0.1 Assignment 2

0.1.1 D5

(a) $SNR_{dB} = 10\log_{10}(SNR) = 5$, hence

$$SNR = 10^{2^{-1}}$$

Spectral Efficiency $\mathcal{E}_{max} = \log_2(1 + SNR)$, and each channel occupies $6^{-1}120 \times 10^3$Hz of bandwidth. Calculating bitrate R_b,

$$R_b = 6^{-1}120 \times 10^3 \log_2(1 + 10^{2^{-1}}) \approx 49400\text{bps}$$

(b) If we have N channel, then we require $N - 1$ guard bands. Suppose now that $B_{guard} = 10^{-1}B_{channel}$, then

$$NB_{channel} + (N-1)B_{guard} = 120 \times 10^3 \implies B_{channel} \approx 18.5\text{kHz}$$

Using the previous bitrate, and scaling by a factor of $(18.5)/20$,

$$R_b \approx 49400\frac{18.5}{20} \approx 45695\text{bps}$$

0.1.2 D7

(a) Let $\mathbf{X}$ be a geometric random variable on an arbitrary space. With $p = 0.2$, then the mean is given by

$$\int_{\Omega} \mathbf{X}dP = \sum_{m\geq 0}(1-p)^m p = p^{-1} = 5 \text{ trials till first success}$$

(b) Load G:

$$G = \ln(0.2)(-2)^{-1} \approx 0.805$$
$$S_{ALOHA} = 0.2G \approx 0.161$$

(c) Decrease load $G \to 2^{-1}$. Maximum achievable rate for a unslotted ALOHA

$$\sup\left\{S_{unslotted}(G)\right\} = S_{unslotted}(G=1) = 2^{-1}e^{-2(2^{-1})} \approx 0.184\text{frames/framelength}$$

In bps,

$$\sup\left\{S_{unslotted}(G)\right\}_{BPS} = (0.1)^{-1}(1000)\cdot 0.184 \approx 1840\text{bps}$$

(d) Maximum achievable rate for slotted ALOHA

$$\sup\left\{S_{Slotted}(G)\right\} = S_{Slotted}(G = 1) = e^{-1} \approx 0.368\text{frames/framelength}$$

Given e^{-2} frames per frame interval, we have $(0.1)^{-1}e^{-2}$ frames per second. Each frame contains 1000 bits, therefore

$$\sup\left\{S_{Slotted}(G)\right\}_{bps} = 10000e^{-1} \approx 3680\text{bps}$$