

Chapter 5

Sequences and Series

1. The sum to infinity of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots$ is [AIEEE-2009]
- (1) 3 (2) 4
(3) 6 (4) 2
2. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \dots = a_{10} = 150$ and a_{10}, a_{11}, \dots are in an AP with common difference -2 , then the time taken by him to count all notes is [AIEEE-2010]
- (1) 24 minutes (2) 34 minutes
(3) 125 minutes (4) 135 minutes
3. Let a_n be the n^{th} term of an A.P.
If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is [AIEEE-2011]
- (1) $\beta - \alpha$ (2) $\frac{\alpha - \beta}{200}$
(3) $\alpha - \beta$ (4) $\frac{\alpha - \beta}{100}$
4. **Statement-1:** The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \dots + (361 + 380 + 400)$ is 8000.
Statement-2: $\sum_{k=1}^n (k^3 - (k-1)^3) = n^3$ for any natural number n . [AIEEE-2012]
- (1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true, statement-2 is **not** a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.
5. If 100 times the 100th term of an AP with non-zero common difference equals the 50 times its 50th term, then the 150th term of this AP is [AIEEE-2012]
- (1) 150 times its 50th term
(2) 150
(3) Zero
(4) -150
6. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,, is [JEE (Main)-2013]
- (1) $\frac{7}{81}(179 - 10^{-20})$ (2) $\frac{7}{9}(99 - 10^{-20})$
(3) $\frac{7}{81}(179 + 10^{-20})$ (4) $\frac{7}{9}(99 + 10^{-20})$
7. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is [JEE (Main)-2014]
- (1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$
(3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$
8. If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + \dots + 10(11)^9 = k(10)^9$, then k is equal to [JEE (Main)-2014]
- (1) 100 (2) 110
(3) $\frac{121}{10}$ (4) $\frac{441}{100}$
9. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is [JEE (Main)-2014]
- (1) $2 - \sqrt{3}$ (2) $2 + \sqrt{3}$
(3) $\sqrt{2} + \sqrt{3}$ (4) $3 + \sqrt{2}$

10. If m is the A.M. of two distinct real numbers l and n ($l, n > 1$) and G_1, G_2 and G_3 are three geometric means between l and n , then $G_1^4 + 2G_2^4 + G_3^4$ equals. [JEE (Main)-2015]

- (1) $4l^2mn$ (2) $4lm^2n$
(3) $4lmn^2$ (4) $4l^2m^2n^2$

11. The sum of first 9 terms of the series

$$\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \dots \text{ is}$$

[JEE (Main)-2015]

- (1) 71 (2) 96
(3) 142 (4) 192

12. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is [JEE (Main)-2016]

- (1) $\frac{4}{3}$ (2) 1
(3) $\frac{7}{4}$ (4) $\frac{8}{5}$

13. If the sum of the first ten terms of the series

$$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \dots,$$

is $\frac{16}{5}m$, then m is equal to [JEE (Main)-2016]

- (1) 101 (2) 100
(3) 99 (4) 102

14. For any three positive real numbers a, b and c ,

$$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c).$$

Then

[JEE (Main)-2017]

- (1) b, c and a are in A.P.
(2) a, b and c are in A.P.
(3) a, b and c are in G.P.
(4) b, c and a are in G.P.

15. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and

$$f(x+y) = f(x) + f(y) + xy, \forall x, y \in R,$$

then $\sum_{n=1}^{10} f(n)$ is equal to [JEE (Main)-2017]

- (1) 165 (2) 190
(3) 255 (4) 330

16. Let $a_1, a_2, a_3, \dots, a_{49}$ be in A.P. such that

$$\sum_{k=0}^{12} a_{4k+1} = 416 \text{ and } a_9 + a_{43} = 66.$$

If $a_1^2 + a_2^2 + \dots + a_{17}^2 = 140m$, then m is equal to

[JEE (Main)-2018]

- (1) 66 (2) 68
(3) 34 (4) 33

17. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \dots$$

If $B - 2A = 100\lambda$, then λ is equal to

[JEE (Main)-2018]

- (1) 232 (2) 248
(3) 464 (4) 496

18. Let a_1, a_2, \dots, a_{30} be an A.P., $S = \sum_{i=1}^{30} a_i$ and

$$T = \sum_{i=1}^{15} a_{(2i-1)}. \text{ If } a_5 = 27 \text{ and } S - 2T = 75, \text{ then}$$

a_{10} is equal to

[JEE (Main)-2019]

- (1) 47 (2) 57
(3) 52 (4) 42

19. If a, b and c be three distinct real numbers in G.P. and $a + b + c = xb$, then x cannot be

[JEE (Main)-2019]

- (1) 2 (2) -3
(3) -2 (4) 4

20. The sum of the following series

$$1 + 6 + \frac{9(1^2 + 2^2 + 3^2)}{7} + \frac{12(1^2 + 2^2 + 3^2 + 4^2)}{9}$$

$$+ \frac{15(1^2 + 2^2 + \dots + 5^2)}{11} + \dots$$

up to 15 terms, is

[JEE (Main)-2019]

- (1) 7830 (2) 7820
(3) 7520 (4) 7510

21. Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also

the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to [JEE (Main)-2019]

- (1) $\frac{1}{2}$ (2) 4
(3) $\frac{7}{13}$ (4) 2

22. Let a_1, a_2, \dots, a_{10} be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals
[JEE (Main)-2019]
(1) 5^3
(2) 5^4
(3) $2(5^2)$
(4) $4(5^2)$
23. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is
[JEE (Main)-2019]
(1) $\frac{1}{3}$
(2) $\frac{2}{9}$
(3) $\frac{2}{3}$
(4) $\frac{4}{9}$
24. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is
[JEE (Main)-2019]
(1) 2 : 1
(2) 1 : 3
(3) 4 : 1
(4) 3 : 1
25. Let $S_n = 1 + q + q^2 + \dots + q^n$ and
$$T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \dots + \left(\frac{q+1}{2}\right)^n$$

where q is a real number and $q \neq 1$.
If $^{101}C_1 + ^{101}C_2 \cdot S_1 + \dots + ^{101}C_{101} \cdot S_{100} = \alpha T_{100}$.
[JEE (Main)-2019]
(1) 200
(2) 202
(3) 2^{99}
(4) 2^{100}
26. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is
[JEE (Main)-2019]
(1) 36
(2) 32
(3) 24
(4) 28
27. Let $S_k = \frac{1+2+3+\dots+k}{k}$. If $S_1^2 + S_2^2 + \dots + S_{10}^2 = \frac{5}{12}A$, then A is equal to
[JEE (Main)-2019]
(1) 303
(2) 156
(3) 301
(4) 283
28. If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \dots$ is equal to $225k$, then k is equal to
[JEE (Main)-2019]
(1) 108
(2) 27
(3) 9
(4) 54
29. The sum of all natural numbers ' n ' such that $100 < n < 200$ and H.C.F. (91, n) > 1 is :
[JEE (Main)-2019]
(1) 3303
(2) 3121
(3) 3203
(4) 3221
30. The sum $\sum_{k=1}^{20} k \frac{1}{2^k}$ is equal to
[JEE (Main)-2019]
(1) $2 - \frac{3}{2^{17}}$
(2) $1 - \frac{11}{2^{20}}$
(3) $2 - \frac{21}{2^{20}}$
(4) $2 - \frac{11}{2^{19}}$
31. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, \dots be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to
[JEE (Main)-2019]
(1) (50, $50 + 46A$)
(2) (A , $50 + 45A$)
(3) (A , $50 + 46A$)
(4) (50, $50 + 45A$)
32. The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \dots$ upto 11th term is
[JEE (Main)-2019]
(1) 916
(2) 946
(3) 945
(4) 915
33. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11th term is
[JEE (Main)-2019]
(1) -36
(2) 25
(3) -25
(4) -35
34. If $a_1, a_2, a_3, \dots, a_n$ are in A.P. and $a_1 + a_4 + a_7 + \dots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to
[JEE (Main)-2019]
(1) 98
(2) 38
(3) 64
(4) 76
35. The sum $\frac{3 \times 1^3}{1^2} + \frac{5 \times (1^3 + 2^3)}{1^2 + 2^2} + \frac{7 \times (1^3 + 2^3 + 3^3)}{1^2 + 2^2 + 3^2} + \dots$ upto 10th term, is
[JEE (Main)-2019]
(1) 620
(2) 600
(3) 680
(4) 660

36. The sum $1 + \frac{1^3 + 2^3}{1+2} + \frac{1^3 + 2^3 + 3^3}{1+2+3} + \dots$
 $+ \frac{1^3 + 2^3 + 3^3 + \dots + 15^3}{1+2+3+\dots+15} - \frac{1}{2}(1+2+3+\dots+15)$

is equal to [JEE (Main)-2019]

- (1) 1860 (2) 620
 (3) 660 (4) 1240

37. Let a , b and c be in G.P. with common ratio r , where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If $3a$, $7b$ and $15c$ are the first three terms of an A.P., then the 4th term of this A.P. is [JEE (Main)-2019]

- (1) $\frac{2}{3}a$ (2) a
 (3) $\frac{7}{3}a$ (4) $5a$

38. Let a_1, a_2, a_3, \dots be an A.P. with $a_6 = 2$. Then the common difference of this A.P., which maximises the product $a_1 a_4 a_5$, is [JEE (Main)-2019]

- (1) $\frac{2}{3}$ (2) $\frac{8}{5}$
 (3) $\frac{3}{2}$ (4) $\frac{6}{5}$

39. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to [JEE (Main)-2019]

- (1) -260 (2) -380
 (3) -320 (4) -410

40. If a_1, a_2, a_3, \dots are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is [JEE (Main)-2019]

- (1) 150 (2) 280
 (3) 200 (4) 120

41. The greatest positive integer k , for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \dots + 49^2 + 49 + 1$, is

[JEE (Main)-2020]

- (1) 65 (2) 60
 (3) 32 (4) 63

42. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is

[JEE (Main)-2020]

- (1) $\frac{21}{2}$ (2) 7
 (3) 27 (4) 16

43. Let a_1, a_2, a_3, \dots be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^9 a_i = 4\lambda$, then λ is equal to [JEE (Main)-2020]

- (1) -513 (2) -171
 (3) $\frac{511}{3}$ (4) 171

44. If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \dots$ is $(102)m$, then m is equal to

[JEE (Main)-2020]

- (1) 5 (2) 20
 (3) 25 (4) 10

45. Let $f: R \rightarrow R$ be such that for all $x \in R$ ($2^{1+x} + 2^{1-x}$), $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is [JEE (Main)-2020]

- (1) 2 (2) 0
 (3) 3 (4) 4

46. If the 10th term of an A.P. is $\frac{1}{20}$ and its 20th term is $\frac{1}{10}$, then the sum of its first 200 terms is

[JEE (Main)-2020]

- (1) $50\frac{1}{4}$ (2) 50
 (3) 100 (4) $100\frac{1}{2}$

47. The product

$$\frac{1}{2^4} \cdot \frac{1}{4^{16}} \cdot \frac{1}{8^{48}} \cdot \frac{1}{16^{128}} \cdot \dots \text{ to } \infty \text{ is equal to}$$

[JEE (Main)-2020]

- (1) $\frac{1}{2^2}$ (2) $\frac{1}{2^4}$
 (3) 2 (4) 1

48. Let a_n be the n^{th} term of a G.P. of positive terms.

If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to

[JEE (Main)-2020]

- (1) 300 (2) 150
(3) 175 (4) 225

49. If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \dots$ is

[JEE (Main)-2020]

- (1) $\frac{x+y+xy}{(1+x)(1+y)}$ (2) $\frac{x+y-xy}{(1-x)(1-y)}$
(3) $\frac{x+y-xy}{(1+x)(1+y)}$ (4) $\frac{x+y+xy}{(1-x)(1-y)}$

50. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in

[JEE (Main)-2020]

- (1) $(-\infty, -9] \cup [3, \infty)$
(2) $[-3, \infty)$
(3) $(-\infty, -3] \cup [9, \infty)$
(4) $(-\infty, 9]$

51. If the sum of first 11 terms of an A.P., a_1, a_2, a_3, \dots is 0 ($a_1 \neq 0$), then the sum of the A.P., $a_1, a_3, a_5, \dots, a_{23}$ is ka_1 , where k is equal to

[JEE (Main)-2020]

- (1) $-\frac{121}{10}$ (2) $-\frac{72}{5}$
(3) $\frac{72}{5}$ (4) $\frac{121}{10}$

52. Let S be the sum of the first 9 terms of the series : $\{x + ka\} + \{x^2 + (k+2)a\} + \{x^3 + (k+4)a\} + \{x^4 + (k+6)a\} + \dots$ where $a \neq 0$ and $x \neq 1$.

If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to

[JEE (Main)-2020]

- (1) -3 (2) 1
(3) -5 (4) 3

53. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is

[JEE (Main)-2020]

- (1) $\frac{1}{6}$ (2) $\frac{1}{4}$
(3) $\frac{1}{7}$ (4) $\frac{1}{5}$

54. If the sum of the series

$20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$ upto n^{th} term is 488 and the n^{th} term is negative, then

[JEE (Main)-2020]

- (1) $n = 41$ (2) n^{th} term is $-4\frac{2}{5}$
(3) $n = 60$ (4) n^{th} term is -4

55. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is

[JEE (Main)-2020]

- (1) 3 : 1 (2) 5 : 3
(3) 9 : 7 (4) 33 : 31

56. If $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \dots + (1 - 20^2 \cdot 19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to

[JEE (Main)-2020]

- (1) (10, 103)
(2) (10, 97)
(3) (11, 97)
(4) (11, 103)

57. Let a_1, a_2, \dots, a_n be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \dots + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to

[JEE (Main)-2020]

- (1) (2490, 249) (2) (2480, 249)
(3) (2490, 248) (4) (2480, 248)

58. If $3^{2 \sin 2\alpha - 1}$, 14 and $3^{4 - 2 \sin 2\alpha}$ are the first three terms of an A.P. for some α , then the sixth term of this A.P. is

[JEE (Main)-2020]

- (1) 65 (2) 78
(3) 81 (4) 66

59. If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \dots + 2 \cdot 3^9 + 3^{10} = S - 2^{11}$, then S is equal to

[JEE (Main)-2020]

- (1) $2 \cdot 3^{11}$ (2) $3^{11} - 2^{12}$
(3) $\frac{3^{11}}{2} + 2^{10}$ (4) 3^{11}

60. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is

[JEE (Main)-2020]

- (1) $\frac{2}{13}(3^{50} - 1)$ (2) $\frac{1}{13}(3^{50} - 1)$
(3) $\frac{1}{26}(3^{49} - 1)$ (4) $\frac{1}{26}(3^{50} - 1)$

61. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \dots$ is 460, then x is equal to **[JEE (Main)-2020]**

- (1) 7^2 (2) e^2
(3) $7^{1/2}$ (4) $7^{46/21}$

62. Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then **[JEE (Main)-2020]**

- (1) a, c, p are in G.P.
(2) a, b, c, d are in A.P.
(3) a, c, p are in A.P.
(4) a, b, c, d are in G.P.

63. The common difference of the A.P. b_1, b_2, \dots, b_m is 2 more than the common difference of A.P. a_1, a_2, \dots, a_n . If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to **[JEE (Main)-2020]**

- (1) -127 (2) -81
(3) 127 (4) 81

64. The sum $\sum_{k=1}^{20} (1 + 2 + 3 + \dots + k)$ is **[JEE (Main)-2020]**

[JEE (Main)-2020]

65. The sum, $\sum_{n=1}^7 \frac{n(n+1)(2n+1)}{4}$ is equal to **[JEE (Main)-2020]**

[JEE (Main)-2020]

66. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is **[JEE (Main)-2020]**

[JEE (Main)-2020]

67. The value of $(0.16)^{\log_{2.5} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{to } \infty \right)}$ is equal to **[JEE (Main)-2020]**

[JEE (Main)-2020]

68. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to **[JEE (Main)-2020]**

[JEE (Main)-2020]

69. Let $A = \{n \in \mathbb{N} : n \text{ is a 3-digit number}\}$

$$B = \{9k + 2 : k \in \mathbb{N}\}$$

$$\text{and } C = \{9k + 1 : k \in \mathbb{N}\} \text{ for some } 1 (0 < 1 < 9)$$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400 , then 1 is equal to **[JEE (Main)-2021]**

[JEE (Main)-2021]

70. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in **[JEE (Main)-2021]**

$$\left(0, \frac{\pi}{2}\right) \text{ is } \underline{\hspace{2cm}}.$$

[JEE (Main)-2021]

71. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices **[JEE (Main)-2021]**

$$(a, c), (2, b) \text{ and } (a, b) \text{ be } \left(\frac{10}{3}, \frac{7}{3}\right). \text{ If } \alpha, \beta \text{ are the}$$

roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is : **[JEE (Main)-2021]**

$$(1) \frac{69}{256} \quad (2) -\frac{71}{256}$$

$$(3) -\frac{69}{256} \quad (4) \frac{71}{256}$$

72. The sum of first four terms of a geometric progression (G.P) is $\frac{65}{12}$ and the sum of their **[JEE (Main)-2021]**

respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α , then 2α is **[JEE (Main)-2021]**

[JEE (Main)-2021]

73. If $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n} \theta$, $y = \sum_{n=0}^{\infty} \sin^{2n} \phi$ and **[JEE (Main)-2021]**

$z = \sum_{n=0}^{\infty} \cos^{2n} \theta \cdot \sin^{2n} \phi$ then : **[JEE (Main)-2021]**

- (1) $xyz = 4$
(2) $xy - z = (x + y)z$
(3) $xy + yz + zx = z$
(4) $xy + z = (x + y)z$

74. Let A_1, A_2, A_3, \dots be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1} . If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is **[JEE (Main)-2021]**

[JEE (Main)-2021]

75. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in \mathbb{R}$ and $a > 0$, is equal to : **[JEE (Main)-2021]**

[JEE (Main)-2021]

$$(1) a + 1 \quad (2) 2\sqrt{a}$$

$$(3) a + \frac{1}{a} \quad (4) 2a$$

76. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th} , 6^{th} and 8^{th} term is equal to :

[JEE (Main)-2021]

- (1) 26 (2) 35
(3) 30 (4) 32

77. The sum of infinite series

$$1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \dots \text{is equal to :}$$

[JEE (Main)-2021]

- (1) $\frac{13}{4}$ (2) $\frac{9}{4}$
(3) $\frac{11}{4}$ (4) $\frac{15}{4}$

78. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ is equal to :

[JEE (Main)-2021]

- (1) $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$
(2) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$
(3) $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$
(4) $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

79. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence $-16, 8, -4, 2, \dots$ satisfy the equation $4x^2 - 9x + 5 = 0$, then $p + q$ is equal to _____.

[JEE (Main)-2021]

80. Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to _____.

[JEE (Main)-2021]

81. Let $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + \dots$ up to n -terms, where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to _____.

[JEE (Main)-2021]

82. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}$, $\frac{1}{b}$, 6 be in A.P., where $a, b > 0$. Then $72(a+b)$ is equal to _____.

[JEE (Main)-2021]

83. $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \dots + \frac{1}{(201)^2-1}$ is equal to

[JEE (Main)-2021]

- (1) $\frac{101}{404}$ (2) $\frac{99}{400}$
(3) $\frac{25}{101}$ (4) $\frac{101}{408}$

84. If α, β , are natural numbers such that $100^\alpha - 199^\beta = (100)(100) + (99)(101) + (98)(102) + \dots + (1)(199)$, then the slope of the line passing through (α, β) and origin is:

[JEE (Main)-2021]

- (1) 550 (2) 530
(3) 540 (4) 510

85. Let S_1 be the sum of first $2n$ terms of an arithmetic progression. Let S_2 be the sum of first $4n$ terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first $6n$ terms of the arithmetic progression is equal to :

[JEE (Main)-2021]

- (1) 7000 (2) 5000
(3) 3000 (4) 1000

86. If $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$,

then the value of α is equal to _____.

[JEE (Main)-2021]

87. For $k \in \mathbb{N}$, let $\frac{1}{\alpha(\alpha+1)(\alpha+2)\dots(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$,

where $\alpha > 0$. Then the value of $100 \left(\frac{A_{14} + A_{15}}{A_{13}} \right)^2$

is equal to _____.

[JEE (Main)-2021]

88. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of

$$47 \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$$
 is equal to _____. [JEE (Main)-2021]

89. Let S_n denote the sum of first n -terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to

[JEE (Main)-2021]

- (1) 1842 (2) 1852
(3) 1862 (4) 1872

90. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value

of $\frac{S_{4n}}{S_{2n}}$ is **[JEE (Main)-2021]**

- (1) 4 (2) 2
(3) 6 (4) 8

91. If the value of

$$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \text{upto } \infty\right)^{\log_{(0.25)}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{upto } \infty\right)}$$

is l , then l is equal to _____.

[JEE (Main)-2021]

92. If $[x]$ be the greatest integer less than or equal to

$$x, \text{ then } \sum_{n=8}^{100} \left[\frac{(-1)^n n}{2} \right] \text{ is equal to}$$

- (1) 2 (2) -2
(3) 0 (4) 4 **[JEE (Main)-2021]**

93. If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to _____.

[JEE (Main)-2021]

94. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to

[JEE (Main)-2021]

- (1) 0 (2) 1
(3) 3 (4) 4

95. The sum of the series

$$\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \dots + \frac{2^{100}}{x^{2^{100}}+1} \text{ when } x = 2 \text{ is}$$

[JEE (Main)-2021]

- (1) $1 + \frac{2^{101}}{4^{2^{101}} - 1}$ (2) $1 - \frac{2^{100}}{4^{2^{100}} - 1}$
(3) $1 + \frac{2^{100}}{4^{2^{101}} - 1}$ (4) $1 - \frac{2^{101}}{4^{2^{101}} - 1}$

96. If the sum of an infinite GP a, ar, ar^2, ar^3, \dots is 15 and the sum of the squares of its each term is 150, then the sum of ar^2, ar^4, ar^6, \dots is :

[JEE (Main)-2021]

- (1) $\frac{5}{2}$ (2) $\frac{9}{2}$
(3) $\frac{25}{2}$ (4) $\frac{1}{2}$

97. If ${}^1P_1 + 2 \cdot {}^2P_2 + 3 \cdot {}^3P_3 + \dots + 15 \cdot {}^{15}P_{15} = {}^nP_r - s$,

$0 \leq s \leq 1$, then ${}^{n+s}C_{r-s}$ is equal to _____.

[JEE (Main)-2021]

98. $\lim_{x \rightarrow 2} \left(\sum_{n=1}^9 \frac{x}{n(n+1)x^2 + 2(2n+1)x + 4} \right)$ is equal to :

[JEE (Main)-2021]

- (1) $\frac{7}{36}$ (2) $\frac{1}{5}$
(3) $\frac{5}{24}$ (4) $\frac{9}{44}$

99. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is _____.

[JEE (Main)-2021]

100. Let a_1, a_2, \dots, a_{10} be an AP with common difference -3 and b_1, b_2, \dots, b_{10} be a GP with common ratio 2. Let $c_k = a_k + b_k, k = 1, 2, \dots, 10$.

If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to

[JEE (Main)-2021]

101. Let $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles

of a triangle ABC . If the lengths of the sides opposite these angles are a, b, c respectively, then

[JEE (Main)-2021]

- (1) a^2, b^2, c^2 are in A.P. (2) $b^2 - a^2 = a^2 + c^2$
(3) b^2, c^2, a^2 are in A.P. (4) c^2, a^2, b^2 are in A.P.

102. If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots$ is equal to

[JEE (Main)-2021]

- (1) $x \left(\frac{1+x}{1-x} \right) + \log_e(1-x)$
(2) $x \left(\frac{1-x}{1+x} \right) + \log_e(1-x)$
(3) $\frac{1+x}{1-x} + \log_e(1-x)$
(4) $\frac{1-x}{1+x} + \log_e(1-x)$

103. If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \dots$, then the value of e^{1+y} at $x = \frac{1}{2}$ is

[JEE (Main)-2021]

- (1) $2e$ (2) $\frac{1}{2}e^2$
(3) $2e^2$ (4) $\frac{1}{2}\sqrt{e}$

104. Three numbers are in an increasing geometric progression with common ratio r . If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d . If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to

[JEE (Main)-2021]

- (1) $7 - 7\sqrt{3}$ (2) $7 + \sqrt{3}$
(3) $7 - \sqrt{3}$ (4) $7 + 3\sqrt{3}$

105. The sum of 10 terms of the series

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \dots \text{ is}$$

[JEE (Main)-2021]

- (1) $\frac{143}{144}$ (2) $\frac{99}{100}$
(3) $\frac{120}{121}$ (4) 1

106. The mean of 10 numbers

$$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, \dots \text{ is } \underline{\hspace{2cm}}.$$

[JEE (Main)-2021]

107. Let a_1, a_2, a_3, \dots be an A.P. If $\frac{a_1 + a_2 + \dots + a_{10}}{a_1 + a_2 + \dots + a_p}$

$$= \frac{100}{p^2}, p \neq 10, \text{ then } \frac{a_{11}}{a_{10}} \text{ is equal to}$$

[JEE (Main)-2021]

- (1) $\frac{19}{21}$ (2) $\frac{100}{121}$
(3) $\frac{21}{19}$ (4) $\frac{121}{100}$

108. If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \dots$, then $160S$ is equal to

JEE (Main)-2021

109. Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \dots$

$$+ (n-1) \cdot 1, n \geq 4.$$

The sum $\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$ is equal to

[JEE (Main)-2021]

- (1) $\frac{e-1}{3}$ (2) $\frac{e}{3}$
(3) $\frac{e}{6}$ (4) $\frac{e-2}{6}$

110. Let a_1, a_2, \dots, a_{21} be an AP such that

$$\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}. \text{ If the sum of this AP is 189,}$$

then $a_6 a_{16}$ is equal to :

JEE (Main)-2021

- (1) 36 (2) 57
(3) 72 (4) 48

111. If $\{a_i\}_{i=1}^n$, where n is an even integer, is an arithmetic progression with common difference 1, and

$$\sum_{i=1}^n a_i = 192, \sum_{i=1}^{n/2} a_{2i} = 120, \text{ then } n \text{ is equal to :}$$

- (1) 48 (2) 96
(3) 92 (4) 104

[JEE (Main)-2022]

112. The sum of all the elements of the set $\{\alpha \in \{1, 2, \dots, 100\} : \text{HCF}(\alpha, 24) = 1\}$ is

[JEE (Main)-2022]

113. If $\frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \dots + \frac{1}{2^{10} \cdot 3} = \frac{K}{2^{10} \cdot 3^{10}}$, then the

remainder when K is divided by 6 is :

- (1) 1 (2) 2
(3) 3 (4) 5

[JEE (Main)-2022]

114. For a natural number n , let $\alpha_n = 19^n - 12^n$. Then, the

$$\text{value of } \frac{31\alpha_9 - \alpha_{10}}{57\alpha_8} \text{ is } \underline{\hspace{2cm}}.$$

[JEE (Main)-2022]

115. The greatest integer less than or equal to the sum of

$$\text{first 100 terms of the sequence } \frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, \dots \text{ is}$$

equal to $\underline{\hspace{2cm}}.$

[JEE (Main)-2022]

116. The sum $1 + 2 \times 3 + 3 \times 3^2 + \dots + 10 \times 3^9$ is equal to

(1) $\frac{2 \cdot 3^{12} + 10}{4}$ (2) $\frac{19 \cdot 3^{10} + 1}{4}$

(3) $5 \cdot 3^{10} - 2$ (4) $\frac{9 \cdot 3^{10} + 1}{2}$

[JEE (Main)-2022]

117. Let $A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$ and $B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}$.

Then $A + B$ is equal to _____.

[JEE (Main)-2022]

118. If $A = \sum_{n=1}^{\infty} \frac{1}{(3 + (-1)^n)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3 + (-1)^n)^n}$, then

$\frac{A}{B}$ is equal to:

(1) $\frac{11}{9}$ (2) 1

(3) $-\frac{11}{9}$ (4) $-\frac{11}{3}$

[JEE (Main)-2022]

119. If $a_1 (> 0)$, a_2, a_3, a_4, a_5 are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_4 = 2a_5$, then $a_2 + a_4 + 2a_5$ is equal to _____.

[JEE (Main)-2022]

120. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$, where a, b, c

are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$, $abc \neq 0$, then :

(1) x, y, z are in A.P.

(2) x, y, z are in G.P.

(3) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

(4) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

[JEE (Main)-2022]

121. If the sum of the first ten terms of the series

$\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots$ is $\frac{m}{n}$, where m

and n are co-prime numbers, then $m + n$ is equal to _____.

[JEE (Main)-2022]

122. Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \dots$. Then $4S$ is equal to

(1) $\left(\frac{7}{3}\right)^2$

(2) $\frac{7^3}{3^2}$

(3) $\left(\frac{7}{3}\right)^3$

(4) $\frac{7^2}{3^3}$

[JEE (Main)-2022]

123. If a_1, a_2, a_3, \dots and b_1, b_2, b_3, \dots are A.P., and $a_1 = 2, a_{10} = 3, a_1 b_1 = 1 = a_{10} b_{10}$, then $a_4 b_4$ is equal to

(1) $\frac{35}{27}$

(2) 1

(3) $\frac{27}{28}$

(4) $\frac{28}{27}$

[JEE (Main)-2022]

124. Let A_1, A_2, A_3, \dots be an increasing geometric progression of positive real numbers. If $A_1 A_3 A_5 A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to

(1) 33

(2) 37

(3) 43

(4) 47

[JEE (Main)-2022]

125. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1 : 7$ and $a + n = 33$, then the value of n is:

(1) 21

(2) 22

(3) 23

(4) 24

[JEE (Main)-2022]

126. Let for $n = 1, 2, \dots, 50$, S_n be the sum of the infinite geometric progression whose first term is n^2 and

whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of

$\frac{1}{26} + \sum_{n=1}^{50} \left(S_n + \frac{2}{n+1} - n - 1 \right)$ is equal to _____.

[JEE (Main)-2022]

127. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$.

Then $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to :

[JEE (Main)-2022]

(1) $\frac{6}{343}$

(2) $\frac{7}{216}$

(3) $\frac{8}{343}$

(4) $\frac{49}{216}$

128. The sum of the infinite series

$$1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \dots \text{ is equal to}$$

- (1) $\frac{425}{216}$ (2) $\frac{429}{216}$
 (3) $\frac{288}{125}$ (4) $\frac{280}{125}$

[JEE (Main)-2022]

129. Let 3, 6, 9, 12, ... upto 78 terms and 5, 9, 13, 17, ... upto 59 terms be two series. Then, the sum of terms common to both the series is equal to _____.

[JEE (Main)-2022]

130. Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to _____.

[JEE (Main)-2022]

131. Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to _____.

[JEE (Main)-2022]

132. Consider two G.Ps. $2, 2^2, 2^3, \dots$ and $4, 4^2, 4^3, \dots$ of 60 and n terms respectively. If the geometric mean of all the $60 + n$ terms is $(2)^{\frac{225}{8}}$, then

$$\sum_{k=1}^n k(n-k) \text{ is equal to}$$

- (1) 560 (2) 1540
 (3) 1330 (4) 2600

[JEE (Main)-2022]

133. The series of positive multiples of 3 is divided into sets: $\{3\}$, $\{6, 9, 12\}$, $\{15, 18, 21, 24, 27\}$, Then the sum of the elements in the 11th set is equal to _____.

[JEE (Main)-2022]

134. Suppose $a_1, a_2, \dots, a_n, \dots$ be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is $5 : 17$ and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to

- (1) 290 (2) 380
 (3) 460 (4) 510

[JEE (Main)-2022]

135. Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \leq 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to _____.

[JEE (Main)-2022]

136. Let the sum of an infinite G.P., whose first term is a and the common ratio is r , be 5. Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first

21 terms of an AP, whose first term is $10ar$, n^{th} term is a and the common difference is $10ar^2$, is equal to _____.

[JEE (Main)-2022]

- (1) $21 a_{11}$ (2) $22 a_{11}$
 (3) $15 a_{16}$ (4) $14 a_{16}$

137. $\frac{2^3 - 1^3}{1 \times 7} + \frac{4^3 - 3^3 + 2^2 - 1^3}{2 \times 11} + \frac{6^3 - 5^3 + 4^3 - 3^3 + 2^3 - 1^3}{3 \times 15}$
 $+ \dots + \frac{30^3 - 29^3 + 28^3 - 27^3 + \dots + 2^3 - 1^3}{15 \times 63}$ is equal to _____.

[JEE (Main)-2022]

138. Consider the sequence a_1, a_2, a_3, \dots such that $a_1 = 1$, $a_2 = 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_n$ for $n = 1, 2, 3, \dots$. If

$$\left(\frac{a_1 + \frac{1}{a_2}}{a_3} \right) \left(\frac{a_2 + \frac{1}{a_3}}{a_4} \right) \left(\frac{a_3 + \frac{1}{a_4}}{a_5} \right) \dots \left(\frac{a_{30} + \frac{1}{a_{31}}}{a_{32}} \right)$$

$= 2^\alpha ({}^{61}C_{31})$, then α is equal to

[JEE (Main)-2022]

- (1) -30 (2) -31
 (3) -60 (4) -61

139. For $p, q \in \mathbf{R}$, consider the real valued function $f(x) = (x - p)^2 - q$, $x \in \mathbf{R}$ and $q > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $|f(a_i)| = 500$ for all $i = 1, 2, 3, 4$, then the absolute difference between the roots of $f(x) = 0$ is

[JEE (Main)-2022]

140. Let $x_1, x_2, x_3, \dots, x_{20}$ be in geometric progression with $x_1 = 3$ and the common ratio $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If \bar{x} is the mean of new data, then the greatest integer less than or equal to \bar{x} is _____.

[JEE (Main)-2022]

141. If $\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \dots + \frac{10240}{3} = 2^n \cdot m$, where m is odd, then $m \cdot n$ is equal to _____.

[JEE (Main)-2022]

142. Let $S = \{1, 2, 3, \dots, 2022\}$. Then the probability, that a randomly chosen number n from the set S such that $\text{HCF}(n, 2022) = 1$, is

[JEE (Main)-2022]

- (1) $\frac{128}{1011}$ (2) $\frac{166}{1011}$
(3) $\frac{127}{337}$ (4) $\frac{112}{337}$

143. Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$, in the increasing powers of $\frac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6} : 1$. If the sixth term from the beginning is $\frac{\alpha}{\sqrt[4]{3}}$, then α is equal to _____.

[JEE (Main)-2022]

144. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1, \forall n \geq 0$.

Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to

[JEE (Main)-2022]

- (1) 483 (2) 528
(3) 575 (4) 624

145. $\sum_{r=1}^{20} (r^2 + 1)(r!)$ is equal to

[JEE (Main)-2022]

- (1) $22! - 21!$ (2) $22! - 2(21!)$
(3) $21! - 2(20!)$ (4) $21! - 20!$

146. If $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$, where m and n are co-prime, then $m + n$ is equal to

[JEE (Main)-2022]

147. Let a_1, a_2, a_3, \dots be an A.P. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to _____.

[JEE (Main)-2022]

148. If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \dots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$,

then the maximum value of a is [JEE (Main)-2022]

- (1) 198 (2) 202
(3) 212 (4) 218

149. The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to

[JEE (Main)-2022]

- (1) $\frac{7}{87}$ (2) $\frac{7}{29}$
(3) $\frac{14}{87}$ (4) $\frac{21}{29}$

150. Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P.'s having at least 3 terms and at most 33 terms is _____.

[JEE (Main)-2022]

151. If $\frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6} + \dots + \frac{1}{100 \times 101 \times 102} = \frac{k}{101}$

then $34k$ is equal to _____.