0.1 B3 Analog Modulation

We will discuss three main modulation techniques, namely

1. Amplitude Modulation (AM),
2. Phase Modulation (PM), and
3. Frequency Modulation (FM)

Let us agree to define $m(t) \iff \mathbf{S}(f)$ as some real-valued, strictly bandlimited, baseband signal. And our carrier wave $A_c \cos 2\pi f_c t$ at carrier frequency f_cHz.

Definition. Coherent Demodulation is when the demodulator requires a reference signal which has exactly the same frequency and phase as the carrier signal.

0.1.1 DSB-LC AM

Transmitted Signal

$$s_{LC}(t) = A_c\Big(1 + km(t)\Big)\cos(2\pi f_c t) \tag{1}$$

$$\mathbf{S}_{LC}(f) = \frac{A_c}{2}\Big[\delta(f - f_c) + \delta(f + f_c)\Big] + \frac{kA_c}{2}\Big[\mathbf{M}(f - f_c) + \mathbf{M}(f + f_c)\Big] \tag{2}$$

Modulation Considerations

- k is chosen such that $1 + km(t) \geq 0$ for all $t \geq 0$ to prevent phase reversal,
- AM Modulation index: $\phi = -km_{min} \leq 1$,
- Percentage Modulation: 100ϕ,
- Under/Over-modulation $\phi < 1$ or $\phi > 1$

Power efficiency

- The unmodulated carrier component has power

$$P_c = \frac{A_c^2}{2}$$

- The information signal power,

$$P_s = \frac{(kA_c)^2}{2} \int |m(t)|^2 dt$$

- The required power,

$$P_t = P_c + P_s$$

- It can be shown that assuming that $m(t)$ is a sinusoid, then the information signal power is bounded above by

$$P_s \leq \frac{1}{3} P_t$$

(Waste power bad!)

What to Remember (DSB-LC AM)**.**

1. Simple and Robust
2. Envelope Detection, does not require coherent demodulation.
3. Bandwidth: $m(t)$ has bandwidth W,then $s_{LC}(t)$ will require $2W$ bandwidth,
4. Low POWER efficiency, because of unmodulated carrier component
5. Bandwidth Overlapping: Require $W \ll f_c$.
6. Within AWGN channel, provides better SNR than DSB-SC, SSB-SC.

0.1.2 DSB-SC AM

Transmitted Signal

$$s_{SC}(t) = A_c k m(t) \cos(2\pi f_c t) \tag{3}$$

$$\mathbf{S}_{SC}(f) = \frac{kA_c}{2} \Big[\mathbf{M}(f - f_c) + \mathbf{M}(f + f_c) \Big] \tag{4}$$

What to Remember (DSB-SC AM)**.**

1. Not as simple as DSB-LC
2. Requires coherent demodulation. Complicated set up.
3. Bandwidth: $2W$, same as DSB-LC
4. Power Efficiency: Higher than DSB-LC

0.1.3 SSB-SC AM

Single-sideband, suppressed carrier. We either choose Upper or Lower side bands (away and towards the origin), because of hermitian symmetry of $\mathbf{S}(f)$.

- Bandwidth: W, improved,
- Hard to realize the phase splitter (unit-step in frequency domain) at baseband, because of the discontinuity at $f = 0$.
- Demodulation is even more complex than DSB-SC

0.1.4 FM

What to Remember.

1. $s_{FM}(t) = A_c \cos(\theta(t))$, with $\theta(t)$ being the 'phase' of the transmitted signal
$$\theta(t) = 2\pi\left(f_c t + k\int_{-\infty}^{t} m(x)dx\right)$$
The instantaneous frequency is therefore
$$\frac{d}{dt}\theta(t) = 2\pi(f_c + km(t))$$
2. Phase proportional to integral of $m(t)$.
3. Peak Frequency Deviation: $\Delta f = k\|m(t)\|_\infty$,
4. FM Index: $\beta = \Delta f/W$, W is the bandwidth of $m(t)$
5. Carson's Rule: required bandwidth for FM $B_{FM} \approx 2(1+\beta)W$

6. FM requires much larger BW than AM,
7. Increasing β increases required BW, and improves $SNR_{out} = SNR_{in}[3\beta^2(1+\beta)/2]$.
8. AM radio systems operate at much lower BW than FM. 500-1700kHz compared to 88-108MHz.

0.1.5 PM

What to Remember.

1. Transmitted Signal

$$s_{PM}(t) = A_c \cos\Bigg(2\pi f_c t + km(t)\Bigg)$$

2. Instant Frequency is proportional to $\frac{d}{dt}m(t)$.
3. Phase proportional to $m(t)$.

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