2018-11-27 Average Lifespan (2.2.3)

2.2.3 Death Rate

Info from last class:

- β = babies born per individual each season (per capita birth rate)
- μ = probability of dying between one season and the next
- N_k = number of individuals at the start of the k^{th} breeding season

3. **What is the probability of dying between time (k - 1) and k?**

a. Expression that **uses** μ = μP(k - 1)

*NOTE: The expression cannot be μ because it is asking specifically for season between k-1 and k, so the individual has to first survive k-1 seasons.

P(alive at season k-1 and die next season) = P(alive at season k-1) * P(die next season) = P(k - 1) * μ

*NOTE: P(alive at season k-1) cannot be 1-P(k) because it includes the death at any season before.

b. Expression that **doesn't use** μ = P(k - 1) - P(k)

e.g: P(74) - P(75)
Probability of living up to 74-years-old is larger than probability of living up to 75-years-old. For example, in a class of 100 people, 30 people will live to age 74, while 25 people will live to age 75. This means that P(74) is 30% and P(75) is 25%, and 5% will die between 74 and 75.

*NOTE: equation a) and equation b) are the same thing

4. **Use previous expressions to find P(k)**
   since part a) and b) are asking for the same thing in question 3, we can equate them.

- Set 3.a) and 3.b) equal to each other:
  - μP(k - 1) = P(k - 1) - P(k)
  - P(0) = 1
- Now solve P(k)
  - Solution: $P(k) = (1-\mu)^k$
  - Solving methods:
    - First, rewrite the equation:
      $P\left(k\right) = \left(1-\mu\right) P\left(k-1\right)$

- Solving by start writing some terms in terms of P(0):

  $P(0) = 1$
  $P(1) = (1-\mu) P(1-1) = (1-\mu) P(0)$
  $P(2) = (1-\mu) P(2-1) = (1-\mu) P(1) = (1-\mu)^2 P(0)$
  $P(3) = (1-\mu) P(3-1) = (1-\mu) P(2) = (1-\mu)^3 P(0)$
  $\dots\dots$
  $P(k) = (1-\mu)^k P(0)\,,\ where\ P(0) = 1$
  $P(k) = (1-\mu)^k$

- Using Gambler's Ruin:

  $\frac{r^k}{r^k} = \frac{(1-\mu)r^{k-1}}{r^k}$
  $1 = (1-\mu)\, r^{-1}$
  $r^2 = (1-\mu)\, r$
  $r\,(r - (1-\mu)) = 0$
  $r_1 = 0,\ r_2 = 1-\mu$
  $P(k) = a(1-\mu)^k$
  $P(0) = 1 = a(1-\mu)^0$
  $a = 1$
  $P(k) = (1-\mu)^k$

5. **What is the probability of the individual dying at age k?**

- = 1 - P(k) ← **WRONG**
- = μP(k - 1) = μ(1 - μ)$^{k-1}$

↑

Comes from 3, shows probability of dying * prob of living at least k-1 years

6. **How do we compute the Average Lifespan ?**

- L = Expected value
- = 1*p(1) + 2*p(2) + 3*p(3) + … = $\sum_{k=1}^{\infty} kP(k)$ ← doesn't work because it uses the probability of living **exactly** k years. Individual who lives for 2 years may also lives for 3 years.

(OR)

- $L = \sum_{k=1}^{\infty} k \cdot (probability\ of\ dying\ at\ age\ k) = \sum_{k=1}^{\infty} k \cdot (probability\ of\ living\ k\ years)$

7. **Find a formula for L = Average Lifespan**

(Hint: $\sum_{k=1}^{\infty} kr^k = \frac{r}{(1-r)^2}$**)**

- $L = \sum_{k=1}^{\infty} k \cdot (probability\ of\ dying\ at\ age\ k) =$
  $\sum_{k=1}^{\infty} k\mu(1-\mu)^{k-1} = \mu(1-\mu)^{-1} \sum_{k=1}^{\infty} k(1-\mu)^{k} = \mu(1-\mu)^{-1}\left(\frac{1-\mu}{\mu^2}\right) = \frac{1}{\mu}$

↑

Use hint

Tutorial #2 - Ebola Epidemic Revisited

Assumptions:

1. Recovery rate = γ = probability of recovering in 1 year
2. Average recovery time is T = 1 / γ

- Recovery rate γ = probability of disease dying between one season and the next = μ
- Average recovery time T = average lifespan of disease in the individual = L
  - L = 1 / μ
  - T = 1 / γ