Assignment 2: Data Visualization

Due beginning of class: Monday September 25

(c) (2 marks) Show that if **P** is an idempotent matrix, then so must be $ ({**I**}__n -{**P**})$.

We first show that P is symmetric:

$$\begin{aligned}
\mathbf{P}^T &= (\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T)^T \\
&= (\mathbf{X}^T)^T(\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1})^T \\
&= \mathbf{X}((\mathbf{X}^T\mathbf{X})^{-1})^T\mathbf{X}^T \\
&= \mathbf{X}((\mathbf{X}^T\mathbf{X})^T)^{-1}\mathbf{X}^T \\
&= \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T \\
&= \mathbf{P}
\end{aligned}$$

$\therefore$ **P** is symmetric.

$$\begin{aligned}
\widehat{\mu}^T\widehat{\mathbf{r}} &= \widehat{\mu}\cdot\widehat{\mathbf{r}} \\
&= \mathbf{P}\mathbf{y}\cdot(\mathbf{I}_n-\mathbf{P})\mathbf{y} \\
&= \mathbf{y}\cdot\mathbf{P}(\mathbf{I}_n-\mathbf{P})\mathbf{y} \\
&= \mathbf{y}^T(\mathbf{P}-\mathbf{P}^2)\mathbf{y} \\
&= \mathbf{y}^T\mathbf{0}\mathbf{y} \\
&= \mathbf{0}
\end{aligned}$$