3.1 Linear Algebra Review

Eigenvector-Eigenvalue

For a linear transformation T, an **eigenvector** for T is a non-zero vector that doesn't change direction when T is applied. That is, $\vec{v} \neq \vec{0}$ is an eigenvector of T if

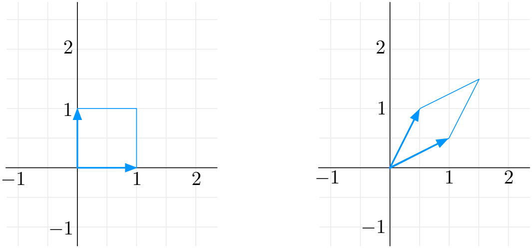
$$T\vec{v} = \lambda\vec{v}$$

for some scalar λ. We call λ the **eigenvalue** of T corresponding to the eigenvector $\vec{v}$.

3.1 Linear Algebra Review

Eigenvector-Eigenvalue: $$T\vec{v} = \lambda\vec{v}$$

The picture shows what the linear transformation T does to the unit square.



1. Give an eigenvector for T. What is the eigenvalue?
2. Can you find another?

For some matrix A,

$$A\begin{bmatrix}3\\3\\1\end{bmatrix} = \begin{bmatrix}2\\2\\\frac{2}{3}\end{bmatrix}.$$

3 Give an eigenvector and corresponding eigenvalue for A.

Consider

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

4. Notice that $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ are eigenvectors for A.

5. Find the eigenvalues of A.

3.1 Linear Algebra Review

Consider

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

6. Find the eigenvalues of A.

7. Find the eigenvectors of A.

Preparation for next lecture

Section 3.3

- How to solve a system of linear ODEs with **real** eigenvalues
  https://youtu.be/YUjdyKhWt6E

- How to sketch a phase portrait for such systems
  https://youtu.be/nyI_JPDrJ_I