大学电磁学 College Electromagnetics

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南京大学 物理学院 物理系

QQ群：128710788 / 南大物理电磁学课程

- 第一类参考书 (后续引用不再注明):

徐游：《电磁学》第二版，科学出版社

赵凯华、陈熙谋：《电磁学》，高等教育出版社

Benjamin Crowell, Electricity and Magnetism, Lightandmatter

Matthew N. O. Sadiku, Elements of Electromagnetics

Jearl Walker, Fundamentals of Physics, Halliday and Resnick

R. Feynman, ********* Sears University Physics**

郭奕玲-沈慧君-物理学史-图片

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- Google, Wiki, …….网络是第二类参考书、百度就暂时算了吧。
  - https://en.wikipedia.org/wiki/Main_Page
  - https://scholar.google.com/ (hosts更新)
  - http://ocw.mit.edu/courses/physics/8-02-physics-ii-electricity-and-magnetism-spring-2007/problem-solving/
  - http://faculty.ece.illinois.edu/rao/FEME/index.html
  - http://www.physics.irfu.se/CED/Book/
  - 。。。。。。。
- IPhone的APP是第三类参考书
  - Apple store

- 成绩：作业30%、期中35%、期末35%
- 三学时主讲课程、一学时提升课程
- 以自学为主、以讲授为辅、以复习概念为线索、以数学运用与拓展为重点
- 微元、微积分、矢量代数(补充讲授)
- 轻视高中那种解题技巧、重视概念的运用与运算能力
- **Are you ready? !**

- 学期课程讲授安排16~18周，每周均 4 学时。主讲课程：

  绪　论：1~2 学时

  第一章：真空中固定电荷，7-8 学时

  第二章：导体周围静电场，6 学时

  第三章：电介质，4 学时

  第四章：恒稳电流，4 学时

  第五章：真空中恒稳电流磁场，8 学时

  第六章：电磁场中电荷运动，4 学时

  第七章：磁介质，4 学时

  第八章：电磁感应，8 学时

  第九章：麦克斯韦方程，2 学时

- 学期课程讲授安排16~18周，每周均 4 学时。提升课程：

- 每两周一次提升课程，孙张吴三位老师指导
- 每次一个专题，围绕课程讲授内容，设计提升题目

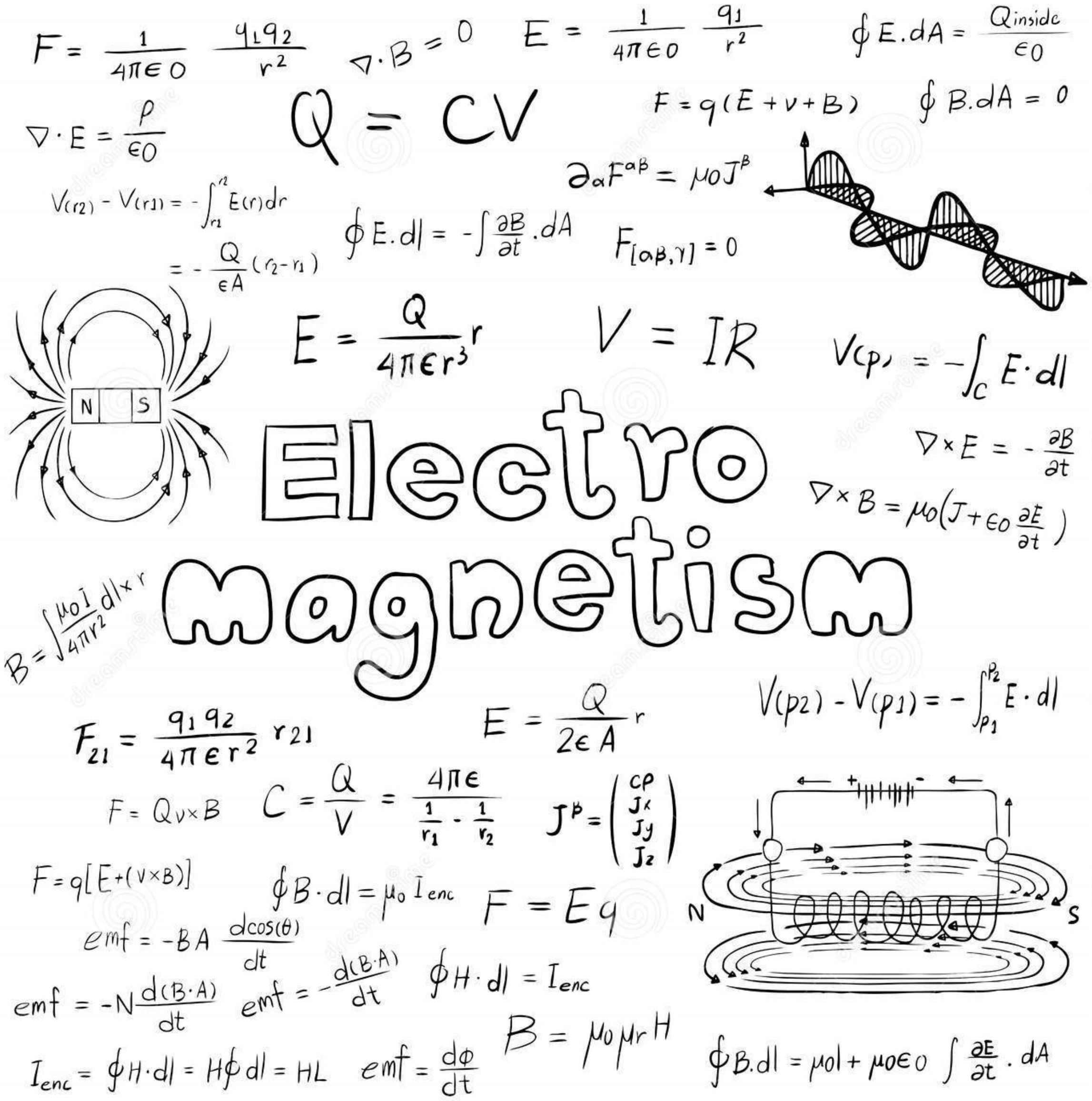
- 形式灵活、追求深度和生动
- 需要充分准备

- 其它时间机动，例如期中考试，节假日等

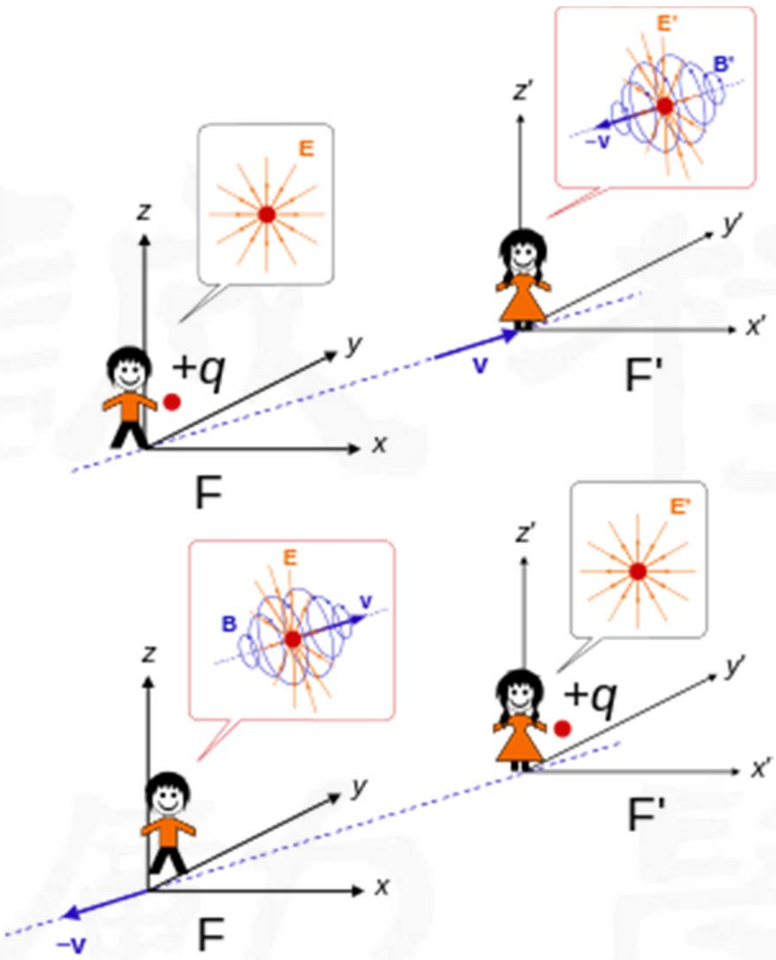
- 授课总计 56~60 学时
- 课外大约 200 学时自学、100 学时作业
- 基础好的学生、基础相对弱的学生
- 课件自成体系，不局限于特定教学参考书
- 课件绝大部分是针对基础相对不是那么雄厚的学生

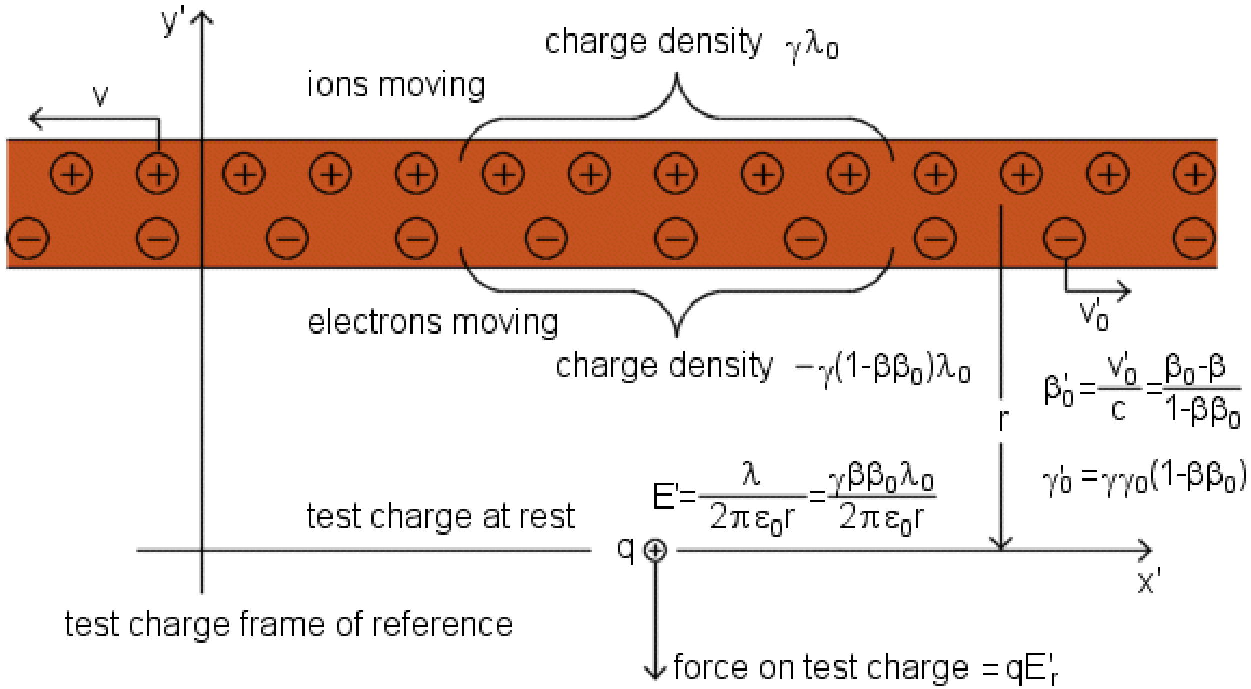
□ 电磁学在近代物理学中的意义：

1600 to 1900	Classical Physics	Mechanics
		Thermodynamics
		Electromagnetism
1900 to 1940	Modern Physics	Relativity Large speeds (≈10^8 m/s).
		Quantum Mechanics Very small scales (≈10^{-10} m).
1940 to present	Current Physics	Particle Physics
		Cosmology

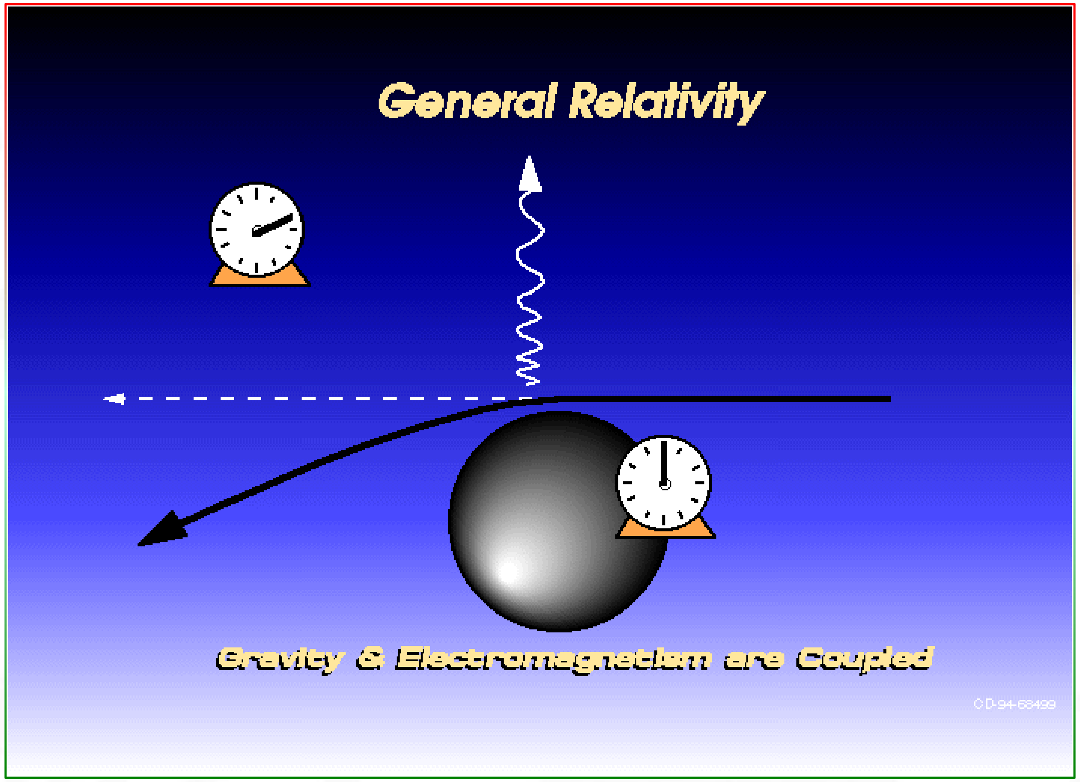


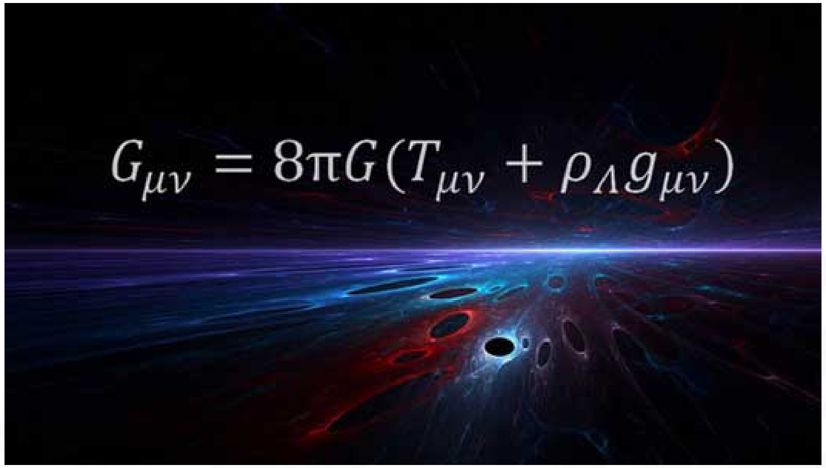
- 在层次上，电磁学与狭义相对论承接：





- 在层次上，电磁学与广义相对论、量子力学相关联：





Einstein's Field Equations

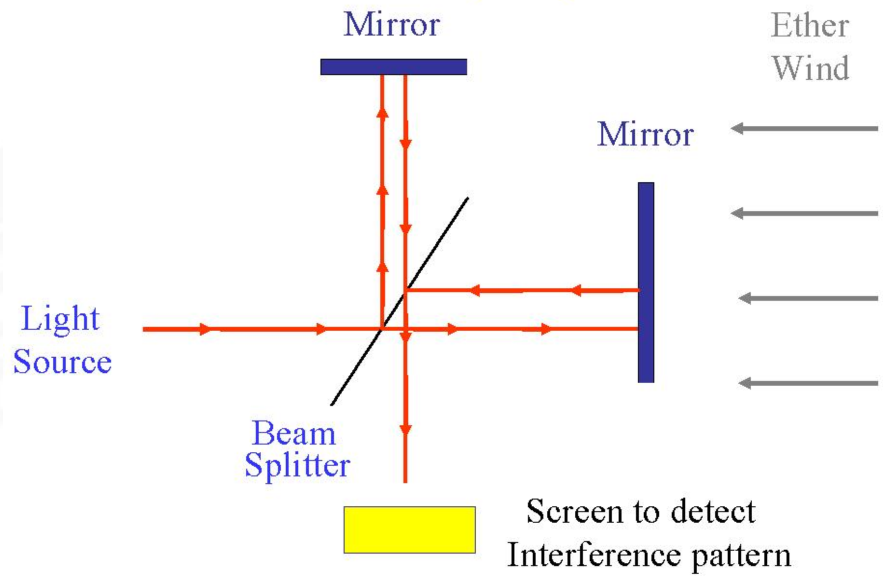
$$R_{\mu\nu} = \frac{\partial\Gamma^{\lambda}_{\mu\lambda}}{\partial x^{\nu}} - \frac{\partial\Gamma^{\lambda}_{\mu\nu}}{\partial x^{\lambda}} + \Gamma^{\beta}_{\mu\lambda}\Gamma^{\lambda}_{\nu\beta} - \Gamma^{\beta}_{\mu\nu}\Gamma^{\lambda}_{\beta\lambda} = 0$$

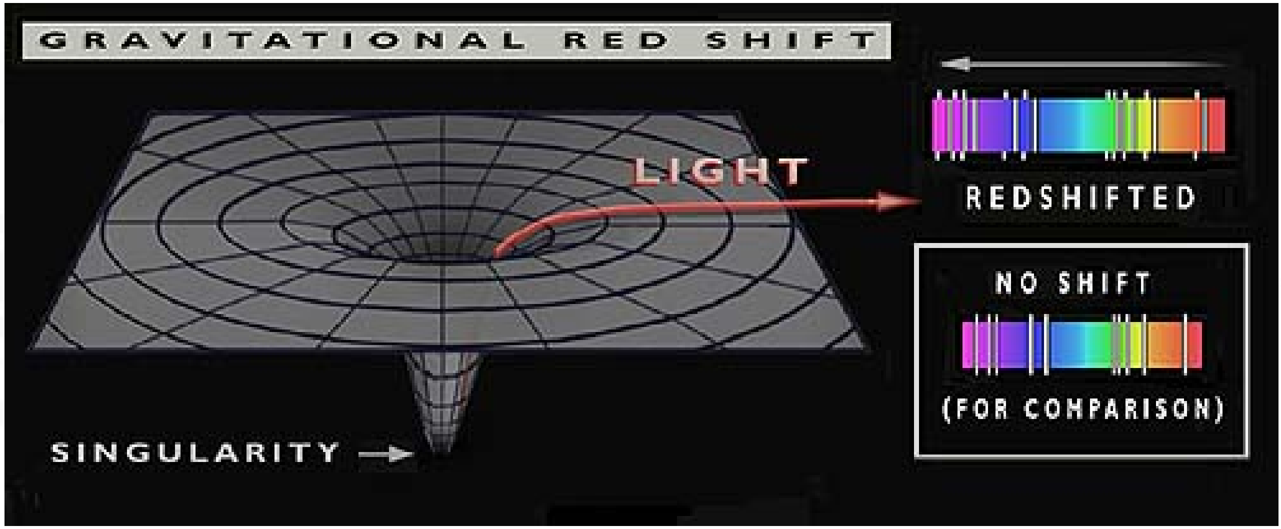
where

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\beta}\left(\frac{\partial g_{\mu\beta}}{\partial x^{\nu}} + \frac{\partial g_{\nu\beta}}{\partial x^{\mu}} + \frac{\partial g_{\mu\nu}}{\partial x^{\beta}}\right)$$

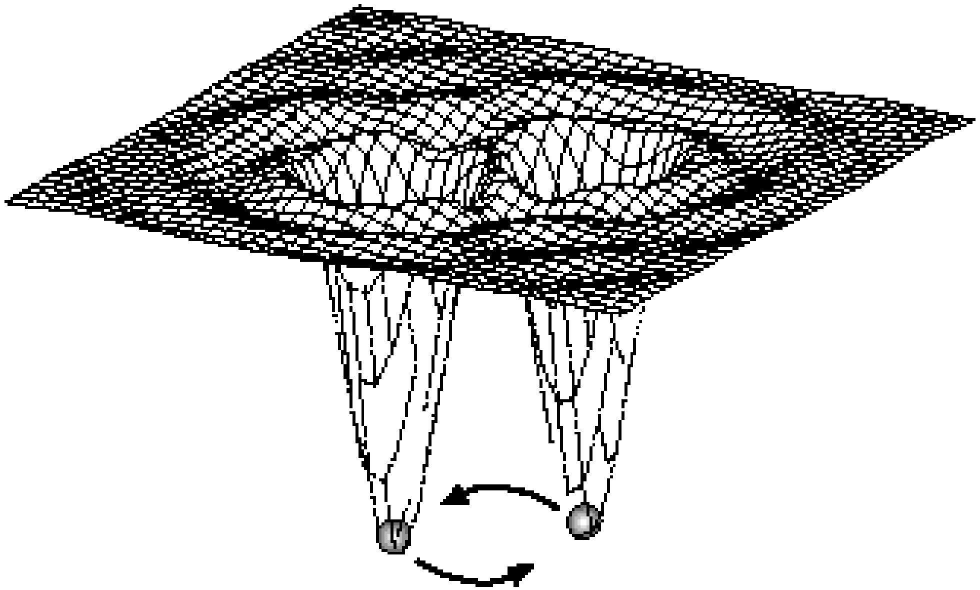
相对论的三个时刻：电磁学与光“视觉”

Michelson-Morley Experiment

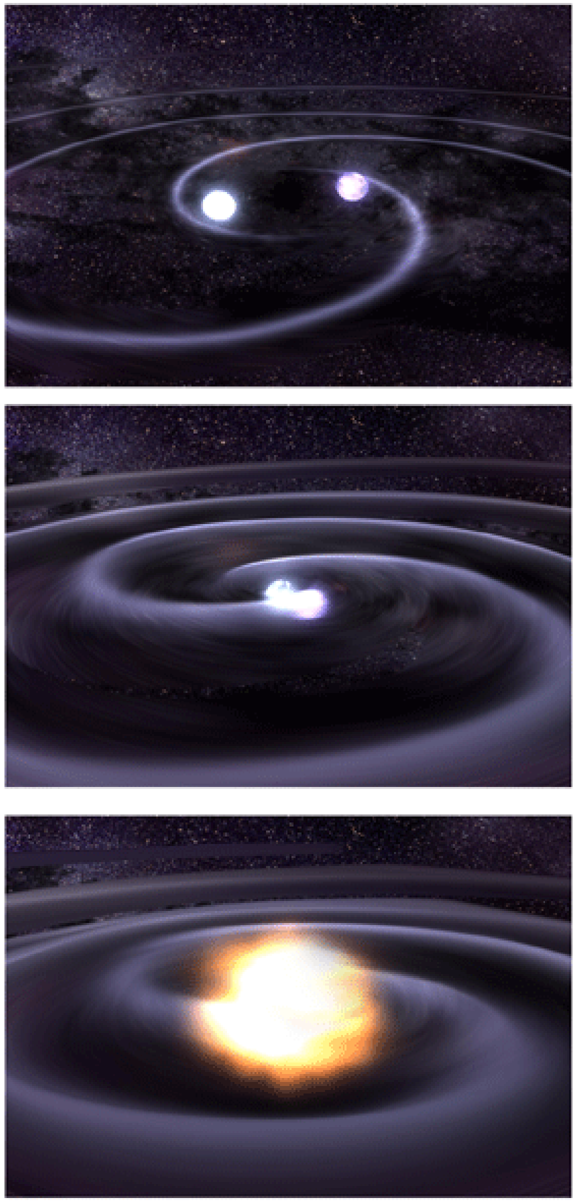




- 相对论的三个时刻：电磁学与引力波“听觉”

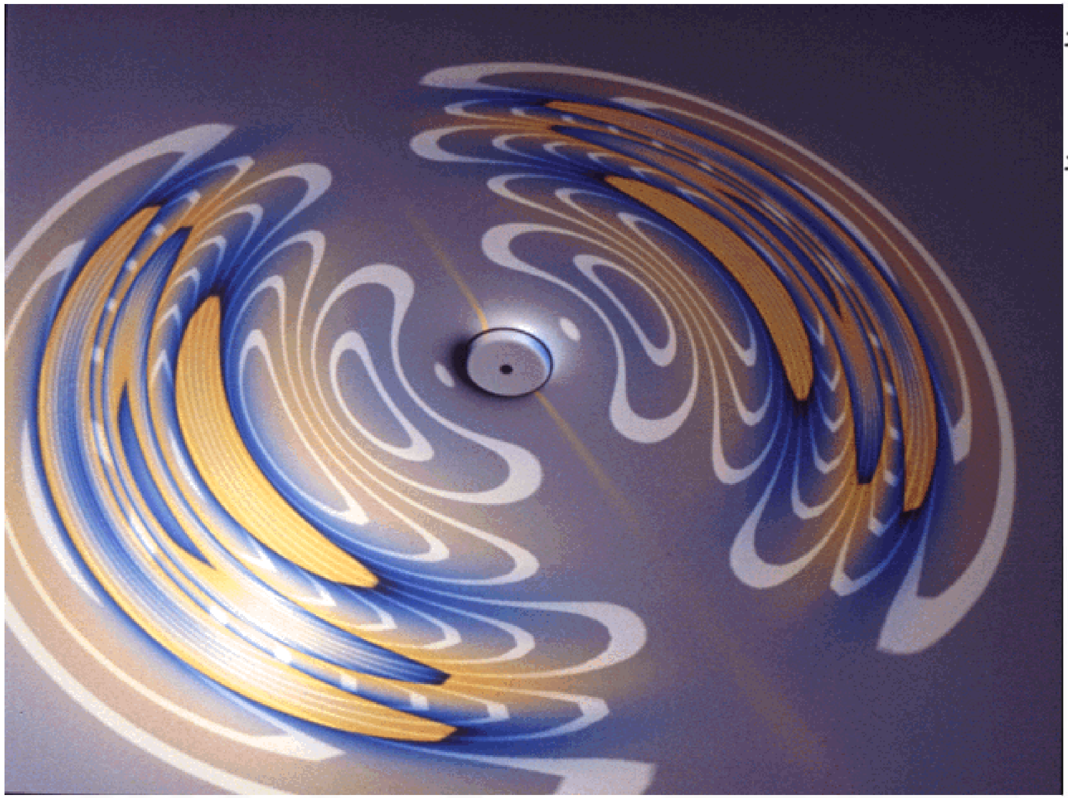


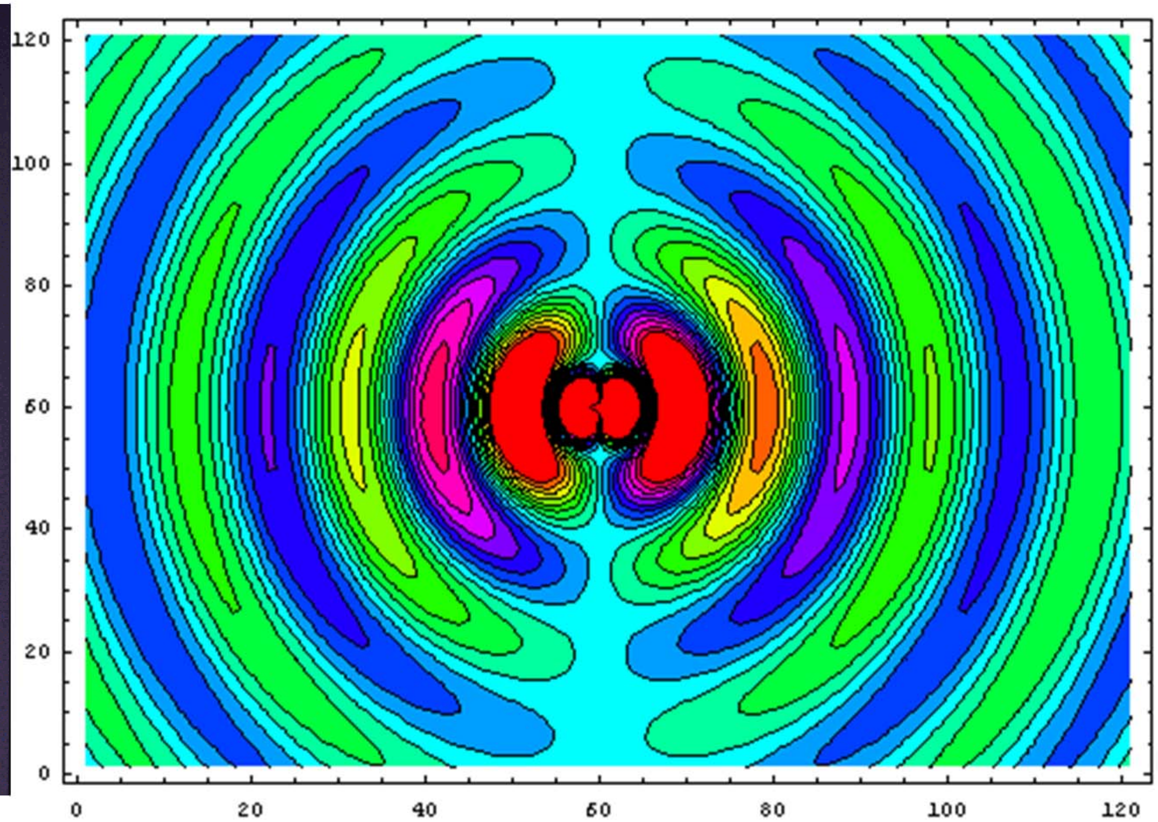
A binary system of compact massive objects rapidly orbiting each other produces ripples in spacetime.

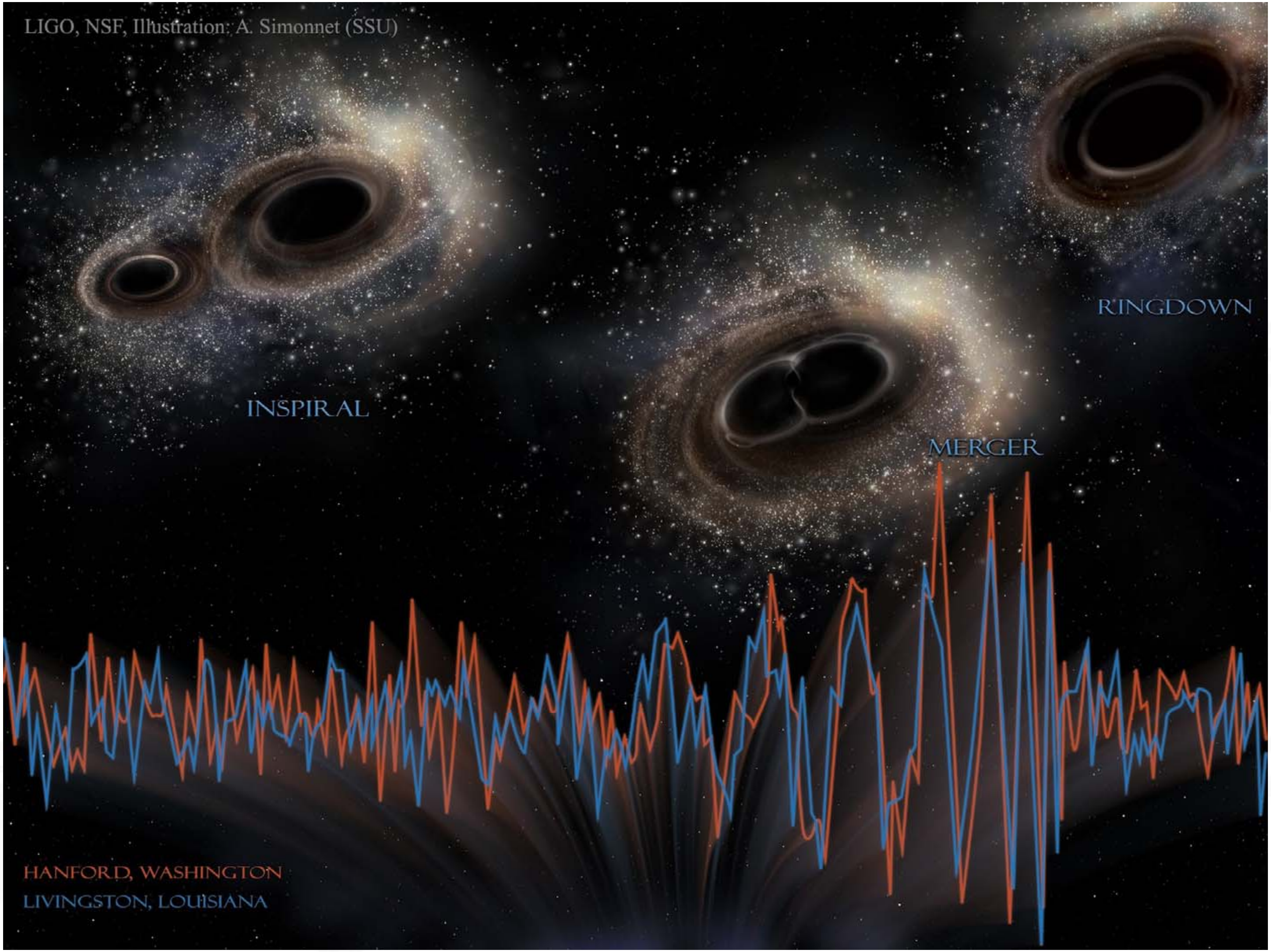


- 引力波模拟视频

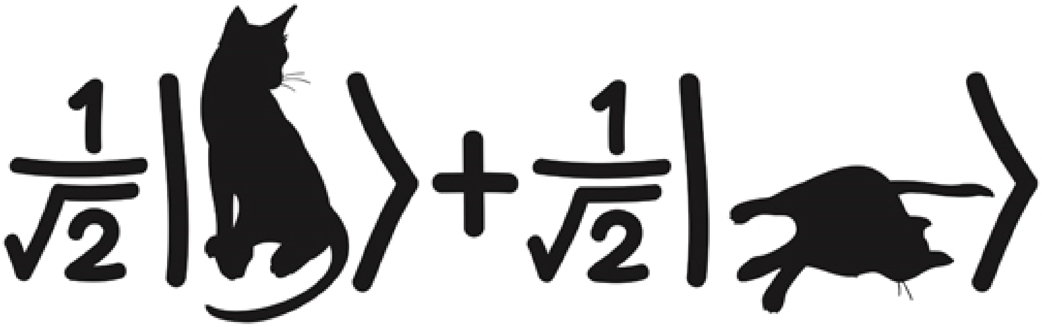
□ 引力波(gravitational wave)与电偶极子振荡产生的电磁波:

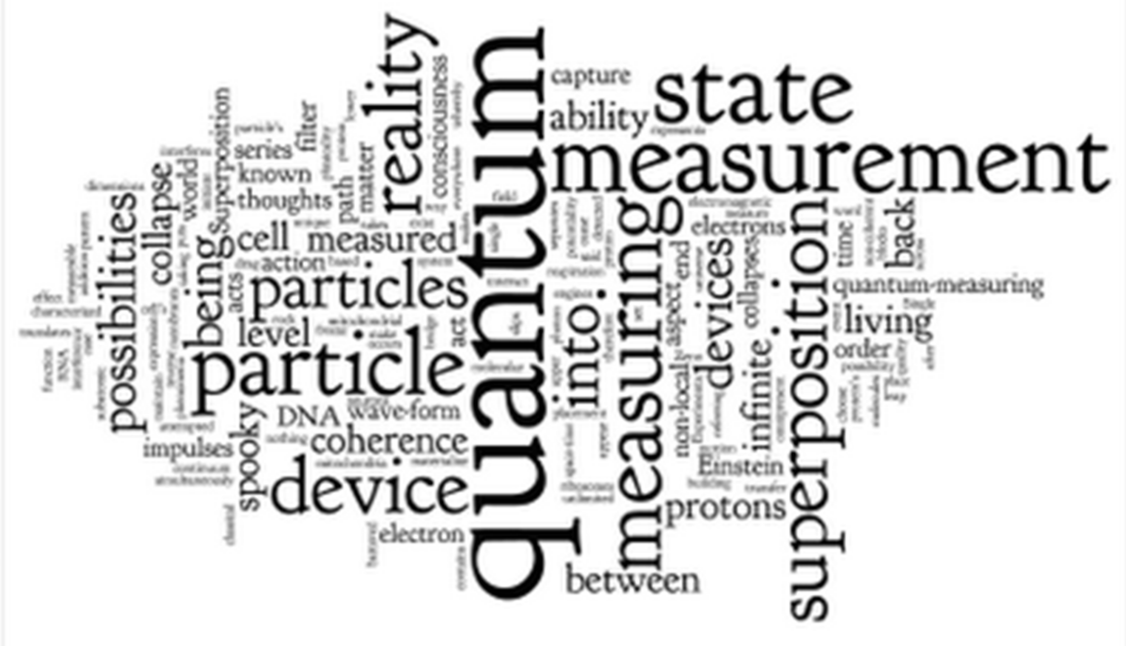


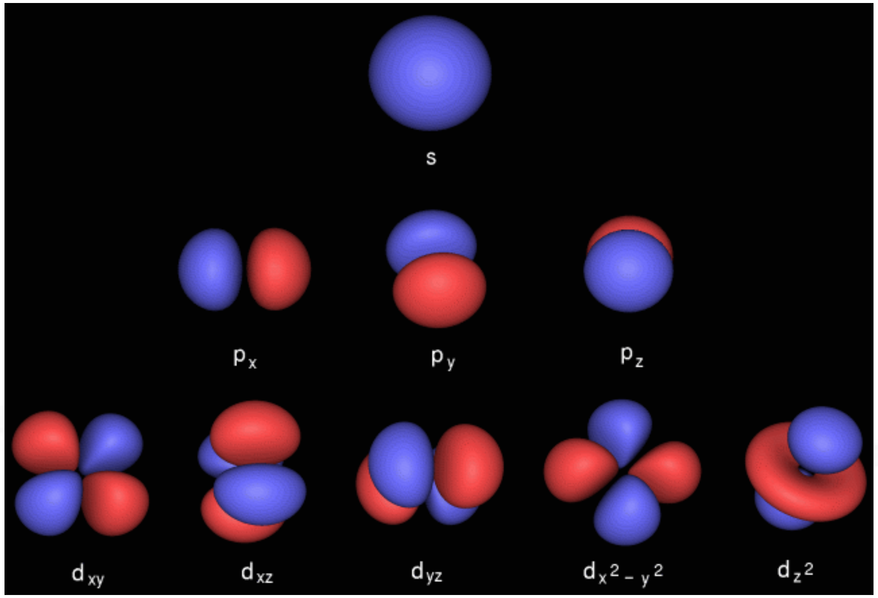


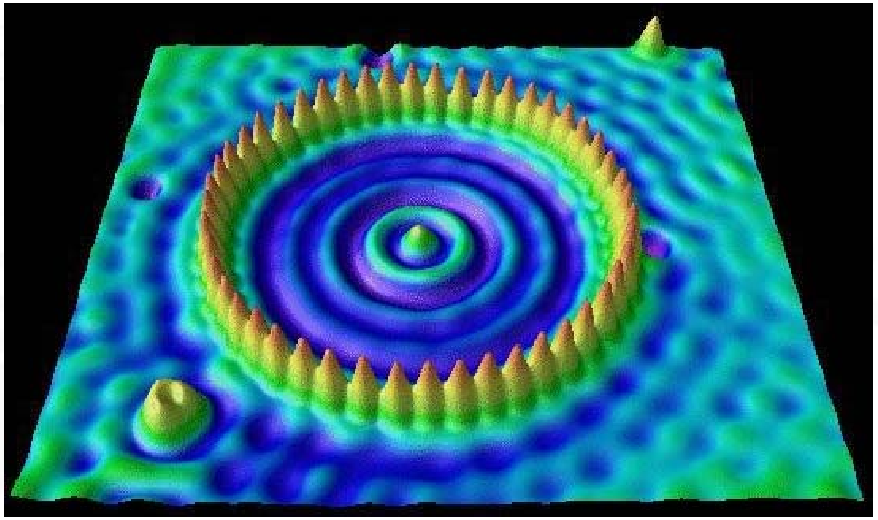

LIGO, NSF, Illustration: A. Simonnet (SSU)
INSPIRAL
MERGER
RINGDOWN
HANFORD, WASHINGTON
LIVINGSTON, LOUISIANA

量子力学重新描述电荷与自旋：



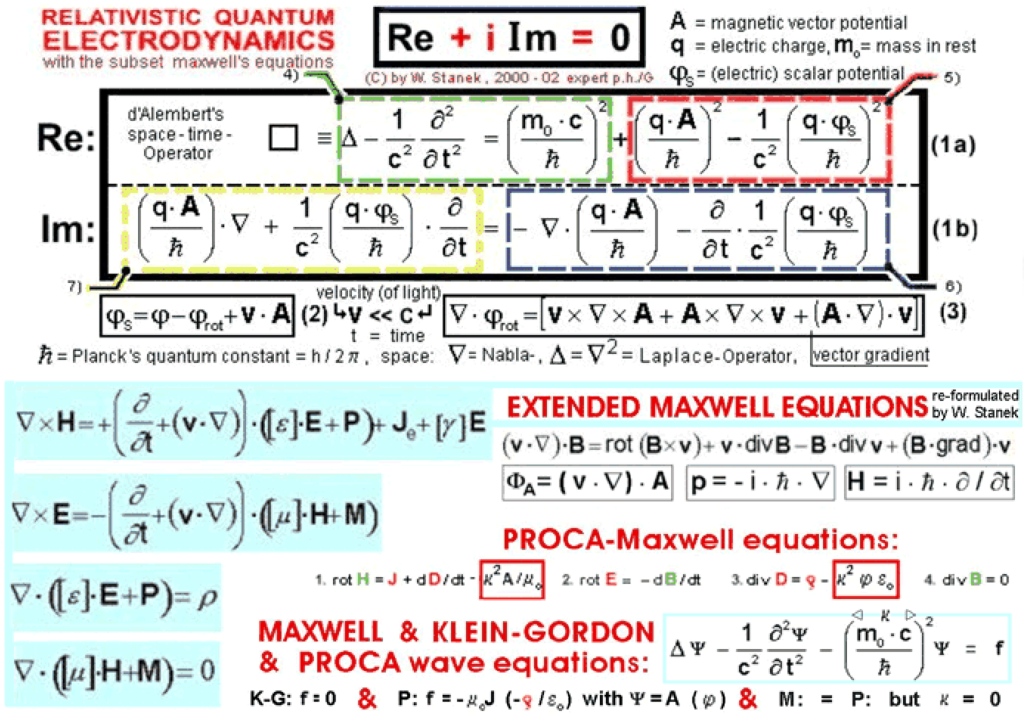






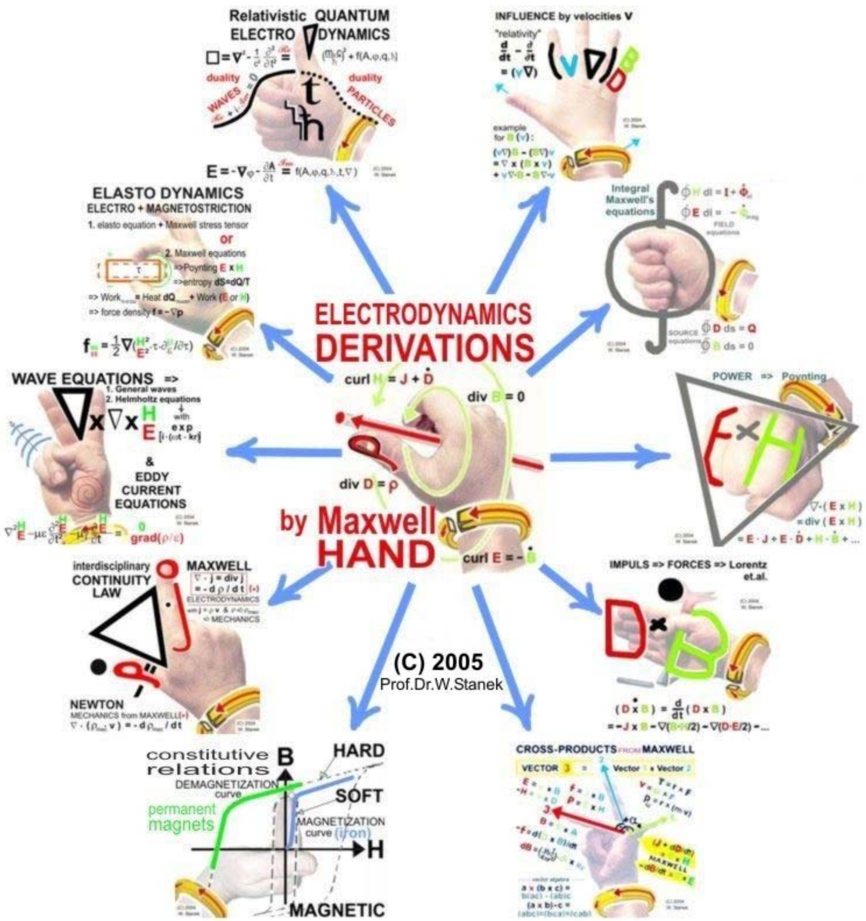
□ 电磁学与光子相互承接：

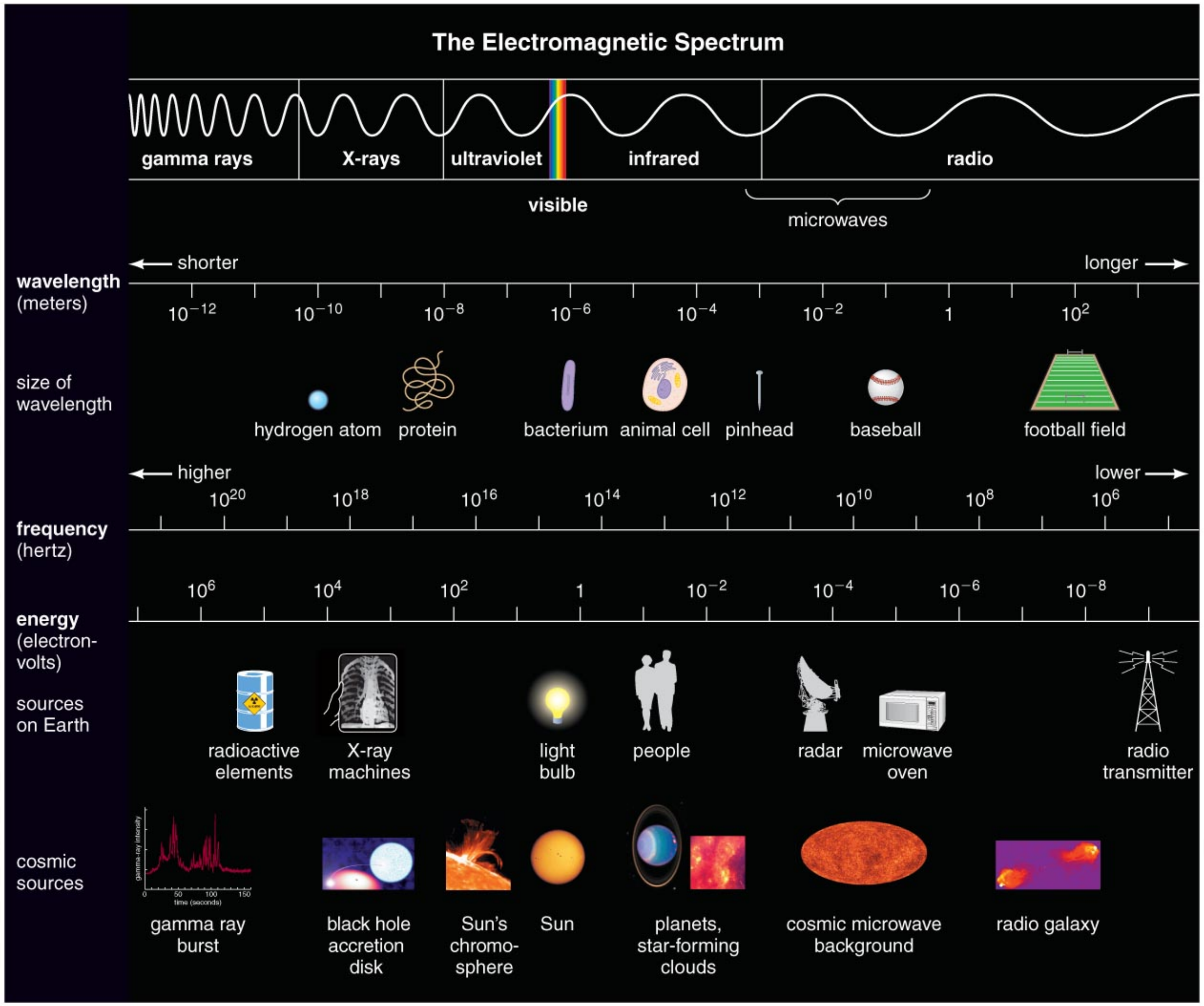

Important Equations in Electrodynamics
Maxwellgleichungen
Elektrodynamischen Potentiale
Elektrostatik
Multipolentwicklung
Magnetostatik
Vektorpotential
Wellengleichungen
Copyright: www.bigel-labs.de


RELATIVISTIC QUANTUM ELECTRODYNAMICS
with the subset maxwell's equations
Re + i Im = 0
(C) by W. Stanek, 2000 - 02 expert p.h./G
EXTENDED MAXWELL EQUATIONS
re-formulated by W. Stanek
PROCA-Maxwell equations:
MAXWELL & KLEIN-GORDON & PROCA wave equations:

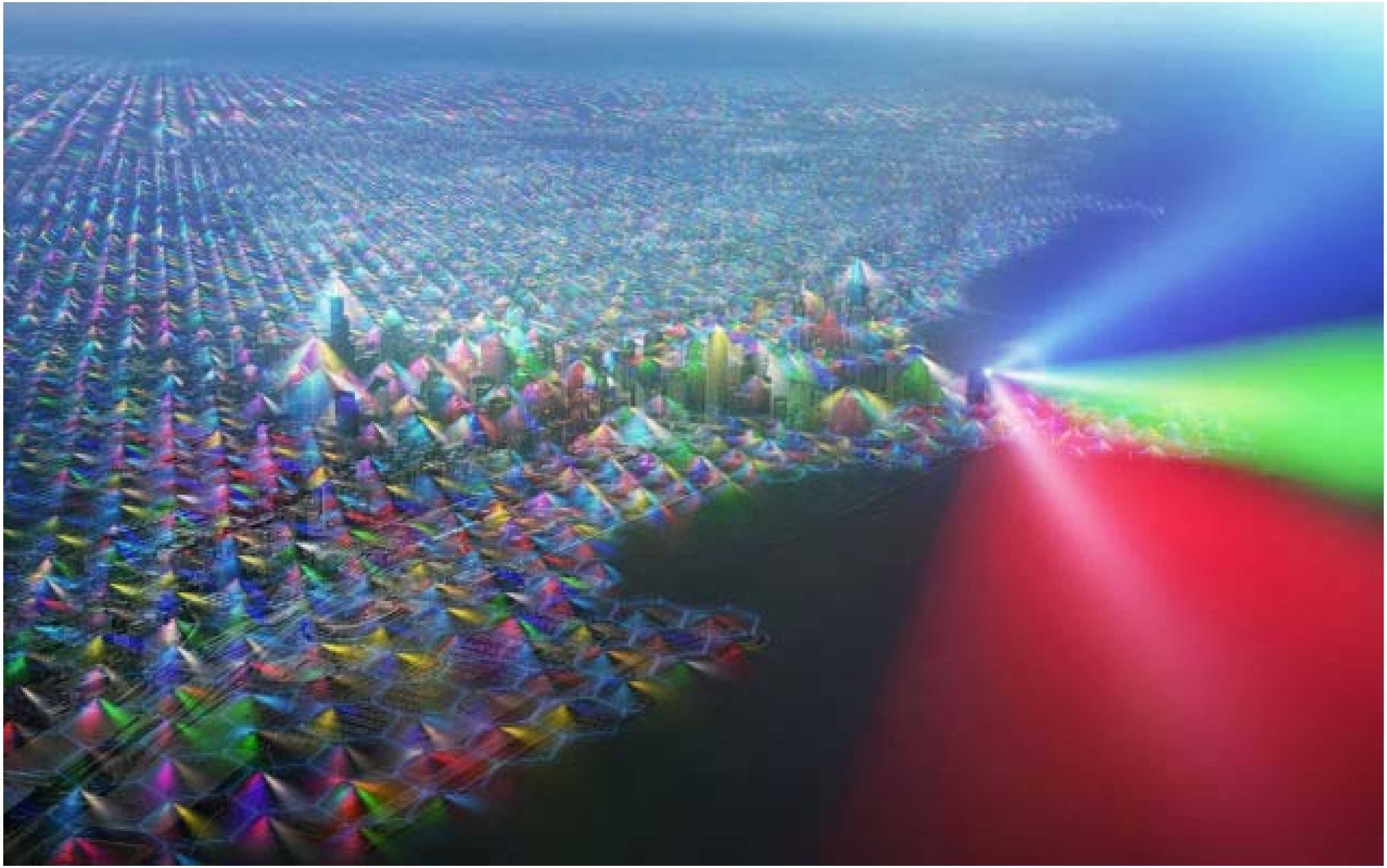
电磁学00-02: 电磁学的地位

- 在尺度上，相对论、电磁学、量子力学相互承接：



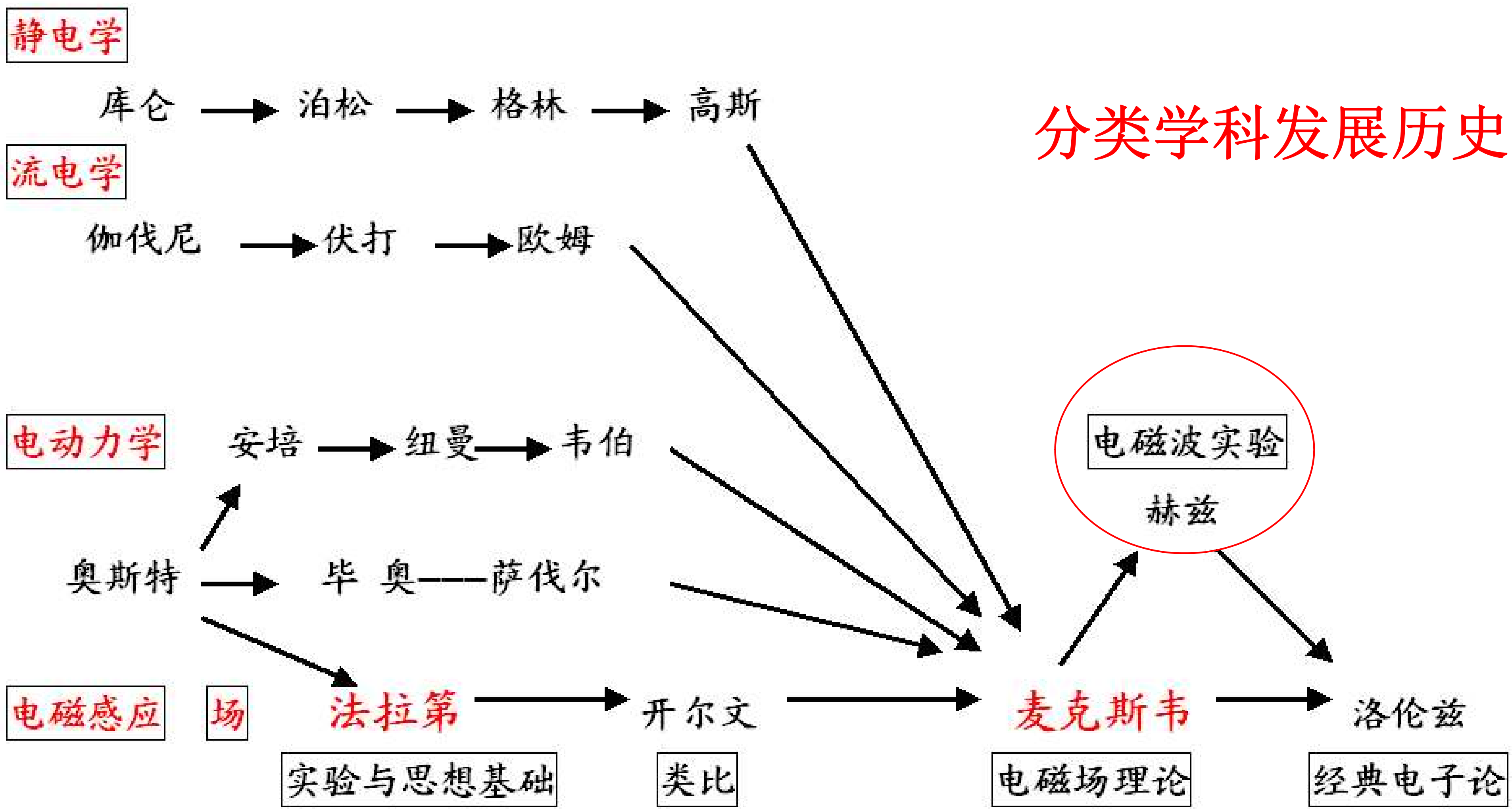


- 电磁波的 3D 打印：



- 电磁波的 3D 打印：

- 电磁学是一门实验学科，诞生与发展依赖于实验现象与分析。



南京大学 NANJING UNIVERSITY

公元前
- 我国春秋战国时代：磁石吸铁
- 约公元前600年希腊的Thales：琥珀摩擦后吸引草屑

17世纪
- 1600年英国的Gilbert：将电力和磁力分开
- 约1660年马德堡的Guericke：发明了第一台摩擦起电机

18世纪前期
- 1729年英国的Gray：发现导体和绝缘体的区别
- 法国的du Fay：发现电有两种；美国的Franklin：提出正电和负电的说法，风筝实验

18世纪后期
- 1785年法国的Coulomb：库仑定律
- 意大利的Volta：发明伏打电池

19世纪
- Oersted、Ampere、Biot、Savart等：电流的磁效应；电流磁效应的应用：电磁铁、电报、电话
- 欧姆定律、电磁感应、麦克斯韦方程；发电机、电动机……

20世纪
- 不计其数的理论发展和应用

学科发展年谱

- 古代春秋战国时期看到的磁石吸铁。(公元前770年~公元前221年)《管子•地数》载：“山上有慈石(即磁石)者，其下有铜金。”
- 司徒南：东汉时期思想家王充写的《论衡》书中“司南之杓，投之于地，其柢指南”的记载。
- 不要太相信中国人对电和磁有多少科学的理解。
- 公元前600年，希腊的**Thales**也有琥珀摩擦吸引草屑的记载。
- 电磁学真正的科学研究来自于英国**William Gilbert**(电磁学之父)对电和磁的实验。
- 吉伯为磁通势单位，用以纪念这位磁学的先驱者。

电磁学发展史参考资料

- 《电磁学发展史》作者: 宋德生 / 李国栋 出版社: 广西人民出版社
- http://history.hyperjeff.net/electromagnetism
- **Prof. S. Errede, A Brief History of The Development of Classical Electrodynamics**

电磁学00-03: 电磁学发展早期简史

□ 附加资料

Antiquity	Many things are known about optics: the rectilinearity of light rays; the law of reflection; transparency of materials; that rays passing obliquely from less dense to more dense medium are refracted toward the perpendicular of the interface; general laws for the relationship between the apparent location of an object in reflections and refractions; the existence of metal mirrors (glass mirrors being a 19th century invention).
5th cent BC	**Empedocles** (b. ca. 492 BC) speculates (based on reason) that the speed of light is finite.
ca 300 BC	Convex lenses in existence at Carthage. **Euclid of Alexandria** (ca. 325 BC – ca. 265 BC) writes, among many other works, *Optics* and *Catoptrica*, dealing with vision theory and perspective.
1st cent BC	Chinese fortune tellers begin using loadstone to construct their divining boards, eventually leading to the first compasses. (Mentioned in Wang Ch'ung's *Discourses weighed in the balance* of 83 B.C.)
1st cent	South-pointing divining boards become common in China.
ca 65	**Lucius Annaeus Seneca** (ca. 4 BC – 65 AD) writes *Naturales quaestiones*, collecting many natural discoveries, including a reference to the production of colors similar those of a rainbow by sunlight passing through glass prisms.
2nd cent	**Hero of Alexandria** (ca. 10 – ca. 75 AD) writes on the topics of mirrors and light, also showing that light rays take the shortest path available. **Claudius Ptolemy** (ca. 85 – ca. 165) writes *Optics*, an experimental and mathematical treatment, extending earlier work on reflection by **Euclid** and **Hero**, including both concave and convex spherical and cylindrical mirrors, and doing original research on refraction.
ca 271	True compasses come into use by this date in China.
6th cent	(China) Discovery that loadstones could be used to magnetize small iron needles.
11st cent	**Abu Ali al-Hasan ibn al-Haitam (Alhazen)** (965–1039) writes *Kitab al-manazir* (translated into Latin as *Opticae thesaurus Alhazeni* in 1270) on optics, dealing with reflection, refraction, lenses, parabolic and spherical mirrors, aberration and atmospheric refraction. He adapts the mathematical extramission theory (which he rejects) to the intramission framework. (China) Iron magnetized by heating it to red hot temperatures and cooling while in south-north orientation.

Alhazen

1086	**Shen Kua** (1031–95)'s *Dream Pool Essays* make the first reference to compasses used in navigation.
1150s	Earliest explicit reference to magnets per se, in *Roman d'Enéas*. (see **reference**)
1190s	**Alexander Neckam** (1157–1217)'s *De naturis rerum* contains the first western reference to compasses used for navigation, and it had by this time been in common use.
13rd cent	**Robert Grosseteste** (ca. 1168–1253) writes *De Iride*, *De Luce*, *De Colore*, and other works on optics and light, lenses and mirrors, describing rectilinear light propagation as a wave phenomenon analogous to sound, and analyzing the optics of the rainbow in terms of refraction. **Witelo** (d. ca. 1281) writes *Perspectiva* around 1270, treating geometric optics, including reflection and refraction. He also reproduces the data given by **Ptolemy** on optics, though was unable to generalize or extend the study. **John Pecham** (ca. 1230–92)'s work on optics and light. **Roger Bacon** (1214–94) writes many works on the nature of light and optics (and some on magnetism). Greatly furthering the work of Grosseteste and Alhazen, and having access to and mastery of the major literature on optics, Bacon develops a unified framework for the understanding of light and geometric optics. **Theodoric of Freiberg** (ca. 1250 – ca. 1310), working with prisms and transparent crystalline spheres, formulates a sophisticated theory of refraction in raindrops which is close to the modern understanding, though it did not become very well known. (**René Descartes** (1596–1650) presents a nearly identical theory roughly 450 years later.) Eyeglasses, convex lenses for the far-sighted, first invented in or near Florence (as early as the 1270s or as late as the late 1280s — concave lenses for the near-sighted appearing in the late 15th century).
1269	**Pierre de Maricourt, aka Petri Pergrinus** (fl. 1269) writes **Letter on the magnet of Peter the Pilgrim of Maricourt to Sygerus of Foucaucourt, Soldier**, the first western analysis of polar magnets and compasses. He demonstrates in France the existence and fundamental role of two poles of a magnet by tracing the directions of a needle laid on to a natural magnet. (First printed in 1558.)
16th cent	**Girolamo Cardano** (1501–76) elaborates the difference between amber and loadstone.
1508	**John of St. Amand** (fl. 1508) suggests the ideas of magnetic poles, the Earth as a magnet, and magnetic currents.
1558	**Giambattista Della Porta** (1535–1615) publishes his major work, *Magia naturalis*, analyzing, among many other things, magnetism.

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1600	**William Gilbert** (1544－1603), after 18 years of experiments with magnetic and electrical materials, finishes his book *De Magnete*. The work included: the first major classification of electric and non-electric materials; gives the name "electric" to the substance behind electrical phenomena; a comarative study of electric and magnetic field effects; the relation of moisture and electrification; showing that electrification effects metals, liquids and smoke; noting that electrics were the attractive agents (as opposed to the air between objects); that heating dispelled the attractive power of electrics; and showing the earth to be a magnet. Gilbert is influenced by, among many others, the mariner **Robert Norman** (fl. 1590).	 Gilbert
1603	**Johannes Kepler** (1571－1630) finishes *Astronomiae Pars Optica*, going over parallax, reflection of mirrors, the inverse square law and related phenomena in astronomy.	
1606	**Della Porta** first describes the heating effects of light rays.	
1618	April 2nd, **Francesco Maria Grimaldi** (1618－63) discovers diffraction patterns of light and becomes convinced that light is a wave-like phenomenon. The theory is given little attention.	
1621	**Willebrord van Roijen Snell** (1580－1626) experimentally determines the law of angles of incidence and reflection for light and for refraction between two media.	
1629	**Nicolo Cabeo** (1586－1650) publishes his observations on electrical repulsion, noting that attracting substances may later repel one another after making contact.	
1630	**Vincenzo Cascariolo** discovers a substance that shines in the dark after exposure to sunslight, the so-called Bologna phosphorus.	
1637	**René Descartes** (1596－1650) publishes his *Dioptics* and *On Meteors* as appendices to his *Discourseins a Method*, detailing a theory of refraction and going over a theory of rainbows which, while containing nothing essentially new, encouraged experimental exploration of the subject.	
1644	**Descartes**' *Principia philosophiae*, describing magnetism as the result of the mechanical motion of channel particles and their displacements, and proposing the absence of both void and action at a distance.	
1646	**Thomas Browne** (1605－82) coins the term "electricity" in his *Pseudodoia Epidemica*.	
1657	**Pierre de Fermat** (1601－65) formulates the principle of least time for understanding the way in which light rays move.	
1660	**Otto von Guericke** (1602－86) builds the first electrical machine, a rotating frictional generator.	

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1661	**Fermat** is able to apply his principle of least time to understand the refractive indices of different materials.	Fermat
1664	**Robert Hooke** (1635－1703) puts forth a wave theory of light in his **Micrographia**, considering light to be a very high speed rectilinear propagation of longitudinal vibrations of a medium in which individual wavelets spherically spread. He also introduces wave-front analysis, the nation of a material's optical density and a theory of color.	
1665	**Francesco Maria Grimaldi** (1618－63)'s *Prysico-mathesis de lumine coloribus et iride* describes experiments with diffraction of light and states his wave theory of light.	
1669	**Erasmus Bartholin** (1625－98) publishes *A Study of Iceland Spar*, about his discovery of double refraction.	
1671	**Isaac Newton** (1642－1727) presents his observations on color and suggests color to be a property of light rays.	
1672	**Newton** presents a corrected account of **Christiaan Huyghens** (1629－95)' discovery of polarization phenomena.	
1675	**Robert Boyle** (1627－91) writes *Experiments and Notes about the Mechanical Origine or Production of Electricity*. Electrical attraction, it was written, was "a Material Effluvium issuing from and returning to, the Electrical Body."	Huyghens
1676	**Ole Christensen Rømer** (1644－1710) demonstrates the finite speed of light via observations of the eclipses of the satellites of Jupiter. While not calculating a speed for light, he estimates the Sun-Earth transit time for light to travel as roughly 11 minutes.	
1677	**Huyghens** extends the wave theory of light in his work *Treatise on Light*, unpublished until 1690.	
1687	**Newton** notes magnetism to be a non-universal force and derives an inverse cubed law for two poles of a magnet.	
1690	Publication of **Huyghens**' work *Treatise on Light* (originally presented to the *Académie* in 1678). This work includes a wave theory of light with a finite speed, a numerical estimate of the speed of light as 180,000,000 toises per second (roughly 131,000 miles per sec), explanations of wave propagation, reflection, single and double refraction, and polarization.	

电磁学00-03: 电磁学发展早期简史

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1699	**Nicolas Malebranche** (1638–1715) proposes monochromatic light to depend on periodic vibrations and that brightness is in proportion to their amplitude.
1704	**Newton**'s research on light culminates in the publication of his *Optics*, describing light both in terms of wave theory and his corpuscular theory.
1709	**Francis Hauksbee** (1666–1713)'s *Physico-Mechanical Experiments on Various Subjects*.
1728	**James Bradley** (1693–1762) discovers the phenomenon of steller aberration, confirming and providing a new method for determining the finite speed of light.
1729	**Stephen Gray** (ca. 1670–1736) shows static electricity to be transported via substances, especially metals.
1733	**Charles-Francois de Cisternai du Fay** (1698–1739) discovers that electric charges are of two types and that like charges repell while unlike charges attract.
1745	**Ewald Georg von Kleist** (1700–48) discovers a method to store electrical charge (known later as a Kleistian or Leyden jar).
1746	**William Watson** (1715–89) suggests conservation of electric charge. **Jean Antoine Nollet** (1700–70)'s *Essai sur l'electricité des corps*.
1747	**Benjamin Franklin** (1706–90) proposes that electricity be modeled by a single fluid with two states of electrification, materials have more or less of a normal amount of electric fluid, independently proposing conservation of electric charge, and introducing the convention of describing the two types of charges as positive and negative. **Watson** passes electrical charge along a two mile long wire.
1750	**John Michell** (1724–93) demonstrates that the action of a magnet on another can be deduced from an inverse square law of force between individual poles of the magnet, published in his work, *A Treatise on Artificial Magnets*.
1759	**Franz Ulrich Theodosius Aepinus** (1724–1802) publishes *An Attempt at a Theory of Electricity and Magnetism*, the first book applying mathematical techniques to the subject.
1764	**Johan Carl Wilcke** (1732–96) invents the electrophorus, a device which can produce relatively large amounts of electric charge easily and repeatedly.
1766	**Joseph Priestley** (1733–1804) deduces the inverse square law for electric charges using the results of experiments showing the absence of electrical effects inside a charged hollow conducting sphere.
1772	**Henry Cavendish** (1731–1810) publishes, "An Attempt to Explain some of the Principal Phenomena of Electricity, by Means of an Elastic Fluid."
1775	**Alessandro Guiseppe Antonio Anastasio Volta** (1745–1827) invents an electrometer, a plate condenser and the electrophorus.

Coulomb

1777	**Charles Augustin de Coulomb** (1736–1806) research sets a new direction in research into electricity and magnetism.
1780s	(early 1780s) **Luigi Galvani** (1737–98) uses the response of animal tissue to begin studies of electrical currents produced by chemical action rather than from static electricity. The mechanical response of animal tissue to contact with two dissimilar metals is now known as galvanism.
1785	**Coulomb** independently invents the torsion balance to confirm the inverse square law of electric charges. He also verifies **Michell**'s law of force for magnets and also suggests that it might be impossible to separate two poles of a magnet without creating two more poles on each part of the magnet.
1799	**Volta** shows that galvanism is not of animal origin but occurred whenever a moist substance is placed between two metals. This discovery eventually leads to the "Volta pile" a year linsert into rst electric batteries.
1800	**Volta** writes a paper on electricity by contact.
1801	**Thomas Young** (1773–1829) work on interference revives interest in the wave theory of light. He also accounts for the recently discovered phenomenon of light polarization by suggesting that light is a vibration in the aether transverse to the direction of propagation. **Johann Georg von Soldner** (1776–1833) makes a calculation for the deflection of light by the sun assuming a finite speed of light corpuscles and a non-zero mass. (The result, 0.85 arc-sec, was rederived independently by **Cavendish** and **Albert Einstein** (1879–1955) (1911), but went unnoticed until 1921.)
1807	**Humphry Davy** (1778–1829)'s lecture, "On Some Chemical Agents of Electricity," drawing close the possible relationships of chemical and electrical forces.
1812	**Simeon-Denis Poisson** (1781–1840) formulates the concept of macroscopic charge neutrality as a natural state of matter and describes electrification as the separation of the two kinds of electricity. He also points out the usefulness of a potential function for electrical systems. **Hans Christian Oersted** (1777–1851) suggests that experiments on galvanism could show the relationship between electricity and magnetism.
1813	Measurements of specific heat of air as a function of pressure by **Delarache** and **Joseph Frédéric Bérard** (1789–1828).
1814	**Augustin-Jean Fresnel** (1788–1827) independently discovers the interference phenomena of light and explains its existence in terms of wave theory.
1817	**Fresnel** predicts a dragging effect on light in the aether.
1818	**Fresnel**'s essay on optics and the aether.

电磁学00-03: 电磁学发展早期简史

1820	(Spring) Oersted notes the deflection of a magnetic compass needle caused by an electric current after giving a lecture demonstration. He then demonstrates that the effect is reciprocal. This initiates the unification program of electricity and magnetism.  Ampère July 27, André Marie Ampère (1775-1836) confirms Oersted's results and presents extensive experimental results to the French Academy of Science. He models magnets in terms of molecular electric currents. His formulation inaugurates the study of electrodynamics independent of electrostatics. Fall, Jean-Baptiste Biot (1774-1862) and Felix Savart (1792-1841) deduce the formula for the strength of the magnitec effect produced by a short segment of current carrying wire.
1825	Ampère's memoirs are published on his research into electrodynamics.
1827	Georg Simon Ohm (1789-1854) formulates the relationship between current to electromotive force and electrical resistance. Ampère publishes *Memoir on the Mathematical Theory of Electrodynamics, Uniquely Deduced from Experiment*.
1828	George Green (1793-1841) introduces the notion of potential and formulates what is now called Green's Theorem relating the surface and volume distributions of charge. (The work goes unnoticed until 1846.)
1831	Michael Faraday (1791-1867) begins his investigations into electromagnetism.
1832	Johann Carl Friedrich Gauss (1777-1855) independently states Green's Theorem without proof. He also reformulates Coulomb's law in a more general form, and establishes experimental methods for measuring magnetic intensities.  Gauss
1835	Gauss formulates separate electrostatic and electrodynamical laws, including "Gauss's law." All of it remains unpublished until 1867.
1838	Faraday explains electromagnetic ininduction, electrochemistry and formulates his notion of lines of force, also criticizing action-at-a-distance theories. Wilhelm Eduard Weber (1804-91) and Gauss apply potential theory to the magnetism of the earth.
1839	The potential theory for magnetism developed by Weber and Gauss extented to all inverse-squared phenomena.
1842	William Thomson (Lord Kelvin) (1824-1907) writes a paper, "On the uniform motion of heat and its connection with the mathematical theory of electricity," based on the ideas of Joseph Fourier (1768-1830). The analogy allows him to formulate a continuity equation of electricity, implying a conservation of electric flux.

1845 to 1850	G T Fechner (1801-87) proposes a connection between Ampère's law and Faraday's law in order to explain Lenz's law. Faraday introduces the idea of "contiguous magnetic action" as a local interaction, instead of the idea of instantaneous action at a distance, using concepts now known as fields. He also estabishes a connection between light and electrodynamics by showing that the transverse polarization direction of a light beam was rotated about the axis of propagation by a strong magnetic field (today known as "Faraday rotation").
1846	Weber proposes a synthesis of electrostatics, electrodynamics and induction using the idea that electric currents are moing charged particles. The interactions are instantaneous forces. Weber's theory contains a limiting velocity of electromagnetic origin with the value *sqrt(2) c*. William Robert Grove (1811-96)'s *Correlation of physical forces*. The partial-drag theory of George Gabriel Stokes (1819-1903) is revived for the explanation of stellar aberration.
1849	Armand Hippolyte Louis Fizeau (1819-96) begins experiments to determine the speed of light.
1851	Fizeau's interferometry experiment confirming Fresnel's theoretical results.
1852	Stokes names and explains the phenomena of fluorescence.
1854	Bernhard Riemann (1826-66) makes unpublished conjectures about an "investigation of the connection between electricity, galvanism, light and gravity."  Riemann
1855	Weber and R Kohlrausch (1809-58) determine a limiting velocity which turns up in Weber's electrodynamic theory, and that it's value is about 439,450 km/s.
1855 to 1868	James Clerk Maxwell (1831-79) completes his formulation of the field equations of electromagnetism. He established, among many things, the connection between the speed of propagation of an electromagnetic wave and the speed of light, and establishing the theoretical understanding of light.
1858	Riemann generalizes Weber's unification program and derives his results via a solution to a wave function of a electrodynamical potential (finding the speed of propagation, correctly, to be *c*). He claimed to have found the connection between electricity and optics. (Results published postumously in 1867.)
1861	Riemann uses Joseph Louis Lagrange (1736-1813)'s theorem to deal with velocity-dependent electrical accelerations.  Gustav Robert Kirchhoff (1824-87) formulates the model of the black body.
1863	John Tyndall (1820-93)'s *Heat Considered as a Mode of Motion*.

电磁学00-03: 电磁学发展早期简史

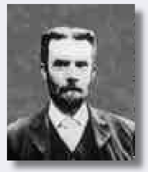
1861	**Riemann** uses **Joseph Louis Lagrange** (1736–1813)'s theorem to deal with velocity-dependent electrical accelerations. **Gustav Robert Kirchhoff** (1824–87) formulates the model of the black body.
1863	**John Tyndall** (1820–93)'s *Heat Considered as a Mode of Motion*.
1864	**Maxwell** publishes *A Dynamical Theory of the Electromagnetic Field*, his first publication to make use of his mathematical theory of fields.
1865	**Maxwell**'s *A Dynamical Theory of the Electromagnetic Field*, formulating an electrodynamical formulation of wave propagation using Lagrangian and Hamiltonian techniques, obtaining the theoretical possibility of generating electromagnetic radiation. (The derivation is independent of the microscopic structures which may underlie such phenomena.)
1870	**Hermann Ludwig Ferdinand von Helmholtz** (1821–94) developes a theory of electricity and shows **Weber**'s theories to be inconsistent with the conservation of energy.
1873	The first edition of **Maxwell**'s *Treatise on Electricity and Magnetism* is published.
1874	**George Johnstone Stoney** (1826–1911) estimates the charge of an electron to be about 10^{-20} Coulombs and introduces the term "electron."
1875	**Hendrik Antoon Lorentz** (1853–1928), in his doctoral thesis, derives the phenomena of reflection and refraction in terms of **Maxwell**'s theory. **W Crookes** (1832–1919) performs experiments on cathode rays.
1879	**Maxwell** suggests that an earth-based experiment to detect possible aether drifts could be performed, but that it would not be sensitive enough.
1881	**Albert Abraham Michelson** (1852–1931) begins his interferometry experiments to detect a luminiferous aether. **Joseph John Thomson** (1856–1940) paper, "On the electric and magnetic effects produced by the motion of electrified bodies" explores inertial effects due to displacement currents.
1884	**Heinrich Rudolf Hertz** (1857–94) develops a reformulation of electrodynamics and shows his and **Helmholtz**'s theories both amount to **Maxwell**'s theory. **John Henry Poynting** (1852–1914) establishes a principle of electromagnetic radiation energy which can be localized and flow (the first such energy localization principle established); not confined to existing only in conductors, but throughout space, independent of matter.
1885 to 1887	**Oliver Heaviside** (1850–1925) writes *Electromagnetic induction and its propagation* over the course of two years, re-expressing **Maxwell**'s results in 3 (complex) vector form, giving it much of its modern form and collecting together the basic set of equations from which electromagnetic theory may be derived (often called "Maxwell's equations"). In the process, He invents the modern vector calculus notation, including the gradient, divergence and curl of a vector.

Maxwell

Lorentz

JJ Thomson

1887	**Hertz** experimentally produces electromagnetic radiation with radio waves in the GHz range, also discovering the photoelectric effect and predicting that gravitation would also have a finite speed of propagation. **W Voight**, working through an analysis of Doppler effects using an elastic model of the luminiferous aether to describe optical properties, produces a set of relations between space and time intervals which are later rediscovered independently by **Lorentz** and now knows as the "Lorentz equations" (first so-called by **Jules Henri Poincaré** (1854–1912) in 1904).
1889	**George Francis FitzGerald** (1851–1901) suggests that bodies contract in the direction of motion against the luminiferous aether by an amount which would account for the null results coming from the Michelson-Morley experiments on aether motion. (A more detailed calculation is performed independently by **Lorentz** in 1895.) FitzGerald also suggests that the speed of light is an upper bound on any possible speed. (This suggestion reappears in 1900 by Lorentz, in 1904 by Poincaré, and again in 1905 by Einstein.) **John William Strutt (Lord Rayleigh)** (1842–1919) presents a model for radiation in terms of wave propagation. **Heaviside**'s "On the electromagnetic effects due to the motion of electrification through a dielectric," proposes part of inertial mass to be electromagnetic in origin and includes dependencies on higher-order terms in *(v/c)*.
1890	**Hertz** publishes his memoirs on electrodynamics, simplifying the form of the electromagnetic equations, replacing all potentials by field strengths, and deduces Ohm's, Kirchoff's and Coulomb's laws.
1892 to 1904	**Lorentz** completes the description of electrodynamics by clearly separating electricity and electrodynamic fields and formulating the equations for charged particles in motion.
1893	**Wilhelm Carl Werner Otto Fritz Franz Wien** (1864–1928) gives his displacement law of blackbody radiation.
1896	**Wien** theoretically derives the radiation distribution law. Discovery of X-rays and Becquerel radiation. Discovery of the Zeeman effect.
1897	**JJ Thomson** experimentally determines the charge-to-mass ratio, *e/m*, of electrons.
1898	**Poincaré** suggests that a complete measurement theory must formulate a notion of distant synchronization and discusses its relevance to the apparent constancy of the speed of light.



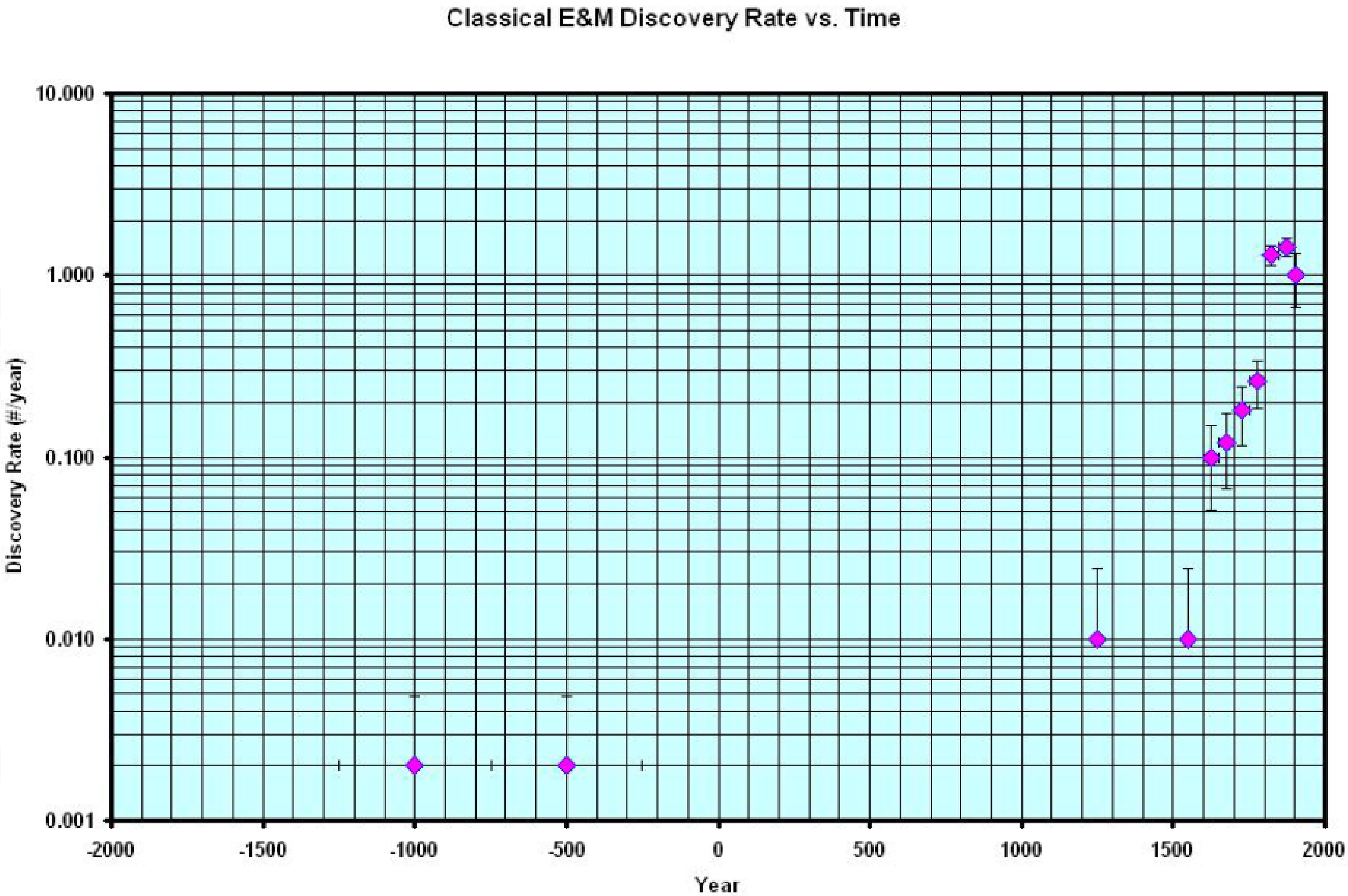
Heaviside

1899	**Lorentz** refines the transformation laws, formulating the notion of local time and local coordinate systems in electrodynamics. **Philipp Eduard Anton von Lénárd** (1862–1947) begin experimental investigations of photoelectric radiation.
1900	**Wien**'s "On the possibility of an electromagnetic foundation of mechanics." **Poincaré**'s paper "The theory of Lorentz and the principle of reaction," showing electromagnetic radiation to have a momentum proportional to a field's Poynting vector, and that the momentum of a recoiling body to be vE/c^2 . **Max Karl Ernst Ludwig Planck** (1858–1947), studying blackbody radiation derives the correct radiation spectrum for blackbodies. Planck proposes the constant, h (Planck's constant), as a quantum of action in phase space.
1902	**Max Abraham** (1875–1922) "The dynamics of electrons," also introducing the concept of electromagnetic momentum. **Walter Kaufmann** (1871–1947) performs experiments on the deflection of electrons by electric and magnetic fields and a determination of the ration e/m, In a second paper, he concludes that the mass of an electron is purely electromagnetic in origin.
1903	**Abraham**'s "Principles of the dynamics of electrons," attempts to show, among other things, the electromagnetic foundation of mechanics.
1904	**Poincaré** uses light signals as a functional technique to establish distant synchronization in application to **Lorentz**'s electron theory, also putting forth the first formulation of a principle of electrodynamic relativity.
1905	**Albert Einstein** (1879–1955) analyzes the phenomena of the photoelectric effect and theorizes that light may be taken to be made up of vast amounts of packets of electromagnetic radiation in discrete units. **Einstein** publishes several papers drawing out the symmetries of Maxwell, Hertz and Lorentz's electromagnetic theory, the underlying connection in measurement theory and the status of the electromagnetic aether.
1907	**Hermann Minkowski** (1864–1909), through considerations of the group properties of the equations of electrodynamics, reinterprets Einstein's relativity theory as a kind of geometry of spacetime, considered as a single medium. **Planck** gives a corrected derivation of the mass-energy relation using Poincaré's radiation momentum.
1908	**Gilbert N Lewis** (1875–1946) publishes "A revision of the fundamental laws of matter and energy," deriving $dE = c^2\ dm$ from considerations of radiation pressure.
1933	Experiments by **Patrick Blackett** (1897–1974) and **Giuseppe Occhialini** (1907–93) on pair production demonstrate the complete aninhilation of matter into electromagnetic energy.

Einstein

Dream Pool Essays (夢溪筆談)

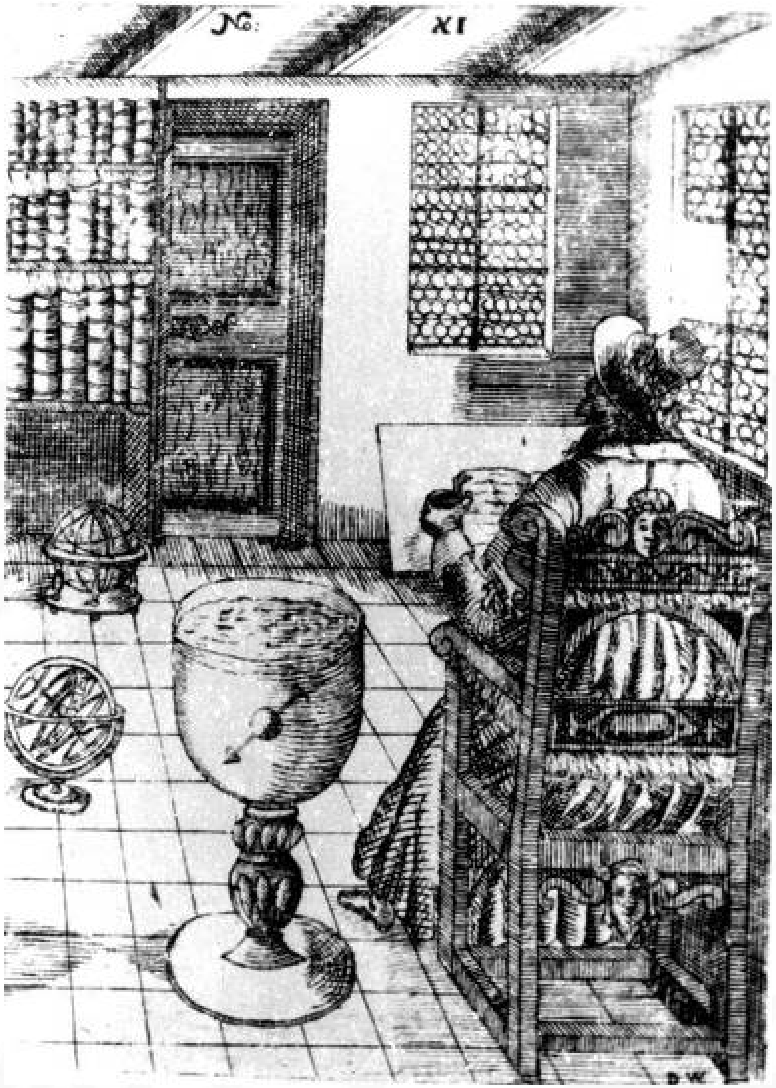
Shen also first described the magnetic needle.


Classical E&M Discovery Rate vs. Time
Discovery Rate (#/year)
10.000
1.000
0.100
0.010
0.001
-2000
-1500
-1000
-500
0
500
1000
1500
2000
Year

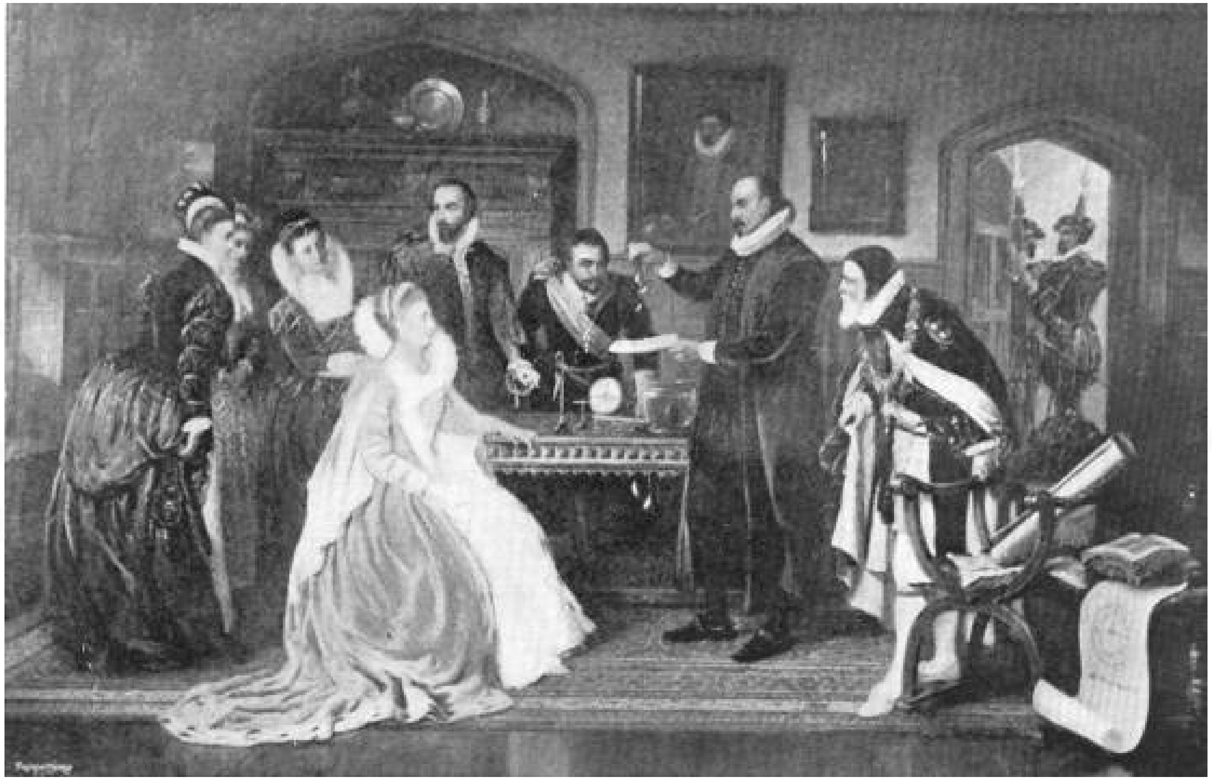
- http://www.hudong.com/wiki/Gilbert%EF%BC%8CWilliam?prd=citiao_right_xiangguancitiao

吉伯关于锻打使铁产生磁性的一幅画

(图中**septentrio**表示北，**avster**表示南)

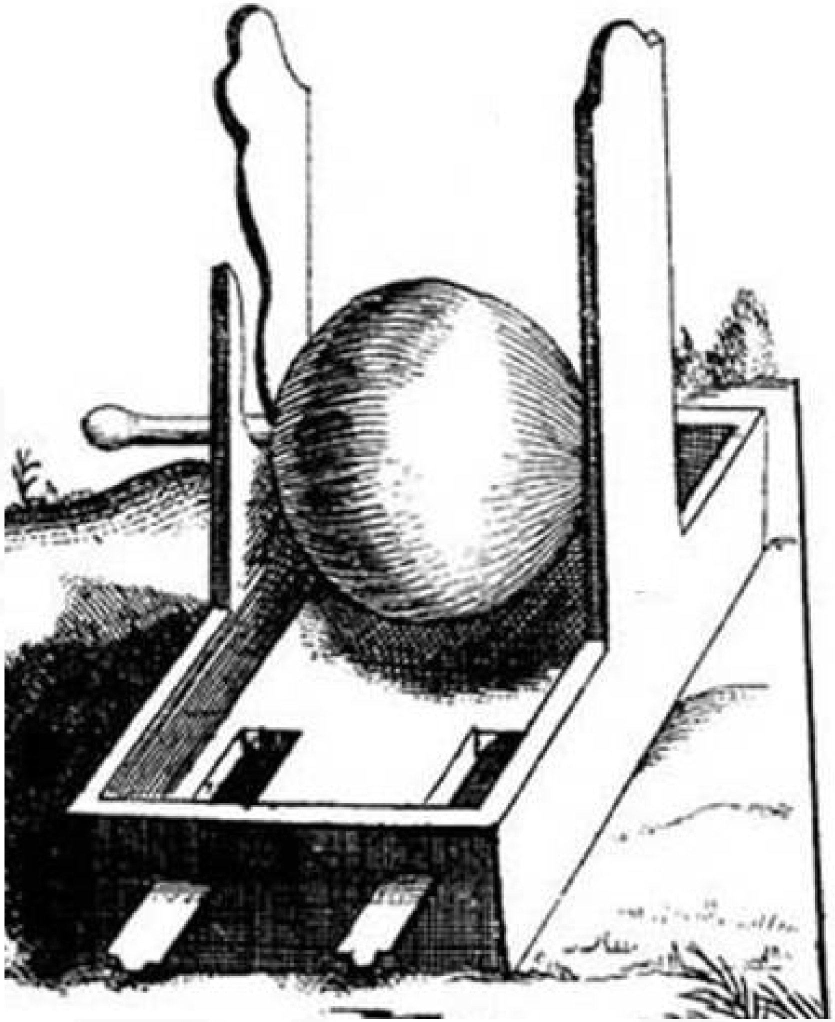


吉伯研究磁倾角

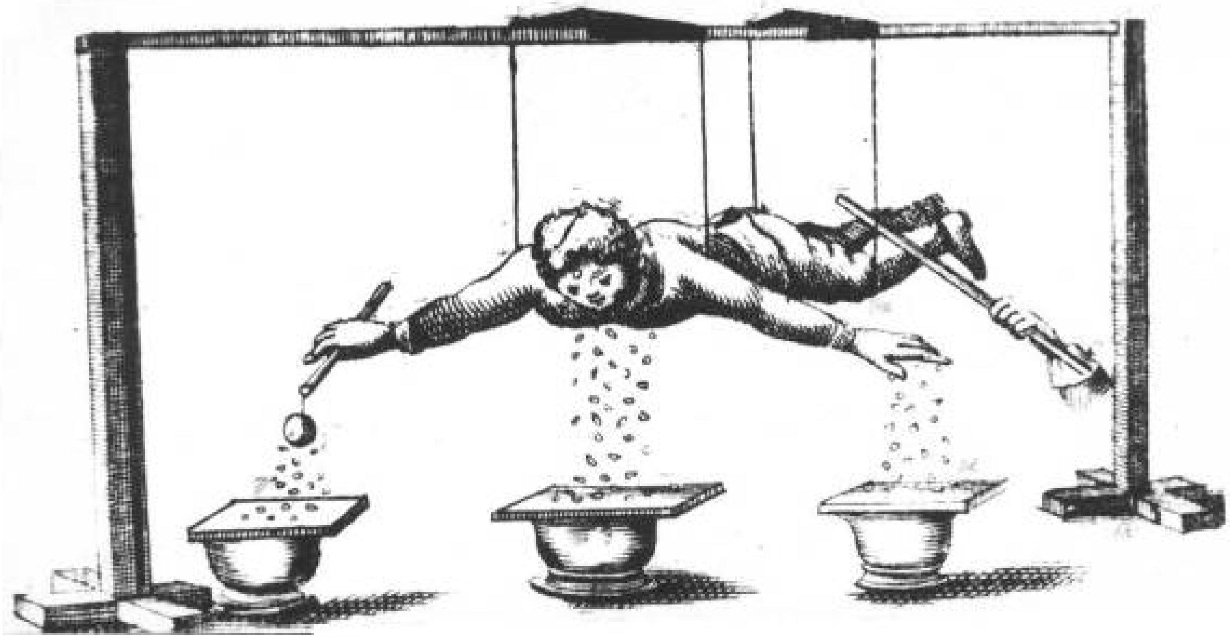


吉伯向伊丽莎白女皇介绍磁学新成果

吉尔伯特把经过摩擦后能吸引小物体的物体叫做**electric**，意思是“琥珀体”，这就是西文中“电”的词根的来源。

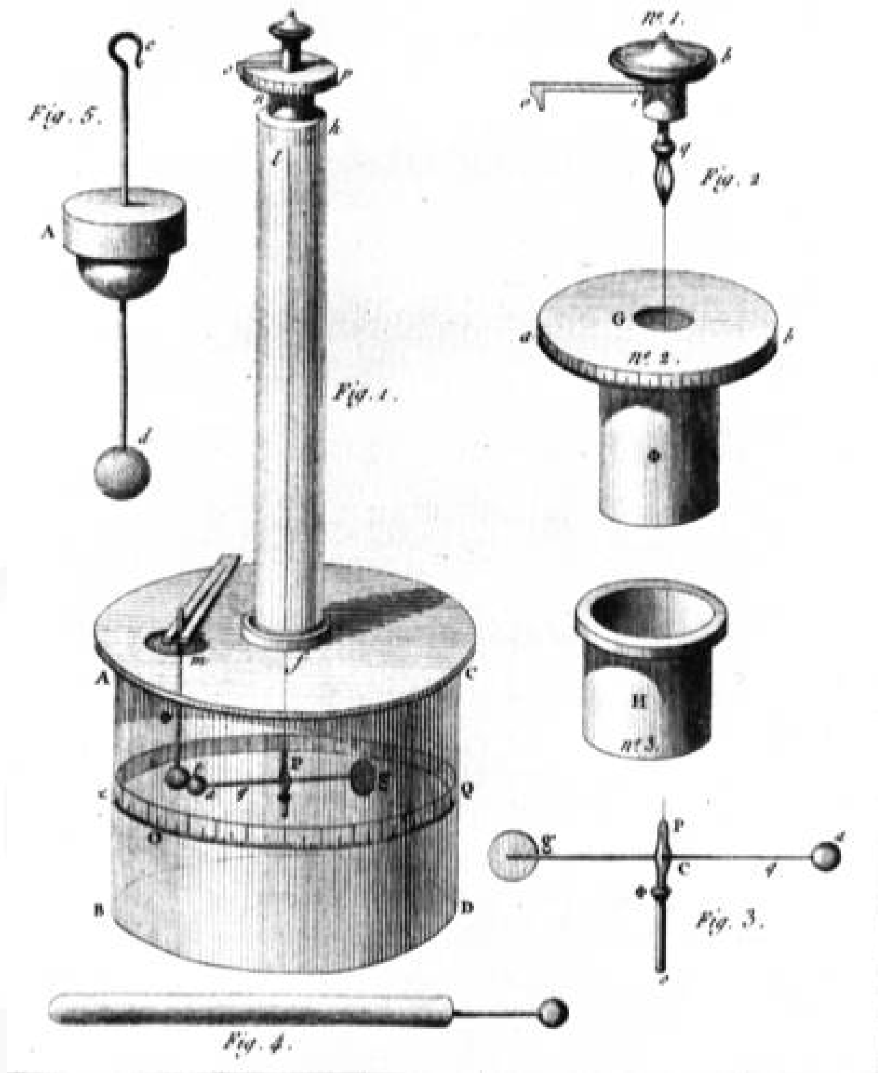


1660年德国·Guericke盖里克的摩擦起电机



1700年代英国的Gray格雷拿“小孩燕子”做实验证明人体可以导电

富兰克林的《电的实验和观察》



库仑的电扭秤实验装置

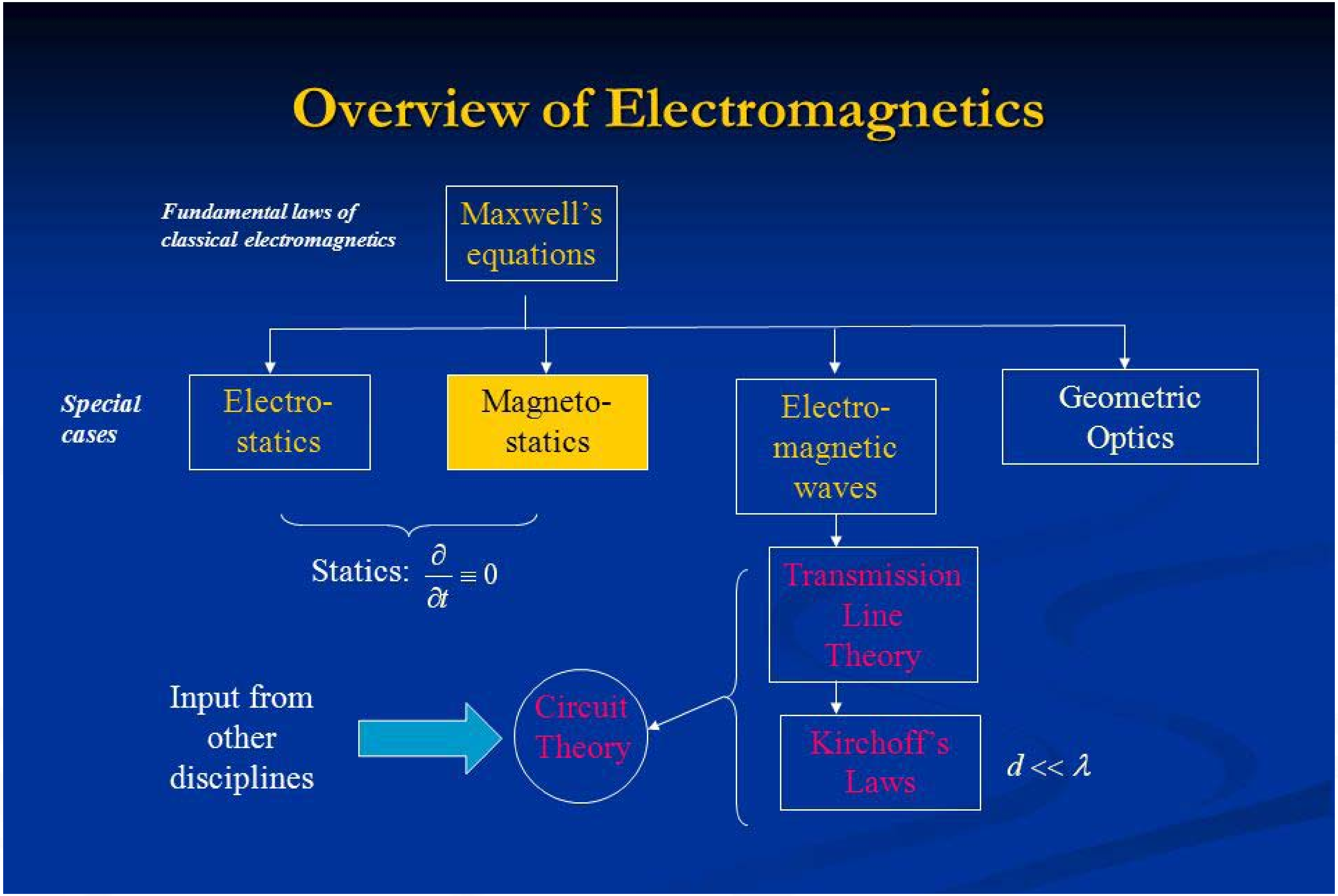
- 忆秦娥@电磁学史

掩长卷，三百年逝从头现。

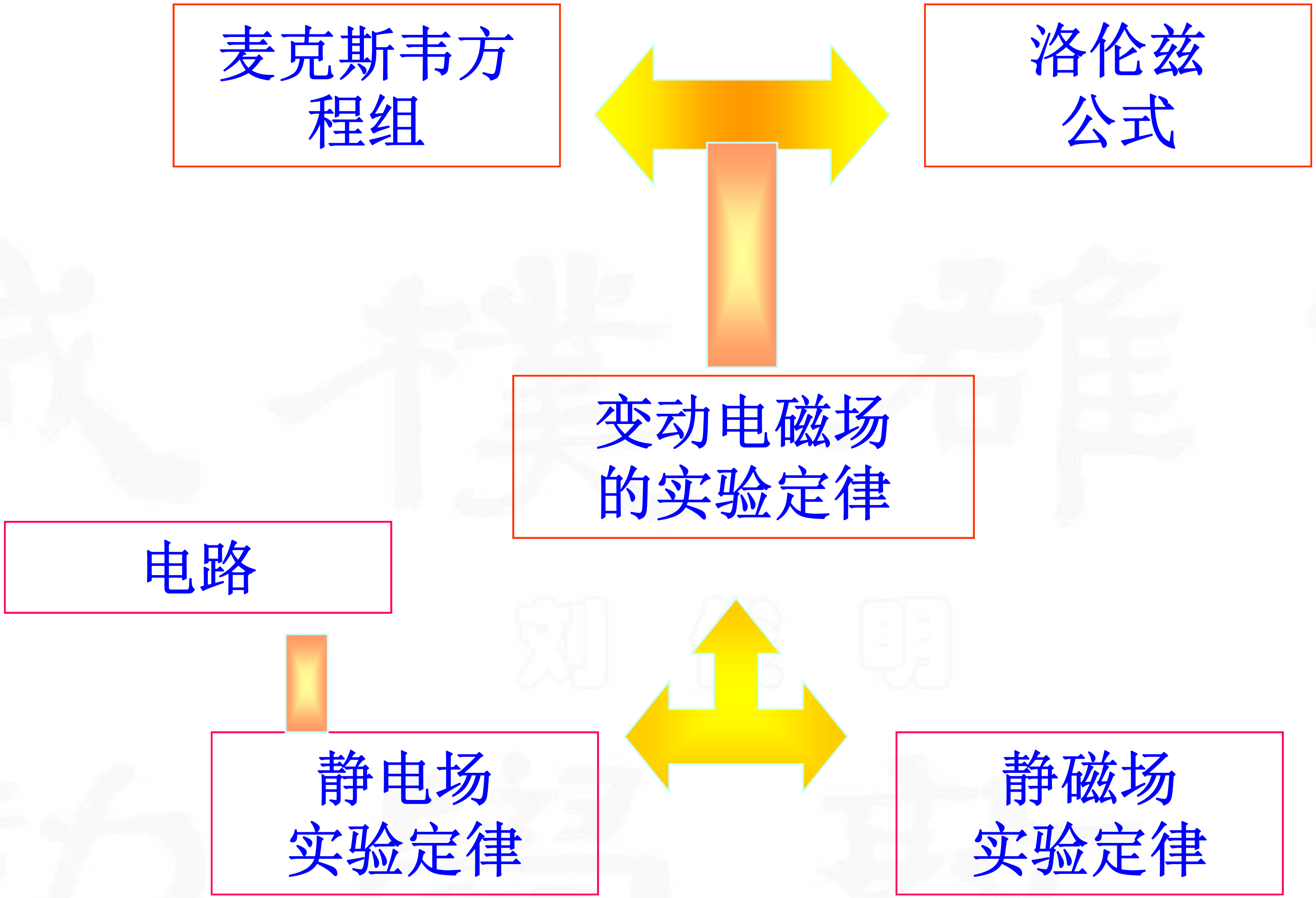
从头现，兰花两表，问谁重撰？

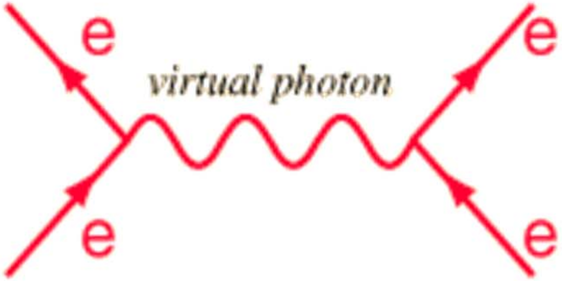
吉伯拆了金銮殿，格雷舍命追飞燕。

追飞燕，库伦沉默，安培潜染。

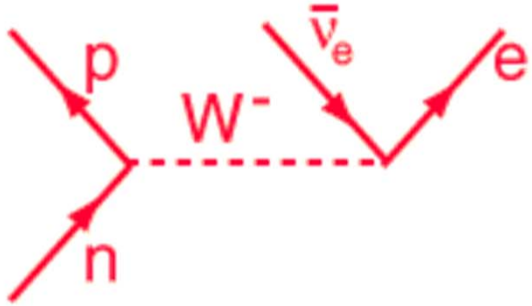


Overview of Electromagnetics
Fundamental laws of classical electromagnetics
Maxwell's equations
Special cases
Electro-statics
Magneto-statics
Electro-magnetic waves
Geometric Optics
Statics: $\frac{\partial}{\partial t} \equiv 0$
Transmission Line Theory
Kirchoff's Laws
$d << \lambda$
Input from other disciplines
Circuit Theory

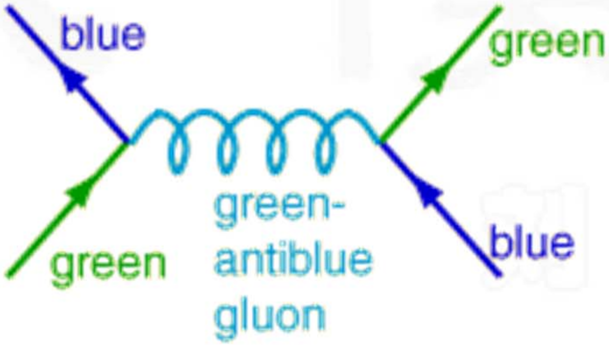

麦克斯韦方程组
洛伦兹公式
变动电磁场的实验定律
电路
静电场实验定律
静磁场实验定律



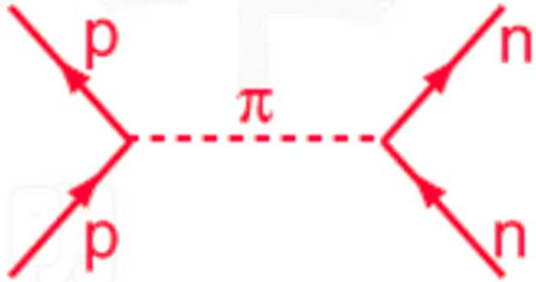
Electromagnetic



Weak



between quarks



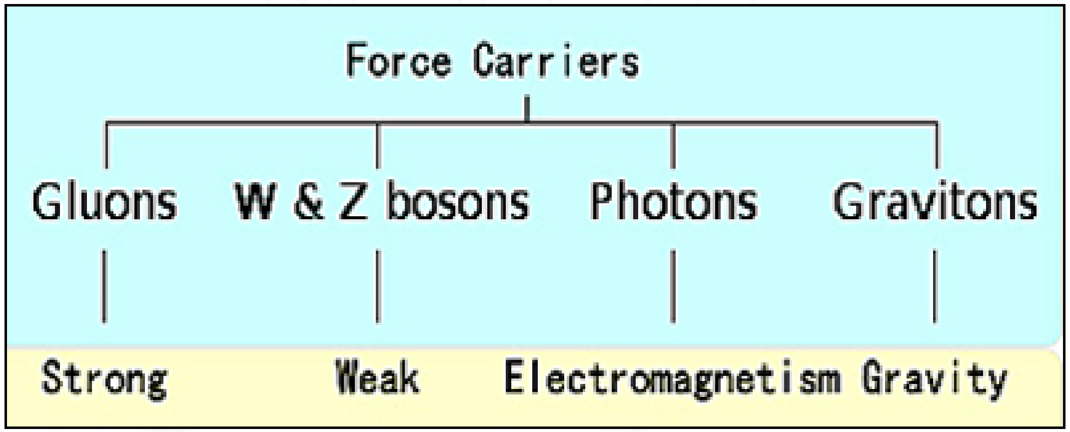
between nucleons

Strong Interaction

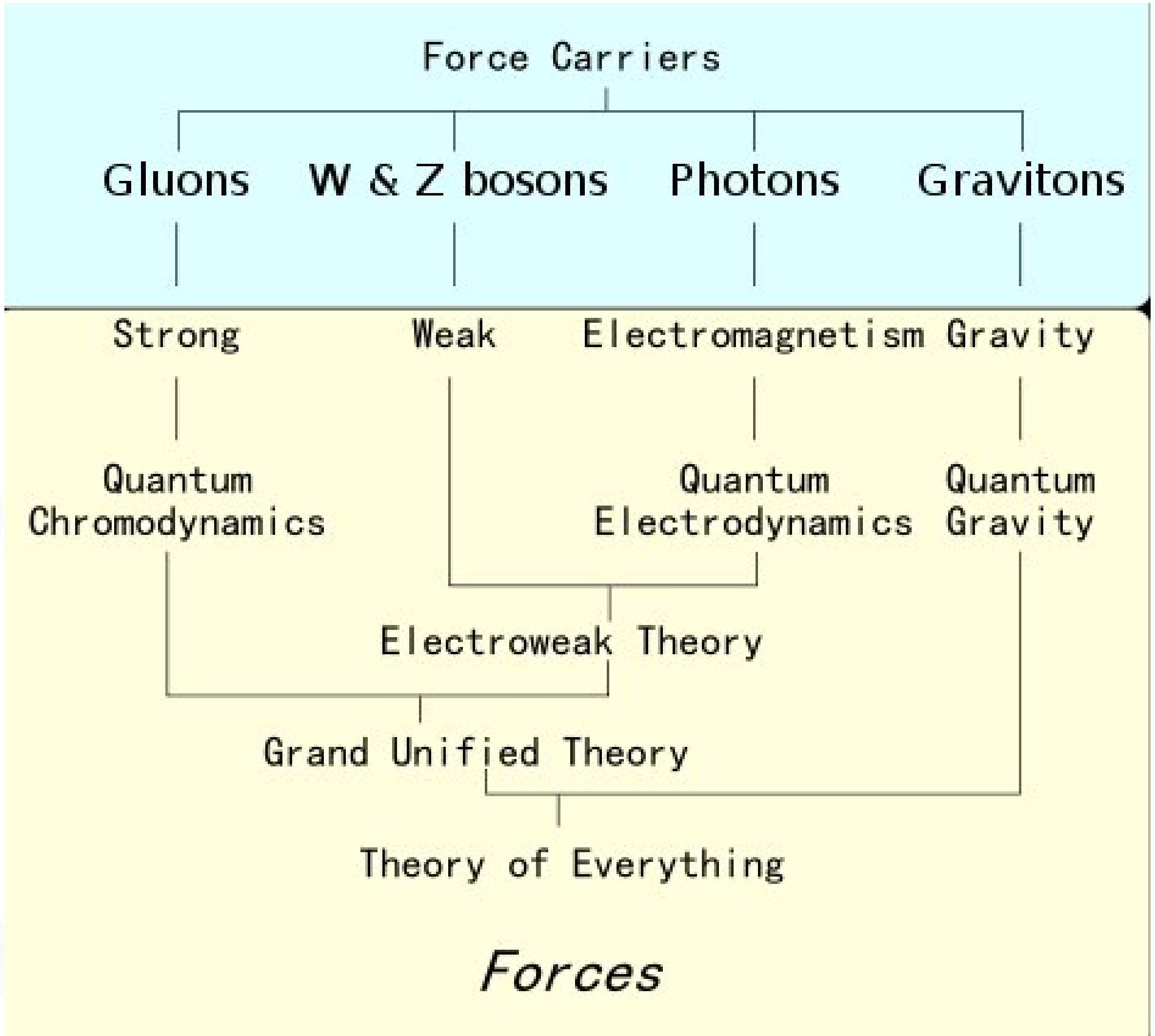
□ 电磁学在近代物理学中的意义：

The electromagnetic interaction is ONE of FOUR known FORCES (or INTERACTIONS) of Nature:

1) Electromagnetic Force – binds electrons & nuclei together to form atoms
   - binds atoms together to form $\underbrace{\text{molecules}}_{\text{gases}}$, liquids, solids. . . .
2) Strong Force – binds protons & neutrons together to form nuclei
3) Weak Force – responsible for radioactivity (e.g. β decay) (weak force important @ high energies)
4) Gravity – binds matter together to form stars, planets, solar systems, galaxies, etc.



电磁学在近代物理学中的意义：



终极理论中电磁学的位置

电磁学在近代物理学中的意义：

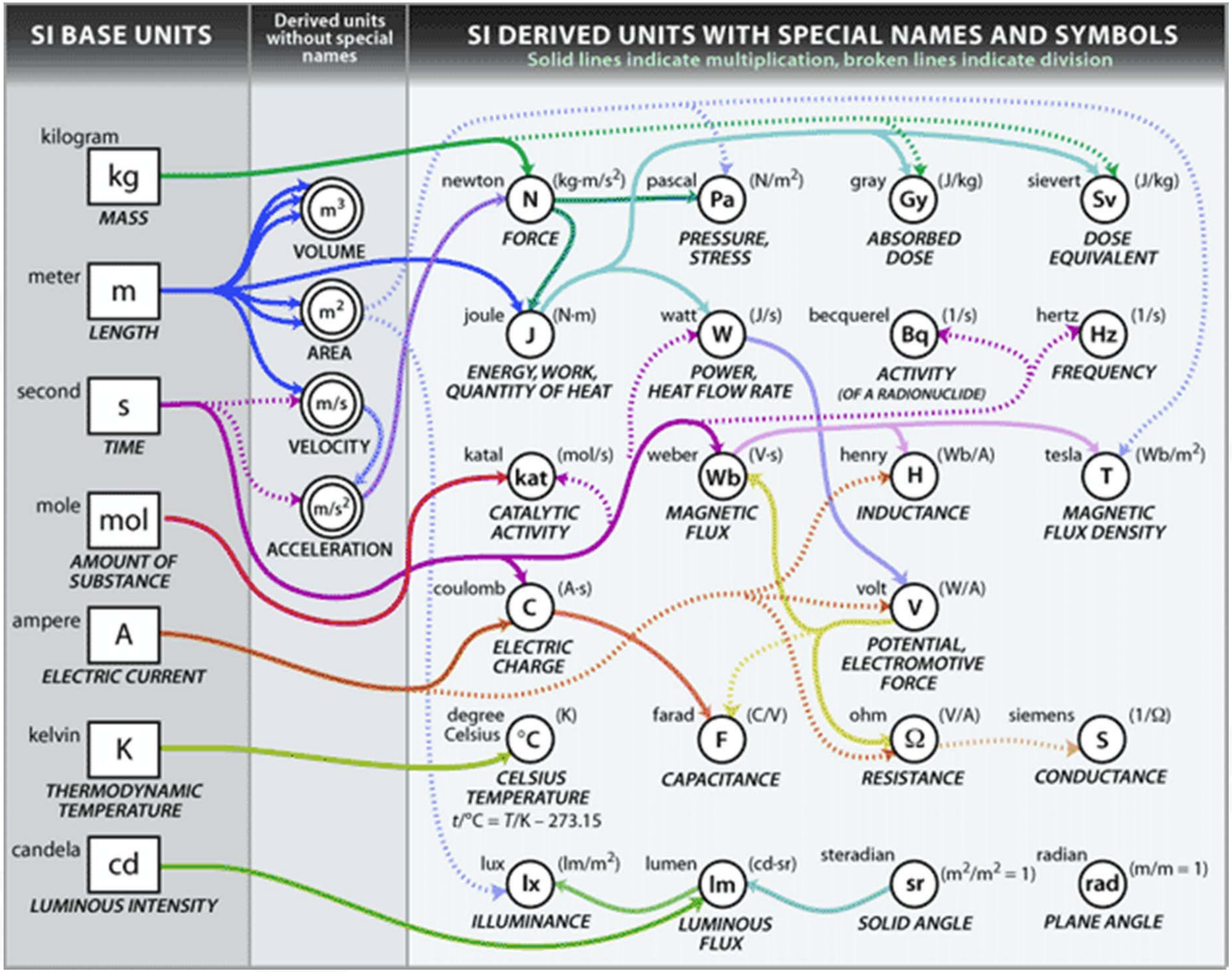
FORCE	"ELECTRIC" FIELD	"MAGNETIC" FIELD
EM	EM – electric	EM – magnetic
STRONG	chromo–electric	chromo–magnetic
WEAK	weak–electric	weak–magnetic
GRAVITY	gravito–electric	gravito–magnetic

Nordvedt Effect
e.g. affects motion of moon's orbit around earth (very small)

- 所有四种力都有“电场”和“磁场”

名称	相对强度(以强相互作用为准)	性质(对距离的作用大小)	作用的范围(米)	传递相互作用的中间玻色子
强相互作用	1	$1/r^7$	10^{-15}	胶子
电磁相互作用	1/137	$1/r^2$	无限大	光子
弱相互作用	10^{-13}	$1/r^{5-7}$	10^{-18}	W及Z玻色子
引力相互作用	10^{-39}	$1/r^2$	无限大	引力子

电磁学00-04: 电磁学相互作用本质



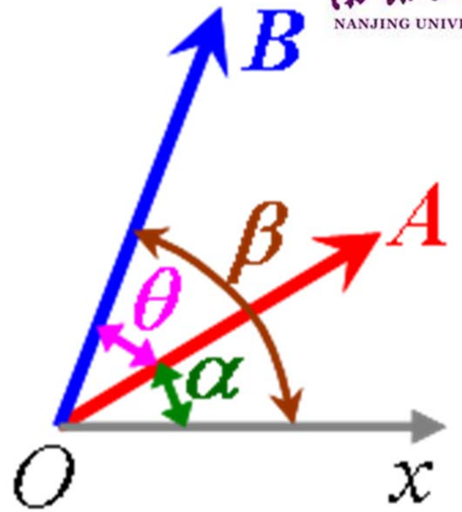
矢量的标积

任意两个矢量 ***A*** 和 ***B***，其标积或点乘定义：

$$\vec{A}=A_x\vec{i}+A_y\vec{j}+A_z\vec{k},\quad \vec{B}=B_x\vec{i}+B_y\vec{j}+B_z\vec{k}$$

$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z\Rightarrow$$

$$\Rightarrow\begin{cases}\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}\\ \vec{A}\cdot(\vec{B}+\vec{C})=\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}\end{cases}$$



标积的几何意义

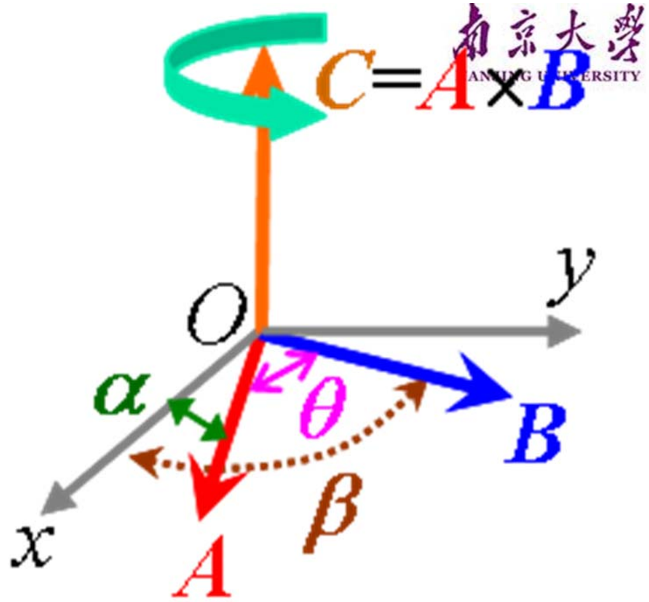
构建一空间坐标，***x-y*** 面在 ***A-B*** 面内。

标积代表一矢量在另一矢量上投影：

$$\left.\begin{aligned} &A_x = A\cos\alpha, A_y = A\sin\alpha \\ &B_x = B\cos\beta, B_y = B\sin\beta \\ &A_z = B_z = 0 \end{aligned}\right\}\left\{\begin{aligned} \vec{A}\cdot\vec{B} &= A_xB_x + A_yB_y \\ &= AB\cos(\beta-\alpha) \\ &= AB\cos\theta \end{aligned}\right.$$

$$\vec{A}\cdot\vec{A} = A^2$$

- 一些特定情况下之标积



矢量的矢积

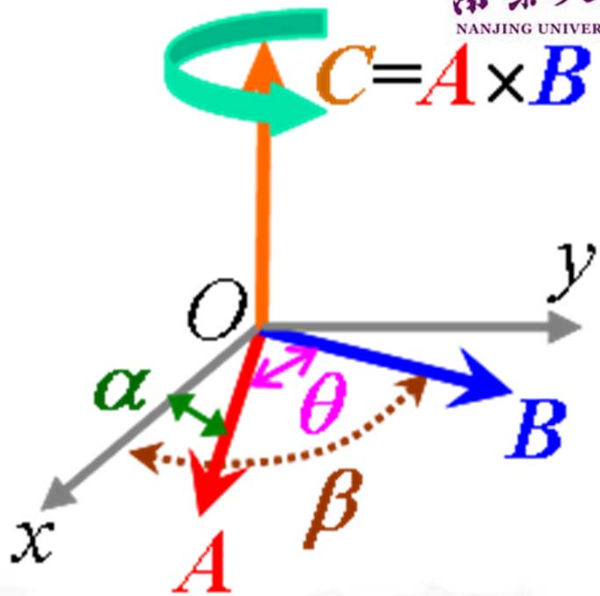
矢积(叉乘)定义为 $\boldsymbol{A}\times\boldsymbol{B}$，其两种表示为：

$$\vec{A}\times\vec{B}=\begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \Rightarrow \begin{cases} \vec{A}\times\vec{B}=-\vec{B}\times\vec{A} \\ \vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C} \end{cases}$$

- 矢积 **C** 按照右手螺旋法则定义方向，恒与 **A** 和 **B** 垂直。

矢量的矢积

$$\vec{A}\times\vec{B}=\begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

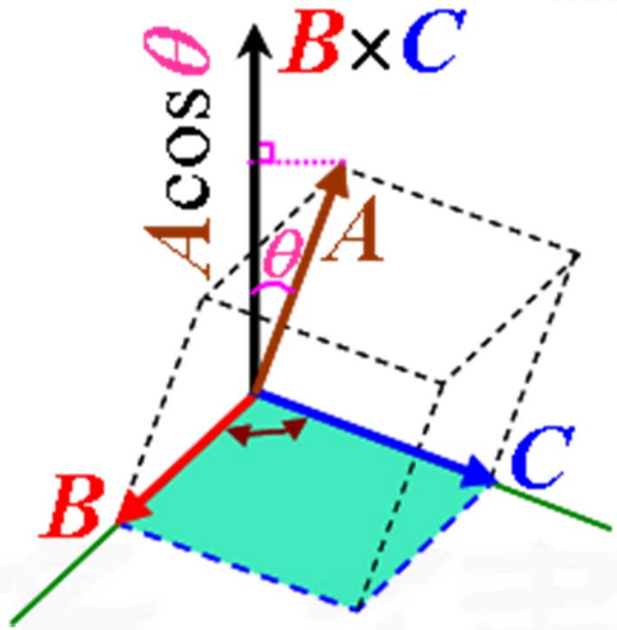


- 矢积 **C** 数值上等于 **A** 和 **B** 组成之平行四边形面积。

$$\left.\begin{aligned} &A_x = A\cos\alpha, A_y = A\sin\alpha \\ &B_x = B\cos\beta, B_y = B\sin\beta \\ &A_z = B_z = 0 \end{aligned}\right\} \left\{\begin{aligned} &\vec{A}\times\vec{B} = (A_x B_y - A_y B_x)\vec{k} \\ &= AB\sin(\beta-\alpha)\vec{k} \\ &= AB\sin\theta\vec{k} \end{aligned}\right.$$

$$\vec{C}=\vec{A}\times\vec{B}=AB\sin\theta\vec{k}, \quad \vec{A}\times\vec{A}=0$$

- 矢量的三重积：三重标积 **A · (B×C)**
  - 标量、绝对值为六面体体积



$$\vec{B}\times\vec{C}=\begin{vmatrix} i & j & k \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

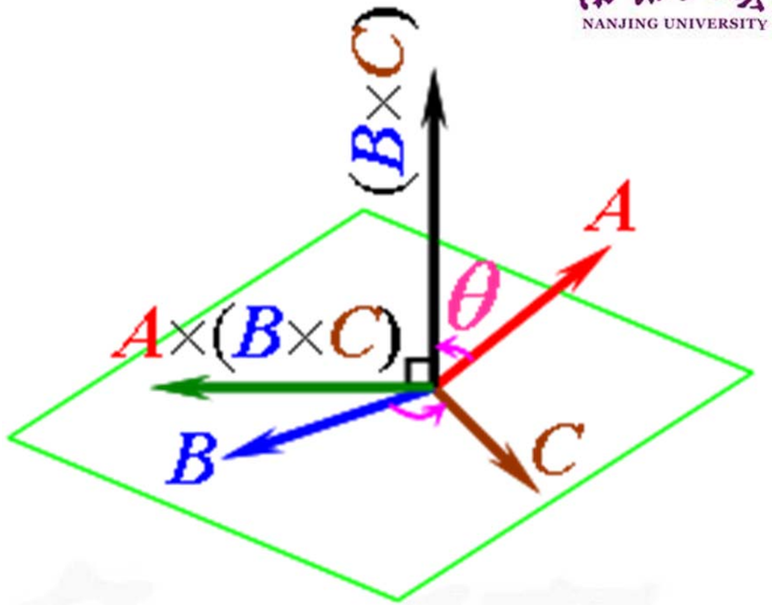
- 矢量的三重标积满足交换律:

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{B}\cdot(\vec{C}\times\vec{A})=\vec{C}\cdot(\vec{A}\times\vec{B})$$
$$=(\vec{A}\times\vec{B})\cdot\vec{C}=(\vec{B}\times\vec{C})\cdot\vec{A}=(\vec{C}\times\vec{A})\cdot\vec{B}$$
$$=-\vec{A}\cdot(\vec{C}\times\vec{B})=-\vec{C}\cdot(\vec{B}\times\vec{A})=-\vec{B}\cdot(\vec{A}\times\vec{C})$$

- 矢量的三重积：三重矢积 ***A*×(*B*×*C*)**

- 是矢量，与 ***B***、***C*** 共面

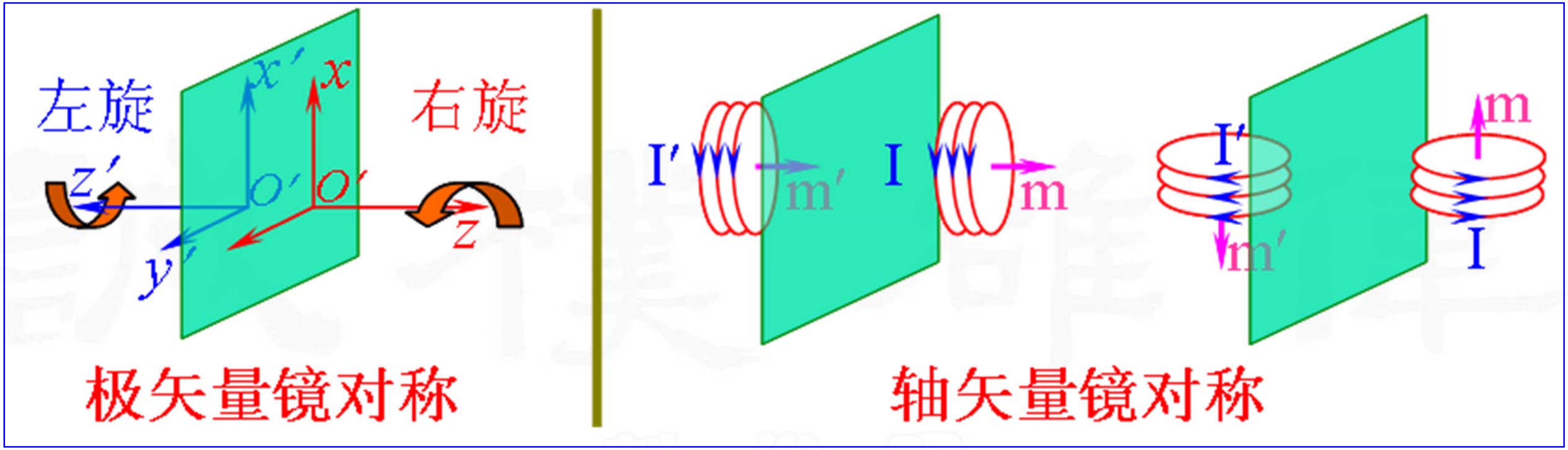


$$\vec{A}\times(\vec{B}\times\vec{C})=a_1\vec{B}+a_2\vec{C}\Rightarrow$$

$$a_1=\vec{A}\cdot\vec{C},\quad a_2=-\vec{A}\cdot\vec{B}$$

$$\vec{A}\times(\vec{B}\times\vec{C})=(\vec{A}\cdot\vec{C})\vec{B}-(\vec{A}\cdot\vec{B})\vec{C}$$

镜像反射对称、极矢量、轴矢量



- 镜面垂直量反向为极矢量，镜面平行量反向为轴矢量
- 空间位矢坐标 $\boldsymbol{r}=(x, y, z)$为极矢量，还有电场、电偶极矩等
- 磁矩 $\boldsymbol{m}$ 为轴矢量，还有磁感应强度等

➢ 两个极矢量叉乘得轴矢量：极矢量 ***a***、***b***，轴矢量 ***c***

$$\vec{a}=(a_x,a_y,a_z),\ \ \vec{b}=(b_x,b_y,b_z),\ \ \vec{c}=(c_x,c_y,c_z)=\vec{a}\times\vec{b}$$

$$\left.\begin{aligned}
c_x &= a_y b_z - a_z b_y \\
c_y &= a_z b_x - a_x b_z \\
c_z &= a_x b_y - a_y b_x \\
c'_x &= a'_y b'_z - a'_z b'_y \\
c'_y &= a'_z b'_x - a'_x b'_z \\
c'_z &= a'_x b'_y - a'_y b'_x
\end{aligned}\right\}
\xrightarrow[b'_x=b_x,\ b'_y=b_y,\ b'_z=-b_z]{a'_x=a_x,\ a'_y=a_y,\ a'_z=-a_z}
\left\{\begin{aligned}
c'_x &= -c_x \\
c'_y &= -c_y \\
c'_z &= c_z
\end{aligned}\right.$$

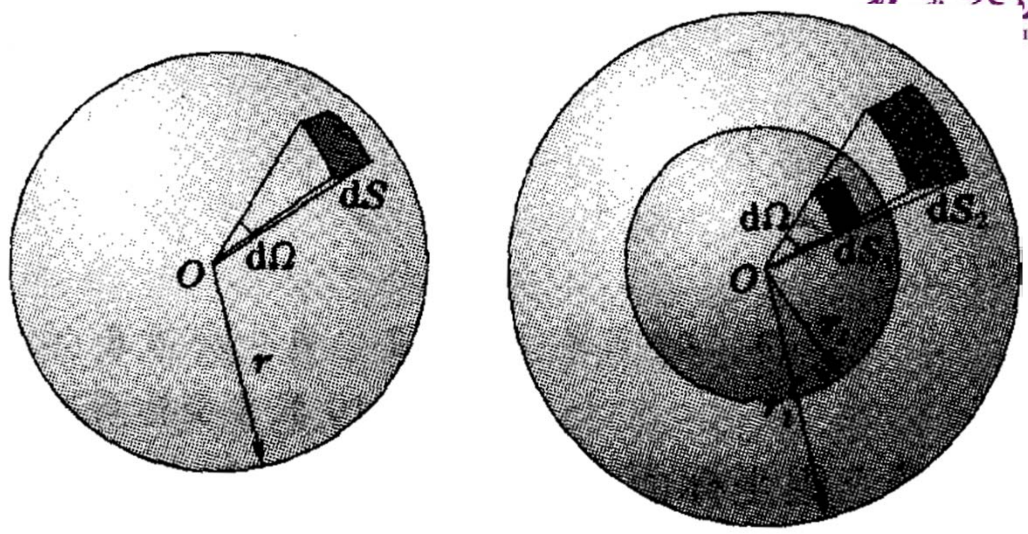
立体角 d*Ω*

$$d\Omega = \frac{dS_1}{r_1^2} = \frac{dS_2}{r_2^2}$$

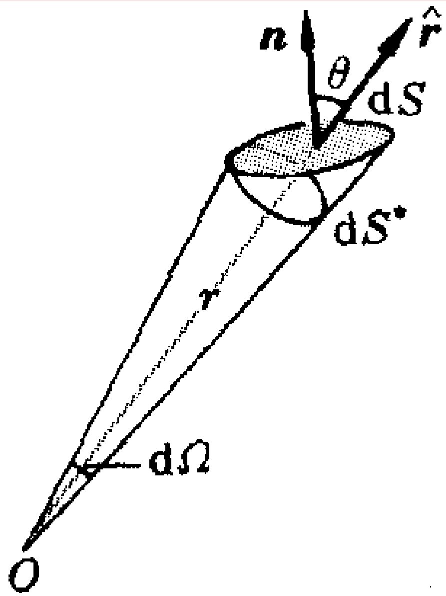
$$d\vec{S} \equiv dS\vec{n}$$

$$d\Omega = \frac{\vec{r}\cdot d\vec{S}}{r^3} = \frac{\hat{\vec{r}}\cdot d\vec{S}}{r^2}$$

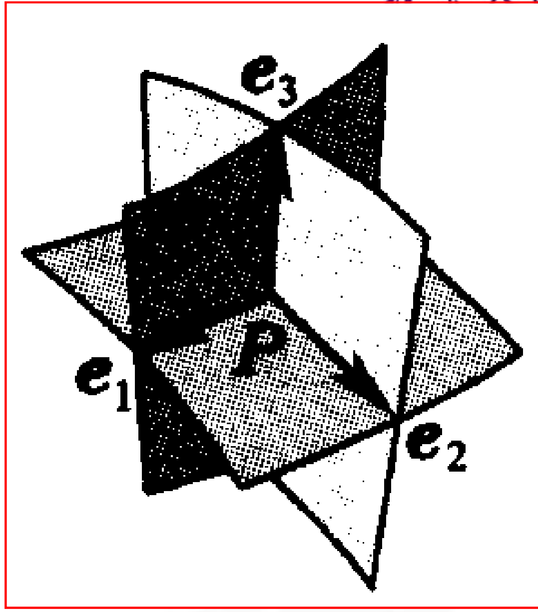
$$= \frac{dS\cos\theta}{r^2} = \frac{dS^*}{r^2}$$



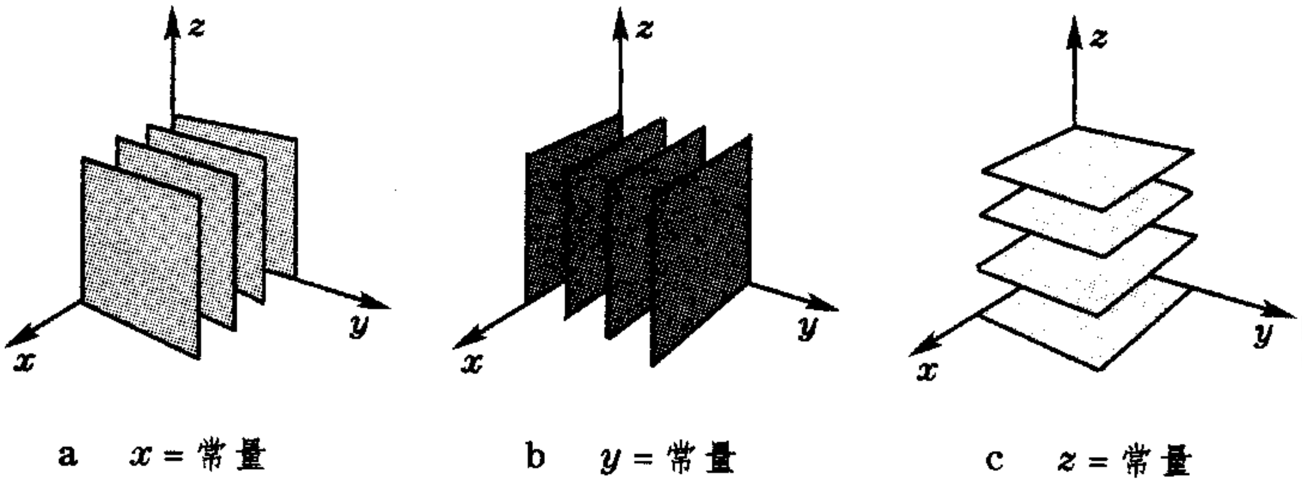
a 球面度　　b d*S* 正比于 r^2



- 复习一下什么是平面角



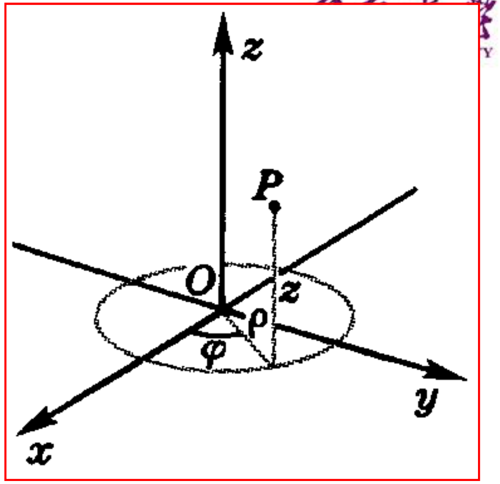
正交曲线坐标系：直角坐标系、坐标面

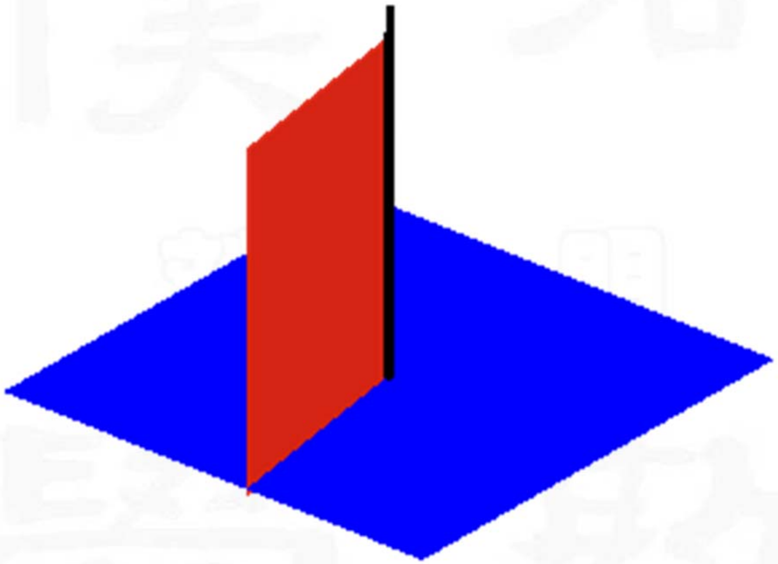


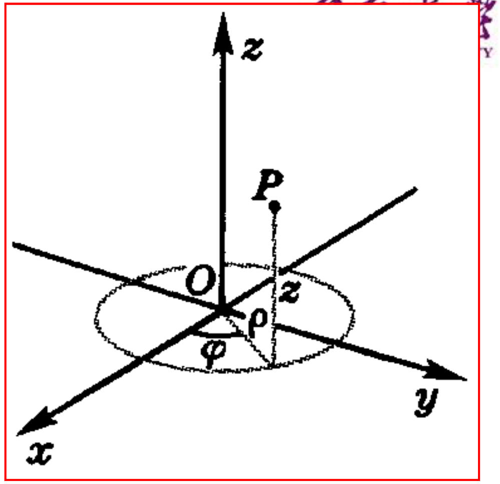
- 一个微元(线元、面积元、体积元)沿三基矢的线段元(dl_1, dl_2, dl_3)与基矢的三坐标变量微分(du_1, du_2, du_3):

$$\left.\begin{aligned} dl_1 &= h_1 du_1 \\ dl_2 &= h_2 du_2 \\ dl_3 &= h_3 du_3 \end{aligned}\right\} \xrightarrow{\text{直角坐标系}} \left\{\begin{aligned} du_1 &= dx \\ du_2 &= dy \\ du_3 &= dz \end{aligned}\right. \Rightarrow \left\{\begin{aligned} h_1 &= 1 \\ h_2 &= 1 \\ h_3 &= 1 \end{aligned}\right.$$

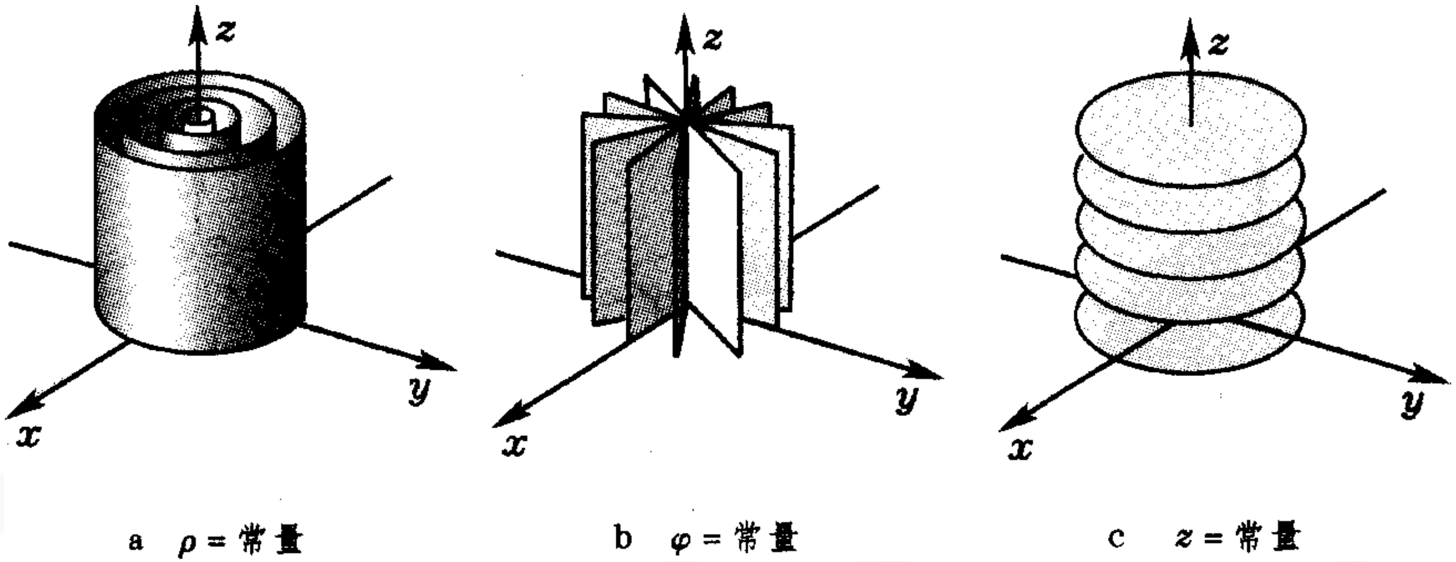
- 正交曲线坐标系：柱坐标系





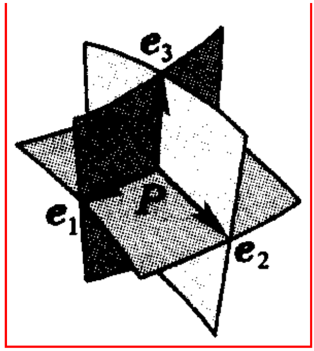


➢ 坐标面：



➢ 与直角坐标系之关系：

$$\left.\begin{aligned} x &= \rho\cos\varphi \\ y &= \rho\sin\varphi \\ z &= z \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \rho &= \sqrt{x^2+y^2} \\ \varphi &= \arctan(y/x) \\ z &= z \end{aligned}\right.$$



- 三基矢(e_1=e_ρ, e_2=e_φ, e_3=e_z)

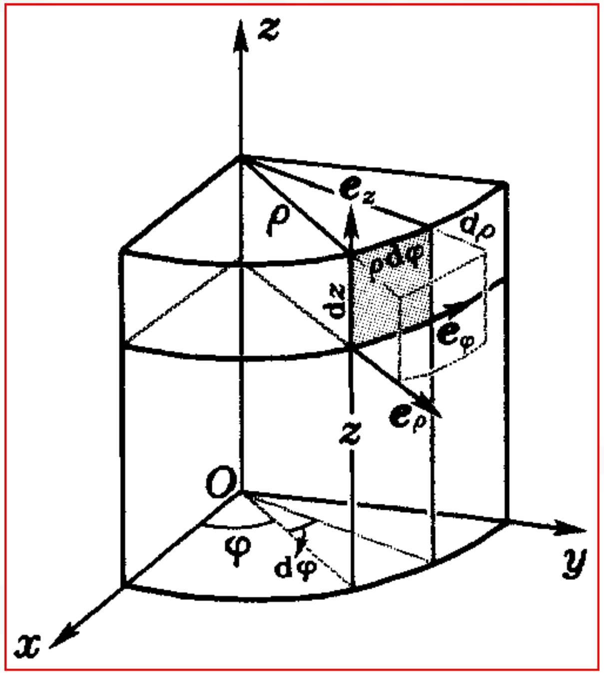
$$\vec{A}=A_\rho\vec{e}_\rho+A_\varphi\vec{e}_\varphi+A_z\vec{e}_z \Rightarrow \left.\begin{aligned} dl_\rho &= h_\rho d\rho \\ dl_\varphi &= h_\varphi d\varphi \\ dl_z &= h_z dz \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} h_\rho &= 1 \\ h_\varphi &= \rho \\ h_z &= 1 \end{aligned}\right.$$

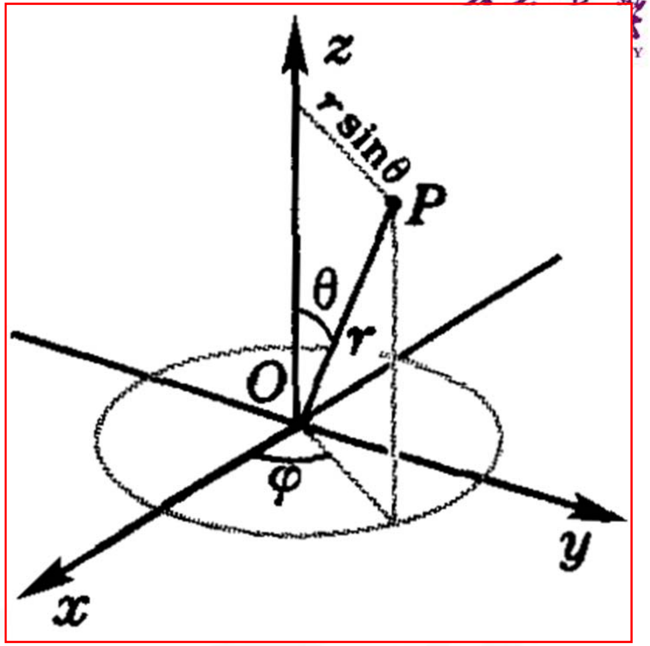
注意： e_φ 只是一个弧度，没有长度量纲

- 柱坐标系之面积元与体积元：

$$dS=dl_\varphi dl_z=\rho d\varphi dz$$

$$dV=dl_\rho dl_\varphi dl_z=\rho d\rho d\varphi dz$$



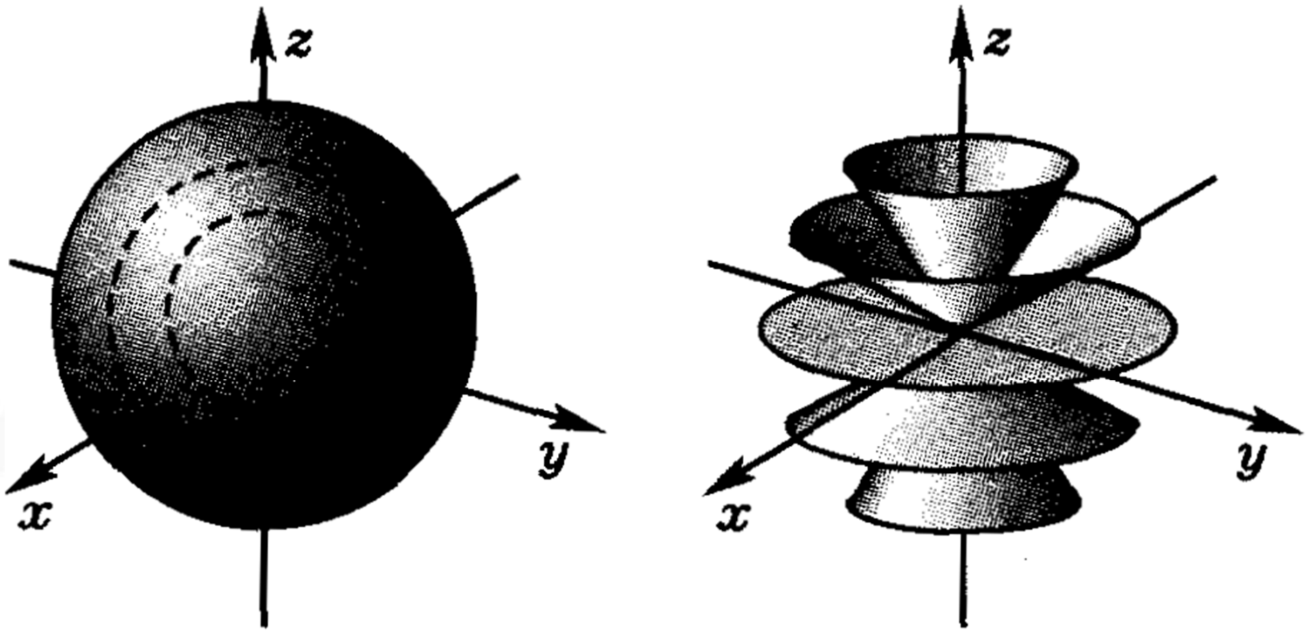


- 正交曲线坐标系：球坐标系

- 与直角坐标系之关系：

$$\left.\begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \\ z &= r\cos\theta \end{aligned}\right\} \Rightarrow \begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \arccos(z/\sqrt{x^2+y^2+z^2}) \\ \varphi = \arctan(y/x) \end{cases}$$

➢ 坐标面:



a　$r=$常量　　b　$\theta=$常量　　c　$\varphi=$常量

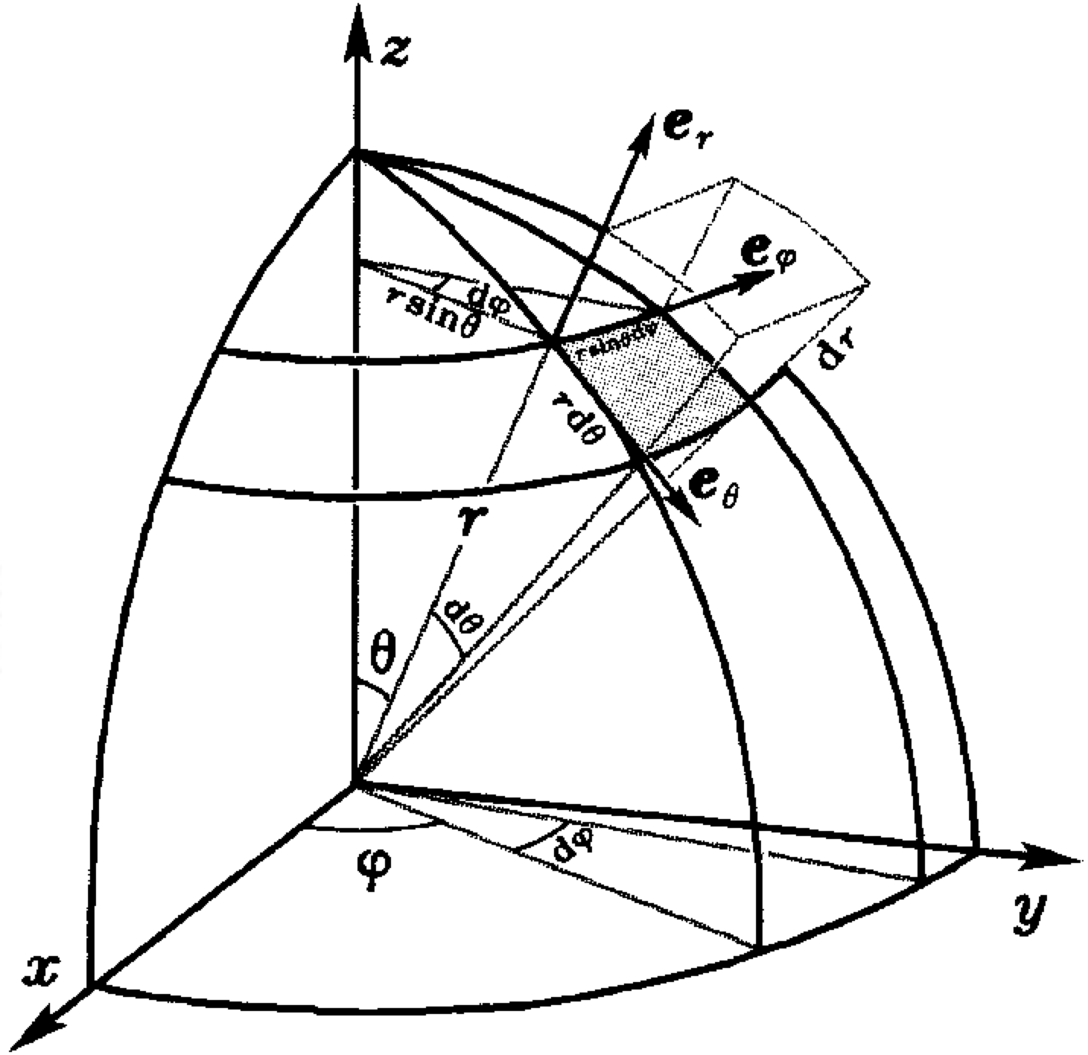
注意： e_θ和e_φ只是弧度基矢，没有长度量纲

➤ 三基矢(e_1=e_r, e_2=e_θ, e_3=e_φ)

$$\vec{A} = A_r\vec{e}_r + A_\theta\vec{e}_\theta + A_\varphi\vec{e}_\varphi$$

$$\Rightarrow \left.\begin{aligned} dl_r &= h_r dr \\ dl_\theta &= h_\theta d\theta \\ dl_\varphi &= h_\varphi d\varphi \end{aligned}\right\}$$

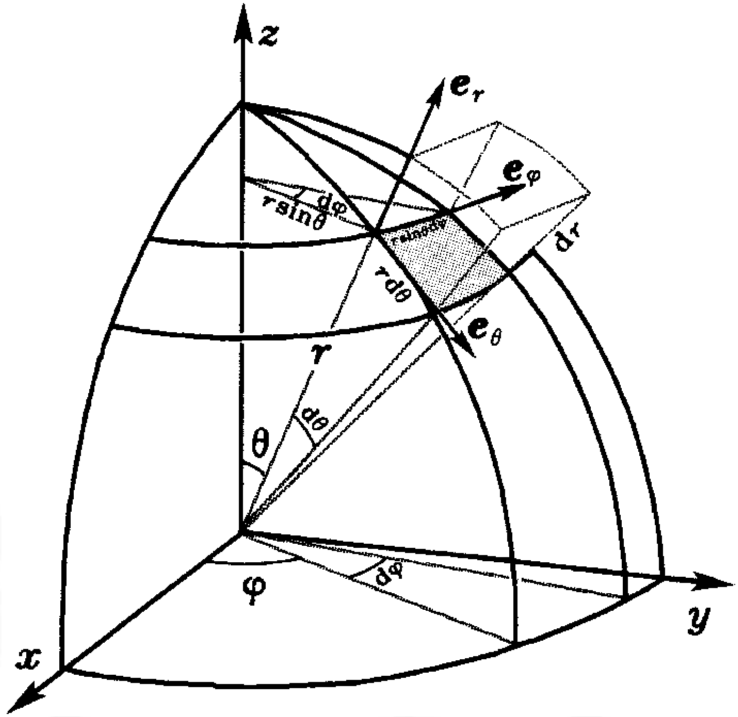
$$\Rightarrow \begin{cases} h_\rho = 1 \\ h_\theta = r \\ h_\varphi = r\sin\theta \end{cases}$$



- 球坐标系之面积与体积元：

$$dS = dl_\theta dl_\varphi = r^2 \sin\theta d\theta d\varphi$$

$$dV = dl_r dl_\theta dl_\varphi = r^2 \sin\theta dr d\theta d\varphi$$



- 求解球面立体角与球体体积(?)

$$d\Omega = \frac{\vec{r}\cdot d\vec{S}}{r^3} = \frac{dS^*}{r^2} = \frac{r^2 \sin\theta d\theta d\varphi}{r^2} = \sin\theta d\theta d\varphi$$

$$\Omega = \oiint \sin\theta d\theta d\varphi = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta = 4\pi$$

标量场与矢量场

- 标量 **Φ** 是空间坐标 $\boldsymbol{r} = (x, y, z)$的函数，称之为标量场
- 与标量场对应有等值面

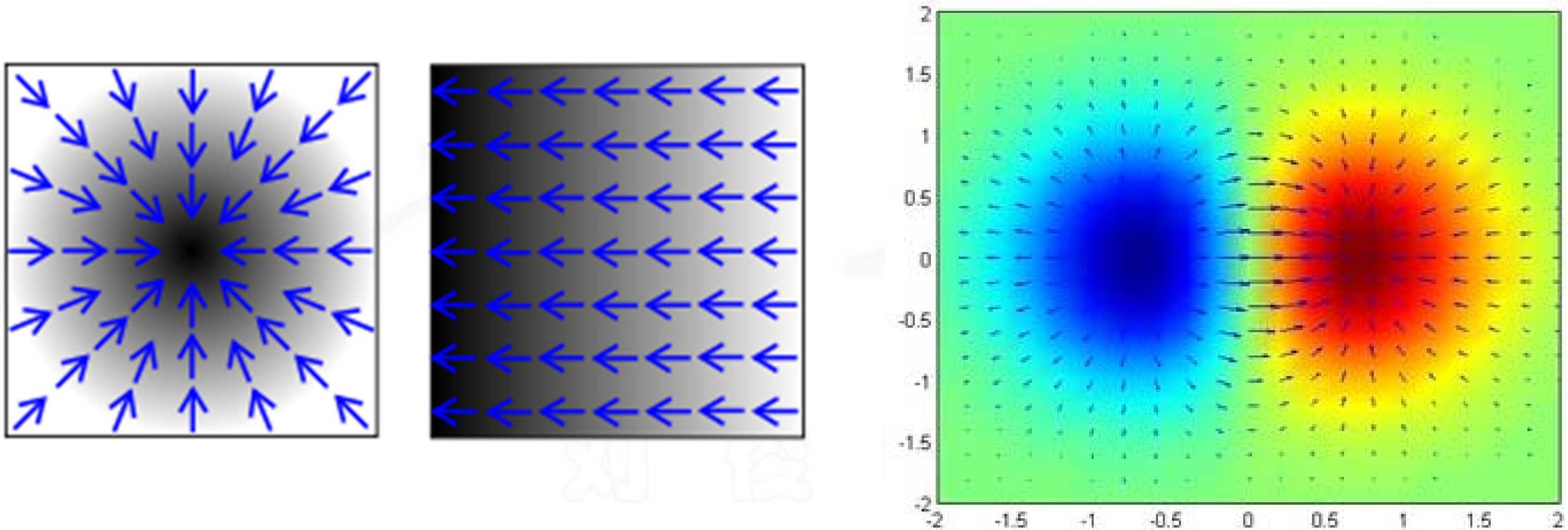
$$\boxed{\Phi = \Phi(x, y, z) \Leftrightarrow \Phi(x, y, z) = cons.}$$

- 矢量 $\boldsymbol{A}$ 是空间坐标 $\boldsymbol{r} = (x, y, z)$的函数，称之为矢量场

$$\boxed{\vec{A} = \vec{A}(x, y, z) \Rightarrow \begin{cases} A_x = A_x(x, y, z) \\ A_y = A_y(x, y, z) \\ A_z = A_z(x, y, z) \end{cases}}$$

标量场的梯度

➢ 梯度针对标量场定义，表示标量场在空间变化的剧烈程度



➢ 上图中衬度表示标量场，箭头表示此标量场之梯度

□ 标量场的梯度

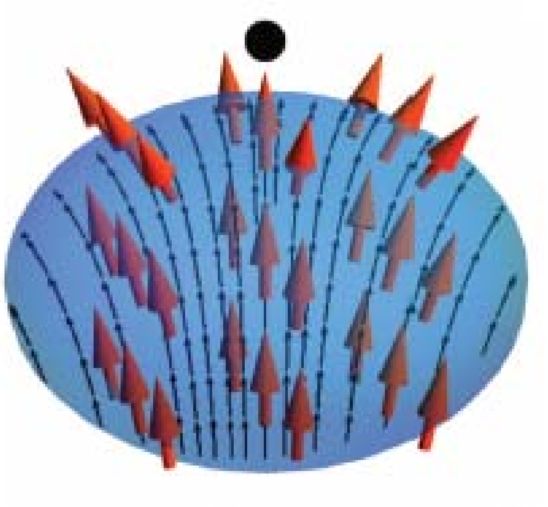
➢ 不同坐标系下标量场 Φ 的梯度表达：

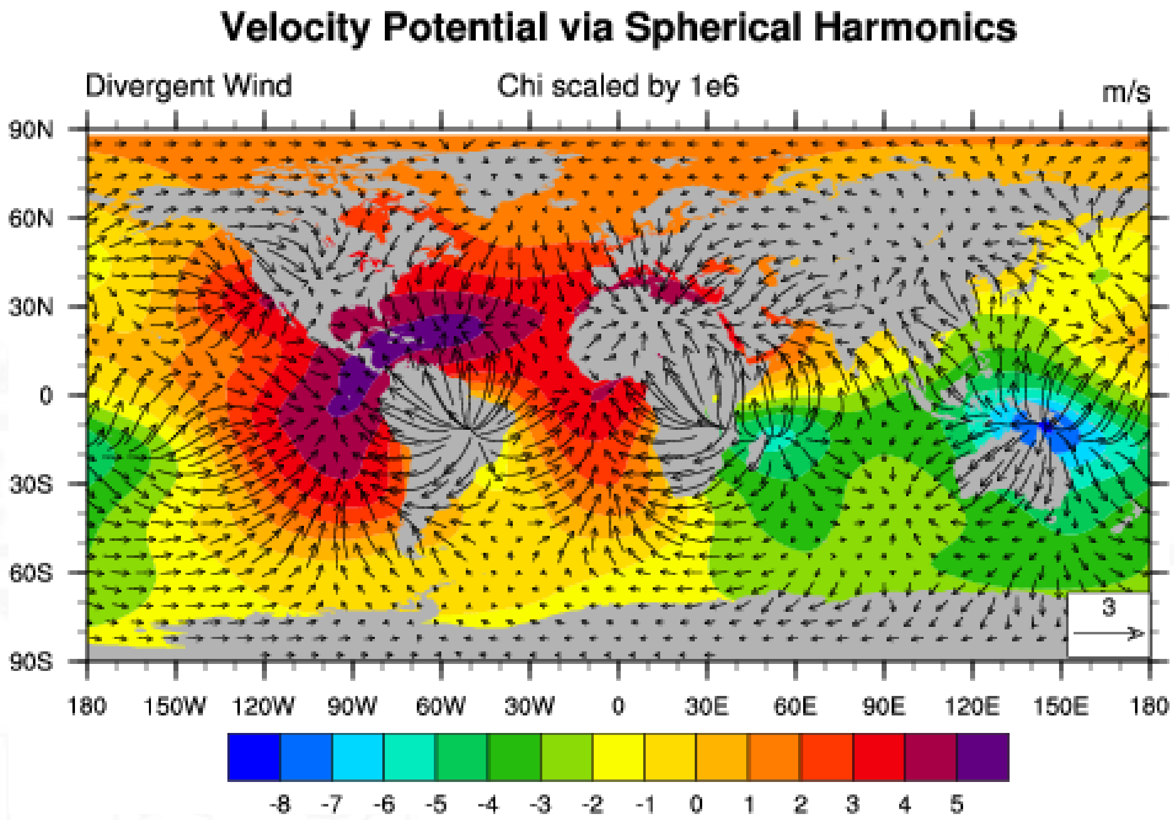
$$\nabla\Phi = \frac{\partial\Phi}{\partial x} i + \frac{\partial\Phi}{\partial y} j + \frac{\partial\Phi}{\partial z} k$$

$$\nabla\Phi = \frac{\partial\Phi}{\partial\rho}\vec{e}_\rho + \frac{1}{\rho}\frac{\partial\Phi}{\partial\varphi}\vec{e}_\varphi + \frac{\partial\Phi}{\partial z}\vec{e}_z$$

$$\nabla\Phi = \frac{\partial\Phi}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\varphi}\vec{e}_\varphi$$

矢量场的通量与散度





- 矢量场 $\boldsymbol{A}$ 通过截面 $\boldsymbol{S}$ 的通量 Φ_A，为标量：

$$\Phi_A = \iint\limits_{(S)} \vec{A} \cdot d\vec{S} = \iint\limits_{(S)} A\cos\theta dS$$

$$\lim_{x\to 0}\frac{\sin x}{x}=?$$

◘ 矢量场的通量与散度

➢ 设 ***S*** 为闭合面，包含体积 **Δ*V***，则矢量场 ***A*** 的散度：

$$\Phi_A=\oiint_{(S)}\vec{A}\cdot d\vec{S}\Rightarrow \Delta V\to 0,\quad \Phi_A\to 0$$

$$div\vec{A}=\nabla\cdot\vec{A}=\lim_{\Delta V\to 0}\frac{\Phi_A}{\Delta V}=\lim_{\Delta V\to 0}\left[\oiint_{(S)}\vec{A}\cdot d\vec{S}\Big/\Delta V\right]$$

$$\Delta V \to 0,\ \Phi_A \to 0$$

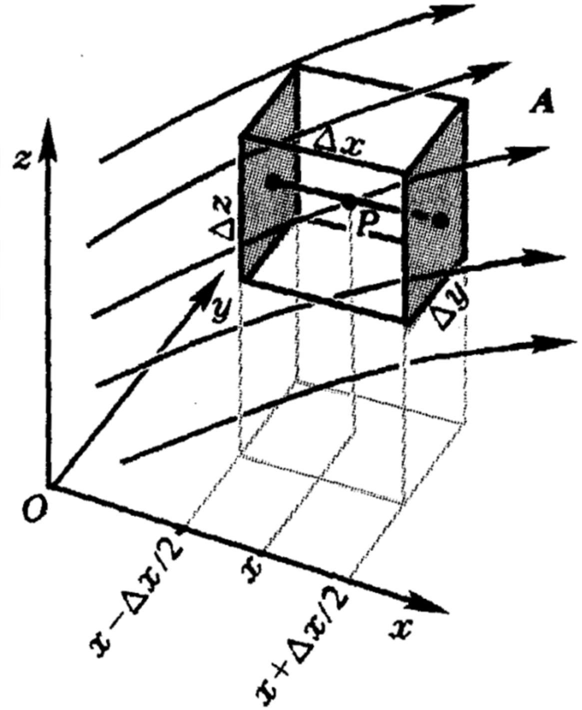
□ 矢量场散度的坐标表达(直角坐标系)

$$\Phi_A = \oiint_{(S)} \vec{A}\cdot d\vec{S}$$

$$= \oiint_{(S)} A_x dS_x + A_y dS_y + A_z dS_z$$

$$= \oiint_{(S)} A_x dydz + \oiint_{(S)} A_y dxdz + \oiint_{(S)} A_z dxdy$$

$$= \Phi_x + \Phi_y + \Phi_z$$



选择一个小长方体单元运算

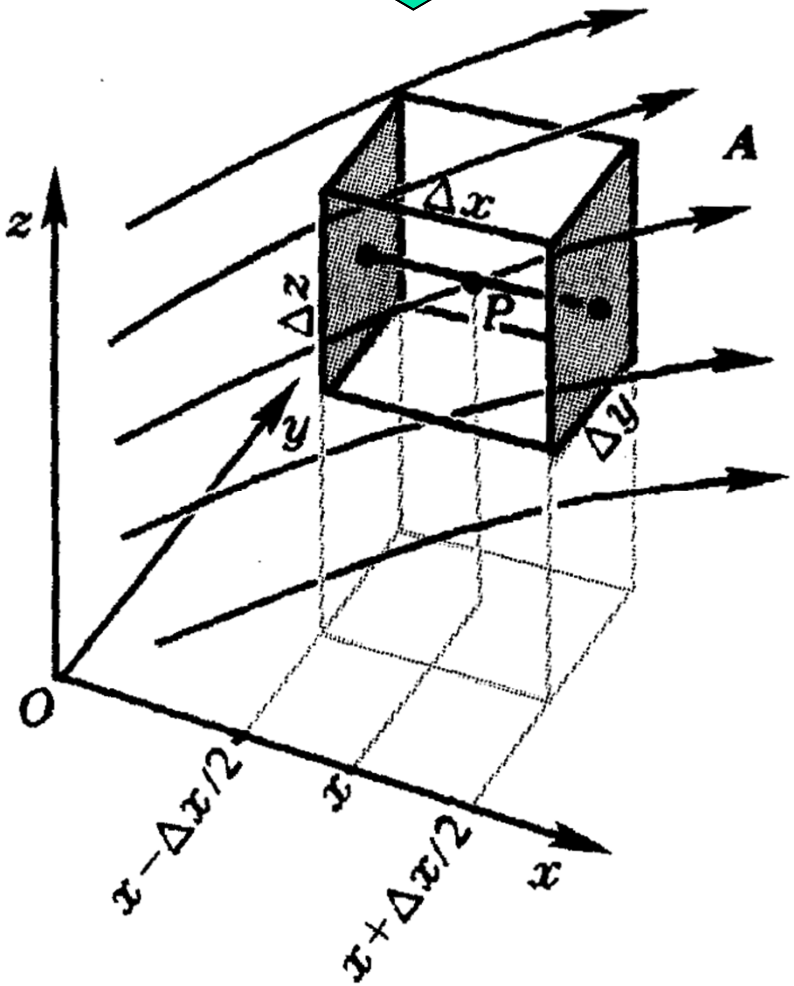
$$\Delta V \to 0, \quad \Phi_A \to 0$$

矢量场散度的坐标表达(直角坐标系)

$$\Phi_x = A_x(x+\Delta x/2, y, z)\Delta y\Delta z - A_x(x-\Delta x/2, y, z)\Delta y\Delta z$$

$$\because A_x(x\pm\Delta x/2, y, z) = A_x(x, y, z) \pm \frac{\partial A_x}{\partial x}\frac{\Delta x}{2} + O(\Delta x^2)$$

$$\therefore \Phi_x = \frac{\partial A_x}{\partial x}\Delta x\Delta y\Delta z + O(\Delta i^4)$$



$$\Phi_i = \frac{\partial A_i}{\partial i}\Delta x\Delta y\Delta z + O(\Delta i^4), \quad (i = x, y, z)$$

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$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

- 矢量场散度(继续)

$$\Phi = \oiint_{\Delta V = \Delta x \Delta y \Delta z} \vec{A} \cdot d\vec{S} = \sum_i \Phi_i$$

$$= \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) \Delta x \Delta y \Delta z + O(\Delta i^2)$$

$$div\vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta V \to 0} \frac{\Phi}{\Delta V} = \lim_{\Delta V \to 0} \left[\oiint_{(S)} \vec{A} \cdot d\vec{S} \Big/ \Delta V \right]$$

$$= \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)$$

➢ 矢量场散度(柱、球坐标系)

$$div\vec{A} = \nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$div\vec{A} = \nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

高斯定理(数学而非物理学)：联系通量与散度

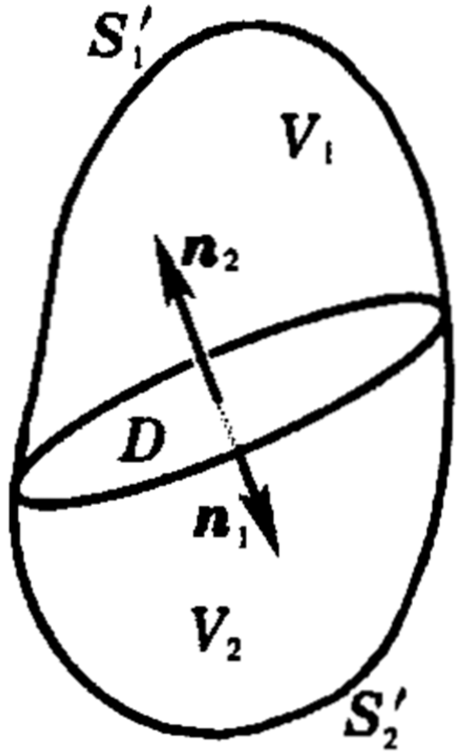
- 体积为 ***V*** 的闭合面 ***S*** 内矢量场 ***A***：

$$\nabla\cdot\vec{A}=\lim_{\Delta V\to 0}\frac{\Phi}{\Delta V}\Rightarrow(\nabla\cdot\vec{A})\Delta V\approx\Phi=\oint\!\!\!\oint_{(S)}\vec{A}\cdot d\vec{S}$$

$$\Downarrow$$

$$\iiint_{(V)}(\nabla\cdot\vec{A})dV=\oint\!\!\!\oint_{(S)}\vec{A}\cdot d\vec{S}$$

- 严格论证：考虑任意闭合曲面 S，体积为 V



- 体积为 *V* 的闭合面 *S* 内矢量场 *A*：

$$\left.\begin{aligned} V &= V_1 + V_2 \\ S &= S_1' + S_2' \end{aligned}\right\} \xrightarrow{D} \left\{\begin{aligned} S_1 &= S_1' + D \\ S_2 &= S_2' + D \end{aligned}\right.$$

$$\Phi_{A1} = \oiint_{(S_1)} \vec{A}\cdot d\vec{S}_1 = \iint_{(S_1')} \vec{A}\cdot d\vec{S}_1 + \iint_{(D)} \vec{A}\cdot d\vec{S}_1$$

$$\Phi_{A2} = \oiint_{(S_2)} \vec{A}\cdot d\vec{S}_2 = \iint_{(S_2')} \vec{A}\cdot d\vec{S}_2 + \iint_{(D)} \vec{A}\cdot d\vec{S}_2$$

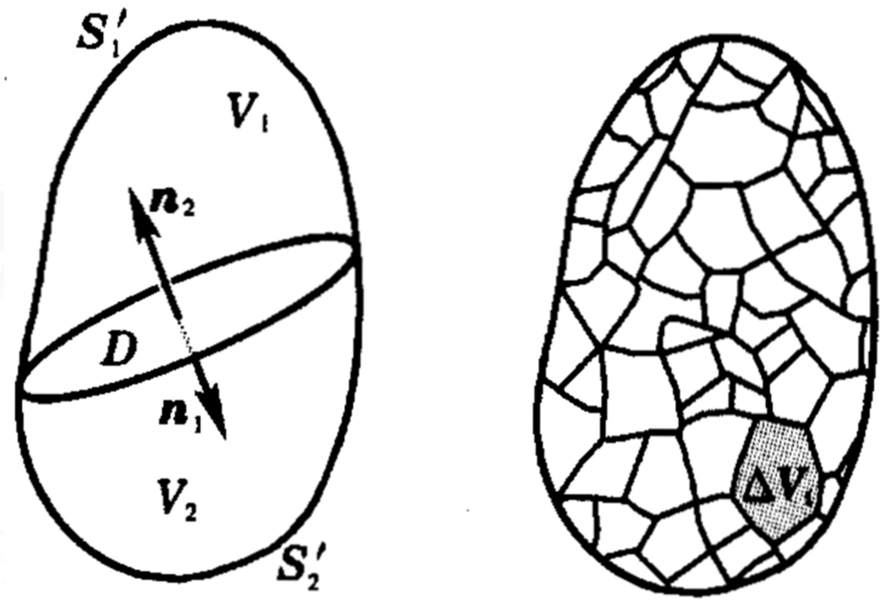
➢ 微分操作

$$\because \iint_{(D)} \vec{A}\cdot d\vec{S}_1 = -\iint_{(D)} \vec{A}\cdot d\vec{S}_2$$

$$\therefore \Phi_A = \Phi_{A1} + \Phi_{A2} =$$

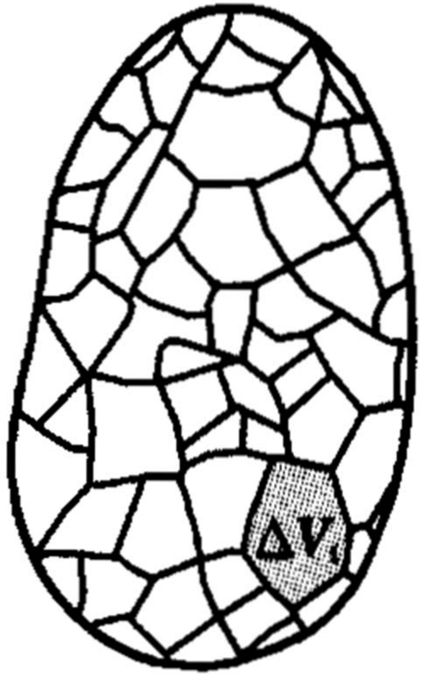
$$= \oint\!\!\!\oint_{(S_1)} \vec{A}\cdot d\vec{S}_1 + \oint\!\!\!\oint_{(S_2)} \vec{A}\cdot d\vec{S}_2$$

$$= \oint\!\!\!\oint_{(S)} \vec{A}\cdot d\vec{S}$$



a b

➢ 微分操作



$$\because \Phi_A = \sum_{i=1}^{n} \Phi_{Ai}$$

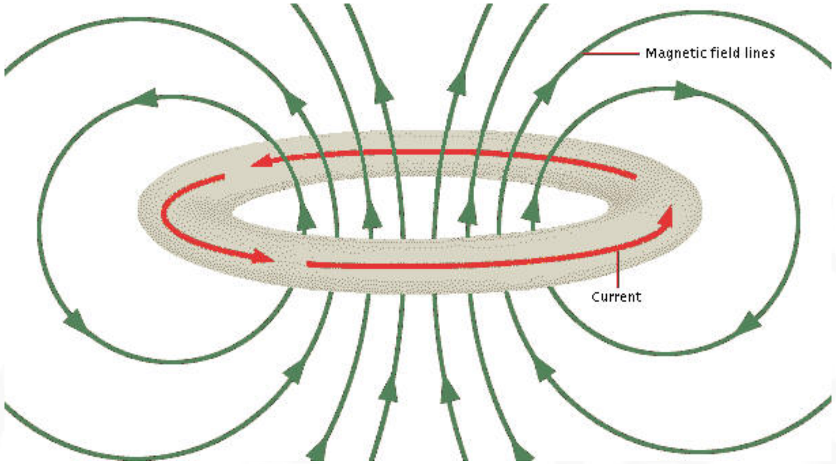
$$\therefore \Phi_{Ai} = \oiint_{(S_i)} \vec{A} \cdot d\vec{S}_i = (div\vec{A})_i \, dV_i$$

$$\therefore \Phi_A = \sum_{i=1}^{n} (div\vec{A})_i \, dV_i = \iiint_{(V)} div\vec{A} dV = \iiint_{(V)} (\nabla \cdot \vec{A}) dV$$

$$\therefore \oiint_{(S)} \vec{A} \cdot d\vec{S} = \iiint_{(V)} (\nabla \cdot \vec{A}) dV$$

矢量的空间守恒性质

矢量场的环量与旋度



- 矢量场 ***A*** 沿闭合回路 ***L*** 之线积分为环量 Γ_A，为标量：

$$\boxed{\Gamma_A = \oint_{(L)} \vec{A}\cdot d\vec{l} = \oint_{(L)} A\cos\theta dl}$$

矢量场的环量与旋度

- 设 ΔS 为 L 包围的面积，n 为 ΔS 右旋单位法向量，则矢量场 A 的旋度定义为 Γ_A 与 ΔS 之极限比。但是这个比值是标量，考虑其在在 n 上的投影，即为矢量：

$$\left.\begin{matrix} rot\vec{A} \\ \nabla\times\vec{A} \end{matrix}\right\}_n \Rightarrow (rot\vec{A})_n = \lim_{\Delta S\to 0}\frac{\Gamma_A}{\Delta S} = \lim_{\Delta S\to 0}\left[\oint_{(L)}\vec{A}\cdot d\vec{l}\Big/\Delta S\right]$$

- 矢量场旋度的坐标表达

$$(rot\vec{A})_n = (rot\vec{A})_x \vec{i} + (rot\vec{A})_y \vec{j} + (rot\vec{A})_z \vec{k}$$

$$= \lim_{\Delta S \to 0} \left[\frac{\oint_{(L_x)} \vec{A} \cdot d\vec{l}}{\Delta S_{(L_x)}} \vec{i} + \frac{\oint_{(L_y)} \vec{A} \cdot d\vec{l}}{\Delta S_{(L_y)}} \vec{j} + \frac{\oint_{(L_z)} \vec{A} \cdot d\vec{l}}{\Delta S_{(L_z)}} \vec{k} \right]$$

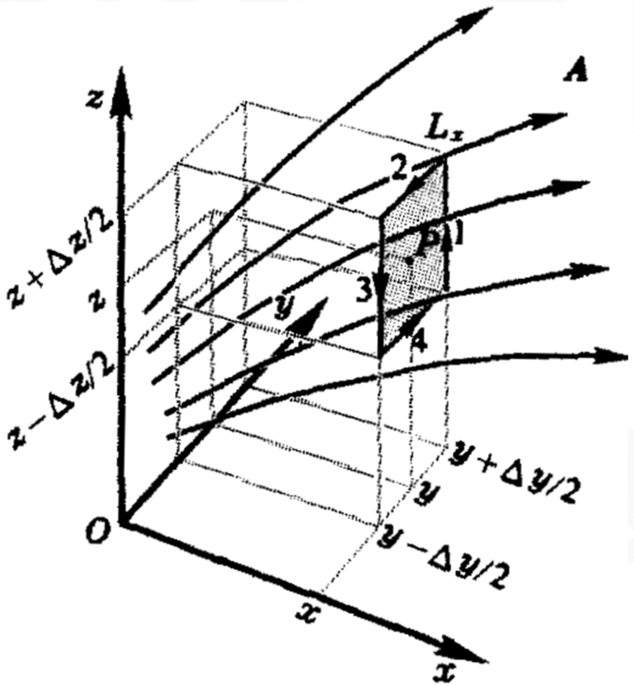
- 注意旋度的定义中 ***n*** 为 **ΔS** 右旋法向量，注定了矢量场 ***A*** 的矢量定义

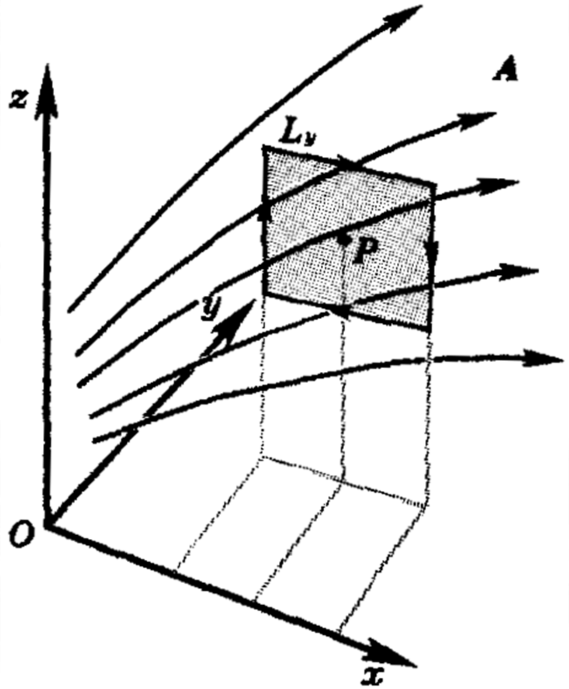
➢ 矢量场旋度的坐标表达

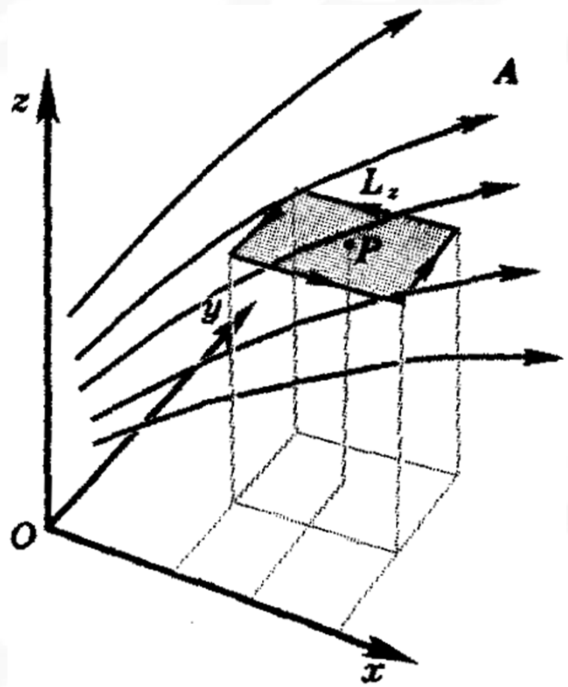
$$\oint_{(L_x)} \vec{A}\cdot d\vec{l}$$

$$\oint_{(L_y)} \vec{A}\cdot d\vec{l}$$

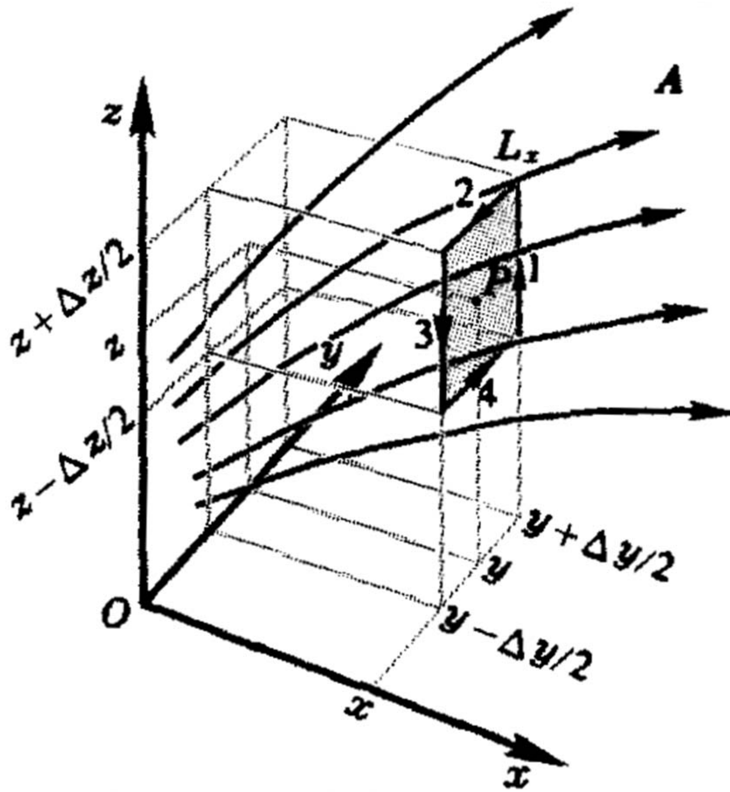
$$\oint_{(L_z)} \vec{A}\cdot d\vec{l}$$







- 闭合回路 L_x 由矩形 1-2-3-4 围绕点 $P(x,y,z)$ 构成，矢量场 A 在四个边 1-2-3-4 的垂直分量 A_x 与各边垂直，环量 Γ 为零；不为零的平行分量分别为：



$$1: A_z(x, y+\Delta y/2, z), \quad 2: -A_y(x, y, z+\Delta z/2)$$

$$3: -A_z(x, y-\Delta y/2, z), \; 4: A_y(x, y, z-\Delta z/2)$$

$$\oint_{(L_x)} \vec{A}\cdot d\vec{l} = A_z(x, y+\Delta y/2, z)\Delta z - A_y(x, y, z+\Delta z/2)\Delta y$$

$$- A_z(x, y-\Delta y/2, z)\Delta z + A_y(x, y, z-\Delta z/2)\Delta y$$

- 围绕点 $P(x,y,z)$ 对 A_y 和 A_z 作级数展开:

$$A_y(x,y,z\pm\Delta z/2)=A_y(x,y,z)\pm\frac{\partial A_y}{\partial z}\frac{\Delta z}{2}+O(\Delta z^2)$$

$$A_z(x,y\pm\Delta y/2,z)=A_z(x,y,z)\pm\frac{\partial A_z}{\partial y}\frac{\Delta y}{2}+O(\Delta y^2)$$

$$\therefore\oint_{(L_x)}\vec{A}\cdot d\vec{l}=\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\Delta y\Delta z+O(\Delta l^3)$$

- 围绕点 **P(x,y,z)** 求 **A** 的旋度在 **x** 方向的投影：

$$(\nabla\times\vec{A})_x=\lim_{\Delta S\to 0}\left[\oint_{(L_x)}\vec{A}\cdot d\vec{l}\Big/\Delta S\right]=$$

$$=\lim_{\substack{\Delta x\to 0\\ \Delta y\to 0\\ \Delta z\to 0}}\left[\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\Delta y\Delta z+O(\Delta l^3)\Big/\Delta y\Delta z\right]$$

$$=\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}$$

- 围绕点 ***P*(*x*,*y*,*z*)** 求 ***A*** 的旋度在 ***x*/*y*/*z*** 方向的投影:

$$\nabla\times\vec{A}=\left\{\begin{aligned}(\nabla\times\vec{A})_x&=\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\\(\nabla\times\vec{A})_y&=\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\\(\nabla\times\vec{A})_z&=\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\end{aligned}\right\}=\begin{pmatrix}i&j&k\\\frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\A_x&A_y&A_z\end{pmatrix}$$

➤ 柱坐标系下旋度的表达：

$$\nabla\times\vec{A}=\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi}-\frac{\partial A_\varphi}{\partial z}\right)\vec{e}_\rho+\left(\frac{\partial A_\rho}{\partial z}-\frac{\partial A_z}{\partial\rho}\right)\vec{e}_\varphi+\frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial\rho}-\frac{\partial A_\rho}{\partial\varphi}\right)\vec{e}_z$$

➢ 球坐标系下旋度的表达:

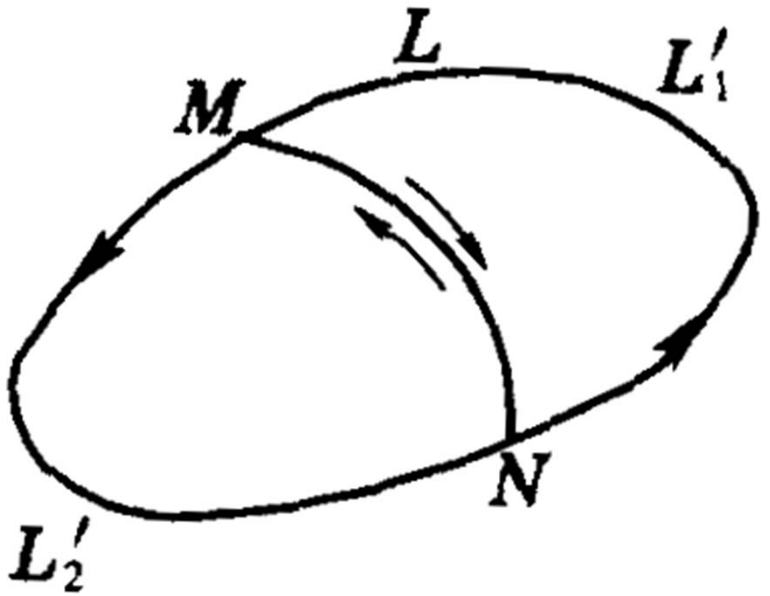
$$\nabla\times\vec{A}=\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta A_\varphi)-\frac{\partial A_\theta}{\partial\varphi}\right)\vec{e}_r+\left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi}-\frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r}\right)\vec{e}_\theta+\frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r}-\frac{\partial A_r}{\partial\theta}\right)\vec{e}_\varphi$$

- **Stokes定理(数学而非物理)：联系环量与旋度**

$$(rot\vec{A})_n = \lim_{\Delta S \to 0} \frac{\Gamma_A}{\Delta S} \Rightarrow (rot\vec{A})_n \Delta S \approx \Gamma_A = \oint_{(L)} \vec{A} \cdot d\vec{l}$$

$$\Downarrow$$

$$\oint_{(L)} \vec{A} \cdot d\vec{l} = \iint_{(S)} (rot\vec{A})_n \cdot d\vec{S} = \iint_{(S)} (\nabla \times \vec{A}) \cdot d\vec{S}$$



- 严格论证：考虑任意闭环路 *L*

- 回路 *L* 被 *MN* 分割

$$L = L_1' + L_2' \xrightarrow{MN} \begin{cases} L_1 = L_1' + MN \\ L_2 = L_2' + MN \end{cases}$$

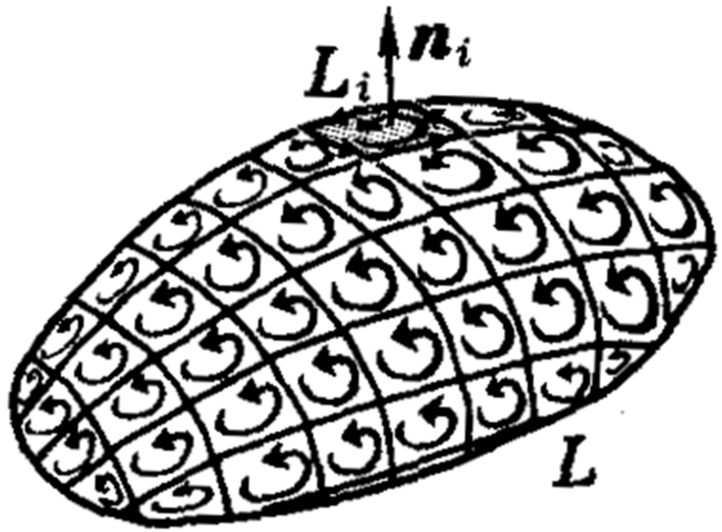
$$\Gamma_{A1} = \oint_{(L_1)} \vec{A}\cdot d\vec{l} = \int_{(L_1')} \vec{A}\cdot d\vec{l} + \int_M^N \vec{A}\cdot d\vec{l}$$

$$\Gamma_{A2} = \oint_{(L_2)} \vec{A}\cdot d\vec{l} = \int_{(L_2')} \vec{A}\cdot d\vec{l} + \int_N^M \vec{A}\cdot d\vec{l}$$

- 继续

$$\Gamma_A = \Gamma_{A1} + \Gamma_{A2} = \int_{(L_1')} \vec{A}\cdot d\vec{l} + \int_{(L_2')} \vec{A}\cdot d\vec{l} = \oint_{(L)} \vec{A}\cdot d\vec{l}$$

- 微分操作



$$(\nabla\times\vec{A})_n = \lim_{\Delta S\to 0}\left[\oint_{(L)}\vec{A}\cdot d\vec{l}\Big/\Delta S\right]$$

- 微分操作继续

$$\because \Gamma_A = \sum_{i=1}^{n}\Gamma_{Ai}$$

$$\therefore \Gamma_{Ai} = \oint_{(L_i)}\vec{A}\cdot d\vec{l} = (\nabla\times\vec{A})_{n_i}\Delta S_i = (\nabla\times\vec{A})\cdot\Delta\vec{S}_i$$

$$\therefore \Gamma_A = \oint_{(L)}\vec{A}\cdot d\vec{l} = \sum_{i=1}^{n}(\nabla\times\vec{A})\cdot\Delta\vec{S}_i = \iint_{(S)}(\nabla\times\vec{A})\cdot d\vec{S}$$

$$\therefore \oint_{(L)}\vec{A}\cdot d\vec{l} = \iint_{(S)}(\nabla\times\vec{A})\cdot d\vec{S}$$

➢ 梯度、散度、旋度：用算符 ∇ 对 Φ、**A**、**A** 计算也可以得到

$$\nabla\Phi,\quad \nabla\cdot\vec{A},\quad \nabla\times\vec{A}$$

$$\nabla = \frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k$$

- 一些矢量公式：一阶微分(注意标量与矢量)

$$\nabla(AB) = (\nabla A)B + A(\nabla B)$$

$$\nabla(\vec{A}\cdot\vec{B}) = (\vec{A}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{A} + \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A})$$

$$\nabla\cdot(A\vec{B}) = \nabla A\cdot\vec{B} + A\nabla\cdot\vec{B}$$

$$\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot\nabla\times\vec{A} - \vec{A}\cdot\nabla\times\vec{B}$$

$$\nabla\times(A\vec{B}) = A\nabla\times\vec{B} + \nabla A\times\vec{B}$$

$$\nabla\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B} + \vec{A}(\nabla\cdot\vec{B}) - \vec{B}(\nabla\cdot\vec{A})$$

- 一些矢量公式：二阶微分(注意标量与矢量)

$$\nabla\times\nabla A=0$$

$$\nabla\cdot\nabla\times\vec{A}=0$$

$$\nabla\times(\nabla\times\vec{A})=\nabla(\nabla\cdot\vec{A})-\nabla\cdot\nabla\vec{A}$$

$$\nabla\cdot\nabla=\nabla^2$$

$$\vec{A}\times(\vec{B}\times\vec{C})=(\vec{A}\cdot\vec{C})\vec{B}-(\vec{A}\cdot\vec{B})\vec{C}$$

- 这些矢量运算应该在高等数学中学习到
- 由于课程安排原因
- 在本课程和未来电动力学中经常用到
- 要找一些题目练习，熟练掌握

- **Baby,** 在这个课程里，你们最好是做事求精、做人中庸

1.5 Given two vectors $\mathbf{A} = \mathbf{a_x} + \mathbf{a_y} + \mathbf{a_z}$ and $\mathbf{B} = 2\mathbf{a_x} + 3\mathbf{a_y} + 6\mathbf{a_z}$, find

(a) the scalar product $\mathbf{A} \bullet \mathbf{B}$
(b) the angle between $\mathbf{A}$ and $\mathbf{B}$
(c) the projection of $\mathbf{A}$ on $\mathbf{B}$ and the projection of $\mathbf{B}$ on $\mathbf{A}$
(d) the vector product $\mathbf{A} \times \mathbf{B}$
(e) the area of the parallelogram spanned by $\mathbf{A}$ and $\mathbf{B}$

In addition, carefully illustrate these vectors if you solve the problem using MATLAB.

1.18 A vector field $\mathbf{A}$ is given as $\mathbf{A}(x, y) = 5x^2 y\mathbf{a}_x + 3x\mathbf{a}_y$. Find

(1) the unit vectors of $\mathbf{A}$ at (1, –2) and (2, 3)
(2) plot A_y versus x for x from –2 to 2 using MATLAB
(3) plot A_x versus x and y for $-2 \le x \le 2$ and $-2 \le y \le 2$ using MATLAB function *surf*
(4) plot $\mathbf{A}$ using MATLAB function *quiver* for $-2 \le x \le 2$ and $-2 \le y \le 2$
(5) draw the contour plot of $|\mathbf{A}|$ using MATLAB function *contour* for $-2 \le x \le 2$ and $-2 \le y \le 2$

1.26 Express vector field $\mathbf{G}(\rho, \phi, z) = \rho\cos\phi\mathbf{a}_\rho + z\mathbf{a}_\phi + 2\tan\phi\mathbf{a}_z$ in Cartesian and spherical systems.

1.28 Express vector field $\mathbf{D}(r, \theta, \phi) = \sin\phi\mathbf{a}_r + r\mathbf{a}_\theta - 3r^2\sin\theta\tan\phi\mathbf{a}_\phi$ in Cartesian and cylindrical systems.

1.39 Evaluate the surface integral $\iint \mathbf{A} \cdot \mathbf{ds}$ if $\mathbf{A} = 3\rho^2 z\cos\phi \mathbf{a}_\rho + 5z\mathbf{a}_z$ and the surface is defined by $\rho = 3$, $15° \le \phi \le 175°$, $2 \le z \le 4$. The normal direction of the surface is $\mathbf{a}_\rho$.

1.43 Evaluate the closed-surface integral of the vector $\mathbf{A} = 2r\mathbf{a}_r + 4\sin\theta \mathbf{a}_\theta + 5r\cos(2\phi)\mathbf{a}_\phi$ if the surface is the outer surface of the volume defined by $2 \le r \le 6.3$, $5° \le \theta \le 87°$, $10° \le \phi \le 95°$.

1.53 Find the gradient of the following scalar field

(1) $V = 4xy^2z^2$

(2) $U = 2\rho^2 z\cos\phi$

(3) $\Phi = r^3\sin(2\theta)\sin\phi$

1.55 Find the divergence of the following vector fields

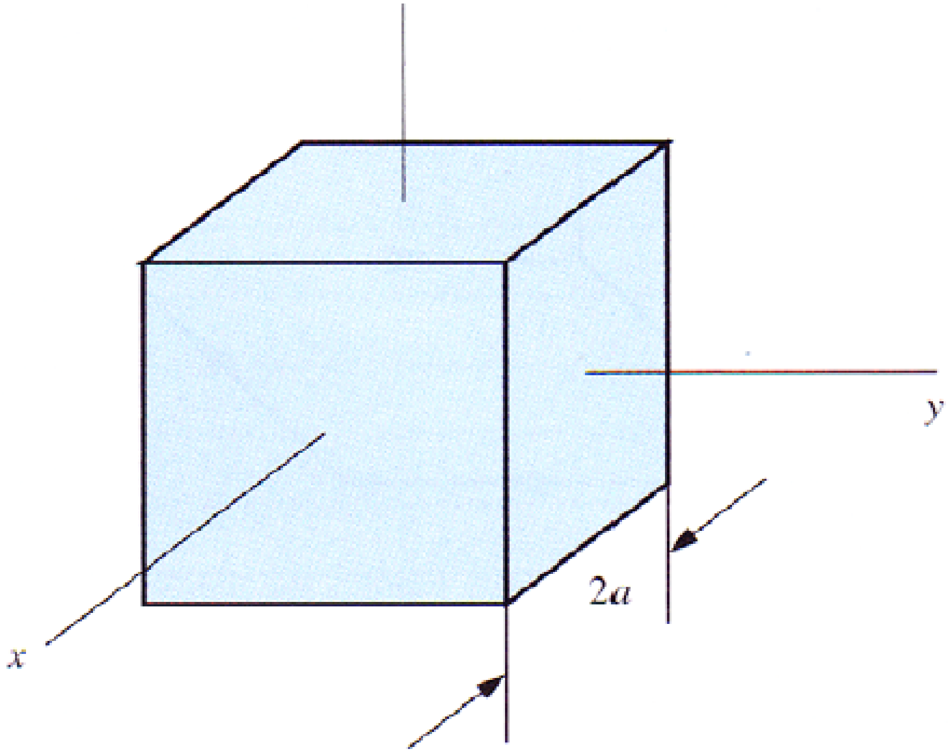
(1) $\mathbf{A} = x^3 y\sin(\pi z)\mathbf{a_x} + xy\sin(\pi z)\mathbf{a_y} + x^2y^2z^2\mathbf{a_z}$

(2) $\mathbf{F} = 2\rho z^2\cos\phi \mathbf{a}_\rho + 4\rho\sin\phi \mathbf{a}_\phi + 5\rho z\mathbf{a}_z$

(3) $\mathbf{G} = 2r\mathbf{a}_r + 4\sin\theta \mathbf{a}_\theta + 5r\cos(2\phi)\mathbf{a}_\phi$

Also evaluate their values at the point (1,1,1).

1.56 Show that the divergence theorem is valid for the cube below, located at the center of a Cartesian coordinate system, for a vector $\mathbf{A} = x\mathbf{a}_x + 2\mathbf{a}_y$.



1.61 Show that $\nabla \times \mathbf{A} = 0$ if $\mathbf{A} = 1/\rho\,\mathbf{a}_\rho$ in cylindrical coordinates.

1.62 Show that $\nabla \times \mathbf{A} = 0$ if $\mathbf{A} = r^2\mathbf{a}_r$ in spherical coordinates.

1.65 In Cartesian coordinates, verify that $\nabla \times (\phi\mathbf{A}) = (\nabla\phi) \times \mathbf{A} + \phi\nabla \times \mathbf{A}$ where $\mathbf{A} = xyz(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$ and $\phi = 3xy + 4zx$ by carrying out the indicated derivatives.