3.1 Linear Algebra Review

3.2 Systems of two ODEs

3.3 Real Eigenvalues

3.4 Complex Eigenvalues

3.5 Repeated Eigenvalues

3.2 Systems of two ODEs

Consider two competing populations:

- Lions $\ell(t)$
- Cheetahs $c(t)$

3.2 Systems of two ODEs

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3.2 Systems of two ODEs

Consider two competing populations:

- Lions $\ell(t)$
- Cheetahs $c(t)$

Properties of their populations:

- In the absence of cheetahs, $\ell'(t) \propto$
- If there are a lot of cheetahs, $\ell'(t)$
- In the absence of lions, $c'(t)$
- If there are a lot of lions, $c'(t)$

1. Obtain a DE for $\ell(t)$ and one for $c(t)$.

3.2 Systems of two ODEs

2. If we include a fixed amount of "harvesting" every year, what is the new system of ODEs?

3. Define $\vec{p}(t) = \begin{pmatrix} \ell(t) \\ c(t) \end{pmatrix}$. Write the system in matrix form.

3.2 Systems of two ODEs

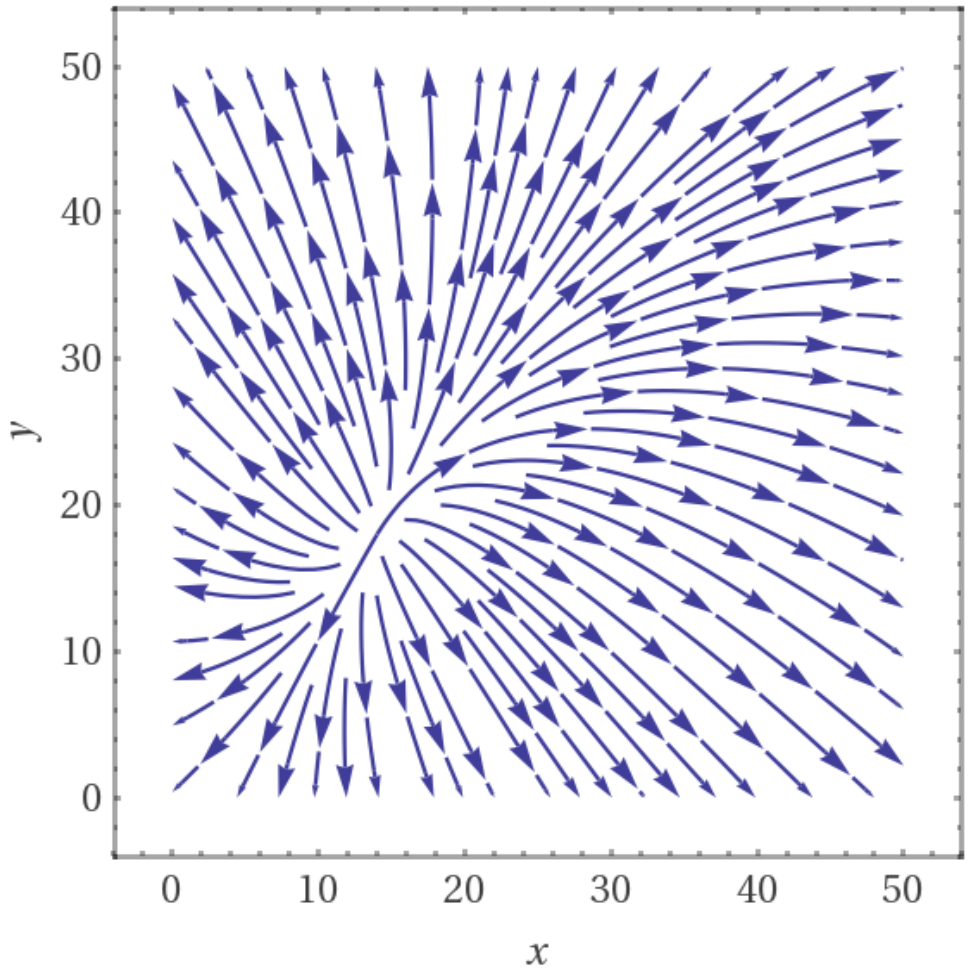
Consider the system

$$\frac{d\vec{p}}{dt} = \begin{pmatrix} 1 & -\frac{1}{6} \\ -\frac{1}{2} & 1 \end{pmatrix} \vec{p} + \begin{pmatrix} -11 \\ -11 \end{pmatrix}$$

4. What is the equilibrium solution?

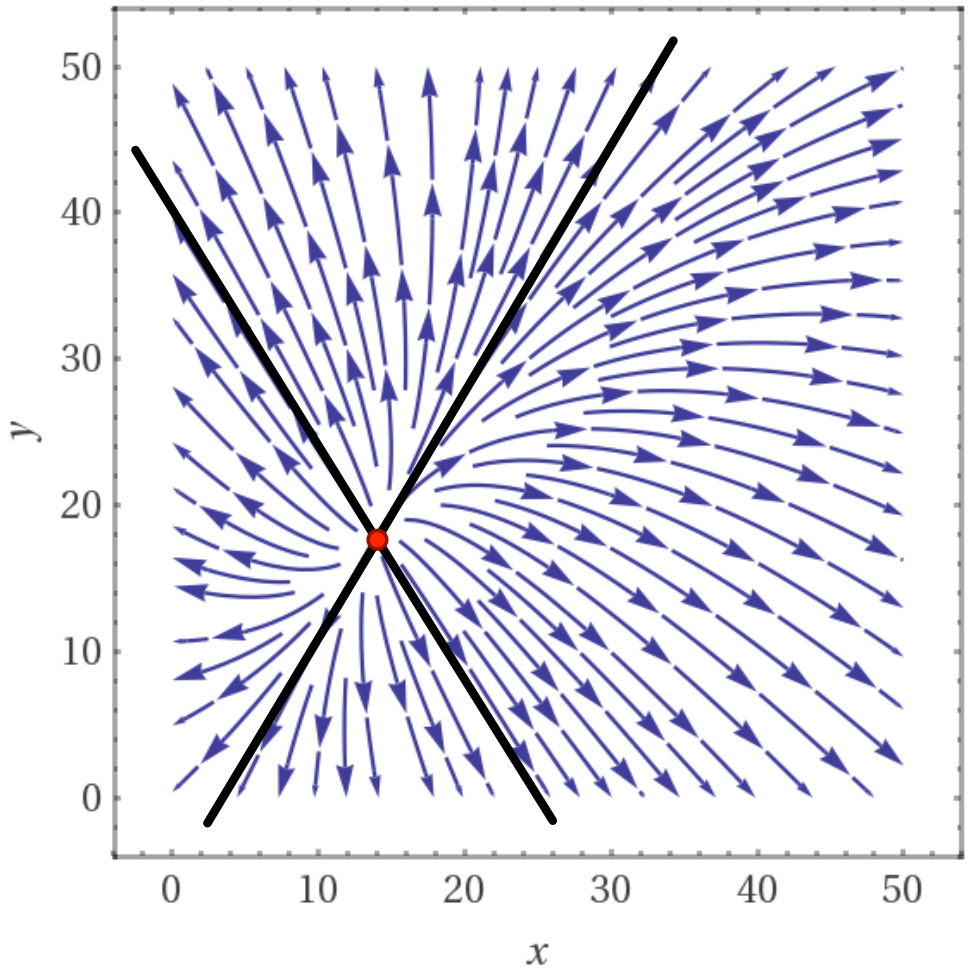
- Direction Field
- Stream Plot

3.2 Systems of two ODEs




```
streamplot (x-y/6-11,-x/2+y-11),x=0..50,y=0..50
```

3.2 Systems of two ODEs

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3.2 Systems of two ODEs

Consider the system

$$\frac{d\vec{p}}{dt} = \begin{pmatrix} 1 & -\frac{1}{6} \\ -\frac{1}{2} & 1 \end{pmatrix} \vec{p} + \begin{pmatrix} -11 \\ -11 \end{pmatrix}$$

Equilibrium is $\vec{p}_{\text{eq}} = \begin{pmatrix} 14 \\ 18 \end{pmatrix}$.

5. Define $\vec{v}(t) = \vec{p}(t) - \vec{p}_{\text{eq}}$.
   Which system of ODEs does $\vec{v}(t)$ satisfy?

Preparation for next lecture

Section 3.3

- How to solve a system of linear ODEs with **real** eigenvalues
  https://youtu.be/YUjdyKhWt6E
- How to sketch a phase portrait for such systems
  https://youtu.be/nyI_JPDrJ_I