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Research Article

MVDR Algorithm Based on Estimated Diagonal Loading for Beamforming

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Beamforming algorithm is widely used in many signal processing fields. At present, the typical beamforming algorithm is MVDR (Minimum Variance Distortionless Response). However, the performance of MVDR algorithm relies on the accurate covariance matrix. The MVDR algorithm declines dramatically with the inaccurate covariance matrix. To solve the problem, studying the beamforming array signal model and beamforming MVDR algorithm, we improve MVDR algorithm based on estimated diagonal loading for beamforming. MVDR optimization model based on diagonal loading compensation is established and the interval of the diagonal loading compensation value is deduced on the basis of the matrix theory. The optimal diagonal loading value in the interval is also determined through the experimental method. The experimental results show that the algorithm compared with existing algorithms is practical and effective.

1. Introduction

The beamformer is used for the purpose of detecting the desired signals and suppressing the interference signals. It filters the interference signals and outputs the desired signals whose mainlobe points to desired direction. Beamformer, which is the key technology in many fields, is widely used in radar, communications, sonar, GPS, and other fields. For example, weather radar system needs the algorithm to get the desired signals [1] from the meteorological information. In MIMO communication system, the beamformer is used to achieve large array gains [2].

MVDR (Minimum Variance Distortionless Response) is a typical beamforming (beamformer) algorithm, which was proposed by Capon in 1967 [3]. It makes the outputting power with minimum interference and noise in the desired direction through adjusting a weight factor. However, the performance of the beamforming algorithm will decline dramatically with the inaccuracy in the steering vector and the covariance matrix [4]. To reduce the dispersive degree of the eigenvalues of the covariance matrix, Carlson proposed diagonal loading by pulsing a constant diagonal loading compensation value to the diagonal elements of the covariance matrix [5]. On this basis of constant diagonal loading compensation, the literature [6] selects the loading value which is higher than the background noise. However, when the snapshots are small, there is a large inaccuracy in the sample covariance matrix, and the constant diagonal loading compensation value cannot correct the inaccuracy. Literatures [7–9] reconstruct the covariance matrix which is also one of the diagonal loadings. In literature [10], an optimal algorithm employing variable diagonal loading of the sample covariance matrix eigenvalues is proposed. In literature [11], an algorithm is used to compute the diagonal loading level automatically from the given data with a shrinkage method of enhancing covariance matrix. Literature [12] uses beam-to-reference ratio which is estimated as a weighting factor for variable diagonal loading. These algorithms have solved the problem that the diagonal loading value is not easy to determine and cannot be adjusted automatically with the sample changing. However, these algorithms are more complex and less efficient in the calculation.

In this paper, we will adopt the diagonal loading method to improve MVDR algorithm. It uses the adaptive diagonal loading compensation method, adjusting the diagonal elements in sample covariance matrix. The sharp decline,

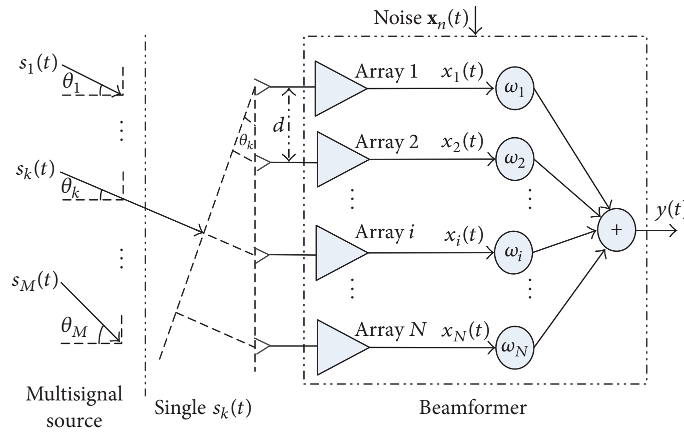

Figure 1: Uniform linear array signal model.

which is caused by the inaccuracy in the covariance matrix, is improved in performance.

The rest of this paper is organized as follows. In Section 2, the beamforming array signal model is introduced. In Section 3, the model with beamforming MVDR algorithm is proposed and the model is solved by Lagrange multiplier. Then, the model with MVDR optimization based on diagonal loading in theory is discussed in Section 4. In Section 5, the method for estimating the diagonal loading value is discussed and analyzed. The experimental results are shown in Section 6. Finally, we conclude this paper in Section 7.

2. Beamforming Array Signal Model

Figure 1 presents uniform linear array signal model. There are M narrow band multisignal sources $s_1(t), s_2(t), \ldots, s_k(t), \ldots, s_M(t)$ (containing both desired signals and interference signals) radiating at the same time, whose wavelengths are all λ. The received antenna is an N linear array with distance d and $M < N$. Taking the weather radar system as an example, the desired signals are usually the meteorological information with different desired directions and the interference signals are the various clutter information reflected by the ground. According to the Shannon communication system model, the noise $\mathbf{x}_n(t)$ is concentrated on the beamformer. In practice, the desired signals are much smaller than the interference signals and the noise, whose Signal-Noise Ratio and Signal/Interference-Noise Ratio are often up to −20 dB. Thus, the beamformer is used to receive narrow band multisignal source. It makes the desired signals without attenuation through adjusting weight factor $\omega_1, \omega_2, \ldots, \omega_i, \ldots, \omega_N$. And at the same time, the interference signals and noise are greatly suppressed.

Referring to Figure 1, single signal is first analyzed. $s_k(t)$ $(k = 1, 2, \ldots, M)$ is radiated to the array parallel, whose desired direction is θ_k $(k = 1, 2, \ldots, M)$. The array filters snapshot data $x_1(t), x_2(t), \ldots, x_i(t), \ldots, x_N(t)$. The snapshot data received have a time delay τ due to the distance of the array, which can be expressed as

$$\tau = \frac{d \sin\theta}{c}, \tag{1}$$

where c is speed of light. The signal filtered by the ith array is expressed as

$$x_i(t) = s(t)\, e^{j2\pi(i-1)d \sin\theta/\lambda} \quad i = 1, 2, \ldots, N. \tag{2}$$

After the single signal $s_k(t)$ and the noise $\mathbf{x}_n(t)$ are got through the beamformer together, the output $y(t)$ is

$$\begin{aligned} y(t) &= \boldsymbol{\omega}^H [\mathbf{x}(t) + \mathbf{x}_n(t)] \\ &= s_k(t)\, \boldsymbol{\omega}^H \mathbf{a}(\theta_k) + \boldsymbol{\omega}^H \mathbf{x}_n(t), \end{aligned} \tag{3}$$

where $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_i(t), \ldots, x_N(t)]^T$, $\mathbf{x}_n(t)$ is column vector with noise, and $\boldsymbol{\omega}^H = [\omega_1, \omega_2, \ldots, \omega_i, \ldots, \omega_N]$, $\mathbf{a}(\theta_k) = [1, e^{j(2\pi/\lambda)d \sin\theta_k}, \ldots, e^{j(2\pi/\lambda)(i-1)d \sin\theta_k}, \ldots, e^{j(2\pi/\lambda)(N-1)d \sin\theta_k}]^T$.

When the M multisignal source radiates at the same time, the output matrix $\mathbf{X}(t)$ of array is

$$\mathbf{X}(t) = \mathbf{A}\mathbf{S}(t) = \mathbf{X}_s(t) + \mathbf{X}_i(t), \tag{4}$$

where $\mathbf{S}(t) = [s_1(t), s_2(t), \ldots, s_k(t), \ldots, s_M(t)]^T$; $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K), \ldots, \mathbf{a}(\theta_M)]$; it is $N \times M$ dimension matrix; $\mathbf{X}(t)$ could be further divided into two parts: output matrix of array with desired signals $\mathbf{X}_s(t)$ and output matrix of array with interference signals $\mathbf{X}_i(t)$.

After the multisignal source $\mathbf{S}(t)$ and the noise $\mathbf{x}_n(t)$ are got through the beamformer together, the output $Y(t)$ is

$$\begin{aligned} Y(t) &= \boldsymbol{\omega}^H [\mathbf{X}(t) + \mathbf{x}_n(t)] \\ &= \boldsymbol{\omega}^H \mathbf{X}_s(t) + \boldsymbol{\omega}^H \mathbf{X}_i(t) + \boldsymbol{\omega}^H \mathbf{x}_n(t). \end{aligned} \tag{5}$$

3. Beamforming MVDR Algorithm

According to the literature [3], MVDR makes the outputting power with minimum interference and noise in the desired direction through adjusting a weight factor, while ensuring the output desired signal without minimum distortion.

From (5), the output power of interference signal adding noise can be expressed as

$$\begin{aligned} E\left\{\left|\boldsymbol{\omega}^H \mathbf{X}_i(t)+\boldsymbol{\omega}^H \mathbf{x}_n(t)\right|^2\right\} &= E\left\{\left|\boldsymbol{\omega}^H \mathbf{X}_{i+n}(t)\right|^2\right\} \\ &= \boldsymbol{\omega}^H E\left\{\mathbf{X}_{i+n}(t)\mathbf{X}_{i+n}^H(t)\right\}\boldsymbol{\omega} = \boldsymbol{\omega}^H \mathbf{R}_{i+n}\boldsymbol{\omega}. \end{aligned} \quad (6)$$

For convenience, (6) $\mathbf{X}_{i+n}(t) = \mathbf{X}_i(t) + \mathbf{x}_n(t)$, $\mathbf{R}_{i+n} = E\{\mathbf{X}_{i+n}(t)\mathbf{X}_{i+n}^H(t)\}$, where $\mathbf{X}_{i+n}(t)$ is the interference signal adding noise. Mathematically, $\mathbf{R}_{i+n}$ is equal to the covariance matrix of $\mathbf{X}_{i+n}(t)$.

Similarly, from (5), in order to guarantee the nondistortion output of the desired signals, we can deduce the necessary and sufficient condition:

$$\boldsymbol{\omega}^H \mathbf{s} = 1, \quad (7)$$

where $\mathbf{s}$ is directional vector with normalizing factor $\mathbf{s} = \mathbf{a}(\theta)/\sqrt{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}$.

Therefore, beamforming MVDR algorithm can be expressed as the following minimization problem:

$$\begin{aligned} \min \quad & \boldsymbol{\omega}^H \mathbf{R}_{i+n}\boldsymbol{\omega} \\ \text{s.t.} \quad & \boldsymbol{\omega}^H \mathbf{s} = 1. \end{aligned} \quad (8)$$

Using Lagrange multiplier, (8) can be changed into the unconstraint equation

$$J(\boldsymbol{\omega}) = \frac{1}{2}\boldsymbol{\omega}^H \mathbf{R}_{i+n}\boldsymbol{\omega} + t\left(\boldsymbol{\omega}^H \mathbf{s} - 1\right). \quad (9)$$

The weighted vector $\boldsymbol{\omega}$ is obtained as

$$\boldsymbol{\omega} = \mathbf{R}_{i+n}^{-1}\left(\frac{\mathbf{s}}{\mathbf{s}^H \mathbf{R}_{i+n}^{-1}\mathbf{s}}\right). \quad (10)$$

4. MVDR Algorithm Based on Diagonal Loading Compensation

According to literature [5], beamforming MVDR algorithm requires an accurate covariance matrix $\mathbf{R}_{i+n}$. In practice, the performance of beamforming MVDR algorithm declines dramatically due to the inaccuracy with the covariance matrix based on sampling $\widehat{\mathbf{R}}_{i+n}$ and the theoretical covariance matrix $\mathbf{R}_{i+n}$. Therefore, literature [5] corrects the inaccuracy of $\widehat{\mathbf{R}}_{i+n}$ by adding a constant value to the diagonal elements of $\widehat{\mathbf{R}}_{i+n}$. As a result, the performance is improved and the robustness is enhanced.

In practice, $\mathbf{R}_{i+n}$ can be estimated using snapshots data with finite order. The method is as below:

$$\widehat{\mathbf{R}}_{i+n} = \mathbf{R}_{i+n} + \varepsilon\mathbf{B} = \frac{1}{K}\sum_{l=1}^{K}\mathbf{X}_{i+n}(t_l)\mathbf{X}_{i+n}^H(t_l), \quad (11)$$

where $\varepsilon\mathbf{B}$ is the inaccuracy term of covariance matrix of sampling data. It is a random matrix with zero mean and unit variance, all the elements of which are far less than 1. $\mathbf{X}_{i+n}(t_l)$ is the sampling data of interference signals adding noise in a moment. K is the sampling snapshots.

$\mathbf{R}_{i+n}$ from (8) can be expressed as covariance matrix with diagonal loading:

$$\widetilde{\mathbf{R}}_{i+n} = \mathbf{R}_{i+n} + \lambda\mathbf{I} + \varepsilon\mathbf{B}, \quad (12)$$

where $\widetilde{\mathbf{R}}_{i+n}$ is covariance matrix with diagonal loading, $\mathbf{I}$ is the unit matrix, and λ is the factor of diagonal loading, which is used to control the diagonal loading value.

Therefore, to get the MVDR optimization problem based on diagonal loading, we amend (8) as

$$\begin{aligned} \min \quad & \widetilde{\boldsymbol{\omega}}^H \widetilde{\mathbf{R}}_{i+n}\widetilde{\boldsymbol{\omega}} \\ \text{s.t.} \quad & \widetilde{\boldsymbol{\omega}}^H \mathbf{s} = 1. \end{aligned} \quad (13)$$

Similarly, using Lagrange multiplier in (13), the weighted vector $\widetilde{\boldsymbol{\omega}}$ is obtained as

$$\widetilde{\boldsymbol{\omega}} = \frac{\widetilde{\mathbf{R}}_{i+n}^{-1}\mathbf{s}}{\mathbf{s}^H \widetilde{\mathbf{R}}_{i+n}^{-1}\mathbf{s}}. \quad (14)$$

5. The Estimated Method with Diagonal Loading Compensation

In practice, λ from (11) is determined manually [6]. It needs strong experience. The covariance matrix on sampling has irreconcilable inaccuracy especially in the small sampling snapshots. The constant value of diagonal loading regulated by experience often fails to work. It also cannot correct the inaccuracy. This paper presents an adaptive diagonal loading method which can change the diagonal loading value with the difference of input signals. It decreases the interval of diagonal loading value depending on the relation between the inaccuracy of covariance matrix on sampling and the diagonal loading value. Therefore, this paper provides theoretical guidance for researchers to select diagonal loading value effectively.

From (12), due to $\mathbf{R}_{i+n}+\lambda\mathbf{I}+\varepsilon\mathbf{B} = (\mathbf{R}_{i+n}+\lambda\mathbf{I})[\mathbf{I}+\varepsilon\mathbf{B}(\mathbf{R}_{i+n}+\lambda\mathbf{I})^{-1}]$, the inverse of covariance matrix on diagonal loading $\widetilde{\mathbf{R}}_{i+n}^{-1}$ can be expressed as

$$\widetilde{\mathbf{R}}_{i+n}^{-1} = \left[\mathbf{I} + \varepsilon\mathbf{B}\left(\mathbf{R}_{i+n}+\lambda\mathbf{I}\right)^{-1}\right]^{-1}\left(\mathbf{R}_{i+n}+\lambda\mathbf{I}\right)^{-1}. \quad (15)$$

The $[\mathbf{I} + \varepsilon\mathbf{B}(\mathbf{R}_{i+n}+\lambda\mathbf{I})^{-1}][\mathbf{I} - \varepsilon\mathbf{B}(\mathbf{R}_{i+n}+\lambda\mathbf{I})^{-1}]$ is unfolded as

$$\begin{aligned} &\left[\mathbf{I} + \varepsilon\mathbf{B}\left(\mathbf{R}_{i+n}+\lambda\mathbf{I}\right)^{-1}\right]\left[\mathbf{I} - \varepsilon\mathbf{B}\left(\mathbf{R}_{i+n}+\lambda\mathbf{I}\right)^{-1}\right] \\ &\quad = \mathbf{I} - \left[\varepsilon\mathbf{B}\left(\mathbf{R}_{i+n}+\lambda\mathbf{I}\right)^{-1}\right]^2. \end{aligned} \quad (16)$$

Because all the elements of $\varepsilon\mathbf{B}$ from (16) are far less than the elements of matrix $\mathbf{R}_{i+n} + \lambda\mathbf{I}$, $[\mathbf{I} + \varepsilon\mathbf{B}(\mathbf{R}_{i+n}+\lambda\mathbf{I})^{-1}][\mathbf{I} - \varepsilon\mathbf{B}(\mathbf{R}_{i+n}+\lambda\mathbf{I})^{-1}] \approx \mathbf{I}$. Then

$$\left[\mathbf{I} + \varepsilon\mathbf{B}\left(\mathbf{R}_{i+n}+\lambda\mathbf{I}\right)^{-1}\right]^{-1} \approx \mathbf{I} - \varepsilon\mathbf{B}\left(\mathbf{R}_{i+n}+\lambda\mathbf{I}\right)^{-1}. \quad (17)$$

Equation (15) can be approximately expressed as

$$\widetilde{\mathbf{R}}_{i+n}^{-1} \approx \left(\mathbf{R}_{i+n} + \lambda \mathbf{I}\right)^{-1} \left[\mathbf{I} - \varepsilon \mathbf{B} \left(\mathbf{R}_{i+n} + \lambda \mathbf{I}\right)^{-1}\right]. \quad (18)$$

$\mathbf{R}_{i+n}$ from (18) can be expressed as

$$\begin{aligned} \mathbf{R}_{i+n} &= \sum_{k=1}^{T} \sigma_k^2 \mathbf{a}\left(\theta_k\right) \mathbf{a}^H\left(\theta_k\right) + E\left\{\mathbf{X}_n\left(t\right) \mathbf{X}_n^H\left(t\right)\right\} \\ &= \mathbf{A}_i \mathbf{\Lambda}_i \mathbf{A}_i^H + \sigma_n^2 \mathbf{I}, \end{aligned} \quad (19)$$

where $\sigma_k^2 = E\{s_k^2\}$ is signal power corresponding to the kth interference signal, $\sigma_n^2 = E\{\mathbf{X}_n(t)\mathbf{X}_n^H(t)\}$ is the noise power, T is the number of interference signals, $\mathbf{A}_i = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K), \ldots, \mathbf{a}(\theta_T)]$ is the directional matrix with interference signal, and $\mathbf{\Lambda}_i = \mathrm{diag}[\sigma_1^2, \ldots, \sigma_k^2, \ldots, \sigma_T^2]$ is diagonal matrix corresponding to the interference signal power.

Using the matrix inversion lemma [13] and substituting (19) into (18), there is

$$\widetilde{\mathbf{R}}_{i+n}^{-1} \approx \left(\mathbf{R}_{i+n} + \lambda \mathbf{I}\right)^{-1} \left\{\mathbf{I} - \frac{\varepsilon}{\lambda + \sigma_n^2} \mathbf{B}\mathbf{C}\right\}, \quad (20)$$

where

$$\mathbf{C} = \left[\mathbf{I} - \mathbf{A}\left(\mathbf{A}^H \mathbf{A} + \left(\sigma_n^2 + \lambda\right) \mathbf{\Lambda}^{-1}\right)^{-1} \mathbf{A}^H\right]. \quad (21)$$

The value of first bracket $(\mathbf{R}_{i+n} + \lambda \mathbf{I})$ from (20) is close to the covariance matrix $\mathbf{R}_{i+n}$; the diagonal loading value λ should be far less than the diagonal elements $\mathbf{R}_{i+n}(i,i)$ of covariance matrix $\mathbf{R}_{i+n}$; that is,

$$\lambda \ll \mathbf{R}_{i+n}\left(i,i\right) \quad i = 1, 2, \ldots, M. \quad (22)$$

It is the reason that performance of MVDR beamforming is degraded by the $(\varepsilon/(\lambda + \sigma_n^2))\mathbf{BC}$ in the curly bracket from (19). And all the elements of $\mathbf{B}$ are far less than 1 and close to 0. To get the optimal performance of MVDR, it should meet the condition $\mathbf{I} - (\varepsilon/(\lambda + \sigma_n^2))\mathbf{BC} \approx \mathbf{I}$. And it can be explained as

$$\lambda \geq \varepsilon - \sigma_n^2. \quad (23)$$

From (22) and (23), there is

$$\varepsilon - \sigma_n^2 \leq \lambda \ll \mathbf{R}_{i+n}\left(i,i\right), \quad (24)$$

where $\mathbf{R}_{i+n}(i,i)$ is the average of diagonal elements in the covariance matrix of sampling data, $\mathbf{R}_{i+n}(i,i) = \mathrm{Trace}(\widehat{\mathbf{R}}_{i+n})/N$. Here N is the number of arrays; Trace is the trace of matrix. ε is the inaccuracy of covariance matrix of sampling data, which can be expressed by obtaining the standard deviation of $\mathbf{R}_{i+n}(i,i)$, $\varepsilon = \mathrm{std}(\mathrm{diag}(\widehat{\mathbf{R}}_{i+n}))$. Here diag is the diagonal elements of matrix; std is the standard deviation of matrix. Moreover, (24) shows that $\mathbf{R}_{i+n}(i,i)$, ε will change with the difference of input signals, and the diagonal loading value will change relatively. Thus, the algorithm is adaptive.

6. Discussion and Simulation

6.1. Evaluation Methodology of Experimental Result

6.1.1. The Output Signal/Interference-Noise Ratio (SINR). The ratio with output power of the desired signals to the interference adding noise signals is defined as the output SINR. When the unit of SINR is dB, it can be rewritten as

$$\mathrm{SINR\,(dB)} = 10\lg\left(\frac{\sigma_s^2}{\boldsymbol{\omega}^H \mathbf{R}_{i+n} \boldsymbol{\omega}}\right), \quad (25)$$

where σ_s is the output power of desired signals.

6.1.2. Beamforming Array Pattern. According to Figure 1, the relation between the absolute value of the output array $|y(t)|$ and the direction angle θ is called beampattern, which is used to evaluate the performance of beamformer.

From (3), the ratio of output to input arrays is defined as $F(\theta)$; that is,

$$F\left(\theta\right) = \frac{\mathbf{y}\left(t\right)}{s\left(t\right)} = \boldsymbol{\omega}^H \mathbf{a}\left(\theta\right). \quad (26)$$

From (26), to get the gain of beampattern in unit dB, it can be rewritten as

$$p\left(\theta\right) = 20\lg\left(\frac{\left|F\left(\theta\right)\right|}{\max\left(\left|F\left(\theta\right)\right|\right)}\right), \quad (27)$$

where $\max(|F(\theta)|)$ is the maximum beampattern. Ideally, when the direction corresponding to its mainlobe is from desired signal, $F(\theta)$ gets the max; the rest of directions are sidelobes. The smaller the sidelobes are, the stronger the suppression for interference adding noise is.

6.2. Simulation and Analysis. It can be observed from (24) that the diagonal loading value λ has been limited to an interval. In this paper, with the value λ increasing gradually in the interval, we try to observe the performance of MVDR on diagonal loading beamformer with the output SINR as the index and get the better diagonal loading value. Then we use (27) to evaluate MVDR algorithms on both constant diagonal loading and estimated diagonal loading λ, where λ is optimal value from the first experiment. In this paper, we design two experiments.

Experiment 1 (optimization and estimation of diagonal loading λ). In this experiment, we employ a uniform 16-element linear array. The distance of elements is half of the wavelength, the direction of the desired signal is 0 degrees, and the direction of the interference signal is 40 degrees. We select three representative input scenarios: the first scenario is that the power of desired signal is far less than the noise and the power of interference signal is greater than the noise. At this point, the input SNR is −5 dB and the input INR is 10 dB. Therefore, input SINR is −15.414 dB. The second scenario is that the power of desired signal is far less than the noise and the power of interference signal is equal to the noise. At this point the input SNR is −5 dB and the input INR is

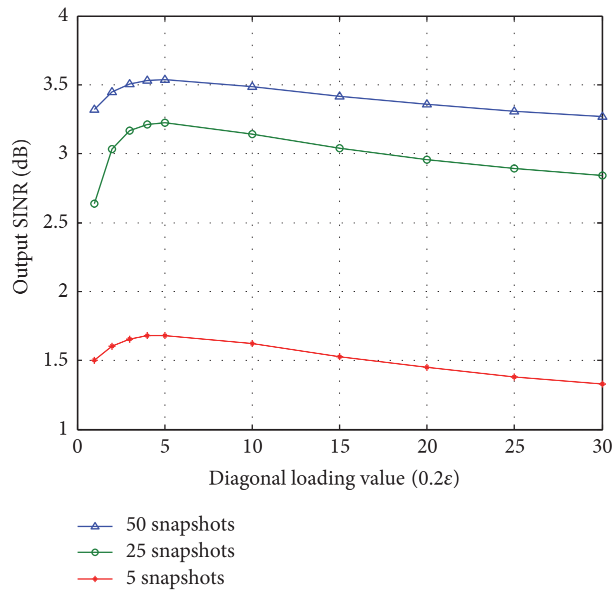

Figure 2: The relations between diagonal loading value and output SINR in different snapshots (input: SNR = −5 dB, INR = 10 dB, and SINR = −15.414 dB).

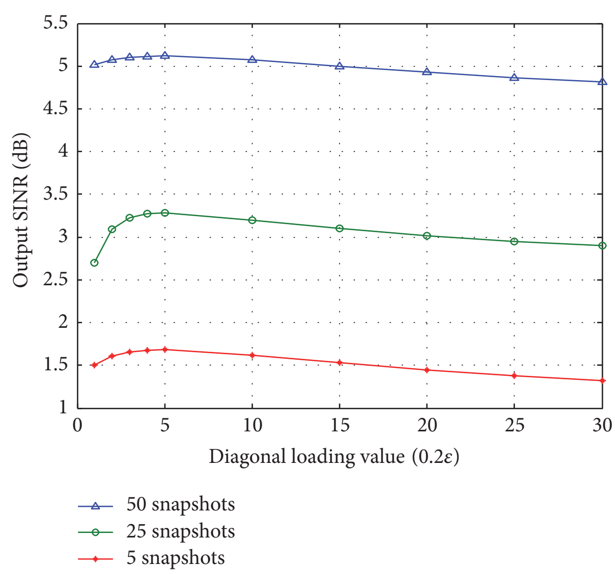

Figure 3: The relations between diagonal loading value and output SINR in different snapshots (input: SNR = −5 dB, INR = 0 dB, and SINR = −8.01 dB).

0 dB. Therefore, input SINR is −8.01 dB. The third scenario is that the power of desired signal is greater than the noise and the power of interference is equal to the noise. The input SNR is 5 dB and the input INR is 0 dB. Therefore, input SINR is 1.990 dB. From (24), 10 points are selected in the interval $\varepsilon - \sigma_n^2 \leq \lambda \ll \mathbf{R}_{i+n}(i,i)$. And we further optimize the diagonal loading λ by using the output SINR. Considering the relations between diagonal loading value and the output SINR in the three scenarios, the snapshots are set as 5, 25, and 50, and the diagonal loading value is set as the unit on 0.2ε. Figures 2, 3, and 4 show that no matter whether

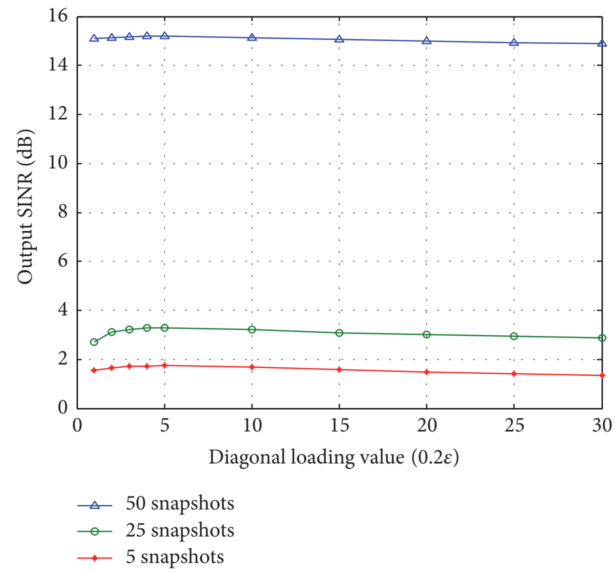

Figure 4: The relations between diagonal loading value and output SINR in different snapshots (input: SNR = 5 dB, INR = 0 dB, and SINR = 1.990 dB).

Table 1: Output SINR comparison of three algorithms in different snapshots (input: SNR = −5 dB, INR = 10 dB, and SINR = −15.414 dB).

Snapshots	Output SINR/dB		
	MVDR	$\text{MVDR}_{\text{cons}}$	MVDR_{est}
5	−10.5200	1.1889	1.6289
15	−8.6626	2.3498	2.7657
25	−0.4494	2.8155	3.1409
50	3.0710	3.5075	3.6187

snapshots are 5, 25, or 50 in various scenarios, when $\lambda = \varepsilon$, the output SINR gets the maximum value, and the performance gets the best. The results were in line with general cognition: diagonal loading factor is a compensation factor. When the compensation factor is equal to the error of the covariance matrix, the disturbance problem on the performance can be counteracted preferably, and the factor is independent of the input SINR.

Experiment 2 (comparison of algorithm performance). The experiment selects the input parameters in the most unfavorable situation (the first scenario) in Experiment 1. The input SNR is −5 dB, INR is 10 dB, and input SINR = −15.414 dB. Under the results of Experiment 1, $\lambda = \varepsilon$, analyzing the MVDR algorithm, MVDR on constant diagonal loading [6] and the MVDR on estimated diagonal loading with the output SINR and the beamforming array pattern in different snapshots (5, 15, 25, and 50). The results are shown in Table 1 and Figures 5 and 6–9.

Table 1 and Figure 5 show that, under the same input conditions, with the increase of the snapshots, the suppression on the interference adding noise of three methods is getting

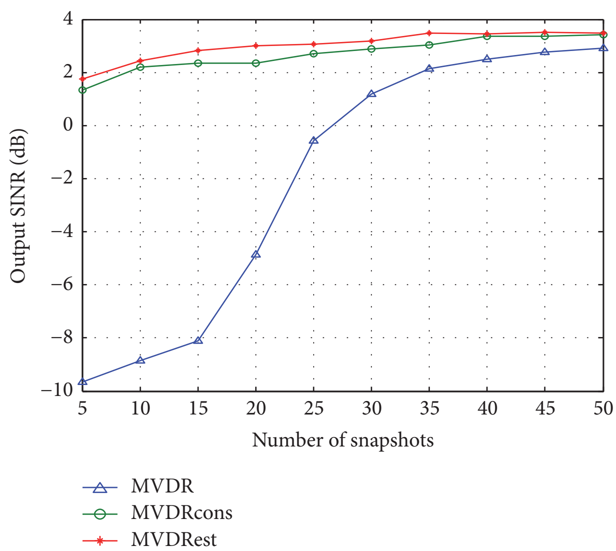

Figure 5: Output SINR comparison of three algorithms in different snapshots (input: SNR = −5 dB, INR = 10 dB, and SINR = −15.414 dB).

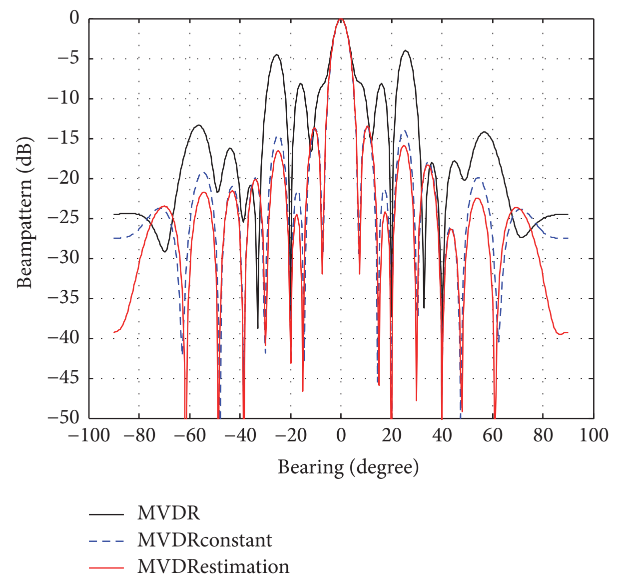

Figure 7: Comparison of three algorithms for beamforming array pattern ($k = 25$).

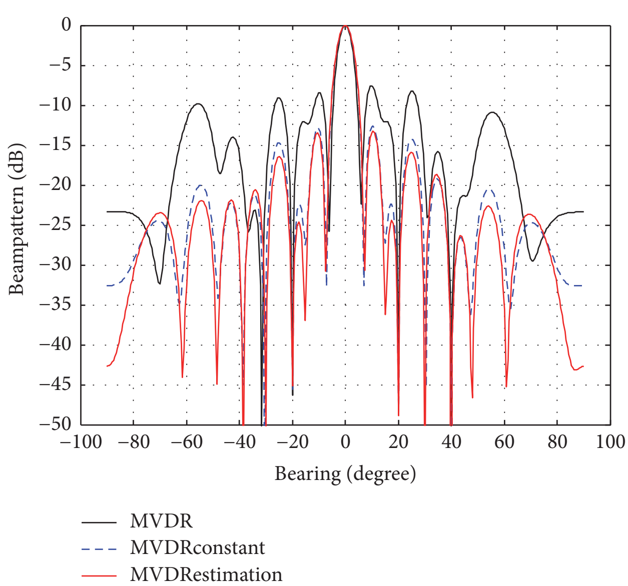

Figure 6: Comparison of three algorithms for beamforming array pattern ($k = 50$).

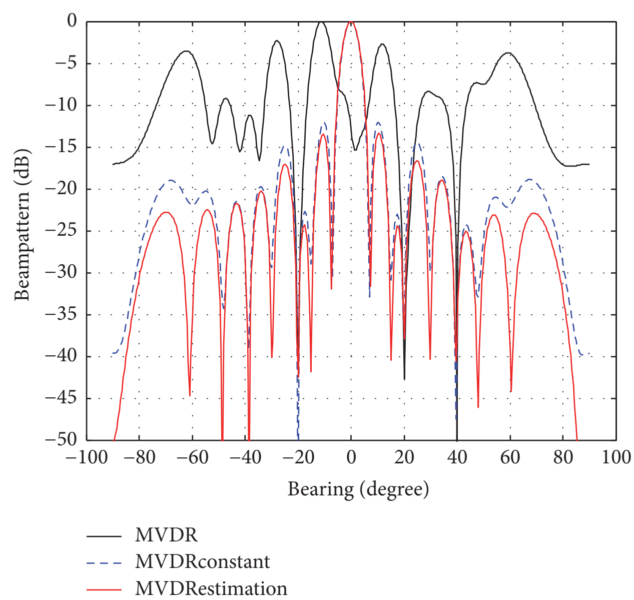

Figure 8: Comparison of three algorithms for beamforming array pattern ($k = 15$).

better. When the snapshots are small, the traditional MVDR algorithm has less suppression on the interference adding noise. And when the snapshots are big, the suppression is tolerable. But it shows poor stability. The algorithm with the paper ($\text{MVDR}_{\text{estmation}}$) and the algorithm with literature [6] ($\text{MVDR}_{\text{constant}}$) are obviously better than MVDR algorithm in the stability and the suppression on the interference adding noise. However, the algorithm with this paper is superior to the algorithm with literature [6] in the stability and the suppression on the interference adding noise.

Figures 6 and 7 show that when the snapshots are big ($K > 20$), the amplitudes of mainlobe in the algorithm with literature [6] and the algorithm with this paper are equal; the amplitude of sidelobe in the algorithm with this paper is below −15 dB, which is slightly superior to the algorithm with literature [6]; the above two methods are both superior to MVDR algorithm.

Figures 8 and 9 show that when snapshots are small ($K < 20$), the algorithm with this paper has obvious advantages of the same direction in pattern. It mainly shows that the amplitudes of sidelobe are superior to the algorithm with

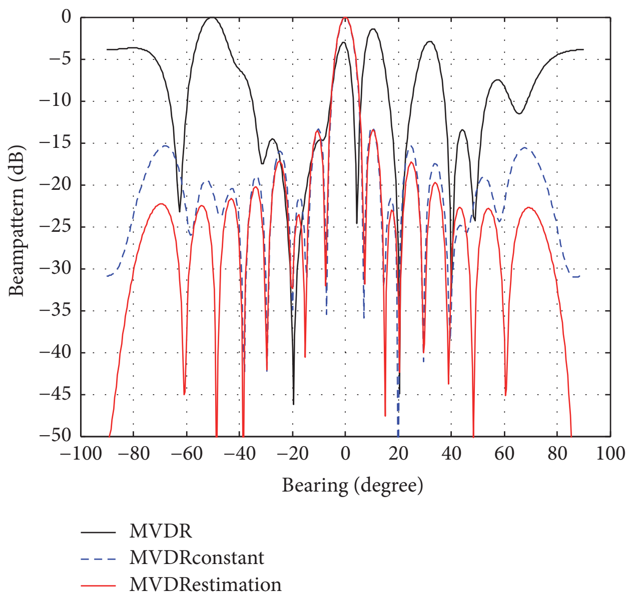

Figure 9: Comparison of three algorithms for beamforming array pattern ($k = 5$).

literature [6], which means that the suppression on the interference and noise is the best.

7. Conclusion

To enhance the robustness of the existing beamforming MVDR algorithms, we propose the MVDR algorithm based on estimated diagonal loading for beamforming. It is on the basis of learning the uniform linear array signal model and beamforming MVDR algorithm. The paper establishes the MVDR optimization model on diagonal loading compensation solved by Lagrange multiplier. The interval of the diagonal loading compensation is deduced on the basis of the matrix theory. And the optimal diagonal loading is also determined through the experimental method. The experimental results show that when the snapshots are small, the traditional MVDR algorithm has less suppression on the interference adding noise. And when the snapshots are big, the suppression is tolerable. It shows poor stability. The algorithm with the paper and the algorithm with literature [6] are obviously better than MVDR algorithm in the stability and the suppression on the interference adding noise. However, the algorithm with this paper is superior to the algorithm with literature [6] in the stability and the suppression on the interference adding noise.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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