3.1 Linear Algebra Review

3.2 Systems of two ODEs

3.3 Real Eigenvalues

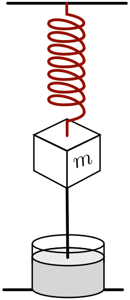
3.4 Complex Eigenvalues

3.5 Repeated Eigenvalues

3.4 Complex Eigenvalues

Exercise for home. Consider a spring with stiffness constant k attached to a mass m and to a dashpot.

- A dashpot provides resistance to movement proportional to the velocity with constant γ.
- Ignore gravity.
- Let $y(t)$ be the position of the mass.



- Find an ODE for $y(x)$.

- Transform that ODE into a system of linear first-order ODEs.

$$\vec{x}' = \begin{bmatrix} 0 & 1 \\ -k & -\gamma \end{bmatrix} \vec{x}$$

Consider no damping: $k = 5$ and $\gamma = 0$.

1. Find one solution $\vec{x}_1(t)$.
2. Write the solution in the form

$$\vec{x}_1(t) = \vec{u}(t) + i\,\vec{v}(t)$$

where $\vec{u}(t)$ and $\vec{v}(t)$ are real-valued.

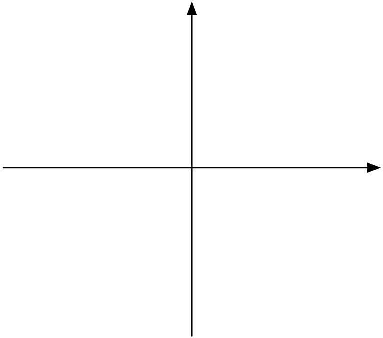
Euler's Formula

$$e^{\theta i} = \cos(\theta) + i\sin(\theta)$$

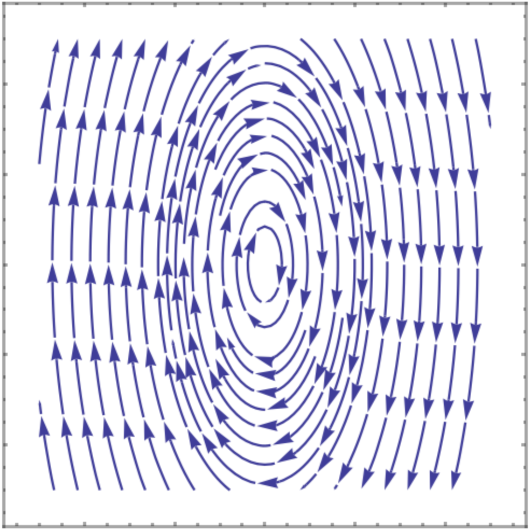
3.4 Complex Eigenvalues

3. Sketch some solutions in the phase plane.

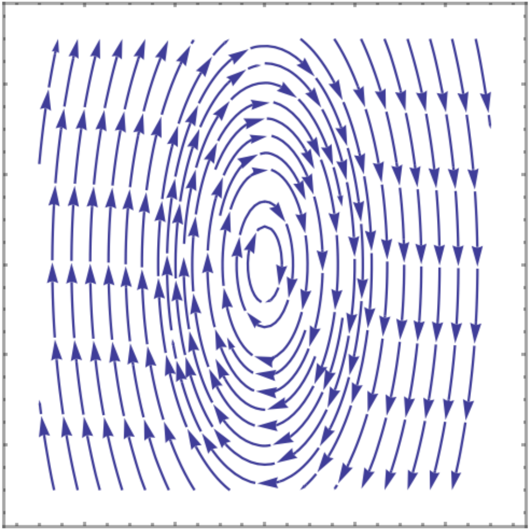
Hint. If the constants are A, B, consider $A = 1, B = 0$.



3.4 Complex Eigenvalues



3.4 Complex Eigenvalues



Centre

Stable

Consider some damping: $k = 5$ and $\gamma = 2$.

$$\vec{x}\,' = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \vec{x}$$

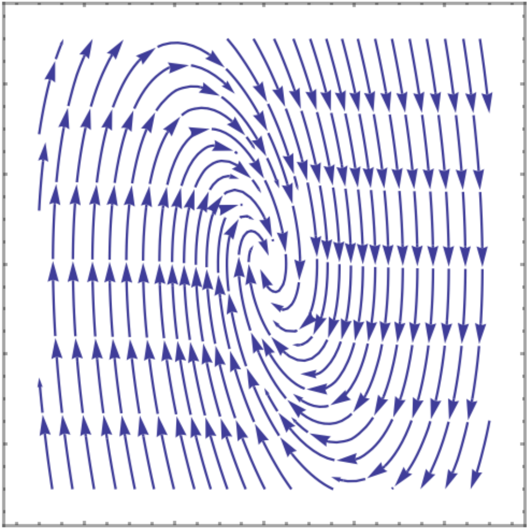
4. Find one solution $\vec{x}_1(t)$.
5. Write the solution in the form

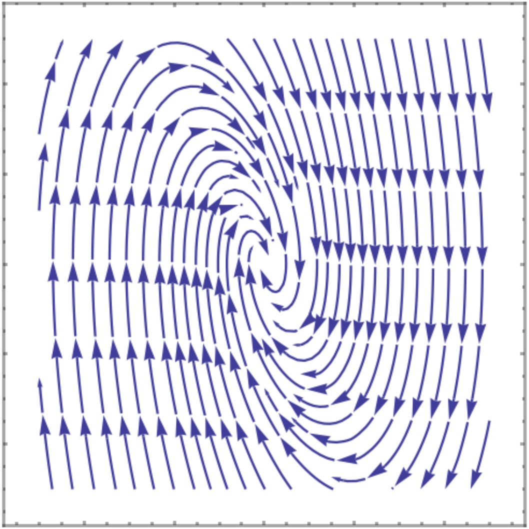
$$\vec{x}_1(t) = \vec{u}(t) + i\,\vec{v}(t)$$

where $\vec{u}(t)$ and $\vec{v}(t)$ are real-valued.
General solution is of the form

$$\vec{x}(t) = A\vec{u}(t) + B\vec{v}(t)$$

6. Sketch some solutions in the phase plane.
7. Clockwise or Counterclockwise?





Spiral Sink

Asymptotically Stable

Preparation for next lecture

Section 3.4

- How to solve a system of linear ODEs with repeated eigenvalues https://youtu.be/hCShTLmeZN4
- How to sketch a phase portrait for such systems. https://youtu.be/dpbRUQ-5YWc