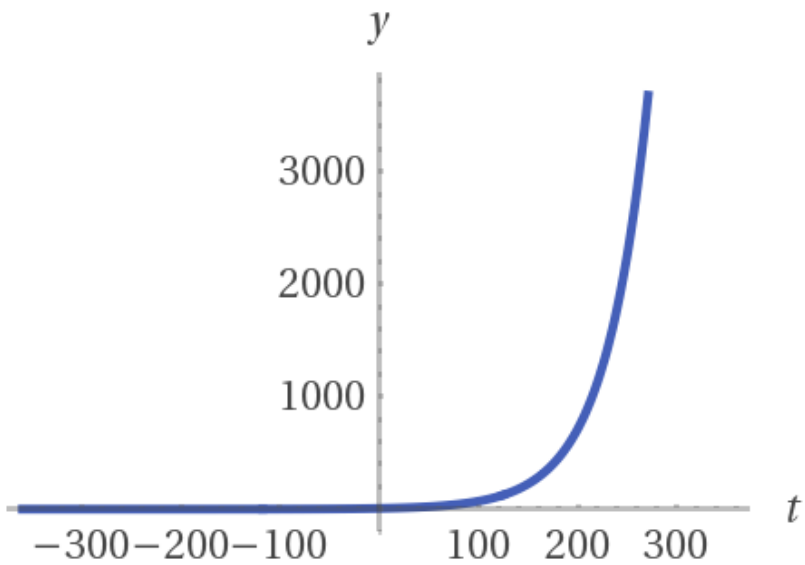
1.1 Basic Models
1.2 Direction Fields
2.3 Modelling with ODEs
2.1 Separable ODEs
2.2 Linear First-Order ODEs
2.4 Linear vs Nonlinear ODEs
2.5 Autonomous ODEs

2.5. Population Growth

Malthusian Growth

Population increases proportionally to its current size.

- $P' = rP \qquad \Rightarrow \qquad P = P_0 e^{rt}$



2.5. Better population model

Idea 1. Growth rate r depends on the population: $r(P)$.
Idea 2. Consider a maximum sustainable population K.
Idea 3. Consider a survivability threshold S (with $S < K$).

Model satisfies:

- If $P > K$, then $r(P)$
- If $S < P < K$, then $r(P)$
- If $P < S$, then $r(P)$

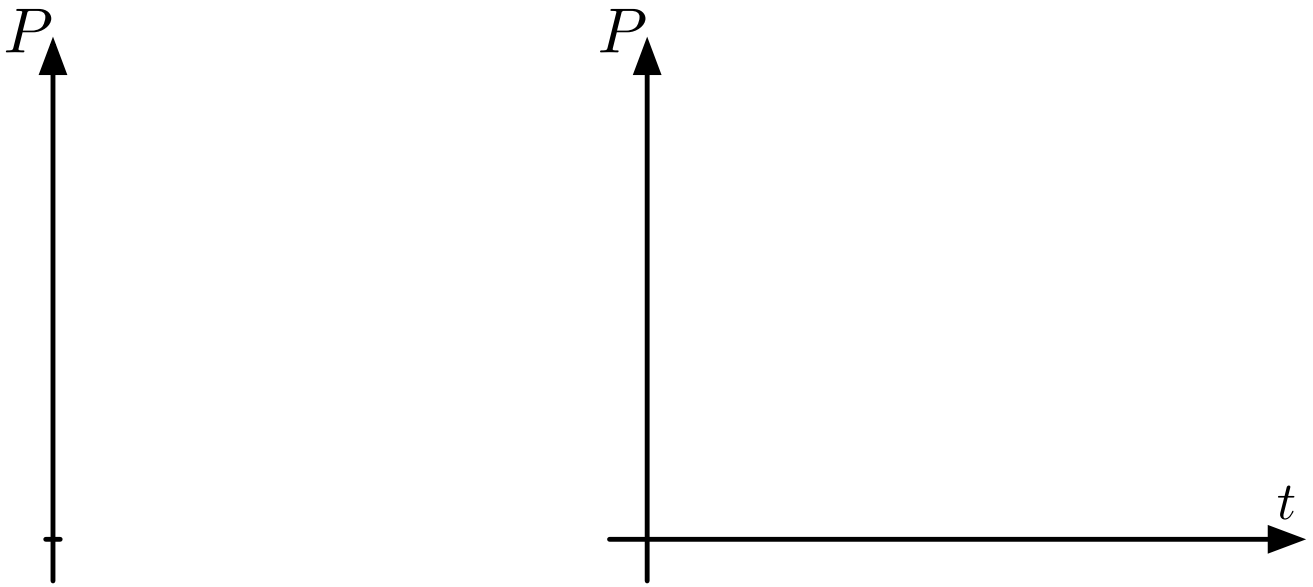
1. Give a function $r(P)$ that will satisfy these conditions.

Hint. Think of the graph of $r(P)$.

2.5. Better population model

2.5. Better population model

$$P' = -kP\left(1 - \frac{P}{S}\right)\left(1 - \frac{P}{K}\right)$$



Phase Plane Solution Graph

2. Sketch the graphs above.
3. What are the critical points? Are they stable, unstable, or semi-stable?

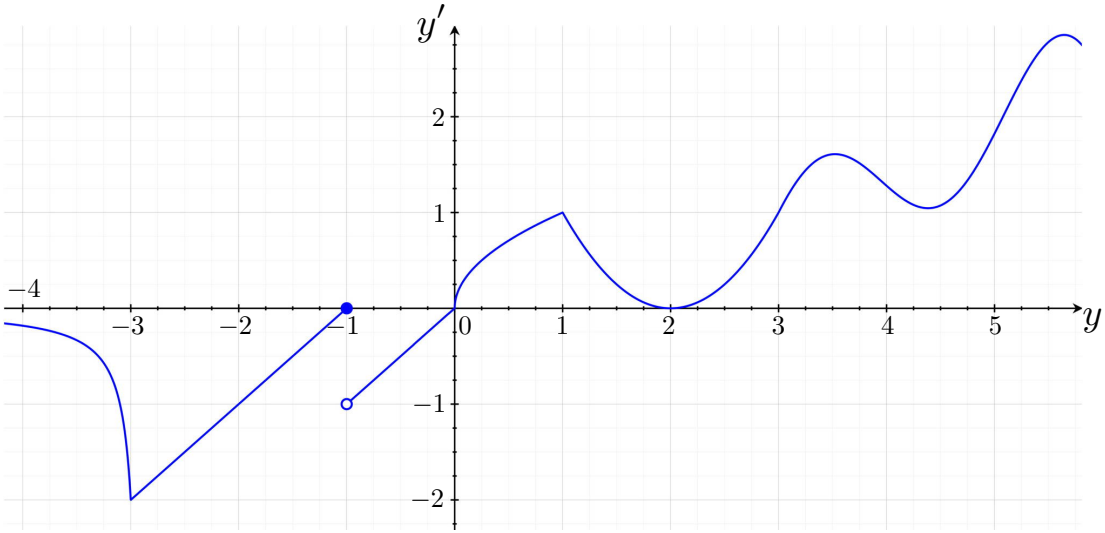
2.5. Better population model

4. Find an Autonomous ODE with a semi-stable equilibrium point.

Don't just sit there, come write it on the board.

Autonomous ODEs

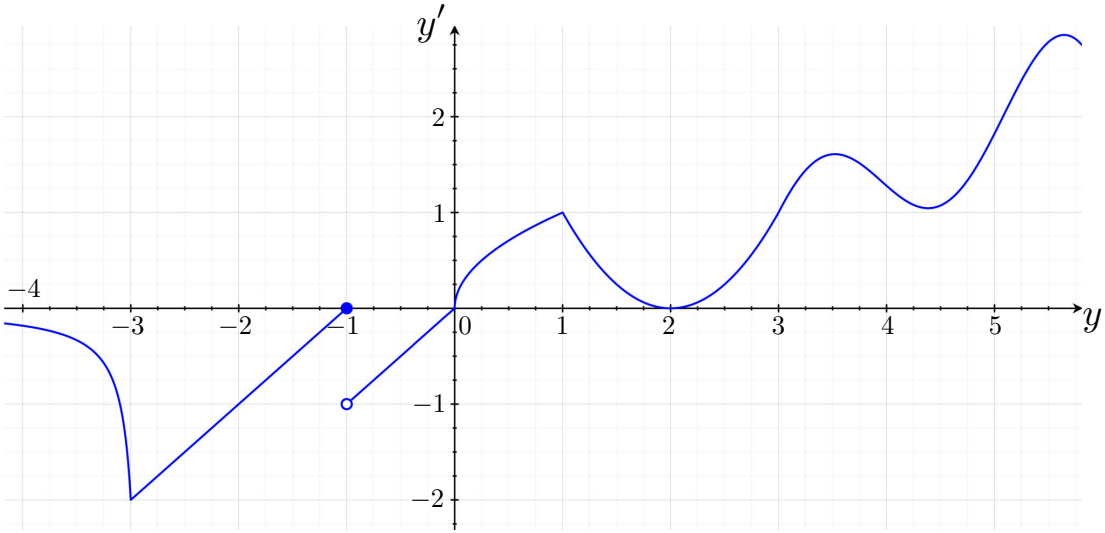
Consider the differential equation $y' = f(y)$.



5. What are the equilibrium solutions?
6. Which equilibrium solutions are
   a. Stable:
   b. Semi-stable:
   c. Unstable:

Autonomous ODEs

Consider the differential equation $y' = f(y)$.

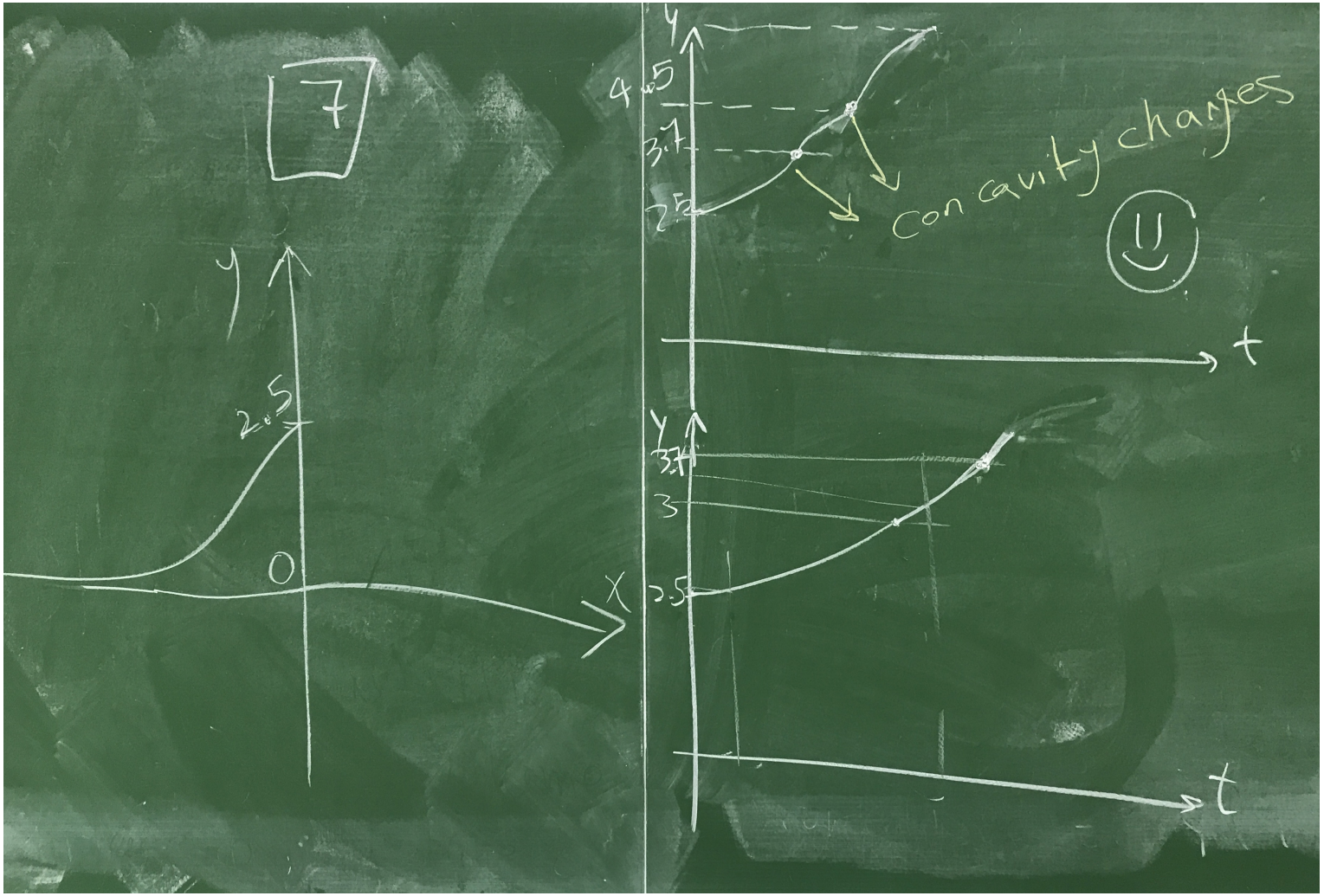


Roughly sketch the graph of a solution with the initial condition:

7 $y(0) = 2.5$. 8 $y(0) = -\frac{1}{4}$. 9 $y(0) = \frac{1}{4}$.

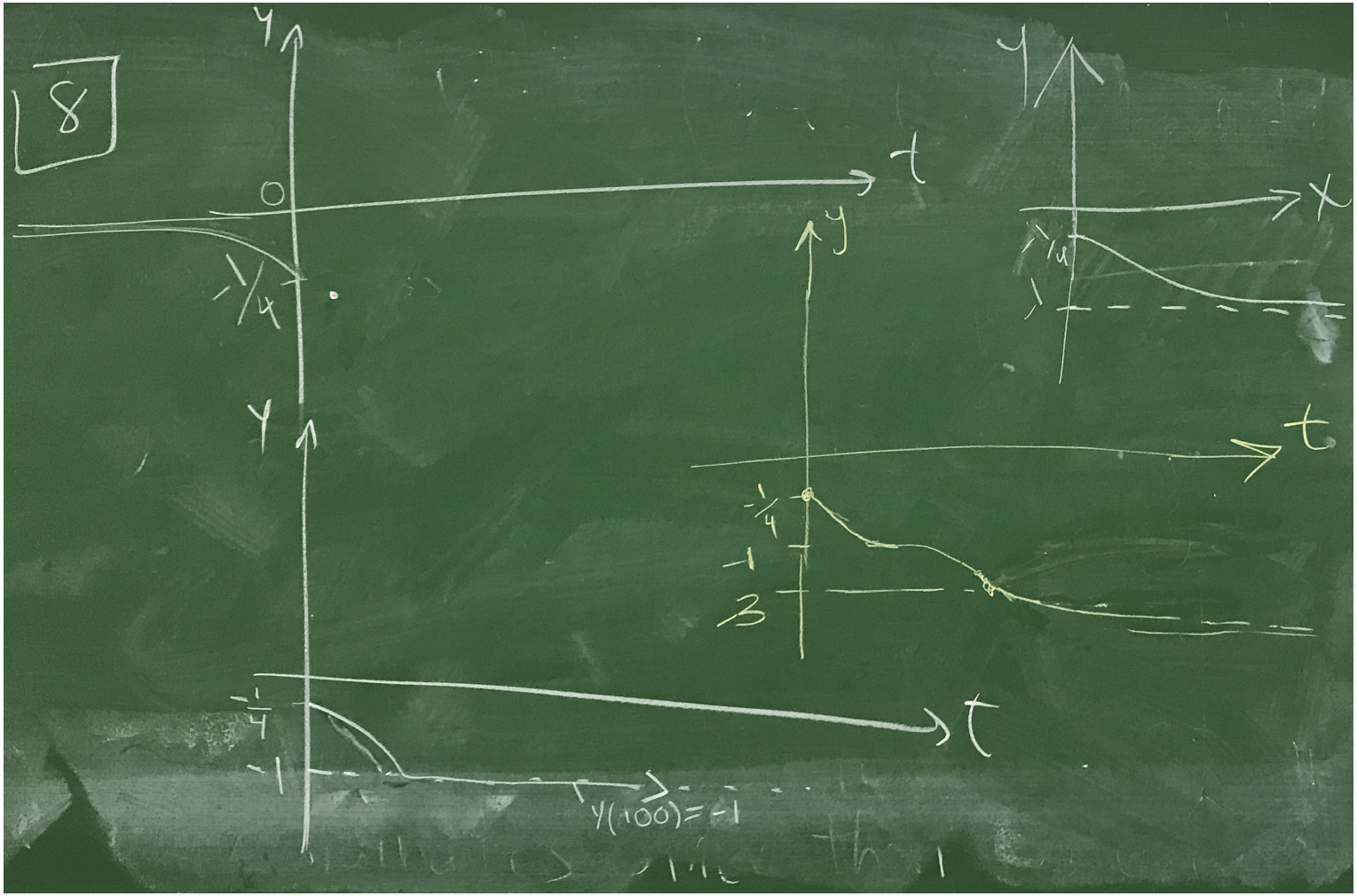
Autonomous ODEs

7



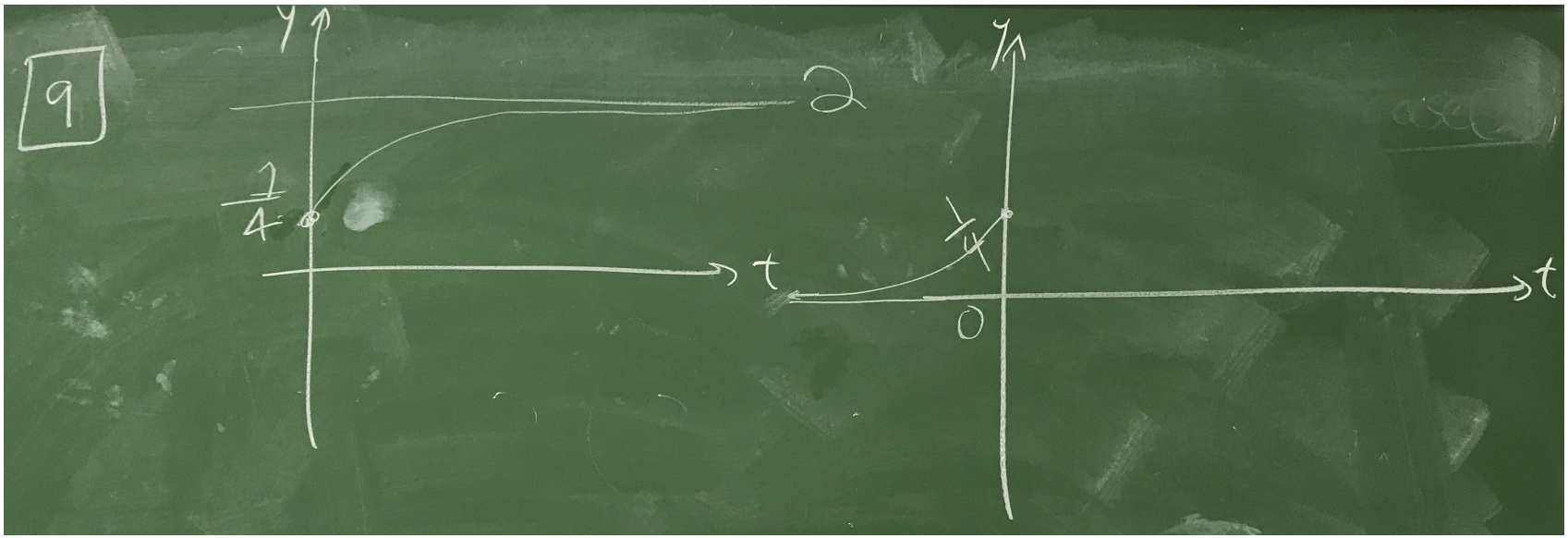
Autonomous ODEs

8



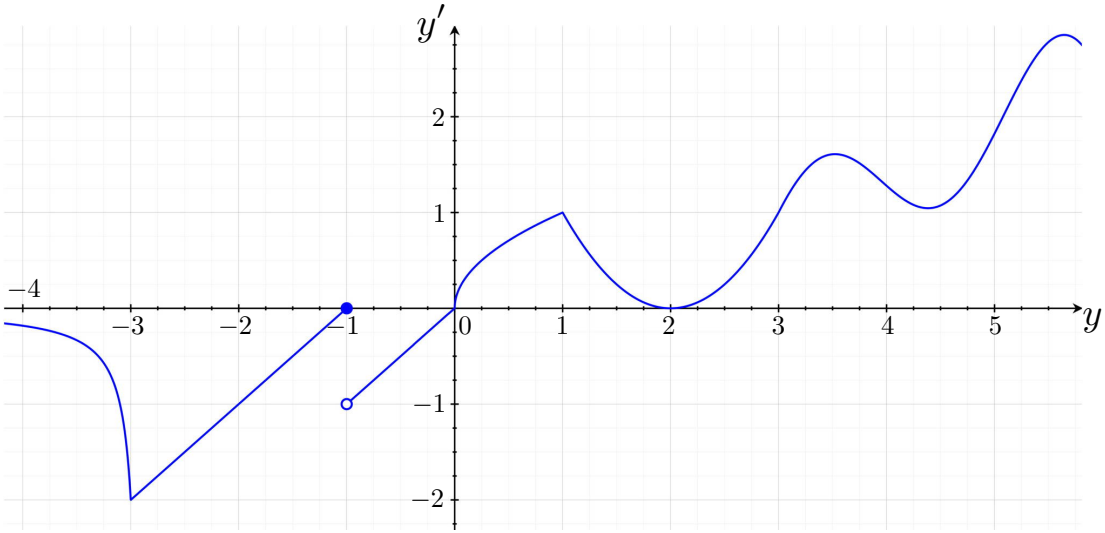
Autonomous ODEs

9



Autonomous ODEs

Consider the differential equation $y' = f(y)$.



10 If $y(0) = 2$, then $y(t) =$

11 If $y(0) = \frac{1}{2}$, then $\lim_{t\to\infty} y(t) =$

12 If $y(0) = -2$, then $\max_{t\in[0,\infty)} y(t) =$

Preparation for next lecture

3.1 Review of Linear Algebra

- Watch https://youtu.be/PFDu9oVAE-g

- What do Eigenvectors and Eigenvalues mean?
- How to compute Eigenvectors and Eigenvalues.
- How to solve Linear Systems of Equations.