Difference Equations

2.1.1 Compound Interest

2.1.2 Loan Repayment

2.1.3 Gambler's Ruin

2.2.2 Exponential Population Growth

2.2.3 Average Lifespan

2.2.$\star$ Rabbit Populations

2.2.4 Nonlinear Population Models

2.2.2 Exponential Population Growth

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1. Find a Difference Equation for N_k.
2. Based on your DE, what happens if $\beta > \mu$?
   Based on your DE, what happens if $\beta < \mu$?

2.2.3. Death Rate

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3. What is the probability of dying between time $(k-1)$ and k?
   a. Find an expression that **uses** μ.
   b. Find an expression that **does not use** μ.

4. Use the previous expressions to find $P(k)$.

5. What is the probability of the individual dying at age k?

2.2.3. Average Lifespan

- Probability of dying at age $k = \mu P(k-1) = \mu(1-\mu)^{k-1}$.

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6 Brainstorm: How do we compute the **Average Lifespan** ?

7 Find a formula for $L =$ Average Lifespan.

Hint. $$\sum_{k=1}^{\infty} kr^k = \frac{r}{(r-1)^2}$$

Tutorial #2 – Ebola Epidemic Revisited

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- Recovery rate γ = probability of disease dying between one season and the next = μ
- Average Recovery time T = average lifespan of disease in the individual = L

$$L = \frac{1}{\mu} \qquad \Leftrightarrow \qquad T = \frac{1}{\gamma}$$

Preparation for next lecture

2.2.3. Average Lifespan

- Figure out the Average Lifespan of this population.