Chapter 16

Relations

1. Consider the following relations :

   $R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$;

   $S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| m, n, p \text{ and } q \text{ are integers such that } n, q \neq 0 \text{ and } qm = pn\right\}$. Then **[AIEEE-2010]**

   (1) R is an equivalence relation but S is not an equivalence relation

   (2) Neither R nor S is an equivalence relation

   (3) S is an equivalence relation but R is not an equivalence relation

   (4) R and S both are equivalence relations

2. If $R = \{(x, y) ; x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is **[JEE (Main)-2020]**

   (1) {0, 1}

   (2) {–2, –1, 1, 2}

   (3) {–1, 0, 1}

   (4) {–2, –1, 0, 1, 2}

3. Let R_1 and R_2 be two relation defined as follows :

   $R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

   $R_2 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$, where Q is the set of all rational numbers. Then **[JEE (Main)-2020]**

   (1) Neither R_1 nor R_2 is transitive.

   (2) R_2 is transitive but R_1 is not transitive.

   (3) R_1 and R_2 are both transitive.

   (4) R_1 is transitive but R_2 is not transitive.

4. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$.

   Then the composition function f(g(x)) is : **[JEE (Main)-2021]**

   (1) neither one-one nor onto

   (2) onto but not one-one

   (3) both one-one and onto

   (4) one-one but not onto

5. Let R = {(P, Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, –1) is the set : **[JEE (Main)-2021]**

   (1) $S = \{(x, y) \mid x^2 + y^2 = 2\}$

   (2) $S = \{(x, y) \mid x^2 + y^2 = 1\}$

   (3) $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$

   (4) $S = \{(x, y) \mid x^2 + y^2 = 4\}$

6. Let A = {2, 3, 4, 5, ..., 30} and '$\simeq$' be an equivalence relation on A × A, defined by $(a, b) \simeq (c, d)$, if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to : **[JEE (Main)-2021]**

   (1) 7 (2) 8

   (3) 5 (4) 6

7. Let N be the set of natural numbers and a relation R on N be defined by

   $R = \{(x, y) \in N \times N : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$.

   Then the relation R is **[JEE (Main)-2021]**

   (1) An equivalence relation

   (2) Reflexive and symmetric, but not transitive

   (3) Reflexive but neither symmetric nor transitive

   (4) Symmetric but neither reflexive nor transitive

8. Let $\mathbb{Z}$ be the set of all integers,

   $A = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + y^2 \leq 4\}$,

   $B = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\}$ and

   $C = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + (y^2-2)^2 \leq 4\}$

   If the total number of relations from A ∩ B to A ∩ C is 2^p, then the value of p is **[JEE (Main)-2021]**

   (1) 16 (2) 49

   (3) 25 (4) 9

9. Which of the following is **not** correct for relation R on the set of real numbers?

[JEE (Main)-2021]

(1) $(x, y) \in R \Leftrightarrow |x - y| \leq 1$ is reflexive and symmetric.

(2) $(x, y) \in R \Leftrightarrow 0\ |x| - |y| \leq 1$ is neither transitive nor symmetric

(3) $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive

(4) $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric

10. Let R_1 and R_2 be relations on the set {1, 2, ..., 50} such that

$R_1 = \{(p, p^n) : p$ is a prime and $n \geq 0$ is an integer$\}$ and $R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or 1$\}$.

Then, the number of elements in $R_1 - R_2$ is _____.

[JEE (Main)-2022]

11. Let $R_1 = \{(a, b) \in \mathbf{N} \times \mathbf{N} : |a - b| \leq 13\}$ and

$R_2 = \{(a, b) \in \mathbf{N} \times \mathbf{N} : |a - b| \text{ “‘ } 13\}$. Then on **N**:

[JEE (Main)-2022]

(1) Both R_1 and R_2 are equivalence relations

(2) Neither R_1 nor R_2 is an equivalence relation

(3) R_1 is an equivalence relation but R_2 is not

(4) R_2 is an equivalence relation but R_1 is not

12. Let a set $A = A_1 \cup A_2 \cup \ldots \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j$, $1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i$, $1 \leq i \leq k\}$. Then, R is :

[JEE (Main)-2022]

(1) reflexive, symmetric but not transitive

(2) reflexive, transitive but not symmetric

(3) reflexive but not symmetric and transitive

(4) an equivalence relation

13. Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by $a\ R_1\ b \Leftrightarrow ab \geq 0$ and $a\ R_2\ b \Leftrightarrow a \geq b$. Then,

[JEE (Main)-2022]

(1) R_1 is an equivalence relation but not R_2

(2) R_2 is an equivalence relation but not R_1

(3) Both R_1 and R_2 are equivalence relations

(4) Neither R_1 nor R_2 is an equivalence relation

14. For $\alpha \in \mathbf{N}$, consider a relation R on **N** given by $R = \{(x, y) : 3x + \alpha y$ is a multiple of 7$\}$. The relation R is an equivalence relation if and only if

[JEE (Main)-2022]

(1) $\alpha = 14$

(2) α is a multiple of 4

(3) 4 is the remainder when α is divided by 10

(4) 4 is the remainder when α is divided by 7

15. Let R be a relation from the set {1, 2, 3, ..., 60} to itself such that $R = \{(a, b) : b = pq$, where $p, q \geq 3$ are prime numbers$\}$. Then, the number of elements in R is : **[JEE (Main)-2022]**

(1) 600 (2) 660

(3) 540 (4) 720

❑ ❑ ❑