Second-Order ODEs

4 Modelling

4.3 Linear Homogeneous

4.5 Method of Undetermined Coeffs.

4.6 Forced Vibrations

4 Modelling

(ve) Keys can only move vertically.

(hl) Each key has a spring to make the key return to its original position after being pressed (Hooke's Law: "the force is proportional to the extension").

(gr) Gravity is much weaker than the spring that keeps the key in place.

(da) Each key must also include some damping, so that it doesn't keep oscillating back and forth once pressed.

(di) A typical letter key is 15mm$\times$15mm and when pressed has a maximum displacement of 0.5mm.

(lo) Keys last 50 million presses on average.

(fo) On average, a person exerts the force of 42 N with one finger on a key.

1. Model the position $y(t)$ of a keypress of one laptop key.

4.3 Linear Homogeneous

$$\begin{cases} my'' = -ky - \gamma y' \\ y(0) = 0.5 \\ y'(0) = 0 \end{cases}$$

Idea to find solution.

Try $y = e^{rt}$

2. Find a formula for r.

3. What kind of number can r be?

4.3 Linear Homogeneous

4. What happens to the key when γ is large? Do we want this?

$$\begin{cases} y'' = -13y - 14y' \\ y(0) = \frac{1}{2} \\ y'(0) = 0 \end{cases}$$

5. What happens to the key when γ is small? Do we want this?

$$\begin{cases} y'' = -13y - 4y' \\ y(0) = \frac{1}{2} \\ y'(0) = 0 \end{cases}$$

4.3 Linear Homogeneous

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$$\begin{cases} y'' = -13y - 4y' \\ y(0) = \frac{1}{2} \\ y'(0) = 0 \end{cases}$$

6. We want a laptop key that doesn't oscillate, but we also don't want too much damping. What is the minimum amount of damping necessary for the key not to oscillate?

4.3 Linear Homogeneous

7 What happens to the key is critically damped?

$$y'' = -9y - 6y'$$

4.3 Linear Homogeneous

7 What happens to the key is critically damped?

$$y'' = -9y - 6y'$$

a. Find one solution $y_1(t)$.

b. How do we find a second solution?

   Look for solutions of the form

$$y(t) = y_1(t)v(t)$$

c. Which ODE does $v(t)$ satisfy?

d. Find $v(t)$. Find $y(t)$.

4.3 Linear Homogeneous

$$\begin{cases} my'' = -ky - \gamma y' \\ y(0) = \frac{1}{2} \\ y'(0) = 0 \end{cases}$$

8. As the key gets older, its spring constant converges to 0. What happens to the key when the spring breaks?

Preparation for next lecture

Section 4.5.

- Know how to use the Method of Undetermined Coefficients.
  https://youtu.be/YRvqem1n0nQ