2.4 Linear vs Nonlinear ODEs

Consider the problem

$$y' = f(t,y) \qquad \text{with} \qquad y(t_0) = y_0.$$

Theorem (Existence and Uniqueness for Nonlinear DEs)

If

- $f(t,y)$ *and* $\frac{\partial f}{\partial y}(t,y)$ *are continuous near* (t_0, y_0)

Then

- *there is one unique solution* $y = \phi(t)$ *defined for* t *near* t_0.

2.4 Linear vs Nonlinear ODEs

Consider the problem

$$\begin{cases} y' = t + \sqrt{y - \pi} \\ y(1) = \cancel{1}\ 4 \end{cases}$$

1. Is there a unique solution? Yes!

2. Without solving, what is its domain?

2.4 Linear vs Nonlinear ODEs

Consider the problem

$$\begin{cases} y' = \sqrt{4-(t^2+y^2)} \\ y(1) = 1 \end{cases}$$

3. Is there a unique solution?

4. Without solving, what is its domain?

The Initial-Value Problem

$$\begin{cases} y' = -\dfrac{x}{y} \\ y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}. \end{cases}$$

has **the solutions**

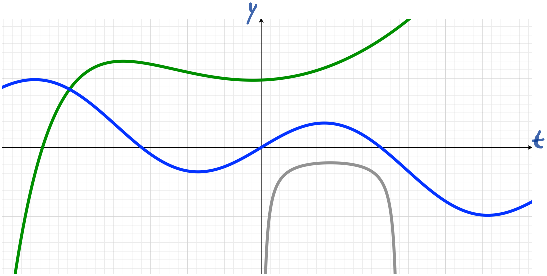
$$y_1 = \cos\left(\arcsin(x)\right) \qquad \text{and} \qquad y_2 = \sqrt{1 - x^2} \qquad .$$

5 Does the problem satisfy the conditions of the Theorem?

6 What can you conclude? $y_1(x) = y_2(x)$

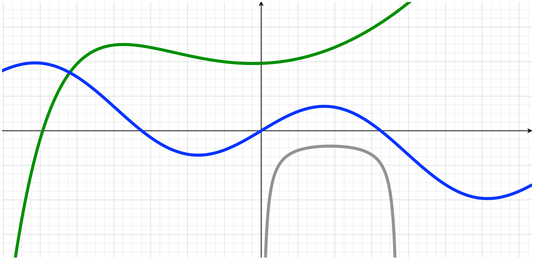
2.4 Linear vs Nonlinear ODEs

Consider the problem $y' = f(t,y)$, where $f(t,y)$ and $\frac{\partial f}{\partial y}(t,y)$ are continuous for all t, y.



7 Could this be the graph of 3 solutions with 3 different initial conditions?

2.4 Linear vs Nonlinear ODEs



8 Which ones could be solutions with different initial conditions?

a. Blue + Green
b. Blue + Gray
c. Green + Gray
d. Only Green
e. Only Blue
f. Only Gray

2.4 Linear vs Nonlinear ODEs

Consider the problem

$$y' = f(t, y)$$

where $f(t, y)$ and $\frac{\partial f}{\partial y}(t, y)$ are continuous for all t, y.

- Assume that $y = \frac{1}{t}$ is a solution for $t > 0$
- Assume that $y = -e^{-t}$ is a solution for all t

9 Let $y = \phi(t)$ be the solution of this ODE with the initial condition $y(1) = \frac{1}{2}$.

Calculate $\lim_{t \to +\infty} y(t)$.

2.4 Linear vs Nonlinear ODEs

Consider the problem

$$y' + p(t)y = g(t) \qquad \text{with} \qquad y(t_0) = y_0.$$

Theorem (Existence and Uniqueness for Linear DEs)

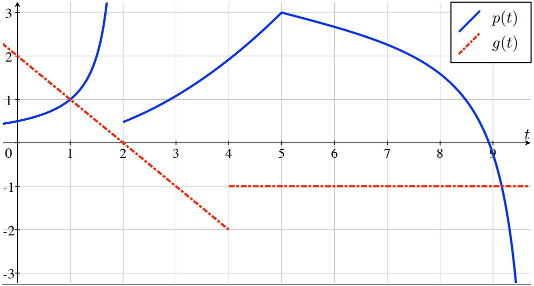
If

- $p(t)$ *and* $g(t)$ *are continuous in* (a, b)
- $t_0 \in (a, b)$

Then

- *There is one unique solution* $y = \phi(t)$ *defined for* $t \in (a, b)$

2.4 Linear vs Nonlinear ODEs



10 There exists a unique solution satisfying $y(3) = 2$ defined for

$$t \in \left(\qquad\qquad , \qquad\qquad \right).$$

11 There exists a unique solution satisfying $y(t_0) = -1$ for

$$t_0 \in \left\{ \qquad\qquad \right\},$$

Preparation for next lecture

2.5 Autonomous ODEs

- Watch https://youtu.be/swt-let4pCI

- Identify Autonomous ODEs
- Sketch phase diagram for an Autonomous ODE
- Identify Equilibrium points: Stable/Unstable/Semi-Stable