Difference Equations

2.1.1 Compound Interest

2.1.2 Loan Repayment

2.1.3 Gambler's Ruin

2.2.2 Exponential Population Growth

2.2.3 Average Lifespan

2.2.$\star$ Rabbit Populations

2.2.4 Nonlinear Population Models

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Goal. Compare nonlinear Differential Equations vs Difference Equations.

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and the discrete Logistic Population Mode

$$p_{n+1} - p_n = Rp_n\left(1 - \frac{p_n}{K}\right)$$

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2. Consider the influenza virus ($R = 3$). Then $\mu = 4$.

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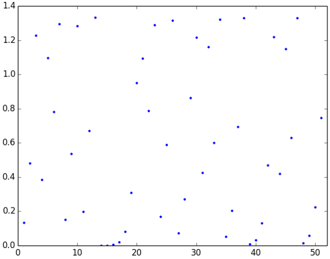

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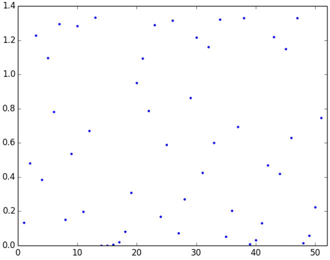

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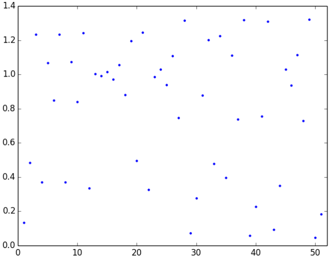

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compare

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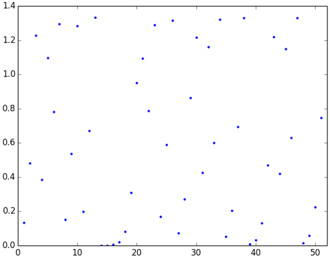

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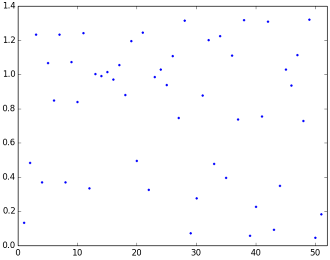

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compare

3 What happens to the continuous model with the same initial conditions?

Logistic and Fractals!

$$x_{n+1} = \mu\, x_n(1 - x_n)$$

4 What happens to x_n if we start with $x_0 = \frac{1}{2}$ and

$$\mu \quad = \quad 1 \quad , \quad 2 \quad , \quad 4 \quad , \quad 8 \quad ?$$

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- Which values of $\mu \in \mathbb{C}$, for which x_n doesn't go to infinity?

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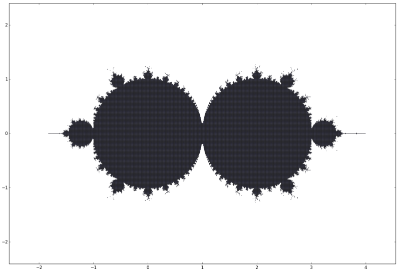

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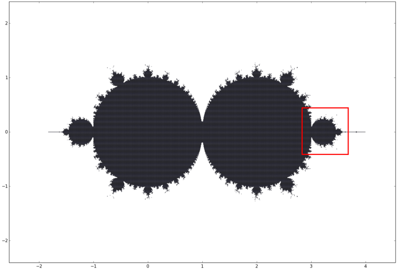

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