Chapter 1

Complex Numbers

1. If $\left|Z - \frac{4}{Z}\right| = 2$, then the maximum value of $|Z|$ is equal to **[AIEEE-2009]**

   (1) $\sqrt{5}+1$ (2) 2

   (3) $2+\sqrt{2}$ (4) $\sqrt{3}+1$

2. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals **[AIEEE-2010]**

   (1) 0 (2) 1

   (3) 2 (4) ∞

3. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies **[AIEEE-2012]**

   (1) On a circle with centre at the origin.

   (2) Either on the real axis or on a circle not passing through the origin

   (3) On the imaginary axis

   (4) Either on the real axis or on a circle passing through the origin

4. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals **[JEE (Main)-2013]**

   (1) $-\theta$ (2) $\frac{\pi}{2}-\theta$

   (3) θ (4) $\pi-\theta$

5. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$ **[JEE (Main)-2014]**

   (1) Is strictly greater than $\frac{5}{2}$

   (2) Is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

   (3) Is equal to $\frac{5}{2}$

   (4) Lies in the interval (1, 2)

6. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a **[JEE (Main)-2015]**

   (1) Straight line parallel to x-axis

   (2) Straight line parallel to y-axis

   (3) Circle of radius 2

   (4) Circle of radius $\sqrt{2}$

7. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is **[JEE (Main)-2016]**

   (1) $\frac{\pi}{6}$ (2) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

   (3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\frac{\pi}{3}$

8. Let $A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary}\right\}$.

   Then the sum of the elements in A is **[JEE (Main)-2019]**

   (1) $\frac{5\pi}{6}$ (2) π

   (3) $\frac{3\pi}{4}$ (4) $\frac{2\pi}{3}$

9. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to **[JEE (Main)-2019]**

   (1) 0 (2) $\frac{\pi}{3}$

   (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

10. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then **[JEE (Main)-2019]**

(1) $\text{Im}(z) = 0$ (2) $\frac{3}{2} \le |z| \le \frac{5}{2}$

(3) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$ (4) $\text{Re}(z) = 0$

11. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z, then **[JEE (Main)-2019]**

(1) $I(z) = 0$

(2) $R(z) > 0$ and $I(z) > 0$

(3) $R(z) < 0$ and $I(z) > 0$

(4) $R(z) = -3$

12. Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x + iy}{27}$ $(i = \sqrt{-1})$, where x and y are real numbers, then $y - x$ equals

[JEE (Main)-2019]

(1) –85 (2) –91

(3) 85 (4) 91

13. Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$). Then $|z|$ is equal to

[JEE (Main)-2019]

(1) $\frac{\sqrt{41}}{4}$ (2) $\frac{5}{4}$

(3) $\frac{5}{3}$ (4) $\frac{\sqrt{34}}{3}$

14. If $\frac{z - \alpha}{z + \alpha}$ $(\alpha \in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is **[JEE (Main)-2019]**

(1) $\sqrt{2}$ (2) 2

(3) $\frac{1}{2}$ (4) 1

15. Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is **[JEE (Main)-2019]**

(1) 0 (2) $\sqrt{2}$

(3) 1 (4) 2

16. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$, then **[JEE (Main)-2019]**

(1) 0 (2) $(-1 + 2i)^9$

(3) –1 (4) 1

17. All the points in the set

$$S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\}(i = \sqrt{-1})$$

lie on a **[JEE (Main)-2019]**

(1) Straight line whose slope is 1

(2) Circle whose radius is $\sqrt{2}$

(3) Circle whose radius is 1

(4) Straight line whose slope is –1

18. Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5 + 3z}{5(1 - z)}$, then **[JEE (Main)-2019]**

(1) $5\,\text{Re}(\omega) > 4$ (2) $5\,\text{Re}(\omega) > 1$

(3) $4\,\text{Im}(\omega) > 5$ (4) $5\,\text{Im}(\omega) < 1$

19. If $a > 0$ and $z = \frac{(1+i)^2}{a - i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to : **[JEE (Main)-2019]**

(1) $-\frac{1}{5} + \frac{3}{5}i$ (2) $-\frac{3}{5} - \frac{1}{5}i$

(3) $\frac{1}{5} - \frac{3}{5}i$ (4) $-\frac{1}{5} - \frac{3}{5}i$

20. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then :

[JEE (Main)-2019]

(1) $z\bar{w} = \frac{1 - i}{\sqrt{2}}$ (2) $\bar{z}w = i$

(3) $z\bar{w} = \frac{-1 + i}{\sqrt{2}}$ (4) $\bar{z}w = -i$

21. The equation $|z - i| = |z - 1|$, $i = \sqrt{-1}$, represents :

[JEE (Main)-2019]

(1) The line through the origin with slope –1

(2) A circle of radius $\frac{1}{2}$

(3) A circle of radius 1

(4) The line through the origin with slope 1

22. Let $z \in C$ with Im(z) = 10 and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n. Then **[JEE (Main)-2019]**

(1) n = 20 and $Re(z)$ = 10

(2) n = 20 and $Re(z)$ = –10

(3) n = 40 and $Re(z)$ = –10

(4) n = 40 and $Re(z)$ = 10

23. If $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on a **[JEE (Main)-2020]**

(1) Circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

(2) Straight line whose slope is $-\frac{2}{3}$

(3) Circle whose diameter is $\frac{\sqrt{5}}{2}$

(4) Straight line whose slope is $\frac{3}{2}$

24. If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is **[JEE (Main)-2020]**

(1) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$ (2) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

(3) $-\tan^{-1}\left(\frac{3}{4}\right)$ (4) $\tan^{-1}\left(\frac{4}{3}\right)$

25. If the equation, $x^2 + bx + 45 = 0$ ($b \in R$) has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then **[JEE (Main)-2020]**

(1) $b^2 - b = 42$ (2) $b^2 - b = 30$

(3) $b^2 + b = 12$ (4) $b^2 + b = 72$

26. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b = \sum_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation **[JEE (Main)-2020]**

(1) $x^2 - 101x + 100 = 0$

(2) $x^2 - 102x + 101 = 0$

(3) $x^2 + 101x + 100 = 0$

(4) $x^2 + 102x + 101 = 0$

27. Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z + 3i|$ is **[JEE (Main)-2020]**

(1) $2\sqrt{3}$ (2) $\frac{7}{2}$

(3) $\sqrt{10}$ (4) $\frac{15}{4}$

28. If z be a complex number satisfying $|\text{Re}(z)| + |\text{Im}(z)| = 4$, then $|z|$ cannot be **[JEE (Main)-2020]**

(1) $\sqrt{10}$ (2) $\sqrt{8}$

(3) $\sqrt{\frac{17}{2}}$ (4) $\sqrt{7}$

29. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is **[JEE (Main)-2020]**

(1) $-\frac{1}{2}(1-i\sqrt{3})$ (2) $\frac{1}{2}(1-i\sqrt{3})$

(3) $\frac{1}{2}(\sqrt{3}-i)$ (4) $-\frac{1}{2}(\sqrt{3}-i)$

30. The imaginary part of $\left(3+2\sqrt{-54}\right)^{1/2} - \left(3-2\sqrt{-54}\right)^{1/2}$ can be **[JEE (Main)-2020]**

(1) $\sqrt{6}$ (2) $-\sqrt{6}$

(3) $-2\sqrt{6}$ (4) 6

31. If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 1|$, $\text{Re}(z_2) = |z_2 - 1|$, and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\text{Im}(z_1 + z_2)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{3}}$

(3) $\frac{2}{\sqrt{3}}$ (4) $2\sqrt{3}$

32. Let $u = \frac{2z+i}{z-ki}$, $z = x+iy$ and $k > 0$. If the curve represented by Re(u) + Im(u) = 1 intersects the y-axis at the point P and Q where $PQ = 5$, then the value of K is **[JEE (Main)-2020]**

(1) 1/2 (2) 3/2

(3) 2 (4) 4

33. If a and b are real numbers such that $(2+\alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$, then $a + b$ is equal to **[JEE (Main)-2020]**

(1) 33 (2) 9

(3) 24 (4) 57

34. If the four complex numbers z, $\bar{z}$, $\bar{z} - 2\text{Re}(\bar{z})$ and $z - 2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to **[JEE (Main)-2020]**

(1) $4\sqrt{2}$ (2) 2

(3) $2\sqrt{2}$ (4) 4

35. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is **[JEE (Main)-2020]**

(1) $-2^{15}i$ (2) -2^{15}

(3) $2^{15}i$ (4) 65

36. The region represented by $\{z = x + iy \in C : |z| - \text{Re}(z) \le 1\}$ is also given by the inequality **[JEE (Main)-2020]**

(1) $y^2 \le x + \frac{1}{2}$ (2) $y^2 \le 2\left(x+\frac{1}{2}\right)$

(3) $y^2 \ge x+1$ (4) $y^2 \ge 2(x+1)$

37. Let $z = x + iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the **[JEE (Main)-2020]**

(1) Line, $y = x$ (2) Imaginary axis

(3) Real axis (4) Line, $y = -x$

38. If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$, $(m, n \in N)$ then the greatest common divisor of the least values of m and n is ________. **[JEE (Main)-2020]**

39. Let p and q be two positive numbers such that p + q = 2 and $p^4 + q^4 = 272$. Then p and q are roots of the equation : **[JEE (Main)-2021]**

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

40. If range of real values of α, for which the equation $z + \alpha|z-1| + 2i = 0$ $\left(z \in C \text{ and } i = \sqrt{-1}\right)$ has a solution, is [p, q) then $4\left(p^2+q^2\right)$ is equal to ______.

[JEE (Main)-2021]

41. Let $i = \sqrt{-1}$. If $\frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}} + \frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}} = k$, and $n = [|k|]$ be the greatest integral part of $|k|$. Then $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to _______.

[JEE (Main)-2021]

42. Let the lines $(2-i)z = (2+i)\bar{z}$ and $(2+i)z + (i-2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is : **[JEE (Main)-2021]**

(1) $3\sqrt{2}$ (2) $\frac{3}{\sqrt{2}}$

(3) $\frac{3}{2\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

43. If $\alpha, \beta \in R$ are such that 1 – 2i (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to: **[JEE (Main)-2021]**

(1) –3 (2) –7

(3) 7 (4) 3

44. The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is _________. **[JEE (Main)-2021]**

45. Let z be those complex numbers which satisfy $|z+5| \le 4$ and $z(1+i) + \bar{z}(1-i) \ge -10$, $i = \sqrt{-1}$. If the maximum value of $|z+1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is ______. **[JEE (Main)-2021]**

46. Let a complex number z, $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to _____. **[JEE (Main)-2021]**

(1) 8 (2) 7
(3) 6 (4) 5

47. Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2, \left|\frac{z+i}{z-3i}\right| = 1$ and Re(w) has minimum value. Then, the minimum value of $n \in N$ for which w^n is real, is equal to **[JEE (Main)-2021]**

48. The least value of |z| where z is complex number which satisfies the inequality exp

$$\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|, i = \sqrt{-1},$$

is equal to : **[JEE (Main)-2021]**

(1) 2 (2) 8
(3) 3 (4) $\sqrt{5}$

49. The area of the triangle with vertices A(z), B(iz) and C(z + iz) is **[JEE (Main)-2021]**

(1) $\frac{1}{2}|z|^2$ (2) $\frac{1}{2}|z+iz|^2$
(3) $\frac{1}{2}$ (4) 1

50. Let S_1, S_2 and S_3 be three sets defined as

$S_1 = \{z \in \mathbb{C} : |z-1| \leq \sqrt{2}\}$

$S_2 = \{z \in \mathbb{C} : \mathrm{Re}((1-i)z) \geq 1\}$

$S_3 = \{z \in \mathbb{C} : \mathrm{Im}(z) \leq 1\}$

Then the set $S_1 \cap S_2 \cap S_3$ **[JEE (Main)-2021]**

(1) Has infinitely many elements
(2) Is a singleton
(3) Has exactly three elements
(4) Has exactly two elements

51. If the equation $a|z|^2 + \overline{\alpha z + \alpha\bar{z}} + d = 0$ represents a circle where a, d are real constants, then which of the following condition is correct?

[JEE (Main)-2021]

(1) $|\alpha|^2 - ad > 0$ and $a \in \mathbf{R} - \{0\}$
(2) $|\alpha|^2 - ad \neq 0$
(3) $\alpha = 0$, a, d $\in \mathbf{R}^+$
(4) $|\alpha|^2 - ad \geq 0$ and $a \in \mathbf{R}$

52. Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of |a| is ________. **[JEE (Main)-2021]**

53. Let a complex number be $w = 1-\sqrt{3}\,i$. Let another complex number z be such that |zw| = 1 and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

[JEE (Main)-2021]

(1) $\frac{1}{2}$ (2) 2
(3) 4 (4) $\frac{1}{4}$

54. If f(x) and g(x) are two polynomials such that the polynomial $P(x) = f(x^3) + x\,g(x^3)$ is divisible by $x^2 + x + 1$, then P(1) is equal to _______.

[JEE (Main)-2021]

55. If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12}-1) + \beta^{96}(\beta^{12}-1)$ is equal to

[JEE (Main)-2021]

(1) 28×3^{25} (2) 56×3^{24}
(3) 52×3^{24} (4) 56×3^{25}

56. If *z* and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is

(Here arg(*z*) denotes the principal argument of complex number *z*) **[JEE (Main)-2021]**

(1) $\frac{3\pi}{4}$ (2) $-\frac{3\pi}{4}$
(3) $\frac{\pi}{4}$ (4) $-\frac{\pi}{4}$

57. Let n denotes the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where *z* is a complex number. Then the value of $\sum_{k=0}^{\infty}\frac{1}{n^k}$ is equal to

[JEE (Main)-2021]

(1) 1 (2) 2
(3) $\frac{4}{3}$ (4) $\frac{3}{2}$

58. Let C be the set of all complex numbers. Let

$S_1 = \{z \in C \mid |z - 3 - 2i|^2 = 8\}$,

$S_2 = \{z \in C \mid \text{Re}(z) \geq 5\}$ and

$S_3 = \{z \in C \mid |z - \bar{z}| \geq 8\}$.

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to **[JEE (Main)-2021]**

(1) 0 (2) 1

(3) 2 (4) Infinite

59. Let $\mathbb{C}$ be the set of all complex numbers. Let

$S_1 = \{z \in \mathbb{C} : |z - 2| \leq 1\}$ and

$S_2 = \{z \in \mathbb{C} : z(1+i) + \bar{z}(1-i) \geq 4\}$.

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to **[JEE (Main)-2021]**

(1) $\frac{5+2\sqrt{2}}{2}$ (2) $\frac{5+2\sqrt{2}}{4}$

(3) $\frac{3+2\sqrt{2}}{4}$ (4) $\frac{3+2\sqrt{2}}{2}$

60. If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to _______.

[JEE (Main)-2021]

61. The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with : **[JEE (Main)-2021]**

(1) Centre at (0, 0) and radius $\sqrt{2}$

(2) Centre at (0, 1) and radius $\sqrt{2}$

(3) Centre at (0, –1) and radius $\sqrt{2}$

(4) Centre at (0, 1) and radius 2

62. Let $z = \frac{1 - i\sqrt{3}}{2}$, $i = \sqrt{-1}$. Then the value of

$$21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \ldots\ldots$$

$$\ldots.. + \left(z^{21} + \frac{1}{z^{21}}\right)^3$$ is _____.

[JEE (Main)-2021]

63. If $\left(\sqrt{3} + i\right)^{100} = 2^{99}(p + iq)$, then p and q are roots of the equation **[JEE (Main)-2021]**

(1) $x^2 - \left(\sqrt{3} - 1\right)x - \sqrt{3} = 0$

(2) $x^2 + \left(\sqrt{3} - 1\right)x - \sqrt{3} = 0$

(3) $x^2 - \left(\sqrt{3} + 1\right)x + \sqrt{3} = 0$

(4) $x^2 + \left(\sqrt{3} + 1\right)x + \sqrt{3} = 0$

64. The least positive integers n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$, is a positive integer, is ________. **[JEE (Main)-2021]**

65. If $S = \left\{z \in \mathbf{C} : \frac{z-i}{z+2i}, \in \mathbf{R}\right\}$, then

(1) S contains exactly two elements

(2) S is a circle in the complex plane

(3) S is a straight line in the complex plane

(4) S contains only one element

[JEE (Main)-2021]

66. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2$ is equal to ______.

[JEE (Main)-2021]

67. If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is **[JEE (Main)-2021]**

(1) $6\sqrt{2}$ (2) $2\sqrt{2}$

(3) $3\sqrt{2}$ (4) $2\sqrt{2} - 1$

68. If for the complex number z satisfying $|z - 2 - 2i| \leq 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then $a + b$ is equal to_____.

[JEE (Main)-2021]

69. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to **[JEE (Main)-2021]**

(1) –1 (2) 0

(3) 1 (4) 2

70. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to

[JEE (Main)-2021]

(1) –512 (2) 512
(3) 256 (4) –256

71. Let $S = \{z \in \mathbb{C} : |z-3| \le 1$ and $z(4 + 3i) + \bar{z}(4-3i) \le 24\}$. If $\alpha + i\beta$ is the point in S which is closest to $4i$, then $25(\alpha + \beta)$ is equal to ______.

[JEE (Main)-2022]

72. Let a circle C in complex plane pass through the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then $\arg(z)$ is equal to:

(1) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \pi$ (2) $\tan^{-1}\left(\frac{24}{7}\right) - \pi$

(3) $\tan^{-1}(3) - \pi$ (4) $\tan^{-1}\left(\frac{3}{4}\right) - \pi$

[JEE (Main)-2022]

73. Let z_1 and z_2 be two complex numbers such that $\overline{z_1} = i\overline{z_2}$ and $\arg\left(\frac{z_1}{\overline{z_2}}\right) = \pi$. Then

(1) $\arg z_2 = \left(\frac{\pi}{4}\right)$ (2) $\arg z_2 = -\frac{3\pi}{4}$

(3) $\arg z_1 = \frac{\pi}{4}$ (4) $\arg z_1 = -\frac{3\pi}{4}$

[JEE (Main)-2022]

74. If $z^2 + z + 1 = 0$, $z \in \mathbb{C}$, then $\left|\sum_{n=1}^{15}\left(z^n + (-1)^n \frac{1}{z^n}\right)^2\right|$ is equal to ________.

[JEE (Main)-2022]

75. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is :

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{3}{2}$ (D) $\frac{3}{4}$

[JEE (Main)-2022]

76. The number of points of intersection of $|z-(4+3i)| = 2$ and $|z| + |z-4| = 6$, $z \in C$, is

(1) 0 (2) 1
(3) 2 (4) 3

[JEE (Main)-2022]

77. Let for some real numbers α and β, $a = \alpha - i\beta$. If the system of equations $4ix + (1 + i)\, y = 0$ and $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$ has more than one solution, then $\frac{\alpha}{\beta}$ is equal to

(1) $-2+\sqrt{3}$ (2) $2-\sqrt{3}$

(3) $2+\sqrt{3}$ (4) $-2-\sqrt{3}$

[JEE (Main)-2022]

78. The number of elements in the set $\{z = a + ib \in \mathbb{C} : a, b \in \mathbb{Z}$ and $1 < |z-3+2i| < 4\}$ is _________.

[JEE (Main)-2022]

79. Let α and β be the roots of the equation $x^2 + (2i-1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to:

(1) 50 (2) 250
(3) 1250 (4) 1500

[JEE (Main)-2022]

80. Let S = $\{z \in \mathbf{C} : |z-2| \le 1,\ z(1+i) + \bar{z}\ (1-i) \le 2\}$. Let $|z-4i|$ attains minimum and maximum values, respectively, at $z_1 \in$ S and $z_2 \in$ S. If $5\left(|z_1|^2 + |z_2|^2\right) = \alpha + \beta\sqrt{5}$, where α and β are integers, then the value of $\alpha + \beta$ is equal to _____.

[JEE (Main)-2022]

81. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to

(1) 1 (2) α
(3) $1 + \alpha$ (4) $1 + 2\alpha$

[JEE (Main)-2022]

82. Let arg(z) represent the principal argument of the complex number z.

Then, $|z| = 3$ and $\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$ intersect

(1) exactly at one point
(2) exactly at two points
(3) nowhere
(4) at infinitely many points

[JEE (Main)-2022]

83. If α, β, γ, δ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to **[JEE (Main)-2022]**

(1) -4 (2) -1

(3) 1 (4) 4

84. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $\text{Re}(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true? **[JEE (Main)-2022]**

(1) $\arg z_2 = \pi - \tan^{-1}3$

(2) $\arg (z_1 - 2z_2) = -\tan^{-1}\left(\frac{4}{3}\right)$

(3) $|z_2| = \sqrt{10}$

(4) $|2z_1 - z_2| = 5$

85. Let the minimum value v_0 of $v = |z|^2 + |z - 3|^2 + |z - 6i|^2$, $z \in \mathbb{C}$ is attained at $z = z_0$. Then $\left|2z_0^2 - \bar{z}_0^3 + 3\right|^2 + v_0^2$ is equal to

[JEE (Main)-2022]

(1) 1000 (2) 1024

(3) 1105 (4) 1196

86. Let $S = \left\{z \in \mathbb{C} : z^2 + \bar{z} = 0\right\}$. Then

$\sum_{z \in S} (\text{Re}(z) + \text{Im}(z))$ is equal to ________.

[JEE (Main)-2022]

87. Let S be the set of all (α, β), $\pi < \alpha, \beta < 2\pi$, for which the complex number $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary and $\frac{1 + i\cos\beta}{1 - 2i\cos\beta}$ is purely real, Let $Z_{\alpha\beta} = \sin 2\alpha + i \cos 2\beta$, $(\alpha, \beta) \in S$. Then $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta} + \frac{1}{i\bar{Z}_{\alpha\beta}}\right)$ is equal to

[JEE (Main)-2022]

(1) 3 (2) $3i$

(3) 1 (4) $2 - i$

88. Let $S_1 = \left\{z_1 \in C : |z_1 - 3| = \frac{1}{2}\right\}$ and $S_2 = \left\{z_2 \in C : \left|z_2 - |z_2 + 1|\right| = \left|z_2 + |z_2 - 1|\right|\right\}$. Then, for $z_1 \in S_1$ and $z_2 \in S_2$, the least value of $|z_2 - z_1|$ is : **[JEE (Main)-2022]**

(1) 0 (2) $\frac{1}{2}$

(3) $\frac{3}{2}$ (4) $\frac{5}{2}$

89. Let $z = a + ib$, $b \neq 0$ be complex numbers satisfying $z^2 = \bar{z} \cdot 2^{1-|z|}$. Then the least value of $n \in N$, such that $z^n = (z + 1)^n$, is equal to _____.

[28-07-2022 Evening]

90. If $z \neq 0$ be a complex number such that $\left|z - \frac{1}{z}\right| = 2$, then the maximum value of $|z|$ is

[JEE (Main)-2022]

(1) $\sqrt{2}$ (2) 1

(3) $\sqrt{2} - 1$ (4) $\sqrt{2} + 1$

91. Let $S = \{z = x + iy : |z - 1 + i| \geq |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x, for which $w = 2x + iy \in S$ for some $y \in$ R, is

[JEE (Main)-2022]

(1) $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$ (2) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

(3) $\left(-\sqrt{2}, \frac{1}{2}\right]$ (4) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

92. If $z = 2 + 3i$, then $z^5 + (\bar{z})^5$ is equal to :

(1) 244 (2) 224

(3) 245 (4) 265

[JEE (Main)-2022]

93. For $z \in \mathbb{C}$ if the minimum value of $\left(|z - 3\sqrt{2}| + |z - p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of p is _________. **[JEE (Main)-2022]**

(1) 3 (2) $\frac{7}{2}$

(3) 4 (4) $\frac{9}{2}$

94. If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z - i| - |z + 5i| = 0$, then [JEE (Main)-2022]

(1) $x + 2y - 4 = 0$ (2) $x^2 + y - 4 = 0$

(3) $x + 2y + 4 = 0$ (4) $x^2 - y + 3 = 0$

95. Let $A = \{z \in \mathbf{C} : 1 \le |z - (1 + i)| \le 2\}$ and $B = \{z \in A : |z - (1 - i)| = 1\}$. Then, B : **[JEE (Main)-2022]**

(1) Is an empty set

(2) Contains exactly two elements

(3) Contains exactly three elements

(4) Is an infinite set

96. Let $A = \left\{z \in \mathbf{C} : \left|\frac{z+1}{z-1}\right| < 1\right\}$

and $B = \left\{z \in \mathbf{C} : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}\right\}$.

Then $A \cap B$ is : **[JEE (Main)-2022]**

(1) A portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

(2) A portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

(3) An empty set

(4) A portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

97. Sum of squares of modulus of all the complex numbers z satisfying $\bar{z} = iz^2 + z^2 - z$ is equal to ________. **[JEE (Main)-2022]**

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