2018-11-26 Exponential Population Growth (2.2.2)

2.2.2 Exponential Population Growth

<u>Assumption</u>: Population reproduces at specific times of the year

β= babies born per individual each season (per capita birth rate)

μ = probability of dying between one season and the next

N_k = number of individuals at the start of the k^{th} breeding season

1) **Find a Difference Equation for N_k?**

   $N_{k+1} = N_k - (N_k*\mu) + (N_k*\beta)$ (Correct because it is per capita) -this assumes all the newborns are alive. We add the beta term because babies do not reproduce.
   (OR)
   $N_k = (1-\mu) N_{k-1} + \beta$
   if we assume newborns die as well we can use $N_k = N_{k-1} (1+ \beta)(1-\mu)$

2) **a) Based on your DE, what happens if $\beta>\mu$?**
   b) Based on your DE, what happens if $\beta<\mu$?

   a) when $\beta>\mu$, population will increase
   $N_{k+1} = N_k - (N_k*\mu) + (N_k*\beta)$ -> $N_{k+1} = N_k*(1-\mu+\beta)$ -> $1-\mu+\beta$ is positive (birth > deaths)
   b) when $\beta<\mu$, population will decrease
   $1-\mu+\beta$ is negative (deaths > birth)

2.2.3 Death Rate

Consider the following:

$P(k)$ = probability that an individual born at $k = 0$ is alive at time k (at the end of k^{th} season)

3) **What is the probability of dying between time (k-1) and k?**
   a. Find an expression that uses μ?
   b. Find an expression that does not use μ?
   <u>Note:</u> The expressions found in a) and b) are equal.

   a) Probability(alive at k-1 and die next season)

= Probability(alive at k-1) *P(die next season)

=P(k-1)*μ

P(alive at k-1) is P(k-1)

P(die next season) is μ, not 1-P(k) or 1-μ

b) P(k-1) – P(k)

30%

25%

It is P(k-1) - P(k) because P(k) should be decreasing, therefore, P(k-1) > P(k), and probability is a number between 0 and 1. This is because you have to be alive at P(k-1) in order to be alive at P(k), but not the other way around.

Example:

K = 75

P(74) - P(75)

30% 25%

If we assume 100 people, we will expect 30 of us to live to age 74 and 5 of us will die between ages 74 to 75. 25 of us will live to age 75.