an accurate method, because the TLE elements that are available in the web pages are for a special time, not for any time that we are interested in tracking a satellite. In this article, we employed a new program-based tracking algorithm to trace satellites. In this algorithm, an available TLE file is used for estimating and creating a new TLE file for any desired time that we are interested in tracing the satellite. This algorithm is a simple inexpensive method that can be used to find the position of a satellite easily. Furthermore, the results indicated that this algorithm is accurate and reliable.

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