2018-10-04 Autonomous ODEs (2.5)

4. Write an autonomous ODE with a semi-stable equilibrium solution

$Y' = (y-1)^2$, always positive, 1 is semi-stable point
$Y' = |y|(1-y)$, 0 is semi-stable point, 1 is stable point
$Y' = (3-y)^2$, always positive, 3 is semi-stable point
$Y' = 3(y-2)^2$, always positive, 2 is semi-stable point
$P' = P(P-k)$, wrong, there are no semi-stable points. 0 is stable point and K is unstable point.

> **Note:** We don't know if P refers to population or not so we cannot consider only values greater than 0.

Exercise:

$$P' = P^3 + C_1P^2 + C_2P$$

$$P' = P\left(P^2 + C_1P + C_2\right)$$

Find C_1, C_2 such that,

(a) there is a semi-stable equilibrium point

(b) there isn't a semi-stable equilibrium point

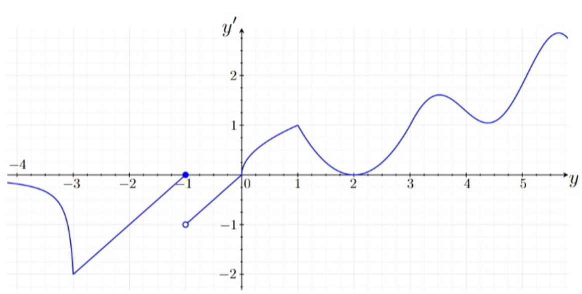
How can you tell if a function is autonomous?

When the function does not explicitly have independent variables (e.g. x or t) in the equation.

For example,

$Y' = (3-y)^2$ only involves y' and y

Consider the differential equation $y' = f(y)$.



5. What are the equilibrium solutions?

Equilibrium points occur when y' = 0, so for this example, the equilibrium solutions occur at -1, 0, and 2.

Is -3 an equilibrium solution? (No, because $y' \neq 0$)

Is $\pm\infty$ an equilibrium solution? (No, the solution tends to go towards 0 at $\pm\infty$ but it does not equal 0)

6. Which equilibrium solutions are stable/semi-stable/unstable?

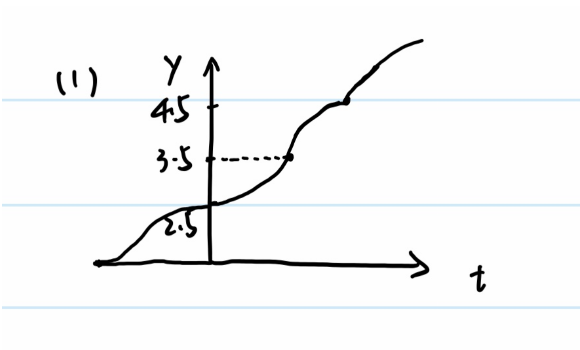
Stable: None

Semi-stable: -1, 2

Unstable: 0

7. Roughly sketch the graph of a solution with the initial condition:

a)$y(0) = 2.5$



- At approximately $2 < y < 3.5$: the slope is increasing so we have a concave up graph
- when $3.5 < y < 4.5$: the slope begins to decrease so the graph concaves down (the graph increases at a decreasing rate)
- $y > 4.5$: the graph concaves upward with an increasing slope.

b) $y(0) = -0.25$

c) $y(0) = 0.25$