Chapter 17

Functions

1. Let $f(x) = (x + 1)^2 - 1$, $x \geq -1$.

Statement-1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.

Statement-2 : f is a bijection.

[AIEEE-2009]

(1) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

2. For real x, let $f(x) = x^3 + 5x + 1$, then

[AIEEE-2009]

(1) f is onto **R** but not one-one

(2) f is one-one and onto **R**

(3) f is neither one-one nor onto **R**

(4) f is one-one but not onto **R**

3. Let y be an implict function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals

[AIEEE-2009]

(1) 1 (2) log 2

(3) –log 2 (4) –1

4. Let f be a function defined by

$f(x) = (x-1)^2 + 1, (x \geq 1)$.

Statement - 1 : The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$.

Statement - 2 : ***f*** is a bijection and $f^{-1}(x) = 1 + \sqrt{x-1}, x \geq 1$.

[AIEEE-2011]

(1) Statement-1 is true, Statement-2 is false

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

5. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

[AIEEE-2012]

(1) No real roots

(2) Exactly one real root

(3) Exactly four real roots

(4) Infinite number of real roots

6. If $a \in R$ and the equation

$-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$

(where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval **[JEE (Main)-2014]**

(1) $(-2, -1)$ (2) $(-\infty, -2) \cup (2, \infty)$

(3) $(-1, 0) \cup (0, 1)$ (4) $(1, 2)$

7. If g is the inverse of a function f and $f'(x) = \dfrac{1}{1+x^5}$, then $g'(x)$ is equal to

[JEE (Main)-2014]

(1) $\dfrac{1}{1+\{g(x)\}^5}$ (2) $1 + \{g(x)\}^5$

(3) $1 + x^5$ (4) $5x^4$

8. If $f(x) + 2f\left(\dfrac{1}{x}\right) = 3x, x \neq 0,$ and

$S = \{x \in R : f(x) = f(-x)\}$; then S

[JEE (Main)-2016]

(1) Contains exactly one element

(2) Contains exactly two elements

(3) Contains more than two elements

(4) Is an empty set

9. The function $f : R \rightarrow \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ defined as $f(x) = \dfrac{x}{1+x^2}$, is **[JEE (Main)-2017]**

(1) Injective but not surjective

(2) Surjective but not injective

(3) Neither injective nor surjective

(4) Invertible

10. For $x \in R - \{0,1\}$, let $f_1(x) = \frac{1}{x}, f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then $J(x)$ is equal to **[JEE (Main)-2019]**

(1) $f_1(x)$ (2) $\frac{1}{x}f_3(x)$

(3) $f_2(x)$ (4) $f_3(x)$

11. Let $A = \{x \in R : x$ is not a positive integer$\}$. Define a function $f : A \to R$ as $f(x) = \frac{2x}{x-1}$, then f is **[JEE (Main)-2019]**

(1) Injective but not surjective

(2) Neither injective nor surjective

(3) Surjective but not injective

(4) Not injective

12. Let N be the set of natural numbers and two functions f and g be defined as

$f, g : N \to N$ such that

$$f(n) = \begin{cases} \frac{n+1}{2} \text{ if } n \text{ is odd} \\ \frac{n}{2} \text{ if } n \text{ is even} \end{cases}$$

and $g(n) = n - (-1)^n$. Then fog is **[JEE (Main)-2019]**

(1) One-one but not onto.

(2) Onto but not one-one.

(3) Neither one-one nor onto.

(4) Both one-one and onto.

13. Let $f : R \to R$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is **[JEE (Main)-2019]**

(1) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(3) $(-1, 1) - \{0\}$ (4) $R - [-1, 1]$

14. Let a function $f : (0, \infty) \to [0, \infty)$ be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then f is **[JEE (Main)-2019]**

(1) Injective only

(2) Both injective as well as surjective

(3) Not injective but it is surjective

(4) Neither injective nor surjective

15. If $f(x) = \log_e\left(\frac{1-x}{1+x}\right), |x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to : **[JEE (Main)-2019]**

(1) $2f(x)$ (2) $2f(x^2)$

(3) $-2f(x)$ (4) $(f(x))^2$

16. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals **[JEE (Main)-2019]**

(1) $2f_1(x)f_1(y)$

(2) $2f_1(x + y)f_1(x - y)$

(3) $2f_1(x + y)f_2(x - y)$

(4) $2f_1(x)f_2(y)$

17. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies $f(x + y) = f(x) f(y)$ for all natural numbers x, y and $f(1) = 2$. Then the natural number a is **[JEE (Main)-2019]**

(1) 2 (2) 3

(3) 16 (4) 4

18. If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to **[JEE (Main)-2019]**

(1) $[0, \infty)$ (2) $R - \{-1\}$

(3) $R - (-1, 0)$ (4) $R - [-1, 0)$

19. The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is **[JEE (Main)-2019]**

(1) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

(2) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(3) $(1, 2) \cup (2, \infty)$

(4) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

20. Let $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R : f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true ? **[JEE (Main)-2019]**

(1) $f(g(S)) = S$ (2) $g(f(S)) = g(S)$

(3) $g(f(S)) \neq S$ (4) $f(g(S)) \neq f(S)$

21. The number of real roots of the equation $5+|2^x-1|=2^x(2^x-2)$ is **[JEE (Main)-2019]**

(1) 4 (2) 2

(3) 1 (4) 3

22. For $x \in R$, let $[x]$ denote the greatest integer $\le x$, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+....+\left[-\frac{1}{3}-\frac{99}{100}\right]$$

is: **[JEE (Main)-2019]**

(1) – 135 (2) –153

(3) –133 (4) –131

23. For $x \in \left(0, \frac{3}{2}\right)$, let $f(x)=\sqrt{x}$, $g(x)=\tan x$ and $h(x)=\frac{1-x^2}{1+x^2}$. If $\phi(x)$ = $((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to : **[JEE (Main)-2019]**

(1) $\tan\frac{5\pi}{12}$ (2) $\tan\frac{\pi}{12}$

(3) $\tan\frac{11\pi}{12}$ (4) $\tan\frac{7\pi}{12}$

24. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to **[JEE (Main)-2020]**

(1) $-\frac{1}{2}$ (2) $\frac{3}{2}$

(3) $\frac{1}{2}$ (4) $-\frac{3}{2}$

25. The inverse function of

$$f(x)=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}, x \in (-1, 1), \text{ is } \_\_\_\_\_\_\_$$

[JEE (Main)-2020].

(1) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(2) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

(3) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

(4) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$

26. Let $f : R \to R$ be a function which satisfies $f(x + y) = f(x) + f(y)\ \forall\ x, y \in R$. If $f(1) = 2$ and $g(n)=\sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which $g(n) = 20$, is **[JEE (Main)-2020]**

(1) 20 (2) 9

(3) 5 (4) 4

27. Let $[t]$ denote the greatest integer $\le t$. Then the equation in x, $[x]^2 + 2[x + 2] - 7 = 0$ has **[JEE (Main)-2020]**

(1) Exactly two solutions

(2) Infinitely many solutions

(3) Exactly four integral solutions

(4) No integral solution

28. If $f(x + y) = f(x)\, f(y)$ and $\sum_{x=1}^{\infty} f(x)=2$, $x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is **[JEE (Main)-2020]**

(1) $\frac{1}{9}$ (2) $\frac{4}{9}$

(3) $\frac{1}{3}$ (4) $\frac{2}{3}$

29. For a suitably chosen real constant a, let a function, $f : R - \{-a\} \to R$ be defined by $f(x)=\frac{a-x}{a+x}$. Further suppose that for any real number $x \ne -a$ and $f(x) \ne -a$, $(fof)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to **[JEE (Main)-2020]**

(1) –3 (2) $\frac{1}{3}$

(3) $-\frac{1}{3}$ (4) 3

30. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f : A \to B \mid 2 \in f(A)$ and f is not one-one$\}$ is ______. **[JEE (Main)-2020]**

31. Suppose that a function $f : R \to R$ satisfies $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(1) = 3$. If $\sum_{i=1}^{n} f(i)=363$, then n is equal to ________. **[JEE (Main)-2020]**

32. The number of function f from {1, 2, 3, ...,20} onto {1, 2, 3, ..., 20} such that $f(k)$ is a multiple of 3, whenever k is a multiple of 4, is

[JEE (Main)-2019]

(1) $5^6 \times 15$ (2) $6^5 \times (15)!$

(3) $5! \times 6!$ (4) $(15)! \times 6!$

33. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is ________.

[JEE (Main)-2021]

34. If $a + \alpha = 1$, $b + \beta = 2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x},\ x\neq 0$, then the value of the expression $\dfrac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is________.

[JEE (Main)-2021]

35. Let f, g : $N \rightarrow N$ such that $f(n+1)=f(n)+f(1)\forall n\in N$ and g be any arbitrary function. Which of the following statements is NOT true ? **[JEE (Main)-2021]**

(1) If g is onto, then fog is one-one

(2) If f is onto, then $f(n)=n\forall n\in N$

(3) f is one-one

(4) If fog is one-one, then g is one-one

36. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then : **[JEE (Main)-2021]**

(1) $2y = 273x$ (2) $2y = 91x$

(3) $y = 273x$ (4) $y = 91x$

37. A function $f(x)$ is given by $f(x)=\dfrac{5^x}{5^x+5}$, then the sum of the series

$$f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+....+f\left(\frac{39}{20}\right)$$ is equal to:

[JEE (Main)-2021]

(1) $\frac{19}{2}$ (2) $\frac{29}{2}$

(3) $\frac{49}{2}$ (4) $\frac{39}{2}$

38. Let A = {1, 2, 3, ..., 10} and f : A $\rightarrow$ A be defined as

$$f(k) = \begin{cases} k+1 & \text{if k is odd} \\ k & \text{if k is even} \end{cases}$$

Then the number of possible functions g : A $\rightarrow$ A such that gof = f is :

[JEE (Main)-2021]

(1) 5^5 (2) 10^5

(3) $5!$ (4) $^{10}C_5$

39. Let $f(x) = \sin^{-1} x$ and $g(x) = \dfrac{x^2-x-2}{2x^2-x-6}$. If $g(2) = \lim\limits_{x\rightarrow 2} g(x)$, then the domain of the function fog is : **[JEE (Main)-2021]**

(1) $(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right)$

(2) $(-\infty,-2]\cup\left[-\frac{3}{2},\infty\right)$

(3) $(-\infty,-1]\cup[2,\infty)$

(4) $(-\infty,-2]\cup[-1,\infty)$

40. If for $x\in\left(0,\frac{\pi}{2}\right)$, $\log_{10}\sin x+\log_{10}\cos x=-1$ and $\log_{10}(\sin x+\cos x)=\frac{1}{2}(\log_{10}n-1), n>0,$ then the value of n is equal to **[JEE (Main)-2021]**

(1) 9 (2) 16

(3) 12 (4) 20

41. The inverse of $y = 5^{\log x}$ is

[JEE (Main)-2021]

(1) $x = y^{\log 5}$ (2) $x = 5^{\log y}$

(3) $x = y^{\frac{1}{\log 5}}$ (4) $x = 5^{\frac{1}{\log y}}$

42. If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$, then what is the common domain of the following functions :

f + g, f – g, f/g, g/f, g – f

where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x)=\dfrac{f(x)}{g(x)}$

[JEE (Main)-2021]

(1) $0 < x < 1$ (2) $0 < x \leq 1$

(3) $0 \leq x \leq 1$ (4) $0 \leq x < 1$

43. Let f : **R** – {3} → **R** – {1} be defined by $f(x)=\frac{x-2}{x-3}$.

Let g : **R** → **R** be given as g(x) = 2x – 3. Then, the sum of all the values of x for which $f^{-1}(x)+g^{-1}(x)=\frac{13}{2}$ is equal to

[JEE (Main)-2021]

(1) 3 (2) 5

(3) 7 (4) 2

44. Let [x] denote the greatest integer ≤ x, where x ∈ R. If the domain of the real valued function $f(x)=\sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is (–∞, a), ∪ [b, c) ∪ (4, ∞), a < b < c, then the value of a + b + c is

[JEE (Main)-2021]

(1) –2 (2) 1

(3) 8 (4) –3

45. If sum of the first 21 terms of the series $\log_{9^{1/2}} x+\log_{9^{1/3}} x+\log_{9^{1/4}} x+...$, where $x > 0$ is 504, then x is equal to **[JEE (Main)-2021]**

(1) 81 (2) 243

(3) 9 (4) 7

46. Let $f:\mathbf{R}-\left\{\frac{\alpha}{6}\right\}\to\mathbf{R}$ be defined by $f(x)=\frac{5x+3}{6x-\alpha}$. Then the value of α for which $(fof)(x) = x$, for all $x\in\mathbf{R}-\left\{\frac{\alpha}{6}\right\}$, is **[JEE (Main)-2021]**

(1) 5 (2) 8

(3) No such α exists (4) 6

47. The number of solutions of the equation $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0$, $x > 0$, is ____. **[JEE (Main)-2021]**

48. If the domain of the function

$$f(x)=\frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$$ is the interval (α, β], then α + β is equal to **[JEE (Main)-2021]**

(1) 1 (2) $\frac{3}{2}$

(3) $\frac{1}{2}$ (4) 2

49. Let [x] denote the greatest integer less than or equal to x. Then, the values of $x\in\mathbf{R}$ satisfying the equation $[e^x]^2+[e^x+1]-3=0$ lie in the interval.

[JEE (Main)-2021]

(1) [0, 1/e) (2) [1, e)

(3) $[0, \log_e 2)$ (4) $[\log_e 2, \log_e 3)$

50. Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions $f: A\to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to ________. **[JEE (Main)-2021]**

51. Let g : N → N be defined as

$g(3n + 1) = 3n + 2$,

$g(3n + 2) = 3n + 3$,

$g(3n + 3) = 3n + 1$, for all n ≥ 0.

Then which of the following statements is true?

[JEE (Main)-2021]

(1) $gogog = g$

(2) There exists an onto function f : N → N such that $fog = f$

(3) There exists a one-one function f : N → N such that $fog = f$

(4) There exists a function f : N → N such that $gof = f$

52. Consider functions f : A → B and g : B → C(A, B, C ⊆ R) such that $(gof)^{-1}$ exists, then

[JEE (Main)-2021]

(1) f is one-one and g is onto

(2) f is onto and g is one-one

(3) f and g both are one-one

(4) f and g both are onto

53. If for $x, y \in \mathbf{R}$, $x > 0$, $y=\log_{10}x+\log_{10}x^{1/3}+\log_{10}x^{1/9}$ +...upto ∞ terms and $\frac{2+4+6+...+2y}{3+6+9+...+3y}=\frac{4}{\log_{10}x}$, then the ordered pair (x, y) is equal to :

[JEE (Main)-2021]

(1) $(10^6, 9)$ (2) $(10^6, 6)$

(3) $(10^4, 6)$ (4) $(10^2, 3)$

54. If $x^2+9y^2-4x+3=0$, $x, y\in\mathbf{R}$, then x and y respectively lie in the intervals

[JEE (Main)-2021]

(1) [1, 3] and [1, 3]

(2) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and $\left[-\frac{1}{3},\frac{1}{3}\right]$

(3) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and [1, 3]

(4) [1, 3] and $\left[-\frac{1}{3},\frac{1}{3}\right]$

55. Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the probability that a randomly chosen onto function g from S to S satisfies $g(3) = 2g(1)$ is **[JEE (Main)-2021]**

(1) $\frac{1}{30}$ (2) $\frac{1}{10}$

(3) $\frac{1}{15}$ (4) $\frac{1}{5}$

56. The sum of the roots of the equation,

$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is

[JEE (Main)-2021]

(1) $\log_2 14$ (2) $\log_2 12$

(3) $\log_2 11$ (4) $\log_2 13$

57. Let $f : N \to N$ be a function such that $f(m + n) = f(m) + f(n)$ for every $m, n \in N$. If $f(6) = 18$, then $f(2)\cdot f(3)$ is equal to **[JEE (Main)-2021]**

(1) 54 (2) 18

(3) 6 (4) 36

58. The range of the function

$$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$$

is **[JEE (Main)-2021]**

(1) $[0, 2]$ (2) $[-2, 2]$

(3) $\left(0, \sqrt{5}\right)$ (4) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$

59. The number of one-one functions $f : \{a, b, c, d\} \to \{0, 1, 2, ..., 10\}$ such that $2f(a) - f(b) + 3f(c) + f(d) = 0$ is ________. **[JEE (Main)-2022]**

60. Let $f : \mathbf{N} \to \mathbf{R}$ be a function such that $f(x + y) = 2f(x)\, f(y)$ for natural numbers x and y. If $f(1) = 2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}(2^{20} - 1) \quad \text{holds, is :}$$

[JEE (Main)-2022]

(1) 2 (2) 3

(3) 4 (4) 6

61. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$f(x) = x^3 + x - 5$

If $g(x)$ is a function such that $f(g(x)) = x, \forall 'x' \in \mathbf{R}$, then $g'(63)$ is equal to_____. **[JEE (Main)-2022]**

(1) $\frac{1}{49}$ (2) $\frac{3}{49}$

(3) $\frac{43}{49}$ (4) $\frac{91}{49}$

62. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined by

$f(x) = \left(2\left(1 - \frac{x^{25}}{2}\right)\left(2 + x^{25}\right)\right)^{\frac{1}{50}}$. If the function $g(x) = f(f(f(x))) + f(f(x))$, then the greatest integer less than or equal to $g(1)$ is ___________.

[JEE (Main)-2022]

63. Let $f(x) = \frac{x-1}{x+1}, x \in R - \{0, -1, 1\}$. If $f^{n+1}(x) = f(f^n(x))$ for all $n \in \mathbf{N}$, then $f^6(6) + f^7(7)$ is equal to :

[JEE (Main)-2022]

(1) $\frac{7}{6}$ (2) $-\frac{3}{2}$

(3) $\frac{7}{12}$ (4) $-\frac{11}{12}$

64. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x - 1$ and $g : \mathbb{R} - \{1, -1\} \to \mathbb{R}$ be defined as $g(x) = \frac{x^2}{x^2 - 1}$.

Then the function fog is: **[JEE (Main)-2022]**

(1) One-one but not onto

(2) Onto but not one-one

(3) Both one-one and onto

(4) Neither one-one nor onto

65. Let $f : \mathbb{R} \to \mathbb{R}$ satisfy $f(x + y) = 2^x f(y) + 4^y f(x)$, $\forall x, y \in \mathbb{R}$. If $f(2) = 3$, then $14\cdot\frac{f'(4)}{f'(2)}$ is equal to ___.

[JEE (Main)-2022]

66. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined by

$$f(x) = \frac{2e^{2x}}{e^{2x}+e}.$$

Then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+\ldots+f\left(\frac{99}{100}\right)$ is equal to ________. **[JEE (Main)-2022]**

67. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Define $f : S ! S$ as

$$f(n) = \begin{cases} 2n & \text{, if } n = 1, 2, 3, 4, 5 \\ 2n-11 & \text{, if } n = 6, 7, 8, 9, 10 \end{cases}.$$

Let $g : S ! S$ be a function such that

$$fog(n) = \begin{cases} n+1 & \text{, if } n \text{ is odd} \\ n-1 & \text{, if } n \text{ is even} \end{cases},$$

Then $g(10)\,(g(1) + g(2) + g(3) + g(4) + g(5))$ is equal to __________. **[JEE (Main)-2022]**

68. Let a function $f : \mathrm{N} \to \mathrm{N}$ be defined by

$$f(n) = \begin{bmatrix} 2n, & n = 2, 4, 6, 8, \ldots\ldots \\ n-1, & n = 3, 7, 11, 15, \ldots\ldots \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \ldots\ldots \end{bmatrix}$$

then, f is **[JEE (Main)-2022]**

(1) One-one but not onto

(2) Onto but not one-one

(3) Neither one-one nor onto

(4) One-one and onto

69. Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \to S : f$ is onto and $f(a, b) = f(b, a) \geq a\ \forall (a, b) \in S \times S\}$ is _____.

[JEE (Main)-2022]

70. Let $c, k \in \mathbf{R}$. If $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ and $f(x + y) = f(x) + f(y) - xy$, for all $x, y \in \mathbf{R}$, then the value of $\left|2\left(f(1)+f(2)+f(3)+\ldots..+f(20)\right)\right|$ is equal to __________. **[JEE (Main)-2022]**

71. The number of solutions of the equation $\sin x = \cos^2 x$ in the interval $(0, 10)$ is _____.

72. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \leq x \leq 4\pi$ is : **[JEE (Main)-2022]**

(1) 4 (2) 6

(3) 8 (4) 12

73. Let $f, g : \mathbb{N} - \{1\} \to \mathbb{N}$ be functions defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes p such that p^α divides a, and $g(a) = a + 1$, for all $a \in \mathbb{N} - \{1\}$. Then, the function $f + g$ is

[JEE (Main)-2022]

(1) one-one but not onto

(2) onto but not one-one

(3) both one-one and onto

(4) neither one-one nor onto

74. The number of functions f, from the set $A = \{x \in N : x^2 - 10x + 9 \leq 0\}$ to the set $B = \{n^2 : n \in N\}$ such that $f(x) \leq (x - 3)^2 + 1$, for every $x \in A$, is _________.

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75. Let α, β and γ be three positive real numbers. Let $f(x) = \alpha x^5 + \beta x^3 + \gamma x$, $x \in \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be such that $g(f(x)) = x$ for all $x \in \mathbb{R}$. If $a_1, a_2, a_3, \ldots, a_n$ be in arithmetic progression with mean zero, then the value of $f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right)$ is equal to

[JEE (Main)-2022]

(1) 0 (2) 3

(3) 9 (4) 27

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