2018-11-20 Gambler's Ruin (2.1.3)

Situation 1

A gambler plays a game at a casino. The game is played one round at a time. Each round, one of two things happens:

- The gambler wins $1 with a probability of q
- The gambler loses $1 with a probability of 1 - q

The gambler will stop playing only if:

- The gambler is ruined (bankrupt)
- The gambler reaches $W.

What is the probability pn that the player will be ruined if he starts gambling with $n?

Situation 2

A drunkard walks at the edge of a cliff. Each step, one of two things happens:

- The drunkard steps to the left with a probability of q (safe)
- The drunkard steps to the right with a probability of 1 - q (dangerous)

What is the probability dn that the drunkard will fall off the cliff if he starts at the step $n ?

1. **How does d_n compare to p_n?**

$d_n = p_n$

Explanation:

This is because the probability of losing $1 in the gambling case is the same as the probability of going to the right for the drunkard case, and P(winning $1) is the same as P(going to the left). Also, the initial and end states are the same.

2. Set this problem up in terms of p_n?

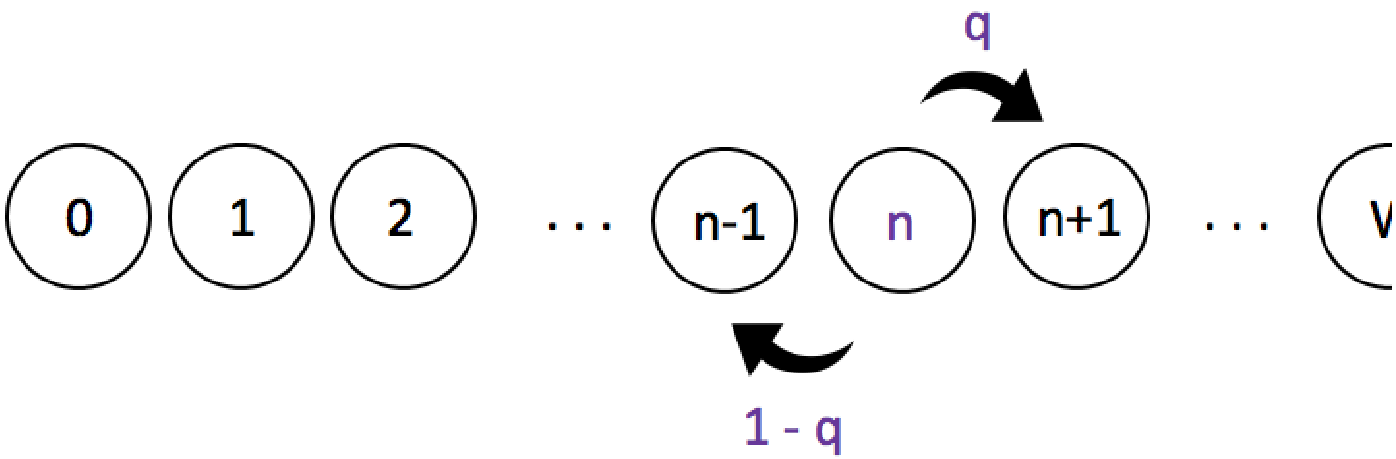
$p_0 = 1$

$p_w = 0$

Explanation:

Here, p_n is the probability that the player will be ruined if he starts gambling with $n. The probability of going bankrupt when starting with 0 dollars is 1 because you are already bankrupt. p_w is zero, since if you already have W, you cannot go bankrupt because you will not gamble.

There are a total of $(W+1)$ states, since we start at zero.



We need an expression relating p_n , p_{n+1} , and p_{n-1}

Hint: Use the total probability theorem

$$P(A) = \sum_{i=1}^{n} P(B_i) . P(A/B_i)$$

B_i mutually exclusive

$p_n = P\,(bankruptcy \mid have\ \$n\ now) = P\,(A)$

B_1 = have $\$n+1$ next round :)

B_2 = have $\$n-1$ next round :(

if we have multiple partition event($B_{n,}$ n=1,2,3.......) of a sample space, for any event A of the same **probability space (https://en.wikipedia.org/wiki/Probability_space)** :

Pr(A)=$\sum$ Pr(A$\cap$$B_n$), for all n

alternatively

Pr(A)=$\sum$ Pr(A | B_n) * Pr(B_n) , for all n.

In this question, the A is the event of going bankrupt, and in order to make this event happen, there are two situations. 1,going bankrupt given having n+1 dollars. 2, going bankrupt given having n-1 dollars.

So,

p_n = P(of going bankrupt | $n+1)*P(going from $n to $n+1)+P(bankruptcy | $n-1)*P(going from $n to $n-1)

P_{n+1} q P_{n-1} 1-q

$$p_n = qp_{n+1} + (1-q)\,p_{n-1}$$

The problem we want to solve is,

$$qp_{n+1} - p_n + (1-q)\,p_{n-1} = 0$$

$$p_0 = 1\,,\; p_w = 0$$

This is a 2nd order Differential equation. It's not easy to find the pattern.

Consider the two previous problems:

$$S_{k+1} = \mu S_k$$

$$D_{k+1} = \mu D_k - R$$

3. What do the solutions look like? What kind of "functions"?

$$S_k = S_0\left(1 + \frac{p\alpha}{100}\right)^k$$

$$D_k = cqk + D_{eq}$$

The solutions of these problems look like exponentials (which we see when we solved for 2nd order DE's)