Second-Order ODEs

4 Modelling

4.3 Linear Homogeneous

4.5 Method of Undetermined Coeffs.

4.6 Forced Vibrations

4.5. Method of Undetermined Coefficie

(ve) Keys can only move vertically.

(hl) Each key has a spring to make the key return to its original position after being pressed (Hooke's Law: "the force is proportional to the extension").

(gr) Gravity is much weaker than the spring that keeps the key in place.

(da) Each key must also include some damping, so that it doesn't keep oscillating back and forth once pressed.

(fo) On average, a person exerts the force of 42 N with one finger on a key.

- Model key being released: $my'' = -ky - \gamma y'$
- If we model key being pressed: $my'' = -ky - \gamma y' - 42$

4.5. Method of Undetermined Coefficients

Solve ODEs of the type

$$a\,y''(t) + b\,y'(t) + c\,y(t) = 42$$

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Let

- $x(t)$ satisfies

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- $z(t)$ satisfies

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1. Then $y(t) = x(t) + z(t)$ satisfies which ODE?

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Idea. Some functions don't change much when we take derivatives.

2. Think of functions that don't change type when differentiated.

3. To solve

$$2\,y''(t) - 3\,y'(t) + 4\,y(t) = \text{polynomial}$$

We need $y(t)$ to be what kind of function?

4.5. Method of Undetermined Coefficients

$$y'' - 4y = 10e^{3t}$$

4. What is the particular solution $y_p(t)$?

5. What is the general solution $y(t)$?

$$y'' - 4y = -e^{2t}$$

6 What is the complementary solution $y_c(t)$?

7 What is the particular solution $y_p(t)$?

8 What is the general solution $y(t)$?

$$y'' + y' - 6y = \sin(t)$$

9 What is the particular solution $y_p(t)$?

10 What is the general solution $y(t)$?

4.5. Method of Undetermined Coefficients

$$y'' + 3y' = 3t$$

11 What is the particular solution $y_p(t)$?

12 What is the general solution $y(t)$?

$$y'' + 3y' = 3t$$
$$y(0) = 0$$
$$y'(0) = 0$$

13 What is the solution $y(t)$?

$$y'''' - 4y''' + 10y'' - 12y' + 5y = te^t + t^2\cos(t) - (2t+1)e^t\sin(t)$$

14 What is the particular solution $y_p(t)$?
Don't find the constants.

Hint. $x^4 - 4x^3 + 10x^2 - 12x + 5 = (x-1)^2\big((x-1)^2 + 4\big)$.

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WolframAlpha Solution

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