# 3.3 Real Eigenvalues

Consider a lions–cheetahs example without "harvesting":

$$\frac{d\vec{p}}{dt} = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \vec{p}$$

Look for solutions that look like

$$\vec{p}(t) = \vec{v}e^{rt}.$$

1. What problem is satisfied by $\vec{v}$ and $r$?
2. Find possible values for $\vec{v}$ and $r$.
3. What is the solution $\vec{p}(t)$?

# 3.3 Real Eigenvalues

$$\frac{d\vec{p}}{dt} = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \vec{p}$$

We obtained two solutions:

$$\vec{p}_1(t) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \qquad \text{and} \qquad \vec{p}_2(t) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{5t}$$

4 Is $\vec{p}_1(t) + \vec{p}_2(t)$ a solution?

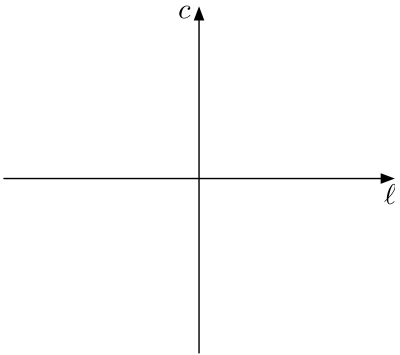
5 Is $\vec{p}_1(t) - \vec{p}_2(t)$ a solution?

6 Is $2\vec{p}_1(t) + 3\vec{p}_2(t)$ a solution?

# 3.3 Real Eigenvalues

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{5t}$$

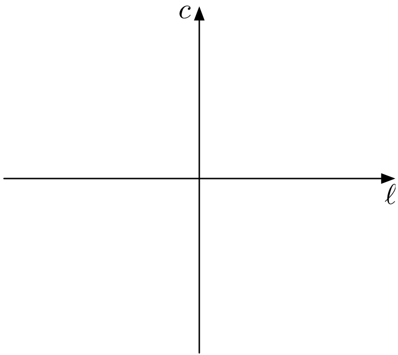
7 Sketch the solution for $A = 1$ and $B = 0$ in the phase plane.



# 3.3 Real Eigenvalues

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{5t}$$

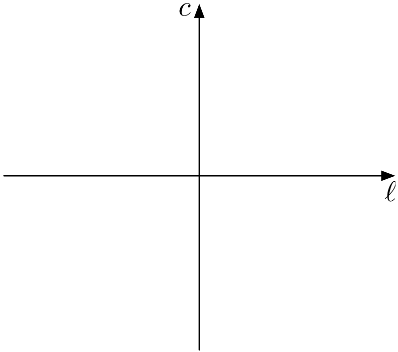
8 Sketch the solution for $A = -1$ and $B = 0$ in the phase plane.



# 3.3 Real Eigenvalues

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{5t}$$

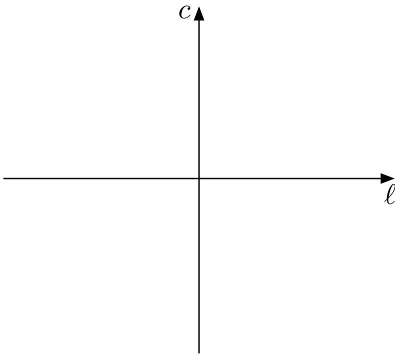
9 Sketch the solution for $A = 0$ and $B = \pm 1$ in the phase plane.



# 3.3 Real Eigenvalues

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{5t}$$

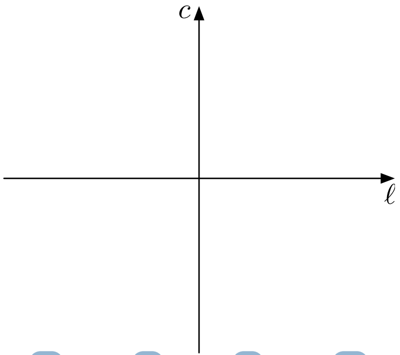
10 Sketch the solution for $A = 1$ and $B = 1$ in the phase plane.



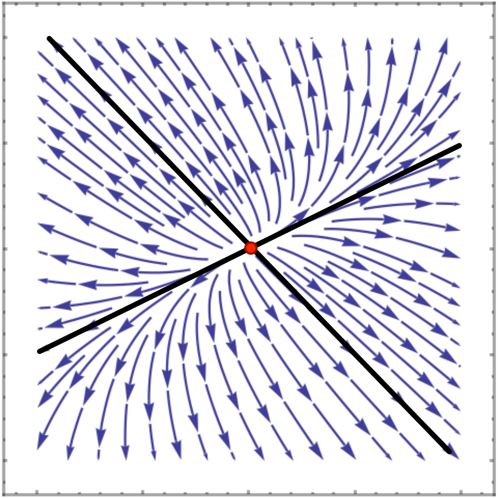
# 3.3 Real Eigenvalues

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{5t}$$

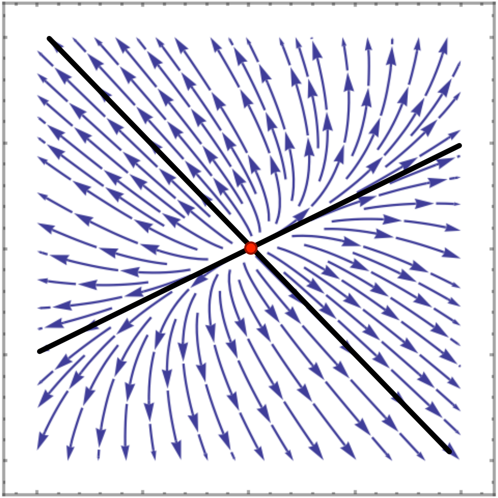
11 Sketch the solution for $A = \pm 1$ and $B = \pm 1$ in the phase plane.



# 3.3 Real Eigenvalues



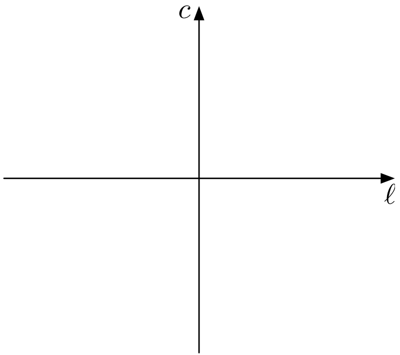
# 3.3 Real Eigenvalues



Source
Unstable

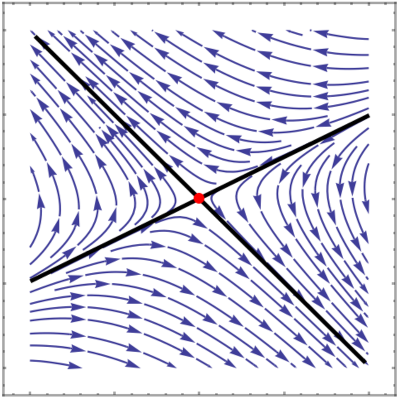
12 Sketch the phase plane if the eigenvalues $r_1 < 0 < r_2$:

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{5t}$$



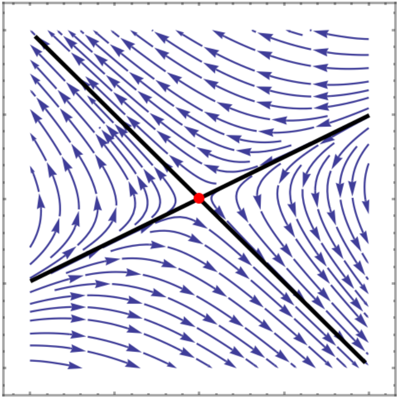
12 Sketch the phase plane if the eigenvalues $r_1 < 0 < r_2$:

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{5t}$$



12 Sketch the phase plane if the eigenvalues $r_1 < 0 < r_2$:

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{5t}$$

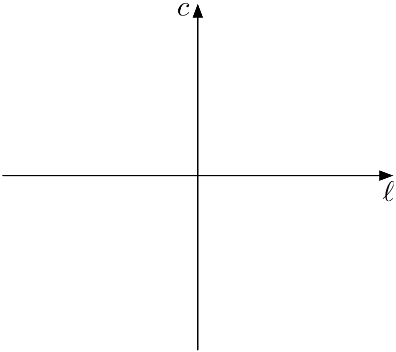


Saddle
Point
Unstable

# 3.3 Real Eigenvalues

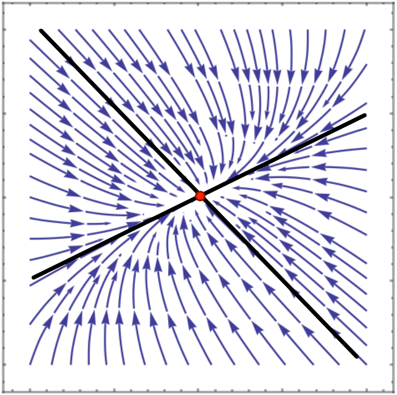
13 Sketch the phase plane if the eigenvalues were negative:

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-5t}$$



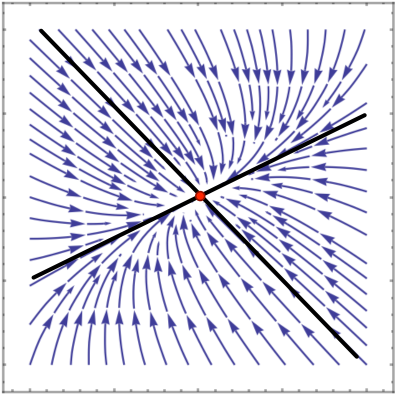
13 Sketch the phase plane if the eigenvalues were negative:

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-5t}$$



13 Sketch the phase plane if the eigenvalues were negative:

$$\vec{p} = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-5t}$$



Sink
Stable

# Preparation for next lecture

## Section 3.4

- How to solve a system of linear ODEs with **complex** eigenvalues

  https://youtu.be/TRVS5Wo9LoM

- How to sketch a phase portrait for such systems: all three types