2018-11-01 Second-Order ODEs (4.3)

Pre-lecture:

$y'' + ay' + by = 0$

When we have equations like this, we look for solutions in the form $y = e^{rt}$ and then see which values of r solve the ODE.

$r^2 e^{rt} + are^{rt} + be^{rt} = 0$

e^{rt} cannot be 0, therefore, $r^2 + ar + b = 0$

This is a quadratic equation and solving it gives us two values of r (r1 and r2). Both values of r work. Solutions to the main equation are any linear combination of the two values, which means:

$y(t) = c_1 e^{r1t} + c_2 e^{r2t}$

Continuation:

1. Model the position y(t) of a keypress of one laptop key

Newton's 2nd Law F = ma

y(t) = position measured from equilibrium t seconds after
a) the finger starts pressing the key
b) the finger lets go of the key

a) key being pressed

F = finger + (gravity) + spring (Hooke's law) + damping

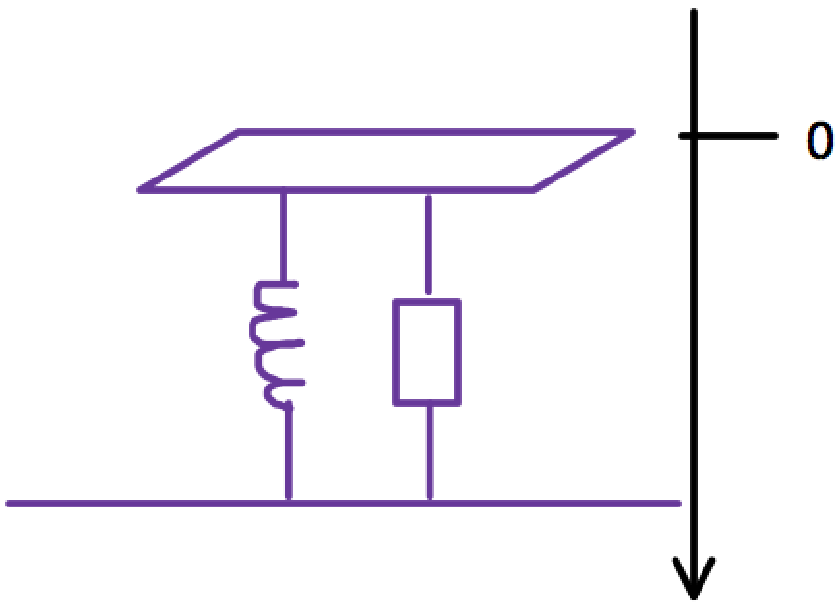
my'' = 42 - ky - γy'

b) key being released

F = (gravity) + spring (Hooke's law) + damping

my'' = -ky - γy'

Forces:



- Hooke's law = -k x(t) or -kΔx or kΔy or -k y(t)
- Damping = -γ y'(t)
- Finger = 42 N

- From the hints: we don't take gravity into account because we know that it is much weaker than the spring that keeps the key in place.

- use the forces above for the equations highlighted in blue

- The extension is y(t) because it is defined as the length of spring from where we defined zero. Otherwise, it would have been Δy.

Q. For a key being released

my" = -ky -γy

y(0) = 0.5

y'(0) = 0

a) Find a formula for r?

try: $y= e^{rt}$

$y'= re^{rt}$

$y''= r^2e^{rt}$

my" = -ky - γy'

$mr^2e^{rt} = -ke^{rt} - \gamma re^{rt}$

$0 = mr^2e^{rt} + ke^{rt} + \gamma re^{rt}$

$0 = e^{rt}(mr^2 + \gamma r + k)$

- use quadratic formula:

$$r = \frac{\left(-\gamma +- \sqrt{\gamma^2 - 4(m)(k)}\right)}{2m}$$

b) What kind of number can r be?

real:

- 2 distinct

- 1 repeated

complex:

- 2 distinct (conjugates)

(NOTE: Just like we see in the case of eigenvalues and eigenvectors where we had two distinct eigenvalues, repeated same eigenvalue and 2 distinct complex (conjugates) eigenvalues)