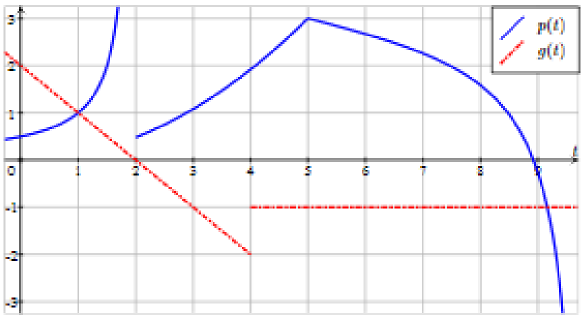
2018-10-01 Autonomous ODEs and Population Growth (2.5)

Q. 

(ii) There exists a unique solution satisfying y(t_0) = -1 defined for

Whether the function is continuous at point t=5?

(There is a sharp change in the line. The function is still continuous at that point because you can draw the function without letting your pen leaving the paper, but the function is not differentiable at that point.)

Continuity Formal Definition: Both the limit to the left and right of the point should be equal to the value at that point. $\lim_{t \longrightarrow a-} f(t) = \lim_{t \longrightarrow a+} f(t) = f(a)$

Derivative Formal Definition: A function f(x) is differentiable at x=a if f'(a) exists. In this case, $f'(a^-) \neq f'(a^+)$. If f(x) is differentiable at x=a, then f(x) is continuous at x=a.

- ANS: $t_0 \in (\text{-}\infty, 2) \cup (4, +\infty)$ and (2,4). (Tip: "U", or union, is the same as writing "and".)
  - Given y(t_0) = -1 .
    - On the left side: At y = -1, p(t) is continuous from (-∞ to 2) and g(t) is continuous from (-∞ to 4); the intersection of these intervals is.$t \in (\text{-}\infty, 2)$
    - Both g(t) and p(t) are continuous within the interval (2,4).
    - On the right side: At y = -1, p(t) is continuous from (2 to +∞) and g(t) is continuous from (4 to +∞); the intersection of these intervals is.$t \in (4, +\infty)$

*Notice:

a. Unlike the solution curves, p(t) and g(t) can intersect.

b. Unlike the solution curves, p(t) and g(t) don't have to go through the y-value, in this case, y(t_0) = -1, but still have a solution for y.

2.5 Population Growth

Population increases proportionally to its current size.

$P' = rP$, $P = P_o e^{rt}$

This is the Malthusian Growth Model. This model can be improved.

Why is this model (Malthusian Growth Model) unrealistic/inaccurate?

- having a fraction of a person/animal will still allow the population to increase.
  (ex. half a zebra will increase to many zebras according to this model)
- other factors that might influence the population (lack of food or space, disease, predators, etc) are not considered
- carrying capacity is not considered (impossible for the population to increase forever)

We can make the model better using the following ideas:

Idea 1. Growth rate r depends on the population $r(P)$

Idea 2. Consider a maximum sustainable population K.

This is the maximum size of the population that can live in the environment before it becomes overpopulated. If the population grows past the maximum sustainable population, the species would have difficulty surviving as they would need to compete for food, water, and other resources. Therefore, the population must be below a certain number (K) for the population to survive.

Idea 3. Consider a survivability threshold S (with $S < K$).

It means that the population is endangered if under the threshold. If left in nature, it will probably go extinct. Therefore, the population must be above a certain number (S) for the population to grow.

Model satisfies:

If, $P > K$ then $r(P) < 0$ (overpopulation)

If, $S < P < K$ then $r(P) > 0$ (normal population)

If, $P < S$ then $r(P) < 0$ (endangered)

1. Give a function that $r(P)$ will satisfy these conditions.

Hint: Think of the graph of $r(P)$

To be continued in the next lecture~