2018-10-18 Systems of ODEs with Real Eigenvalues (3.3)

Q1. Now consider this function and sketch the phase plane if eigenvalues $r_1 < 0 < r_2$:

$$\vec{p} = A\begin{bmatrix}2\\1\end{bmatrix}e^{-2t} + B\begin{bmatrix}-1\\1\end{bmatrix}e^{5t}$$

Method:

1. **Draw easy solutions where A and B are both 1**

- Draw eigen vectors
- Then, try to draw the graph when A = B =1, and as t goes from $-\infty$ to $+\infty$:
- **Use easy t values to get direction and behaviour:** It will pass the point when t = 0. (Which is easy to find out)
- When t =$-\infty$, [2, 1] part becomes very large (b/c e^{-2t} -> ∞), and [-1, 1] part becomes very small (b/c e^{5t} -> 0). As a result, the curve becomes nearly parallel to the line [2, 1].
- When t = $+\infty$, [2, 1] part becomes very small (b/c e^{-2t} -> 0), and [-1, 1] part becomes very large (b/c e^{5t} -> ∞). As a result, the curve becomes nearly parallel to the line [-1, 1].
- **Other methods to find direction:**
  - 1. Plot t=0, and t= 1and draw a vector from point 0 to point 1
  - 2. Follow the direction of eigen vectors, similar to phase planes.

2. Then we get the graph when A = B =1. (Similar to $y = \frac{1}{x}$)

3. Finally, we use the properties of symmetry to finish it.

Q2: What is A and B given I(0) = 200 and c(0) =100?

$$\vec{p} = A\begin{bmatrix}2\\1\end{bmatrix}e^{-2t} + B\begin{bmatrix}-1\\1\end{bmatrix}e^{5t}$$

- Set t=0
- 200 = 2 A + -1 B
- 100 = 1 A + 1 B
- Therefore, A=100 , B=0

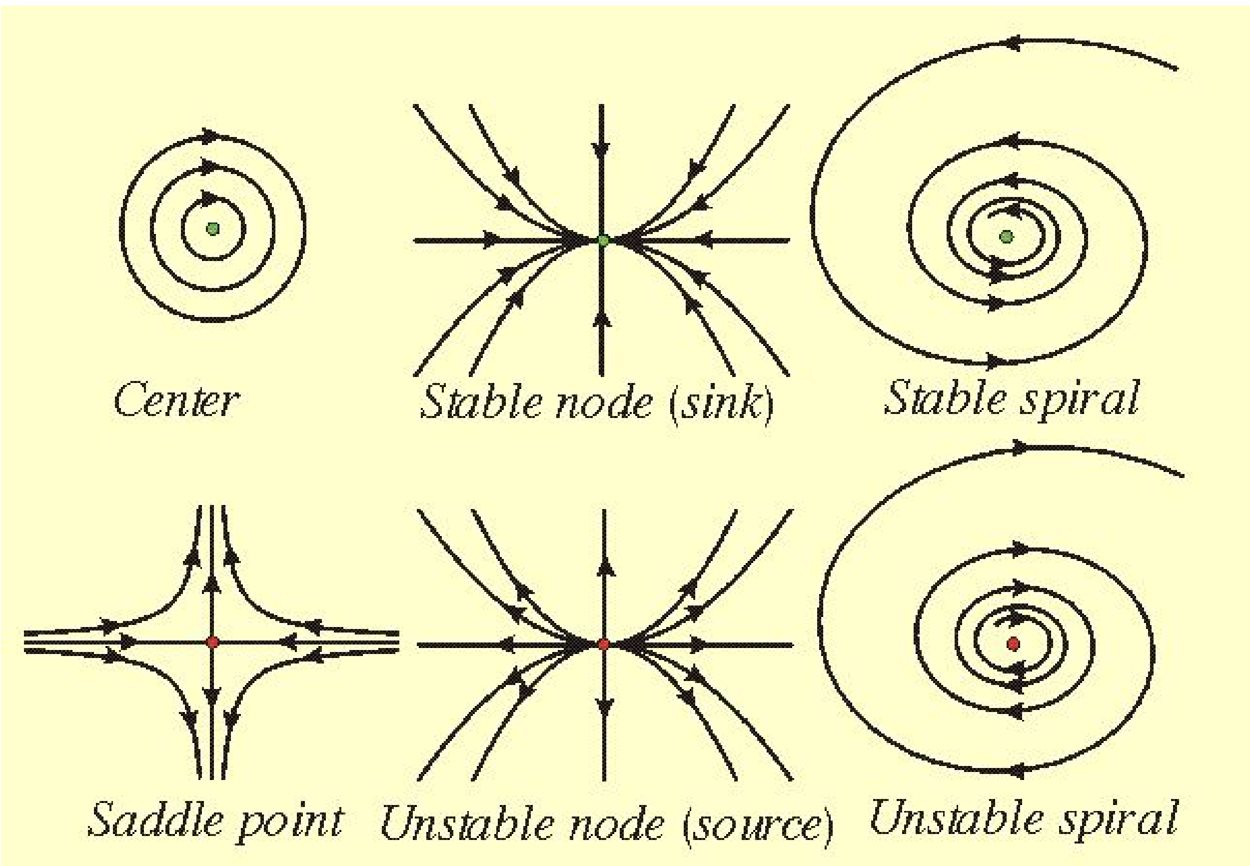
Q3: What kind of critical point is (0,0)?

Ans: Saddle point which is unstable.

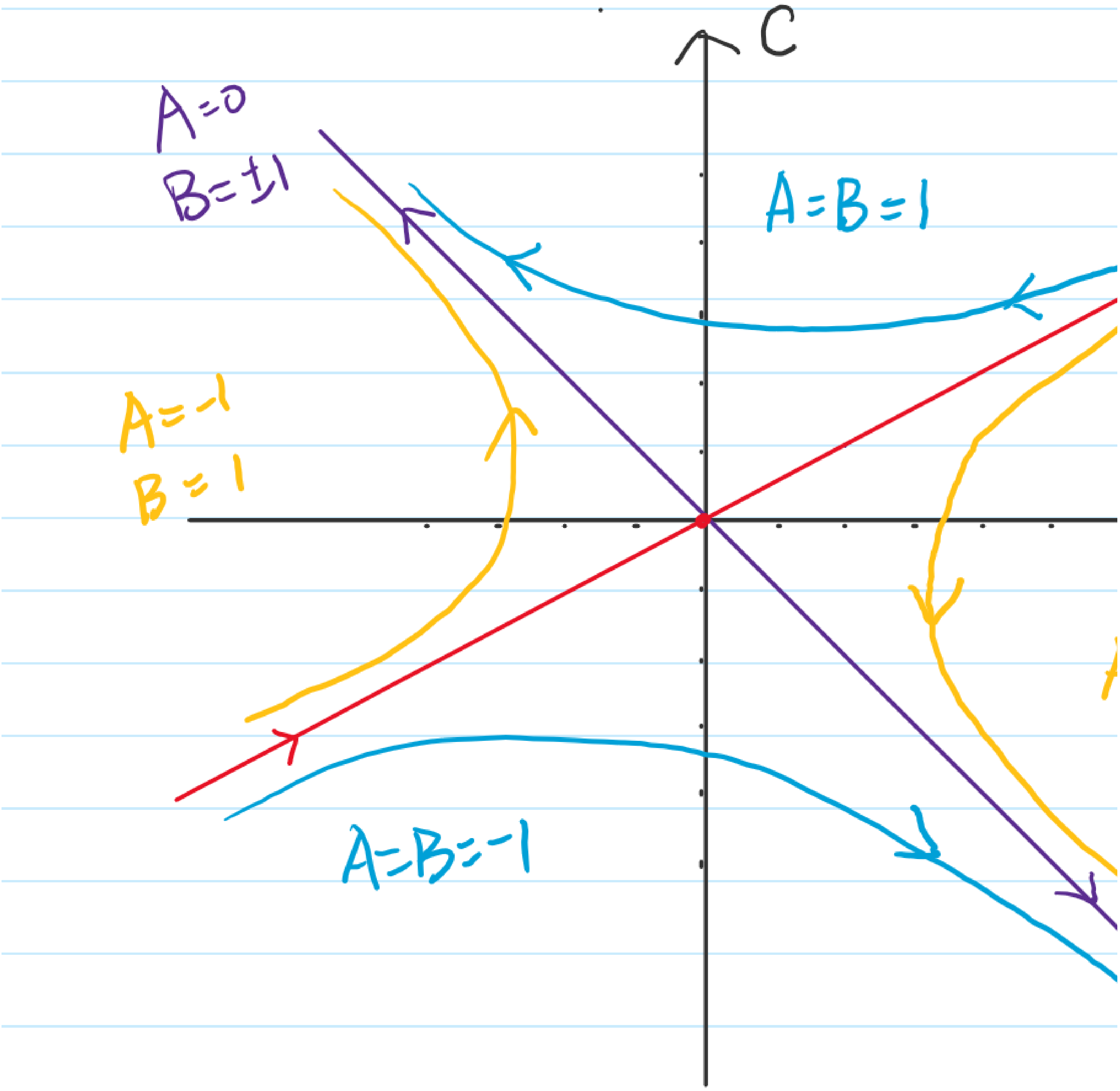
Not stable because solutions do not converge to (0,0) but instead, pass by it.

- Other Resources:

The classification of critical points: **Link (http://staff.www.ltu.se/~larserik/applmath/chap9en/part7.html)**



From the above method, we know the $(0,\ 0)$ point in $\vec{p} = A\begin{bmatrix}2\\1\end{bmatrix}e^{-2t} + B\begin{bmatrix}-1\\1\end{bmatrix}e^{5t}$ is unstable.



Saddle point

*Note:

- the solution curves move in consistent directions. E.g. the yellow and blue curves move in the same direction; otherwise, they would crash into each other.
- the probability of hitting the red curve is zero, because the area of the curve is 0.

Q. Now consider this function and sketch the phase plane:

$$p(t) = A \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{(-2t)} + B \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{(-5t)}$$

The graph of p(t) would be the same as,

$$p(t) = \begin{bmatrix} 1 \\ -1 \end{bmatrix} c_1 e^{5t} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} c_2 e^{2t}$$

but with opposite arrow directions.