Chapter 8

Trigonometric Functions

1. Let A and B denote the statements :

   A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

   B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

   If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then **[AIEEE-2009]**

   (1) A is false and B is true

   (2) Both A and B are true

   (3) Both A and B are false

   (4) A is true and B is false

2. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then tan 2$\alpha$ = **[AIEEE-2010]**

   (1) $\frac{25}{16}$ (2) $\frac{56}{33}$

   (3) $\frac{19}{12}$ (4) $\frac{20}{7}$

3. For a regular polygon, let *r* and *R* be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is **[AIEEE-2010]**

   (1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

   (2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

   (3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

   (4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

4. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **[AIEEE-2011]**

   (1) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

   (2) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

   (3) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

   (4) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

5. In a ΔPQR, if 3 sin *P* + 4 cos *Q* = 6 and 4 sin *Q* + 3 cos *P* = 1, then the angle *R* is equal to **[AIEEE-2012]**

   (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$

   (3) $\frac{3\pi}{4}$ (4) $\frac{5\pi}{6}$

6. *ABCD* is a trapezium such that *AB* and *CD* are parallel and *BC* $\perp$ *CD*. If $\angle ADB = \theta$, *BC* = *p* and *CD* = *q*, then *AB* is equal to **[JEE (Main)-2013]**

   (1) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (2) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$

   (3) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (4) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

7. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as **[JEE (Main)-2013]**

   (1) sin*A* cos*A* + 1

   (2) sec*A* cosec*A* + 1

   (3) tan*A* + cot*A*

   (4) sec*A* + cosec*A*

8. Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$ where $x\in R$ and $k\geq 1$. Then $f_4(x) - f_6(x)$ equals **[JEE (Main)-2014]**

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$

(3) $\frac{1}{6}$ (4) $\frac{1}{3}$

9. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point *O* on the ground is 45°. It flies off horizontally straight away from the point *O*. After one second, the elevation of the bird from *O* is reduced to 30°. Then the speed (in m/s) of the bird is **[JEE (Main)-2014]**

(1) $20\sqrt{2}$ (2) $20\left(\sqrt{3}-1\right)$

(3) $40\left(\sqrt{2}-1\right)$ (4) $40\left(\sqrt{3}-\sqrt{2}\right)$

10. If the angles of elevation of the top of a tower from three collinear points *A*, *B* and *C*, on a line leading to the foot of the tower, are 30º, 45º and 60º respectively, then the ratio, *AB* : *BC*, is **[JEE (Main)-2015]**

(1) $\sqrt{3}$: 1 (2) $\sqrt{3}$: $\sqrt{2}$

(3) 1 : $\sqrt{3}$ (4) 2 : 3

11. If $0\leq x<2\pi$, then the number of real values of *x*, which satisfy the equation $\cos x+\cos 2x+\cos 3x+\cos 4x=0$, is **[JEE (Main)-2016]**

(1) 5 (2) 7

(3) 9 (4) 3

12. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point *A* on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from *A* in the same direction, at a point *B*, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from *B* to reach the pillar, is **[JEE (Main)-2016]**

(1) 10 (2) 20

(3) 5 (4) 6

13. If 5 ($\tan^2 x - \cos^2 x$) = 2cos 2*x* + 9, then the value of cos 4*x* is **[JEE (Main)-2017]**

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$

(3) $-\frac{7}{9}$ (4) $-\frac{3}{5}$

14. Let a vertical tower *AB* have its end *A* on the level ground. Let *C* be the mid-point of *AB* and *P* be a point on the ground such that *AP* = 2*AB*. If $\angle BPC = \beta$ then tan β is **[JEE (Main)-2017]**

(1) $\frac{1}{4}$ (2) $\frac{2}{9}$

(3) $\frac{4}{9}$ (4) $\frac{6}{7}$

15. If sum of all the solutions of the equation $8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in [0, π] is $k\pi$, then *k* is equal to : **[JEE (Main)-2018]**

(1) $\frac{2}{3}$ (2) $\frac{13}{9}$

(3) $\frac{8}{9}$ (4) $\frac{20}{9}$

16. *PQR* is a triangular park with *PQ* = *PR* = 200 m. A T.V. tower stands at the mid-point of *QR*. If the angles of elevation of the top of the tower at *P*, *Q* and *R* are respectively 45º, 30º and 30º, then the height of the tower (in m) is **[JEE (Main)-2018]**

(1) 100 (2) 50

(3) $100\sqrt{3}$ (4) $50\sqrt{2}$

17. For any $\theta\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals **[JEE (Main)-2019]**

(1) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(2) $13 - 4\cos^2\theta + 6\cos^4\theta$

(3) $13 - 4\cos^6\theta$

(4) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

18. If $0\leq x<\frac{\pi}{2}$, then the number of values of *x* for which $\sin x - \sin 2x + \sin 3x = 0$, is **[JEE (Main)-2019]**

(1) 2 (2) 3

(3) 1 (4) 4

19. Consider a triangular plot *ABC* with sides *AB* = 7 m, *BC* = 5 m and *CA* = 6 m. A vertical lamp-post at the midpoint *D* of *AC* subtends an angle 30° at *B*. The height (in m) of the lamp-post is **[JEE (Main)-2019]**

(1) $2\sqrt{21}$ (2) $7\sqrt{3}$

(3) $\frac{2}{3}\sqrt{21}$ (4) $\frac{3}{2}\sqrt{21}$

20. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is **[JEE (Main)-2019]**

(1) $\frac{5\pi}{4}$ (2) $\frac{\pi}{2}$

(3) $\frac{3\pi}{8}$ (4) π

21. If 5, $5r$, $5r^2$ are the lengths of the sides of a triangle, then r **cannot** be equal to **[JEE (Main)-2019]**

(1) $\frac{3}{2}$ (2) $\frac{7}{4}$

(3) $\frac{3}{4}$ (4) $\frac{5}{4}$

22. With the usual notation, in ΔABC, if $\angle A + \angle B = 120°$, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio $\angle A : \angle B$, is **[JEE (Main)-2019]**

(1) 7 : 1 (2) 3 : 1

(3) 9 : 7 (4) 5 : 3

23. The value of $\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}.\ldots.\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$ is **[JEE (Main)-2019]**

(1) $\frac{1}{512}$ (2) $\frac{1}{256}$

(3) $\frac{1}{2}$ (4) $\frac{1}{1024}$

24. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{-1}{12}$ (2) $\frac{1}{12}$

(3) $\frac{5}{12}$ (4) $\frac{1}{4}$

25. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triplet (α, β, γ) has a value **[JEE (Main)-2019]**

(1) (3, 4, 5) (2) (7, 19, 25)

(3) (19, 7, 25) (4) (5, 12, 13)

26. The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is **[JEE (Main)-2019]**

(1) $\sqrt{34}$ (2) $\sqrt{19}$

(3) $\frac{\sqrt{79}}{2}$ (4) $\sqrt{31}$

27. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is **[JEE (Main)-2019]**

(1) 45 (2) 50

(3) 42 (4) 60

28. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to **[JEE (Main)-2019]**

(1) $\sqrt{2}$ (2) $-\sqrt{2}$

(3) –1 (4) 0

29. If $\cos(\alpha+\beta) = \frac{3}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{21}{16}$ (2) $\frac{63}{52}$

(3) $\frac{33}{52}$ (4) $\frac{63}{16}$

30. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is **[JEE (Main)-2019]**

(1) 16 (2) 18

(3) 15 (4) 12

31. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is **[JEE (Main)-2019]**

(1) 4 : 5 : 6 (2) 3 : 4 : 5

(3) 5 : 9 : 13 (4) 5 : 6 : 7

32. Let $S = \{\theta \in [-2\pi, 2\pi] ; 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is

[JEE (Main)-2019]

(1) π (2) 2π

(3) $\frac{13\pi}{6}$ (4) $\frac{5\pi}{3}$

33. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is

[JEE (Main)-2019]

(1) $\frac{3}{4}$ (2) $\frac{3}{2}(1+\cos 20°)$

(3) $\frac{3}{2}$ (4) $\frac{3}{4}+\cos 20°$

34. Two poles standing on a horizontal ground are of heights 5 m and 10 m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is

[JEE (Main)-2019]

(1) $5\left(2+\sqrt{3}\right)$ (2) $10\left(\sqrt{3}-1\right)$

(3) $5\left(\sqrt{3}+1\right)$ (4) $\frac{5}{2}\left(2+\sqrt{3}\right)$

35. The value of $\sin 10° \sin 30° \sin 50° \sin 70°$ is

[JEE (Main)-2019]

(1) $\frac{1}{18}$ (2) $\frac{1}{32}$

(3) $\frac{1}{16}$ (4) $\frac{1}{36}$

36. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the equation

[JEE (Main)-2019]

(1) $\sin x = |\sin y|$ (2) $\sin x = 2 \sin y$

(3) $2 \sin x = \sin y$ (4) $2|\sin x| = 3 \sin y$

37. ABC is a triangular park with $AB = AC = 100$ metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is :

[JEE (Main)-2019]

(1) 20 (2) $10\sqrt{5}$

(3) 25 (4) $\frac{100}{3\sqrt{3}}$

38. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq.cm) of this triangle is :

[JEE (Main)-2019]

(1) $\frac{2}{\sqrt{3}}$ (2) $4\sqrt{3}$

(3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$

39. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/s, then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is

[JEE (Main)-2019]

(1) $\frac{25}{3}$ (2) $25\sqrt{3}$

(3) $\frac{25}{\sqrt{3}}$ (4) 25

40. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is

[JEE (Main)-2019]

(1) 3 (2) 5

(3) 4 (4) 7

41. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is

[JEE (Main)-2019]

(1) $15\,(3+\sqrt{3})$ (2) $15\,(1+\sqrt{3})$

(3) $15\,(3-\sqrt{3})$ (4) $15\,(5-\sqrt{3})$

42. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to

[JEE (Main)-2019]

(1) [1, 4] (2) R

(3) [2, 6] (4) [3, 7]

43. The value of

$$\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right) \text{ is}$$

[JEE (Main)-2020]

(1) $\frac{1}{4}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{2}}$

44. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval

[JEE (Main)-2020]

(1) $\left[-1,-\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2},-\frac{5}{4}\right]$

(3) $\left(-\frac{1}{2},-\frac{1}{4}\right]$ (4) $\left(-\frac{5}{4},-1\right)$

45. Two vertical poles *AB* = 15 m and *CD* = 10 m are standing apart on a horizontal ground with points *A* and *C* on the ground. If *P* is the point of intersection of *BC* and *AD*, then the height of *P* (in m) above the line *AC* is

[JEE (Main)-2020]

(1) 6 (2) 20/3

(3) 10/3 (4) 5

46. If $L=\sin^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$ and

$M=\cos^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$, then

[JEE (Main)-2020]

(1) $L=-\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$

(2) $M=\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$

(3) $M=\frac{1}{4\sqrt{2}}+\frac{1}{4}\cos\frac{\pi}{8}$

(4) $L=\frac{1}{4\sqrt{2}}-\frac{1}{4}\cos\frac{\pi}{8}$

47. The angle of elevation of a cloud *C* from a point *P*, 200 m above a still lake is 30°. If the angle of depression of the image of *C* in the lake from the point *P* is 60°, then *PC* (in m) is equal to

[JEE (Main)-2020]

(1) 400 (2) $400\sqrt{3}$

(3) 100 (4) $200\sqrt{3}$

48. The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is

[JEE (Main)-2020]

(1) $\frac{1}{\sqrt{3}+1}$ (2) $\frac{1}{\sqrt{3}-1}$

(3) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (4) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

49. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$, $\alpha, \beta \in \left(0,\frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to _____.

[JEE (Main)-2020]

50. The number of distinct solutions of the equation, $\log_{\frac{1}{2}}|\sin x| = 2-\log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$, is _____.

[JEE (Main)-2020]

51. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is _____.

[JEE (Main)-2020]

52. If $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+\ldots\infty\right)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$ is :

[JEE (Main)-2021]

(1) $\frac{3}{2}$ (2) $\sqrt{3}$

(3) $2\sqrt{3}$ (4) $\frac{1}{2}$

53. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is : **[JEE (Main)-2021]**

(1) $\mathbf{25\sqrt{3}}$ (2) 30

(3) 25 (4) $\mathbf{20\sqrt{3}}$

54. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is : **[JEE (Main)-2021]**

(1) $3600\sqrt{3}$ m (2) $2400\sqrt{3}$ m

(3) $1800\sqrt{3}$ m (4) $1200\sqrt{3}$ m

55. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in : **[JEE (Main)-2021]**

(1) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(2) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(3) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(4) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

56. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At the point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is : **[JEE (Main)-2021]**

(1) $10\sqrt{3}$ (2) 10

(3) $10\left(\sqrt{3}+1\right)$ (4) $10\left(\sqrt{3}-1\right)$

57. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{1+\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{2}$ (4) $\frac{1-\sqrt{3}}{2}$

58. The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k+1$ has a solution, $k \in R$ is _____. **[JEE (Main)-2021]**

59. If $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3}-1\right) \cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is _____. **[JEE (Main)-2021]**

60. Let A(1, 4) and B(1, –5) be two points. Let P be a point on the circle $(x - 1)^2 + (y - 1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points, P, A and B lie on : **[JEE (Main)-2021]**

(1) an ellipse (2) a parabola

(3) a straight line (4) a hyperbola

61. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to **[JEE (Main)-2021]**

(1) 4 (2) 2

(3) 8 (4) 3

62. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to _____. **[JEE (Main)-2021]**

63. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to _____. **[JEE (Main)-2021]**

64. The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is : **[JEE (Main)-2021]**

(1) 2 (2) 4

(3) 3 (4) 5

65. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the area of ΔPAB and ΔCAB is : **[JEE (Main)-2021]**

(1) 2 : 1 (2) 3 : 1

(3) 11 : 4 (4) 9 : 4

66. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is _____. **[JEE (Main)-2021]**

67. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line x + y = 3. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then (R + r) is equal to : **[JEE (Main)-2021]**

(1) $3\sqrt{2}$ (2) $2\sqrt{2}$

(3) $\frac{9}{\sqrt{2}}$ (4) $7\sqrt{2}$

68. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to : **[JEE (Main)-2021]**

(1) $\frac{2\sqrt{3}}{3}$ (2) $\frac{1}{\sqrt{3}}$

(3) $2\sqrt{3}$ (4) $\sqrt{3}$

69. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in \mathbf{R}$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to : **[JEE (Main)-2021]**

(1) 350 (2) 250

(3) 400 (4) 500

70. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is : **[JEE (Main)-2021]**

(1) $10 + 6\sqrt{2}$ (2) $6 + 8\sqrt{3}$

(3) $8 + 2\sqrt{2}$ (4) $4 + 2\sqrt{3}$

71. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{\sqrt{5}+1}{4}$ (2) $\frac{\sqrt{2}-1}{2}$

(3) $\frac{\sqrt{5}-1}{2}$ (4) $\frac{\sqrt{5}-1}{4}$

72. Consider a triangle having vertices A(–2, 3), B(1, 9) and C(3, 8). If a line L passing through the circum-center of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is _____. **[JEE (Main)-2021]**

73. The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to : **[JEE (Main)-2021]**

(1) 5 (2) 7

(3) 11 (4) 9

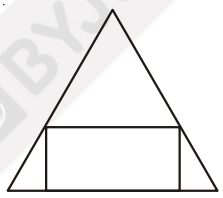
74. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 *k*, then *k* is equal to _____. **[JEE (Main)-2021]**

75. The value of $\cot\frac{\pi}{24}$ is **[JEE (Main)-2021]**

(1) $3\sqrt{2} - \sqrt{3} - \sqrt{6}$ (2) $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$

(3) $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$ (4) $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

76. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is _____. **[JEE (Main)-2021]**



77. If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to **[JEE (Main)-2021]**

(1) 23 (2) –23

(3) 27 (4) –27

78. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, \frac{\pi}{4}\right\}$ is : **[JEE (Main)-2021]**

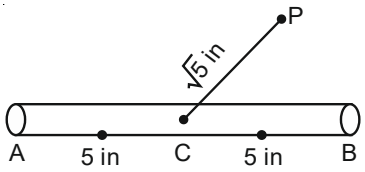
(1) $\frac{\pi}{10}$ (2) $-\frac{7\pi}{30}$

(3) $-\frac{11\pi}{30}$ (4) $-\frac{\pi}{15}$

79. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that $PC = \sqrt{5}$ inches and $\angle PCB = \tan^{-1}(2)$.

The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is

[JEE (Main)-2021]



(1) $\tan^{-1}\left(\frac{1}{2}\right)$ (2) $\tan^{-1}\left(\frac{3}{4}\right)$

(3) $\tan^{-1}(1)$ (4) $\tan^{-1}\left(\frac{4}{3}\right)$

80. The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

is **[JEE (Main)-2021]**

(1) $\frac{1}{8\sqrt{2}}$ (2) $\frac{1}{4\sqrt{2}}$

(3) $\frac{1}{8}$ (4) $\frac{1}{4}$

81. Let $A(a, 0)$, $B(b, 2b + 1)$ and $C(0, b)$, $b \neq 0$, $|b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is **[JEE (Main)-2021]**

(1) $\frac{2b}{b+1}$ (2) $\frac{-2b^2}{b+1}$

(3) $\frac{2b^2}{b+1}$ (4) $\frac{-2b}{b+1}$

82. Two poles, AB of length a metres and CD of length $a + b$ ($b \neq a$) metres are erected at the same horizontal level with bases at B and D. If $BD = x$ and $\tan\angle ACB = \frac{1}{2}$, then **[JEE (Main)-2021]**

(1) $x^2 - 2ax + a(a + b) = 0$

(2) $x^2 + 2(a + 2b)x - b(a + b) = 0$

(3) $x^2 + 2(a + 2b)x + a(a + b) = 0$

(4) $x^2 - 2ax + b(a + b) = 0$

83. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$.

Then $\frac{8S}{\pi}$ is equal to _____.

[JEE (Main)-2021]

84. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

[JEE (Main)-2021]

(1) $8\sqrt{10}$ (2) $12\sqrt{10}$

(3) $12\sqrt{15}$ (4) $6\sqrt{10}$

85. The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81$, $0 \le x \le \frac{\pi}{4}$ is **[JEE (Main)-2021]**

(1) 3 (2) 1

(3) 2 (4) 0

86. If n is the number of solutions of the equation

$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1,\ x \in [0, \pi]$$

and S is the sum of all these solutions, then the ordered pair (n, S) is : **[JEE (Main)-2021]**

(1) $\left(3, \frac{5\pi}{3}\right)$ (2) $\left(3, \frac{13\pi}{9}\right)$

(3) $\left(2, \frac{2\pi}{3}\right)$ (4) $\left(2, \frac{8\pi}{9}\right)$

87. Let the points of intersections of the lines $x - y + 1 = 0$, $x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is _____.

[JEE (Main)-2021]

88. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$f(x+y)+f(x-y)=2f(x)f(y)$, $f\left(\frac{1}{2}\right)=-1$. Then, the value of $\sum_{k=1}^{20}\frac{1}{\sin(k)\sin(k+f(k))}$ is equal to

[JEE (Main)-2021]

(1) $\text{cosec}^2(21)\cos(20)\cos(2)$

(2) $\sec^2(21)\sin(20)\sin(2)$

(3) $\text{cosec}^2(1)\,\text{cosec}(21)\sin(20)$

(4) $\sec^2(1)\sec(21)\cos(20)$

89. If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$,

for $0 < \theta < \frac{\pi}{4}$, then **[JEE (Main)-2021]**

(1) $x(1 - y) = 1$ (2) $y(1 + x) = 1$

(3) $y(1 - x) = 1$ (4) $x(1 + y) = 1$

90. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is **[JEE (Main)-2021]**

(1) $8\left(2+2\sqrt{3}+\sqrt{2}\right)$ (2) $8\left(\sqrt{6}-\sqrt{2}+2\right)$

(3) $8\left(\sqrt{2}+2+\sqrt{3}\right)$ (4) $8\left(\sqrt{6}+\sqrt{2}+2\right)$

91. The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to : **[JEE (Main)-2021]**

(1) 11π (2) 9π

(3) 8π (4) 12π

92. Let

$$S = \left\{\theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}.$$

If $T = \sum_{\theta \in S} \cos 2\theta$, then $T + n(S)$ is equal to

[JEE (Main)-2022]

(1) $7+\sqrt{3}$ (2) 9

(3) $8+\sqrt{3}$ (4) 10

93. The number of solutions of the equation

$$\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}\cos^2 2x, \quad x \in [-3\pi, 3\pi]$$

is **[JEE (Main)-2022]**

(1) 8 (2) 5

(3) 6 (4) 7

94. Let a, b and c be the length of sides of a triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of $\frac{R}{r}$ is equal to **[JEE (Main)-2022]**

(1) $\frac{5}{2}$ (2) 2

(3) $\frac{3}{2}$ (4) 1

95. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\text{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is _____. **[JEE (Main)-2022]**

96. The value of $2\sin(12°) - \sin(72°)$ is **[JEE (Main)-2022]**

(1) $\frac{\sqrt{5}\left(1-\sqrt{3}\right)}{4}$ (2) $\frac{1-\sqrt{5}}{8}$

(3) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{2}$ (4) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

97. If $\sin^2(10°)\sin(20°)\sin(40°)\sin(50°)\sin(70°) = \alpha - \frac{1}{16}\sin(10°)$, then $16 + \alpha^{-1}$ is equal to _____. **[JEE (Main)-2022]**

98. $16\sin(20°)\sin(40°)\sin(80°)$ is equal to **[JEE (Main)-2022]**

(1) $\sqrt{3}$ (2) $2\sqrt{3}$

(3) 3 (4) $4\sqrt{3}$

99. The value of $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$ is equal to **[JEE (Main)-2022]**

(1) –1 (2) $-\frac{1}{2}$

(3) $-\frac{1}{3}$ (4) $-\frac{1}{4}$

100. α = sin 36º is a root of which of the following equation? **[JEE (Main)-2022]**

(1) $16x^4 - 10x^2 - 5 = 0$ (2) $16x^4 + 20x^2 - 5 = 0$

(3) $16x^4 - 20x^2 + 5 = 0$ (4) $16x^4 - 10x^2 + 5 = 0$

101. Let *AB* and *PQ* be two vertical poles, 160 m apart from each other. Let *C* be the middle point of *B* and *Q*, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from *C* to *P* and *A*, respectively. If the height of pole *PQ* is twice the height of pole *AB*, then $\tan^2\theta$ is equal to **[JEE (Main)-2022]**

(1) $\frac{3-2\sqrt{2}}{2}$ (2) $\frac{3+\sqrt{2}}{2}$

(3) $\frac{3-2\sqrt{2}}{4}$ (4) $\frac{3-\sqrt{2}}{4}$

102. If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are **[JEE (Main)-2022]**

(1) $-\frac{1}{7}$ and IVth quadrant

(2) 7 and Ist quadrant

(3) –7 and IVth quadrant

(4) $\frac{1}{7}$ and Ist quadrant

103. The number of elements in the set

$S = \{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$ is _____. **[JEE (Main)-2022]**

104. The number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$ in **R** is equal to _____. **[JEE (Main)-2022]**

105. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is **[JEE (Main)-2022]**

(1) $15\sqrt{3}$ (2) $20\sqrt{3}$

(3) $20 + 10\sqrt{3}$ (4) 30

106. A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that QR = 15 m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is :

(1) $5\left(2\sqrt{3}+3\right)$ m (2) $5\left(\sqrt{3}+3\right)$ m

(3) $10\left(\sqrt{3}+1\right)$ m (4) $10\left(2\sqrt{3}+1\right)$ m

[JEE (Main)-2022]

107. Let a vertical tower *AB* of height 2*h* stands on a horizontal ground. Let from a point *P* on the ground a man can see upto height *h* of the tower with an angle of elevation 2α. When from *P*, he moves a distance *d* in the direction of $\overrightarrow{AP}$, he can see the top *B* of the tower with an angle of elevation α. if $d = \sqrt{7}\,h$, then $\tan\alpha$ is equal to

(1) $\sqrt{5}-2$ (2) $\sqrt{3}-1$

(3) $\sqrt{7}-2$ (4) $\sqrt{7}-\sqrt{3}$

[JEE (Main)-2022]

108. The angle of elevation of the top *P* of a vertical tower *PQ* of height 10 from a point *A* on the horizontal ground is 45°, Let *R* be a point on *AQ* and from a point *B*, vertically above *R*, the angle of elevation of *P* is 60°. If $\angle BAQ = 30°$, *AB* = *d* and the area of the trapezium *PQRB* is α, then the ordered pair (d, α) is :

(1) $\left(10(\sqrt{3}-1), 25\right)$ (2) $\left(10(\sqrt{3}-1), \frac{25}{2}\right)$

(3) $\left(10(\sqrt{3}+1), 25\right)$ (4) $\left(10(\sqrt{3}+1), \frac{25}{2}\right)$

[JEE (Main)-2022]

109. Let $S = \left\{0 \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right)\sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$.

Then **[JEE (Main)-2022]**

(1) $S = \left\{\frac{\pi}{12}\right\}$ (2) $S = \left\{\frac{2\pi}{3}\right\}$

(3) $\sum_{\theta \in S} \theta = \frac{\pi}{2}$ (4) $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$

110. A horizontal park is in the shape of a triangle OAB with $AB = 16$. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15°$ and $\angle PCO = 45°$, where C is the midpoint of AB. Then $(OP)^2$ is equal to

(1) $\frac{32}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (2) $\frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$

(3) $\frac{16}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (4) $\frac{16}{\sqrt{3}}\left(2-\sqrt{3}\right)$

[JEE (Main)-2022]

111. Let $S=\left[-\pi, \frac{\pi}{2}\right)-\left[-\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}\right]$. Then the number of elements in the set $A=\left\{\theta \in S : \tan\theta\left(1+\sqrt{5}\tan(2\theta)\right)=\sqrt{5}-\tan(2\theta)\right\}$ is _____.

[JEE (Main)-2022]

112. The number of elements in the set

$S=\left\{x\in\mathbb{R} : 2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}\right\}$ is :

(1) 1 (2) 3

(3) 0 (4) Infinite

[JEE (Main)-2022]

113. If the sum of solutions of the system of equations $2\sin^2\theta - \cos2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to _____.

[JEE (Main)-2022]

114. Let $S=\left\{\theta\in[0,2\pi] : 8^{2\sin^2\theta}+8^{2\cos^2\theta}=16\right\}$. Then $n(S)+\sum_{\theta\in S}\left(\sec\left(\frac{\pi}{4}+2\theta\right)\text{cosec}\left(\frac{\pi}{4}+2\theta\right)\right)$ is equal to **[JEE (Main)-2022]**

(1) 0 (2) – 2

(3) – 4 (4) 12

115. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to : [JEE (Main)-2022]

(1) $\frac{3}{16}$ (2) $\frac{1}{16}$

(3) $\frac{1}{32}$ (4) $\frac{9}{32}$

116. Let $S = \{\theta \in (0, 2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\}$. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$, $\theta \in S$, is _____. [JEE (Main)-2022]

117. The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$.

If the distance of the point B from the tower is 15 units, then $\cot\alpha$ is equal to :

(1) $\frac{6}{5}$ (2) $\frac{9}{5}$

(3) $\frac{4}{3}$ (4) $\frac{7}{3}$

[JEE (Main)-2022]

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