1.1 Basic Models
1.2 Direction Fields
2.3 Modelling with ODEs
2.1 Separable ODEs
2.2 Linear First-Order ODEs
2.4 Linear vs Nonlinear ODEs
2.5 Autonomous ODEs

2.3 Modelling with ODEs

The equations that describe the boulder's altitude are

$$mh'' = -mg - \gamma h'$$
$$h(0) = 0 \quad , \quad h'(0) = 10$$

The solution is

$$h(t) = -\frac{mg}{\gamma}t - \left(\frac{m^2 g}{\gamma^2} + \frac{10m}{\gamma}\right)\left(e^{-\frac{\gamma}{m}t} - 1\right)$$

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1. Can you think of ways/experiments to measure γ based on using this formula?
2. Can you also measure m? And g?

2.3 Modelling with ODEs

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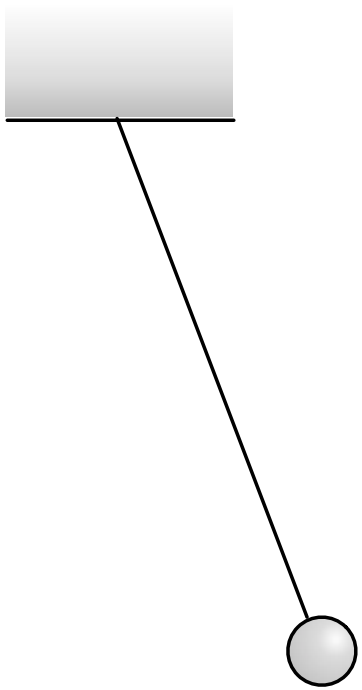
- Define variables

- Start with Basic Principles

2.3 Modelling with ODEs

- Define variables
- Start with Basic Principles
- Models are approximations – require assumptions

2.3 Modelling with ODEs



2.3 Modelling with ODEs

3. Which basic principle should we use?

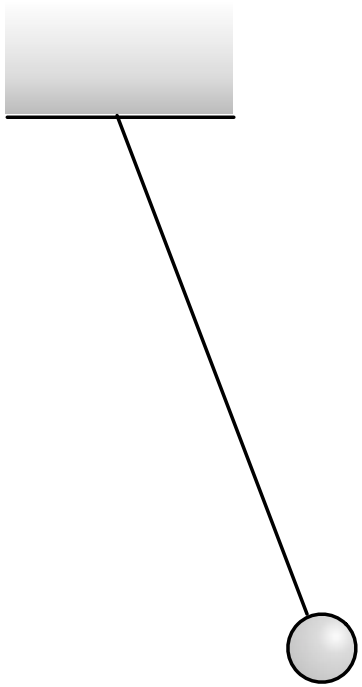
   a. Conservation of Angular Momentum

   b. Newton's 2^{nd} Law

   c. Rate of Change = rate in − rate out

   d. Conservation of Linear Momentum

   e. Conservation of Energy

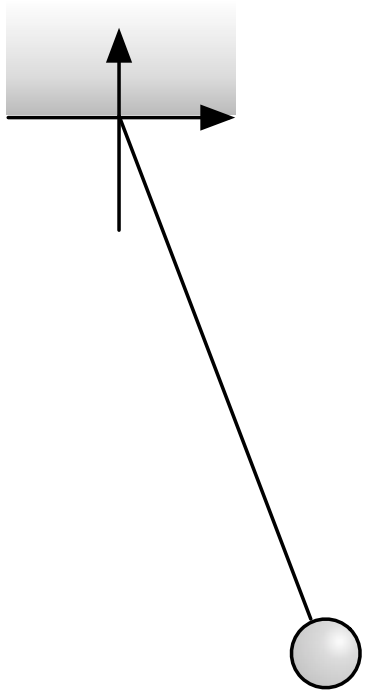


Newton's 2nd Law: $F = ma$

We need to know:

4. Force =

5. Acceleration =

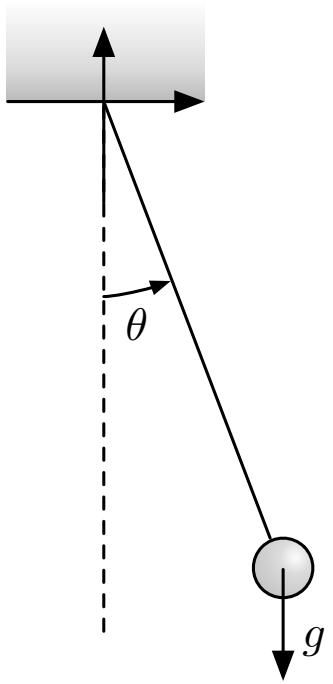


Position

Idea. Use the angle to define position.

Then

6 $\vec{r}(t) = \left(\qquad , \qquad \right)$



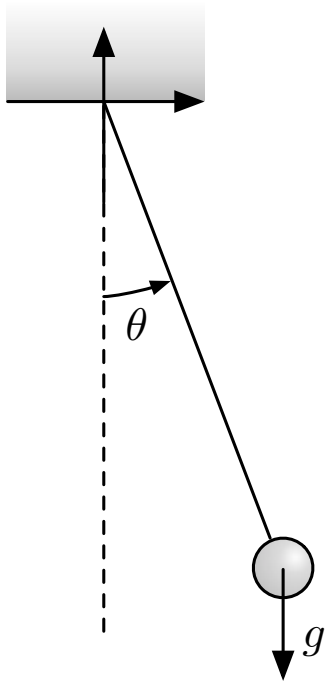
Position

Idea. Use the angle to define position.

Then

6. $\vec{r}(t) = \left(\quad L\sin\theta(t) \quad , \quad -L\cos\theta(t) \quad \right)$

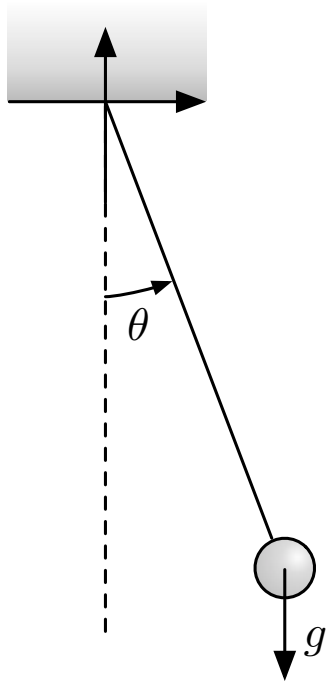
7. Newton's 2nd Law implies ...



Analysis of the Model

Pendulum Equation

$$\theta'' + \frac{g}{L}\sin\theta = 0$$



8 Where is m? What does this mean?

9 Constants g and L appear only as $\frac{g}{L}$. What does this imply?

9* We want to know how a pendulum on the Moon with length 1m swings. We need to build a pendulum on Earth with length $L = ?$

Preparation for next lecture

2.1 Separable ODEs

- Watch https://youtu.be/txtFH89HwOA

- Identify a Separable Equation
- Know how to solve a Separable Equation