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STAT 442: Data Visualization

Assignment 1

1) Numbers

Part (a)

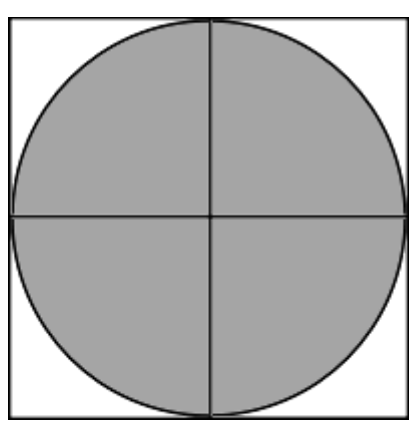
Part i.

HHHΔII

Part ii.

Part iii.

Part (b)



2) Evolution of the Eye

Part (a)

Part i.

The degree of binocular overlap is the area where both eyes of the creature can see, i.e. the area where the fields of vision of both eyes overlap.

The purpose of binocular overlap served for a Tyrannosaurus Rex was to allow it to properly gauge the distance of targets in its field of vision and determine how the rest of the world was moving as the Tyrannosaurus moved forward.

Part ii.

Dr. Stevens first modelled a Tyrannosaurus's head and pointed a laser in each of its eyes, then he drew out the field of vision for each eye based on this. The overlap was then determined by examining the area of overlap of these fields of vision. The binocular overlap for a Tyrannosaurus is over 55 degrees.

Part iii.

The binocular overlap for an Allosaurus is about 20 degrees. This suggests that these creatures had to ambush their prey as their style of hunting due to their lack of awareness, an assumption that is supported by the fact that modern predators that have similarly lateral vision tend to be ambush predators as a result.

Part (b)

The features of mammal eyeballs that Kirk suggests to be adaptations for night vision are:

- The large diameter of the corneas, which allows more light to enter the eye
- The large diameter of the whole eye, which allows the mammal to have both highly sensitive and acute vision
- Eye shine, which is produced by the tapetum lucidum behind the retina that reflects any light that is not absorbed at first

Part (c)

Part i.

Primates have a large overlap because it allows them to better make use of their hand-eye coordination, and it gives them better acuity and depth perception.

Part ii.

As a result of this larger overlap, birds of prey exerted more selector pressure on primates and took advantage of the fact that primates could no longer view objects behind them, i.e. the primates' limited field of vision led to a drop in their safety.

To adapt to this pressure, primates began to depend on and live with others, i.e. group living, which led to an evolution of larger brains in order to better remember all the faces within the group. Eye sockets and optic nerve canals also grew larger as brain size increased. Color vision and binocular vision improved, and they became extremely sensitive to the slightest changes of expression in other primates' faces. Overall, this increased need for visual processing fueled the development of intelligence in primates.

3) Steroscopic Displays

Part (a)

Part i.

A. stereo, eyed, Wide

B. Since everything but the window appears to be floating, the view of the tree, some of the flowers and the sun seem to be in front of the window.

Part ii.

A. stereo, eyed, Wide

B. The view of the tree, flowers and sun is completely behind the window.

Part iii.

The view behind the window is shifted to the left in picture, and the objects in the view have less space between them than the one on the right. This makes the left image's view through the window tilted to the left of the whole images while the right image's view is straight, allowing us to fuse the images properly using the wide-eyed method.

Part iv.

The two images' backgrounds (the tree, flowers, and sun) are not facing somewhat away from each other, which is why the images fail to fuse when using the cross-eyed method. Changing the tilt of the right image's background by shifting the characters slightly to the right and reducing the space between the characters will fix this.

Part (b)

The highest point is at the mountain tops in the top-center, which appear to be sticking out of the image. The lowest point is the concave cliff face in the bottom left of the image.

Part (c)

In the second (cross-eyed) image, the word "data" is somewhat slanted, in all lowercase, and is convex to the background, i.e. it is embossed on the background.

4) Orthogonal Projections

Part (a)

$$\begin{aligned}
proj_{\mathbf{x}}\mathbf{y} &= \frac{\mathbf{x}\cdot\mathbf{y}}{||\mathbf{x}||^2}\mathbf{x} \\
&= \frac{\mathbf{x}\cdot\mathbf{y}}{||\mathbf{x}||\ ||\mathbf{x}||\ \cos 0}\mathbf{x} \quad && since\ \ \cos 0 = 1 \\
&= \frac{\mathbf{x}\cdot\mathbf{y}}{\mathbf{x}^T\mathbf{x}}\mathbf{x} && since\ \theta = 0\, for\ any\ vector \text{ and itself} \\
&= \mathbf{x}(\mathbf{x}^T\mathbf{x})^{-1}\mathbf{x}^T\mathbf{y} && \text{by definition of the usual inner product}
\end{aligned}$$

Part (b)

Let $\mathbf{r}$ be a vector in $\Re^n$ such that

$$\mathbf{r}\ =\ perp_{\mathbf{x}}\mathbf{y}\ =\ \mathbf{y} - \frac{\mathbf{x}\cdot\mathbf{y}}{||\mathbf{x}||^2}\mathbf{x}$$

Then:

$$\begin{aligned}
\mathbf{r}\cdot\mathbf{x} &= (\mathbf{y} - \frac{\mathbf{x}\cdot\mathbf{y}}{||\mathbf{x}||^2}\mathbf{x})\cdot\mathbf{x} \\
&= \mathbf{y}\cdot\mathbf{x} - \frac{\mathbf{x}\cdot\mathbf{y}}{||\mathbf{x}||^2}(\mathbf{x}\cdot\mathbf{x}) \\
&= \mathbf{y}\cdot\mathbf{x} - \frac{\mathbf{x}\cdot\mathbf{y}}{||\mathbf{x}||^2}||\mathbf{x}||^2 \quad && \text{by definition of the norm} \\
&= (\mathbf{y}\cdot\mathbf{x}) - (\mathbf{x}\cdot\mathbf{y}) \\
&= 0
\end{aligned}$$

$\therefore$ $\mathbf{r}$ is orthogonal to $\mathbf{x}$ by definition of orthogonality.

Part (c)

Part i.

We first prove that $\mathbf{u}\ and\ \mathbf{v}$ are both unit vectors, i.e. normalized:

$$||\mathbf{u}||^2 = \mathbf{u}\cdot\mathbf{u} = (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 = 1$$

$$||\mathbf{v}||^2 = \mathbf{v}\cdot\mathbf{v} = (\frac{-1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 = 1$$

$$\therefore\ ||\mathbf{u}|| = ||\mathbf{v}|| = 1$$

We then prove $\mathbf{u}\ and\ \mathbf{v}$ are orthogonal to each other:

$$\mathbf{u}\cdot\mathbf{v} = \frac{1}{\sqrt{2}}(\frac{-1}{\sqrt{2}}) + \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}) = \frac{-1}{2} + \frac{1}{2} = 0$$

$\therefore$ $\mathbf{u}\ and\ \mathbf{v}$ are orthonormal by definition.

Part ii.

$$\begin{aligned} proj_{\mathbf{u}}\mathbf{y} &= \frac{\mathbf{y}\cdot\mathbf{u}}{||\mathbf{u}||^2}\mathbf{u} \\ &= (\tfrac{1}{\sqrt{2}}y_1 + \tfrac{1}{\sqrt{2}}y_2)\mathbf{u} \end{aligned}$$

$$\begin{aligned} proj_{\mathbf{v}}\mathbf{y} &= \frac{\mathbf{y}\cdot\mathbf{v}}{||\mathbf{v}||^2}\mathbf{v} \\ &= (\tfrac{-1}{\sqrt{2}}y_1 + \tfrac{1}{\sqrt{2}}y_2)\mathbf{v} \end{aligned}$$

Note that since $\mathbf{u}$ *and* $\mathbf{v}$ are orthonormal, they form an orthogonal basis for $\mathfrak{R}^2$. Hence:

$$\begin{aligned} \mathbf{y} &= proj_{\mathbf{u}}\mathbf{y} + proj_{\mathbf{v}}\mathbf{y} \\ &= (\tfrac{1}{\sqrt{2}}y_1 + \tfrac{1}{\sqrt{2}}y_2)\mathbf{u} + (\tfrac{-1}{\sqrt{2}}y_1 + \tfrac{1}{\sqrt{2}}y_2)\mathbf{v} \end{aligned}$$

$\therefore\ a = (\frac{1}{\sqrt{2}}y_1 + \frac{1}{\sqrt{2}}y_2)$ and $b = (\frac{-1}{\sqrt{2}}y_1 + \frac{1}{\sqrt{2}}y_2)$.