# 2018-09-18 Separable ODEs (2.1)

## 2.1 Separable ODEs

## Q: Which of these ODEs are Separable ODEs? Why?

a. 
$$heta'' = rac{-g}{l} sin\left( heta
ight)$$
 Second order (Not Separable.)  $rac{1}{\sin( heta)} d^2 heta = -rac{g}{l} dt^2$ 

b. 
$$v'=-g$$
 Yes.  $dv=-gdt$ 

c. 
$$v'=rac{-g-r}{mv}$$
 Yes.  $rac{1}{\left(rac{-g-r}{mv}
ight)}dv=dt$ 

d. 
$$y'=rac{-gt-r}{my+10}$$
 Yes.  $rac{1}{my+10}dy=\left(-gt-r
ight)dt$ 

e. 
$$y' = -gt - \frac{\gamma}{m}y + 10$$
 Not Separable.

[ Observe if the differential equation can be written as  $\frac{dy}{dx}=g\left(x\right)\cdot h\left(y\right)$  then it is separable. e.g. equation c. can be written as  $v'=\frac{-g-r}{m}\cdot\frac{1}{v}$  ]

b:

$$v' = -g$$

$$\frac{dv}{dt} = -g$$

$$dv = -g dt$$

$$\int dv = \int -g \, dt$$

## Let us recall the model for the altitude of a boulder thrown by a catapult:

$$v'\left(t
ight)=rac{-g-r}{m\cdot v(t)}$$

Initially the boulder has velocity  $oldsymbol{v_0}$ 

## 1. What is v(t)?

$$rac{dv}{dt}=\,-g-rac{\gamma}{m}v\left(t
ight)$$

$$\frac{dv}{dt} = \frac{-(gm + \gamma v(t))}{m}$$

$$rac{1}{(gm+\gamma v(t))}dv = -rac{1}{m}dt$$

$$\int rac{1}{(gm+\gamma v(t))} dv \ = \int -rac{1}{m} dt$$
 (integrate both sides)

$$rac{\ln(|\gamma v(t)+gm|)}{\gamma} = -rac{1}{m}t + c$$

$$\ln(\gamma v\left(t
ight)+gm) \ = -rac{\gamma}{m}t+c$$

$$\gamma v\left(t
ight)+gm\,=e^{-rac{r}{m}t+c}$$

$$v\left(t
ight)=rac{\left(ce^{-rac{\gamma}{m}t}-gm
ight)}{\gamma};\;\;v_{0}=rac{\left(ce^{0}-gm
ight)}{\gamma}\;;\;c=gm+\gamma v_{0}$$

$$v\left(t
ight)=rac{\left(\left(gm+\gamma v_{0}
ight)e^{-rac{\gamma}{m}t}-gm
ight)}{\gamma}$$

## 2. What is y(t)?

$$v\left( t\right) =y^{\prime}\left( t\right)$$

$$y(t) = \int v(t) dt$$

$$y\left(t
ight)=\intrac{\left(\left(gm+\gamma v_{0}
ight)e^{-rac{y}{m}t}-gm
ight)}{\gamma}\;dt$$

$$y\left(t
ight)=rac{-m\left(\gamma v_{o}+mg
ight)e^{rac{-\gamma t}{m}}}{\gamma^{2}}-rac{mgt}{\gamma}+c$$

Note:  $\frac{m(rv_0+mg)}{r^2}$  in above equation is the C value, found by using initial value y(0)=0.

$$0=rac{-m(\gamma v_0+mg)}{\gamma^2}+c$$

$$y\left(t
ight)=rac{-m(\gamma v_{0}+gm)e^{-rac{\gamma}{m}t}}{\gamma^{2}}\,-rac{gm}{\gamma}t\,+rac{m(\gamma v_{0}+mg)}{\gamma^{2}}$$