

# Chapter 4

## Permutations and Combinations

1. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is **[AIEEE-2009]**
  - (1) At least 500 but less than 750
  - (2) At least 750 but less than 1000
  - (3) At least 1000
  - (4) Less than 500
2. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is **[AIEEE-2010]**
  - (1) 3
  - (2) 36
  - (3) 66
  - (4) 108
3. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is **[AIEEE-2012]**
  - (1) 629
  - (2) 630
  - (3) 879
  - (4) 880
4. Let  $T_n$  be the number of all possible triangles formed by joining vertices of an  $n$ -sided regular polygon. If  $T_{n+1} - T_n = 10$ , then the value of  $n$  is **[JEE (Main)-2013]**
  - (1) 7
  - (2) 5
  - (3) 10
  - (4) 8
5. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is **[JEE (Main)-2015]**
  - (1) 216
  - (2) 192
  - (3) 120
  - (4) 72
6. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is **[JEE (Main)-2016]**
  - (1) 59<sup>th</sup>
  - (2) 52<sup>nd</sup>
  - (3) 58<sup>th</sup>
  - (4) 46<sup>th</sup>
7. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is **[JEE (Main)-2017]**
  - (1) 468
  - (2) 469
  - (3) 484
  - (4) 485
8. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is **[JEE (Main)-2018]**
  - (1) At least 1000
  - (2) Less than 500
  - (3) At least 500 but less than 750
  - (4) At least 750 but less than 1000
9. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is **[JEE (Main)-2019]**
  - (1) 200
  - (2) 350
  - (3) 500
  - (4) 300
10. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repetition of digits allowed) is equal to **[JEE (Main)-2019]**
  - (1) 374
  - (2) 375
  - (3) 250
  - (4) 372
11. If  $\sum_{r=0}^{25} \left\{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \right\} = K \left( {}^{50}C_{25} \right)$ , then K is equal to **[JEE (Main)-2019]**
  - (1)  $2^{25} - 1$
  - (2)  $(25)^2$
  - (3)  $2^{25}$
  - (4)  $2^{24}$

12. The value of  $r$  for which  ${}^{20}C_r {}^{20}C_0 + {}^{20}C_{r-1} {}^{20}C_1 + {}^{20}C_{r-2} {}^{20}C_2 + \dots + {}^{20}C_0 {}^{20}C_r$  is maximum, is

[JEE (Main)-2019]

- (1) 10 (2) 20  
(3) 15 (4) 11

13. Consider three boxes, each containing 10 balls labelled 1, 2, ..., 10. Suppose one ball is randomly drawn from each of the boxes. Denote by  $n_i$ , the label of the ball drawn from the  $i^{\text{th}}$  box, ( $i = 1, 2, 3$ ). Then, the number of ways in which the balls can be chosen such that  $n_1 < n_2 < n_3$  is

[JEE (Main)-2019]

- (1) 240 (2) 120  
(3) 164 (4) 82

14. There are  $m$  men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of  $m$  is

[JEE (Main)-2019]

- (1) 9 (2) 7  
(3) 11 (4) 12

15. If  ${}^nC_4$ ,  ${}^nC_5$  and  ${}^nC_6$  are in A.P., then  $n$  can be

[JEE (Main)-2019]

- (1) 12 (2) 9  
(3) 14 (4) 11

16. The sum of the series  $2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + 11 \cdot {}^{20}C_3 + \dots + 62 \cdot {}^{20}C_{20}$  is equal to

[JEE (Main)-2019]

- (1)  $2^{23}$  (2)  $2^{25}$   
(3)  $2^{24}$  (4)  $2^{26}$

17. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is

[JEE (Main)-2019]

- (1) 180 (2) 175  
(3) 162 (4) 160

18. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is :

[JEE (Main)-2019]

- (1) 360 (2) 306  
(3) 288 (4) 310

19. A committee of 11 members is to be formed from 8 males and 5 females. If  $m$  is the number of ways the committee is formed with at least 6 males and  $n$  is the number of ways the committee is formed with at least 3 females, then

[JEE (Main)-2019]

- (1)  $m = n = 68$  (2)  $m + n = 68$   
(3)  $m = n = 78$  (4)  $n = m - 8$

20. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is

[JEE (Main)-2019]

- (1) 157 (2) 225  
(3) 262 (4) 190

21. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is

[JEE (Main)-2019]

- (1) 72 (2) 48  
(3) 60 (4) 36

22. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is

[JEE (Main)-2019]

- (1) 210 (2) 180  
(3) 170 (4) 190

23. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is

[JEE (Main)-2019]

- (1)  $\frac{3}{20}$  (2)  $\frac{1}{5}$   
(3)  $\frac{3}{10}$  (4)  $\frac{1}{10}$

24. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is

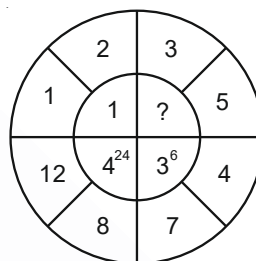
[JEE (Main)-2019]

- (1)  $2^{20} + 1$  (2)  $2^{21}$   
(3)  $2^{20} - 1$  (4)  $2^{20}$

25. A group of students comprises of 5 boys and  $n$  girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then  $n$  is equal to  
[JEE (Main)-2019]
- (1) 24 (2) 27  
(3) 25 (4) 28
26. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is  
[JEE (Main)-2020]
- (1)  $\frac{1}{2}(6!)$  (2)  $\frac{5}{2}(6!)$   
(3)  $5^6$  (4)  $6!$
27. If the number of five digit numbers with distinct digits and 2 at the  $10^{\text{th}}$  place is  $336k$ , then  $k$  is equal to  
[JEE (Main)-2020]
- (1) 8 (2) 6  
(3) 7 (4) 4
28. Let  $n > 2$  be an integer. Suppose that there are  $n$  Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of  $n$  is  
[JEE (Main)-2020]
- (1) 199 (2) 201  
(3) 101 (4) 200
29. The value of  $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \dots$  up to  $51^{\text{th}}$  term)  $+ (1! - 2! + 3! - \dots$  up to  $51^{\text{th}}$  term) is equal to  
[JEE (Main)-2020]
- (1) 1 (2)  $1 + (52)!$   
(3)  $1 - 51(51)!$  (4)  $1 + (51)!$
30. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is  
[JEE (Main)-2020]
- (1) 2250 (2) 3000  
(3) 1500 (4) 2255
31. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?  
[JEE (Main)-2020]
- (1)  $2! 3! 4!$  (2)  $(3!)^3 \cdot (4!)$   
(3)  $(3!)^2 \cdot (4!)$  (4)  $3! (4!)^3$
32. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is \_\_\_\_\_.  
[JEE (Main)-2020]
33. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is \_\_\_\_\_  
[JEE (Main)-2020]
34. The total number of 3-digit numbers, whose sum of digits is 10, is \_\_\_\_\_.  
[JEE (Main)-2020]
35. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is \_\_\_\_\_.  
[JEE (Main)-2020]
36. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is \_\_\_\_\_.  
[JEE (Main)-2020]
37. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is \_\_\_\_\_.  
[JEE (Main)-2020]
38. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :  
[JEE (Main)-2021]
- (1) 560 (2) 1050  
(3) 1625 (4) 575
39. Let  $M$  be any  $3 \times 3$  matrix with entries from the set  $\{0, 1, 2\}$ . The maximum number of such matrices, for which the sum of diagonal elements of  $M^T M$  is seven is \_\_\_\_\_.  
[JEE (Main)-2021]
40. The students  $S_1, S_2, \dots, S_{10}$  are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is \_\_\_\_\_.  
[JEE (Main)-2021]

41. The total number of positive integral solutions  $(x, y, z)$  such that  $xyz = 24$  is : **[JEE (Main)-2021]**  
 (1) 36 (2) 30  
 (3) 45 (4) 24
42. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is \_\_\_\_\_. **[JEE (Main)-2021]**
43. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is : **[JEE (Main)-2021]**  
 (1) 77 (2) 42  
 (3) 82 (4) 35
44. A natural number has prime factorization given by  $n = 2^x 3^y 5^z$ , where  $y$  and  $z$  are such that  $y + z = 5$  and  $y^{-1} + z^{-1} = \frac{5}{6}$ ,  $y > z$ . Then the number of odd divisors of  $n$ , including 1, is : **[JEE (Main)-2021]**  
 (1) 12 (2)  $6x$   
 (3) 11 (4) 6
45. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is \_\_\_\_\_. **[JEE (Main)-2021]**
46. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let  $\alpha$  be the number of triangles having these points from different sides as vertices and  $\beta$  be the number of quadrilaterals having these points from different sides as vertices. Then  $(\beta - \alpha)$  is equal to: **[JEE (Main)-2021]**  
 (1) 1890 (2) 717  
 (3) 795 (4) 1173
47. Team 'A' consists of 7 boys and  $n$  girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then  $n$  is equal to **[JEE (Main)-2021]**  
 (1) 5 (2) 6  
 (3) 2 (4) 4
48. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to : **[JEE (Main)-2021]**  
 (1) 240 (2) 364  
 (3) 360 (4) 333

49. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is : **[JEE (Main)-2021]**  
 (1) 122234 (2) 122664  
 (3) 22264 (4) 26664
50. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is \_\_\_\_\_. **[JEE (Main)-2021]**
51. The missing value in the following figure is \_\_\_\_\_. **[JEE (Main)-2021]**



52. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is \_\_\_\_\_. **[JEE (Main)-2021]**
53. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to \_\_\_\_\_. **[JEE (Main)-2021]**
54. Let  $M = \left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\} \right\}$ . Define  $f : M \rightarrow \mathbb{Z}$ , as  $f(A) = \det(A)$ , for all  $A \in M$ , where  $\mathbb{Z}$  is set of all integers. Then the number of  $A \in M$  such that  $f(A) = 15$  is equal to \_\_\_\_\_. **[JEE (Main)-2021]**
55. If  ${}^n P_r = {}^n P_{r+1}$  and  ${}^n C_r = {}^n C_{r-1}$ , then the value of  $r$  is equal to **[JEE (Main)-2021]**  
 (1) 4 (2) 3  
 (3) 2 (4) 1
56. Let  $S = \{1, 2, 3, 4, 5, 6, 7\}$ . Then the number of possible functions  $f : S \rightarrow S$  such that  $f(m \cdot n) = f(m) \cdot f(n)$  for every  $m, n \in S$  and  $m \cdot n \in S$  is equal to \_\_\_\_\_. **[JEE (Main)-2021]**
57. The point  $P(a, b)$  undergoes the following three transformations successively :  
 (a) Reflection about the line  $y = x$ .  
 (b) Translation through 2 units along the positive direction of  $x$ -axis.  
 (c) Rotation through angle  $\frac{\pi}{4}$  about the origin in the anti-clockwise direction.



If the co-ordinates of the final position of the point P are  $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ , then the value of  $2a + b$  is equal to

[JEE (Main)-2021]

- (1) 7 (2) 9  
(3) 5 (4) 13

58. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is \_\_\_\_\_ [JEE (Main)-2021]

59. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is \_\_\_\_\_ [JEE (Main)-2021]

60. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is \_\_\_\_\_ [JEE (Main)-2021]

61. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is \_\_\_\_\_ [JEE (Main)-2021]

62. Let  $P_1, P_2, \dots, P_{15}$  be 15 points on a circle. The number of distinct triangles formed by points  $P_i, P_j, P_k$  such that  $i + j + k \neq 15$ , is \_\_\_\_\_ [JEE (Main)-2021]

- (1) 455 (2) 12  
(3) 419 (4) 443

63. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is \_\_\_\_\_ [JEE (Main)-2021]

64. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set  $A \times B$ , each having at least three elements is \_\_\_\_\_ [JEE (Main)-2021]

- (1) 219 (2) 256  
(3) 275 (4) 510

65. Let n be a non-negative integer. Then the number of divisors of the form " $4n + 1$ " of the number  $(10)^{10} \cdot (11)^{11} \cdot (13)^{13}$  is equal to \_\_\_\_\_ [JEE (Main)-2021]

67. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, -2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways

a student appearing in the examination gets 5 marks is \_\_\_\_\_ [JEE (Main)-2022]

68. The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is \_\_\_\_\_ [JEE (Main)-2022]

69. The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is \_\_\_\_\_ [JEE (Main)-2022]

70. The total number of three-digit numbers, with one digit repeated exactly two times, is \_\_\_\_\_ [JEE (Main)-2022]

71. There are ten boys  $B_1, B_2, \dots, B_{10}$  and five girls  $G_1, G_2, \dots, G_5$  in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both  $B_1$  and  $B_2$  together should not be the members of a group, is \_\_\_\_\_ [JEE (Main)-2022]

72. The total number of 3-digit numbers, whose greatest common divisor with 36 is 2, is \_\_\_\_\_ [JEE (Main)-2022]

73. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is \_\_\_\_\_ [JEE (Main)-2022]

74. The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is \_\_\_\_\_ [JEE (Main)-2022]

- (1) 36 (2) 48  
(3) 60 (4) 72

75. The number of ways to distribute 30 identical candies among four children  $C_1, C_2, C_3$  and  $C_4$  so that  $C_2$  receives atleast 4 and atmost 7 candies,  $C_3$  receives atleast 2 and atmost 6 candies, is equal to: \_\_\_\_\_ [JEE (Main)-2022]

- (1) 205 (2) 615  
(3) 510 (4) 430

76. Let  $b_1 b_2 b_3 b_4$  be a 4-element permutation with  $b_i \in \{1, 2, 3, \dots, 100\}$  for  $1 \leq i \leq 4$  and  $b_i \nmid b_j$  for  $i < j$ , such that either  $b_1, b_2, b_3$  are consecutive integers or  $b_2, b_3, b_4$  are consecutive integers. Then the number of such permutations  $b_1 b_2 b_3 b_4$  is equal to \_\_\_\_\_ [JEE (Main)-2022]

77. The total number of four digit numbers such that each of first three digits is divisible by the last digit, is equal to \_\_\_\_\_ [JEE (Main)-2022]

78. The total number of functions,  
 $f : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4, 5, 6\}$   
such that  $f(1) + f(2) = f(3)$ , is equal to  
[JEE (Main)-2022]  
(1) 60 (2) 90  
(3) 108 (4) 126
79. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is \_\_\_\_\_.  
[JEE (Main)-2022]
80. The number of 5-digit natural numbers, such that the product of their digits is 36, is \_\_\_\_\_.  
[JEE (Main)-2022]
81. Let  $S$  be the set of all passwords which are six to eight characters long, where each character is either an alphabet from  $\{A, B, C, D, E\}$  or a number from  $\{1, 2, 3, 4, 5\}$  with the repetition of characters allowed. If the number of passwords in  $S$  whose at least one character is a number from  $\{1, 2, 3, 4, 5\}$  is  $\alpha \times 5^6$ , then  $\alpha$  is equal to \_\_\_\_\_.  
[JEE (Main)-2022]
82. A class contains  $b$  boys and  $g$  girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then  $b + 3g$  is equal to \_\_\_\_\_.  
[JEE (Main)-2022]
83. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is \_\_\_\_\_.  
[JEE (Main)-2022]
84. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is \_\_\_\_\_.  
[JEE (Main)-2022]
85. Let  $S = \{4, 6, 9\}$  and  $T = \{9, 10, 11, \dots, 1000\}$ . If  $A = \{a_1 + a_2 + \dots + a_k : k \in \mathbf{N}, a_1, a_2, a_3, \dots, a_k \in S\}$ , then the sum of all the elements in the set  $T - A$  is equal to \_\_\_\_\_.  
[JEE (Main)-2022]

