Chapter 18

Limits

1. Let $f : R \to R$ be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$ **[AIEEE-2010]**

(1) 1 (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) 3

2. Let $f : R \to [0, \infty)$ be such that $\lim_{x\to 5} f(x)$ exists and $\lim_{x\to 5} \frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$. Then $\lim_{x\to 5} f(x)$ equals **[AIEEE-2011]**

(1) 2 (2) 3

(3) 0 (4) 1

3. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to **[JEE (Main)-2013]**

(1) $-\frac{1}{4}$ (2) $\frac{1}{2}$

(3) 1 (4) 2

4. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to **[JEE (Main)-2014]**

(1) $-\pi$ (2) π

(3) $\frac{\pi}{2}$ (4) 1

5. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to **[JEE (Main)-2015]**

(1) 4 (2) 3

(3) 2 (4) $\frac{1}{2}$

6. Let $p = \lim_{x\to 0+} \left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to **[JEE (Main)-2016]**

(1) 1 (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) 2

7. $\lim_{n\to\infty} \left(\frac{(n+1)(n+2)...3n}{n^{2n}}\right)^{1/n}$ is equal to **[JEE (Main)-2016]**

(1) $\frac{27}{e^2}$ (2) $\frac{9}{e^2}$

(3) 3 log3 – 2 (4) $\frac{18}{e^4}$

8. $\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2\pi)^3}$ equals **[JEE (Main)-2017]**

(1) $\frac{1}{16}$ (2) $\frac{1}{8}$

(3) $\frac{1}{4}$ (4) $\frac{1}{24}$

9. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then **[JEE (Main)-2018]**

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$$

(1) Is equal to 0

(2) Is equal to 15

(3) Is equal to 120

(4) Does not exist (in R)

10. $\lim\limits_{y\to 0}\dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$ **[JEE (Main)-2019]**

(1) Exists and equals $\dfrac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

(2) Does not exist

(3) Exists and equals $\dfrac{1}{4\sqrt{2}}$

(4) Exists and equals $\dfrac{1}{2\sqrt{2}}$

11. For each $x \in R$, let $[x]$ be the greatest integer less than or equal to x. Then $\lim\limits_{x\to 0^-}\dfrac{x([x]+|x|)\sin[x]}{|x|}$ is equal to **[JEE (Main)-2019]**

(1) –sin1 (2) 1

(3) sin1 (4) 0

12. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then,

$$\lim_{x\to 1+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{x}{2}[1-x]\right)}{|1-x|[1-x]}$$

[JEE (Main)-2019]

(1) Equals 0 (2) Equals 1

(3) Equals –1 (4) Does not exist

13. Let $[x]$ denote the greatest integer less than or equal to x. Then

$$\lim_{x\to 0}\frac{\tan(\pi\sin^2 x)+(|x|-\sin(x[x]))^2}{x^2}$$

[JEE (Main)-2019]

(1) Equals 0 (2) Equals π + 1

(3) Equals π (4) Does not exist

14. $\lim\limits_{x\to 0}\dfrac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to **[JEE (Main)-2019]**

(1) 2 (2) 4

(3) 1 (4) 0

15. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cot^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ is **[JEE (Main)-2019]**

(1) $8\sqrt{2}$ (2) 4

(3) $4\sqrt{2}$ (4) 8

16. $\lim\limits_{x\to 1}\dfrac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to **[JEE (Main)-2019]**

(1) $\sqrt{\dfrac{2}{\pi}}$ (2) $\sqrt{\dfrac{\pi}{2}}$

(3) $\sqrt{\pi}$ (4) $\dfrac{1}{\sqrt{2\pi}}$

17. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals **[JEE (Main)-2019]**

(1) $\sqrt{2}$ (2) $2\sqrt{2}$

(3) 4 (4) $4\sqrt{2}$

18. Let $f : R \to R$ be a differentiable function satisfying $f'(3) + f'(2) = 0$. Then $\lim\limits_{x\to 0}\left(\dfrac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to **[JEE (Main)-2019]**

(1) e (2) 1

(3) e^2 (4) e^{-1}

19. If $f(x) = [x] - \left[\dfrac{x}{4}\right]$, $x \in R$, where $[x]$ denotes the greatest integer function, then **[JEE (Main)-2019]**

(1) $\lim\limits_{x\to 4+} f(x)$ exists but $\lim\limits_{x\to 4-} f(x)$ does not exist

(2) f is continuous at $x = 4$

(3) $\lim\limits_{x\to 4-} f(x)$ exists but $\lim\limits_{x\to 4+} f(x)$ does not exist

(4) Both $\lim\limits_{x\to 4-} f(x)$ and $\lim\limits_{x\to 4+} f(x)$ exist but are not equal

20. If $\lim\limits_{x\to 1}\dfrac{x^4-1}{x-1}=\lim\limits_{x\to k}\dfrac{x^3-k^3}{x^2-k^2}$, then k is **[JEE (Main)-2019]**

(1) $\dfrac{4}{3}$ (2) $\dfrac{3}{2}$

(3) $\dfrac{8}{3}$ (4) $\dfrac{3}{8}$

21. If $\lim\limits_{x\to 1}\dfrac{x^2-ax+b}{x-1}=5$, then $a + b$ is equal to **[JEE (Main)-2019]**

(1) 5 (2) –4

(3) 1 (4) –7

22. $\lim\limits_{x\to 0}\dfrac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is

[JEE (Main)-2019]

(1) 3 (2) 6

(3) 1 (4) 2

23. Let $f(x) = 5 - |x - 2|$ and $g(x) = |x + 1|$, $x \in R$. If f(x) attains maximum value at α and $g(x)$ attains minimum value at β, then

$\lim\limits_{x\to -\alpha\beta}\dfrac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to

[JEE (Main)-2019]

(1) 1/2 (2) –1/2

(3) –3/2 (4) 3/2

24. $\lim\limits_{x\to 0}\left(\dfrac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to **[JEE (Main)-2020]**

(1) e (2) $\dfrac{1}{e}$

(3) $\dfrac{1}{e^2}$ (4) e^2

25. $\lim\limits_{x\to 0}\left(\tan\left(\dfrac{\pi}{4}+x\right)\right)^{\frac{1}{x}}$ is equal to

(1) e^2 (2) 1

(3) e (4) 2

26. Let $[t]$ denote the greatest integer $\le t$. If for some $\lambda \in R - \{0, 1\}$, $\lim\limits_{x\to 0}\left|\dfrac{1-x+|x|}{\lambda-x+[x]}\right| = L$, then L is equal to **[JEE (Main)-2020]**

(1) 2 (2) $\dfrac{1}{2}$

(3) 0 (4) 1

27. $\lim\limits_{x\to a}\dfrac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}$ $(a \ne 0)$ is equal to

[JEE (Main)-2020]

(1) $\left(\dfrac{2}{9}\right)\left(\dfrac{2}{3}\right)^{\frac{1}{3}}$ (2) $\left(\dfrac{2}{3}\right)\left(\dfrac{2}{9}\right)^{\frac{1}{3}}$

(3) $\left(\dfrac{2}{3}\right)^{\frac{4}{3}}$ (4) $\left(\dfrac{2}{9}\right)^{\frac{4}{3}}$

28. Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function such that $f(1) = e$ and

$\lim\limits_{t\to x}\dfrac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$

If $f(x) = 1$, then x is equal to **[JEE (Main)-2020]**

(1) $2e$ (2) e

(3) $\dfrac{1}{2e}$ (4) $\dfrac{1}{e}$

29. If α is the positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim\limits_{x\to\alpha^+}\dfrac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to

[JEE (Main)-2020]

(1) $\dfrac{1}{\sqrt{2}}$ (2) $\dfrac{1}{2}$

(3) $\dfrac{3}{\sqrt{2}}$ (4) $\dfrac{3}{2}$

30. $\lim\limits_{x\to 0}\dfrac{x\left(e^{(\sqrt{1+x^2+x^4}-1)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$ **[JEE (Main)-2020]**

(1) Is equal to 0 (2) Is equal to $\sqrt{e}$

(3) Is equal to 1 (4) Does not exist

31. $\lim\limits_{x\to 1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$ **[JEE (Main)-2020]**

(1) Does not exist (2) Is equal to 0

(3) Is equal to 1 (4) Is equal to $\dfrac{1}{2}$

32. $\lim\limits_{x\to 2}\dfrac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}$ is equal to _____.

[JEE (Main)-2020]

33. If $\lim\limits_{x\to 1}\dfrac{x+x^2+x^3+\ldots+x^n-n}{x-1}=820$, $(n \in N)$ then the value of n is equal to _____

[JEE (Main)-2020]

34. If $\lim\limits_{x\to 0}\left\{\dfrac{1}{x^8}\left(1-\cos\dfrac{x^2}{2}-\cos\dfrac{x^2}{4}+\cos\dfrac{x^2}{2}\cos\dfrac{x^2}{4}\right)\right\}=2^{-k}$, then the value of k is _____.

[JEE (Main)-2020]

35. If $\lim\limits_{x\to 0}\dfrac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$ exists and is equal to b, then the value of a –2b is _____.

[JEE (Main)-2021]

36. The value of $\lim\limits_{x\to 0}\left(\dfrac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ is equal to **[JEE (Main)-2021]**

(1) 4 (2) –4

(3) –1 (4) 0

37. If α, β are the distinct roots of $x^2 + bx + c = 0$, then $\lim\limits_{x\to\beta}\dfrac{e^{2\left(x^2+bx+c\right)}-1-2\left(x^2+bx+c\right)}{(x-\beta)^2}$ is equal to : **[JEE (Main)-2021]**

(1) $b^2 - 4c$ (2) $b^2 + 4c$

(3) $2(b^2 + 4c)$ (4) $2(b^2 - 4c)$

38. If $\lim\limits_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is **[JEE (Main)-2021]**

(1) $\left(1, \dfrac{1}{2}\right)$ (2) $\left(-1, -\dfrac{1}{2}\right)$

(3) $\left(-1, \dfrac{1}{2}\right)$ (4) $\left(1, -\dfrac{1}{2}\right)$

39. $\lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$ is equal to : **[JEE (Main)-2021]**

(1) 4π (2) π^2

(3) $4\pi^2$ (4) $2\pi^2$

40. Let a be an integer such that $\lim\limits_{x\to 7}\dfrac{18-[1-x]}{[x-3a]}$ exists, where $[t]$ is greatest integer $\le t$. Then a is equal to : **[JEE (Main)-2022]**

(1) –6 (2) –2

(3) 2 (4) 6

41. Let $[t]$ denote the greatest integer $\le t$ and $\{t\}$ denote the fractional part of t. The integral value of α for which the left hand limit of the function

$f(x) = [1 + x] + \dfrac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$ at $x = 0$ is

equal to $\alpha-\dfrac{4}{3}$, is _____.

[JEE (Main)-2022]

42. If $\lim\limits_{x\to 1}\dfrac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b}=-2$, then the value of $(a - b)$ is equal to _____.

[JEE (Main)-2022]

43. The value of $\lim\limits_{x\to 1}\dfrac{\left(x^2-1\right)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to

[JEE (Main)-2022]

(1) $\dfrac{\pi^2}{6}$ (2) $\dfrac{\pi^2}{3}$

(3) $\dfrac{\pi^2}{2}$ (4) π^2

44. $\lim\limits_{x\to 0}\left(\dfrac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)}{(x+2)^3+2(x+2)^2+3\sin(x+2)}\right)^{\frac{100}{x}}$ is equal to _____.

[JEE (Main)-2022]

45. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ is equal to

(1) 14 (2) 7

(3) $14\sqrt{2}$ (4) $7\sqrt{2}$

[JEE (Main)-2022]

46. $\lim\limits_{n\to\infty}\dfrac{1}{2^n}\left(\dfrac{1}{\sqrt{1-\frac{1}{2^n}}}+\dfrac{1}{\sqrt{1-\frac{2}{2^n}}}+\dfrac{1}{\sqrt{1-\frac{3}{2^n}}}+.....+\dfrac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$ is equal to

(1) $\dfrac{1}{2}$ (2) 1

(3) 2 (4) –2

[JEE (Main)-2022]

47. $\lim\limits_{x\to\frac{\pi}{2}}\tan^2 x\left(\dfrac{\left(2\sin^2 x+3\sin x+4\right)^{\frac{1}{2}}}{-\left(\sin^2 x+6\sin x+2\right)^{\frac{1}{2}}}\right)$ is equal to

[JEE (Main)-2022]

(1) $\dfrac{1}{12}$ (2) $-\dfrac{1}{18}$

(3) $-\dfrac{1}{12}$ (4) $\dfrac{1}{6}$

48. $\lim\limits_{x\to\frac{1}{\sqrt{2}}} \dfrac{\sin\left(\cos^{-1}x\right)-x}{1-\tan\left(\cos^{-1}x\right)}$ is equal to :

[JEE (Main)-2022]

(1) $\sqrt{2}$ (2) $-\sqrt{2}$

(3) $\dfrac{1}{\sqrt{2}}$ (4) $-\dfrac{1}{\sqrt{2}}$

49. $\lim\limits_{x\to 0} \dfrac{\cos\left(\sin x\right)-\cos x}{x^4}$ is equal to:

[JEE (Main)-2022]

(1) $\dfrac{1}{3}$ (2) $\dfrac{1}{4}$

(3) $\dfrac{1}{6}$ (4) $\dfrac{1}{12}$

50. If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$, then $8(\alpha+\beta)$ is equal to **[JEE (Main)-2022]**

(1) 4 (2) –8

(3) –4 (4) 8

51. If $\lim\limits_{x\to 0}\dfrac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\dfrac{2}{3}$, where $\alpha, \beta, \gamma \in \mathbf{R}$, then which of the following is **NOT** correct?

[JEE (Main)-2022]

(1) $\alpha^2+\beta^2+\gamma^2=6$

(2) $\alpha\beta+\beta\gamma+\gamma\alpha+1=0$

(3) $\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=0$

(4) $\alpha^2-\beta^2+\gamma^2=4$

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