3.1 Linear Algebra Review
3.2 Systems of two ODEs
3.3 Real Eigenvalues
3.4 Complex Eigenvalues
3.5 Repeated Eigenvalues

3.5 Repeated Eigenvalues

Consider the same problem but with $k = 4$.

$$\vec{x}' = \begin{bmatrix} 0 & 1 \\ -4 & -\gamma \end{bmatrix} \vec{x}$$

1. Find the eigenvalues of this matrix.
2. What happens for $\gamma < 4$?
3. What happens for $\gamma > 4$?
4. What happens for $\gamma = 4$?

3.5 Repeated Eigenvalues

Consider the same problem but with $k = 4$.

$$\vec{x}' = \begin{bmatrix} 0 & 1 \\ -4 & -\gamma \end{bmatrix} \vec{x}$$

1. Find the eigenvalues of this matrix.
2. What happens for $\gamma < 4$?
3. What happens for $\gamma > 4$?
4. What happens for $\gamma = 4$?

   This is called a **critically damped** spring-mass system.

3.5 Repeated Eigenvalues

Consider the critically damped problem: $\gamma = 4$:

$$\vec{x}' = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} \vec{x}$$

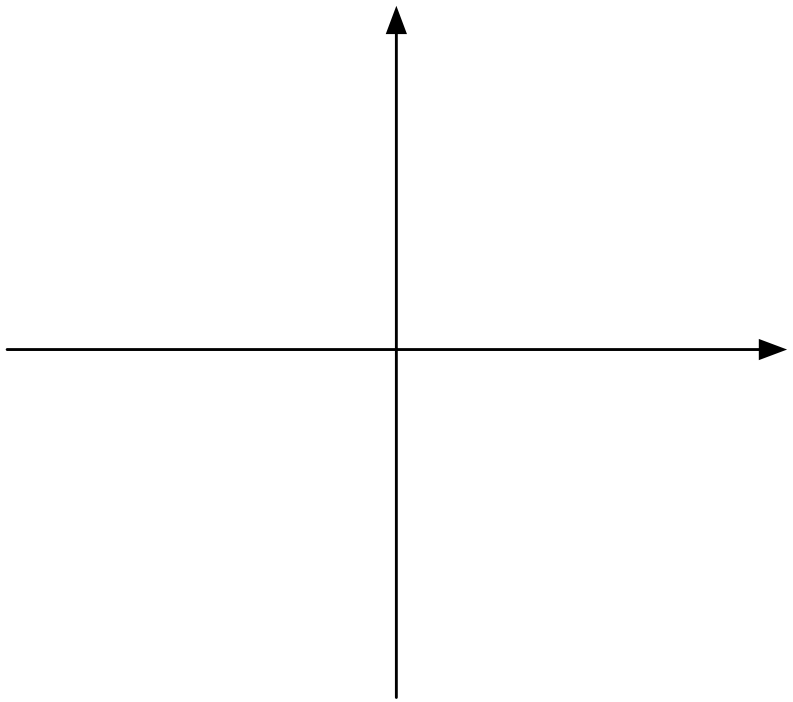
5. Find one solution $\vec{x}_1(t)$.

6. There are no more eigenvalues, so how do we find a second solution $\vec{x}_2(t)$?

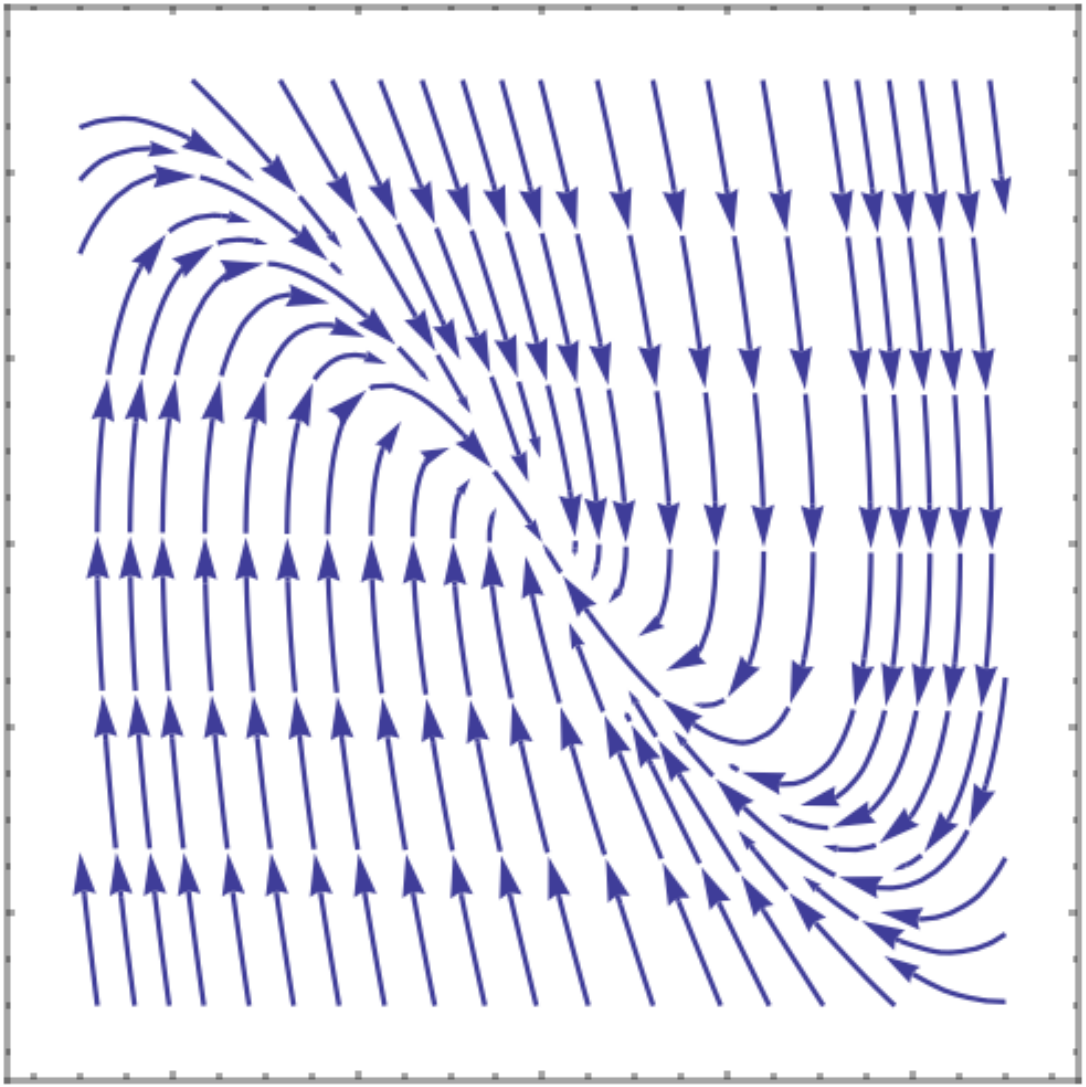
3.5 Repeated Eigenvalues

$$\vec{x}(t) = A \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-2t} + B \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} t \right) e^{-2t}$$

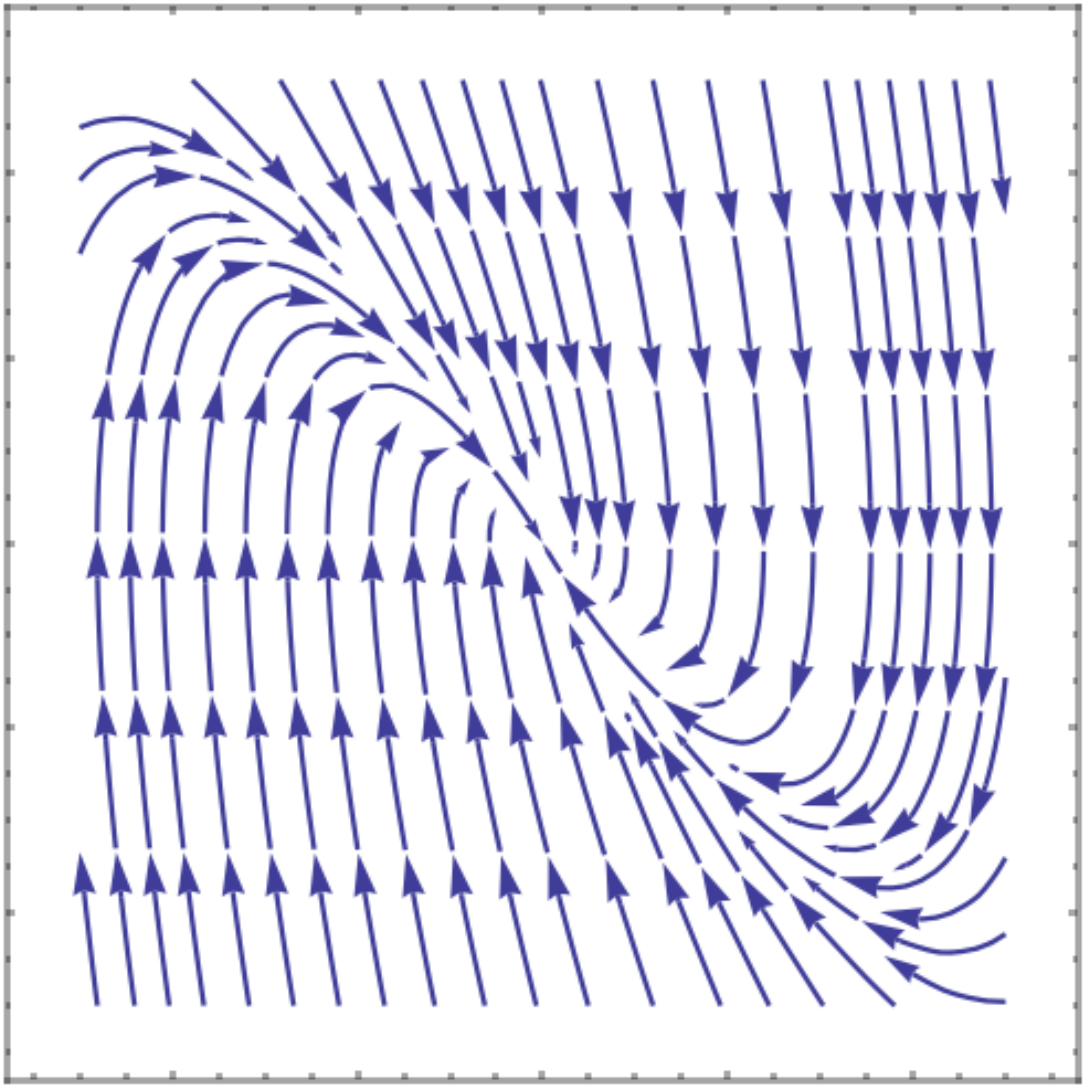
7. Sketch the solutions for $A = \pm 1$ and $B = 0$ in the phase plane.
8. Sketch the solutions for $A = 0$ and $B = \pm 1$ in the phase plane.



3.5 Repeated Eigenvalues



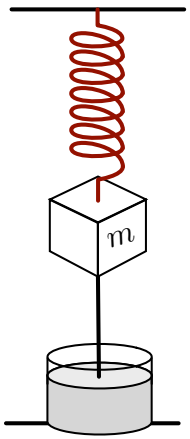
3.5 Repeated Eigenvalues

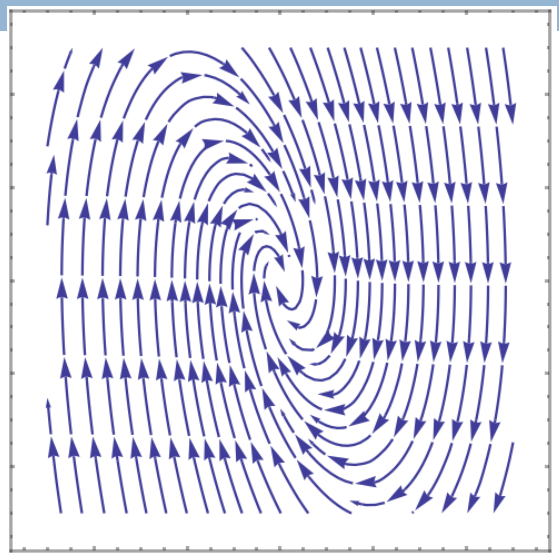


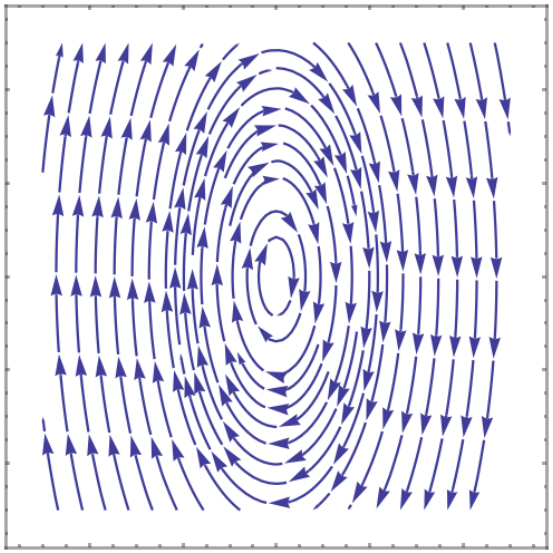
Improper Node

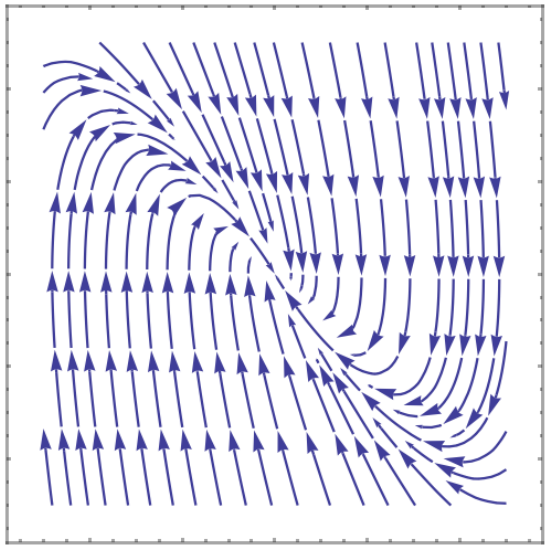
Asymptotically Stable

Spring-Mass-Dashpot









Preparation for next lecture

Section 3.4

- How to solve a system of linear ODEs with repeated eigenvalues https://youtu.be/hCShTLmeZN4
- How to sketch a phase portrait for such systems. https://youtu.be/dpbRUQ-5YWc