1.1 Basic Models

1.2 Direction Fields

2.3 Modelling with ODEs

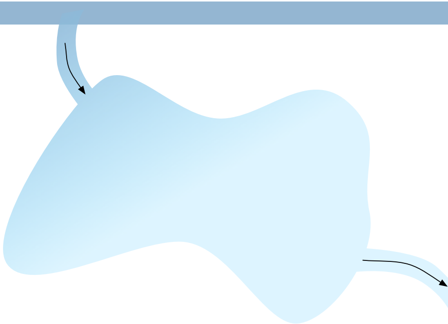
2.1 Separable ODEs

2.2 Linear First-Order ODEs

2.4 Linear vs Nonlinear ODEs

2.5 Autonomous ODEs

Modelling pollution in a lake



- Lake contains V L of fresh water
- Water flows into and out of the pond at the rate of r L/year
- Incoming water is polluted with

$$\gamma(t) = 2 + \sin(2t) \quad \text{kg/L of pollutant}$$

1. Obtain an ODE for the amount of pollutant (in kg) at time t. (check units)

2. Which assumptions are made in this model?

2.2 Linear First-Order ODEs

3. What is the solution of the DE?

Hint. $$\int e^{\frac{r}{V}t}\sin(2t)\,dt = -Ve^{\frac{r}{V}t}\frac{2V\cos(2t) - r\sin(2t)}{r^2 + 4V^2} + C$$

2.2 Linear First-Order ODEs

4 Calculate $\big(\sin(x)f(x)\big)'$

5 Find the general solution of

$$\sin(x)y' + \cos(x)y = \sqrt{x}$$

6 What is the integrating factor for

$$y' + \frac{\cos(x)}{\sin(x)}y = \frac{\sqrt{x}}{\sin(x)}$$

6' What happens when you multiply the ODE by the integrating factor?

7 Find the solution of $$\begin{cases} y' - \dfrac{y}{2(x+4)} = \dfrac{1}{2(x+4)} \\ y(0) = -5 \end{cases}$$

7a What is the integrating factor $p(t)$?

7b What is the domain of the solution?

8 Find the solution of $$\begin{cases} y' - \dfrac{y}{2(x+4)} = \dfrac{1}{2(x+4)} \\ y(-5) = -5 \end{cases}$$

9 Find the general solution of

$$2\ln(x)e^{2y}y' + \frac{e^{2y}}{x} = 4x^3$$

Hint. If the right-hand side is the result of a product rule $(f(x)g(x))'$, then what are $f(x)$ and $g(x)$?

Preparation for next lecture

2.4 Linear vs Nonlinear ODEs

- Watch https://youtu.be/53BPf9JrFcU
- Watch https://youtu.be/GV1gFLZ7V18

- Know how to calculate $\frac{\partial f}{\partial y}$
- Understand the idea of the Existence and Uniqueness Theorem in the first video (the example video should help)