2018-09-11 Direction Fields (1.2)

1.2 Direction Fields

We need to know:

1. Acceleration
2. Force = Gravity + Friction

   $F = -mg \pm \gamma v$

   We then talked about the fact that friction always opposes the motion of an object.

   Therefore, we decided to use the negative sign for **friction** formula:

   => **F = -mg - γv**

- Gravity is pointing down in relation to our defined axis, and friction always acts in the opposite direction of velocity

  $mv' = -mg - rv$ (when expressed in terms of v)

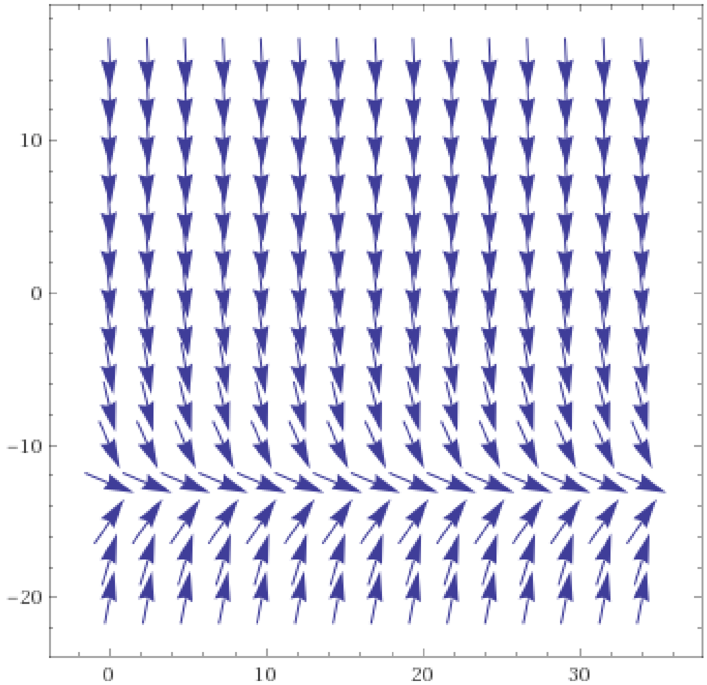
  $mh'' = -mg - rh'$ (when expressed in terms of h)

3. What do we know about the boulder at t = 0?

- $v(0) = V_o$ (May be helpful to think of this as time $v\left(0^{+}\right)$ as this is the notation used in our previous classes) (Note V_0 is not equal to 0 because if it was we would get v' = -g which is not possible because the boulder is not falling)
- $h(0) = 0$ (As we defined our axis to be fixed at the center of gravity of the boulder at time 0, so by definition we are at the origin at time 0)

Direction field for: $v'(t) = -g - rv(t)$ (Divide both sides of the equation, $mv' = -mg - rv$, by m. Since γ is a constant that represents all the factors affecting the motion of the boulder, we denote γ /m=r. This causes **m** notation to be canceled from the formula)

Graph:



Note: WolframAlpha graphing inputs of this example are shown below. You should always try to make your graph square by adjusting the variable range.

- **vector field**: $\left\{ \frac{(1,-9.8-0.75y)}{\sqrt{(1+(-9.8-0.75y)^2)}}, x \right\}$
- **variable 1**: x
- **lower limit 1**: 0
- **upper limit 1**: 34
- **variable 2**: y
- **lower limit 2**: -20
- **upper limit 2**: 15

4. Label the axes (x-axis is t, and y-axis is v)

5. What will happen to the object as time increases?

- When t = 0, v is already -5
- When t > 0, the velocity increases in the downward direction (known because it will follow the arrows in the diagram which have a downward trend towards the horizontal asymptote).
- The acceleration will eventually approach 0 due to friction, causing it to reach terminal velocity (acceleration = 0). This is seen as the line of horizontal arrows on the graph, which makes a horizontal asymptote.
- We can regard this scenario as launching a satellite into outer space if the catapult example cannot relate to your life experience. When the satellite reaches a certain height, the friction and the gravity will cancel out, so that the satellite can maintain a uniform motion. You could also relate it to skydiving. As you jump out of the helicopter, your initial descending speed will be very fast. However, gradually, due to air resistance, your speed will slow down until it reaches a constant speed.

6. What do the horizontal arrows mean?

- The slope is zero, or in this case, acceleration is zero.
- This is the point where terminal velocity is reached.

$v'(t) = -g - rv(t)$ with $v(0) = 10$

7. Integrate both sides. What do you obtain?

$$v(t) = h'(t) = -gt - rh + C$$

And v(0) = 10

$$\therefore C = Vo$$

A few notes on direction fields (not from lecture)

- The equilibrium solution (when the slope is zero) separates increasing from decreasing solutions and it can be:
  - unstable--solutions diverge from equilibrium solution
  - stable--solutions converge to the equilibrium solution
- if the equation you are given is a subtraction:
  - if the variable comes after the number (ie. 3-y) the solutions will converge
  - if the variable comes before the number (ie. y-3) the solutions will diverge
- if the equation is addition:
  - if the variable comes after the number (ie. 2y+3) the solutions will diverge from equilibrium
  - if the variable comes before the number the solution will converge from equilibrium

Variables	Convergence?
When the variable is Positive	Diverge from equilibrium
When the variable is Negative	Converge to equilibrium

- it is possible to have more than one equilibrium solution (ex. y(y-3))