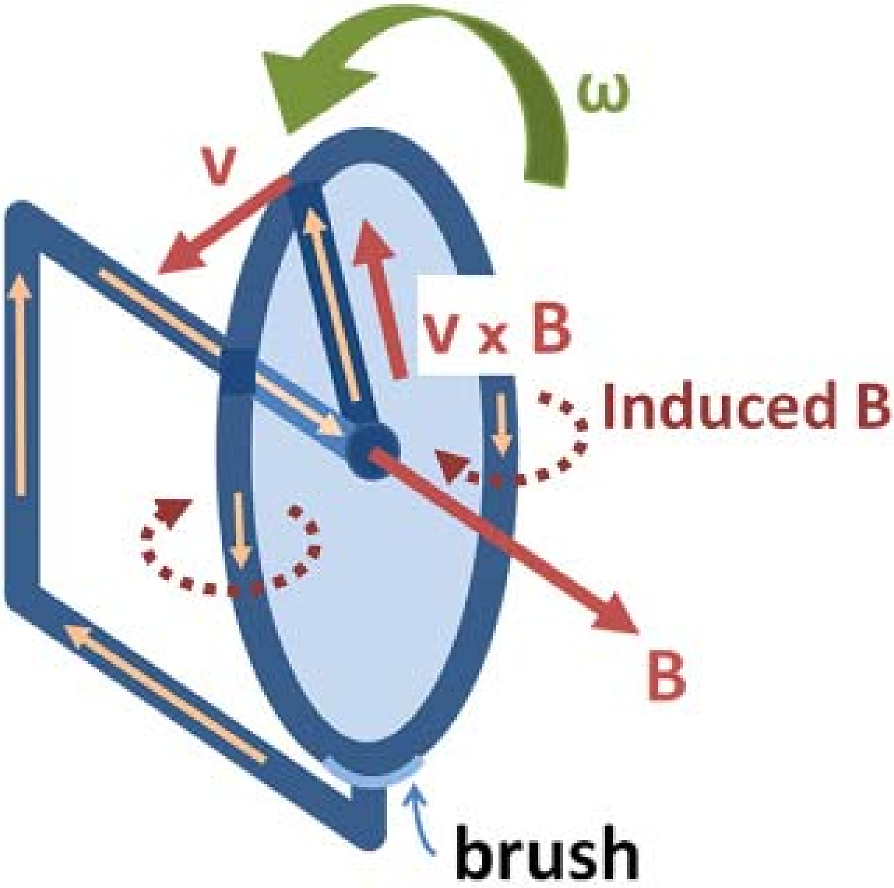
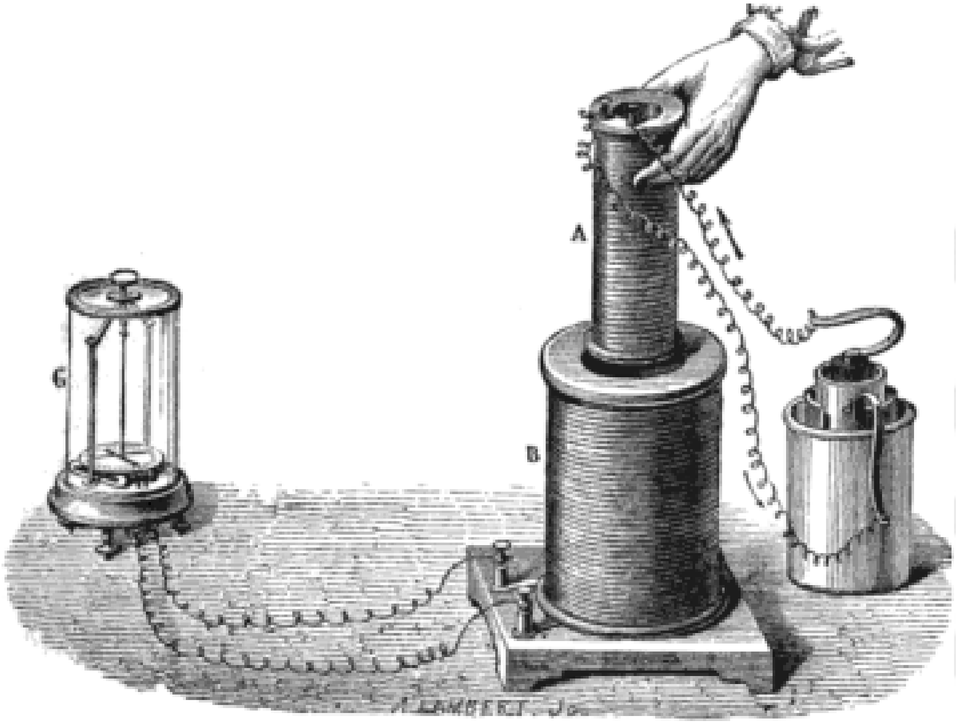
上课是不是照着ppt念？！

第八章 电磁感应

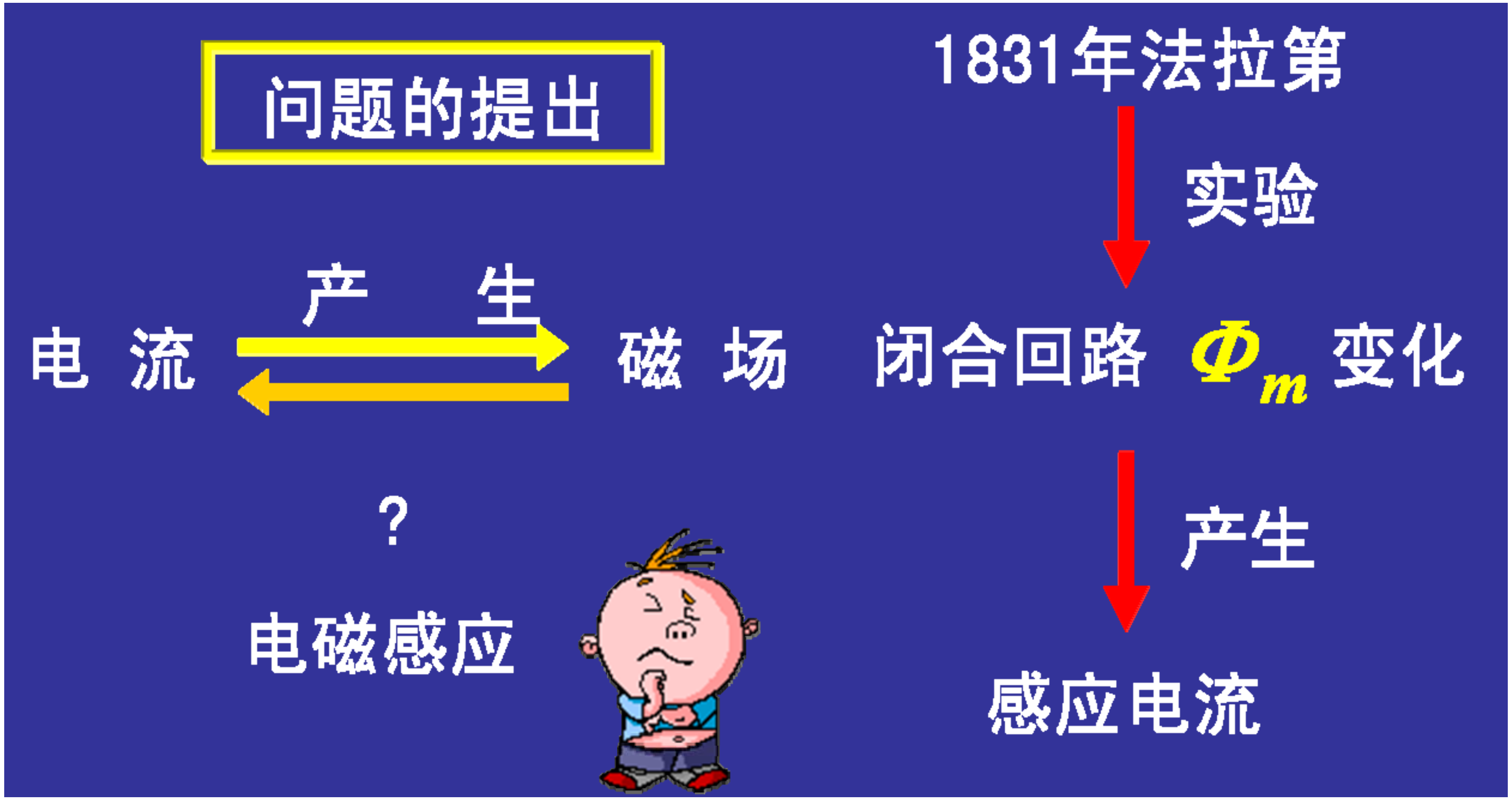


法拉第的圆盘发电机

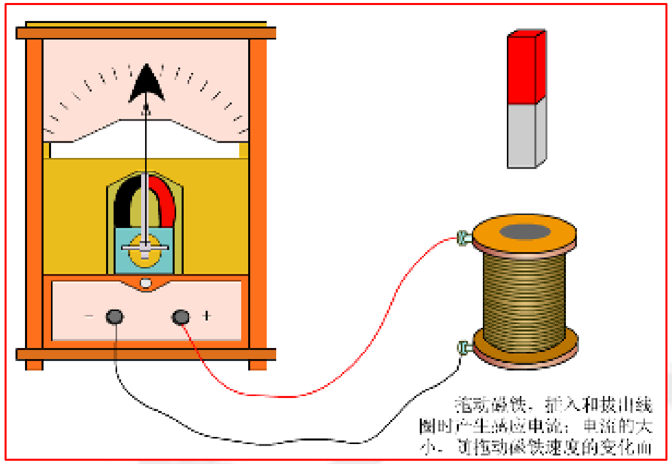


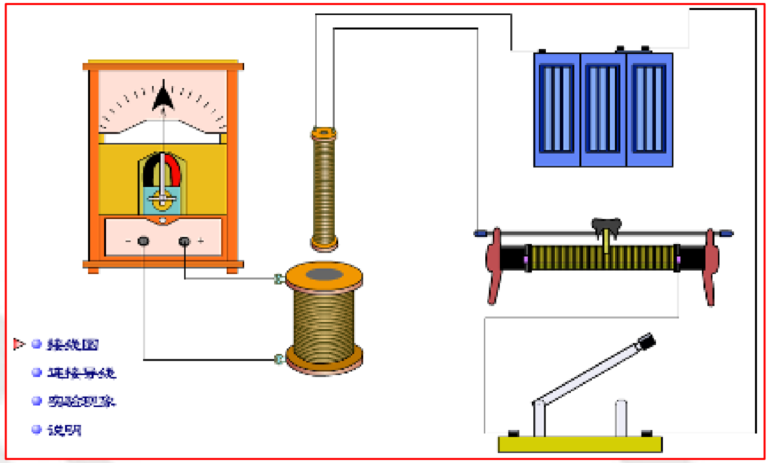
法拉第的电磁感应实验装置

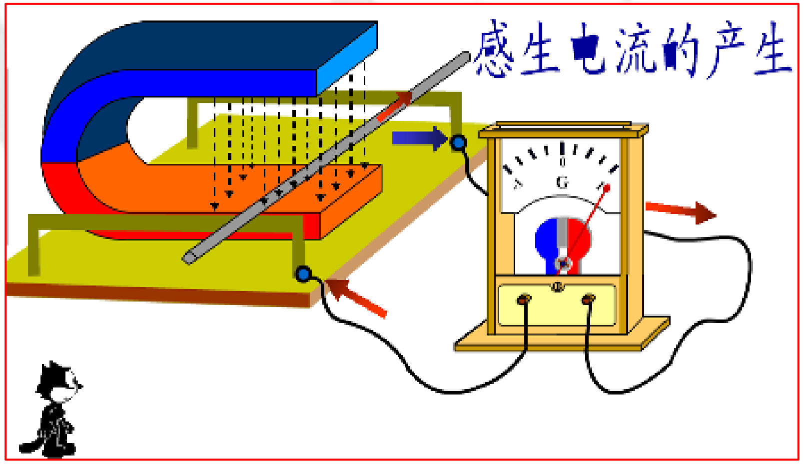
□ 电磁感应：



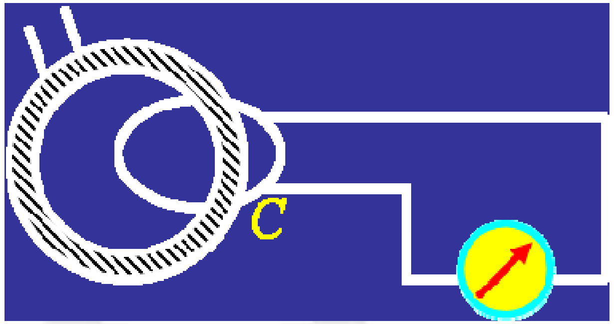
□ 电磁感应：


移动磁铁，插入和拔出线
圈时产生感应电流；电流的大
小，随拖动磁铁速度的变化而

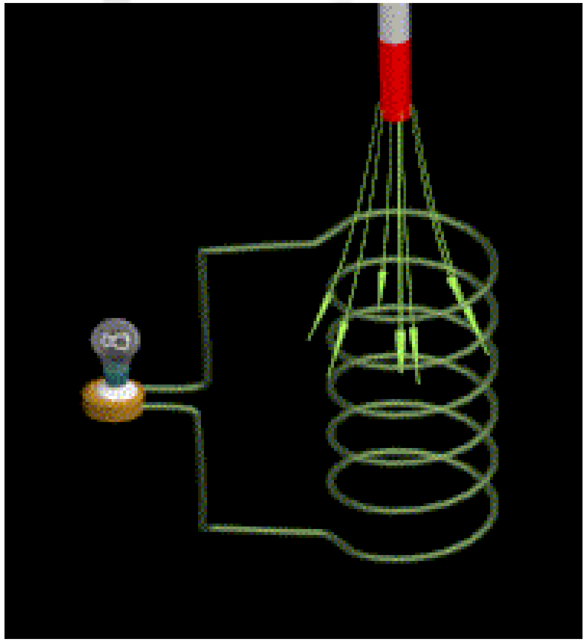

接线图
连接导线
实验现象
说明

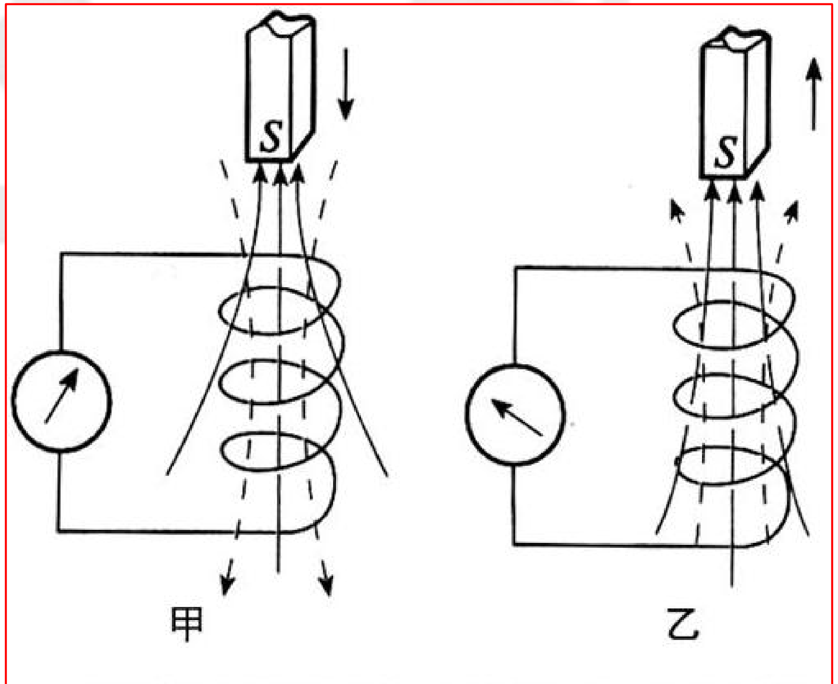

感生电流的产生
G

- 电磁感应：含有铁芯的线圈与未含铁芯的线圈产生的感应电流非常不同，意味著是 ***B*** 而非 ***H*** 的变化导致电流产生。



- 电磁感应现象--当回路磁通发生变化时在回路中产生感应电动势的现象，其电流叫感应电流。



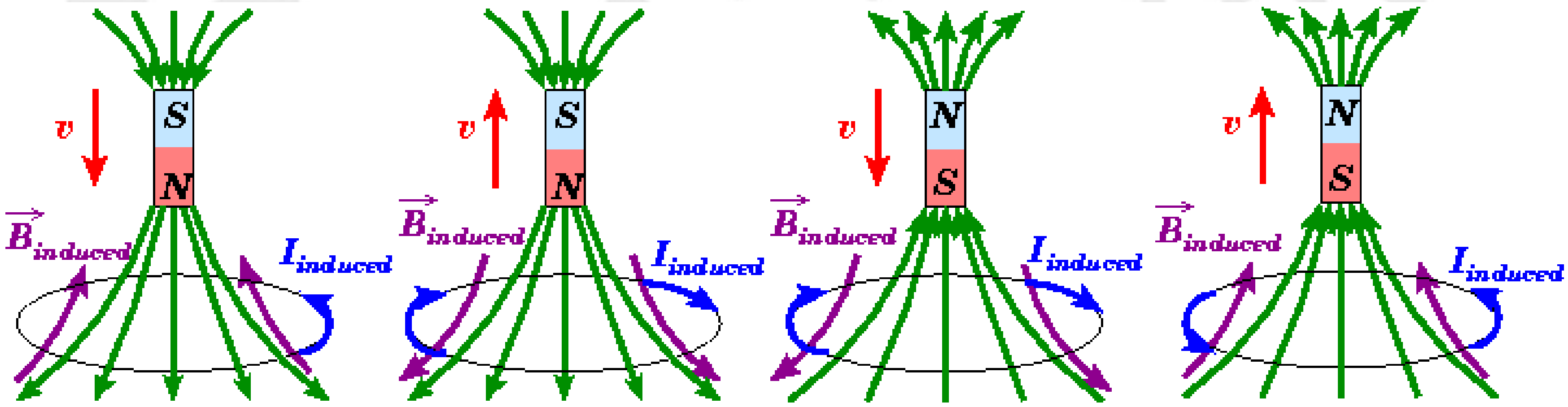


实线表示磁铁的磁感线，虚线表示感应电流的磁感线.

Some movies for demonstration

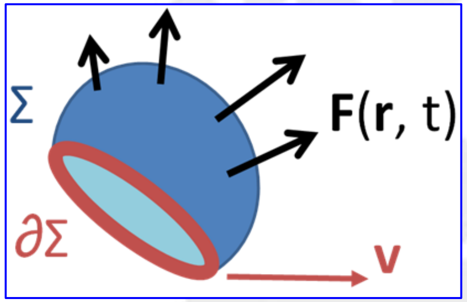
- Back emf in a Large Solenoid
- Galvanometer Principle
- Jacob's Ladder and the Melting Nail
- Lenz's Law
- Pendulum and Magnet

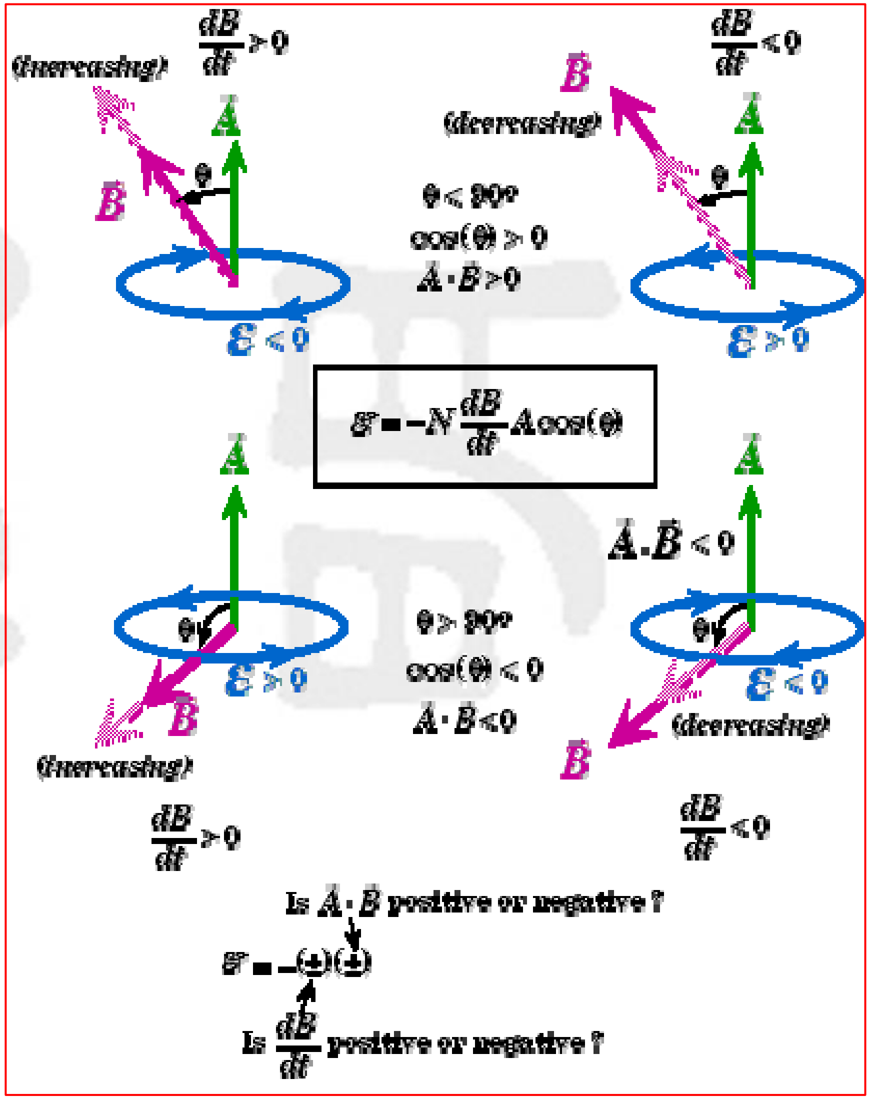
- 当回路磁通变化时，感应电流所产生的感应磁通总是力图阻止原磁通的变化。**(An induced current is always in such a direction as to oppose the motion or change causing it)**
- 感应电流所起作用是对抗产生感应的那个因素，存在相互能量转换，与能量守恒定律一致。
- 楞次定律含有惯性的意象。



- 当导体回路中磁通发生变化时，回路中的感应电动势与穿过此回路的磁通变化率成正比。
- 感应电动势

$$\Phi_B = \iint\limits_{\Sigma(t)} \mathbf{B}(\mathbf{r}, t) \cdot d\mathbf{A} \, ,$$



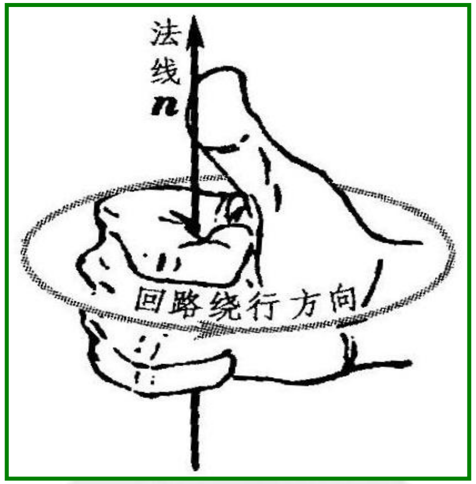


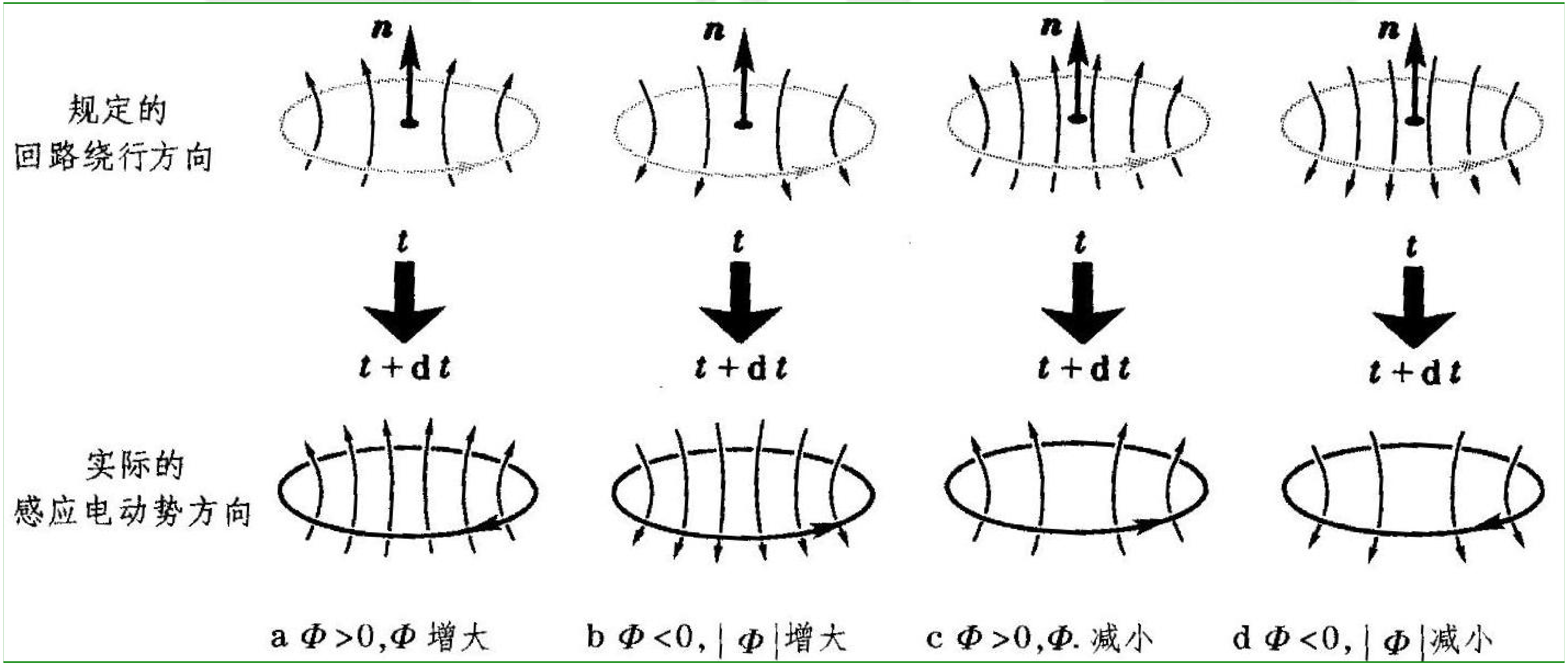
Faraday's Law of Induction

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad (for\ one\ loop)$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \quad (for\ multiple\ loops)$$

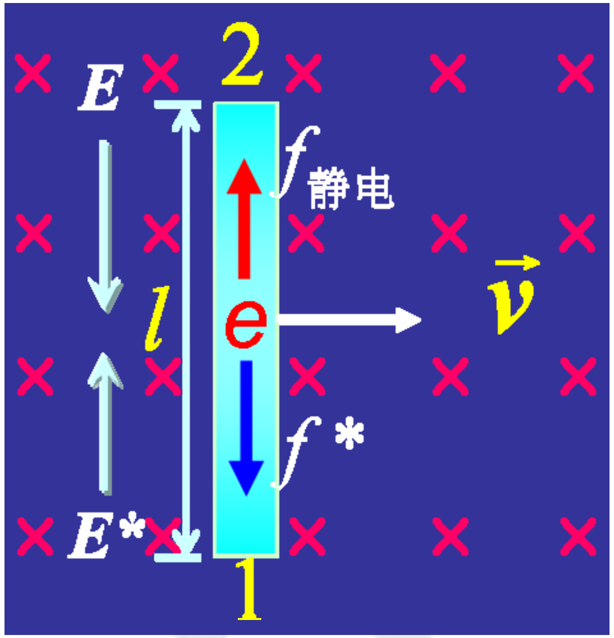
- 感应电动势的方向问题：定义与变化





- 磁通变化产生感应电动势的方法有两类：
  - (1) 磁感应强度不变、磁感应线贯穿的曲面空间变化；
  - (2) 磁感应线贯穿的曲面空间不变，磁感应强度变化；
  - 两类方法微观机制不同，但殊途同归 ⇒ 电磁感应现象。

(1) 磁感应强度不变、磁感应线贯穿的曲面空间变化



Lorentz force:

$$f^* = -e(\vec{v}\times\vec{B}) \Rightarrow \vec{E}^* = f^* / -e = \vec{v}\times\vec{B}$$

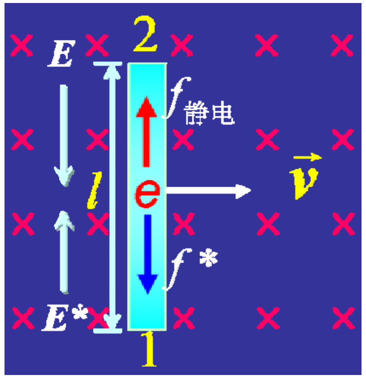
Static force:

$$\vec{f}_{静电} = \int_2^1 \vec{E}\cdot de$$

$$\vec{f}^* = -\vec{f}_{静电} \Rightarrow \vec{E} = -\vec{E}^*$$

$$\Sigma = \int_1^2 \vec{E}^*\cdot d\vec{l} = \int_1^2 (\vec{v}\times\vec{B})\cdot d\vec{l}$$

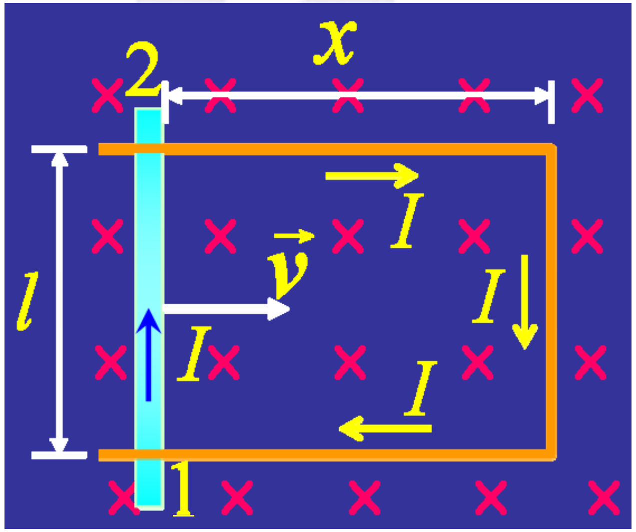
感应电动势为非库仑力场 ***E**** (其大小和方向均等于单位正电荷所受的洛伦兹力)沿电路自低电势端到高电势端的线积分。



- 导体中真实的电场、电势是洛伦兹力驱动电荷运动建立起来的静电场、静电势，***E****是虚拟等效电场。
- 对于闭合回路：

$$\boxed{\Sigma = \oint_L \vec{E}^* \cdot d\vec{l} = \oint_L (\vec{v}\times\vec{B})\cdot d\vec{l}}$$

- 最简单的回路：两种方法



$$(1):\ \Sigma = \oint_L \vec{E}^* \cdot d\vec{l} = \oint_L (\vec{v}\times\vec{B})\cdot d\vec{l}$$

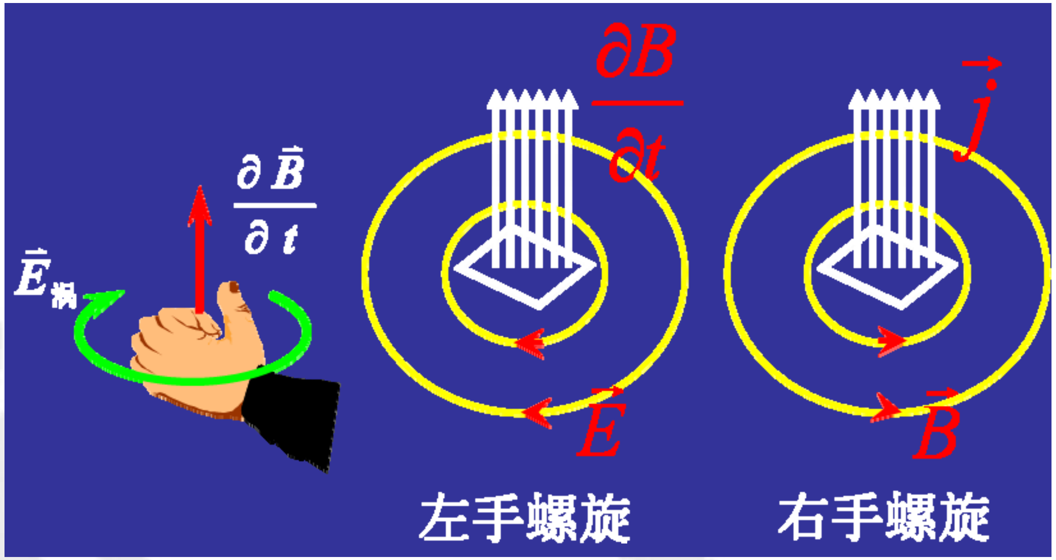
$$= vBl$$

$$(2):\ \Phi_B = Blx \Rightarrow$$

$$\Sigma = -\frac{d\Phi_B}{dt} = -\frac{dx}{dt}Bl = vBl$$

感应电动势方向问题

- (2) 磁感应线贯穿的曲面空间不变，磁感应强度变化：完全不同的物理机制。
- 与电流产生磁场类比。
- 磁感应强度随时间变化激发周围空间形成漩涡电场 $E_{感}$

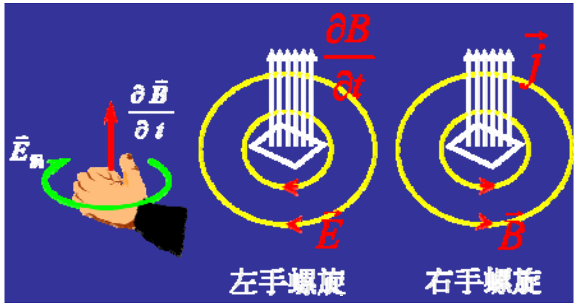


- 漩涡感应电动势：沿漩涡电场方向的“回路”线积分，这个回路可以是虚拟的，即空间存在电场、电势。

$$\Sigma = \oint_L \vec{E}_{感} \cdot d\vec{l} \Leftarrow \text{左手螺旋法则}$$

$$\Sigma = -\frac{d\Phi_B}{dt} \Rightarrow \oint_L \vec{E}_{感} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\iint_S \vec{B} \cdot dS\right)$$

电磁学08-02: 法拉第电磁感应定律



- 随时间变化磁场激发的漩涡电场沿任意闭合曲线 **L** 的线积分等于通过曲线 **L** 所张曲面 **S** 的磁通量之时间变化率之负数。

$$\because \partial(S,L)/\partial t=0,\quad \therefore \oint_L \vec{E}_{感}\cdot d\vec{l}=-\iint_S\left(\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{S}$$

$$\Sigma=\int_1^2 \vec{E}_{感}\cdot d\vec{l}$$

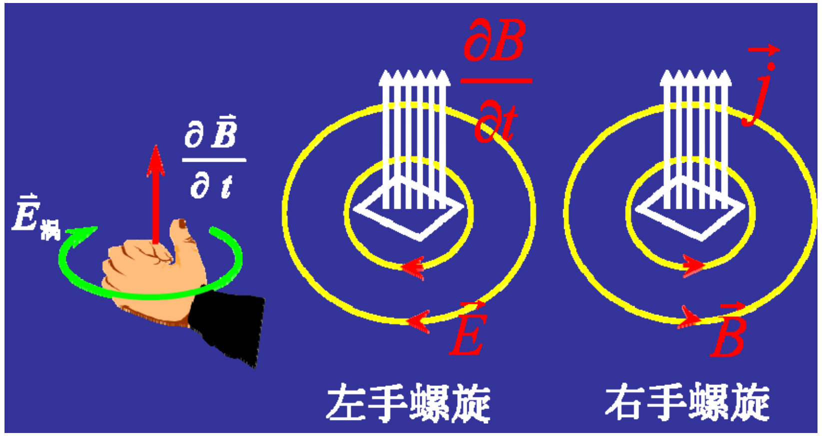
- 微分形式与扩展形式:

$$\oint_L \vec{E}_{感}\cdot d\vec{l}=-\iint_S\left(\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{S}\Rightarrow rot\vec{E}_{感}=-\frac{\partial \vec{B}}{\partial t}$$

$$\because \oint_L \vec{E}_{静}\cdot d\vec{l}=0\Rightarrow rot\vec{E}_{静}=0$$

$$\oint_L \vec{E}\cdot d\vec{l}=\oint_L(\vec{E}_{静}+\vec{E}_{感})\cdot d\vec{l}=-\iint_S\left(\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{S}\Rightarrow rot\vec{E}=-\frac{\partial \vec{B}}{\partial t}$$

普遍情况下电场为非保守场，无电势



- 从磁矢势 ***A*** 角度看：
- 构建一任意面元 ***S***，其边界为为 ***L***：

$$\Sigma = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S} = -\frac{d}{dt}\oint_L \vec{A}\cdot d\vec{l} = -\oint_L \frac{\partial \vec{A}}{\partial t}\cdot d\vec{l}$$

$$\because \Sigma = \oint_L \vec{E}_{感}\cdot d\vec{l} \Rightarrow \therefore \vec{E}_{感} = -\frac{\partial \vec{A}}{\partial t}$$

$$\vec{E} = \vec{E}_{静} + \vec{E}_{感} = -\nabla U - \frac{\partial \vec{A}}{\partial t}$$

感应电动势与感应电场电势【例8.1.7】:

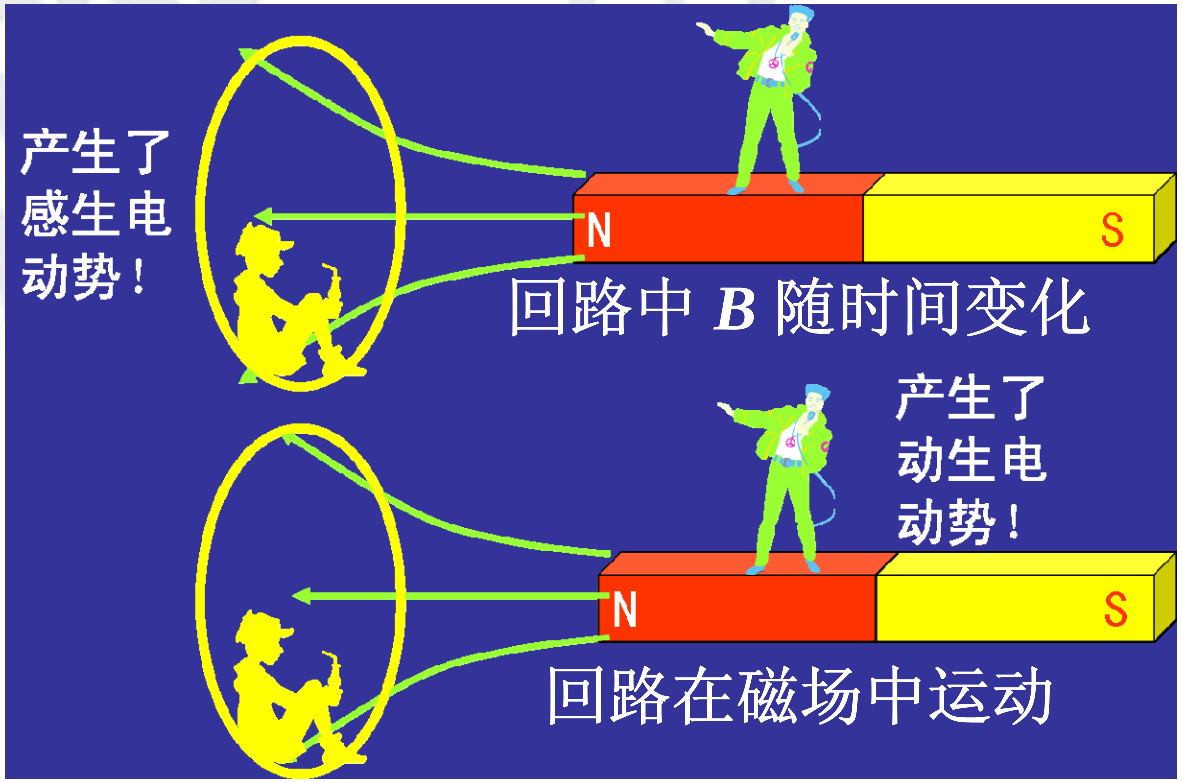
- 涡旋电场(感生电场)是非保守力场，不能引入电势和电势差；而静电场是库仑场，可以引入电势和电势差。
- 如果涡旋电场中有导体，其上两点电势差有意义----导体内与涡旋电场大小相等方向相反的静电场导致两点电势差。

$$\left\langle \begin{array}{l|l} \oint_L \vec{E}_{库} \cdot d\vec{l} = 0 & \oint_L \vec{E}_{感} \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \\ \nabla \times \vec{E}_{库} = 0 & \nabla \times \vec{E}_{感} = -\frac{\partial \vec{B}}{\partial t} \end{array} \right\rangle$$

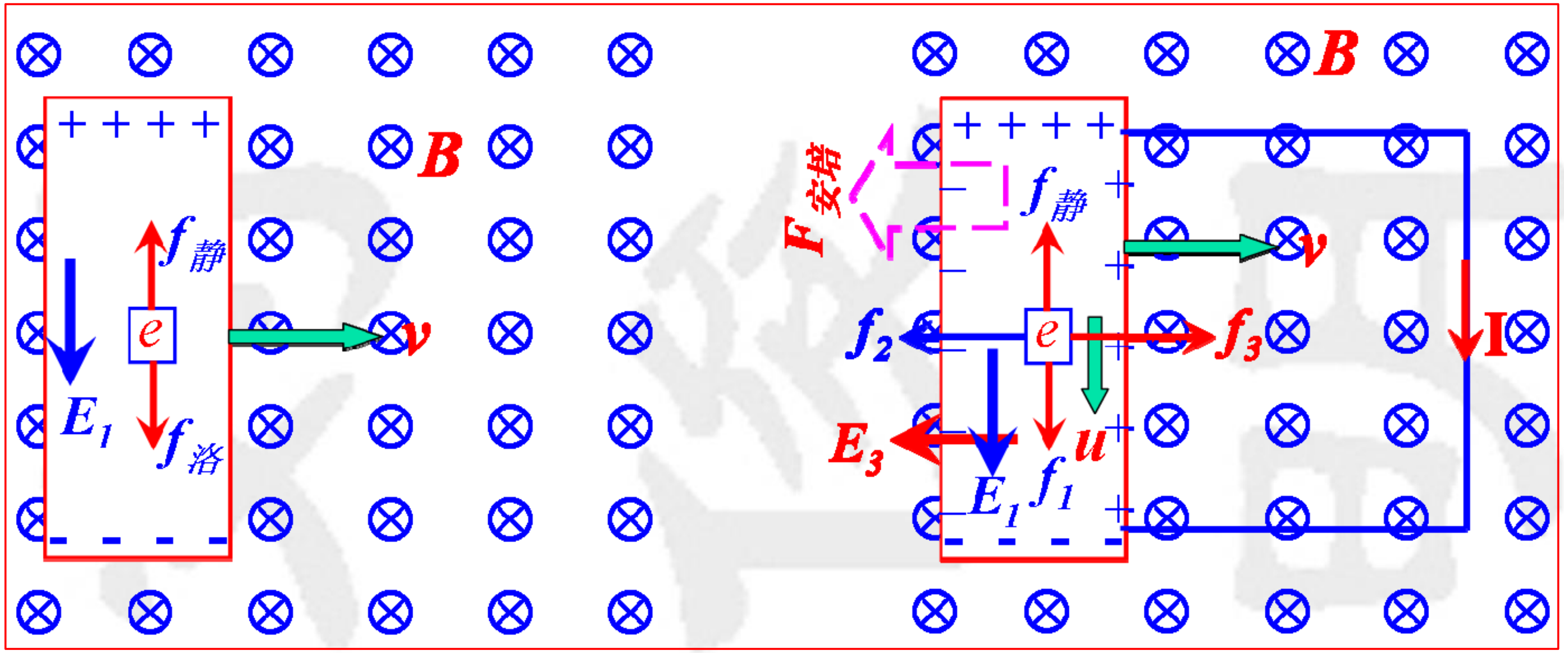
- (3) 感应电动势一般形式：

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S} = -\iint_S \left(\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{S} + \oint_L (\vec{v}\times\vec{B})\cdot d\vec{l}}$$

- 如导线运动和磁场变化同时存在，则感应定律中 Φ 的变化应理解为磁场变化和导线移动所引起的效应之叠加。
- 将感应电动势看成 动生 电动势与 感生 电动势是相对不同参考系而言的。

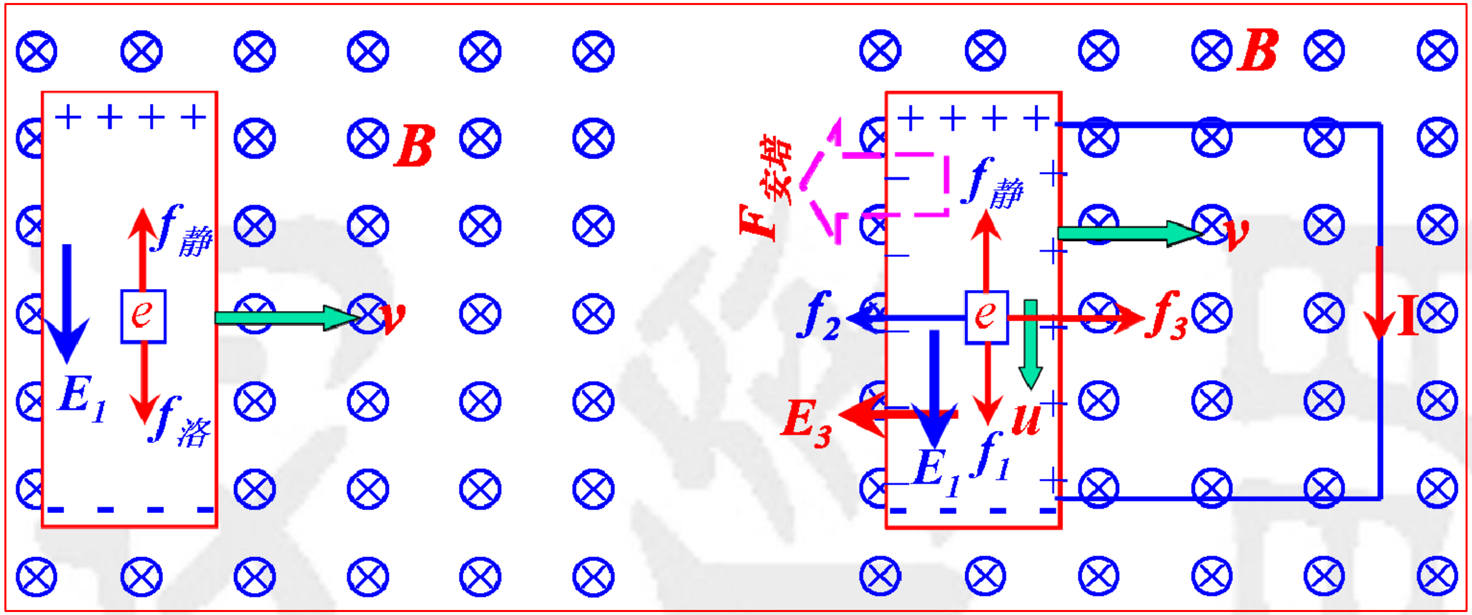


- (4) 能量问题：洛伦兹力并不做功，功能关系？以导体运动为例

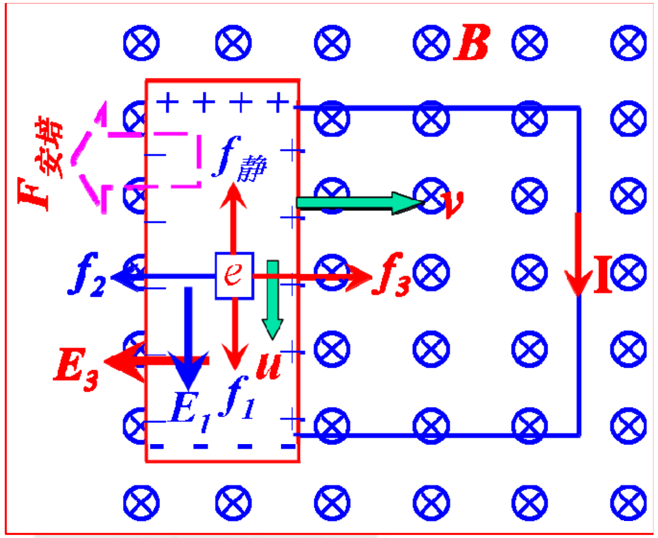


- 左边：未形成回路，洛伦兹力 $f_{洛} \Leftrightarrow$ 静电力 $f_{静}$ (静电场 E_1)
- 最初一瞬间有功能转换，达到稳态后 $f_{洛}$ 不做功

- 右侧：形成回路，有焦耳热释放，因此有功能转换



- 导体运动 (v) 导致的洛伦兹力 $f_1 \Leftrightarrow$ 静电力 $f_{静}$ (电场 E_1)
- 电流流动 (u) 导致的洛伦兹力 $f_2 \Leftrightarrow$ 静电力 f_3 (电场 E_3)
- 电流流动将 f_3 通过电荷-晶格作用传递给导体，形成导体整体的安培力 $F_{安培}$，否则电子会跑出导体了。^_^



受力的数学关系:

$$\vec{f}=\vec{f}_1+\vec{f}_2=-e\vec{v}\times\vec{B}-e\vec{u}\times\vec{B}$$
$$=-e(\vec{v}+\vec{u})\times\vec{B}$$
$$\left.\begin{array}{l}\vec{f}_1=-e\vec{v}\times\vec{B}\\ \vec{f}_2=-e\vec{u}\times\vec{B}\end{array}\right\}\Rightarrow\left\{\begin{array}{l}\vec{f}_{静}=-\vec{f}_1\\ \vec{f}_3=-\vec{f}_2\end{array}\right\}\Rightarrow\sum_{导体}\vec{f}_2=F_{安培}$$

功/能的数学关系:

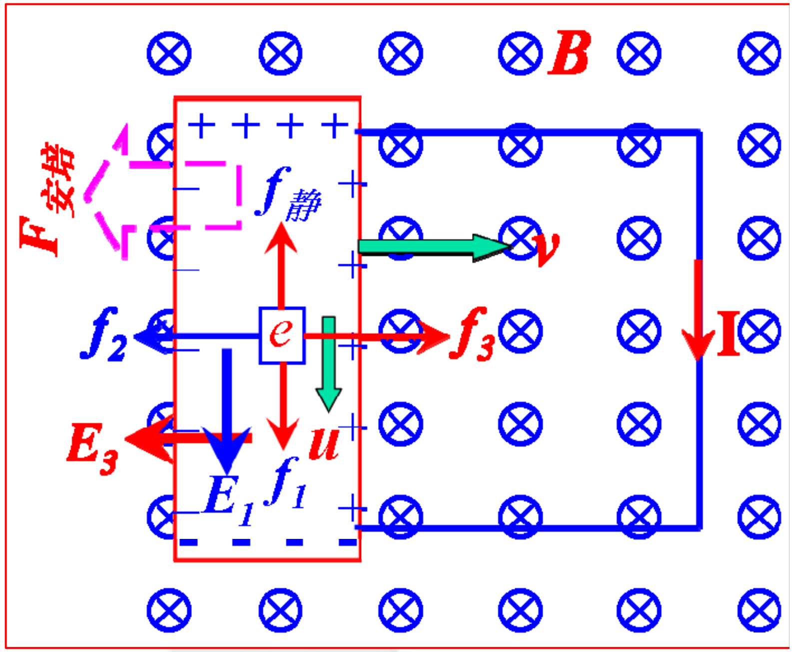
$$\because I=\Sigma/R=vBl/R,\ u=\frac{j}{ne}=\frac{I}{neS}=\frac{vBl}{neSR}$$
$$\therefore\left\{\begin{array}{l}\vec{f}_1\Rightarrow p_1=(evB)u\Rightarrow P_1=\sum_{nSl}p_1=\dfrac{v^2B^2l^2}{R}\\ \vec{f}_2\Rightarrow p_2=-(euB)v\Rightarrow P_2=\sum_{nSl}p_2=-\dfrac{v^2B^2l^2}{R}\end{array}\right\}\Rightarrow P_1+P_2=0$$

两个洛伦兹力做功之和为零

- 考虑与 f_2 对应的静电力 f_3 做功问题:

$$\vec{f}_3 \Rightarrow p_3 = -p_2 \Rightarrow P_3 = \sum_{nSl} p_3 = \frac{v^2B^2l^2}{R}$$

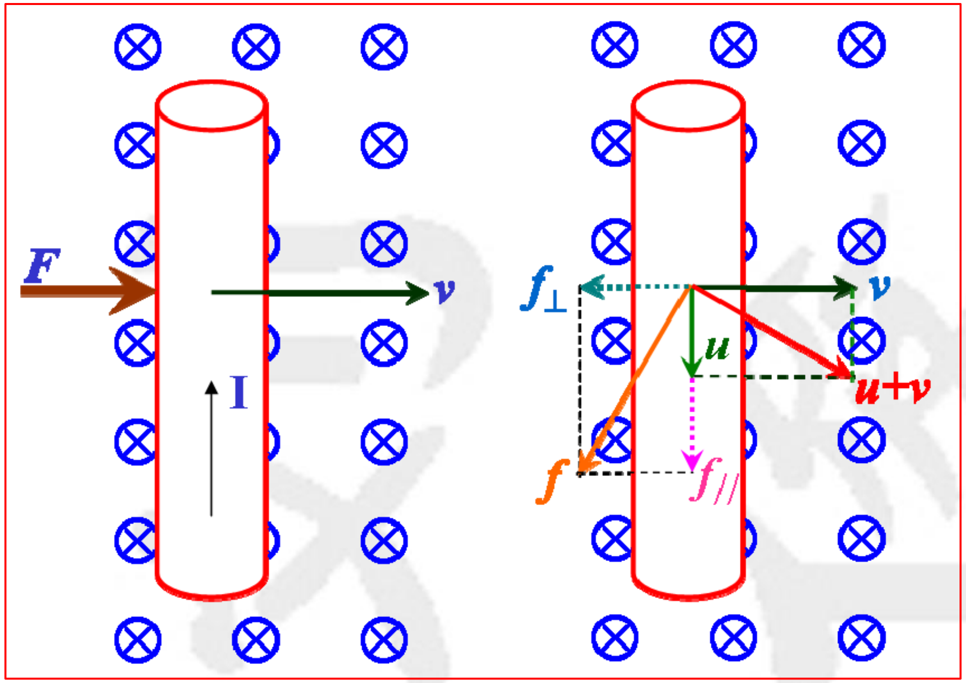
$$\sum_{nSl} \vec{f}_3 \Rightarrow \vec{F}_{ampere} \Rightarrow P_{ampere} = P_3 = \frac{v^2B^2l^2}{R}$$



- 外力必须克服 $F_{安培}$,对系统做功,再通过功能关系转化焦耳热:

$$P_3 = \frac{v^2B^2l^2}{R} \Rightarrow P = I^2R = \left(\frac{vBl}{R}\right)^2 R = \frac{v^2B^2l^2}{R} \Rightarrow P = P_3$$

(5) 做功问题的另外一种简化解释【例8.1.9】:



在实验室静止坐标系:

$$\vec{f}_{Lorentz} = -e(\vec{u}+\vec{v})\times\vec{B}$$

$$\because\ \vec{f}_{Lorentz} \perp (\vec{u}+\vec{v})$$

$$\therefore\ dW = \vec{f}_{Lorentz}\cdot d\vec{l}_{(\vec{u}+\vec{v})} = 0$$

$$\because\ \vec{f}_{Lorentz} = \vec{f}_{\parallel} + \vec{f}_{\perp}$$

$$\therefore \begin{cases} \vec{f}_{\parallel}\cdot(\vec{u}+\vec{v}) = \vec{f}_{\parallel}\cdot\vec{u} > 0 \\ \vec{f}_{\perp}\cdot(\vec{u}+\vec{v}) = \vec{f}_{\perp}\cdot\vec{v} < 0 \end{cases}$$

$f_{//}$对感应电流自由电子做正功

$f_\perp$对它们做负功

总功为零

总结:

	动生电动势 $\varepsilon_i = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$	感生电动势 $\varepsilon_i = \oint \vec{E}_{涡} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$
特点	磁场不变，闭合电路的整体或局部在磁场中运动导致回路中磁通量的变化	闭合回路的任何部分都不动，空间磁场发生变化导致回路中磁通量变化
原因	由于S 的变化引起回路中 Φ_B 变化	由于 B 的变化引起回路中 Φ_B 变化
非库仑力来源	洛仑兹力	感生电场力(磁矢势A)

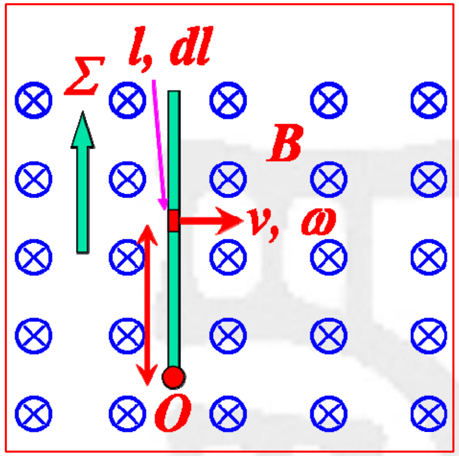
总结:

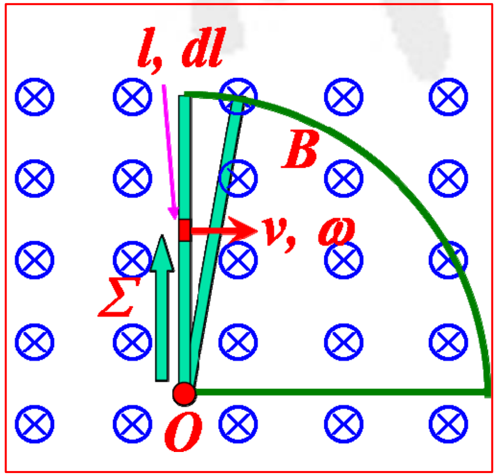
静电场(库仑场)	感生电场(旋涡电场)
具有电能、对电荷有作用力	具有电能、对电荷有作用力
由静止电荷产生	由变化磁场产生
$E_{静}$线是“有头有尾”的，起于正电荷而终于负电荷 $\oint_S \vec{E}_{库} \cdot d\vec{S} = \frac{1}{\varepsilon_0}\sum q_i$	$E_{感}$线是“无头无尾”的，是一组闭合曲线 $\oint_S \vec{E}_{涡} \cdot d\vec{S} = 0$
$\oint_L \vec{E}_{库} \cdot d\vec{l} = 0$	$\oint_L \vec{E}_{涡} \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$

- 【例2 p.312】长度为 L 的金属棒围绕一端以角速度 ω 在与其垂直的均匀磁场 B 中转动，求其感应电动势。

$$d\Sigma = (\vec{v}\times\vec{B})\cdot d\vec{l} = l\omega B dl$$

$$\Sigma = \int_O^A d\Sigma = \int_0^L l\omega B dl = \frac{1}{2}B\omega L^2$$





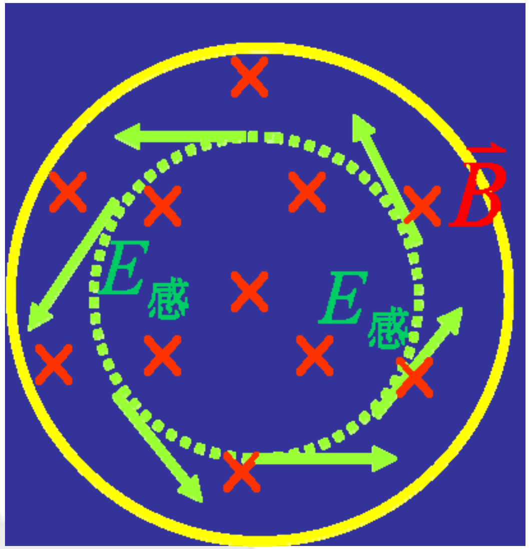
$$d\Phi = -B\frac{1}{2}L^2 d\theta \Rightarrow$$

$$\Sigma = -\frac{d\Phi}{dt} = B\frac{1}{2}L^2\frac{d\theta}{dt} = \frac{1}{2}B\omega L^2$$

方向的判断----是个绕弯弯的活！

- 【例3 p.313】均匀磁场 **B** 以速率 ***dB/dt*** 增加，求空间漩涡电场。

  根据左手螺旋法则，知漩涡电场如图取向。根据法拉第定律也可以判断这一点。



$$\Phi = B(\pi r^2) \quad \text{at } r \le R$$

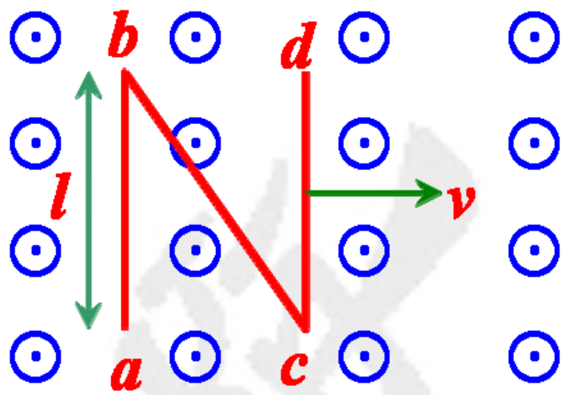
$$\oint \vec{E}_{感} \cdot d\vec{l} = -\frac{d\Phi}{dt} = -\pi r^2 \frac{d\vec{B}}{dt}$$

$$2\pi r \vec{E}_{感} = -\pi r^2 \frac{d\vec{B}}{dt} \Rightarrow E_{感} = -\frac{1}{2} r \frac{dB}{dt}$$

$$\Phi = B(\pi R^2) \quad \text{at } r > R$$

$$E_{感}(2\pi r) = -\frac{d\Phi}{dt} = -(\pi R^2)\frac{d\vec{B}}{dt} = -\frac{1}{2}\frac{R^2}{r}\frac{d\vec{B}}{dt}$$

【例8.1.26】求端 *a* 与端 *d* 的电势差 U_{ad}



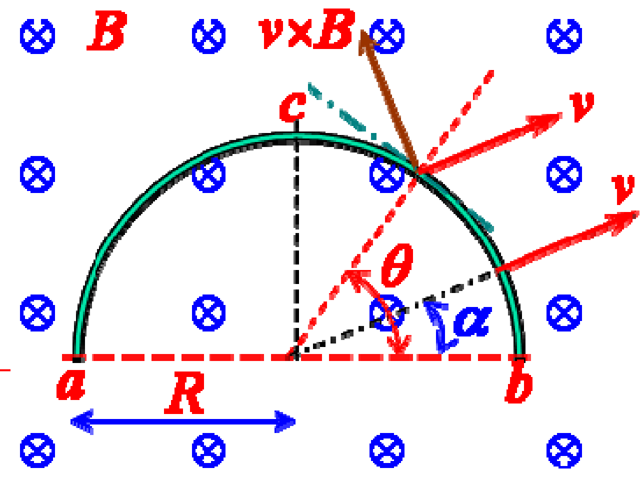
$$U_{ad}=U_a-U_d=\int_a^d \vec{E}_s\cdot d\vec{l}=\int_a^d(-\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$=-\int_a^b(\vec{v}\times\vec{B})\cdot d\vec{l}-\int_b^c(\vec{v}\times\vec{B})\cdot d\vec{l}-\int_c^d(\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$\because \int_a^b(\vec{v}\times\vec{B})\cdot d\vec{l}+\int_b^c(\vec{v}\times\vec{B})\cdot d\vec{l}=vBl-vBl=0$$

$$\therefore U_{ad}=-\int_c^d(\vec{v}\times\vec{B})\cdot d\vec{l}=vBl$$

电动势与电势差是相反的！



电子电荷为负，受力与
$v\times B$方向相反

- 【例8.1.30】求导线感应电动势、*ab* 端电势差，*cb* 端电势差，*ac* 端电势差

$$\Sigma=\int_b^a \vec{E}_i\cdot d\vec{l}=\int_b^a(\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$\because (\vec{v}\times\vec{B})\cdot d\vec{l}=vB(Rd\theta)\cos(\theta-\alpha)$$

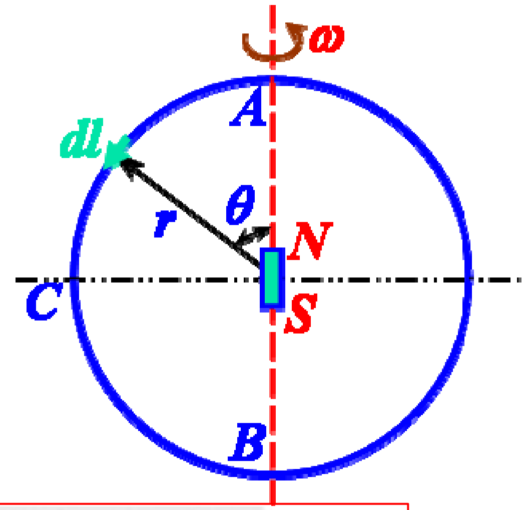
$$\therefore \Sigma=\int_b^a(\vec{v}\times\vec{B})\cdot d\vec{l}=vBR\int_0^\pi\cos(\theta-\alpha)d\theta=vBR\sin(\theta-\alpha)\Big|_0^\pi=2vBR\sin\alpha$$

$$U_{ab}=U_a-U_b=\int_a^b\vec{E}_s\cdot d\vec{l}=\int_a^b(-\vec{E}_i)\cdot d\vec{l}=\int_b^a\vec{E}_i\cdot d\vec{l}=\Sigma=2vBR\sin\alpha$$

$$U_{cb}=\int_c^b\vec{E}_s\cdot d\vec{l}=\int_c^b(-\vec{E}_i)\cdot d\vec{l}=\int_b^c\vec{E}_i\cdot d\vec{l}=vBR\int_0^{\pi/2}\cos(\theta-\alpha)d\theta$$

$$=vBR\sin(\theta-\alpha)\Big|_0^{\pi/2}=vBR(\cos\alpha+\sin\alpha)$$

$$U_{ac}=U_a-U_c=\int_a^c\vec{E}_s\cdot d\vec{l}=\int_a^c(-\vec{E}_i)\cdot d\vec{l}=vBR(\sin\alpha-\cos\alpha)\Rightarrow 0\Big|_{\alpha=\pi/4}$$



- 【例8.1.47】金属丝圆环转动，小磁矩 **m** 与转动角速度 **ω** 同向，求 **AC** 间感应电动势和 **AB** 间电势差

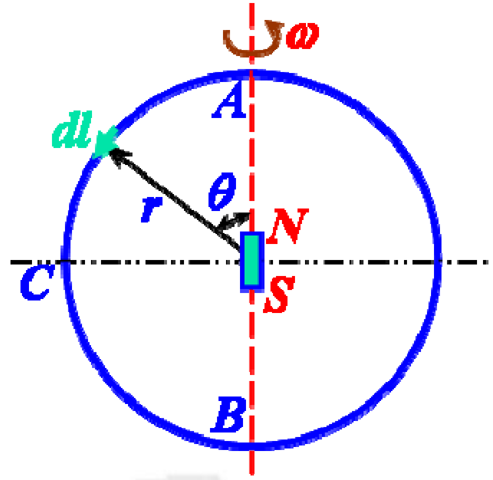
$$\Sigma_{AC}=\int_A^C \vec{E}_i\cdot d\vec{l}=\int_A^C(\vec{v}\times\vec{B})\cdot d\vec{l}=\int_A^C\left[(\vec{\omega}\times\vec{r})\times\vec{B}\right]\cdot d\vec{l}$$

$$=\int_A^C\left[\left(\vec{\omega}\cdot\vec{B}\right)\vec{r}-\left(\vec{r}\cdot\vec{B}\right)\vec{\omega}\right]\cdot d\vec{l}\xrightarrow{\because\ \vec{r}\cdot d\vec{l}=0}-\int_A^C(\vec{r}\cdot\vec{B})\vec{\omega}\cdot d\vec{l}$$

$$=-\int_A^C(\vec{r}\cdot\vec{B})\cos\left(\frac{\pi}{2}+\theta\right)\omega dl=\omega r\int_A^C(\vec{r}\cdot\vec{B})\sin\theta d\theta$$

$$\because\ \vec{B}=\frac{\mu_0}{4\pi r^3}\left[\frac{3(\vec{m}\cdot\vec{r})}{r^2}\vec{r}-\vec{m}\right]\Rightarrow\vec{r}\cdot\vec{B}=\frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\vec{r})-\vec{r}\cdot\vec{m}\right]$$

$$=\frac{\mu_0}{2\pi r^3}\vec{m}\cdot\vec{r}=\frac{\mu_0 m}{2\pi r^2}\cos\theta$$



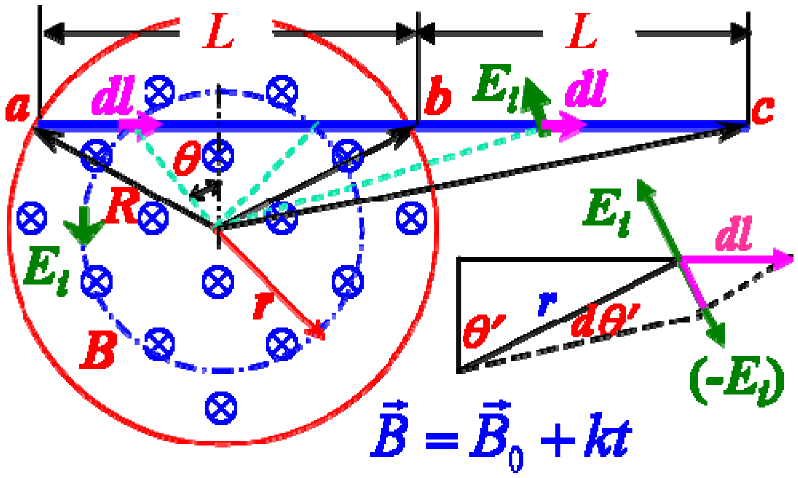
□ 继续

$$\therefore \Sigma_{AC} = \omega r\int_A^C \frac{\mu_0 m}{2\pi r^2}\cos\theta\sin\theta d\theta$$
$$= \frac{\mu_0 m\omega}{4\pi r}\left[\sin^2\theta\right]_0^{\pi/2} = \frac{\mu_0 m\omega}{4\pi r}$$

$$U_{AC} = U_A - U_C = -\Sigma_{AC} = -\frac{\mu_0 m\omega}{4\pi r}$$

$$U_{AB} = U_A - U_B = \int_A^B \vec{E}_s\cdot d\vec{l} = \int_A^B(-\vec{E}_i)\cdot d\vec{l} = \int_B^A \vec{E}_i\cdot d\vec{l} = \int_B^A(\vec{v}\times\vec{B})\cdot d\vec{l}$$
$$= \omega r\int_B^A \frac{\mu_0 m}{2\pi r^2}\cos\theta\sin\theta d\theta = \frac{\mu_0 m\omega}{4\pi r}\left[\sin^2\theta\right]_\pi^0 = 0$$

$$U_{CB} = U_C - U_B = U_C - U_A + U_A - U_B = -U_{AC} + U_{AB} = \frac{\mu_0 m\omega}{4\pi r}$$



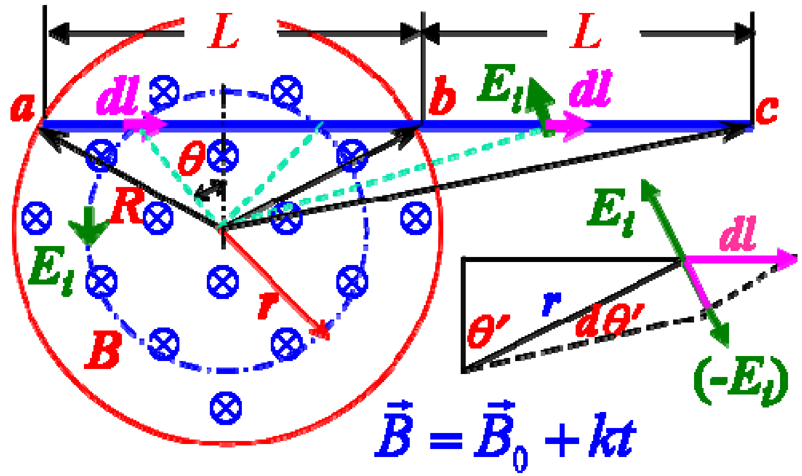
- 【例8.1.63】导体棒***abc***，磁场 ***B(t)*** 只在半径为 ***R*** 的圆环区域内存在，求导体 ***abc*** 的电势差 U_{ac}
- 涡旋电场为 E_i，方向如图

$$\oint \vec{E}_i\cdot d\vec{l} = E_i\cdot 2\pi r = -\frac{d\Phi}{dt} = \begin{cases} -\pi r^2\dfrac{dB}{dt} = -\pi r^2 k, & r<R \\ -\pi R^2\dfrac{dB}{dt} = -\pi R^2 k, & r>R \end{cases}$$

$$E_i = \begin{cases} -\dfrac{1}{2}kr, & r<R \\ -\dfrac{kR^2}{2r}, & r>R \end{cases}$$

$$U_{ac} = U_a - U_c = \int_a^c \vec{E}_s\cdot d\vec{l} = \int_a^c (-\vec{E}_i)\cdot d\vec{l} = \int_a^b (-\vec{E}_i)\cdot d\vec{l} + \int_b^c (-\vec{E}_i)\cdot d\vec{l}$$

继续



$$\int_a^b (-\vec{E}_i)\cdot d\vec{l} \xrightarrow[d\vec{l}\ \text{and}\ (-\vec{E}_i)\ \text{is}\ \theta]{\text{The angle between}}$$

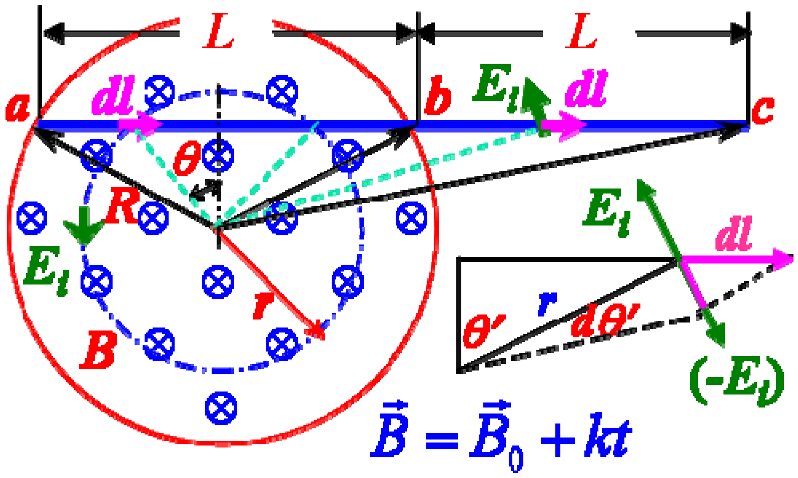
$$\int_a^b (-E_i)\cos\theta dl = \int_a^b \frac{1}{2}kr\cos\theta dl \xrightarrow[\cos\alpha=\sqrt{R^2-(L/2)^2}/R]{r\cos\theta=R\cos\alpha}$$

$$= \int_a^b \frac{1}{2}k\sqrt{R^2-(L/2)^2}dl = \frac{1}{2}k\sqrt{R^2-(L/2)^2}\int_0^L dl = \frac{1}{2}kL\sqrt{R^2-(L/2)^2}$$

$$\int_b^c (-\vec{E}_i)\cdot d\vec{l} \xrightarrow[d\vec{l}\ \text{and}\ (-\vec{E}_i)\ \text{is}\ \theta']{\text{The angle between}}$$

$$\int_b^c (-E_i)\cos\theta' dl = \int_b^c \frac{kR^2}{2r}\cos\theta' dl = \frac{kR^2}{2}\int_b^c \frac{\cos\theta'}{r}dl \xrightarrow{dl\cos\theta'=rd\theta'}$$

$$= \frac{kR^2}{2}\int_b^c d\theta' = \frac{kR^2}{2}\theta'\Big|_{\arcsin(\frac{L/2}{R})}^{\arcsin(\frac{3L/2}{\sqrt{2L^2+R^2}})} = \frac{kR^2}{2}\left(\arcsin(\frac{3L/2}{\sqrt{2L^2+R^2}})-\arcsin(\frac{L}{2R})\right)$$



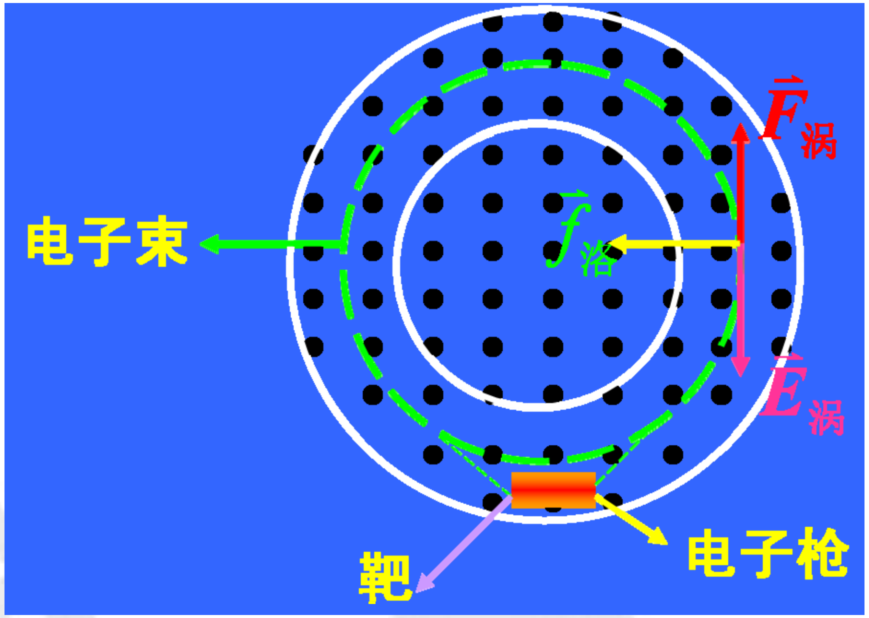
□ 继续

$$U_{ac}=U_a-U_c$$

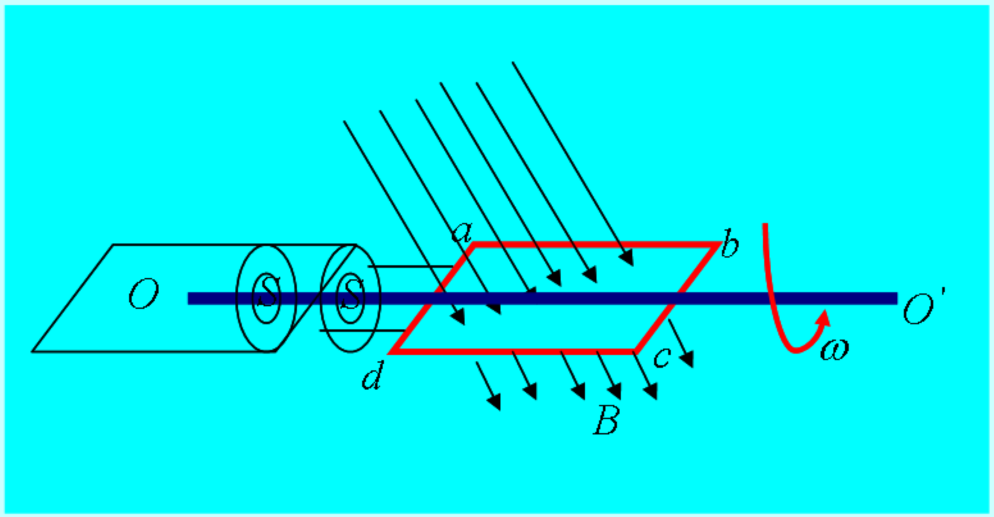
$$=\frac{1}{2}kL\sqrt{R^2-(L/2)^2}+\frac{kR^2}{2}\left(\arcsin\left(\frac{3L/2}{\sqrt{2L^2+R^2}}\right)-\arcsin\left(\frac{L}{2R}\right)\right)$$

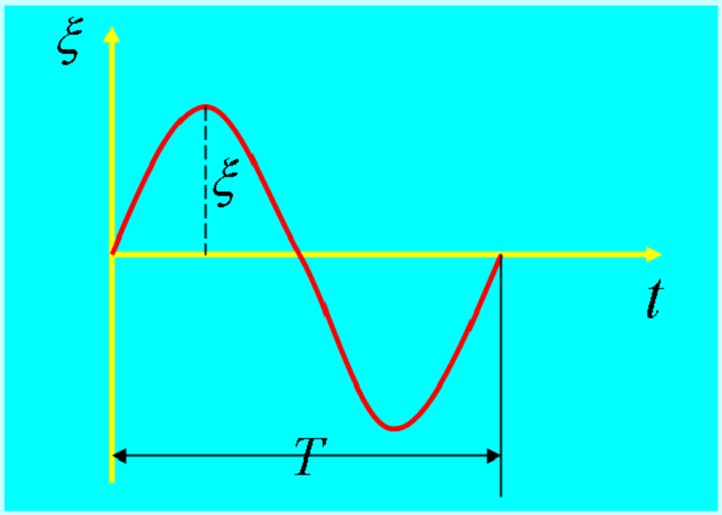
$$=\frac{3\sqrt{3}+\pi}{12}kR^2\Big|_{L=R}$$

- 电子感应加速器

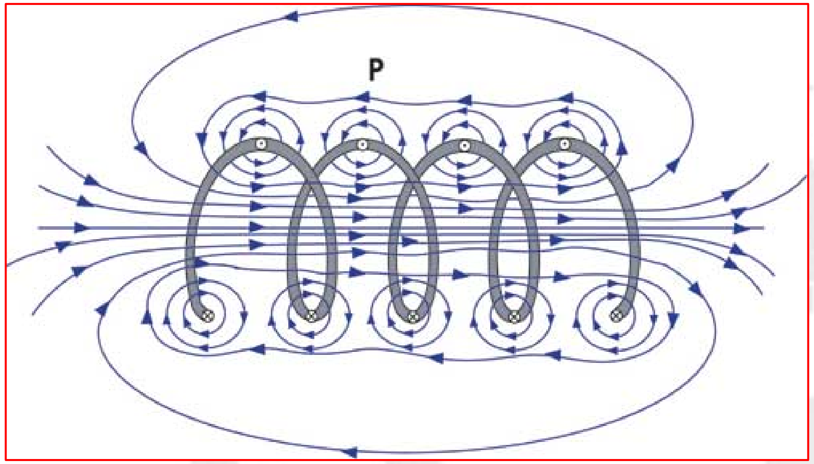


- 发电机

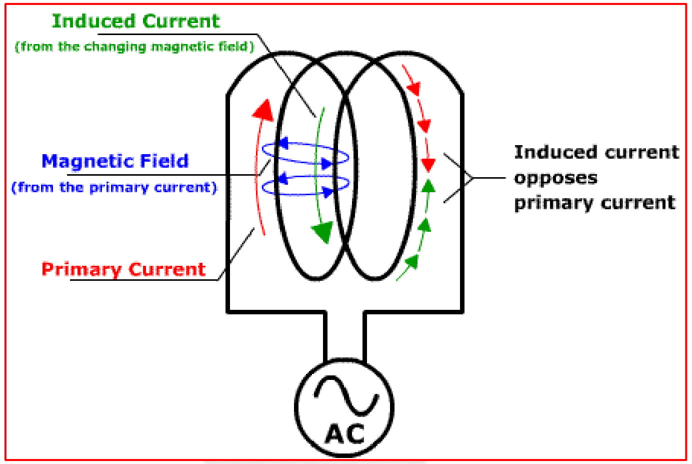




- 自感现象：由于回路自身电流、回路形状、或回路周围磁介质发生变化时，穿过该回路自身的磁通量随之改变，从而在回路中激发感应电动势。



这种自感现象会不会一级一级无穷无尽？！



$$\because d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3},\quad \therefore \vec{B}\propto I \Rightarrow \Phi_B=\iint_S \vec{B}\cdot d\vec{S}\propto I = L\cdot I$$

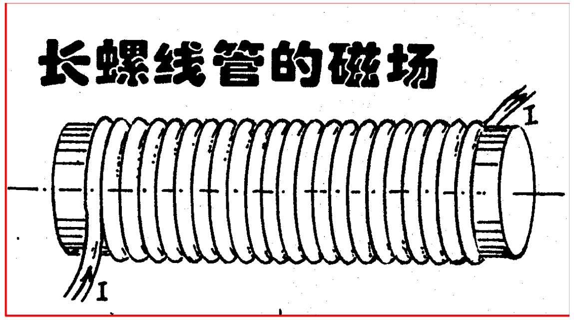
$$\Sigma=-\frac{d\Phi}{dt}=-\frac{d}{dt}(L\cdot I)=-\left(L\frac{dI}{dt}+I\frac{dL}{dt}\right)\xrightarrow{\text{if } L=\text{const}}-L\frac{dI}{dt}$$

- 方向问题：回路电流参考方向与回路所张平面法线满足右手螺旋法则，此时自感系数 L 恒为正；
- 量纲问题：单位--亨利(H)=[Φ]/[I]=V·s/A

自感电动势将反抗回路中电流的改变

- 真空中的电势，自感系数的量值仅决定于回路的几何形状。
- 若存在磁介质，则自感系数决定于回路的几何形状、周围介质的性质及所通电流。

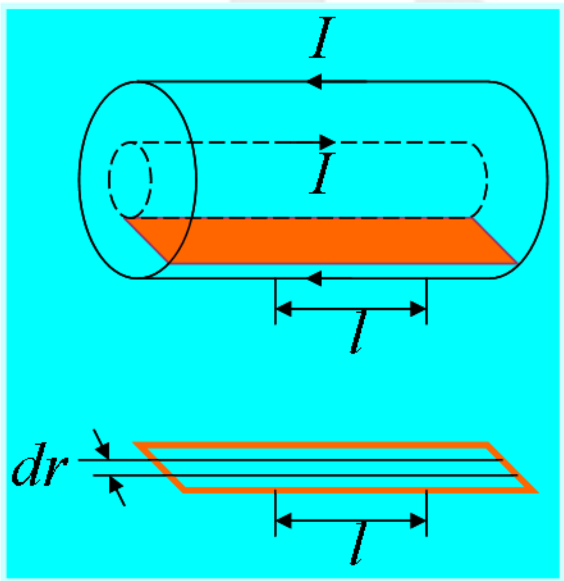
电磁学08-04: 自感与互感问题



【例1 p.320】计算螺绕环的自感系数

$$\left.\begin{aligned} B &= \frac{N}{l}\mu_0\mu_r I \\ \Psi &= N\Phi_B = \frac{\mu_0\mu_r N^2 SI}{l} \end{aligned}\right\} \Rightarrow L = \frac{\Psi}{I} = \frac{\mu_0\mu_r N^2 S}{l} = \frac{\mu_0\mu_r N^2 (Sl)}{l^2} = \mu_0\mu_r n^2 V$$

【例2 p.321】同轴电缆单位长度自感系数，内部填充了磁介质。



$$B = \frac{\mu_0\mu_r I}{2\pi r}, \quad d\Phi = BdS = B(ldr) = \frac{\mu_0\mu_r Il}{2\pi}\frac{dr}{r}$$

$$\Phi = \int d\Phi = \int_{R_1}^{R_2} \frac{\mu_r\mu_0 Il}{2\pi}\frac{dr}{r} = \frac{\mu_r\mu_0 Il}{2\pi}\ln\frac{R_2}{R_1}$$

$$\because \Phi = LI, \quad \therefore L = \frac{1}{l}\frac{\Phi}{I} = \frac{\mu_r\mu_0}{2\pi}\ln\frac{R_2}{R_1}$$

相位问题:

$$\cos\theta = \sin(\frac{\pi}{2} - \theta),\ \sin\theta = \cos(\frac{\pi}{2} - \theta),\quad \frac{d\sin\theta}{d\theta} = \cos\theta,\quad \frac{d\cos\theta}{d\theta} = -\sin\theta$$

$$U \Rightarrow U(t) \Rightarrow Q(t) = CU(t) \Rightarrow i = \frac{dQ}{dt}$$

$$\Rightarrow \text{phase difference } \frac{\pi}{2} \Rightarrow RC \text{ circuit relaxation}$$

$$I \Rightarrow I(t) \Rightarrow B(t) \Rightarrow \Phi(t) = SB(t) \Rightarrow \Sigma = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

$$\Rightarrow \text{phase difference } \frac{\pi}{2} \Rightarrow RL \text{ circuit relaxation}$$

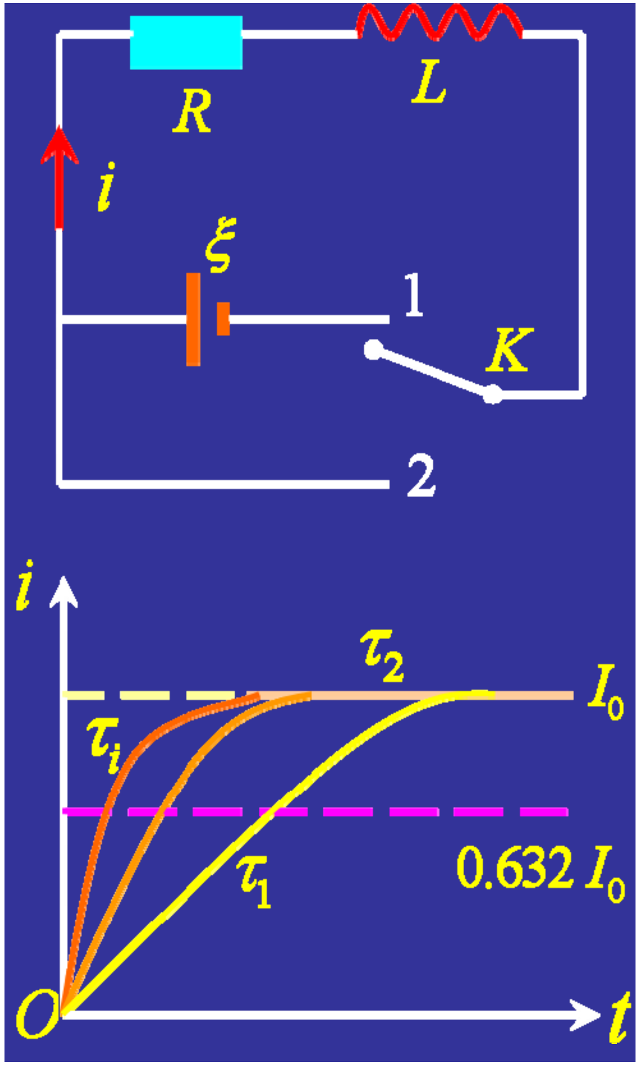
自感电路的过渡过程：电路接通

$$i=0 \text{ at } t=0, \quad \therefore \xi - L\frac{di}{dt} = iR \Rightarrow \frac{di}{\frac{\xi}{R}-i} = \frac{R}{L}dt$$

$$\Rightarrow \int_0^i \frac{di}{\frac{\xi}{R}-i} = \int_0^t \frac{R}{L}dt \Rightarrow i = \frac{\xi}{R}(1-e^{-\frac{R}{L}t})$$

$$i = I_0(1-\tfrac{1}{e}) \text{ at } t = L/R$$

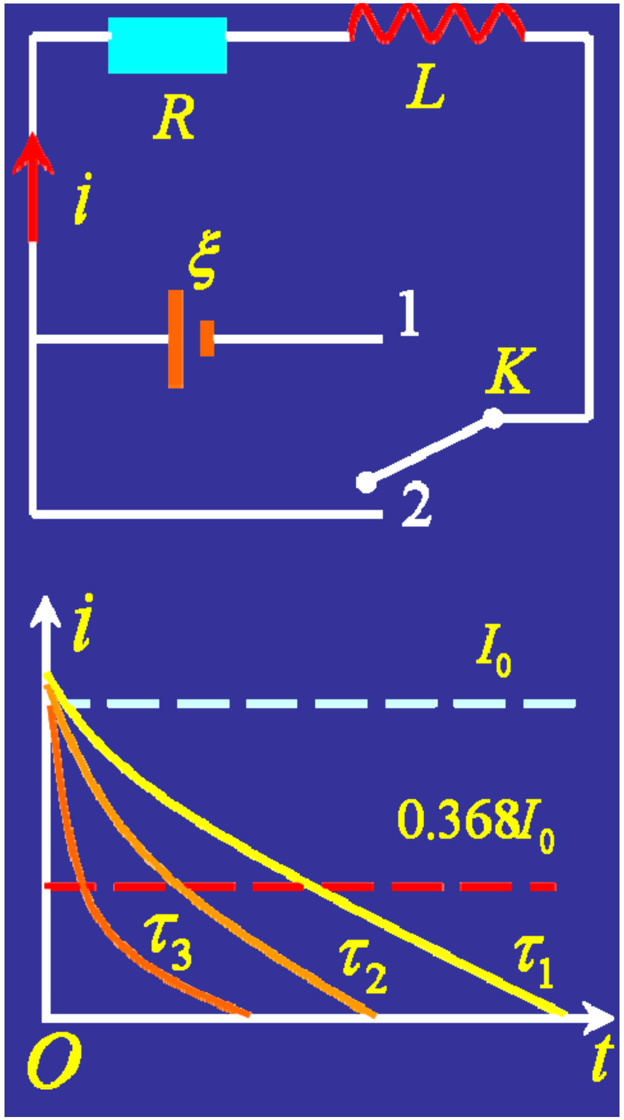
回路时间系数
弛豫时间



- 自感电路的过渡过程：电路断开

$$i = I_0 = \frac{\xi}{R} \text{ at } t = 0, \quad -L\frac{di}{dt} = iR$$

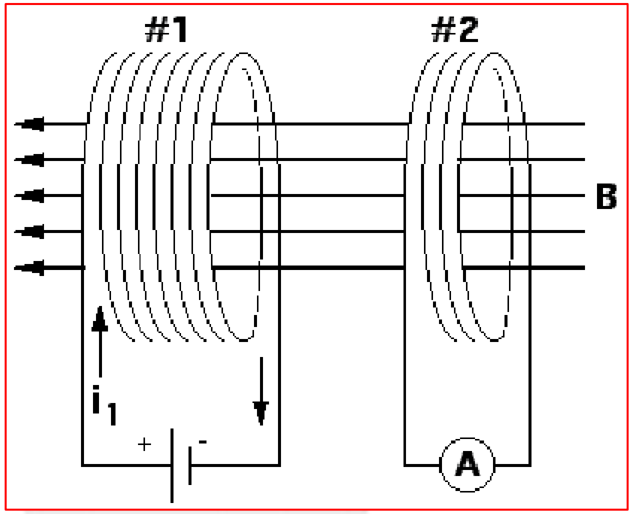
$$\Rightarrow \int_{I_0}^{i} \frac{di}{i} = \int_0^t -\frac{R}{L} dt \Rightarrow i = I_0 e^{-\frac{R}{L}t}$$

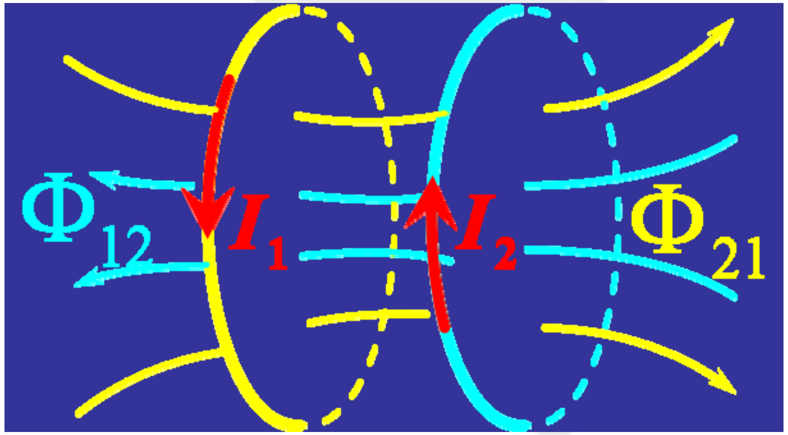


- 自感电路的危害与应用

- <u>**Resonant RLC circuit**</u>

- 互感现象：一对相互靠近的感应线圈，因一个或两个载流线圈的电流变化而在对方线圈中激起感应电动势，这一现象称为互感应现象。





for loop 1: $\Phi_1 = \int_{S_1} (\vec{B}_1 + \vec{B}_2) \cdot d\vec{S}_1$

$\because \Phi_{11} = \int_{S_1} \vec{B}_1 \cdot d\vec{S}_1, \quad \Phi_{12} = \int_{S_1} \vec{B}_2 \cdot d\vec{S}_1$

$\therefore \Phi_1 = \Phi_{11} + \Phi_{12} = L_1 I_1 + M_{12} I_2$

$$M_{12} = \frac{\Phi_{12}}{I_2}$$

for loop 2: $\Phi_2 = \int_{S_2} (\vec{B}_1 + \vec{B}_2) \cdot d\vec{S}_2$

$\because \Phi_{21} = \int_{S_2} \vec{B}_1 \cdot d\vec{S}_2, \quad \Phi_{22} = \int_{S_2} \vec{B}_2 \cdot d\vec{S}_2$

$\therefore \Phi_2 = \Phi_{21} + \Phi_{22} = M_{21} I_1 + L_2 I_2$

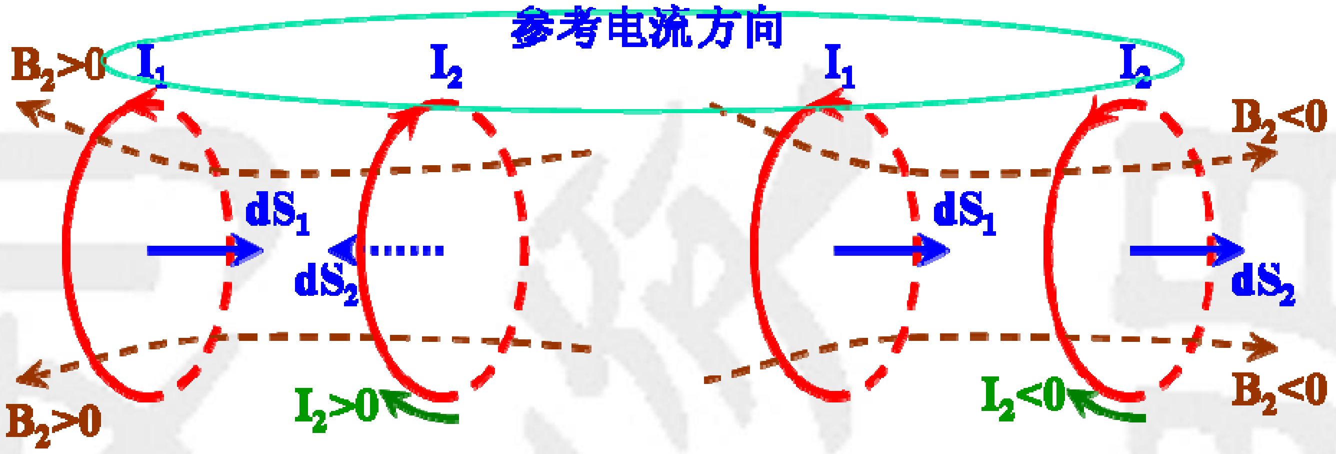
$$M_{21} = \frac{\Phi_{21}}{I_1}$$

回路2对1的互感系数 M_{12}

回路1对2的互感系数 M_{21}

$$\boxed{M_{12} = M_{21} = M}$$

- 符号问题：每个回路电流参考方向与回路所张曲面法线方向满足右手螺旋法则；一旦实际电流与参考电流相反，则取负号。



$$M_{12}=\frac{\Phi_{12}}{I_2}=\frac{1}{I_2}\iint_{S_1}\vec{B}_2\cdot d\vec{S}_1$$

$$\text{if } I_2>0,\ \angle(\vec{B}_2\cdot\vec{S}_1)\sim[\pi/2,3\pi/2],\ \iint_{S_1}\vec{B}_2\cdot d\vec{S}_1<0\Rightarrow M_{12}<0$$

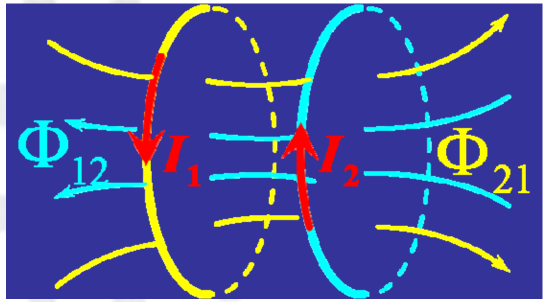
$$\text{if } I_2<0,\ \vec{B}_2<0,\ \iint_{S_1}\vec{B}_2\cdot d\vec{S}_1<0\Rightarrow M_{12}>0,\ \ (M_{12}I_2)\text{ keeps its sign!}$$

- 互感系数和两回路的几何形状、尺寸，它们的相对位置，以及周围介质的磁导率有关
- 互感系数的大小反映了两个线圈磁场的相互影响程度。

$$\text{for loop 1: } \Sigma_1 = -\frac{d\Phi_1}{dt} = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt} = \Sigma_{11} + \Sigma_{12}$$

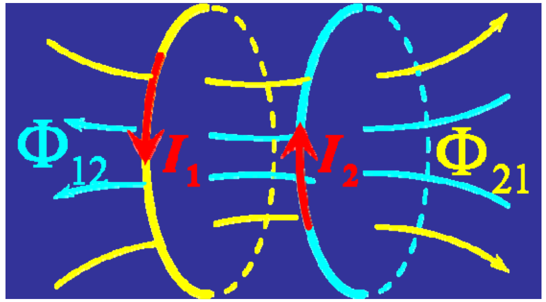
$$\text{for loop 2: } \Sigma_2 = -\frac{d\Phi_2}{dt} = -M\frac{dI_1}{dt} - L_2\frac{dI_2}{dt} = \Sigma_{21} + \Sigma_{22}$$

$$\because \Sigma_{12} = -M\frac{dI_2}{dt}, \quad \therefore |\Sigma_{12}| = |M| \text{ at } \frac{dI_2}{dt} = 1A/s$$



- 互感系数在数值上等于当第二个回路电流变化率为每秒一安培时，在第一个回路所产生的互感电动势的大小。

电磁学08-04: 自感与互感问题



- 互感系数相等证明(两线圈电流从零变化到 I_1、I_2)
- 从做功角度切入：

$$\text{for loop } 1\text{: } \Sigma_1 = -L_1 \frac{di_1}{dt} - M_{12} \frac{di_2}{dt},\ dW_1 = \left|\Sigma_1\right| i_1 dt$$

$$\text{for loop } 2\text{: } \Sigma_2 = -M_{21} \frac{di_1}{dt} - L_2 \frac{di_2}{dt},\ dW_2 = \left|\Sigma_2\right| i_2 dt$$

$$dW = dW_1 + dW_2 = L_1 i_1 di_1 + L_2 i_2 di_2 + M_{12} i_1 di_2 + M_{21} i_2 di_1$$

$$\because i_1 di_2 = d(i_1 i_2) - i_2 di_1,\ \ \therefore dW = L_1 i_1 di_1 + L_2 i_2 di_2 + M_{12} d(i_1 i_2) + (M_{21} - M_{21}) i_2 di_1$$

- 假定 (i_1, i_2) 各自独立变化到 (I_1, I_2)，则总功跟过程无关：

$$W = \int_0^{I_1, I_2} dW = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2 + (M_{21} - M_{12}) \int_0^{I_1, I_2} i_2 di_1$$

- 如果互感问题不涉及铁磁质，即功为单值函数，则

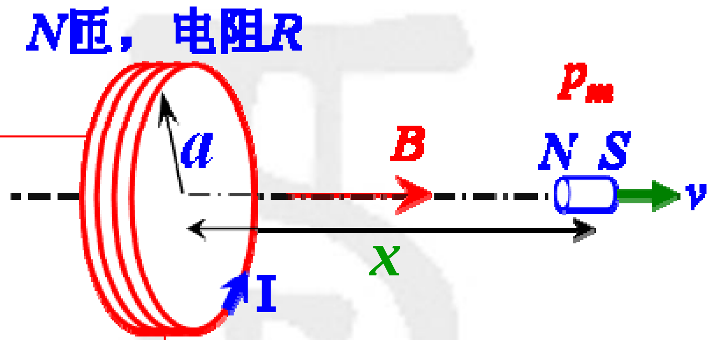
$$\boxed{(M_{21}-M_{12})\int_0^{I_1,I_2} i_2 di_1 = 0 \Rightarrow \because \int_0^{I_1,I_2} i_2 di_1 \neq 0,\quad \therefore M_{21}=M_{12}}$$

- 如果互感问题涉及铁磁质，则磁化过程与路径相关，总功 W 不再是单值函数，则：

$$\boxed{M_{21} \neq M_{12}}$$

- 实际电工学中上式都是成立的，互感系数不能视为相等。

- 【例8.3.25】一大圆线圈半径为 a，共有 N 匝，电阻为 R。一磁偶极矩为 p_m 的小磁棒沿它的轴线抽出，在离中心为 x 处时，抽出速度为 v。求大线圈中产生的感应电动势；当小磁棒从线圈中心移到无穷远处，大线圈中流过的电荷量。



- 设在 x 处有一同轴小线圈 S：

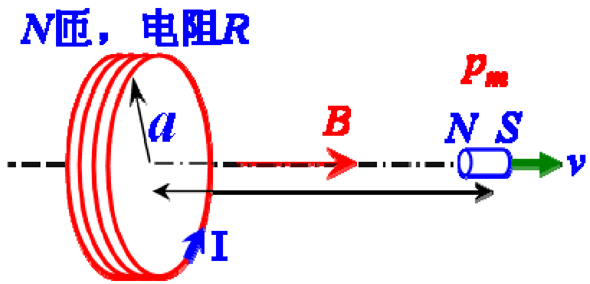
$$\text{at point } x: \ B=\frac{\mu_0 N a^2 I}{2(x^2+a^2)^{3/2}} \Rightarrow$$

$$\Phi = BS = \frac{\mu_0 N a^2 IS}{2(x^2+a^2)^{3/2}} \Rightarrow M_{小大} = \frac{\Phi}{I} = \frac{\mu_0 N a^2 S}{2(x^2+a^2)^{3/2}}$$

互感系数的等价与转换

- 设同轴小线圈 S 有电流 i 流过：

$$p_m = \mu_0 iS \Rightarrow \Psi = M_{大小} i \xrightarrow{M_{大小}=M_{小大}} \frac{N a^2 p_m}{2(x^2+a^2)^{3/2}}$$



- 由此，小线圈引起的大线圈感应电动势：

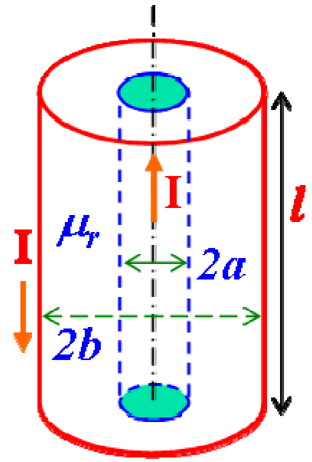
$$\Sigma=-\frac{d\Psi}{dt}=-\frac{d}{dt}\left(\frac{Na^2p_m}{2(x^2+a^2)^{3/2}}\right)=-\frac{Na^2p_m}{2}\frac{d}{dt}\left(\frac{1}{(x^2+a^2)^{3/2}}\right)$$

$$=\frac{3Na^2p_m x}{2(x^2+a^2)^{5/2}}\frac{dx}{dt}=\frac{3Na^2p_m x}{2(x^2+a^2)^{5/2}}v$$

- 大线圈的电流：

$$I=\frac{dq}{dt}=\frac{\Sigma}{R}=-\frac{Na^2p_m}{2R}\frac{d}{dt}\left(\frac{1}{(x^2+a^2)^{3/2}}\right)$$

$$dq=-\frac{Na^2p_m}{2R}d\left(\frac{1}{(x^2+a^2)^{3/2}}\right)\Rightarrow q=-\frac{Na^2p_m}{2R}\frac{1}{(x^2+a^2)^{3/2}}\bigg|_{x=0}^{x=\infty}=\frac{Np_m}{2aR}$$



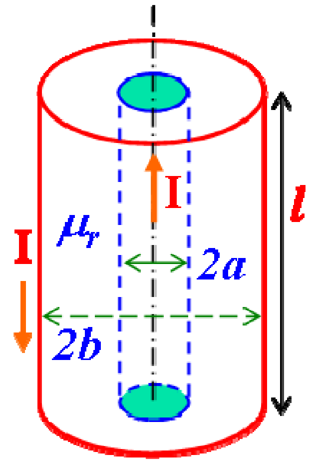
- 磁链的概念：
- 【例8.3.28】同轴电缆有直导线(半径为a，磁导率μ_1)和外圆筒(半径为 b)组成，中间充满介质(μ_2)。电流在电缆内均匀流动。求电缆单位长度自感 L。
- 磁场局限在外圆筒内部：导线内磁链为 Ψ_i，磁介质内磁链为 Ψ_e，则：

$$L_i = \Psi_i / I,\ L_e = \Psi_e / I \Rightarrow L = L_i + L_e$$

$$\Psi_e = \iint_S \vec{B}\cdot d\vec{S} = \int_a^b Bldr = \frac{\mu_0\mu_2 Il}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0\mu_2 Il}{2\pi}\ln\frac{b}{a} \Rightarrow L_e = \frac{\mu_0\mu_2 l}{2\pi}\ln\frac{b}{a}$$

- 对导线而言：

$$\oint_i \vec{H}\cdot d\vec{l} = 2\pi r H = \frac{I}{\pi a^2}\pi r^2 \Rightarrow H = \frac{\mu_0 I r}{2\pi a^2} \Rightarrow B = \frac{\mu_0\mu_1 I r}{2\pi a^2}$$



- 继续：以轴心为原点，取半径 **r** 处的面积元 **dS=ldr**

$$d\Phi_i = BdS = \frac{\mu_0\mu_1 Il}{2\pi a^2} rdr$$

$$Id\Psi_i = I_i d\Phi_i$$

磁链的概念

- 这部分磁通只是包围了导线内半径为 **r** 的部分，其电流为：

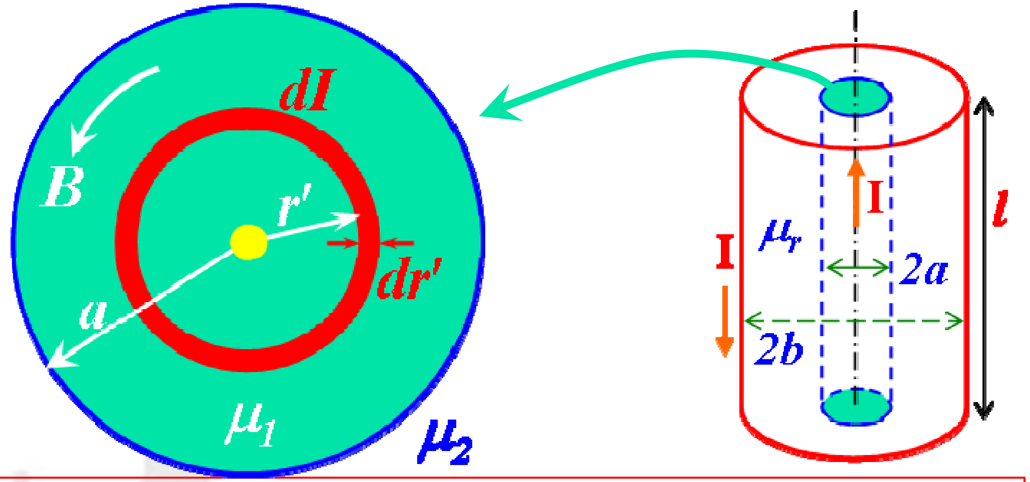
$$I_i = \frac{r^2}{a^2} I \Rightarrow \eta = \frac{I_i}{I} = \frac{r^2}{a^2}$$

分数匝数

$$d\Psi_i = \frac{I_i}{I} d\Phi_i = \frac{\mu_0\mu_1 Il}{2\pi a^4} r^3 dr \Rightarrow \Psi_i = \int_0^a d\Psi_i = \frac{\mu_0\mu_1 Il}{8\pi} \Rightarrow L_i = \frac{\Psi_i}{I} = \frac{\mu_0\mu_1 l}{8\pi}$$

$$L = L_e + L_i = \frac{\mu_0\mu_2 l}{2\pi}\ln\frac{b}{a} + \frac{\mu_0\mu_1 l}{8\pi} \Rightarrow \frac{L}{l} = \frac{\mu_0}{4\pi}\left(2\mu_2 \ln\frac{b}{a} + \frac{1}{2}\mu_1\right)$$

- 对导线部分换一个角度求解：对截面取电流圆环 **dI**



$$dI = \frac{I}{\pi a^2} 2\pi r'dr' = \frac{2I}{a^2} r'dr'$$

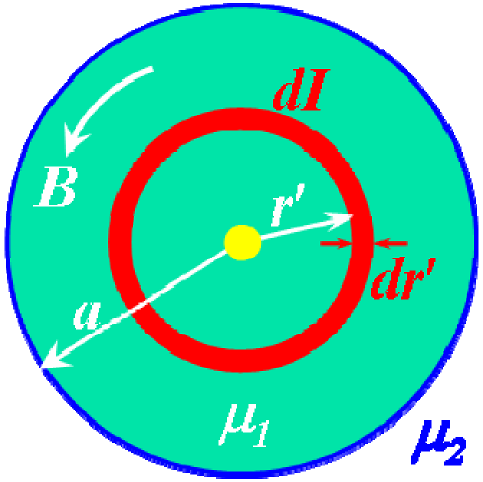
$$\oint_i \vec{H}_{dI} \cdot d\vec{l}\,\Big|_{r>r'} = 2\pi r H_{dI}\Big|_{r>r'} = dI \Rightarrow B_{dI} = \frac{\mu_0\mu_1}{2\pi r} dI = \frac{\mu_0\mu_1}{\pi r}\frac{I}{a^2} r'dr'\Big|_{r>r'}$$

$$d\Phi_{dI} = \iint_{S_{dI}} B_{dI} dS_{dI} \xrightarrow{dS_{dI}=ldr} \frac{\mu_0\mu_1 I}{\pi a^2} r'dr' \int_{r'}^{a} \frac{1}{r} ldr = \frac{\mu_0\mu_1 lI}{\pi a^2} r'(\ln\frac{a}{r'})dr'$$

$$\Psi_i = \int_0^a d\Phi_{dI} = \frac{\mu_0\mu_1 lI}{\pi a^2}\int_0^a r'(\ln\frac{a}{r'})dr' = \frac{\mu_0\mu_1 Il}{4\pi} \Rightarrow L_i = \frac{\Psi_i}{I} = \frac{\mu_0\mu_1 l}{4\pi}$$

??????看起来不对！错误在哪里？

也可以从能量角度求！即【例2, p.328】，后面会涉及！



- 对导线部分再换一个角度求解：一系列电流环所产生自感的“叠加和” $L_i=\Sigma dL_i$
- 注意：在对 **dL** 进行求和时，我们化其为对 **dr'/a** 的积分，以保证量纲一致。

$$dI=\frac{I}{\pi a^2}2\pi r'dr'=\frac{2I}{a^2}r'dr',\quad d\Phi_{dI}=\frac{\mu_0\mu_1 Il}{\pi a^2}r'dr'\int_{r'}^{a}\frac{1}{r}dr=\frac{\mu_0\mu_1 lI}{\pi a^2}r'(\ln\frac{a}{r'})dr'$$

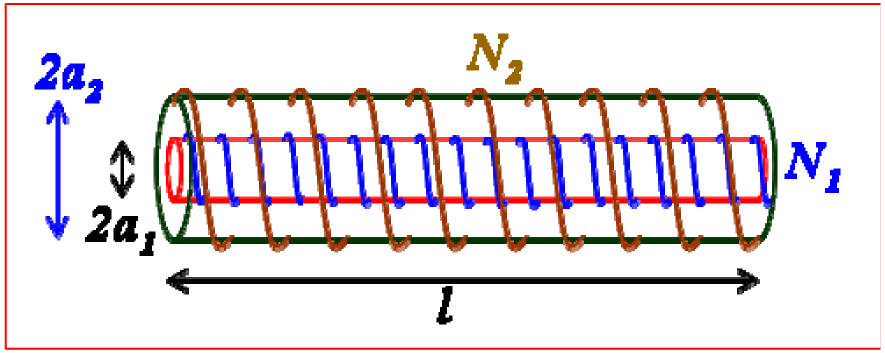
$$dL\Big|_{dI}=\frac{d\Phi_{dI}}{dI}=\frac{\mu_0\mu_1 l}{2\pi}\int_{r'}^{a}\frac{1}{r}dr\Rightarrow r'dL\Big|_{dI}=adL_i$$

$$L_i=\int_0^a\frac{r'}{a}\left(dL\Big|_{dI}\right)\frac{dr'}{a}=\frac{\mu_0\mu_1 l}{2\pi a^2}\int_0^a r'dr'\int_{r'}^{a}\frac{1}{r}dr=\frac{\mu_0\mu_1 l}{4\pi}$$

$$\Psi_i=IL_i=\frac{\mu_0\mu_1 lI}{4\pi}$$

也可以从能量角度求！即【例2, p.328】，后面会涉及！

- 【例8.3.7】两螺线管相套，略去边缘效应，求自感、互感及相互关系。



$$B_1 = \mu_0 n_1 I_1 = \mu_0 \frac{N_1}{l} I_1 \Rightarrow \Phi_1 = B_1 S_1 = \frac{\pi\mu_0 N_1 a_1^2}{l} I_1$$

$$\Psi_1 = N_1 \Phi_1 = \frac{\pi\mu_0 N_1^2 a_1^2}{l} I_1 \Rightarrow L_1 = \frac{\Psi_1}{I_1} = \frac{\pi\mu_0 N_1^2 a_1^2}{l}$$

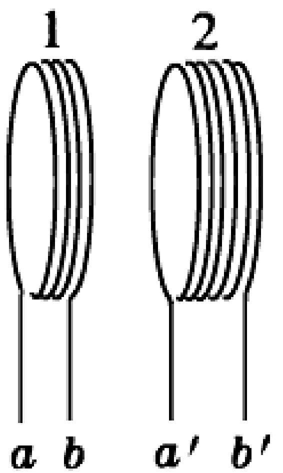
$$L_2 = \frac{\pi\mu_0 N_2^2 a_2^2}{l}$$

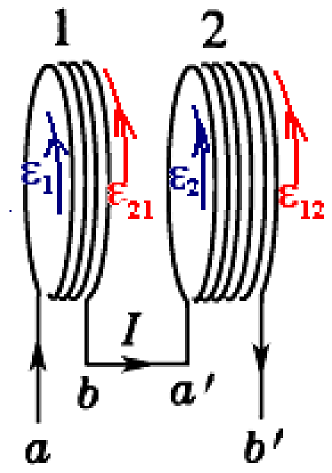
$$\Psi_{21} = N_2 \Phi_{21} = N_2 \left[B_1 S_1 + B_1(=0)(S_2 - S_1) \right] = N_2 B_1 S_1 = \frac{\pi\mu_0 N_1 N_2 a_1^2}{l} I_1 \Rightarrow$$

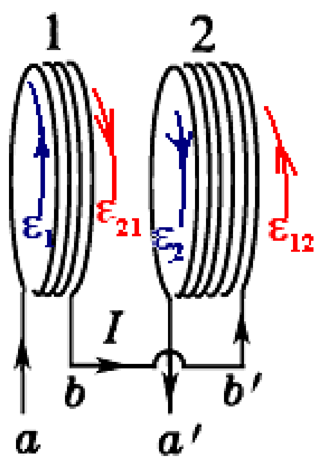
$$M = \frac{\Psi_{21}}{I_1} = \frac{\pi\mu_0 N_1 N_2 a_1^2}{l} \Rightarrow M = \frac{a_1}{a_2}\sqrt{L_1 L_2}$$

注意小螺线管外和大螺旋管内空间没有 B_1，或者说 $B_1=0$。

电感串联问题:







$$\Sigma_1 + \Sigma_{21} = -\left(L_1 \frac{dI}{dt} + M\frac{dI}{dt}\right), \quad \Sigma_2 + \Sigma_{12} = -\left(L_2 \frac{dI}{dt} + M\frac{dI}{dt}\right)$$

$$\Sigma = \Sigma_1 + \Sigma_{21} + \Sigma_2 + \Sigma_{12} \Rightarrow L = L_1 + L_2 + 2M$$

$$\Sigma = \Sigma_1 - \Sigma_{21} + \Sigma_2 - \Sigma_{12} \Rightarrow L = L_1 + L_2 - 2M$$

- 通过感应电路来讨论磁场能量的宏观形式
- 电路接通后的过渡过程：

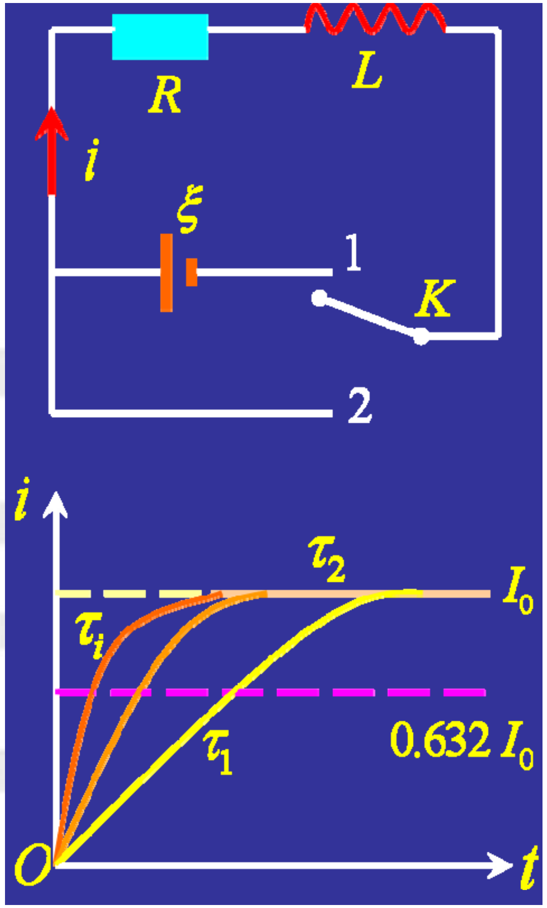
$$\xi - L\frac{di}{dt} = iR \Rightarrow$$

$$\xi i dt = i^2 R dt + L i di$$

dt内电源对电路做功

dt内电阻产生的焦耳热

dt内电流变化在线圈内产生磁通变化，转化为与电流反向的感应电动势，电流抵抗电动势做功



$$dW = Lidt \Rightarrow W_m = \int_0^{I_0} Lidi = \frac{1}{2} L I_0^2$$

- 电路切断后的过渡过程：

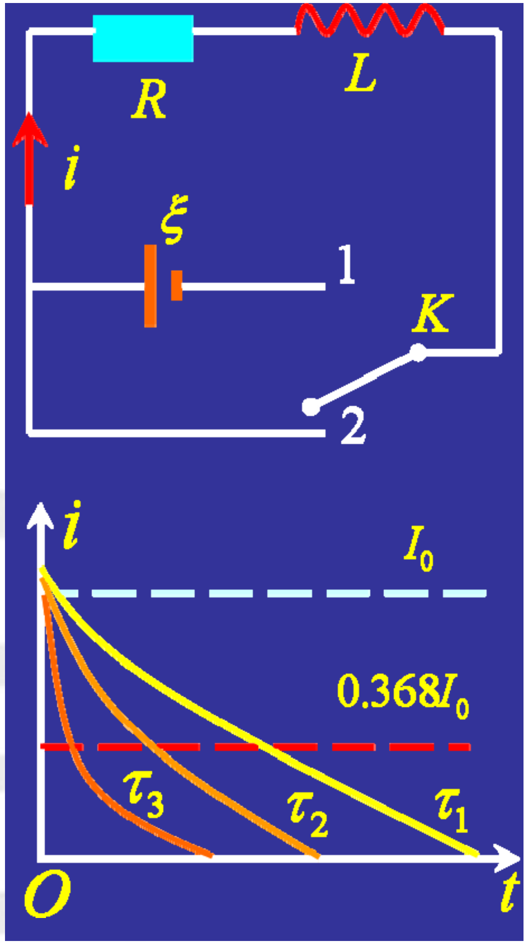
$$-L\frac{di}{dt}=iR\Rightarrow$$

$$i^2Rdt+Lidi=0$$

电路切断，dt内磁通释放在电路中产生电流，转化为焦耳热

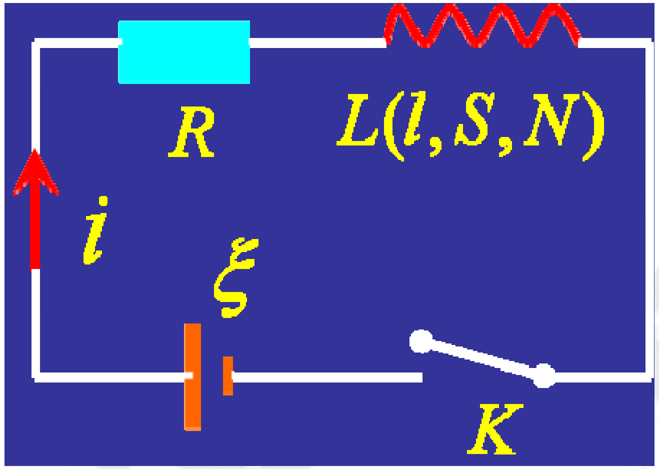
dt内电阻产生的焦耳热

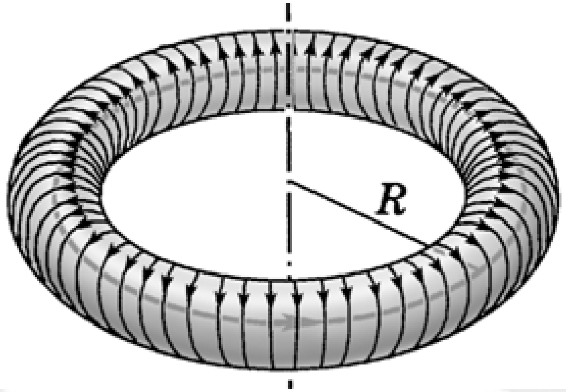
$$\int_0^\infty i^2Rdt=I_0^2R\int_0^\infty e^{-\frac{2R}{L}t}dt=\frac{1}{2}LI_0^2$$



- 电流所消耗的能量
- 以上推导仅适合于线圈中无铁磁质，不适用于一般情况

磁场储能一般情况讨论：还是螺绕线圈





$$\Phi = nlBS \Rightarrow \xi - nlS\frac{dB}{dt} = iR \Rightarrow \xi idt = i^2Rdt + nlSidB$$

$$\because \oint_L \vec{H}\cdot d\vec{l} = \sum_{L内} I_0 \Rightarrow Hl = nil \Rightarrow H = ni$$

$$\therefore \xi idt = i^2Rdt + lSHdB \Rightarrow w_m = lSHdB/V(=lS) = HdB$$

在螺绕圈内磁场已经为 ***H*** 时，电源再对螺绕圈做功 ***HdB***

- 电源供给螺绕圈的能量变为磁场能量，螺绕圈内部磁场体积近似为 ***Sl***，则单位体积磁能为：

$$W_m = \int_0^B H dB \xrightarrow{anisotropic\ media} \int_0^B \vec{H} \cdot d\vec{B}$$

$$\because \vec{B} = \mu_r \mu_0 \vec{H},\ W_m = \int_0^H \vec{H} \cdot d(\mu_r \mu_0 \vec{H})$$

$$= \begin{cases} \dfrac{1}{2}\mu_r \mu_0 H^2 = \dfrac{1}{2} BH, \text{ if } \mu_r = const \\ \mu_0 \displaystyle\int_0^H \vec{H} \cdot d(\mu_r \vec{H}), \text{ if } \mu_r = \mu_r(H) \end{cases}$$

- 对于存在互感的一般情况讨论： 在建立电流过程中， 电源除了供给线圈中产生焦耳热的能量和反抗自感电动势做功外，还要反抗互感电动势做功。
- 反抗互感电动势做的功： $$\boxed{A_b = \int(-\Sigma_{12} I_1 dt - \Sigma_{21} I_2 dt)}$$
- 如果考虑互感系数 ***M*** 为常数(无磁介质)，则：

$$A_b = \int(MI_1 dI_2 + MI_2 dI_1) = \int_0^{I_1 I_2} Md(I_1 I_2) = MI_1 I_2$$

$$\therefore W_m = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + MI_1 I_2 = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + \frac{1}{2}M_{12} I_1 I_2 + \frac{1}{2}M_{21} I_2 I_1$$

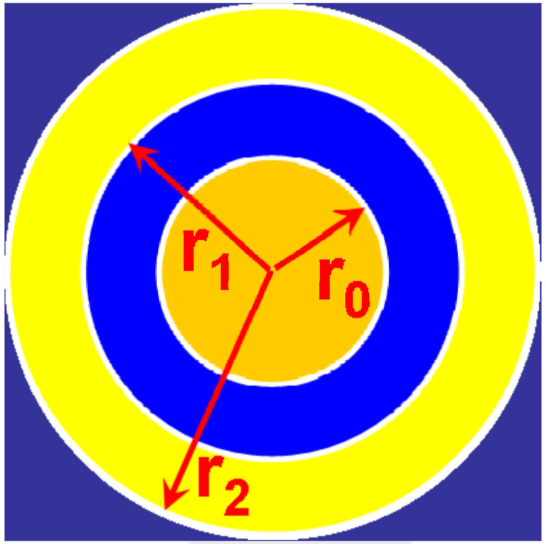
for n indution coils: $W_m = \frac{1}{2}\sum_{i=1}^{n} L_i I_i^2 + \frac{1}{2}\sum_{\substack{i,j=1 \\ i\neq j}}^{n} M_{ij} I_i I_j = \frac{1}{2}\sum_{i=1}^{n} I_i \Phi_i$

- 功能原理：

$$\boxed{\sum_k \Sigma_k I_k dt = \sum_j I_j^2 R_j dt + dA + dW_m}$$

- 电源供给的能量=释放的焦耳热+磁场对外所作功+磁能的增加

- 【例2 p.328】一同轴电缆，电流从中间导体圆柱面流入，从外层圆柱面流出构成闭合回路。试计算单位长度电缆内的磁场能以及自感系数 *L*。



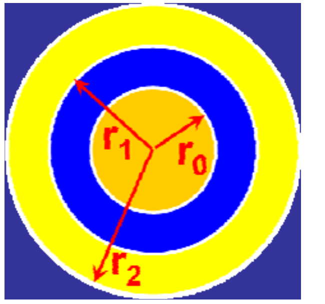
$$W_m = \frac{1}{2}LI^2$$

$$\begin{cases} H_1 = \dfrac{Ir}{2\pi r_0^2}, & (r \le r_0) \\ H_2 = \dfrac{I}{2\pi r}, & (r_0 < r \le r_1) \\ H_3 = \dfrac{I(r_2^2 - r^2)}{2\pi r(r_2^2 - r_1^2)}, & (r_1 < r \le r_2) \\ H_4 = 0, & (r_2 < r) \end{cases}$$

- 长度为 *l* 的一段电缆内磁能为：

$$W_L = \int W_m dV = \int (\frac{1}{2}\mu_0\mu_r H^2) dV$$

$$= \frac{1}{2}\mu_0\mu_r \int_0^{r_2} H^2 \cdot 2\pi r l dr$$



- 继续:

$$W=\frac{W_L}{l}=\frac{1}{2}\mu_0\mu_r\left[\int_0^{r_0}H_1^2 2\pi r dr+\int_{r_0}^{r_1}H_2^2 2\pi r dr+\int_{r_1}^{r_2}H_3^2 2\pi r dr\right]$$

$$=\frac{1}{4\pi}\mu_0\mu_r I^2\left[\frac{1}{4}+\ln\frac{r_1}{r_0}+\frac{1}{(r_2^2-r_1^2)}\left(\frac{r_2^4}{(r_2^2-r_1^2)}\ln\frac{r_2}{r_1}-\frac{3r_2^2-r_1^2}{4}\right)\right]$$

$$L=\frac{2W}{I^2}=\frac{1}{2\pi}\mu_0\left[\frac{1}{4}+\ln\frac{r_1}{r_0}+\frac{1}{(r_2^2-r_1^2)}\left(\frac{r_2^4}{(r_2^2-r_1^2)}\ln\frac{r_2}{r_1}-\frac{3r_2^2-r_1^2}{4}\right)\right]$$

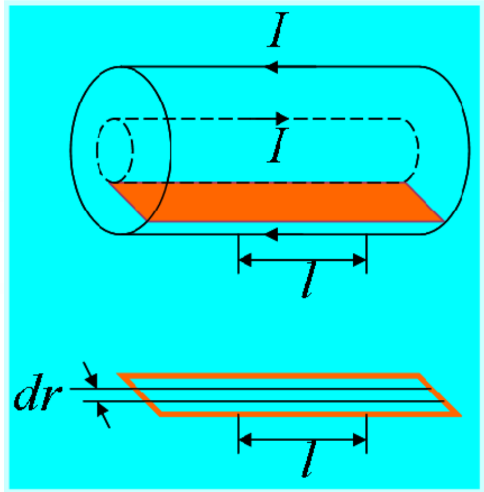
- 上式中 ***L*** 为单位长度的自感系数

- 类似的问题如果应用到【例2 p.321】:

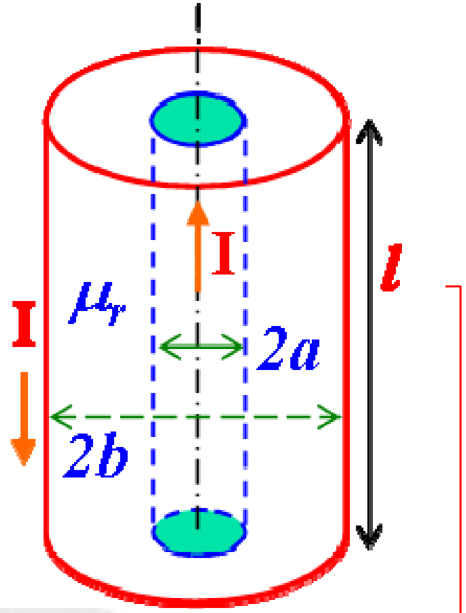
$$W=\frac{1}{2}\mu_0\mu_r\int_{R_1}^{R_2}H_2^2\,2\pi r dr$$

$$=\frac{1}{2}\mu_0\mu_r\int_{R_1}^{R_2}\left(\frac{Ir}{2\pi r}\right)^2 2\pi r dr=\frac{\mu_0\mu_r I^2}{4\pi}\ln\left(\frac{R_2}{R_1}\right)$$

$$\because W=\frac{1}{2}LI^2,\quad \therefore L=\frac{\mu_r\mu_0}{2\pi}\ln\frac{R_2}{R_1}$$



- 类似的问题如果应用到【例8.3.28】(ppt.46)。设电缆来回流动电流为 I:



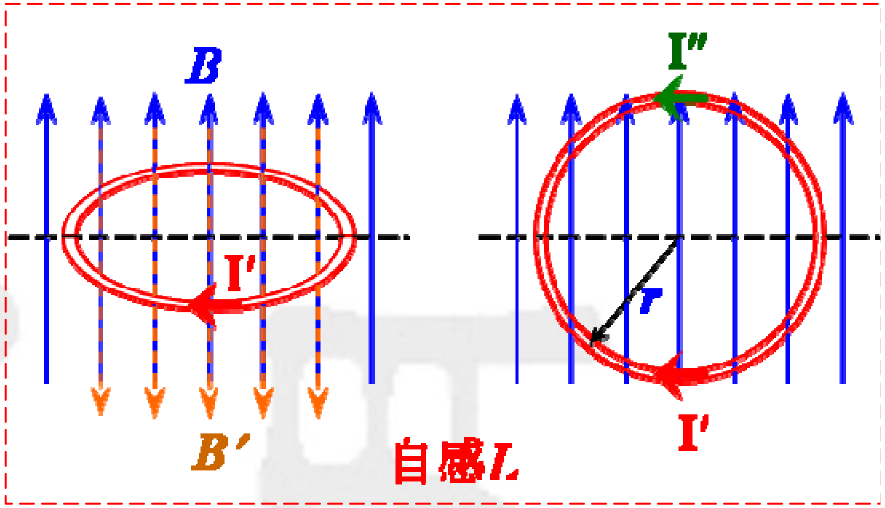
$$H=\begin{cases}\dfrac{I}{2\pi a^2}r, & r<a\\ 0, & r>b\\ \dfrac{I}{2\pi r}, & a<r<b\end{cases}$$

$$W_m=\frac{1}{2}\mu_0\iiint_V\mu_r H^2 dV=\frac{1}{2}\mu_0 l\iint_S\mu_r H^2 dS=\frac{1}{2}\mu_0 l\int_0^b\mu_r H^2 2\pi r dr$$

$$=\frac{1}{2}\mu_0 l\left[\int_0^a\mu_1\left(\frac{I}{2\pi a^2}r\right)^2 2\pi r dr+\int_a^b\mu_2\left(\frac{I}{2\pi r}\right)^2 2\pi r dr\right]=\frac{\mu_0\mu_1 I^2 l}{16\pi}+\frac{\mu_0\mu_2 I^2 l}{4\pi}\ln\frac{b}{a}$$

$$\because W_m=\frac{1}{2}LI^2,\quad\therefore L=\frac{\mu_0 l}{4\pi}\left(2\mu_2\ln\frac{b}{a}+\frac{1}{2}\mu_1\right)\Rightarrow\frac{L}{l}=\frac{\mu_0}{4\pi}\left(2\mu_2\ln\frac{b}{a}+\frac{1}{2}\mu_1\right)$$

- 【例8.4.3/8.4.6】超导环自感为 L，初始 $T<T_c$，然后在 Δt_1 内开启磁场 B 与环轴线平行，求环内电流及磁通表达式。如果再将环在 Δt_2 转到与 B 平行方向，如图，求环内电流和外力做功。



- 开启磁场，引起感应电动势：

$$\boxed{LI' + \Delta\Phi = 0}$$

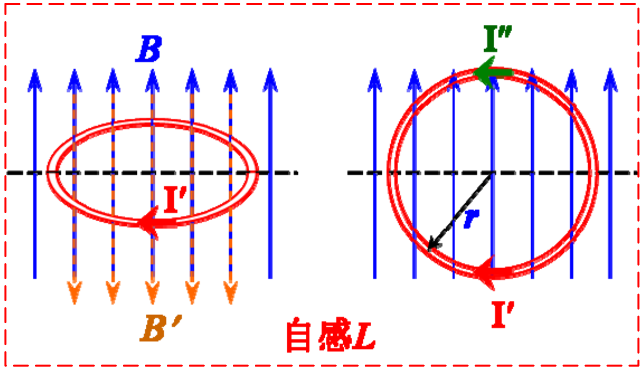
$$\Sigma = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\iint_S \vec{B}\cdot d\vec{S}\right) = -\pi r^2 \frac{dB}{dt}$$

$$\Sigma - L\frac{dI'}{dt} = RI' \xrightarrow{R=0} -\pi r^2 \frac{dB}{dt} = L\frac{dI'}{dt} \Rightarrow LdI' + \pi r^2 dB = 0$$

$$LI' + \pi r^2 B = 0 \Rightarrow I' = -\frac{\pi r^2}{L}B, \quad \text{independent of time } \Delta t_1$$

- $I'<0$ 意味着感应磁场与 B 反向，为 B'

电磁学08-05: 磁场能量问题



继续：圆环转动90º：

$$\Sigma = -\frac{d\Phi}{dt} = -\frac{\Delta\Phi}{\Delta t} \qquad \Sigma - L\frac{dI'}{dt} = 0$$

$$\Delta\Phi = -\pi r^2 B$$

$$\because\ LI'' + \Delta\Phi = 0$$

$$\therefore\ I'' = -\Delta\Phi / L = I'' = \frac{\pi r^2}{L}B,\quad \text{independent of time } \Delta t_2$$

$$I = I' + I'' = 0$$

转动圆环做功问题：

$$W = \frac{1}{2}LI''^2 = \frac{\pi^2 r^4 B^2}{2L}$$

- 磁能与电能的比较:
- 对铁磁质，μ_r=1000，磁能密度 10^4~10^5J/m^3；对顺磁(μ_r小)等，磁能更大(针对 **B** 固定而言)。
- 对电介质，ε_r=10，电能密度约10^4J/m^3。

$$W_m=\int_0^B HdB=\frac{1}{2\mu_0\mu_r}B^2 \xrightarrow{B=10T}$$

$$=\frac{100N^2/(A\cdot m)^2}{2\times 1000\times 4\pi\times 10^{-7}N/A^2}$$

$$=39432J/m^3$$

$$W_E=\frac{1}{2}\varepsilon_0\varepsilon_r E^2 \xrightarrow{E=10^7 V/m}$$

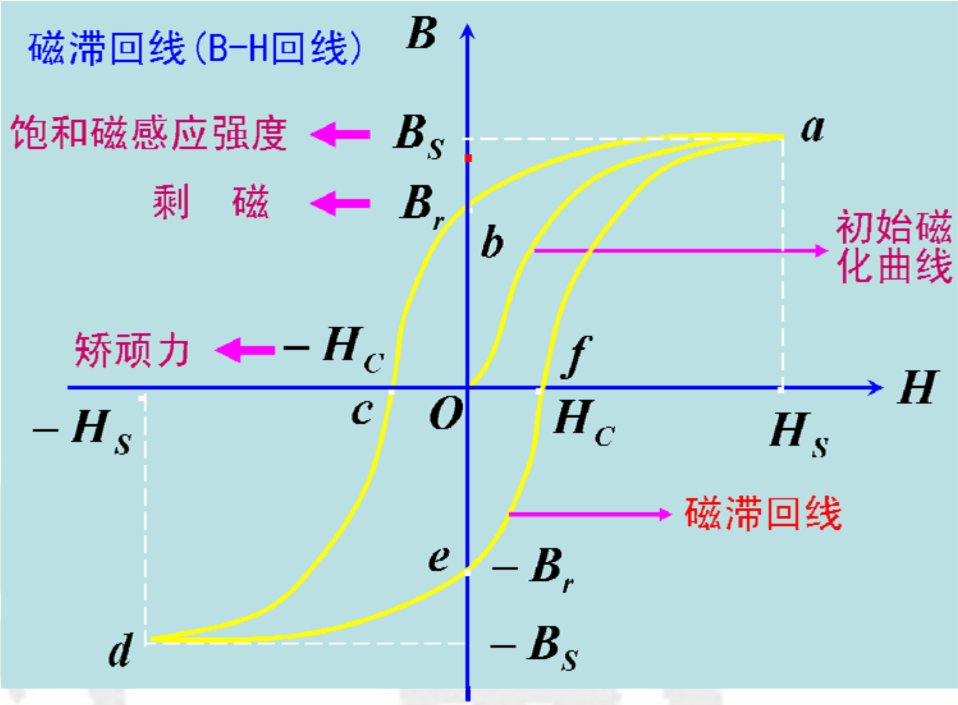
$$=8900J/m^3$$

- 但并非总是如此，原子或电子轨道磁矩产生的磁场所负载的能量，比静电能要小很多。

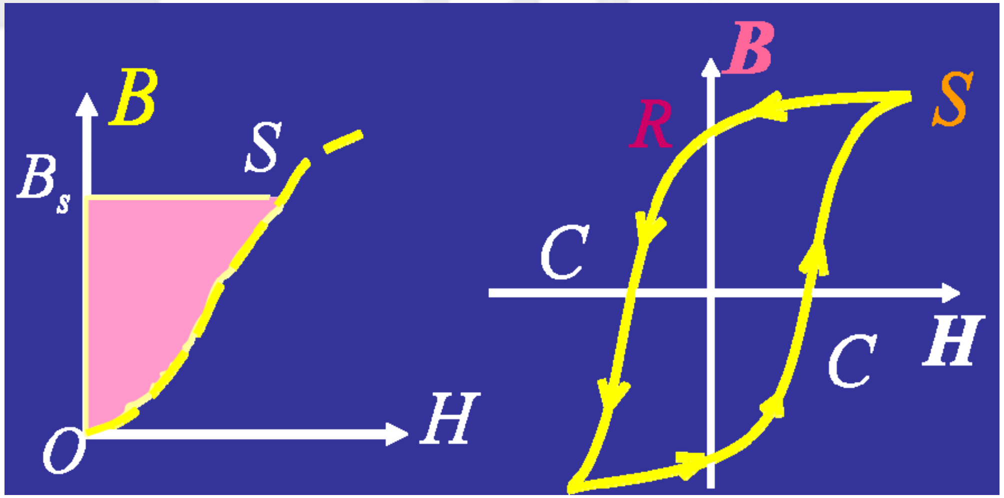
- 再看磁滞回线：

$$W_m = \int_0^B \vec{H} \cdot d\vec{B} = f(H_{\max}, \omega)$$

$$dW_m = \vec{H} \cdot d\vec{B}$$



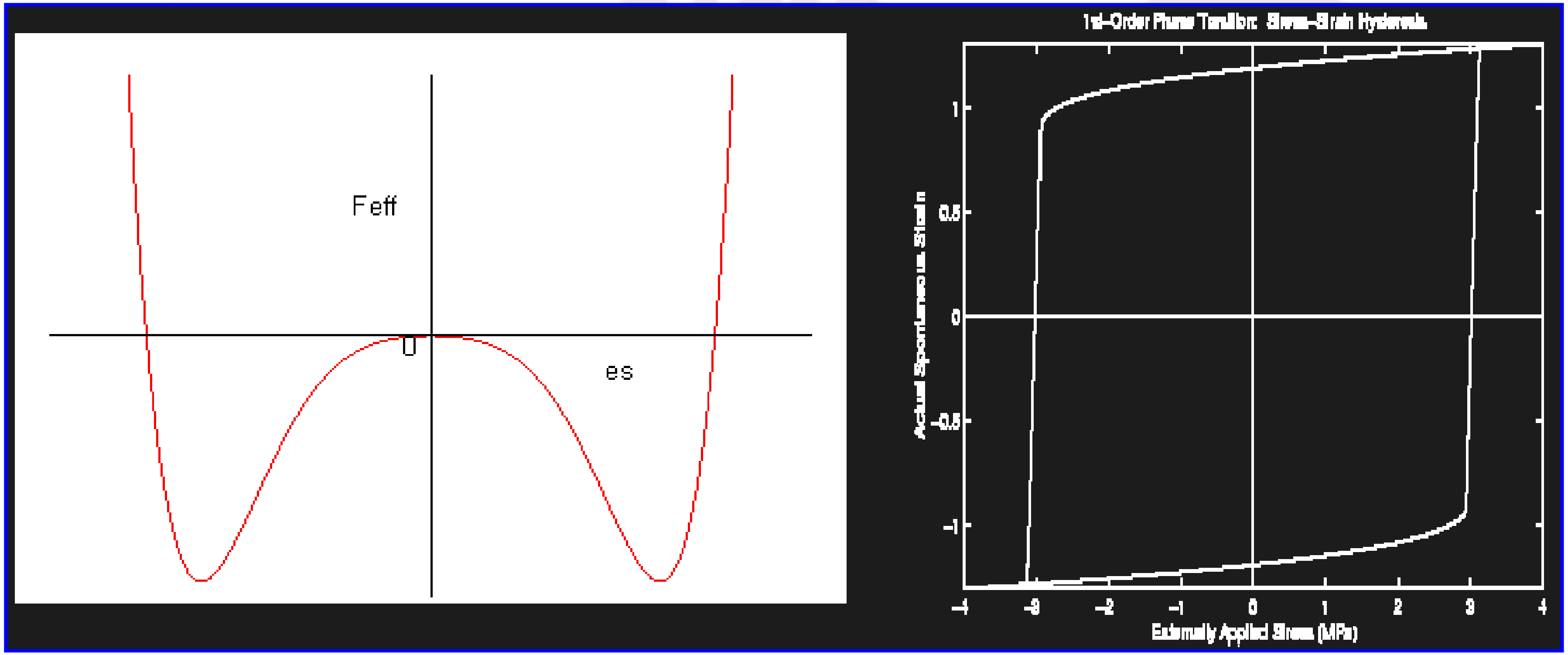
- 如果是磁滞介质，则升磁和退磁两个过程不重合，产生能量交换，转化为热量。
- 对磁滞回线，其面积就是循环一周所交换的能量。
- **Dynamic hysteresis**
- 对不同应用，这种能量损失有利有弊。



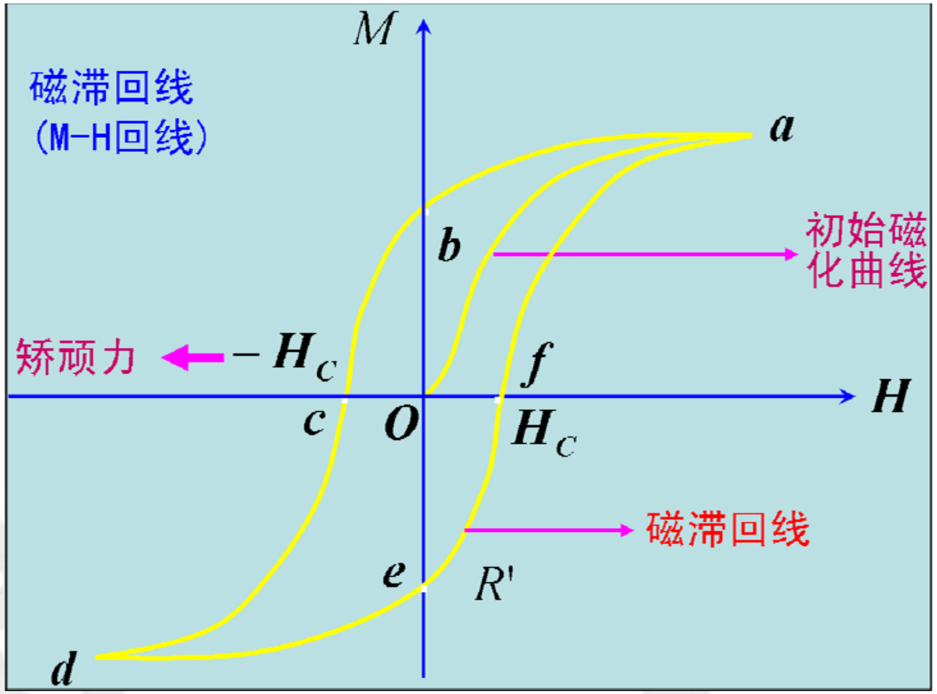
回线动力学的一些现象：

$$\frac{\partial \Phi_\alpha}{\partial t} = -\Gamma \frac{\delta F}{\delta \Phi_\alpha} + \varphi_\alpha$$

$$\beta F = \int d^3x \left[\frac{1}{2} J \left(\nabla \Phi_\alpha \cdot \nabla \Phi_\alpha \right) + \frac{r}{2} \left(\Phi_\alpha \Phi_\alpha \right) + \frac{u}{4N} \left(\Phi_\alpha \Phi_\alpha \right)^2 - \sqrt{N} H_\alpha \Phi_\alpha \right]$$



- 回线动力学的一些现象:



Heisenburg model

$$H = -\sum_{<i,j>} \vec{S}_i \vec{S}_j - h\sum_i \vec{S}_i$$

Potts model

$$H = -\sum_{<i,j>} S_i^n S_j^n - h\sum_i S_i^n$$

where n is the number of chosen value of S, $2<n<\infty$

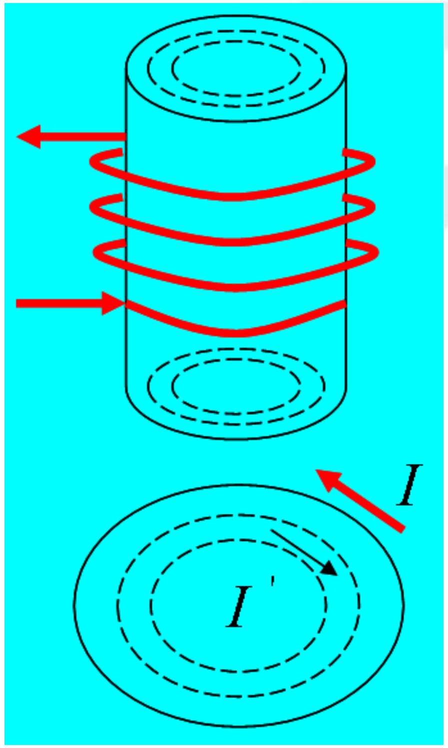
Ising model

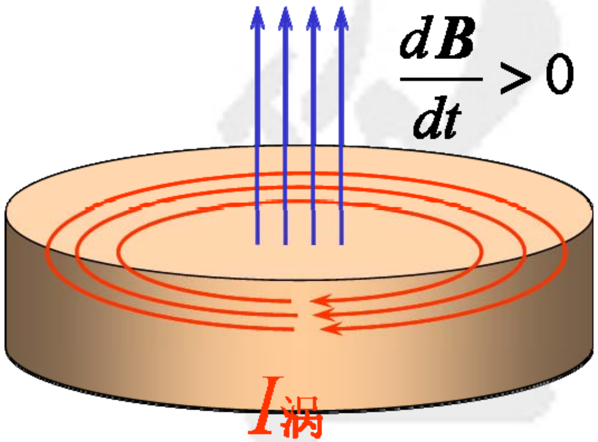
$$H = -\sum_{<i,j>} S_i S_j - h\sum_i S_i$$

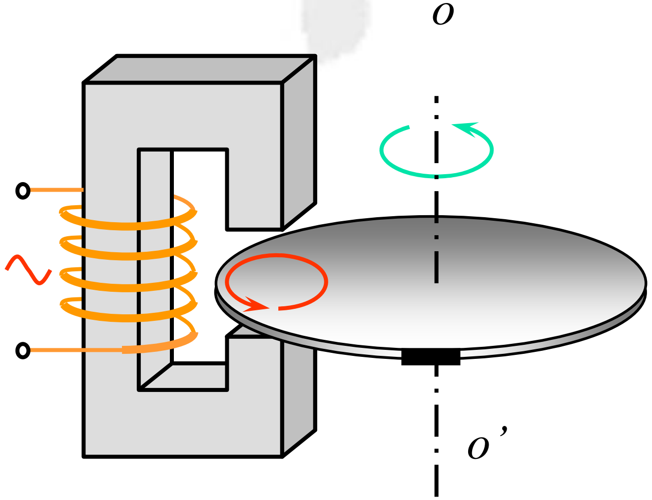
where S_i, S_j=+1, -1

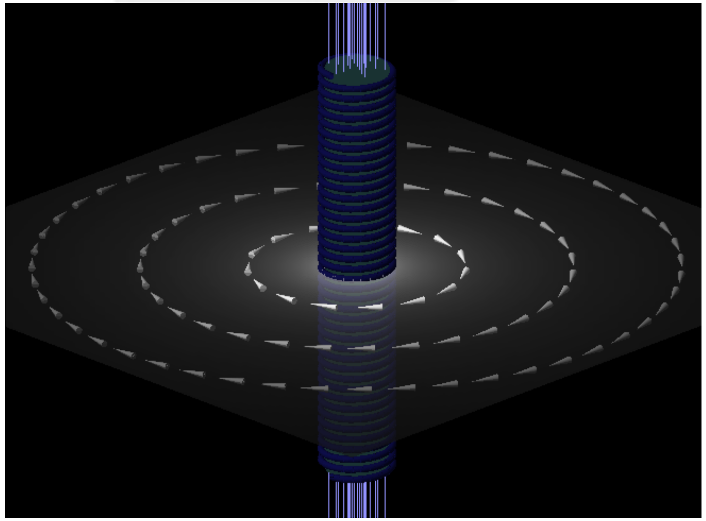
电磁学08-07: 涡电流(vortex flow, eddy current)

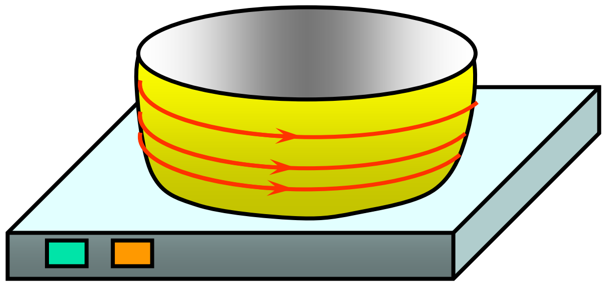
- 块状金属置于变化着的磁场中或在磁场中运动体内也产生感应电流，称涡电流或傅科电流。
- 涡电流损耗；叠片式铁芯；粉末状的铁芯；高频感应冶金炉；电动阻尼器等。







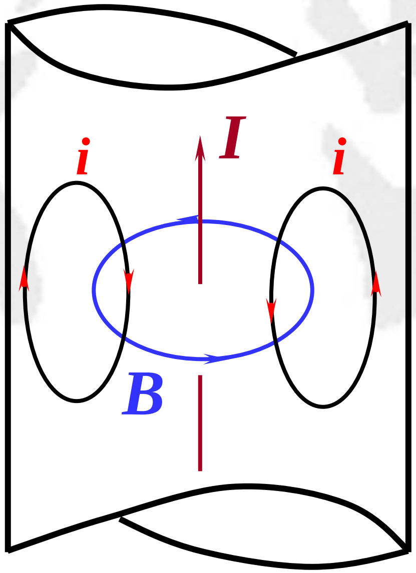


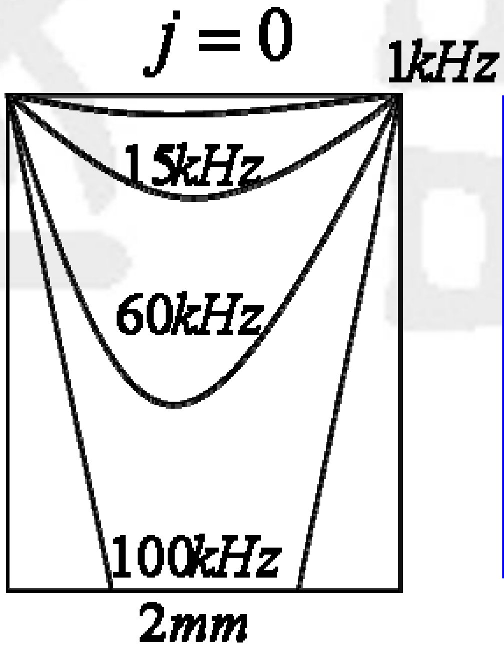


Steady State Magnetic Levitation

- 交流电通过导体时，导体本身涡流导致等效交变电流趋向于导体外表面。
- 趋肤电流与交流电频率相关。
- 应用与危害。

越是在外侧，磁场强度越大，引起的感应涡流也越大，导致更大趋肤效应。





频率越高，磁场强度越大，引起的感应涡流也越大，导致更大趋肤效应。

- **Skin effect**

- 基本完成静电学和静磁学的内容，静磁学中稍微涉及涡流和交变磁场。
- 归纳总结基本关系式：

$$\oint_L \vec{E}\cdot d\vec{l} = 0$$

$$\oint_S \vec{D}\cdot d\vec{S} = \int_V \rho_0 d\tau$$

$$\oint_L \vec{H}\cdot d\vec{l} = \iint_S \vec{j}_0 \cdot d\vec{S}$$

$$\oint_S \vec{B}\cdot d\vec{S} = 0$$

静电场：环路定理、高斯定理

静磁场：安培环路、高斯定理

$$\oint_L \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

法拉第电磁感应定律、楞次定律

- 存在的问题：从静态走向动态？

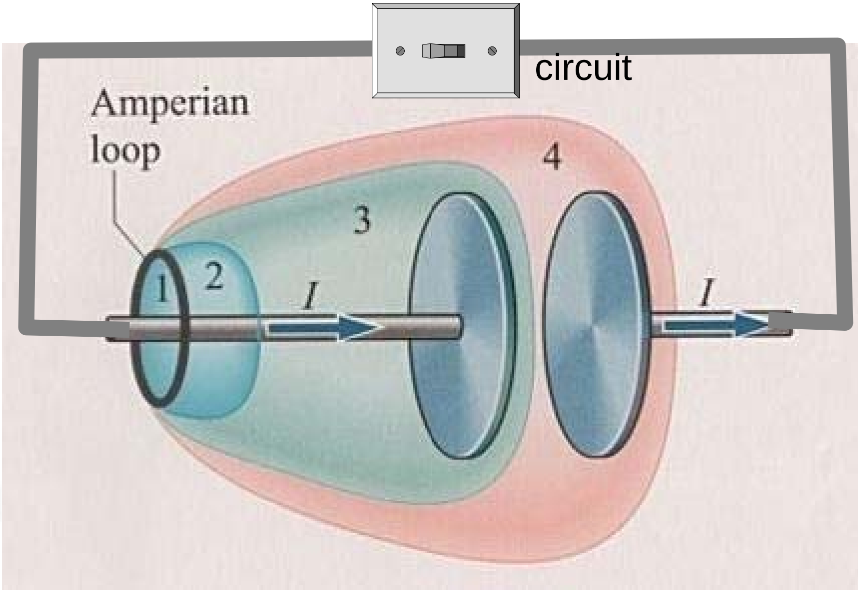
$$\left.\begin{aligned}&\oint_L \vec{E}\cdot d\vec{l}=0\\&\oint_S \vec{D}\cdot d\vec{S}=\int_V \rho_0 d\tau\end{aligned}\right\}\Rightarrow\begin{cases}\vec{D}(t),\ \vec{E}(t),\ \vec{P}(t),\ \varepsilon(t)\text{????}\\ \rho_0(t)\text{????}\end{cases}$$

- 在经典电磁学框架下，只是进行推广，不存在矛盾

$$\left.\begin{aligned}&\oint_L \vec{H}\cdot d\vec{l}=\iint_S \vec{j}_0\cdot d\vec{S}\\&\oint_S \vec{B}\cdot d\vec{S}=0\end{aligned}\right\}\Rightarrow\begin{cases}\vec{H}(t),\ \vec{B}(t),\ \vec{M}(t),\ \mu(t)\text{????}\\ \vec{j}_0(t)\text{????}\end{cases}$$

- 在经典电磁学框架下，可以进行推广，但存在矛盾

- 问题(1)：非恒稳电流 *I(t)* 下的安培环路定理



$$\oint_{L(\text{Amperian loop})} \vec{H}\cdot d\vec{l}$$

$$= \iint_{S1} \vec{j}_0 \cdot d\vec{S} = \iint_{S2} \vec{j}_0 \cdot d\vec{S} \neq I$$

$$\neq \iint_{S3} \vec{j}_0 \cdot d\vec{S} = 0 \neq \iint_{S4} \vec{j}_0 \cdot d\vec{S} = I$$

在静磁学中，上式满足的条件是恒温闭合电流I，而不是中间插入一个电容器。因此，这里的类比未必十分严格。

注意：因为平板电容充放电，S3包围的体积内部存在自由电荷 $\rho_0(t)$，

静电场：环路定理、高斯定理

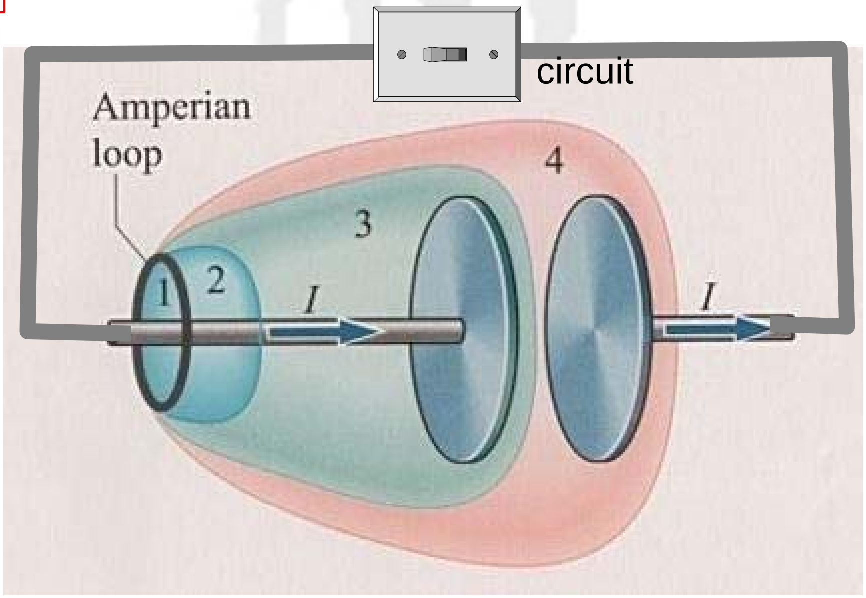
- 在经典电磁学框架下，无论电流是否恒稳，电荷守恒是满足的

$$\oint_S \vec{j}_0(t)\cdot d\vec{S} = -\frac{\partial}{\partial t}\iiint_V \rho_0(t)d\tau \Rightarrow$$

$$\oint_S \vec{j}_0(t)\cdot d\vec{S} + \frac{\partial}{\partial t}\iiint_V \rho_0(t)d\tau = 0$$

注意：我们偷偷将S面积分换成闭合面积分了。有没有问题？

电容器问题就不可能是恒温电流，我们称之为似稳电流。



引入高斯定理(非恒稳情况下也成立):

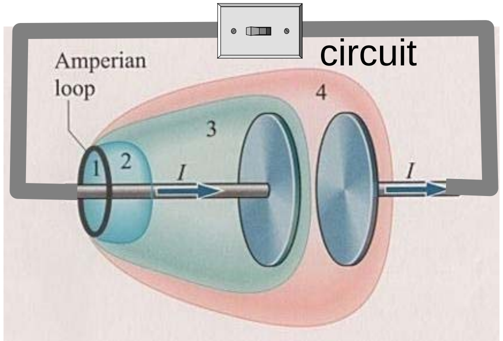
$$\left.\begin{aligned}&\oint_S \vec{j}_0(t)\cdot d\vec{S}+\frac{\partial}{\partial t}\iiint_V \rho_0(t)d\tau=0\\&\oint_S \vec{D}\cdot d\vec{S}=\iiint_V \rho_0(t)d\tau\end{aligned}\right\}\Rightarrow \oint_S \vec{j}_0(t)\cdot d\vec{S}+\frac{\partial}{\partial t}\oint_S \vec{D}\cdot d\vec{S}=0$$

$$\oint_S \vec{j}_0(t)\cdot d\vec{S}+\oint_S\left(\frac{\partial \vec{D}}{\partial t}\cdot d\vec{S}+\vec{D}\cdot\partial\left(\frac{d\vec{S}}{dt}\right)\right)=0\xrightarrow{\frac{d\vec{S}}{dt}=0}$$

$$\oint_S\left(\vec{j}_0(t)+\frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{S}=0\Rightarrow \vec{j}_t(t)=\vec{j}_0(t)+\vec{j}_D(t)=\vec{j}_0(t)+\frac{\partial \vec{D}}{\partial t}$$

安培环路定理:

$$\boxed{\oint_L \vec{H}\cdot d\vec{l}=\int_S \vec{j}_t\cdot d\vec{S}=\int_S\left(\vec{j}_0(t)+\frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{S}}$$



回答问题(1):

$$\because \oint_S \left(\vec{j}_0(t) + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S} = 0 \xrightarrow{\text{For this special case}} \frac{\partial \vec{D}}{\partial t} = \vec{j}_0$$

$$\therefore \oint_L \vec{H} \cdot d\vec{l} = \int_{S1} \vec{j}_0 \cdot d\vec{S} = \int_{S2} \vec{j}_0 \cdot d\vec{S} = \int_{S3} \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} = \int_{S4} \vec{j}_0 \cdot d\vec{S} = I$$

问题(2)：变化磁场产生涡电流，变化电场产生磁场？

$$\oint_L \vec{H} \cdot d\vec{l} = \int_S \left(\vec{j}_0(t) + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S} \xrightarrow{\vec{j}_0 = 0} \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$$

$$\oint_L \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

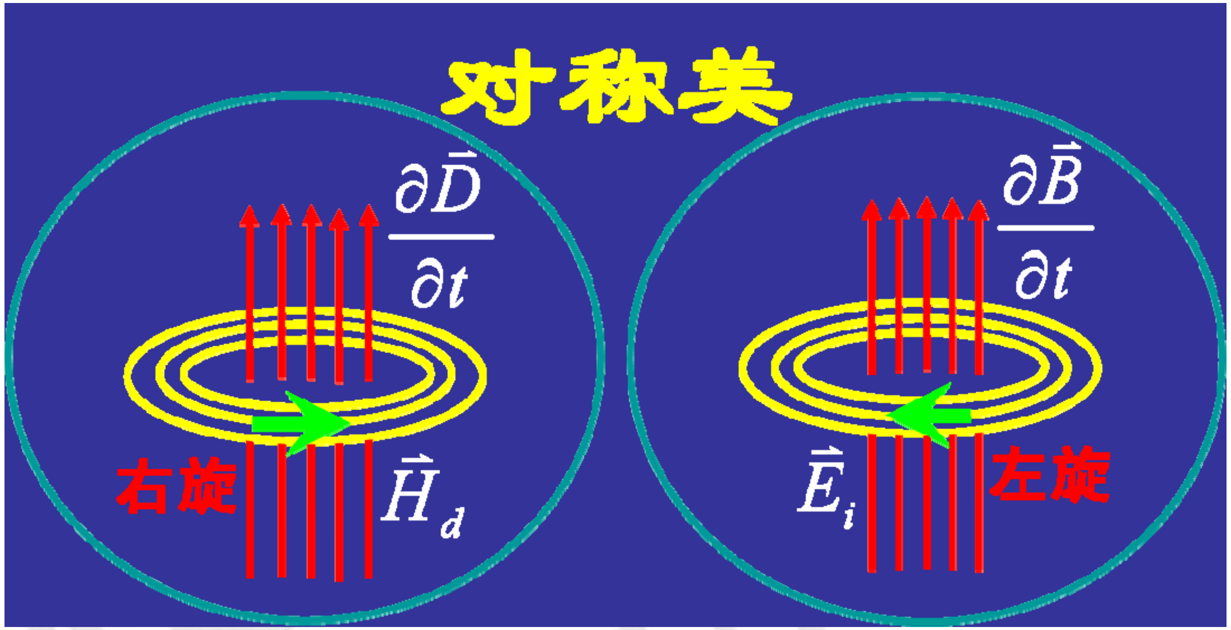
极化电流：

$$\because \vec{D} = \varepsilon_0 \vec{E} + \vec{P}, \quad \therefore \frac{\partial \vec{D}}{\partial t} = \frac{\partial(\varepsilon_0 \vec{E})}{\partial t} + \frac{\partial \vec{P}}{\partial t}$$

$$\therefore \oint_L \vec{H} \cdot d\vec{l} = \oint_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} = \oint_S \frac{\partial(\varepsilon_0 \vec{E})}{\partial t} \cdot d\vec{S} + \oint_S \frac{\partial \vec{P}}{\partial t} \cdot d\vec{S}$$

真空中的
位移电流密度

介质中的
极化电流密度

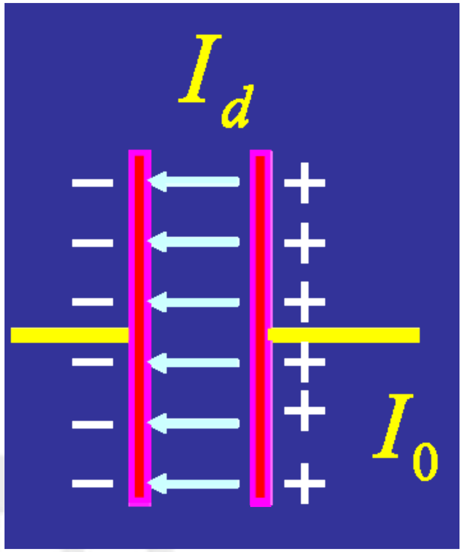


- 欣赏一下：

传导电流	位移电流
电荷的定向移动	电场的变化
通过电流产生焦耳热	真空中无热效应、介质有热效应

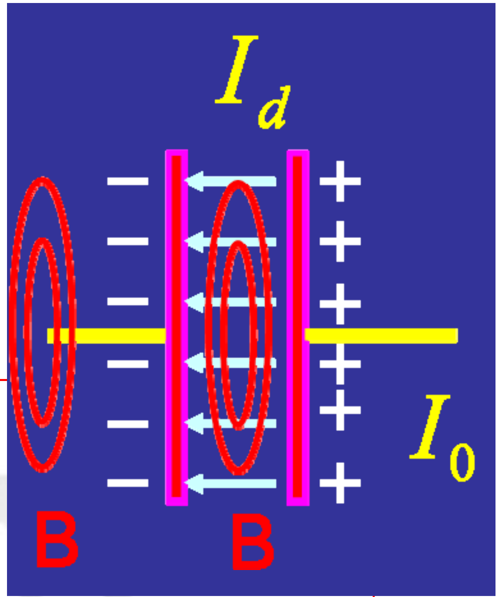
传导电流和位移电流在激发磁场上是等效

- 【例1 p.336】真空平行板电容器充放电，求位移电流



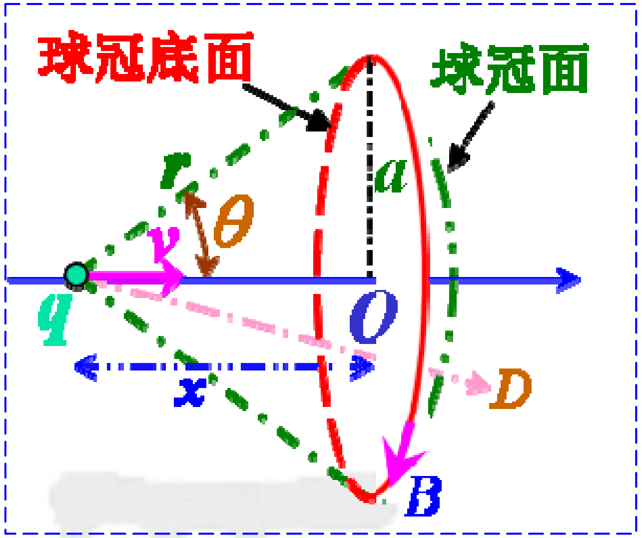
$$\left.\begin{array}{l} I_0 = \dfrac{dQ}{dt} \\[2ex] E = \dfrac{Q}{\varepsilon_0 S} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} j_d = \dfrac{dD}{dt} = \dfrac{\partial}{\partial t}\left(\dfrac{\varepsilon_0 Q}{\varepsilon_0 S}\right) = \dfrac{1}{S}\dfrac{dQ}{dt} \\[2ex] I_d = j_d S = \dfrac{dQ}{dt} = I_0 \end{array}\right.$$

- 【例2 p.336】真空平行板电容器充放电，求电容器内磁场



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \frac{\partial \vec{D}}{\partial t}\cdot d\vec{S} = \mu_0\varepsilon_0 \int \frac{d\vec{E}}{dt}\cdot d\vec{S}$$

$$\begin{cases} \text{at } r<R,\ 2\pi r B = \mu_0\varepsilon_0 \dfrac{dE}{dt}\pi r^2 \Rightarrow \\ \qquad B = \dfrac{1}{2}\mu_0\varepsilon_0 r\dfrac{dE}{dt} = \dfrac{1}{2}\mu_0\varepsilon_0 r\dfrac{1}{\varepsilon_0 \pi R^2}\dfrac{dQ}{dt} = \dfrac{1}{2}\mu_0\dfrac{r}{\pi R^2}I_0 \\ \text{at } r>R,\ B = \dfrac{\mu_0 I_0}{2\pi r} \end{cases}$$



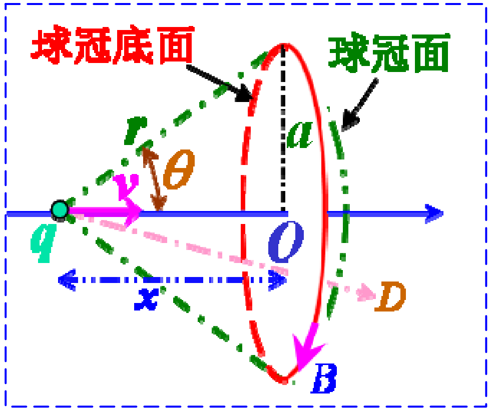
- 【例6】电荷 q 以速度 v 向 O 点运动，q 到 O 点的距离以 x 表示。在 O 点处作一半径为 a 的圆(球冠底面)，圆面与 v 垂直。试求通过该圆面的位移电流和圆周上各点处的磁感应强度。

- 电荷运动使空间电通量变化，引发磁场：

$$\text{穿过圆面的电通量：}\Phi_D = \iint_{\text{底面}S} \vec{D}\cdot d\vec{S} = \iint_{\text{球冠面}S'} \vec{D}\cdot d\vec{S}'$$

$$\because v << c,\quad \therefore \Phi_D = \iint_{\text{球冠面}S'} \vec{D}\cdot d\vec{S}' = D\cdot S' = D\cdot 2\pi r(r - r\cos\theta)$$

$$\because D = \varepsilon_0 E = \varepsilon_0 \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2},\quad \therefore \Phi_D = \frac{q}{2}(1-\cos\theta) = \frac{q}{2}\left(1 - \frac{x}{\sqrt{a^2+x^2}}\right)$$



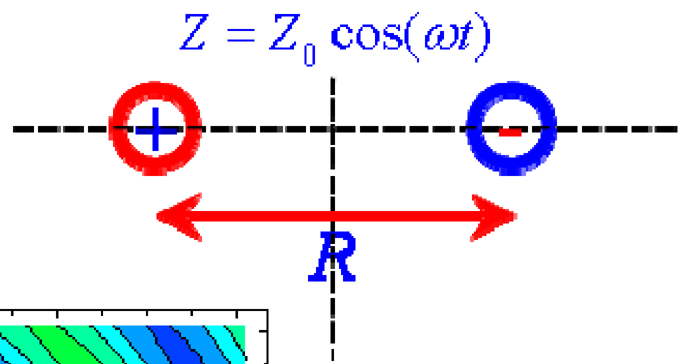
□ 继续:

$$\text{位移电流}: I_D = \frac{d\Phi_D}{dt} = -\frac{q}{2}\frac{a^2}{(a^2+x^2)^3}\frac{dx}{dt} = \frac{qa^2}{2r^3}v$$

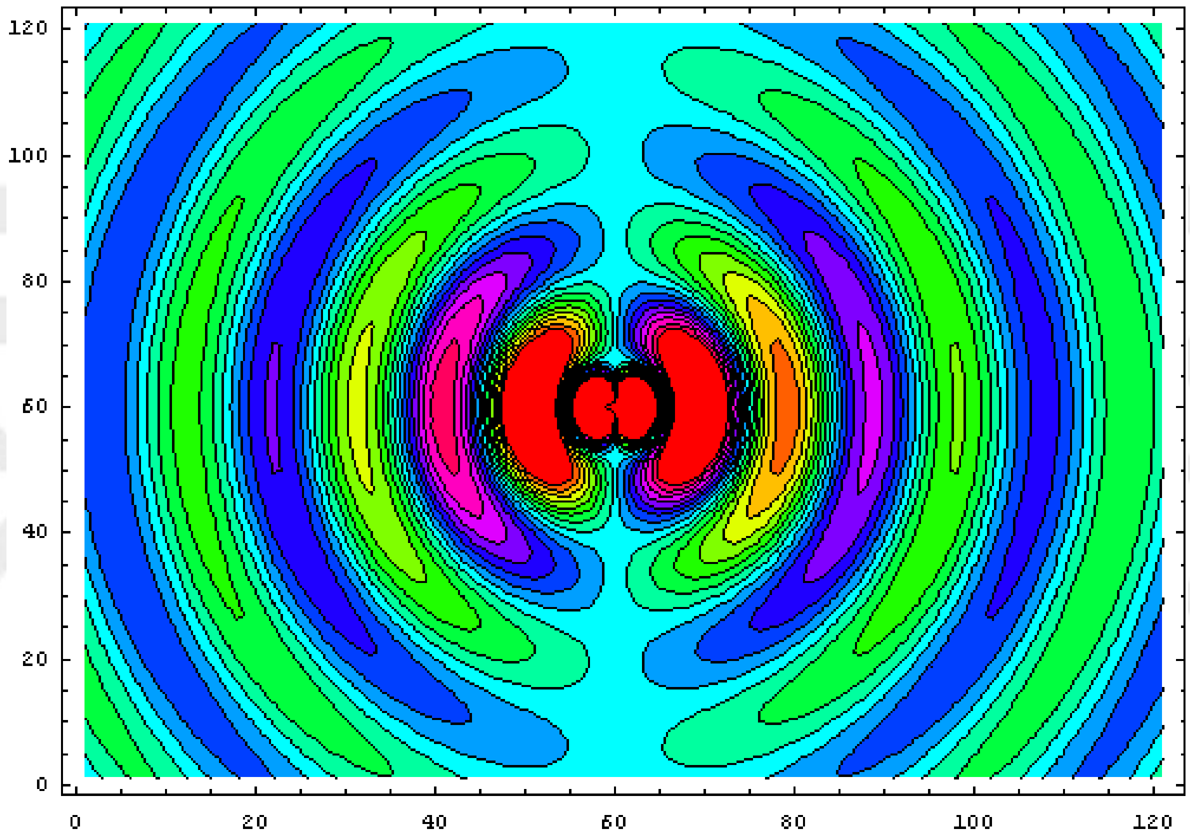
$$\text{安培回路定理}: \oint_L \vec{H}\cdot d\vec{l} = I_D = \frac{qa^2}{2r^3}v$$

$$\therefore 2\pi aH = \frac{qa^2}{2r^3}v \Rightarrow B = \frac{qa}{4\pi\mu_0 r^3}v$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$$

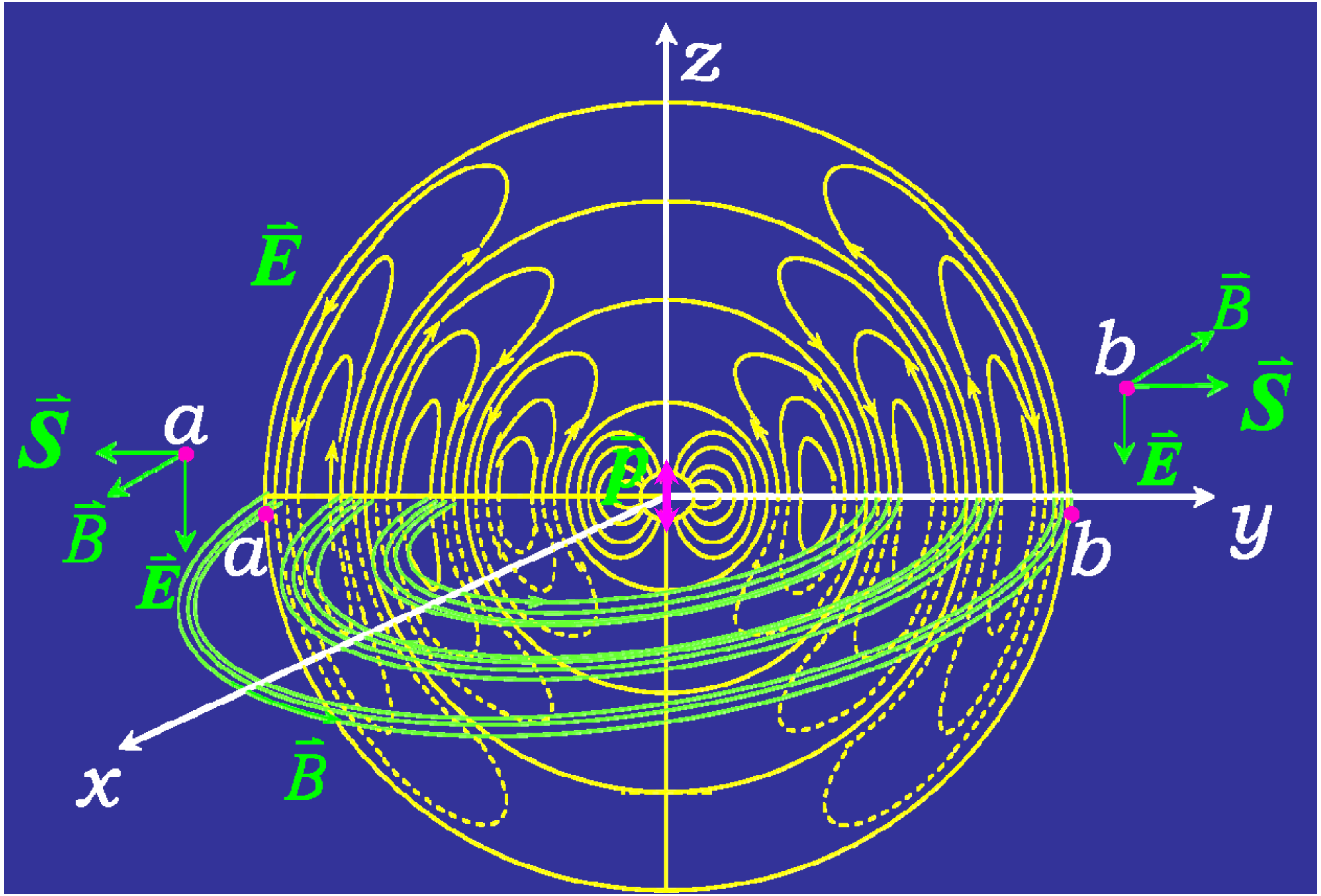


- 振荡电偶极子的电磁场



- 第九章将详细处理这个问题

电磁学08-10: 电磁感应的若干问题



- 本章习题:

P354: 8.1, 8.2
P355: 8.8, 8.9, 8.12
P356: 8.19, 8.25
P357: 8.26