Exploring the Dynamical Behaviour of Spin Waves Through Energy Absorption Interferometry

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Abstract

The layout suggested is: 1. Introduction 2. Review 3. Methodology 4. Theory 5. Results sections 6. Discussion 7. Conclusion 8. References 9. Appendices

1 Introduction

1.1 Spin Waves

Spin waves are coherent excitations of magnetic dipoles, with scale lengths from $O(10^{-4})$m to $O(10^{-8})$m and characteristic frequencies from gigahertz to terahertz.

At low frequencies they are mediated by dipole-dipole interactions and at high frequencies exchange interactions. They can be created by applying a large static field and a small time varying field to a magnetic material - the static field causes the dipoles to align, or at least form domains, and the time-varying field causes the dipoles to precess in the form of a disturbance that propagates as a waves. They were originally studied in bulk materials but recently focus has shifted towards engineering microscopic band gaps to perform operations such as data manipulation and storage. The unusual properties of single and multiple nanoparticles are of particular interest.

1.2 Ferromagnetic Resonance

The current primary technique for investigation of spin wave systems is ferromagnetic resonance. In this technique -.... We model such a etc

1.3 Energy Absorption Interferometry

The general idea is that the magnetic material is placed in a large static magnetic field, and a pair of phase-locked, time-varying sources - say magnetic dipoles -

are placed nearby. The total power absorbed is measured as a function of the phase difference between the sources, giving a fringe. The complex visibility of this fringe is measured for pairs of source locations, using which the technique is able to reconstruct the dynamical modes and their relative responsivities. Power absorbed can be measured very accurately for a thin film by placing it on a thermally isolated Si or SiN_x membrane and using bolometer technology.

The most interesting thing about the technique is that only the total absorbed power needs to be measured, making this applicable to a large number of systems, for example electric dipoles - as has already been demonstrated [S. Withington and C. N. Thomas, Probing the dynamical behavior of surface dipoles through energy-absorption interferometry, Phys Rev A, 86, 043835, 2012].

1.4 In Practice

A practical system for performing EAI might look like: ????

Bolometer technology is used to measure the power absorped through very accurate temperature sensing

2 Theory

Short Note on Notation

Vectors in bold font, with hats for unit vectors: $\mathbf{u}, \mathbf{m}_j, \hat{\mathbf{k}}$

Scalars are (slightly) italicised: H, ω

Dyadics (outer products of vectors): bold with a double overbar: $\overline{\overline{\boldsymbol{\chi}}}, \overline{\overline{\mathbf{H}}}$

Matrix representations of dyadics and scalars: normal font: $\mathrm{h}, \mathrm{H}^{\mathrm{tot}}, \chi$

2.1 Spin Waves

We shall model the system as a discrete system of magnetic dipoles. This may be done for any system using the discrete dipole approximation (DDA) - the equation then represents averages over finite macroscopic regions (check/write more????). A single dipole with magnetic dipole moment $\mathbf{m_j}$ will evolve according the Landau-Lifshitz-Gilbert equation:

$$\frac{d\mathbf{m_j}}{dt} = -\gamma_j \mu_0 \mathbf{m_j} \times \mathbf{H}_j^{tot} + \frac{\alpha_j}{|\mathbf{m_j}|}(\mathbf{m_j} \times \frac{d\mathbf{m_j}}{dt}) \tag{1}$$

j is an index which specifies a dipole, $\mathbf{H}_j^{tot}$ is the total magnetic field at the dipole, γ_j (positive and negative values corresponding to opposite rotation directions) is the gyromagnetic ratio of the dipole and α_j is a damping coefficient (in FMR, for the transverse components of the dynamic part of the magnetization). This equation could be generalized to a Bloch equation with different longitudinal and transverse dampings but this will not be done to simplify the analysis.

We move to the steady state by writing $\mathbf{H}_j^{tot} = \mathbf{H}_j^{tot\,(0)} + \mathbf{H}_j^{tot\,(t)}$ which represent the non-time dependent and time dependent terms respectively, provided by the large static field and the smaller RF sources. Thus when only the static field is present, the system equilibrates into a state $\mathbf{m_j}^{(0)}$ with no torque on dipoles (from LLG). Generally, the dipoles can form non-aligned domains but we assume they are in the saturated state: all totally aligned with the static field, so $\mathbf{m_j}^{(0)} = m^s\hat{\mathbf{k}}$, and $\mathbf{H}^{tot(0)} \equiv H^0\hat{\mathbf{k}} \approx \mathbf{H}^{ext(0)}$. We also make the vital assumption that in the steady state, all of the absorbed (or indeed extracted) power is transferred from substrate (physically, its phonon system).

We use periodic RF sources so that $\mathbf{H}_j^{tot\,(t)} = \mathbf{H}_j^{tot\,(1)}e^{-i\omega t}$. Substituting into LLG and discarding non-linear terms, effectively the approximation that the dynamical component $\mathbf{m_j}$ is much smaller than the static component in the same direction (we keep only the transverse dynamical components):

$$i\frac{\omega}{\gamma\mu_0}\mathbf{m}^{(1)} = m^s\hat{\mathbf{k}} \times \mathbf{H}^{tot\,(1)} + \mathbf{m}^{(1)} \times H^0\hat{\mathbf{k}} + i\frac{\omega\alpha}{\gamma\mu_0}\hat{\mathbf{k}} \times \mathbf{m}^{(1)}$$

Non-Interacting Case

We may invert the above equation to write the susceptibility of a system of non-interacting, precessing dipoles with position vectors $\mathbf{r}_{0j}$ as:

$$\overline{\overline{\boldsymbol{\chi}}}(\boldsymbol{r}) = \sum_j \frac{\gamma_j\mu_0 m_j^s\omega}{(\omega_{0j}^2 - \omega^2) - i\omega\Gamma_j}\overline{\overline{\boldsymbol{S}}}_j\delta(\boldsymbol{r} - \boldsymbol{r_{0j}}) \tag{2}$$

with

$$\overline{\overline{\mathrm{S}}}_j = \frac{\omega_{0j}}{\omega}\hat{\boldsymbol{i}}\hat{\boldsymbol{i}} + i\hat{\boldsymbol{j}}\hat{\boldsymbol{i}} - i\hat{\boldsymbol{i}}\hat{\boldsymbol{j}} + \frac{\omega_{0j}}{\omega}\hat{\boldsymbol{j}}\hat{\boldsymbol{j}}, \quad S_j = \begin{pmatrix} \omega_0/\omega & -i \\ i & \omega_0/\omega \end{pmatrix}$$

so that the spatial dipole moment is $\mathbf{M}^{(1)}(\mathbf{r}) = \overline{\overline{\chi}}(\mathbf{r}) \cdot \mathbf{H}^{(1)}(\mathbf{r})$. Note here there is no spatial integral or $\mathbf{r}'$ dependence as the non-interacting case is totally local. S_j is the matrix form in the usual representation. The $\mathbf{m}_j^{(1)}$ are time independent in the plane perpendicular to $\mathbf{H}^{(0)}$. The natural frequency of precession is $\omega_{0j} = \gamma_j\mu_0 H_j^{(0)}$ and the damping rate is $\Gamma_j = 2\alpha_j\omega_{0j}$, and we have assumed low loss (small α_j).

The natural modes of the isolated dipole are found from the eigenvectors and eigenvalues (diagonalisation) of $\overline{\overline{\mathbf{S}}}$:

$$\mathbf{u}_+ = \frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + i\hat{\mathbf{j}}), \quad \lambda_+ = \frac{\omega_0}{\omega} + 1$$

$$\mathbf{u}_- = \frac{1}{\sqrt{2}}(\hat{\mathbf{i}} - i\hat{\mathbf{j}}), \quad \lambda_- = \frac{\omega_0}{\omega} - 1$$

corresponding to rotation in the the natural precession direction, and in the reverse direction respectively.

Interacting Case

We can write $\mathbf{H}_j^{tot} = \mathbf{H}_j^{ext} + \mathbf{H}_j^{xch} + \mathbf{H}_j^{dip}$ where the terms represent the applied field to the system, the effective field from exchange interaction with the other dipoles, and the scattered field from the other dipoles.

The exchange interaction effective field can be written as:

$$\mathbf{H}_j^{xch} = J \sum_{k=NN} \mathbf{m}_k \tag{3}$$

where NN indicates the nearest neighbours.

The dipole scattering term is:

$$\mathbf{H}^{dip(1)}(\mathbf{r}) = \int \overline{\overline{\mathbf{G}}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{M}^{(1)}(\mathbf{r}') d^3\mathbf{r}'$$

using the non-retarded magnetostatic greens dyadic

$$\overline{\overline{\mathbf{G}}}(\mathbf{r}, \mathbf{r}') = \frac{1}{|\mathbf{r} - \mathbf{r}'|^3} \left(3\overline{\overline{\mathbf{R}}}(\mathbf{r}, \mathbf{r}') - \bar{\bar{\mathbf{I}}} \right), \qquad \overline{\overline{\mathbf{R}}}(\mathbf{r}, \mathbf{r}') = ???? \tag{4}$$

which may be derived from (reference, also what is R, and is it correct in my code????).

We may now combine these terms and the non interacting succeptebility to give:

$$\mathbf{H}^{ext(1)}(\mathbf{r}_i) = \sum_j \overline{\overline{\mathbf{T}}}_{ji} \cdot \mathbf{H}^{tot(1)}(\mathbf{r}_j) \tag{5}$$

$$\overline{\overline{\mathbf{T}}}_{ij} = \bar{\bar{\mathbf{I}}}\delta_{ij} - \frac{\gamma_i \mu_0 m_i^s \omega}{(\omega_{0i}^2 - \omega^2) - i\omega\Gamma_i} \left\{ J\delta_{i,NN} \overline{\overline{\mathbf{S}}}_i + (1 - \delta_{ij}) \overline{\overline{\mathbf{G}}}(\mathbf{r}_j, \mathbf{r}_i) \cdot \overline{\overline{\mathbf{S}}}_i \right\}$$

which can be written as a matrix equation in the dipole locations, and inverted to give the total field at each dipole in terms of the applied field.

Power Absorption

Since EAI uses power absorption measurements, calculation of them is needed. In the explicitly time dependent and real case, the instantaneous power absorption is:

$$P(t) = \mu_0 \int \mathbf{H}(\mathbf{r}, t) \cdot \frac{\partial \mathbf{M}(\mathbf{r}, t)}{\partial t} d^3\mathbf{r}$$

and thus the time average power absorption is

$$P(\nu) = \frac{\omega}{2} \mu_0 \mathrm{Im} \int_V \mathbf{H}^*(\mathbf{r}) \cdot \mathbf{M}(\mathbf{r}) d^3\mathbf{r} = \frac{\omega}{2} \mu_0 \mathrm{Im} \int_V \int_V \mathbf{H}^*(\mathbf{r}) \cdot \overline{\overline{\boldsymbol{\chi}}}(\boldsymbol{r}, \mathbf{r}') \cdot \mathbf{H}(\mathbf{r}') d^3\mathbf{r} d^3\mathbf{r}' \tag{6}$$

which is over the volume of the sample, V, since the succeptibility elsewhere is zero.

For our system of dipoles we thus find a power absorption:

$$P(\nu) = \sum_j \frac{\gamma_j \mu_0^2 m_j^s}{2} \left[\frac{\omega \Gamma_j}{\omega^2 (\frac{\omega_{0j}^2}{\omega^2} - 1)^2 + \Gamma_j^2} \right] \times \mathbf{H}_j^{tot(1)*} \cdot \overline{\overline{\mathbf{S}_j}} \cdot \mathbf{H}_j^{tot(1)}$$

If the system is rotationally excited, $\mathbf{H}^{tot(1)}(\mathbf{r}_i) = a\mathbf{u}_\pm$, the absorbed power will be

$$< P(a\mathbf{u}_\pm) >= \frac{1}{2} a^2 \Gamma \gamma \mu_0^2 m_s \frac{\omega(\omega_0/\omega \pm 1)}{\omega^2(\omega_0^2/\omega^2 - 1)^2 + \Gamma^2}$$

We see from the numerator that the power is always positive for rotation with the natural precession direction, and reaches a peak at $\omega = \omega_0$. However, for the opposite direction rotations, the power is zero at $\omega = \omega_0$, and work may be done by the system for $\omega > \omega_0$, taking energy out of the dipole. Physically this corresponds to ????

2.2 Theory of EAI

After a number of steps from the above forms for the power we may arrive at the most useful form:

$$< P(\nu) >= \frac{\omega}{2} \mu_0 \int_V \int_V \overline{\overline{\mathbf{C}}}^{tot(1)}(\boldsymbol{r}, \mathbf{r}') \; \cdot \cdot \, \overline{\overline{\boldsymbol{\chi}}}^{R}(\boldsymbol{r}, \mathbf{r}') d^3\mathbf{r} d^3\mathbf{r}' \tag{7}$$

where we have defined $\overline{\overline{\boldsymbol{\chi}}}^{R}(\boldsymbol{r}, \mathbf{r}') = -i\overline{\overline{\boldsymbol{\chi}}}^{A}(\boldsymbol{r}, \mathbf{r}')$ with $\overline{\overline{\boldsymbol{\chi}}}^{A}$ the antihermitian part of $\overline{\overline{\boldsymbol{\chi}}}$ (this makes the power real and discards the non-absorping hermitan part of $\overline{\overline{\boldsymbol{\chi}}}$); and the magnetic field correlation dyadic (the expectation value across an ensemble of systems, or no such step required for coherent sources):

$$\overline{\overline{\mathbf{C}}}(\boldsymbol{r}, \mathbf{r}') =< \mathbf{H}(\mathbf{r}) \mathbf{H}^*(\mathbf{r}') > \tag{8}$$

and the double dot is a full contraction to a scalar.

This clearly has the form of an inner product in the abstract vector space of dyadic fields. The power absorbed is the projection of the magnetic field correlation dyadic onto the succeptibility dyadic.

Non-Scattered Case

If a source at $\mathbf{r}_n$ produces the field $\mathbf{h}_n(\mathbf{r})$, and we have N possible source positions, we produce a basis $\mathbb{A} = \{\mathbf{h}_n(\mathbf{r}), \forall n \in 1...N\}$. We can then write the matrix elements of $\overline{\overline{\boldsymbol{\chi}}}^{R}(\boldsymbol{r}, \mathbf{r}')$ in the basis, and approximately (or exactly, if the source fields span the basis of absorption) reconstruct it using the dual vector set:

$$\chi_{nm} = \int_V \int_V \mathbf{h}_n^*(\mathbf{r}) \cdot \overline{\overline{\boldsymbol{\chi}}}^{R}(\boldsymbol{r}, \mathbf{r}') \cdot \mathbf{h}_m(\mathbf{r}') d^3\mathbf{r} d^3\mathbf{r}'$$

$$\overline{\overline{\boldsymbol{\chi}}}^{R}(\boldsymbol{r},\mathbf{r}') \approx \sum_{nm} \chi_{nm} \tilde{\mathbf{h}}_m(\mathbf{r}) \tilde{\mathbf{h}}_m^*(\mathbf{r}') \tag{9}$$

Since we can find the dual set $\tilde{\mathbb{A}}$ numerically from knowing the forms of the impressed fields, if we know the matrix elements χ_{nm} we can find the response dyadic and diagonalise it to find the natural modes of the system.

It turns out we can do this using only power measurements: this is the beauty of EAI. We illuminate the sample with two fully coherent sources so that (???? tot or ext)

$$\mathbf{H}^{ext????}(\mathbf{r}_j) = \mathbf{h}_n + \mathbf{h}_m e^{-i\Delta\phi}$$

where $\Delta\phi$ is a phase that we may rotate. We then find that the absorbed power becomes:

$$< P(\nu) >= \frac{k_0 Z_0}{2} \left\{ \chi_{nn} + \chi_{mm} + 2|\chi_{nm}| \cos(\Delta\phi + \arg(\chi_{nm})) \right\}$$

Thus χ_{nm} is a hermitan matrix, and we may measure the on diagonal terms directly from the power absorbance from each source, and the off diagonal terms by sweeping the phase between the sources and measuring the resulting fringe in the power.

Scattering

In the scattered case????

2.3 Matrix Formulation

Now that our model is fully discretised, we describe it in terms of matricies. The column vectors $\mathrm{h}^{\mathrm{ext}}, \mathrm{h}^{\mathrm{tot}} \in \mathbb{C}^{2J}$ contain the complex amplitudes of the transverse (to the static field, now placed so $\hat{\mathbf{k}} = \hat{\mathbf{z}}$, also where is time dependence, have to take real part????) cartesian field components of the external and total fields at the position of the dipoles. Clearly from 5 we can write $\mathrm{h}^{\mathrm{tot}} = \mathrm{T}^{-1}\mathrm{h}^{\mathrm{ext}}$, and also:

$$P = \frac{\omega}{2} \mu_0 (\mathrm{h}^{\mathrm{tot}})^\dagger \chi^R \mathrm{h}^{\mathrm{tot}}$$

$$< P >= \mathrm{Tr}\left[\mathrm{C}^{\mathrm{tot}}\mathrm{N}\right], \qquad \mathrm{N} = \frac{\omega}{2}\mu_0 \chi^R$$

where we have taken then trace of a number and used the cyclic property to write $\mathrm{C}^{\mathrm{tot}} =< \mathrm{h}^{\mathrm{tot}} (\mathrm{h}^{\mathrm{tot}})^\dagger >$. Conceptually, this is equivalent to equation 7.

We may also write the power in terms of the external field:

$$P = \frac{\omega}{2}\mu_0 (\mathrm{h}^{\mathrm{tot}})^\dagger \chi^R \mathrm{h}^{\mathrm{tot}} = \frac{\omega}{2}\mu_0 (\mathrm{h}^{\mathrm{ext}})^\dagger (\mathrm{T}^{-1})^\dagger \chi^R (\mathrm{T}^{-1}) \mathrm{h}^{\mathrm{ext}} = (\mathrm{h}^{\mathrm{ext}})^\dagger \mathrm{L} \mathrm{h}^{\mathrm{ext}}$$

$$< P >= \mathrm{Tr}\left[\mathrm{C}^{\mathrm{ext}}\mathrm{L}\right] \tag{10}$$

Since $\mathrm{C}^{\mathrm{ext}}$ and L are hermitian, they may be diagonalised: $\mathrm{C}^{\mathrm{ext}} = \sum_i \alpha_i \mathrm{f}_i \mathrm{f}_i^\dagger$, $\mathrm{L} = \sum_i \beta_i \mathrm{g}_i \mathrm{g}_i^\dagger$. The power is then

$$< P >= \sum_{ij} \alpha_i \beta_j \left| \mathrm{f}_i^\dagger \mathrm{g}_j \right|^2$$

which describes how the impressed field modes project onto the system modes.

Closer examination of L

We may look more closely at exactly where L comes from and how it may be incrementally built up. We have a column vector $\mathrm{m} \in \mathbb{C}^{2J}$ containing the transverse components of the dipole moments. If exchange interactions are present then:

$$\mathrm{m} = \chi \mathrm{h}^{\mathrm{ext}} + \chi \mathrm{G}^{\mathrm{xch}} \mathrm{m} = (\chi^{-1} - \mathrm{G}^{\mathrm{xch}})^{-1} \equiv \xi \mathrm{h}^{\mathrm{ext}}$$

where $\mathrm{G}^{\mathrm{xch}}$ is the exchange interaction from 3 and χ is simply the local succeptibility.

Next we include dipole-dipole scattering, with 4 giving us $\mathrm{G}^{\mathrm{dip}}$:

$$\mathrm{h}^{\mathrm{tot}} = \mathrm{h}^{\mathrm{ext}} + \mathrm{G}^{\mathrm{dip}} \mathrm{m} = (1 - \mathrm{G}^{\mathrm{dip}} \xi)^{-1} \equiv \kappa \mathrm{h}^{\mathrm{ext}}$$

defining κ, and giving $m = \xi \kappa h^e$.

Thus

$$\mathrm{L} = \frac{\omega}{2} \mu_0 \kappa^\dagger \xi^R \kappa \tag{11}$$

where ξ^R is the hermitian part of $-i\xi$.

Finally, for convinience of computation, we also introduce the matrix:

$$\mathrm{M} = \frac{\omega}{2} \mu_0 \xi^R$$

2.4 Reconstruction with EAI

If the matrix $\mathrm{H}^{\mathrm{ext}} \in \mathbb{C}^{2J \times 2N}$ (???? naming?) is made up of columns containing the fields $\mathrm{h}^{\mathrm{ext}}$ associated with particular possible sources. If these impressed fields span the absorption modes, the response matrix can be written in terms of the duals:

$$\mathrm{L} = \sum_{nm} a_{nm} \tilde{\mathrm{h}}_{\mathrm{n}}^{\mathrm{ext}} \tilde{\mathrm{h}}_{\mathrm{m}}^{\mathrm{ext}\dagger} = \tilde{\mathrm{H}} \mathrm{A} \tilde{\mathrm{H}}^\dagger \tag{12}$$

so A gives the response in the basis of the sources.

The power absorbed due to two sources is given by:

$$P = [\mathrm{h}_1^{\mathrm{ext}\dagger} e^{-i\phi_1} + \mathrm{h}_2^{\mathrm{ext}} e^{-i\phi_2}] \, \mathrm{L} \, [\mathrm{h}_1^{\mathrm{ext}} e^{i\phi_1} + \mathrm{h}_2^{\mathrm{ext}\dagger} e^{i\phi_2}]$$

and subbing in the response:

$$P_{nm} = a_{nn} + a_{mm} + a_{nm}e^{i\Delta\phi} + a_{mn}e^{-i\Delta\phi}$$

so we may measure a fringe in the power by rotating the phase, and hence find the matrix elements of A.

Deconvolution

The matrix A contains the intrinsic absorbances of the dipoles, N, convolved with the scattering and exchange interactions, T^{-1}, and finally the greens function taking the field at the sources to the dipoles. Generally only the final step may be deconvolved, as if we know the source properties we may calculate the forms of the impressed fields and invert them (we are allowed to do this because in general we do not only use the matrix forms but the dyadic ones above - we just use the dipole positions as part of our simulation????).

The dual vectors are defined by $\mathrm{H}^{\mathrm{ext}\dagger}\tilde{\mathrm{H}}^{\mathrm{ext}} = 1$, but inversion is not possible if J is not equal to N. We use the SVD pseudo-inverse, defined by:

$$\mathrm{H} = \mathrm{U}\Sigma\mathrm{V}^{\dagger} \implies \tilde{\mathrm{H}} = \mathrm{U}\Sigma^{-1}\mathrm{V}^{\dagger}; \qquad \mathrm{H}^{\dagger}\tilde{\mathrm{H}} = \mathrm{U}\Sigma\Sigma^{-1}\mathrm{U}^{\dagger}$$

which correctly accounts for numbers of source positions and sample points. If the basis is complete, or overcomplete, the final relation will equal the identity. If it is undercomplete, the modes are projected onto a measurement space, a filter is applied (which may be impossible to get rid of due to noise) and then the modes are reconstructed.

The whole process may be done using incremental SVD, where each new measurement increases information until all degrees of freedom are found.

3 Modeling

3.1 Response Matrix

All calculations are done in the basis with the fields or moments at the dipoles unless otherwise stated. Firstly we calculate the local succeptibility, called X rather than χ for clarity, from 2. Next we apply the coupling and scattering from ...

4 Results and Discussion

4.1 General Dipole Systems

Ferromagnetic resonance first, then scan source frequency, then show different source locations, then fringes, then coherence lengths/volumes